Title: Field Theory 5

Date: Jun 10, 2006 03:20 PM

URL: http://pirsa.org/06060045

Abstract:

Maxwell-Chern-Simons Q-balls

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hep-th/0604036
to be accepted, Phys Rev D

Outline

1. Basic (ungauged) Q-balls
2. Gauged Q-balls
3. M-C-S Q-balls

   (intrinsically 2+1 d)

Motivation

Why study MCS Q-balls?

Chern-Simons term is:

1) interesting mathematically
 - topological field theory
 - knot theory

2) interesting physically
 - massive gauge field
 - fractional statistics
 - FQHE
 - topological quantum computing??

So it's worth exploring anything and everything in CS/MCS theory.

1. Basic Q-balls

[S. Coleman Nucl Phys B262 (1985) 263]

Scalar field theory with conserved global charge

Q: what is the lowest-energy config. of a given charge? (particle number)

Naive : "Q" widely-separated particles ~~of~~ at rest

$$E = Q \cdot M$$

If there are attractive forces:

possibility of bound states

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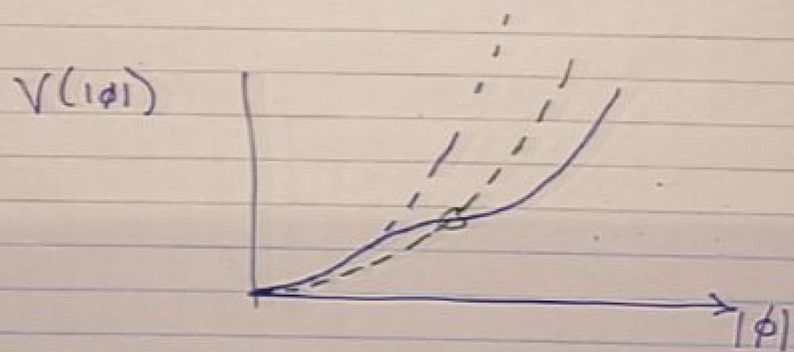
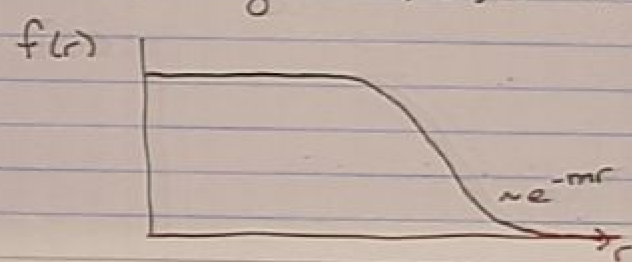


Q-ball ansatz: $\phi(\vec{r},t) = f(r)\, e^{-i\omega t}$



Can show: • $\omega \uparrow \Rightarrow Q \downarrow$

• $\exists$ config. with any charge Q of energy $< QM$

• Large Q: $f \simeq$ constant inside, $E \propto Q$
(Edelman) & indep. of Q)

• Small Q: Kusenko Phys Lett 404B (1997) 285

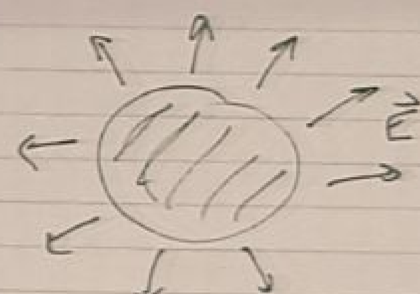
• 2+1d: no significant difference

2. Gauged Q-balls.

Can we find Q-balls if the conserved charge is gauged.

Yes. (3+1d) K. Lee et al. Phys Rev **D39** 1665 (1989)

Gauge field enters energetics



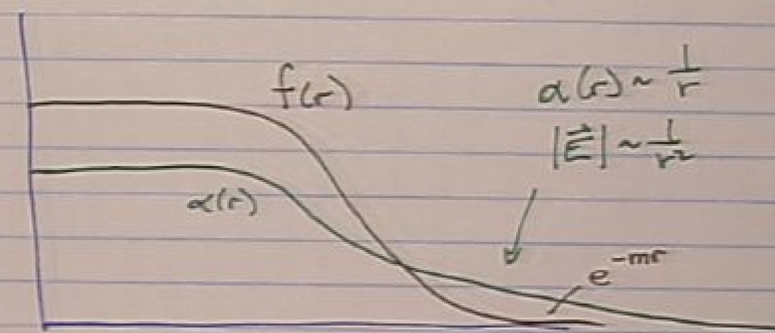
electric field energy: harder to compete with ordinary matter.

$\Rightarrow$ maximal charge.

Ansatz: $\phi(\vec{r},t) = f(r)\, e^{-i\omega t}$

$A^0(\vec{r},t) = \alpha(r)$

$A^i(\vec{r},t) = 0$



- "mirror image" of superconducting vortex : $|\phi| \neq 0$ inside, $\to 0$ outside

Q-ball superconducting inside Q-ball

2+1d: $\vec{E} \sim \frac{1}{r} \Rightarrow \int d^2r\, \vec{E}^2$

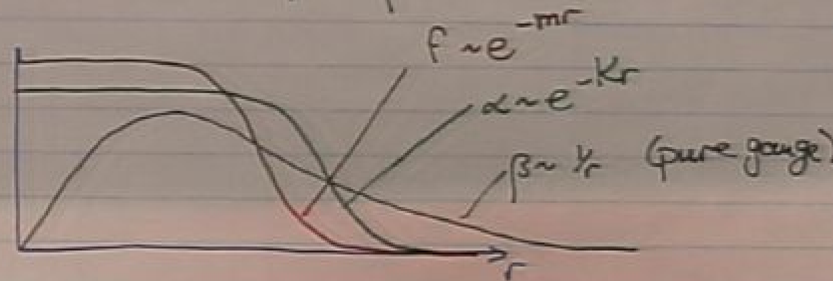
diverges! Can't compete w/ ordinary matter

Ansatz: with CS term, static charge density $\Rightarrow \vec{E}$ and $\vec{B}$.

$$\phi(\vec{r},t) = f(r)\,e^{-i\omega t}$$

$$A^0(\vec{r},t) = \alpha(r)$$

$$A^i(\vec{r},t) = \frac{\epsilon^{ij} r_j}{r}\,\beta(r)$$



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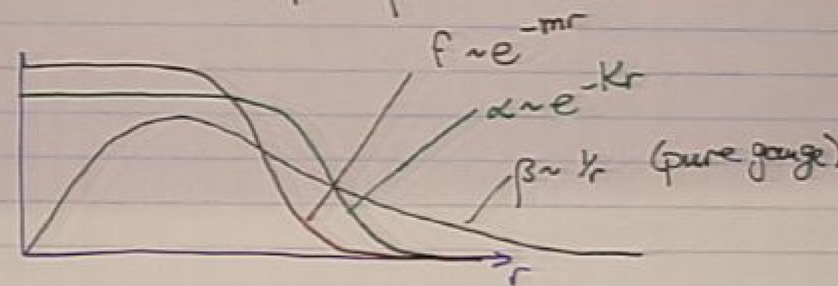

Numerical investigation of Q-balls in MCS theory.

2 relevant parameters: e, K.

Expect: • as $e \uparrow$, electromagnetic field energy $\uparrow$; Q balls get more and more difficult.

• as $K \uparrow$, photon mass $\uparrow$; photon decays faster; electromagnetic energy $\downarrow$; Q balls get easier.

Look for Q-balls using <u>relaxation</u>.

- initial (discretized) guess
- estimate by how much this guess is <u>not</u> a solution
- compute better guess; repeat until desired accuracy achieved.

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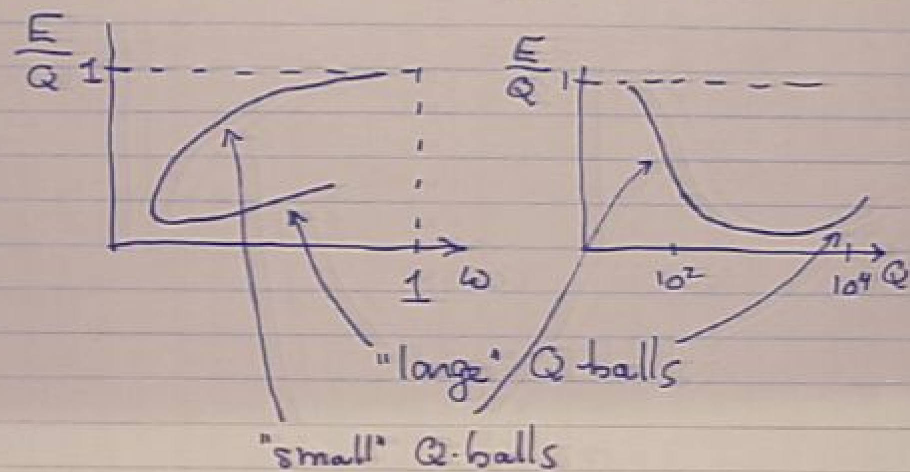
Solutions can be found for a wide variety of parameter values.

For most ω: $\underline{\underline{2}}$ Q-balls.



As parameters vary: behaviour as expected:

$e \uparrow$ more difficult

$K \uparrow$ easier

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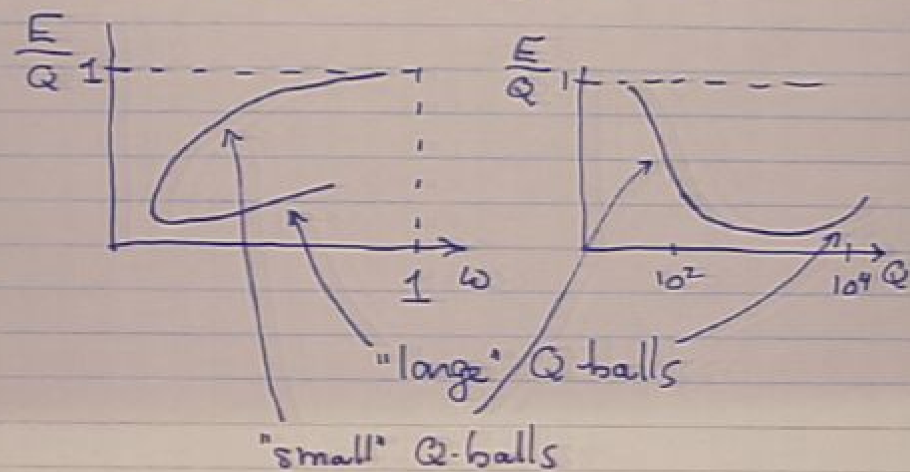

As parameters vary: behaviour as expected:

$e \uparrow$	more difficult
$K \uparrow$	easier

(11)

Conclusions

- Ungauged Q-balls exist in any dimension
- Gauged Q-balls exist in 3d but not 2d
- In 2d, Chern-Simons term renders finite the (otherwise-divergent) electric field energy
- Found solutions in MCS theory for a wide variety of parameters.