Title: Quantum Information Theory 5

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Abstract:

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Quantum Information Science
at the University of Calgary

# Decoherence-free subspaces and spontaneous emission cancellation: necessity of Dicke limit

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Theory CANADA 2, Perimeter Institute
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# People



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# Outline

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- Why decoherence-free subspaces (DFS)?
- Dark states and spontaneous emission cancellation
- Decoherence-free subspaces
- A theorem about limitations of DFS
- Conclusions

# Why DFS?

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In quantum information processing qubits should be well isolated from their environment

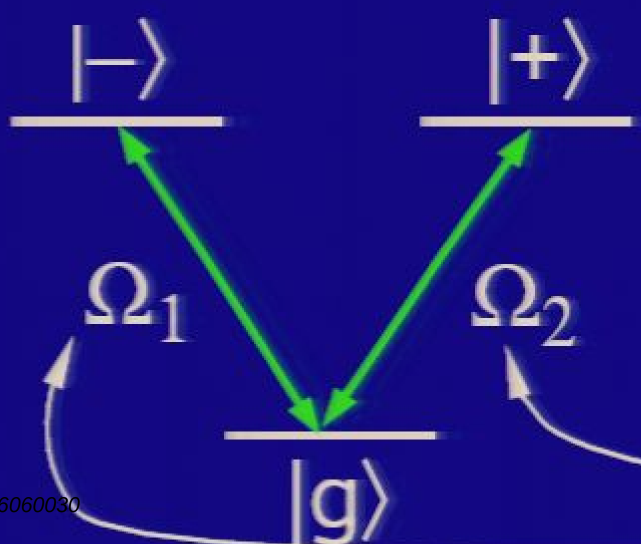
Decoherence-free subspaces consist of states that are not coupled to the environment

DFS were first suggested by Duan & Guo '97, Zanardi & Rasetti '97, Lidar, Chuang & Whaley '98

They could be used as an alternative/addition to quantum error correction

Decoherence-Free Subspaces appear if a particular superposition of states is not coupled to the environment

Consider a V-system with two excited states




The state

$$|D\rangle = \Omega_2 \,|-\rangle - \Omega_1 \,|+\rangle$$

is not coupled to the ground state by *this* particular super-position of laser light

# DFS

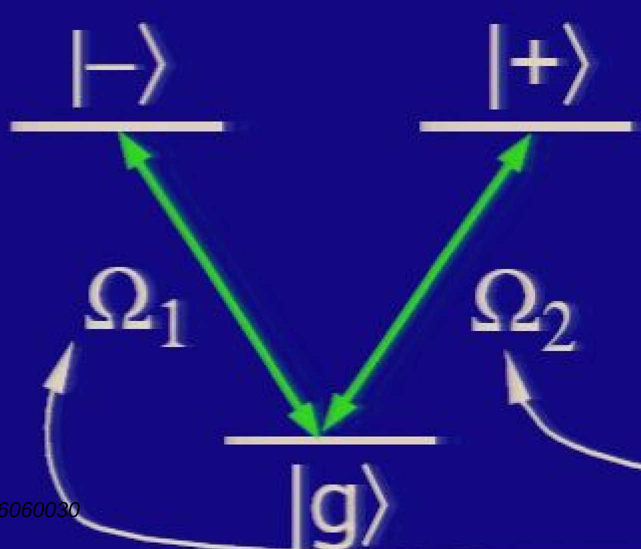


Hamiltonian: $H_{int} = |e\rangle \left( \Omega_1 \langle -| + \Omega_2 \langle +| \right) + H.c.$

$\Omega_i$ = light field amplitude

Obviously $H_{int}\,|D\rangle = H_{int}\,(\,\Omega_2\,|-\rangle - \Omega_1\,|+\rangle) = 0$

(very much like dark states)

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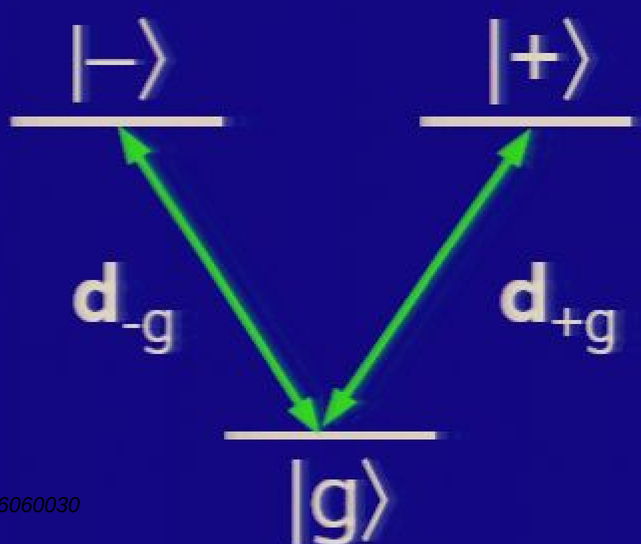
To be immune against decoherence $|D\rangle$ must not couple to *any* mode configuration

This is possible if the transition matrix elements are the same: $\mathbf{d}_{-g} = \mathbf{d}_{+g} \Leftrightarrow$
spontaneous emission cancellation [ Zhu & Scully '96]




However, selection rules forbid this for atoms

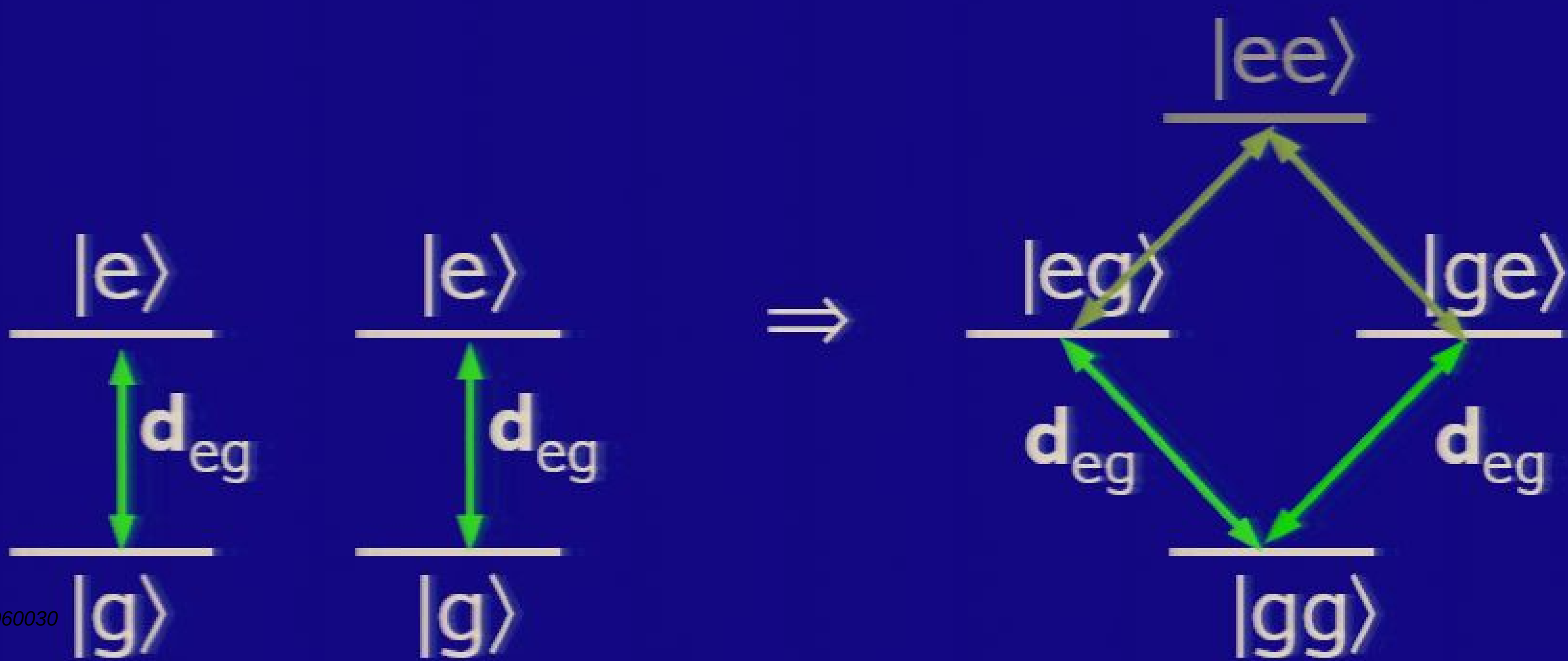
Molecules seem to be necessary, but ....

# DFS



... another trick to achieve such a cancellation is to use more than one atom. Consider two 2-level atoms

The energy eigenstates form a V-system with equal dipole matrix elements. $|eg\rangle - |ge\rangle$ forms a DFS



This procedure can be generalized to many N-level atoms [Duan & Guo '97, Zanardi & Rasetti '97, Lidar, Chuang & Whaley '98]

Theories usually employ master equations and Lie groups or Dicke states

However, there's a catch:

How far can the atoms be apart? Surely there's a limit

$|e\rangle$

$\mathbf{d}_{eg}$

$|g\rangle$

$|e\rangle$

$\mathbf{d}_{eg}$

$|g\rangle$

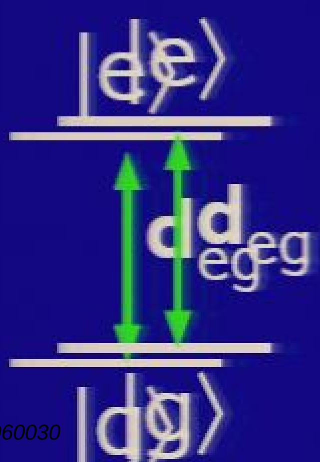
# DFS



We have proven that, under very general conditions, a real DFS can only be obtained in the Dicke limit

Dicke limit: particles are co-located

Main assumptions of the theorem:




- System is composed of particles with a finite-dimensional internal Hilbert space located at a fixed position
- Markovian reservoir (no memory)
- Reservoir invariant under translations
- Energy is exchanged in system-reservoir interaction

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Some details: $H_{int} = \Sigma_n \Sigma_i E^i(x_n) d_{i,n}$
with reservoir operators $E^i$ and system operators $d_{i,n}$

Translational invariance is used to employ the Lehmann representation for reservoir operators

$$\hat{E}^i(\mathbf{x}_0 + \mathbf{x}) \;=\; e^{i\hat{\mathbf{P}}\cdot\mathbf{x}/\hbar}\hat{E}^i(\mathbf{x}_0)e^{-i\hat{\mathbf{P}}\cdot\mathbf{x}/\hbar}$$

The Markovian master equation can the be written as

$$\dot{\rho} = -\hat{\Gamma}\rho - \rho\hat{\Gamma} - i[H,\rho] + \text{jump terms}$$

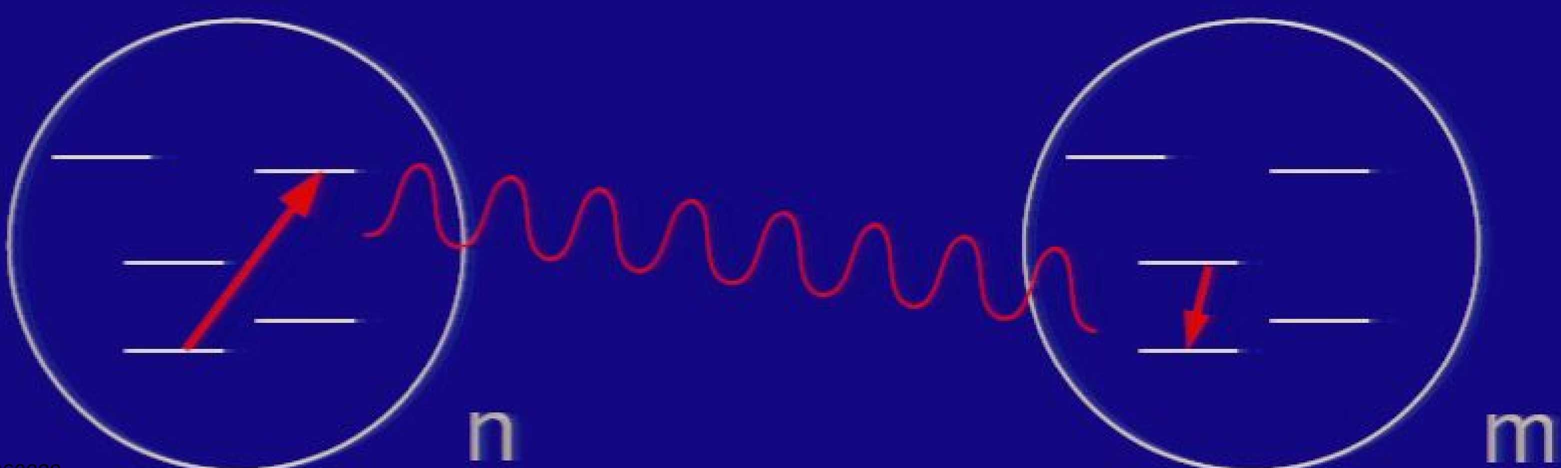
where the decoherence matrix $\hat{\Gamma}$ describes the (de-) excitation of any particle in the system

# DFS



Energy conservation (or time averaging) is needed to keep the master equation consistent [Dumcke & Spohn 1979]

Without it the system would gain or loose energy in processes like photon-exchange between two atoms.



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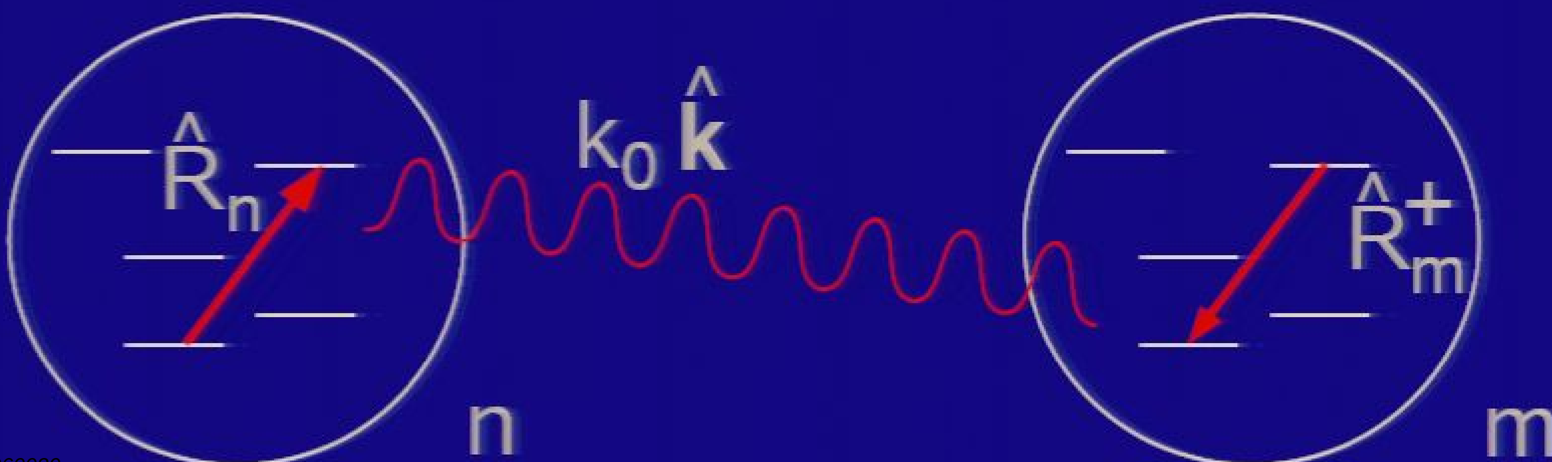
After lengthy calculations $\hat{\Gamma}$ can be written as

$$\hat{\Gamma} = \sum_{\Delta E} \int \mathrm{d}^2\hat{k} \sum_{n,m} e^{ik_0 \hat{\mathbf{k}}\cdot(\mathbf{x}_n - \mathbf{x}_m)} \hat{R}_n(\Delta E)\, \hat{R}^\dagger_m(\Delta E)$$

- $\hat{\mathbf{k}}$ is a unit vector (direction of the reservoir momentum)
- $\hat{R}_n(\Delta E)$ changes the energy of particle n by $\Delta E$



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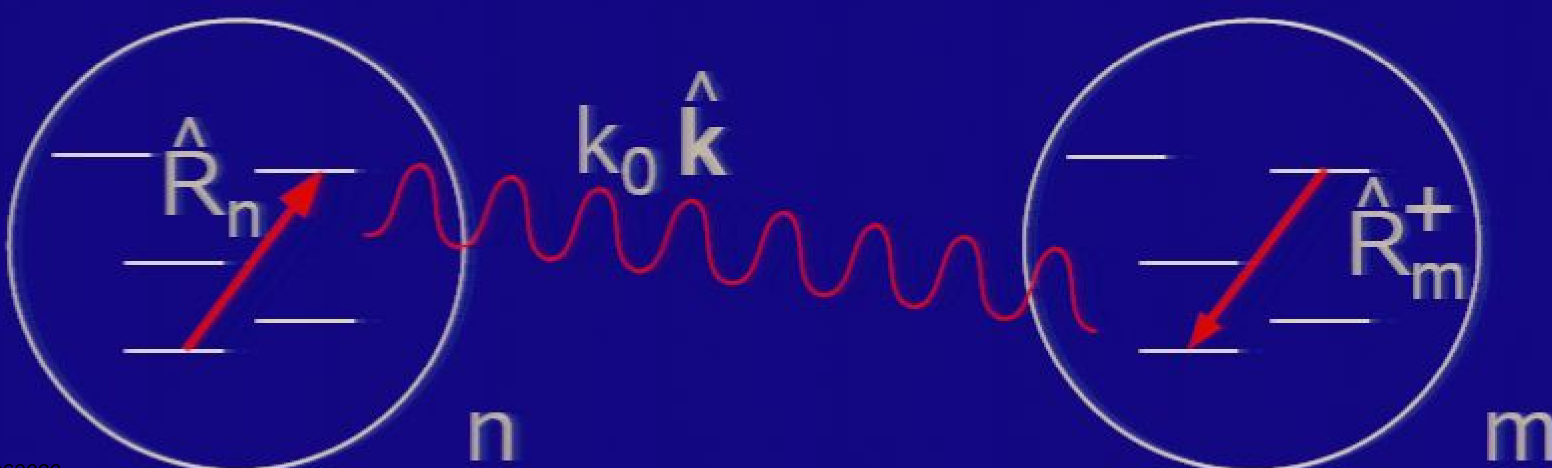
$$\hat{\Gamma} = \sum_{\Delta E} \int d^2\hat{k} \sum_{n,m} e^{ik_0 \hat{\mathbf{k}}\cdot(\mathbf{x}_n - \mathbf{x}_m)} \hat{R}_n(\Delta E)\, \hat{R}^\dagger_m(\Delta E)$$

Our proof exploits that $\hat{\Gamma}$ can only have zero eigenvalues for all reservoir modes $\hat{\mathbf{k}}$ if $\mathbf{x}_n = \mathbf{x}_m$, i.e., in the Dicke limit

Otherwise the integral will always contain nonzero parts

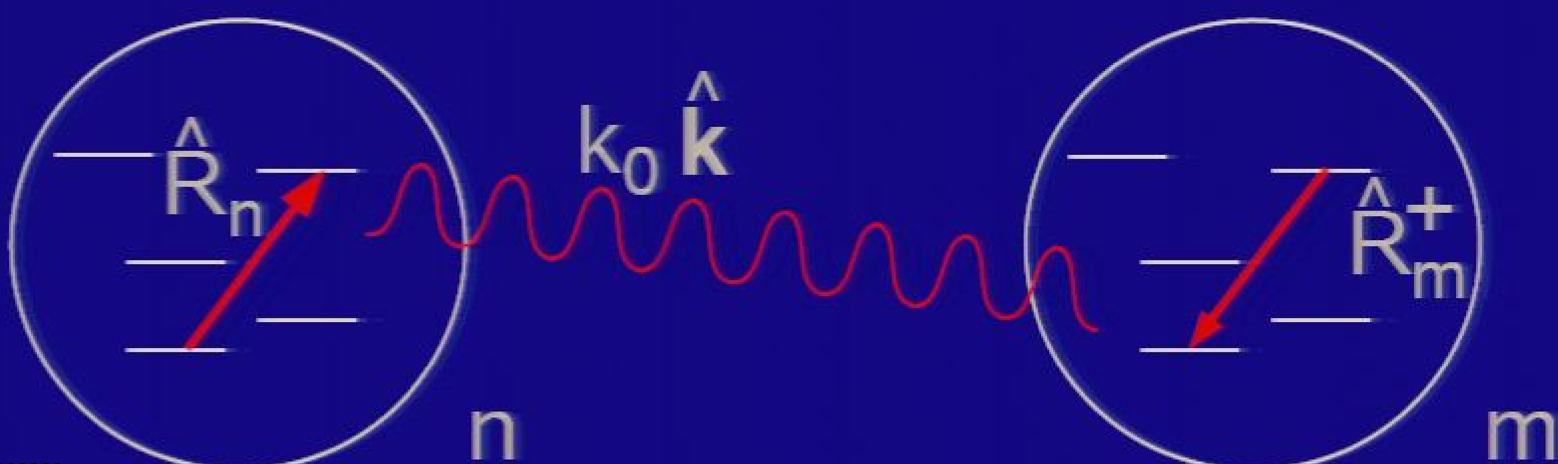


# DFS



## This result points out where DFS may be realized:

- Single-particle DFS (spontaneous emission cancellation)
- The integral disappears for a 1D reservoir. Waveguides may therefore allow for ordinary DFS outside the Dicke limit



# DFS

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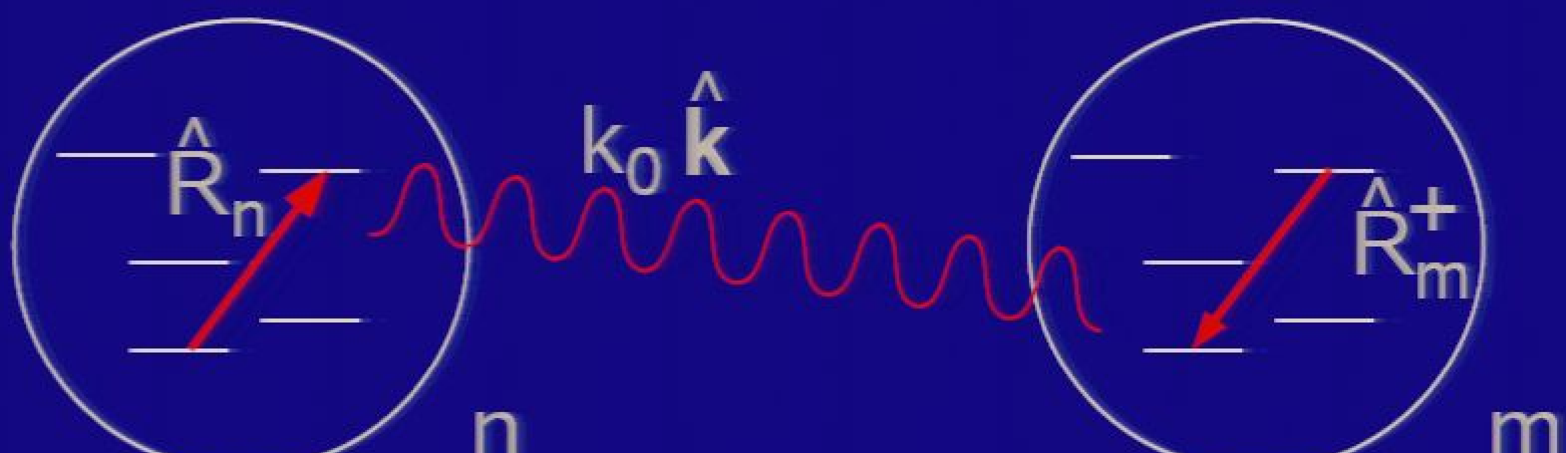

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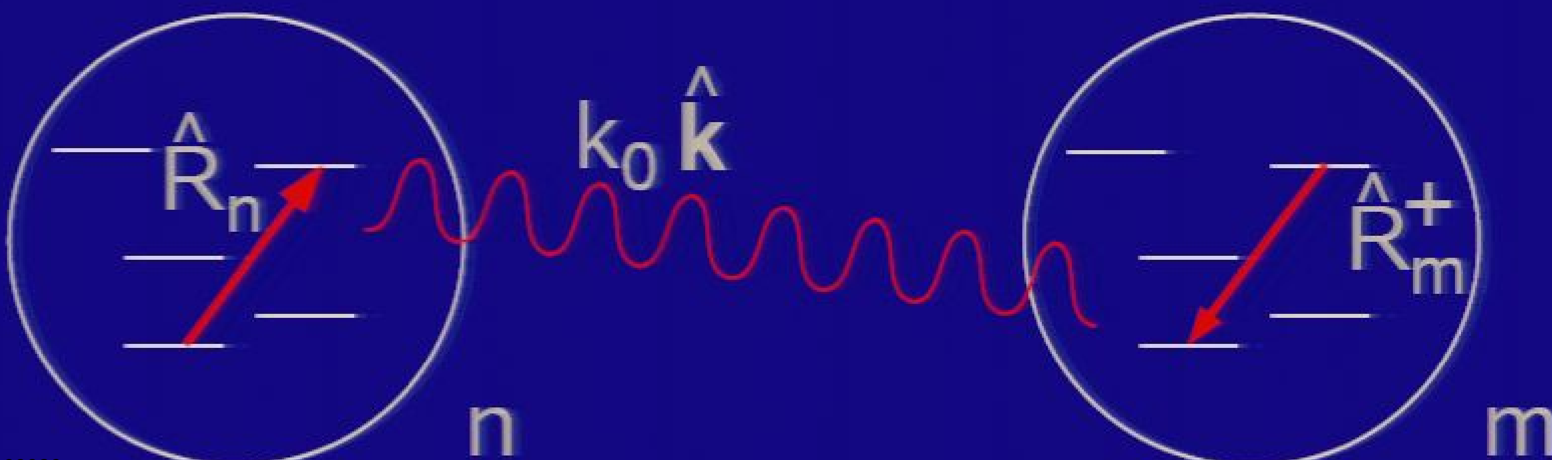

# Conclusions

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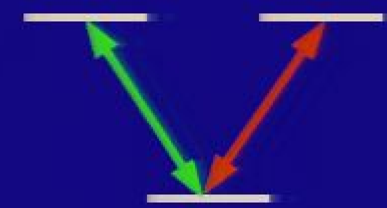
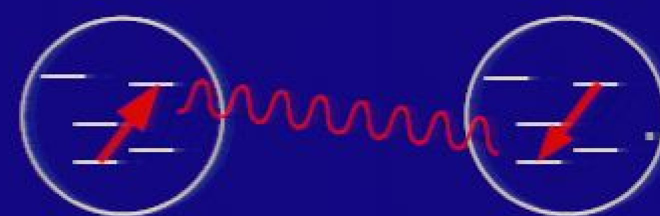
- DFS are a tool to suppress decoherence



- DFS can only exist in Dicke limit,



- single-particle DFS, or 1D reservoirs may be a way around this problem

# Thanks

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