

Title: Nonlocal boxes and C*-algebras

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Abstract: Clifton, Bub, and Halvorson claim to be able to derive quantum mechanics from information-theoretic axioms. However, their derivation relies on the auxiliary assumption that the relevant probabilities for measurement outcomes can be represented by the observables (self-adjoint operators) and states of a C*-algebra. There are legitimate probability theories that are not so representable --- in particular, the nonlocal boxes of Popescu and Rohrlich. We explain the impact of nonlocal boxes on the interpretation of the CBH derivation, and we discuss possible generalizations of the CBH derivation in the framework of these more general probability theories.

Math	Physics	Info. Theory

Math	Physics	Info. Theory
Noncommutativity		

Math	Physics	Info. Theory
Noncommutativity (Nondistributive)		

$$p \wedge (q \vee r) \neq (p \wedge q) \vee (p \wedge r)$$

Math	Physics	Info. Theory
Noncommutativity (Nondistributive)		
Bell inequality violation.		

$$P \wedge (Q \vee R) \neq (P \wedge Q) \vee (P \wedge R)$$

$$r = a_1 b_1 + a_2 b_2 + a_3 b_1 - b a_2 b_2$$

$$p \wedge (q \vee r) \neq (p \wedge q) \vee (p \wedge r)$$


$$r = a_1 b_1 + a_1 b_2 + a_2 b_1 - b a_2 b_2$$

$$|\omega(r)| > 2$$

$$p \wedge (q \vee r) \neq (p \wedge q) \vee (p \wedge r)$$


$$r = a_1 b_1 + a_2 b_2 + a_3 b_1 - b_2$$

$$\left| \underline{\omega}(R) \right| > 2$$

$$\omega(R) = \langle x, Rx \rangle$$

$$P \wedge (Q \vee R) \neq (P \wedge Q) \vee (P \wedge R)$$

Ex: $r = a_1 b_1 + a_1 b_2 + a_2 b_1 - k a_2 b_2$

$$|\underline{\omega(F)}| > 2$$

Math	Physics	Info. Theory
Noncommutativity (Nondistributive)	Uncertainty Principle	
Bell inequality, violation.		

Math

Noncommutativity
(Nondistributive)

Bell inequality
violation.

Physics

Uncertainty
Principle

Entanglement

Info. Theory



Math	Physics	Info. Theory
Noncommutativity (Nondistributive)	Uncertainty Principle	No cloning
Bell inequality violation.	Entanglement	Steering

Math	Physics	Info. Theory
Noncommutativity (Nondistributive)	Uncertainty Principle	No cloning
Bell inequality validation.	Entanglement	Steering (No B.f Commitment)

①

"Characteristic of QM"

a_1, a_2, \dots, a_n



Math	Physics	Info. Theory
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Noncommutativity
(Nondistributive)

Bell inequality
violation.

Physics

Uncertainty
Principle

Entanglement

Info. Theory

No cloning

Steering
(No B.f
(Commitment))

Hughston → No Bit
Josza
Wootters Thm
Conciliant

No Bit
Commitment

Hughston Josza

Schmidt → → Wootters Thm

Decomposition

$$\psi = \sum_i c_i (x_i \otimes y_i)$$

$$B(H_A) \otimes B(H_B) \xrightarrow{\text{"type I"}}$$

No. B, t

Hughston

Josza

Wootters Thm

Schmidt

Decomposition

$$\Psi = \sum c_i |x_i\rangle \otimes |y_i\rangle$$

①

"Characteristic" of QM

②

D. IT principles hold more generally

① "Characteristic" of QM

② D-TT principles hold more generally?

① "Characteristics of QM"

② Do IT principles hold more generally?

B A^* I

by C^* algebras.

Systems (probabilities, etc.,) are represented

BA⁺I

by C^* algebras.



$\alpha(O_B)$

$$\begin{array}{l} \text{Alice} \quad B(H_A) \otimes I = A \\ \text{Bob} \quad I \otimes B(H_B) = B \end{array}$$

$$\begin{array}{ll} \text{Alice} & B(H_A) \otimes I = A \\ \text{Bob} & I \otimes B(H_B) = B \end{array} \quad \left. \begin{array}{c} A \\ B \end{array} \right\} C$$

Observable : $a^* = a$

$$\begin{array}{ll} \text{Alice} & B(H_A) \otimes I = A \\ B_{\text{Bob}} & I \otimes B(H_B) = B \end{array} \Bigg\} C$$

Observable : $a^* = a$, $\text{sp}(a) \subseteq \mathbb{R}$

$$\begin{array}{ll} \text{Alice} & B(H_A) \otimes I = A \\ B_{\text{Bob}} & I \otimes B(H_B) = B \end{array} \left. \right\} C$$

Observable : $a^* = a$, $\text{sp}(a) \subseteq \mathbb{R}$

State : $d \in C \rightarrow [a \mapsto \text{Tr}(da)]$

$$\left. \begin{array}{l} \text{Alice} \quad B(H_A) \otimes I = A \\ \text{Bob} \quad I \otimes B(H_B) = B \end{array} \right\} \subset$$

Observable : $a^* = a$, $\text{sp}(a) \subseteq \mathbb{R}$

State : $d \in C \rightarrow [a \mapsto \text{Tr}(da)]$

$$\left. \begin{array}{l} \text{Alice} \quad B(H_A) \otimes I = A \\ \text{Bob} \quad I \otimes B(H_B) = B \end{array} \right\} C$$

Observable : $a^* = a$, $\text{sp}(a) \subseteq \mathbb{R}$

State : $d \in C \rightarrow [a \xrightarrow{\omega} \text{Tr}(da)]$
 $\omega(a) \geq 0$ if $\text{sp}(a) \subseteq \mathbb{R}^+$

State: $a \in C \rightarrow [a \xrightarrow{\omega} \text{Tr}(da)]$
 $\omega(a) \geq 0$ if $\text{sp}(a) \subseteq \mathbb{R}^+$
 $\omega(I) = 1$

Dynamics $\Psi \in H_A \otimes H_B \rightarrow u\Psi$

State: $d \in C \rightarrow [a \xrightarrow{\omega} \text{Tr}(da)]$
 $\omega(a) \geq 0 \text{ if } \text{sp}(a) \subseteq \mathbb{R}^+$
 $\omega(I) = 1$

Dynamics $\Psi \in H_A \otimes H_B \rightarrow u\Psi$
 $d \xrightarrow{\hspace{1cm}} u du^*$

State: $d \in C \rightarrow \left[a \xrightarrow{\omega} \text{Tr}(da) \right]$

 $\omega(a) \geq 0 \text{ if } \text{sp}(a) \subseteq \mathbb{R}^+$
 $\omega(I) = 1$

Dynamics

$$\Psi \in H_A \otimes H_B \rightarrow u\Psi$$

$$d \longrightarrow u d u^*$$

$$a \longrightarrow$$

State: $d \in C \rightarrow [a \xrightarrow{\omega} \text{Tr}(da)]$

$$\omega(a) \geq 0 \text{ if } \text{sp}(a) \subseteq \mathbb{R}^+$$

$$\omega(I) = 1$$

Dynamics

$$\Psi \in H_A \otimes H_B \rightarrow u\Psi$$

$$d \longrightarrow u du^*$$

$$a \longrightarrow u^* au$$

$B(H)$ "like \mathbb{C} "

$\phi : C \rightarrow C$ bit

ϕ is linear if $\phi(\lambda a + b) = \lambda \phi(a) + \phi(b)$

ϕ is completely positive.

$\phi \otimes \text{id}$:

$B(H)$ "type I"

$\phi : C \rightarrow C$

ϕ is linear ; $\phi(\lambda a + b) = \lambda \phi(a) + \phi(b)$

ϕ is completely positive.

$\phi \otimes \text{id} : C \otimes M_n \rightarrow C \otimes M_n$

$\phi : C \rightarrow C$ is "Type I"

ϕ is linear if $\phi(\lambda a + b) = \lambda \phi(a) + \phi(b)$

ϕ is completely positive.

$\phi \otimes \text{id} : C \otimes M_n \rightarrow C \otimes M_n$

$\phi(I) = I$.

$$\phi(I) = I.$$

$$\alpha \rightarrow \underline{\omega(\omega)}$$



$$\phi(I) = I.$$

$$\alpha \rightarrow \underline{\omega(\omega)}$$

$$P = P^* = P^2 \quad sp(p) \subseteq \{0, 1\}$$

$X \rightarrow$ Hausdorff
Topological
Space



$X \rightarrow$ Hausdorff
Topological
Space | \mathbb{R}^n

$X \rightarrow$ Hausdorff
Topological
Space | \mathbb{R}^n

$B(X) =$ Borel functions

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{R}

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{R}
 (f, g)

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{C}
 \mathbb{R} (f, g) $f + ig$

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{C}
 \mathbb{R} $(f, g) \rightarrow f + ig$

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{C}
 \mathbb{R} (f, g) $f + ig$

$\mu : \Sigma(X) \rightarrow \mathbb{R}^+$

$X \rightarrow$ Hausdorff
Topological
Space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{R}
 $(f, g) \rightarrow f + ig$

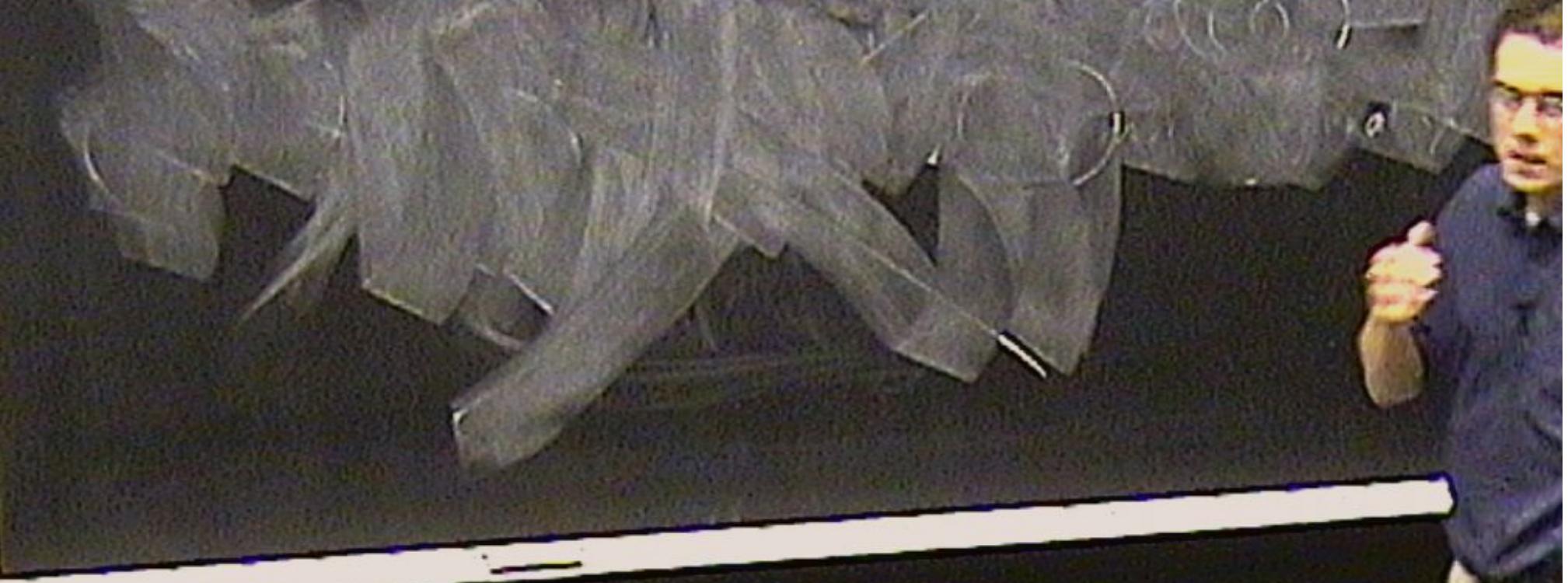
$\mu : \Sigma(X) \rightarrow \mathbb{R}^*$

$f \rightarrow \int f d\mu$

$$f \in \mathcal{B}(\Sigma)$$

$$f^*(x) \doteq \overline{f(x)}$$

is called
→



$f \in B(\bar{X})$

$$f^*(x) = \overline{f(x)}$$

$$f^* = f \iff \text{sp}(f) \subseteq \mathbb{R}$$

$f \in B(X)$

$$f^*(x) \doteq \overline{f(x)}$$

$$f^* = f \iff \text{sp}(f) \subseteq \mathbb{R}$$

Projections; $f = f^* = f^2 \iff \text{sp}(f) \subseteq \{0,1\}$

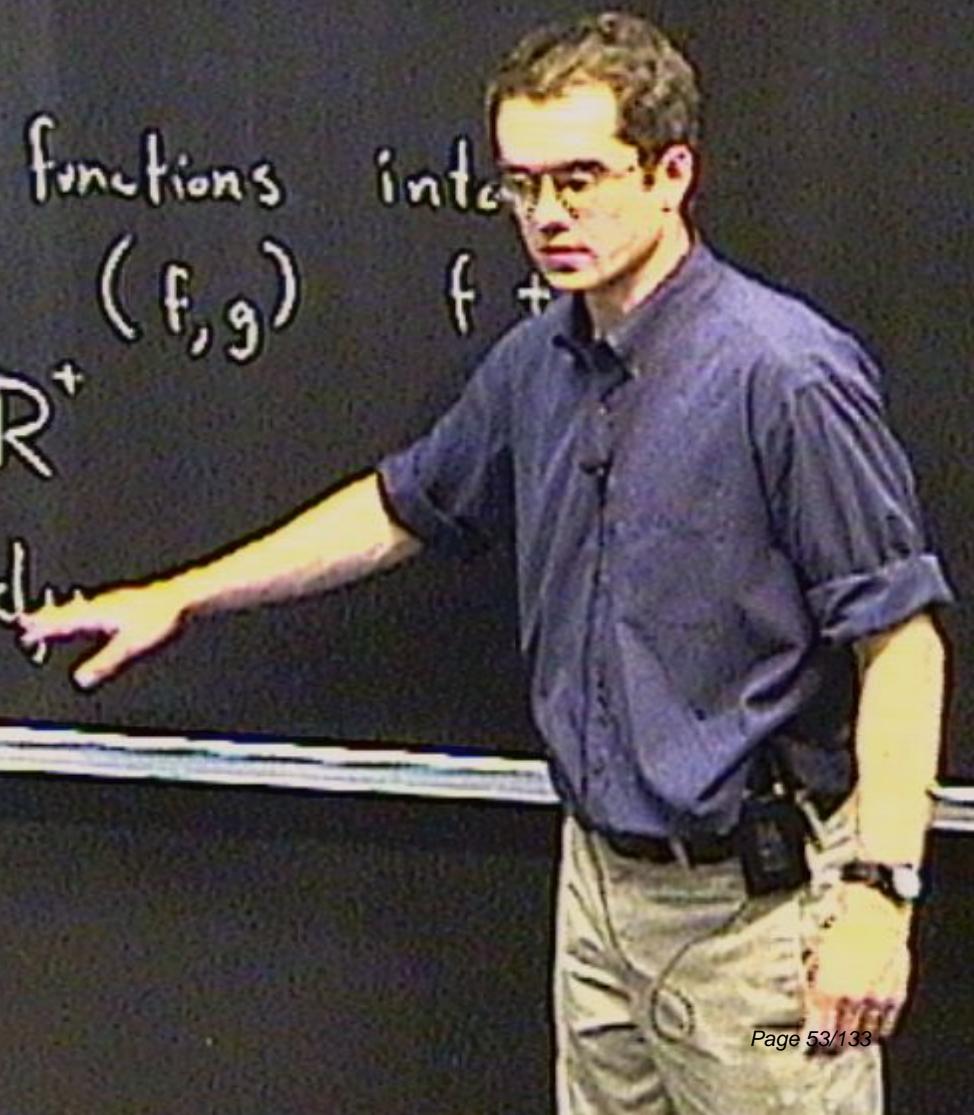
$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into

\mathbb{R} (f, g)

$\mu: \Sigma(X) \rightarrow \mathbb{R}^+$

$f \rightarrow \int f d\mu$



$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B(X) =$ Borel functions into \mathbb{R}
 $(f, g) \in f + ig$

$\mu: \Sigma(X) \rightarrow \mathbb{R}^+$

$$f \mapsto \int f d\mu$$

Observable : $a^* = a$ $s_a(a) \subset \mathbb{R}$

$$f \in B(\overline{X})$$

$$f^*(x) \stackrel{\text{def}}{=} \overline{f(x)}$$

$$f^* = f \iff \text{sp}(f) \subseteq \mathbb{R}$$

Projections: $f = f^* = f^2 \iff \text{sp}(f) \subseteq \{0,1\}$

$$f = \chi_S$$

Dynamics: "Flow" on X

Observable : $a^* = a \in \text{sp}(a) \subset \mathbb{R}$

$$f \in B(\overline{X})$$

$$f^*(x) \stackrel{\text{def}}{=} \overline{f(x)}$$

$$f^* = f \iff \text{sp}(f) \subseteq \mathbb{R}$$

Projections: $f = f^* = f^2 \iff \text{sp}(f) \subseteq \{0,1\} \iff f = \chi_S$

Dynamics: "Flow" on \overline{X}

$$\phi: \overline{X} \rightarrow \overline{X}$$

$$\bar{\phi}(f)(x) = f(\phi(x))$$

$$\mathcal{B}(H_A) \otimes \mathcal{B}(H_B)$$


Type I

$$B(H_A) \otimes B(H_B) \supseteq B(H_A \otimes H_B)$$



$B(H_A \otimes H_B)$

$X \rightarrow$ Hausdorff
topological
space | \mathbb{R}^n

$B_{\text{Borel}}(X) = \text{Borel functions into } \mathbb{C}$
 $\mathbb{R} \quad (f, g) \quad f + ig$
 $\mu : \Sigma(X) \rightarrow \mathbb{R}^+$
 $f \longrightarrow \int f \, d\mu$

$$\mathcal{B}(H_A) \otimes \mathcal{B}(H_B) = \mathcal{B}(H_A \otimes H_B)$$
$$C(X_A \times X_B)$$

$$\mathcal{B}(H_A) \otimes \mathcal{B}(H_B) \supseteq \mathcal{B}(H_A \otimes H_B)$$

$$C(X_A) \otimes C(X_B) = C(X_A \times X_B)$$

$$\mathcal{B}(H_A) \otimes \mathcal{B}(H_B) = \mathcal{B}(H_A \otimes H_B)$$

$$C(X_A) \otimes C(X_B) = C(X_A \times X_B)$$

$$B(H_A) \otimes B(H_B) = B(H_A \oplus H_B)$$
$$C(X_A) \otimes C(X_B) = C(X_A \times X_B)$$

{0,1,2}

$$B(H_A) \otimes B(H_B) \cong B(H_A \otimes H_B)$$
$$C(X_A) \otimes C(X_B) = C(X_A \times X_B)$$
$$\{0, 1, 2\}$$

Commitment /

$$M_1 \oplus \dots \oplus M_n$$



Commitment /

$$M_1 \oplus \cdots \oplus M_n \in R^n$$

$$4 \times 2 \times 2$$

$$+ \mu$$

$$M_1 \oplus \dots \oplus M_n$$

$$\mathbb{C}^{2 \times 2}$$

(a_1, \dots, a_n)
 (b_1, \dots, b_n)
 $(a_1 b_1, \dots, a_n b_n)$



$$\mathbb{M}_1 \oplus \dots \oplus \mathbb{M}_n \mathbb{R}^n$$

\downarrow

2×2

$n \times n$

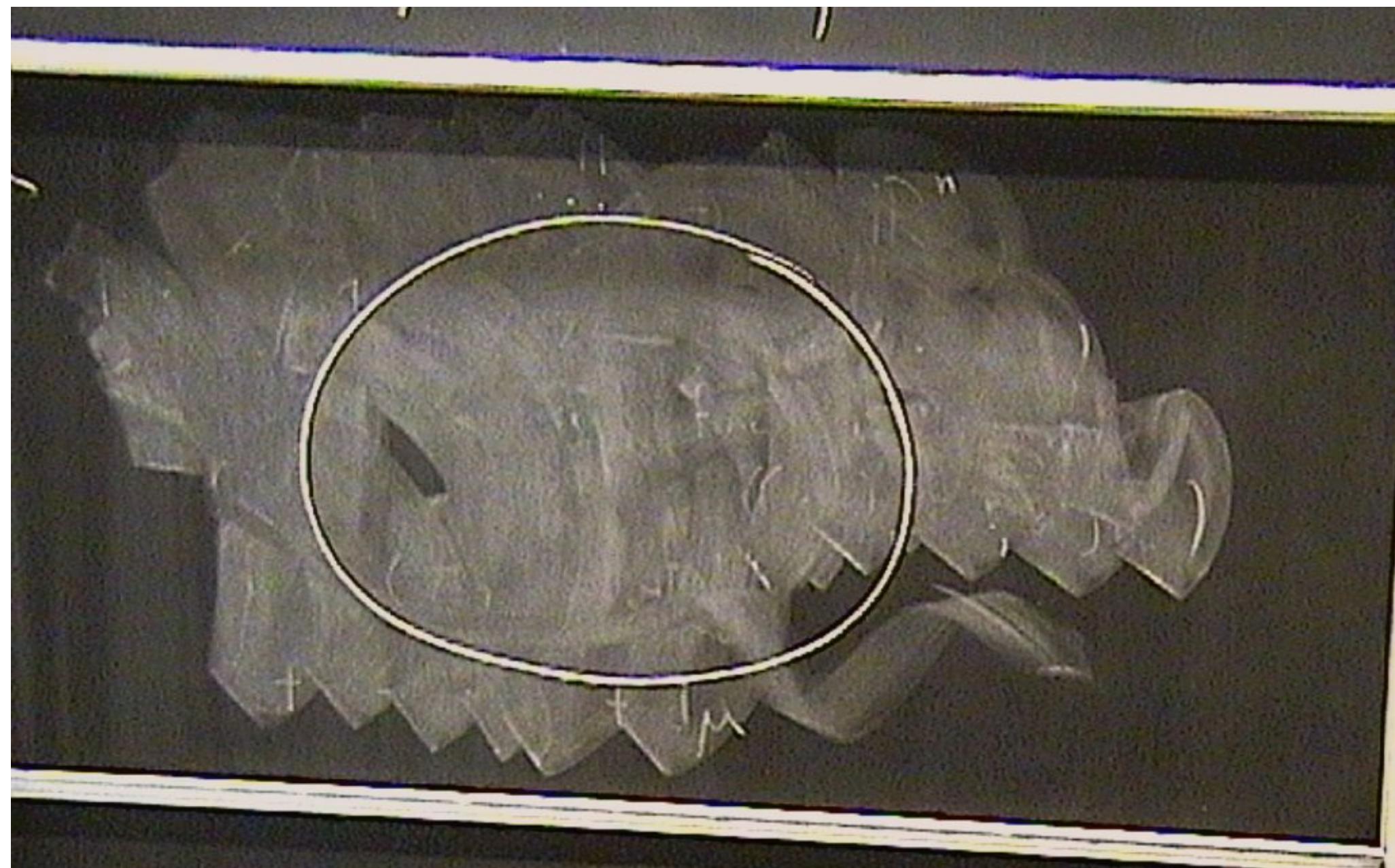
(a_1, \dots, a_n)
 (b_1, \dots, b_n)
 $(a_1 b_1, \dots, a_n b_n)$

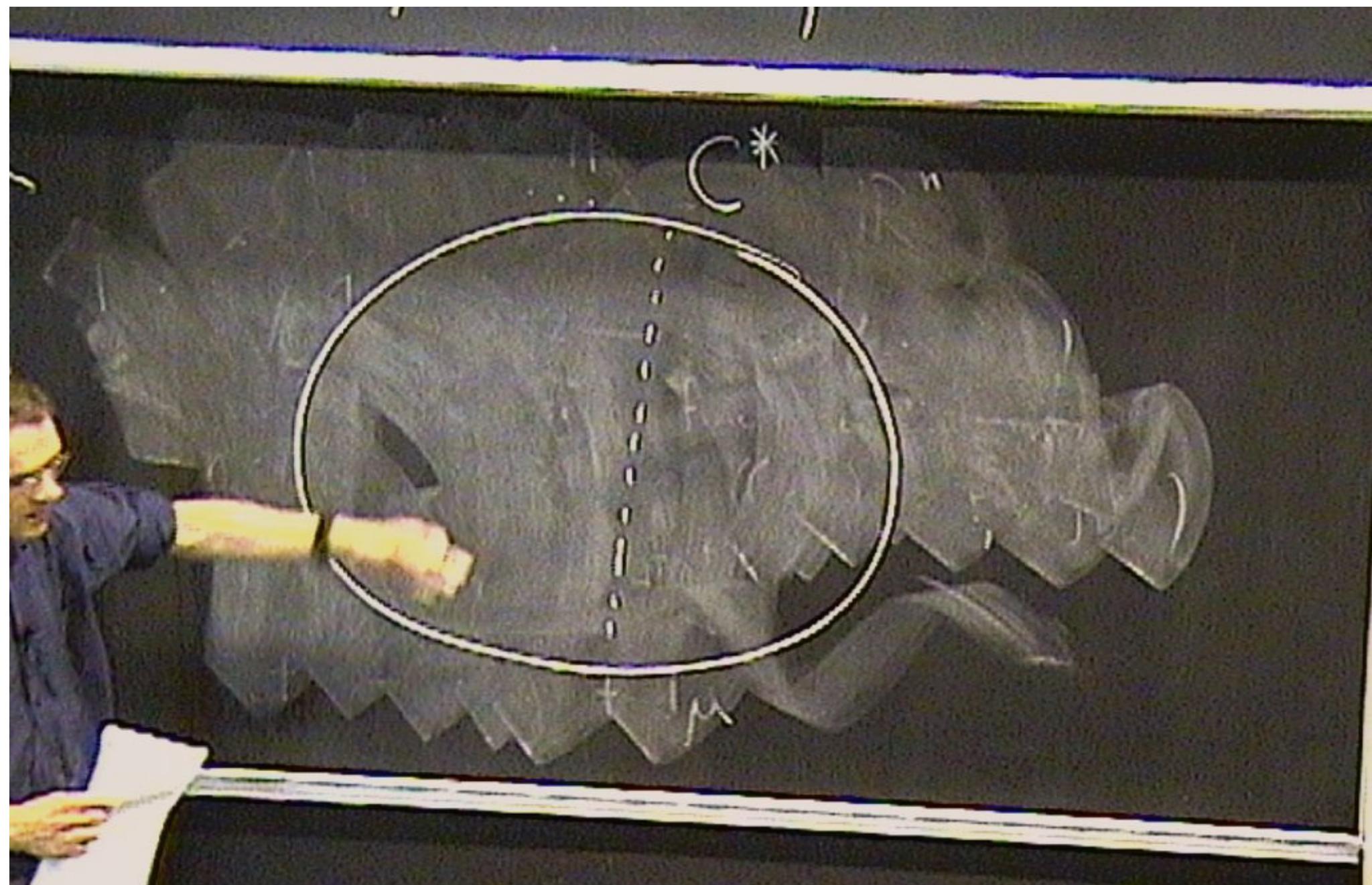
$$\mathbb{M}_i \oplus \dots \oplus \mathbb{M}_n \mathbb{R}^n$$

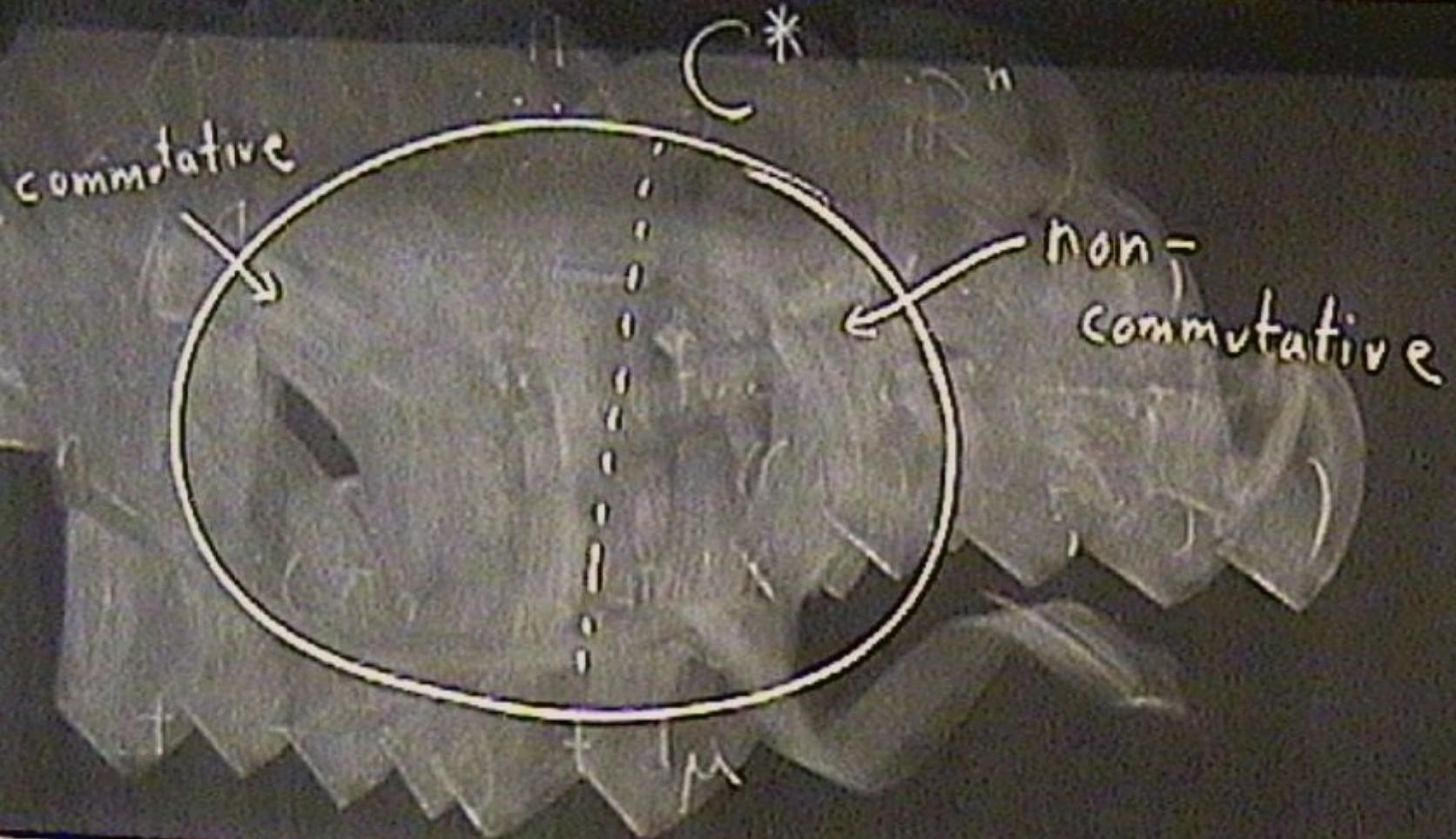
2×2

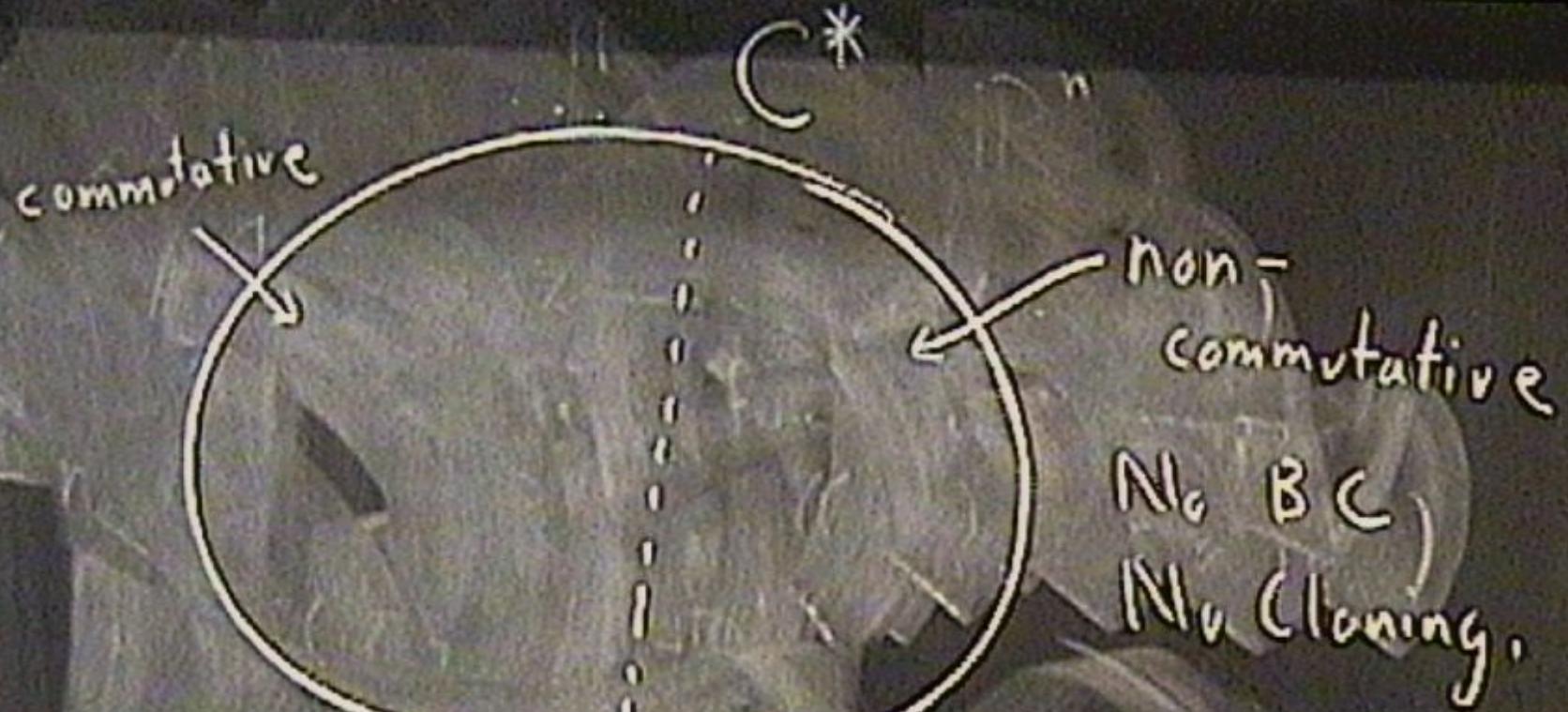
$$\begin{pmatrix} a_1, \dots, a_n \\ b_1, \dots, b_n \end{pmatrix}$$
$$(a_1b_1, \dots, a_nb_n)$$

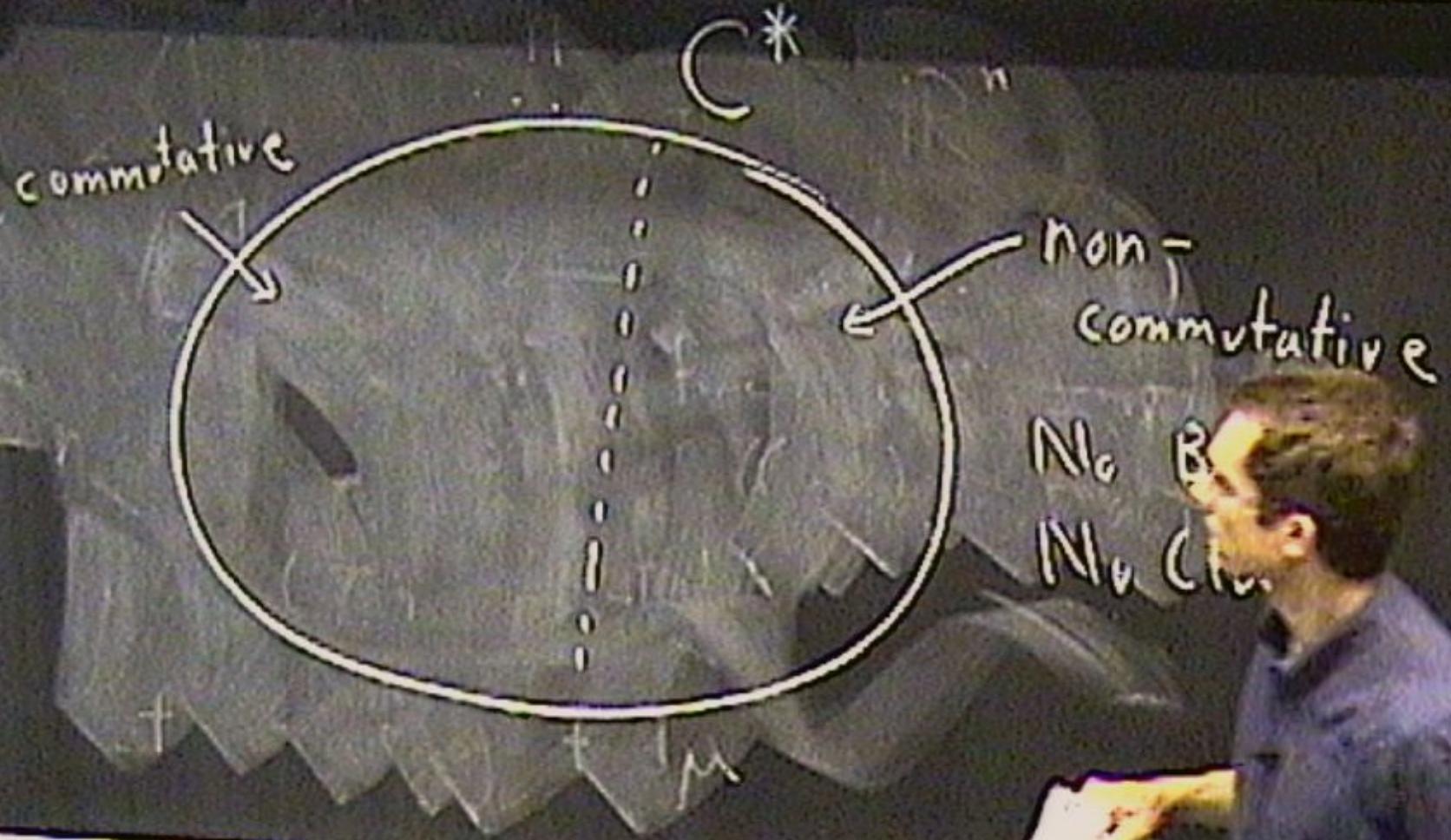












$A \otimes B$

commutative

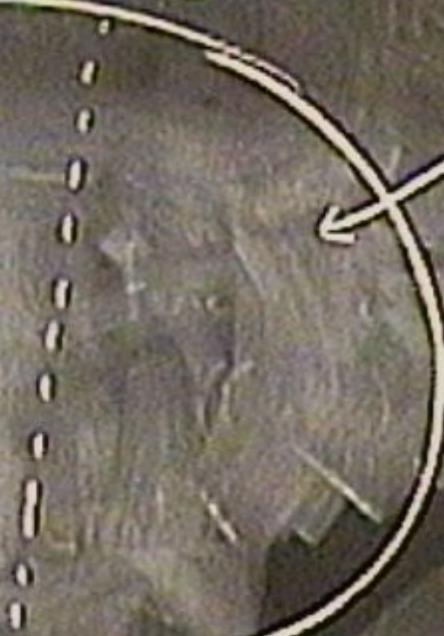


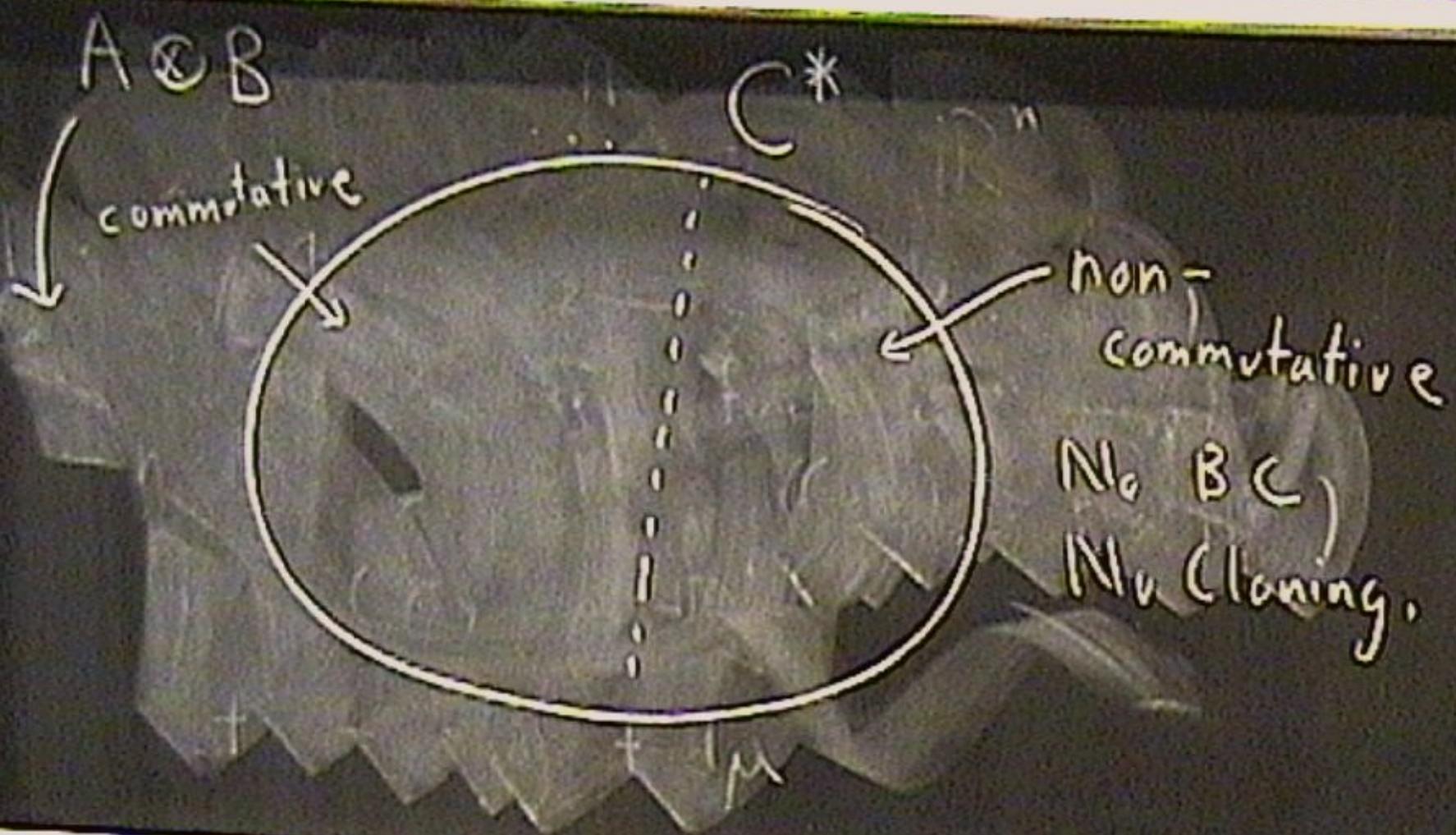
C^*

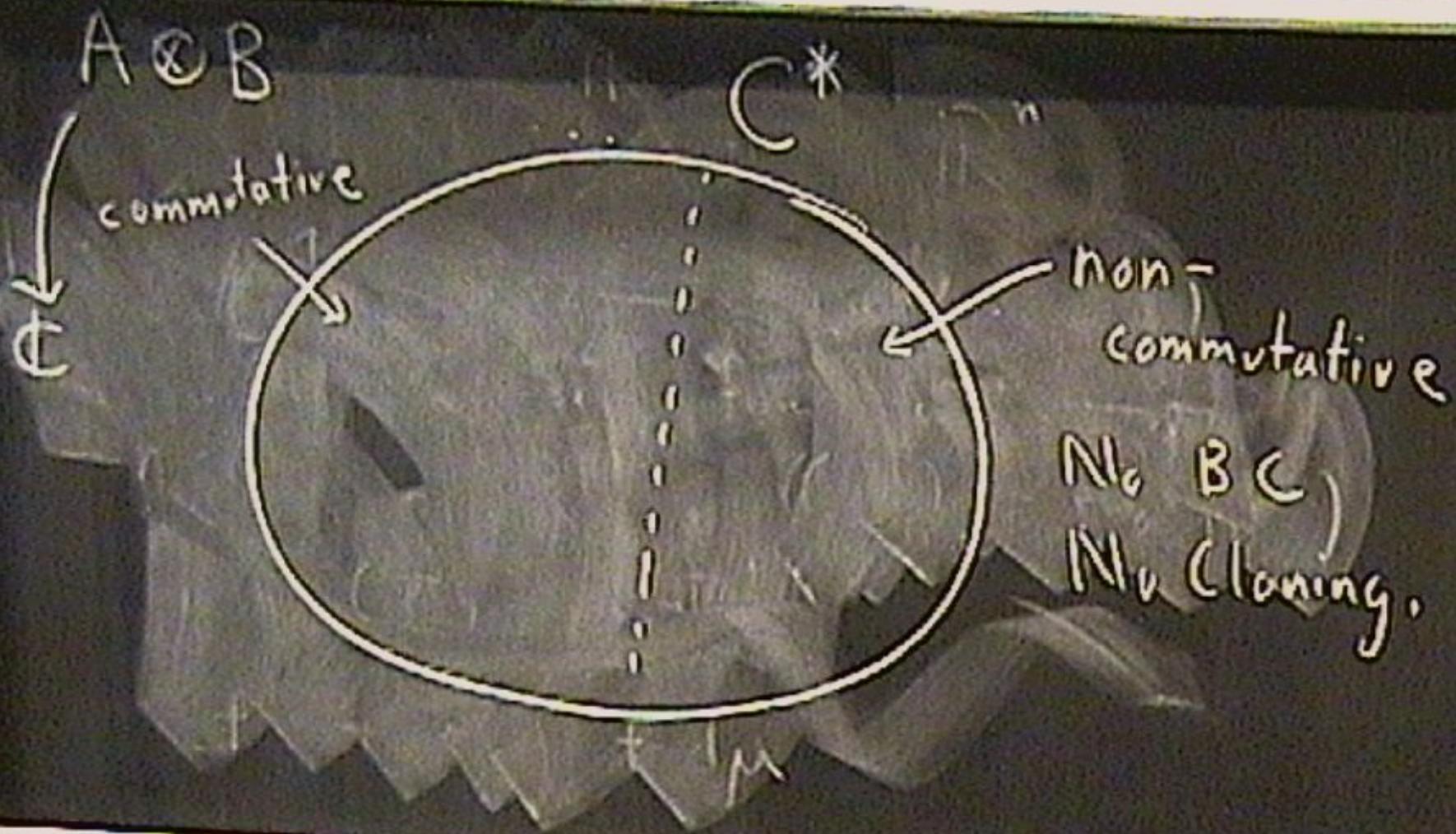
non-commutative

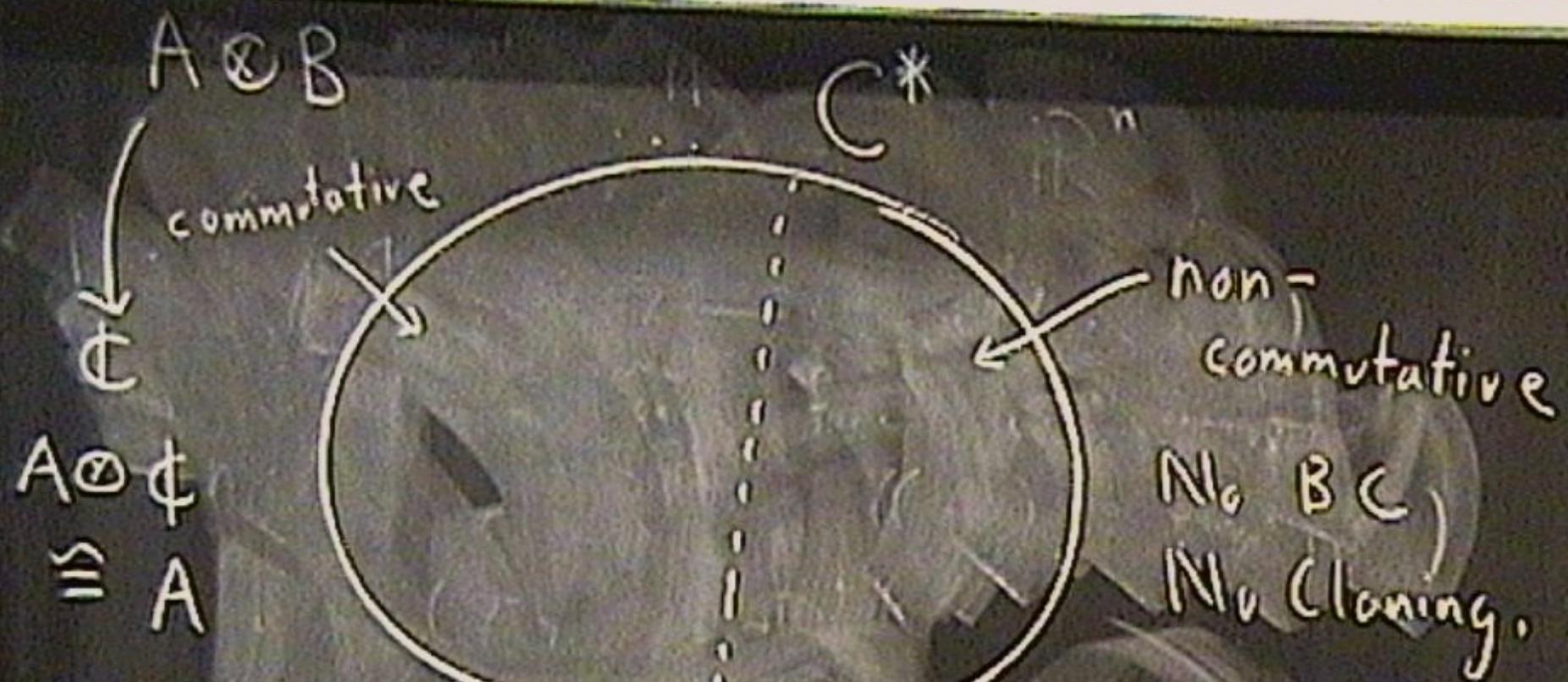
No $B(C)$

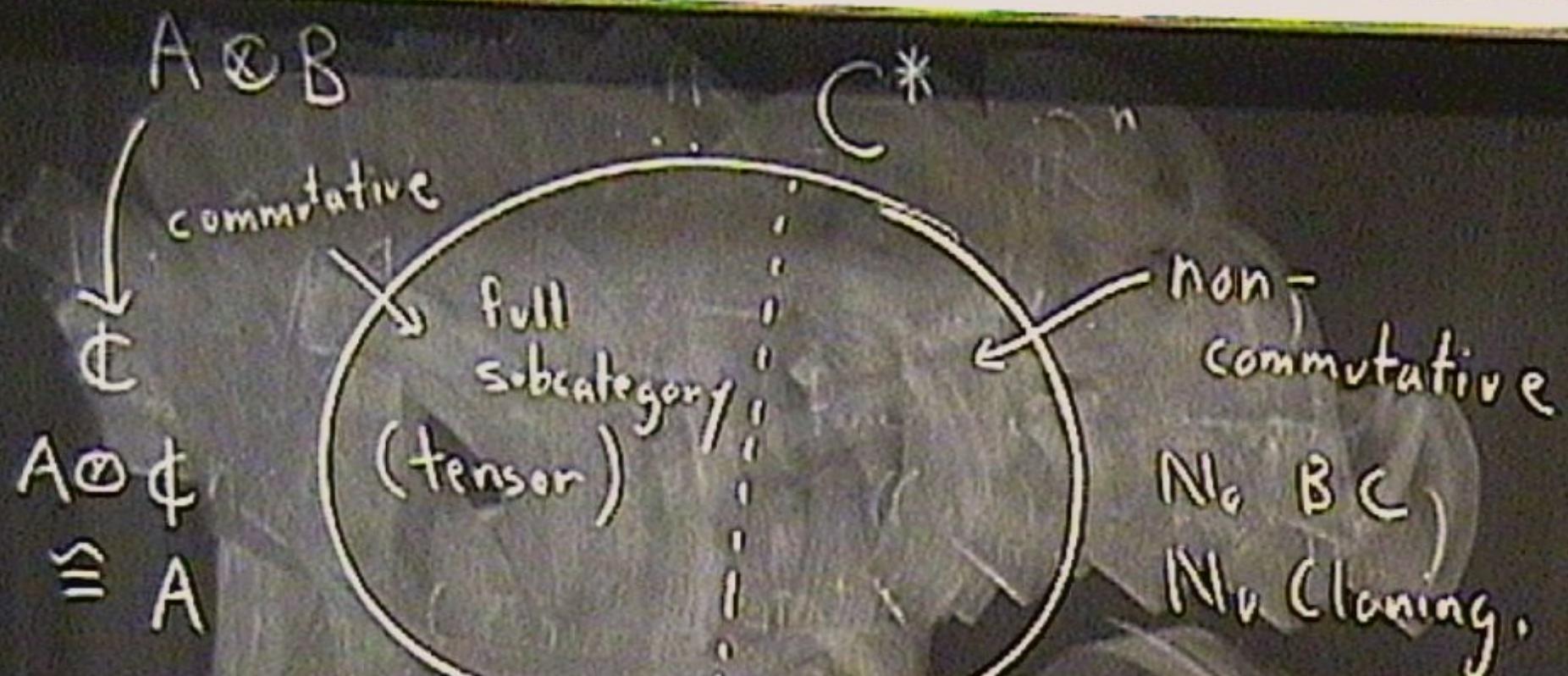
No Cloning,

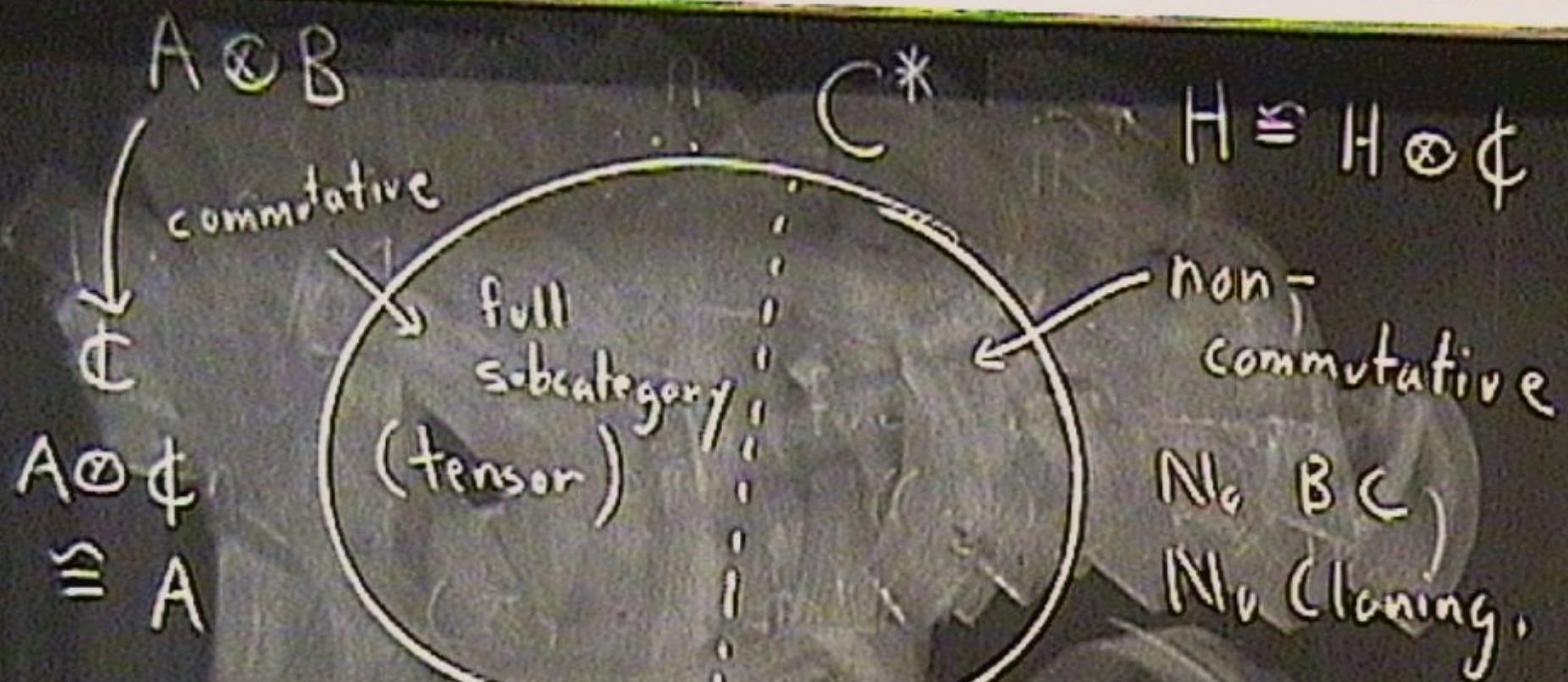












Math

Noncommutativity
(Nondistributive)

Bell inequality
violation.

Info. Theory

Uncertainty
Principle

Entanglement

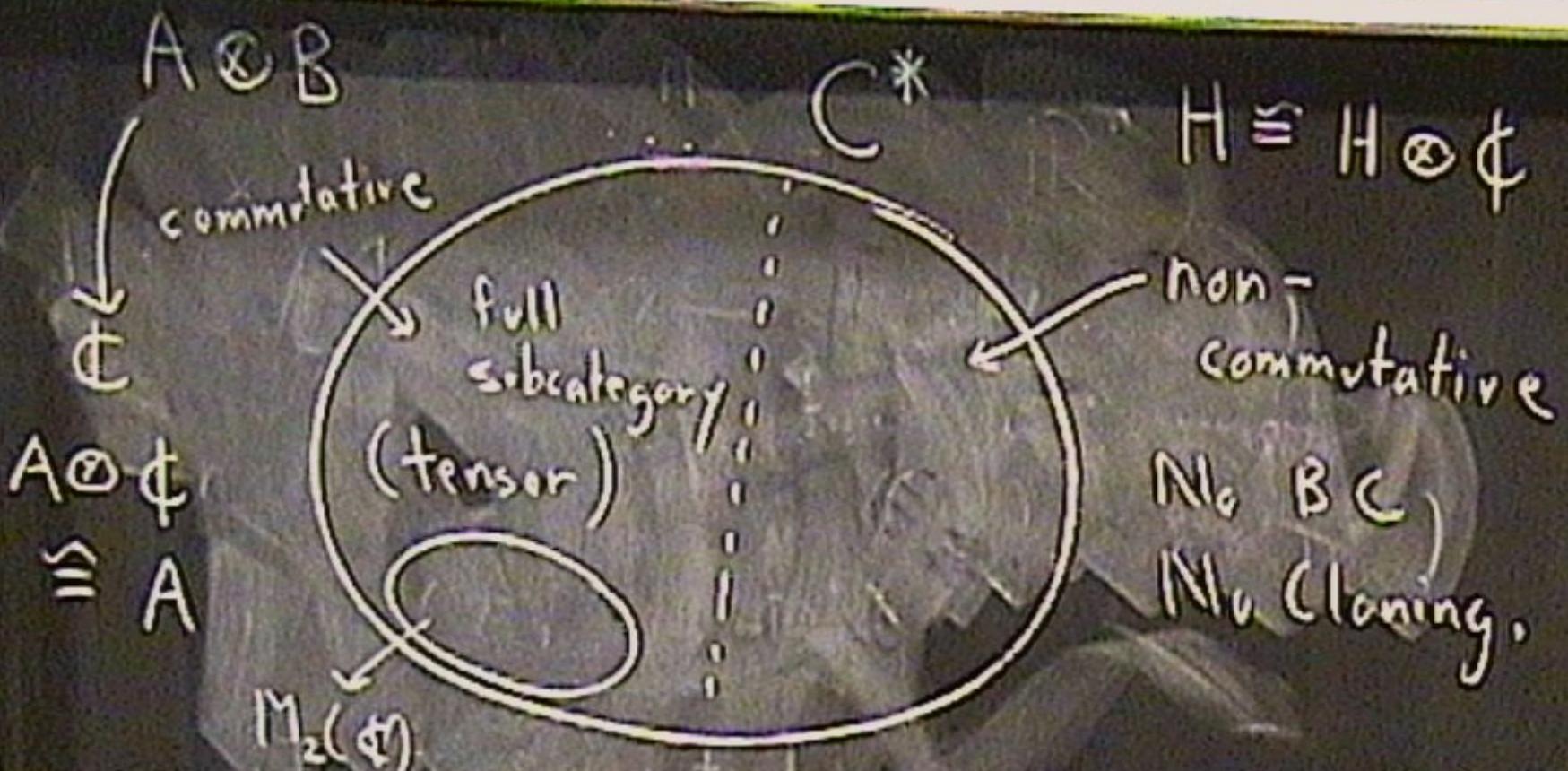
No cloning

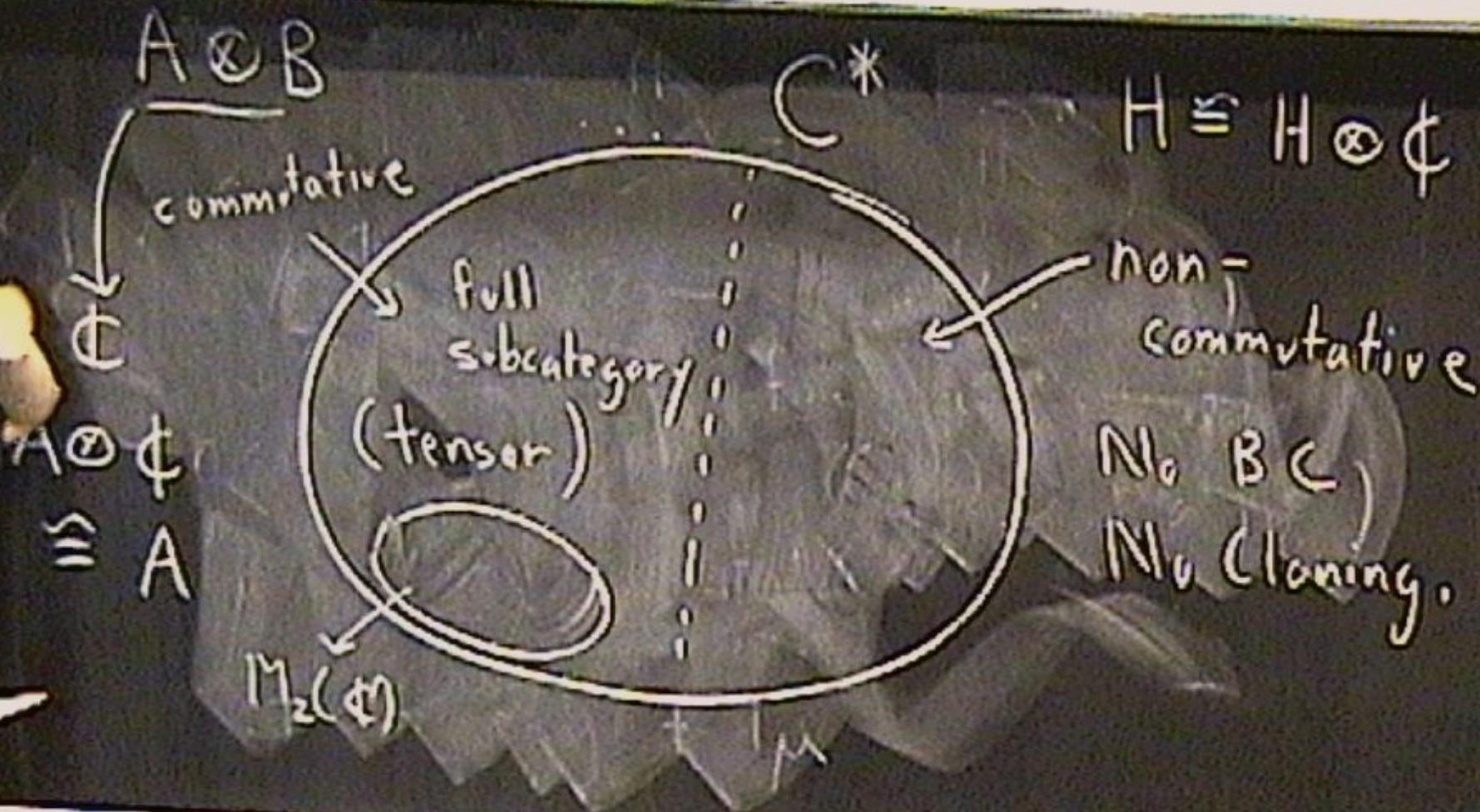
Steering
(N. Bit
Commitment)

$A \otimes B$

C^*

$H \cong H \otimes C$





Observe that $\phi_1 \circ \phi_2 = \phi$

$$\begin{aligned}\phi_1 : A_1 &\rightarrow B_1 \\ \phi_2 : A_2 &\rightarrow B_2\end{aligned}$$

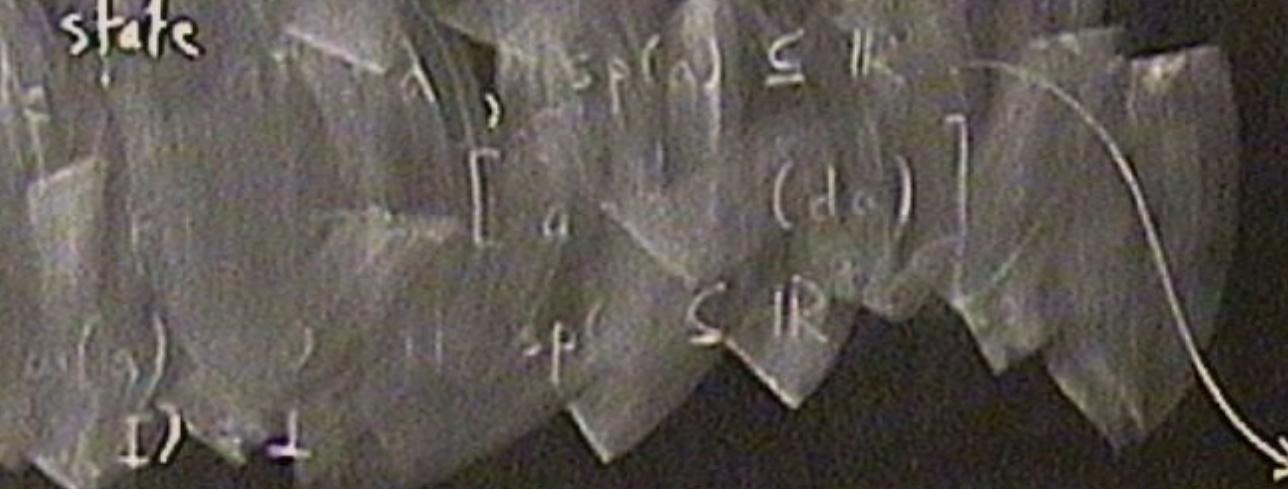
$$\phi_1 \circ \phi_2$$

Observation: $\alpha^* = \alpha$ and $\beta^* = \beta$

$$\begin{array}{l} \phi_1: A_1 \rightarrow B_1 \\ \phi_2: A_2 \rightarrow B_2 \end{array}$$

$$\left. \begin{array}{c} \overline{\phi_1} \\ \overline{\phi_2} \end{array} \right\} \phi_1 \otimes \phi_2: A_1 \otimes A_2 \xrightarrow{\quad} B_1 \otimes B_2$$

$$\phi(f)(x) = s(\phi(x))$$



state

$$\omega \otimes \sigma \rightarrow \omega$$

$$D_{\perp}$$

$$p(\alpha) \subseteq \mathbb{B}$$

$$[a, p(\alpha)]$$

$$-p(\alpha) \subseteq \mathbb{B}$$

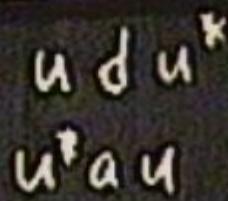
d

$\rightarrow u d u^*$

$\rightarrow u^* a u$

$$\omega \rightarrow \omega \otimes \sigma \rightarrow \omega$$

state



$$\omega \xrightarrow{\text{?}} \omega \otimes \sigma \xrightarrow{\phi} \text{state}$$

A person in a blue shirt is writing on a chalkboard. The chalkboard contains the following text:

$\omega \xrightarrow{\text{?}} \omega \otimes \sigma \xrightarrow{\phi} \text{state}$

Below the first two arrows, there is a diagram consisting of several overlapping circles. One circle contains the letter 'A'. Another circle contains the letter 'B'. A third circle contains the letters 'H' and 'P'. A curved arrow points from the right side of the board towards the bottom right corner.

d
a

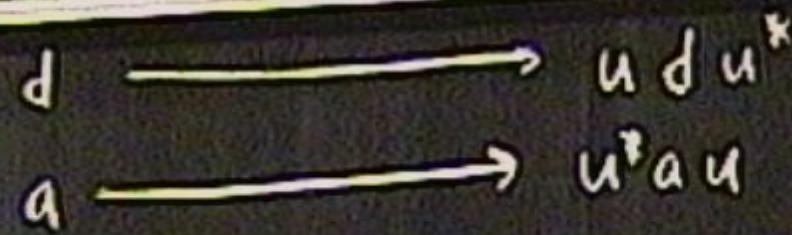
du^k
au

state

$$\omega \xrightarrow{\iota} \omega \otimes \sigma \xrightarrow{\phi} r_1(\phi(\omega \otimes \sigma)) = (\omega)$$

$$r_2(\phi(\omega \otimes \sigma)) = \omega$$

$\Gamma, \lambda \vdash \perp$



$$\Delta : A \rightarrow A \otimes A$$

$$r_1 \circ \Delta = id_A$$

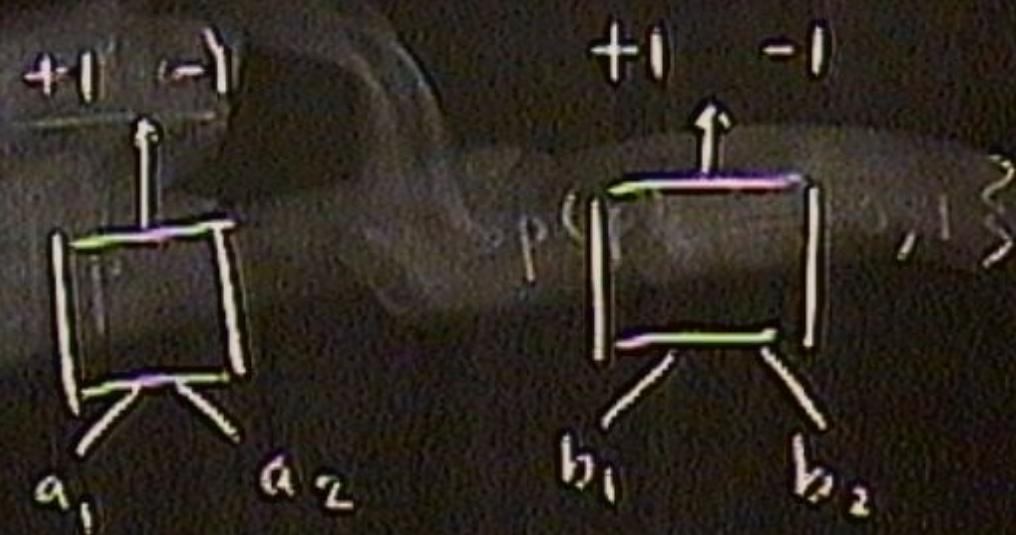
$$r_2 \circ \Delta = id_A$$

$$\Delta : A \rightarrow A \otimes A$$

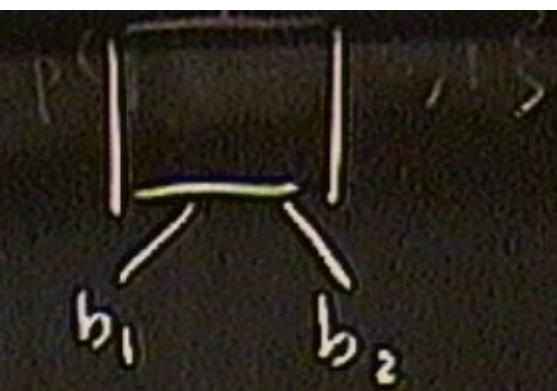
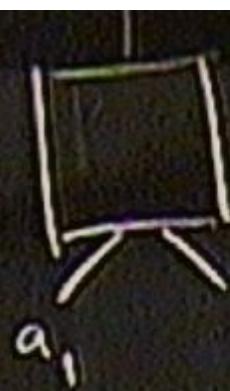
$$r_1 \circ \Delta = id_A$$

$$r_2 \circ \Delta = id_A$$

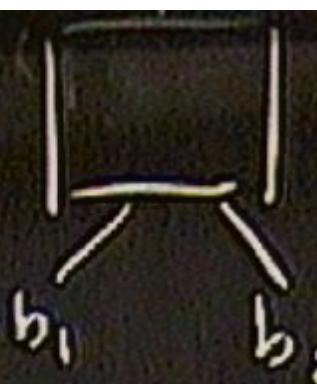
Δ exists for $A \iff A$ is commutative.



Pirsa:06050024
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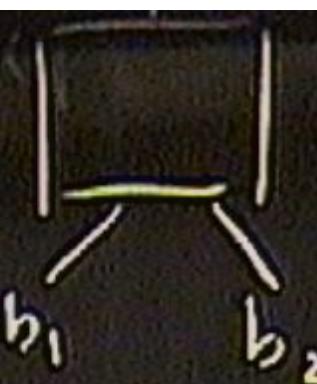
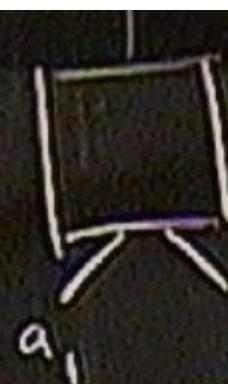
	-1	-1	+1	+1
a1, b1	-1	-1	+1	+1
a1, b2	-1	-1	+1	+1
a2, b1	-1	-1	+1	+1
a2, b2	-1	+1	+1	-1


 a_2

 b_2

$\omega(a_1, b_1) = 1$

a_1, b_1	-1	-1	+1	+1
a_1, b_2	-1	-1	+1	+1
a_2, b_1	-1	-1	+1	+1
a_2, b_2	-1	+1	+1	-1





$$\omega(a_1b_1 + a_1b_2 + a_2b_1 - a_2b_2) = 4$$

a_1, b_1	-1	-1	+1	+1
a_1, b_2	-1	-1	+1	+1
a_2, b_1	-1	-1	+1	+1
a_2, b_2	-1	+1	+1	-1

$$| \omega(a_1b_1 + a_1b_2 + a_2b_1 - a_2b_2) | = 4$$

	-1	-1	+1	+1
a ₁ , b ₁	-1	-1	+1	+1
a ₁ , b ₂	-1	-1	+1	+1
a ₂ , b ₁	-1	-1	+1	+1
a ₂ , b ₂	-1	+1	+1	-1

$$a \otimes b \wedge \cong \frac{1}{2}(\gamma ab + b_a)$$

YIS

10A

matrices.

$$a \otimes b \wedge \simeq \frac{1}{2}(\gamma_{ab} + b_a)$$

$$a \circ b = b \circ a$$



$$a \diamond b = \frac{1}{2}(\rho_{ab} + b_a)$$

$$a \circ b = b \circ a$$

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

X13

Jordan algebras

$$a \triangleleft b := \frac{1}{2}(ab + ba)$$

$$a \circ b = b \circ a$$

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

x15



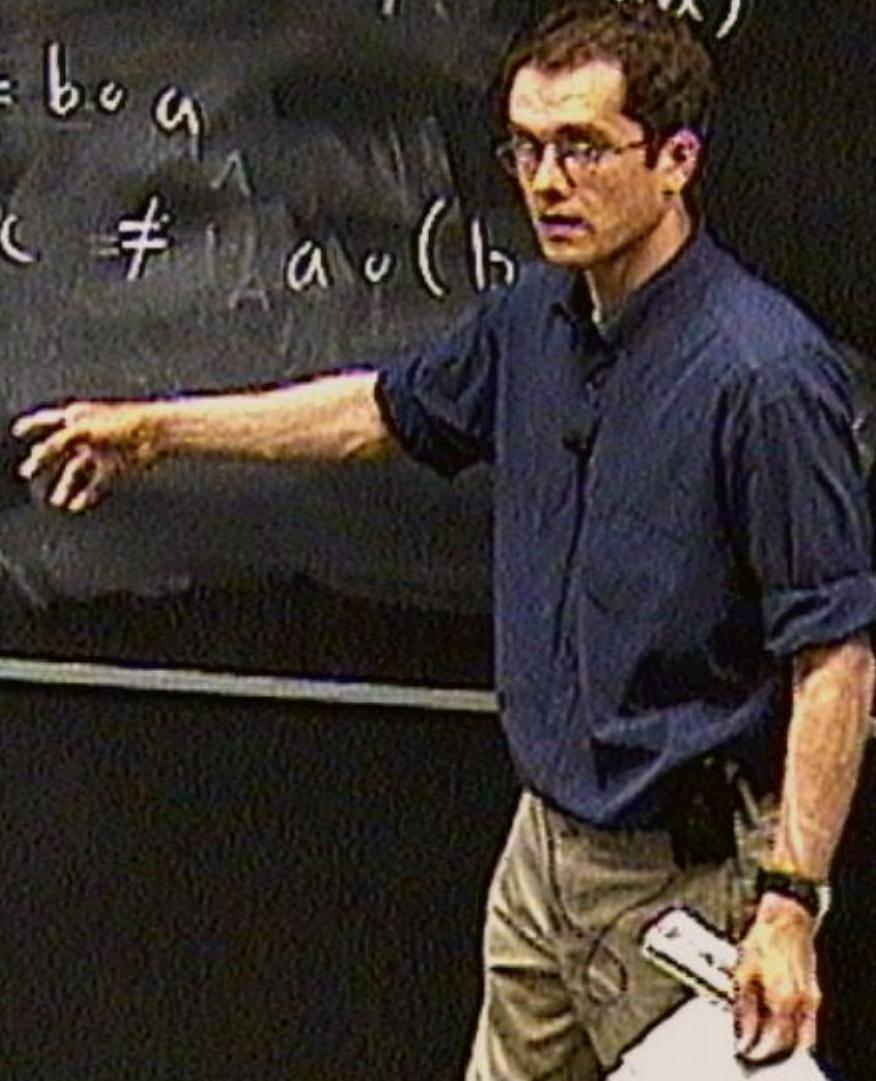
Jordan algebras

$$a \triangle b = \frac{1}{2}(ab + ba)$$

$$a \circ b = b \circ a$$

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

$$\mathbb{M}_2(\mathbb{R})$$



Jordan algebras

$$a \triangleleft b := \frac{1}{2}(ab + ba)$$

$$a \circ b = b \circ a$$

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

ex. $M_2(\mathbb{R}) \rightarrow M_2(\mathbb{H})$

Jordan algebras

$$a \triangleleft b := \frac{1}{2}(ab + ba)$$

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$$\text{axis } M_2(\mathbb{R})$$

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$$\cong M_2(\mathbb{R})$$

Hanche-Olsen: $A \otimes M_2(\mathbb{C})$

$\phi \circ id$



$$|\omega(a_1b_1 + a_1b_2 + a_2b_1 - a_2b_2)| = 4$$

$$a_1 \circ (b_1 + b_2) = a_1 \circ b_1 + a_1 \circ b_2$$



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$$\begin{aligned} & |\omega(a_1 \circ (b_1 + b_2) + a_2 \circ (b_1 - b_2))| \\ & |\langle a_1, b_1 + b_2 \rangle + \langle a_2, b_1 - b_2 \rangle| \end{aligned}$$

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$$|\omega(a_1 \circ (b_1 + b_2) + a_2 \circ (b_1 - b_2))| \leq 2\sqrt{z}$$

Finslerson
Bavnel

$$r_2(\phi(w \otimes \sigma)) = \omega$$

(Turing) Segal Algebra

$$r_2(\phi(w \otimes \sigma)) = w$$

String Segal Algebra

\cong Jordan algebra - distributivity

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String Segal Algebra

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 $\cong a \in A, a = \lambda_1 e_1 + \dots + \lambda_n e_n$

state

String Segal Algebra

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$\cong a \in A, a = \lambda_1 e_1 + \dots + \lambda_n e_n$

Landau For any set of non-signalling correlation data, there is a Segal algebra A and state w of A s.t.

$M_2(\mathbb{R})$

$M_2(\mathbb{H})$

correlation data, which is a Segal algebra A
and state ω of A . $\phi(f) = \omega(a^* f)$

$$\Pr(a_1=1 \wedge b_1=1) = \omega(a_1 \circ b_1)$$

To this -

Lancum
Correlation data, there is a Segal algebra
and state ω of A s.t. $\omega(a \circ b) = \langle a | b \rangle$

$$\Pr(a_1 = 1 \wedge b_1 = 1) = \omega(a_1 \circ b_1)$$

$$\circ : a, b \rightarrow$$

$$\omega(a \circ b)$$

$$| \tau_a \rangle \langle b | - \text{etc.}$$

$$\Pr(a_1 = 1 \wedge b_1 = 1) = \omega(a_1 \circ b_1)$$

$$\circ : a, b \rightarrow$$

$$\omega(a \circ b)$$

$$\begin{array}{c} A \\ \circ \\ B \end{array}$$
$$a \circ b = \underline{ab}$$

Convex set (states)

$$H = \bigcap_{n \in \mathbb{N}} C_n$$

B
C
D
E
F
G

Convex set (states)

Observables =

Convex set (states) K

Observables = Affine functions on K .

Convex set (states) K

Observables = Affine functions on K .

f^z

g^z

Convex set (states) K

Observables = linear functions on K .
 K is spectral

Convex set (states) \mathcal{K}

Observables = Affine functions on \mathcal{K}

\mathcal{K} is "spectral"

$$f = \lambda_1 c_1 + \dots +$$



Convex set (states) K



Observables = Affine functions on K

K is "spectral"

$$f = \lambda_1 e_1 + \dots + \lambda_n e_n$$



Convex set (states) K



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Convex set (states) K



Observables = Affine functions on K .

K is spectral

$$f = \lambda_1 e_1 + \dots + \lambda_n e_n$$

$$f^2 =$$

$$f \circ g = \frac{1}{4} \left\{ (f+g) \circ (f+g) - (f-g) \circ (f-g) \right\}$$

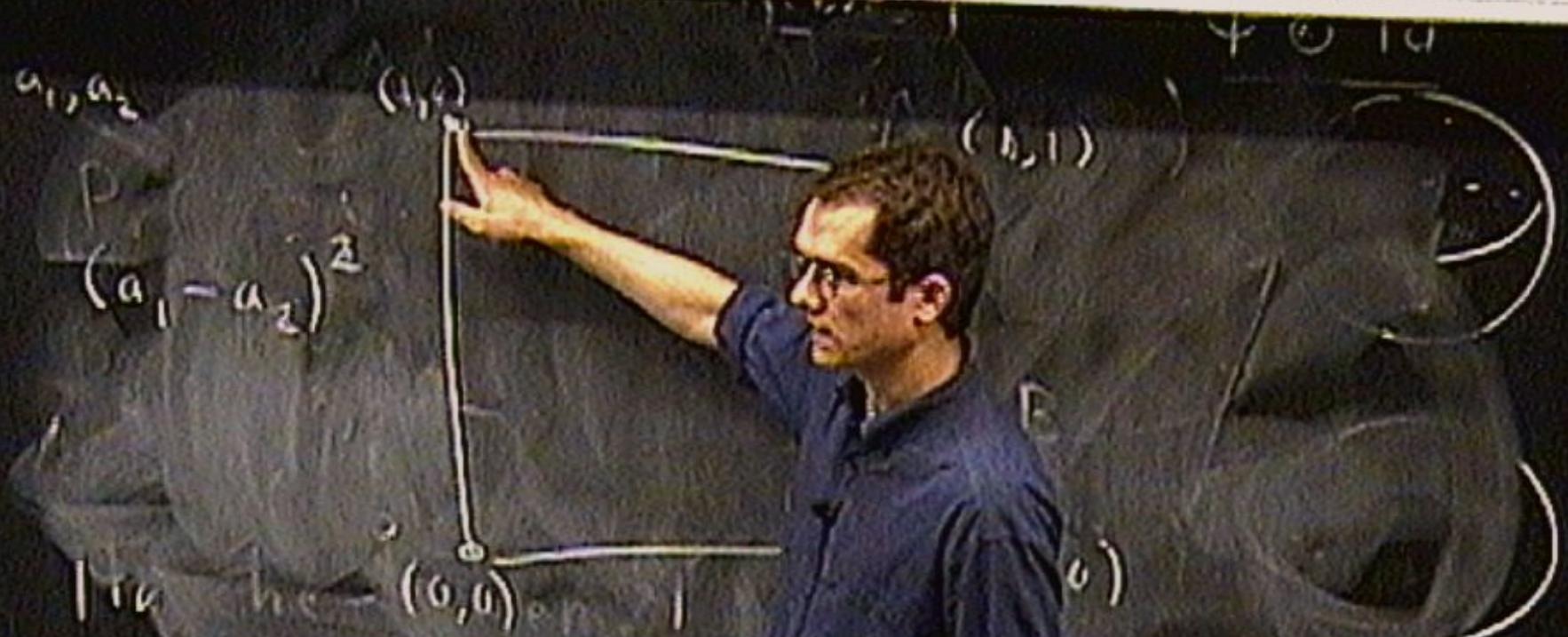
Convex set (states) K

Observables = Affine functions
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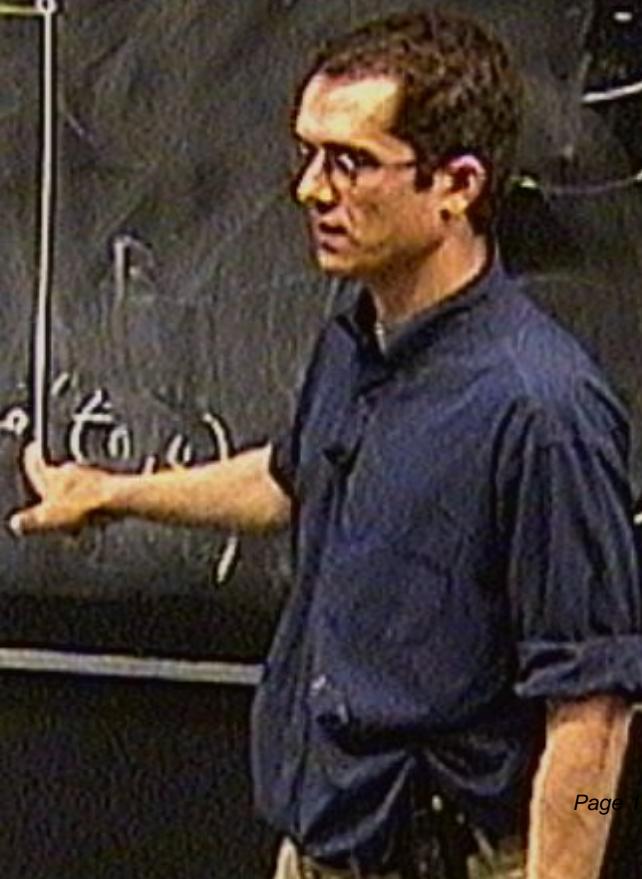
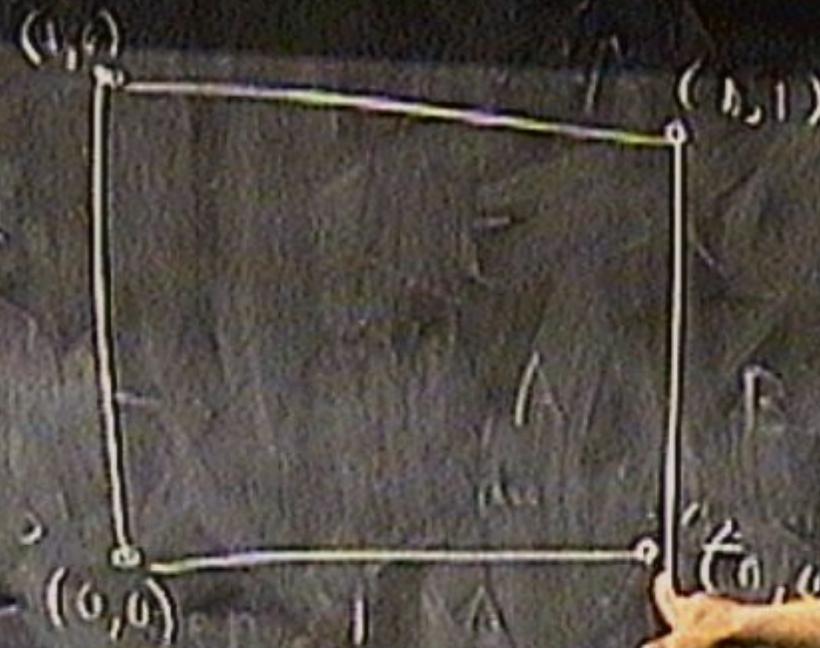
$$f \circ f = f^2 = \lambda_1^2 c_1^2 + \dots + \lambda_n^2 c_n^2$$

correlation data, there is a Segal algebra A
and state ω of A s.t. $\phi(\cdot) = \omega(\phi(\cdot))$



correlation data, there is a Segal algebra A
and state ω of A s.t.

$$\begin{aligned} & a_1, a_2 \\ & P \\ & (a_1 - a_2)^2 \\ & \text{H}_0: a_1 = a_2 \end{aligned}$$



correlation data, there is a Segal algebra A ,
and state ω of A s.t. $\phi_{\omega}(d(A))$

