Title: Watching the adiabatic quantum computer work to learn more about physics

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Abstract: Adiabatic Quantum Computation is not only a possibly more robust alternative to standard quantum computation. Since it considers a continuous-time evolution of the system, it also provides a natural bridge towards studying the dynamics of interacting many-particle quantum systems, quantum phase transitions and other issues in fundamental physics. After a brief review of adiabatic quantum computation, I will show our recent results on the dynamics of entanglement and fidelity for the search and Deutsch algorithms including several variations and optimization. I will show how these studies led to suggesting an alternative definition of entanglement and compare the results, and discuss possible implications for considering entanglement a resource. I will conclude with an outlook on further applications and extensions of adiabatic quantum computation.

Why?

- AQC itself is interesting, possibly more robust against errors than standard circuit QC, but equivalent with regard to computational power

- seemingly simple systems illuminate fundamental problems

- AQC allows to track dynamical quantities easily

- AQC provides possible connection to quantum field theory; natural description of spin models

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How?

- study dynamics of entanglement and other physical quantities during adiabatic algorithms

- starting with small systems that allow analytical calculations

- D.A., Proc. Theory Canada I, Can. J. Phys. 83, 2005

- more systematic "experimental" approach to a wider variety of cases, numerics

- K. Choy, G. Passante, D.A., M. Carrington, T. Fugleberg, R. Kobes, and G. Kunstatter, quant-ph/0605040

Adiabatic Quantum Computer: The idea

$$|\Psi_0\rangle \Longrightarrow |\Psi_1\rangle$$

input → solution

(easy to build)

continuous time evolution!

constructing the adiabatic quantum computer:

$$|\Psi_0\rangle \overset{H(t)}{\Longrightarrow} |\Psi_1\rangle$$

ground states of

$$H_0 = I - |\Psi_0\rangle\langle\Psi_0| \qquad H_1 = I - |\Psi_1\rangle\langle\Psi_1|$$

$$H(t) = f(t)H_0 + g(t)H_1$$

drives QC from initial to final instantaneous ground stat

condition for staying in ground state: slow evolution

$$H(t)|E_k;t\rangle = E_k(t)|E_k;t\rangle$$

instantaneous energy eigenstates

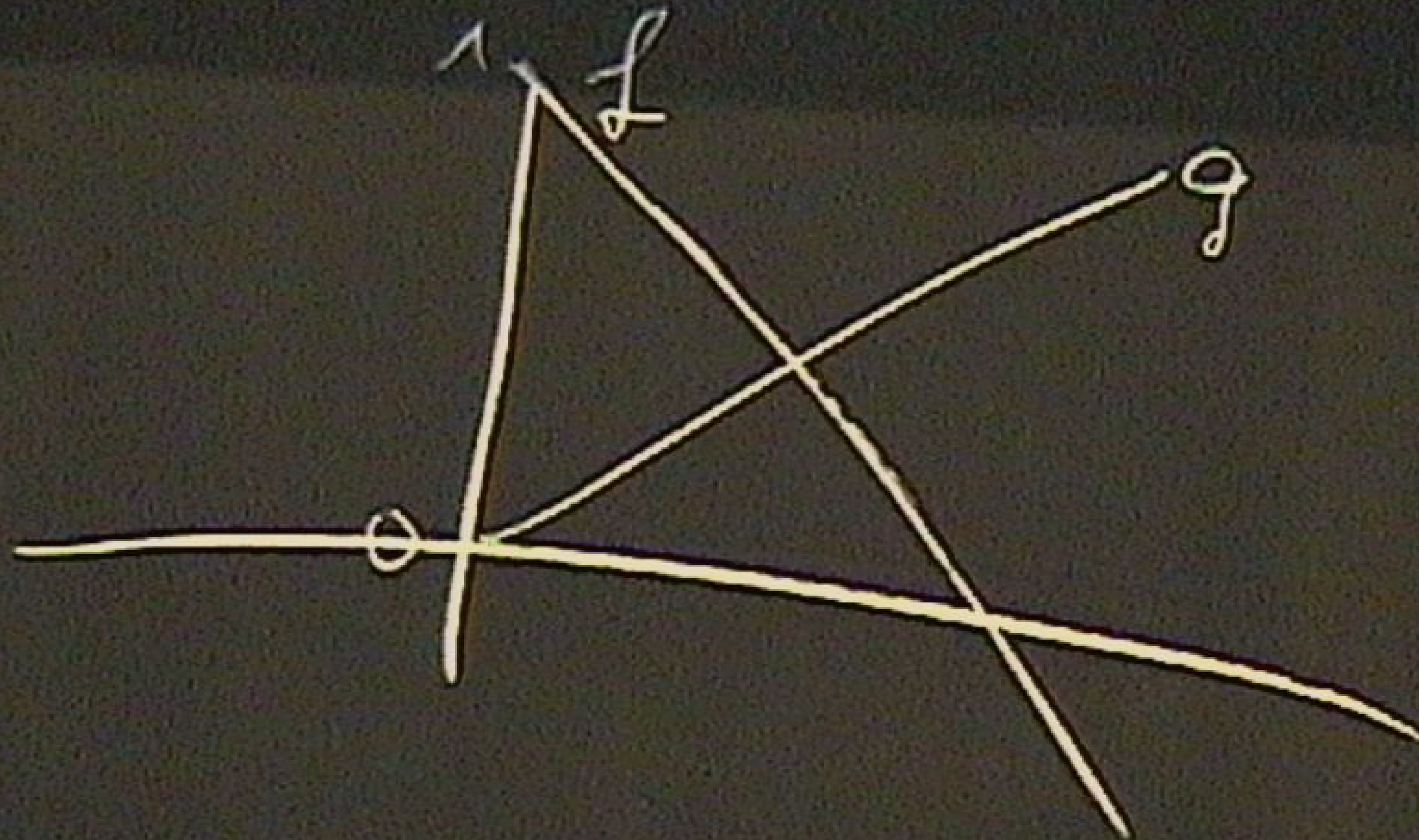
$$g_{min} = \min_{0\leq t\leq T}[E_1(t) - E_0(t)]$$

min. gap between two lowest eigenstates

$$\frac{\max_{0\leq t\leq T}|\langle E_1;t|\frac{dH}{dt}|E_0;t\rangle|}{g_{min}^2} \leq \varepsilon$$

Adiabatic Theorem

after running time T, AQC in solution state $|\Psi_1\rangle$ with $P = |\langle E_0;T|\Psi(T)\rangle|^2 \geq 1-\varepsilon^2$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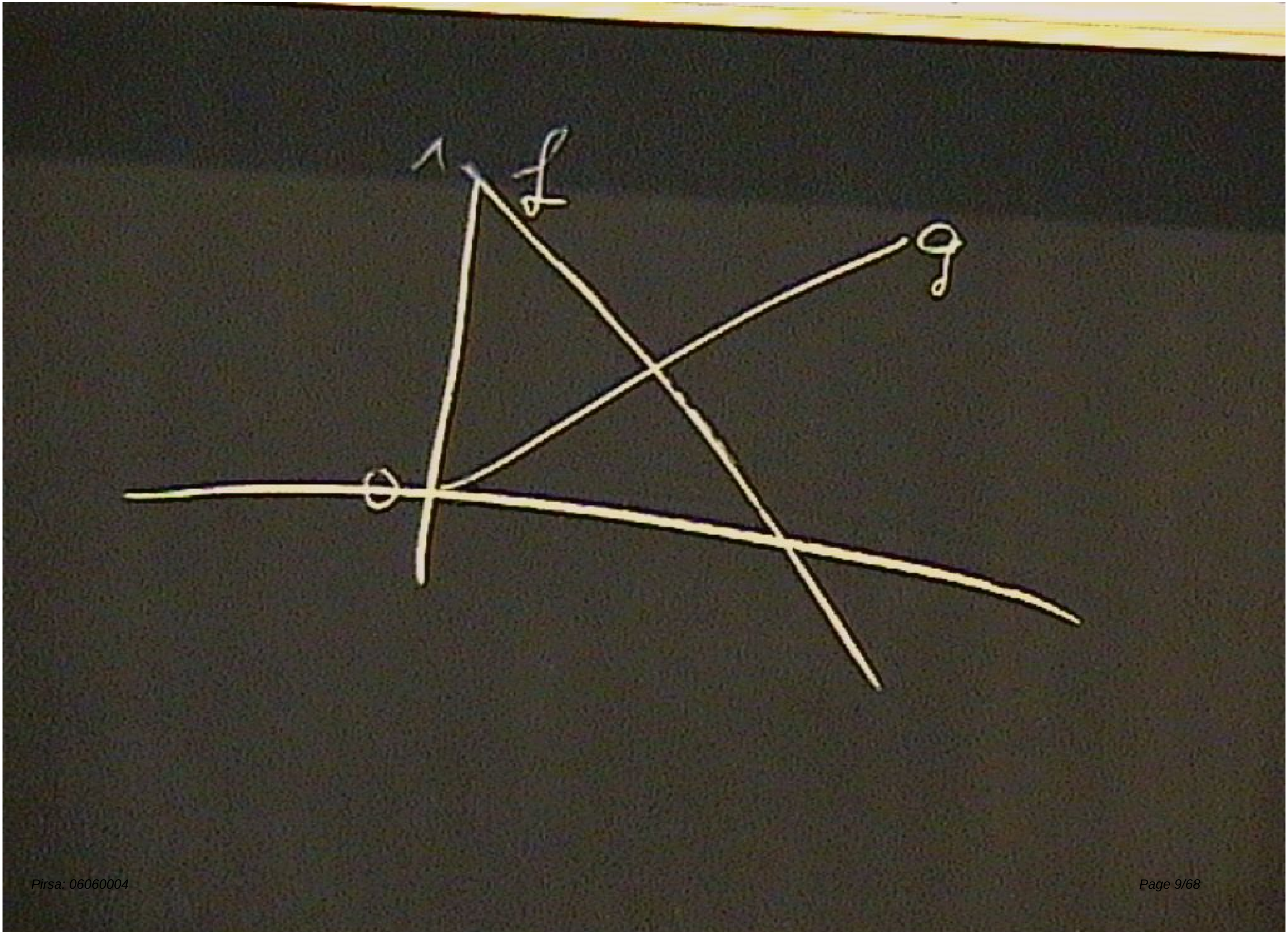

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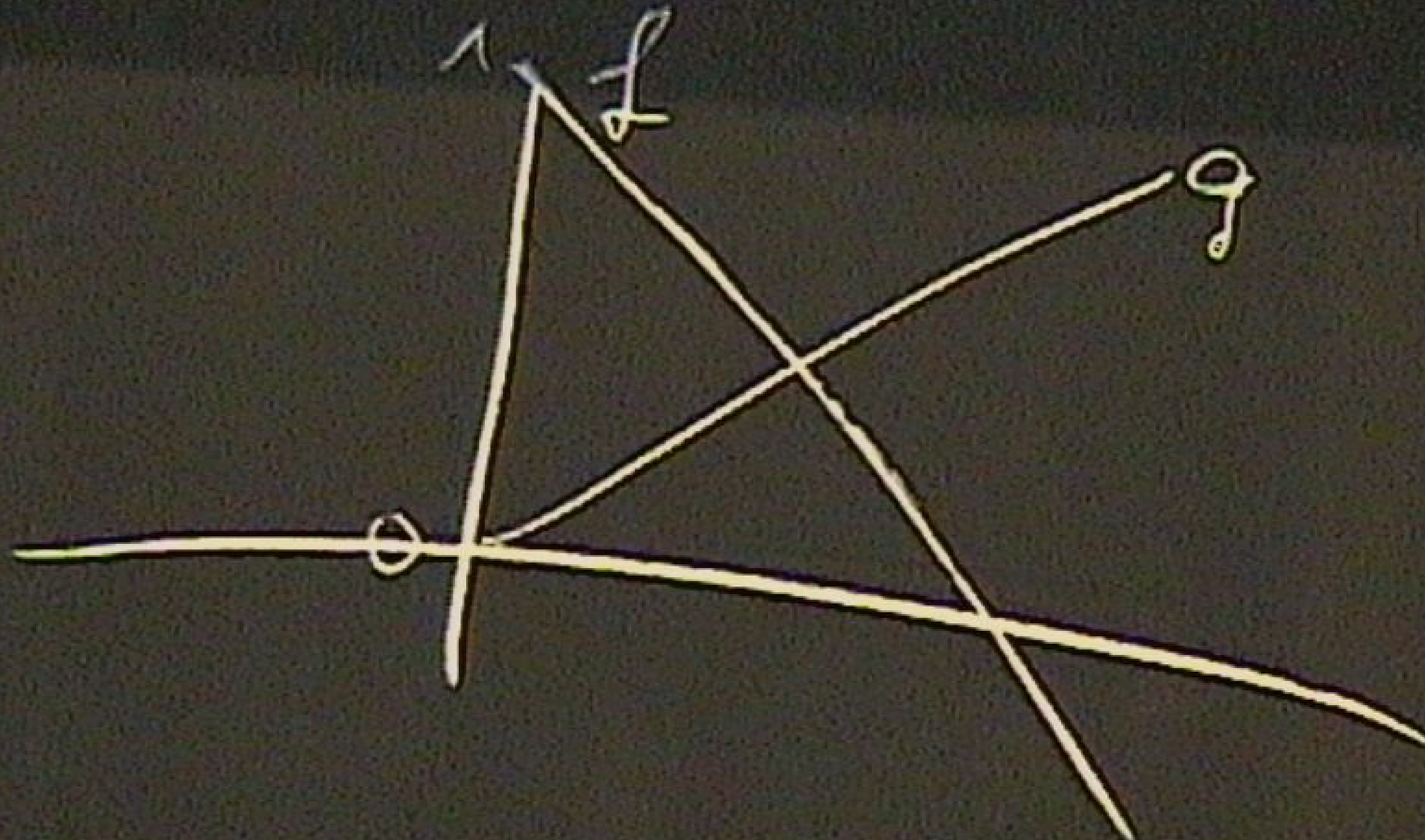
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Standard Model QC vs. Adiabatic QC

- qubits (or continuous variables)
- sequence of gates
- general purpose computer
- software separate from hardware
- classical input, quantum algorithm, measurement, classical output

- qubits (or continuous variables)
- H(t)
- special purpose computer
- software, hardware not separate
- output could be non-classical?

Example: Adiabatic Quantum Search

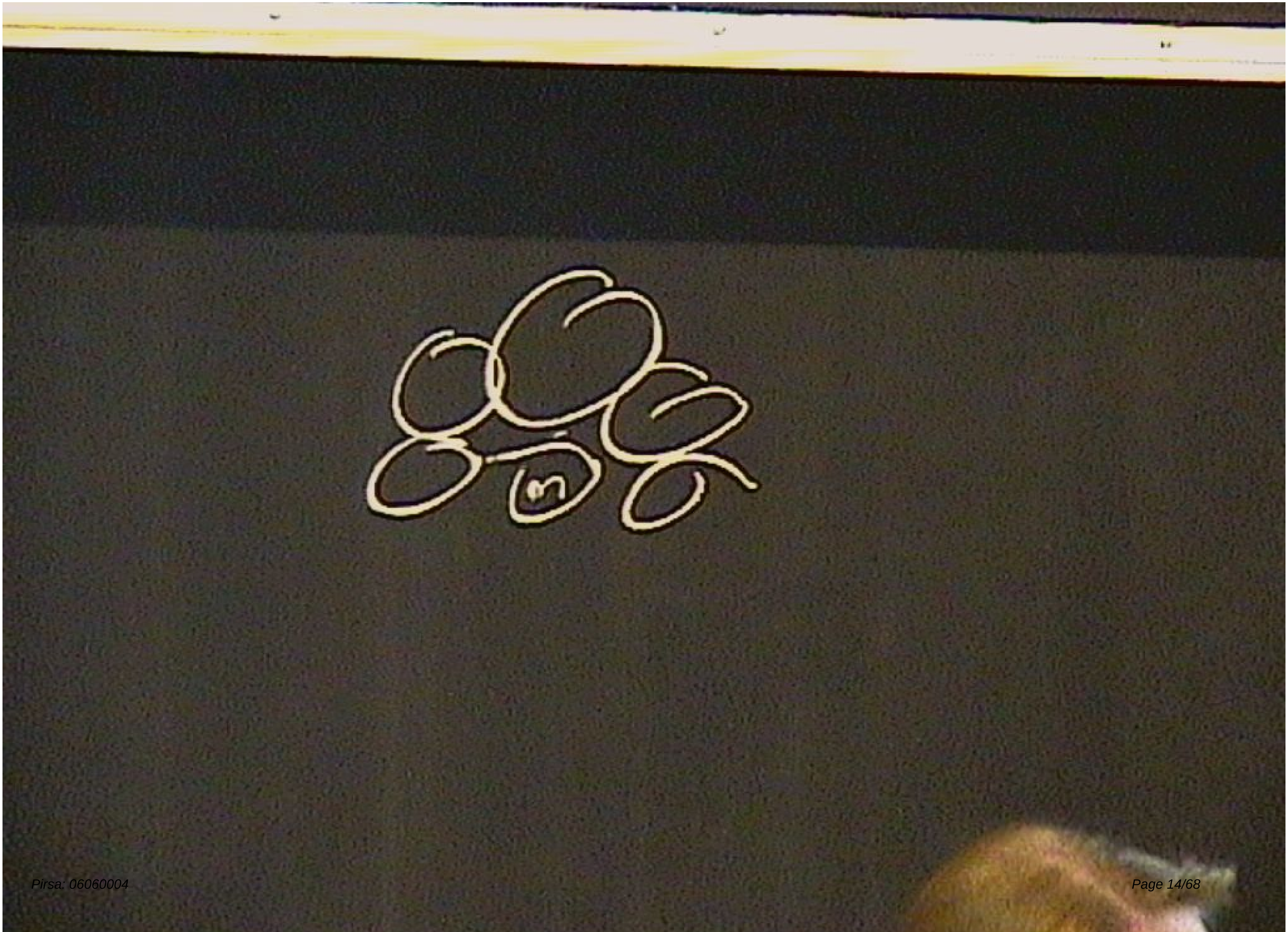
Task: in the unsorted database $|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle$ *"haystack"*

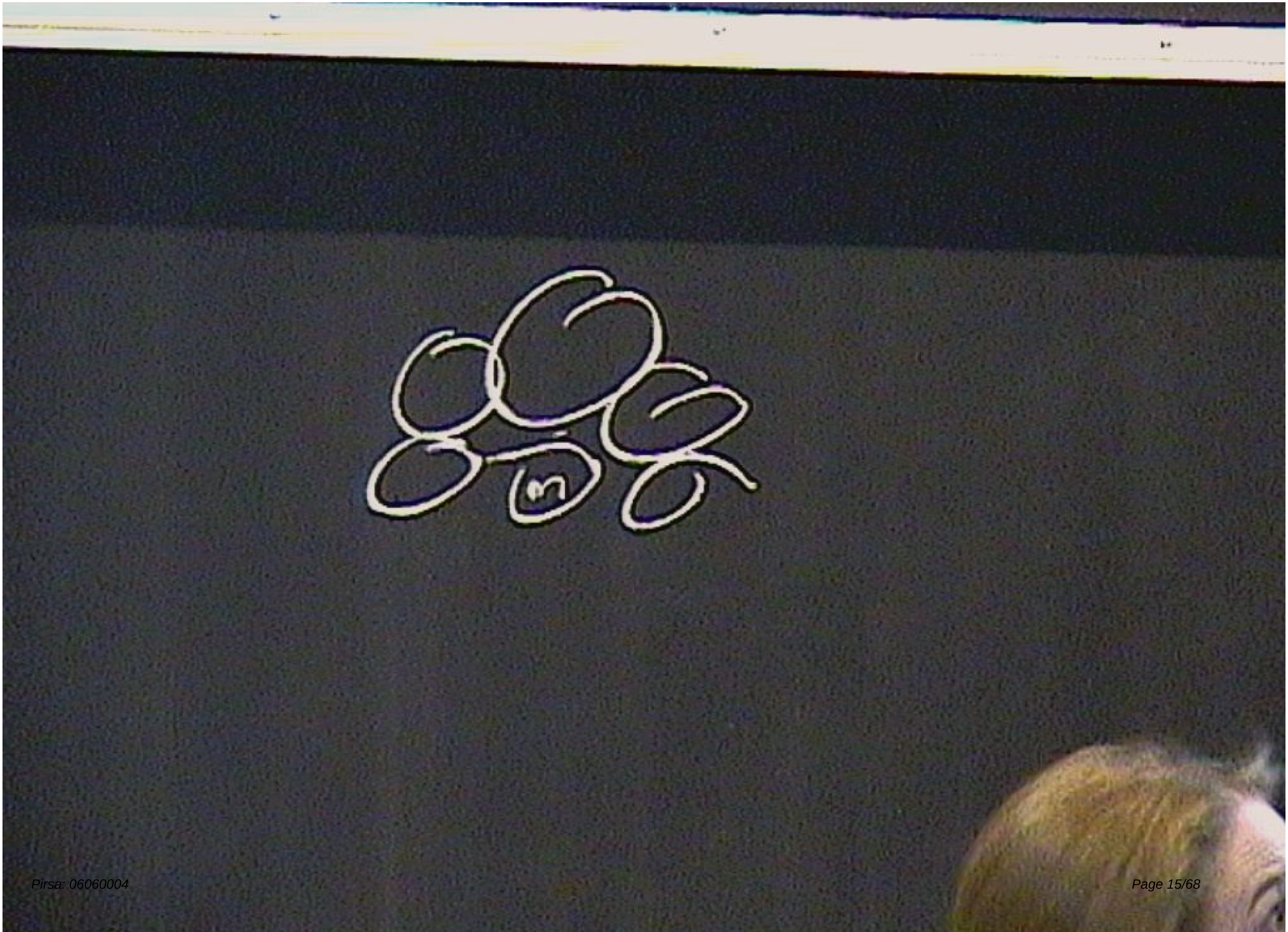
find the marked state $|\Psi_1\rangle = |m\rangle$ *"needle"*

"... as quick as possible

$$H(t) = f(t)(I - |\Psi_0\rangle\langle\Psi_0|) + g(t)(I - |m\rangle\langle m|)$$

without loss of generality, we choose $|m\rangle = |0\rangle = (1,0,0,0)$





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Adiabatic Quantum Search

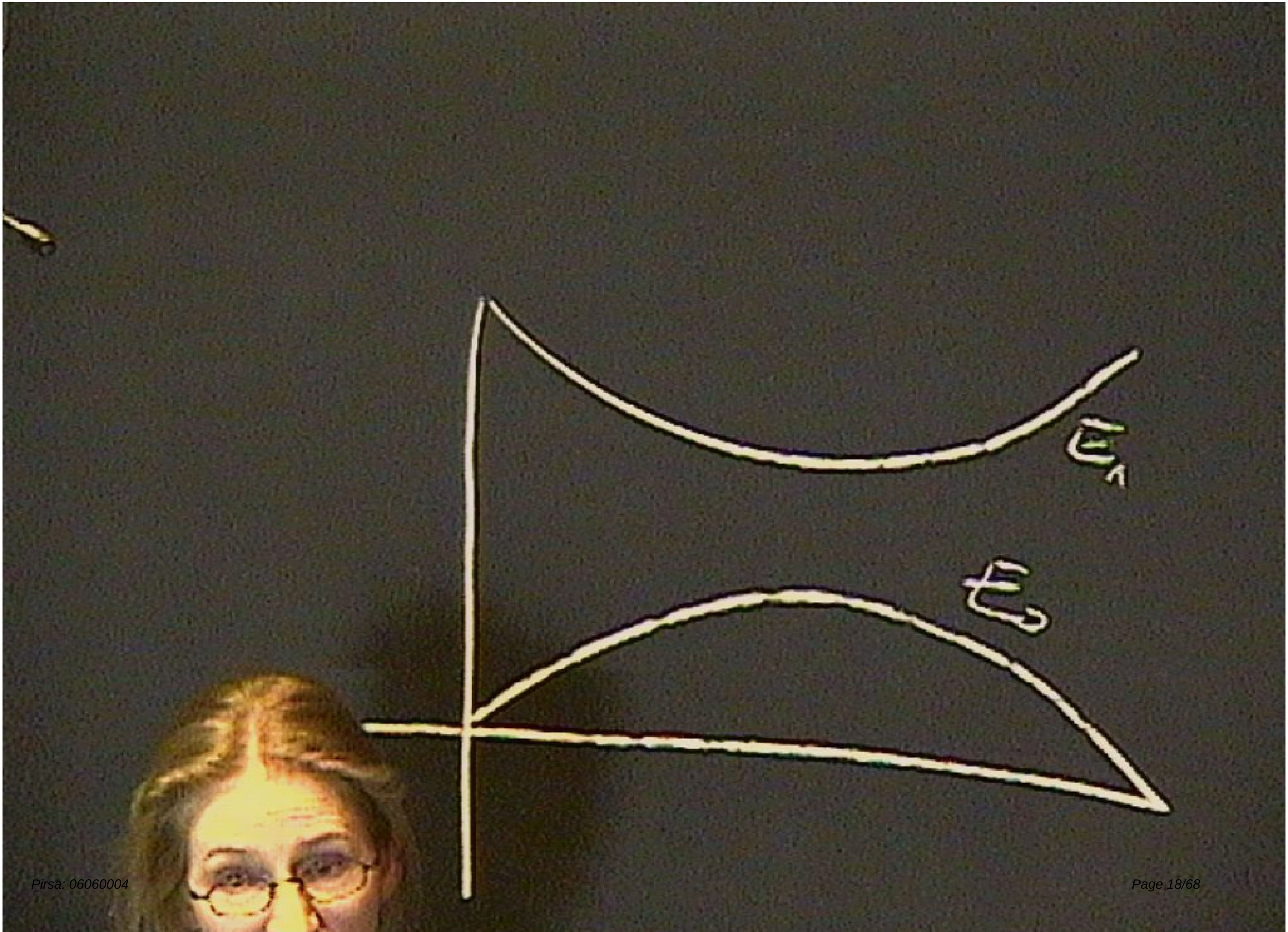
$$H(t) = \begin{pmatrix} f(1-\frac{1}{N}) & -\frac{f}{N} & \cdots & \cdots & -\frac{f}{N} \\ -\frac{f}{N} & f(1-\frac{1}{N})+g-\frac{f}{N} & & \cdots & -\frac{f}{N} \\ \vdots & -\frac{f}{N} & \cdots & \cdots & -\frac{f}{N} \\ \vdots & \vdots & \vdots & f(1-\frac{1}{N})+g & -\frac{f}{N} \\ -\frac{f}{N} & -\frac{f}{N} & \cdots & -\frac{f}{N} & f(1-\frac{1}{N})+g \end{pmatrix}$$

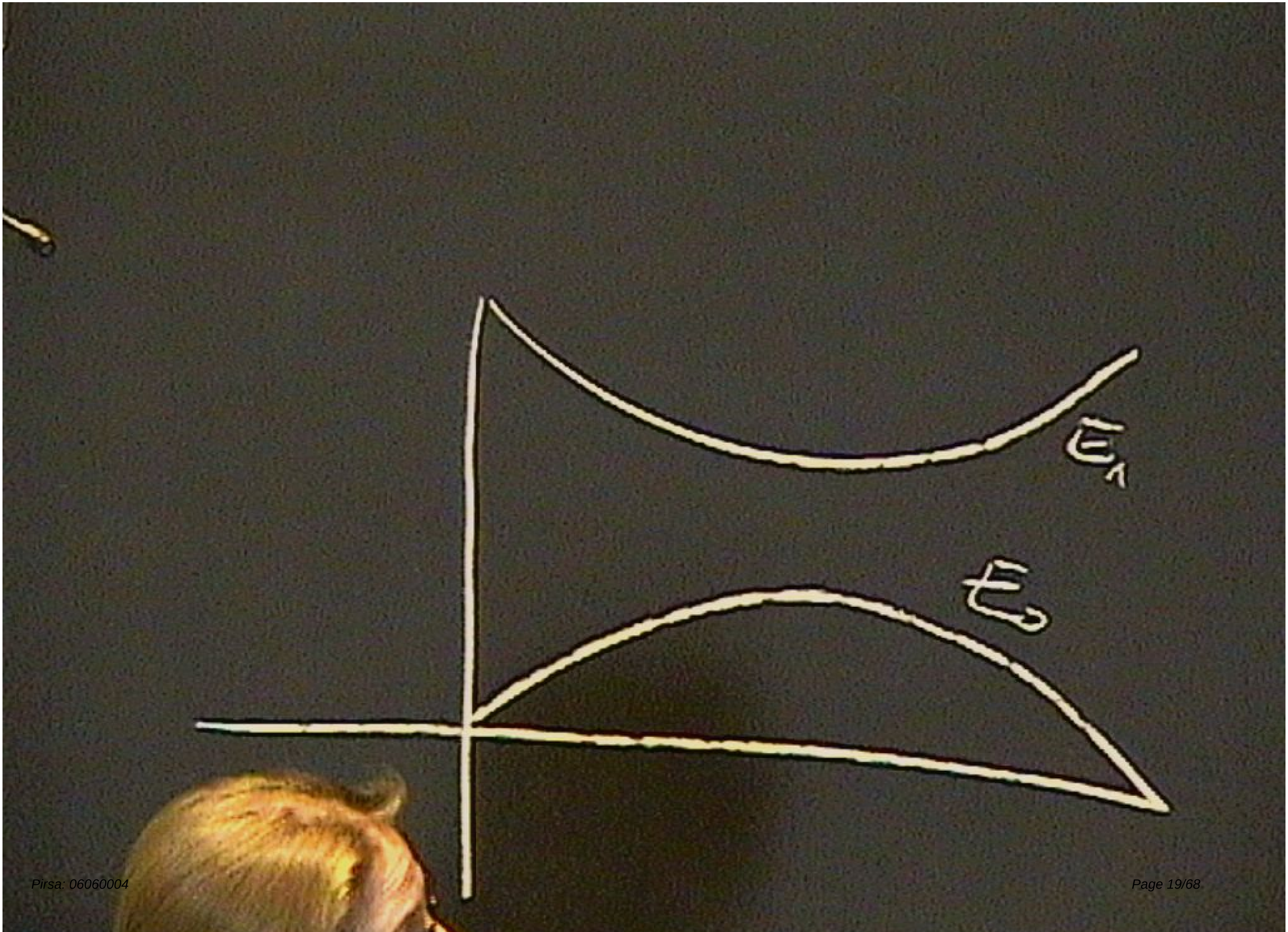
find eigenvectors and eigenvalues, two lowest:

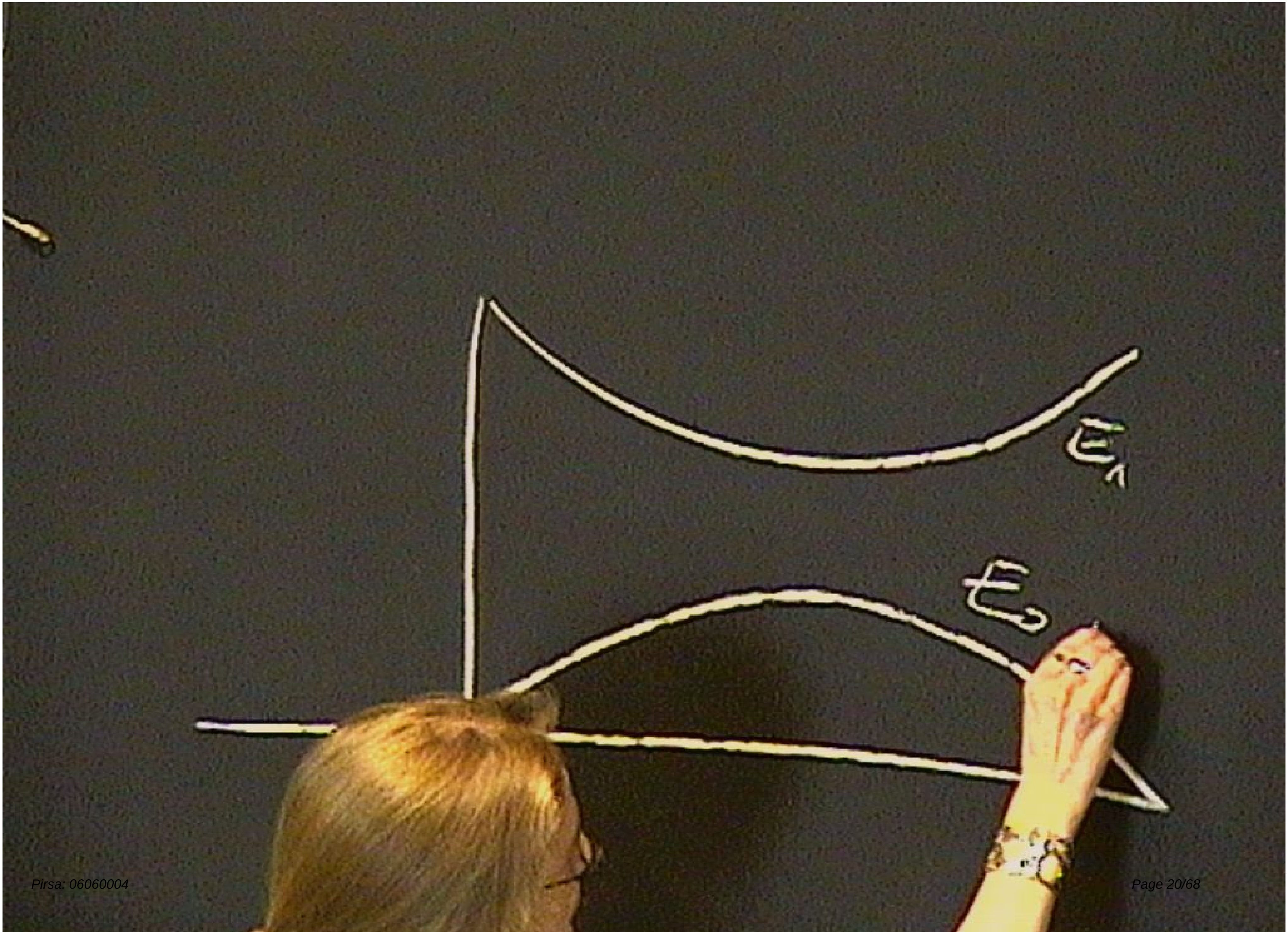
$$E_{\pm}(t) = \frac{1}{2}\left((f+g) \pm \sqrt{(f-g)^2 + \frac{4}{N} fg}\right)$$

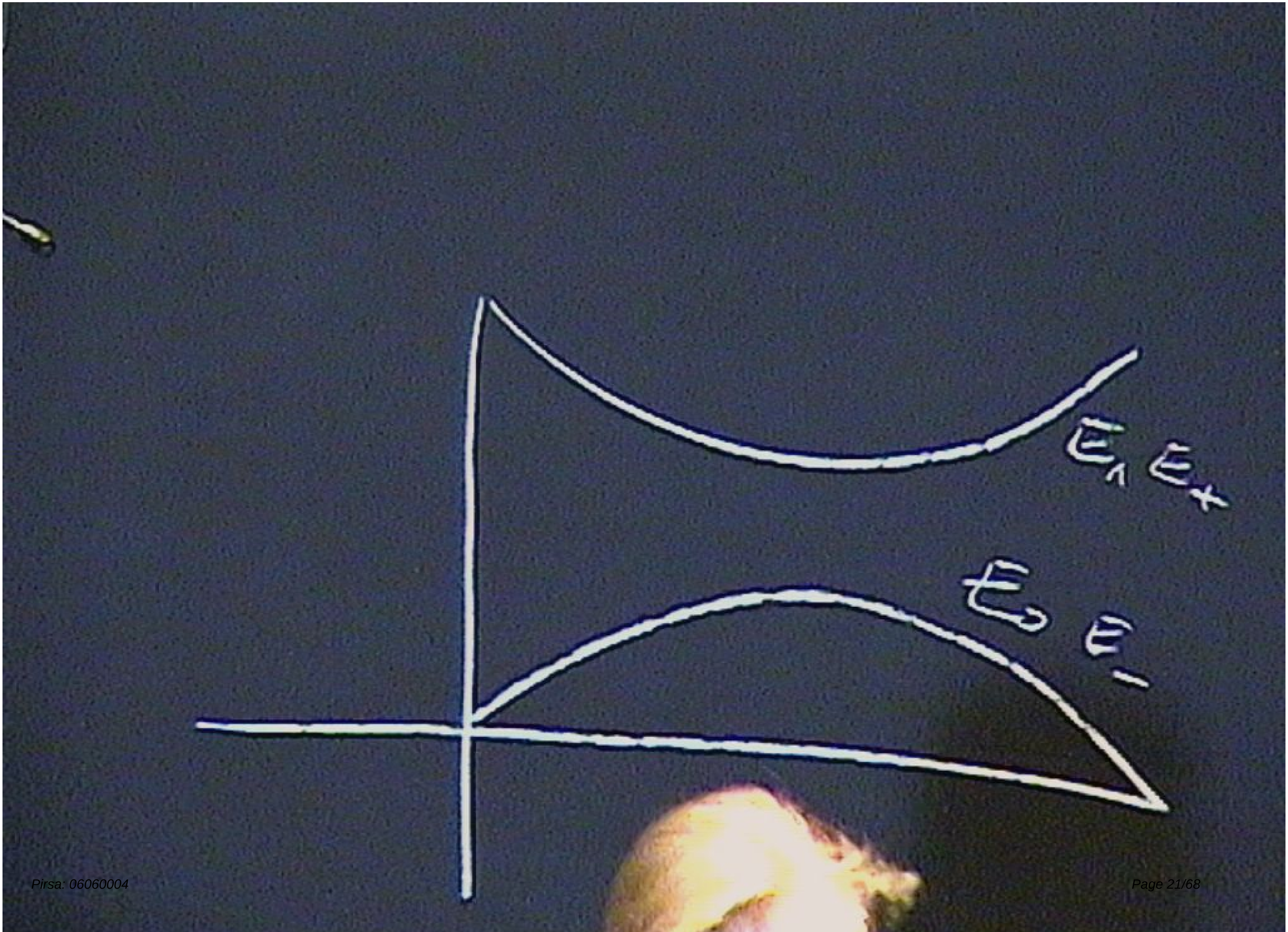
determine running time from adiabaticity condition

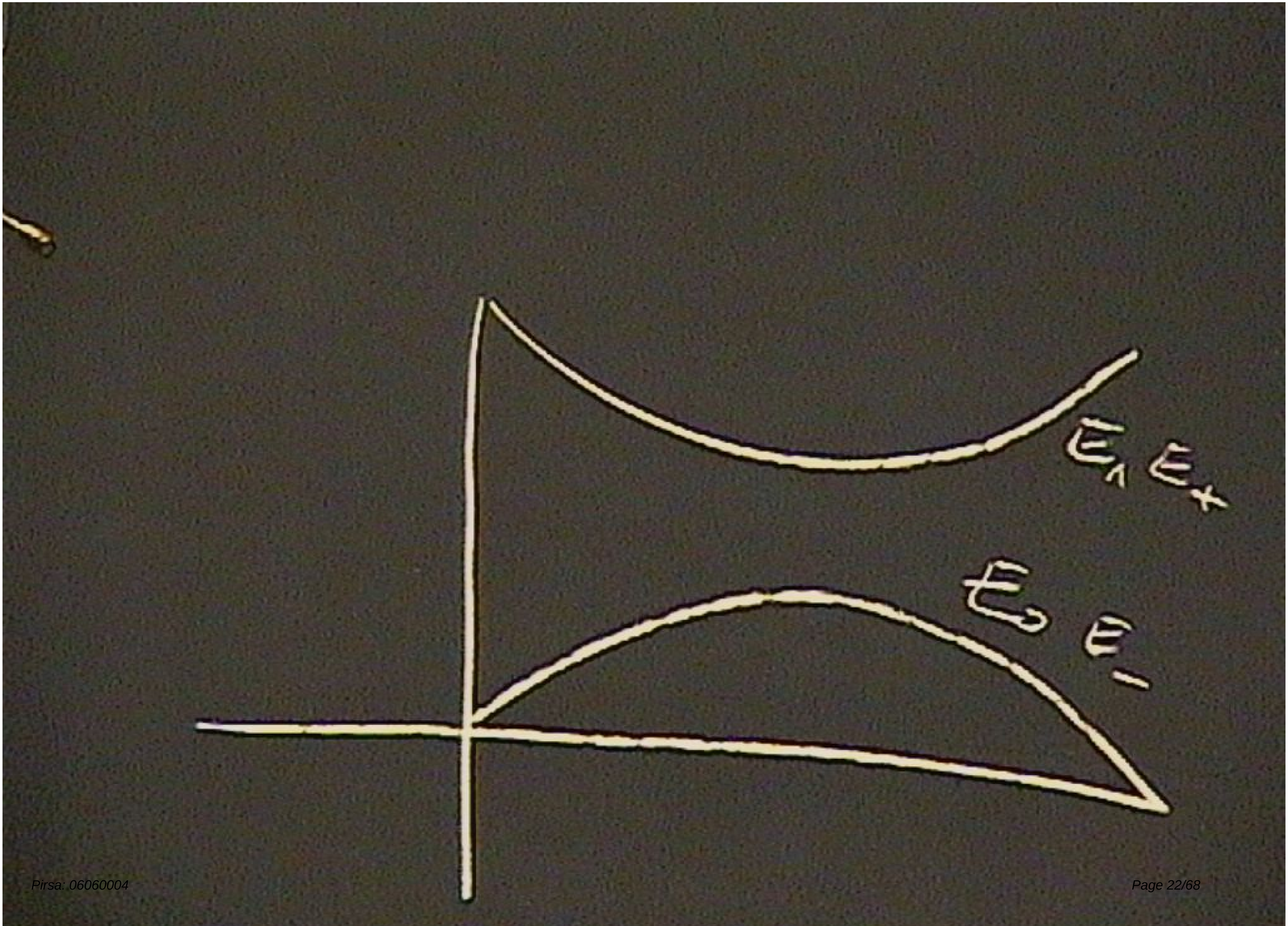
$$\frac{\max_{0 \leq t \leq T} |\langle E_1;t|\frac{dH}{dt}|E_0;t\rangle|}{g_{min}^2} \leq \varepsilon$$

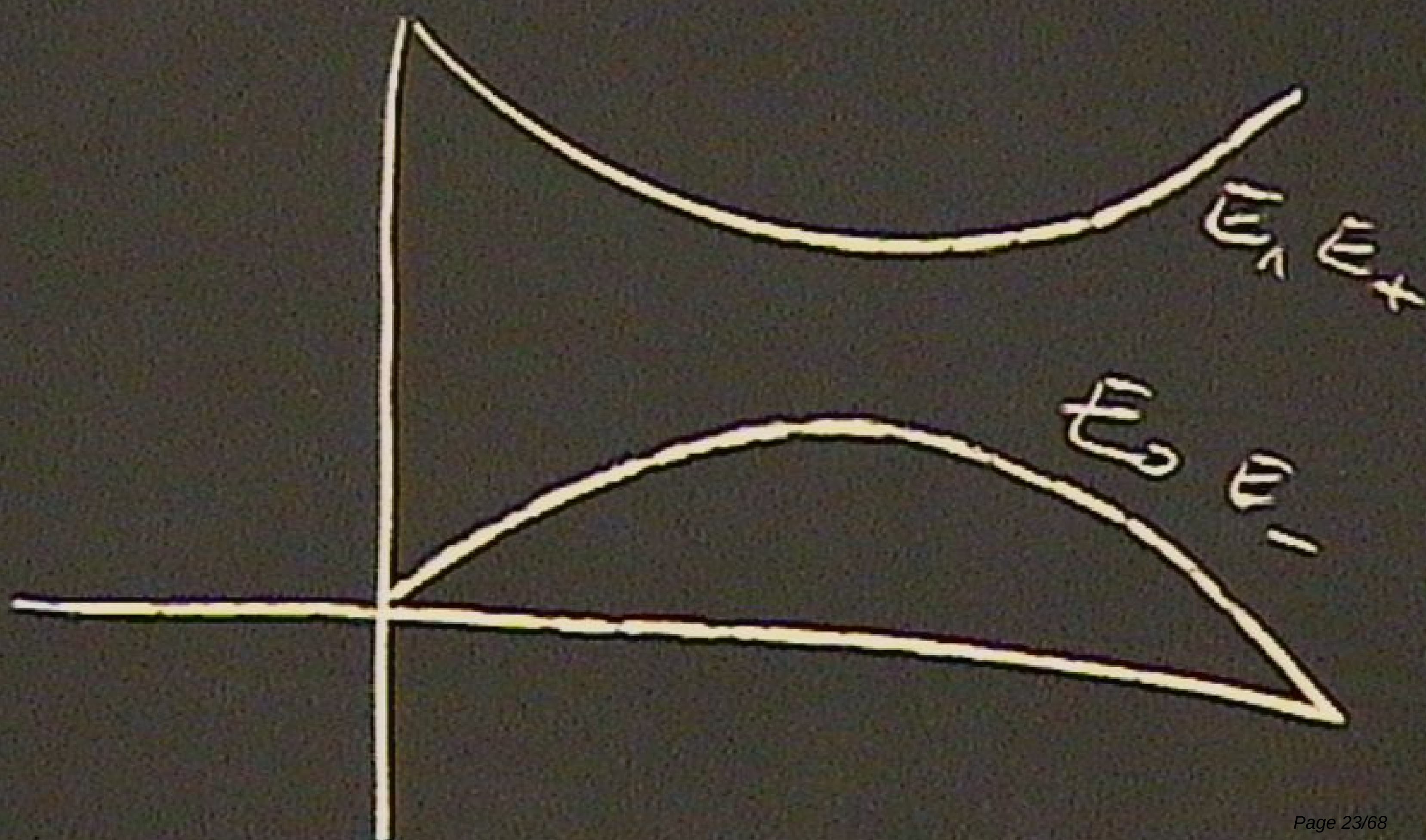

E_1
E_0


E_1
E_0


E_1
E_0


E_1 E_+
E_2 E_-


E_1 E_+
E_2 E_-


E_+
E_-

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Possible resources for (adiabatic) quantum computation

- energy, time `see Das, Kunstatter, Kobes, J.Phys.A 36 (2003) 2839`
- Hilbert space structure, superposition, “parallelism”
- entanglement, non-locality
- measurement

Entanglement

a state that cannot be written as a product of single qubit states, e.g.

$$|\Psi\rangle_{Bell} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B) \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$ *is **entangled***

state of whole system is completely known,
but state of each subsystem is not,
i.e. it is in mixed state

use this for information-based definition of entanglement :

Entropy of entanglement

for bipartite systems in pure state

$$E(\rho_{AB}) = S_{vN}(\rho_A) = -Tr(\rho_A \log \rho_A) \quad \text{with} \quad \rho_A = Tr_B(\rho_{AB})$$
$$= S_{vN}(\rho_B) = -Tr(\rho_B \log \rho_B) \quad \rho_B = Tr_A(\rho_{AB})$$

gives amount of information of one qubit
that can be obtained by making measurement
on the other qubit of a pair

$$S_{vN}(\rho) = -Tr(\rho \log \rho) = -\sum_n \lambda_n \log \lambda_n$$

calculated from eigenvalues

Toy model quantum computer: 2 qubits

$$|\Psi\rangle = \sum_{i=0}^{3} c_i|i\rangle \equiv (c_0, c_1, c_2, c_3)$$

general state in computational basis

$$\begin{aligned}
|0\rangle_2 &= |00\rangle = |0\rangle_1 \otimes |0\rangle_1 = (1,0,0,0)\\
|1\rangle_2 &= |01\rangle = |0\rangle_1 \otimes |1\rangle_1 = (0,1,0,0)\\
|2\rangle_2 &= |10\rangle = |1\rangle_1 \otimes |0\rangle_1 = (0,0,1,0)\\
|3\rangle_2 &= |11\rangle = |1\rangle_1 \otimes |1\rangle_1 = (0,0,0,1)
\end{aligned}$$

$$\rho_{AB} = \begin{pmatrix} c_0c_0^* c_0c_1^* c_0c_2^* c_0c_3^* \\ c_1c_0^* c_1c_1^* c_1c_2^* c_1c_3^* \\ c_2c_0^* c_2c_1^* c_2c_2^* c_2c_3^* \\ c_3c_0^* c_3c_1^* c_3c_2^* c_3c_3^* \end{pmatrix}$$

reduced density matrix

$$\rho_A = \begin{pmatrix} c_0c_0^* + c_1c_1^* & c_0c_2^* + c_1c_3^* \\ c_2c_0^* + c_3c_1^* & c_2c_2^* + c_3c_3^* \end{pmatrix}$$

eigenvalues

$$\mu_\pm = \frac{1}{2}(1 \pm \sqrt{1 - 4|c_0c_3 - c_1c_2|^2})$$

Toy Model **Adiabatic** Quantum Computer

$$|\psi(t)\rangle = \sum_{i=0}^{3} c_i(t)|i\rangle = (c_0(t), c_1(t), c_2(t), c_3(t))$$

instantaneous eigenstate

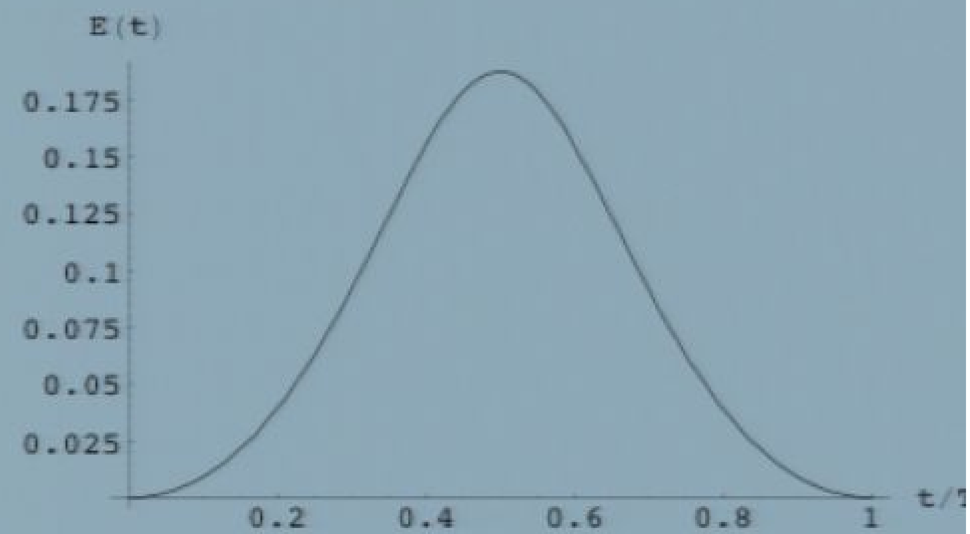
eigenvalues of reduced density matrix: $$\mu_\pm(t) = \frac{1}{2}(1 \pm \sqrt{1-4|c_0(t)c_3(t)-c_1(t)c_2(t)|^2})$$

calculate entropy of entanglement as function of time:

$$\mathcal{E}(t) = -\left[\mu_+(t)\log\mu_+(t) + \mu_-(t)\log\mu_-(t)\right]$$

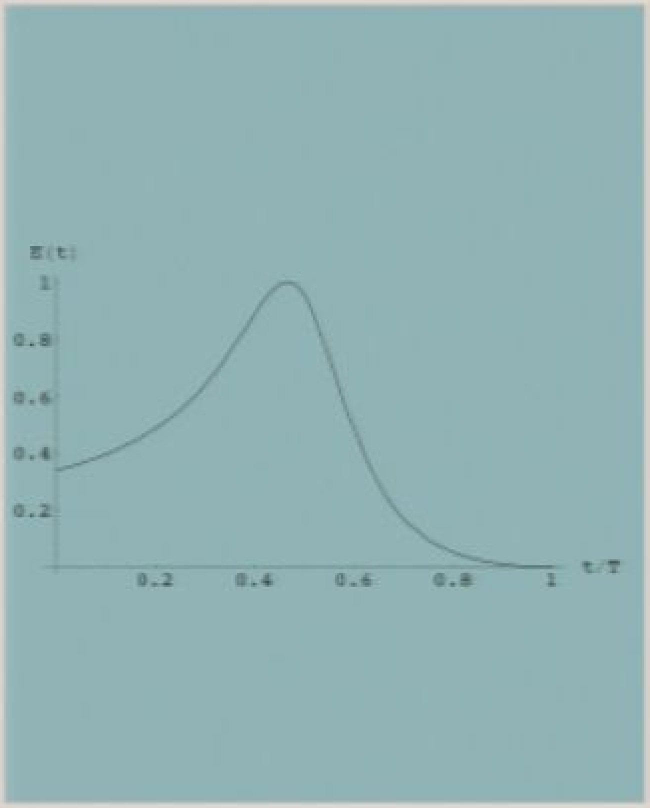
a first result:

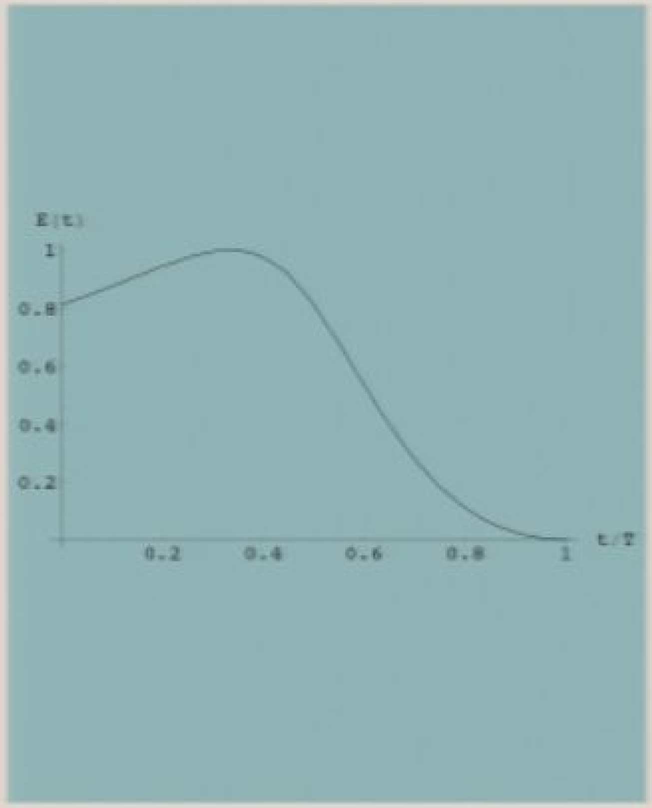
- entanglement is produced during the computation
- max value far below 1
- is entanglement **necessary** for algorithm to work, and for speed-up?
- or is it just a **byproduct** of time evolution?

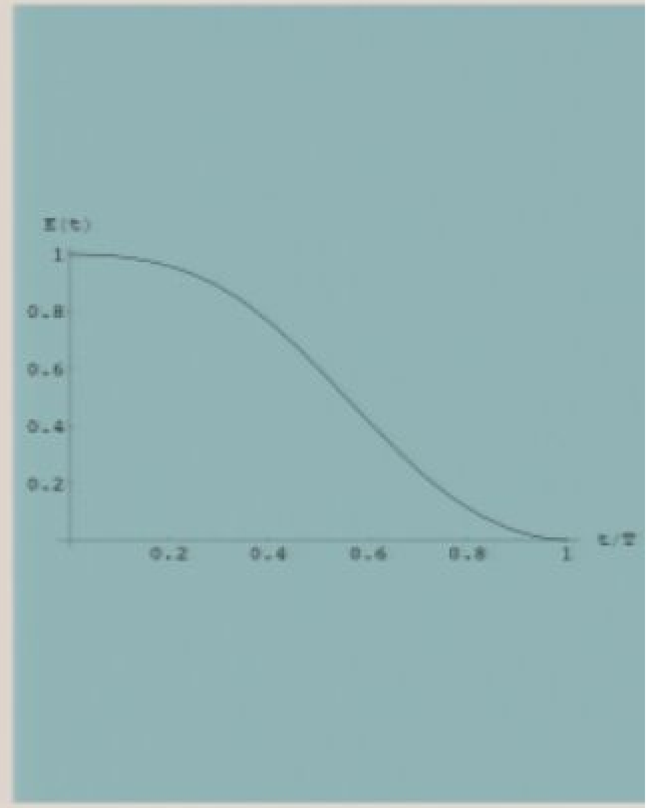


varying the initial state (changing the Hamiltonian):

two non-maximally entangled states — Bell state







for $c_0 \geq c_3$ (states close to $|m\rangle$), $\mathcal{E}$ approaches 0 monotonically

for $c_3 \gg c_0$ (states further away from $|m\rangle$), difference between initial and maximum $\mathcal{E}$ larg

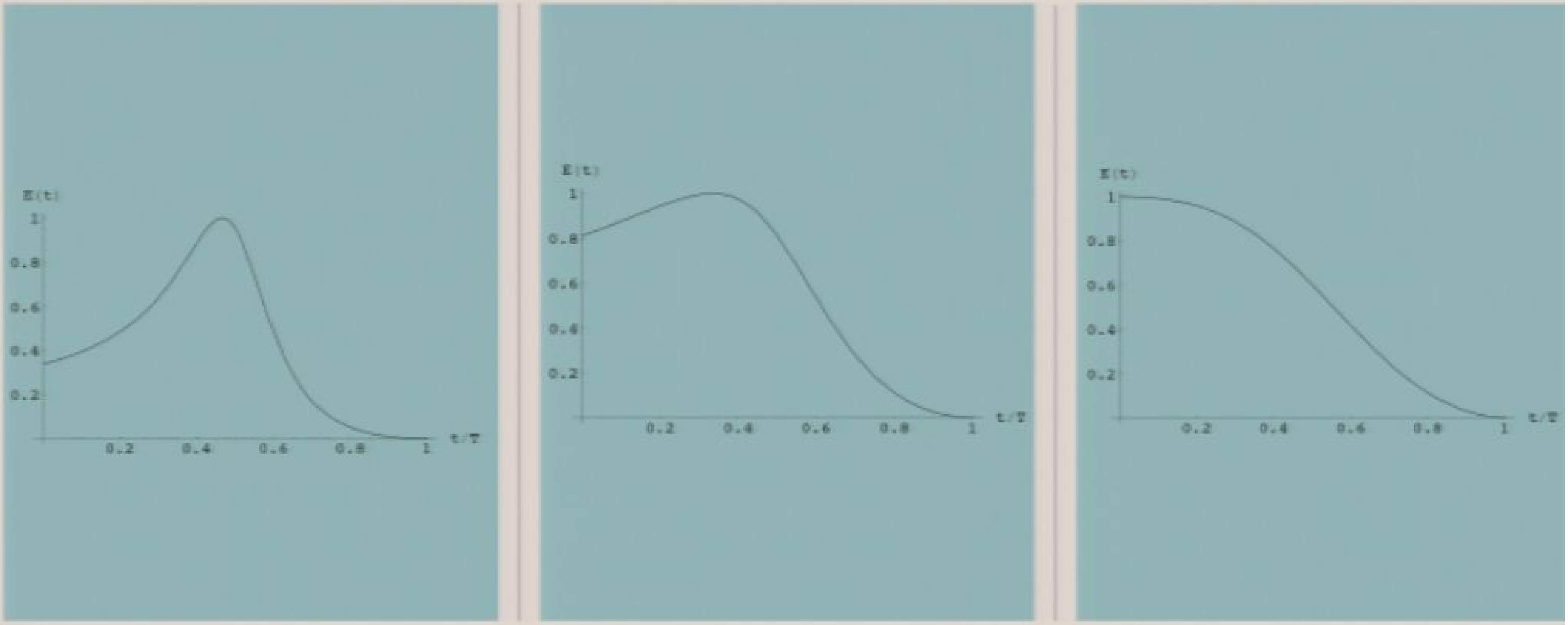
more results and more *questions:*

- non-maximally entangled state in fig.a: Hamiltonian with the same spectrum as equally weighted superposition, same running time
- *running time correlated with spectrum (as adiabatic theorem suggests), and not so much with entanglement?*
- for states further away from marked state, more entanglement created during search; speed-up related to overlap of initial and final state $1/N = \langle \Psi_0 | m \rangle$
- *relevance of overlap (fidelity) for algorithm?*
- *larger n; and Large n*
- *other algorithms*

varying the initial state (changing the Hamiltonian):

two non-maximally entangled states

Bell state



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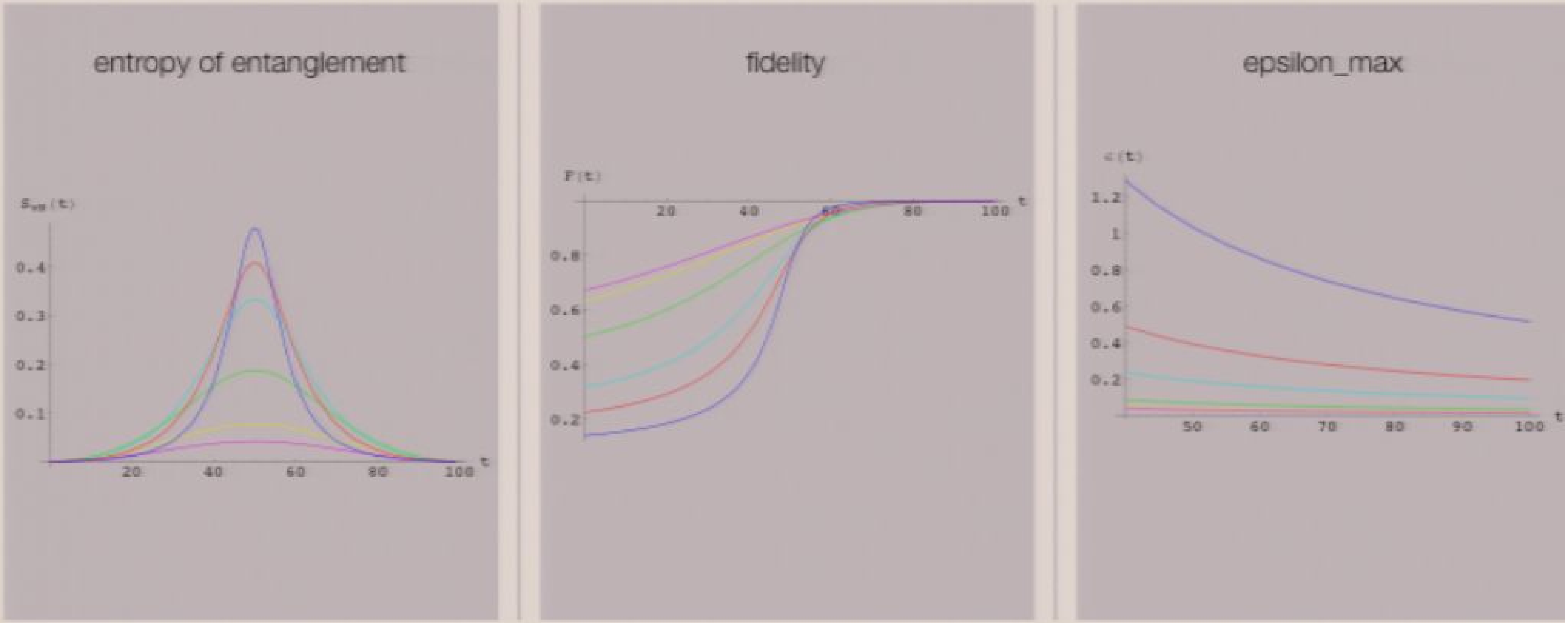
search with different initial states, n=2 qubits



$|Y\rangle = N(2,2,1,1)$

$|C\rangle = N(1,1,2,2)$

$|G\rangle = N(1,1,1,1)$

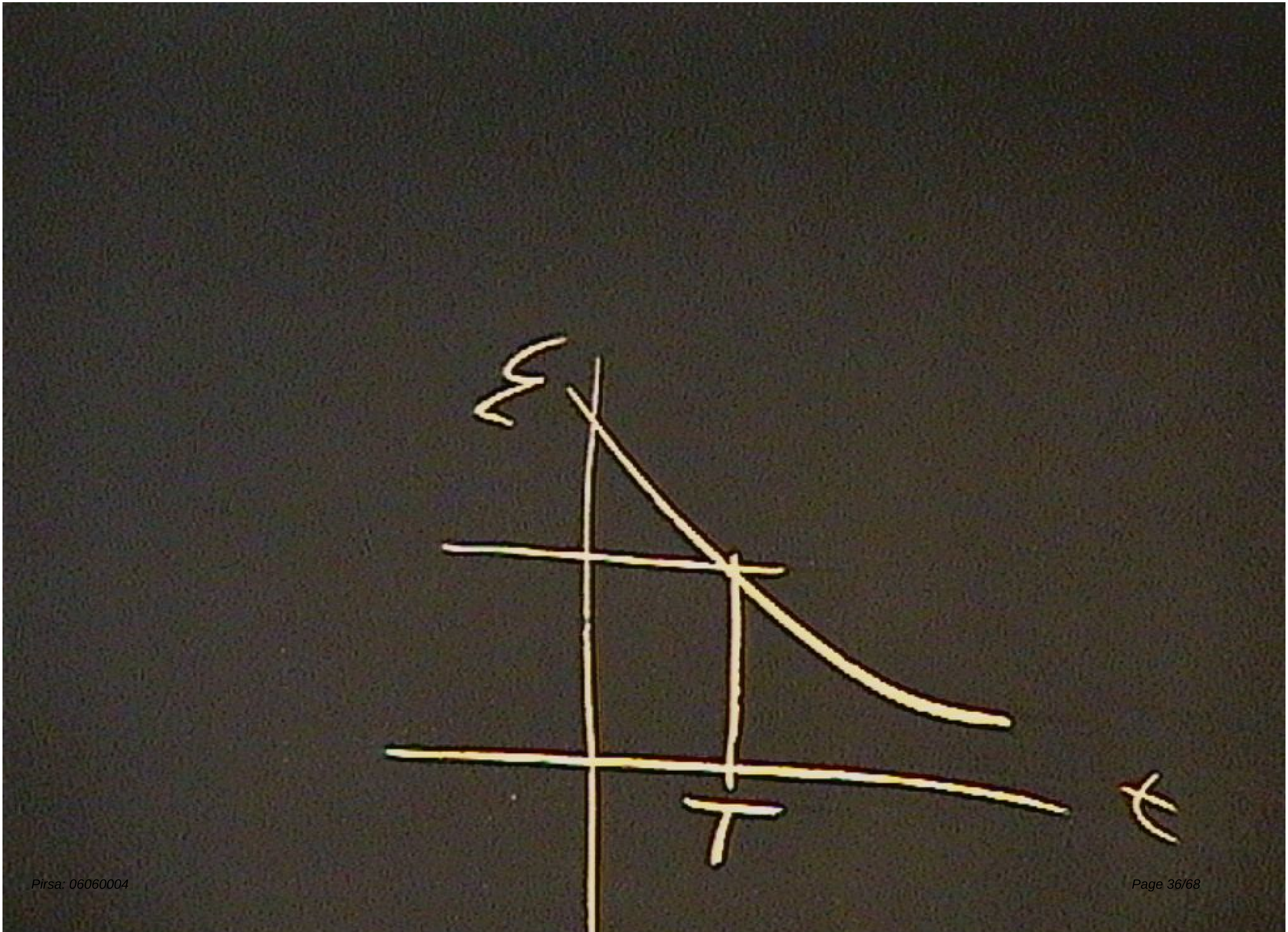
$|R\rangle = N(1,3,1,3)$

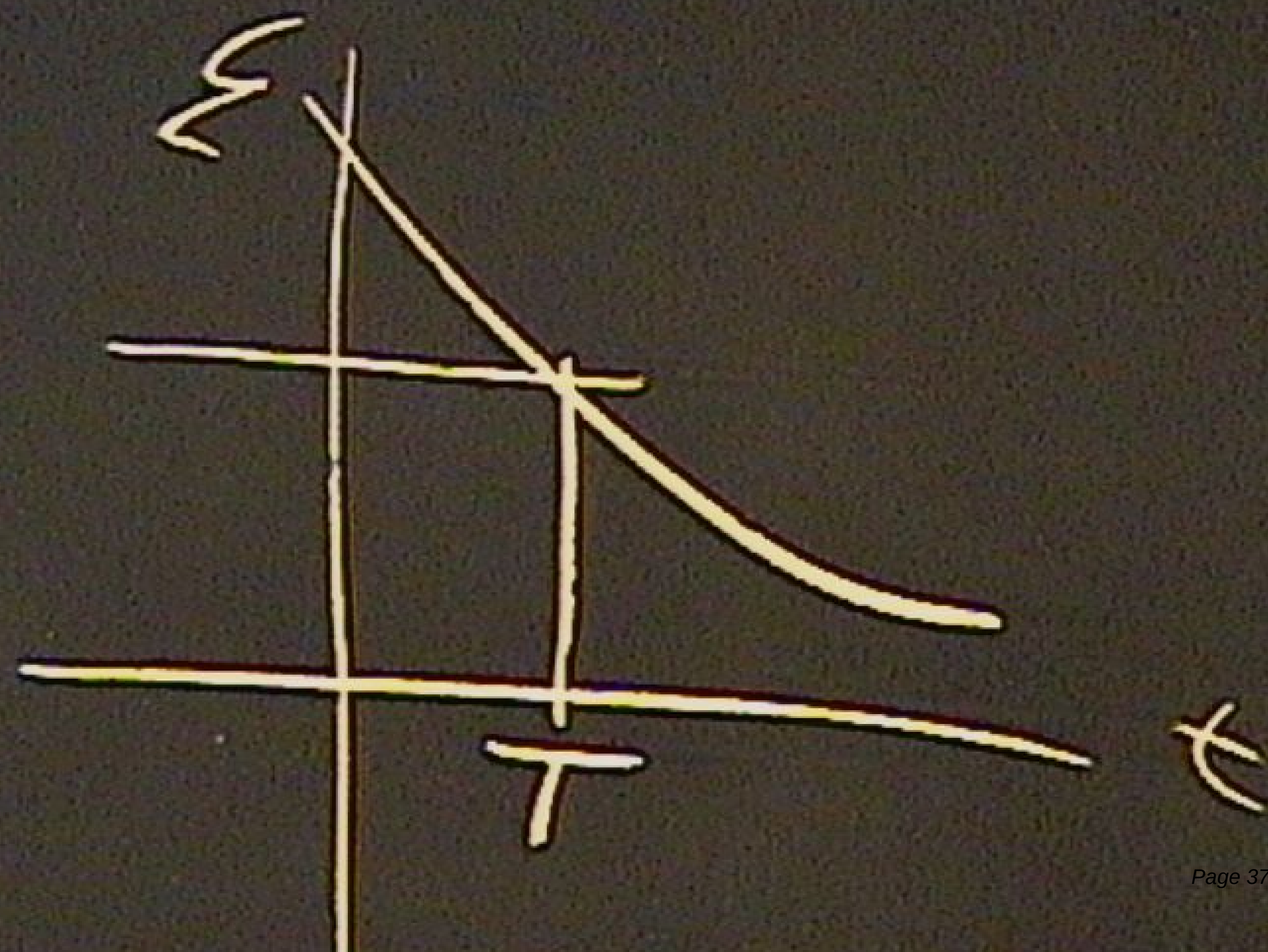
$|P\rangle = N(3,1,3,1)$

$|B\rangle = N(1,5,1,5)$

correlations: large maximum entanglement - large running time
small initial fidelity - large running time

but...


ε
T
t


ε
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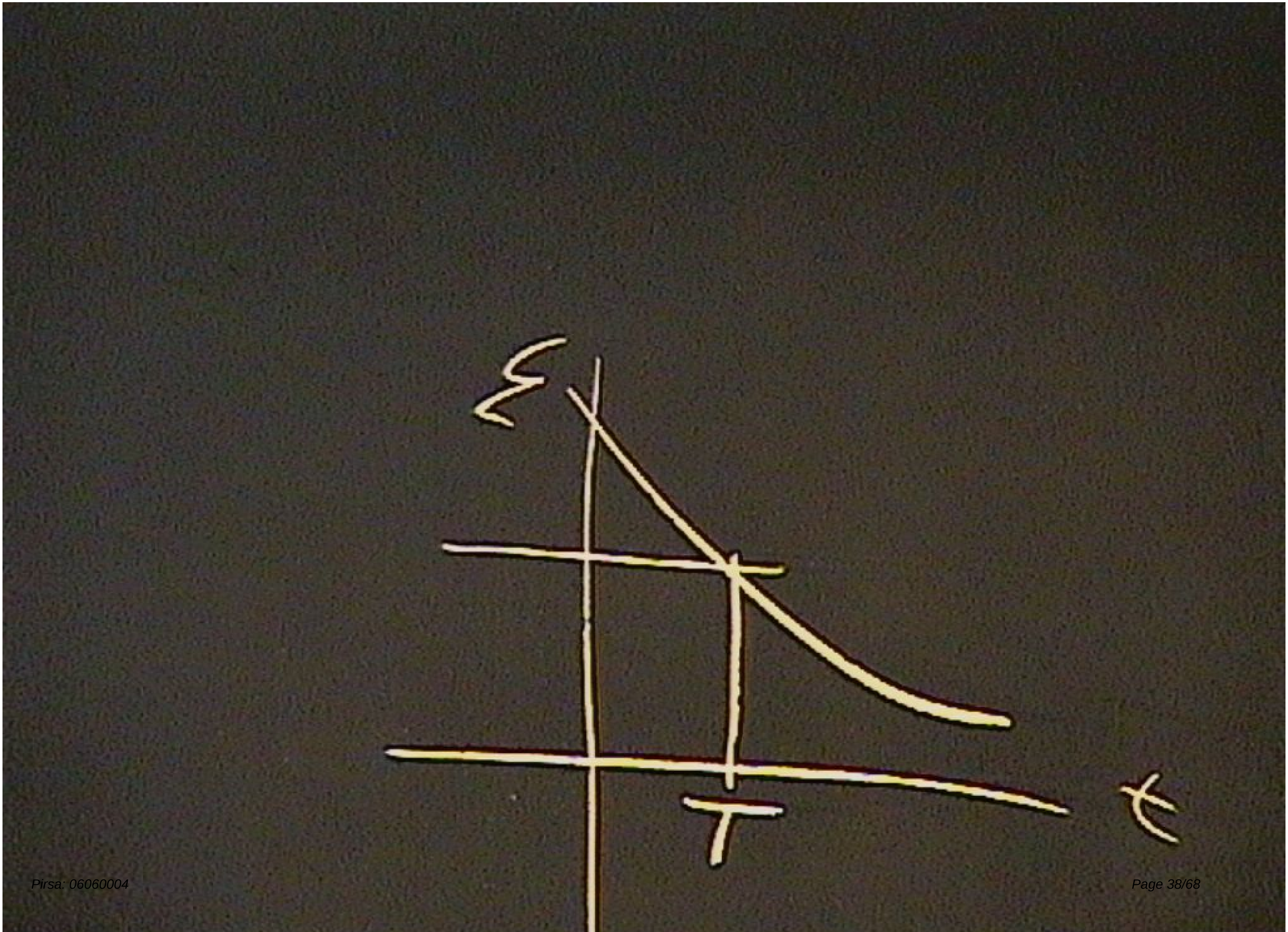
ε
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t

Deutsch-Jozsa Algorithm

determine whether a function $F : \{0,1\}^n \rightarrow \{0,1\}$

is constant or balanced

e.g. for n=1: four possible outcomes

$F(0)=F(1)=0$
$F(0)=F(1)=1$
constant (all outputs identical)

$F(0)=0; F(1)=1$
$F(0)=1; F(1)=0$
balanced (number of zeros=number of ones)

classically: need to measure F(0) and F(1)

quantum: one measurement yields result

Deutsch-Jozsa algorithm: adiabatic version

Das, Kobes, Kunstatter, quant-ph/0111032

$$|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle \quad \Rightarrow \quad |\Psi_1\rangle = \alpha|0\rangle + \frac{\beta}{\sqrt{N-1}} \sum_{i=0}^{N-1} |i\rangle$$

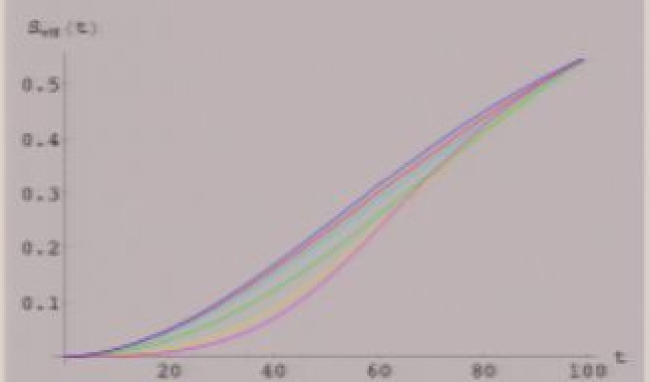
$$\alpha = \frac{1}{N}\left|\sum_{x\in\{0,1\}^n} (-1)^{F(x)}\right|; \beta = 1-\alpha$$

if measurement yields $|0\rangle$, F is constant, otherwise balanced

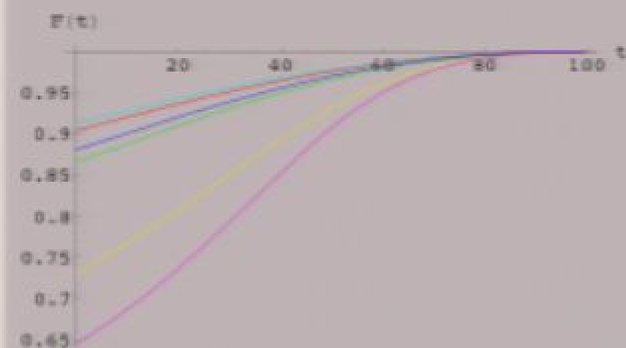
note: $\alpha = 1, \beta = 0$ coincides with search algorithm for $|m\rangle = |0\rangle$

Deutsch-Jozsa algorithm, n=2

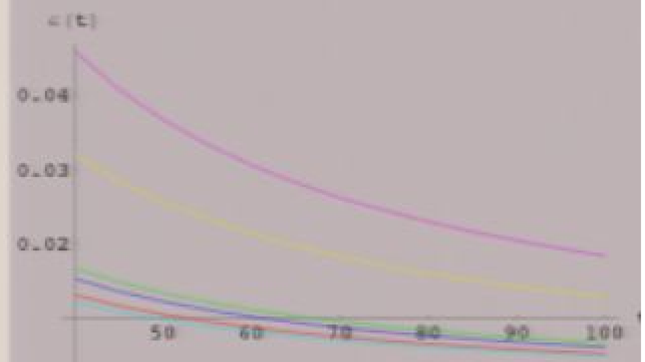
entropy of entanglement



fidelity



epsilon_max



correlations: small initial fidelity - large running time

try “larger” system, n=3

- how should we calculate entropy of entanglement?

results depend on which qubits are traced over

- check if results on running time etc. change with a different definition of entanglement

- alternative definition of entanglement: not mathematical or axiomatic, or operational, but physically motivated

measuring entanglement geometrically

as the distance $|d\rangle = |c\rangle - |p\rangle$

between the composite state under consideration $|c\rangle = (c_0, c_1, c_2, c_3)$

and the closest product state

$$\begin{aligned} |p\rangle &= (a_0|0\rangle + a_1|1\rangle) \otimes (b_0|0\rangle + b_1|1\rangle) \\ &= a_0 b_0|00\rangle + a_0 b_1|01\rangle + a_1 b_0|10\rangle + a_1 b_1|11\rangle \\ &= (a_0 b_0, a_0 b_1, a_1 b_0, a_1 b_1) \end{aligned}$$

i.e. minimize the function

$$D(a_0, a_1, b_0, b_1) = c_0^2 + c_1^2 + c_2^2 + c_3^2 + (a_0^2 + a_1^2)(b_0^2 + b_1^2) - 2(a_0 b_0 c_0 + a_0 b_1 c_1 + a_1 b_0 c_2 + a_1 b_1 c_3)$$

remarks

- advantage over entropy of entanglement: generalization to more than two subsystems straightforward, problem of how to partition the system does not occur
- D is very similar to the Hilbert-Schmidt distance, but faster to calculate numerically
- minimization of D with unnormalized product states gives condition that might suggest geometric interpretation of von Neumann entropy of entanglement for n=2: length of closest non-normalized product state determines one of the terms

$$(N_a N_b)^2 - N_a N_b + |c_0 c_3 - c_1 c_2|^2 = 0$$

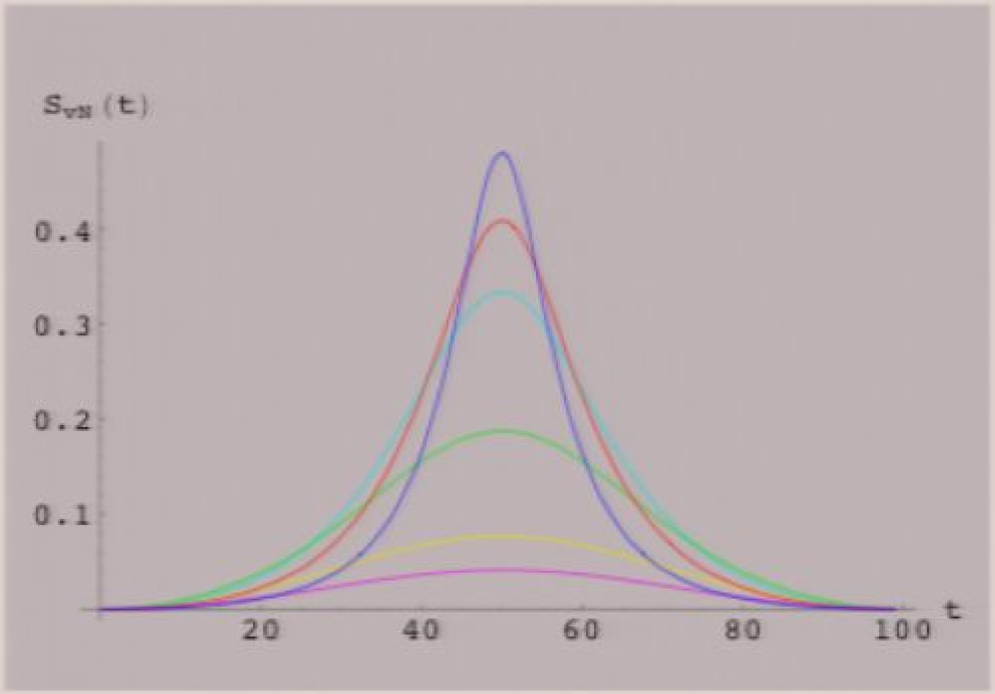
$$\mu^2 - \mu + |c_0 c_3 - c_1 c_2|^2 = 0$$

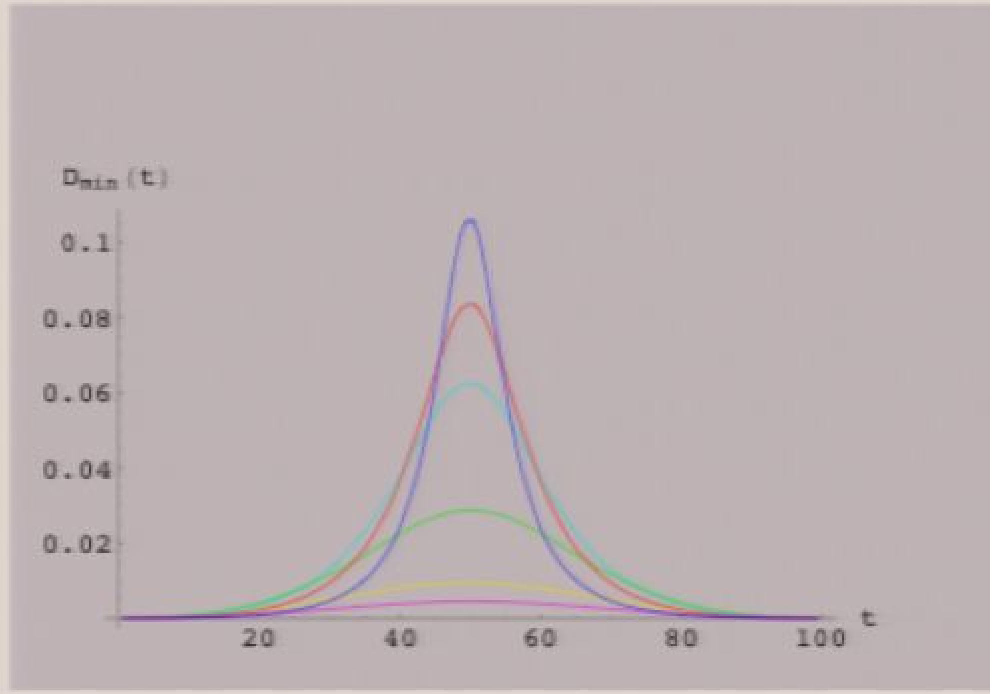
with $N_a N_b = (a_0^2 + a_1^2)(b_0^2 + b_1^2)$

geometrical interpretation:

- space of all normalized 4-dim. states is a 3-sphere
- space of normalized *product* states is a subset on the surface of this 3-sphere
- unnormalized *product* states lie on radial lines that intersect the 3-sphere
- the closest *product* state to an arbitrary normalized state will lie on such a line in the interior of the 3-sphere, i.e. it will be unnormalized
- the only case where the closest *product* state will be normalized is if the arbitrary state is a product state itself

Entanglement for the search algorithm, n=2





another distance measure

we compare the function

$$G(t) = \sum_{i=0}^{N-1} ||c_i(t)| - |c_i(T)||$$

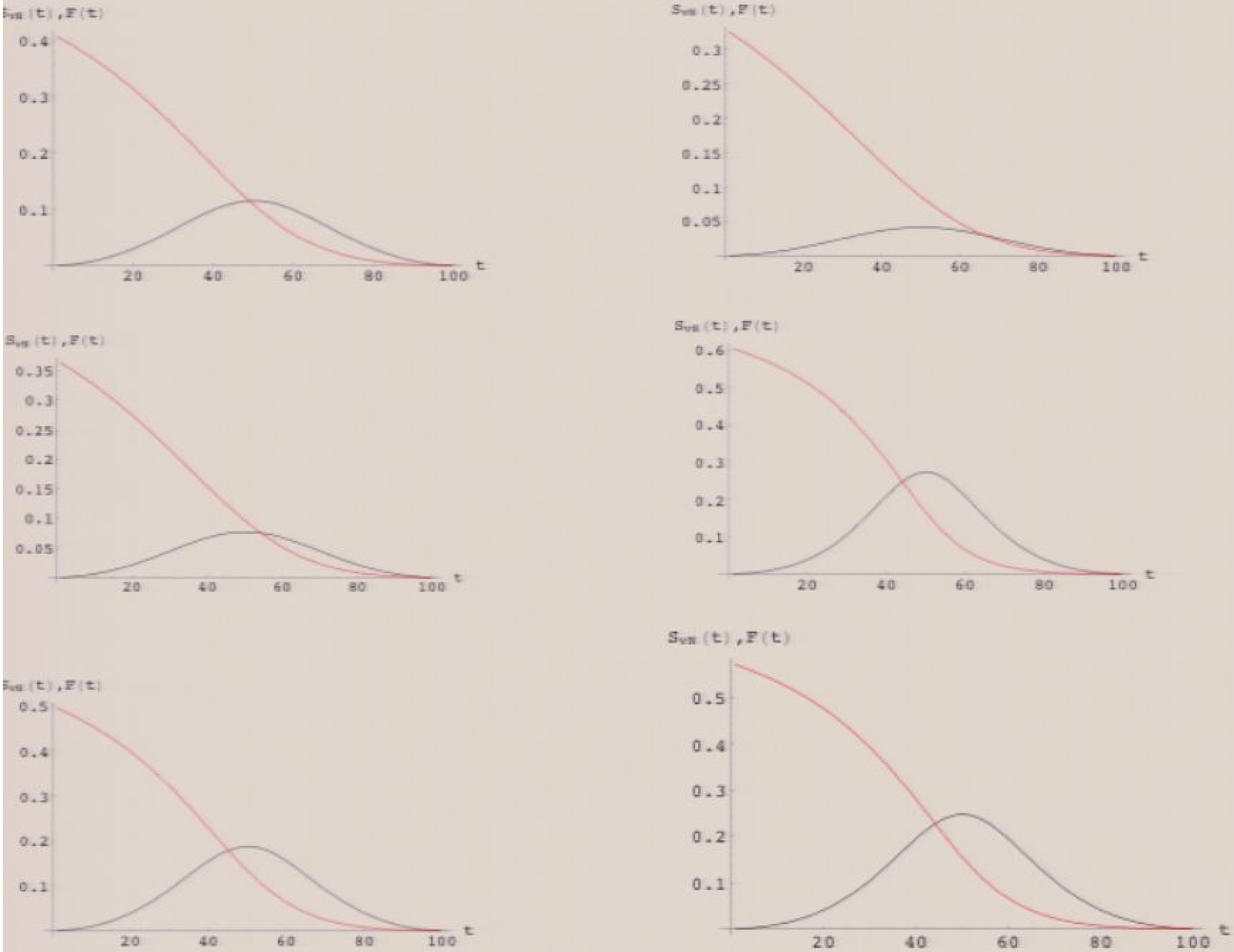
which measures the distance between instantaneous ground state and final state componentwis

with the function

$$F(t) = 1 - \tilde{F}(t)$$

which is essentially the negative of the fidelity

$$\tilde{F}(t) = |\langle c(t)|c(T)\rangle| = |\sum_{i=0}^{N-1} c_i^*(t)c_i(T)|$$



SvN(t),F(t)
0.4
0.3
0.2
0.1
20 40 60 80 100 t
SvN(t),F(t)
0.3
0.25
0.2
0.15
0.1
0.05
20 40 60 80 100 t
SvN(t),F(t)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
20 40 60 80 100 t
SvN(t),F(t)
0.6
0.5
0.4
0.3
0.2
0.1
20 40 60 80 100 t
SvN(t),F(t)
0.5
0.4
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SvN(t),F(t)
0.5
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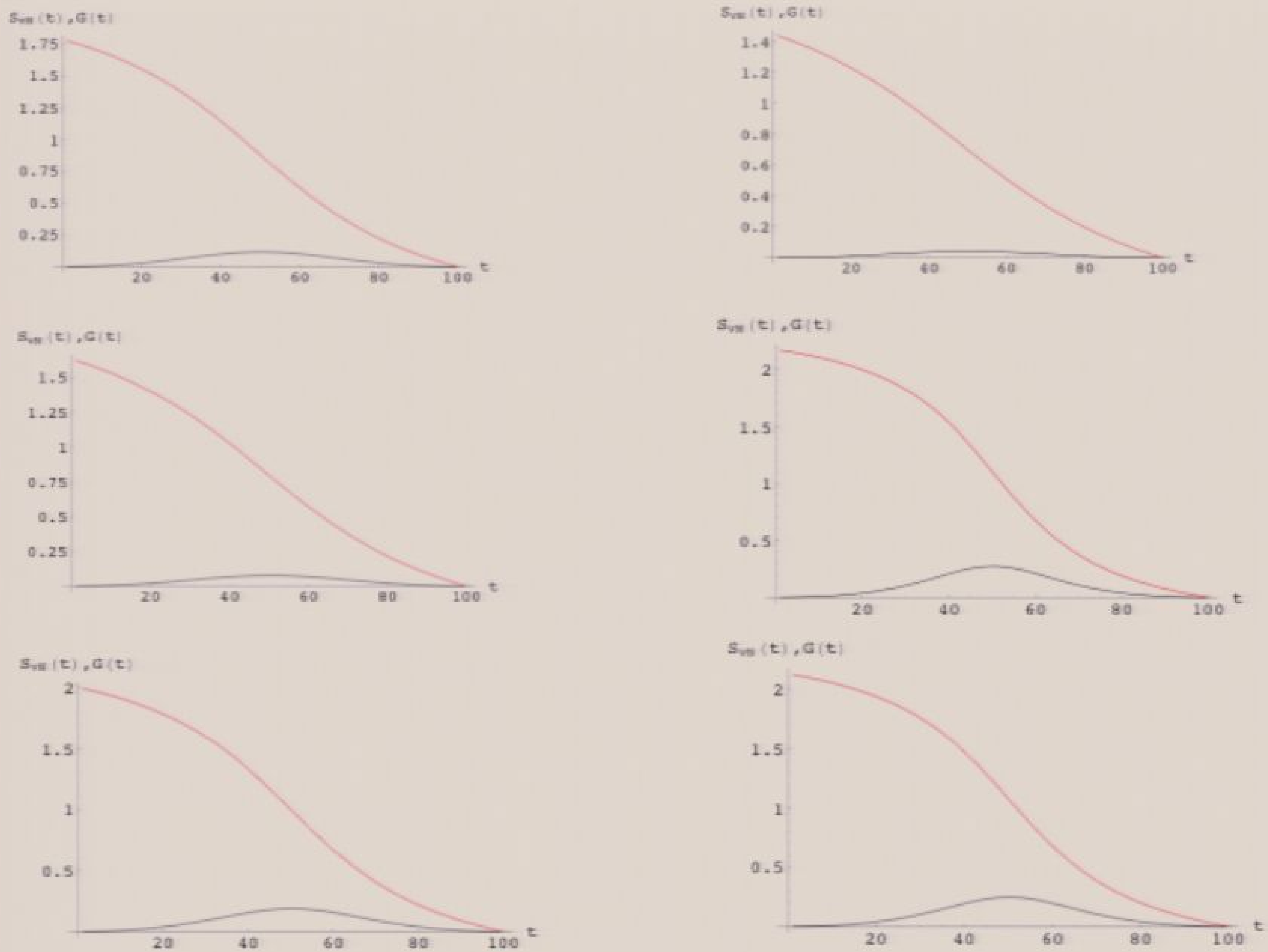
0.1
20 40 60 80 100 t



correlation entanglement - “speed” F’ or G’?

Optimizing the adiabatic evolution

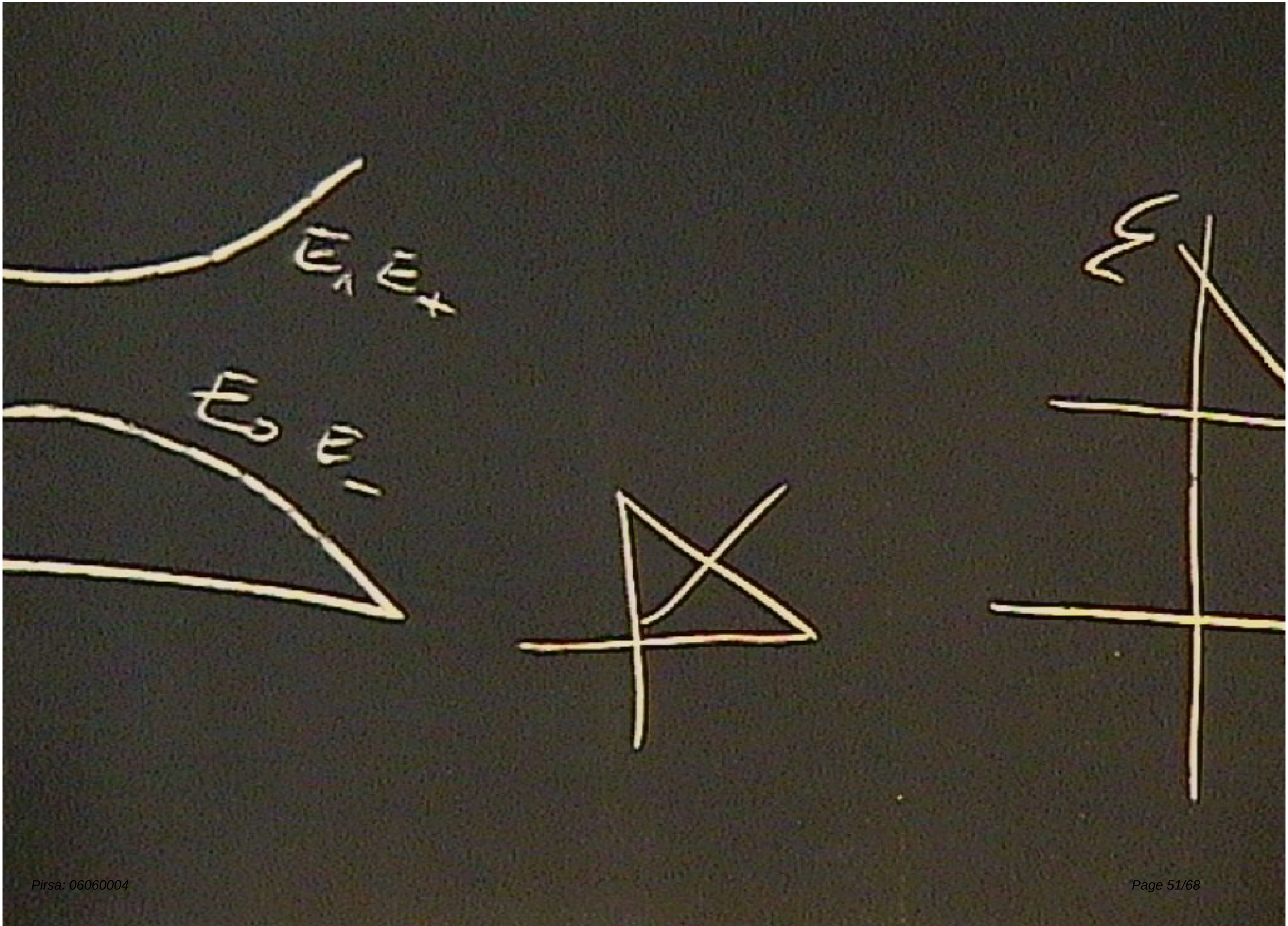
- adiabaticity condition $\varepsilon \ll 1$ for $\varepsilon = \max_{0 \le t \le T} \left| \frac{\langle E_+ | \frac{dH}{dt} | E_- \rangle}{(E_+ - E_-)^2} \right|$
- running time for a particular value of ε $T = \max_{0 \le s \le 1} \left| \frac{\langle E_+ | \frac{dH}{ds} | E_- \rangle}{\varepsilon (E_+ - E_-)^2} \right|$

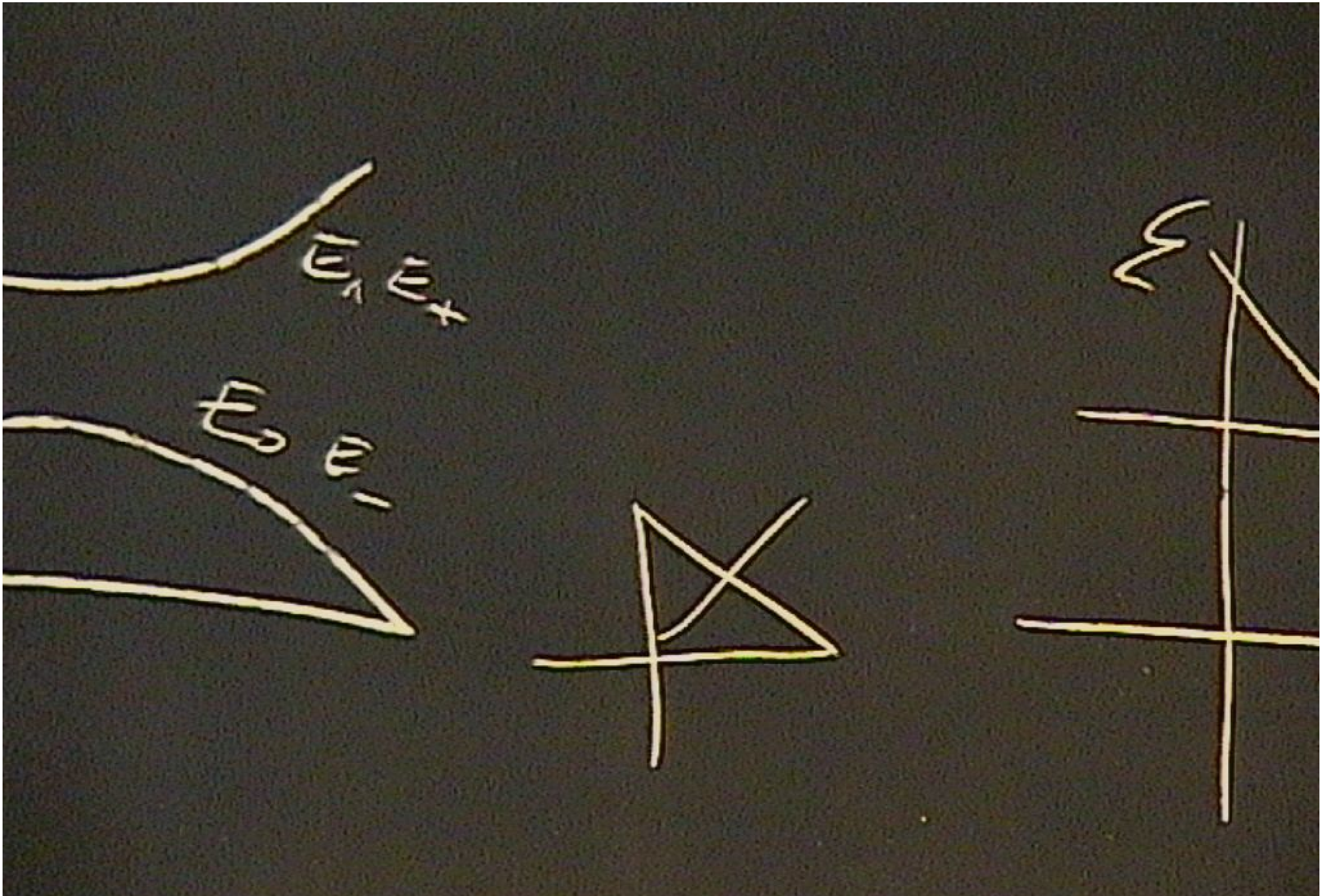
with dimensionless time variable $s = t/T$

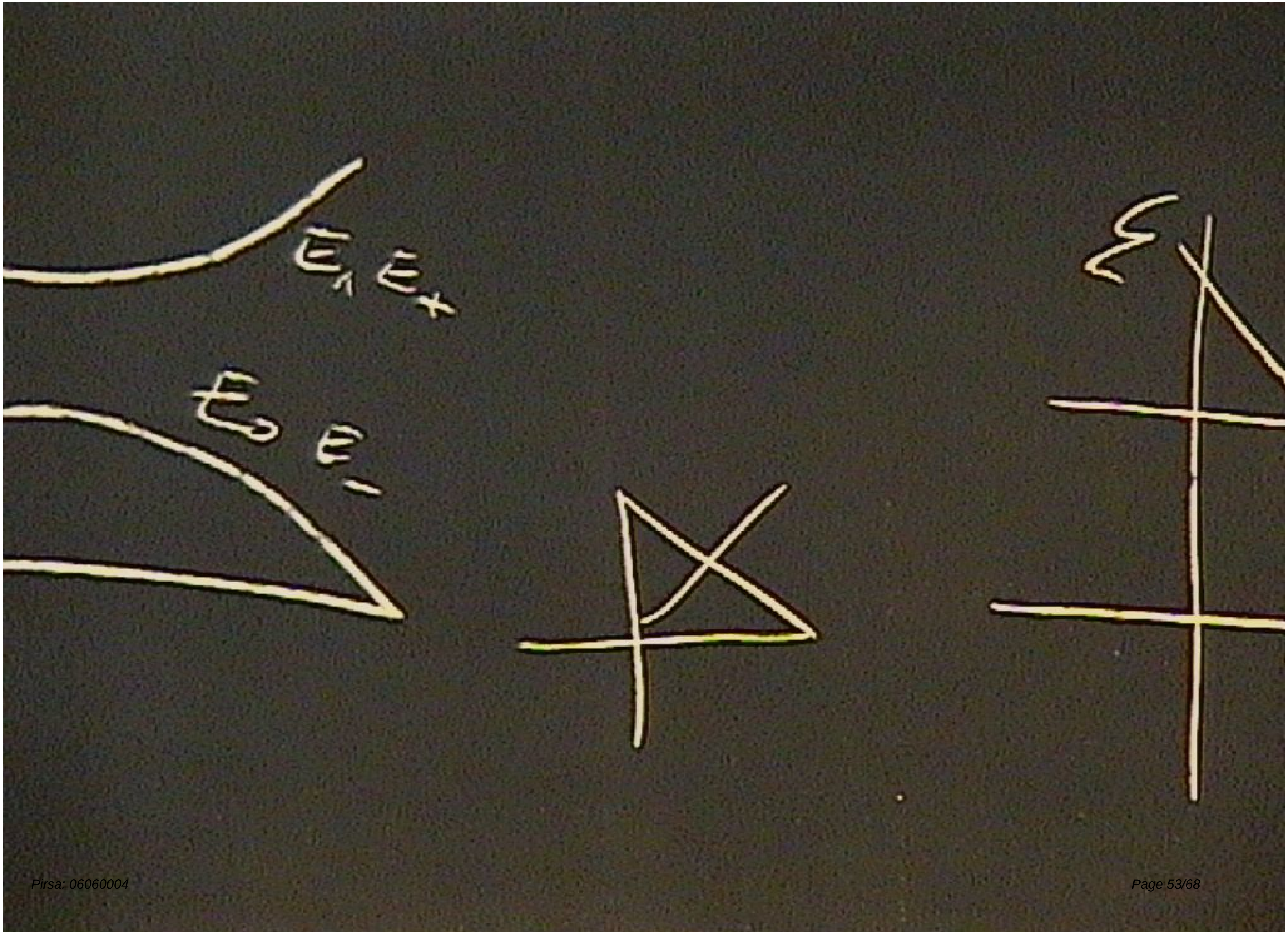
- optimization: apply condition locally to each infinitesimal time step dt, i.e. replace s(t) by non-linear function that changes fast where energy gap is large, and slow only where energy gap is small

$$dt = ds \left| \frac{\varepsilon (E_+ - E_-)^2}{\langle E_+ | \frac{dH}{ds} | E_- \rangle} \right|$$

$$T_{opt} = \varepsilon \int_0^1 ds \left| \frac{(E_+ - E_-)^2}{\langle E_+ | \frac{dH}{ds} | E_- \rangle} \right|$$ optimized running time


E_1 E_+
E_0 E_-
E


ε_1 ε_+
ε_0 ε_-
ε


E1 E+
E0 E-
ε

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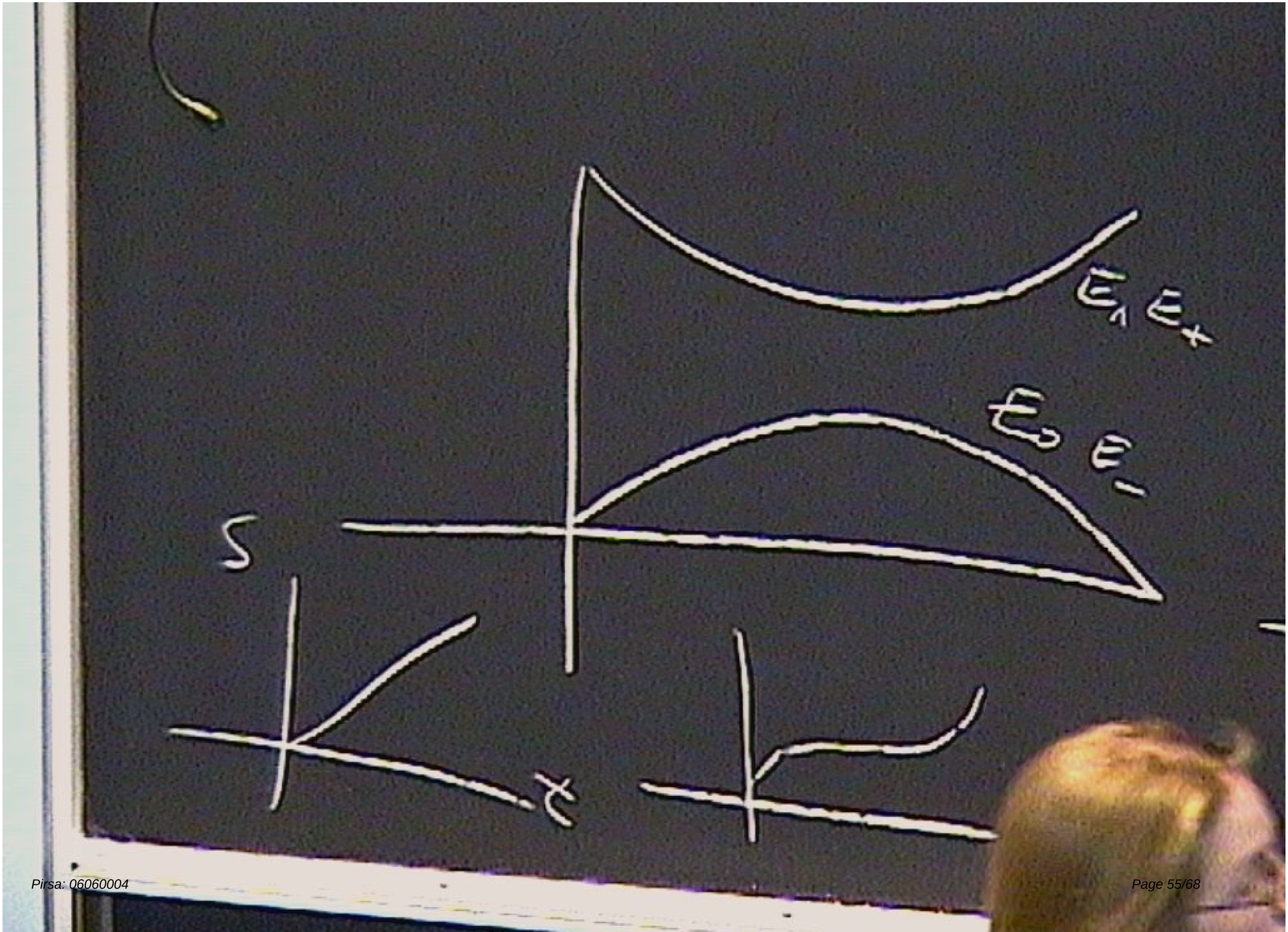
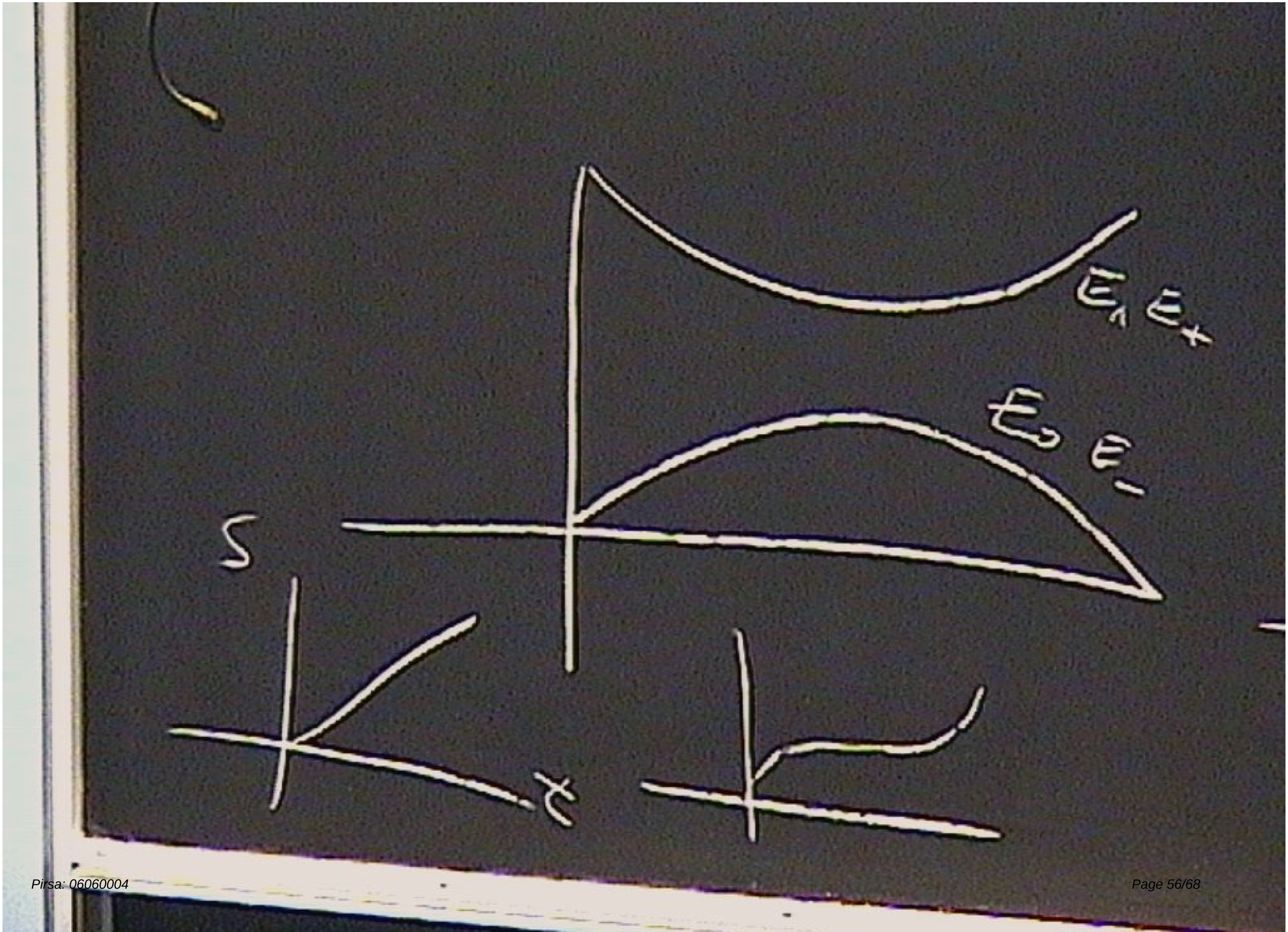
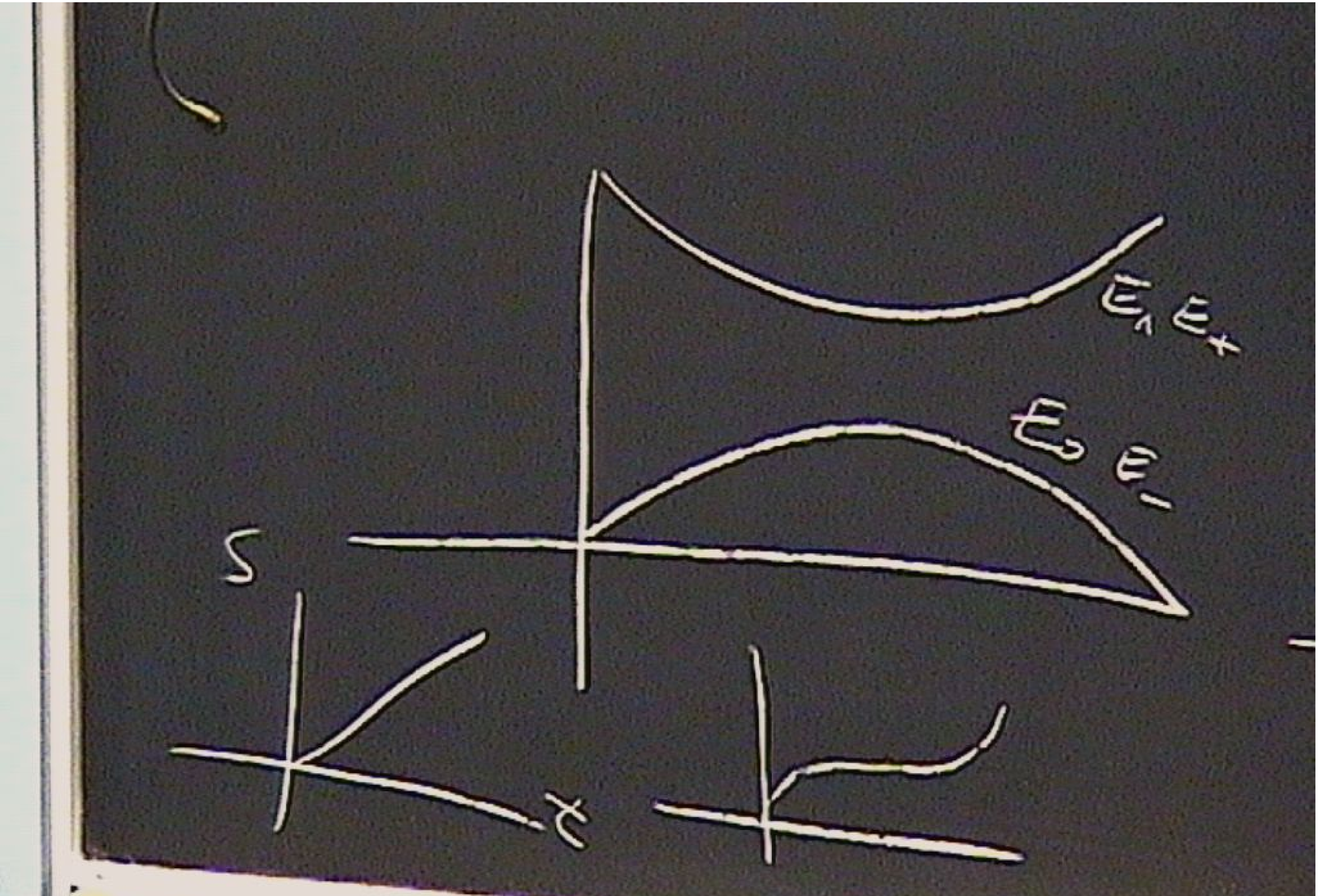


s
t


E_1 E_+
E_0 E_-
S
t


E+
E−
s
t

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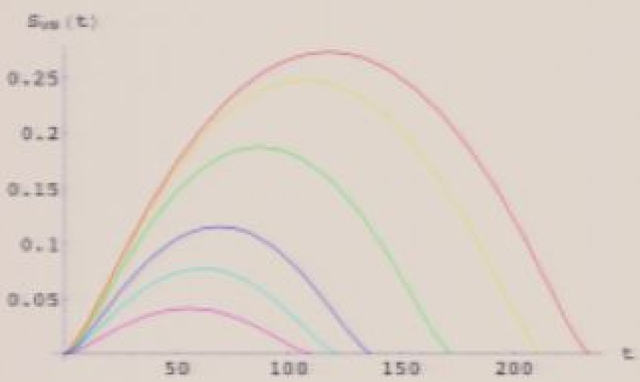
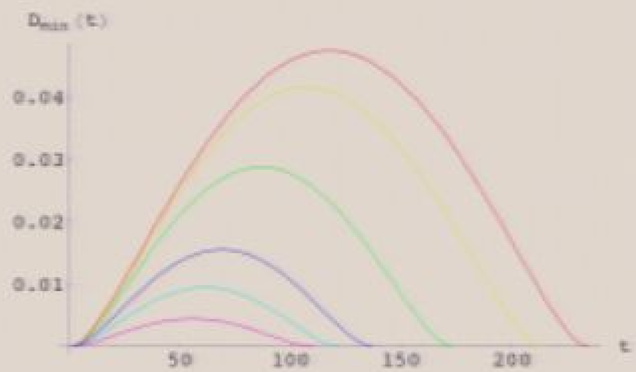
adiabatic search with 2 qubits and optimized time

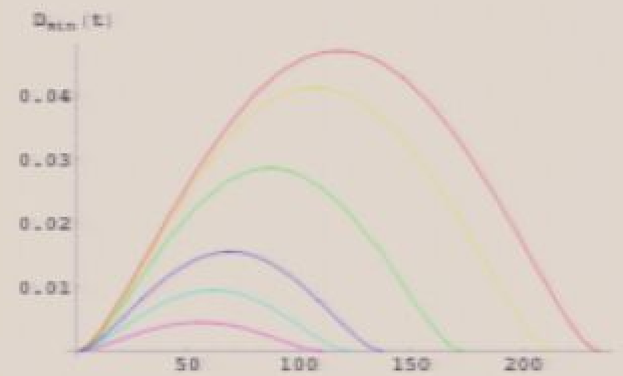
entropy of entanglement



D_min (normalized)



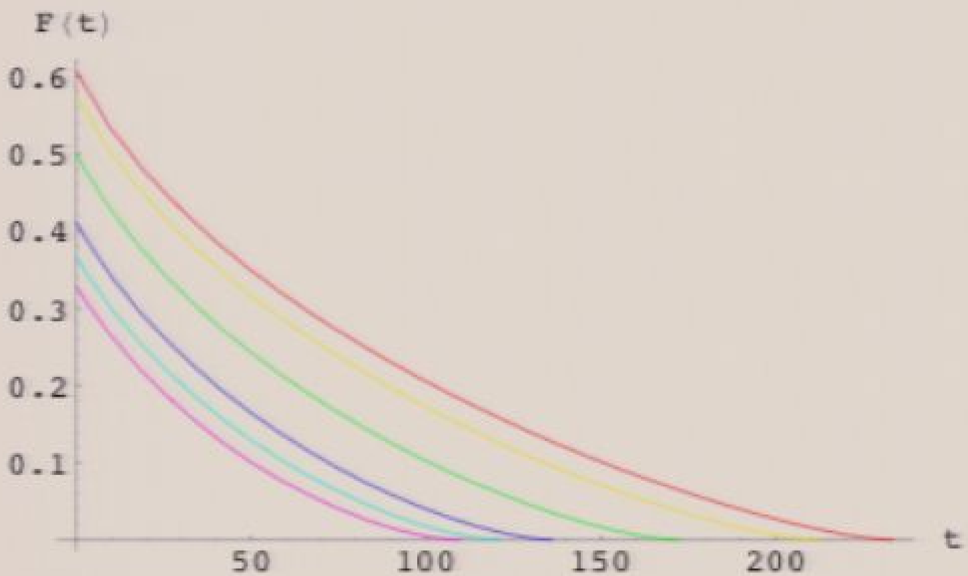
D_min (unnormalized)

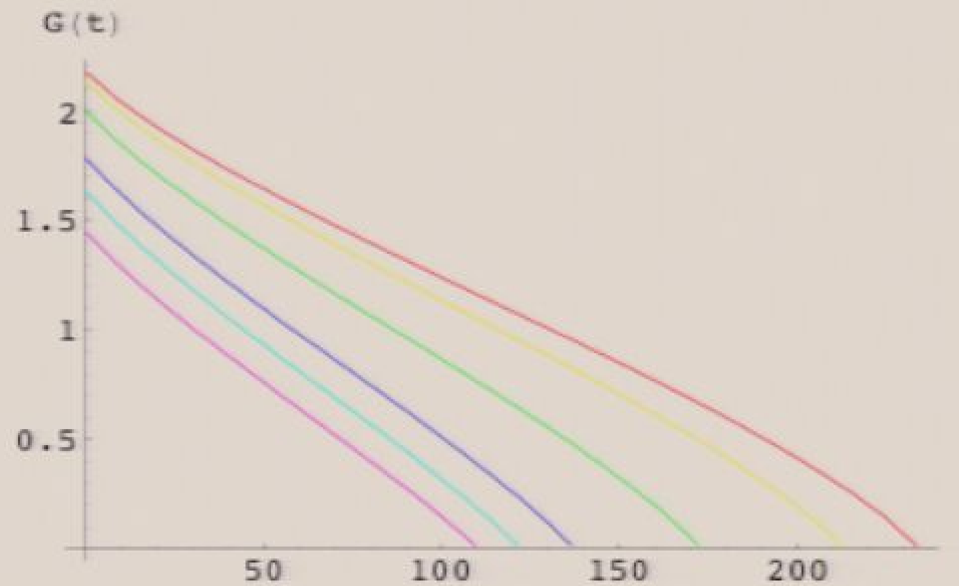


entropy of entanglement and geometric entanglement show qualitatively the same behaviou

largest maximum value of entanglement corresponds to largest running time

adiabatic search with 2 qubits and optimized time

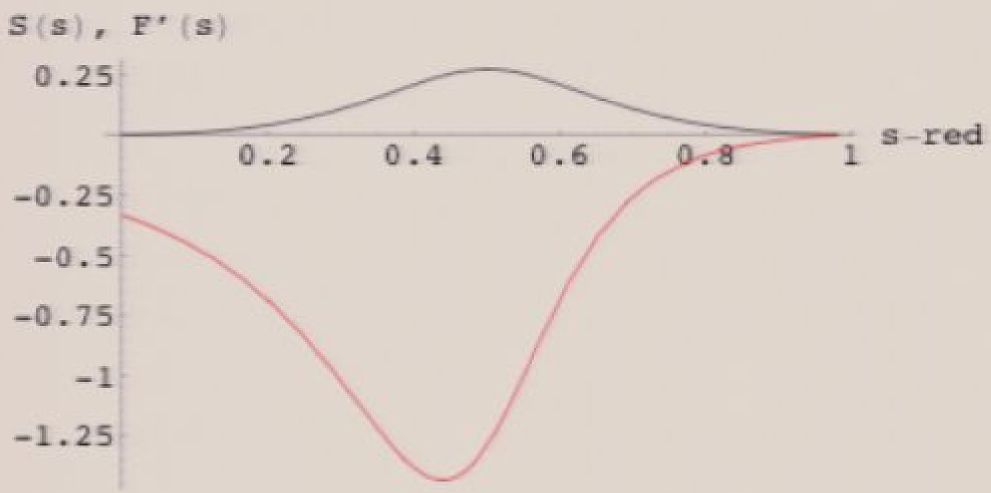




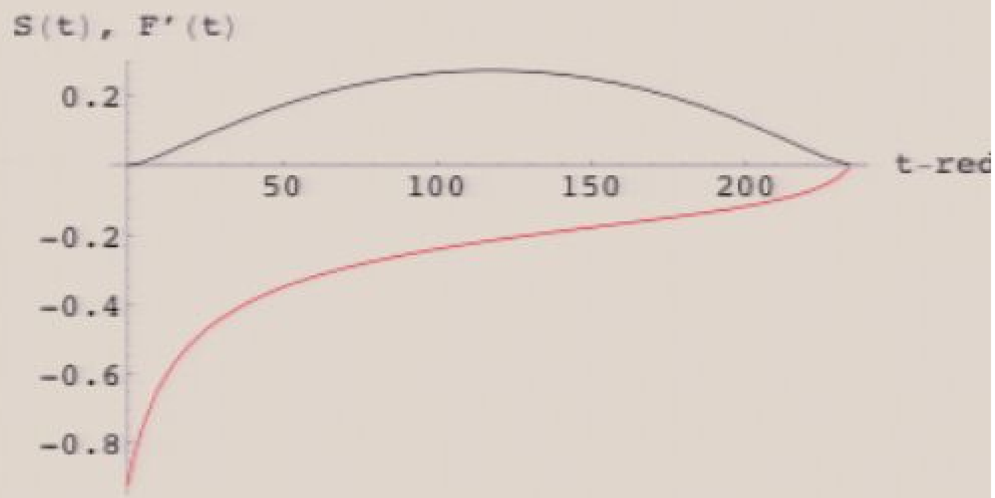
argest initial distance (small fidelity) corresponds to largest running tim

adiabatic search for n=2 qubits

global adiabatic evolution



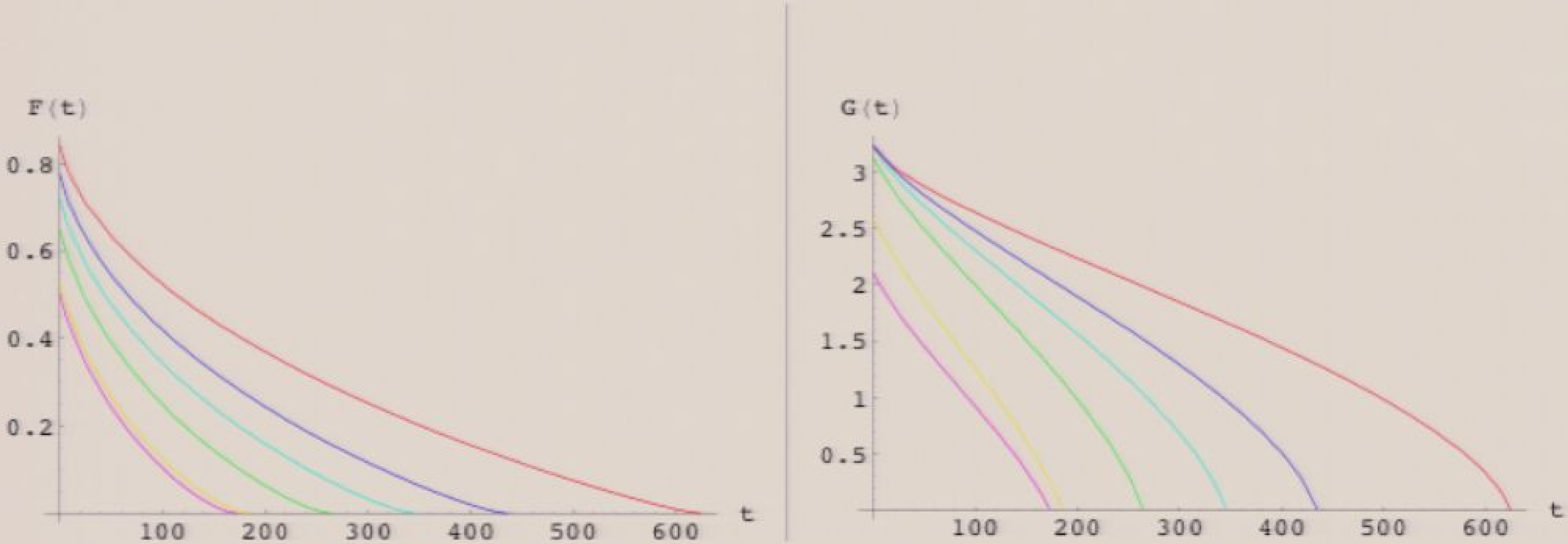
optimized running time



rate of change of fidelity is largest where entanglement is largest;

optimizing the running time flattens the rate of change of the fidelity; decreases maximum value of entanglement

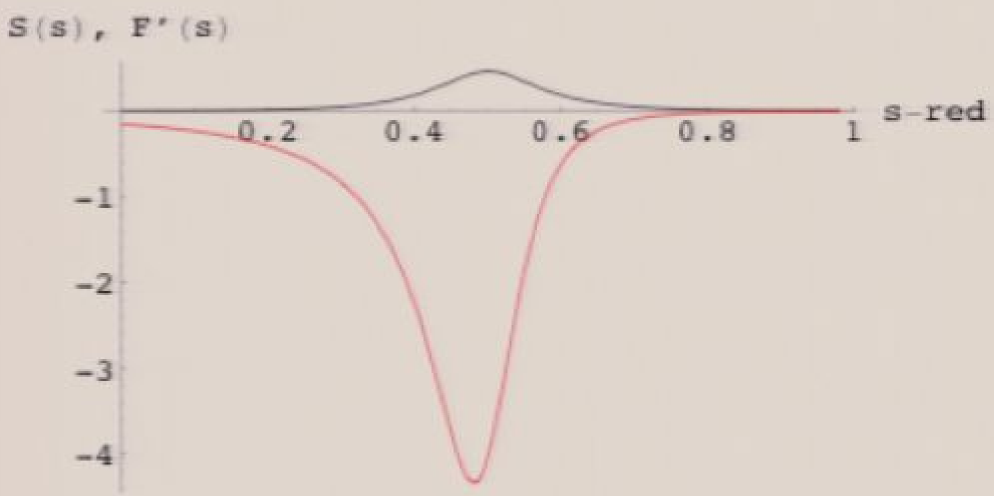
adiabatic search for n=3 qubits, optimized time



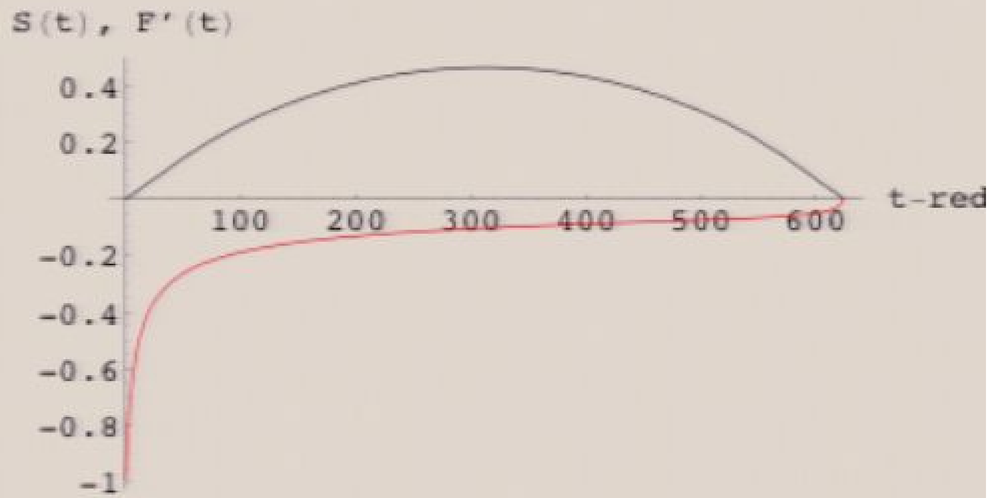
largest initial distance (smallest fidelity) corresponds to largest running time

adiabatic search for n=3 qubits

global adiabatic evolution



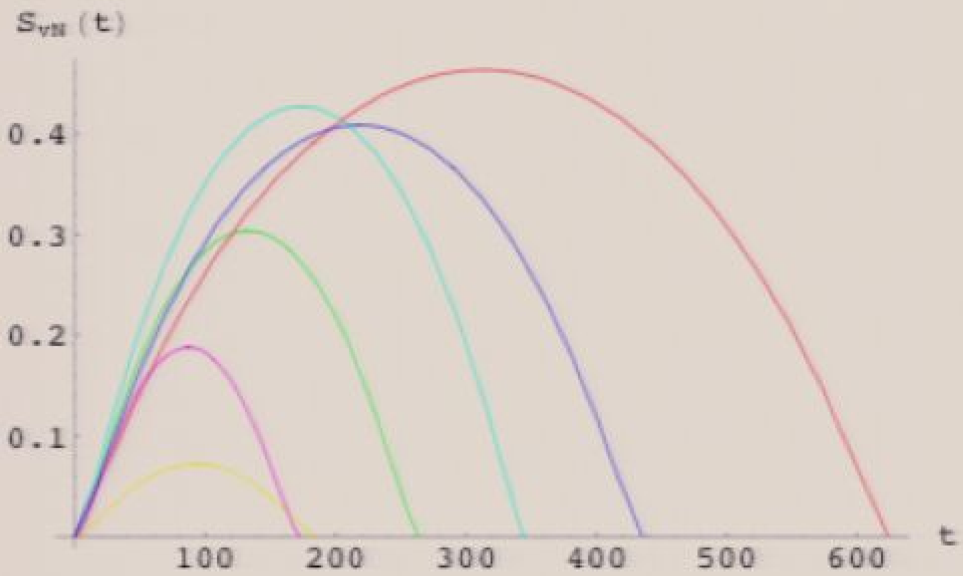
optimized running time

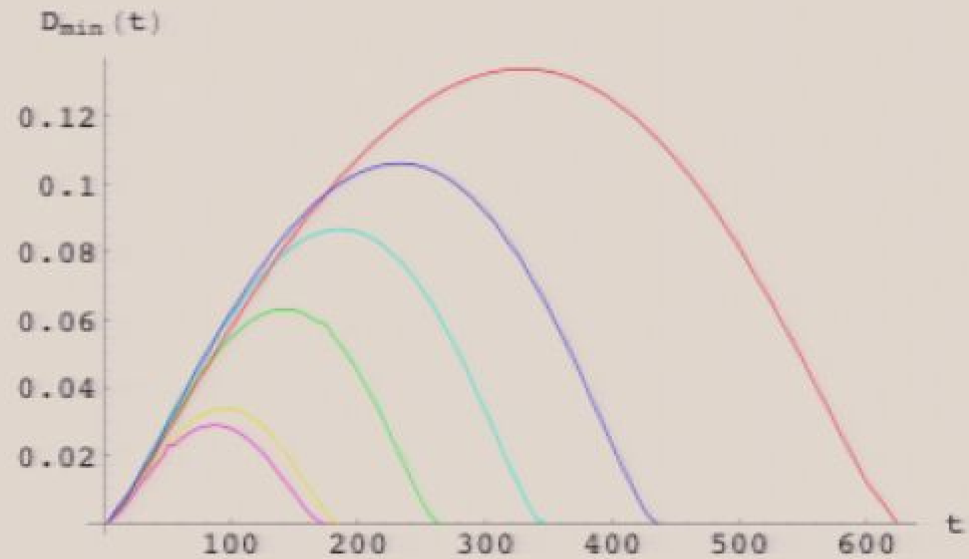


rate of change of fidelity is largest where entanglement is largest;

optimizing the running time flattens the rate of change of the fidelity; decreases maximum value of entanglement

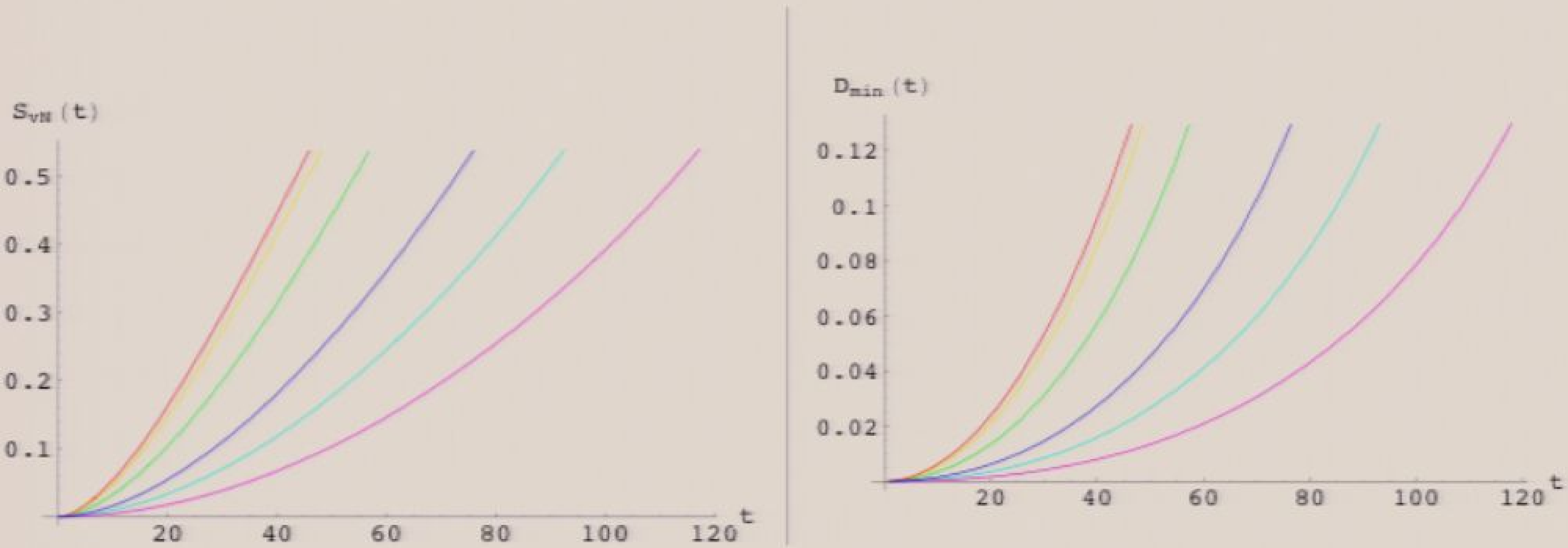
adiabatic quantum search for n=3 qubits, optimized time





note: order of maxima is not the same for the two measures of entanglement (same result for unoptimized time)

adiabatic Deutsch algorithm for n=2 qubits



states that generate entropy faster have a shorter running time

Results

- larger initial fidelity yields smaller running time, same result for G
- for search algorithm with unoptimized time, F' is largest where entanglement production is largest
- optimizing the time variable flattens out the rate of change of the fidelity
- using optimized time, initial states that produce more entanglement have larger running time for n=2 (for n=3, result only holds for geometric entanglement)
- for Deutsch's algorithm, states that generate entropy more quickly have a shorter running time

questions

- does the initial fidelity or the production of entanglement determine the running time? or both?
- different initial states have different initial entropy, but also produce different interactions and therefore different entanglement -- how to separate effects?
- does the geometric measure of entanglement allow conclusions about the partitioning problem?

wish list

- larger systems
- more algorithms
- e.g. structured search
- from combinatorial to spatial search
- role of the Hamiltonian, interactions, relation to quantum phase transitions, and from there to spin models
- definition of entanglement for multipartite systems, identical particles, thermal systems etc.