Title: Geometric Flows and String Theory

Date: Jun 13, 2006 02:00 PM

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Abstract: Geometric flows, especially the Ricci flow, have been used with considerable success in recent years to address the Poincare and Thurston conjectures for 3-manifolds. In this talk, I will briefly introduce these geometric flows, and describe how they appear in a completely different context in the physics of string theory. I will then outline how recently developed techniques in geometric flows could be used to address questions of importance in string theory.

GEOMETRIC FLOWS AND STRING THEORY:

V. Suneeta,
U. of New Brunswick.

Based on:

T. Oliynyk, V.S, E. Woolgar,
Phys. Lett. B610 (2005) 115; hep-th/0410001.

T. Oliynyk, V.S, E. Woolgar,
NPB 739 (2006) 441; hep-th/0510239.

J. Gegenberg, V.S, hep-th/0605230.

What **geometric flows** will appear in this talk . . . . .

RG flows of closed (bosonic) string theory obtained from the sigma model:

$$S = \int_\Sigma d^2\sigma \sqrt{\gamma}\, \gamma^{\alpha\beta} g_{ij}(x) (\partial_\alpha X^i)(\partial_\beta X^j) + \underbrace{\ldots\ldots}_{\text{terms from other fields}}$$

(M, g)

(Σ, γ)

$X : \Sigma \to M$ are scalar fields for sigma model

$g_{ij}(x)$: coupling "constant"

Therefore, if we start with a sigma model with some arbitrary metric g_{ij} & do a renormalization group (RG) transformation,

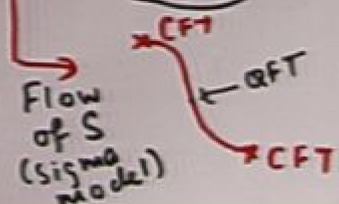
$$-\frac{\partial g_{ij}}{\partial t} = -\alpha' R_{ij} + \underbrace{\ldots\ldots}_{\text{other terms could be}}$$

$t = -\log k$, k: 2d scale

other terms could be

⊗ higher order (in α') corrections (neglected if $R \ll \alpha'^{-1}$)

⊛ involving other fields B_{ij}, Φ, $T(x)$ etc.



So, a "geometric flow" is a PDE of the form:

$$\frac{\partial g_{ij}}{\partial t} = -\alpha' R_{ij} + \cdots\cdots$$

along with the stipulation that we really want a flow through geometries, i.e of metrics mod. diffeomorphisms.

⊗ Interest has been mainly in fixed points – sigma model is a CFT.

⊗ However, recently — LOT OF INTEREST IN SOLUTIONS TO RG FLOW (not just fixed points) in the hope that they may approxi-mate off-shell/on-shell string dynamics in various situations. (eg tachyon condensation) (Banks, Vafa, ... etc.)

⊗ In fact, explicit solutions computed.

So, motivated by this interest, theme for this talk is "a study of geometric RG flows and their fixed points."

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Interestingly, one geometric flow studied independently in math:

$$\frac{\partial g_{ij}}{\partial t} = -\alpha' R_{ij} \qquad \text{"Ricci flow"}$$

⊗ Introduced by R Hamilton in 1982 to address Thurston's Geometrization Conjecture.

Recent developments:

⊗ Ricciflow used to successfully resolve (at least a part of) Thurston's Conjecture.

⊗ Made possible by recent techniques/results of Perelman (2002):

- Ricci flow is monotonic on compact $\mathcal{M}$.
- Better understanding of singular behaviour.

Q// Can these techniques/results be used to understand string theory RG flow better?

Let us focus on one of Perelman's achievements:

"No periodic solutions to Ricci flow on compact manifolds (except ones that change throughout by diffeos)." (i.e no periodic solns. in geometry)

Realising that Ricci flow is a (first-order) RG flow, this statement sounds familiar...

Recall: RG scale transformation corresponds to a coarse-graining of description of a physical system. So intuitively ....

⊗ They ought to be irreversible. (exceptions known)

⊗ Usually flow to nice, simple fixed points. (definitely not true in general!)

Therefore no general proof of above exists...

Except — for 2d renormalizable QFTs

"Zamolodchikov's c-theorem"

c-theorem :

Consider RG flow through 2d rotationally invariant renormalizable QFTs, & define a function on this space.

"C-function" $\qquad r = \sqrt{z\bar{z}}$

$$C(r) = 2F(r) - G(r) - \frac{3H(r)}{8}$$

$$\langle T_{zz}(z)\, T_{zz}(0) \rangle = \frac{F(r)}{z^4}$$

$$\langle T_{z\bar{z}}(z)\, T_{zz}(0) \rangle = \frac{G(r)}{4 z^3 \bar{z}}$$

$$\langle T_{z\bar{z}}(z)\, T_{z\bar{z}}(0) \rangle = \frac{H(r)}{16 z^2 \bar{z}^2}$$

Then, under a scale transformation

$$\frac{dC}{d(\ln r)} = -\frac{3}{4} H$$

log(RG length scale) = t

Facts :

* In a unitary field theory, $H \geq 0$. So C changes $\boxed{\text{monotonically}}$ under an RG flow.
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* In a unitary field theory, $H \geq 0$. So C changes $\boxed{\text{monotonically}}$ under an RG flow.
* If the QFT = CFT, $T_{z\bar{z}} = 0$. So $H = 0$ & $C = c$ (central charge). So CFT's are fixed pts. of flow of C.

QUESTION:

* Is the converse true??

Is every solution to $\frac{dC}{dt} = 0$ a CFT?

If YES, then since every fixed point of RG flow is a CFT,

RG flow fixed pt. $\Longleftrightarrow$ $C(r) = c$ (constant)

Also, since $C(r)$ is then strictly decreasing under RG flow, it is an "entropy".

So 2d RG flow is then irreversible.

PROBLEM: Converse need not always be true!

May have a non-trivial RG flow, but $dC/dt = 0$. So C-function is then insensitive to flow, & tells us nothing about irreversibility.

EXAMPLE?

The world-sheet sigma model on generic non-compact target spaces.

Why? For the sigma model, embedding coordinates of the sheet in target space are the "fields".

What does, for eg $\langle T_{zz}(z)\, T_{zz}(0)\rangle$ look like?

$\langle T_{zz}(z)\, T_{zz}(0) \rangle$

$$= \frac{\int_{\mathcal{M}} \mathcal{D}x^i \, e^{-S[x]} \, T_{zz}[x(z)]\, T_{zz}[x(0)]}{\int_{\mathcal{M}} \mathcal{D}x^i \, e^{-S[x]}}$$

$\int \longrightarrow$ would be prop. to $V(\mathcal{M})$

NOTE: C-function is a combination of such correlators.

As pointed out: (Polchinski/Vafa/Harvey/Kutasov ....)

Numerator may be finite.

Generally, denominator $\propto V(\mathcal{M}) \to \infty$ on non-compact target spaces $\mathcal{M}$.

Then $\langle$correlators$\rangle \to 0$

so C function $= 0$ (even though sigma model may not be a CFT!!)

When does this happen? (Tseytlin)

eg when $\phi = B_{ij} = 0$, numerator is finite if $\int_{\mathcal{M}} R \, dV$ is finite.

So in RG flow through spaces where happens, C-theorem breaks down.

When target space is compact:

Situation no better (although for a different reason)

C-function for NLSM computed by Tseytlin:
Computation of C requires knowledge of the $\boxed{\text{full flow}}$ from $-\infty < t < \infty$.

Problem! Generically geometric flows lead to singular behaviour (shrinking of portions of $\mathcal{M}$) somewhere between $t = -\infty$ and $t = +\infty$.
Shrinking means $R \propto 1/\alpha'$ (curvature). So $\boxed{\text{all}}$ orders in beta function become equally important.
(So even to compute C-function for a small range of t when $R < 1/\alpha'$ requires knowledge of all order corrections!)

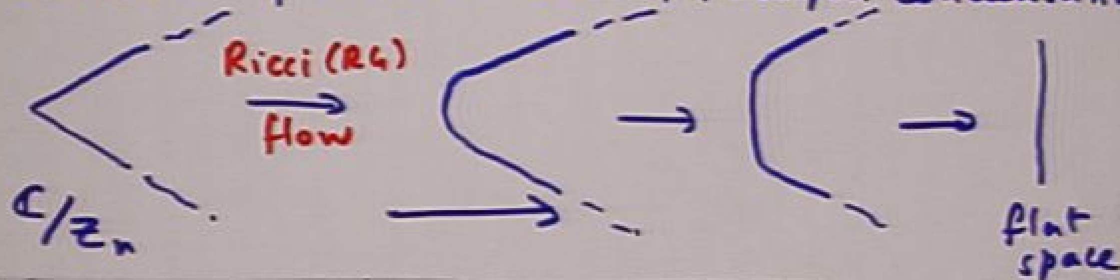
So $\boxed{\text{use}}$ of c-theorem for string theory RG flow is unclear.

Q: HOWEVER, IF RG FLOW APPROXIMATES STRING DYNAMICS, WE NEED TO KNOW IF IT EXHIBITS PATHOLOGICAL BEHAVIOUR, LIKE PERIODIC SOLUTIONS....
(eg closed string tachyon condensation, which it should approximate, is supposed to be irreversible)

Some suggestions to prove irreversibility of RG flows through non-compact target spaces:

(Gutperle / Headrick / Minwalla / Schomerus)

Looked at particular case of tachyon condensation:



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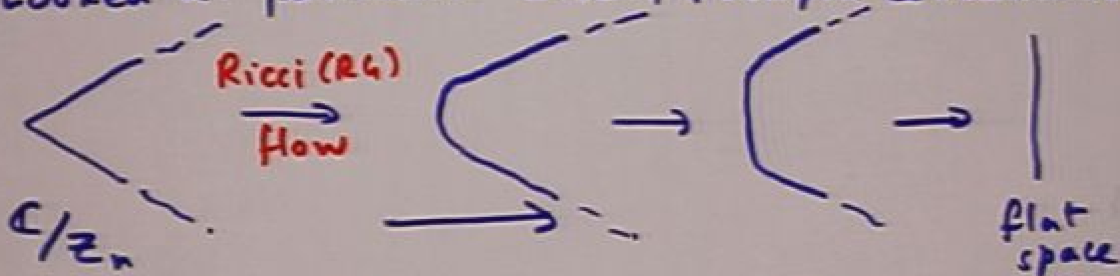

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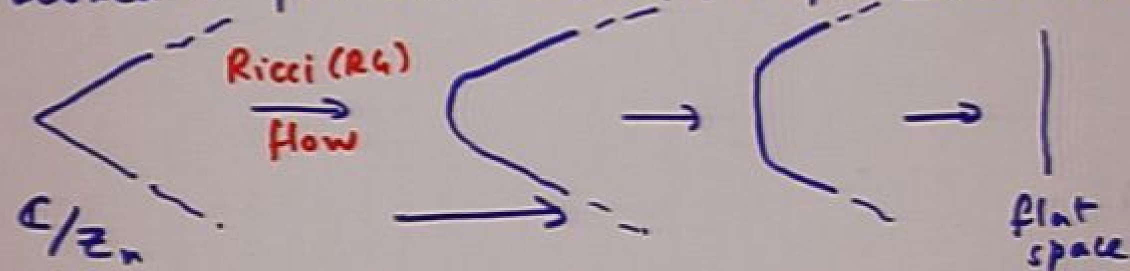

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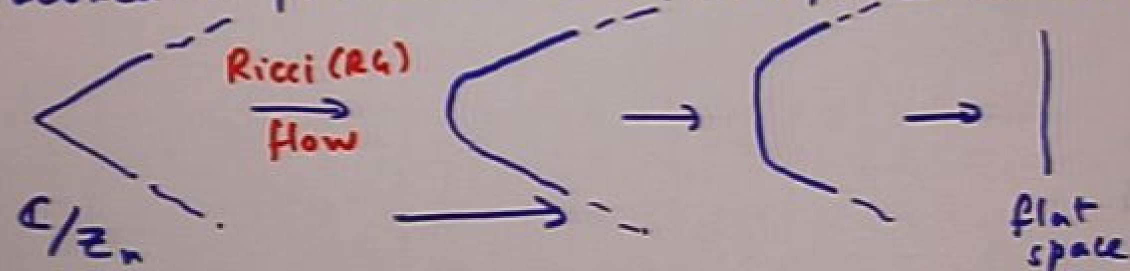

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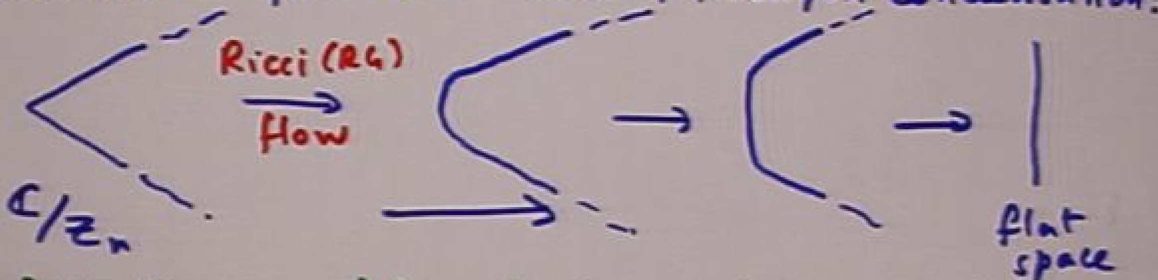

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Because one doesn't know what an arbitrary diffeomorphism will do to boundary of box.

Also, this may give a "spurious" mass increase for a solution changing only by diffeomorphisms.

What we find: A Preview

⊗ Indeed possible to construct diffeo-invariant entropies for RG flows.
(some of which actually equal central charge at fixed points)
So — closest analogue to c-theorem.

⊗ In fact, entropies can be used to develop a "target space" language for describing perturbations of fixed points.
(i.e relevant/irrelevant etc.)

⊗ Standard geometric techniques can be used to tell us when non-trivial fixed points to these flows exist.

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From $\dfrac{\partial g_{ij}}{\partial t} = -\alpha' R_{ij}$

volume element

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Integrate $\uparrow$ in $[t_0, t]$

$$\sqrt{g}(t) - \sqrt{g}(t_0) = -\frac{\alpha'}{2} \int_{t_0}^{t} R \sqrt{g}\, ds$$

Define
$$\mu(t) := \int_M \sqrt{g}(t) - \sqrt{g}(t_0)\, d^D x$$

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$\delta g_{\mu\nu} > 0$

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① $R \geqslant 0$ everywhere on $\mathcal{M}$ all along the RG flow.

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& $R \geq 0$.

$$\boxed{\text{So } d\mu/dt \geq 0}$$

So for flows where $R \geq 0$:

$$\mu(t) = \int_M \left(\sqrt{g(t)} - \sqrt{g(t_0)}\right) d^D x$$ "Regularized volume"

is non-decreasing in $[t_0, t]$.

$\mu(t_0) = 0$ (obviously). $\mu(t) > \mu(t_0)$ unless $R(\hat{t}) = 0$ for all $\hat{t} \in [t_0, t]$

$R(\hat{t}) = 0 \Rightarrow R_{ij}(\hat{t} = 0)$ from

$$\frac{\partial R}{\partial t} = \alpha' (\Delta R + |R_{ij}|^2)$$

$\mu(t) = 0 \Leftrightarrow R_{ij} = 0$ (fixed points of Ricci flow)

Otherwise $\mu(t)$ increases monotonically.

So regularised volume is an entropy!

Can be used to rule out periodic solutions in geometry

$$g(t) = \varphi_* g(t_0)$$

(metrics at t & t_0 related by a diffeomorphism)

PROVIDED — diffeo. "falls off" asymptotically.

Can't do better! 2d Witten black hole ($R_{ij} \neq 0$)

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$$(-\Delta + \underline{R})$$

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Case ② $R < 0$ somewhere on $\mathcal{M}$, some time along the flow.

Entropy in this case is:

Lowest eigenvalue of $(-\Delta[g(t)] + \kappa R(t))$ on $\mathcal{M}$. $(\kappa \gtrsim 1)$ "Perelman-like entropy"

Proof of above: Non trivial generalisation of Perelman's proof. A control of how fall-offs are preserved under the flow required.

This entropy rules out solutions in some $[t_0, t]$ such that $g(t_0) = \varphi^* g(t)$ unless $R_{ij} = 0$ all along the flow (fixed point).

So - MODULO FALL-OFF CONDITIONS ON THE DIFFEO. & $\int R\, dV < \infty$: Analogue of c-theorem

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$$\frac{\partial \Phi}{\partial t} = \alpha' \left[\frac{1}{2} \Delta \Phi - |\nabla \Phi|^2 + \frac{|H|^2}{24} \right]$$

Under this flow, lowest eigenvalue of $(-4\Delta + R - |H|^2/12)$ is an entropy.

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Ⓡ Of course, proves irreversibility of this flow.

Ⓡ <u>MORE IMPORTANTLY</u>: This equals the central charge at fixed points!!

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$$\frac{\partial g_{ij}}{\partial t} = -\alpha' \left[R_{ij} + 2\nabla_i \nabla_j \Phi - \frac{1}{4} H_{ikl} H_j{}^{kl} \right]$$

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$$\frac{\partial \Phi}{\partial t} = \alpha' \left[\frac{1}{2} \Delta \Phi - |\nabla \Phi|^2 + \frac{|H|^2}{24} \right]$$

Under this flow, lowest eigenvalue of $\left(-4\Delta + R - |H|^2/12\right)$ is an entropy.

⊛ Of course, proves irreversibility of this flow.

⊛ MORE IMPORTANTLY: This equals the central charge at fixed points!!

⁎ So indeed seems analogous to c-theorem..

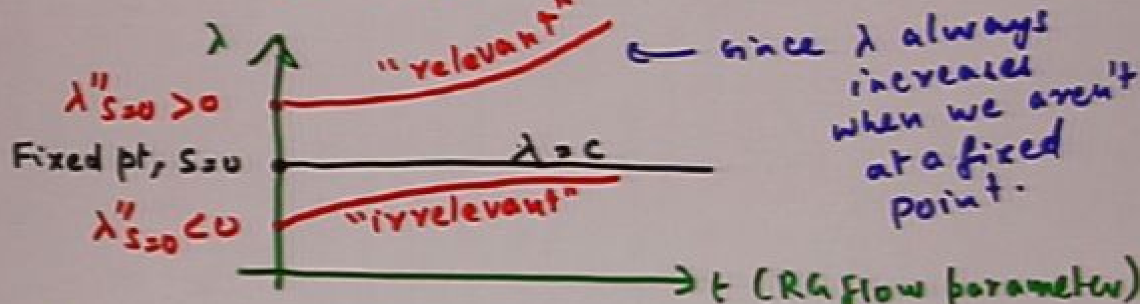
⊛ Entropy can be used to make a statement on linear stability of a perturbation of a fixed point.

Idea: At $t=0$, perturb a fixed point, say (g°, H°) w.r.t parameter s, $s=0$ corresponds to fixed point.

Entropy $:= \lambda$

At fixed point, $\lambda =$ constant, & $d\lambda/ds\,\big|_{s=0} = 0$

Compute $\frac{d^2\lambda}{ds^2}\Big|_{s=0} = \lambda''_{s=0}$



So sign of $\lambda''|_{s=0}$ expresses classification of relevant, marginal or irrelevant perturbations of CFT in language of target space geometry.

So $\lambda''_{s=0} \leq 0$ defines a notion of linear stability.

Math literature: (Ricci flow)

Linear stability + further assumptions $\Rightarrow$ Dynamical stability → info for all t.

Can we extend this to case of B-field?

Other results on fixed points to flows:

Q: For RG flow with just metric & nonzero dilaton, what are all the fixed points on compact target spaces?

A: (As we all know) to first order in α', $R_{ij} = 0$, $\Phi = \text{Const.}$
eg C-Y mflds, flat tori etc.

We want only a fixed point in $\boxed{\text{geometry}}$.

Why can't we have an exotic solution with $R_{ij} \neq 0$ & metric changes only by diffeos.?

Solves

$$R_{ij} + \nabla_i v_j + \nabla_j v_i = 0$$

v_i : Generates diffeos. (could be gradient of dilaton)

Result: (Bourguignon) Not possible.
No solutions other than $R_{ij} = 0$, $v_j = 0$.

→ obtain

$$\Delta R = -2(\nabla^m R) v_m - 2 |R_{ij}|^2$$

On $\mathcal{M}$, consider $p \in \mathcal{M}$ where R is minimum.

$$\Delta R|_p \geq 0 \quad \& \quad \nabla^m R|_p = 0$$

→ So, at p, LHS ≥ 0, RHS ≤ 0.

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So, at p, LHS ≥ 0, RHS ≤ 0.

$\therefore$ LHS = RHS = 0. $R_{ij}|_p = 0 \Rightarrow R|_p = 0$

Since $p \in \mathcal{M}$ is where R is minimum, everywhere else, $R \geq 0$.

Now $R + \nabla \cdot v = 0$

Integrate:

$$\int_{\mathcal{M}} R \, dv = -\int_{\mathcal{M}} \nabla \cdot v \, dv = 0$$

$\mathcal{M} \to$ compact!!

$$\int_{\mathcal{M}} R \, dv = 0 \ \& \ R \geq 0 \Rightarrow R = 0$$
$$\Rightarrow R_{ij} = 0$$

Q What happens to such a result when we have other fields? eg tachyon (bulk)

Can we have exotic non-Ricci-flat fixed points on compact target spaces?

(*) RG fixed points with a bulk tachyon: (J Gegenberg, VS)

$$v_i = \nabla_i \Phi$$

$$R_{ij} = \nabla_i v_j + \nabla_j v_i + (\partial_i T)(\partial_j T)$$

$$\Delta T - V'(T) + 2 v_i (\nabla^i T) = 0$$

derivative of tachyon potential $V(T)$.

Without assuming a form for tachyon potential, & applying minimum principle arguments, we get:

Existence of solutions on compact target spaces (other than $R_{ij} = 0$, $T = c$) linked to sign of $V''(T)$.

In fact: sign of $V''(T)$ at a pt. $p \in M$ where $(R - |\nabla T|^2)$ takes its minimum.

If $V''(T)\big|_p > 0$, no nontrivial solutions.

If $V''(T)\big|_p < 0$, nontrivial solutions POSSIBLE!!

So, if $V(T)$ has only leading order term $V(T) = -m^2 T^2$, non-Ricci-flat solutions are not ruled out!!

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But a physical justification needed!

Higher-order stringy effects? Winding tachyons?