Title: Nuclear Theory/Heavy Ions 2

Date: Jun 10, 2006 09:30 AM

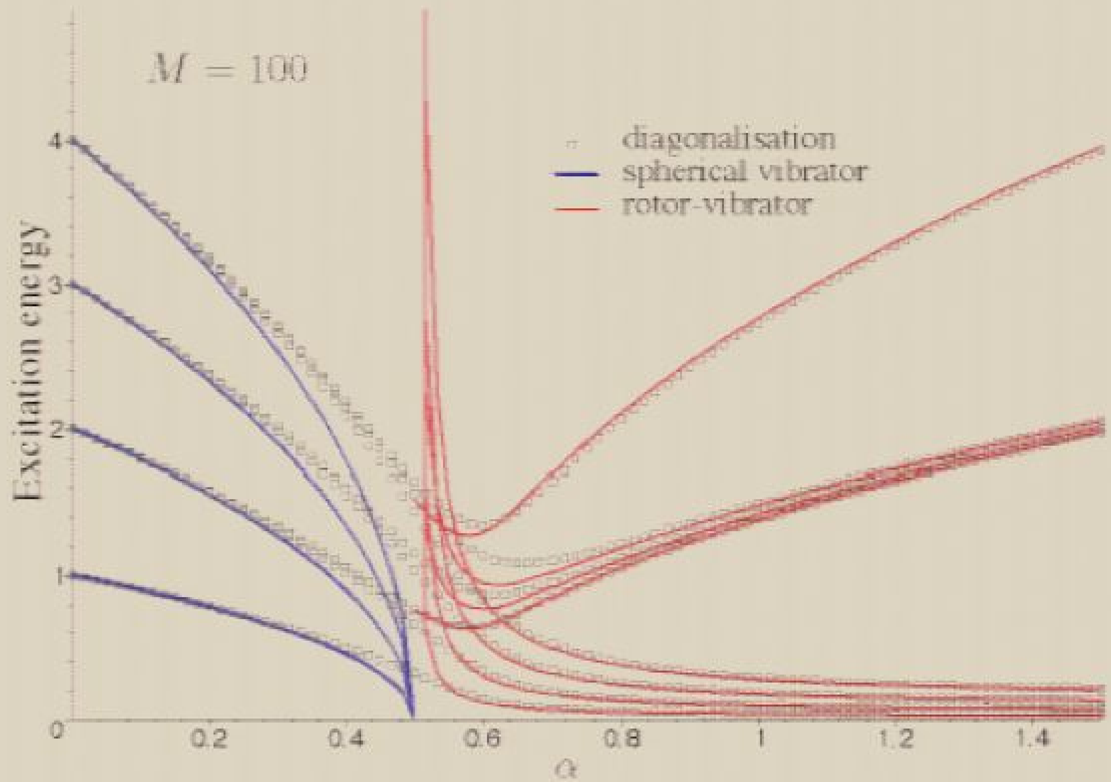
URL: http://pirsa.org/06060034

Abstract:

# Critical phenomena and quasi symmetries

Phases are usually associated
with
(dynamical) symmetries

In a finite system, phase transitions take place continuously. This means we can study how such a system progresses through a critical point.


$M = 100$
diagonalisation
spherical vibrator
rotor-vibrator
Excitation energy
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0.8
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1.4
$\alpha$

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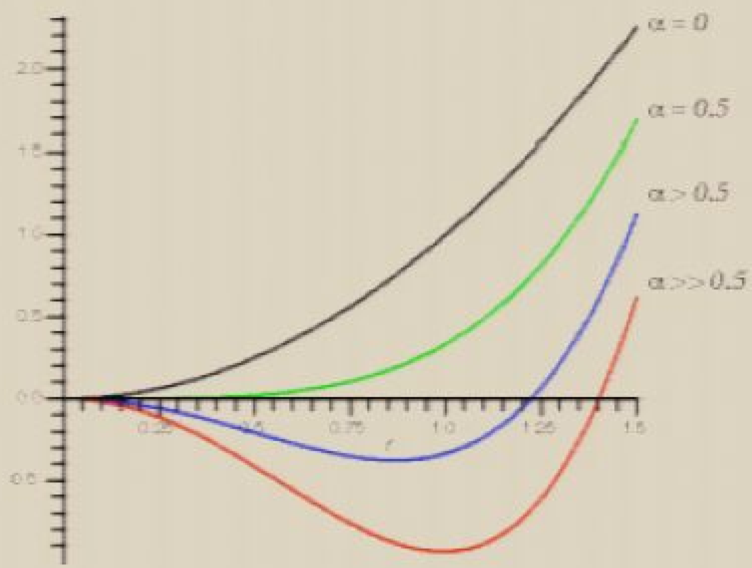


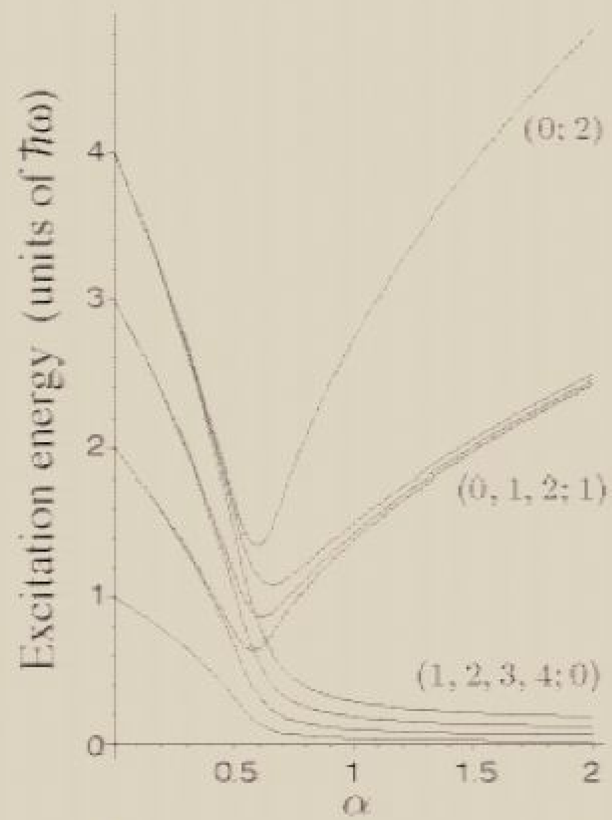


Turner and Rowe
Nucl. Phys. A756, 333 (2005)

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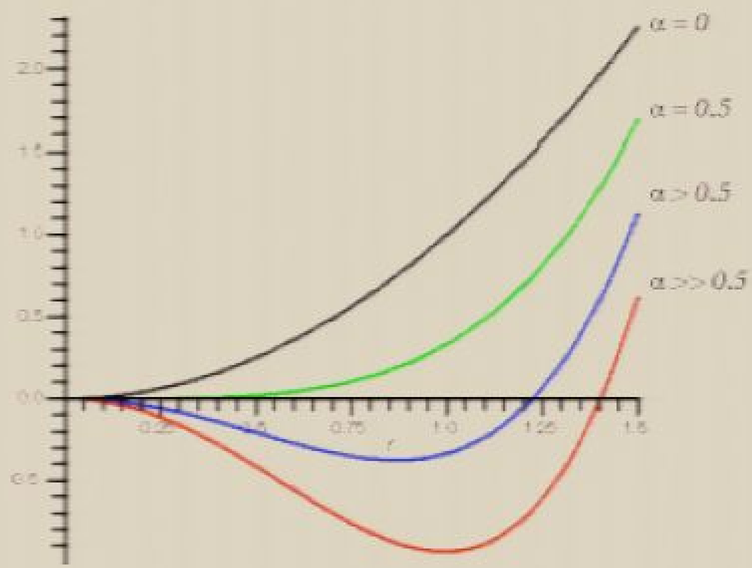


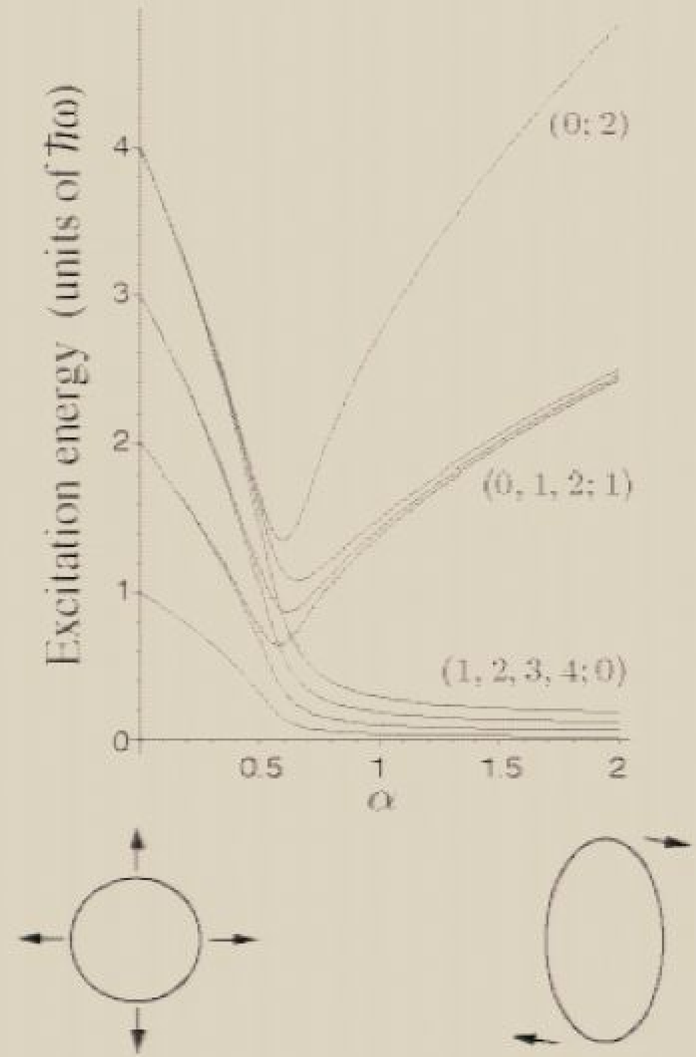


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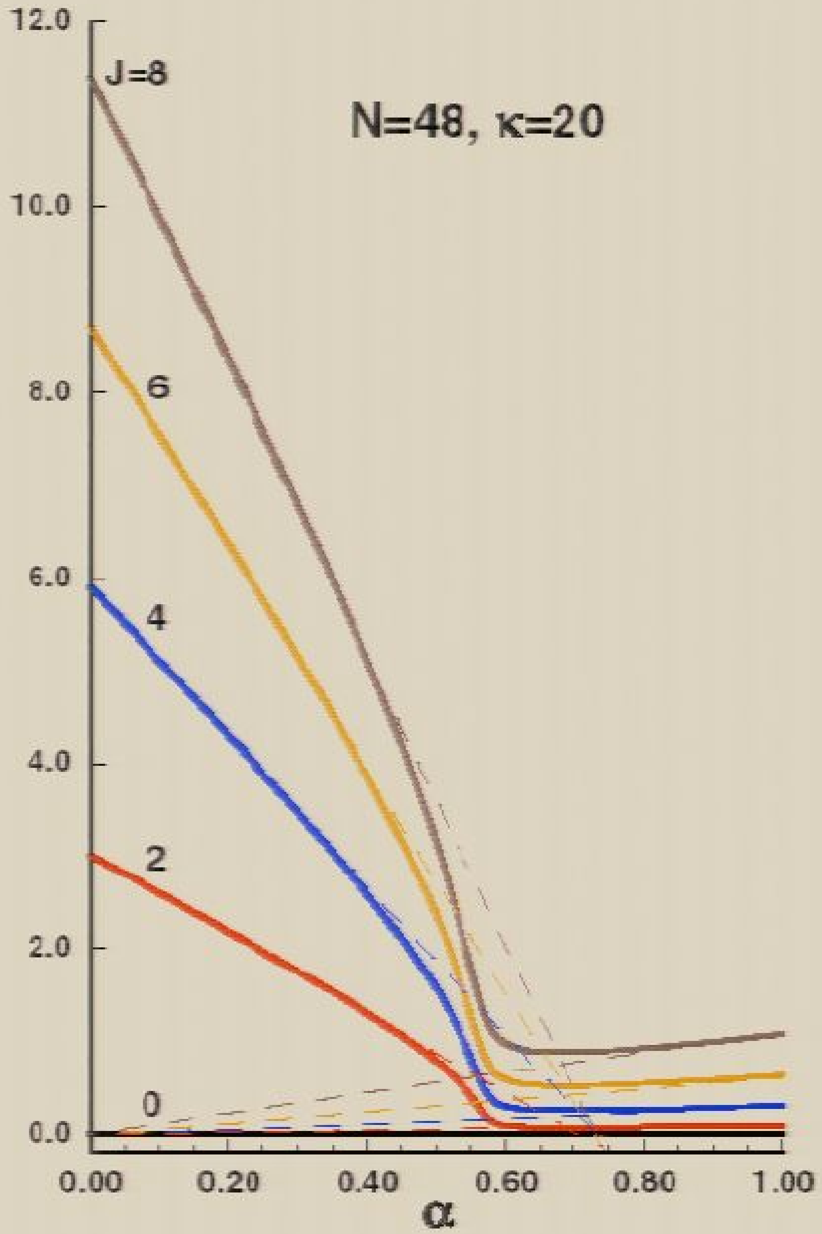


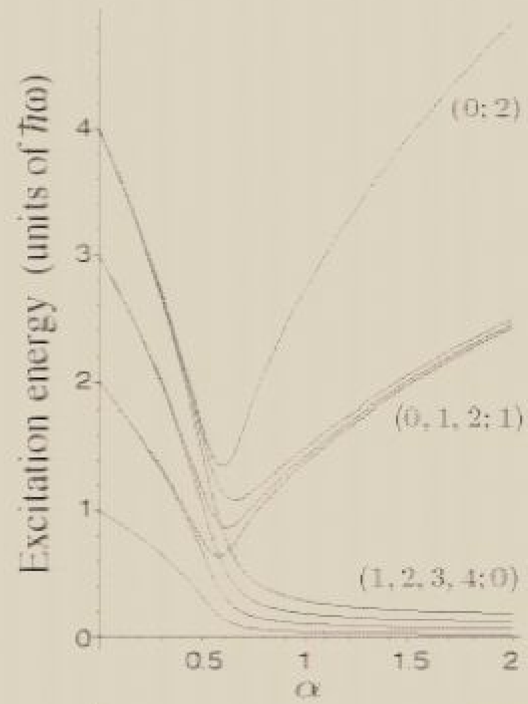


Bahri, Rowe and Wijesundera
Phys. Rev. C58, 1539 (1998)

# Spherical superconductor to deformed rotor phase transition

**Many-fermion model**

$$H(\alpha) = (1-\alpha)V_{\mathrm{SU(2)}} + \alpha V_{\mathrm{SU(3)}}$$

$H(\alpha=0)$ has the SU(2) dynamical symmetry of a spherical superconductor

$H(\alpha=1)$ has the SU(3) dynamical symmetry of a deformed rotor

For $0 < \alpha < 1$, solutions are generally too complicated to solve because there is no simple subgroup that contains both SU(2) and SU(3).

A model problem: suppose SU(2) and SU(3) are non-commuting subgroups of USp(6)

USp(6)

SU(2) U(3)

$$H_{\mathrm{SU(2)}} = \chi_1 S_+ S_-$$

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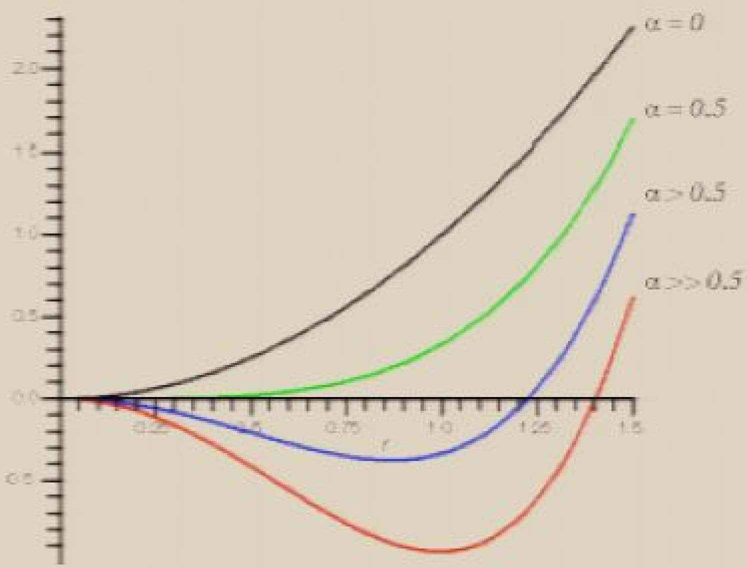


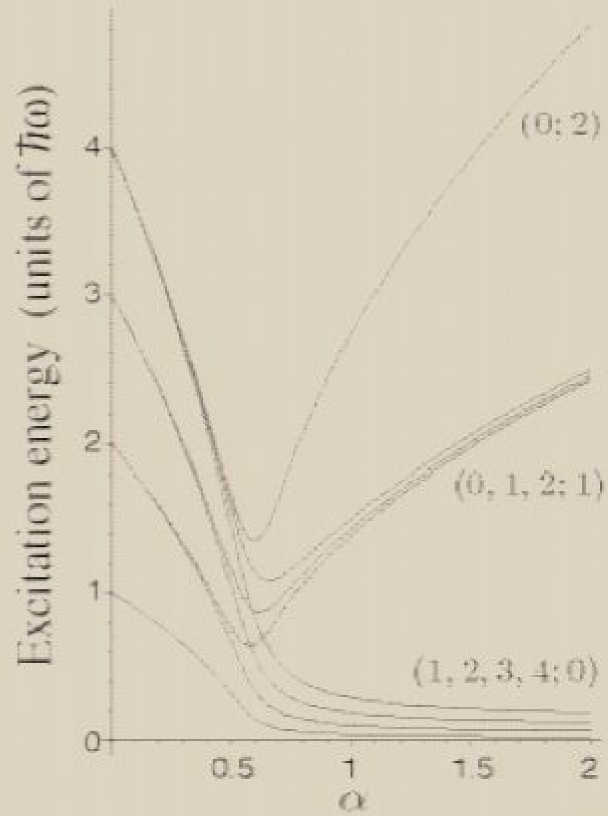


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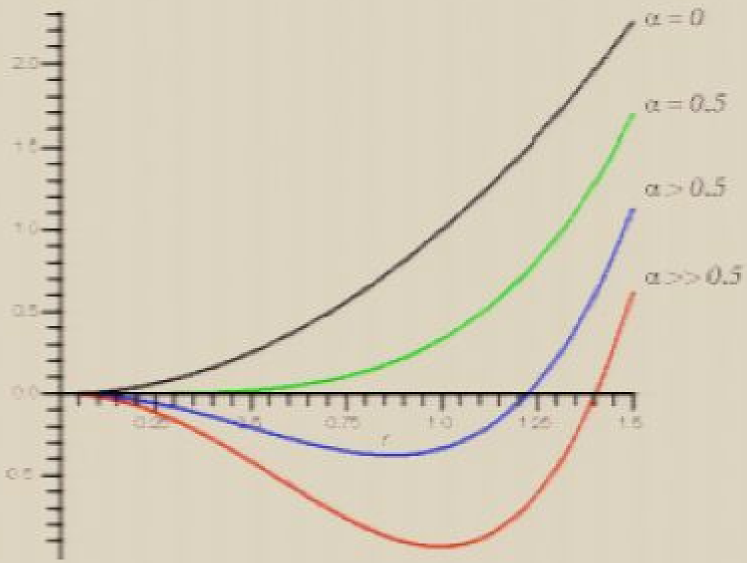


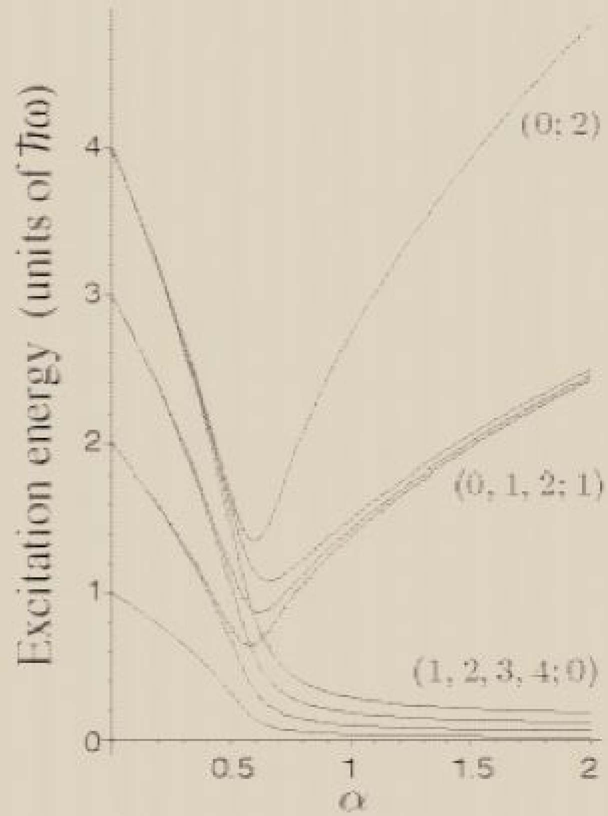


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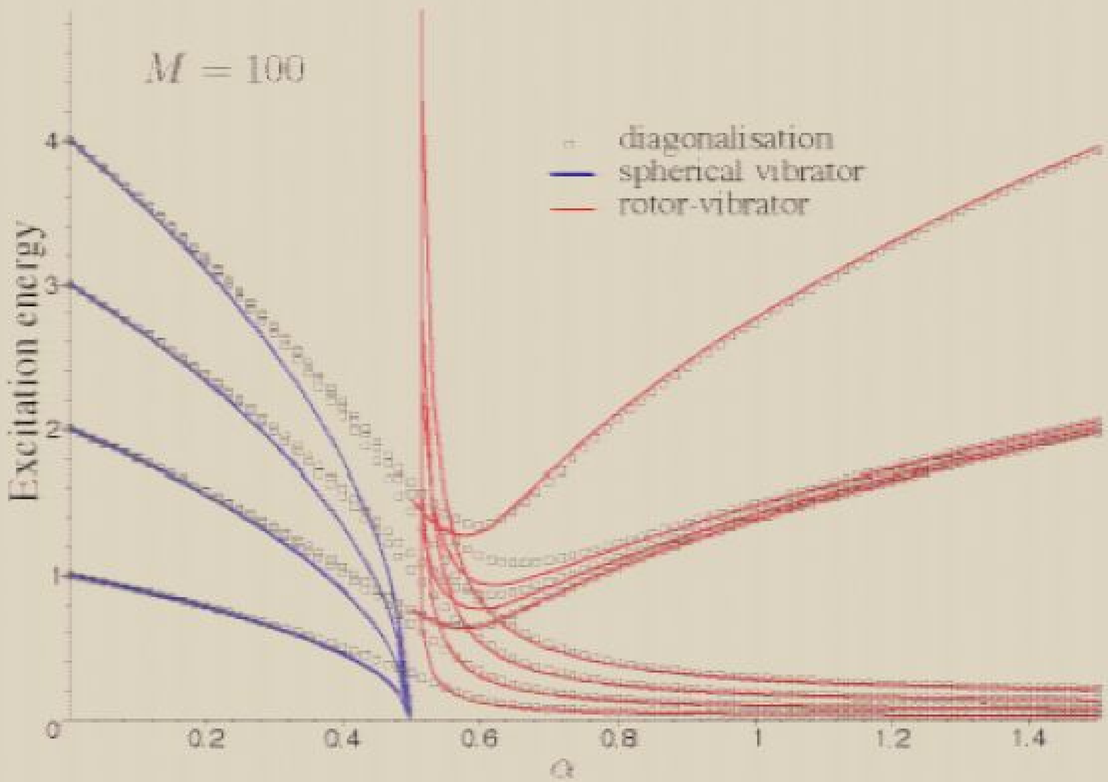

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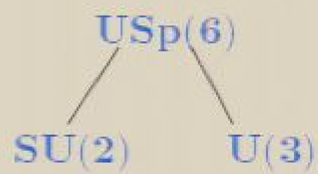


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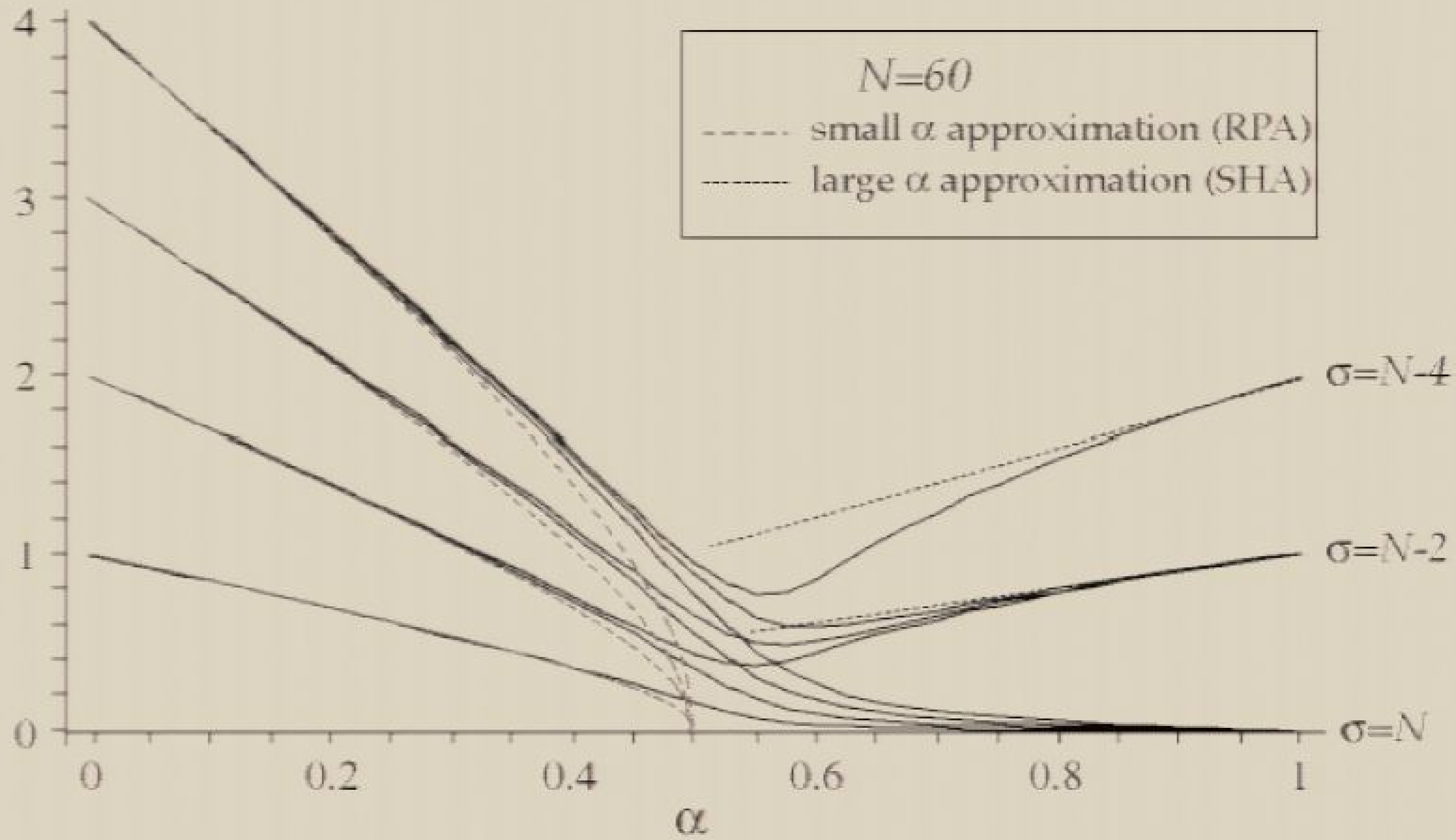
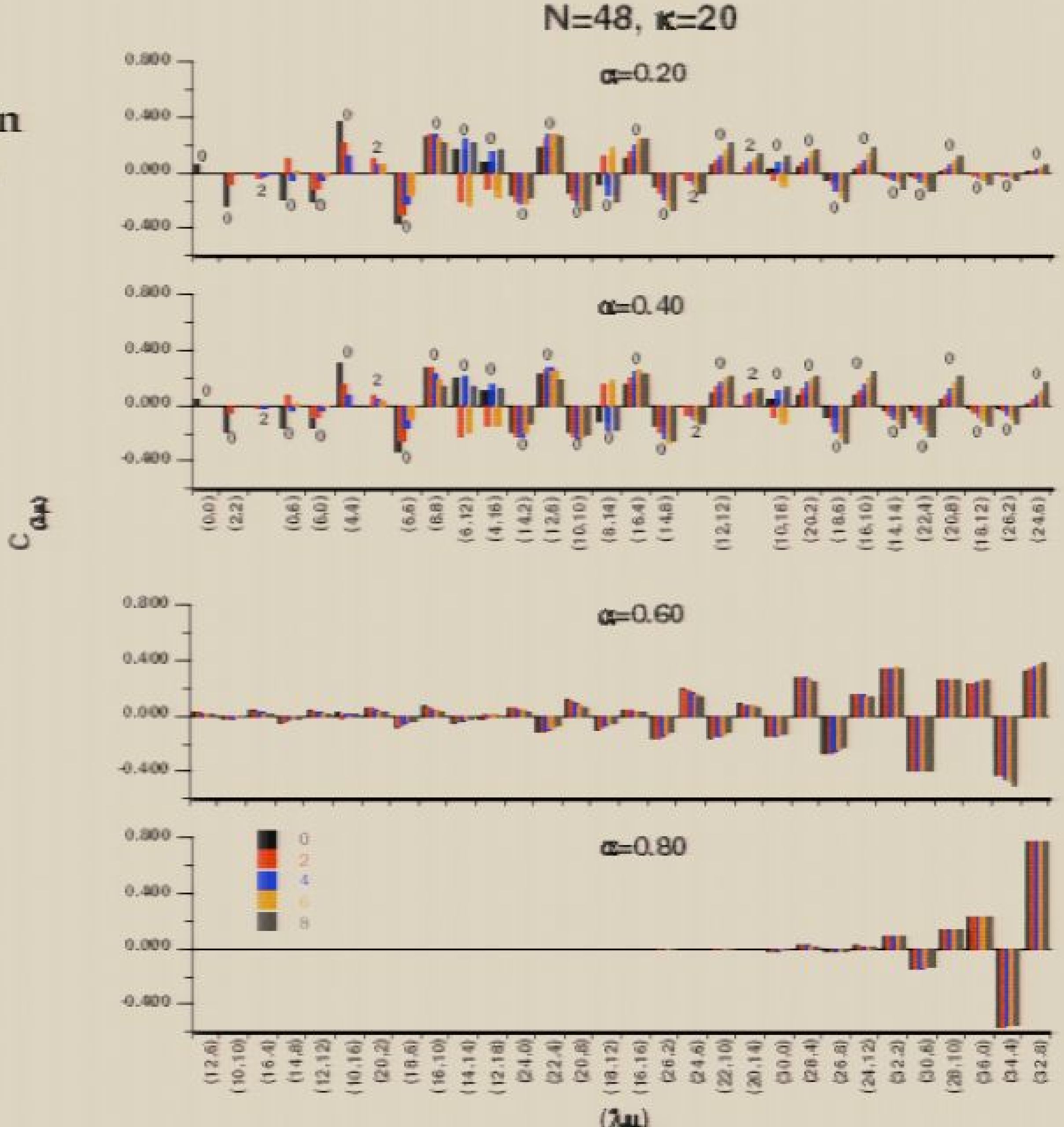
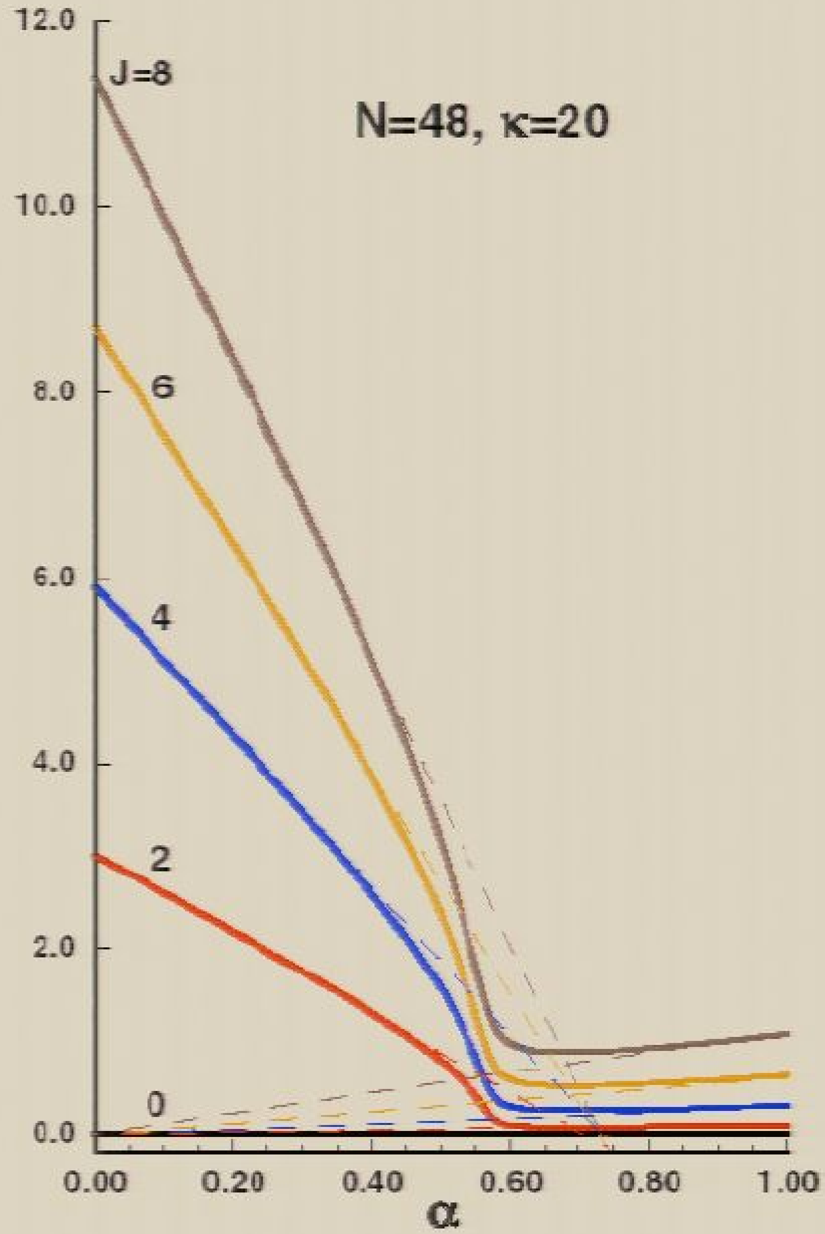


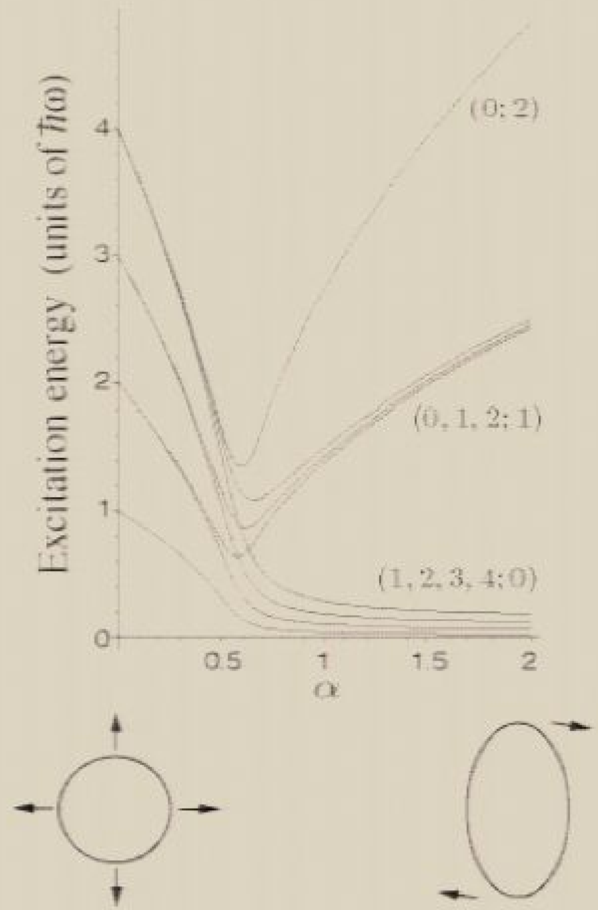

A sharp phase transition in the $N \to \infty$ limit

Djr, Nucl. Phys. A745, 47 (2004).

Wave functions in an SU(3) > SO(3) basis



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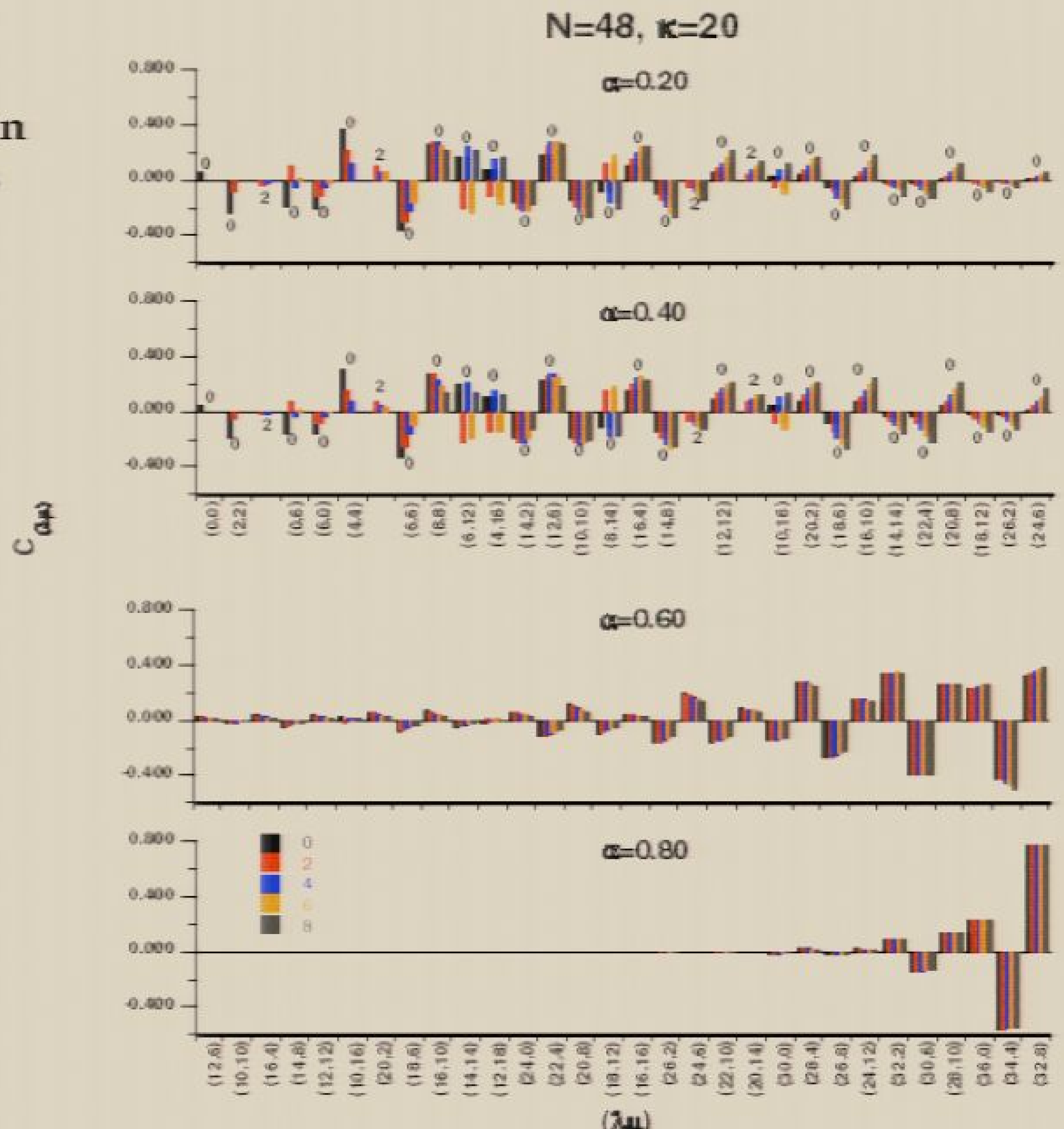

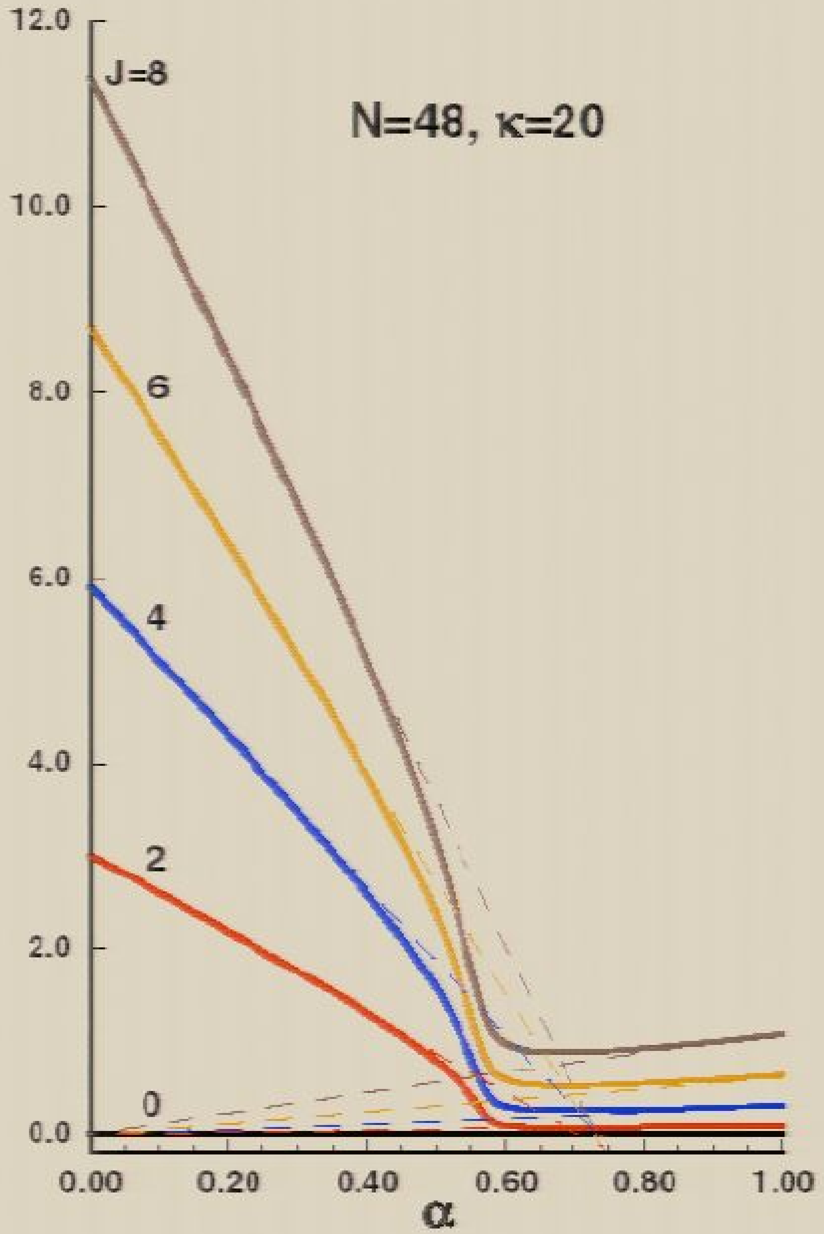


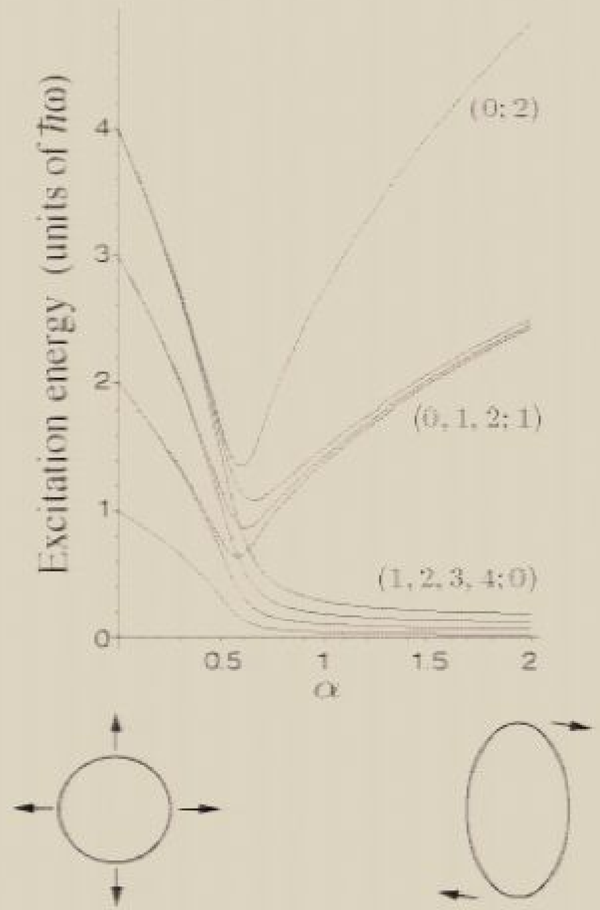


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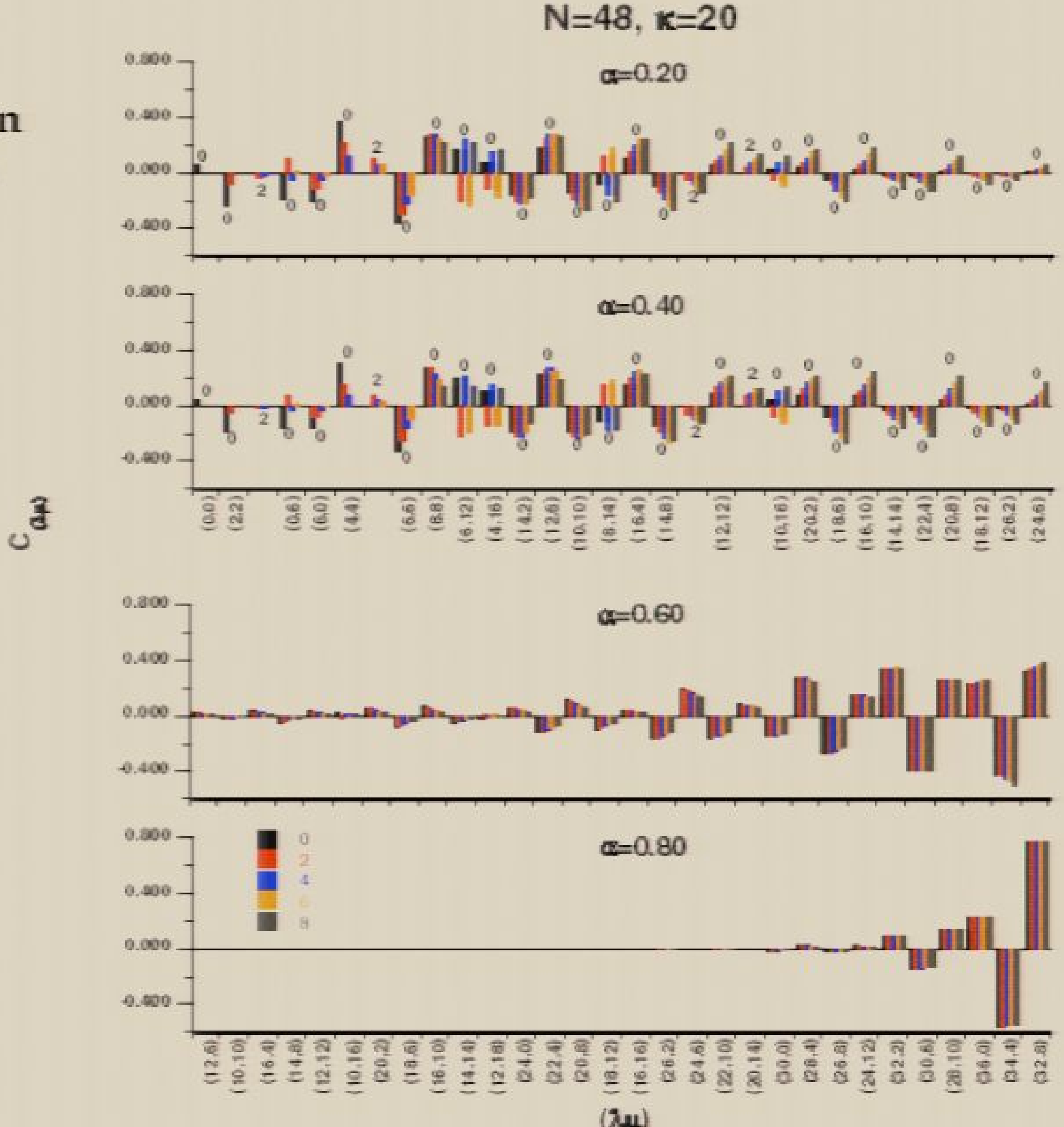


# An interacting boson model phase transition

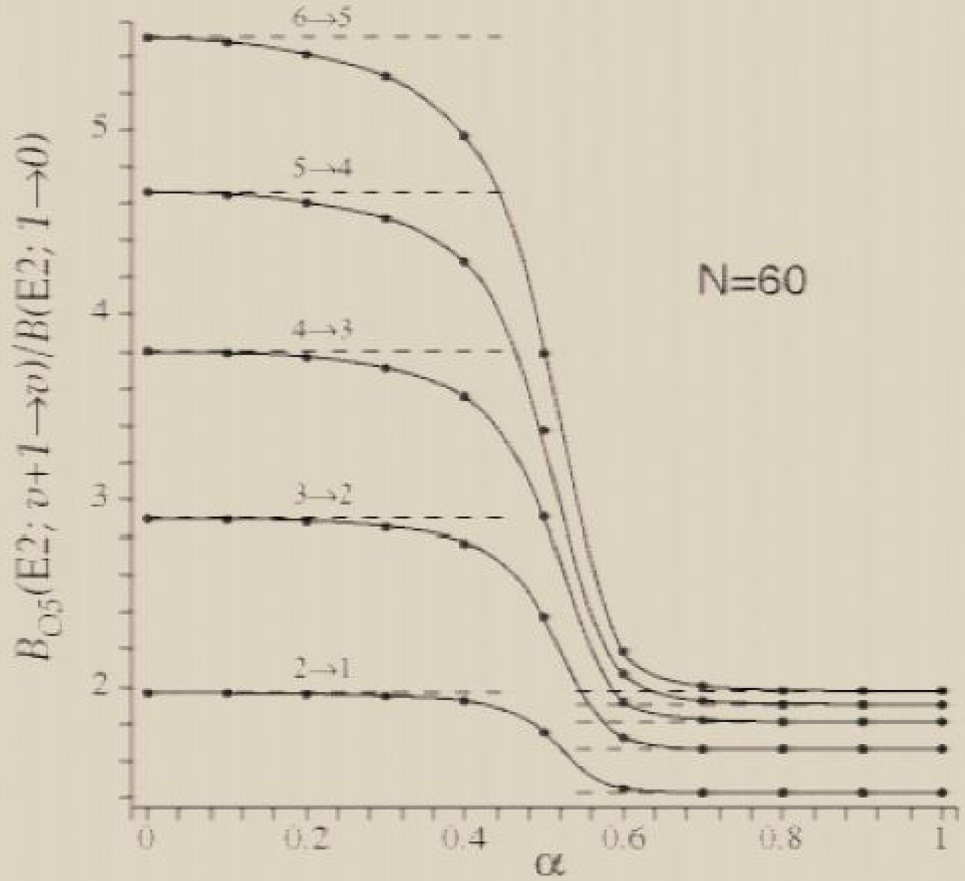
$$H = (1-\alpha)H_{\mathrm{U(5)}} + \alpha H_{\mathrm{O(6)}}$$

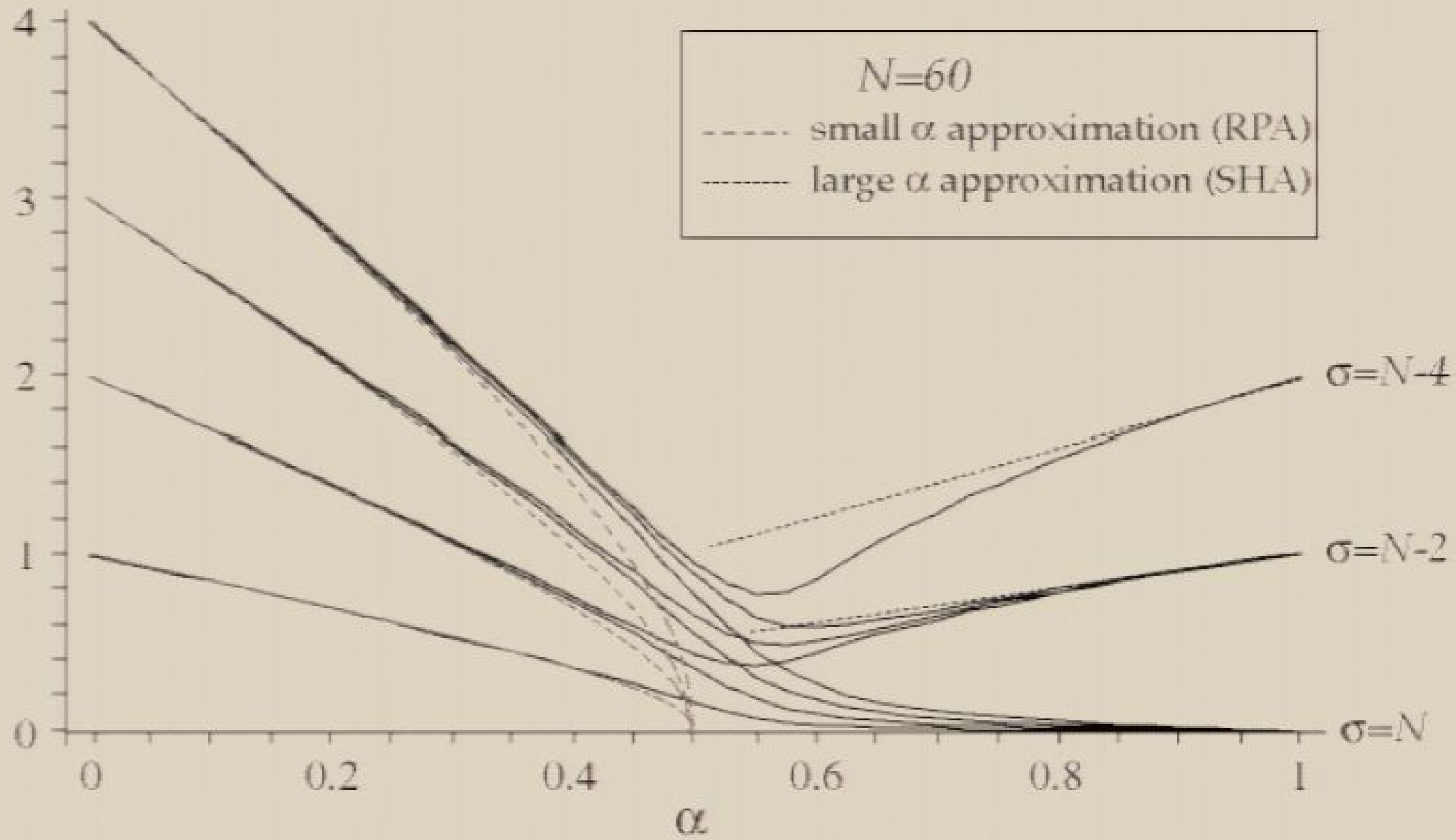
$$\begin{array}{ccccc} \mathbf{U(6)} & \supset & \mathbf{U(5)} & & \\ \cup & & \cup & & \\ \mathbf{O(6)} & \supset & \mathbf{SO(5)} & \supset & \mathbf{SO(3)} \end{array}$$

The Hamiltonians are essentially the Casimir operators of the U(5) and O(6) groups.

It is SO(5) and SO(3) invariant

## Electromagnetic transition ratios



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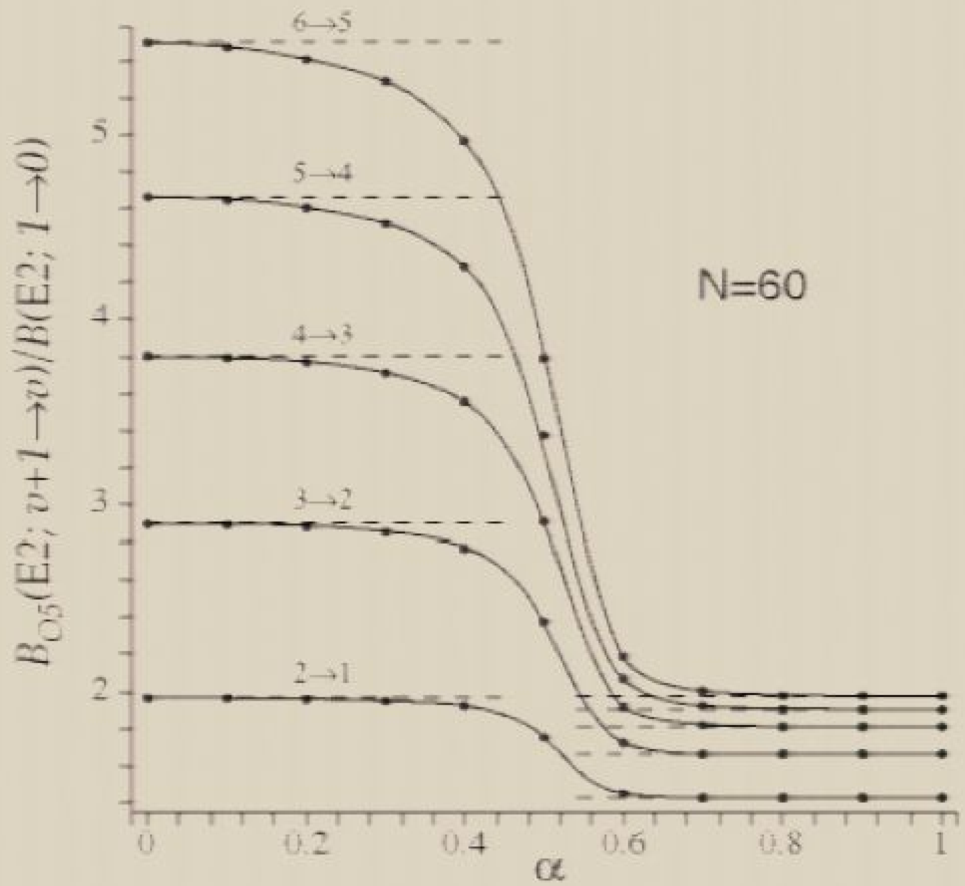
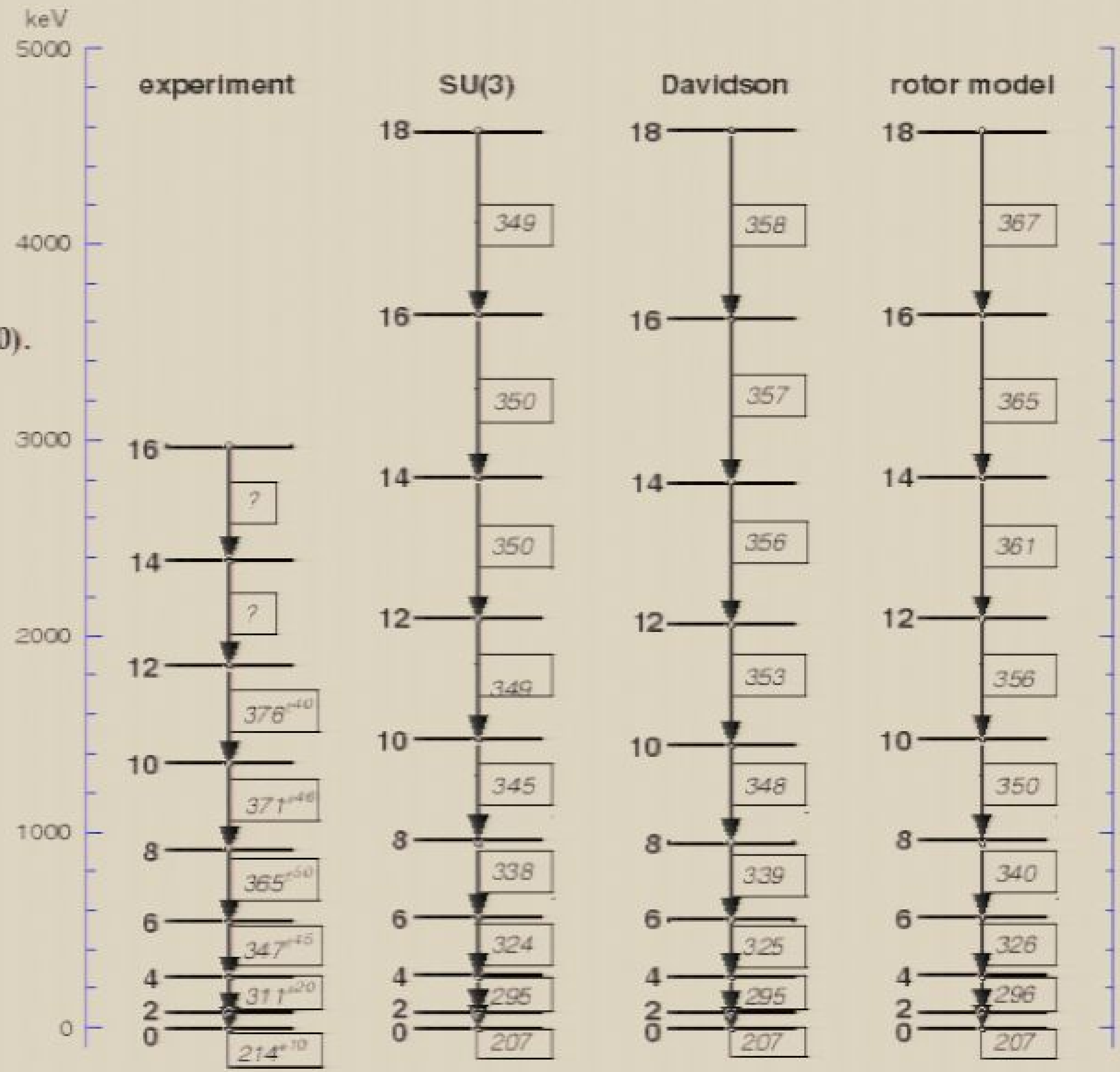
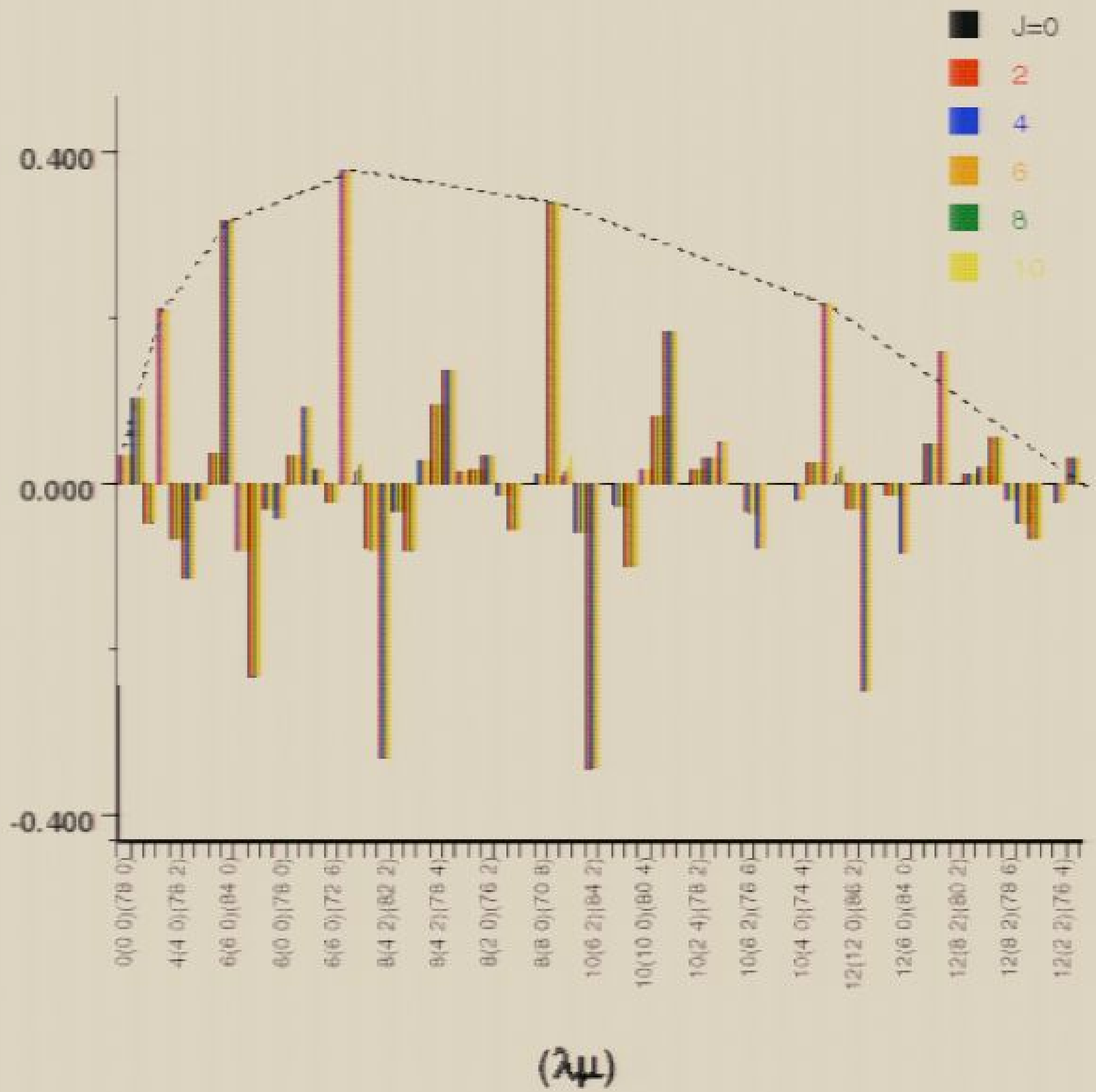


# Ground-state rotational band of $^{166}$Er

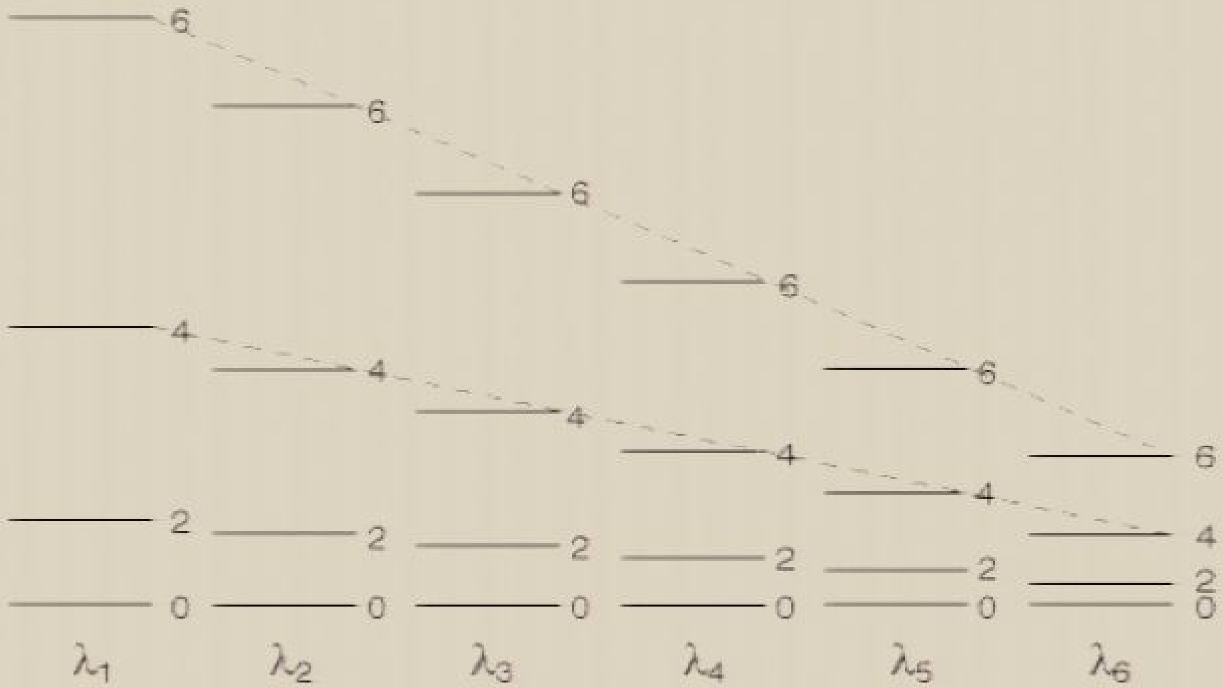
Bahri & Rowe
Nucl. Phys. A662, 125 (2000).



Davidson-model model wave functions expanded on an SU(3) basis



## Scale related irreps




$$\Phi_{KLM} = \sum_{\lambda} C_{\lambda} \Psi_{KLM}^{(\lambda)}$$

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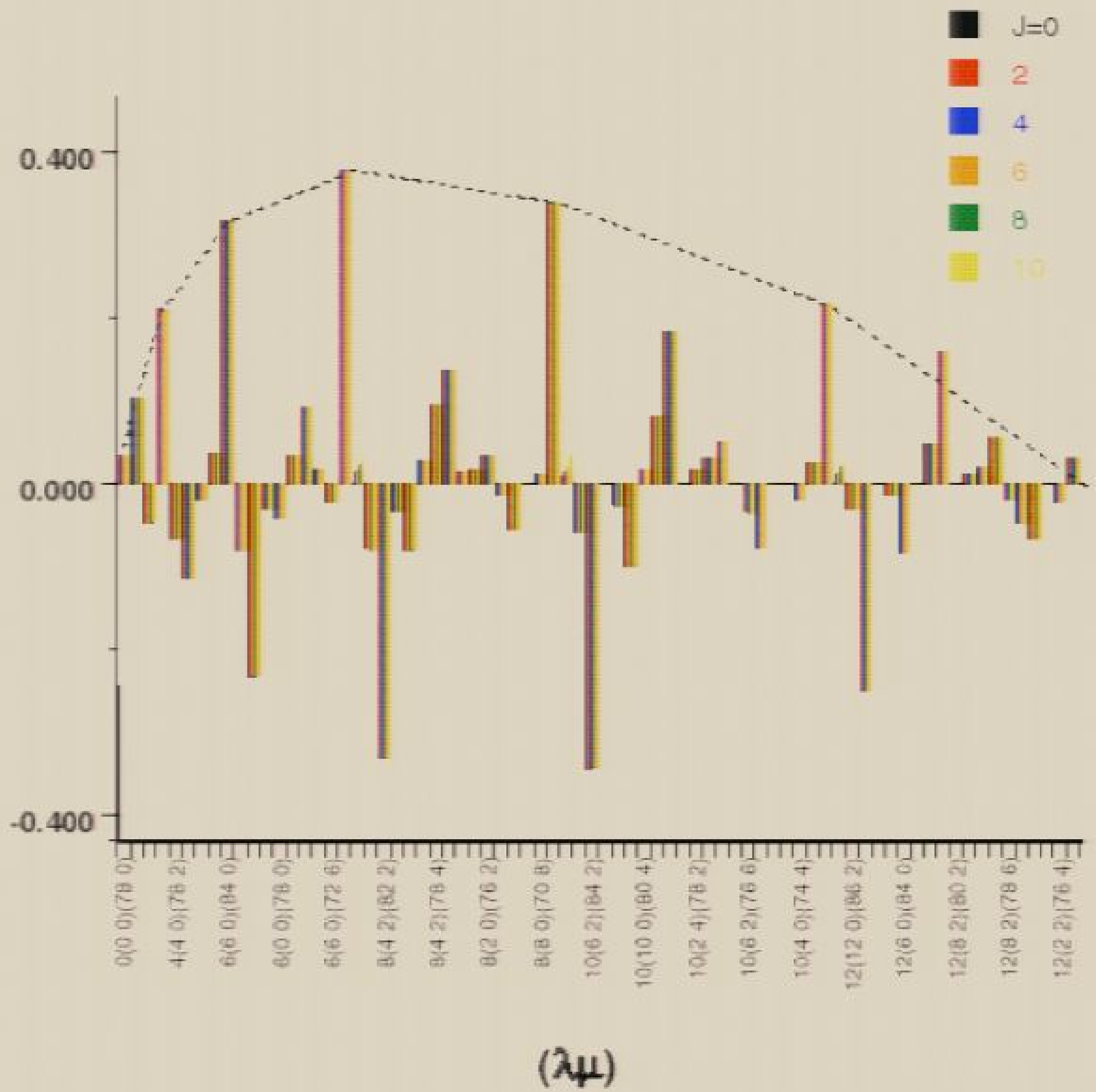

# Embedded representation

## Definition:

If H is the Hilbert space for a reducible unitary representation U of

a Lie algebra g and $H_0 < H$ is a subspace, then if the submatrices of U corresponding to the projection of $H \rightarrow H_0$ are those of a representation $U_0$ of g there are two possibilities:

- $H_0$ is an invariant subspace of H and $U_0$ is a subrepresentation of U.
- $H_0$ is not an invariant subspace of H and $U_0$ is an embedded representation.

Rowe, Rochford & Repka, J. Math. Phys. 29, 572 (1988).

## Mixing irreps

Equivalent irreps

$$\Phi_{i\lambda\nu} = \sum_{\alpha} C_{i\alpha} \Psi_{\alpha\lambda\nu}$$

Quasi-equivalent irreps

$$\Phi_{i\nu} = \sum_{\alpha\lambda} C_{i\alpha\lambda} \Psi_{\alpha\lambda\nu}$$

All unitary irreps of a vibrator (Heisenberg-Weyl) group are equivalent.

All unitary irreps of a translation group (Abelian) are quasi -equivalent (scale related)

All infinite-dimensional unitary irrep of a rotor group (semi -direct product of an abelian and orthogonal group) are quasi-equivalent.

"Almost" all groups contract in the limit of infinite-dimensional irreps to combinations of the above. Thus, they have asymptotic embedded representations.

# Mixing of quasi-equivalent representations

**All irreps of a rigid rotor are quasi equivalent (matrix elements are linearly related)**

**They can be mixed to form quasi-representations**

J. Carvalho, R. Le Blanc, M. Vassanji, D.J. Rowe and J. McGrory, 1986. "The Symplectic Shell Model Theory of Collective States". Nucl. Phys. A**452** 240-262 (1986).

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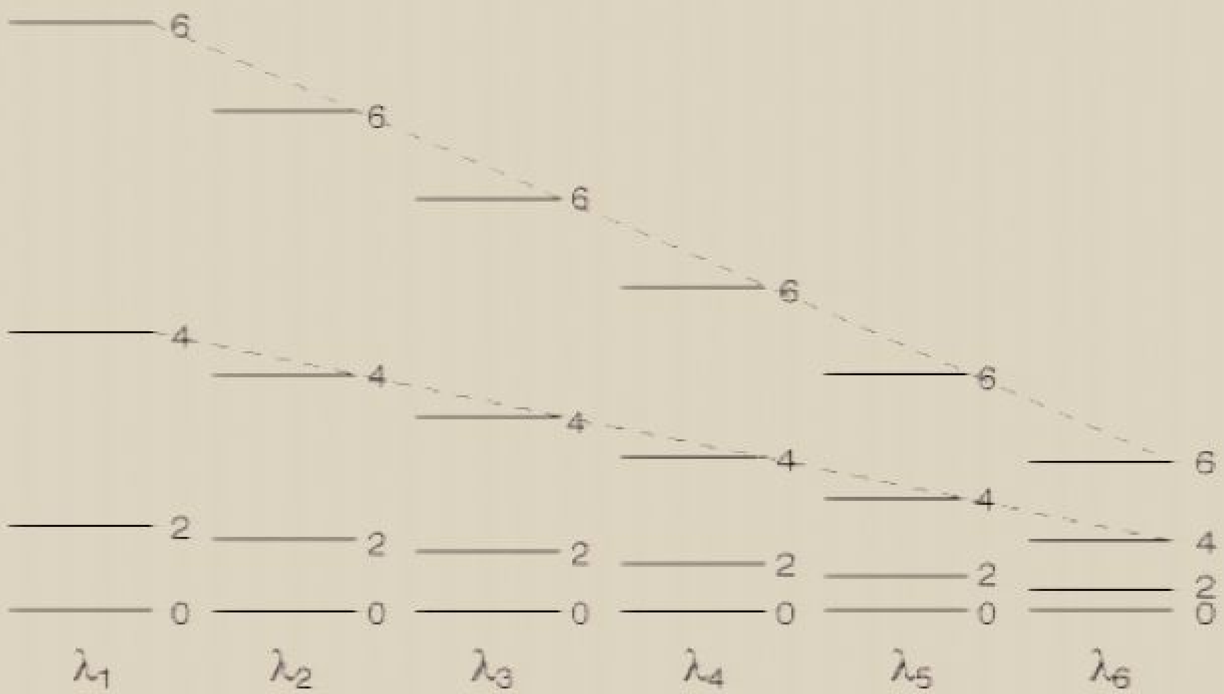


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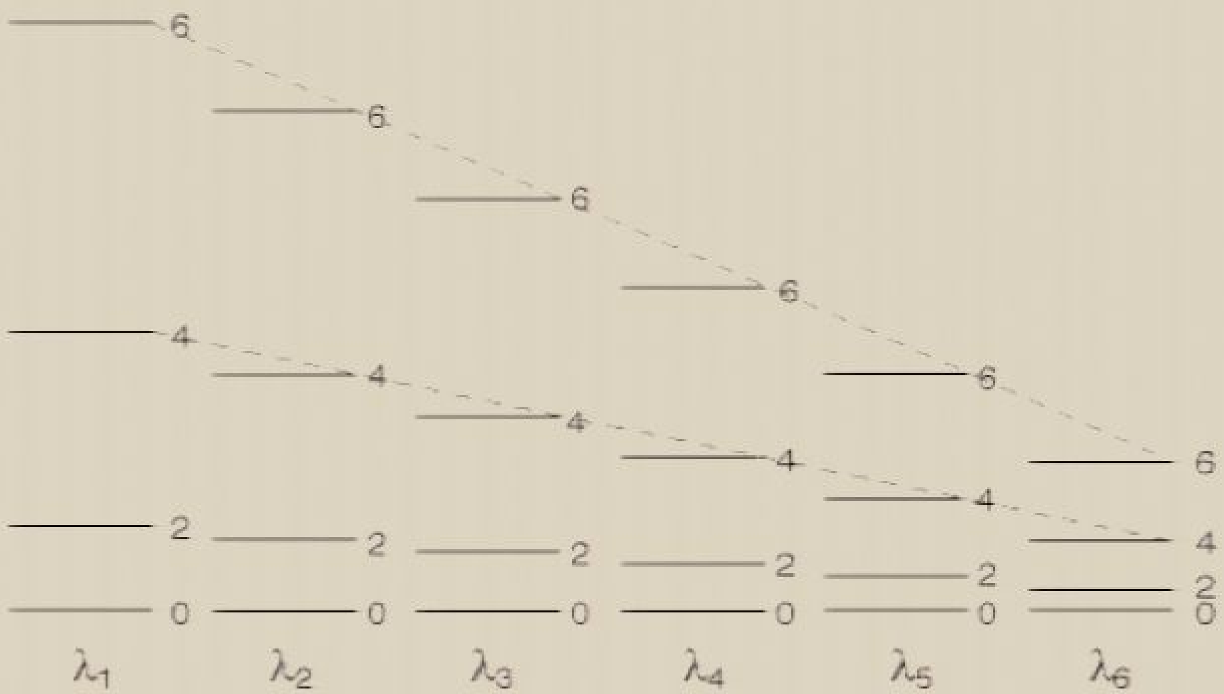


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# Properties of quasi-dynamical symmetry for a rotor

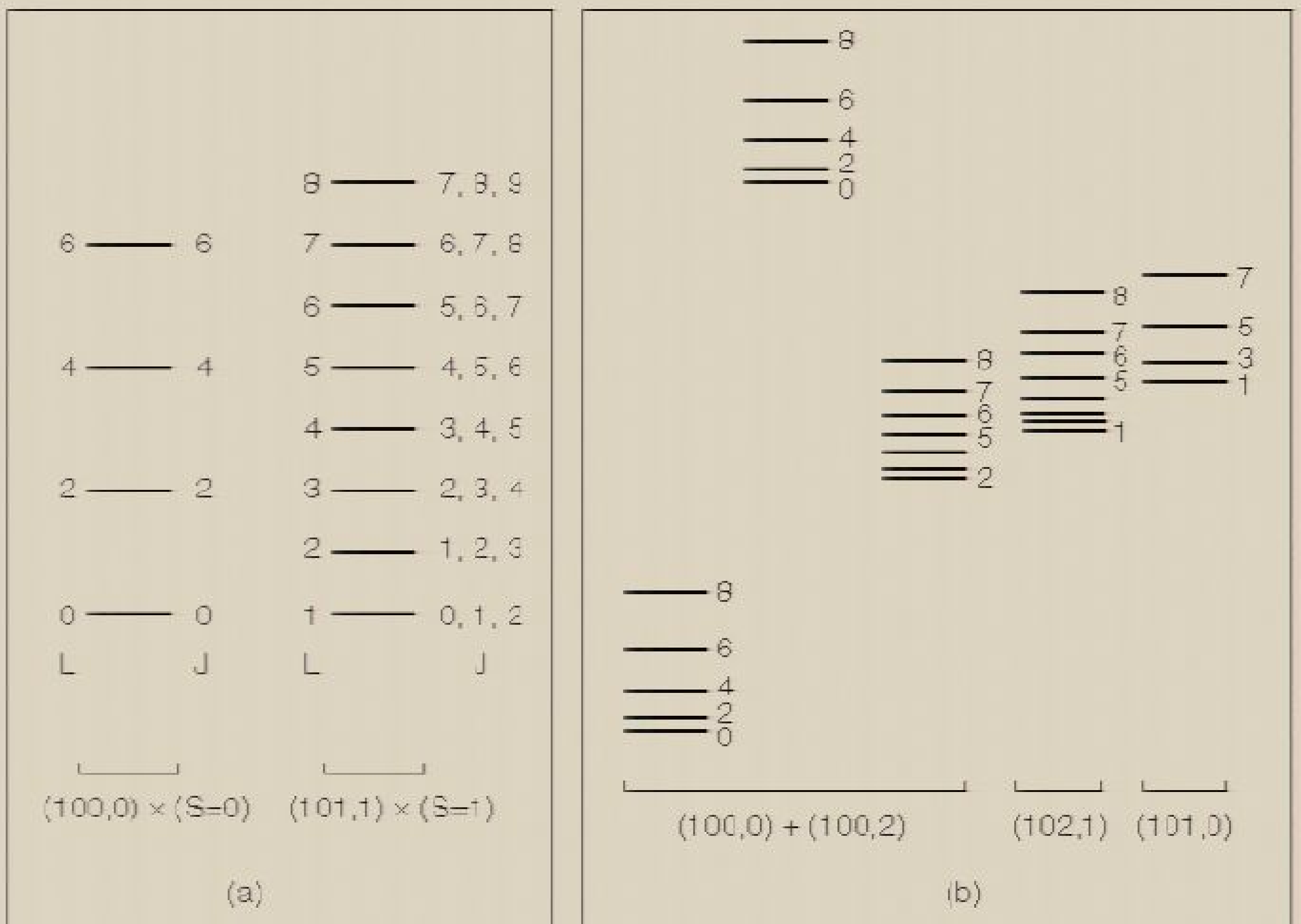
If we add a spin-orbit interaction to the rotor model, we expect that (if it is strong enough) the spin will become strongly coupled to the orbital rotational angular momentum:

If we add rotationally invariant interactions to the rotational model we may change its intrinsic structure but so long as the rotational motions remain adiabatic relative to the intrinsic excitations we will continue to get a soft rotational spectrum.

What breaks quasi-dynamical symmetry for a rotor are the Coriolis and the centrifugal forces which become negligible in the adiabatic limit.

SU(3) contracts to the rigid rotor algebra as $2\lambda+\mu$ becomes large.
Therefore we expect similar results for SU(3) in the limit.

# SU(3) mixing with a spin-orbit interaction




The spin-orbit mixed irreps are indistinguishable from pure irreps at the 1% level

# Summary

- Nature likes to hold onto its symmetries.
- The way in which it does so is demonstrated in a phase transition in which a system goes from one symmetry state to another with increase of a control parameter $\alpha$
  - As $\alpha$ is increased the symmetry is progressively distorted into a quasisymmetry.
  - Eventually when the symmetry-breaking interactions become too strong, the system gives up and goes into a transition region.
  - As $\alpha$ is further increased, the system emerges from the transition region with a new quasi-symmetry which becomes a real symmetry in the large $\alpha$ limit.
- Quasi-dynamical symmetries are probably present whenever a system appears to exhibit a symmetry.
- Quasi-dynamical symmetry shows up much more dramatically and more generally than we had anticipated.

How can we make use of it? Another talk!