Title: Strings/Quantum Gravity 5

Date: Jun 08, 2006 03:50 PM

URL: http://pirsa.org/06060016

Abstract:

1. Discreteness *can respect* Lorentz-transformations

   (Kinematic randomness plays a role – Poisson processes)

2. But locality must be abandoned

   Implies radical nonlocality at fundamental level (*micro*-scale l)

3. One can recover locality approximately at large scales (*macro*-scale)

4. But residual nonlocality survives at *intermediate* length-scales (*meso*-scale, below λ_0)

5. An effective meso-theory would be *continuous* but *nonlocal*

Illustrate these claims with scalar field ϕ on a *fixed* causet C:

Recovery of $\Box\phi$.

($\delta\Lambda$ is also a nonlocal effect of discreteness; I'll not discuss it)

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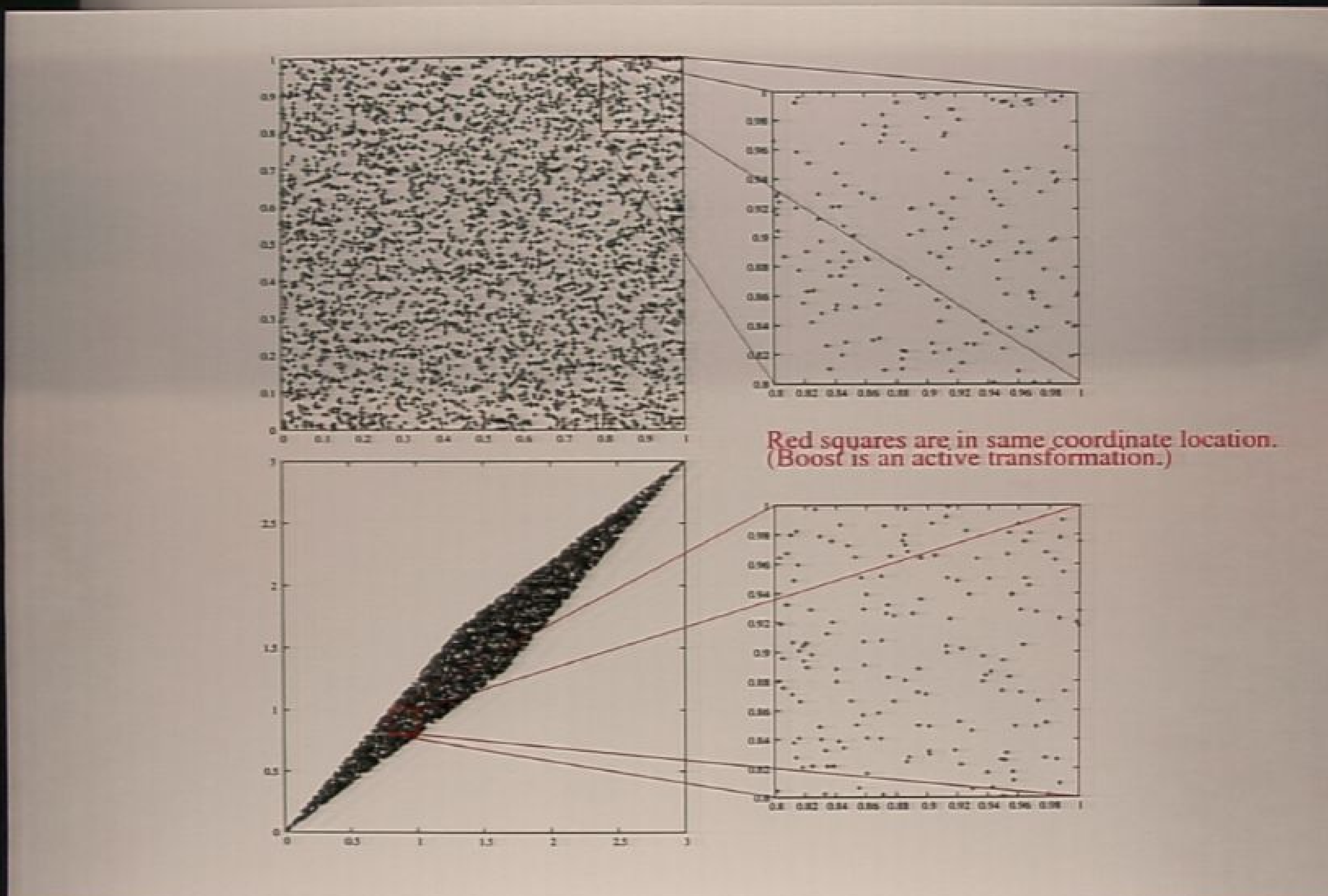
Red squares are in same coordinate location.
(Boost is an active transformation.)

A theorem on Poisson processes

Ω = space of all sprinklings of $\mathbb{M}^d$ (sample space)

Poisson process induces a measure μ on Ω

Let f be a rule for deducing a direction from a sprinkling
$f : \Omega \to H$ = unit vectors in $\mathbb{M}^d$

Require f *equivariant* ($f\Lambda = \Lambda f$, $\Lambda \in$ Lorentz)

Assume that f is measurable (hardly an assumption)

THEOREM *No such f exists*
(not even on a partial domain of positive measure)

(So with probability 1, a sprinkling will not determine a frame.)

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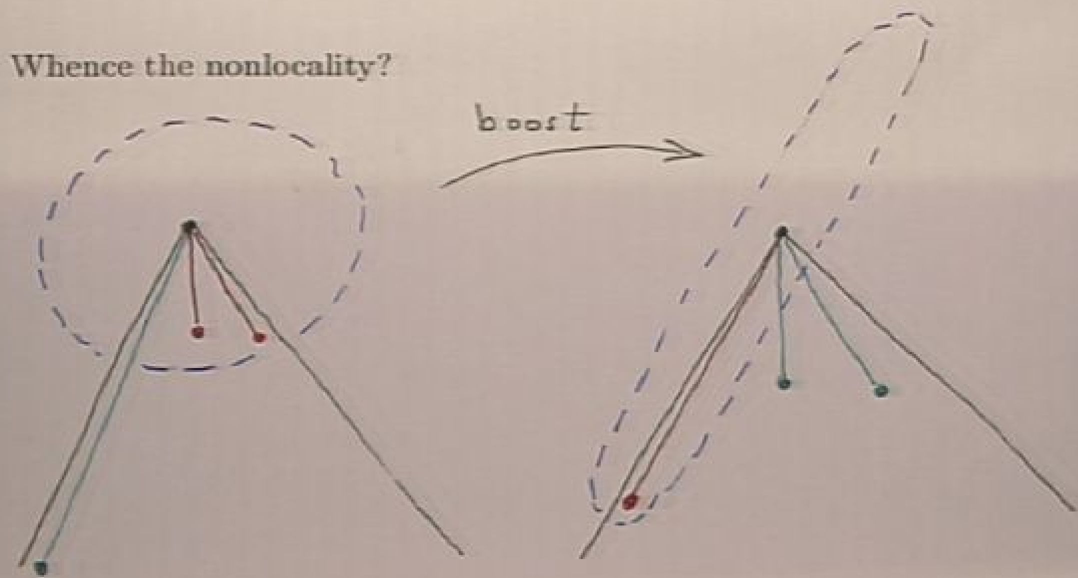
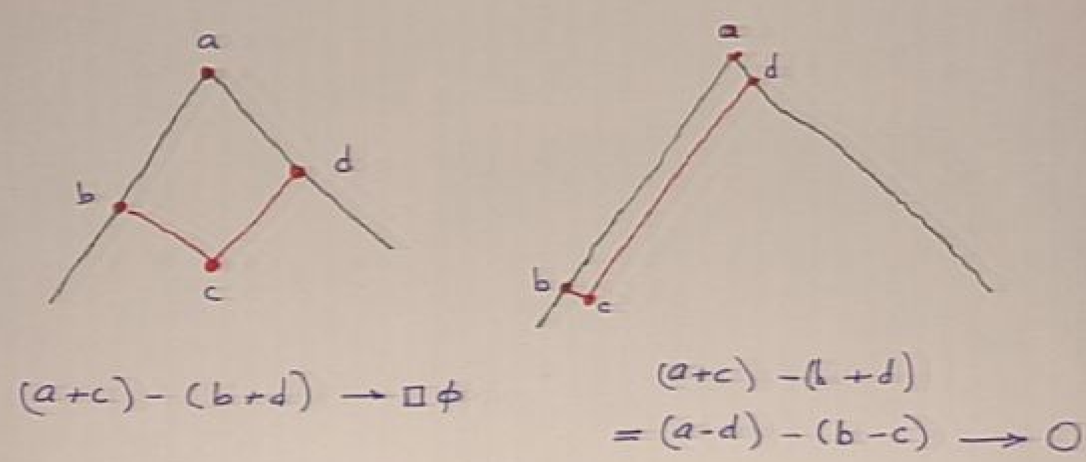


Needs a miracle. (consider eg $\phi = t^2 - x^2$, invariance $\Rightarrow \infty$?)



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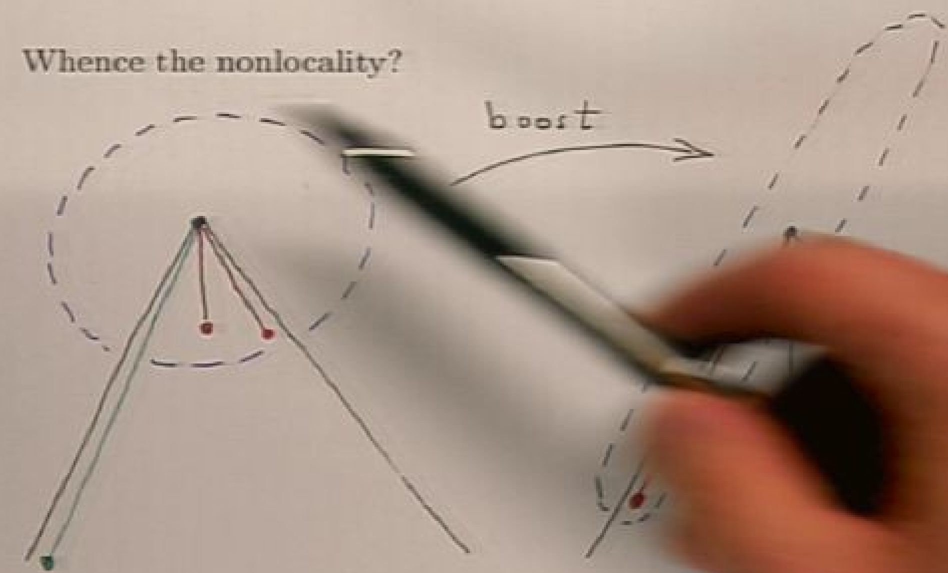

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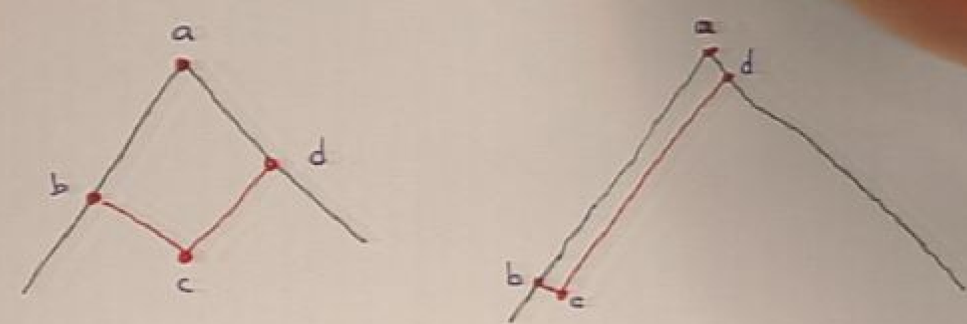

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$(a+c) - (b+d)$
$= (a-d) - (b-c) \longrightarrow 0$

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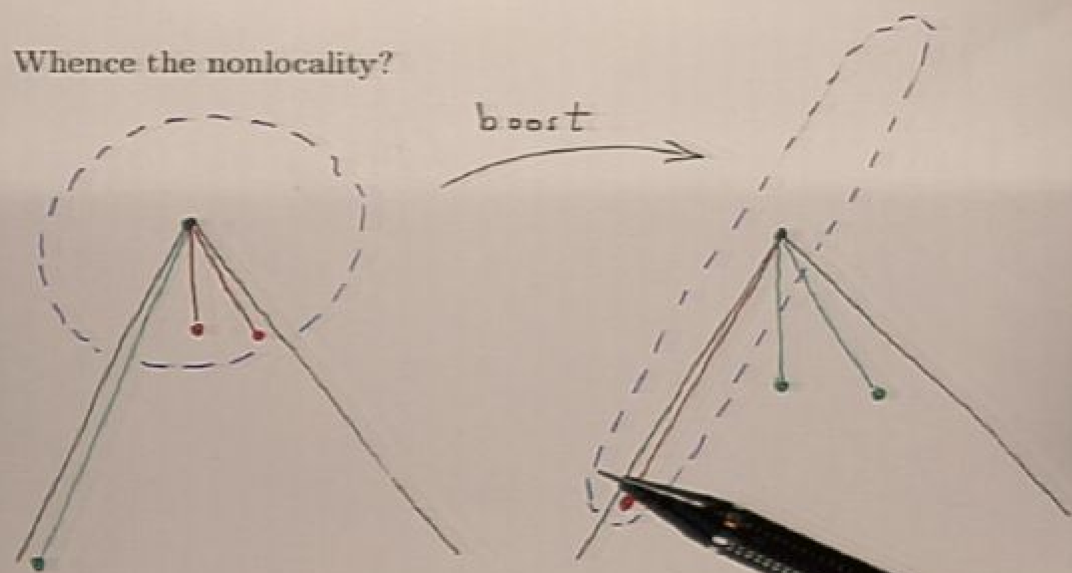

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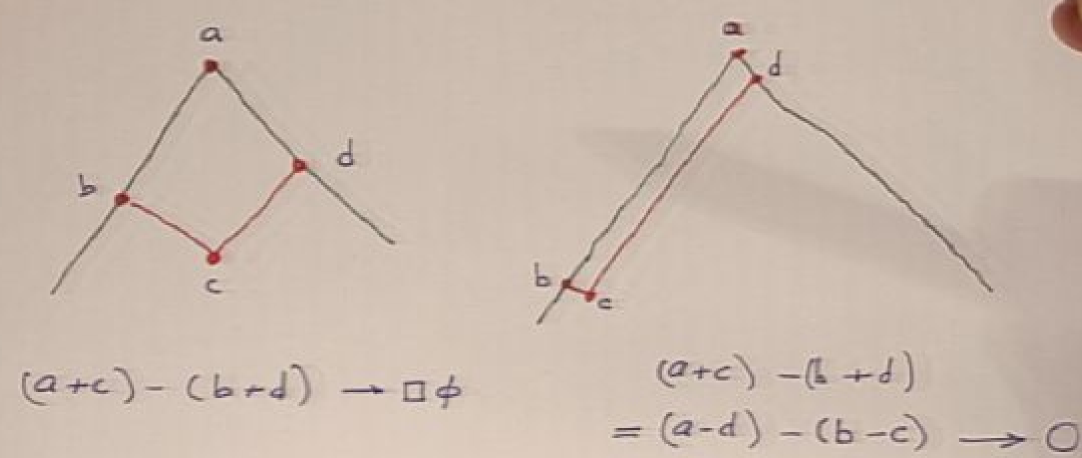

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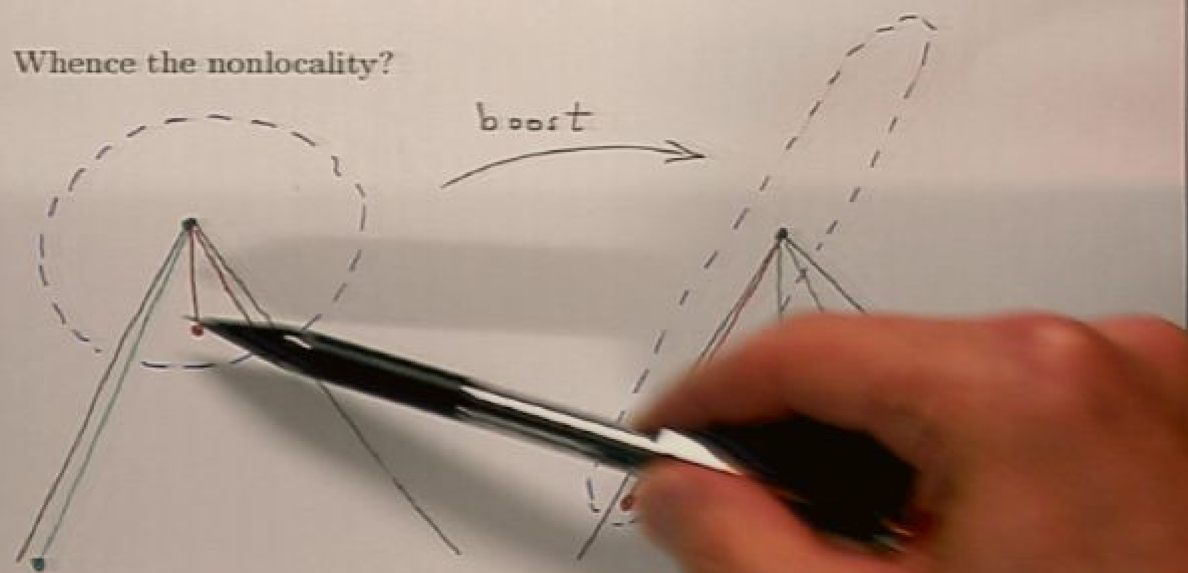

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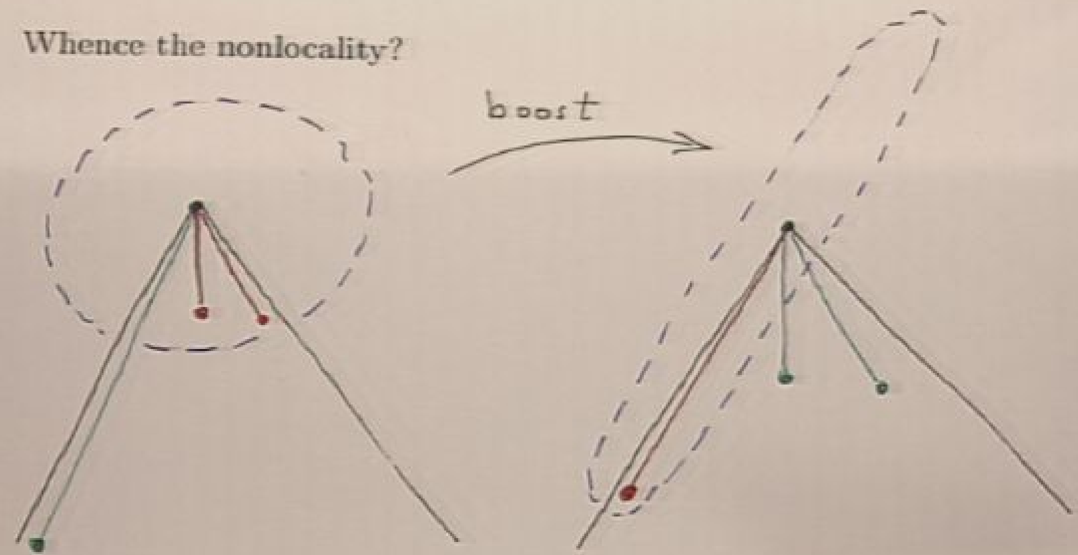

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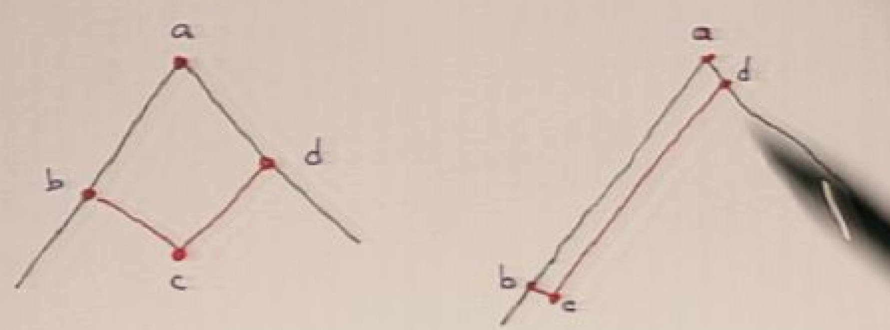

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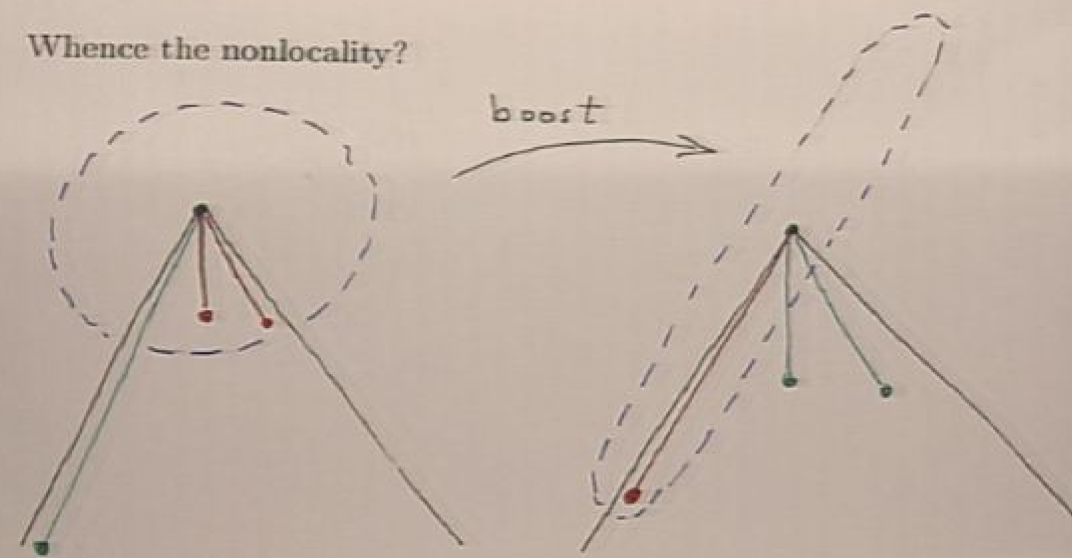

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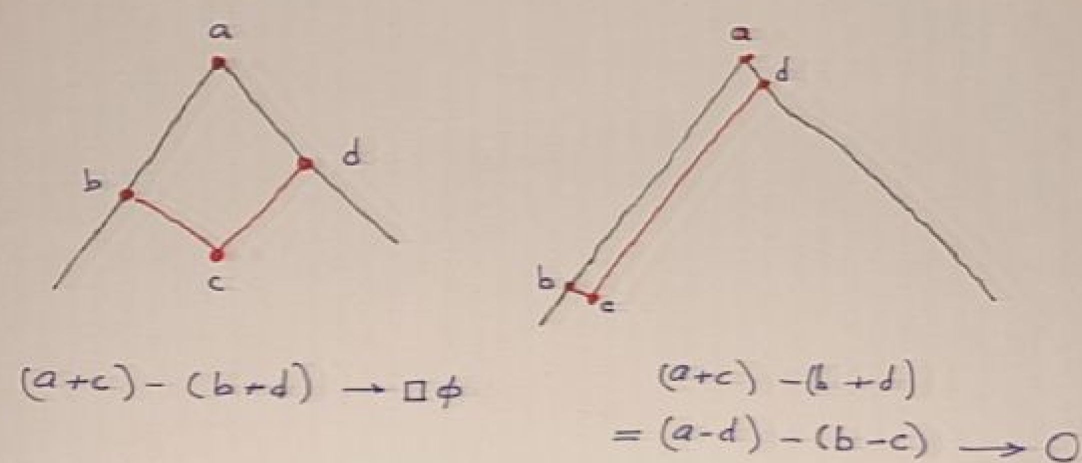
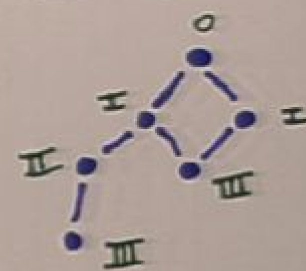


These ideas lead to expressions like

$$\frac{4}{l^2}\left(-\frac{1}{2}\phi(0)+\sum_{x\in I}\phi(x)-2\sum_{x\in II}\phi(x)+\sum_{x\in III}\phi(x)\right)$$

i.e.

$$\Box\phi(i)\leftrightarrow\sum_k B(i,k)\phi(k)$$



where

$$\frac{l^2}{4}B(i,k)=\begin{cases}-\frac{1}{2} \text{ if } i=k\\ \;\;1 \text{ if } i\prec k \text{ is a link (NN) } |<i,k>|=0\\ -2 \text{ if } i\prec k \text{ and (NNN) } \; |<i,k>|=1\\ \;\;1 \text{ if } i\prec k \text{ and (NNNN) } |<i,k>|=2\end{cases}$$

Can prove that, as $l\to 0$

$$S\equiv \mathbf{E}\sum_k B_{ik}\phi_k \qquad\qquad \to\ \Box\phi(x_i)$$

using e.g.

$$\mathbf{E}\sum_{x\in I}\phi(x)=\int\frac{dudv}{l^2}\exp\{-uv/l^2\}\ \phi(u,v)$$

Problem: $\Delta S\to\infty$ (fluctuations) as $l\to 0$!

IDEA: Our averaged sum is a *continuum* expression,

$$\int B(x-x')\,\phi(x')\,d^2x'\ ,$$

where

$$B(x) = \theta(x)\left(-2K\delta(x) + 4K^2\,p(\xi)\,e^{-\xi}\right)\ ,$$

where $\xi = Kuv$ with $K = 1/l^2$.

But can *decouple* K from l^2. We get a nonlocal continuum analog of the D'alembertian! Call it $\Box_K$.

Umkehren: approximate $\int$ by $\sum$ over sprinkled points!
This produces the causet expression,

$$\frac{4\varepsilon}{l^2}\left(-\frac{1}{2}\phi(y) + \varepsilon \sum_{x \prec y} p(\xi)\,e^{-\xi}\,\phi(x)\right)\ ,$$

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Remarks and applications

- Analogous expressions exist in other dimensions. In $d = 4$

$$p(\xi) = 1 - 3\xi + (3/2)\xi^2 - (1/6)\xi^3$$

$$\leftrightarrow \sum_{I} - 3\sum_{II} + 3\sum_{III} - \sum_{IV}$$

- Can now study propagation on sprinkled causet (Rideout)
cf. swerves

- The continuum theory's free field is *stable*: $(\ker \Box_K = \ker \Box)$
But response to sources differs

- Quantum Field Theory version? New approach to renormalization? Our nonlocality does *not* remove ∞'s, but perhaps it will allow an invariant (Lorentzian) cutoff.

- How big is λ_0? Must balance fluctuations vs. nonlocality.
$L =$ Hubble^{-1}, $l =$ Planck length.

$$\lambda_0 \gtrsim (l^2 L)^{1/3}$$

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