Title: Self-interacting scalar fields and the Eot-Wash experiment

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Abstract: Experiments have ruled out unit-strength scalar-mediated fifth forces on scales ranging from 0.1 mm to 10,000 AU. However, allowing the scalar to have a quartic self-interaction weakens these constraints considerably. This weakening is due to the "chameleon mechanism", which gives the scalar field an effective mass that depends on the local matter density. I will describe the chameleon mechanism and discuss experimental constraints on self-interacting scalar fields. In particular, I will compare the chameleon-mediated self interaction to constraints from the Eot-Wash experiment, at the University of Washington, which comes closest to detecting such a scalar field today. It will be shown that a quartic self interaction of unit strength is just out of reach of the current Eot-Wash experiment, but will be readily visible to their next-generation instrument.

Outline

- Overview of fifth force constraints
- Phenomenology of self-interacting scalar fields
- Eöt-Wash: a test of gravitational inverse square law on small scales
- Computation of fifth force from self-interacting scalar
- Can we see the chameleon field?

Fifth-force constraints

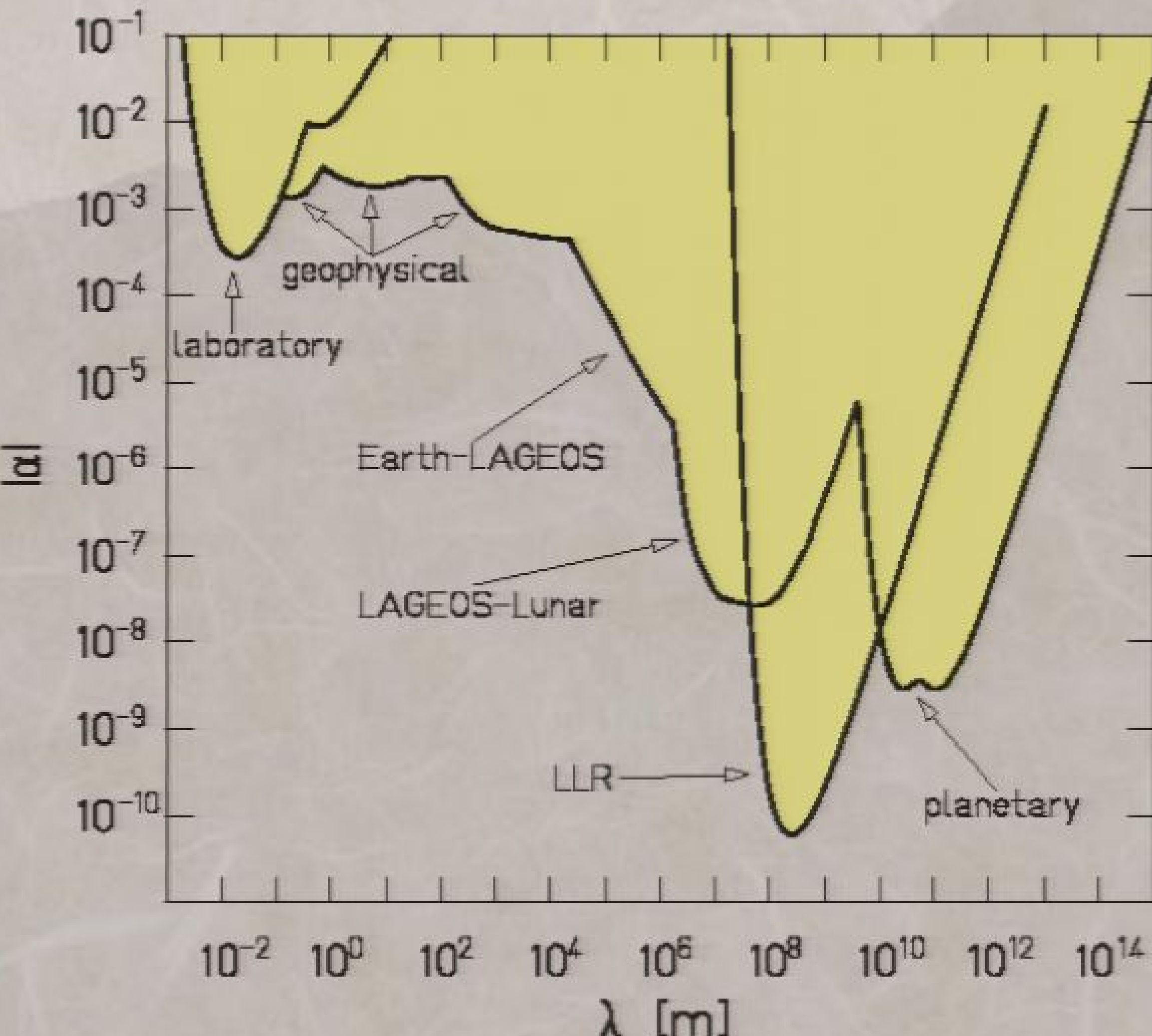
In the low energy, nonrelativistic limit, a massive scalar field coupled to matter through a Yukawa interaction

$$\mathcal{L}_{int} = \beta m \phi \bar{\psi} \psi / M_{Pl}$$

will result in a fifth-force correction to the gravitational inverse-square law:

$$V(r) = -\frac{G m_1 m_2}{r} \left(1 + \alpha e^{-m_\phi r}\right).$$

Fifth-force constraints



Self interacting scalar field

Add a quartic self interaction,

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{\beta \phi \rho}{M_{Pl}} \right)$$

where $\rho(\vec{x}) = \sum_i m_i \bar{\psi}\psi.$

The equation of motion is

$$-\partial_\mu \partial^\mu \phi = m_\phi^2 \phi + \frac{\lambda}{3!}\phi^3 - \frac{\beta}{M_{Pl}}\rho(\vec{r}) = \frac{dV_{eff,\rho}}{d\phi},$$

where the effective potential is given by

$$V_{eff,\rho}(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda}{4!}\phi^4 - \frac{\beta}{M_{Pl}}\rho\phi.$$

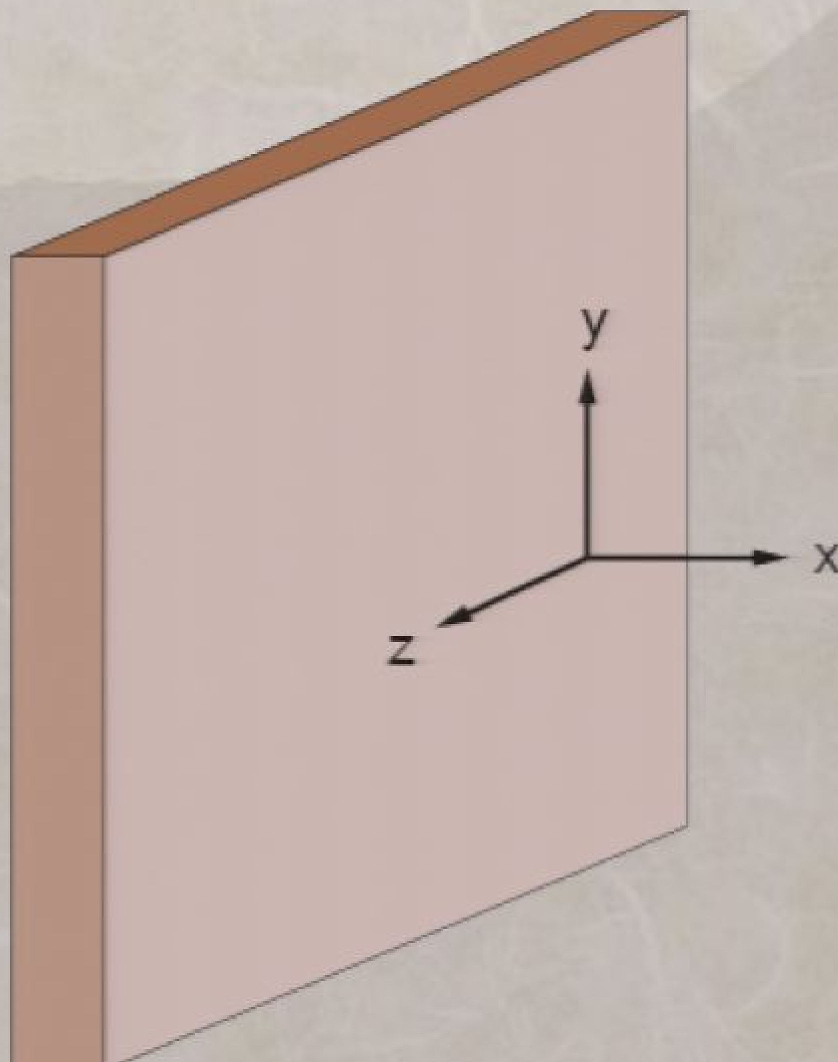
Chameleon mechanism

In bulk matter of uniform density, the effective potential is minimized at some $\phi_\rho = 0$, and the scalar field picks up an effective mass

$$m^2_{eff,\rho} = \left. \frac{d^2 V_{eff,\rho}(\phi)}{d\phi^2} \right|_{\phi_\rho} = m^2_\phi + \frac{1}{2}\lambda\phi^2_\rho.$$

Even with $m_\phi = 0$, the effective mass is nonzero, and the field becomes a short-range interaction with an effective length scale $m^{-1}_{eff,\rho} \sim \beta^{-1/3}\lambda^{-1/6}\rho^{-1/3}$. In conventional units, this length scale is approximately 0.1 mm in a material of density 1g/cm^3.

Chameleon mechanism



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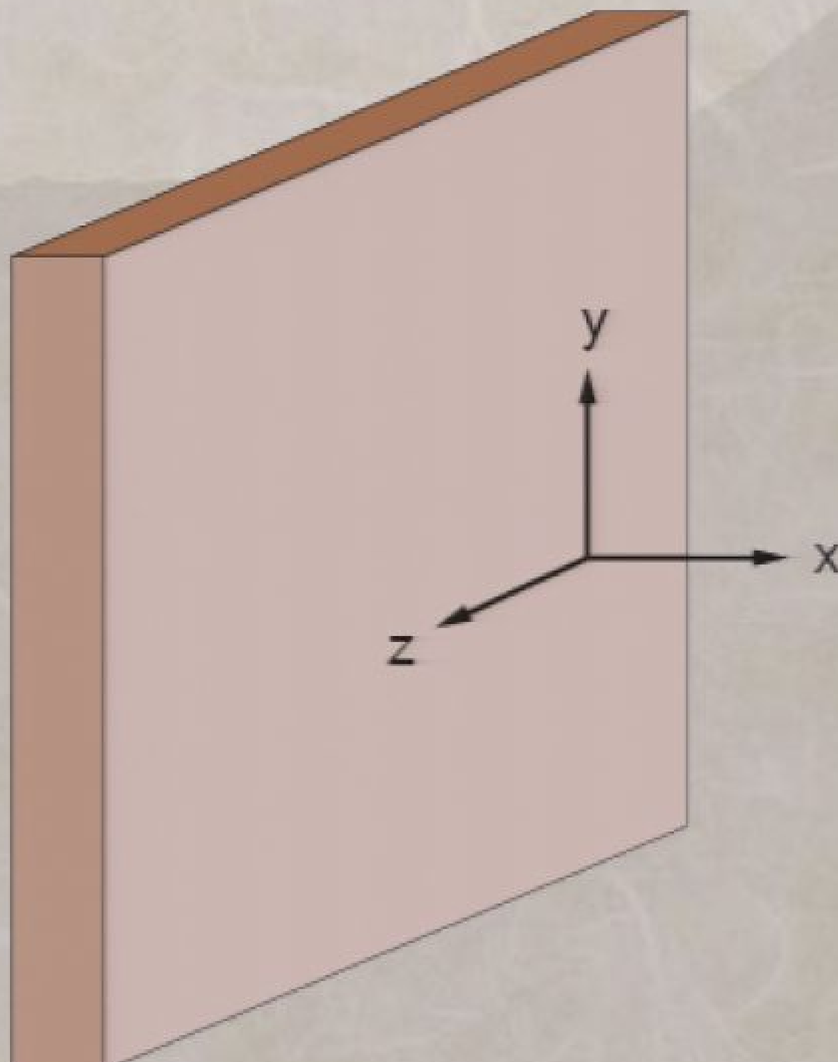


Chameleon mechanism

Outside the plane, in the vacuum, the equation of motion becomes

$$\frac{d^2\phi}{dx^2} = \frac{\lambda}{3!}\phi^3.$$

The appropriate solution is $\phi(x) = \frac{\sqrt{12/\lambda}}{x-b}.$

A test particle in the vacuum will feel a fifth force

$$F_{test} = \int \beta \rho_{test} \nabla \phi d^3x = \frac{-\beta m_{test}\sqrt{12/\lambda}}{(x-b)^2}.$$

Thin shell effect

For a massless scalar, the effective potential has a minimum

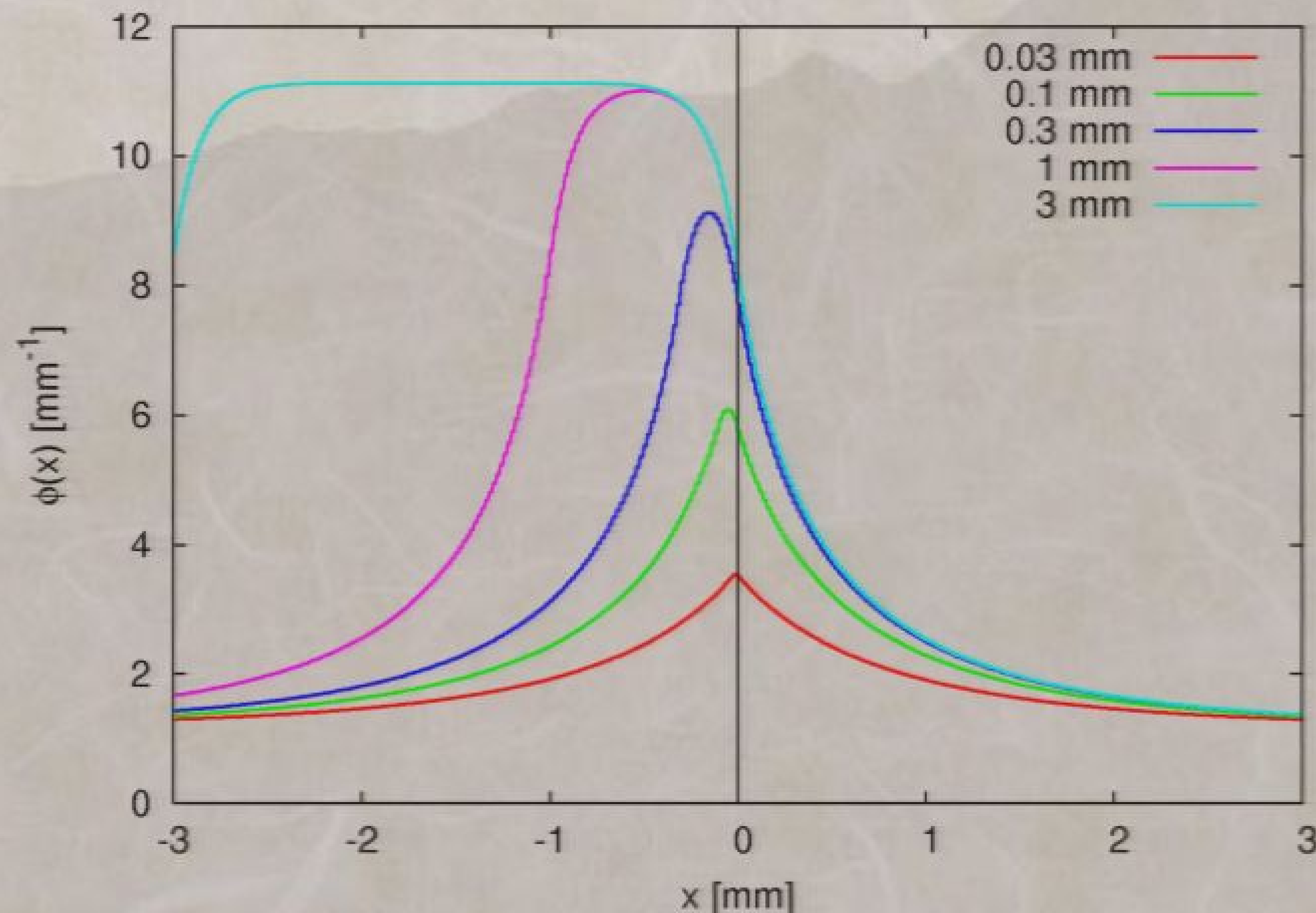
$$\frac{\partial V_{eff,\rho}}{\partial \phi} = \frac{\lambda}{3!}\phi^3 - \beta\rho = 0$$

only if the field has a self interaction, $\lambda \neq 0.$

The self interaction prevents the field inside a material of density ρ from increasing beyond

$$\phi_\rho = 6\beta^{1/3}\lambda^{-1/3}\rho^{1/3}.$$

Thin shell effect



Thin shell effect

thickness $R_t[mm]$	$\nabla\phi[mm^{-2}]$ at $x=0$	F_ϕ/F_{grav}
0.03	-3.0	1.8
0.1	-8.0	1.4
0.3	-13.9	0.81
1.0	-15.4	0.27
3.0	-15.5	0.090
10.0	-15.5	0.027

Attractive force theorem

It can be shown for a large range of effective potentials and mass distributions that the fifth force between two masses must be attractive. To show this, we begin with the Hamiltonian,

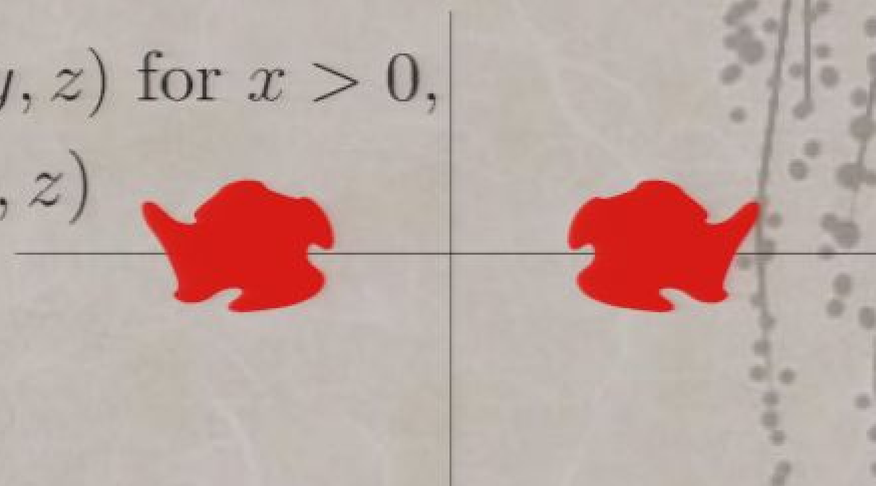
$$H = \int d^3x \left[\frac{1}{2} \left| \vec{\nabla} \phi \right|^2 + V_{eff,\rho}(\phi) \right].$$

For a static mass distribution, the field configuration which solves the equations of motion also minimizes the Hamiltonian.

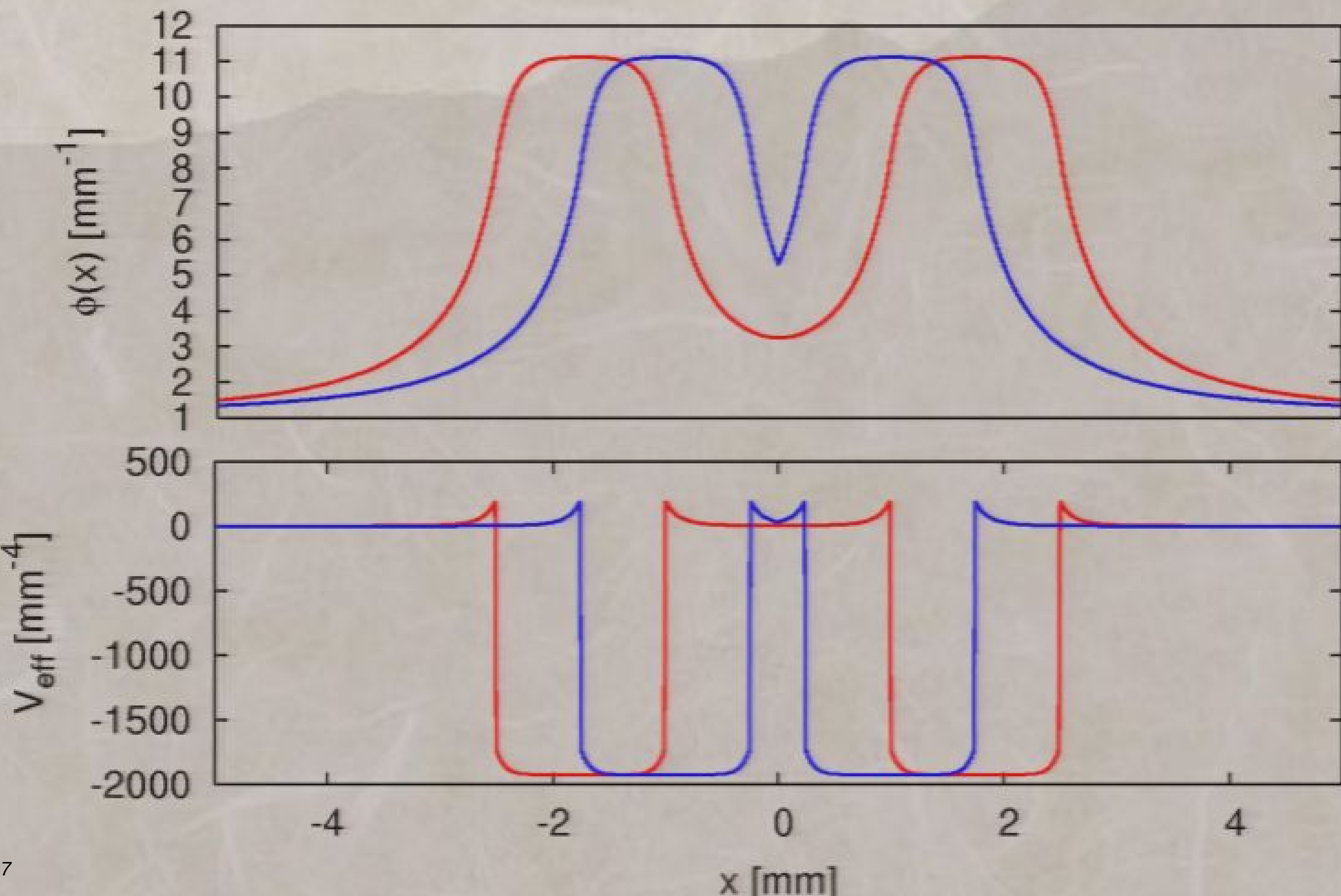
Attractive force theorem

$\phi_1(x, y, z)$

$$\phi_2(x, y, z) = \phi_1(x + a, y, z) \text{ for } x > 0,$$
$$\phi_2(-x, y, z) = \phi_2(x, y, z)$$



Attractive force theorem



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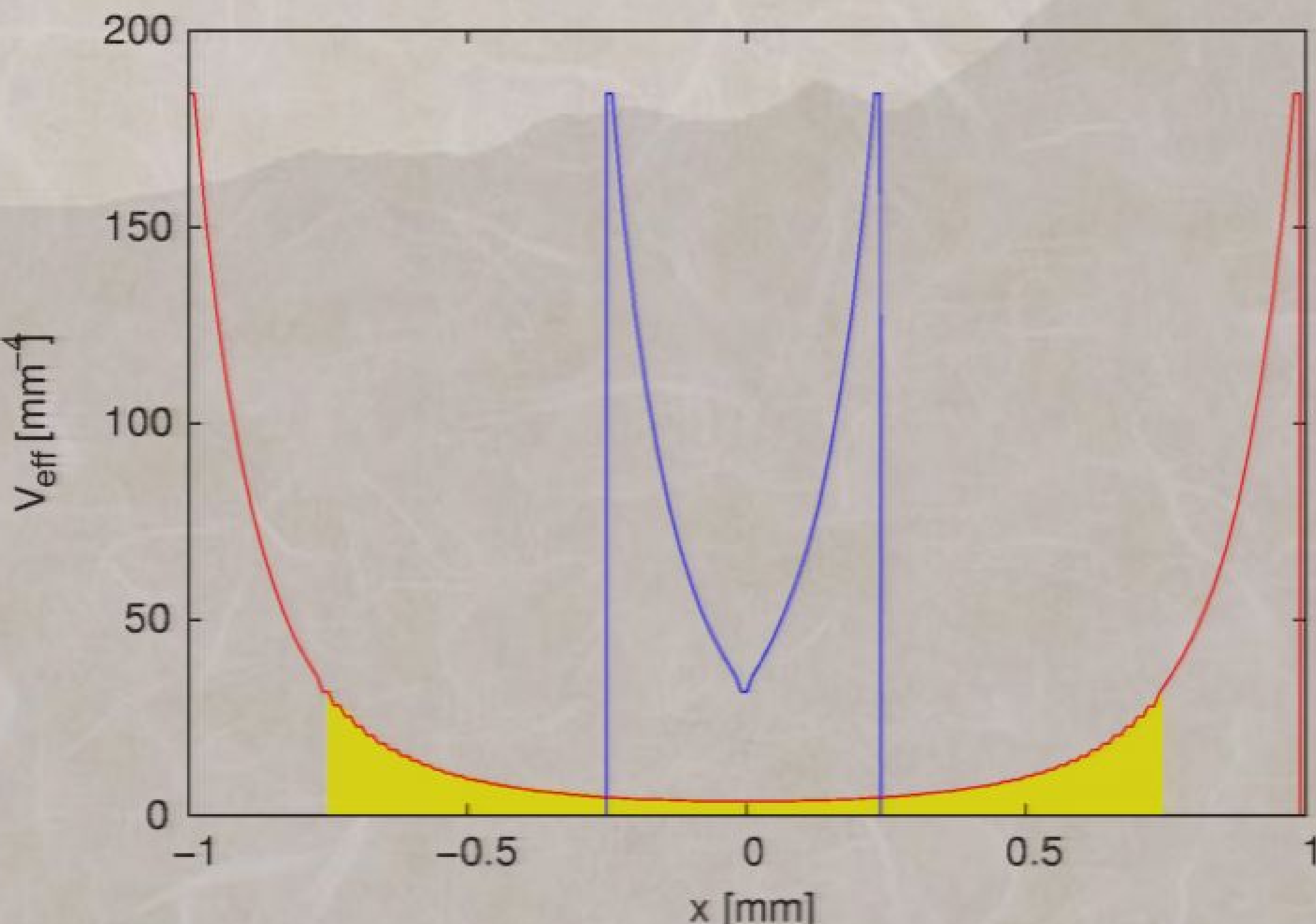

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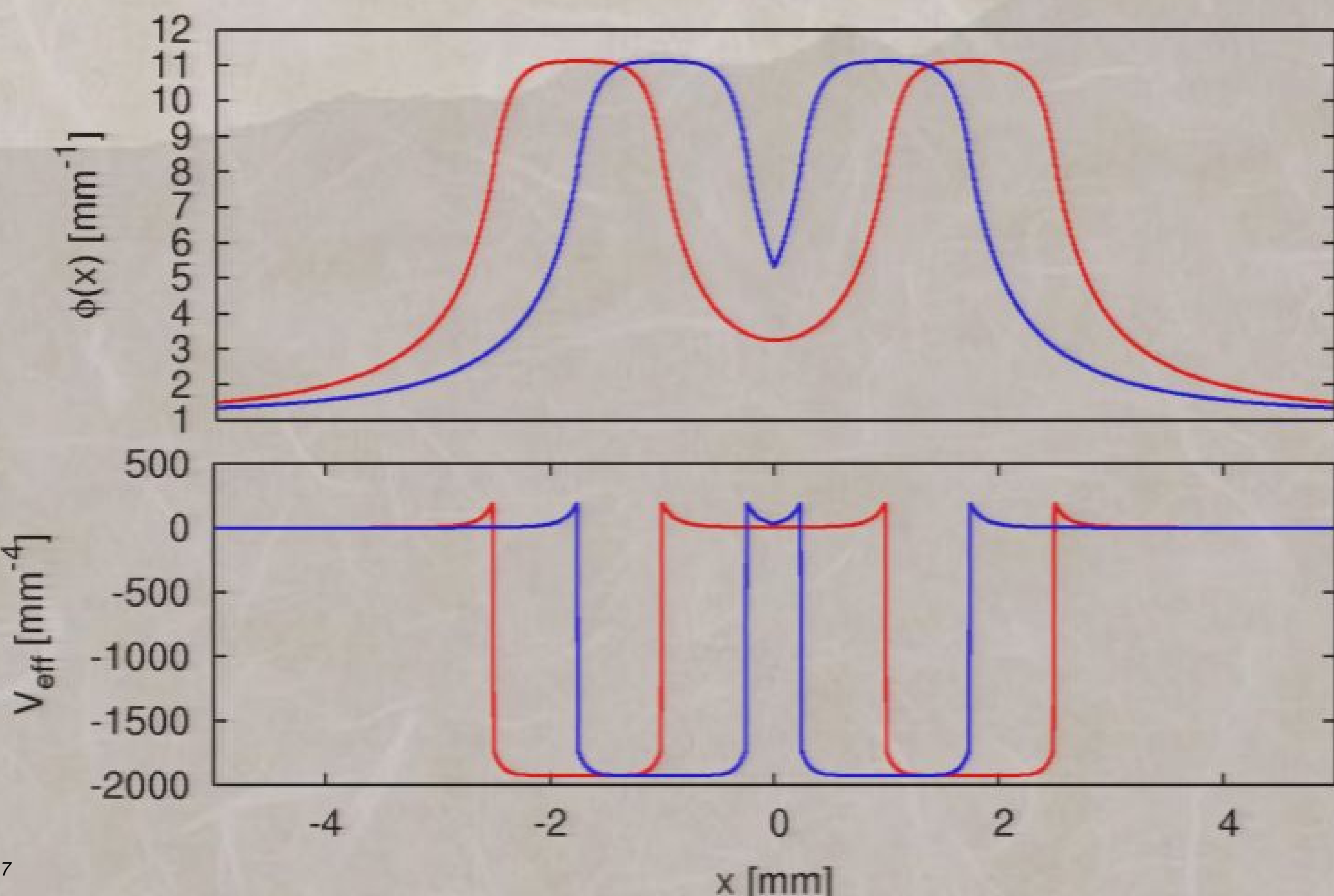

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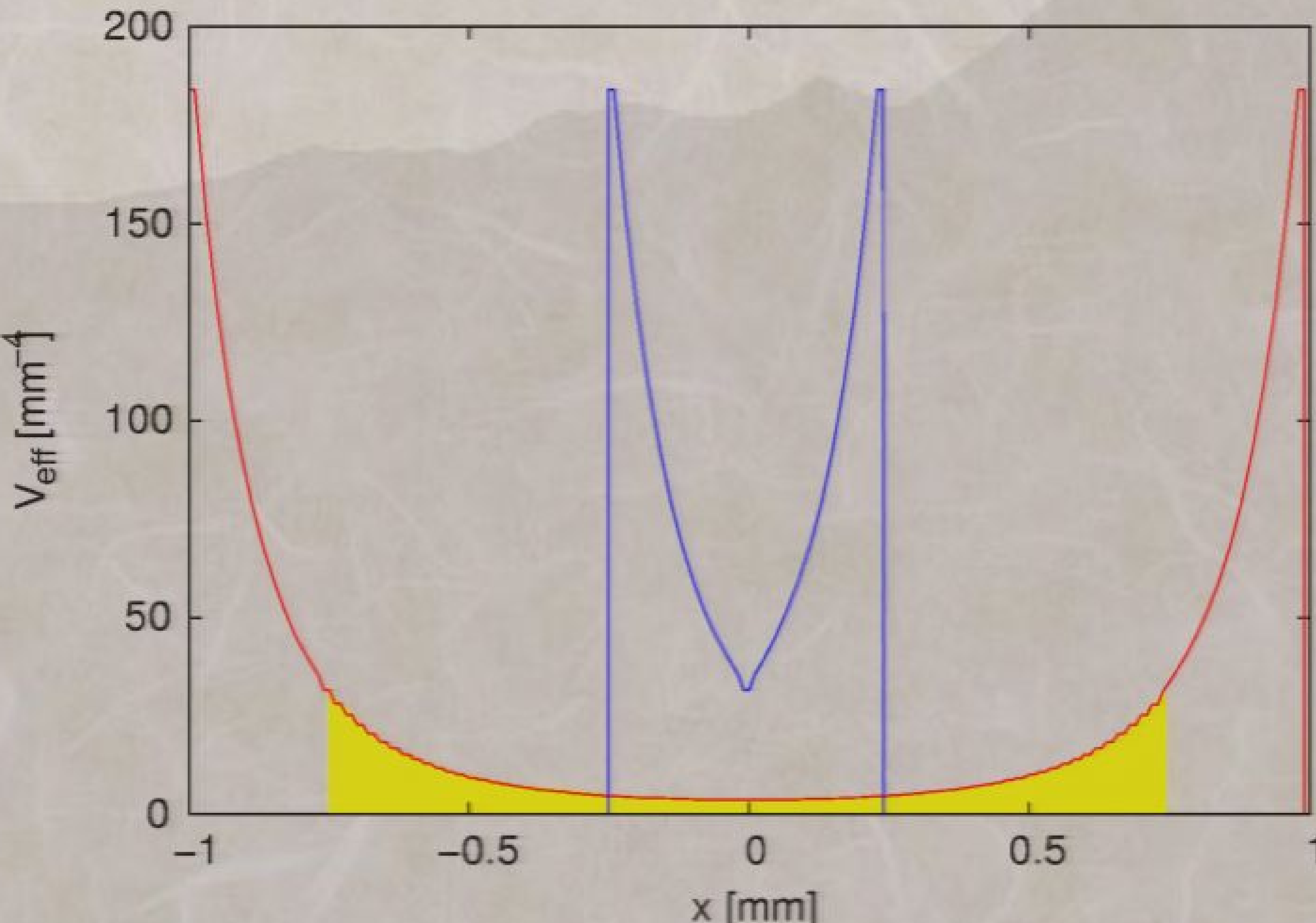


Attractive force theorem

When the two objects are moved closer together, the energy must decrease.

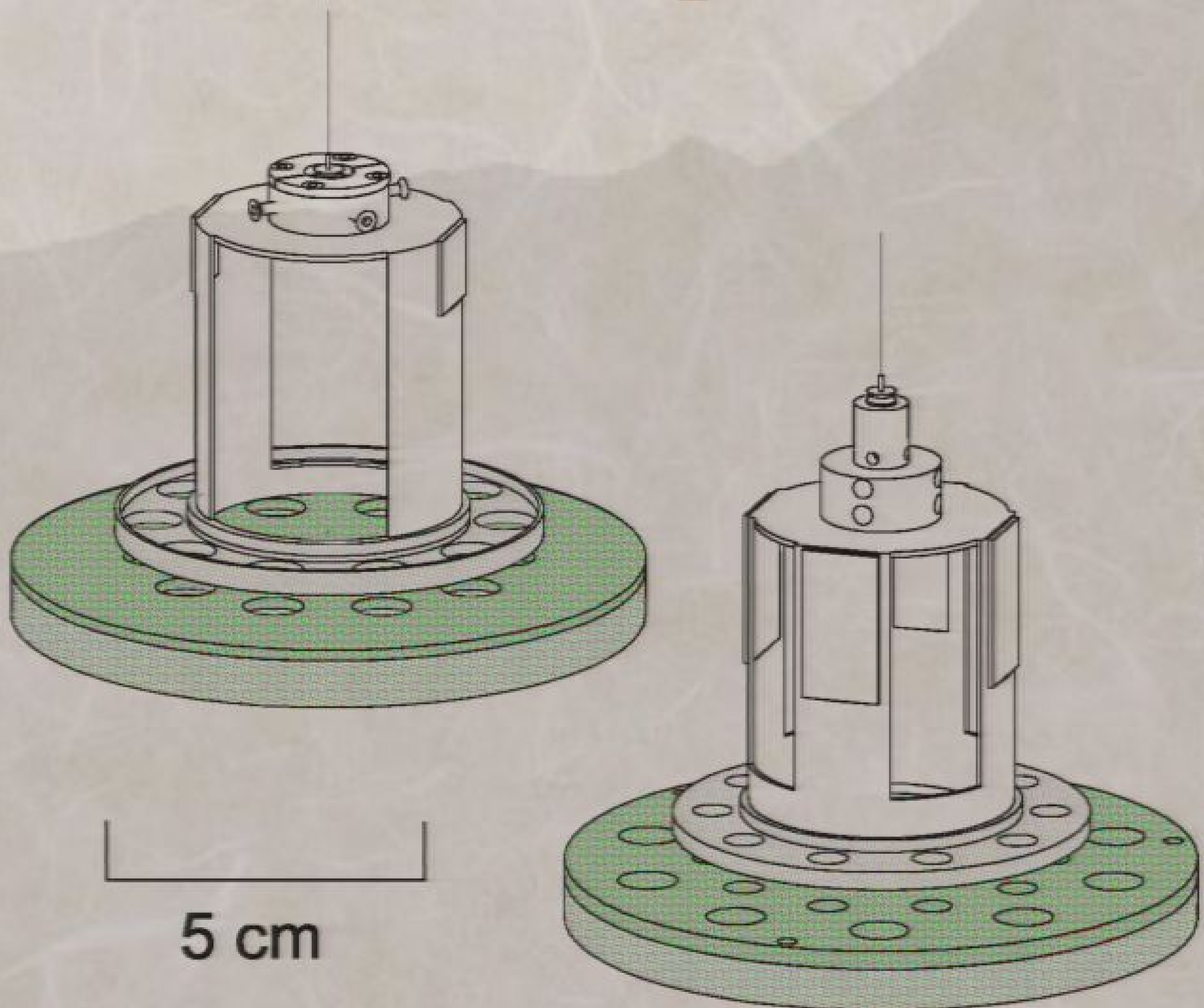
Given an effective potential bounded from below, and a mass distribution symmetric under reflection, the force between an object and its image object must be attractive.

Summary of chameleon physics

The fifth force due to the chameleon field:

- must be attractive;
- has a characteristic length scale $m_{eff,\rho}^{-1}$;
- falls off more rapidly than gravity at distances larger than this length scale;
- couples only to shells of thickness $\sim m_{eff,\rho}^{-1}$ for objects larger than this length.

Eöt-Wash experiment



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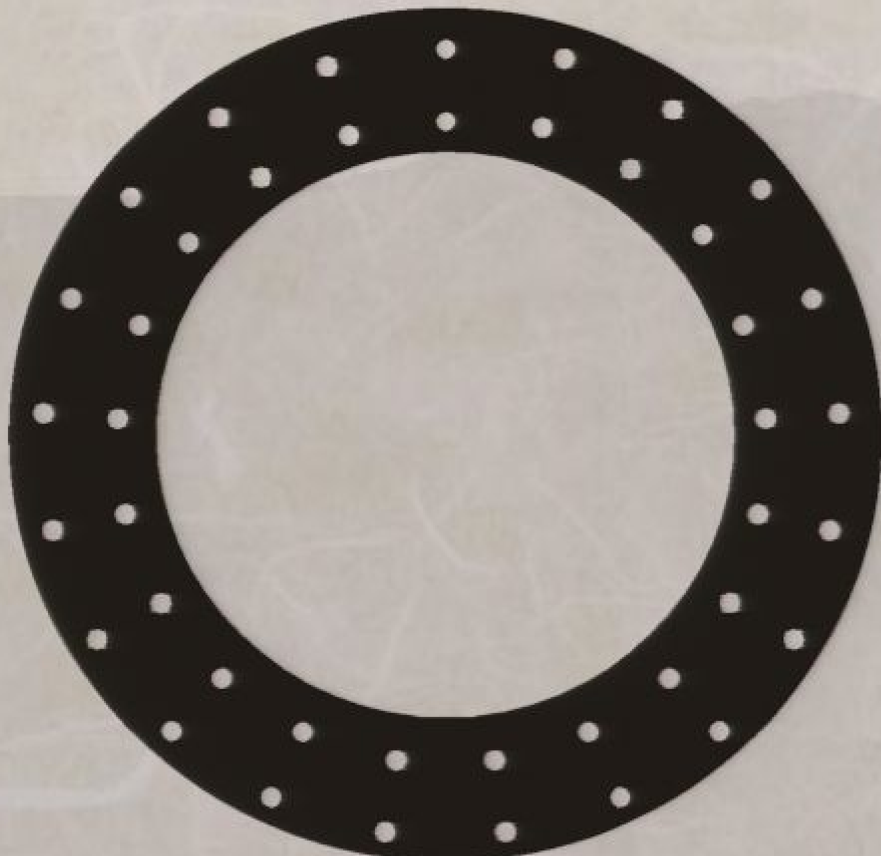
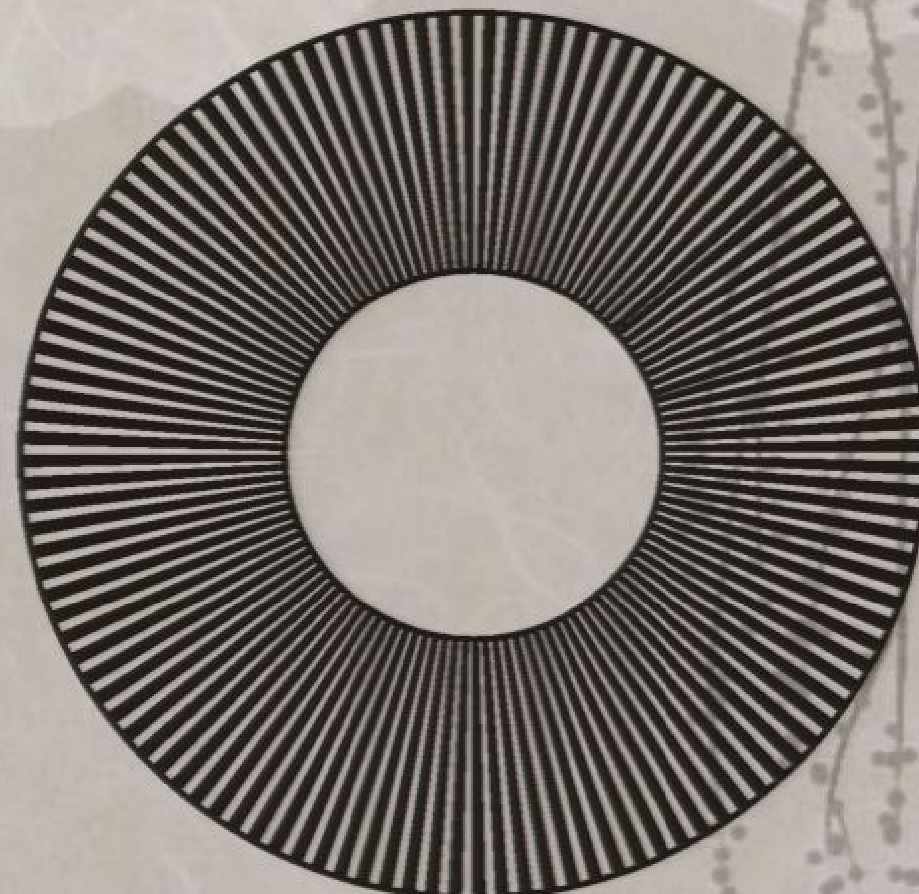


current



next-generation

Eöt-Wash experiment

The Eöt-Wash experiment is sensitive to torques as low as 0.1 fNm at distances down to 50-60 μm.

Solving the equation of motion

Discretize space:

$$\{x, y, z\} \rightarrow \{x_i, y_j, z_k\}$$

Approximate Hamiltonian on lattice:

$$H \approx \sum_i \sum_j \sum_k \left[\frac{1}{2} \left(\left(\frac{\Delta\phi}{\Delta x} \right)^2 + \left(\frac{\Delta\phi}{\Delta y} \right)^2 + \left(\frac{\Delta\phi}{\Delta z} \right)^2 \right) + \frac{1}{2} m_\phi^2 \phi^2 \ldots \right.$$
$$\left. + \frac{\lambda}{4!}\phi^4 - \beta\rho(x_i, y_j, z_k)\phi(x_i, y_j, z_k) \right] \Delta x \Delta y \Delta z.$$

where $\frac{\Delta\phi}{\Delta x} = \frac{\phi_{(i+1),j,k} - \phi_{(i-1),j,k}}{x_{i+1} - x_{i-1}}$

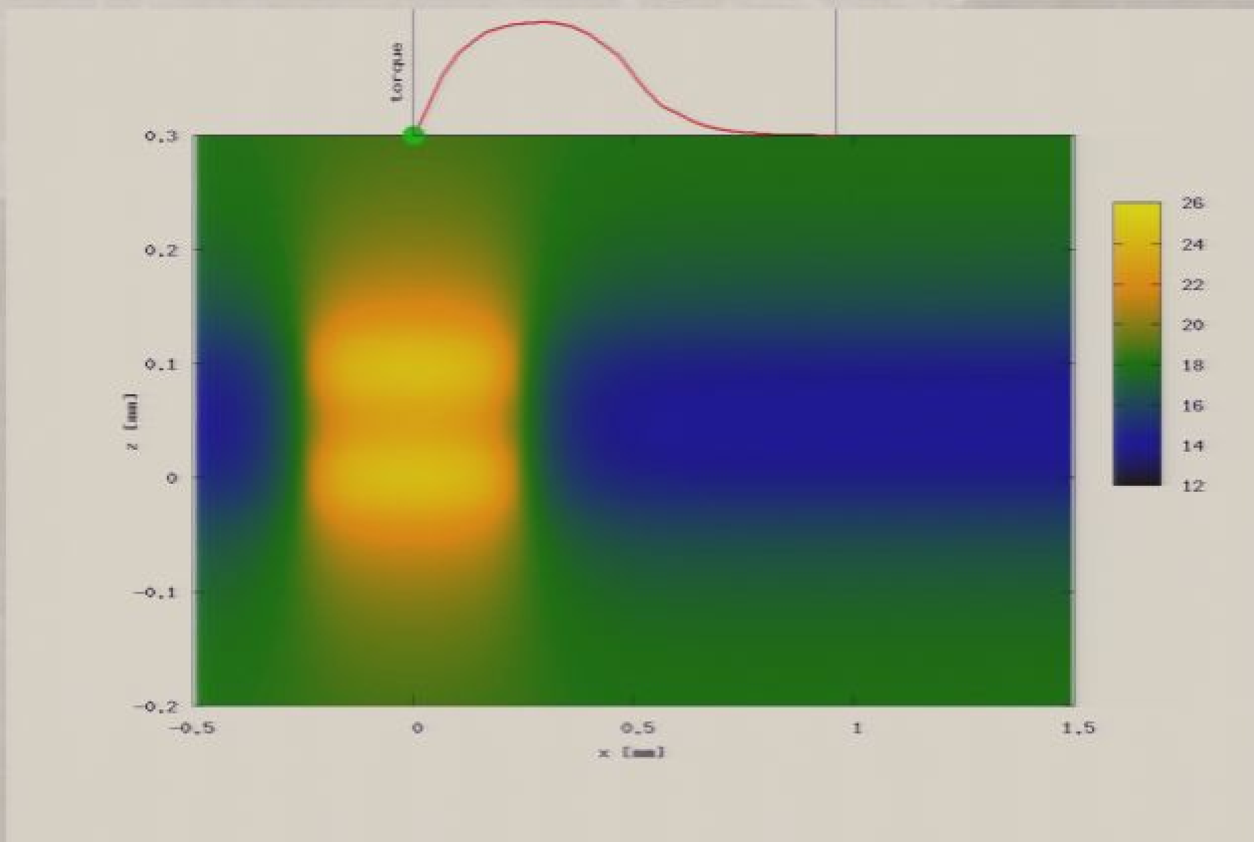
Solving the equation of motion

- Hamiltonian $H[\phi(x,y,z)] \longrightarrow H(\{\phi_{ijk}\})$
  (becomes a function of $N_x N_y N_z$ variables)

- Compute gradient of Hamiltonian
  (derivatives with respect to each ϕ_{ijk})

- Minimize H using a conjugate gradient minimization

Solving the equation of motion



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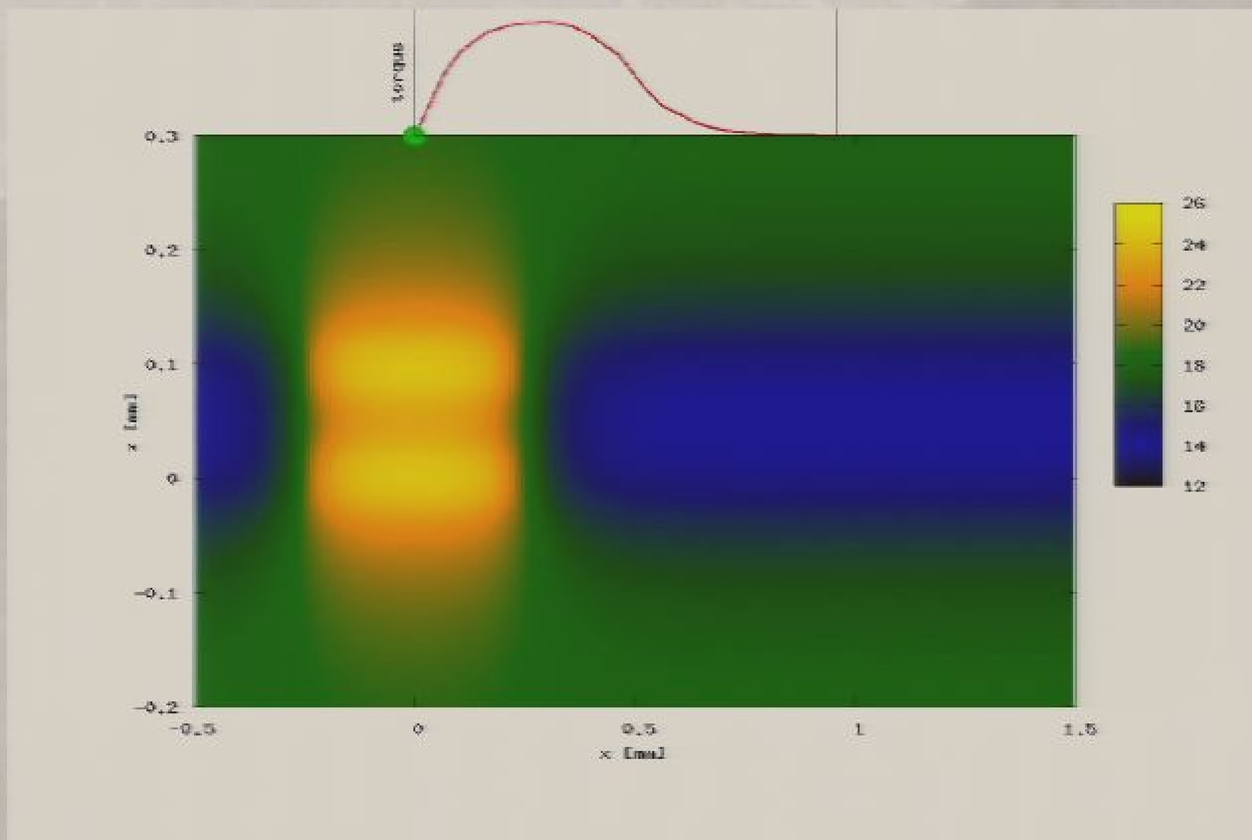

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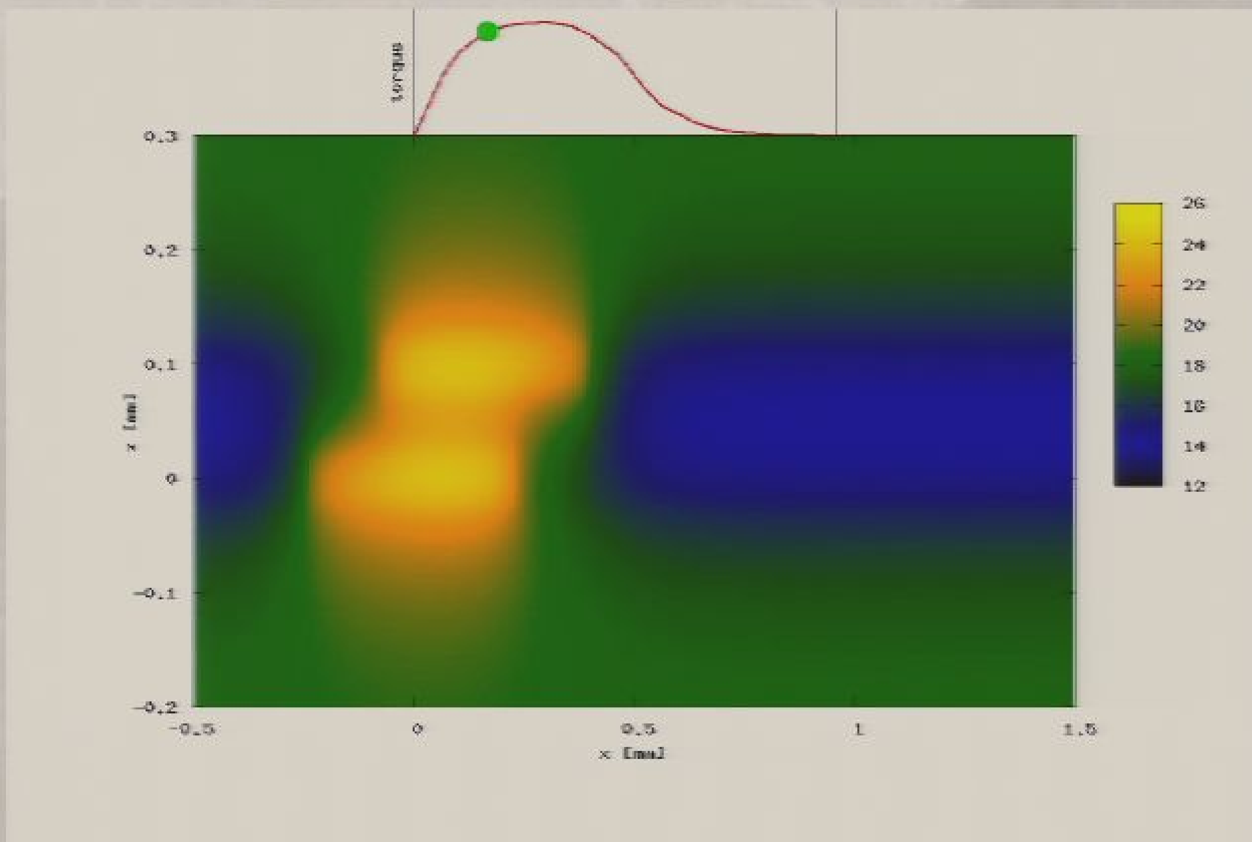

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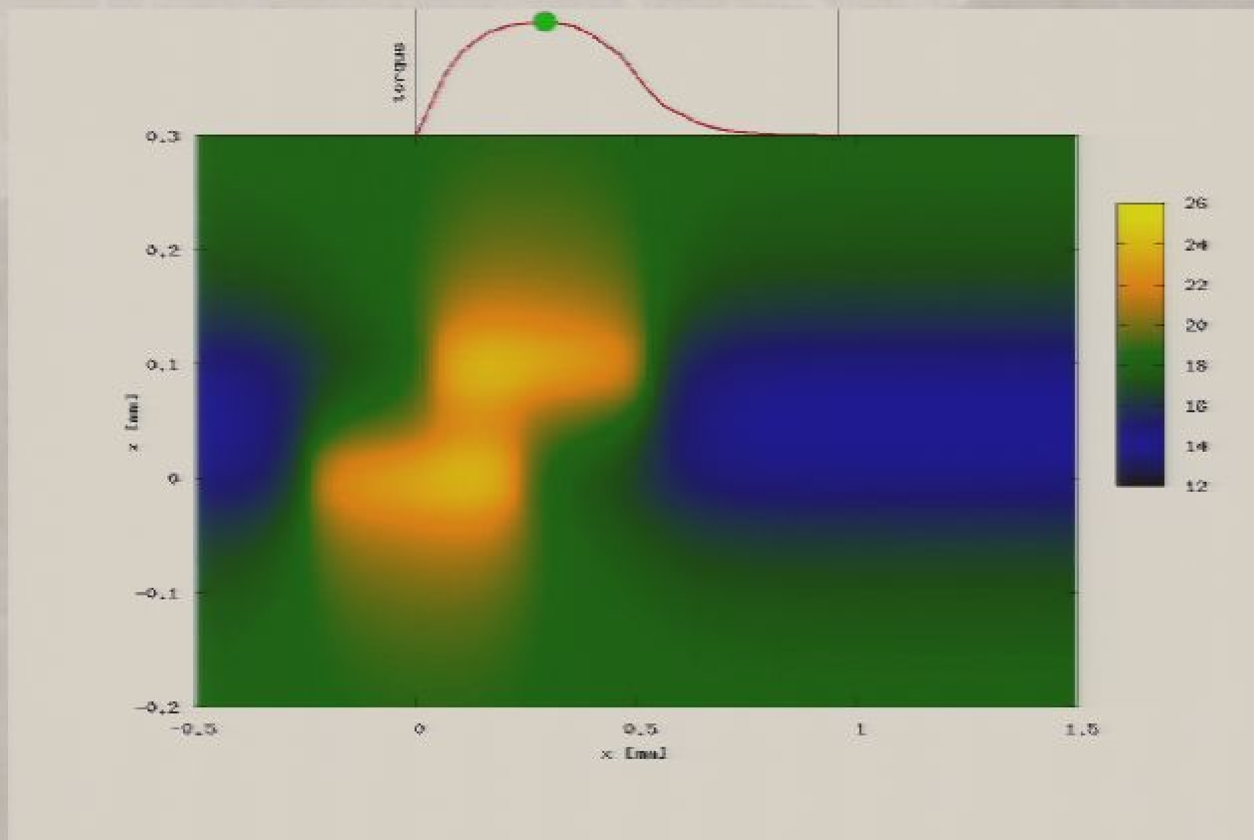

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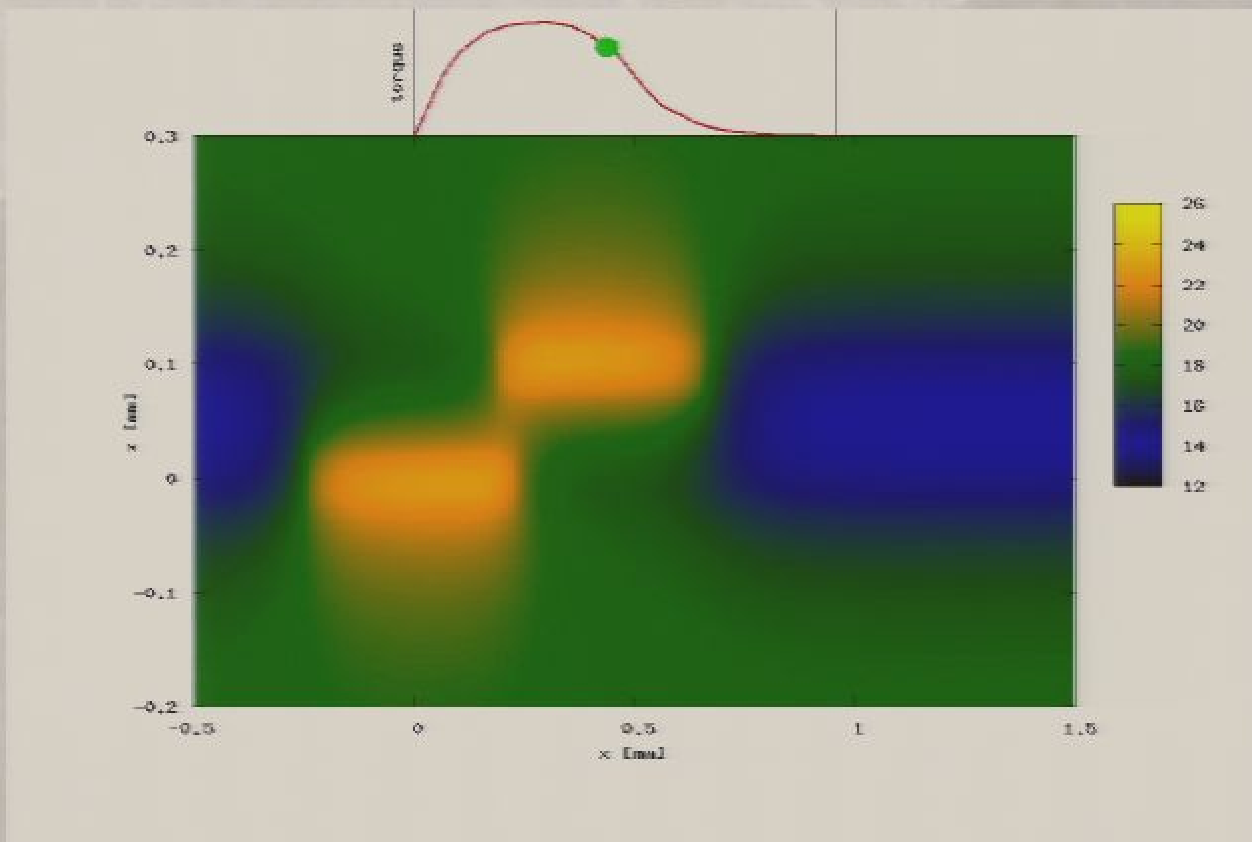

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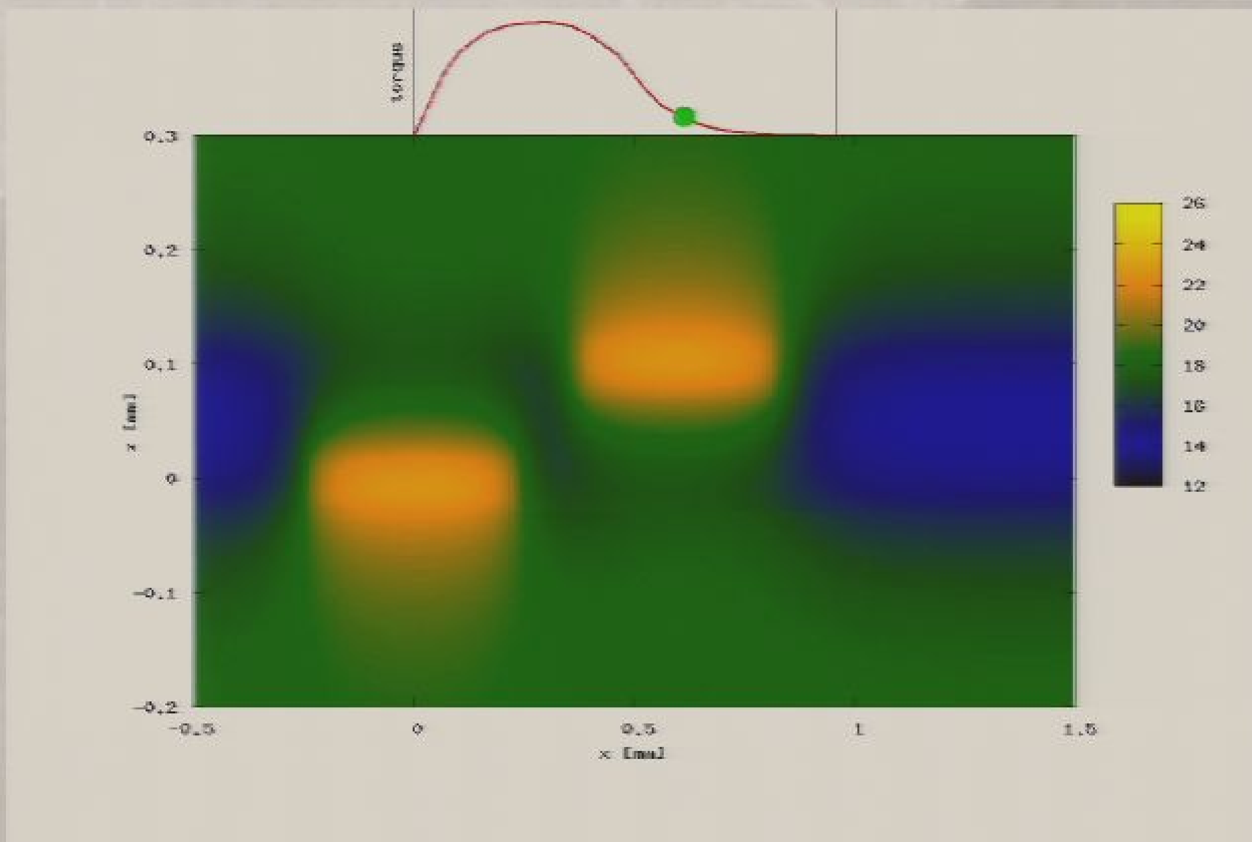

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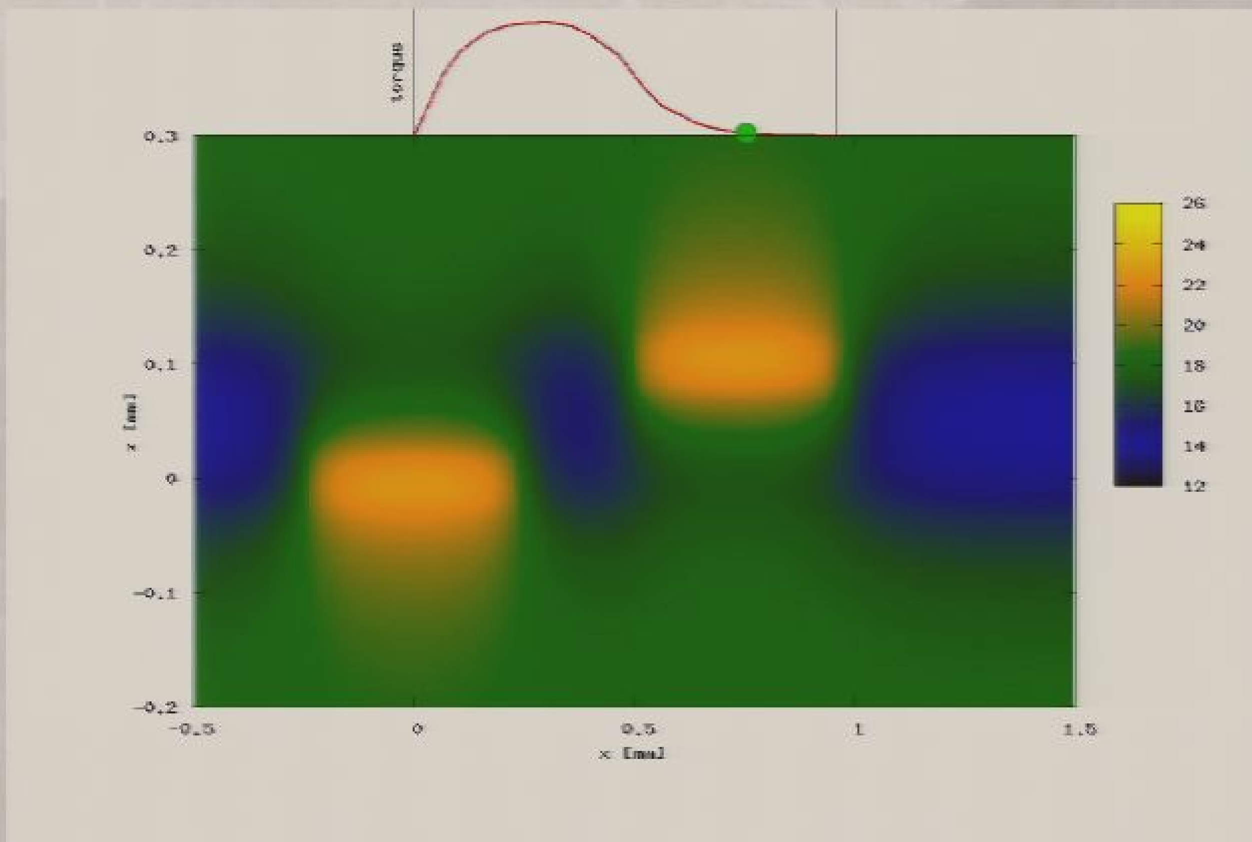

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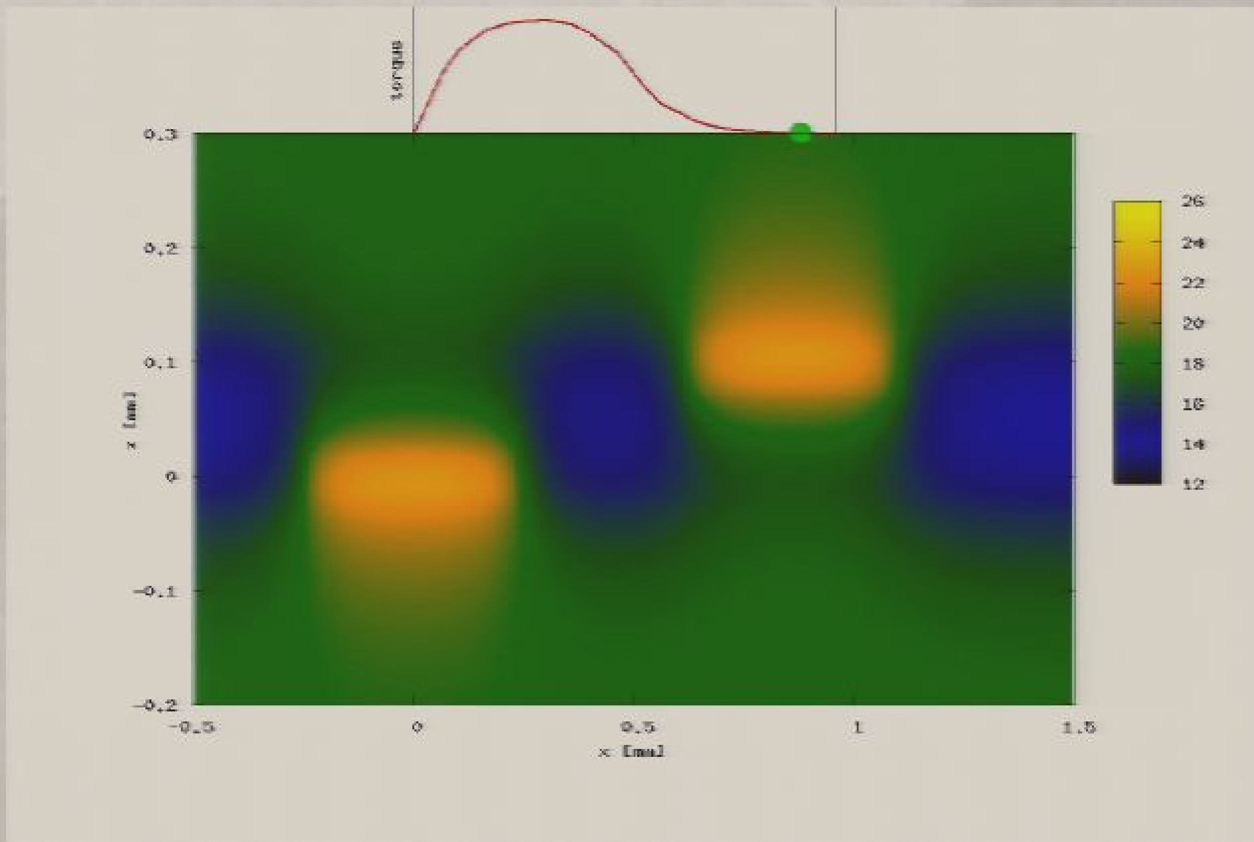

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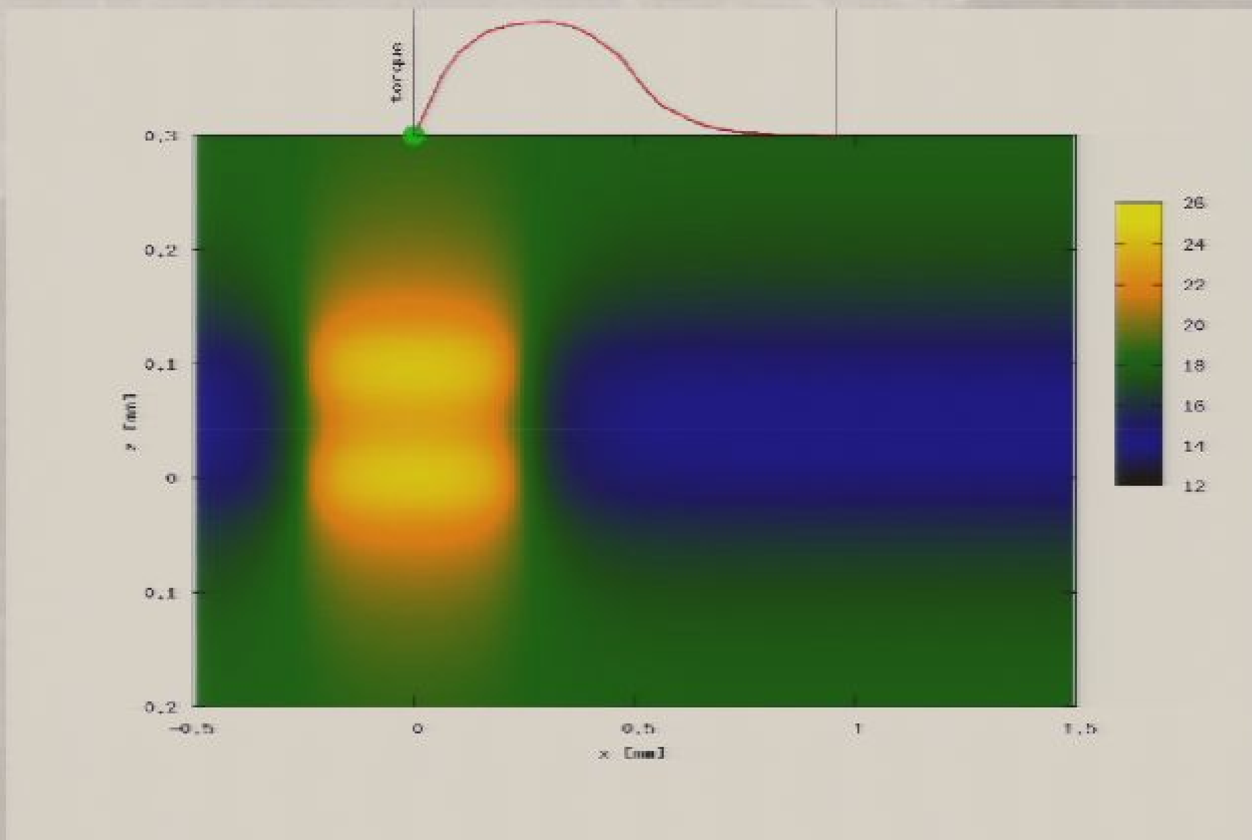

Error estimates

torque	$N = 0.38$ fNm
decrease lattice spacing	$\delta N = 3.9 \times 10^{-3}$ fNm
force from nearby wedges	$\delta N = 1.6 \times 10^{-3}$ fNm
wedges vs. rectangular ridges	$\delta N = 4.3 \times 10^{-4}$ fNm
two attractor wedges (nonlinear)	$\delta N = 2.1 \times 10^{-4}$ fNm
spurious torque at $\theta = 0$	$\delta N = 7 \times 10^{-6}$ fNm
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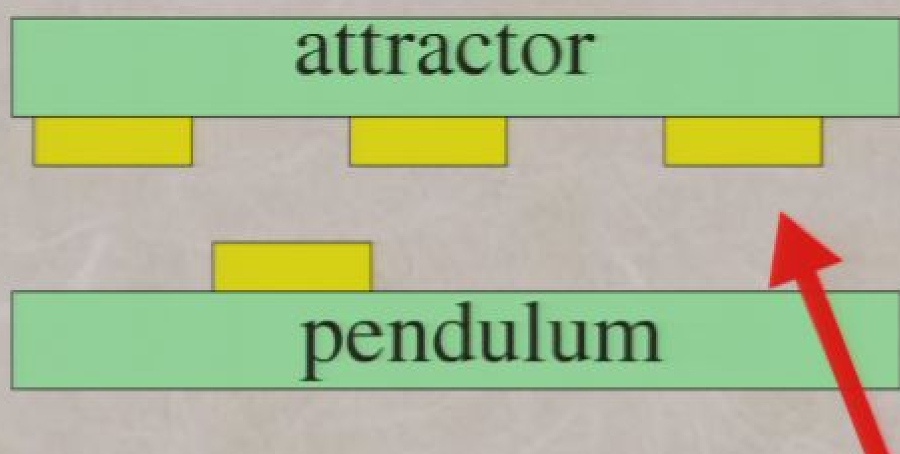

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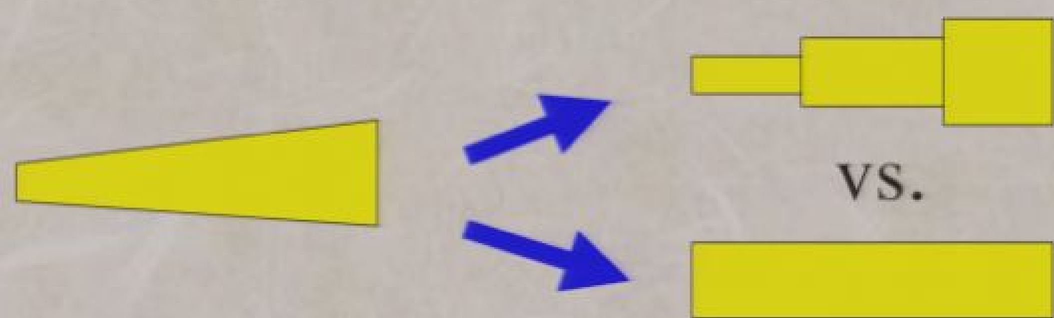

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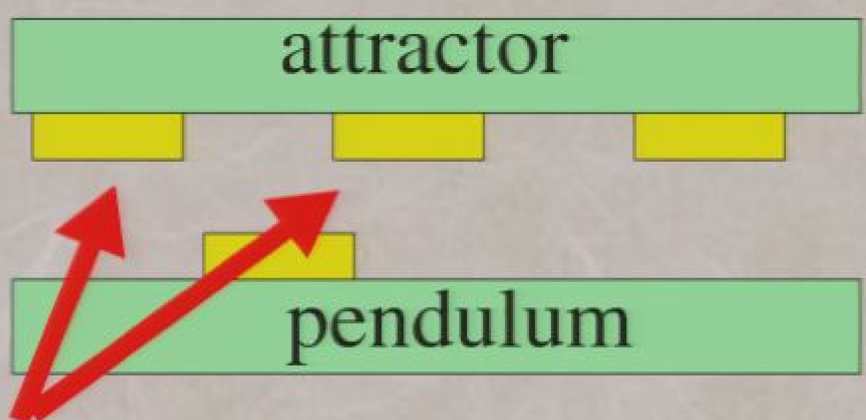

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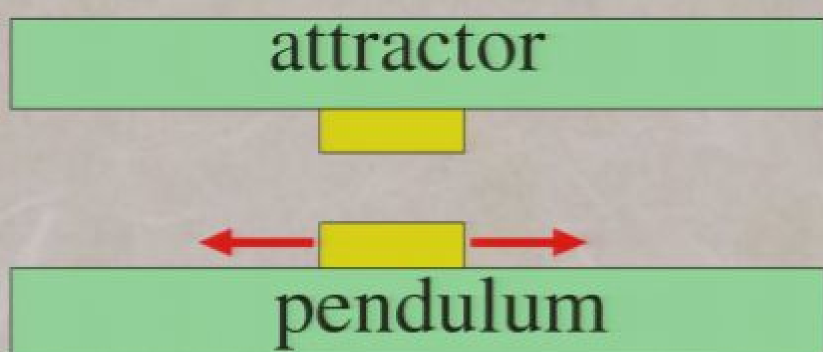

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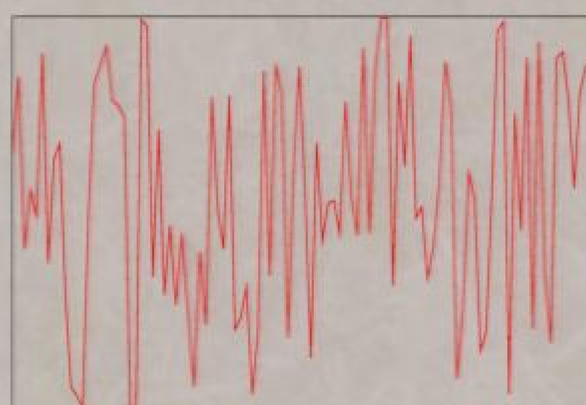
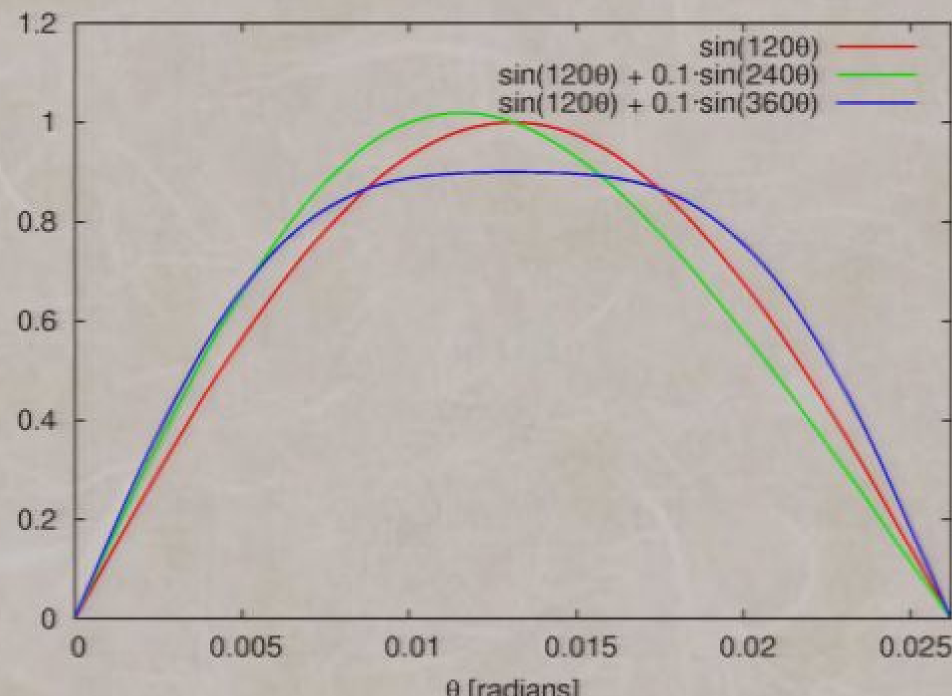


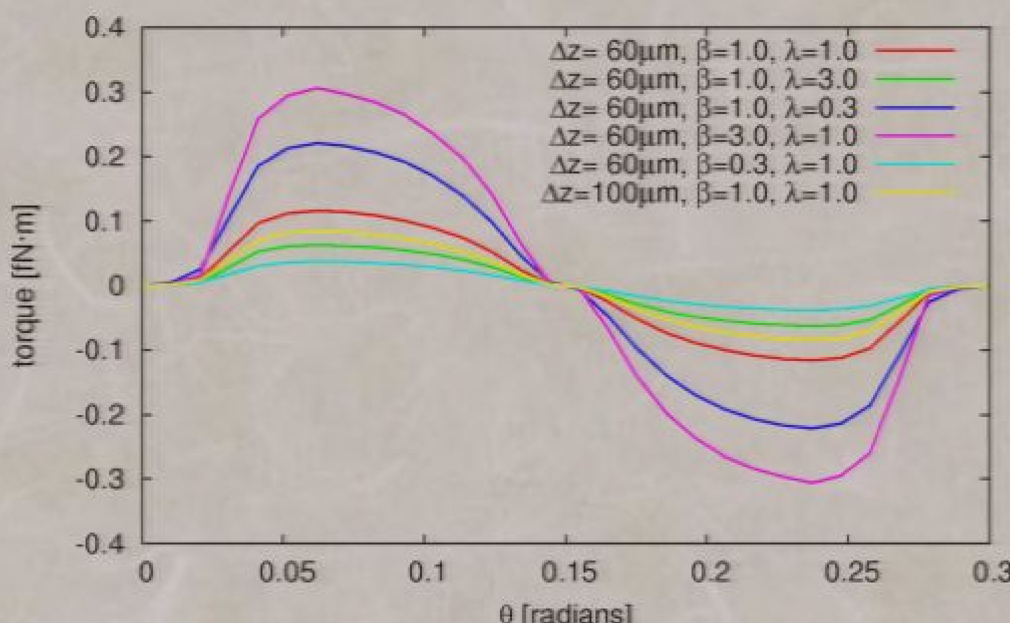
Results

Eöt-Wash will report the first three nonzero Fourier coefficients in the expansion of the torque, $N(\theta) = \sum N_n \sin(n\theta)$.



Results

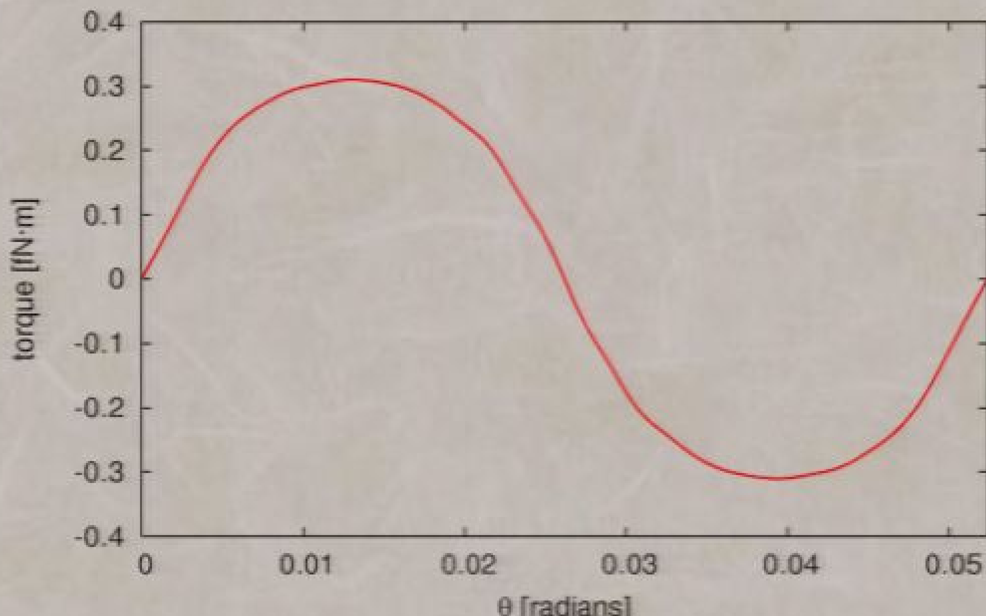
The current (42 hole) Eöt-Wash experiment will not be able to detect a chameleon-mediated fifth force with unit couplings.



$$N_{21} = 0.11\beta^{0.91}\lambda^{-0.55}$$ at $\Delta z = 60\mu m$

Results

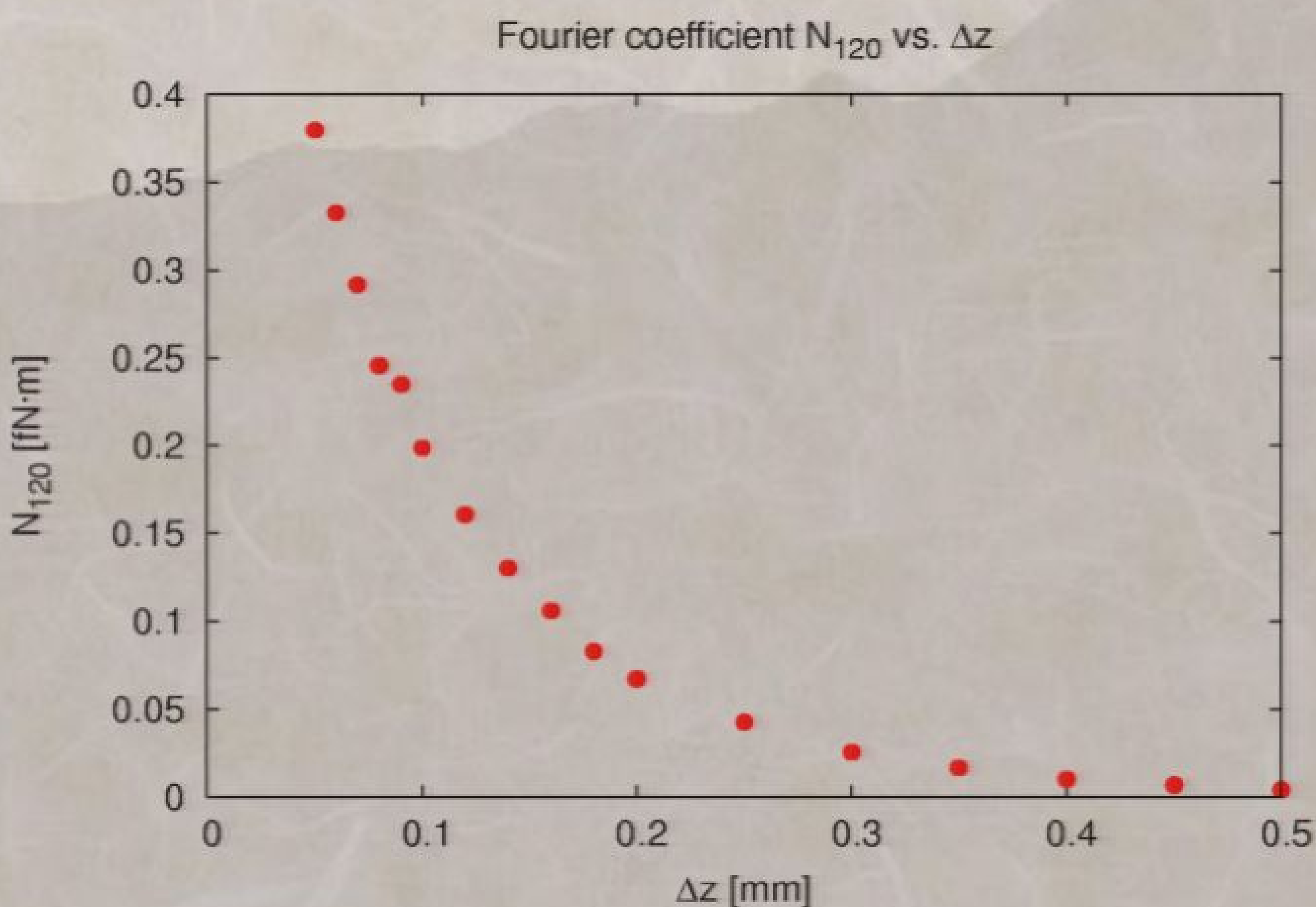
The next-generation (120 wedge) Eöt-Wash experiment should see the chameleon.



$$N_{120} = 0.38\beta^{1.34}\lambda^{-0.33}$$

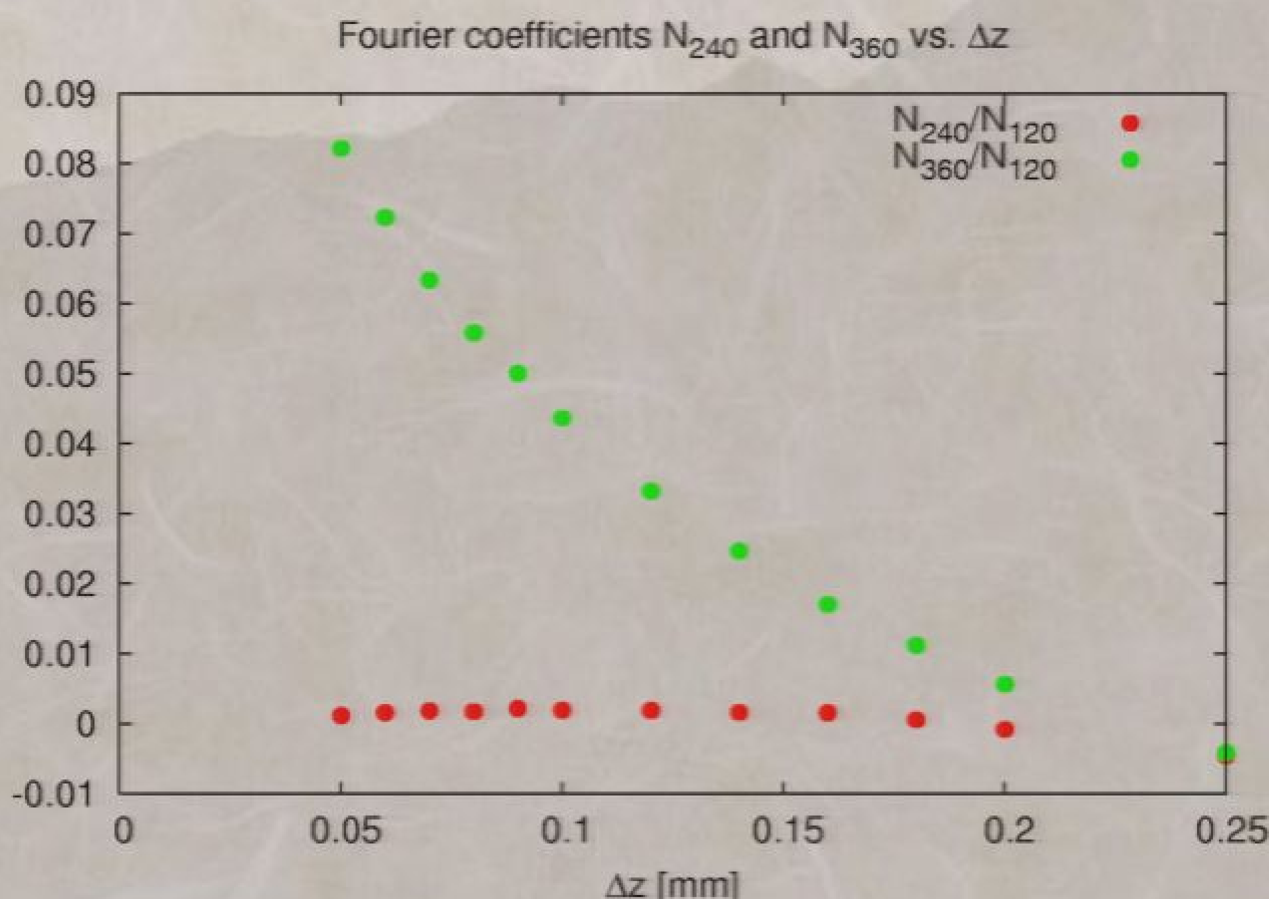
$$N_{360} = 0.031\beta^{1.68}\lambda^{-0.16}$$ at $\Delta z = 50\mu m$

Results



Fourier coefficient N_{120} vs. Δz
N_{120} [fN·m]
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
Δz [mm]

Results



Fourier coefficients N240 and N360 vs. Δz
N240/N120
N360/N120
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
0.05
0.1
0.15
0.2
0.25
Δz [mm]

Conclusions

- A chameleon scalar field, with a ϕ^4 self interaction, will give rise to an attractive force which falls off rapidly with distance and couples only to thin shells of matter.
- The current Eöt-Wash experiment is beginning to constrain the most interesting region of parameter space.
- The next-generation Eöt-Wash experiment will be able to detect a chameleon-mediated fifth force with unit-strength couplings.

Error estimates

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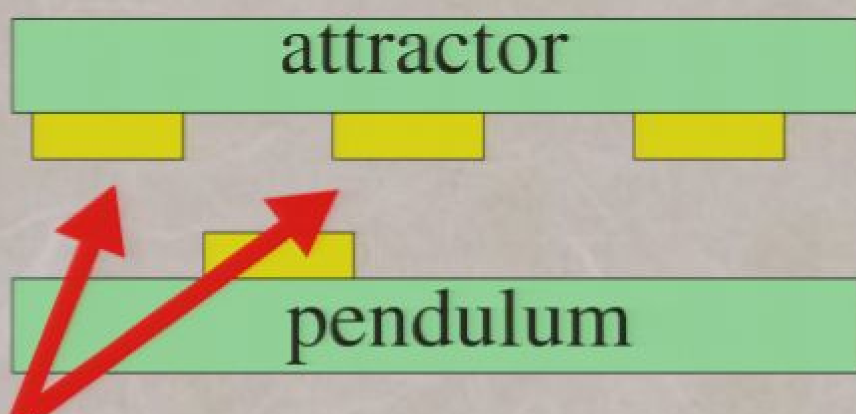
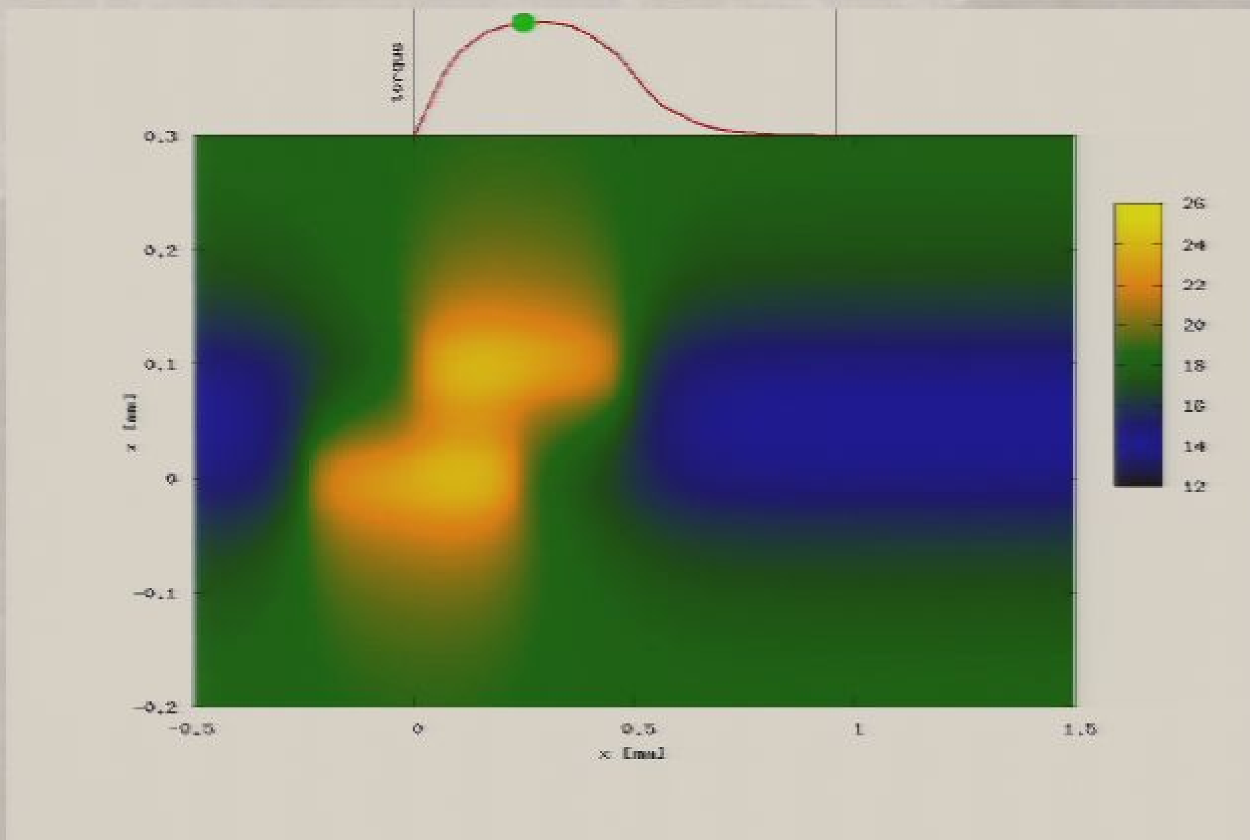


Solving the equation of motion



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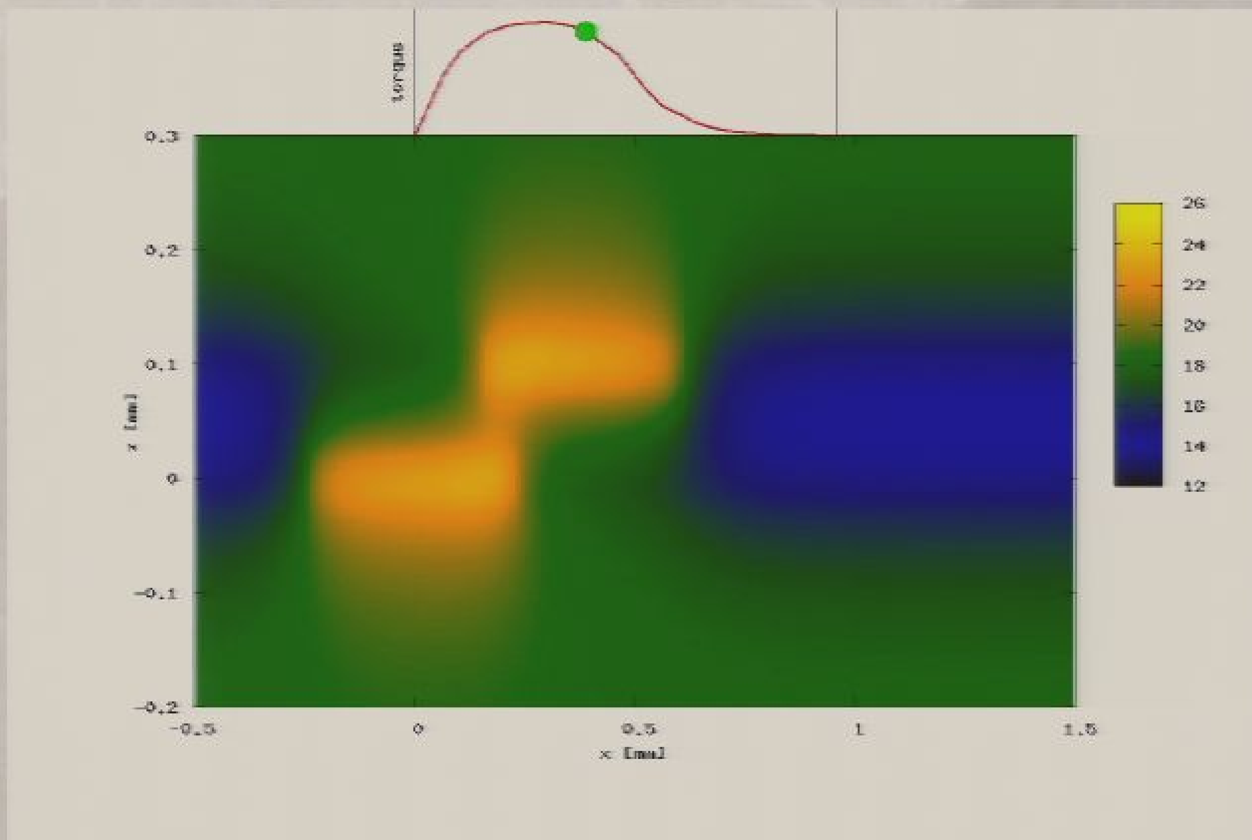

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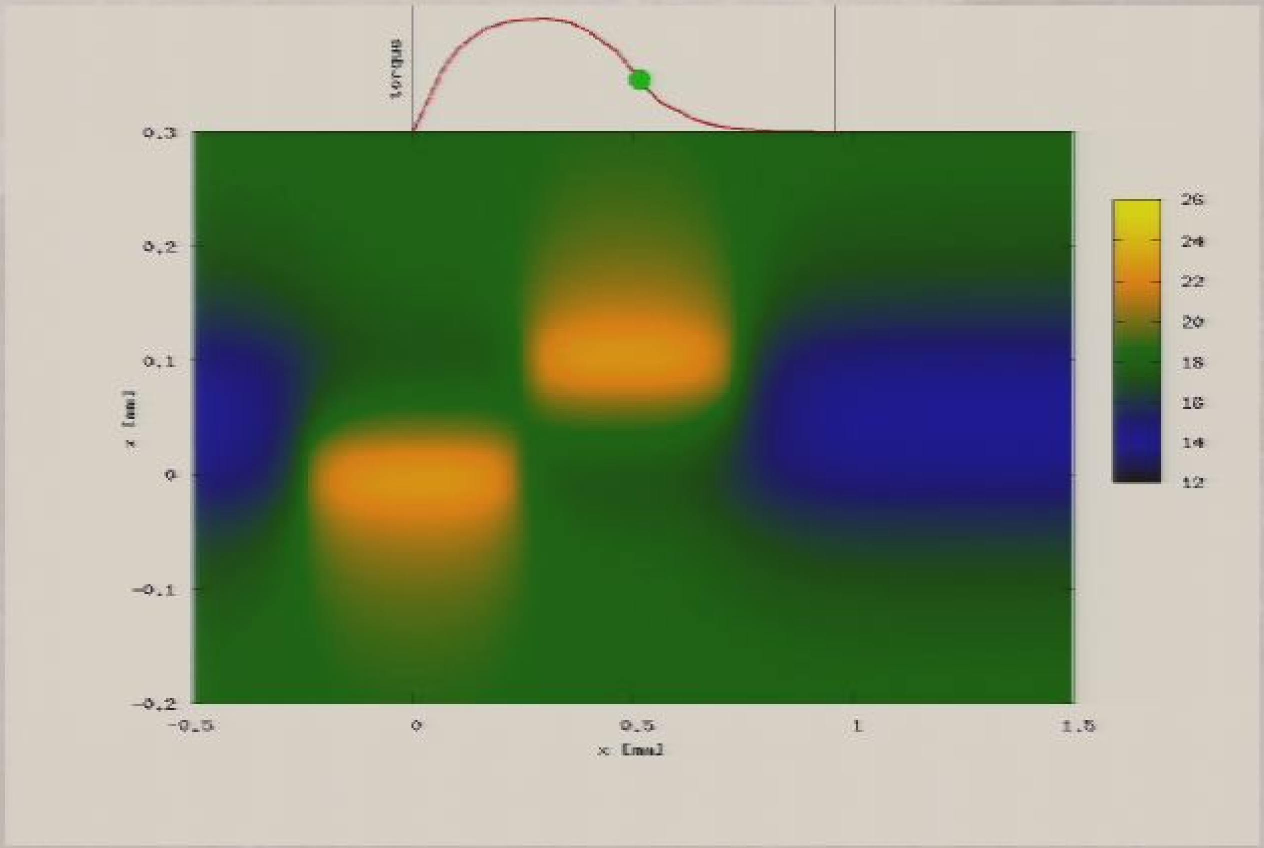

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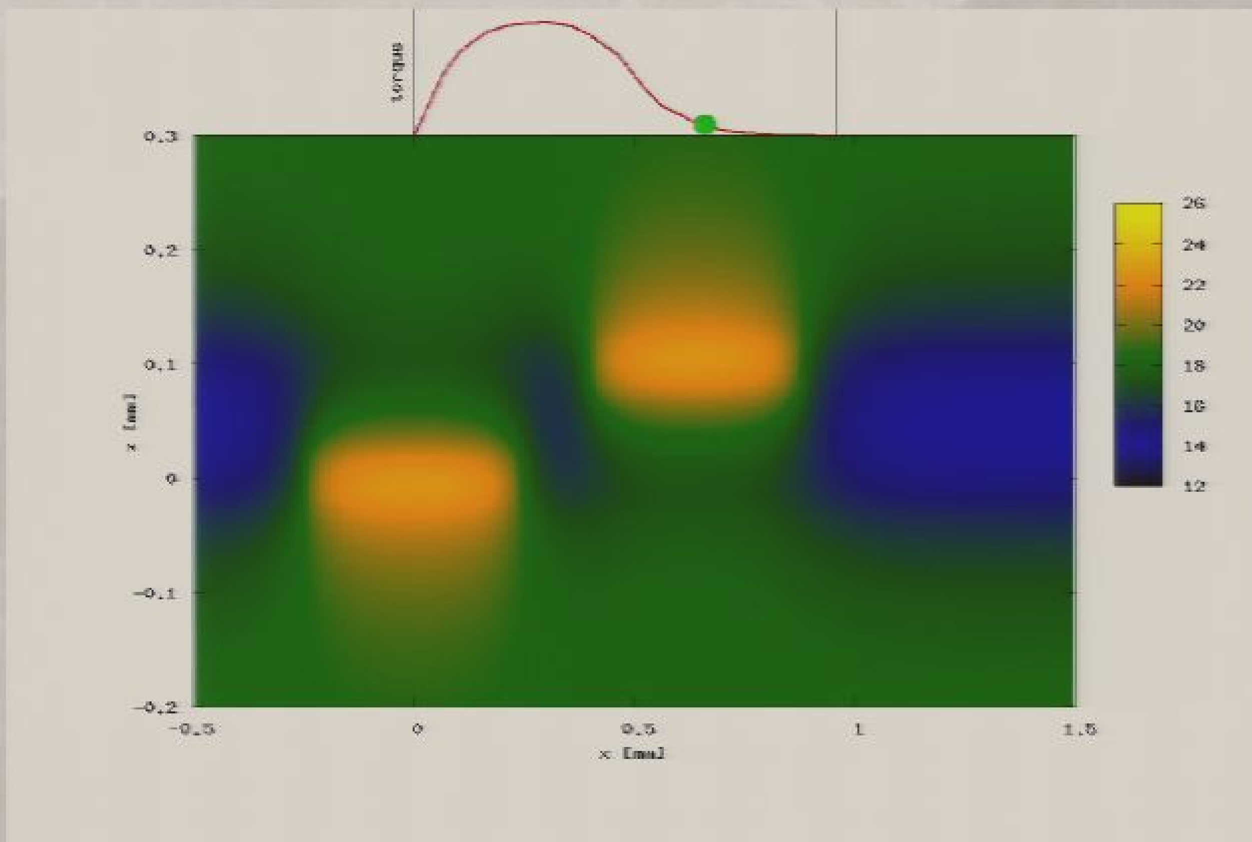

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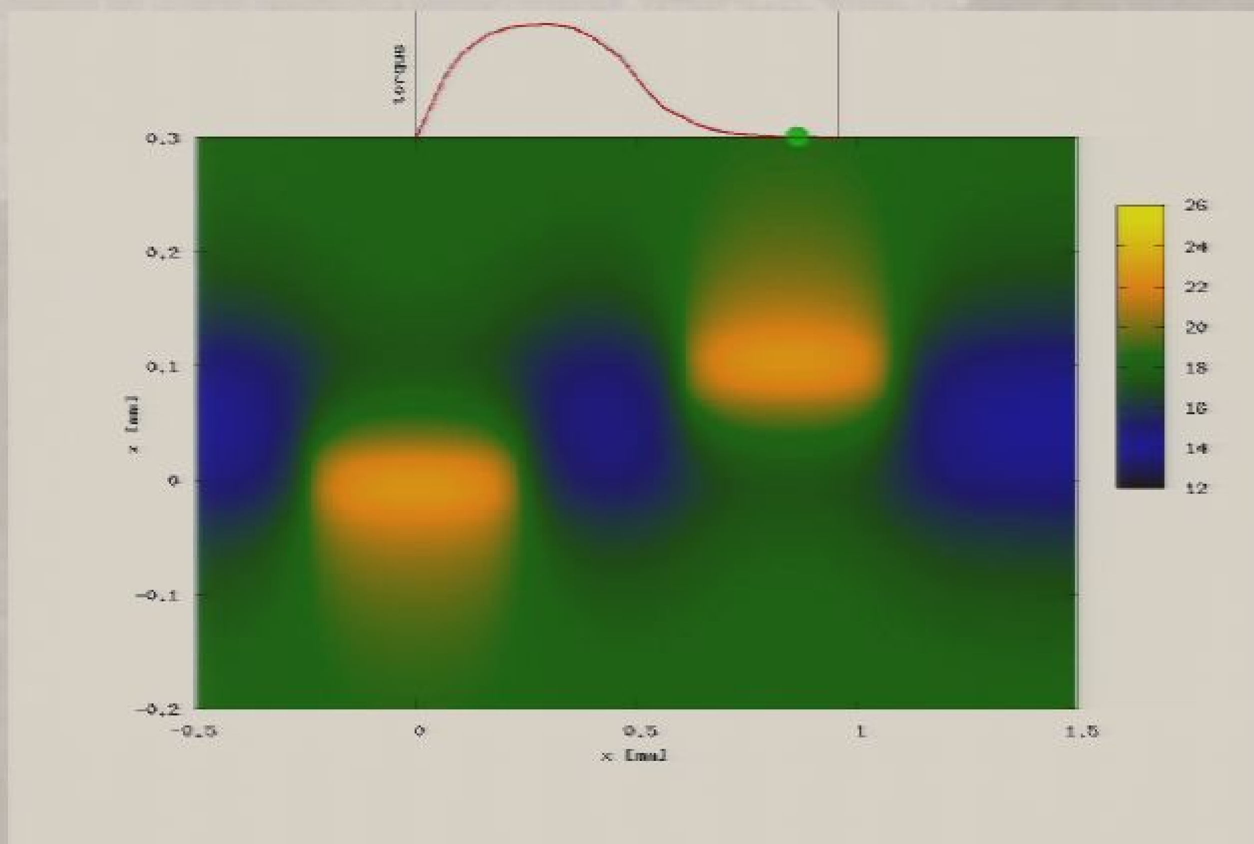

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