Title: General relativity as a quantum effective field theory

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Abstract: At low energy and small curvature, general relativity has the form of an effective field theory. I will describe the structure of the effective field theory, and show how it can be used to calculate low energy quantum effects.

Goals

1) The quantum correction to gravitational interaction

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

*

Donoghue 1994
Bjerem-Bohr
JFD, Holstein 2002
Khriplovich, Kirilin 2002

2) The quantum theory of general relativity at ordinary energies exists and is of the form called an "effective field theory"

Effective field theory is a standard technique:

- calculate quantum effects at a given energy scale
- shifts focus from U.V. to I.R.
- handles main obstacle
  – quantum effects involve **all** scales

Completion of program of Feynman, De Witt,..Weinberg…

….. 't Hooft, Veltman ……

Previously: Quantization and divergence structure

E.F.T $\Longrightarrow$ Extraction of quantum predictions

Known vs unknown physics

Why do quantum calculations work?

<u>**The problem**</u>: QM says to sum over **all** intermediate states

$$\sum_I \frac{<f|V|I><I|V|i>}{E_i - E_I}$$

But, physics is an experimental science

-know particles and interactions up to some energy scale

So, how can you sum over all states if you don't know what they are or how they interact??

<u>**Some possible solutions:**</u>

1) The energy denominator suppresses high energy states ✕

$$\sum_I \frac{1}{E_i - E_I} \rightarrow \int d^3 p_I \frac{1}{E_i - \frac{p_I^2}{2m}}$$

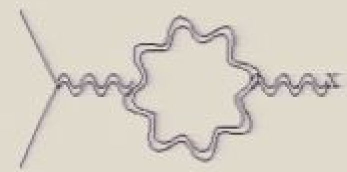
2) Perhaps matrix elements are small to high energy states ✕

All observables sensitive to high energy at some order in PT

The solution: the uncertainty principle

High energy effects look local - very short range
$\Longrightarrow$Look like some term in a local Hamiltionian/Lagrangian

Mass term or charge coupling
Shift in mass or coupling
We measure **total** mass and coupling



Applequist Carrazone theorem:
-effects of high energy either absorbed in "coupling constants" or suppressed by powers of the heavy scale

Renormalization Program

Effects of high energy go into measured values of the parameters
-including unknown physics and potential divergences

<u>Renormalizable Field Theory</u>

- finite number of parameters sensitive to high energy
- terms (in Lagrangian) suppressed by powers of heavy scale are not allowed

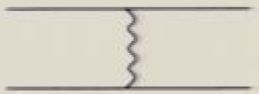
<u>Effective Field Theory</u>

- allow terms suppressed by powers of heavy scale
- quantum effects from low energy D.O.F. only
- more general

Key Steps

1) High energy effects are local (when viewed at low E)

Example = W exchange



=> local 4 Fermi interaction

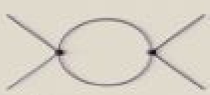
Even loops

=> local mass counterterm

Low energy particle propagate long distances:

Photon:

Not local $\quad V \sim \frac{1}{q^2} \sim \frac{1}{r}$



Even in loops – cuts, imag. parts….

Result: High energy effects in **local** Lagrangian

$$L = g_1 L_1 + g_2 L_2 + g_3 L_3 + \ldots$$

But local Lagrangian is not enough

- low energy effects are distinct

2) Energy Expansion

Order lagrangians by powers of (low scale/high scale)^N

Only a finite number needed to a given accuracy

Then:

Quantization: use lowest order Lagrangian

Renormalization: **

-U.V. divergences are **local**

- can be absorbed into couplings of local Lagrangian

Remaining effects are predictions

General Procedure

1) Identify Lagrangian

-- most general (given symmetries)

-- order by energy expansion

2) Calculate and renormalize

-- start with lowest order

-- renormalize parameters

3) Phenomenology

-- measure parameters

-- residual relations are predictions

Note: Two differences from textbook renormalizable field theory:

1) no restriction to renormalizable terms only

2) energy expansion

Effective Field Theory in Action:

Chiral Perturbation Theory

-QCD at very low energies –pions and photons

Non-linear lagrangian required by symmetry:

$$\mathcal{L} = F_\pi^2 Tr(D_\mu U D^\mu U^\dagger) + L_1[Tr(D_\mu U D^\mu U^\dagger)]^2 + ...$$

$$\boxed{U = exp[i\frac{\tau \cdot \phi}{F_\pi}]}$$

Very well studied: Theory and phenomenology

- energy expansion, loops, symmetry breaking, experimental constraints, connection to QCD.

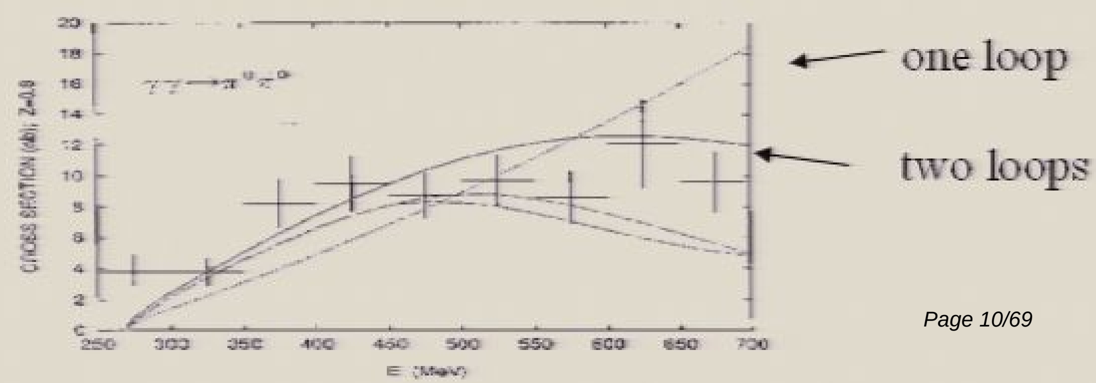
$$\gamma\gamma \longrightarrow \pi^0\pi^0$$

mple calculation:

o direct couplings at low energy

- pure loops

ssentially parameter free at low energy



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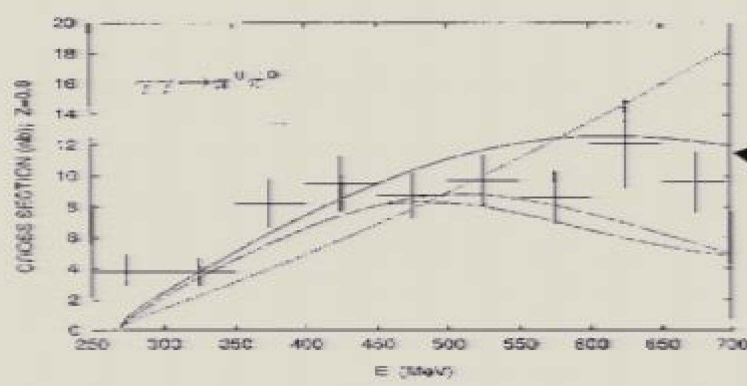

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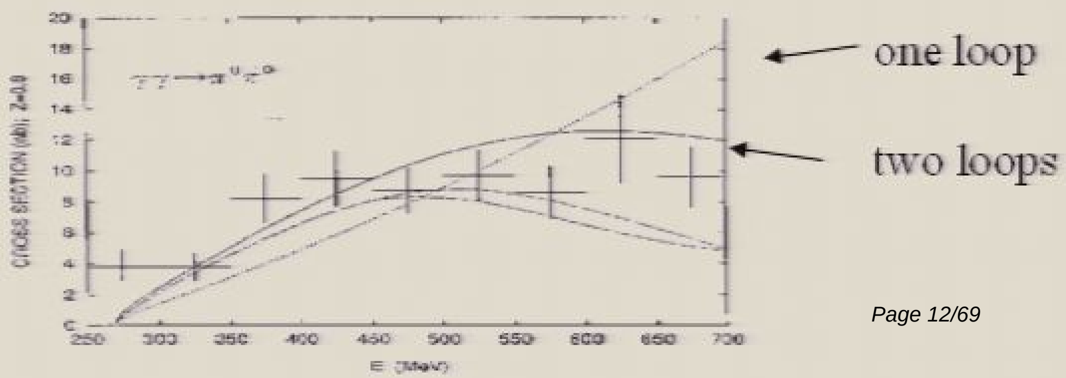


We have come to think of all of our theories as effective field theorie

Theories are tested over some distance/energy scale
 -we know D.O.F. and interactions for that scale
 -can do calculations at that scale

But, there likely are new particles and new interactions at higher energy
 -these do not propagate at low energy
 -only give suppressed local interactions

All theories likely modified as we go to higher energy

Gravity as an effective theory

Weinberg
JFD

Both General Relativity and Quantum Mechanics known and tested over common range of scales

Is there an incompatibility **at those scales** ?

Or are problems only at uncharted high energies?

Need to study GR with a careful consideration of scales

The general Lagrangian

The Einstein action:

$$S_{grav} = \int d^4x \sqrt{-g} \left[\frac{2}{\kappa^2} R \right]$$

$\kappa^2 = 32\pi G$, $g = det g_{\mu\nu}$, $g_{\mu\nu}$ is the metric tensor and $R = g^{\mu\nu} R_{\mu\nu}$

$$\begin{aligned} R_{\mu\nu} &= \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\lambda\sigma} \\ \Gamma^\lambda_{\alpha\beta} &= \frac{g^{\lambda\sigma}}{2} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta} \right) \end{aligned}$$

But this is not the most general lagrangian consistent with general covariance.

Key: R depends on <u>two derivatives</u> of the metric
$\Longrightarrow$ Energy expansion – expansion in number of derivatives

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

Parameters

$$S_{grav} = \int d^4x\sqrt{-g}\left\{\Lambda + \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + \ldots\right\}$$

1) Λ = cosmological constant

$$\Lambda = (1.2 \pm 0.4) \times 10^{-123} M_P^4$$

$$M_P = 1.22 \times 10^{19}\ \mathrm{GeV}$$

-this is observable only on cosmological scales
-neglect for rest of talk
-interesting aspects

2) Newton's constant

$$\kappa^2 = 32\pi G$$

3) Curvature –squared terms c_1, c_2

- studied by Stelle
- modify gravity at very small scales
-essentially unconstrained by experiment

$$c_1, c_2 \leq 10^{74}$$

Matter couplings

Spinless heavy particle:

$$\begin{aligned} \mathcal{L}_{m0} &= \frac{1}{2}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right] \\ \mathcal{L}_{m2} &= d_1 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R\left(d_2\partial_\mu\phi\partial^\mu\phi + d_3 m^2\phi^2\right) \end{aligned}$$

Parameters d_i like charge radii – non-universal

Quantization

"**Easy" to quantize gravity**:

-Covariant quantization   Feynman deWitt

-gauge fixing

-ghosts fields

-Background field method   't Hooft Veltman

-retains symmetries of GR

-path integral

Background field:

$$
\begin{aligned}
g_{\mu\nu} &= \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \\
g^{\mu\nu} &= \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \ldots
\end{aligned}
$$

Expand around this background:

$$
S_{grav} = \int d^4x \sqrt{-\bar{g}} \left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \ldots \right]
$$

$$
\begin{aligned}
\mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa} \left[\bar{g}^{\mu\nu} \bar{R} - 2\bar{R}^{\mu\nu} \right] \\
\mathcal{L}_g^{(2)} &= \frac{1}{2} h_{\mu\nu;\alpha} h^{\mu\nu;\alpha} - \frac{1}{2} h_{;\alpha} h^{;\alpha} + h_{;\alpha} h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha} h^{\mu\alpha;\beta} \\
&\quad + \bar{R} \left(\frac{1}{4} h^2 - \frac{1}{2} h_{\mu\nu} h^{\mu\nu} \right) + \left(2h^{\lambda}_{\mu} h_{\nu\lambda} - h h_{\mu\nu} \right) \bar{R}^{\mu\nu}
\end{aligned}
$$

Linear term vanishes by Einstein Eq. $\bar{R}^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$

Gauge fixing:

-harmonic gauge

$$\mathcal{L}_{gf} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}$$

$$h \equiv h^{\lambda}_{\lambda}$$

Ghost fields:

$$\mathcal{L}_{ghost} = \sqrt{-\bar{g}}\eta^{*\mu}\left\{\eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu}\eta^{\nu}\right\}$$

vector fields
anticommuting,
in loops only

Interesting note:
Feynman introduced
ghost fields in GR
before F-P in YM

Quantum lagrangian:

$$S_{eff} = \int d_x^4 \sqrt{\bar{g}} \left\{ \bar{\mathcal{L}}(\bar{g}) - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta\gamma\delta} h_{\gamma\delta} + \ldots \right\}$$

with

$$\begin{aligned} D^{\alpha\beta\gamma\delta} &= I^{\alpha\beta,\mu\nu} d_\lambda d^\lambda I_{\mu\nu,}{}^{\gamma\delta} - \frac{1}{2} \bar{g}^{\alpha\beta} d_\lambda d^\lambda \bar{g}^{\gamma\delta} + \bar{g}^{\alpha\beta} d^\gamma d^\delta + d^\alpha d^\beta \bar{g}^{\gamma\delta} \\ &\quad -2 I^{\alpha\beta,\mu\nu} d_\sigma d_\lambda I_\mu{}^{\sigma,\gamma\delta} + \bar{R} \left(I^{\alpha\beta,\gamma\delta} - \frac{1}{2} g^{\alpha\beta} g^{\gamma\delta} \right) \\ &\quad + \left(\bar{g}^{\alpha\beta} \bar{R}^{\gamma\delta} + \bar{R}^{\alpha\beta} \bar{g}^{\gamma\delta} \right) - 4 I^{\alpha\beta,\lambda\mu} \bar{R}_{\mu\nu} I_\lambda{}^{\nu,\gamma\delta} \end{aligned}$$

and

$$I^{\alpha\beta,\gamma\delta} = \frac{1}{2} \left(\bar{g}^{\alpha\gamma} \bar{g}^{\beta\delta} + \bar{g}^{\alpha\delta} \bar{g}^{\beta\gamma} \right)$$

Propagator around flat space:

$$iD_{\mu\nu\alpha\beta}(q) = \frac{i}{q^2 - i\epsilon} P_{\mu\nu,\alpha\beta}$$

$$P_{\mu\nu,\alpha\beta} = \frac{1}{2} \left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta} \right]$$

Feynman rules:

A.1 Scalar propagator

The massive scalar propagator is:

$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:

$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]$$

A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:

$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2}\left[p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left((p \cdot p') - m^2\right)\right]$$

A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:

$$= \tau_2^{\eta\lambda\rho\sigma}(p, p', m)$$

$$\tau_2^{\eta\lambda\rho\sigma}(p, p') = i\kappa^2\Bigg[\left\{I^{\eta\lambda\alpha\delta}I^{\rho\sigma\beta}{}_{\delta} - \frac{1}{4}\left\{\eta^{\eta\lambda}I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma}I^{\eta\lambda\alpha\beta}\right\}\right\}(p_\alpha p'_\beta + p'_\alpha p_\beta)$$
$$- \frac{1}{2}\left\{I^{\eta\lambda\rho\sigma} - \frac{1}{2}\eta^{\eta\lambda}\eta^{\rho\sigma}\right\}\left[(p \cdot p') - m^2\right]\Bigg] \tag{61}$$

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]

$$= \tau_{3\,\alpha\beta\gamma\delta}^{\mu\nu}(k, q)$$

where

$$\tau_{3\,\alpha\beta\gamma\delta}^{\mu\nu}(k, q) = -\frac{i\kappa}{2} \times \Bigg(\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right]$$
$$+ 2q_\lambda q_\sigma\left[I_{\alpha\beta}{}^{\sigma\lambda}I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda}I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta}{}^{\nu\lambda}\right]$$
$$+ \Big[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\lambda}\right)$$
$$- q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\nu}\right) - \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\sigma\lambda}\right)\Big]$$
$$+ \Big[2q_\lambda\big(I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\nu}(k-q)^\mu + I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\mu}(k-q)^\nu - I_{\gamma\delta}{}^{\lambda\sigma}I_{\alpha\beta\sigma}{}^{\nu}k^\mu - I_{\gamma\delta}{}^{\lambda\sigma}$$
$$+ q^2\left(I_{\alpha\beta\sigma}{}^{\mu}I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma}I_{\gamma\delta\sigma}{}^{\mu}\right) + \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}{}^{\lambda\rho}I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho}I_{\alpha\beta\rho}{}^{\sigma}\right)\Big]$$
$$+ \Big\{(k^2 + (k-q)^2)\left[I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta\sigma}{}^{\nu} + I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\right]$$
$$- \left(I_{\gamma\delta}{}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}{}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right)\Big\}\Bigg) \tag{62}$$

Performing quantum calculations

Quantization was straightforward, but what do you do next?
- calculations are not as simple

<u>Next step: Renormalization</u>
-divergences arise at high energies
-not of the form of the basic lagraingian

Solution: Effective field theory and renormalization
- renormalize divergences into parameters of the most general lagrangian ($c_1, c_2 \ldots$)

Power counting theorem: (pure gravity, $\Lambda=0$)
-each graviton loop$\Longrightarrow$2 more powers in energy expansion
-1 loop $\Longrightarrow$ Order $(\partial g)^4$
-2 loop $\Longrightarrow$ Order $(\partial g)^6$

Renormalization

One loop calculation: 't Hooft and Veltman

$$Z[\phi, J] = Tr ln D$$

Divergences are local:

dim. reg. preserves symmetry

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\} \quad \epsilon = 4 - d$$

Renormalize parameters in general action:

$$c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$$
$$c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Pure gravity "one loop finite" since $R_{\mu\nu}=0$

Note: Two loop calculation known in pure gravity Goroff and Sagnotti

$$\Delta\mathcal{L}^{(2)} = \frac{209\,\kappa}{2880(16\pi^2)^2}\frac{1}{\epsilon}\sqrt{-g}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

Order of six derivatves

More formal study – Gomis and Weinberg

"Are non-renormalizable gauge theories renormalizeable"

Gauge theories could present separate problems
 -gauge fixing
-are there potential coefficients in general Lagrangian
 to renormalize all divergences?

Proven for Yang-Mills and gravitation
 - structural constraints and cohomology theorems

Undecided for theories with U(1) symmetry
 -no counter examples exist

What are the quantum predictions?

Not the divergences

-they come from the Planck scale

-unreliable part of theory

Not the parameters

-local terms in L

-we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

-not the same as terms in the Lagrangian

- most always non-analytic dependence in momentum space

-can't be Taylor expanded – can't be part of a local Lagrangian

-long distance in coordinate space

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein,
Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses.

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Potential found using from

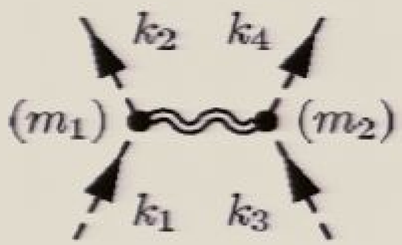
$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

Lowest order:

one graviton exchange



$$iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1, k_2, m_1)\left[\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2}\right]\tau_1^{\alpha\beta}(k_3, k_4, m_2)$$

Non-relativistic reduction:

$$M_{1(a)}(\vec{q}) = -\frac{4\pi G m_1 m_2}{\vec{q}^2}$$

Potential:

$$V_{1(a)}(r) = -\frac{G m_1 m_2}{r}$$

What to expect:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Relation to momentum space:

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}} \ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

short range

Non-analytic

analytic

Parameter free and divergence free

Recall: divergences like local Lagrangian $\sim R^2$

Also unknown parameters in local Lagrangian $\sim c_1, c_2$

But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

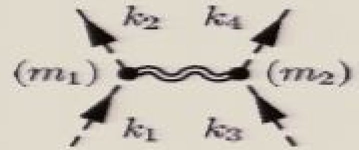
Then:
Treating R^2 as perturbation

$$V_{R^2} \sim G^2 Mm \;\; \frac{1}{q^2}\, q^4\, \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm\delta^3(x)$$

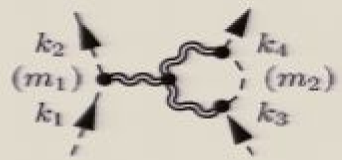
Local lagrangian gives only short range terms

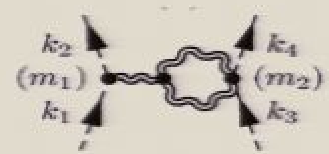
The calculation:

Lowest order:

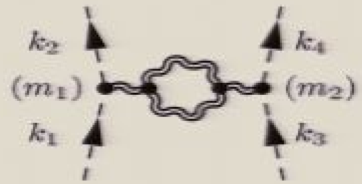


Vertex corrections:

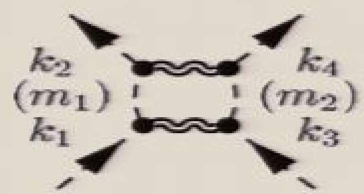


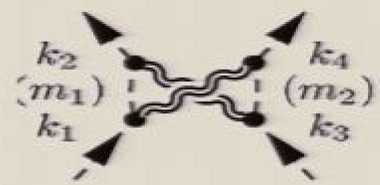


Vacuum polarization:
(Duff 1974)

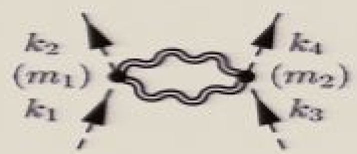


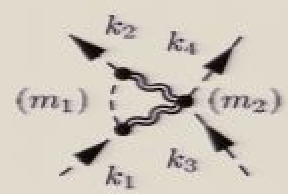
Box and crossed box





Others:





Results:

Pull out non-analytic terms:
-for example the vertex corrections:

$$
\begin{aligned}
M_{5(a)+5(b)}(\vec{q}) &= 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2 \right) \\
M_{5(c)+5(d)}(\vec{q}) &= -\frac{52}{3} G^2 m_1 m_2 \log \vec{q}^2
\end{aligned}
$$

Sum diagrams:

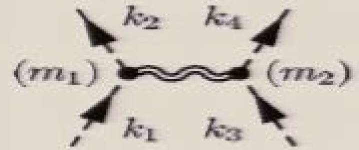
$$
\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}
$$

Gives precession
of Mercury, etc
(Iwasaki ;
Gupta + Radford)

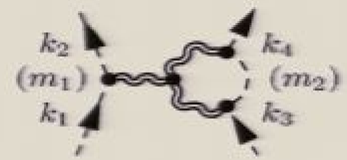
Quantum
correction

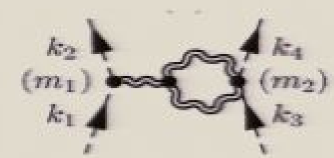
The calculation:

Lowest order:

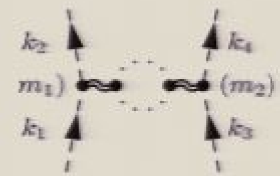


Vertex corrections:





Vacuum polarization:
(Duff 1974)



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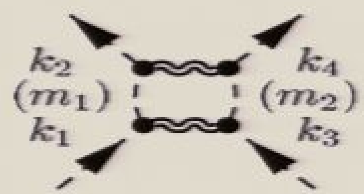

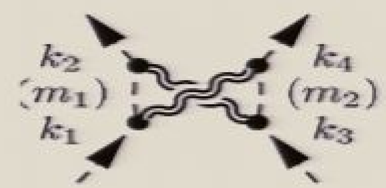

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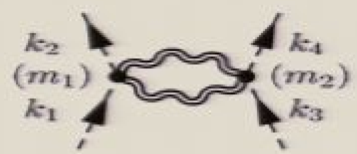

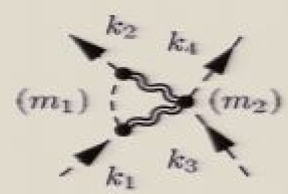

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Comments

1) Both classical and quantum emerge from a one loop calculation!
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1) Unmeasurably small correction:
   - best perturbation theory known(!)

3) Quantum loop well behaved - no conflict of GR and QM

4) Other calculations
   (Duff; JFD; Muzinich and Vokos; Hamber and Liu; Akhundov, Bellucci, and Sheikh)
   -other potentials or mistakes

5) Why not done 30 years ago?
   - power of effective field theory reasoning

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JFD, Holstein
2004 PRL

Field theory folk lore:

Loop expansion is an expansion in $\hbar$

"Proofs" in field theory books

This is not really true.

- numerous counter examples – such as the gravitational potential

- can remove a power of $\hbar$ via kinematic dependence

$$\sqrt{\frac{m^2}{-q^2}} = \frac{m}{\hbar\sqrt{-k^2}}$$

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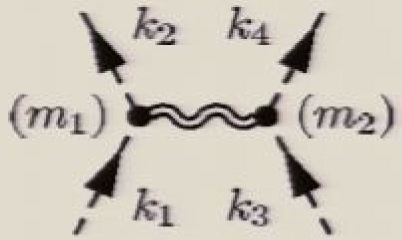
Lowest order:

one graviton exchange



$$iM_{1(a)}(\vec{q}) = \tau_1^{\mu\nu}(k_1, k_2, m_1)\left[\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2}\right]\tau_1^{\alpha\beta}(k_3, k_4, m_2)$$

Non-relativistic reduction:

$$M_{1(a)}(\vec{q}) = -\frac{4\pi G m_1 m_2}{\vec{q}^2}$$

Potential:

$$V_{1(a)}(r) = -\frac{G m_1 m_2}{r}$$

Corrections to Newtonian Potential

JFD 1994
JFD, Holstein,
Bjerrum-Bohr 2002
Khriplovich and Kirilin
Other references later

Here discuss scattering potential of two heavy masses.

$$\begin{aligned} \langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle \end{aligned}$$

Potential found using from

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

Classical potential has been well studied

Iwasaki
Gupta-Radford
Hiida-Okamura

Parameter free and divergence free

Recall: divergences like local Lagrangian $\sim R^2$

Also unknown parameters in local Lagrangian $\sim c_1, c_2$

But this generates only "short distance term"

Note: R^2 has 4 derivatives $R^2 \sim q^4$

Then:
Treating R^2 as perturbation

$$V_{R^2} \sim G^2 Mm \ \frac{1}{q^2}\, q^4\, \frac{1}{q^2} \sim \text{const.} \rightarrow G^2 Mm\delta^3(x)$$

Local lagrangian gives only short range terms

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- classical behavior seen when massless particles are involved

Dispersive treatment of quantum potential

JFD
Holstein
Ross
Thorn

Quantum physics without loops

Interesting alternative method for low energy calculations:

- microcausality and crossing
- independent of perturbation theory

Interactions from multiplying tree amplitudes

- only involves on-shell gravitons
- classical and quantum corrections emerge from zero-energy limit of tree amplitudes

High energy behavior only generates analytic terms

- subtractions in dispersion relation
- can't modify classical/quantum terms

Basic dispersive framework:

diagrams satisfy analyticity requirements
leads to a dispersive representation

$$V_2(s,q^2) = \frac{1}{\pi}\int_0^\infty \frac{dt}{t-q^2}\rho(s,t) \; + \; (\text{L.H.cut} = \text{short range})$$

Spectral functions calculated via Cutkosky rules
- on shell intermediate states

quantum
without loops

At low energy:

$$\rho(s,t) = a_2(s)\frac{1}{\sqrt{t}} + a_3(s) + \ldots.$$

$$V_2(s,q^2) = a_2(s)\frac{1}{\sqrt{-q^2}} - \frac{a_3(s)}{\pi}\ln(-q^2) + \ldots.$$

$$V_2(r) = \frac{1}{8\pi^2 m_A m_B}\left[\frac{a_2}{r^2} + \frac{a_3}{r^3}\right]$$

Note: leading non-analytic structures – independent of possible subtractions

High energy end - subtractions

Upper end of dispersive integral can give analytic terms

Suppress upper end by **subtractions**

$$V_2(s,q^2) = V(s,0) + \frac{q^2}{\pi}\int_0^\infty \frac{dt}{t(t-q^2)}\rho(s,t)$$

$$V_2(s,q^2) = V(s,0) + a(s)q^2 + \frac{q^4}{\pi}\int_0^\infty \frac{dt}{t^2(t-q^2)}\rho(s,t)$$

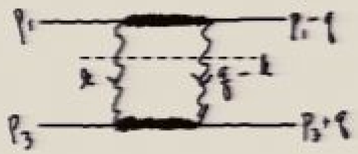
Subtraction constants equivalent to parameters
in effective Lagrangian (c_1 , c_2 ....)

Nonanalytic terms not modified – come from t->0 end of integral

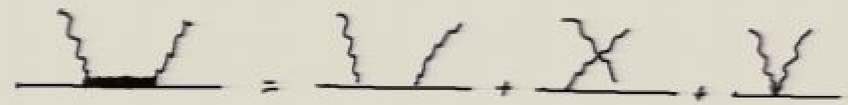
Corrections to the electromagnetic potential:

Feinberg
Sucher

- cut involves the Compton amplitude



with



$$\rho(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\,\mathcal{M}_{A\mu\nu}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\mu\nu}(p_3,-k,p_3+q,k-q)$$

Multiply amplitudes, integrate over angles, expand in t:

$$\rho(s,t) = a_2(s)\frac{1}{\sqrt{t}} + a_3(s) + ...$$

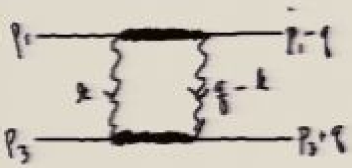
Threshold expansion: $\quad a_2 = -e^4\frac{(m_A+m_B)}{4} \qquad a_3 = -e^4\frac{7}{12\pi}$

$$V_{EM}(r) = \frac{e^2}{4\pi r}\left[1 + \frac{e^2}{8\pi r}\,\frac{(m_A+m_B)}{m_A m_B c^2} - \frac{7e^2}{24\pi^2}\,\frac{\hbar}{m_A m_B c^3 r^2}\right]$$

Subtleties about iteration of lowest order – quantum part is not modified

Gravitational potential via dispersive techniques:

- cut involves gravitational Compton amplitude



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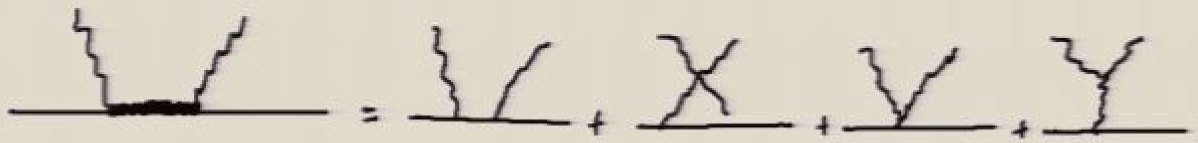


$$\rho_g(s,t) = \frac{-1}{32\pi}\int \frac{d\Omega_k}{4\pi}\,\mathcal{M}_A^{\mu\nu,\lambda\sigma}(p_1,-k,p_1-q,q-k)\mathcal{M}_B^{\alpha\beta,\gamma\delta}(p_3,-k,p_3+q,k-q)P_{\mu\nu,\alpha\beta}P_{\lambda\sigma}$$

Amplitudes are more complicated, but procedure is the same:

Reproduce usual result –diagram by diagram

Ghosts done by hand for now

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3}\right]$$

VERY strong check on loop calculation

Universality of the quantum corrections:

We have been calculating other spins

- quantum corrections seem universal

<u>Can we prove universality?:</u> Weinberg

Electromagnetic amplitude has multipole expansion

- at low energy, E1 transitions dominate – fixed tensor structure
- E1 transition has fixed $q^2 \to 0$ limit, normalized to charge

This low energy limit determines $\sqrt{t}$ and constant terms in ρ

Reasons to expect that gravitational interaction is similar

- universal form - factorization
- low energy is square of E1 amplitude – fixed form

$\Rightarrow$all gravity spectral functions have the same low energy structure

$\Rightarrow$same classical and quantum corrections

Coordinate redefinitions:

Possibility of further coordinate changes

$$r \to r\left[1+\alpha\frac{G(m_1+m_2)}{r}\right]$$

which changes the classical potential

$$\frac{Gm_1m_2}{r}\left[1+c\frac{G(m_1+m_2)}{r}\right] \to \frac{Gm_1m_2}{r}\left[1+(c-\alpha)\frac{G(m_1+m_2)}{r}\right]$$

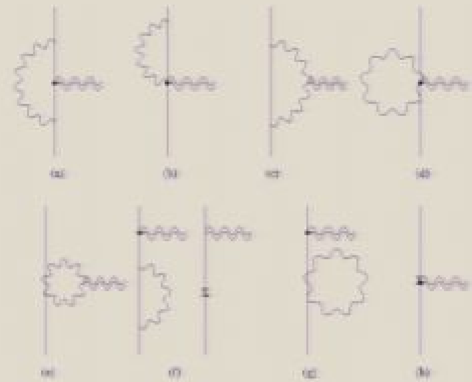
However, in Hamiltonian treatment this is compensated by change of other terms

$$\begin{aligned} H &= \left(\frac{\mathbf{p}^2}{2m_1}+\frac{\mathbf{p}^2}{2m_2}\right)-\left(\frac{\mathbf{p}^4}{8m_1^3}+\frac{\mathbf{p}^4}{8m_2^3}\right) \\ &- \frac{Gm_1m_2}{r}\left[1+a\frac{\mathbf{p}^2}{m_1m_2}+b\frac{(\mathbf{p}\cdot\hat{\mathbf{r}})^2}{m_1m_2}+c\frac{G(m_1+m_2)}{r}\right] \end{aligned}$$

$$\begin{aligned} a &\to \frac{1}{2}\left[1+(3+2\alpha)\frac{(m_1+m_2)^2}{m_1m_2}\right] \\ b &\to \frac{1}{2}-\alpha\frac{(m_1+m_2)^2}{m_1m_2} \\ c &\to -\frac{1}{2}-\alpha \end{aligned}$$

Einstein Infeld Hoffmann coordinates

Calculation:



Boson:

$$< p_2|T_{\mu\nu}(x)|p_1 >= \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_2E_1}}\left[2P_\mu P_\nu F_1(q^2) + (q_\mu q_\nu - g_{\mu\nu}q^2)F_2(q^2)\right]$$

$$\begin{aligned} F_1(q^2) &= 1 + \frac{\alpha}{4\pi}\frac{q^2}{m^2}\left(-\frac{8}{3} + \frac{3}{4}\frac{m\pi^2}{\sqrt{-q^2}} + 2\log\frac{-q^2}{m^2} - \frac{4}{3}\log\frac{\lambda}{m}\right) + \ldots \\ F_2(q^2) &= -\frac{1}{2} + \frac{\alpha}{4\pi}\left(-\Omega - \frac{26}{9} + \frac{m\pi^2}{2\sqrt{-q^2}} + \frac{4}{3}\log\frac{-q^2}{m^2}\right) + \ldots \end{aligned}$$

where

$$\Omega = \frac{2}{\epsilon} - \gamma - \log\frac{m^2}{4\pi\mu^2}$$

Fermion:

$$\begin{aligned} < p_2|T_{\mu\nu}|p_1 > &= \bar{u}(p_2)\Big[F_1(q^2)P_\mu P_\nu\frac{1}{m} \\ &- F_2(q^2)(\frac{i}{4m}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4m}\sigma_{\nu\lambda}q^\lambda P_\mu) \\ &+ F_3(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)\frac{1}{m}\Big]u(p_1) \end{aligned}$$

Results:

-reproduce classical terms (harmonic gauge)

-quantum terms common to fermions, bosons

$$\begin{aligned} g_{00} &= 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\alpha\hbar}{3\pi m r^3} + \ldots \\ g_{0i} &= (\frac{2G}{r^3} - \frac{G\alpha}{mr^4} + \frac{2G\alpha\hbar}{\pi m^2 r^5})(\vec{S}\times\vec{r})_i \\ g_{ij} &= -\delta_{ij} - \delta_{ij}\frac{2Gm}{r} + G\alpha\frac{r_i r_j}{r^2} + \frac{4G\alpha\hbar}{3\pi m r^3}\left(\frac{r_i r_j}{r^2} - \delta_{ij}\right) + \ldots \end{aligned}$$

Quantum corrections to Reissner-Nordstrom and Kerr-Newman metrics

Metric around charged bodies,
without (RN) or with (KN) angular momentum

JFD
Holstein
Garbrecht
Konstantin

Quantum Electrodynamics calculation
-gravity is classical here
-but uses EFT logic

Metric determined by energy momentum tensor:

$$\Box h_{\mu\nu} = -16\pi G(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T)$$

harmonic gauge

$$h_{\mu\nu}(x) \;\; = \;\; -16\pi G \int d^3y D(x-y)(T_{\mu\nu}(y) - \frac{1}{2}\eta_{\mu\nu}T(y))$$

Logic:
looking for non-analytic terms again:
-long range propagation of photons

$$\begin{aligned} \text{metric} \;\; &\sim \;\; Gm\int \frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\frac{1}{\vec{q}^2}\left[1 - b\alpha\frac{\vec{q}^2}{m^2}\sqrt{\frac{m^2}{\vec{q}^2}} - \frac{\vec{q}^2}{m^2}\log(\vec{q}^2) - c\alpha\frac{\vec{q}^2}{m^2} + \ldots\right] \\ &\sim \;\; Gm\left[\frac{1}{r} + \frac{a\alpha}{mr^2} + \frac{b\alpha\hbar}{m^2r^3} + \frac{c\alpha}{m^2}\delta^3(x) + \ldots\right] \qquad (7) \end{aligned}$$

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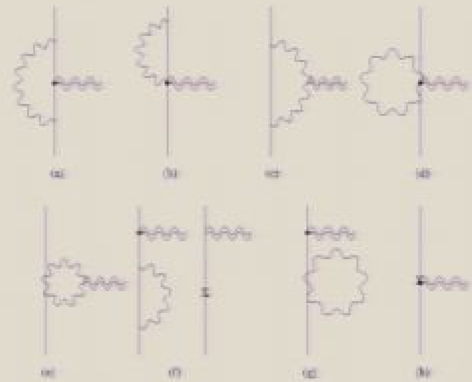

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Physical intepretation:

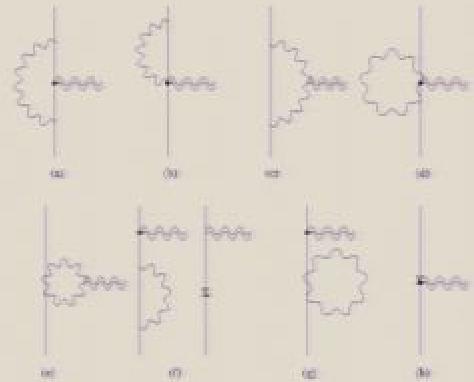
- classical terms are just the classical field around charged particl

$$
\begin{aligned}
T_{00}^{EM}(\vec{r}) &= \frac{1}{2}E^2 = \frac{e^2}{32\pi^2 r^4} \\
T_{0i}^{EM}(\vec{r}) &= 0 \\
T_{ij}^{EM}(\vec{r}) &= -E_i E_j + \frac{1}{2}\delta_{ij}E^2 = -\frac{e^2}{16\pi^2 r^4}\left(\frac{r_i r_j}{r^2} - \frac{1}{2}\delta_{ij}\right)
\end{aligned}
$$

-reproduced in the loops expansion

- quantum terms are fluctuations in the electromagnetic fields

Calculation:



Boson:

$$< p_2|T_{\mu\nu}(x)|p_1 >= \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_2E_1}}\left[2P_\mu P_\nu F_1(q^2) + (q_\mu q_\nu - g_{\mu\nu}q^2)F_2(q^2)\right]$$

$$\begin{aligned} F_1(q^2) &= 1 + \frac{\alpha}{4\pi}\frac{q^2}{m^2}\left(-\frac{8}{3} + \frac{3}{4}\frac{m\pi^2}{\sqrt{-q^2}} + 2\log\frac{-q^2}{m^2} - \frac{4}{3}\log\frac{\lambda}{m}\right) + \ldots \\ F_2(q^2) &= -\frac{1}{2} + \frac{\alpha}{4\pi}\left(-\Omega - \frac{26}{9} + \frac{m\pi^2}{2\sqrt{-q^2}} + \frac{4}{3}\log\frac{-q^2}{m^2}\right) + \ldots \end{aligned}$$

where

$$\Omega = \frac{2}{\epsilon} - \gamma - \log\frac{m^2}{4\pi\mu^2}$$

Fermion:

$$\begin{aligned} < p_2|T_{\mu\nu}|p_1 > &= \bar{u}(p_2)\left[F_1(q^2)P_\mu P_\nu\frac{1}{m}\right. \\ &- F_2(q^2)\left(\frac{i}{4m}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4m}\sigma_{\nu\lambda}q^\lambda P_\mu\right) \\ &+ \left. F_3(q^2)(q_\mu q_\nu - g_{\mu\nu}q^2)\frac{1}{m}\right]u(p_1) \end{aligned}$$

Results:

-reproduce classical terms (harmonic gauge)

-quantum terms common to fermions, bosons

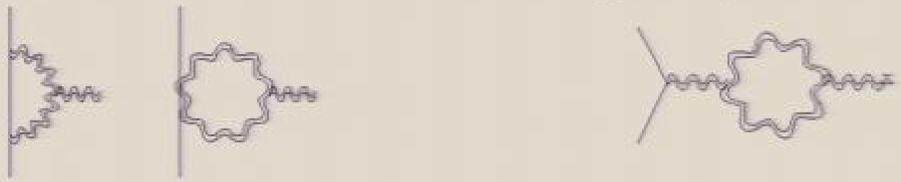
$$\begin{aligned} g_{00} &= 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\alpha\hbar}{3\pi mr^3} + \ldots \\ g_{0i} &= \left(\frac{2G}{r^3} - \frac{G\alpha}{mr^4} + \frac{2G\alpha\hbar}{\pi m^2r^5}\right)(\vec{S}\times\vec{r})_i \\ g_{ij} &= -\delta_{ij} - \delta_{ij}\frac{2Gm}{r} + G\alpha\frac{r_ir_j}{r^2} + \frac{4G\alpha\hbar}{3\pi mr^3}\left(\frac{r_ir_j}{r^2} - \delta_{ij}\right) + \ldots \end{aligned}$$

Quantum Correction to Schwarzschild and Kerr Metrics

Interpretation less clear – but calculation well-defined
incomplete quantum calculation

JFD
Holstein
Bjerrum-Bohr

Similar methods as RN and KN metrics:
- need to add in vacuum polarization diagram
-again reproduce harmonic gauge classical terms
-again boson, fermion results agree (non-trivial)



$$
\begin{aligned}
g_{00} &= 1-2\frac{Gm}{r}+2\frac{G^2m^2}{r^2}+\frac{62G^2m\hbar}{15\pi r^3}+\ldots \\
g_{0i} &= \left(\frac{2G}{r^3}-\frac{2G^2m}{r^4}+\frac{36G^2\hbar}{15\pi r^5}\right)(\vec{S}\times\vec{r})_i+\ldots \\
g_{ij} &= -\delta_{ij}\left(1+2\frac{Gm}{r}+\frac{G^2m^2}{r^2}+\frac{14G^2m\hbar}{15\pi r^3}\right)-\frac{r_ir_j}{r^2}\left(\frac{G^2m^2}{r^2}+\frac{76G^2m\hbar}{15\pi r^3}\right)+
\end{aligned}
$$

Note: Duff and Liu only consider vac. pol.
-vertex corrections are the same order
-is there a "classical source" for gravity
-interactions do not decouple as m $\longrightarrow\infty$

Graviton –graviton scattering

Fundamental quantum gravity process

Lowest order amplitude:

$$\mathcal{A}^{tree}(++;++) = \frac{i}{4}\frac{\kappa^2 s^3}{tu}$$

Cooke;
Behrends Gastmans
Grisaru et al

One loop:

Incredibly difficult using field theory

Dunbar and Norridge –string based methods! (just tool, not full string theory)

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right) \\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--) \\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right. \\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right) \\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u} \\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4},
\end{aligned} \tag{4}$$

Infrared safe:

JFD + Torma

The $1/\epsilon$ is from infrared

-soft graviton radiation

-made finite in usual way

$1/\epsilon$ -> ln(1/resolution) (gives scale to loops)

-cross section finite

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2} f\left(\frac{-t}{s}, \frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

*

finite

Beautiful result:

-low energy theorem of quantum gravity

Hawking Radiation

Hambli, Burgess

Exploratory calculation

-remove high energy contributios
-Pauli Villars regulators

-flux from local limit of Green's function

$$\mathcal{F} \equiv -\langle T_t{}^r \rangle = -\langle T_{tr^\star} \rangle$$
$$= -\frac{1}{2} \lim_{x' \to x} \left(\frac{\partial}{\partial t'} \frac{\partial}{\partial r^\star} + \frac{\partial}{\partial r^{\star\prime}} \frac{\partial}{\partial t} \right) G(x, x').$$

-dependence on regulator vanishes exponentially

-radiation appears to be property
of the low energy theory

Limitations of the effective field theory

Corrections grow like $Amp \sim A_0\left[1 + Gq^2 + Gq^2 \ln q^2\right]$

Overwhelm lowest order at $q^2 \sim M_P^2$

Also sicknesses of $R+R^2$ theories beyond M_P
(J. Simon)

Effective theory predicts its own breakdown at M_P
-could in principle be earlier

Needs to be replaced by more complete theory
at that scale

Treating quantum GR beyond the Planck scale
is likely not useful

The extreme IR limit

Singularity theorems:

-most space times have singularities

-EFT breaks down near singularity

Can we take extreme IR limit?

-wavelength greater than distance to nearest singularity?

-$r \longrightarrow \infty$ past black holes?

Possible treat singular region as source

-boundary conditions needed

deSitter horizon in IR

Reformulate problem of quantum gravity

Old view: GR and Quantum Mechanics incompatible

Shocking!

New view: We need to find the right "high energy" theory which includes gravity

Less shocking:

-not a conflict of GR and QM

-just incomplete knowledge

THIS IS PROGRESS!

Dueling quotations:

But for gravity, renormalization theory fails, because of the inherent nonlinearities in general relativity. So we come to a key puzzle: The existence of gravity clashes with ou description of the rest of physics by quantum fields.

Edward Witten
Physics Today
April 1996

A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data.

Frank Wilczek
Physics Today
August 2002

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Another thoughtful quote:

"I also question the assertion that we presently have no quantum field theory of gravitation. It is true that there is no *closed*, internally consistent theory of quantum gravity valid at all distance scales. But such theories are hard to come by, and in any case, are not very relevant in practice. But as an *open* theory, quantum gravity is arguably our *best* quantum field theory, not the worst. ….

{Here he describes the effective field theory treatment}

From this viewpoint, quantum gravity, when treated –as described above- as an effective field theory, has the largest bandwidth; it is credible over 60 orders of magnitude, from the cosmological to the Planck scale of distances."

J.D. Bjorken

Possible applications

Singularity theorems

Cosmology – early universe

Possible tests???
 -long distance propagation
 -quantum effects on photons
 -frequency dispersion

Comparison to numerical methods
 -lattice gravity

EFT in presence of Λ
 -quantization and divergences known (Christensen and Duff)
 -power counting modified

de Sitter "instability":
 -understanding Tsamis Woodard effect

Summary

We have a quantum theory of general relativity

- -quantization and renormalization
- -perturbative expansion

It is an effective field theory

- -valid well below the Planck scale
- -corrections are very well behaved

Effective field theory techniques allow predictions

- -finite
- -parameter free
- -due to low energy (massless) propagation

EFT may be full quantum content of GR

- -points to breakdown by $E = M_P$

Need full theory at or before Planck scale

- -many interesting questions need full theory
- -not conflict between QM and GR, but lack of knowledge