

Title: Quantum Information Theory 3

Date: Jun 09, 2006 03:00 PM

URL: <http://pirsa.org/06060028>

Abstract:

Entanglement of zero angular momentum mixtures *and black hole entropy*

Danny Terno

joint work with

Etera Livine

Nucl. Phys. B **741**,
131 (2006)

Phys. Rev. A **72**,
022307 (2005)



Loop quantum gravity

Quantization of GR in 3+1 dimensions

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Action $S = \frac{1}{2k_M} \int d^4x \sqrt{|\det g|} R \quad \rightarrow \quad \Omega = d\omega + \omega \wedge \omega$

$$S(e_{\mu}^I, \omega_{\mu J}^I) = \frac{1}{4k_M} \int \varepsilon_{IJKL} e^I \wedge e^J \wedge \Omega^{KL} - \frac{1}{2\gamma k_M} \int e^I \wedge e^J \wedge \Omega_{IJ}$$

$$e_{\mu}^I e_{\nu}^J = g_{\mu\nu}$$

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Canonical variables

$$A_a^i = \Gamma_a^i - \gamma K_a^i$$

$$P_i^a = E_i^a / k\gamma$$

$$E_i^a E_j^a = h_{ij}$$

Poisson brackets $\{A_a^i(\mathbf{x}), P_j^b(\mathbf{y})\} = \delta_j^i \delta_b^a \delta(\mathbf{x} - \mathbf{y})$

Constraints: Gauss/gauge, vector/diffeo, scalar/Hamiltonian

Gauge freedom: $\mathfrak{su}(2)$

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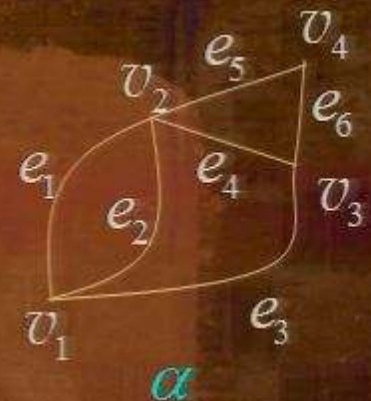
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Quantization

Configuration space: graphs & holonomies

$$\bar{A}(e_i) = e^{i \int_{e_i} A dx} \in \text{SU}(2)$$



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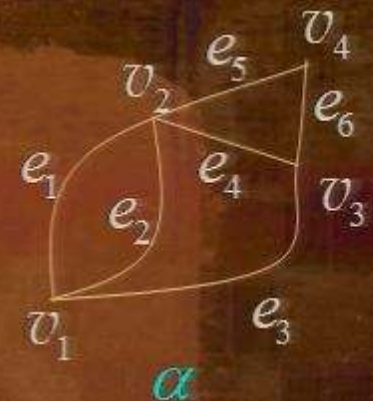
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Phase space: holonomies & fluxes

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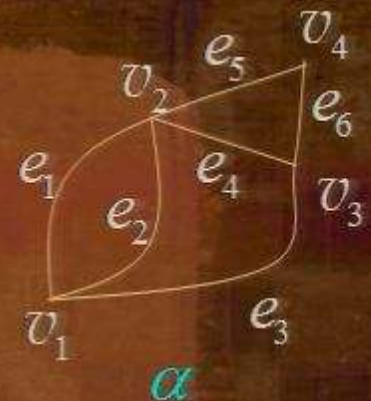
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Phase space: holonomies & fluxes

Hilbert space: functions of holonomies, basic operators

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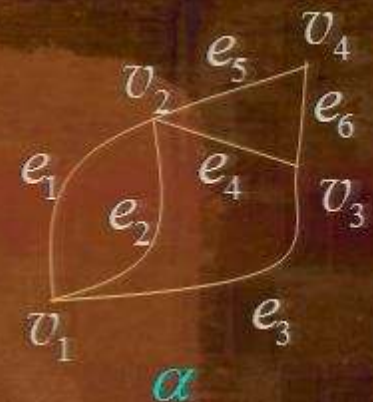
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Quantization

Configuration space: graphs & holonomies

$$\bar{A}(e_i) = e^{P \int_{e_i} A dx} \in \text{SU}(2)$$



More math

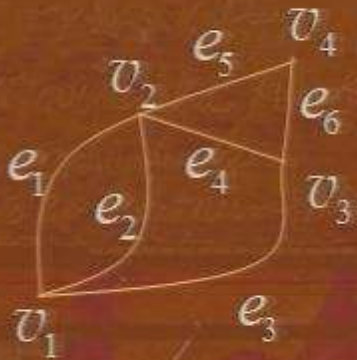
Phase space: holonomies & fluxes

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Decomposition: example

Peter-Weyl

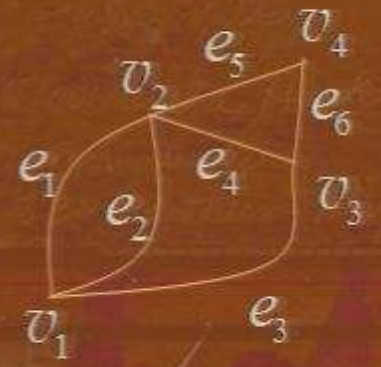
$$L^2(G, d\mu_H) = \bigoplus_j V_j \otimes V_j^* \equiv \bigoplus_j H_j$$



Decomposition $H = \bigoplus_{\alpha} H_{\alpha} = \bigoplus_{\alpha} H_{|\alpha|}$

Gauge & diffeomorphism constraints: $\mathbf{I} = 0$

Intertwiners:



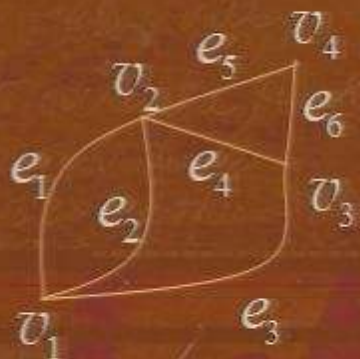
Coupling theory

spin networks

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Coupling theory

spin networks

Fixed graph:

$$L^2(SU(2)^E / SU(2)^V)$$



$$I_1 I_2 \dots I_V$$

contraction

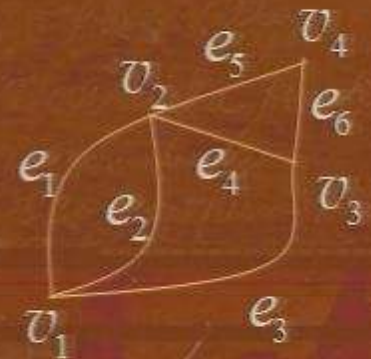
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Area operator

Cutting edges

Volume operator

Counting vertices

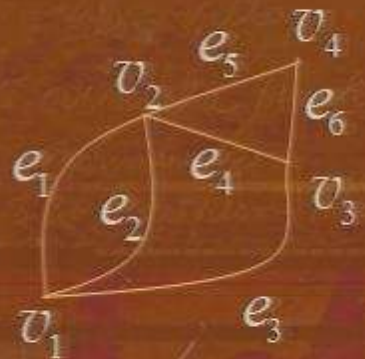
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Problems...

Black holes

PHILOSOPHICAL TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

L O N D O N.

VOL. LXXIV. For the Year 1784.

PART I.



L O N D O N,

SOLD BY LOCKYER DAVIS, AND PETER ELMSLY,
PRINTERS TO THE ROYAL SOCIETY.

MDCCLXXXIV.

42 Mr. MICHELL on the Means of discovering the

16. Hence, according to article 10, if the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

17. But if the semi-diameter of the sun, was that of the sun, though such a body, would always suffer some diminution of the said velocity, may be easily found to represent the semi-diameter of a sphere of the same density with the sun, from what has been the difference between the force of gravity, will be always proportional to the square of the distance after it has suffered all the diminution of the velocity from this cause; and the whole velocity of light above, will be the diminution of the velocity of light, account of it's gravitation, what less than a 497th part would have had if no diminution of the velocity of light had occurred.

2

[35]

VII. *On the Means of discovering the Distance, Magnitude, &c. of the Fixed Stars, in consequence of the Diminution of the Velocity of their Light, in case such a Diminution should be found to take place in any of them, and such other Data should be procured from Observations, as would be farther necessary for that Purpose.* By the Rev. John Michell, B. D. F. R. S. In a Letter to Henry Cavendish, Esq. F. R. S. and A. S.

Read November 27, 1783.

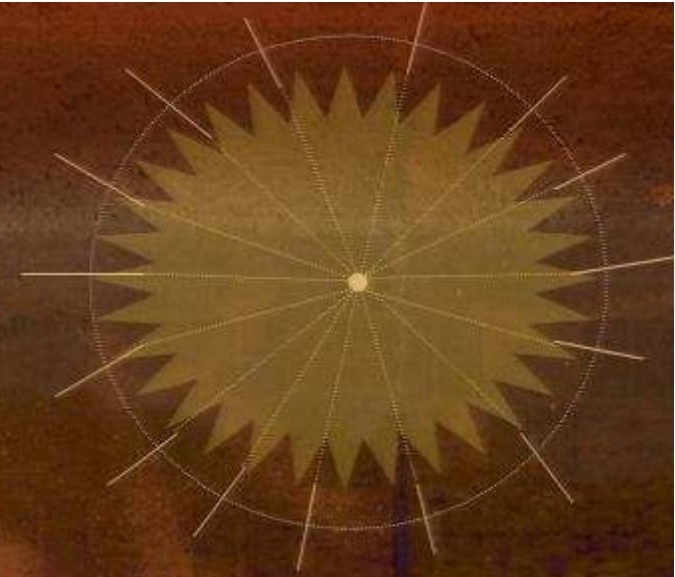
DEAR SIR,

Thornhill, May 26, 1783.

THE method, which I mentioned to you when I was last in London, by which it might perhaps be possible to find the distance, magnitude, and weight of some of the fixed stars, by means of the diminution of the velocity of their light, occurred to me soon after I wrote what is mentioned by Dr. PRIESTLEY in his History of Optics, concerning the diminution of the velocity of light in consequence of the attraction of the sun; but the extreme difficulty, and perhaps impossibility, of procuring the other data necessary for this purpose appeared to me to be such objections against the scheme, when I first thought of it, that I gave it then no farther consideration. As some late observations, however, begin to give us a little more chance of procuring some at least of these data, I thought it would not be amiss, that astronomers should be apprized of the method, I propose (which, as far as I know,

F 2

Black hole in LQG

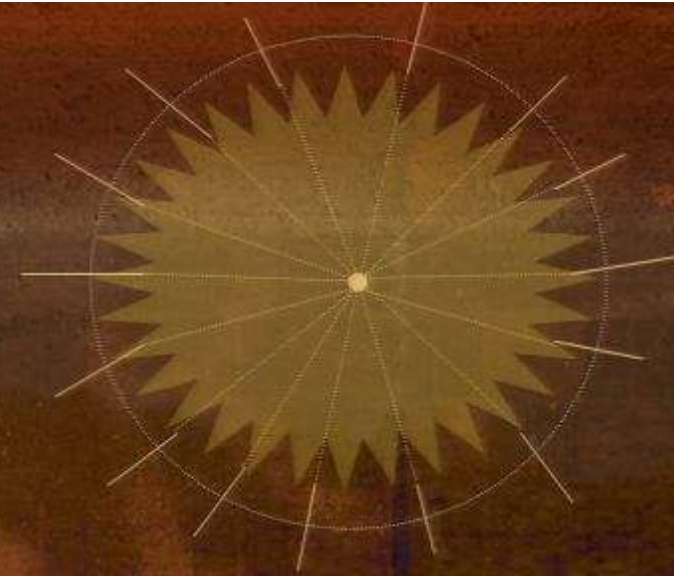


Black hole in LQG

Object: static black hole

Comment 1: no dynamics

Comment 2: closed 2-surface



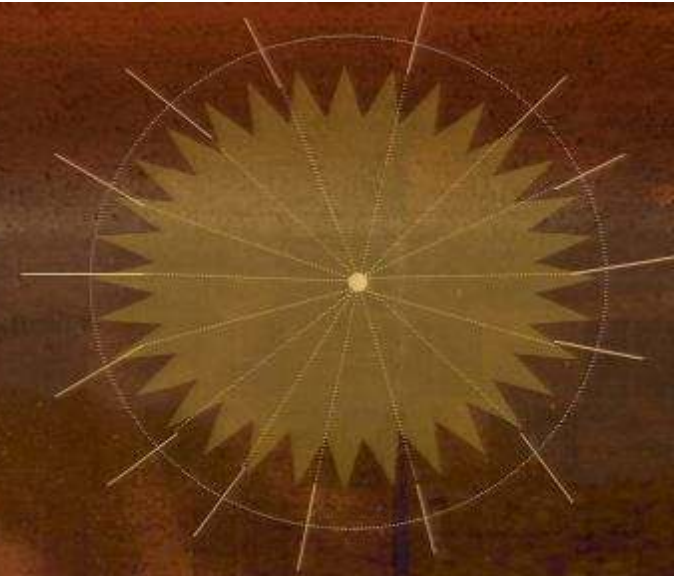
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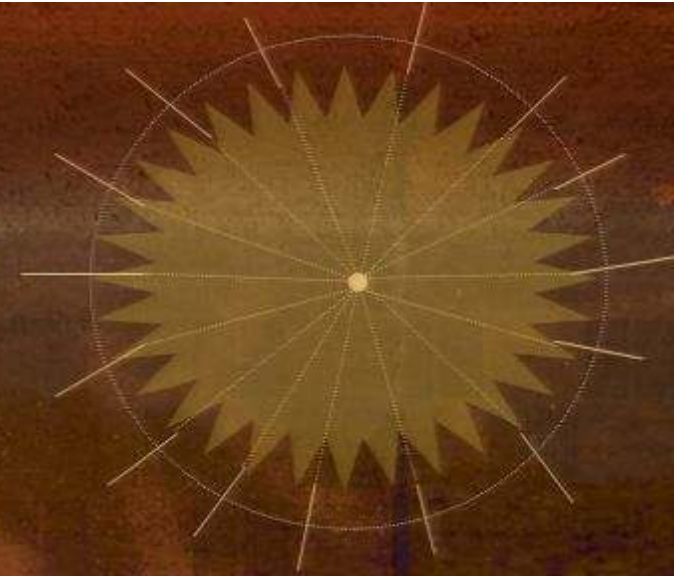
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States: spin network that crosses the horizon



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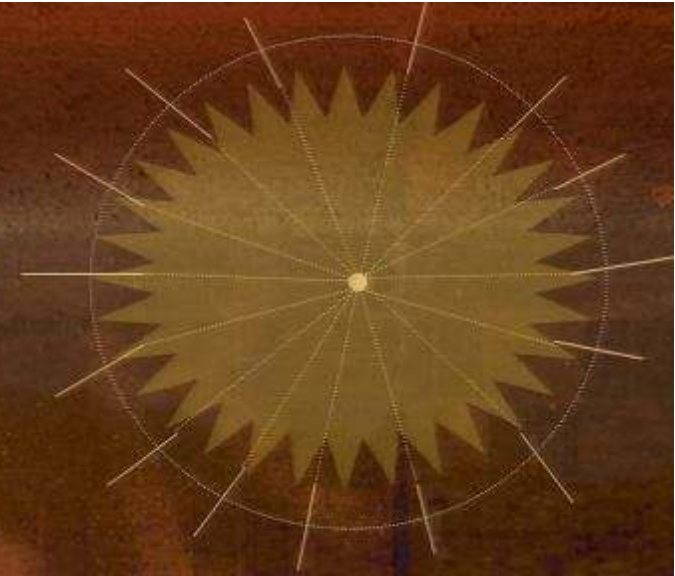
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Definition of a “black hole”:

complete coarse-graining of the spin network inside

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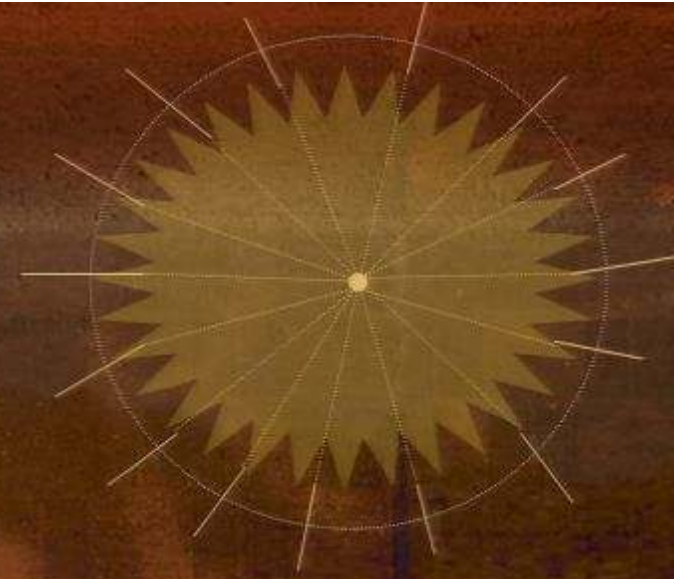
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Microscopic states: intertwiners $J = 0$

Features & assumptions

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The probing scale

$$j = \frac{1}{2}, 1, \frac{3}{2}, \dots$$

Area spectrum

$$a_j \propto \begin{cases} \sqrt{j(j+1)} \\ j + \frac{1}{2} \end{cases}$$

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We work at fixed j

The flow: scaling and invariance
of physical quantities

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For starters:
a qubit black hole

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Standard counting story

$2n$ spins

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Fancy counting story

density matrix $\rho = \frac{1}{N} \sum_{k=1}^N |\Psi_k\rangle \langle \Psi_k|$

entropy $S = -\text{tr} \rho \log \rho = \log N$

Combinatorics

Schur's duality $\otimes_{j=0}^{2n} \mathbb{C}^2 \cong \bigoplus_{j=0}^n H_j \equiv \bigoplus_{j=0}^n V^j \otimes \sigma_{n,j}$

$\sigma_{n,j}$ is the irrep $[n+j, n-j]$ of the permutation group

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$d_j^{(2n)} = \dim \sigma_{n,j} = \#$ irreps in the universal rep
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$$N = d_j^{(2n)}$$

$$H_0 \cong V^0 \otimes \sigma_{n,0}$$

Entanglement calculation

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$$S(\{\Psi\}) = \sum_i w_i S(\text{tr}_\bullet |\Psi_i\rangle\langle\Psi_i|)$$

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Entanglement calculation

- Alternative decomposition: linear combinations
- Its reduced density matrices: mixtures
- Entropy: concavity

$$\rho = \sum_{\alpha} w_{\alpha} |\Psi_{\alpha}\rangle \langle \Psi_{\alpha}|$$

$$|\Psi_{\alpha}\rangle = \sum_{j a_j b_j} c_{\alpha, j a_j b_j} |j\rangle_{AB} \otimes |a_j\rangle_{D^{A_j}} \otimes |b_j\rangle_{D^{B_j}}$$

Clever notation (2): $\lambda_{\alpha j a_j a'_j} = \sum_{b_j} c_{\alpha j a_j b_j} c_{\alpha j a'_j b_j}^* \quad \pi_{\alpha j} = \sum_{a_j} \lambda_{\alpha j a_j a_j}$

Clever notation (3): $\rho_A(\alpha) = \sum_{a_j} \pi_{\alpha j} \rho_j \otimes \Lambda_j(\alpha)$

Coup de grâce: $\sum_{\alpha} w_{\alpha} S(\rho_A(\alpha)) \geq \sum_{\alpha j} w_{\alpha} \pi_{\alpha j} [S(\rho_j) + S(\Lambda_j(\alpha))] \geq$

$$\sum_{\alpha j} w_{\alpha} \pi_{\alpha j} S(\rho_j) = \frac{1}{N} \sum_j c_j^A c_j^B S(\rho_j)$$



Entanglement

calculation:: distillation

$$\rho = \frac{1}{N} \sum_{j, a_j, b_j} |j\rangle_{AB} \langle j| \otimes |a_j\rangle_{\sigma_A} \langle a_j| \otimes |b_j\rangle_{\sigma_B} \langle b_j|$$



Bi-local non-destructive identification

Entanglement

calculation:: distillation

$$\rho = \frac{1}{N} \sum_{j, a_j, b_j} |j\rangle_{AB} \langle j| \otimes |a_j\rangle_{\sigma_A^j} \langle a_j| \otimes |b_j\rangle_{\sigma_B^j} \langle b_j|$$



Bi-local non-destructive identification

spin j singlet: $P(j) = \frac{d_A^j d_B^j}{N}$ $E(j) = \log(2j + 1)$

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and since $\langle S \rangle \geq E_F \geq E_D$

we are done

Entanglement

2 vs $2n-2$



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States

$$|0,0,a_0\rangle \otimes |0,0,b_0\rangle$$

degeneracy indices

$$\frac{1}{\sqrt{3}}(|1,-1,a_1\rangle \otimes |1,1,b_1\rangle - |1,0,a_1\rangle \otimes |1,0,b_1\rangle + |1,1,a_1\rangle \otimes |1,-1,b_1\rangle)$$

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Unentangled fraction $f_0 : \frac{1}{4}$

Entanglement $E(\rho | 2) : \frac{3}{4} \log 3$



n vs n



n vs n $E(\rho) : \frac{1}{2} \log n$



$$n \text{ vs } n \quad E(\rho) : \frac{1}{2} \log n$$

Entropy of the whole vs. sum of its parts

$$S(\rho) \leq S(\rho_A) + S(\rho_B)$$



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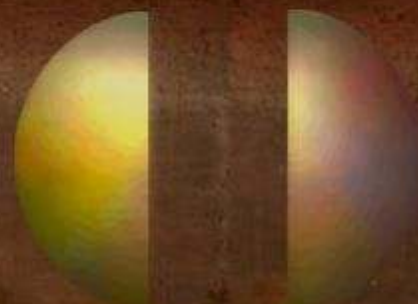
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$$S(\rho_A) = S(\rho_B) : n \log 2$$

$$S(\rho) : 2S(\rho_{\text{half}}) - 3E(\rho)$$



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BH is not made
from independent qubits,
but...

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Universality and the random walks

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Entropy

$$S(\rho) : 2n_j \log(2j + 1) - \frac{3}{2} \log n_j$$

Explanation: a random walk with a mirror

$$\frac{3}{2} \log$$

Open questions



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Dynamics: evolution of entanglement
dynamical evolution of evaporation
" $H=0$ " section & the number of states



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Semi-classicality: requiring states to represent
semi-classical BH
rotating BH



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Quantum info: is there anything beyond
local distinguishability?

