Title: Particle Physics 3

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Abstract:

Introduction

The area of a black hole horizon behaves as an adiabatic invariant
(Christodoulu, Ruffini, Bekenstein)
Quantization of horizon area
Bekenstein, later Mukhanov, Kogan (strings).

Simple-minded arguments: $A = 8\pi\gamma l_p^2 N\,; \quad l_p^2 = \hbar k/c^3,$

N generalized quantum number (the same power as $\hbar$).

No sound arguments in favor of integer N, or equidistant spectrum.

$A = 8\pi\gamma l_p^2 \sum_{jm} a(j)\nu_{jm}$

$S = A/4l_p^2 = 2\pi\gamma \sum_{jm} a(j)\nu_{jm} = 2\pi\gamma \sum_j a(j)\nu_j$

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Radiation spectrum of quantized black holes

Due to quantum effects, the minimum change of the horizon area under adiabatic process is (Bekenstein)

$$(\Delta A)_{\min} = \xi l_p^2.$$

Thus, there are no "combinatorial" frequencies,
i.e., the radiation occurs when a site of given j disappears:

$$\Delta N = a(j), \quad \omega_j = \mu T a(j).$$

Discrete spectrum! $\omega_{\min} = \mu\, T\, a(j_{\min})$.

Even finite number of lines: since $\nu_{jm} \geq 1$,
$a(j_{\max}) = \ln \nu / \mu$. $\omega_{\max} = T \ln(A/l_p^2)$.
However, anyway, exponential decrease.

We need maximum S for given N ("microcanonical" entropy).
With simple calculations, we obtain:

$$\nu_{jm} = \nu\,\mathrm{e}^{-\mu a(j)}.$$

$$\sum_{jm} \mathrm{e}^{-\mu a(j)} = \sum_j g(j)\,\mathrm{e}^{-\mu a(j)} = 1$$

(a secular equation for the Lagrange multiplier μ).

$$S_{\max} = \mu N = \frac{\mu}{8\pi\gamma l_p^2} A. \quad \gamma = \mu/2\pi.$$

$$A = 8\pi\,\gamma\, l_p^2\,\nu \sum_j \mathrm{e}^{-\mu a(j)}\, g(j)\, a(j).$$

Number of quantum states

$S = \ln K$,
K is the total number of quantum states, which depends essentially on the assumptions related to the distinguishability of the sites.
The only reasonable assumption:
only sites with the same jm are indistinguishable. Then

$$S = \ln \left[\nu! \prod_{jm} 1/\left(\nu_{jm}!\right) \right] .$$

But what is in common with

$$A \sim N = \sum_{jm} a(j) \nu_{jm} ?$$

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Holographic bound

(Bekenstein, 't Hooft, Susskind)

Entropy S of any spherical nonrotating body confined inside a sphere of area A is bounded as follows:

$S \leq A/4l_p^2$,

with the equality attained only for a body that is a black hole.

To prove it, let this body collapse into a black hole. Then, we have

$S \leq S_{bh} = A_{bh}/4l_p^2 \leq A/4l_p^2$.

Quite unexpected,

but for common objects too mild to contradict the common experience.

Alternative formulation:

Among the spherical surfaces of a given area, it is the surface of a black hole horizon that has the maximum entropy.

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Natural assumption: $\Gamma_j \sim \nu_j = \nu g(j) \mathrm{e}^{-\omega_j/T}$. Then
$I_j \sim \omega_j \nu_j = \nu \omega_j g(j) \mathrm{e}^{-\omega_j/T}$
Exponential Wien profile (almost) free!

With an obvious extra factor (the "oscillator strength") we obtain
$I_j \Delta\omega_j = A \frac{\omega_j^3}{4\pi^2} \mathrm{e}^{-\omega_j/T} \Delta\omega_j = A \frac{T^3}{4\pi^2} [\mu a(j)]^3 \mathrm{e}^{-\mu a(j)} \Delta\omega_j$;
$\Delta\omega_j = T\, \mu(\partial a/\partial j) \Delta j$.

Compare the line width $\Gamma_j \Delta\omega_j = I_j \Delta\omega_j / \omega_j$ with the line separation $\Delta\omega_j$.
Their ratio is

$\Gamma_j = \frac{1}{16\pi^3} [\mu a(j)]^2 \mathrm{e}^{-\mu a(j)} \leq 10^{-3}$! $\quad (x^2 \mathrm{e}^{-x} \leq 0.5.)$

Spectrum is really discrete!

The total intensity

$$I = \sum_j I_j \Delta\omega_j \simeq \frac{AT^4}{4\pi^2} \int_{\mu a_{\min}}^{\infty} dx x^3 \mathrm{e}^{-x} \approx 0.14 AT^4 \quad \text{for} \quad \mu a_{\min} \sim 1.$$

The constant 0.14 here is close to the Stefan - Boltzmann one $\pi^2/60 = 0.164$in the common Planck spectrum.
There is no Rayleigh - Jeans region here since $\omega_{\min} \sim T$ $(T \sim \hbar!)$.
But its contribution to the total intensity is small anyway.

The result refers directly to photons and gravitons with 2 polarizations.
For other particles (fermions included!) one should change, if necessary, the statistical weight only.

Is Radiation of Quantized Black Holes Observable?

Analysis of observational data for secular perihelion precession of Earth and Mars results in upper limit on dark matter density ρ in Solar system (Khriplovich, Pitjeva):

$\rho < 3 \times 10^{-19}$ g/cm^3 .

Estimates for the expected signal from PBHs (Khriplovich, Produit) are performed under the (optimistic) assumption that their density is

$\rho \simeq 10^{-19}$ g/cm^3.

PBH with initial mass $m \lesssim m_0 = 5 \times 10^{14}$ g cannot survive till our time due to their radiation. For masses larger than 10^{17} g, the signal gets hopelessly small. The estimates for reasonable masses are as follows:

m, g	n, cm^{-3}	$\bar{r}$, cm	T, MeV	N, ph s^{-1}	ν, ph cm^{-2} s^{-1}
5×10^{14}	2×10^{-34}	1.7×10^{11}	20	6×10^{19}	1.6×10^{-4}
2×10^{15}	5×10^{-35}	2.7×10^{11}	5	1.5×10^{19}	1.6×10^{-5}
10^{16}	10^{-35}	0.5×10^{12}	1	3×10^{18}	10^{-6}
10^{17}	10^{-36}	10^{12}	0.1	3×10^{17}	2×10^{-8}

Table 1: Predictions for radiation of primordial black holes in Solar system

The typical signature of radiating PBH would be 2 — 3 relatively strong lines in the spectrum, for instance, with energies about 5, 10, 15 MeV for PBH with mass $\mathbf{2 \times 10^{15}}$ g.

In particular, the plot below demonstrates that the sensitivity of SPS spectrometer aboard the INTEGRAL satellite is in principle sufficient to observe the line around 5 MeV.

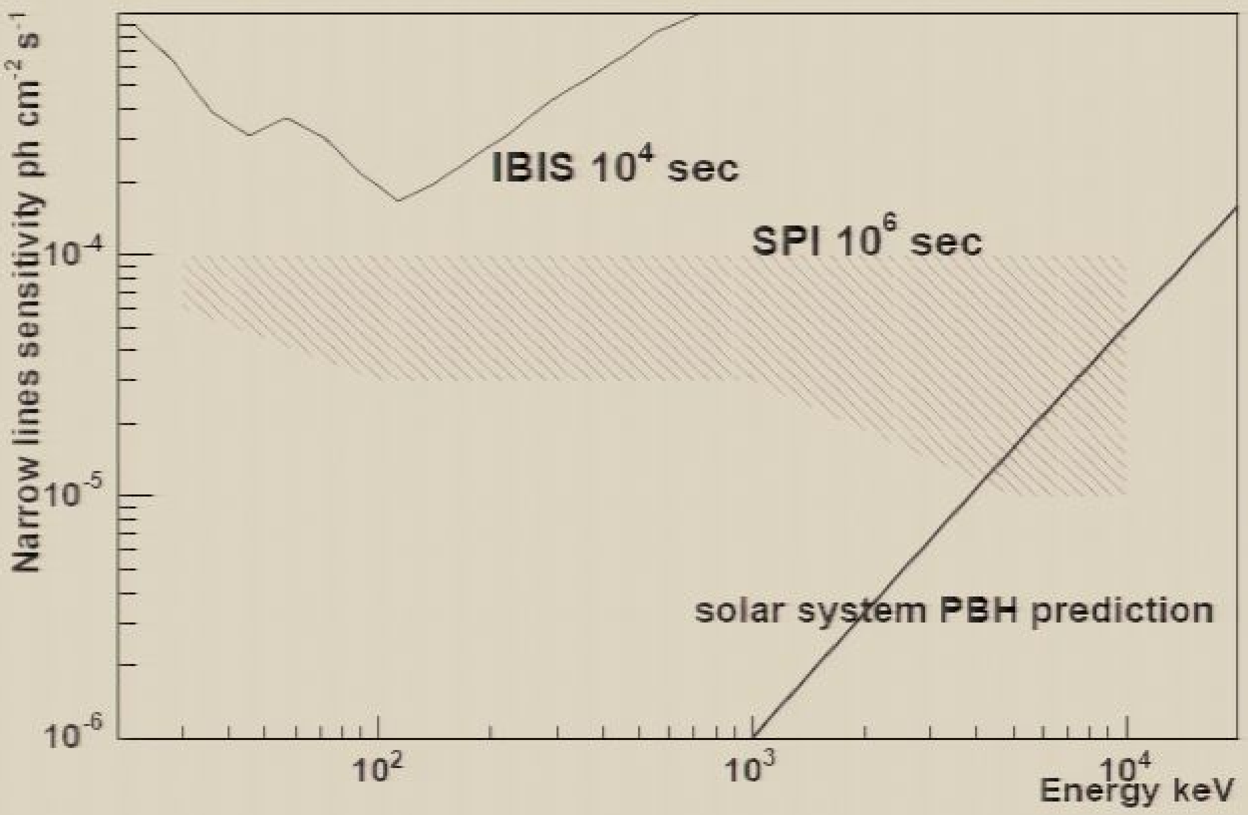

Figure 1: Sensitivity of IBIS imager for narrow line of point-like source exposed for 10^4 s and of the SPI spectrometer for point-like source exposed for 10^6 s

It can be demonstrated that our assumption on the density of PBHs in the Solar system does not contradict the best observational upper limits on it.

These limits do not preclude
the searches for quantized PBHs in the Solar system.

One should try! (Telegdi)