Title: Quantum Information Theory 1

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Abstract:

Probabilistic theories with

non-fixed causal structure

– A framework for Quantum Gravity

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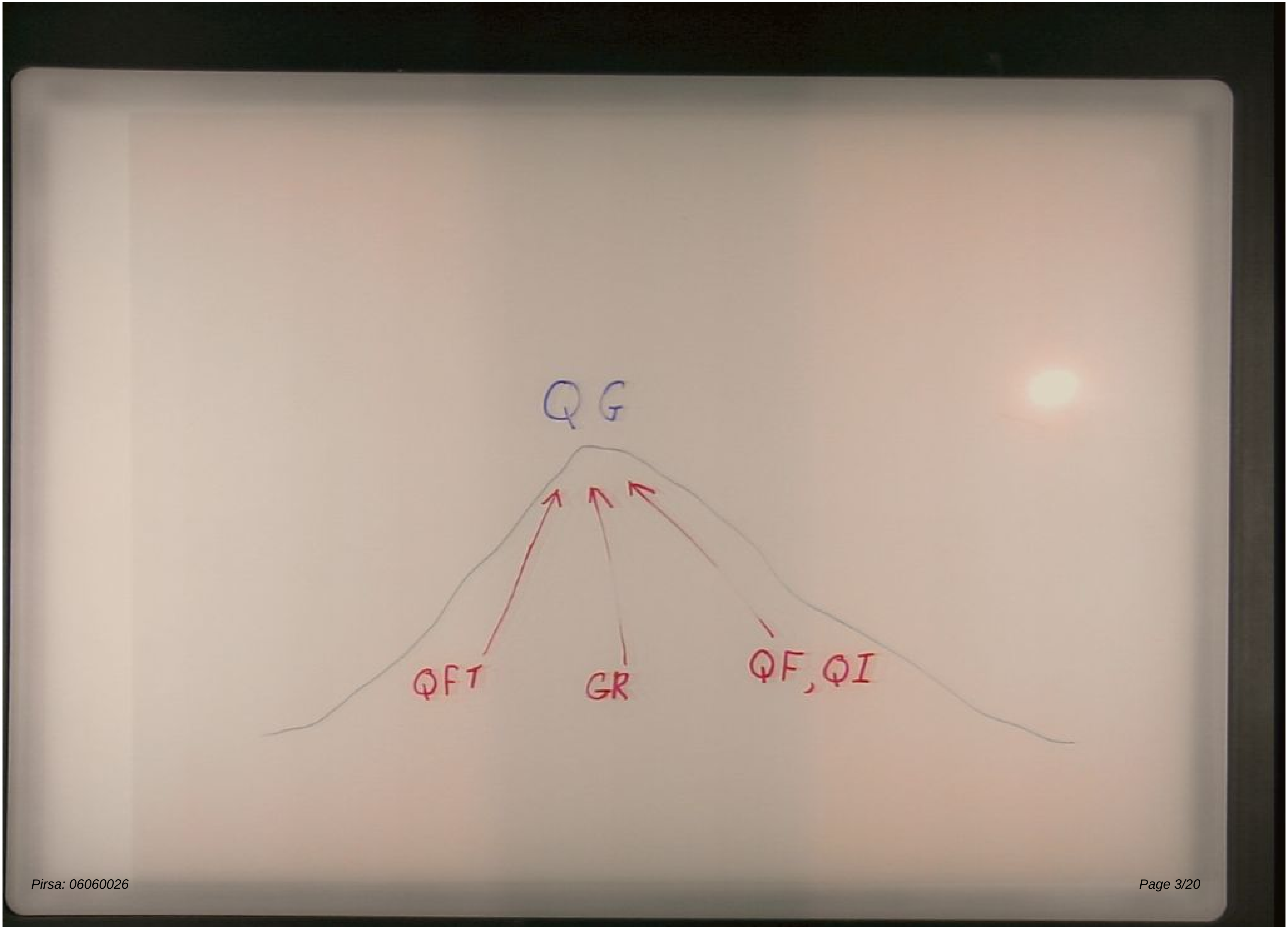
talk based on gr-qc/0509120

see also 5 Axioms work quant-ph/0111068


QG
QFT
GR
QF, QI

GR

δx^μ

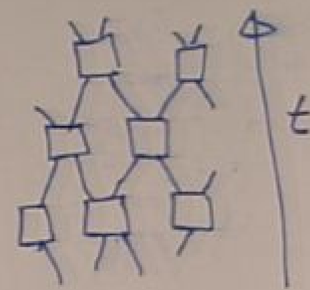
Causal structure (time like or space like) not fixed in advance

Must solve Einstein's field eqns.

$\Rightarrow$ $g_{\mu\nu}$

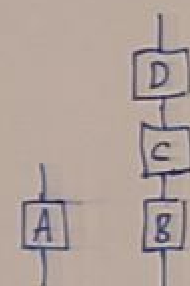
$\Rightarrow$ causal structure.

QT



have background time t

$U(t)|\psi(0)\rangle$



	causal-structure	compression
$\hat{A} \otimes \hat{B}$	space-like	none
$\hat{C}\,\hat{B}$	time-like	yes
$[D?B]$	time-like (with gap)	none

Combine radical elements.

GR

deterministic

non-fixed causal structure

QT

fixed causal structure

probabilistic.

The causaloid framework

probabilistic

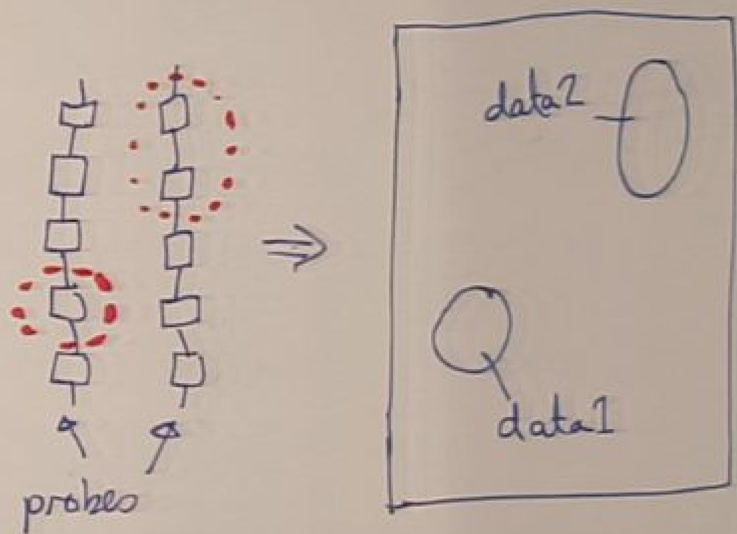
non-fixed causal structure

QT $\Longrightarrow$ The causaloid framework

GR? $\Longrightarrow$ The causaloid framework

The causaloid framework $\Longrightarrow$ QG?

Assertion: A physical theory, whatever else it does, must correlate recorded data.



Is

prob(data2 | data1)

well defined?

If so what is it?

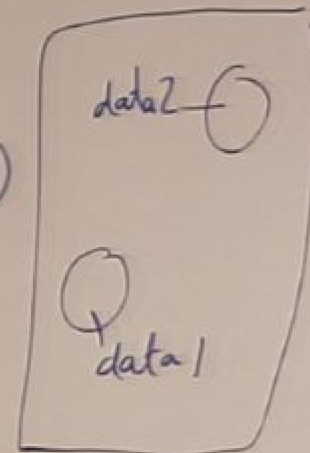
Usual approaches

1) time evolution

2) solve local eqns.

3) histories.

prob(data2|data1)



New approach.

Write down expressions which directly tell us

(a) if prob(data2 | data1) is well defined

(b) If it is well defined then write an expression which says what it is equal to.

Ability to make predictions is not
referred to some fixed causal background.

Key idea!: compression due to physical theory

Example: Spin half particle

$$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \iff \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

$$a = p_{x+} - i\,p_{y+} - \frac{(1-i)}{2}\left(p_{z+} + p_{z-}\right)$$

Can we $\underline{p}$ to calculate spin along arbitrary dir$^{\underline{n}}$ $\hat{\underline{n}}$

$$\begin{pmatrix} \vdots \\ p_{\hat{\underline{n}}+} \\ \vdots \end{pmatrix}_{\alpha} \xrightarrow{\text{linear compression}} \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

Fiducial set (l)

The physical theory (QT) compresses down to just four probabilities in this case.

In QT

$$\text{prob} = \operatorname{tr}(\hat{A}\,\hat{\rho})$$

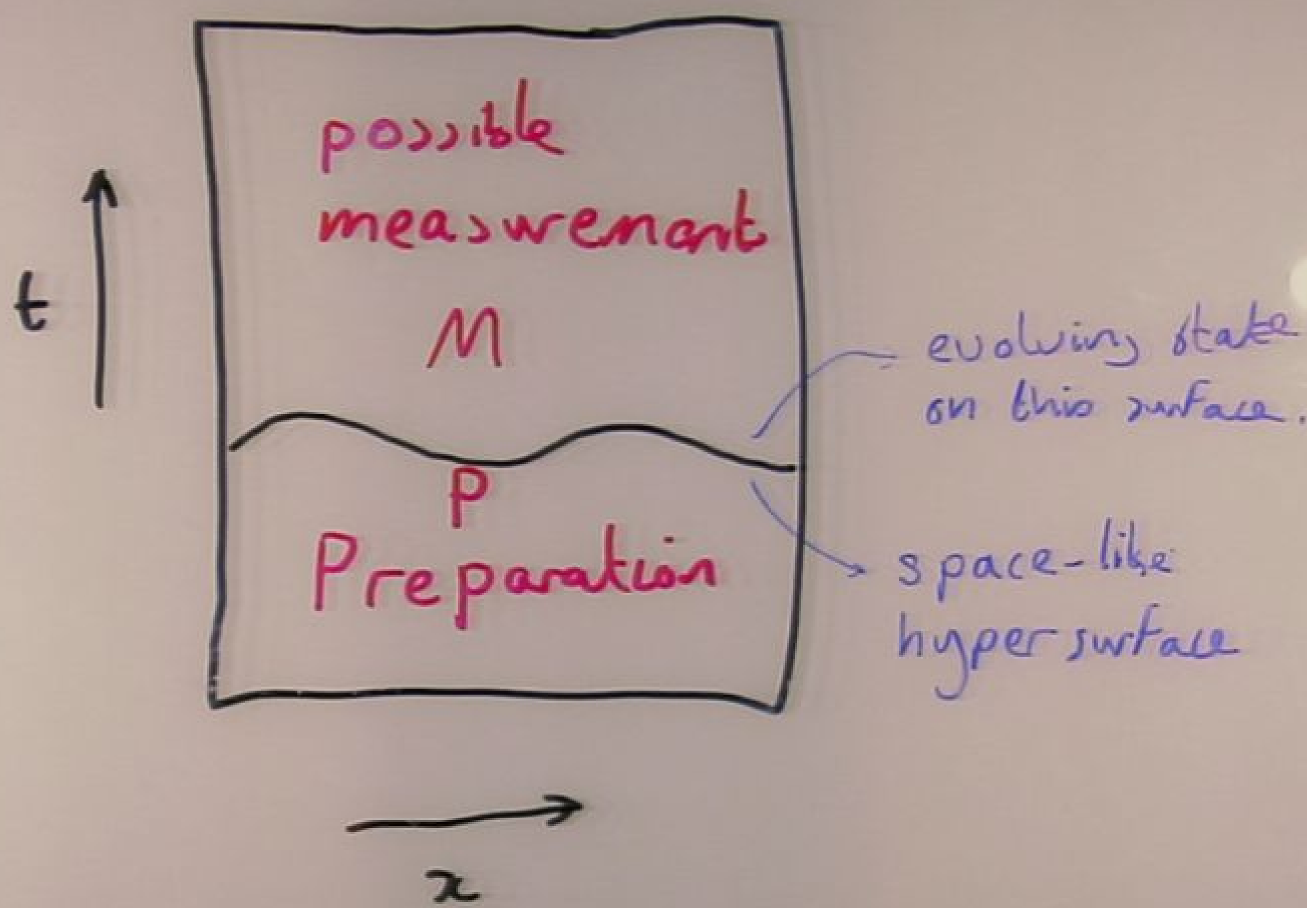
$$= \underline{\Gamma} \cdot \underline{p}$$

$$\hat{A} \iff \underline{\Gamma}$$

$$\hat{A} \otimes \hat{B} \qquad \hat{A}\hat{B} \qquad [\hat{A} ? \hat{B}] \iff \underline{\Gamma}_A \overset{\wedge}{\otimes} \underline{\Gamma}_B$$

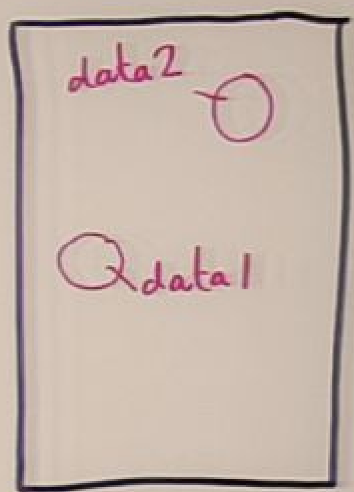
The causaloid product.

Usual view


possible
measurement
M
P
Preparation
t
x
evolving state
on this surface.
space-like
hyper surface

New approach: Find eqns that directly correlate data

(without imposing any particular fixed causal structure)



Want to be able to calculate

$$\text{prob}(\text{data2} \mid \text{data1})$$

directly, if it is well defined, for any data1 and data2.

Have a framework for this

$\Rightarrow$ The causaloid formalism

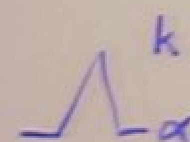
Three levels of linear compression

st level

State (P) $\Longleftrightarrow \begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \\ \vdots \end{pmatrix} \Longleftrightarrow \underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{pmatrix}$

$$p_\alpha = \underline{r}_\alpha \cdot \underline{p}$$

determine



First level compression matrices.

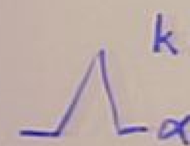
Three levels of linear compression

First level

$$\text{State } (P) \iff \begin{pmatrix} \vdots \\ p_\alpha \\ \vdots \\ \vdots \end{pmatrix} \iff \underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{pmatrix}$$

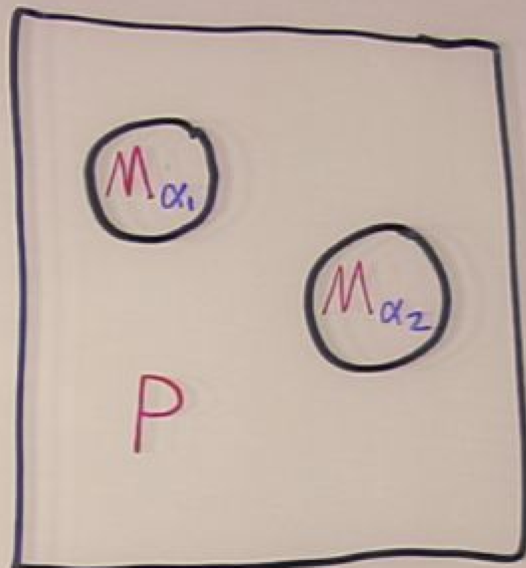
$$p_\alpha = \underline{r}_\alpha \cdot \underline{p}$$

determine



First level compression matrices.

Second level linear compression.



$$\text{state} \Leftrightarrow \begin{pmatrix} \vdots \\ P_{\alpha_1,\alpha_2} \\ \vdots \end{pmatrix} \Longleftrightarrow \underline{P} = \begin{pmatrix} \vdots \\ P_{k_1 k_2} \\ \vdots \end{pmatrix}$$

$$P_{\alpha_1,\alpha_2} = \underline{\Gamma}_{\alpha_1,\alpha_2} \cdot \underline{P}$$

$$\underline{\Gamma}_{\alpha_1,\alpha_2}\Big|_{k_1,k_2} = \sum_{l_1,l_2} \underbrace{\Lambda^{k_1\,k_2}_{l_1,\,l_2}} \Lambda^{l_1}_{\alpha_1} \Lambda^{l_2}_{\alpha_2}$$

second level linear compression matrix.

Multi region composites also at second level.

Second level $\Rightarrow$ causaloid product

$$\underline{\Gamma}_{\alpha_1\alpha_2}\Big|_{k_1 k_2} = \sum_{l_1 l_2} \Lambda^{k_1 k_2}_{l_1 l_2} \; \underline{\Gamma}_{\alpha_1}\Big|_{l_1} \; \underline{\Gamma}_{\alpha_2}\Big|_{l_2}$$

$$\underline{\Gamma}_{\alpha_1\alpha_2} = \underline{\Gamma}_{\alpha_1} \otimes^{\Lambda} \underline{\Gamma}_{\alpha_2}$$

2nd level compression $\Leftrightarrow$ causal adjacency.

Third level. (actually not strictly linear)

Have

$$\begin{bmatrix} \Lambda^{k}_{\alpha} & \text{first level} \\ \Lambda^{k_1 k_2}_{l_1 l_2} & \\ \Lambda^{k_1 k_2 k_3}_{l_1 l_2 l_3} & \\ \vdots & \end{bmatrix} \overset{\text{compress}}{\Longrightarrow} \begin{bmatrix} \Lambda(j) \\ \text{plus} \\ \text{RULES} \end{bmatrix}$$

second level

the causaloid

To make predictions.

Define

$$\underline{v} = \underline{\Gamma}_{data2} \otimes \underline{\Gamma}_{data1}$$

$$\underline{u} = \left(\underline{\Gamma}_{data2} + \underline{\Gamma}_{\overline{data2}} \right) \otimes \underline{\Gamma}_{data1}$$

$\text{prob}(\text{data}2 \mid \text{data}1) = p$

p well defined iff $\underline{v}$ parallel to $\underline{u}$

Then $\underline{v} = p\,\underline{u} \Rightarrow$ gives p.

The Causaloid Formalism

The causaloid (theory dependent

Some basic equations (theory independent)

can predict all probabilit like

$$\text{prob}(X_{R_1} \mid X_{R_2}, F_{R_1}, F_{R_2})$$

The causaloid formalism.

1) treats all pairs of elementary regions on an equal ~~footi~~ footing – it provides a unification of $\hat{A} \otimes \hat{B}$ and $\hat{A}\hat{B}$ in Q.T.

2) We can formulate Q.T. in this formalism

3) We can hope to formulate GR in framework.

4) Provides a mathematical setting for QG.