Title: String Cosmology in the Hagedorn Phase

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Abstract: It has recently been proposed by Nayeri, Brandenberger and Vafa, that the thermodynamics of strings in the early universe can provide us with a causal mechanism to generate a scale invariant spectrum of primordial density fluctuations, without requiring an intervening epoch of inflation. We will review this mechanism, and report on more recent work which has uncovered several observational consequences of the NBV mechanism, some of which in principle, will be distinguishable from the generic predictions of inflation.

String Cosmology in the Hagedorn Phase

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May 30th 2006

hep-th/0511140
hep-th/0604126
hep-th/0606???

w/ Robert Brandenberger
Ali Nayeri
Cumrun Vafa

Q) What is SGC?

First it helps to ask:

(Q) What is BBC?)

Big Band Cosmology is

- A theory of Gravity

$$S = \frac{1}{16\pi G_D}\int \sqrt{-g}\, d^{D+1}x\, R[g]$$

\+ A theory of Matter

$$T^\mu{}_\nu = T^\mu{}_\nu(\cdots, A_\mu, \psi_\alpha, \phi)$$

\+ Certain Symmetries

Thermodynamics + Democratic initial conditions

Homogeneity + Isotropy

... Has to be supplemented w/ Inflation, Dark energy, Dark matter, a solution to C.C. problem, etc.

A) SGC is: All of the above <u>but</u> w/ assumption that String theory is the true description of nature.

- Theory (Theories) of Gravity (SUGRA)

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x \sqrt{-g}\, e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi\right) + \ldots$$

\+ Strings, branes & the fields neccesary to describe consistent backgrounds on which these propagate.

$$T^\mu_\nu = T^\mu_\nu[g_{\alpha\beta}, \phi, B_{\alpha\beta} \ldots]$$

\+ Same old symmetries, but with new twists, and then some...

↙ Thermodynamis + Democratic i.c.'s

↘ Homogeneity + Isotropy

\+ dualities + new features to thermodynamics

Q) So What?

- Might dynamically generate a 3+1d universe? Brandenberger, Vafa
  Nucl. Phys. B287 (1987) 402

- Can stabilize moduli in a phenomenologically consistent manner

  S.P., R.B, hep-th/0401037
  S.P, R.B, hep-th/0502069
  S.P, hep-th/0504145

- Contains within it:

  dark energy candidate: S.Räsänen & F. Ferrer hep-th/0509225
  solution to the size problem: N. Shumaker & R.B hep-th/~~[illegible]~~ 0511299
  solution to the horizon & structure problem: R.B., A.N., C.V., hep-th/0511140

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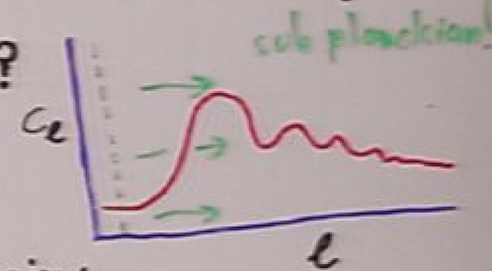
• SGC in Hagedorn Phase:

solves horizon, entropy & structure problem but <u>not</u> flatness problem.

But Why look for an alternative to inflation? (it is after all, very succesful)

- We don't know what the inflaton is! (RHB - "paradigm in search of a theory")
- transplanckian problem?



- initial singularity remains.
- Cosmological Const. problem?
- very hard to falsify (!)

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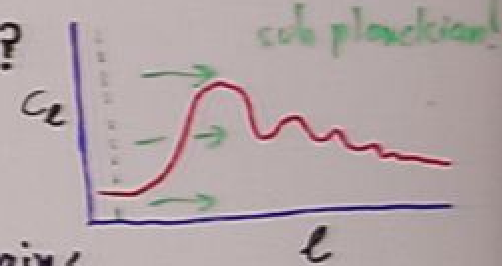

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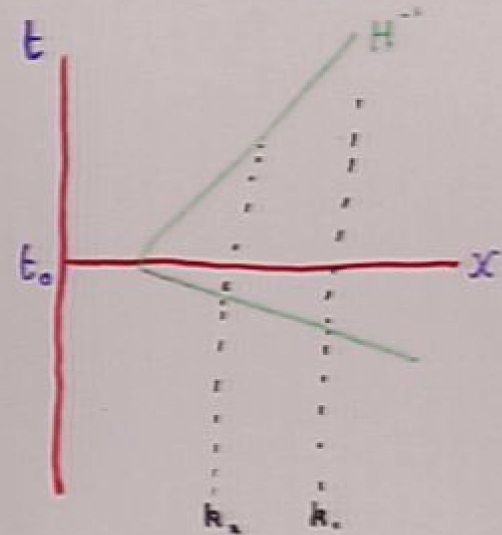
Outline

- Overview of NBV mechanism
  - Quasi-static initial states & the Horizon problem
  - Strings at T_H & scale invariance
- open issues of NBV
- spectral tilt, running, tensor modes, adiabaticity, Gaussianity, squeezing...
- compare & contrast w/ inflation... observational prospects?

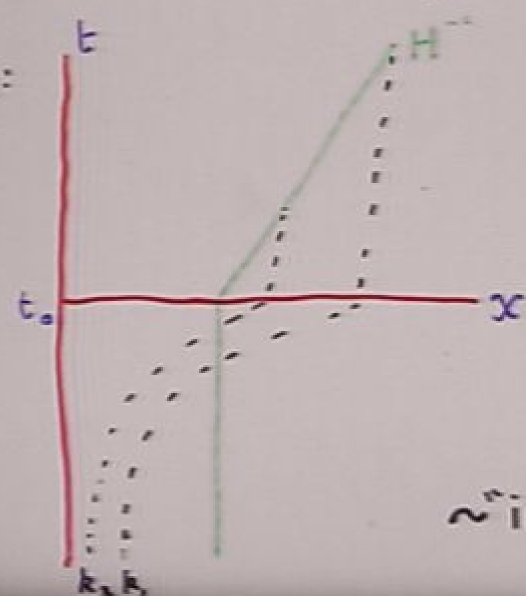
Quasi-static initial state:



(aka loitering)

(modes exit during transition, enter later)

c.f.:



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~"inverse" of inflation

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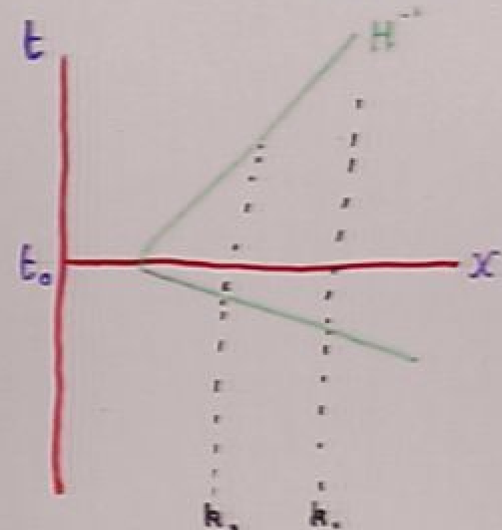

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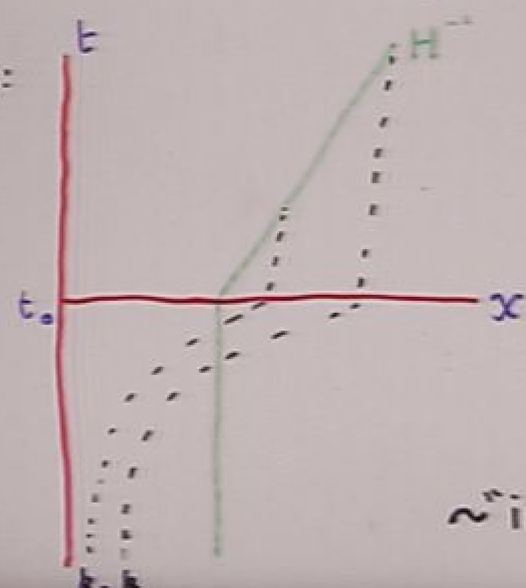

(" ")

~ "inverse" of inflation

$$ds^2 = -(1+2\phi)\,dt^2 + a^2(t)(1-2\phi)\,dx_i dx^i$$

$\Rightarrow$ 00 constraint eqn

$$\nabla^2 \phi = 4\pi G\, \delta\rho$$

.. we want to evaluate dimensionless power spectrum: $P_\phi(k) := k^3 \langle |\phi^2(k)| \rangle$

$$:= \sim k^{n(k)-1}$$

$n(k)$ is the spectral index

using constraint eqn: $P_\phi(k) = 16\pi^2 G^2 k^2 \langle \delta M^2 \rangle_{R=k^{-1}}$

consider $Z = \sum e^{-\beta E}$; $\langle E \rangle = \frac{1}{Z} \sum E e^{-\beta E}$

$$\Rightarrow \langle \delta E^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = T^2 \left(\frac{\partial \langle E \rangle}{\partial T} \right)_V$$

$$= T^2 C_V$$

$$\Rightarrow \quad P_\phi(k) = 16\pi^2 G^2 k^2 \langle \delta M^2 \rangle_{R=k^{-1}}$$

$$= 16\pi^2 G^2 k^2 T^2 C_V$$

Tan, Jain, Deo, Narayan ('89 → '92)

w/ 3+6 D static background:

$$C_V \simeq \frac{R^2}{\alpha'^{3/2} T} \frac{1}{1 - T/T_H} \qquad (g_s \ll 1)$$

$$\Rightarrow \quad P_\phi(k) = 16\pi^2 G^2 \alpha'^{-3/2} \frac{T}{1 - T/T_H}\Big|_{t_i(k)}$$

... scale invariant!

$$P_\phi(k) = 16\pi^2 \left(\frac{\ell_{pl}}{\ell_s}\right)^4 \frac{T/T_H}{1 - T/T_H}\Big|_{t_i(k)}$$

$$\Rightarrow \quad P_\phi(k) = 16\pi^2 G^2 k^2 \langle \delta M^2 \rangle_{R=k^{-1}}$$

$$= 16\pi^2 G^2 k^2 T^2 C_V$$

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$\Rightarrow \quad \dfrac{\ell_{pl}}{\ell_s} \sim g_s \sim 10^{-3}$ ($\Leftarrow$ same postdiction as in inflation)

(consistent w/ weak coupling)

- open issues

  $\Rightarrow$ 3+6 + quasi static?

  $\Rightarrow$ dilaton? (entropy perturbations, $O(1)$ effects)

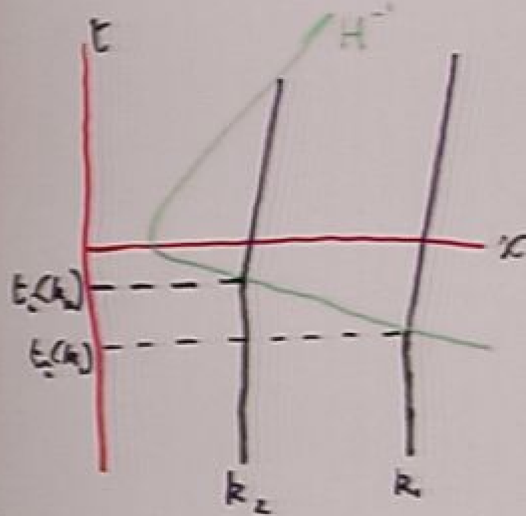
  $\Rightarrow$ calculational control vis à vis string theory

  $\Rightarrow$ flatness? (symmetries? naturalness?)

... at the very least, might show the way forward for more examples...

⇒ Has many interesting (observable?) consequences + no room to fine tune...

tilt + running

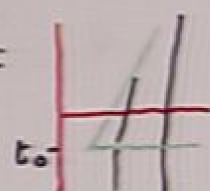


⇒ $P_\phi(k)$ to be evaluated for each k at Hubble radius crossing

⇒ larger k's exit later

$$P_\phi(k) = 16\pi^2 G^2 \alpha'^{-3/2} \frac{T(t_i(k))}{1 - T/T_H}$$

consider:



← $t_i(k) = t_0 \; \forall k$

spectrum is flat

$\Rightarrow$ since $T \propto 1/a(t)$ & $a(t)$ is increasing during transition,

$$\frac{T(k)}{1 - T(k)/T_H} \searrow \quad \text{as} \quad k \nearrow$$

... expect a slight red tilt...

using: $P_\phi(k) \sim \left(k/k_0\right)^{[n(k)-1]}$

$$\frac{d \ln P_\phi(k)}{d \ln k} = n(k) - 1 + \ln(k/k_0)\frac{dn(k)}{d\ln(k)}$$

now using $T \propto 1/a(t)$, we find

$$\frac{-kH[t_i(k)]}{1 - \frac{T[t_i(k)]}{T_H}}\frac{dt_i(k)}{dk} = n(k) - 1 + \ln(k/k_0)\frac{dn(k)}{d\ln(k)}$$

... can be integrated w/ integration factor $1/k$

$$\Rightarrow\ n(k)-1 = \frac{-1}{\ln(k/k_0)}\int_{k_0}^{k} \frac{H[t_i(k')]}{1-\frac{T[t_i(k')]}{T_H}}\,\frac{dt_i(k')}{dk'}\,dk'$$

$$\Rightarrow\ n(k)-1 = \frac{-1}{\ln(k/k_0)}\int_{t_i(k_0)}^{t_i(k)} \underbrace{\frac{H(t_i)}{1-T(t_i)/T_H}}\,dt_i$$

background determines the tilt... assumed nothing yet...

if tree level string process, $T \sim 1/\sqrt{\alpha'}$

$$\Rightarrow \Delta t \sim \sqrt{\alpha'}$$

$$H^{-1}(t_i) \sim \frac{M_{pl}}{T_H^2}$$

$$\Rightarrow\ n(k)-1 \sim \frac{T_H^2\sqrt{\alpha'}}{M_{pl}} \sim \frac{1}{\sqrt{\alpha'}M_{pl}} \sim \frac{\ell_{pl}}{\ell_s} \sim g_s\,O(?)$$

similarly, $\frac{dn}{d\ln k} \sim g_s O(?)$

- Adiabatic if no dilaton (also; no μ)
- Gaussian (central limit theorem)
- Hubble damping → standing wave before re-entry (squeezing) ⇒ acoustic peaks

⇒ Tensor modes

$$P_h(k) = \left(\frac{\ell_{pl}}{\ell_s}\right)^4 (1 - T/T_H) \ln^2\left[\frac{1}{\ell_s^2 k^2}(1 - T/T_H)^{-4}\right]$$

... blue tilt!

scalar/tensor ratio: $r \sim (1 - T/T_H)^2 \ln^2\left[\frac{1}{\ell_s^2 k^2}(1 - T/T_H)\right]$

... typically quite large...

$$\Rightarrow \quad n(k)-1 = \frac{-1}{\ln(k/k_0)} \int_{k_0}^{k} \frac{H[t_i(k')]}{1 - \frac{T[t_i(k')]}{T_H}} \frac{dt_i(k')}{dk'} dk'$$

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$$\langle \delta T^{i}_{j} \rangle$$

$$i \neq 1$$

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⇒ need a more precise realization of background to proceed, but results so far should be a strong motivation.

⇒ could determine cosmological parameters in terms of g_s, α'.

⇒ might be an example of something much more generic (w/ Anupam Mazumdar)

⇒ is easily falsifyable

⇒ is more fun to work on than inflation.