Title: Supersymmetric Black Holes in AdS5

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Abstract:

SUPERSYMMETRIC BLACK HOLES IN AdS_5

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Black Holes in Quantum Gravity

- Classically characterized by an event horizon
- Event horizon of stationary BH is a Killing horizon, a null hypersurface $\mathcal{H}$ with a Killing vector V normal to it, i.e. if $R = const$ is null and $V_\mu dx^\mu \sim dR$ on $\mathcal{H}$
- Associated with V is a scalar κ; this measures the local acceleration felt by an observer on the horizon
- Black holes carry conserved charges (M, Q_i, J_i); in $D = 4$ at least, these uniquely specify them
- These conserved charges satisfy relations in complete analogy with thermodynamics provided we identiy

$$T_H = \frac{\hbar\kappa}{2\pi} \qquad S = \frac{A}{4G\hbar}$$

- Classically, black holes are boring but quantum mechanically radiate at temperature T_H
- Need a *microscopic* derivation of S

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$D \geq 5$ Black Holes

- Static, asymptotically flat BH have S^n topology (Gibbons et al.)
- in general number of independent angular momenta is given by [(D-1)/2], the dimension of the Cartan subalgebra of $SO(D-1)$
- No uniqueness theorems in (A)dS
- in $D = 5$ one has at minimum 3 parameters (M, J_1, J_2) in general
- S^3 Black holes (Myers-Perry) have $\eta \equiv \frac{27\pi J^2}{32M^3} < 1$, i.e. J is bounded from above
- $S^2 \times S^1$ Black Holes 'black rings' (Reall-Emparan) have $\eta > \frac{27}{32}$, i.e. J is bounded from below
- Although there is a Myers-Perry AdS solution (Hawking, Hunter, Taylor-Robinson) there is no analogous ring solution

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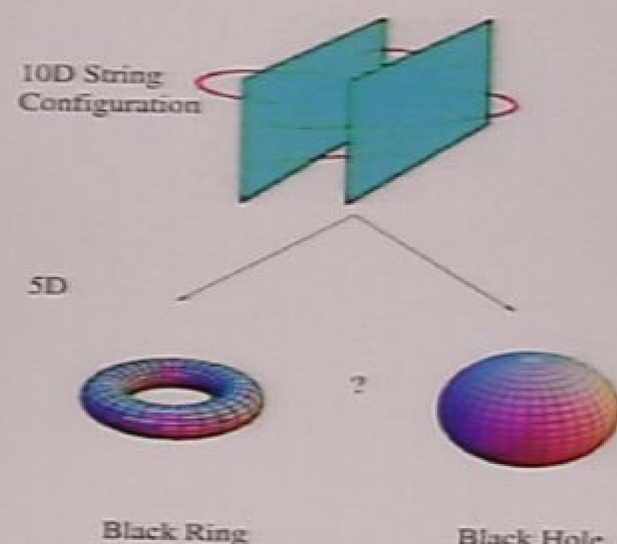
BPS Black Holes in String Theory

- String theory has supplied such a derivation; i.e. $D1 - D5 - P$ system reduced on $T^4 \times S^1$
- in g_s small,$\alpha' \to 0$ limit described by strings streching between the $D1$ and $D5$; get S from Cardy's formula with $c = \sqrt{6N_1N_5}$
- at $g_s >> 1$, system is a 3-charge BPS Black Hole in $D = 5$



- Supersymmety *crucial* to this calculation

Asymptotically Flat BPS BH

- Near horizon analysis shows only BPS black holes with S^3 and $S^2 \times S^1$ are allowed
- BMPV: isometry $R \times U(1) \times SU(2)$, S^3 horizon: $J_1 = J_2$, $M = Q_1 + Q_2 + Q_3 \to 4$ parameters
- Note $J_1 = J_2 = 0$ is allowed, i.e. *static* S^3 BH exist
- Black Rings (EEMR): isometry $R \times U(1) \times U(1)$, $S^2 \times S^1$ horizon; $J_1 > J_2$, $M = Q_1 + Q_2 + Q_3$ and three non-conserved dipole charges $\to 7$ parameters
- for $Q_i = Q$, BMPV has 2 paramters, BR has 3
- In minimal theory, there is no overlap of conserved charges; in $U(1)^3$ theory, there are BPS black rings with the same conserved charges but different entropy...

AdS_{d+1} Black Holes

- in principle have an *exact* defintion of quantum gravity in asymptotically AdS spacetimes in terms of a dual CFT living on the boundary
- (Brown,Henneaux) *Any* theory of QG in AdS_3 is a $2d$ CFT with $c = \frac{3\ell}{2G}$
- BTZ black hole has boundary $R \times S^1$, mass M and angular momentum J. In the limit of large charges, $n_L \sim \frac{\ell M + J}{2}$ and $n_R \sim \frac{\ell M - J}{2}$. Then

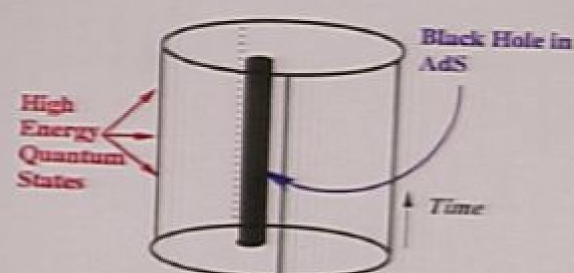
$$S_{CFT} = 2\pi\sqrt{\frac{cn_L}{6}} + 2\pi\sqrt{\frac{cn_R}{6}}$$

gives precise agreement with $\frac{A}{4}$ (Strominger)

- Consistent with D1-D5-P result, as they have near horizon geometry $AdS_3 \times S^3 \times T^4$. Effective $3D$ theory has ℓ given in terms of the charges N_1 and N_5, and $G_3 = \frac{G_{10}}{Vol(S^3 \times T^4)}$.
- d=4: (Kostelecky, Perry) Curiously, BPS AdS_4 black holes depend on only *one* parameter; but $d = 3$ CFT is not understood

$AdS_5 \times S^5$ Black Holes

- dual CFT is $\mathcal{N}=4$ Super Yang Mills with gauge group $SU(N)$ on boundary $R \times S^3$ of AdS_5
- for large N the gauge theory coupling is $\lambda \equiv g_s N$. Worldsheet σ-model coupling constant $\alpha' \sim \lambda^{-\frac{1}{2}}$ Classical supergravity $\Leftrightarrow \lambda \to \infty$
- $Z_{CFT}(h) = Z_{sugra}(h) = e^{-I_s(g)}$
- Non-extemal BH in AdS $\Leftrightarrow$ CFT on $S^1 \times S^3$ with $T_{CFT} = T_{BH}$ (Witten)
- can show $I_s \sim (\ell^2 r_+^3 - r_+^5)$ (Hawking,Page)
- $T < T_c$, thermal AdS dominates - confined phase, $T > T_c$, BH with $r_+ >> \ell$ dominate - phase transition to deconfined phase
- Free field computation gives $S_{YM} = \frac{4}{3} S_{BH}$



BPS AdS_5 Black Holes

- dual CFT states belong to short superconformal multiplets; degeneracy for states wtih given charges not expected to vary as λ varied $\rightarrow$ counting at weak coupling should yield right entropy
- CFT bosonic subalgebra $SO(2,4) \times SU(4)$: states labelled by irreps $SO(2)$ (energy), $SO(4) \sim SU(2) \times SU(2)$: (J_L, J_R) (spins) and $SO(6)$: (Q_1, Q_2, Q_3) R charges
- Reduce Type IIB sugra on S^5 and to get $D = 5$ gauged supergravity with gauge group $U(1)^3$

$$S = \frac{1}{16\pi G} \int R_5 \star 1 - Q_{IJ} F^I \wedge \star F^J - Q_{IJ} dX^I \wedge \star dX^J - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K + 2g^2 \mathcal{V} \star 1$$

- Maximally supersymmetric vacuum is AdS_5 (all other fields zero)
- Complication: BPS BH in this theory *must* rotate!

BPS solutions

- apply systematic method (Tod) to find solutions assuming a Killing spinor exists (Gutowski and Reall); given KS ϵ, construct $f^2 \sim \bar{\epsilon}\epsilon$, $V \sim \bar{\epsilon}\gamma\epsilon$, so $V^2 \sim -f^2$ Wlog write

$$ds^2 = -f^2 (dt + \omega)^2 + f^{-1} h_{mn} dx^m dx^n$$

- $V = \partial/\partial t$, h_{mn} is a metric on a 4-dimensional Riemannian "base space" B and ω a 1-form on B. Supersymmetry implies B is Kähler
- EOM all reduce to (complicated) differential and algebraic equations on B
- $f \sim -\frac{1}{R} \rightarrow$ need *singular* B for BPS black holes
- B that gives AdS_5 is the Bergmann manifold

$$d\rho^2 + a^2((\sigma_L^1)^2 + (\sigma_L^2)^2) + (2aa')^2(\sigma_L^3)^2$$

with $a = \frac{\ell}{2} \sinh \frac{\rho}{\ell}$

- Need singular deformations of Bergmann manifolds

Comments on Near Horizon Analysis

Method for finding horizon *geometries* (Reall) This is valid for *Degenerate* horizons

$$ds^2 = -r^2\Delta^2 du^2 + 2dudr + 2rh_A dudx^A + \gamma_{AB}dx^A dx^B$$

- $V = \partial_u$ is Killing. Near horizon limit is $r = \epsilon\hat{r}, u = \bar{u}/\epsilon$ as $\epsilon \to 0$. Here γ is metric on H
- In this limit h_A, γ, Δ are only functions of x^A

Can show using supersymmetry that

$$d\Delta \wedge \star d\Delta = dA_2 + Bl^{-1}$$

- if $l \to \infty$ then can integrate over a compact H to show Δ is constant.
- More complicated in the AdS case
- Further one can show h is Killing on H in the $l \to \infty$ case
- Assuming $\Delta =$ constant one can make progress - led to first examples of BPS AdS(5) black holes (Reall and Gutowski)

Previous Solutions

- First examples: (Gutowski, Reall): cohomogeniety-1 deformation of Bergmann with isometry $SU(2) \times U(1) \times R \rightarrow J_1 = J_2$ Paramterized by (Q_1, Q_2, Q_3) using systematic Near-horizon analysis:

$$g = -R^2\Delta^2 du^2 + 2\Gamma(x^A) du dR + 2Rh_A du dx^A + \gamma_{AB} dx^A dx^B$$

  with $R = 0$ the horizon. Impose BPS constraints to find possible horizon geometries. Assumed Δ constant.

- Chong, Cvetic, Lu, Pope find four-parameter *non-extremal* solution with $Q_i = Q$. BPS limit leads to 2-parameter solution with $J_1 \neq J_2$. Also find 2-parameter BPS solution to $U(1)^3$ theory

- Very curious: in AF case, BPS black holes with toplogy S^3 *must* have $J_1 = J_2$ (BMPV)

- Expect 5 paramter solution: need a more general base...

$$
\begin{aligned}
h \;=\; & (r^2-r_0^2)\left\{\frac{dr^2}{\Delta_r}+\frac{d\theta^2}{\Delta_\theta}+\frac{\cos^2\theta}{\Xi_b^2}\left[\Xi_b+\cos^2\theta\left(\rho^2g^2+2(1+bg)(a+b)g\right)\right]d\psi^2\right.\\
& +\frac{\sin^2\theta}{\Xi_a^2}\left[\Xi_a+\sin^2\theta\left(\rho^2g^2+2(1+ag)(a+b)g\right)\right]d\phi^2\\
& \left.+\frac{2\sin^2\theta\cos^2\theta}{\Xi_a\Xi_b}\left[\rho^2g^2+2(a+b)g+(a+b)^2g^2\right]d\psi d\phi\right\},
\end{aligned}
$$

where $\rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta$ with Kähler form and Ricci scalar

$$
J=-d\left[\frac{(r^2-r_0^2)}{2}\left(\frac{\cos^2\theta d\psi}{\Xi_b}+\frac{\sin^2\theta d\phi}{\Xi_a}\right)\right] \qquad R\sim F(\rho^2)(r^2-r_0^2)^{-1}
$$

for r_0 fixed in terms of (a,b) at special values.

- **Actually Kähler for all r_0**
- Is this a 3-parameter deformation of Bergmann? NO
- Change of coordinates shows r_0 can be removed; it is pure gauge

$$
\begin{aligned}
h \;=\; & d\sigma^2+g^{-2}\sinh^2(g\sigma)\left(\cosh^2(g\sigma)-\Delta_\theta\right)\left(\cos^2\theta\frac{d\psi}{B^2}+\sin^2\theta\frac{d\phi}{A^2}\right)^2\\
& +\;g^{-2}\sinh^2(g\sigma)\left(\frac{d\theta^2}{\Delta_\theta}+\Delta_\theta\cos^2\theta\frac{d\psi^2}{B^4}+\Delta_\theta\sin^2\theta\frac{d\phi^2}{A^4}\right).
\end{aligned}
$$

- Depends only on two parameters (A,B)
- Singular at $\sigma=0$; isometry $U(1)\times U(1)$. When $A=B$, has isometry $SU(2)\times U(1)$
- Using this base and general ansatz yield solution

- Find solution

$$ds^2 = -(H_1H_2H_3)^{-2/3}(dt + \omega_\phi d\phi + \omega_\psi d\psi)^2 + (H_1H_2H_3)^{1/3} h$$

where

$$\omega_\psi \sim \cos^2\theta \times \text{quadratic in } \rho^2$$
$$\omega_\phi \sim \sin^2\theta \times \text{quadratic in } \rho^2$$

and

$$H_I = 1 + \frac{\sqrt{\Xi_a \Xi_b}(1 + g^2\mu_I) - \Xi_a \cos^2\theta - \Xi_b \sin^2\theta}{g^2 r^2},$$

The scalars are

$$X^I = \frac{(H_1H_2H_3)^{1/3}}{H_I}.$$

The vectors are:

$$A^I = H_I^{-1}(dt + \omega_\psi d\psi + \omega_\phi d\phi) + U_\psi^I d\psi + U_\phi^I d\phi$$

with

$$U_\psi^I \sim \cos^2\theta \times \text{linear in } \rho^2$$
$$U_\phi^I \sim \sin^2\theta \times \text{linear in } \rho^2$$

- Parameterized by $(\mu_1, \mu_2, \mu_3, a, b)$ with ONE constraint:

$$\mu_1 + \mu_2 + \mu_3 = F(a, b)$$

- To show it is regular, transform to new coordinates $(v, R, \theta, \phi', \psi')$ where $R = gr^2$,

$$dv = dt - \left(\frac{A_0}{g^2R^2} + \frac{A_1}{gR}\right) dR, \qquad d\psi' = d\psi - \frac{B_0\Xi_b}{R} dR, \qquad d\phi' = d\phi - \frac{C_0\Xi_a}{R} dR$$

- Smooth horizon at $R = 0$

$$V_\mu dx^\mu|_{R=0} \sim \pm [dR]_{R=0}.$$

The electric charges are

$$Q_I = \frac{1}{8\pi}\int_{S^3}\left(Q_{IJ}\star_5 F^J + \frac{1}{4}C_{IJK}A^J\wedge F^K\right)$$

over an S^3 at infinity.

$$\begin{aligned} Q_1 &= \frac{\pi}{4G}\left[\mu_1 + \frac{g^2}{2}(\mu_1\mu_2+\mu_1\mu_3-\mu_2\mu_3)\right] \\ Q_2 &= \frac{\pi}{4G}\left[\mu_2 + \frac{g^2}{2}(\mu_2\mu_3+\mu_2\mu_1-\mu_1\mu_3)\right] \\ Q_3 &= \frac{\pi}{4G}\left[\mu_3 + \frac{g^2}{2}(\mu_3\mu_2+\mu_1\mu_3-\mu_1\mu_2)\right]. \end{aligned}$$

It is possible to set one, but not two, of these charges to zero. The angular momenta are given by

$$\begin{aligned} J_1 &= \frac{\pi}{4G}\left(\frac{1}{2}g(\mu_1\mu_2+\mu_2\mu_3+\mu_1\mu_3)+g^3\mu_1\mu_2\mu_3+\frac{(B-A)}{Ag^3}\mathcal{J}\right), \\ J_2 &= \frac{\pi}{4G}\left(\frac{1}{2}g(\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3)+g^3\mu_1\mu_2\mu_3+\frac{(A-B)}{Bg^3}\mathcal{J}\right). \end{aligned}$$

where $\mathcal{J}>0$. Use BPS equality to get mass

$$M = g|J_1| + g|J_2| + |Q_1| + |Q_2| + |Q_3|,$$

and is of the form

$$M = \frac{\pi}{4G}\left[M(\mu_1,\mu_2,\mu_3) + \frac{(A-B)^2}{g^2AB}\mathcal{J}\right]$$

where $M(\mu_1,\mu_2,\mu_3)$ is the mass of the corresponding self-dual black hole with $A=B$.

- In the asymptotically static coordinates $(\bar{t}, \bar{\phi}, \bar{\psi})$, the supersymmetric Killing field is

$$V = \frac{\partial}{\partial \bar{t}} + g\frac{\partial}{\partial \bar{\phi}} + g\frac{\partial}{\partial \bar{\psi}}.$$

so the angular velocities of the event horizon with respect to the static frame at infinity are

$$\Omega_{\psi} = \Omega_{\phi} = g = \frac{1}{l}$$

i.e. 1 in AdS units.

- Spatial cross sections of the event horizon have the geometry of a deformed S^3 with area

$$A_H = 2\pi^2 \sqrt{\left(\frac{A_{\mu_1,\mu_2,\mu_3}}{2\pi^2}\right)^2 - \frac{(A-B)^2}{g^6 AB}\mathcal{J}}$$

where

$$\left(\frac{A_{\mu_1,\mu_2,\mu_3}}{2\pi^2}\right)^2 = [1+g^2(\mu_1+\mu_2+\mu_3)]\mu_1\mu_2\mu_3 - \frac{g^2(\mu_1\mu_2+\mu_1\mu_3+\mu_2\mu_3)^2}{4}$$

- It now remains for the field theory to reproduce this...

Field theory Calculations

- Oxidize BPS BH to Type IIB: have 2 supercharges, i.e. $\frac{1}{16}$ BPS
- Recall $G_5 = G_{10}/Vol(S^5) = \frac{\pi}{2N^2} \rightarrow$ all charges, and area go as $O(N^2)$
- no 1/8-BPS BH. From field theory can show (Roiban, Reall) that $S \sim N \log N$ (too small)
- (Kinney,Maladacena, Minwalla, Raju) construct most general superconformal index from group theory, counts BPS states that cannot combine into long reps on $R \times S^3$
- at large N limit is of O(1); counts precisely KK states but does not see BPS BH
- comes with $(-1)^F$, so bosonic and fermion states cancel
- Partition function $Z \sim Tre^{-\phi_i Q^i}$ over 1/16 BPS states in free SYM gives $S_{field}/S_{gravity} \sim 1.3$ but in general depends on 5 parameters
- expect dynamics would remove BPS states

Open Issues

- BPS BH have only 4 parameters instead of 5. Either there are more general ones or there is some unknown constraint in the large N limit
- in AF case S^3 horizon implies $J_1 = J_2$; we have derived an analogous constraint in AdS_5
- Expect a general non-extremal BH with $M, J_1, J_2, Q_1, Q_2, Q_3$ free
- If there are more general BPS S^3 BH, and a similar 2-parameter loss happens in the BPS limit, then the non-extemal BH would not be determined in uniquely in terms of (M, J_i, Q_i) (!)
- in AF case, BPS $S^2 \times S^1$ BH have more parameters than S^3 BH
- In both vacuum and BPS case, black rings 'fill in' regions of J-space which cannot be occupied by BH
- All idle talk, have to do a proper Near Horizon analysis...