Title: Born-Infeld Black Holes

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Abstract:

Born-Infeld black holes

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Born-Infeld electrodynamics

$$L_{BI} = 4b^2\left[\sqrt{g} - \sqrt{\left|\det\left(g_{\mu\nu} + \frac{F_{\mu\nu}}{b}\right)\right|}\,\right]$$

In $d=4$

$$L_{BI} = 4b^2\sqrt{g}\left(1 - \sqrt{1 + \frac{F^2}{2b^2}}\right)$$

$$b \to \infty \qquad L_{BI} \to -F^2$$

Field due to a point charge

$$E = \frac{Q}{\sqrt{r^4 + \frac{Q^2}{b^2}}}$$

$$|E| \leq b$$

Born-Infeld electrodynamics

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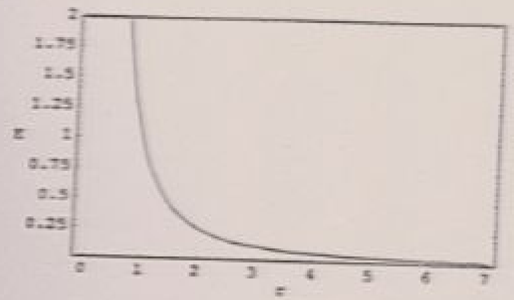
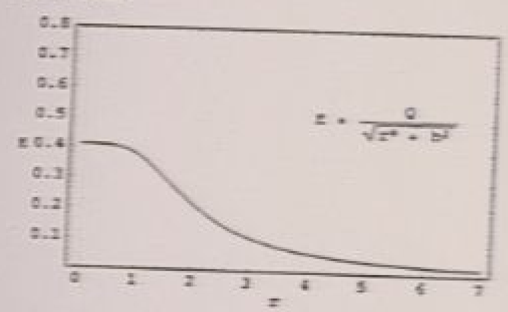
Maxwell E field



BI E field



* Born-Infeld electrodynamics first introduced in 1930's to obtain finite energy for a point charge.

* Loop calculations in open superstring theory lead to a low energy effective action of the Born-Infeld type

* Low energy effective actions for D-branes, Born-Infeld type

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BI equations

$$G^{\mu\nu} = \frac{F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2b^2}}}$$

$$\left.\begin{aligned} \nabla_\mu G^{\mu\nu} &= 0 \implies d * G = 0 \\ \nabla_{[\mu} F_{\nu\sigma]} &= 0 \implies dF = 0 \end{aligned}\right\} \text{BI}$$

$$\left.\begin{aligned} d * F &= 0 \\ dF &= 0 \end{aligned}\right\} \text{Maxwell}$$

Electric-Magnetic duality preserved

$$F \longleftrightarrow * G$$

Black holes in Born-Infeld gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + L(F) \right]$$

$$L(F) = 4b^2 \left(1 - \sqrt{1 + \frac{F^2}{2b^2}} \right)$$

equations of motion

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

$$T_{\mu\nu} = -2 \left(\frac{F_\mu{}^\sigma F_{\nu\sigma}}{\sqrt{1 + \frac{F^2}{2b^2}}} + \frac{g_{\mu\nu}}{4} L(F) \right)$$

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\Lambda r^2}{3} + \frac{2b}{r}\int_r^\infty \left(\sqrt{x^4 b^2 + Q^2} - bx^2\right) dx$$

$b \to \infty$

$$f(r) \longrightarrow 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{\Lambda r^2}{3}$$

the integral can be simplified

$$f(r) = 1 - \frac{2M}{r} + \frac{2br^2}{3}\left(1 - \sqrt{1 + \frac{Q^2}{r^4 b^2}}\right)$$

$$+ \frac{4Q^2}{3r^2}\; {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{b^2 r^4}\right)$$

Hypergeometric functions

$F(a, b, c, z)$

The hypergeometric differential equation

$$z(1-z)\frac{d^2F}{dz^2} + [c-(a+b+1)z]\frac{dF}{dz} - abF = 0$$

$$F(1,1,2,z) = -\frac{\ln(1-z)}{z}$$

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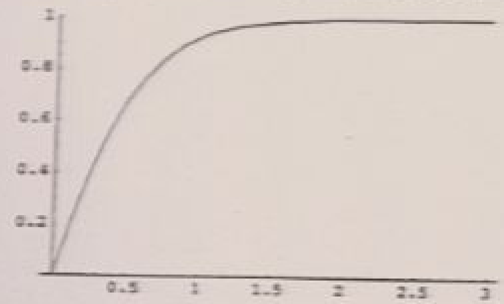


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Out[1]= - Graphics -
```

when $r \to \infty$

$$F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{-Q^2}{b^2 r^4}\right) \to \text{constant}$$

$$f(r) \to 1 - \frac{2M}{r} + \frac{\Lambda r^2}{3}$$

when r is small

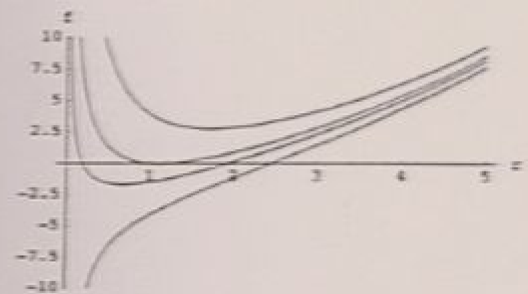
$$f(r) \approx 1 - \frac{(2M-A)}{r} - 2Qb + \frac{2b^2r^2}{3} + \frac{b^2r^4}{5} + \frac{\Lambda r^2}{3}$$

$$A = \sqrt{\frac{b}{\pi}}\, Q^{3/2}\, \Gamma(1/4)^2$$

if $2M - A > 0 \Rightarrow$ SchAdS type

$2M - A < 0 \Rightarrow$ RNAdS type

f (r) for various values of M (b, Q fixed)



Extreme black holes

$$f(r_e) = 0 \qquad f'(r_e) = 0$$

$$r_e^4\left(\Lambda^2 - 4b^2\Lambda\right) + r_e^2\left(-2\Lambda + 4b^2\right) + \left(1 - 4Q^2b^2\right) = 0$$

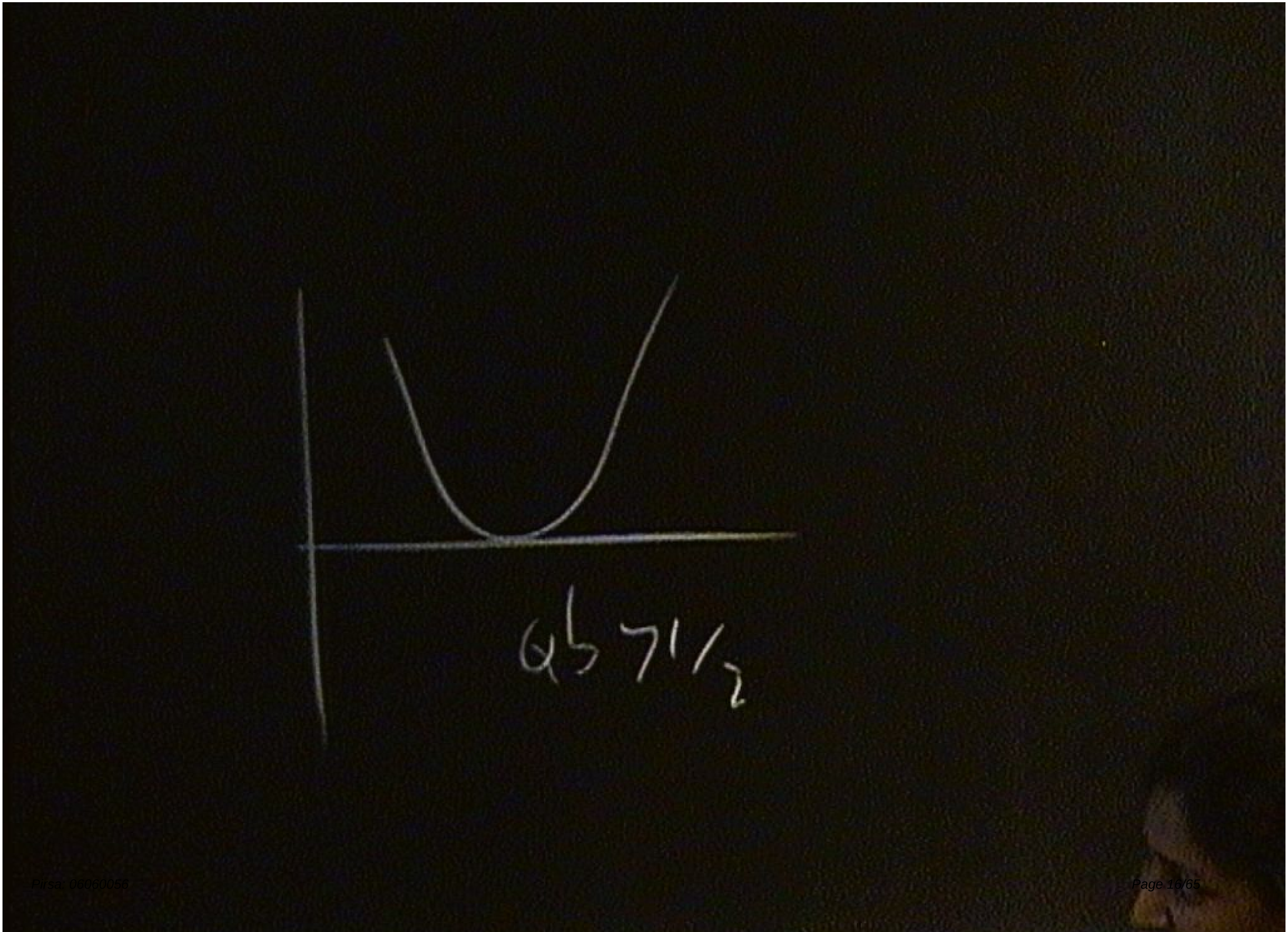
Solution possible if

$$\boxed{Qb > 1/2}$$

$$r_e(\Lambda, Q, b)$$

$$M_{ex} = \left[1 + \left(\frac{2b^2}{3} + \frac{\Lambda}{3}\right)r_e^2 - 2b\sqrt{Q^2 + r_e^4 b^2} + \frac{4Q^2}{3r_e^2}F\right]\frac{r_e}{2}$$

$$M_{ex}(Q, \Lambda, b)$$



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$$M_{ex}(Q, \Lambda, b)$$

$\boxed{Qb > 1/2}$

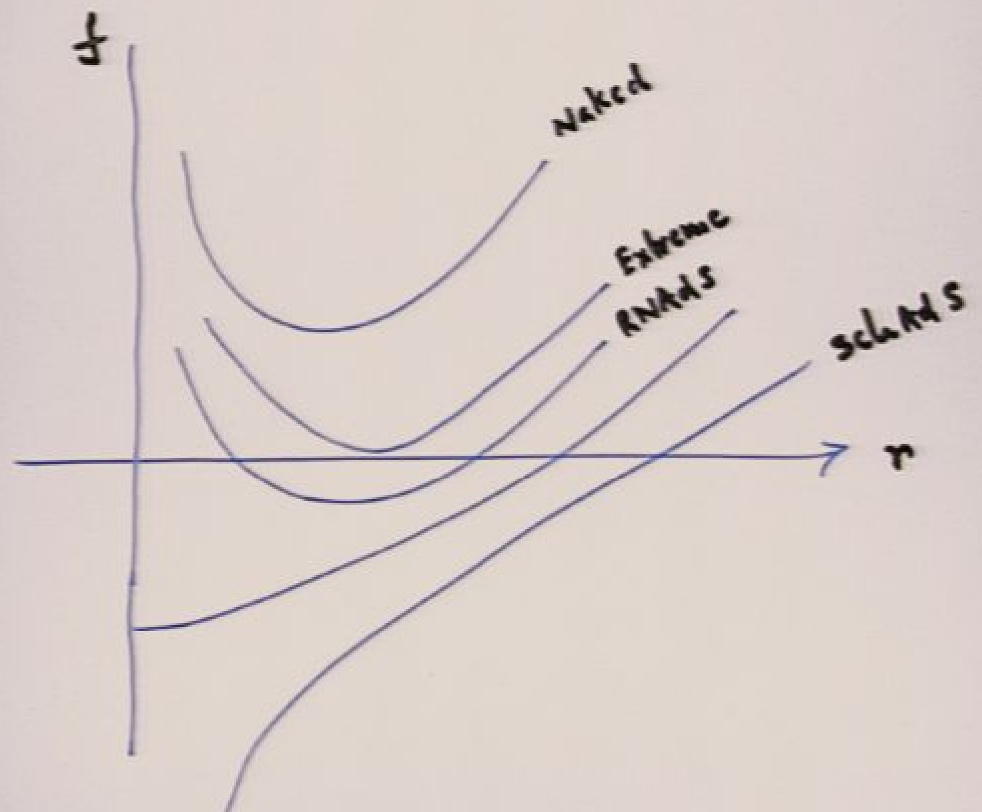
$M > \frac{A}{2} \Rightarrow$ SchAdS type BH

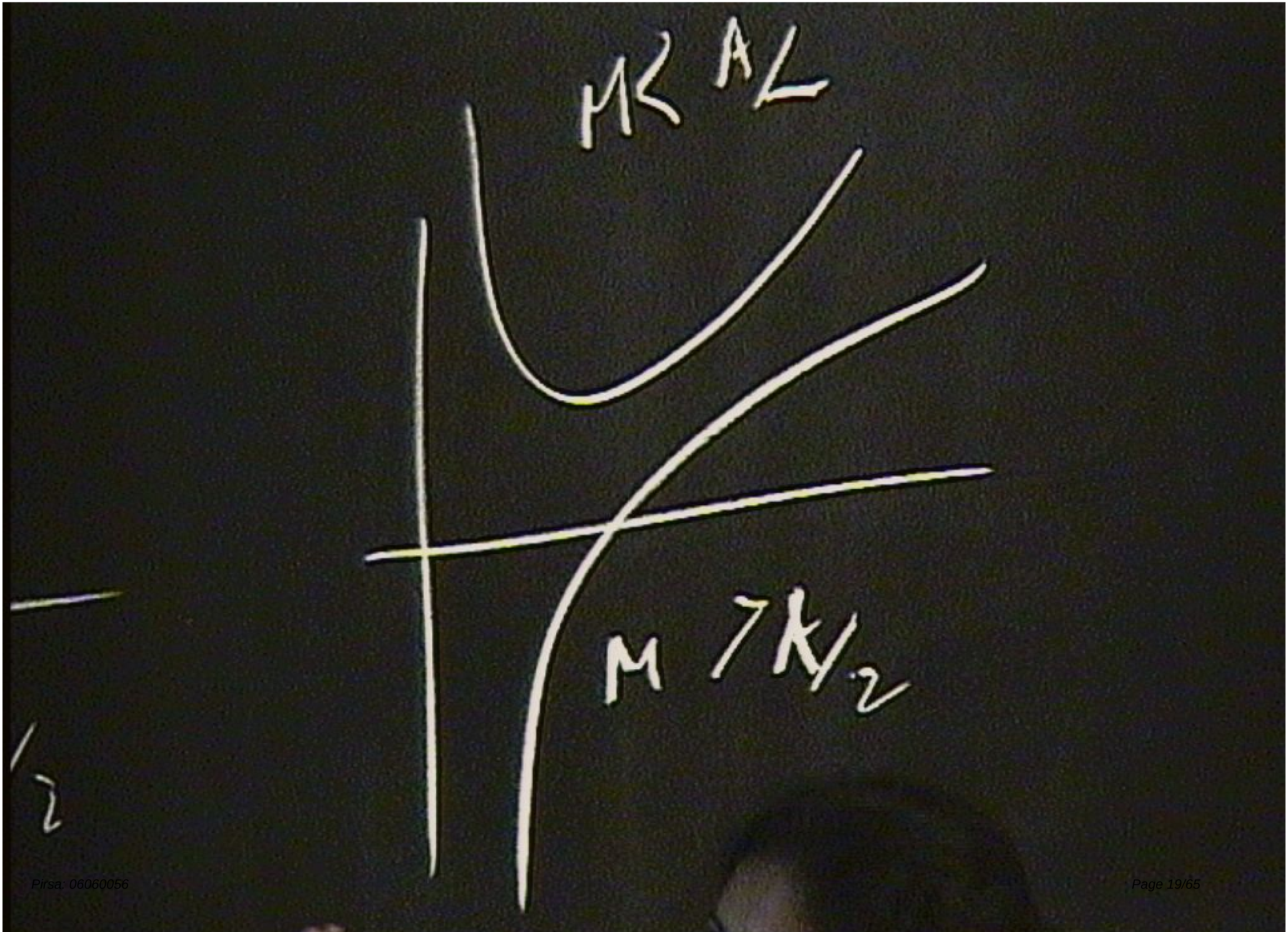
$M = \frac{A}{2} \Rightarrow$ SchAdS type BH

$M_{ex} < M < \frac{A}{2} \Rightarrow$ RNAdS BH

$M = M_{ex} \Rightarrow$ RNAdS extreme BH

$M > M_{ex} \Rightarrow$ Naked singularity





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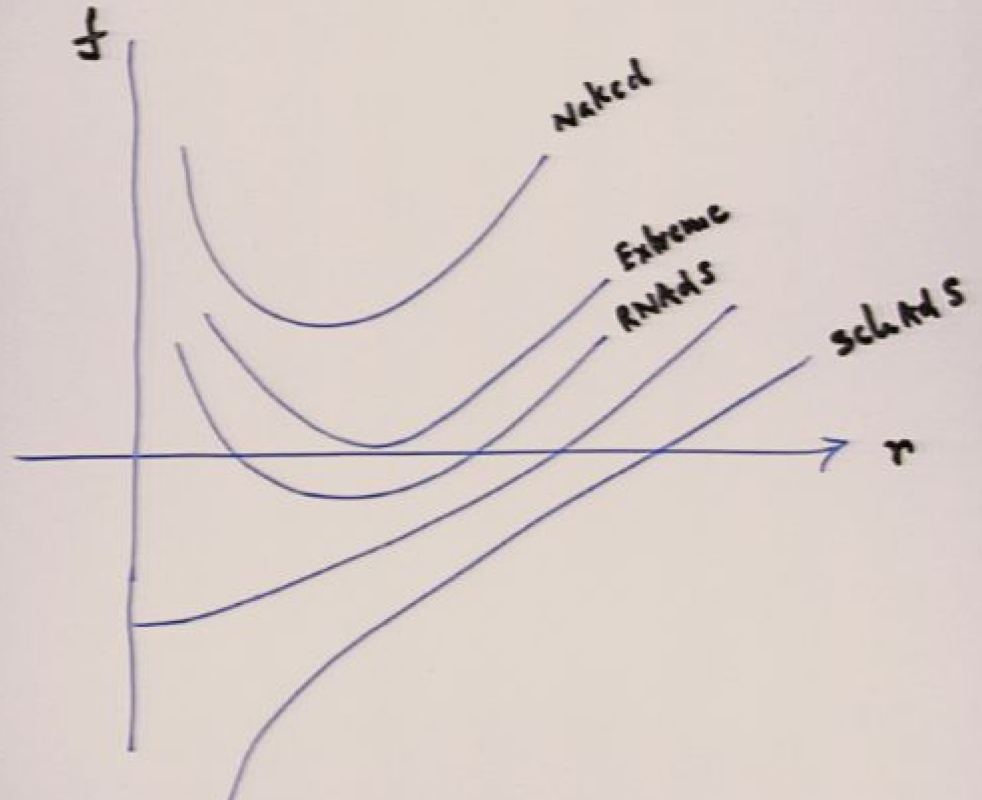
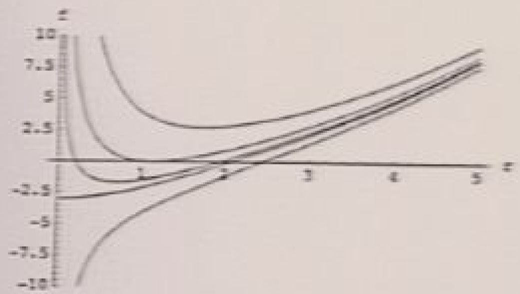
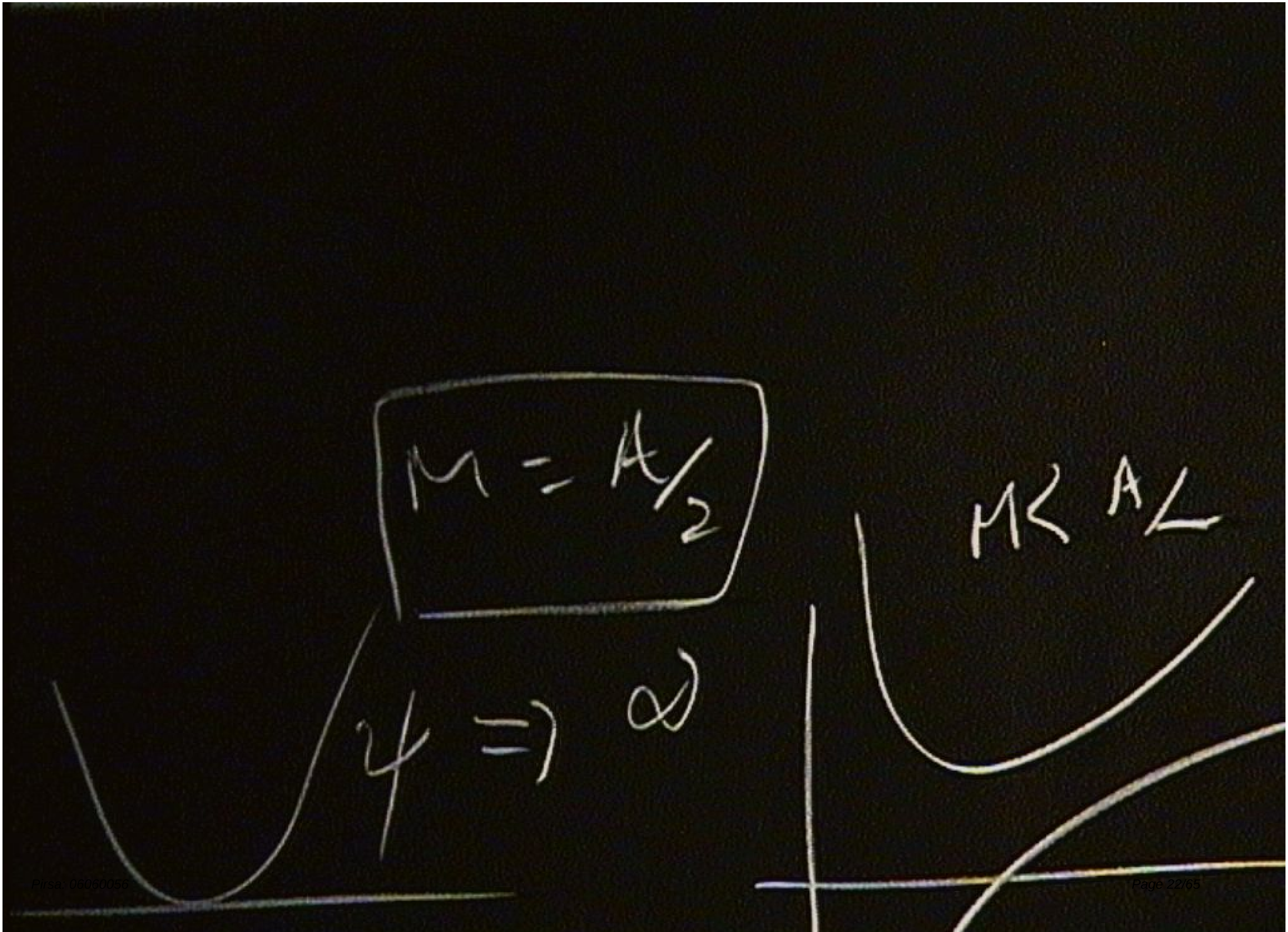
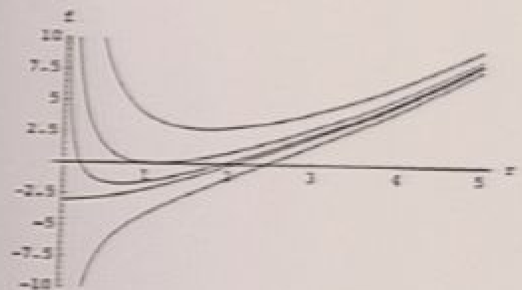


Qb > 1/2




M = A/2
M < A

Q b > 1 / 2

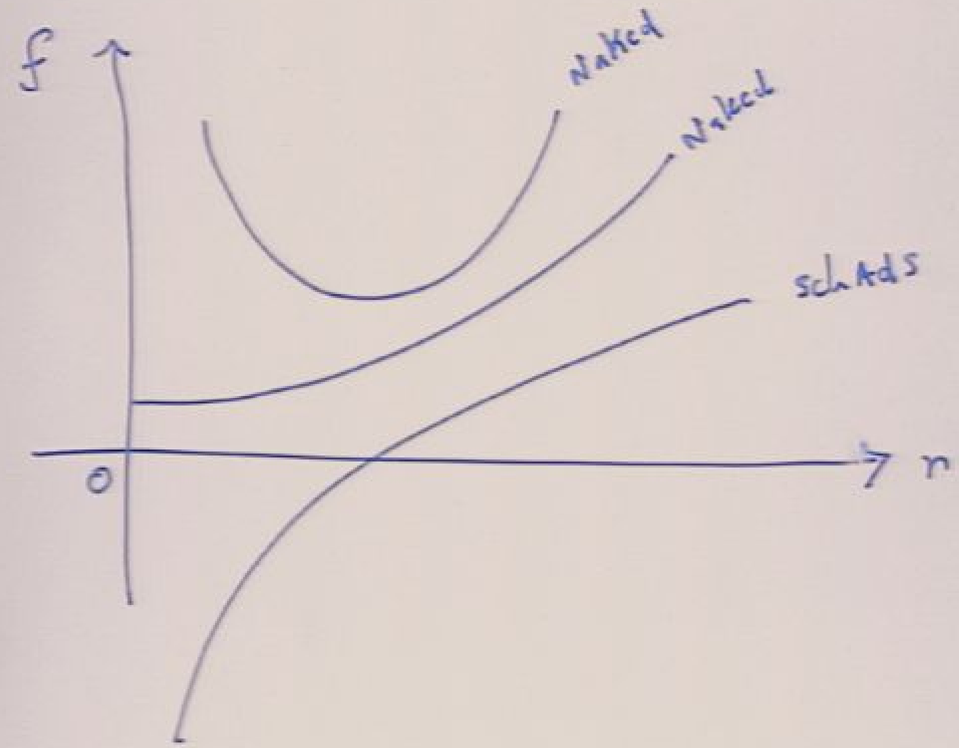


$\boxed{Qb < 1/2}$

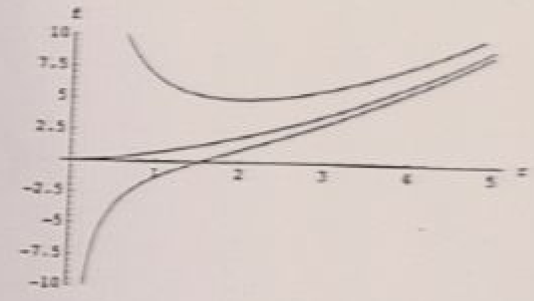
$M > A/2 \Rightarrow$ SchAdS BH

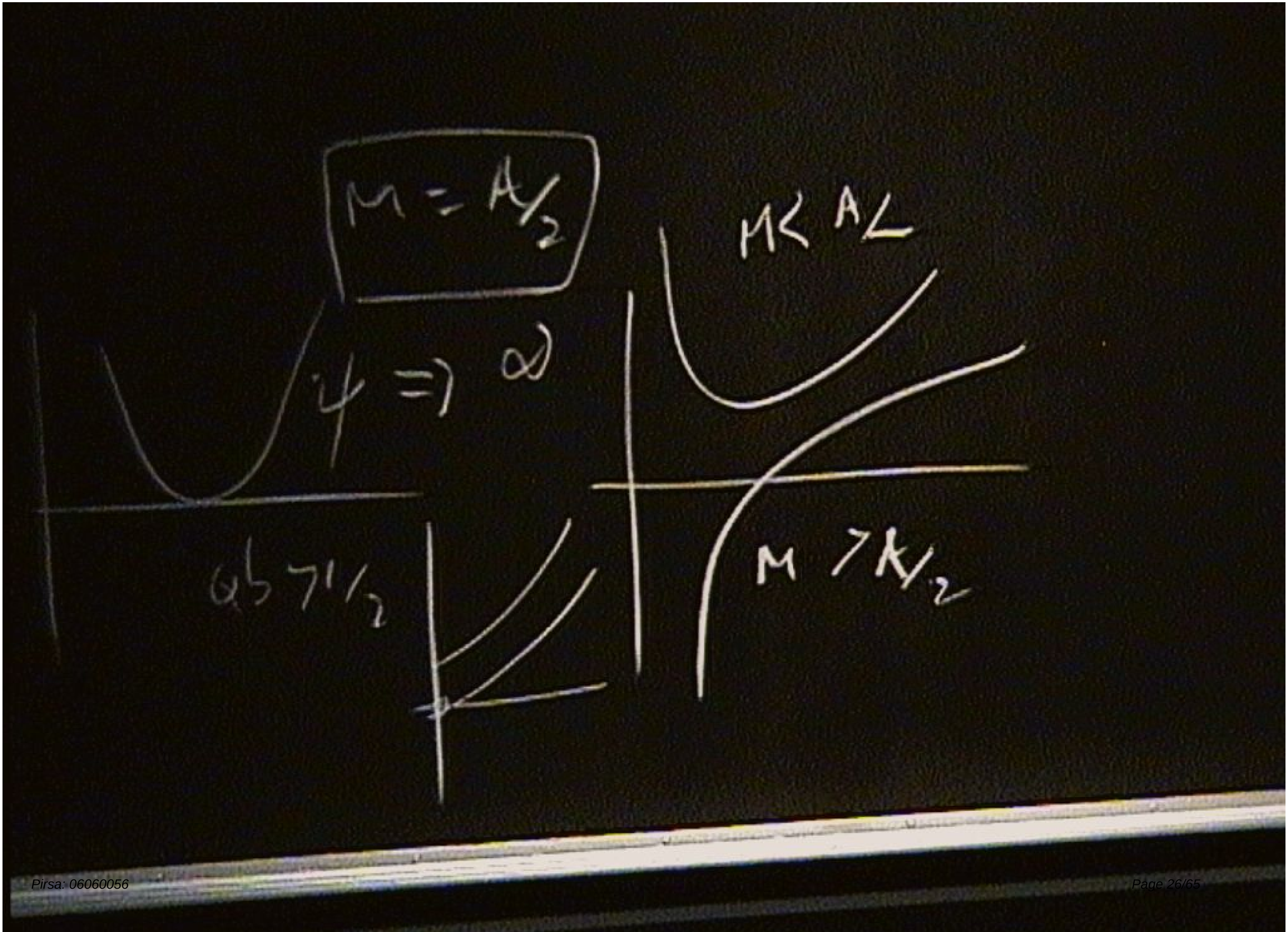
$M = A/2 \Rightarrow$ Naked singularity

$M < A/2 \Rightarrow$ Naked singularity



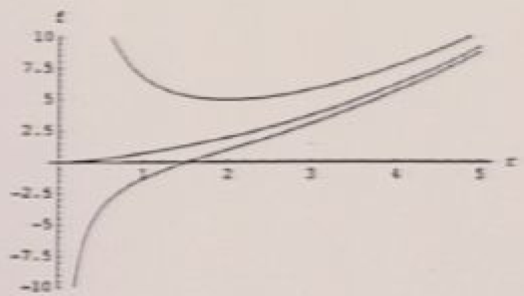
function f when Q b = 1 / 2

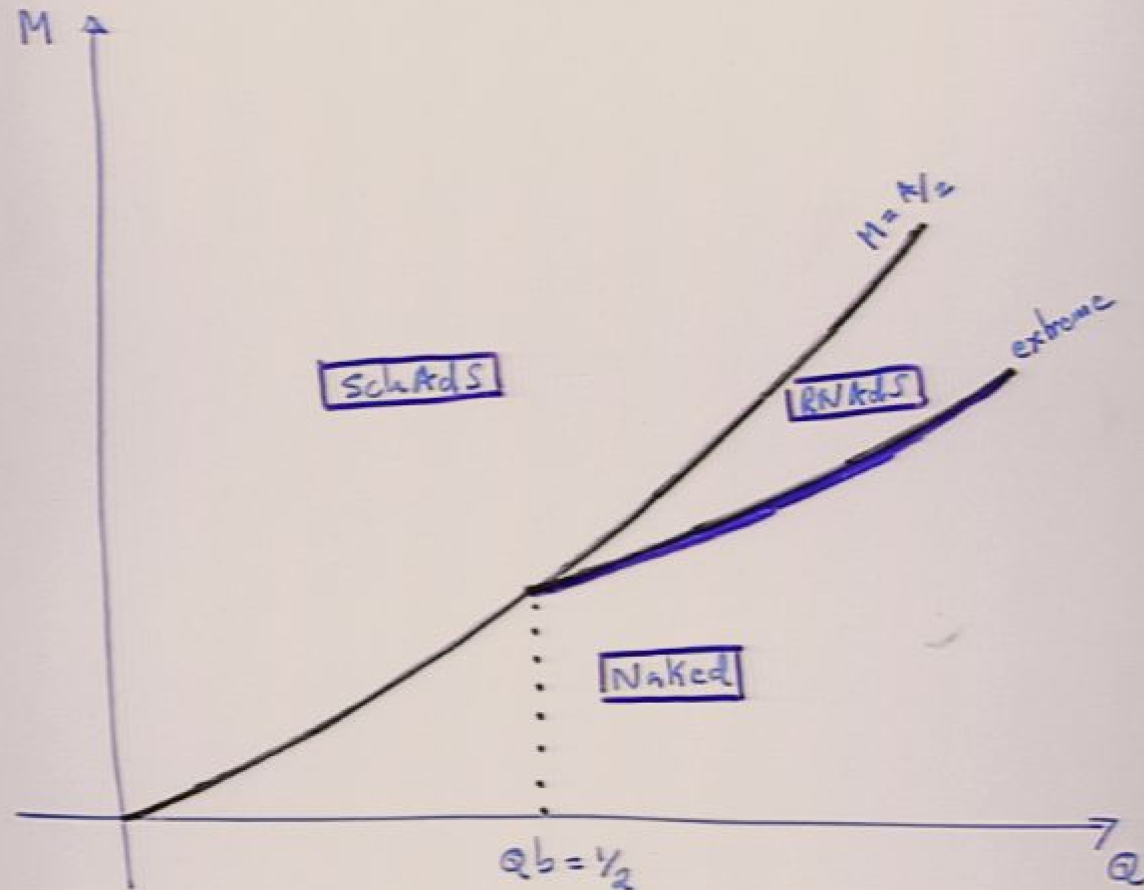




M = A/2
⇒ ∞
αb > 1/2
M < A/2
M > A/2

function f when Q b = 1 / 2





$A \sim Q^{3/2}$

RNAdS black holes

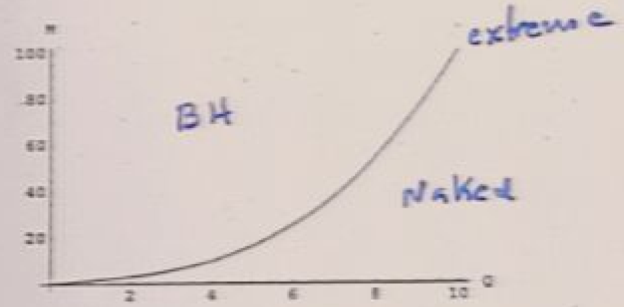
$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{\ell^2}$$

$$r_{ex} = \frac{1}{6}\left[-\ell^2 + \ell\sqrt{\ell^2 + 12Q^2}\right]$$

$$M_{ex} = \left(1 + \frac{Q^2}{r_{ex}^2} + \frac{r_{ex}^2}{\ell^2}\right)\frac{r_{ex}}{2}$$

$$= M_{ex}(Q, \Lambda)$$

RNAdS M vs Q



r (exterene) = $\frac{1}{6}\left(-l^2 + l\sqrt{l^2 + 12 Q^2}\right)$

mass (extreme) =

$$-0.083\left(-1.` l^2 + l\sqrt{l^2 + 12.` Q^2}\right)\left(-1.` - \frac{36.` Q^2}{\left(-1.` l^2 + l\sqrt{l^2 + 12.` Q^2}\right)^2} - \frac{0.027\left(-1.` l^2 + l\sqrt{l^2 + 12.` Q^2}\right)^2}{l^2}\right)$$

Thermodynamic stability

Global stability

Grand canonical ensemble

Fixed electric potential

Canonical ensemble

Fixed electric charge

Possible geometries

BI black holes & Pure AdS_4

Grand canonical ensemble

ϕ = electrostatic potential between horizon & infinity

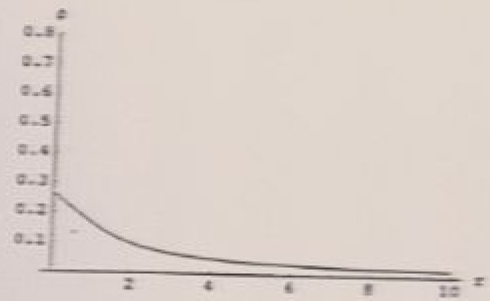
RN AdS $\Rightarrow$ $A_t = -\frac{Q}{r} + \phi$

$$\boxed{\phi = \frac{Q}{r_+}}$$

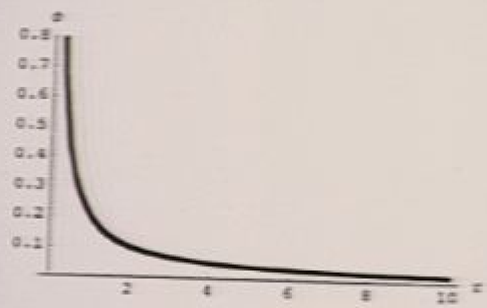
BI AdS $\Rightarrow$ $A_t = -\frac{Q}{r} F + \Phi$

$$\boxed{\Phi = \frac{Q}{r_+} F\left(\tfrac{1}{4}, \tfrac{1}{2}, \tfrac{5}{4}, -\frac{Q^2}{b^2 r_+^4}\right)}$$

Potential vs r for BIAdS



Potential vs r for RNAdS

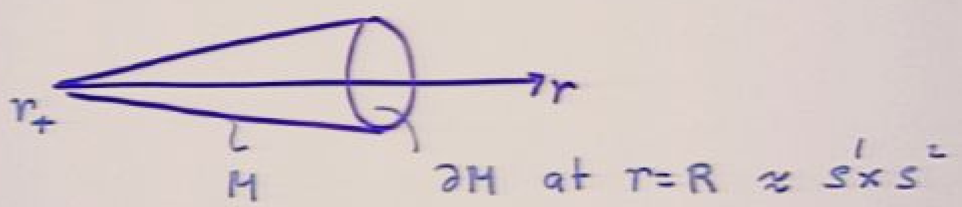


Euclidean Action

$t = i\tau$

$$ds^2 = f\,d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

τ has the character of an angular coordinate with period $\beta = \frac{1}{T}$



$$I_{BH} = I_{bulk} + I_{surface}$$

$$= -\frac{1}{16\pi G}\int_{M} d^4x\,\sqrt{g}\,\left[\,R - 2\Lambda + L_{BI}\,\right]$$

$$- \frac{1}{8\pi G}\int_{\partial M} d^3x\,\sqrt{h}\;K$$

$$I_E = \lim_{R\to\infty}\left(I_{BH} - I_{AdS} \right)$$

Finally!

$$I_E = \frac{\omega\beta}{16\pi G}\left[r_+ - \frac{r_+^3}{l^2} - \frac{2b^2 r_+^3}{3} + \frac{2br_+}{3}\sqrt{r_+^4 b^2 + Q^2} - \frac{4Q^2}{3r_+}F\right]$$

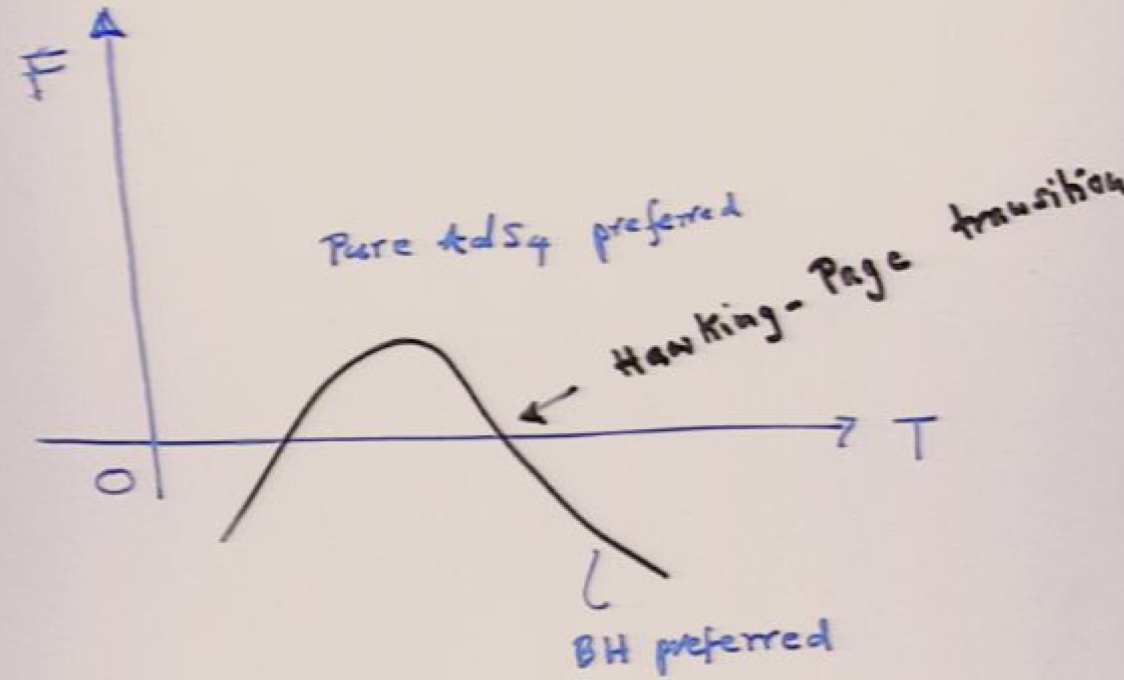
Take $\lim b \to \infty$

$$I_{RNAdS} = \frac{\omega\beta}{16\pi G l^2}\left[l^2 r_+ - r_+^3 - \frac{Q^2 l^2}{r_+}\right]$$

Gibbs free energy

$$F = \frac{I}{\beta}$$

$$= \frac{\omega}{16\pi G}\left[r_+ - \frac{r_+^3}{l^2} - \frac{2b^2 r_+^3}{3} + \frac{2br_+}{3}\sqrt{r_+^4 b^2 + Q^2} - \frac{4Q^2}{3r_+}F\right]$$


F
Pure AdS$_4$ preferred
Hawking-Page transition
T
0
BH preferred

Temperature

$$T = \frac{1}{4\pi}\left[\frac{1}{r_+} + \left(2b^2 + 3/l^2\right) r_+ - \frac{2b}{r_+}\sqrt{r_+^4 b^2 + Q^2}\right]$$

$$\beta = \frac{1}{T}$$

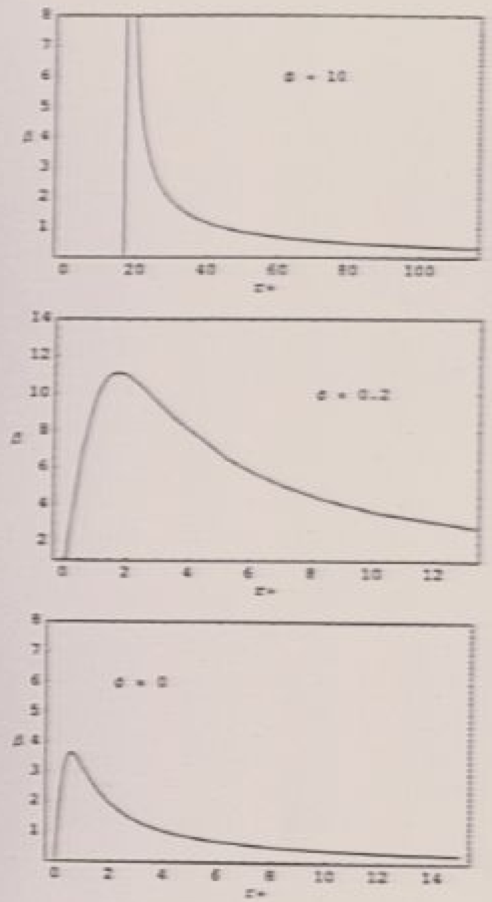
$$x = \frac{Q}{r_+^2} \Rightarrow \phi = x\,F(x)\,r_+$$

$$I(\phi, x)$$

$$\beta(\phi, x)$$

$$F(\phi, x)$$

Inverse temperature Vs horizon (BI)

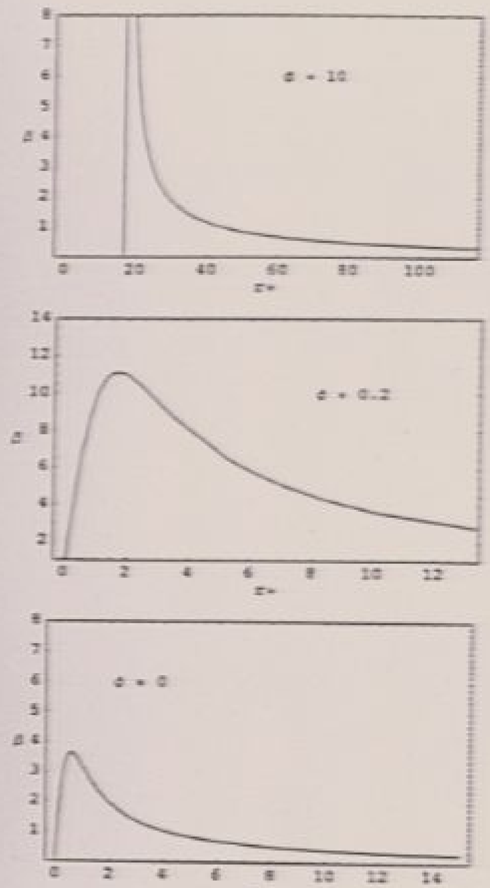


extreme BH
T = 0

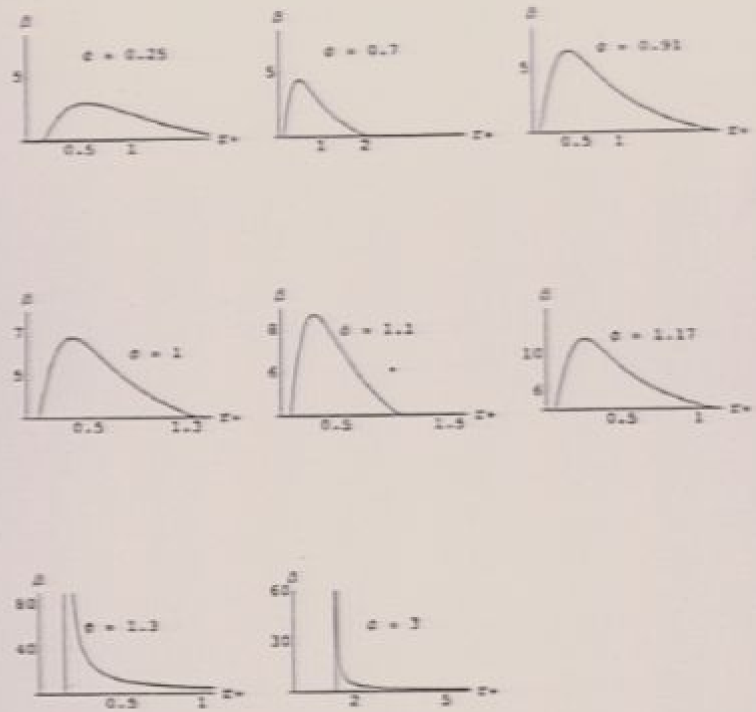
ab $7^{1/2}$

$$\frac{1}{T} = \beta$$

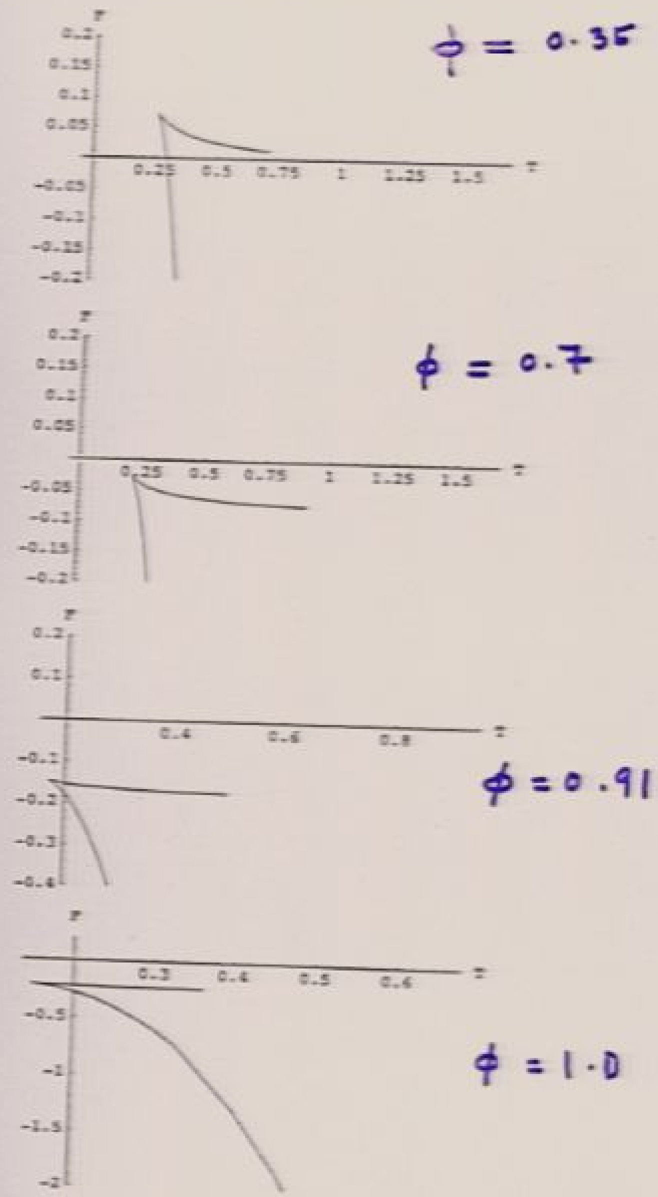
Inverse temperature Vs horizon (BI)



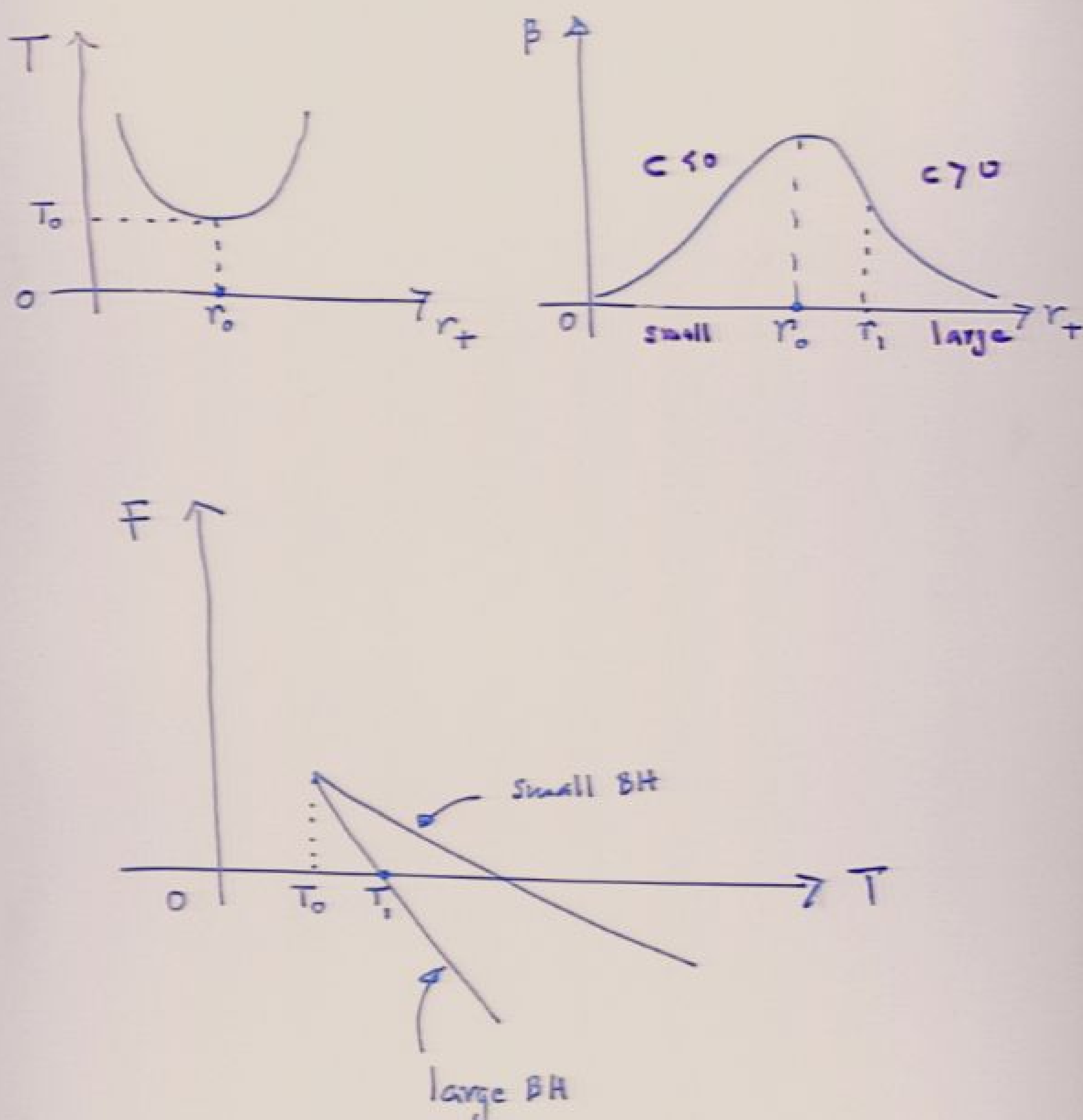
extreme BH
T = 0



Free energy vs Temp for BI black holes



Hawking-Page transition

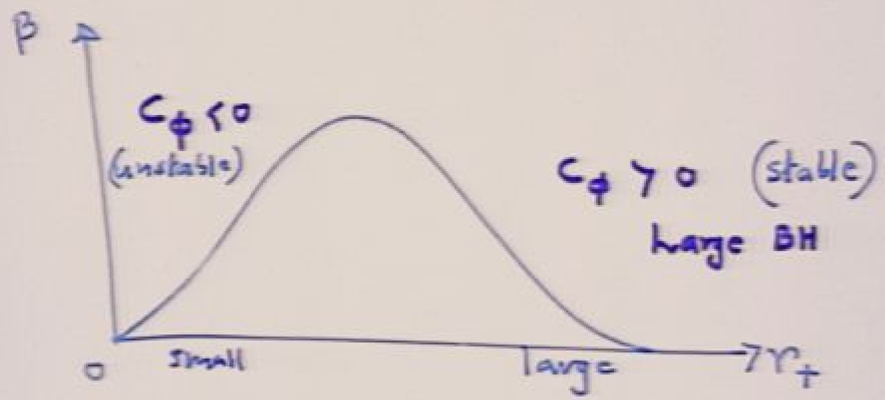


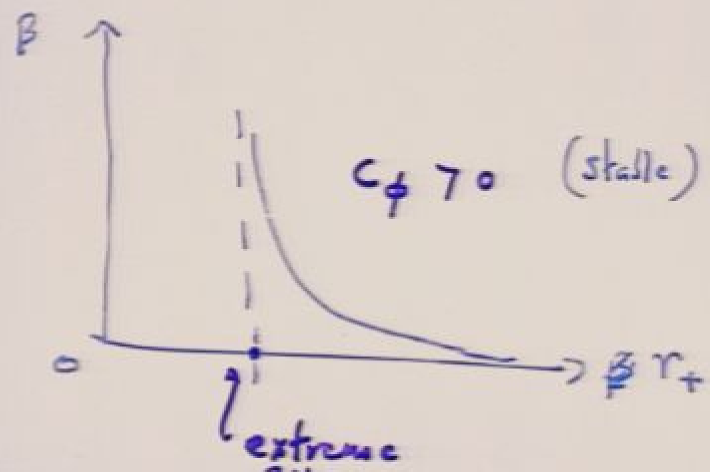
Local Stability

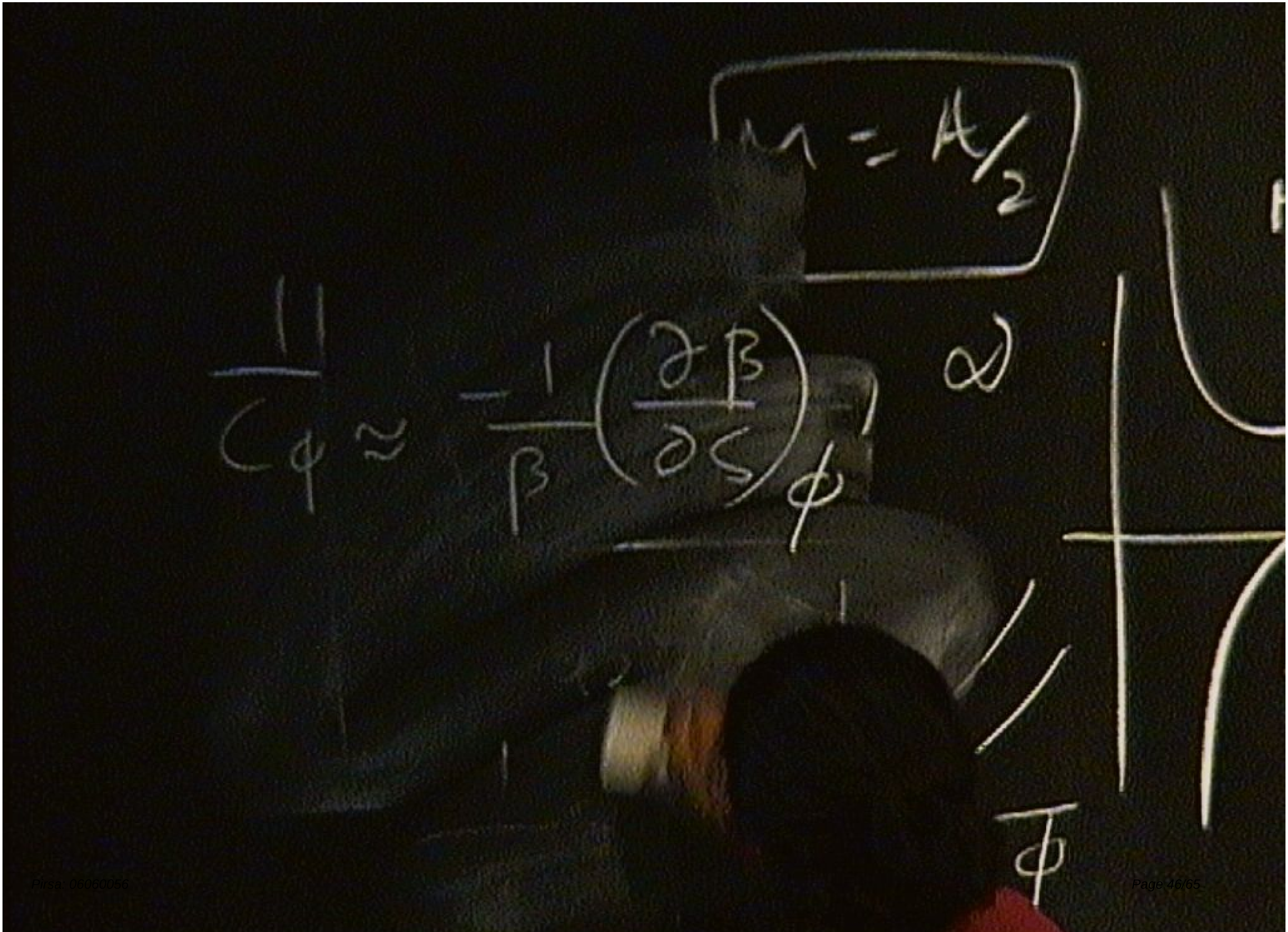
C_ϕ positive ?

$$\frac{1}{C_\phi} = -\,{}^{1}\!/_{\beta} \left(\frac{\partial \beta}{\partial S}\right)_\phi$$

$$S = \frac{r_+^2}{16\pi G}$$






$M = A/2$
$\frac{1}{C_\phi} \approx -\frac{1}{\beta}\left(\frac{\partial \beta}{\partial S}\right)_\phi$
α
Φ

$$\frac{1}{C_\phi} \approx -\frac{1}{\beta}\left(\frac{\partial \beta}{\partial S}\right)_\phi$$

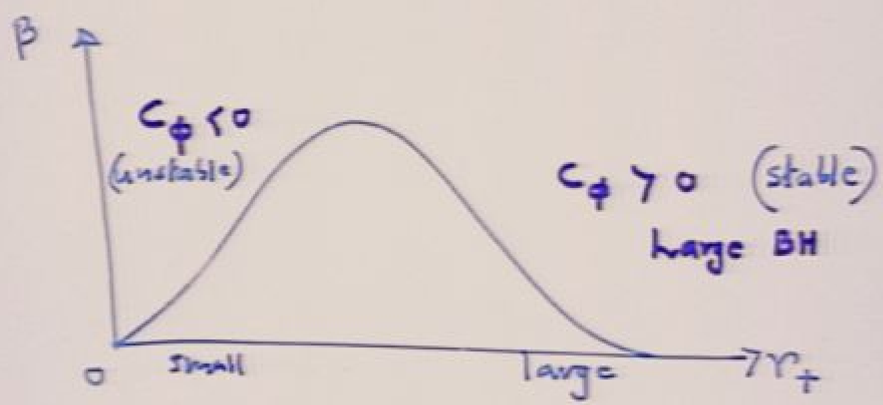
$$S \sim 4\pi r_+^2$$

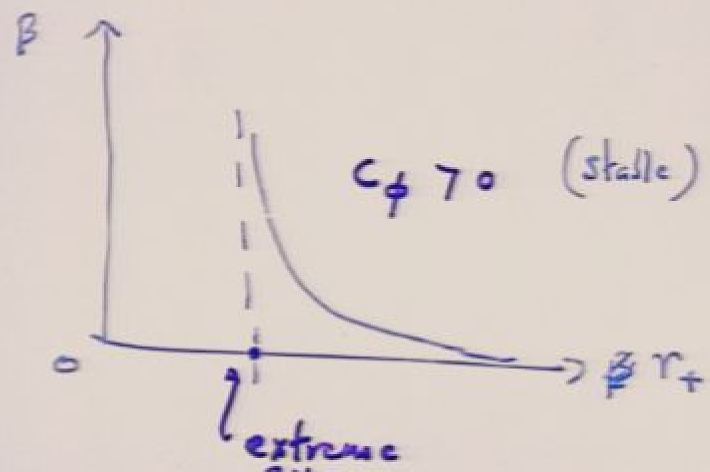
Local Stability

C_ϕ positive ?

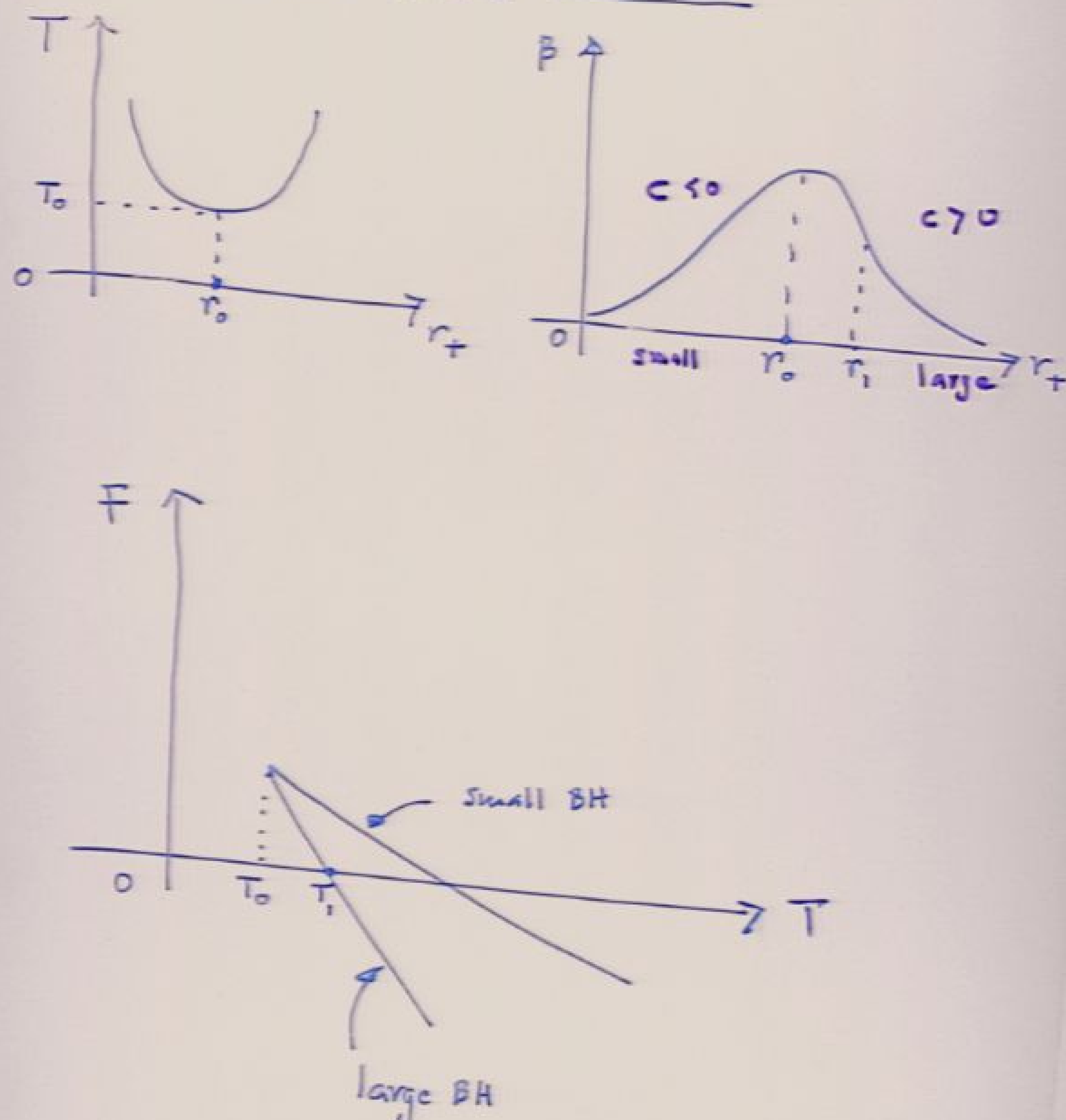
$$\frac{1}{C_\phi} = -\frac{1}{\beta}\left(\frac{\partial \beta}{\partial S}\right)_\phi$$

$$S = \frac{r_+^2}{16\pi G}$$

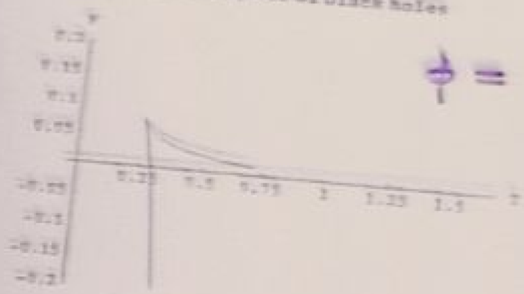




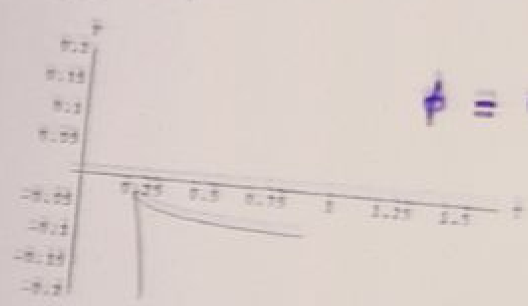
Hawking-Page transition



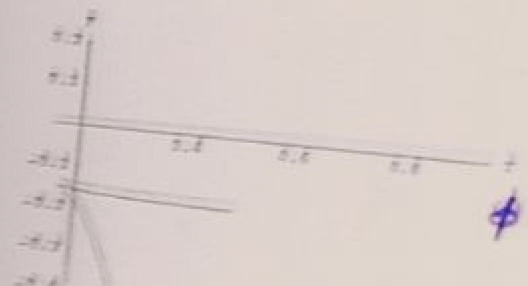
Free energy vs Temp for S2 black holes



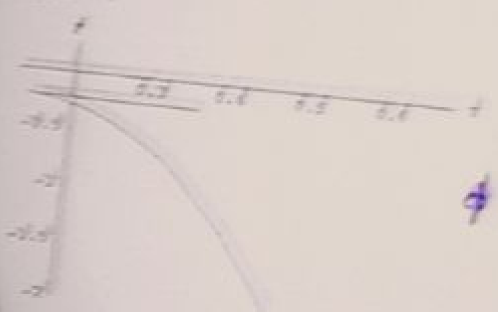
$\phi = 0.35$



$\phi = 0.7$

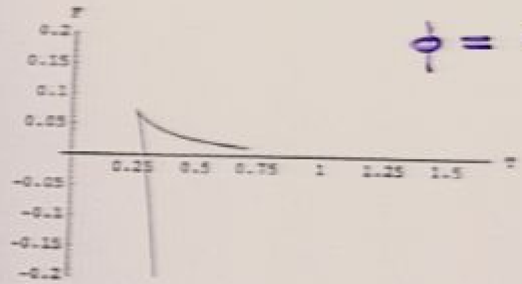


$\phi = 0.91$



$\phi = 1.8$

Free energy vs Temp for BI black holes



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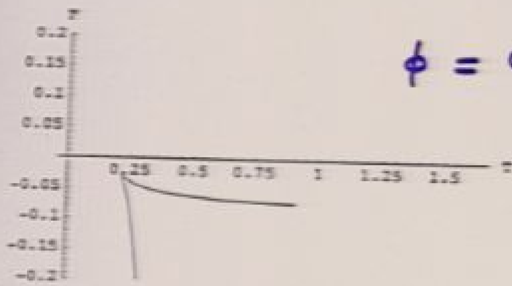
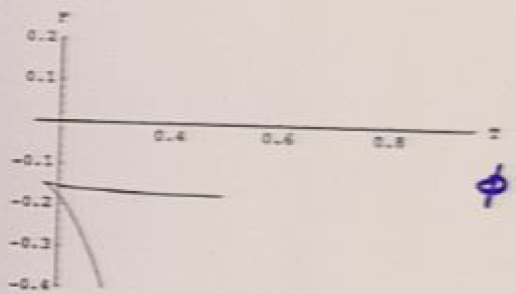


$\phi = 0.7$



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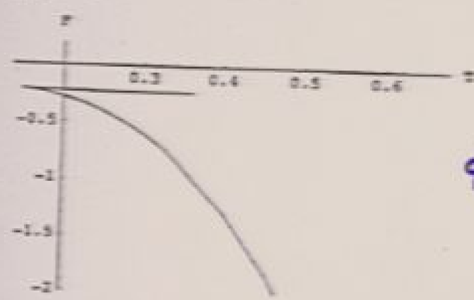
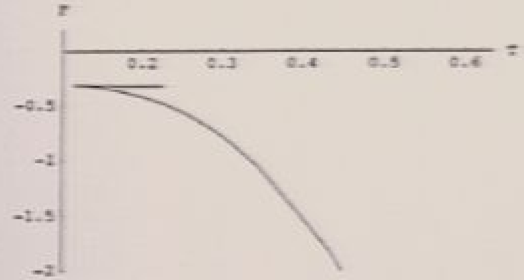
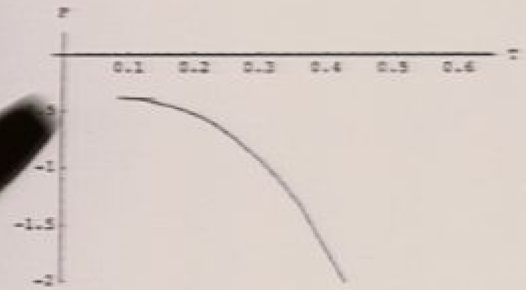


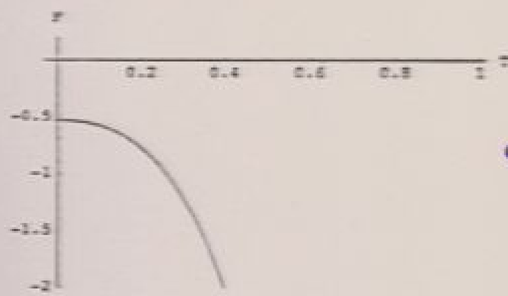
$\phi = 1.0$



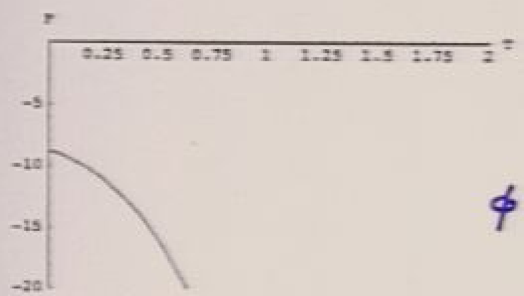
$\phi = 1.1$



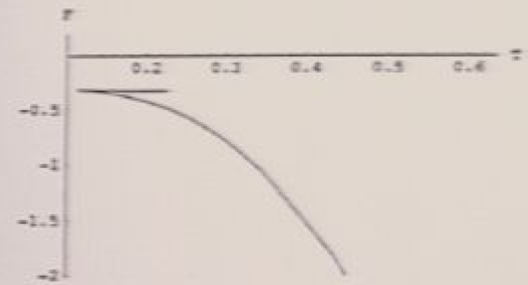
$\phi = 1.17$



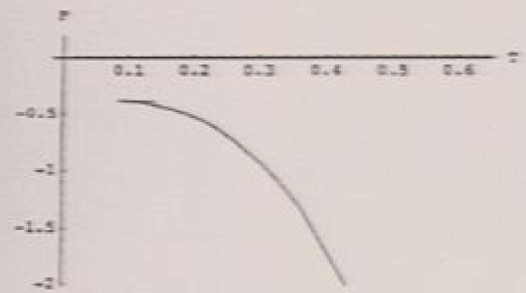
$\phi = 1.3$



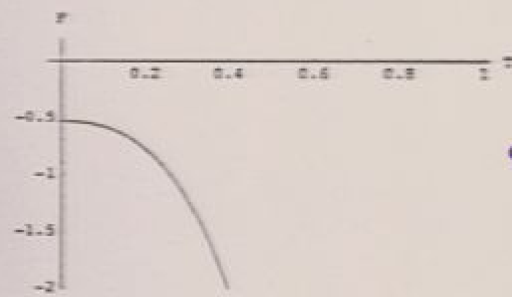
$\phi = 3$



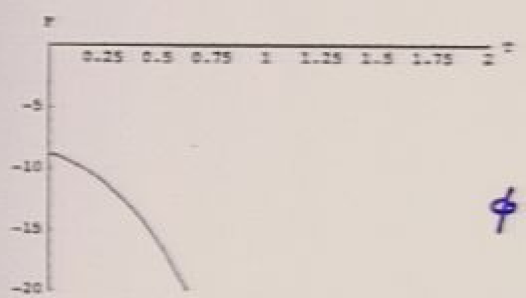
$\phi = 1\cdot 1$



$\phi = 1\cdot 17$

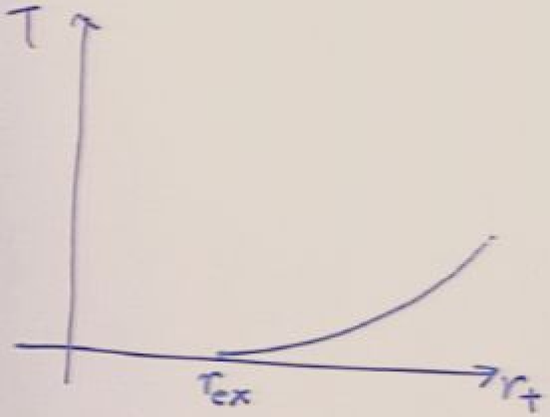


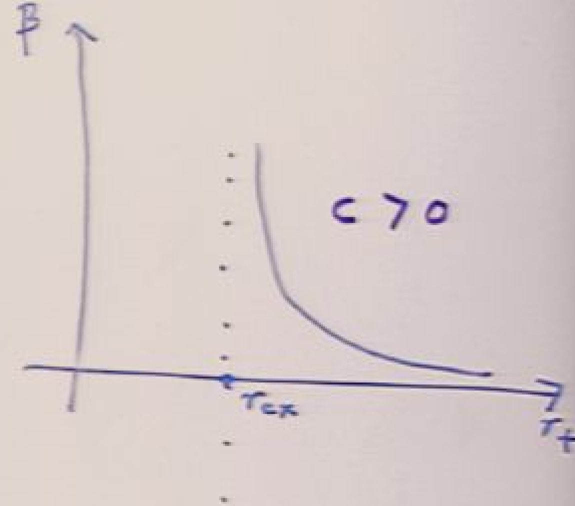
$\phi = 1\cdot 3$

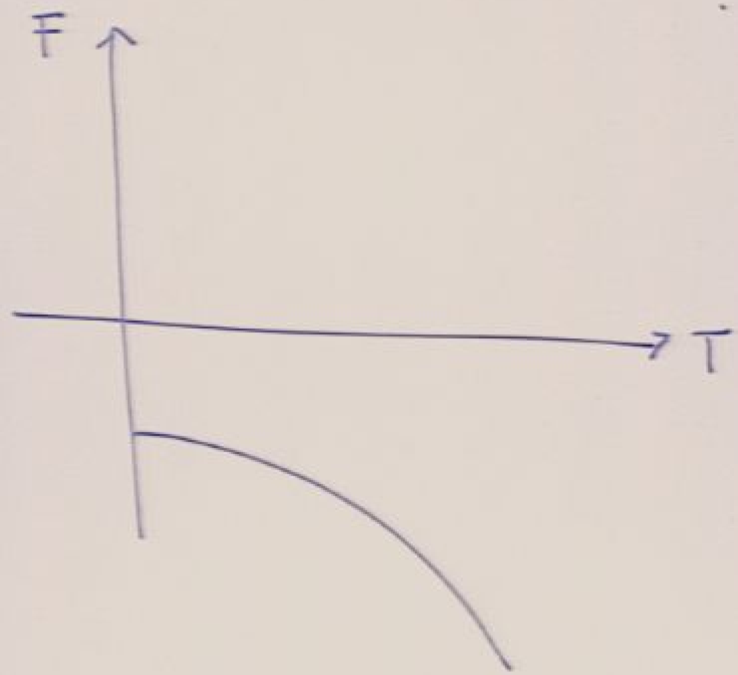


$\phi = 3$

extreme BH







RNAdS case

$$I = \frac{\omega\beta}{16\pi G \ell^2}\left[\ell^2 r_+ (1-\Phi^2) - r_+^3\right]$$

$$\beta = \frac{1}{T} = \frac{4\pi \ell^2 r_+}{\ell^2(1-\phi^2) + 3r_+^2}$$

$I(r_+, \phi)$

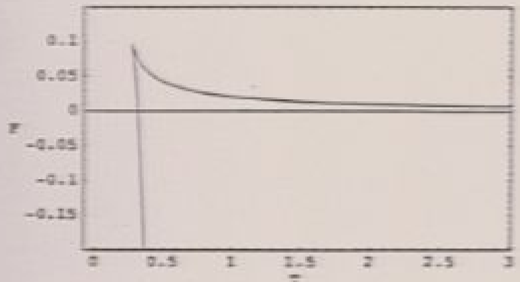
$\beta(r_+, \phi)$

Gibbs potential $F = \frac{I}{\beta}$

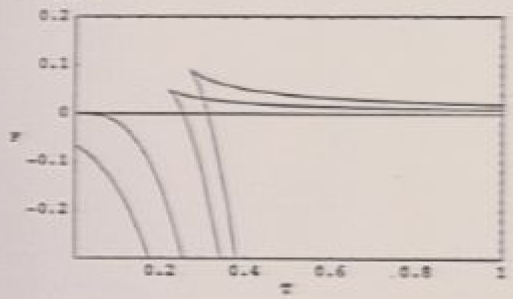
$$F = \frac{\omega}{16\pi G \ell^2}\left[\ell^2 r_+ (1-\phi^2) - r_+^3\right]$$

Freeenergy Vs T for RN - Ads case

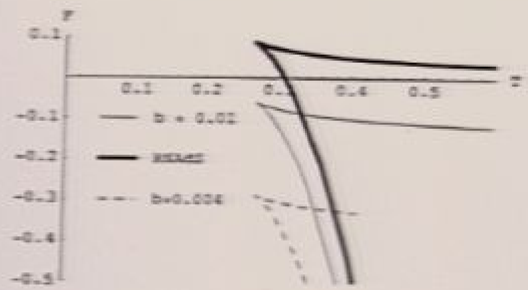
uncharged



Charged



Free energy with non - linear parameter



$\phi = 0.25$

Canonical ensemble

RNAdS case

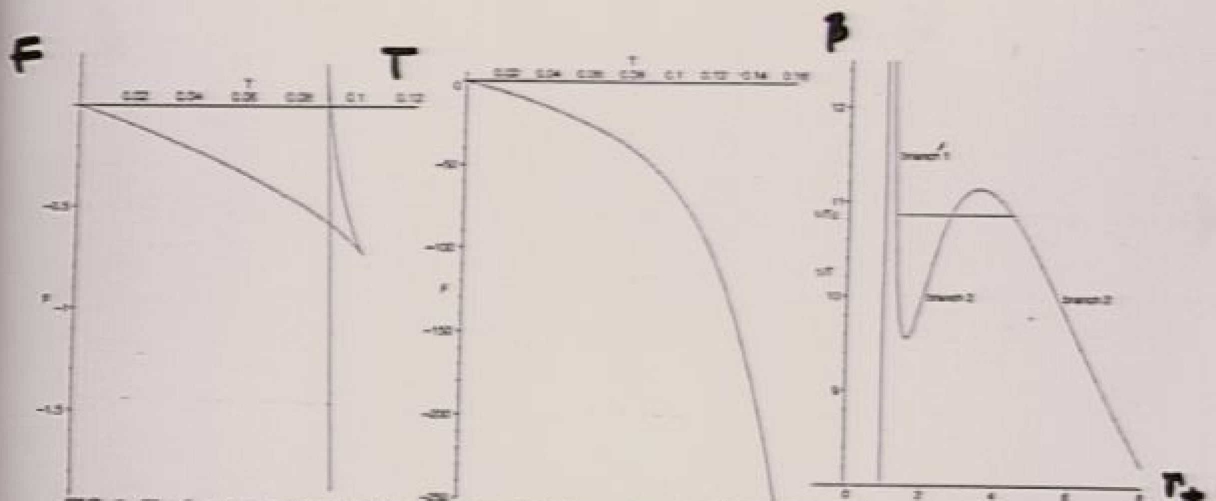


FIG. 5. The first two graphs show the free energy *vs.* temperature for the fixed charge ensemble. The situation for $q<q_{crit}$ and $q\geq q_{crit}$, respectively, are plotted. (The values $n=4$, $G=1$, $l=5$ and $q=1,25$ have been used here.) The first graph is the union of three branches. Branch 1 emanates from the origin, and merges with branch 2 at a cusp. Branch 3 forms a cusp with the other end of branch 2, and continues towards the bottom right. The graph on the right shows how the branches arise from the inverse temperature curves of eqn. (23). (See text for discussion of critical temperature T_c.)

That there are three branches for the small charge case follows from the second graph in figure 3, which is magnified and labeled in fig. 5, on the right. From there, it is clear that for low temperature there can only be one solution ("branch 1") for the black hole radius. At some temperature $T_1=1/\beta_1$, the origin of two new branches ("branches 2 and 3") of solutions appears ($T_1=0.089, \beta_1=11.15$ for the chosen parameters in the plot.). Above this temperature

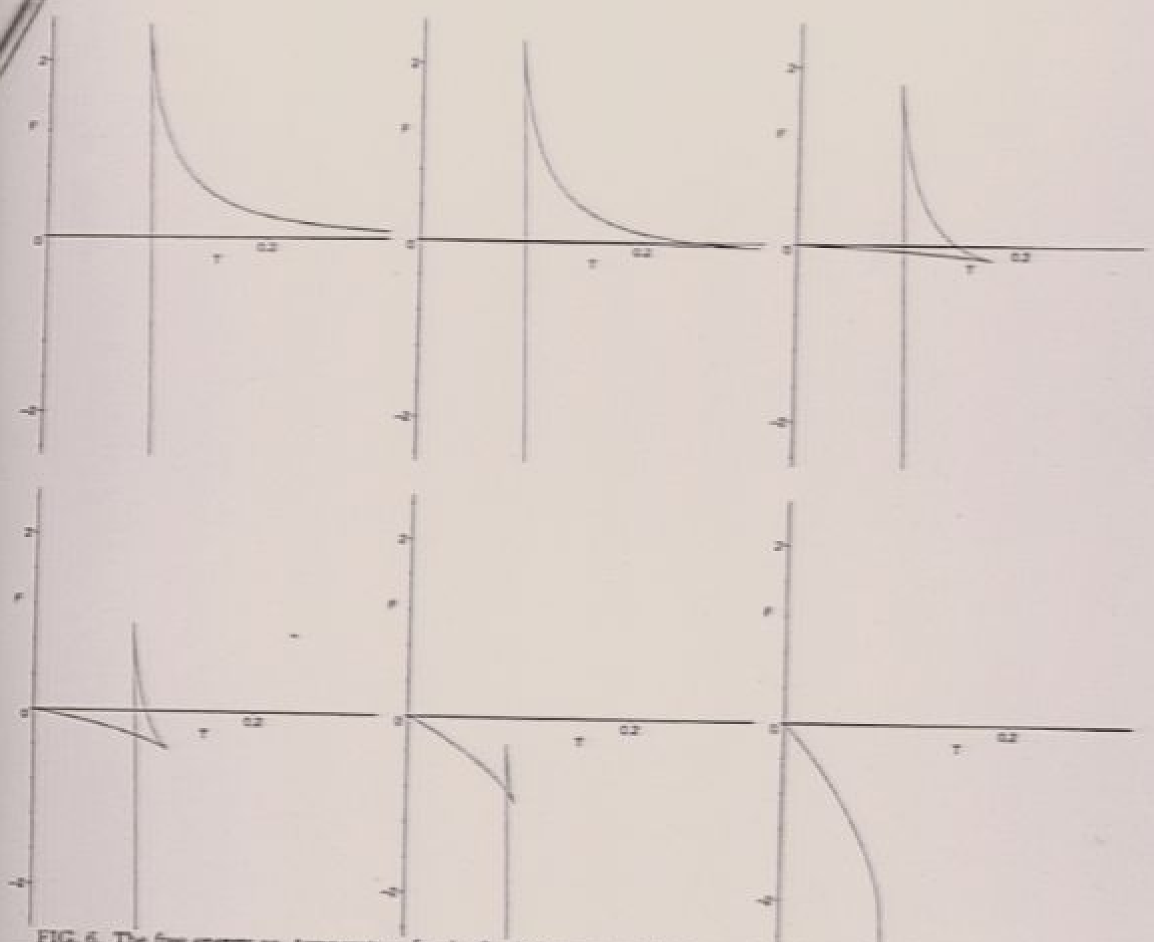


FIG. 6. The free energy *vs.* temperature for the fixed charge ensemble, in a series of snapshots for varying charge, starting from (near) zero charge (top left) and finishing with large charge (bottom left). The values l=5, G=1, and n=4 are used here. This complete evolution describes the two dimensional "swallowtail" catastrophe.

The resulting thermodynamic phase structure for the fixed charge ensemble is summarized in the diagram on the right in figure 1.

VI. CATASTROPHIC HOLOGRAPHY?

We cannot refrain from further general comments upon the meaning and structure of the curves that we have uncovered in the previous sections. Although we plotted only the cases for the n=4 case, representing AdS$_5$ (and hence four dimensional field theory), the same universal structures appear in the cases n=3 and 6 as well, giving the same pleasing phase structure for the fixed charge ensemble.

The phase structure that we uncovered for the fixed charge ensemble should remind the reader of the classic van der Waals–Maxwell behaviour, modeling the liquid–gas system. Indeed, they are isomorphic. The $\beta(r_+)$ curve (the middle graph of figure 3) should recall the graph of the $P(V)$ van der Waals equation of state, where P (the pressure) is replaced here by β and V (volume) by [illegible]

Metric perturbations

time dependent & axially symmetric

$$ds^2 = -e^{2\nu} dt^2 + e^{2\psi}(d\varphi - q_2 dr - q_3 d\theta - \omega dt)^2$$

$$+ e^{2\mu_2} dr^2 + e^{2\mu_3} d\theta^2$$

Unperturbed metric (BI black hole)

$$e^{2\nu} = e^{-2\mu_2} = f(r)$$

$$e^{\mu_3} = r \qquad e^{\psi} = r\sin\theta$$

$$\omega = q_2 = q_3 = 0$$

$$F_{02} = -\frac{Q}{r^2}$$

$$R_{00} = -R_{22} = -2b^2\left(1 - \frac{1}{\sqrt{1 - \frac{E^2}{b^2}}}\right)$$

$$R_{11} = R_{33} = 2b^2\left(1 - \sqrt{1 - \frac{E^2}{b^2}}\right)$$

$$R_{01} = R_{02} = R_{03} = R_{12} = R_{13} = R_{23} = 0$$

Perturbations

$\omega, q_2, q_3 \neq 0 \Rightarrow$ Axial perturbations

$\delta\nu, \delta\mu_2, \delta\mu_3, \delta\psi \neq 0 \Rightarrow$ Polar perturbations

Perturbed equations are obtained by linearising blackhole around unperturbed metric.

Ref: S. Fernando hep-th/0407062
Gen. Rel. Grav. 37 (2005) 585

$$e^{(1)}_{\mu} = (-\omega e^{\psi}, e^{\psi}, -q_2 e^{\psi}, -q_3 e^{\psi}) \tag{25}$$

$$e^{(2)}_{\mu} = (0, 0, e^{\mu_2}, 0) \tag{26}$$

$$e^{(3)}_{\mu} = (0, 0, 0, e^{\mu_3}) \tag{27}$$

Tensors in coordinate and orthonormal basis are related to each other by the tetrads. For example the field strength $F_{\mu\nu} = F_{ab} e^a_{\mu} e^b_{\nu}$.

3.1 Born-Infeld equations

As mentioned in the previous section, there are total of eight equations resulting from the Bianchi identities and the equations of motion for Born-Infeld electrodynamics. The four equations resulting from Bianchi identities $\nabla_{[\mu} F_{\nu\sigma]} = 0$ are as follows,

$$\left(e^{\psi+\mu_2} F_{12}\right)_{,3} + \left(e^{\psi+\mu_3} F_{31}\right)_{,2} = 0 \tag{28}$$

$$\left(e^{\psi+\nu} F_{01}\right)_{,2} + \left(e^{\psi+\mu_2} F_{12}\right)_{,0} = 0 \tag{29}$$

$$\left(e^{\psi+\nu} F_{01}\right)_{,3} + \left(e^{\psi+\mu_3} F_{13}\right)_{,0} = 0 \tag{30}$$

$$\begin{aligned} &\left(e^{\nu+\mu_2} F_{02}\right)_{,3} - \left(e^{\nu+\mu_3} F_{03}\right)_{,2} + \left(e^{\mu_2+\mu_3} F_{23}\right)_{,0} \\ &= e^{\psi+\nu} F_{01} Q_{23} + e^{\psi+\mu_2} F_{12} Q_{03} - e^{\psi+\mu_3} F_{13} Q_{02} \end{aligned} \tag{31}$$

There are four equations resulting from $\nabla_{\mu} G^{\mu\nu} = 0$ in the orthonormal basis as follows,

$$\left(e^{\psi+\mu_3} G_{02}\right)_{,2} + \left(e^{\psi+\mu_2} G_{03}\right)_{,3} = 0 \tag{32}$$

$$-\left(e^{\psi+\mu_3} G_{23}\right)_{,2} + \left(e^{\psi+\mu_2} G_{03}\right)_{,0} = 0 \tag{33}$$

$$\left(e^{\psi+\nu} G_{23}\right)_{,3} + \left(e^{\psi+\mu_3} G_{02}\right)_{,0} = 0 \tag{34}$$

$$\begin{aligned} &\left(e^{\mu_2+\mu_3} G_{01}\right)_{,0} + \left(e^{\nu+\mu_3} G_{12}\right)_{,2} + \left(e^{\nu+\mu_2} G_{13}\right)_{,3} \\ &= e^{\psi+\mu_3} G_{02} Q_{02} + e^{\psi+\mu_2} G_{03} Q_{03} - e^{\psi+\nu} G_{23} Q_{23} \end{aligned} \tag{35}$$

Here, the partial derivative of a function g is given with the notation,

$$(g)_{,a} = \frac{\partial g}{\partial x^a} \tag{36}$$

The function Q_{AB} are given by,

$$Q_{A0} = \frac{\partial q_A}{\partial x^0} - \frac{\partial \omega}{\partial x^A} \quad \text{and} \quad Q_{AB} = \frac{\partial q_A}{\partial x^B} - \frac{\partial q_B}{\partial x^A} \quad (A, B = 2, 3) \tag{37}$$

Pirsa: 06060056

$$R_{22} = -e^{-2\mu_3}\left[(\psi+\nu+\mu_3)_{,2,2} + \psi_{,2}(\psi-\mu_2)_{,2} - \mu_{3,2}(\mu_3-\mu_2)_{,2} + \nu_{,2}(\nu-\mu_2)_{,2}\right]$$
$$-e^{-2\mu_3}\left[\mu_{2,3,3} + \mu_{2,3}(\psi+\nu+\mu_2-\mu_3)_{,3}\right] + e^{-2\nu}\left[\mu_{2,0,0} + \mu_{2,0}(\psi-\nu+\mu_2+\mu_3)_{,0}\right]$$
$$-\frac{1}{2}e^{2\psi-2\mu_2}\left[e^{-2\mu_3}Q_{23}^2 - e^{-2\nu}Q_{20}^2\right] \tag{48}$$

$$R_{01} = -\frac{1}{2}e^{-2\psi-\mu_2-\mu_3}\left[(e^{3\psi-\nu-\mu_2+\mu_3}Q_{20})_{,2} + (e^{3\psi-\nu-\mu_3+\mu_2}Q_{30})_{,3}\right] \tag{49}$$

$$R_{12} = -\frac{1}{2}e^{-2\psi-\nu+\mu_3}\left[\left(e^{3\psi+\nu-\mu_2+\mu_3}Q_{23}\right)_{,3} + \left(e^{3\psi-\nu+\mu_3-\mu_2}Q_{02}\right)_{,0}\right] \tag{50}$$

$$R_{02} = -e^{-\mu_2-\nu}\left[(\psi+\mu_3)_{,2,0} + \psi_{,2}(\psi-\mu_2)_{,0} + \mu_{3,2}(\mu_3-\mu_2)_{,0} - (\psi+\mu_3)_{,0}\nu_{,2}\right]$$
$$+\frac{1}{2}e^{2\psi-\nu-2\mu_3-\mu_2}Q_{23}Q_{30} \tag{51}$$

$$R_{23} = -e^{-\mu_2-\mu_3}\left[(\psi+\nu)_{,2,3} - \mu_{2,3}(\psi+\nu)_{,2} - \mu_{3,2}(\psi+\nu)_{,3} + \psi_{,2}\psi_{,3} + \nu_{,2}\nu_{,3}\right]$$
$$+\frac{1}{2}e^{2\psi-2\nu-\mu_2-\mu_3}Q_{20}Q_{30} \tag{52}$$

The other components R_{33}, R_{13} and R_{03} are not given here. They can be obtained by interchanging the indices 2 and 3 in R_{22}, R_{12} and R_{02}. The Ricci tensor for the Born-Infeld electrodynamics is given by,

$$R_{ab} = -2\left[\eta^{cd}\frac{F_{ac}F_{bd}}{\sqrt{1+\frac{F^2}{2\beta^2}}} + \eta_{ab}\left\{\beta^2\left(1-\sqrt{1+\frac{F^2}{2\beta^2}}\right)\right\}\right] \tag{53}$$

Since the expressions for the perturbed Ricci tensor is given in terms of the metric functions, one has to compute the changes to the Ricci tensor via the energy momentum tensor to obtain the complete equations. Therefore the perturbed components of the Ricci tensor for the Born-Infeld case are computed as follows;

$$\delta R_{ab} = -2\left[\frac{4F_{02}\delta F_{02}}{\sqrt{1-\frac{F_{02}^2}{\beta^2}}}\left(\frac{\eta_{ab}}{4} + \frac{\eta^{nm}F_{an}F_{bm}}{4\beta^2(1+\frac{F^2}{2\beta^2})}\right) + \eta^{nm}\left(\frac{\delta F_{an}F_{bm} + F_{an}\delta F_{bm}}{\sqrt{1+\frac{F^2}{2\beta^2}}}\right)\right] \tag{54}$$

Considering the fact that only non-zero component of F_{ab} before the perturbation is F_{02}, the exact expressions for δR_{ab} can be computed as,

$$\delta R_{00} = -\delta R_{22} = -\frac{2Q}{r^2}\frac{\delta F_{02}}{(1+\frac{F^2}{2\beta^2})} \tag{55}$$

$$\delta R_{11} = \delta R_{33} = -\frac{2Q}{r^2}\delta F_{02} \tag{56}$$

$$\delta R_{01} = -\frac{2Q}{r^2}\delta F_{12}, \quad \delta R_{03} = \frac{2Q}{r^2}\delta F_{23} \tag{57}$$

$$\delta R_{12} = \frac{2Q}{r^2}\delta F_{01}, \quad \delta R_{23} = \frac{2Q}{r^2}\delta F_{03} \tag{58}$$

$$\delta R_{13} = \delta R_{02} = 0 \tag{59}$$

Finally !

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right) Z = V(r_*)\, Z$$

Sch → Regge & Wheeler

RN → Moncrief & Zerilli

Kerr → Teukolsky

charged dilaton →

Future work

* Canonical ensemble
* Extension to higher dimensions
* Born-Infeld-de-Sitter
* Thermodynamic stability of hyperbolic charged black holes.