Title: On graviton production by moving branes

Date: Jun 27, 2006 11:00 AM

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Abstract: In this talk I will discuss some aspects of graviton production by moving branes. After a brief introduction to braneworld cosmology I will focus on braneworlds in a five-dimensional bulk, where cosmological expansion is mimicked by motion through AdS_5. The moving brane acts naturally as a time-dependent boundary for the five-dimensional graviton (five-dimensional tensor perturbations) leading to graviton production out of quantum vacuum fluctuations. This effect is related to the so-called dynamical Casimir effect, i.e. the generation of real photons out of vacuum fluctuations of the quantized electromagnetic field in dynamical cavities. By applying the formalism used to study the dynamical Casimir effect I will show explicitly that the five-dimensional graviton reduces to the four-dimensional one in the late time approximation of such braneworlds. In the last part of the talk I will study a (toy) model where two branes approach each other in a radiation dominated phase, bounce off and move apart from each other afterwards. Thereby generation of massive gravitons takes place caused by the coupling of the Kaluza-Klein modes to the gravitational zero mode which exhibits a blue spectrum. At the end I will discuss possible applications of the formalism to more interesting scenarios (braneworld inflation etc).

On Graviton Production By Moving Branes

Marcus Ruser
Université de Genève

In collaboration with Cyril Cartier and Ruth Durrer

Perimeter Institute, Waterloo, 27 June 2006

Overview of talk

Study of tensor perturbations / gravity waves in braneworld cosmology

„Signatures of extra dimensions in CMB"

Model

Methods

Dynamical Casimir effect = generation of photons out of vacuum fluctuations of the quantized electromagnetic field in dynamical cavities

New insights

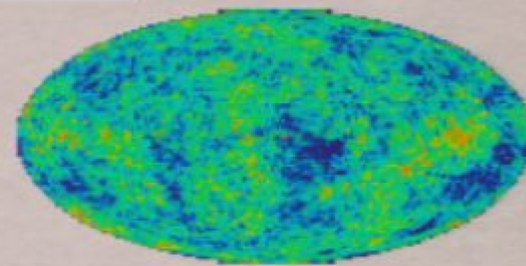
I will show explicitly that the five-dimensional graviton reduces to the four-dimensional one in the late-time approximation.

I will study a (toy) model of bouncing branes to show basic effects and to demonstrate the advantages of the method. This second part is work in progress!

Warning: This might be a rather technical talk! -> „No direct link to obervations"

Intention: Trigger discussions, comments, etc.

Concordance Model



Precise measurements of the anisotropies in the CMB have confirmed the so-called

concordance model / standard cosmological model.

Universe is spatially flat

Hubble constant $H_0 \simeq 70 km/s Mpc$

Energy density is dominated by vacuum energy (Λ), dark energy $\Omega_\Lambda \simeq 0.7$

Cold dark matter $\Omega_{\mathrm{CDM}} \simeq 0.3$

Baryons $\Omega_b \simeq 0.02$

massive neutrinos less than Baryons

Structures in the universe (galaxies, clusters, voids) have formed out of small initial fluctuation which have been generated during inflation. This is the success of inflation.

OPEN QUESTIONS AND STRING THEORY

Even though the concordance model is supported by most cosmological data, on the theoretical side there are still a lot of open questions:

What is dark matter? What is dark energy?

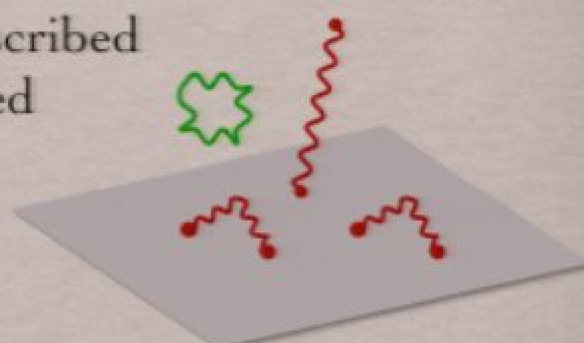
What is the physics of inflation? (What is the inflaton?)

How can one resolve the Big Bang singularity of classical general relativity?

In order to answer (some of) these questions we need a theory of quantum gravity which unifies all fundamental interactions, i.e. gauge interactions and gravity.

Nowadays, the most promising candidate is string theory (-> M-theory) which manifests itself at very high energies of order of Planck scale $\simeq 3 \times 10^{18} GeV$ and predicts that spacetime is ten-dimensional.

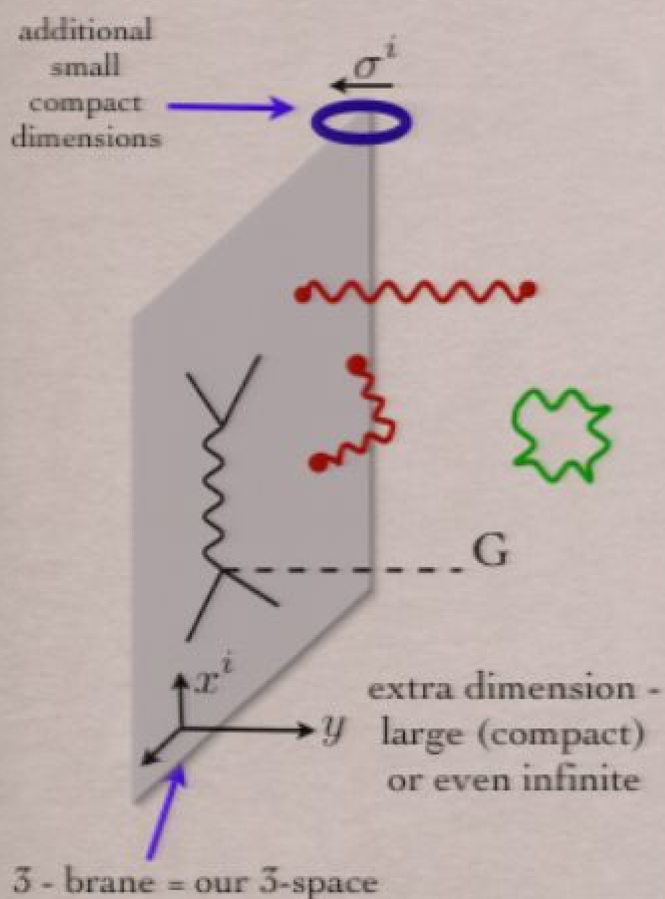
In string theory, particles (gauge fermions, gauge fields) are described by excitations of open strings while gravity is described by closed strings (loops).



The discovery of so-called Dp - branes, p+1 - dimensional sub-manifolds on which open strings have to end **[Polchinski 1995]**, has led to the idea of braneworlds.

[Reviews: R.Maartens, Living Rev. Relativity 7, 7 (2004), R.Durrer, AIP Conf. Proc. 782, 202 (2005)]

THE BRANEWORLD PICTURE



The universe is described as a 3-brane. All the standard model fields confined to a 3+1-hypersurface and only gravity is allowed to propagate in the bulk (entire spacetime).

String theory predicts the existence of so-called Dp-branes (Polchinski 1995) onto which standard model particles are confined (excitations of open strings with their endpoints attched to Dp-branes).

The bulk spacetime around a Dp - brane can be probed only by gravity (excitations of closed strings) and not by standard model fields.

Gravity (Newtons law) has been tested only down to scales of 0.1mm. In the braneworld picture where only gravity can probe the extra dimensions, these can be as large as 0.1mm. This can be used to address the hierarchy problem.
[Arkani-Hamed, Dimopoulos and Dvali, 1998]

Particularly the so-called Randall - Sundrum braneworld models **[Randall and Sundrum, 1999]** consisting of one (two) brane(s) in an five-dimensional anti-de Sitter spacetime have attracted lot of attention during recent years. In this talk I will concentrate on these 5D - models.

However, note that first „brane worlds" in the context of domain walls known much longer **[Rubakov and Shaposhnikov 1983]**

The Randall Sundrum Geometry

RS model: five-dimensional Anti-de Sitter (AdS) bulk, i.e. $\Lambda_5 < 0$, with Minkowski brane(s).

In Poincare coordinates the bulk - metric is given by

$$\boxed{ds_5^2 = \frac{l^2}{y^2}\left[-dt^2 + \delta_{ij}dx^i dx^j + dy^2\right]} \qquad \boxed{\Lambda_5 = -6/l^2}$$

l is the curvature radius of AdS

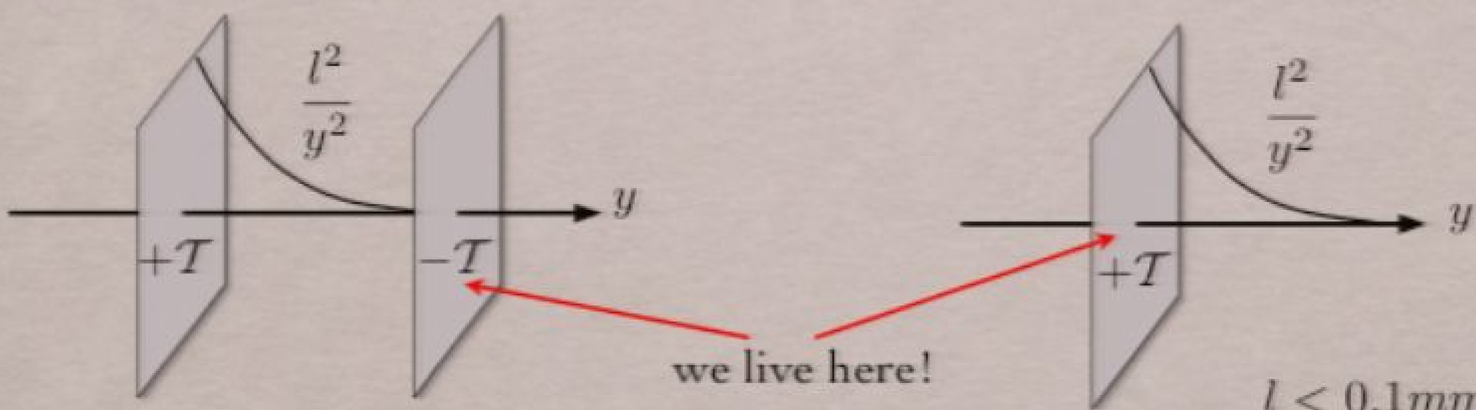
warp factor, this is new compared to other scenarios

$$l = \frac{6}{\kappa_5 \mathcal{T}} \quad (\Lambda_4 = 0)$$

$$\kappa_5 = 6\pi^2 G_5 = \frac{1}{M_5^3}$$

RS I = 2-brane model

RS II = 1-brane model



$y = l$

SM/TeV brane

Planck brane, hidden brane

$y = l$

$$M_4^2 = M_5^3 l$$

$$l < 0.1mm \;\Rightarrow\; \mathcal{T} > (1Tev)^4$$

$$\Rightarrow\; M_5 = (M_4^2/l)^{1/3} > 10^5 TeV$$

Fine tuning of brane tension (self gravity) : $\mathcal{T} = \frac{3M_4^2}{4\pi l^2}$

=> zero effective cosmological constant on the brane

=> brane has induced geometry of Minkowski space time

Junction conditions

The Einstein equation at the position of a brane $y = y_b$ is singular, it contains a δ - function:

$$G_{AB} + \Lambda_5 \, g_{AB} = \kappa_5 \, T_{AB}^{\text{brane}} \, \delta(y - y_b)$$

confines the matter to the brane

Integrating Einsteins equation over the extra dimension yield the so-called Israel-Darmois junction conditions [Darmois 1927, Israel 1966].

1st one: Induced metric be continous across the brane.

2nd one: Links the extrinsic curvature of the hypersurface and the brane energy momentum tensor. It replaces the 4D Einstein equation.

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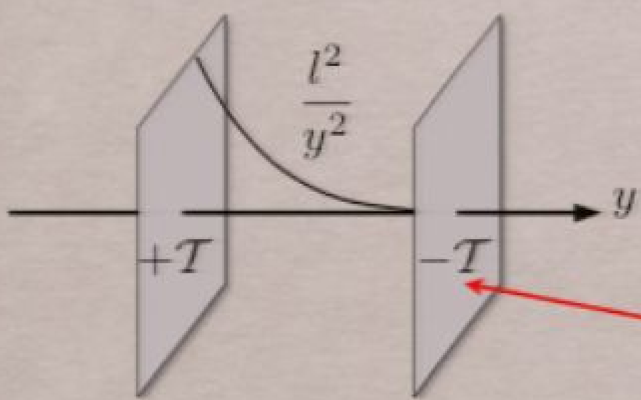

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we live here!

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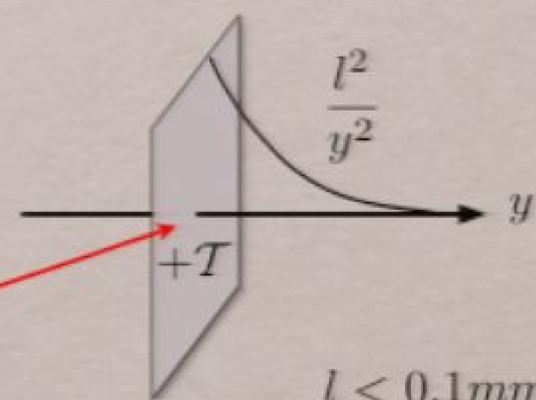

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Braneworld cosmology

In braneworld cosmology the brane is moving in the bulk, i.e. $y_b = y_b(t)$.
A homogeneous and isotropic brane moving through AdS with position $y_b(\eta)$ where η is the conformal of an oberver on the brane has the Friedman-Robertson Walker metric

$$ds^2 = a^2(\eta)[-d\eta^2 + \delta_{ij}dx^i dx^j]$$

The scale factor is related to the brane position via

$$y_b(\eta) = \frac{l}{a(\eta)}$$

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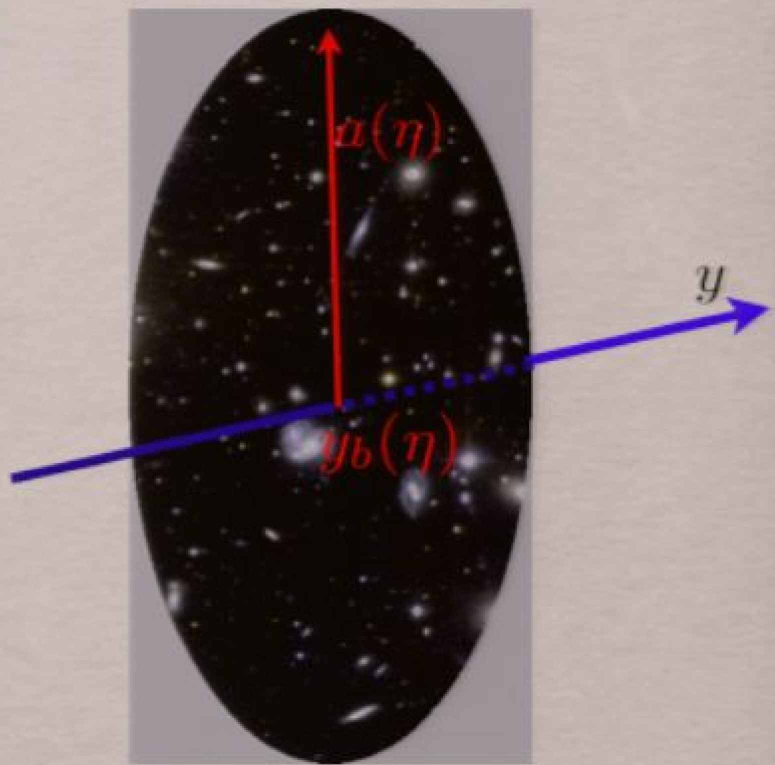

The expansion of the universe is mimicked by the motion of the brane through AdS.

Thereby the dynamics of the scale factor is governed by the so-called modified Friedmann equation.

Modified Friedmann equation

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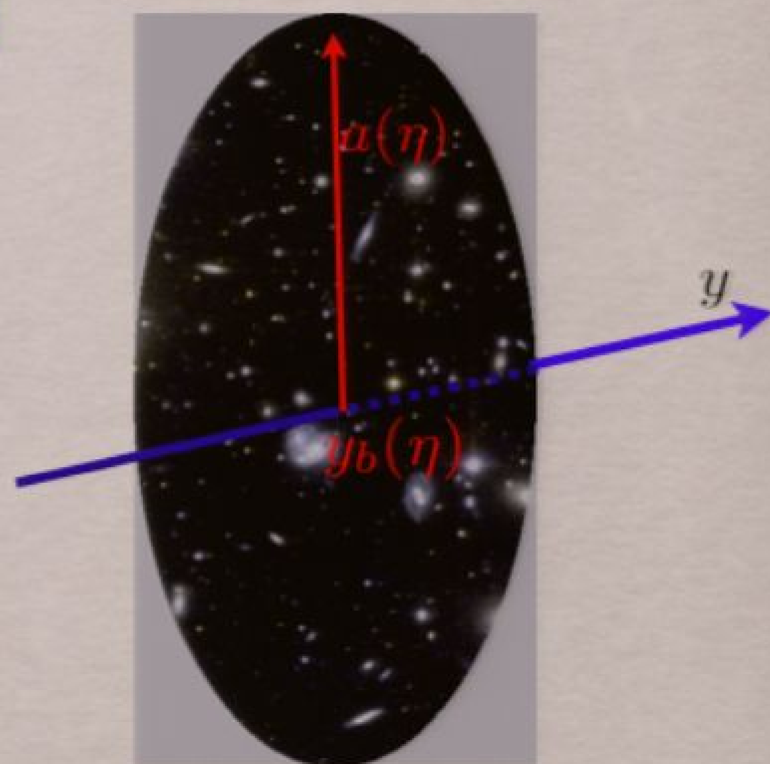

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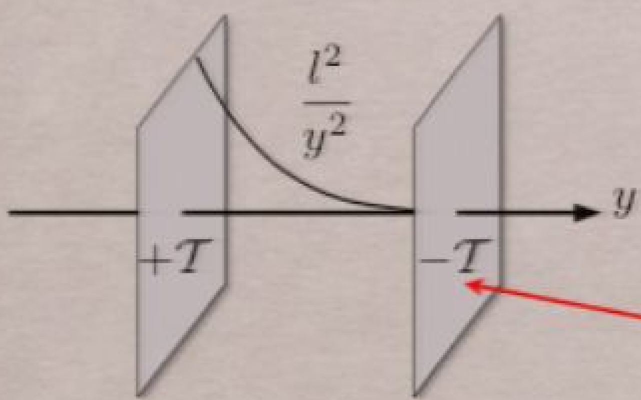

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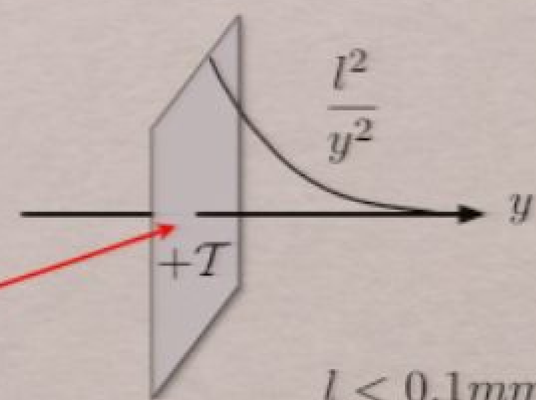

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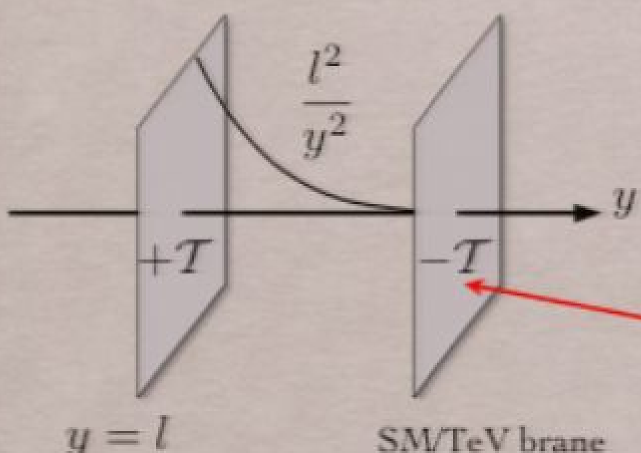

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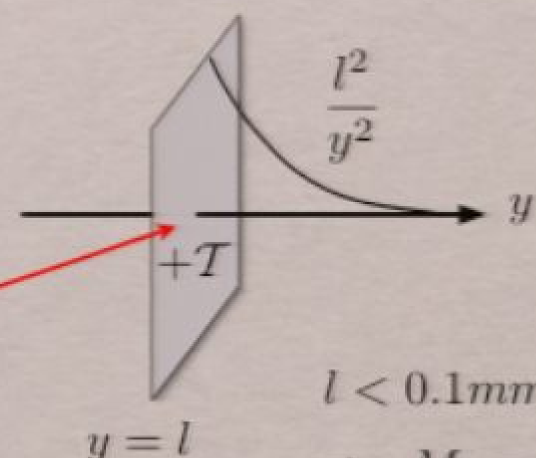

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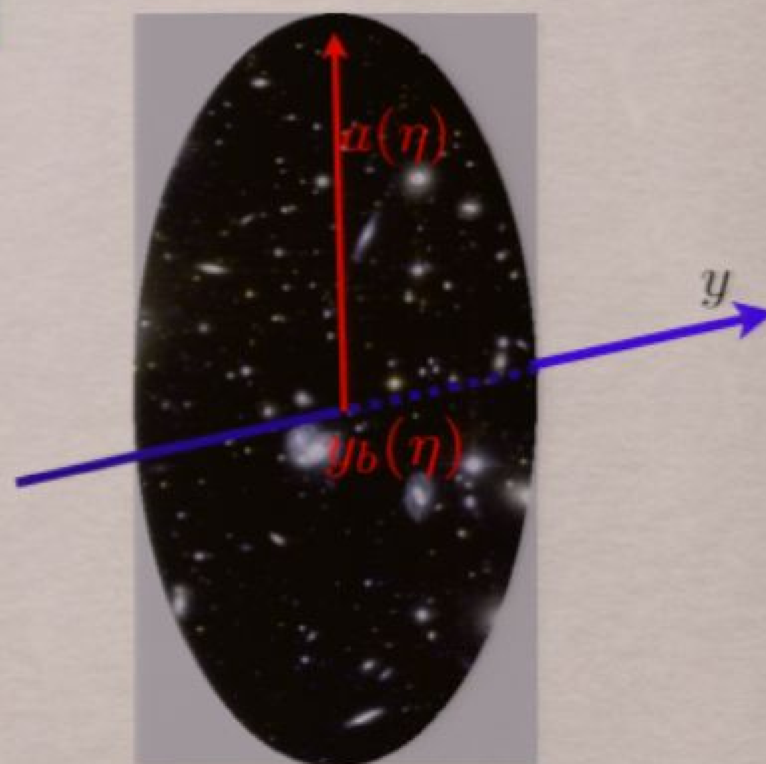

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Tensor perturbations

Linear perturbation of the five-dimensional bulk allowing for tensor perturbations:

$$ds^2 = \frac{l^2}{y^2}\left[-dt^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j + dy^2\right]$$

polarization tensors

Decomposition into spatial Fourier modes: $h_{ij} = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_{\bullet=+,\times} e^{i\mathbf{kx}} e^{\bullet}_{ij} h_\bullet(t,y;k)$

The perturbed Einstein equations yield the equation of motion for the amplitude $h_\bullet$:

$$\left[\partial_t^2 + k^2 - \partial_y^2 + \frac{3}{y}\partial_y\right] h_\bullet(t,y;k) = 0$$

Klein-Gordon equation in AdS

In addition, the second junction condition implies a boundary condition for the tensor modes at the brane position:

$$(v\partial_t - \partial_y)h_\bullet|_{y_b(t)} = 0$$

with brane velocity $v = \frac{lH}{\sqrt{1+l^2H^2}}$ and $H = \partial_\eta a/a^2$

time -dependent!

Neglect possible anisotropic stress perturbations in the brane energy momentum tensor.

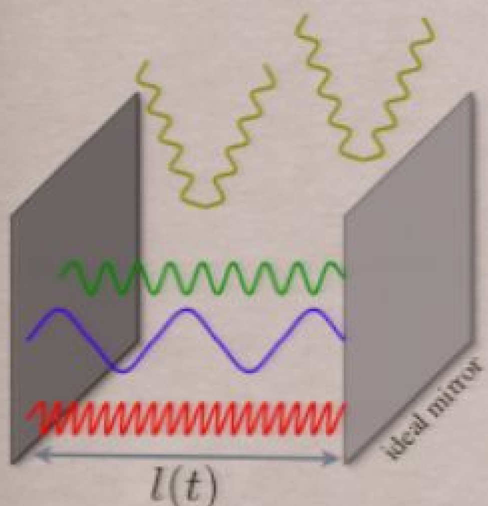
In the late-time / low-energy limit $lH \ll 1 \Rightarrow v \ll 1$ the BC reduces to $\partial_y h_\bullet(t,y)|_{y_b(t)} = 0$

The time evolution of tensor perturbations in moving braneworlds is given by a Klein-Gordon equation and subject to a time dependent bounday condition.

Dynamical Casimir Effect

In quantum field theory time-dependent (classical) external fields can lead to particle creation from vacuum, e.g. cosmological particle creation, oscillating inflaton field, ...

Also time-dependent BC's can be considered as (sharp) classical background fields yielding particle creation from vacuum fluctuations.



Dynamical Casimir Effect (motion induced radiation): creation of photons out of the vacuum fluctuations of the quantized electromagnetic field in dynamical cavities (moving mirrors).

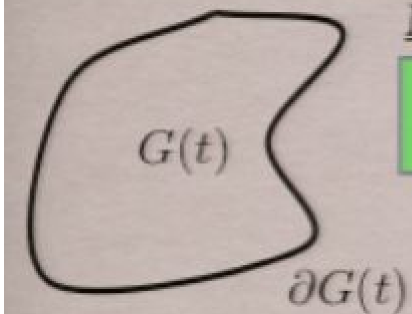
Thereby effects like parametric resonance can yield to exponential particle creation. However, this effect is quite different to, e.g. resonant particle creation due to the oscillating inflaton. The difference is that the time-dependent boundary conditions for the field at the mirrors (boundaries) lead to intermode couplings.

Note that the static Casimir effect = attractive force between the mirrors has been verified experimentally with high accuracy, which proves the reality of vacuum fluctuations!
[Lamoreaux 1997, Mohideen and Roy 1998, Bressi et al 2002,...]

What I have done: Numerical formalism for studying the dynamical Casimir effect.
[MR, J.Opt.B. Quantum Semiclass. Opt. 7 (2005), J. Phys. A: Math Gen. 39 (2006), Phys. Rev. A 73 (2006)]

In braneworld cosmology we have a similar situation: a field equation for tensor modes and time-dependent boundary conditions. Consequently, graviton production due to the brane motion happens. We are applying the formalism of the dynamical Casimir effect to study the evolution of tensor perturbations and graviton production by moving branes.

Field theory with moving boundaries I

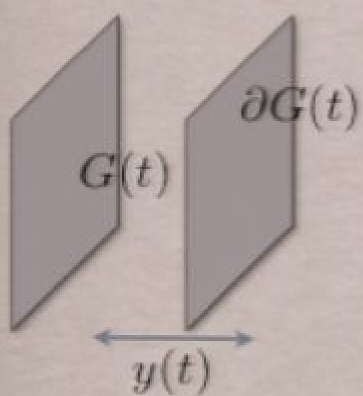


Mathematical problem:

$$\boxed{\text{wave equation } \; \Box\Phi = 0 \; \text{ in } \; G(t) \; \text{ and BC } \; (A + B\partial_x)\,\Phi = 0 \; \text{ at } \; \partial G(t)}$$

Integrating over the non-dynamical dimensions the wave equation reduces to

$$[\partial_t^2 - \triangle_y + k^2]\Phi(t, y; k) = 0 \qquad\qquad [a\Phi + b\,\partial_y\Phi]_{y=\{0,y(t)\}} = 0$$



It is then possible to introduce a set of instantaneous (time-dependent) eigenfunctions $\phi_n(t,x)$ of $-\triangle_y$ satisfying the BC's at any time t.

$$\boxed{[-\Delta_y + k^2]\phi_n(t,y) = \omega_n^2(t)\phi_n(t,y)}$$

time-dependent frequeny

time-dependent eigenfunctions

This is a Sturm - Liouville problem $\forall t$,
hence the set is complete and orthonormal $\forall t$:

$$\boxed{\Phi(t,x) = \sum_n q_n(t)\phi_n(t,x)}$$

canonical variables

Terms like $(\partial_t\Phi)^2$ in the action or $(\partial_t^2\Phi)\phi_m$ in the equation of motion lead to mode couplings described by a coupling matrix:

$$\boxed{M_{nm}(t) = \int_{G(t)} d\mu\, \dot{\phi}_n(t,x)\,\phi_m(t,x)} \qquad \boxed{N = MM^T}$$

Field theory with moving boundaries II

Equations of motion for canonical variables:

$$\ddot{q}_n + \Omega_n^2 q_n + \sum_m \left\{ \left[M_{mn}(t) - M_{nm}(t) \right] \dot{q}_m + \left[\dot{M}_{mn}(t) - N_{nm}(t) \right] q_m \right\} = 0$$

Corresponding Hamiltonian:

$$H(t) = \frac{1}{2} \sum_n \left[p_n^2(t) + \Omega_n^2(t) q_n^2(t) \right] + \sum_{nm} q_n(t)\, M_{nm}(t)\, p_m(t)$$

Momentum:

$$p_n = \dot{q}_n + \sum_m q_m M_{mn}$$

time-dependent frequency
= Squeezing Effect

time-dependent coupling matrix =
Acceleration Effect (boundary effect!)

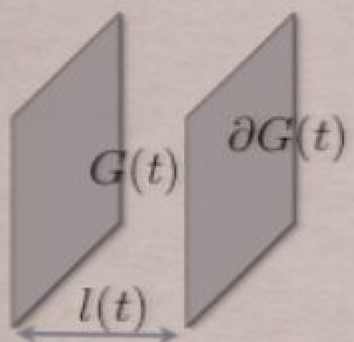
The dynamics of the boundary yields time - dependent mode couplings. Together with the time-dependent frequency one has therefore two external time-dependences entering the equations of motion. Consequently, in the quantum description, two sources of particle creation are present.

We will see that for the evolution of tensor perturbations in braneworld cosmology, the mode couplings are much more important than the time-dependent frequency.

FIELD THEORY WITH MOVING BOUNDARIES III

Quantization, Vacuum And Particle Definition

Consider the two mirror Dynamical Casimir effect situation.



We assume an In - Out - scenario, i.e. for $t \leq t_0$ the mirrors are at rest. At t_0 they start to move and are at rest again for $t \geq t_1$. Canonical quantization via replacing $\{q_n, p_n\} \rightarrow \{\hat{q}_n, \hat{p}_n\}$ and imposing the usual commutation relations. Furthermore, we adopt the Heisenberg picture. Before and after the motion the Hamiltonian is diagonalizable and the solutions of the EOM are plane waves of frequency $\Omega_n^0 = \Omega_n(t \leq t_0)$ and $\Omega_n^1 = \Omega_n(t \geq t_1)$, respectively.

The particle concept can then be introduced without ambiguity. The initial vacuum state $\hat{a}_n|0, t \leq t_0\rangle = 0$ and the final vacuum state $\hat{A}_n|0, t \geq t_1\rangle = 0$ are related via a Bogoliubov transformation

$$\hat{A}_n = \sum_m \left[\mathcal{A}_{mn}(t_1)\hat{a}_m + \mathcal{B}^*_{mn}(t_1)\hat{a}^\dagger_m\right]$$

and the number of created (final state) particles is given by

$$N_n = \langle 0, t \leq t_0|\hat{A}^\dagger_n \hat{A}_n|0, t \leq t_0\rangle = \sum_m |\mathcal{B}_{mn}(t_1)|^2$$

The Bogoliubov coefficient $\mathcal{B}_{nm}$ can be calculated numerically by solving a system of coupled first order differential equations **[MR, J. Phys. A: Math. Gen. 39 (2006)]** .

Field theory with moving boundaries IV

Bogoliubov Transformation And First Order System

[MR, J. Phys. A: Math. Gen. 39 (2006)]

Parametrization of the time evolution of $\hat{q}_n(t)$ in a particular way yields

$$\mathcal{B}_{mn}(t_1) = \frac{1}{2}\sqrt{\frac{\Omega_n^1}{\Omega_m^0}}\left[\Delta_n^-(t_1)\,\xi_n^{(m)}(t_1) + \Delta_n^+(t_1)\,\eta_n^{(m)}(t_1)\right],\quad \Delta_n^\pm(t) = \frac{1}{2}\left[1 \pm \frac{\Omega_n^0}{\Omega_n(t)}\right]$$

and a similar expression for $\mathcal{A}_{nm}$. The functions $\xi_n^{(m)}$, $\eta_n^{(m)}$ satisfy an infinite system of coupled first order differential equations:

$$\dot{\xi}_n^{(m)} = -i\left[a_{nn}^+\xi_n^{(m)} - a_{nn}^-\eta_n^{(m)}\right] - \sum_k\left[c_{nk}^-\xi_k^{(m)} + c_{nk}^+\eta_k^{(m)}\right]$$
$$\dot{\eta}_n^{(m)} = -i\left[a_{nn}^-\xi_n^{(m)} - a_{nn}^+\eta_n^{(m)}\right] - \sum_k\left[c_{nk}^+\xi_k^{(m)} + c_{nk}^-\eta_k^{(m)}\right]$$

Vacuum initial conditions:

$$\xi_n^{(m)}(t_0) = 2\delta_{nm}$$
$$\eta_n^{(m)}(t_0) = 0$$

Squeezing Effect

Acceleration Effect (boundary motion)

$$a_{nn}^\pm(t) = \frac{\Omega_n^0}{2}\left\{1 \pm \left[\frac{\Omega_n(t)}{\Omega_n^0}\right]^2\right\},\quad c_{nk}^\pm(t) = \frac{1}{2}\left[M_{kn}(t) \pm \frac{\Omega_k^0}{\Omega_n^0}M_{nk}(t)\right]$$

Only $\Omega_n(t)$, $M_{nm}(t)$ enters but neither $\dot{M}_{nm}(t)$ nor $N_{nm}(t)$.

FIELD THEORY WITH MOVING BOUNDARIES V

Observable Quantities

From the particle number $N_n(k)$ quantities like the energy density and the power spectrum can be constructed.

Energy density:

in general:

$$\rho = \int \frac{d^3k_{\rm phys}}{(2\pi)^3} \sum_n (\Omega_n)_{\rm phys} N_n(k)$$

$$k_{\rm phys} = \frac{k}{a(y_b)}$$

$$(\Omega_n)_{\rm phys} = \frac{\Omega_n}{a(y_b)}$$

in braneworld cosmology:

$$\rho = \frac{1}{a^4(y_b)} \int \frac{d^3k}{(2\pi)^3} \sum_n \Omega_n N_n(k)$$

[Gorbunov, Rubakov, Sibiryakov, 2001]

we are working with co-moving momentum and frequency!

Power spectrum:

[Kobayashi and Tanaka, 2005]

$$\mathcal{P}_\Phi(k) = k^3 \langle\langle \hat{\Phi}^2 \rangle\rangle$$

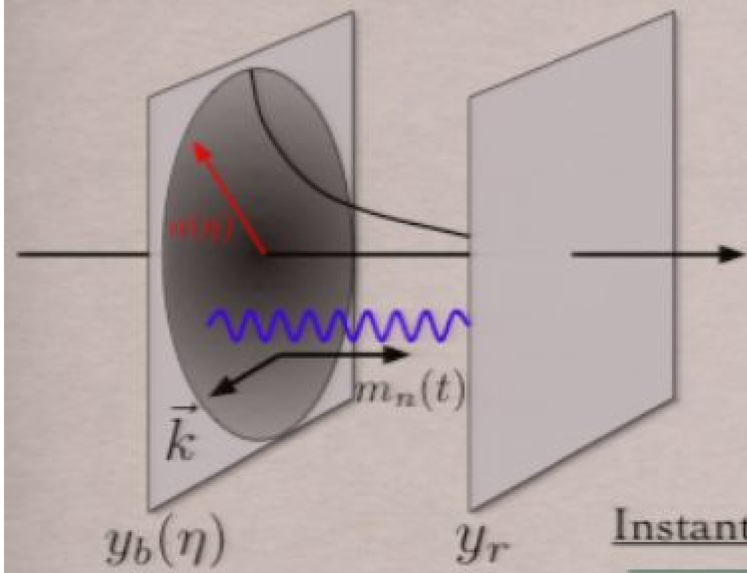
includes spatial averaging + quantum expectation value

$$\mathcal{P}_\Phi(k) \propto k^3 \langle 0, \mathrm{in} | \hat{q}_n^2 | 0, \mathrm{in} \rangle \propto k^3 \sum_n \frac{N_n(k)}{\Omega_n}$$

Given by the particle number alone!

After these technicalities lets go back to braneworlds...

Mode Expansion in AdS Braneworld I



Consider a two brane setup (RS II plus regulator brane) with branes at $y_b(\eta)$ (our universe) and y_r (static brane) at low energies/late times, i.e. v << 1 (-> $t \simeq \eta$).

Hence, time-dependent Neumann BC:

$$\boxed{\partial_y h_\bullet|_{y=\{y_b(t),y_r\}} = 0}$$

<u>Instantaneous mode expansion:</u>

$$\boxed{h_\bullet(t,y;k) = \sqrt{\frac{\kappa_5}{l^3}}\left[q_0(t;k)\phi_0(t;k) + \sum_{\alpha=1}^{\infty} q_\alpha(t;k)\phi_\alpha(t,y;k)\right]}$$

Normalization

4D graviton

Kaluza-Klein modes
-> massive 4D gravitons

Corresponding Sturm - Liouville problem:

$$\boxed{\left[-\partial_y^2 + \frac{3}{y}\partial_y\right]\phi_\alpha(t,y) = m_\alpha^2(t)\phi_\alpha(t,y)}$$

time-dependent Kaluza-Klein masses

The solutions are Bessel functions. The Kaluza - Klein masses are determined by the boundary condition.

Freq. of one mode: $$\boxed{\Omega_n(k) = \sqrt{k^2 + m_n^2(t)}}$$

3-momentum

Kaluza-Klein mass

Mode Expansion in AdS braneworld II

The complete set of solutions: **[Cyril Cartier, Ruth Durrer, MR, Phys. Rev. D 72 (2005)]**

zero mode (4d graviton)

$$\phi_0(t) = \frac{\sqrt{2}\, y_b(t)\, y_r}{\sqrt{y_r^2 - y_b^2(t)}}$$

Kaluza-Klein modes

$$\phi_\alpha(t,y) = A_i(m_i\, y) C_2(m_i\, y)$$

$$C_\nu(m_i y) = J_\nu(m_i\, y) + B_i Y_\nu(m_i\, y)$$

BC at $y_b(t)$:

$$B = -\frac{J_1(m\, y_b)}{Y_1(m\, y_b)}$$

BC at y_r (discret KK tower):

$$J_1(m\, y_r) + BY_1(m\, y_r) = 0$$

Normalization:

$$(A_i\, m_i^2\, y)^2 C_2^2(m_i\, y)|_{y_b}^{y_r} = 2$$

All these qunatities have to be calculated numerically. For the coupling matrix one finds:

The vanishing of the zero-mode - KK - coupling is generic for Neumann BC's

$$M_{00} = \hat{y}_b \frac{y_r^2}{y_r^2 - y_b^2} \qquad M_{ii} = \hat{m}_i \qquad M_{i0} = 2\sqrt{\hat{m}_i M_{00}} \qquad M_{0j} = 0$$

$$M_{ij} = - A_i A_j \hat{m}_i m_i^3 m_j^2 \int_{y_b}^{y_r} y^2 C_1(m_i y) C_0(m_j y) dy - \frac{2m_i^2}{m_i^2 - m_j^2} \sqrt{\hat{m}_i \hat{m}_j} \left[1 + \frac{\hat{m}_i}{\hat{y}_b}\right] \left[\sqrt{\frac{\hat{m}_i(\hat{y}_b + \hat{m}_j)}{\hat{m}_j(\hat{y}_b + \hat{m}_i)}} - 1\right]$$

where: $\hat{m}_i = \dfrac{\partial_t m_i}{m_i}$

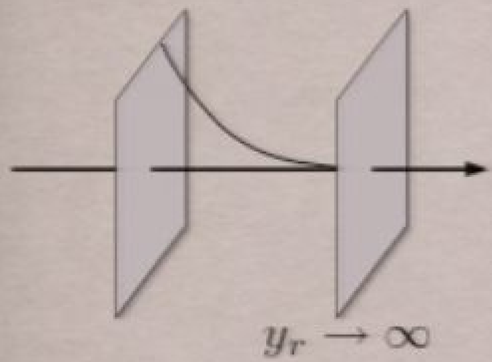
$$\hat{y}_b = \frac{\partial_t y_b}{y_b} \simeq H\, a = -\frac{\dot{a}}{a} = -\mathcal{H}$$

With this at hand the evolution of tensor perturbations and graviton generation in moving AdS braneworlds can now be investigated analytically and numerically.

RECOVERING 4D-GRAVITY IN RS II

[Cyril Cartier, Ruth Durrer, MR, Phys. Rev. D 72 (2005)]

Studying the evolution of tensor perturbations in RS II fully numerically by solving the Klein Gordon equation, it was found that at late times the evolution seems to be identical to the 4D evolution [Koyama, 2004]. We can prove this with our formalism.



If we send the 2nd brane to infinity we go back to RS II.

Setting $\epsilon = y_b/y_r$ one finds $\hat{m}_i \simeq \hat{y}_b \epsilon^2$ for $\epsilon \to 0$

To lowest order in ϵ and in the low energy limit $v \ll 1$ the coupling matrices reduce to

$$\begin{array}{ll} M_{00} \simeq -\mathcal{H}(1+\mathcal{O}(\epsilon)) & M_{0j} = 0 \\ M_{ii} = M_{ij} = \mathcal{H}\mathcal{O}(\epsilon^2) & M_{i0} = \mathcal{H}\mathcal{O}(\epsilon) \end{array}$$

For $\epsilon \to 0$ no coupling of the zero mode to Kaluza - Klein modes. The system of differential equations for the zero mode (4d - graviton) reduces to $\ddot{q}_0 + [k^2 - \dot{\mathcal{H}} - \mathcal{H}^2]q_0 = 0$.

With $h_\bullet \simeq h_{\bullet 0} \propto q_0/a$ we find for the amplitude of the tensor perturbation:

$$\ddot{h} + 2\mathcal{H}\dot{h} + k^2 h = 0$$

usual equation for 4D gravity waves!

This proves that at low energies / late times (v<<1) the homogeneous tensor perturbation equation in brane cosmology reduces to the 4-dimensional tensor perturbation equation. At late times, no „signature“ of the 5th dimension, i.e. no modification of the 4D gravity wave equation, is observed. Massive gravitons (KK modes) are not excited by the brane motion.

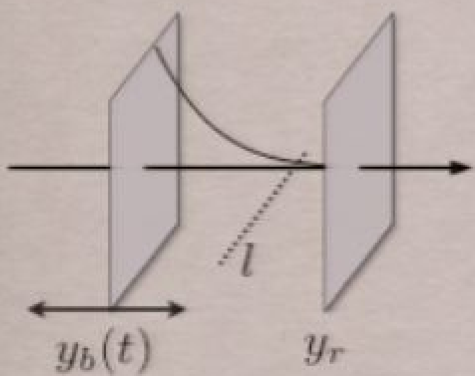
Let me now finally discuss a toy-model for studying the quantum generation of gravitons - massless and massive.

The following is work in progress and all results are preliminary!

Radiation collaps - The model

[MR, Ruth Durrer (2006), in preparation]

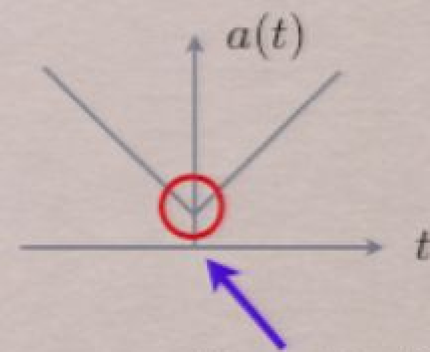
In order to study quantum generation of gravitons, let us consider the following (toy) model with two branes: two branes approach each other in a radiation dominated low energy phase with v << 1 (i.e. $t \simeq \eta$), bounce off and move apart from each other afterwards.



brane motion: $y_b(t) = \dfrac{l}{t_b + |t|}$

scale factor: $a(t) = t_b + |t|$

$v \ll 1 \Rightarrow v_{\max} = l\, t_b^{-2} \ll 1$



The advantage of this scenario is the existence of well defined in - and out - vacuum states because the couplings vanish $M_{ij} \to 0$ if $t \to \pm\infty$, i.e. when the brane system returns asymptotically to its initial configuration.

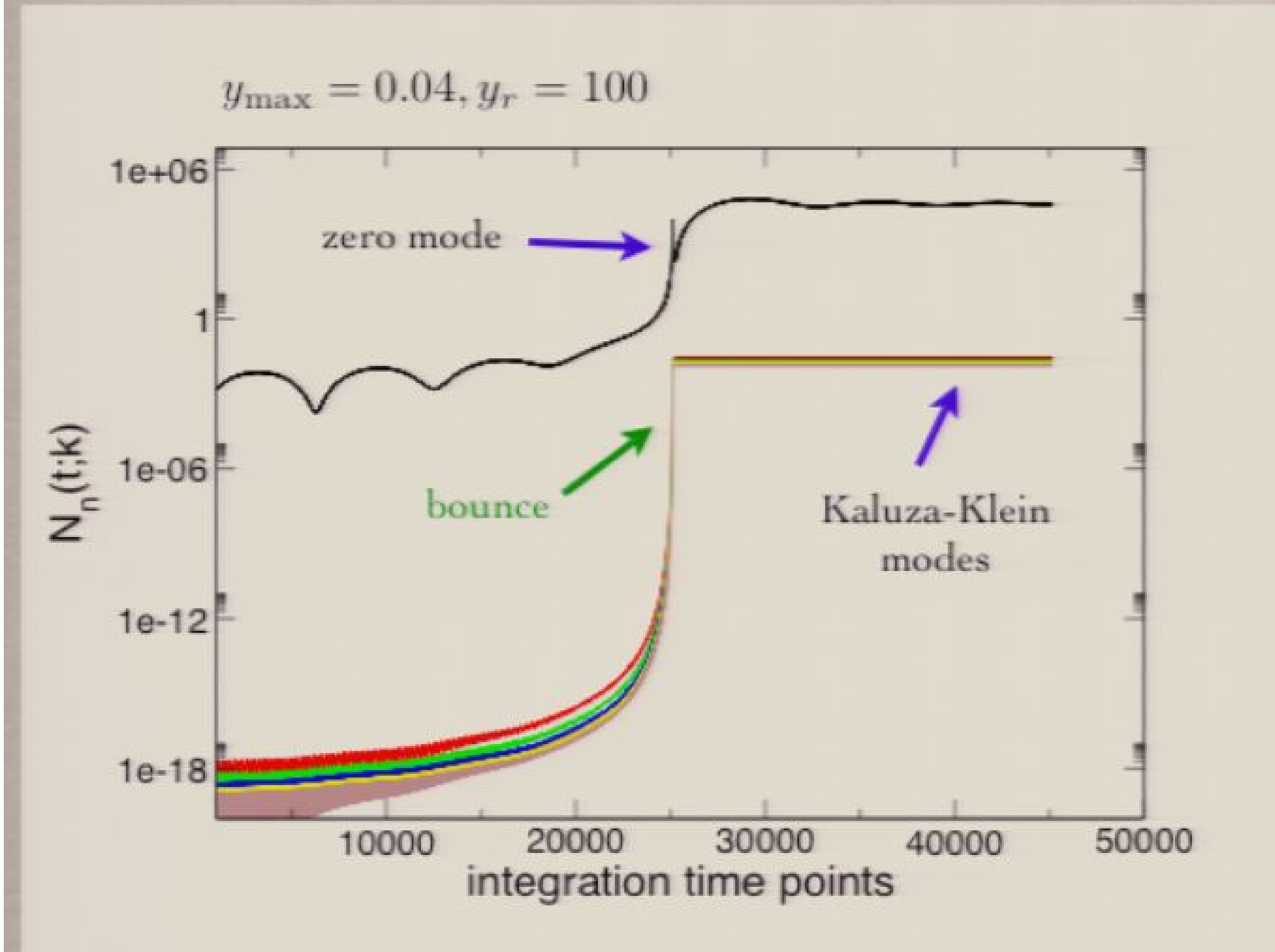
Possible interesting implications for ekpyrotic models, for example. However, I'd like to see it more as a (toy) model to study principle effects.

We are studying this problem fully numerically with the method outlined before.

Let me show you and discuss some preliminary results.

RADIATION COLLAPS -NUMERICS

[MR, Ruth Durrer (2006), in preparation]



Our formalism allows to study the couplings between the modes; we can switch-off particular couplings by hand and compare the numerical results!

Observations:

1) The zero mode is not affected by the KK modes. Solving for it independently gives the same!

2) The KK modes do (practically) not couple to each other! We can neglect M_{ij} and in addition M_{ii} . Taking into account their coupling to the zero mode only M_{i0} gives the same results!

The numerics is under control: stability, accuracy, initial time to avoid spurious effects and so on.

RADIATION COLLAPS - THE ZERO MODE

[MR, Ruth Durrer (2006), in preparation]

Numerics: no coupling of the zero mode (4D graviton) to Kaluza - Klein modes.

In this case we can solve for the zero mode analytically. The time-evolution for the 4D graviton is effectively given by the scattering problem:

$$\ddot{\epsilon}(t) + V(t)\epsilon(t) = 0 \quad \text{with } \delta\text{ - function potential} \quad V(t) = k^2 + 2\sqrt{v_{\max}}\,\delta(t)$$

Scattering theory arguments then lead to the simple result:

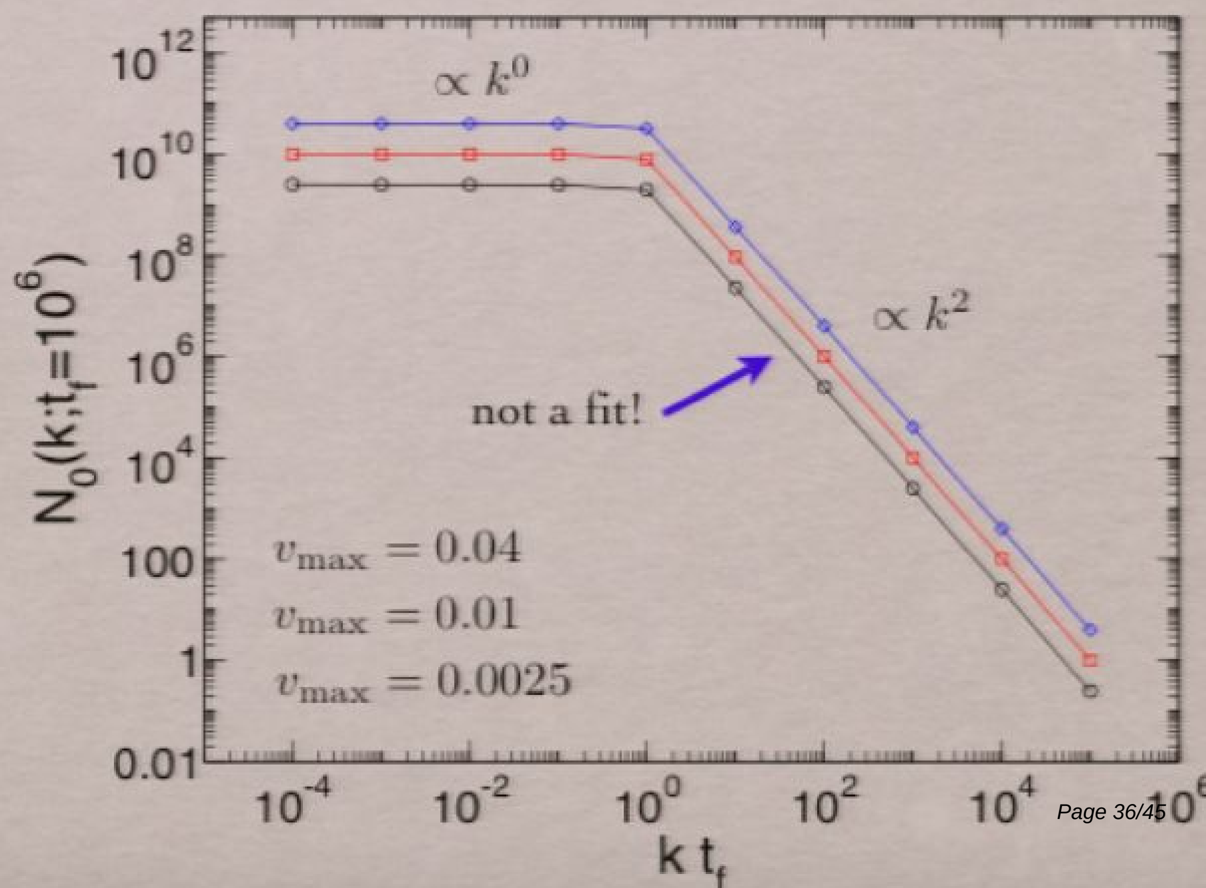
$$N(k, kt \gg 1) = \frac{v_{\max}}{k^2}$$

Power spectrum:

$$P_0(k) \propto k^2 N_0(k)$$

$$\propto v_{\max} k^0 \quad kt \gg 1$$

$$\propto v_{\max} k^2 \quad kt \ll 1$$



Radiation collaps - KK-modes

[MR, Ruth Durrer (2006), in preparation]

Numerics: the dominant term for the Kaluza-Klein modes is their coupling to the zero mode.

Taking this into account, the system of equations for the Bogoliubov coefficients decouples and can be solved in closed form. For the number of created massive gravitons, one finds:

$$N_n(k) = \frac{1}{16}\frac{k}{\omega_n^0}|\widehat{S}(\omega_n^0)|^2$$

with the Fourier transformed

$$\widehat{S}(\omega_n^0) = \int_{-\infty}^{+\infty} dt S_n(t;k) e^{-i\omega_n^0 t}$$

of the source:

$$S_n(t;k) = \frac{2i}{k} M_{n0}(\dot{\epsilon} + M_{00}\epsilon)$$

zero mode self coupling

Kaluza-Klein - zero mode coupling

zero-mode evolution

The coupling of the 4D graviton to the Kaluza-Klein modes is entirely due to the time-dependent BC!

The production of Kaluza-Klein particles (massive gravitons) takes place entirely due to the coupling of these modes to the zero mode. The 4D graviton (zero mode) acts as a source for the generation of massive gravitons out of vacuum fluctuations. Without coupling to the zero mode, Kaluza-Klein modes are not excited.

Radiation collaps - KK-modes

[MR, Ruth Durrer (2006), in preparation]

The Fourier transformation can be carried out analytically for the case $m_n y_b \ll 1, y_r \gg 1$
The solutions can be expressed in terms of exponential integrals. Perfect agreement with numerics.

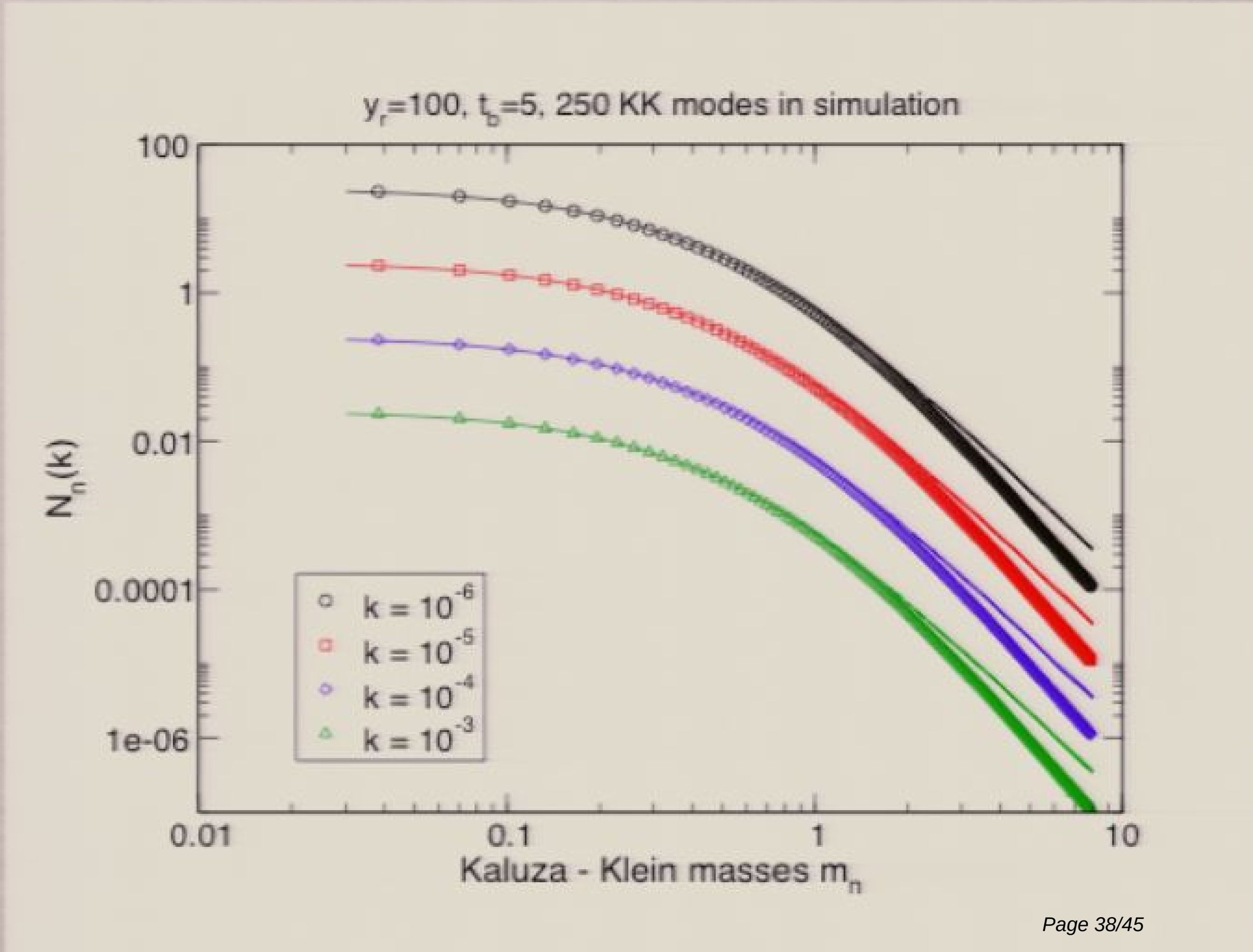
From general properties of Fourier transformations one finds that in the asymptotic limit $m_n \to \infty$ the number of generated massive gravitons for a fixed wave number k behaves like

$$N_n(k = \text{fixed}) \propto \frac{1}{m_n^5}$$

For a fixed KK mode we have

$$N_{n=\text{fixed}}(k) \propto \frac{1}{k^2}$$

Consequently, the energy density of massive gravitons produced from vacuum fluctuations for a fixed wave number k converges.



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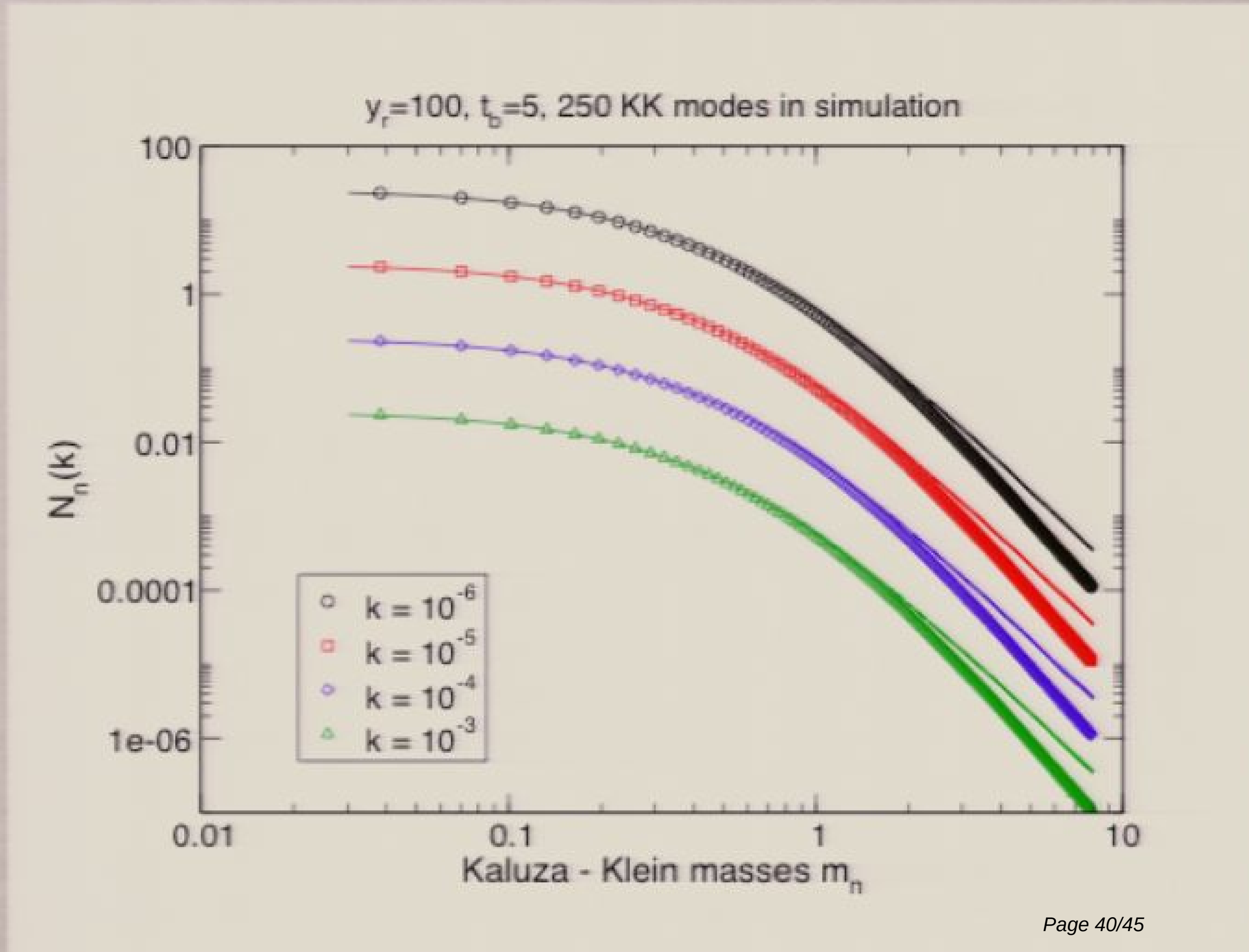

Summary / Conclusions

We have a formalism to study (numerically) the evolution of tensor perturbations in moving braneworlds for general situations keeping track of the intermode coupling.

This formalism can also be applied to vector perturbations (in progress).

We have shown that the 5D tensor perturbation equation reduces to the 4D one at low energies and late times. In addition we have studied a (toy) model of bouncing branes -> Kaluza-Klein particles are created due to the coupling to the zero mode! This work is still to be finished.

It is also possible to use practically the same formalism to study the high energy regime, braneworld inflation (in progress - numerically a bit more involved due to the more complicated boundary condition).
-> Observable consequences?

Thanks a lot for your time and attention!

$$\left(\cancel{v\,\partial_t} - \partial_r\right) h \Big|_{r_\xi(t)} = 0$$

$$v \ll 1$$

$$v = \frac{\ell H}{\sqrt{1 + (\ell H)^2}}$$

$$\left(\cancel{v\partial_t} - \partial_z\right) h\Big|_{z_b(t)} = 0$$

$$v \ll 1$$

$$v = \frac{\ell H}{\sqrt{1+(\ell H)^2}}$$

$$\ell H \ll 1$$

$$\left(\cancel{\sqrt{\partial_t}} - \partial_y\right) h \Big|_{z_b(t)} = 0$$

$$v \ll 1$$

$$\ell H \ll 1$$

$$v = \frac{\ell H}{\sqrt{1 + (\ell H)^2}}$$

$$y_h \gg \ell$$