Title: Strings/Quantum Gravity 1

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Abstract:

Theory CANADA 2, PI, June 2006

Twistor Space, String Theory and Gauge Theory: A Powerful Mix

Freddy Cachazo
Perimeter Institute for Theoretical Physics

Introduction

The goal of this talk is to give you a snapshot of the status and open problems of the developments in perturbative gauge theory that were triggered by the introduction of twistor string theory (Witten Dec. 2003).

(Previously thought impossible calculations are now simple exercises!)

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Motivations I (String Theory)

Twistor string theory (Witten Dec.2003): a string theory (topological B-model) with target twistor space (Penrose 1967). New description of $\mathcal{N}=4$ SYM at weak coupling. Complementary to the AdS/CFT correspondence.

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Motivations II (Twistor Theory)

- Formulation of interacting quantum fields in twistor space.
- Explicit lagrangian formulations in twistor space.

Motivation III (Field Theory)

- Why do we compute perturbative QCD or $SU(N)$ amplitudes?
  Background in Hadron colliders like Tevatron and LHC.

- In principle perturbation theory is under control: Feynman diagrams!
  But not in practice or conceptually.

  In practice: $\#$ of FD grows very rapidly with $\#$ gluons and $\#$ of loops.

  Conceptually: After simplifying a huge $\#$ of FD the final answer is often simple and elegant.

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Our Target: Amplitudes of gluons. Each gluon carries $\{p_i, h_i = \pm, a_i\}$.

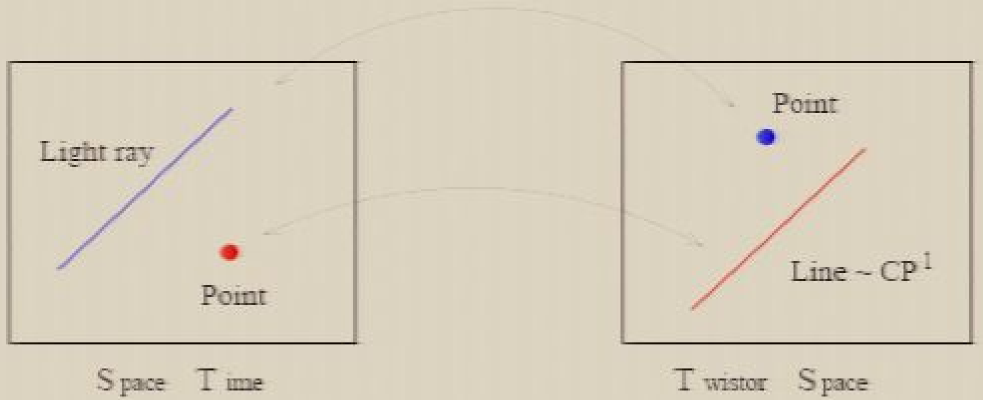
$$\mathcal{A}_n^{\rm tree}(\{p_i, h_i, a_i\}) =$$

$$g^{n-2}\delta^{(4)}(p_1+p_2+\ldots+p_n)\,{\rm Tr}(T^{a_1}T^{a_2}\ldots T^{a_n})A_n(1^{h_1},2^{h_2},\ldots,n^{h_n})+\ldots$$

Twistor String Theory

Twistor String Theory: (Witten 12/2003)

Twistor space (Penrose 1967):



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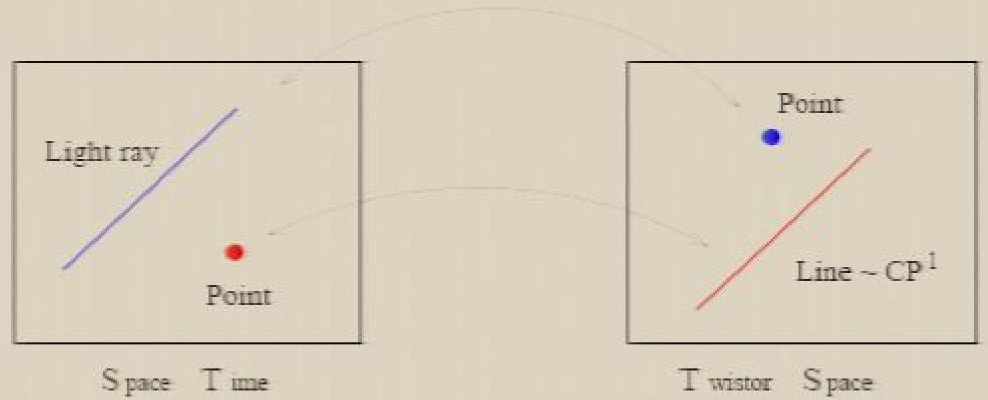

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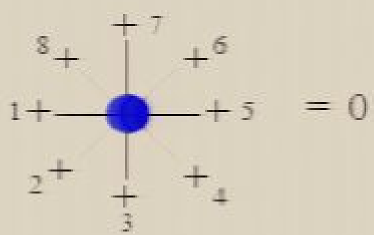
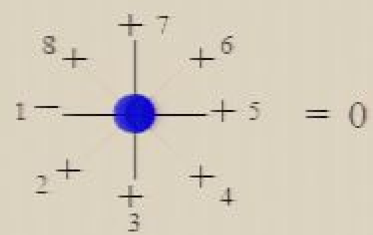


n = 8 m = 0



n = 8 m = 1

These would be called "Maximally Helicity Violating" (All lines incoming or outgoing) (Late 80's Parke, Taylor, Berends, Giele)

Twistor String Theory: (Witten 2003)

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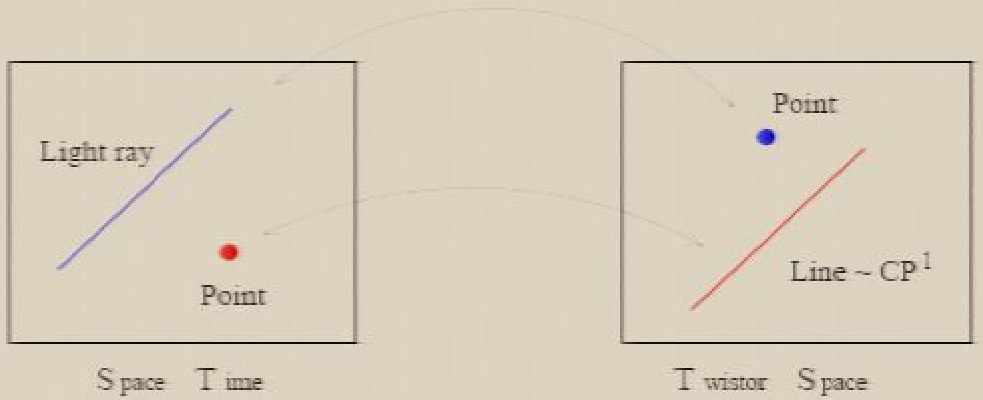

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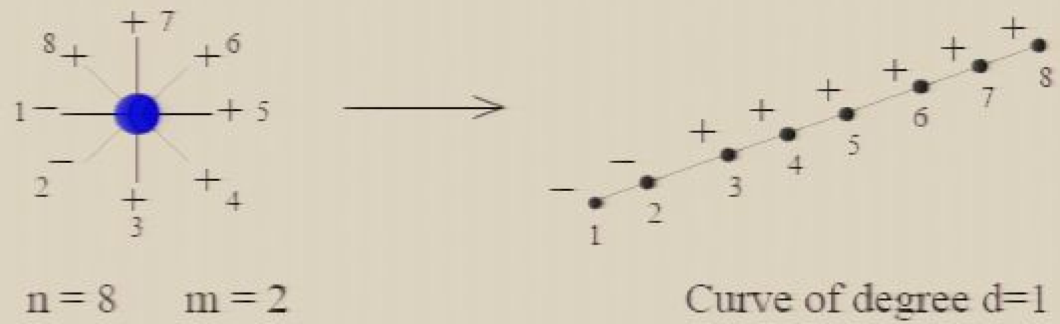

"Maximally Helicity Violating (MHV)" (Abstract line Nair 1988) (Late 80's Parke, Taylor, Berends, Giele)

Twistor String Theory: (Witten 2003)

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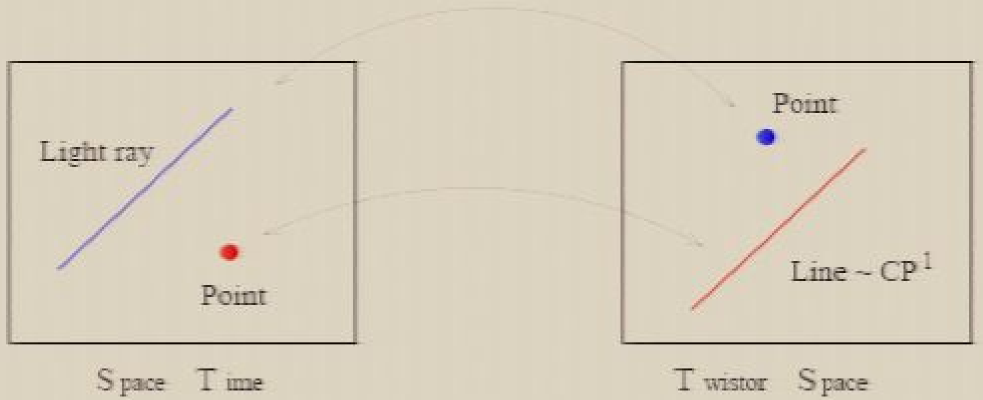

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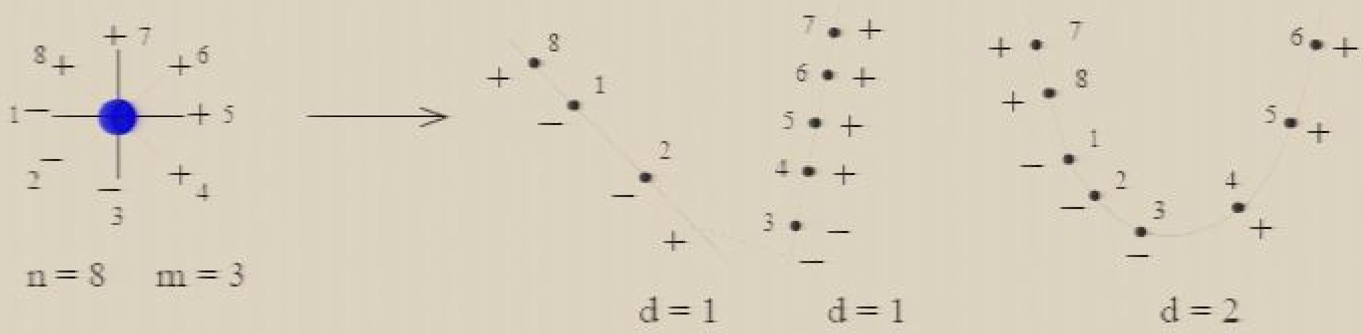

Next-to-Maximally Helicity Violating (NMHV). Obs: $d = m - 1$

Interpretation as a String Theory:

- P: Usual string theories live in 10 D but twistor space is 6 D!

(Problems that Witten had to overcome!)

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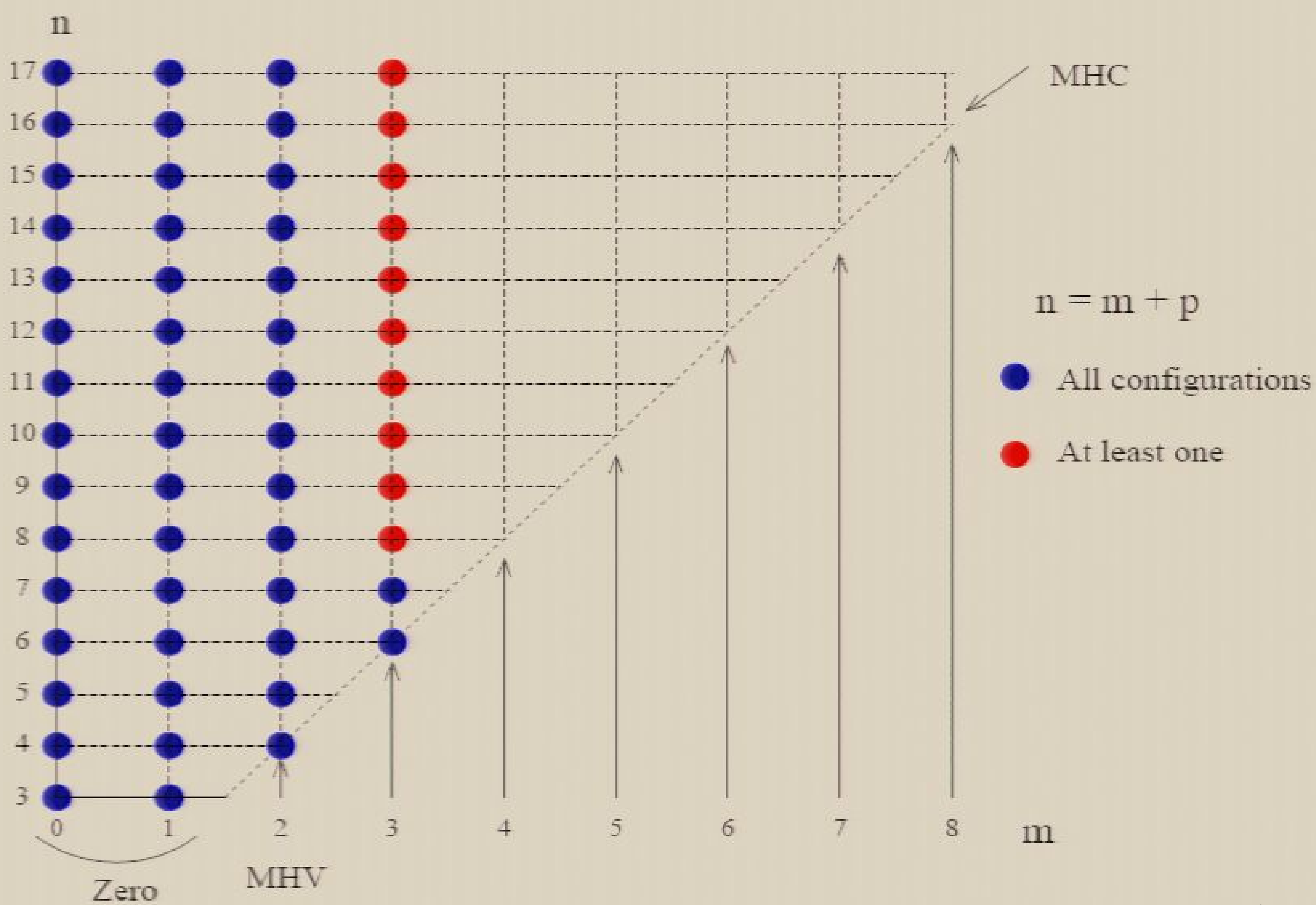
This construction led to many new and powerful ideas to solve old problems in scattering amplitude calculations. In order to support this claim let us make the standard "Before and After" test.

Tree-level Amplitudes of Gluons: Before 2004



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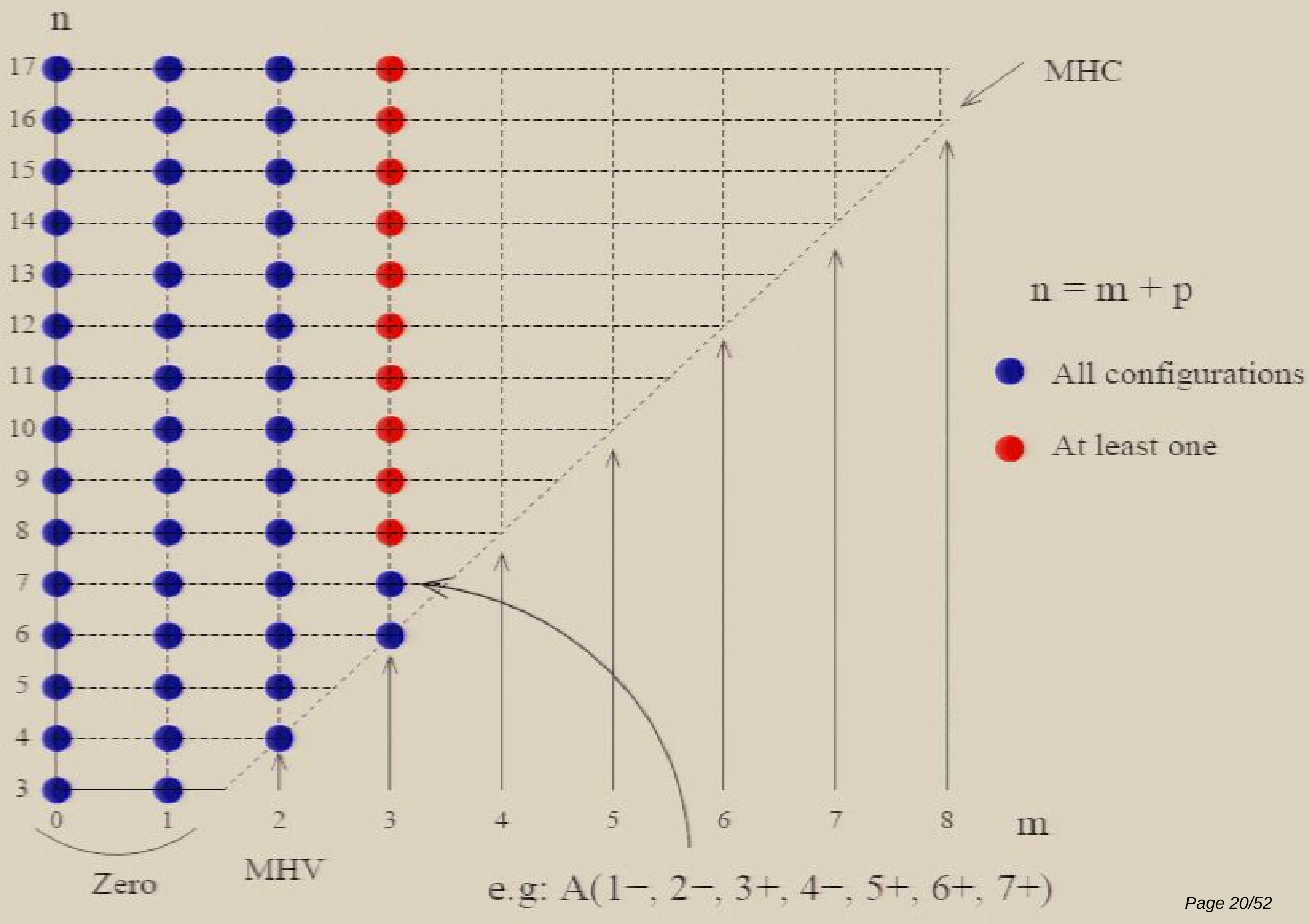

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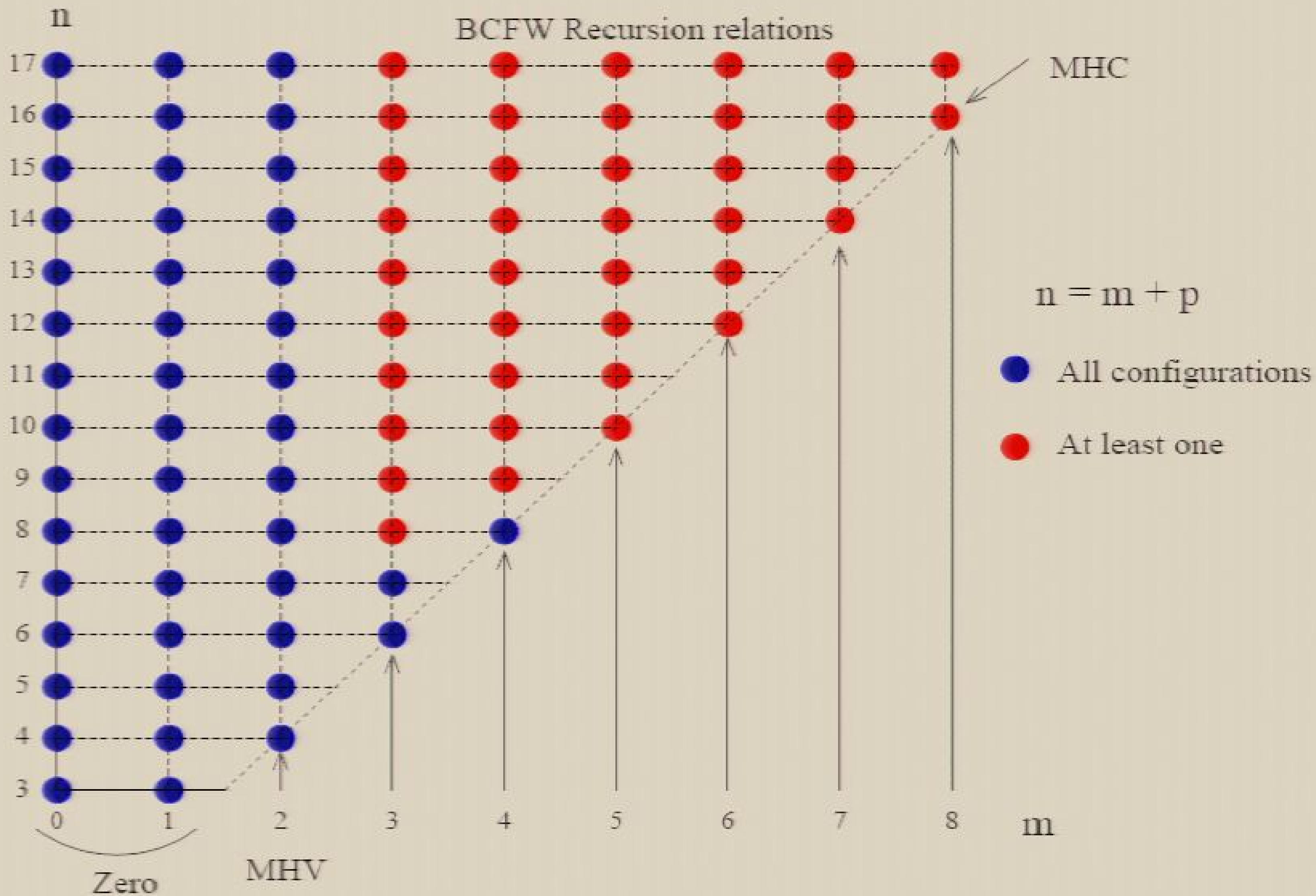

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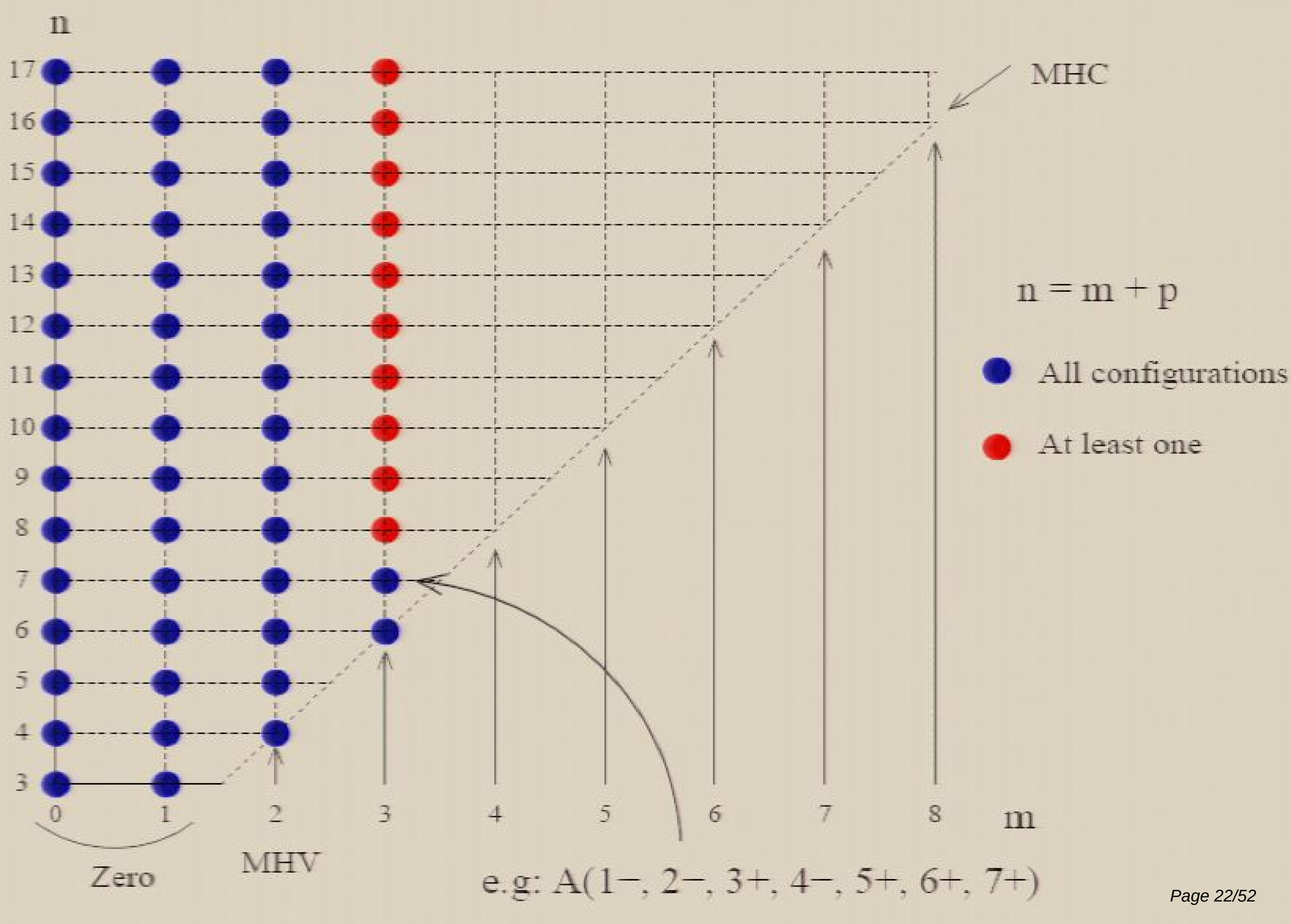

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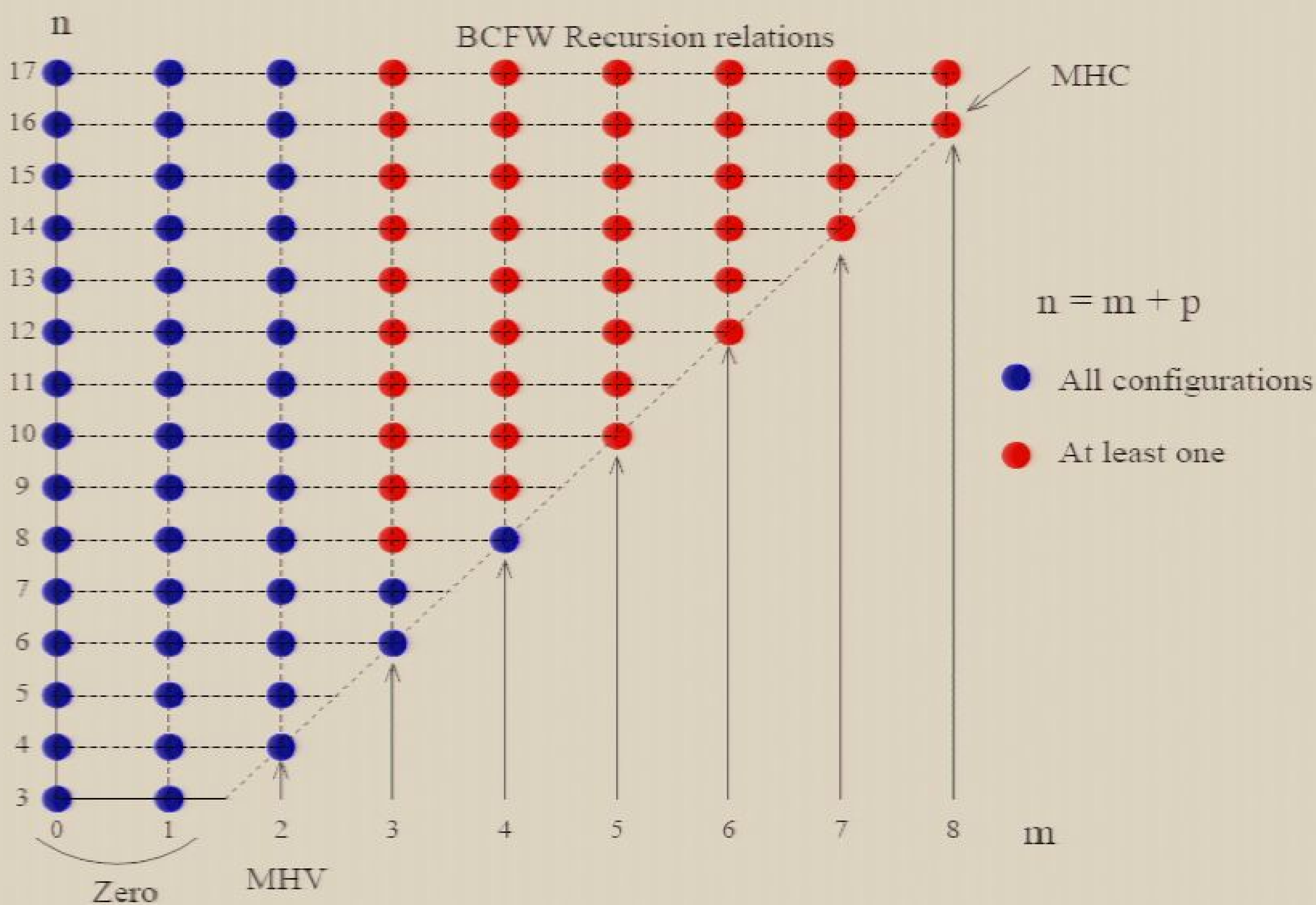
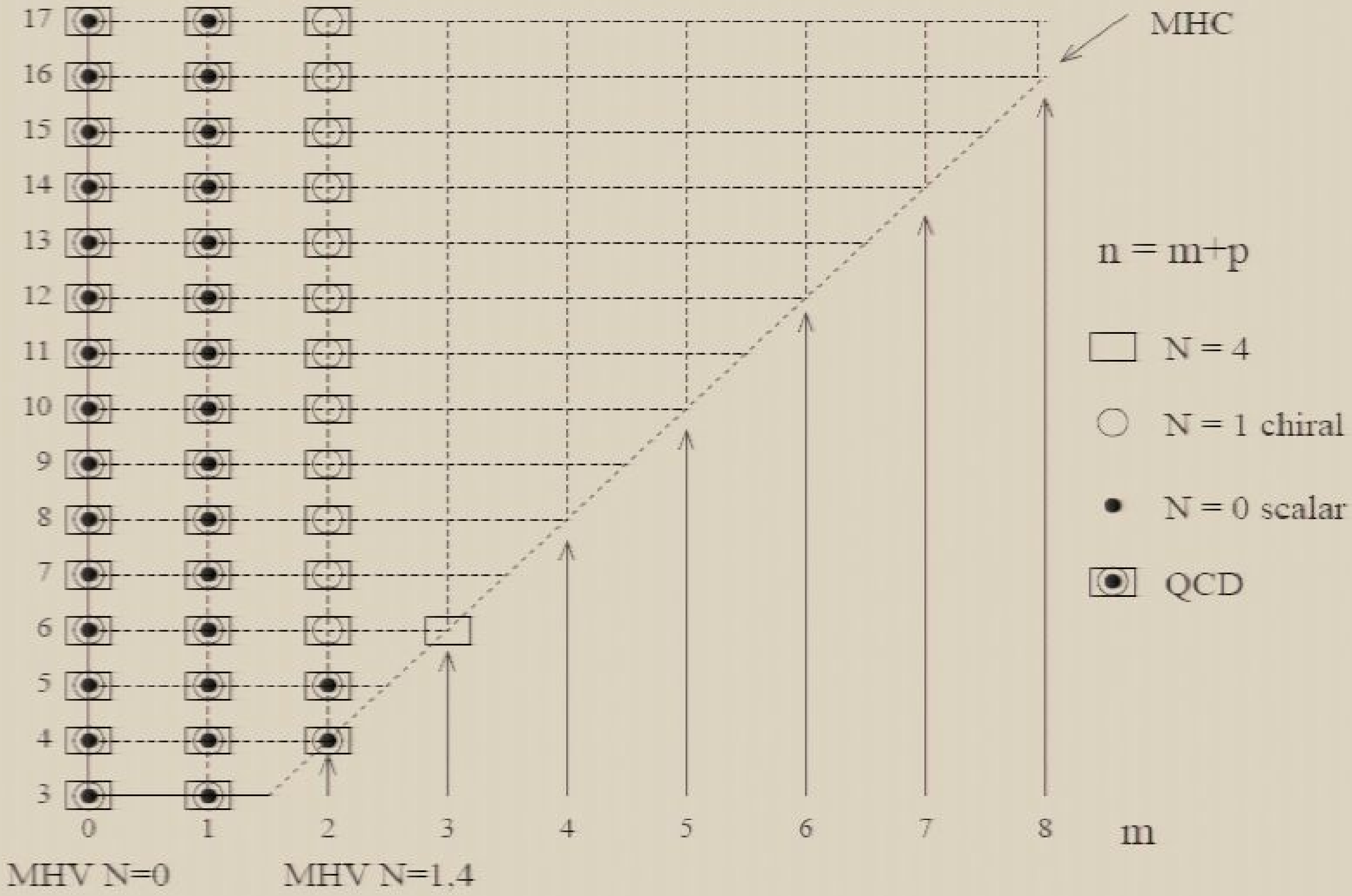


One-Loop Amplitudes of Gluons: Before 2004

$$g = (g+4f+3s) - 4(f+s) + s \Rightarrow A^{QCD} = A^{\mathcal{N}=4} - 4A_{\Phi}^{\mathcal{N}=1} + A^{\text{scalar}}$$



One-Loop Amplitudes of Gluons: Now

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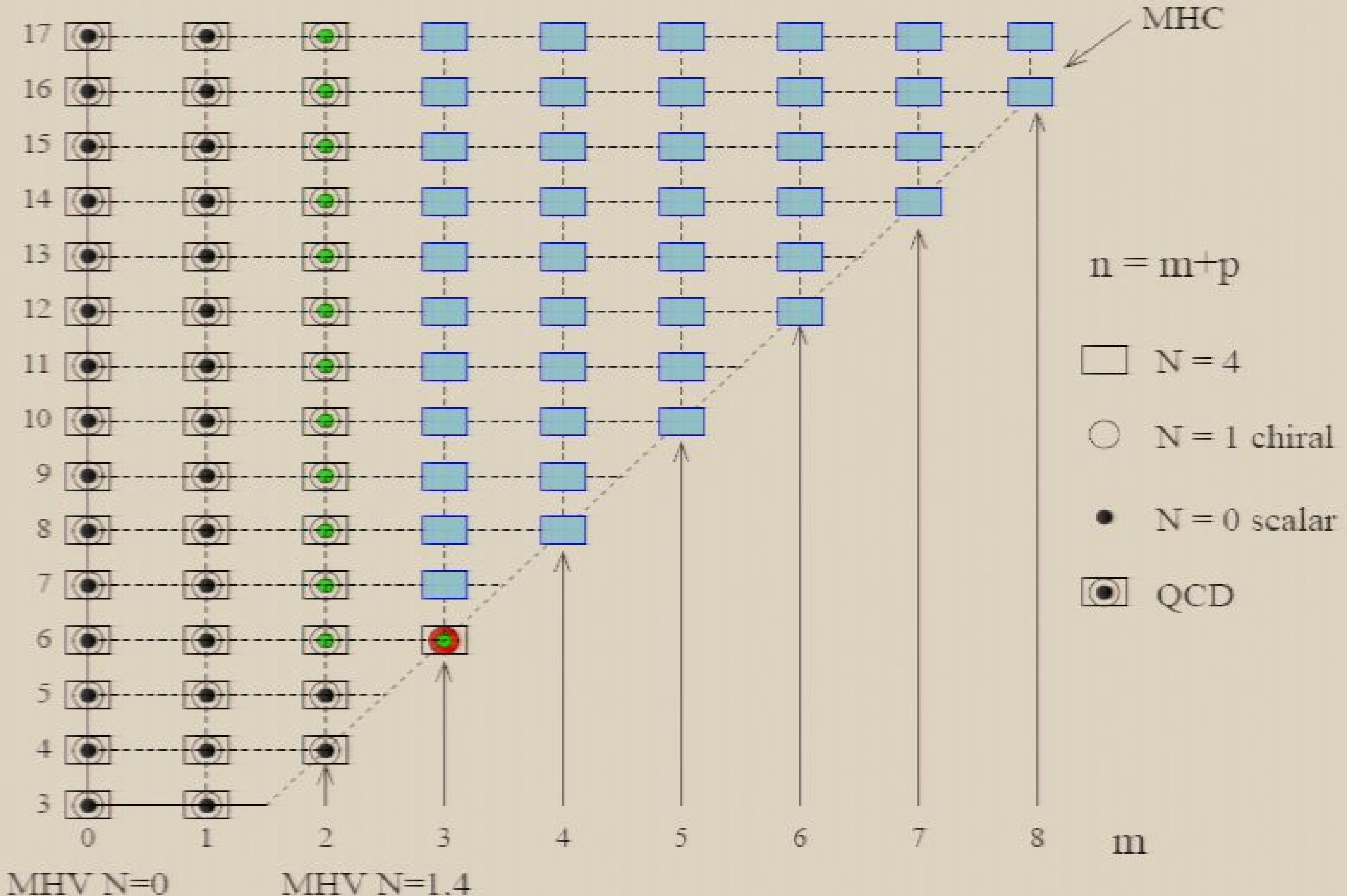

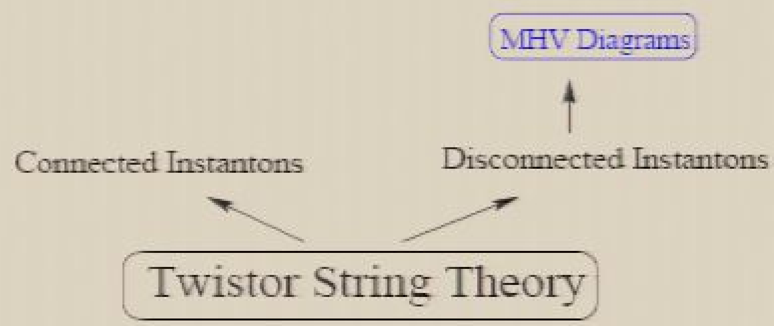
MHV Diagrams
Connected Instantons
Disconnected Instantons
Twistor String Theory

Disconnected prescription, CSW rules or MHV diagrams

(F.C., Svrček, Witten 03/2004)

Obs:

$$D_R(x-y) \sim \delta^{(+)}((x-y)^2) \quad \Rightarrow \quad D_R(P) \sim \frac{1}{P^2}$$

$$A(1^-,2^-,3^-,4^+,\ldots,8^+) \to A(8^+,1^-,2^-,P^+)\frac{1}{P^2}A(P^-,3^-,4^+,5^+,6^+,7^+)$$

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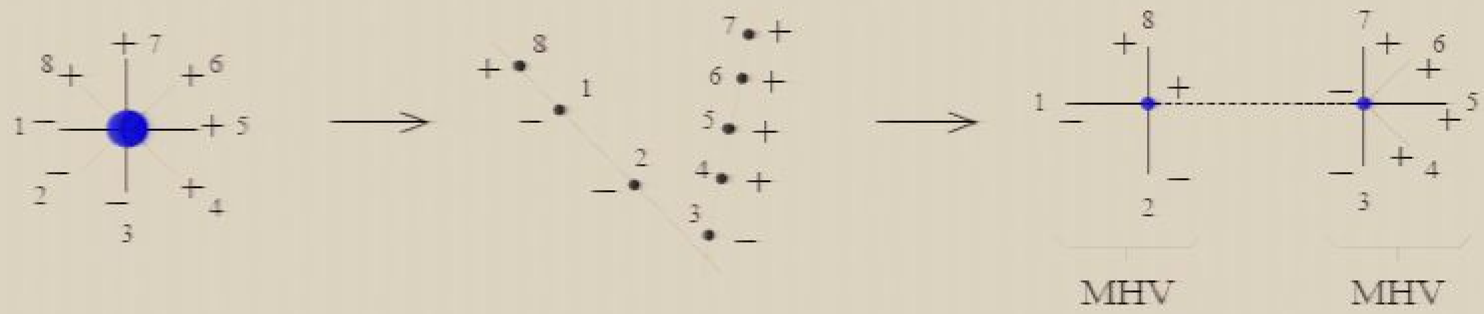

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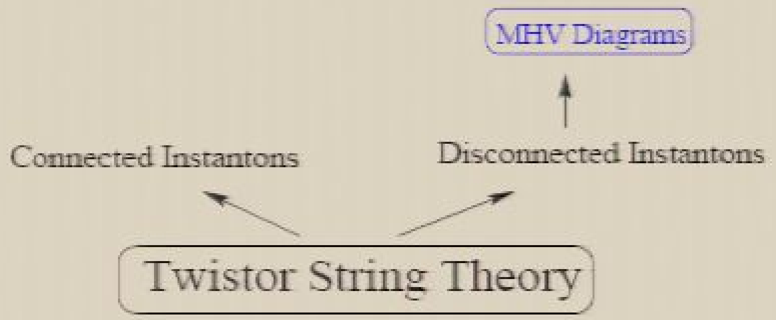
Complain 1: Feynman diagrams are computed using the Feynman propagator $D_F(P) = 1/(P^2+i\epsilon)$ not $D_R(P)$. Where is the $i\epsilon$?

Complain 2: $p_i^2 = 0$ but $P^2 = (p_8+p_1+p_2)^2 \neq 0$! S: Use a projection!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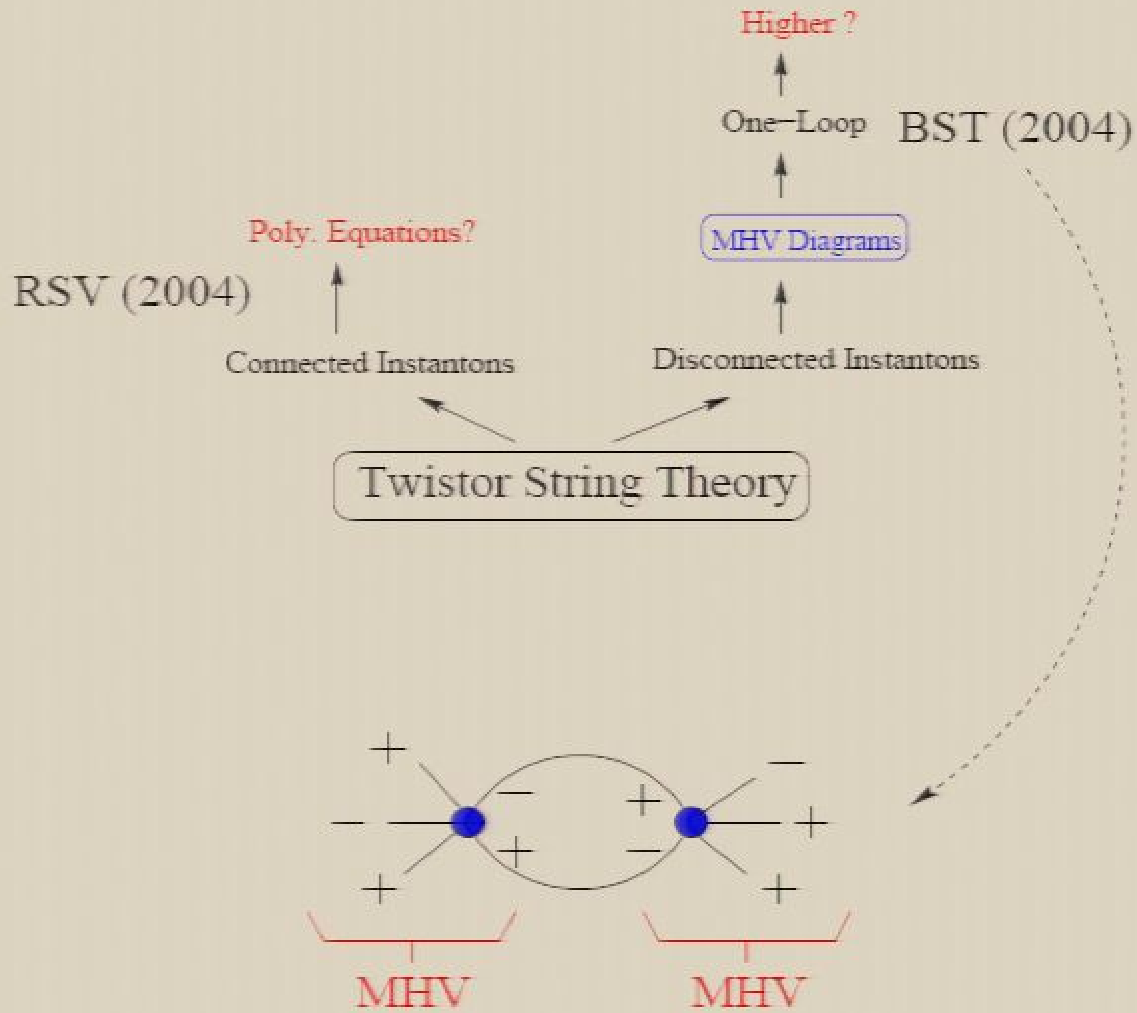
Higher ?
One−Loop
BST (2004)
Poly. Equations?
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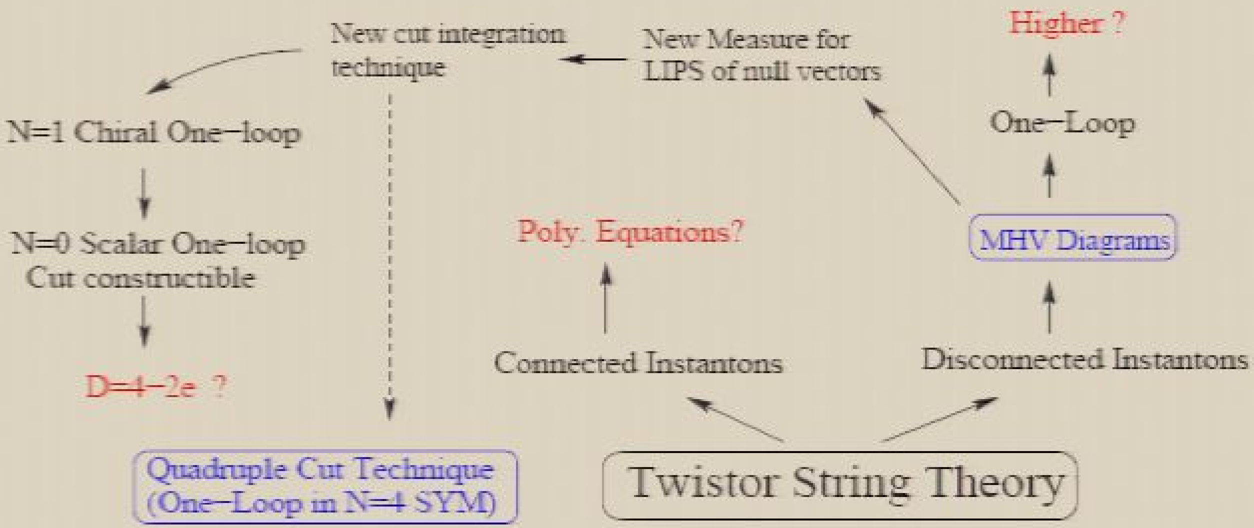


$$\int d^4\ell\delta^{(+)}(\ell^2)\bullet = \oint \bullet$$

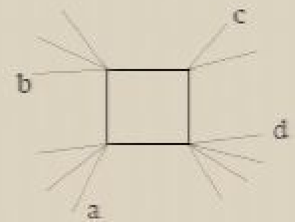
Measure is over $R^+ \times CP^1 \times CP^1$ and the contour is the diagonal CP^1.

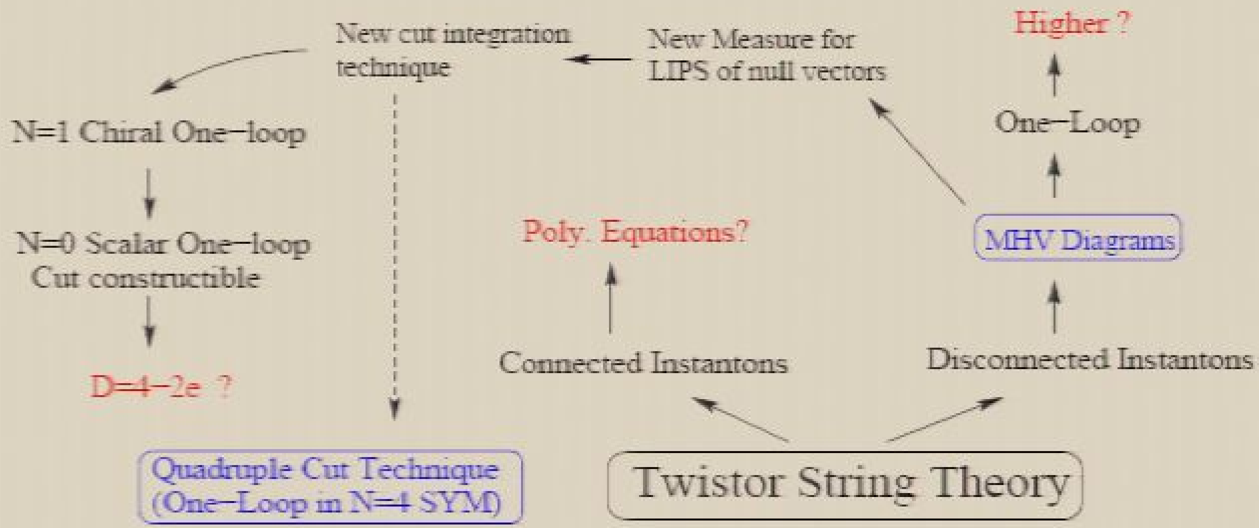
Quadruple cuts: Taming One-Loop Amplitudes in $\mathcal{N} = 4$

(Britto, F.C., Feng 12/2004)

Any n-gluon one-loop amplitude in $\mathcal{N} = 4$ SYM can be written as: (Bern, Dixon, Kosower 1993 & with Dunbar 1994)

$$A_n^{1-\mathrm{loop}} = \sum_{1<a<b<c<d<n} B_{abcd} \times$$





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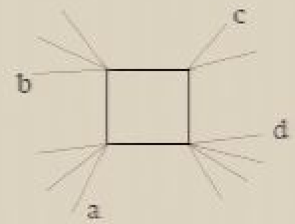

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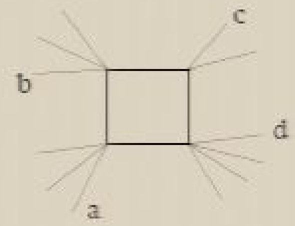

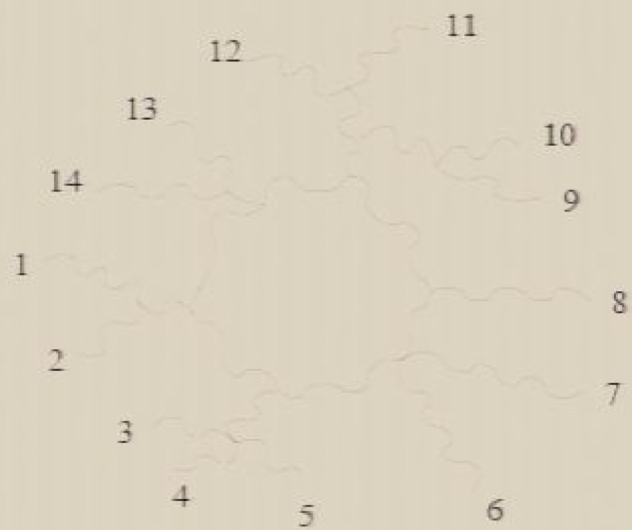

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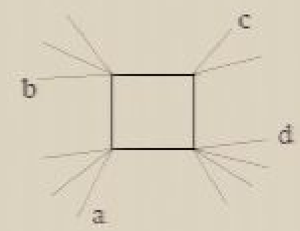

Observation: The problem of computing $\mathcal{N}=4$ one-loop amplitudes is reduced to that of computing the coefficients B_{abcd}.

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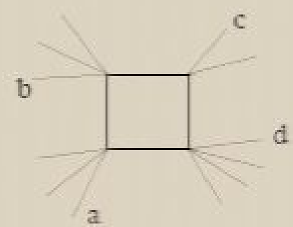

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Analogy: Think about each box as a vector $\hat{e}_i$. The set of all $\hat{e}_i$ form a basis of some vector space. An amplitude is a vector $\vec{V} = \sum_i c_i \hat{e}_i$.

All we need is a way to project out the vector $\vec{V}$ along the subspace spanned by one of the $\hat{e}_i$.

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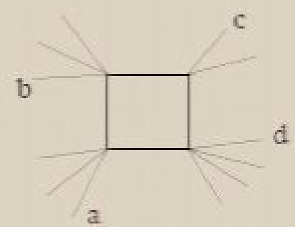

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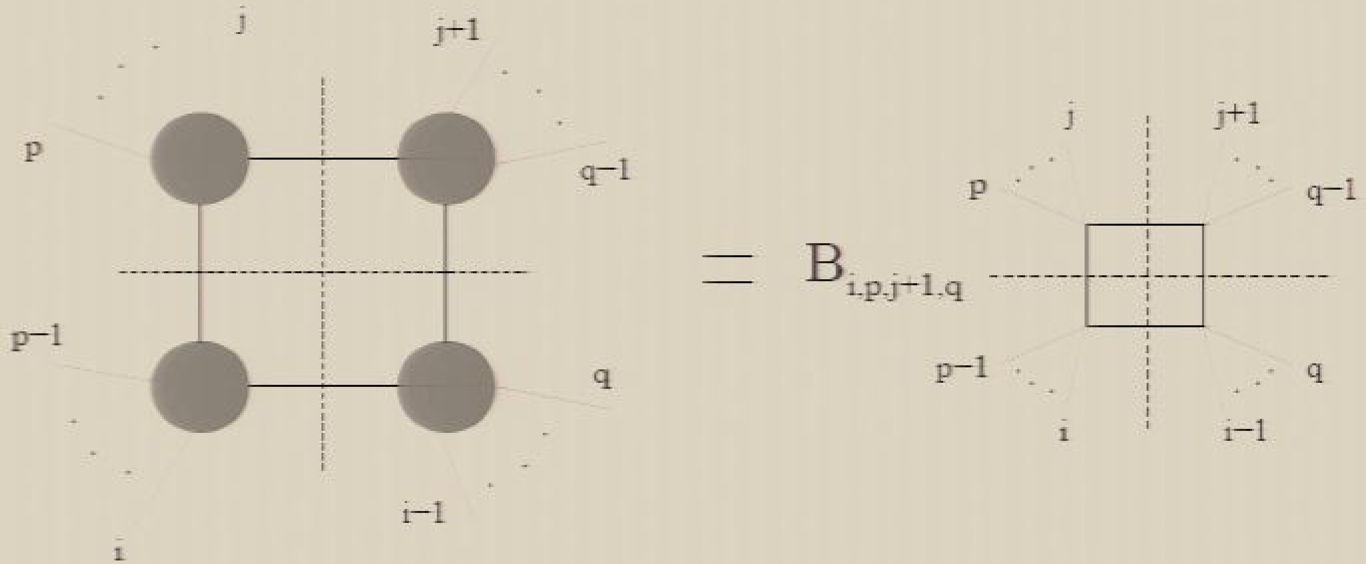
Clue: It turns out that each scalar box integral has a unique singularity! The discontinuity across it is computed by a quadruple cut!



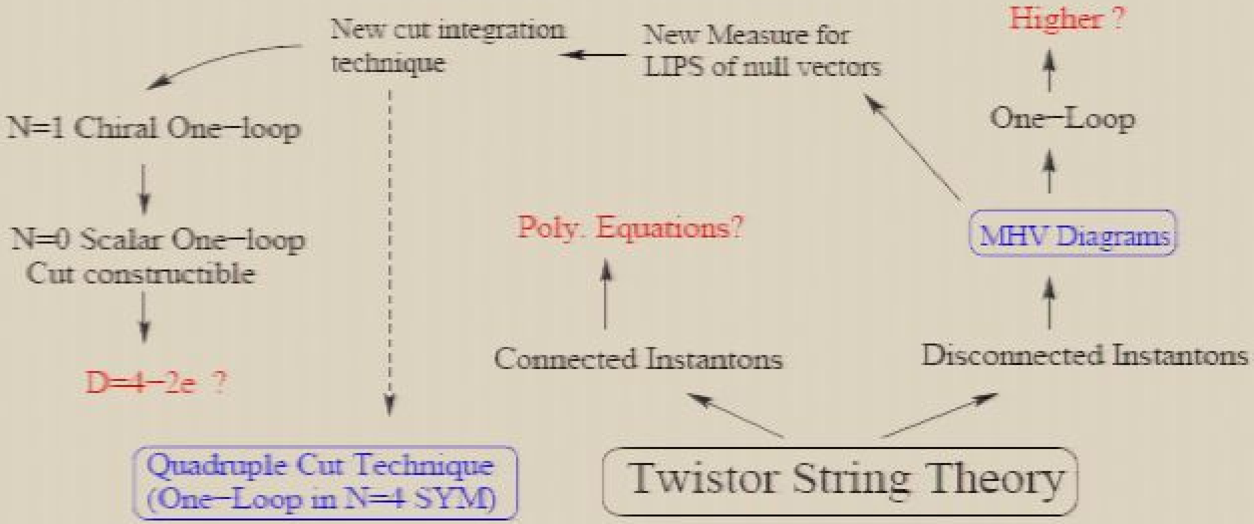
$$\int d\mu A_1^{\text{tree}} A_2^{\text{tree}} A_3^{\text{tree}} A_4^{\text{tree}} = B \int d^4\ell \delta^{(+)}(\ell_1^2)\delta^{(+)}(\ell_2^2)\delta^{(+)}(\ell_3^2)\delta^{(+)}(\ell_4^2)$$

$$\vec{V} \star \hat{e}_i = B(\hat{e}_i \star \hat{e}_i)$$

Final Formula:

$$B_{abcd} = \frac{1}{2} \sum_{\mathcal{S}} \sum_{h} A_1^{\text{tree}} A_2^{\text{tree}} A_3^{\text{tree}} A_4^{\text{tree}}$$

$$\mathcal{S} = \{\, \ell \,|\, \ell^2 = 0,\ (\ell - K_1)^2 = 0,\ (\ell - K_1 - K_2)^2 = 0,\ (\ell + K_4)^2 = 0 \}$$


New cut integration technique
New Measure for LIPS of null vectors
Higher ?
N=1 Chiral One−loop
One−Loop
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Poly. Equations?
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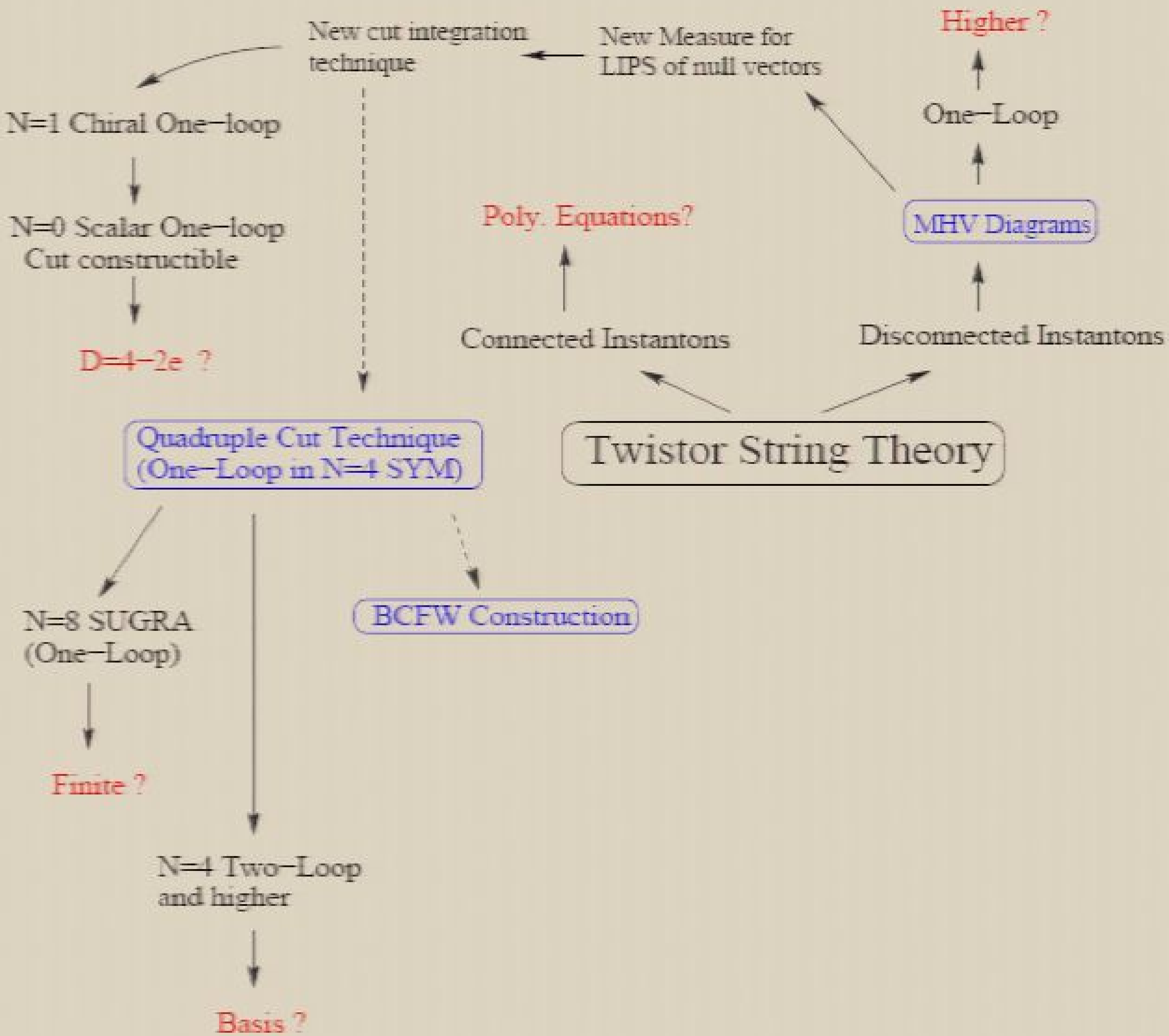
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BCFW Construction

(Britto, F.C., Feng, Witten 01/2005)

Q: Is there a systematic way of constructing an amplitude from its singularities?

Consider any amplitude of gluons: $A(p_1, p_2, \ldots, p_n)$

Define the following function of a complex variable z:

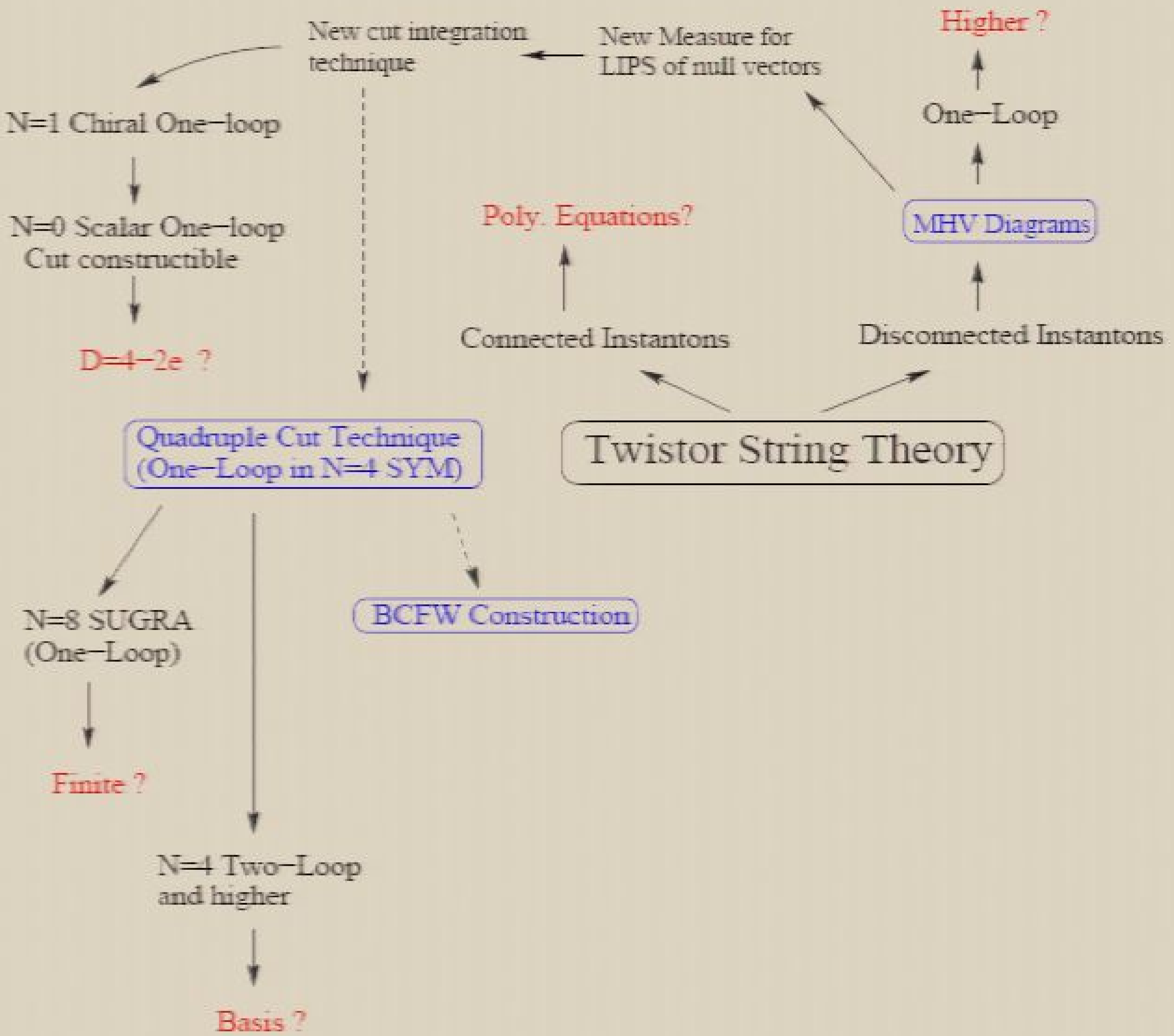
$$A(z) = A(p_1, \ldots, p_{k-1}, p_k(z), p_{k+1}, \ldots, p_{n-1}, p_n(z))$$

where

$$p_k(z) = p_k - zM, \qquad p_n(z) = p_n + zM.$$

We want $A(z)$ to be a physical amplitude for all z:

$$p_i^2(z) = 0 \quad \forall i \qquad \text{and} \qquad \sum_{j=1}^{n} p_j(z) = 0$$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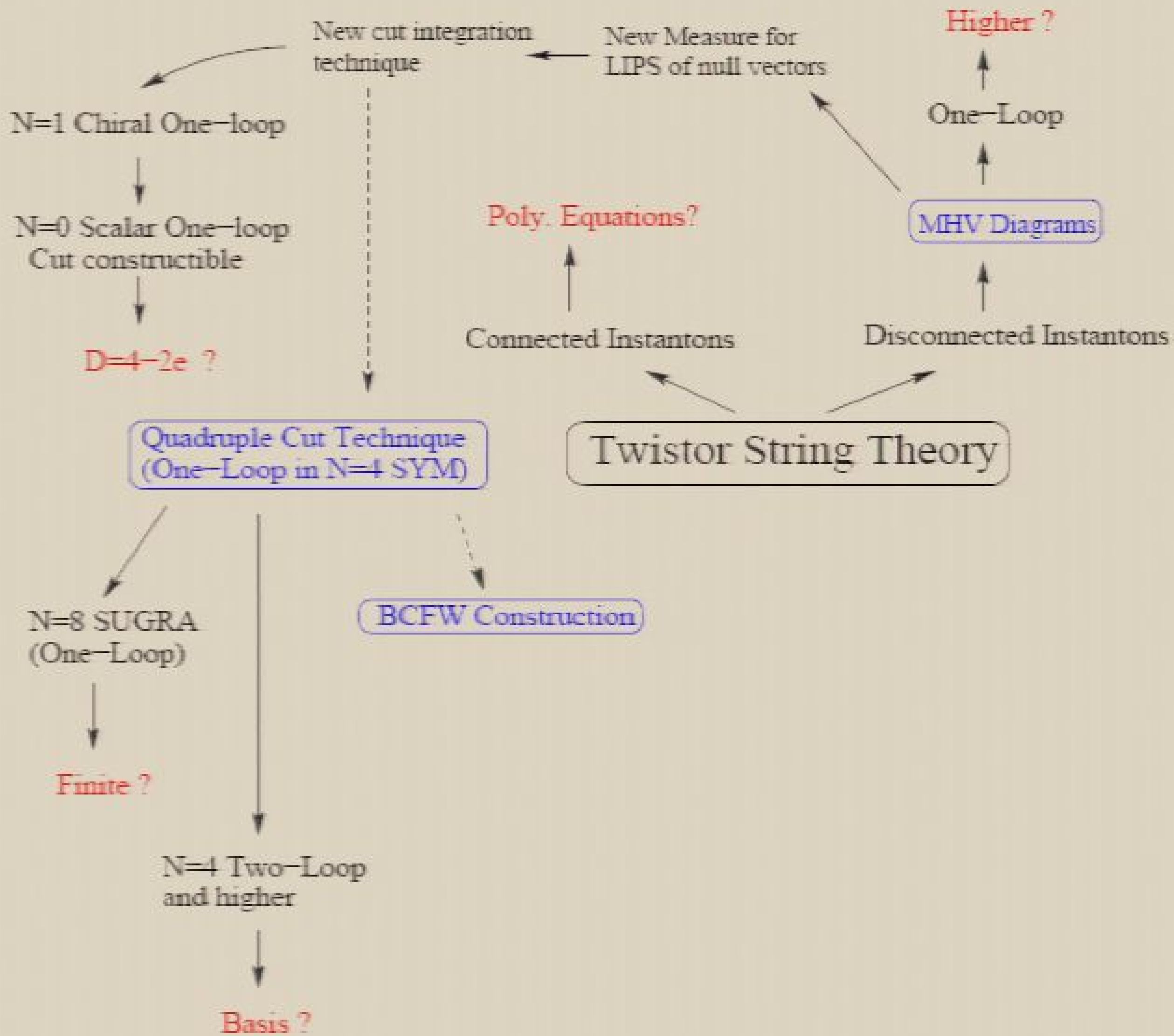
where

$$p_k(z) = p_k - zM, \qquad p_n(z) = p_n + zM.$$

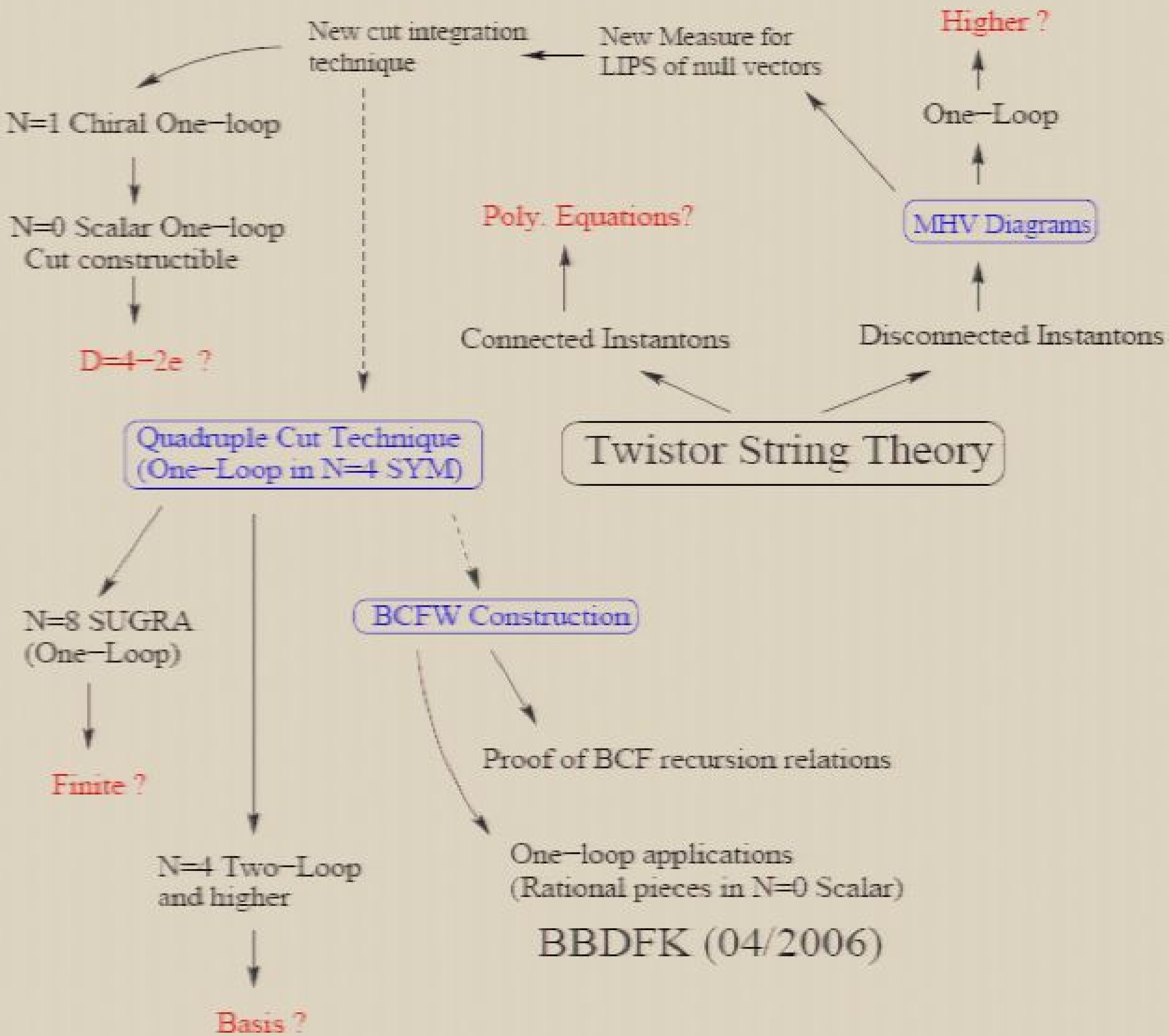
Observation: $(p_k)_\mu \sigma^\mu_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \qquad (p_n)_\mu \sigma^\mu_{a\dot{a}} = \eta_a \tilde{\eta}_{\dot{a}}$

Mysterious (or magic or mix) vector: $M_\mu \sigma^\mu_{a\dot{a}} = \lambda_a \tilde{\eta}_{\dot{a}}$

All we need is good control of the analytic structure of $A(z)$ which comes from physical singularities. (At tree level: Only simple poles!)



New cut integration technique
New Measure for LIPS of null vectors
Higher ?
N=1 Chiral One−loop
One−Loop
N=0 Scalar One−loop Cut constructible
Poly. Equations?
MHV Diagrams
D=4−2e ?
Connected Instantons
Disconnected Instantons
Quadruple Cut Technique (One−Loop in N=4 SYM)
Twistor String Theory
N=8 SUGRA (One−Loop)
BCFW Construction
Finite ?
N=4 Two−Loop and higher
Basis ?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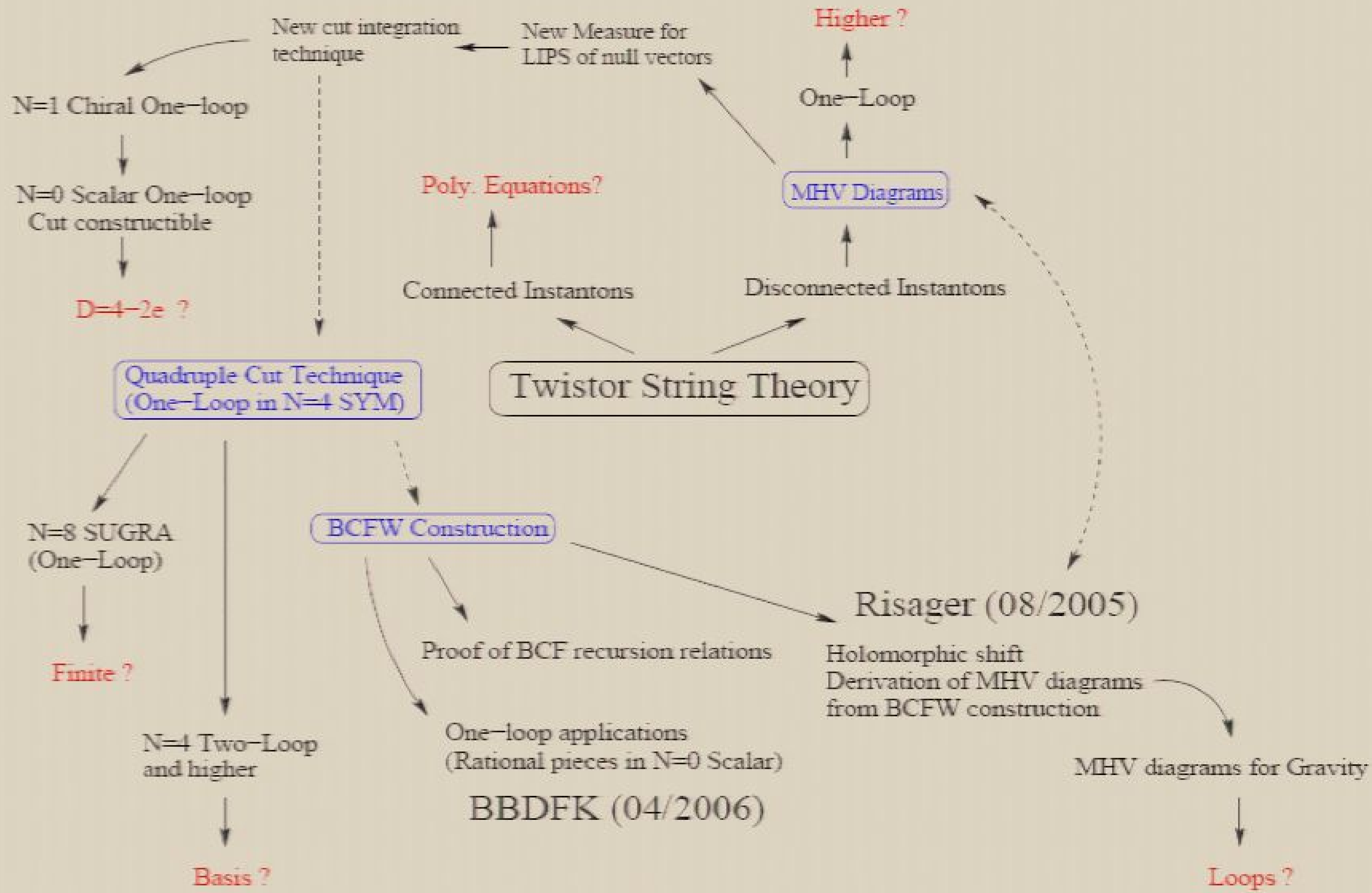
BBDFK (04/2006)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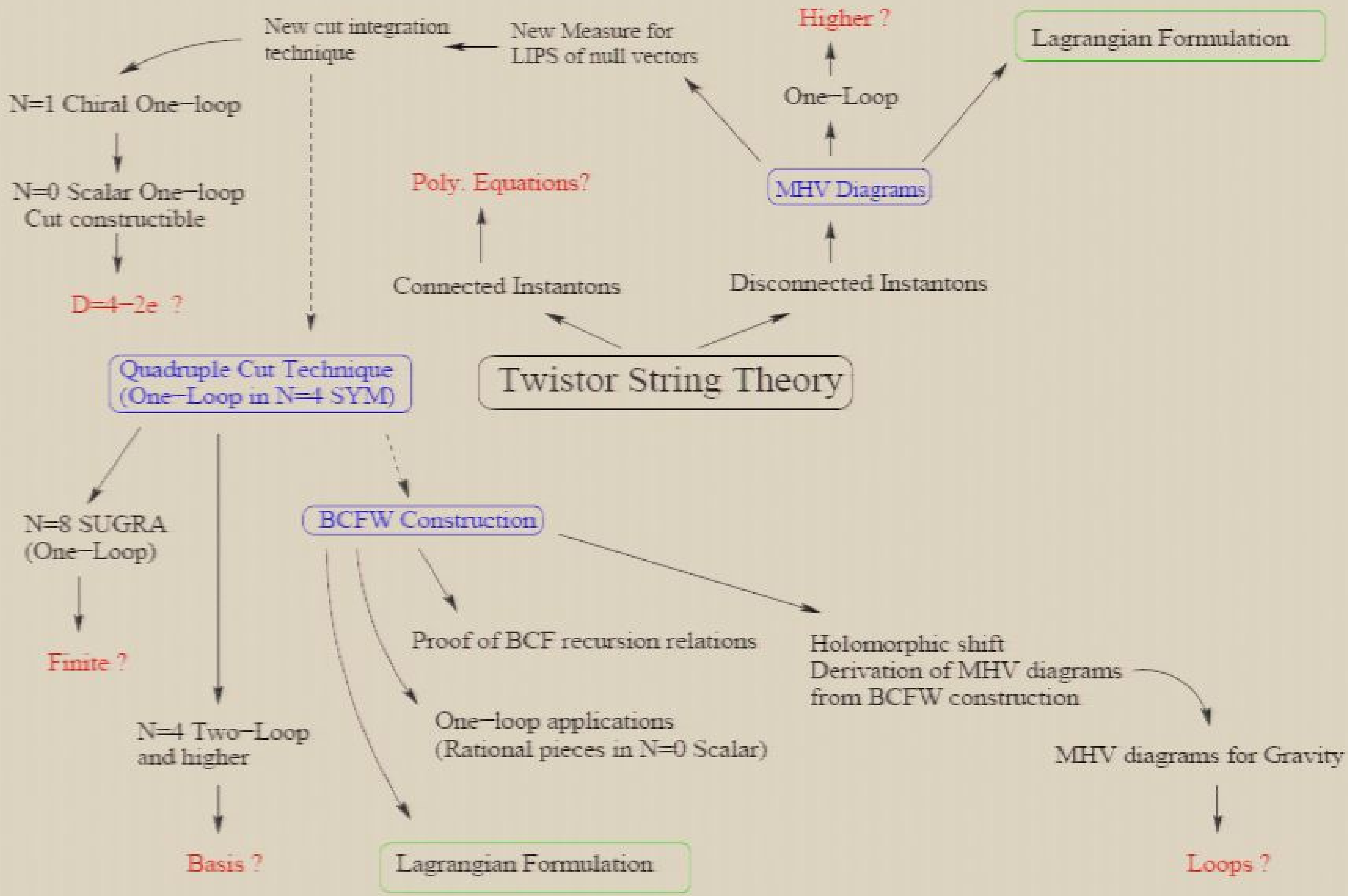
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Lagrangian Description (MHV Vertices)

Mansfield (11/2005); Gorsky, Rosly (10/2005), Ettle, Morris (05/2006); Mason, Skinner (10/2005)

Consider Yang-Mills in the light-cone gauge. $n^\mu A_\mu = 0$

$$L = L^{-+}[A] + L^{++-}[A] + L^{--+}[A] + L^{--++}[A]$$

Consider a canonical change of variables such that:

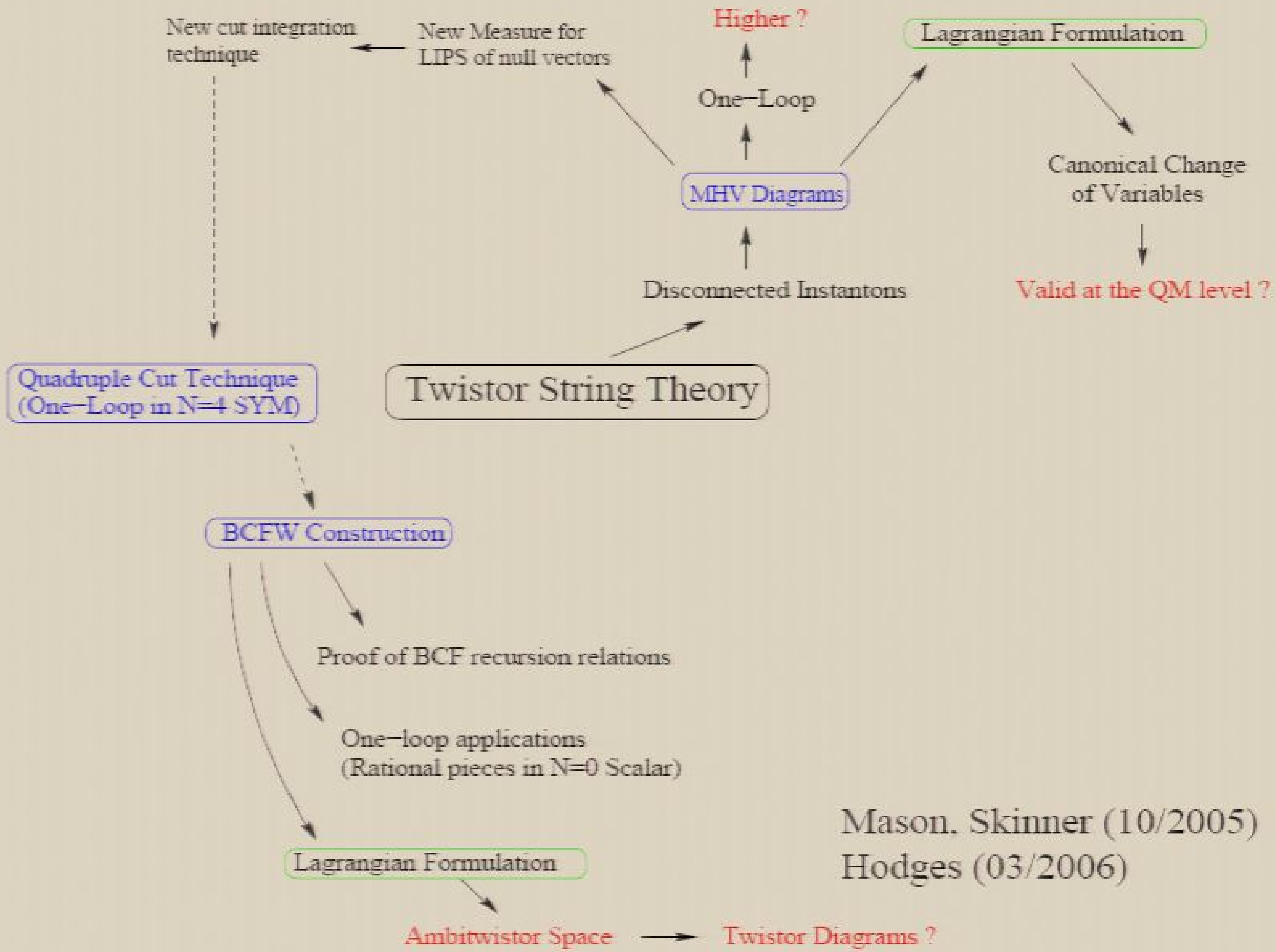
$$L^{-+}[B] = L^{-+}[A] + L^{++-}[A]$$

Obs: The new L is quadratic in B_- and is given as an infinite series in B_+.

$$L = L^{-+}[B] + L^{--+}[B] + L^{--++}[B] + \ldots + L^{--+\cdots+}[B] + \ldots$$

Q1: Can we make the choice of A or B a gauge choice of some sort of extended gauge symmetry?

Q2: Can we make $L^{-+}[B] = L^{-+}[A] + L^{--++}[A]$ such that we get the a lagrangian formulation of the BCF RR?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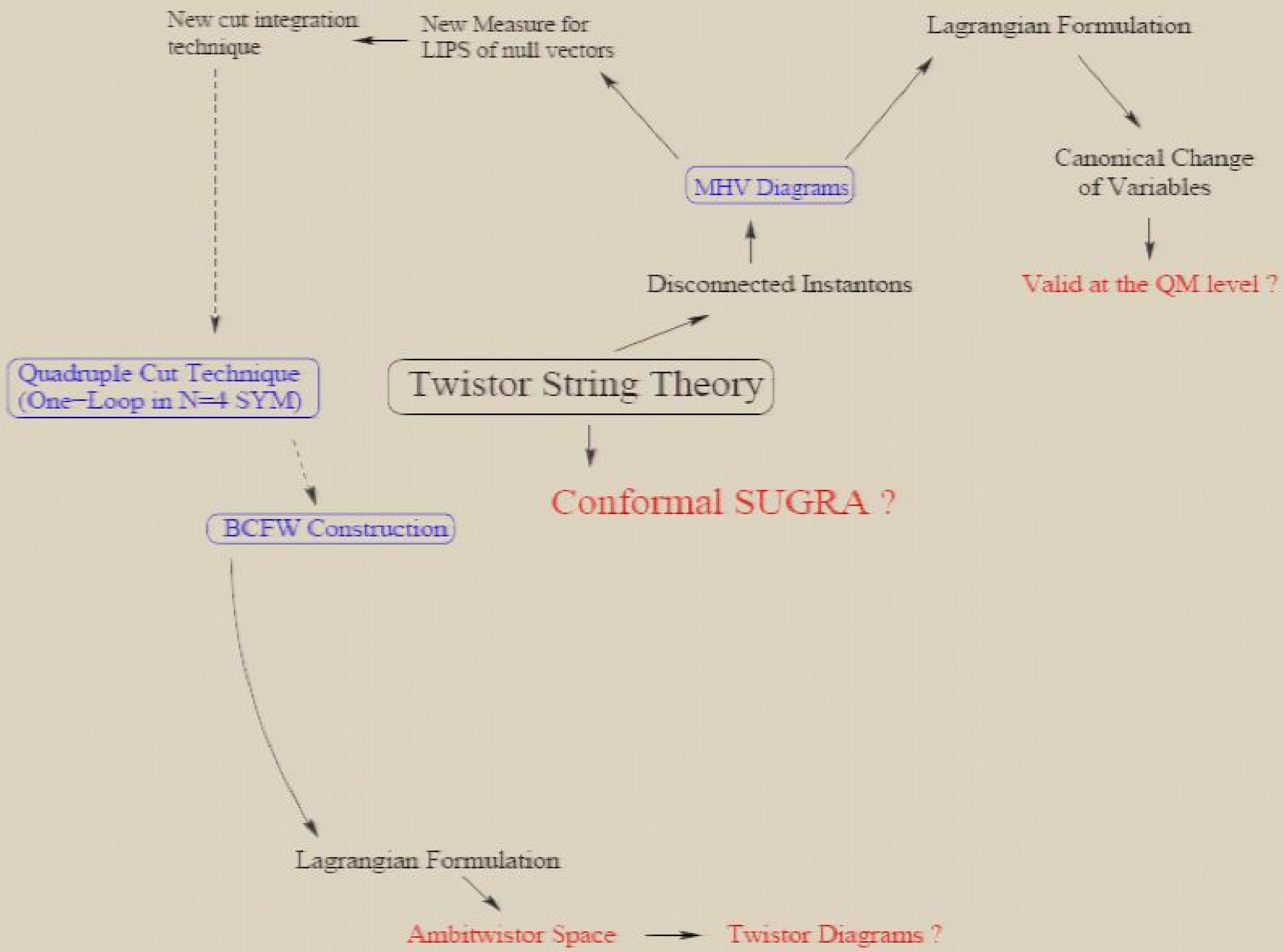
Ambitwistor Space
Twistor Diagrams ?
Mason, Skinner (10/2005)
Hodges (03/2006)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