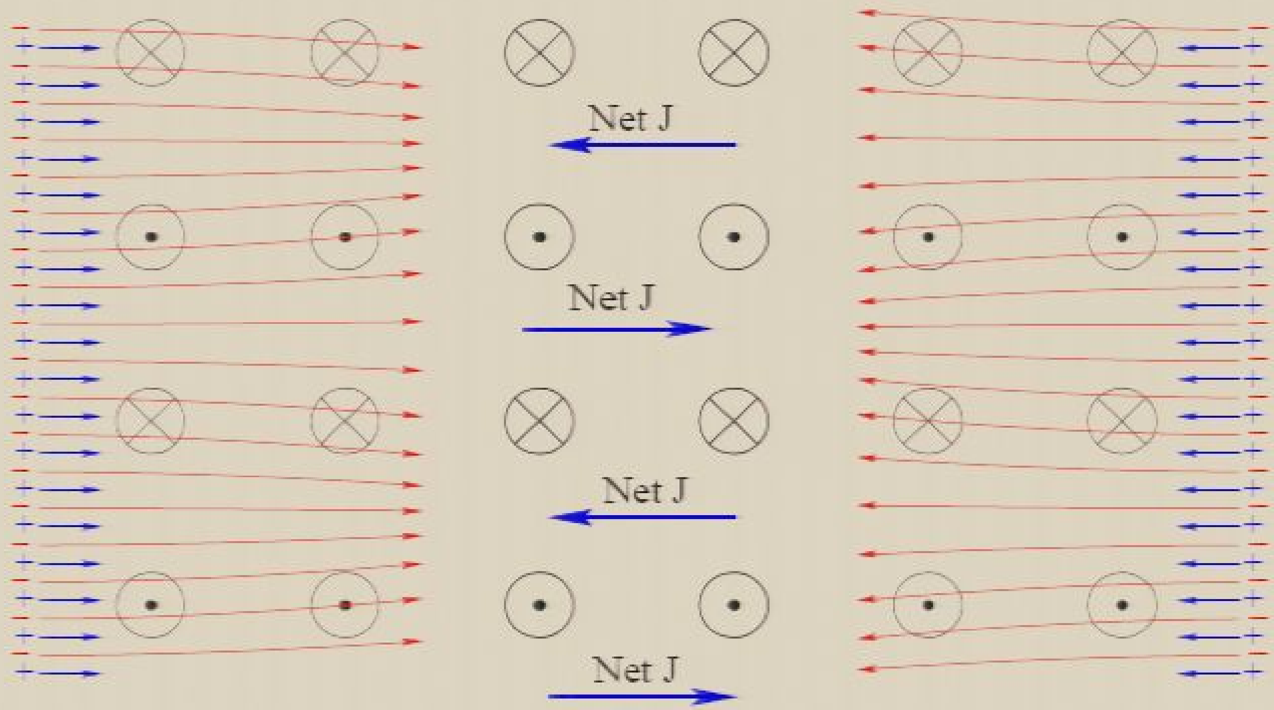
Title: Nuclear Theory/Heavy Ions 5

Date: Jun 10, 2006 11:10 AM

URL: http://pirsa.org/06060038

Abstract:

Negative charges:



Induced B *adds* to seed B. Exponential Weibel instability

Linearized analysis: B grows until bending angles become large.

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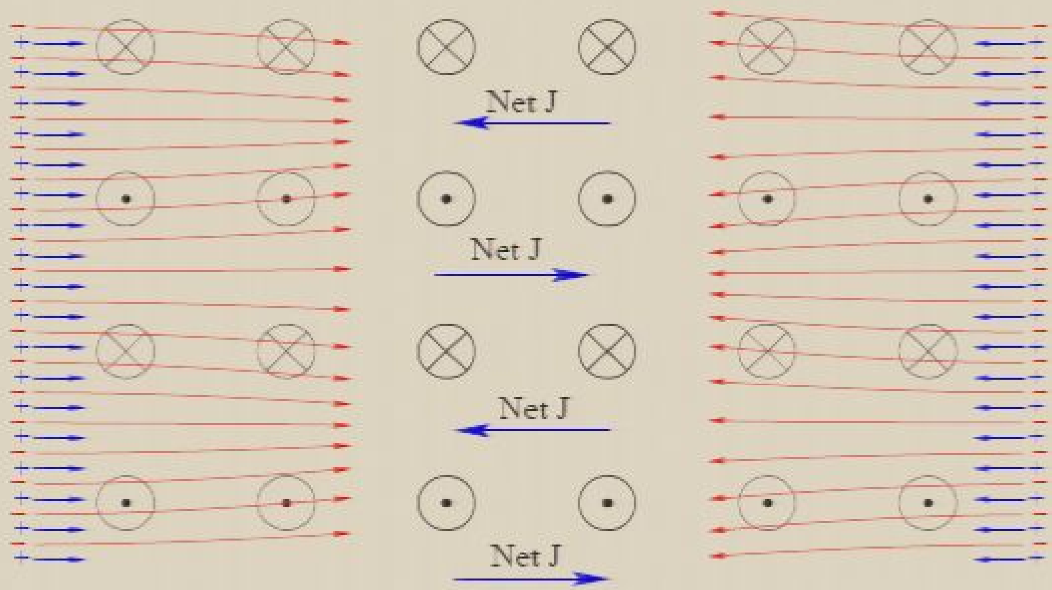

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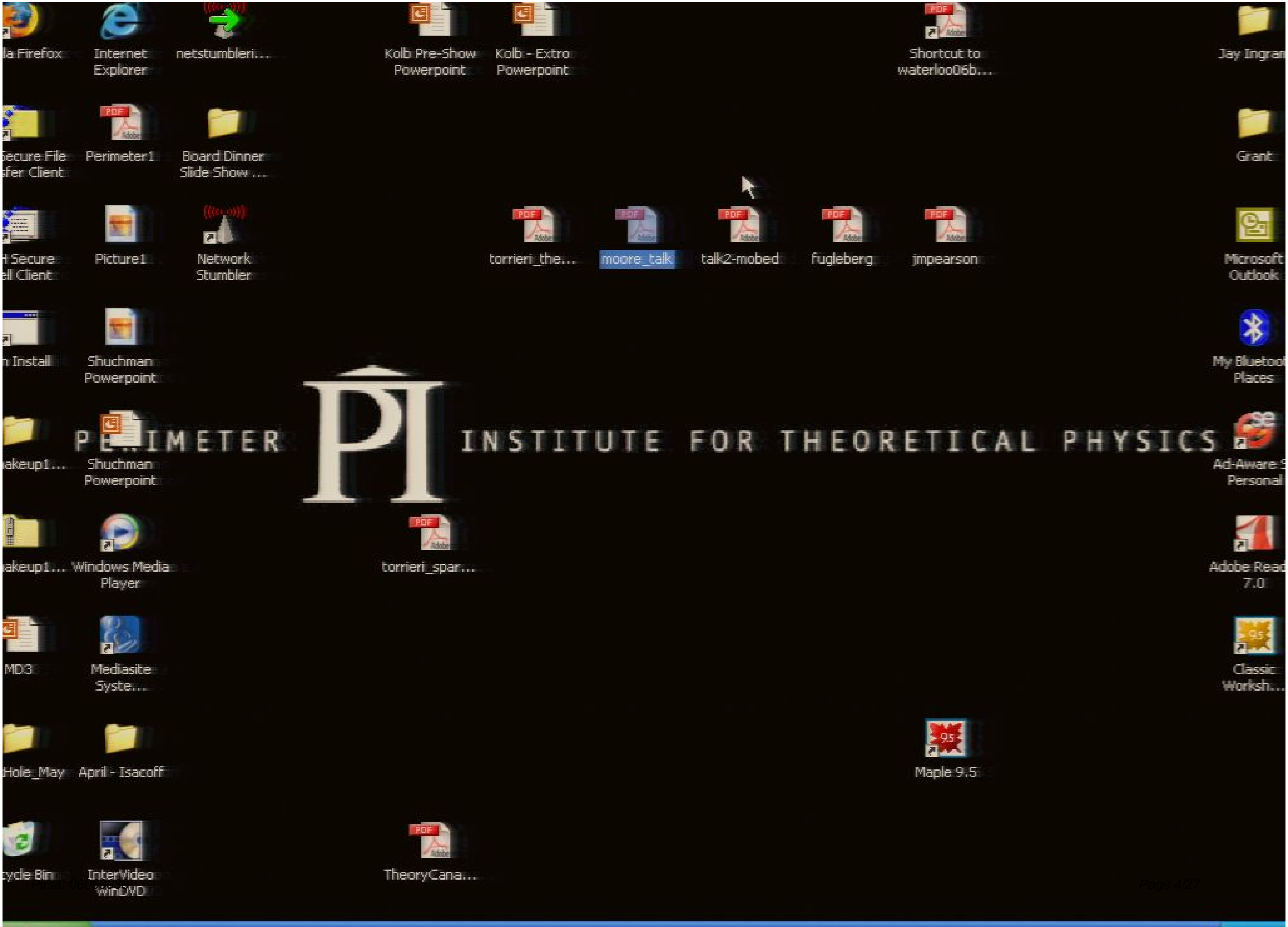
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Convergence Issues in Baryon Chiral Perturbation Theory for the Reaction $N(\pi, 2\pi)N$

Nader Mobed

Convergence Issues in Baryon Chiral Perturbation Theory for the Reaction $N(\pi, 2\pi)N$

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University of Regina

Work done in collaboration with
J. Zhang and D. Singh

Motives

- Aspects of chiral symmetry and its spontaneous breaking
  - $\pi\pi$ scattering
  - Non-linear realization of chiral symmetry

- In principle, the $\pi\pi$ scattering amplitude can be extracted from the reaction $\pi + N \longrightarrow \pi + \pi + N$.

- Chiral Perturbation Theory (χPT) is the effective low-energy theory of QCD.

- In the Baryon sector of Chiral Perturbation Theory (BχPT) the correspondence between the loop-expansion and the chiral expansion is destroyed. Alternative formulations of BχPT are: HBχPT, IR, EOMS.

- How well does HBχPT reproduce the experimental data for the reaction $\pi + N \longrightarrow \pi + \pi + N$?

- How fast does HBχPT converge?

  - How large are the Low Energy Constants (LECs) of the theory?
  - How significant are the loop effects (unitarity corrections)?

- How high in energy can one go before HBχPT breaks down?

Building Blocks

Meson Sector

$$
\begin{aligned}
U &= u^2 = e^{i\vec{\tau}\cdot\vec{\pi}/F} \\
D_\mu U &= \partial_\mu U - iUr_\mu + il_\mu U \longrightarrow \partial_\mu U \\
\chi &= 2B_0(S + iP) = (m_u + m_d)B_0 \longrightarrow m_\pi^2 \\
\chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u \longrightarrow m_\pi^2(U^\dagger \pm U) \\
u_\mu &= i\Big(u^\dagger\big(\partial_\mu - ir_\mu\big)u - u\big(\partial_\mu - il_\mu\big)u^\dagger \Big) \longrightarrow i(u^\dagger \partial_\mu U u^\dagger)
\end{aligned}
$$

$\mathcal{L}_\pi$	# of LECs	$N(\pi, 2\pi)N$	Status of LECs [1]
$\mathcal{L}_\pi^{(2)}$	2	2	known (F_π and m_π)
$\mathcal{L}_\pi^{(4)}$	7	4	known ($\bar{l}_i$)

[1] The renormalized scale-independent LECs of $\mathcal{O}(q^4)$ are defined as:

$$
\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i - 2\ln\frac{m_\pi}{\mu} - \left\{ \frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 + \Gamma'(1)] \right\}
$$

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Formalism

- The chiral expansion of the effective Lagrangian reads:

$$\mathcal{L}_{\text{eff}} = \underbrace{\mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \cdots}_{\Downarrow}$$

Transition amplitudes of $\mathcal{O}(q)$, $\mathcal{O}(q^2)$, $\mathcal{O}(q^3)$ or higher

- In HBχPT the chiral loops first appear at $\mathcal{O}(q^3)$.

- The short distance dynamics of QCD are encoded in the LECs of χPT.

Building Blocks

Meson Sector

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\begin{aligned}
U &= u^2 = e^{i\vec{\tau}\cdot\vec{\pi}/F} \\
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[1] The renormalized scale-independent LECs of $\mathcal{O}(q^4)$ are defined as:

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i - 2\ln\frac{m_\pi}{\mu} - \left\{ \frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 + \Gamma'(1)] \right\}$$

Nucleon Sector

The building blocks involve U, u_μ, and

$$\nabla_\mu = \partial_\mu + \Gamma_\mu - i v_\mu^{(s)} \longrightarrow \partial_\mu + \frac{1}{2}\left\{ u^\dagger \partial_\mu u + u \partial_\mu u^\dagger \right\}.$$

$$S^\mu = \frac{i}{2}\gamma_5 \sigma^{\mu\nu} v_\nu$$

The heavy-field transformation is defined as:

$$N_v = e^{iMv_\mu x^\mu} P_v^+ \Psi,$$

$$P_v^+ = \frac{1}{2}(1 + \not{v}), \qquad v^2 = 1.$$

$\mathcal{L}_{\pi N}$	# of LECs	$N(\pi, 2\pi)N$	Status of LECs [1]
$\mathcal{L}_{\pi N}^{(1)}$	2	2	known (g_A and m_N)
$\mathcal{L}_{\pi N}^{(2)}$	7	5	known (a_i)
$\mathcal{L}_{\pi N}^{(3)}$	23	13	7 known (b_i) 6 unknown (b_i)

[1] The scale-independent renormalized LECs of $\mathcal{O}(q^3)$ are defined as:

$$\bar{b}_i = b_i - \beta_i \left\{ \frac{1}{d-4} - \frac{1}{2}[\ln(4\pi) + 1 + \Gamma'(1)] + \ln\frac{m_\pi}{\mu} \right\}$$

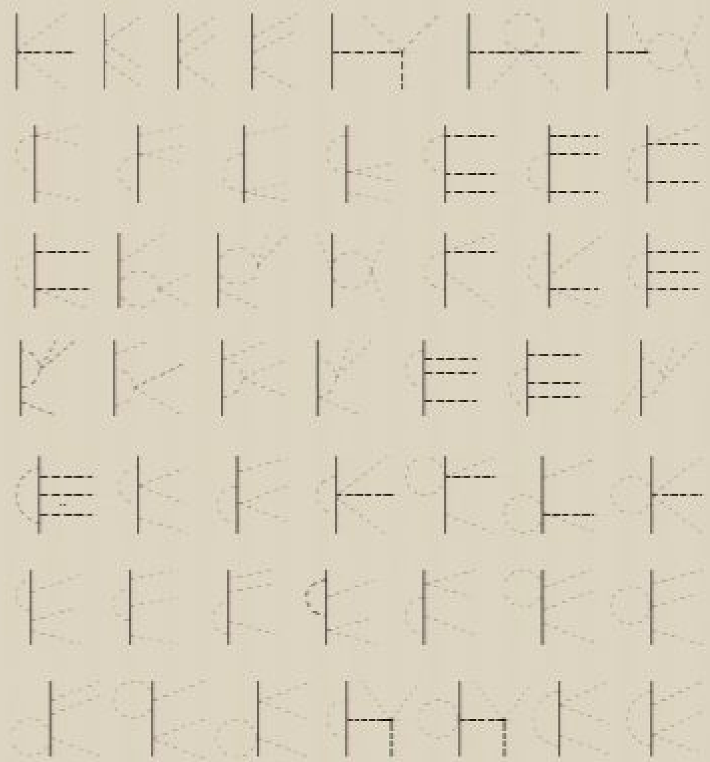

Figure 1: Topologically distinct Feynman graphs

Calculations

There are five experimentally accessible channels:

$$\begin{aligned} \pi^- p &\longrightarrow \pi^0 \pi^0 n \\ \pi^\pm p &\longrightarrow \pi^\pm \pi^+ n \\ \pi^\pm p &\longrightarrow \pi^\pm \pi^0 p \end{aligned}$$

The unknown LECs in $\mathcal{L}^{(3)}_{\pi N}$ were determined by fitting

- $\sigma_{N(\pi,2\pi)N}$
- $\dfrac{d\sigma_{N(\pi,2\pi)N}}{dT_\pi d\Omega_\pi}$

LECs of $\mathcal{O}(q^3)$ determined in this work[1,2]

(The fitting energy range 170 MeV $\leq (T_\pi)_{\rm lab} \leq$ 260 MeV)

$\tilde{b}_5$	$\tilde{b}_{11}$	$\tilde{b}_{12}$
2.6 ± 3.3	-25.1 ± 6.2	-10.2 ± 4.7

$\tilde{b}_{13}$	$\tilde{b}_{14}$	$\tilde{b}_{17}$
22.3 ± 5.7	-8.4 ± 2.7	-5.3 ± 1.1

[1] The RGE were employed. [2] The χ^2/dof of the fit is 3.4.

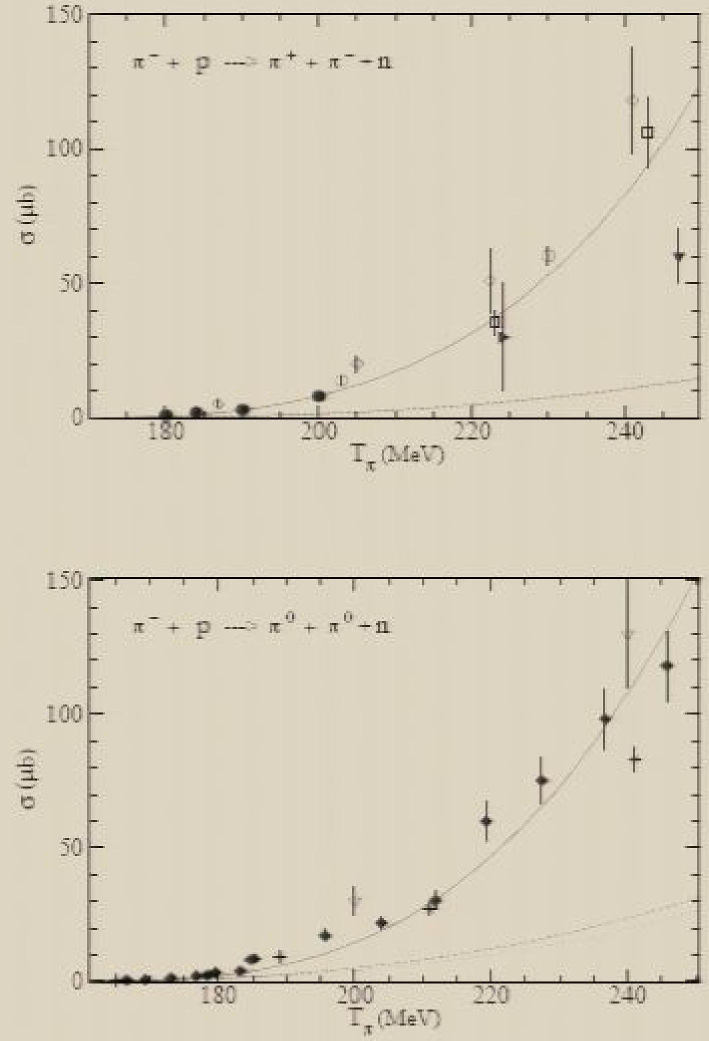

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Figure 2: The total cross-section σ.

Dashed curve: $\mathcal{O}(q)$, Dotted curve: $\mathcal{O}(q^2)$, Solid curve: $\mathcal{O}(q^3)$.

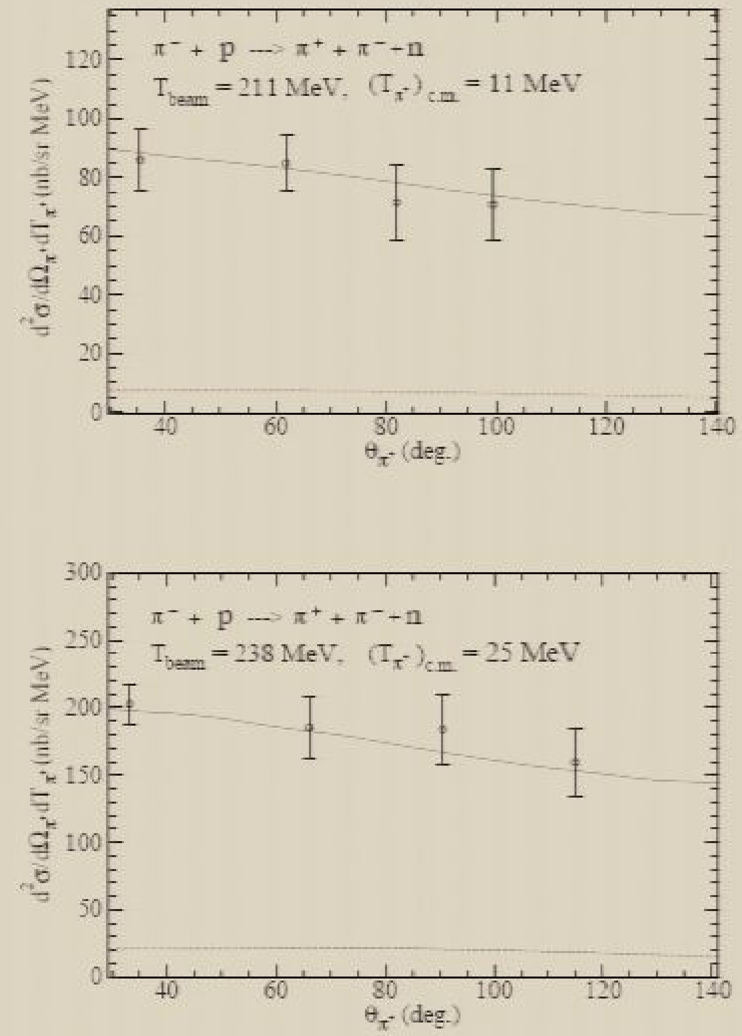

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Figure 3: The double-differential cross-section $d\sigma/d\Omega dT$.

Dashed curve: $\mathcal{O}(q)$, Dotted curve: $\mathcal{O}(q^2)$, Solid curve: $\mathcal{O}(q^3)$.

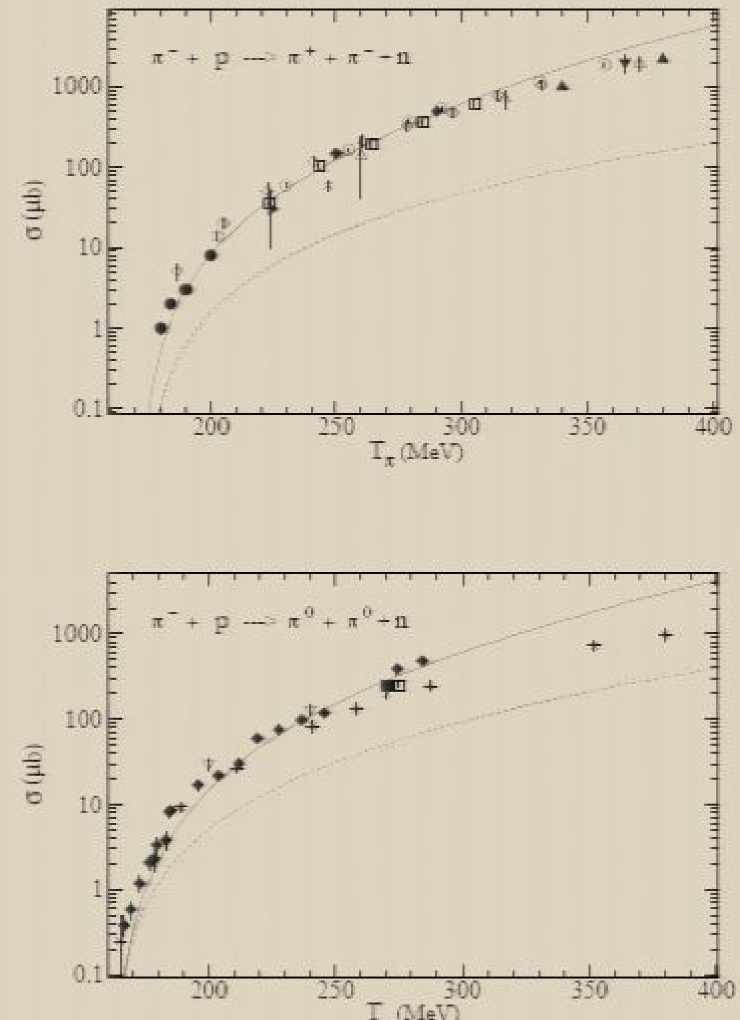

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Figure 4: The total cross section σ.

Dashed curve: $\mathcal{O}(q)$, Dotted curve: $\mathcal{O}(q^2)$, Solid curve: $\mathcal{O}(q^3)$.

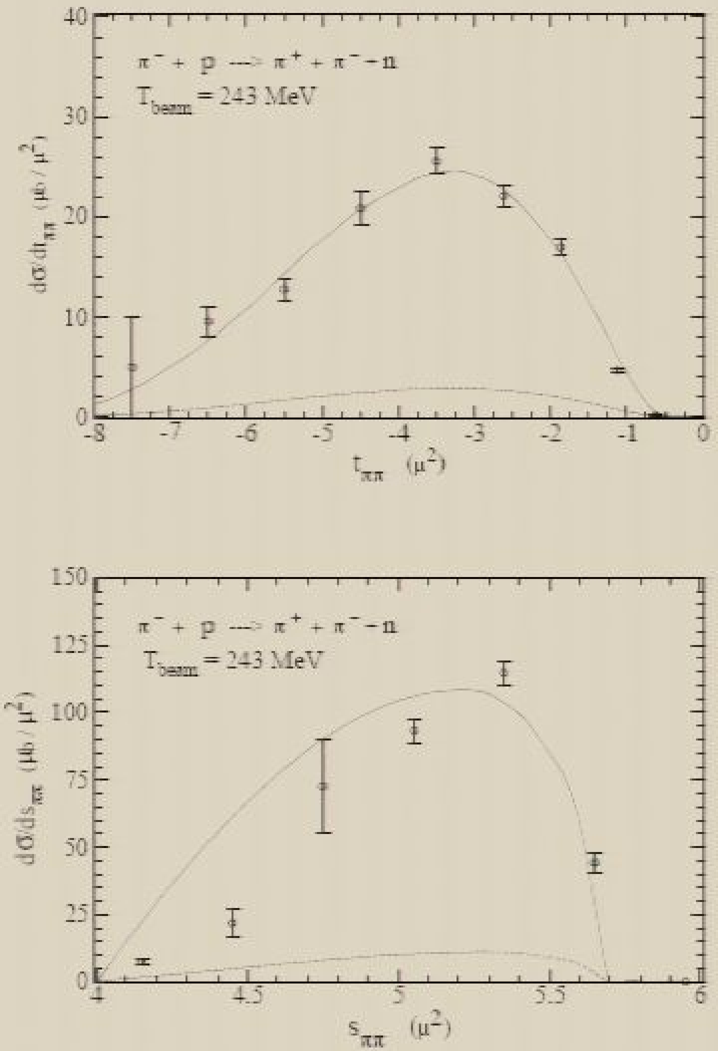

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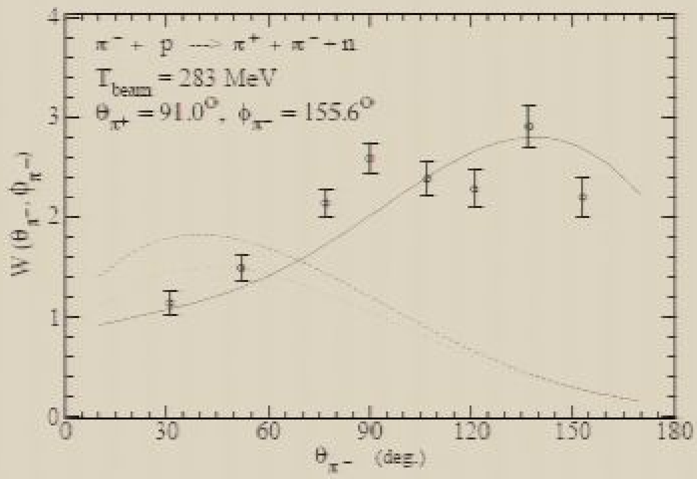
Figure 5: The invariant cross sections: $d\sigma/dt$ and $d\sigma/ds$.

Dashed curve: $\mathcal{O}(q)$, Dotted curve: $\mathcal{O}(q^2)$, Solid curve: $\mathcal{O}(q^3)$.



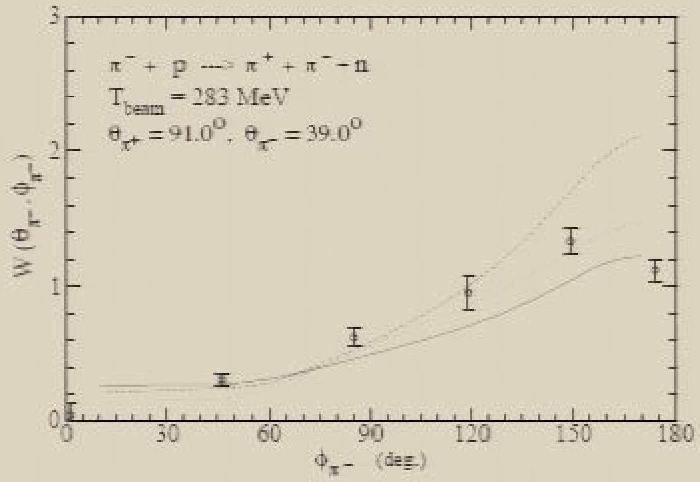


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Figure 6: The angular correlation function $W(\theta, \phi)$.

Dashed curve: $\mathcal{O}(q)$, Dotted curve: $\mathcal{O}(q^2)$, Solid curve: $\mathcal{O}(q^3)$.

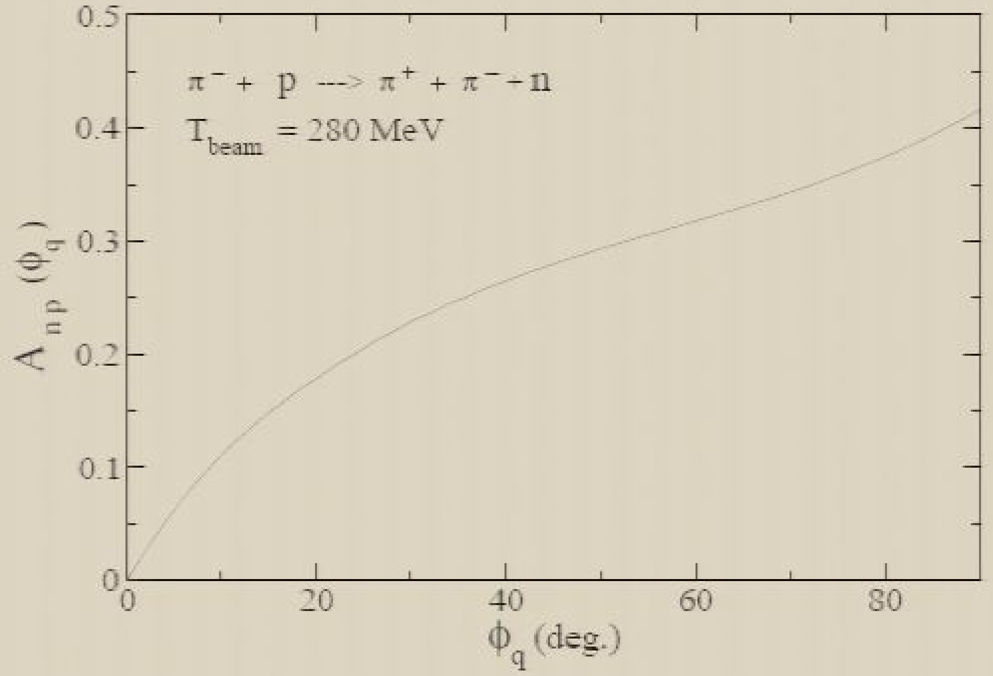

Figure 7: The polarization observable $A_{np}(\phi_q)$ vanishes identically in the absence of loop contributions (unitarity corrections).

Threshold Amplitudes

The information regarding S-wave $\pi\pi$ scattering lengths is contained in the threshold isospin amplitudes, D_1 and D_2, for the reactions:

$$D_1 : \quad \pi^+ p \longrightarrow \pi^+ \pi^+ n$$
$$D_2 : \quad \pi^- p \longrightarrow \pi^0 \pi^0 n$$

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	Expt. [1]
D_1(fm^3)	2.50	2.14	1.90 $\pm$ 0.43	2.26 $\pm$ 0.12
D_2(fm^3)	-7.70	-6.96	-11.44 $\pm$ 2.60	-9.05 $\pm$ 0.36

	Present work	BKM [2]	Expt. [1]
D_1(fm^3)	1.90 $\pm$ 0.43	2.65 $\pm$ 0.24	2.26 $\pm$ 0.12
D_2(fm^3)	-11.44 $\pm$ 2.60	-9.06 $\pm$ 1.05	-9.05 $\pm$ 0.36

[1] Burkhardt *et al.* Phys. Rev. Lett. **72**, 2622 (1991)
[2] Bernard *et al.* Nucl. Phys. **B457**, 146 (1995)

Unitarity Corrections

The amplitude for the reaction $N(\pi, 2\pi)N$ reads:

$$\mathcal{M}_{fi} = \bar{u}(p_f)\gamma_5\left[f_1 + f_2\not{q}_1 + f_3\not{q}_2 + f_4\not{q}_1\not{q}_2\right]u(p_i)$$

The imaginary parts of the invariant amplitudes $f_1, f_2, f_3,$ and f_4 originate entirely form loops and are hence parameter free quantities.

Threshold amplitudes for the reaction $\pi^- p \longrightarrow \pi^0\pi^0 n$

$\mathcal{O}(q^n)$	$f_1 \times m_\pi^{-2}$	$f_2 \times m_\pi^{-3}$	$f_3 \times m_\pi^{-3}$	f_4
$\mathcal{O}(q)$	-44.30	3.90	3.90	0
$\mathcal{O}(q^2)$	-39.51	3.81	3.81	0
$\mathcal{O}(q^3)$	-38.69 $+i$ 47.88	3.20 $-$ i 5.22	3.20 $-$ i 5.22	0

Threshold amplitudes for the reaction $\pi^+ p \longrightarrow \pi^+\pi^+ n$

$\mathcal{O}(q^n)$	$f_1 \times m_\pi^{-2}$	$f_2 \times m_\pi^{-3}$	$f_3 \times m_\pi^{-3}$	f_4
$\mathcal{O}(q)$	30.75	-1.44	-1.44	0
$\mathcal{O}(q^2)$	25.36	-1.71	-1.71	0
$\mathcal{O}(q^3)$	-23.71 $-$ $i3.89$	$1.67 + i0.03$	$1.67 + i0.03$	0

CONCLUSIONS

Comparison with Data

The calculated values of different observables are in reasonable agreement with experimental data. In most cases contributions of $\mathcal{O}(q^3)$ are essential in order to reproduce the data.

Issues

- Some of the LECs of $\mathcal{O}(q^3)$ are substantially larger than their expected "Natural Size".
- In some reaction channels the unitarity corrections are sizable.
- Is HBχPT a (rapidly) converging series?
  Slow convergence is observed also in other chiral processes.
  [1] Mojzis, Eur. Phys. J. C2, 181 (1998)
  [2] Djukanovic, Gegelia, Scherer, hep-ph/0604164

Possibilities

- The short distance dynamics of QCD are encoded in the LECs of χPT. Calculate amplitudes of $\mathcal{O}(q^4)$ in HBχPT for the reaction under consideration in order to complete the one loop analysis.

- Include the low-lying resonances as dynamical degrees of freedom in the chiral Lagrangian.[1]

  [1] Bernard, Hemmert, and Meissner, Nucl. Phys. **A686**, 290 (2001)

- Calculate the process under consideration within the framework of the infrared[2] or extended on-mass-shell renormalization[3] schemes. These methods are manifestly Lorentz covariant, provide a consistent chiral power counting scheme, and may offer an improved convergence rates for the chiral series.

  [2] Becher and Leutwyler, JHEP **06**, 017 (2001)

  [3] Schindler, Gegelia, Scherer, Nucl. Phys. **B682**, 367 (2004)