Title: Fast Optimization or the Radiation Therapy of Tumors - the Impossible Possible

Date: May 27, 2006 02:00 PM

URL: http://pirsa.org/06050020

Abstract: <kw> Radiation Delivery, cancer, radiation therapy, electron, isocentre, planned target volume, adaptive radiotherapy, multi-leaf collimator, energy deposition, multi-beam delivery, intensity modulation, optimal radiation treatment, matrix inversion, inverse planning optimization, symmetries, fast inverse dose optimization </kw>

Pedro Goldman

Dept. of Physics
Ryerson University

http://www.physics.ryerson.ca

RYERSON UNIVERSITY

$$\delta E = \frac{4\alpha^3}{3}\left[\frac{\langle \delta(r_1) + \delta(r_2) \rangle}{Z^3}\right]\left(-2\ln\alpha + \frac{19}{30} - \ln\frac{k}{Z^2}\right)$$

$$\ln k = \frac{\sum_n \int \left|\langle \psi_n | \vec{r}_1 + \vec{r}_2 | \psi_0 \rangle\right|^2 (E_n - E_0)^3 \ln|E_n - E_0|}{\sum_n \int \left|\langle \psi_n | \vec{r}_1 + \vec{r}_2 | \psi_0 \rangle\right|^2 (E_n - E_0)^3}$$

$$\beta = \sum_n \int \left|\langle \psi_n | \vec{r} | \psi_0 \rangle\right|^2 (E_n - E_0)^3 \ln|E_n - E_0|$$

β = 2.2909 8137 5205 5523 0134 2545 0657 1 a.u.

$$\sum_n \int \left|\langle \psi_n | \vec{r} | \psi_0 \rangle\right|^2 (E_n - E_0)^3 = 2\pi Z \langle \delta(\vec{r}) \rangle_0$$

$$\lambda_k = \exp\left[(a x_{k-1})^b\right] \qquad \Phi_{ik} \propto e^{-\lambda_k r} r^{n_i} Y_{l_i m_i}(\theta, \varphi)$$

Fast Optimization of the Radiation Therapy of Tumours - the Impossible Possible –

S. P. Goldman
Dept. of Physics
Ryerson University

Fast Optimization of the Radiation Therapy of Tumours - the Impossible Possible –

S. P. Goldman
Dept. of Physics
Ryerson University

Fast Optimization of the Radiation Therapy of Tumours - the Impossible Possible –

S. P. Goldman
Dept. of Physics
Ryerson University

RYERSON UNIVERSITY

Co-Authors
Mentors

Jerry Battista

Jeff Chen

London Regional Cancer Centre
London, Ontario

Co-Authors
Mentors

Jerry Battista

Jeff Chen

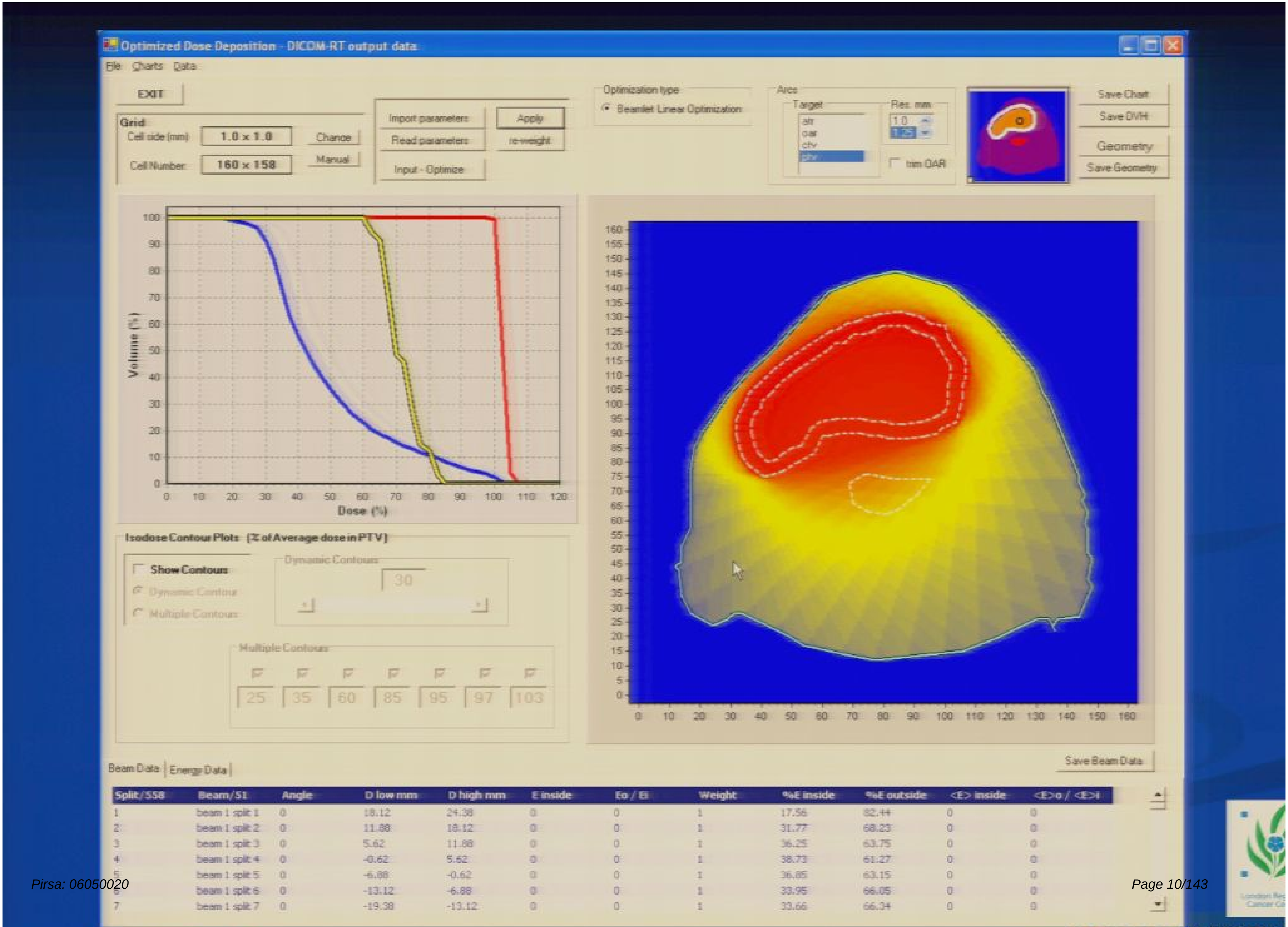
London Regional Cancer Centre
London, Ontario

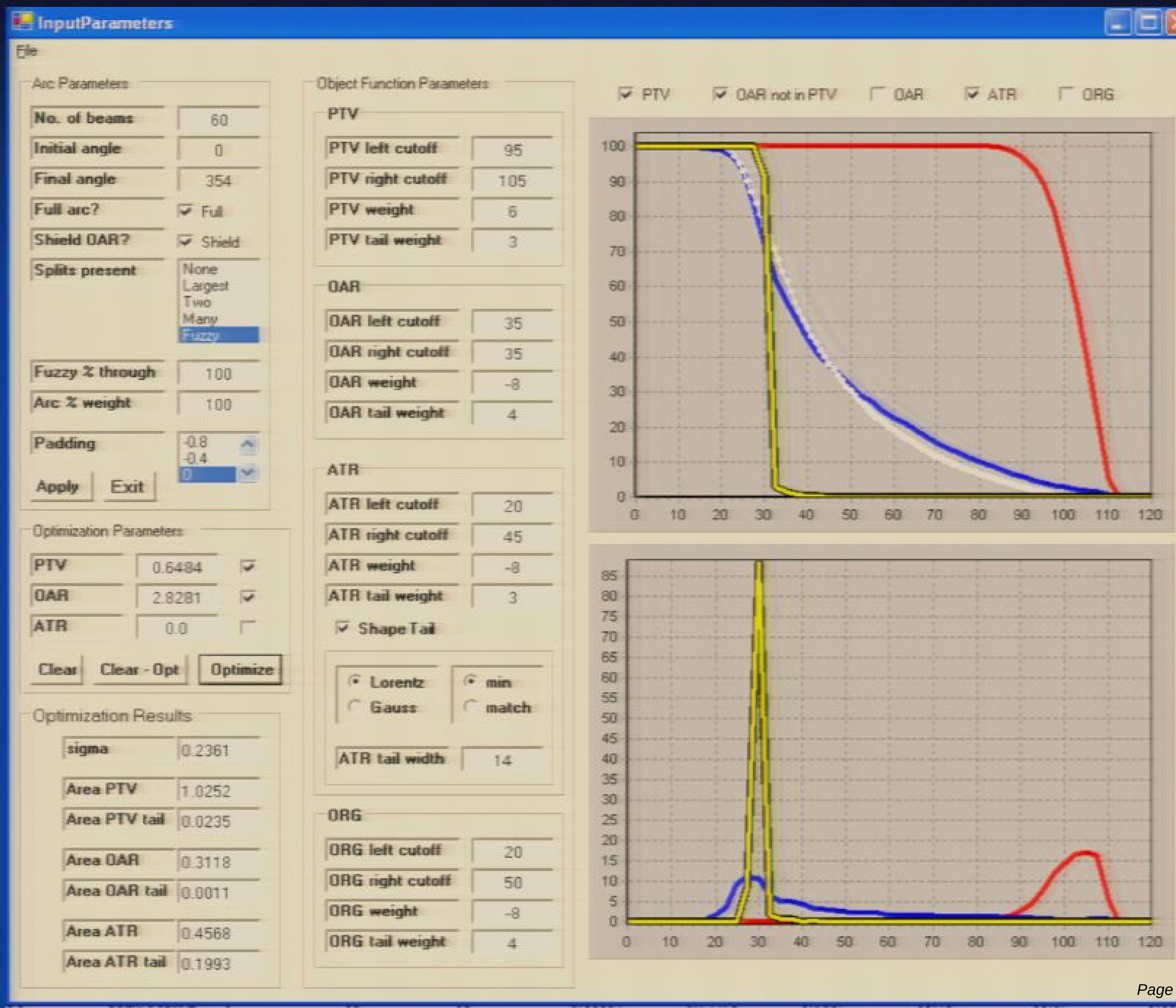
London Regional Cancer Centre

Research Associates

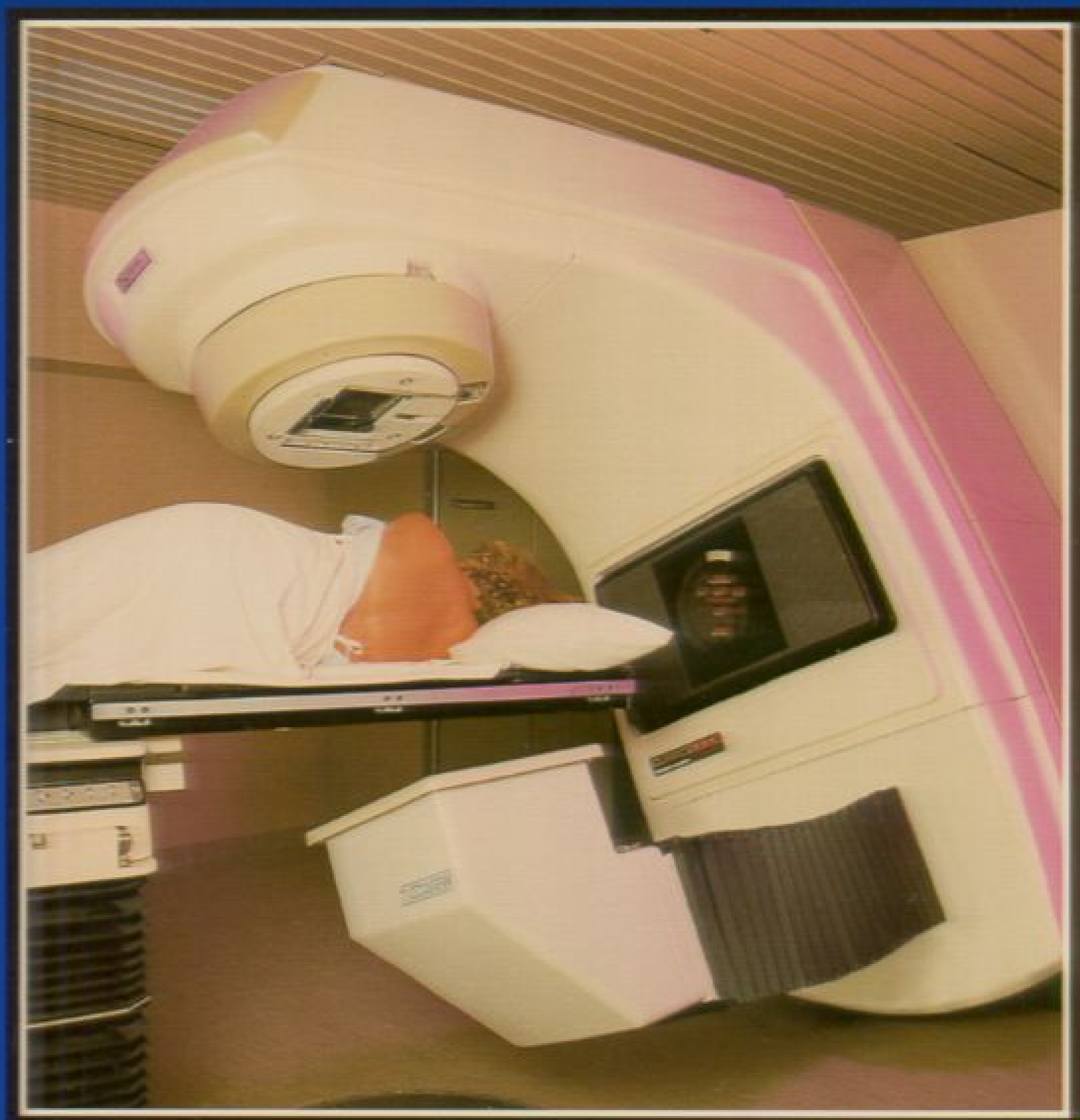
Carol Johnson
David Turnbull
Thomas Neff

University of Western Ontario
London, Ontario


Optimized Dose Deposition - DICOM-RT output data
File Charts Data
EXIT
Grid
Cell side (mm) 1.0 x 1.0
Cell Number 160 x 158
Change
Manual
Import parameters
Read parameters
Input - Optimize
Apply
re-weight
Optimization type
Beamlet Linear Optimization
Arcs
Target
Res. mm
trim OAR
Save Chart
Save DVH
Geometry
Save Geometry
Volume (%)
Dose (%)
Isodose Contour Plots (% of Average dose in PTV)
Show Contours
Dynamic Contour
Multiple Contours
Dynamic Contours
30
Multiple Contours
25 35 60 85 95 97 103
Beam Data
Energy Data
Save Beam Data
Split/SSB Beam/S1 Angle D low mm D high mm E inside Eo / Ei Weight %E inside %E outside <E> inside <E>o / <E>i
1 beam 1 split 1 0 18.12 24.38 0 0 1 17.56 82.44 0 0
2 beam 1 split 2 0 11.88 18.12 0 0 1 31.77 68.23 0 0
3 beam 1 split 3 0 5.62 11.88 0 0 1 36.25 63.75 0 0
4 beam 1 split 4 0 -0.62 5.62 0 0 1 38.73 61.27 0 0
5 beam 1 split 5 0 -6.88 -0.62 0 0 1 36.85 63.15 0 0
6 beam 1 split 6 0 -13.12 -6.88 0 0 1 33.95 66.05 0 0
7 beam 1 split 7 0 -19.38 -13.12 0 0 1 33.66 66.34 0 0

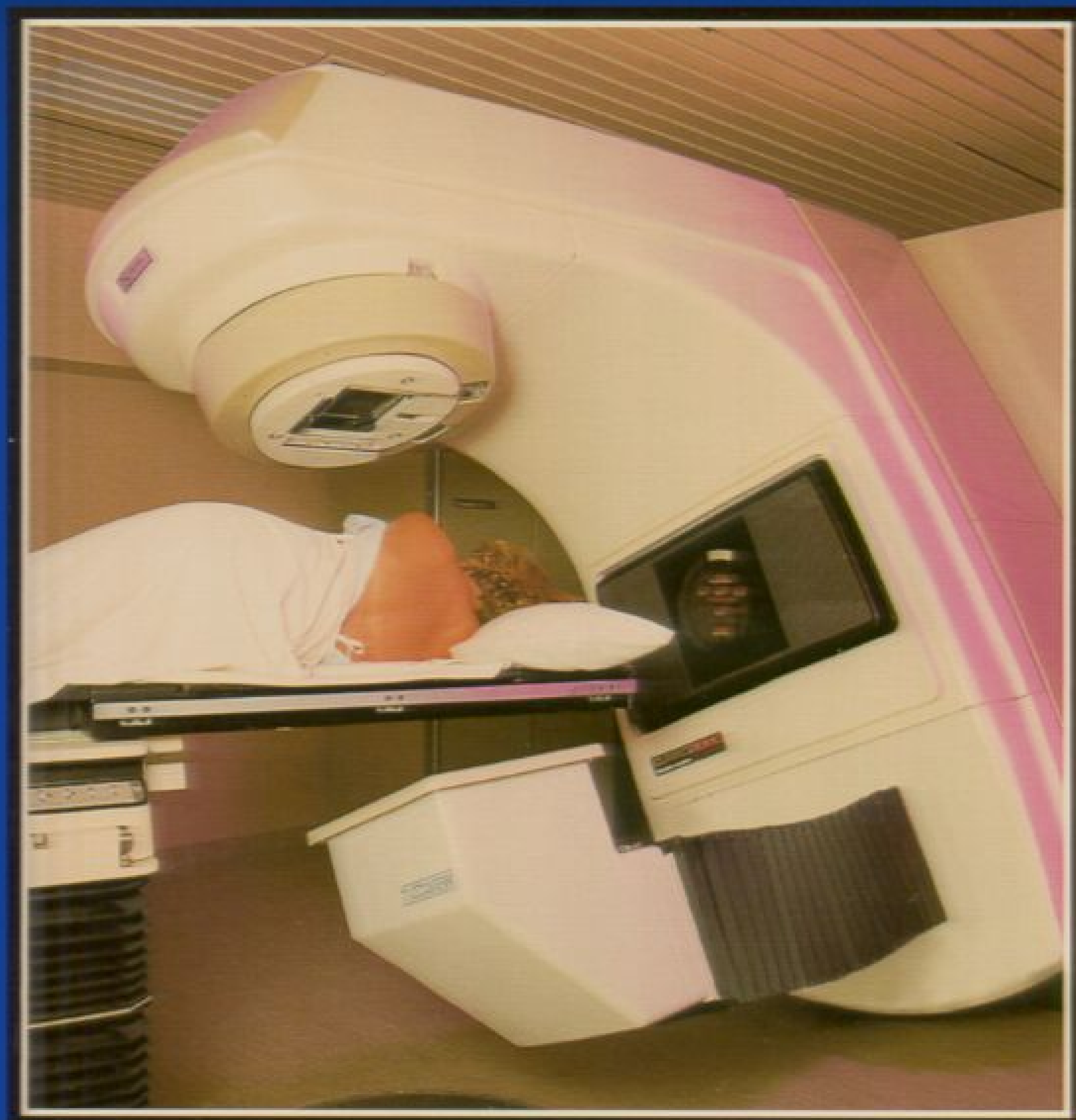

InputParameters
File
Arc Parameters
No. of beams 60
Initial angle 0
Final angle 354
Full arc? Full
Shield OAR? Shield
Splits present
None
Largest
Two
Many
Fuzzy
Fuzzy % through 100
Arc % weight 100
Padding -0.8 -0.4 0
Apply
Exit
Optimization Parameters
PTV 0.6484
OAR 2.8281
ATR 0.0
Clear
Clear - Opt
Optimize
Optimization Results
sigma 0.2361
Area PTV 1.0252
Area PTV tail 0.0235
Area OAR 0.3118
Area OAR tail 0.0011
Area ATR 0.4568
Area ATR tail 0.1993
Object Function Parameters
PTV
PTV left cutoff 95
PTV right cutoff 105
PTV weight 6
PTV tail weight 3
OAR
OAR left cutoff 35
OAR right cutoff 35
OAR weight -8
OAR tail weight 4
ATR
ATR left cutoff 20
ATR right cutoff 45
ATR weight -8
ATR tail weight 3
Shape Tail
Lorentz
Gauss
min
match
ATR tail width 14
ORG
ORG left cutoff 20
ORG right cutoff 50
ORG weight -8
ORG tail weight 4
PTV
OAR not in PTV
OAR
ATR
ORG

Radiation Delivery



Radiation Delivery

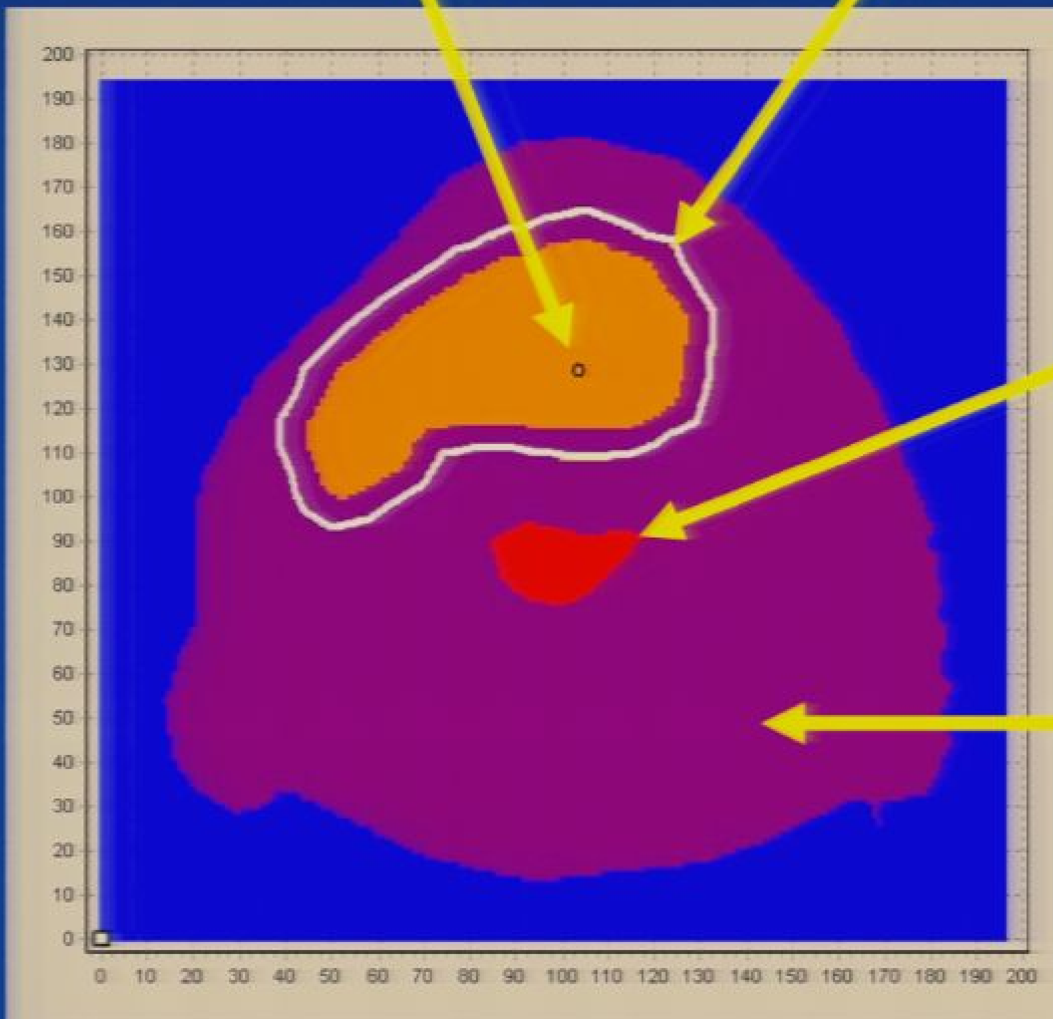
Radiation Delivery



Radiation Delivery

Isocentre

PTV:
Planned Target Volume



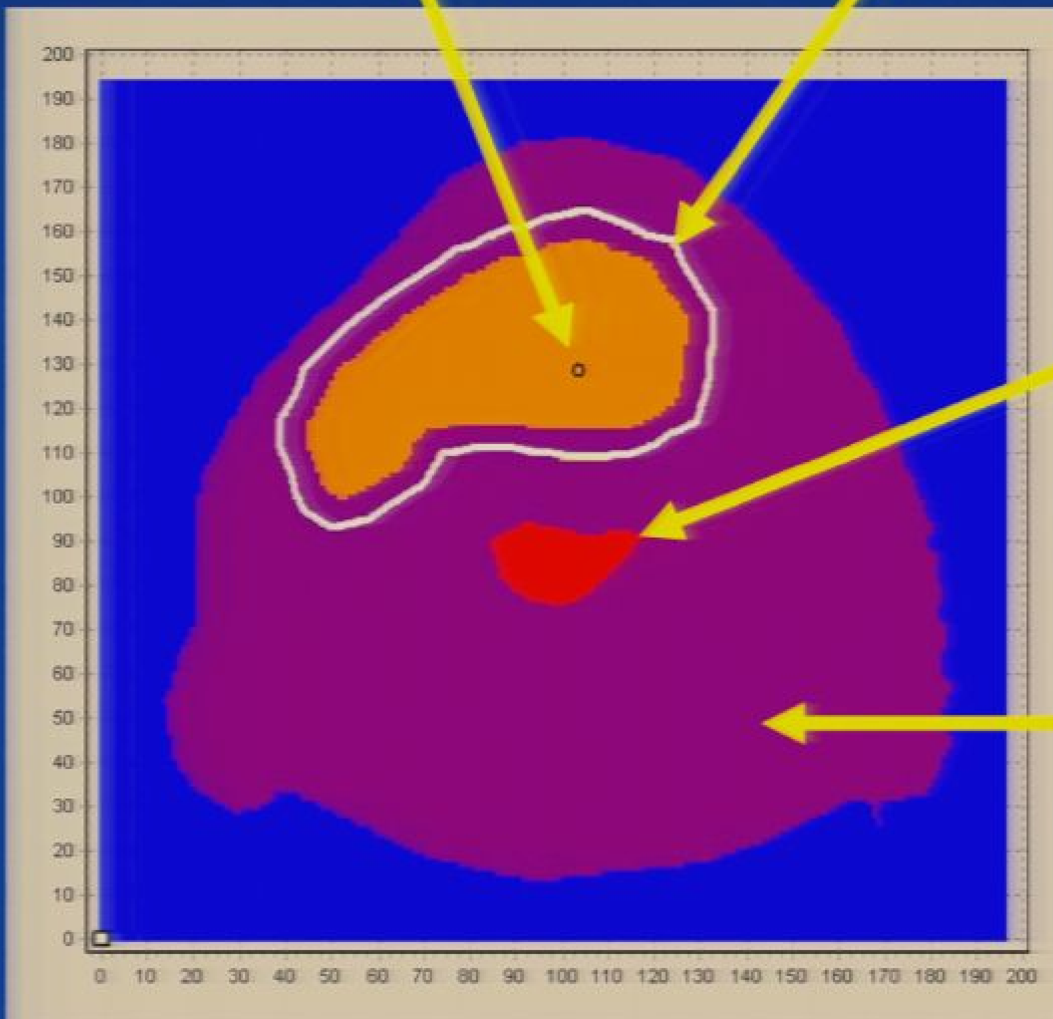
OAR:
Organ at Risk

ATR:
All the Rest

Radiation Delivery

Isocentre

PTV: Planned Target Volume



OAR: Organ at Risk

ATR: All the Rest

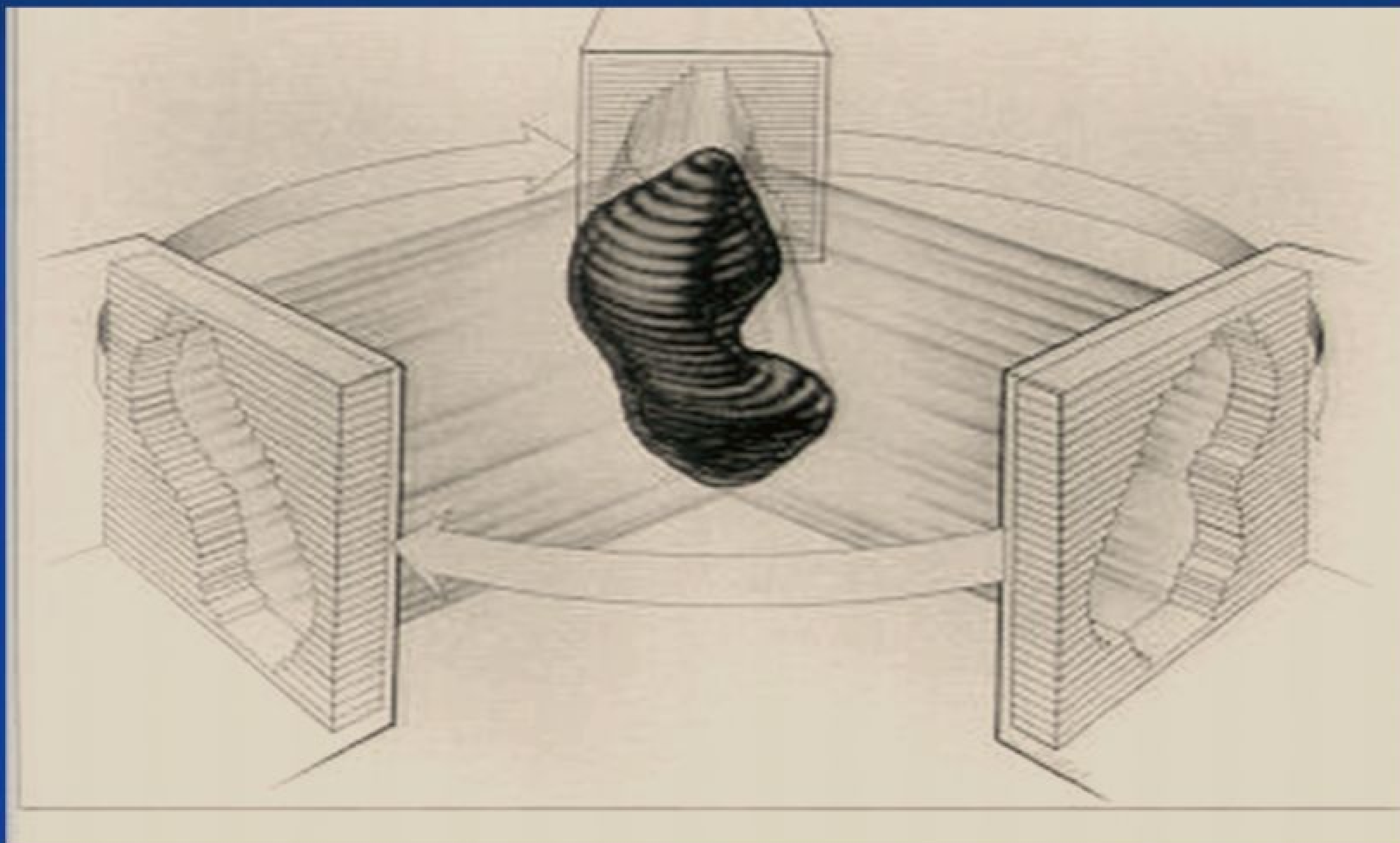
Adaptive Radiotherapy

- Radiation delivery plan is calculated once before treatment starts
- Wouldn't it be nice to be able to readapt the treatment plan each day before treatment?

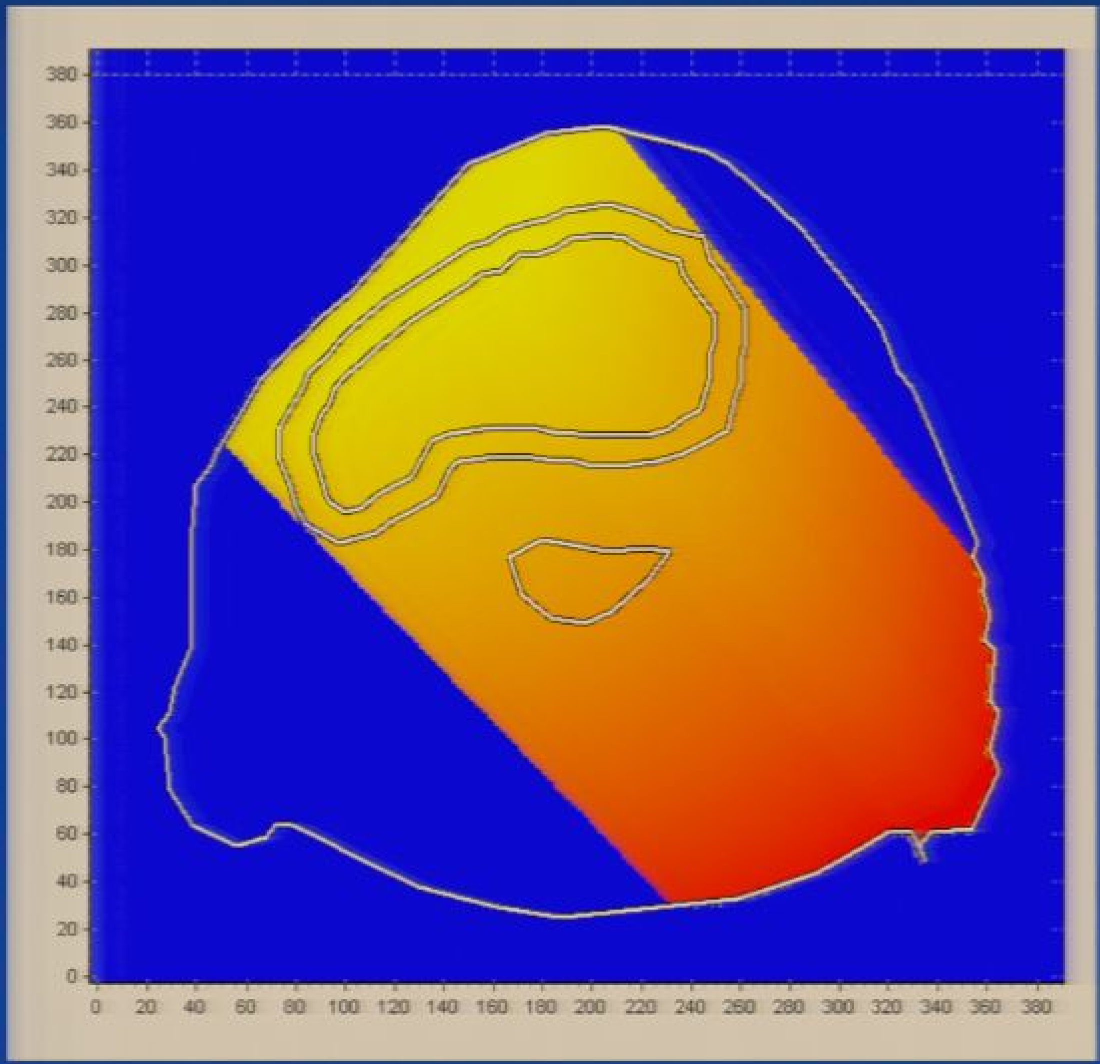
Multi-Leaf Collimator

Multileaf-Collimator

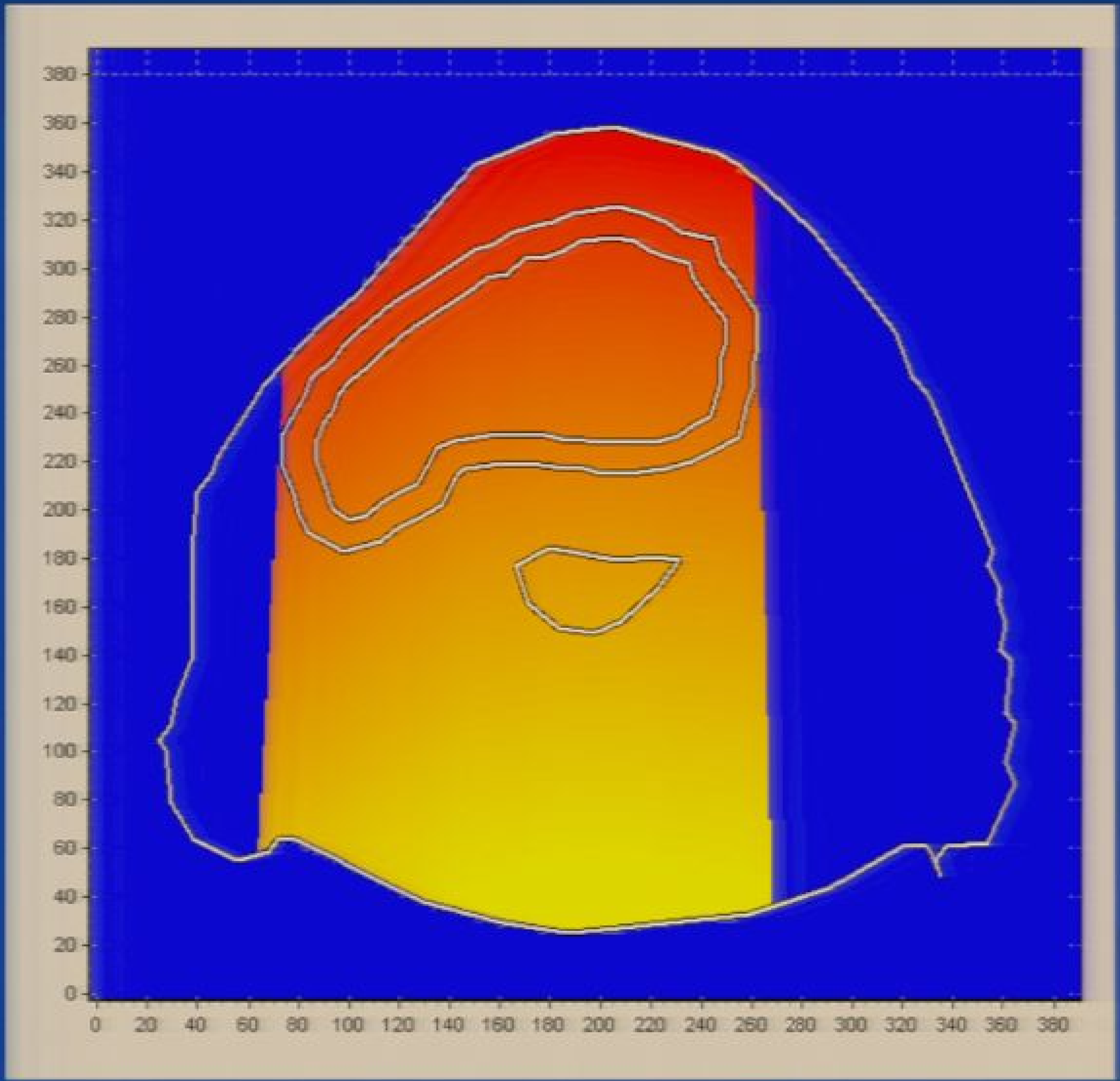
Radiation Delivery



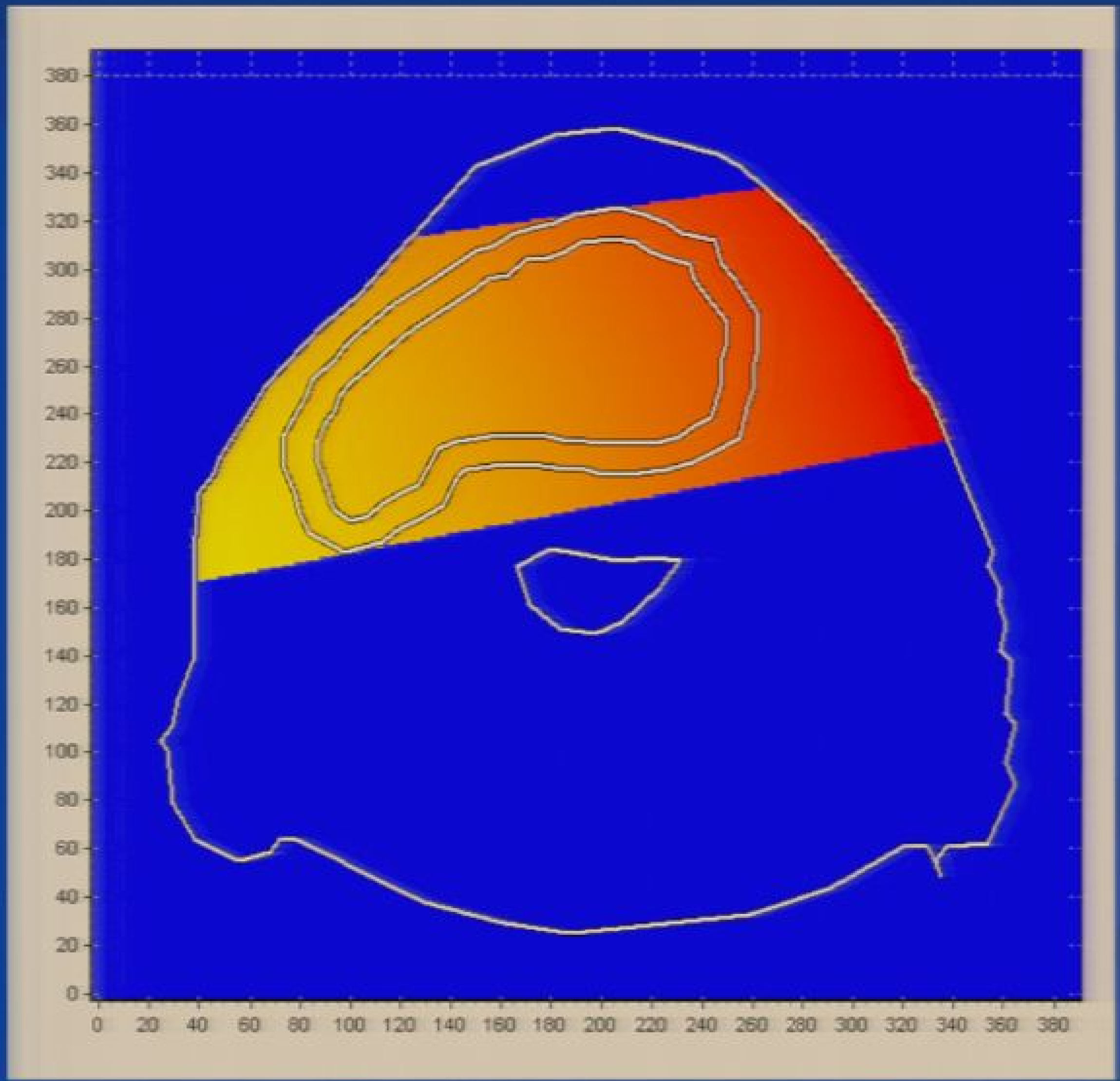
Beam: Energy Deposition



Beams: Outer Boundaries

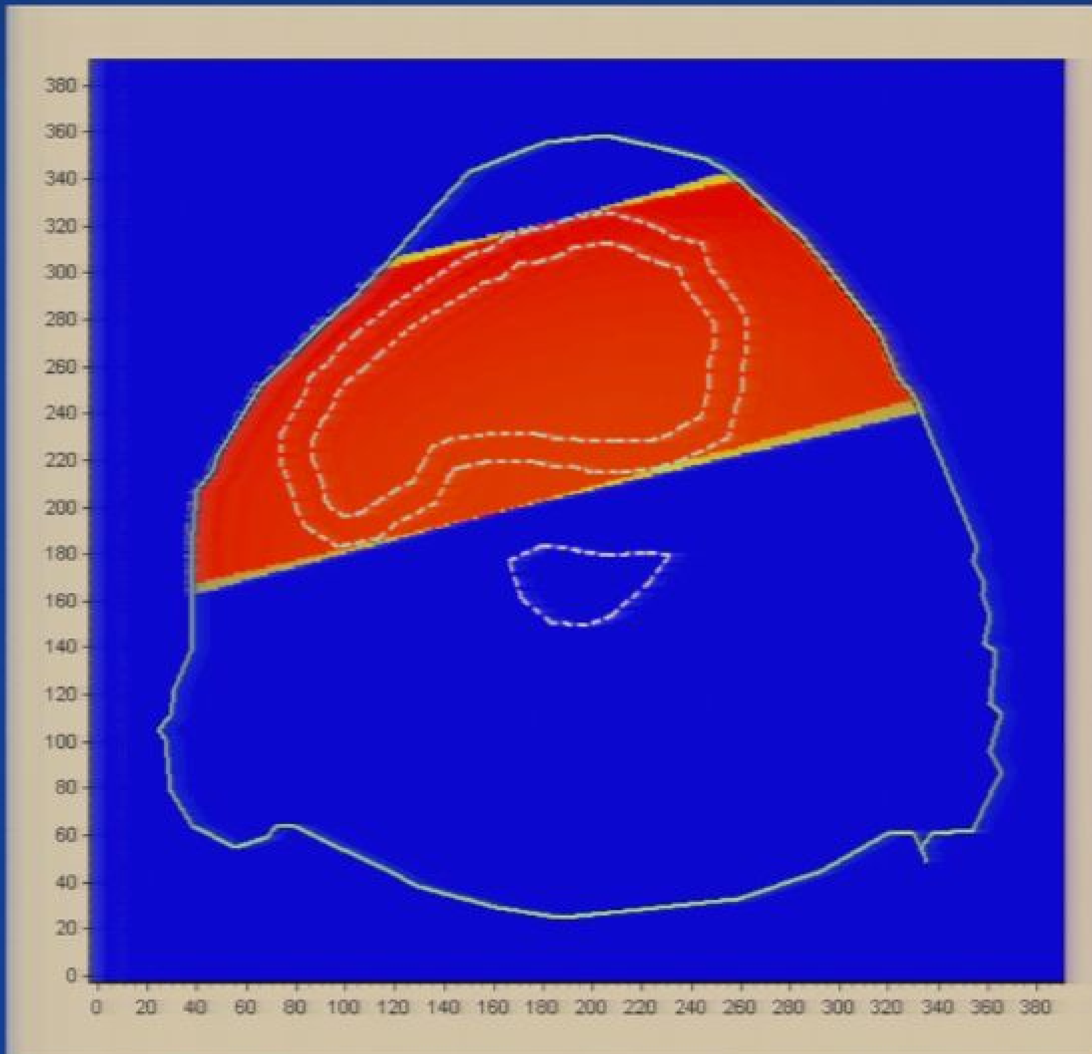


Beams: Outer Boundaries

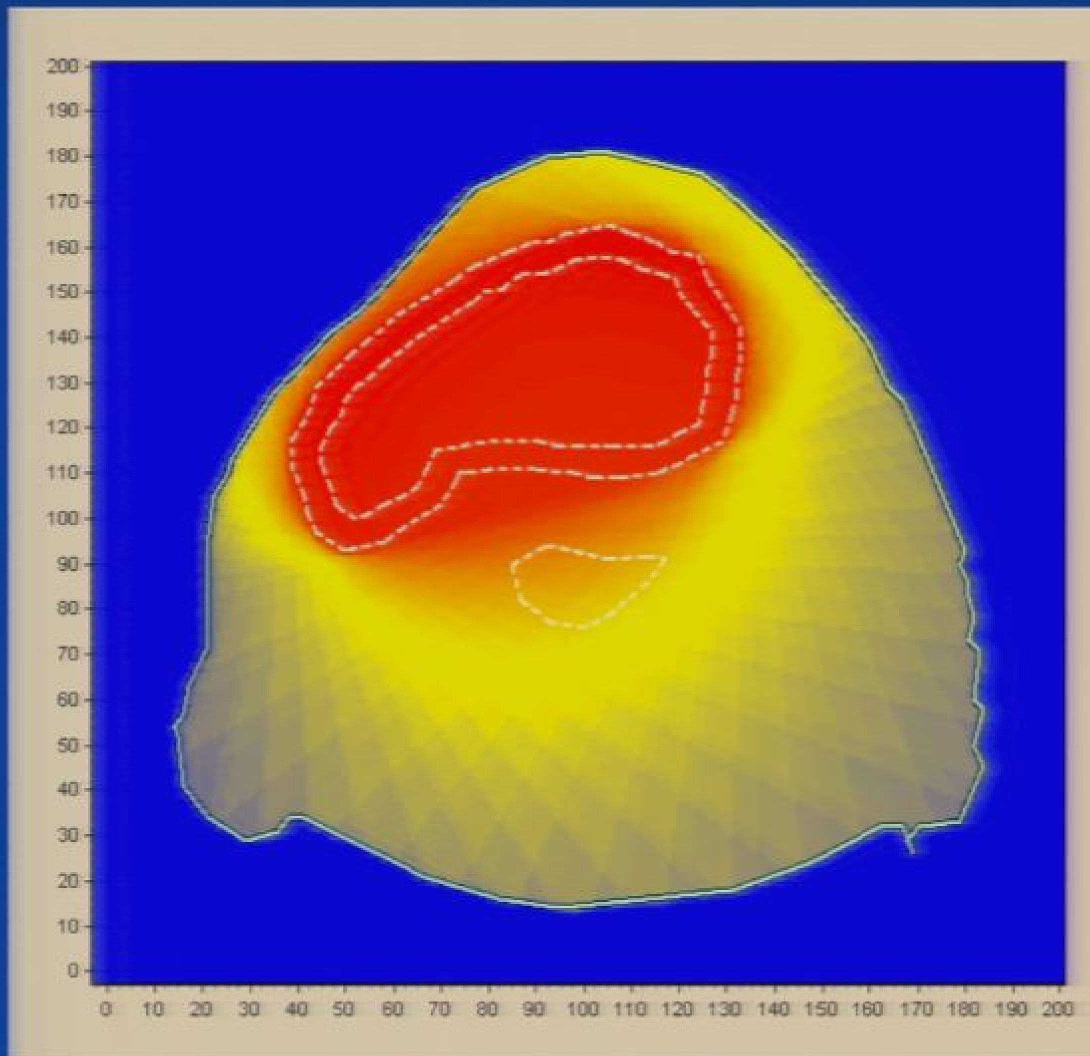


Radiation Delivery

two beams, same intensity

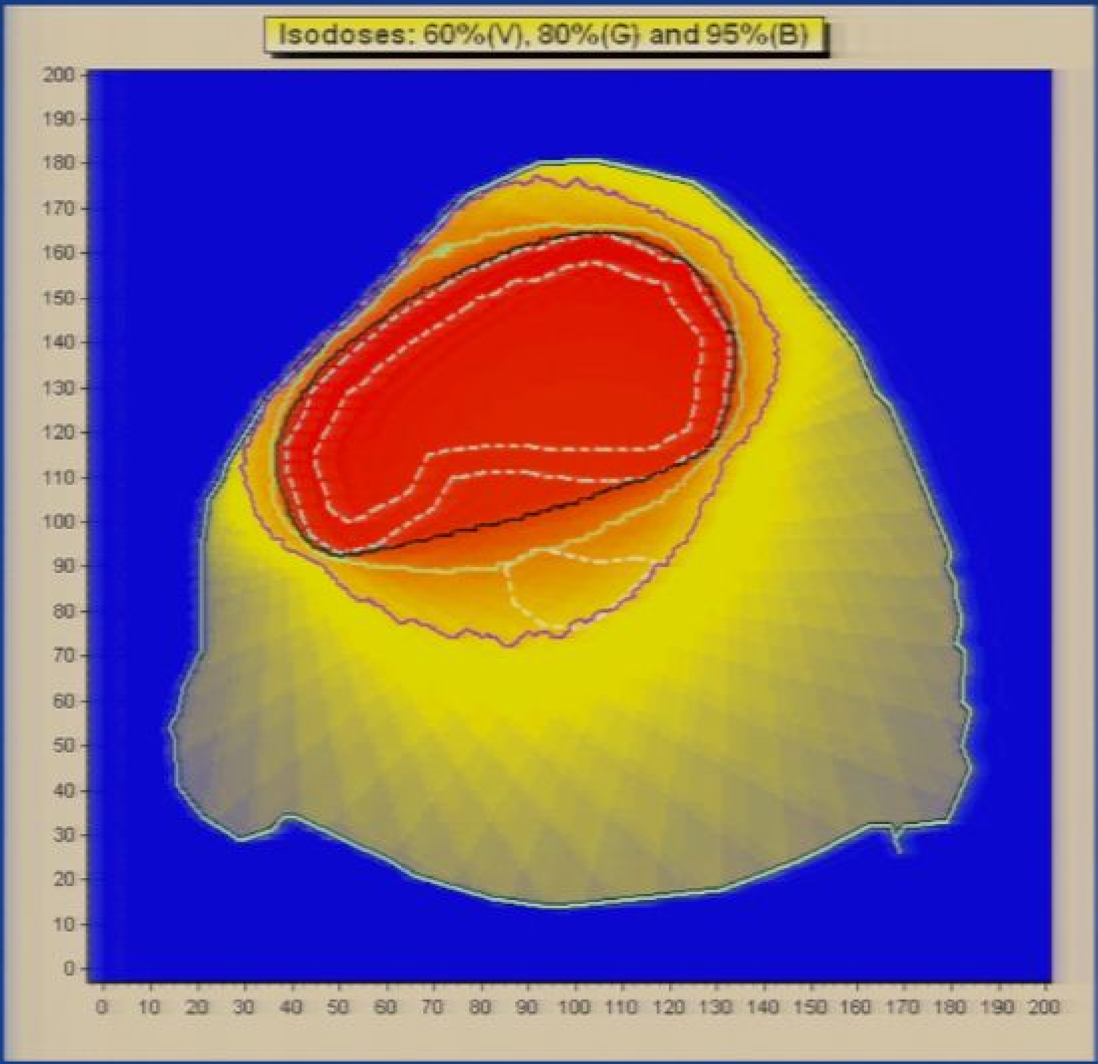


Radiation Delivery
fifty beams, same intensity

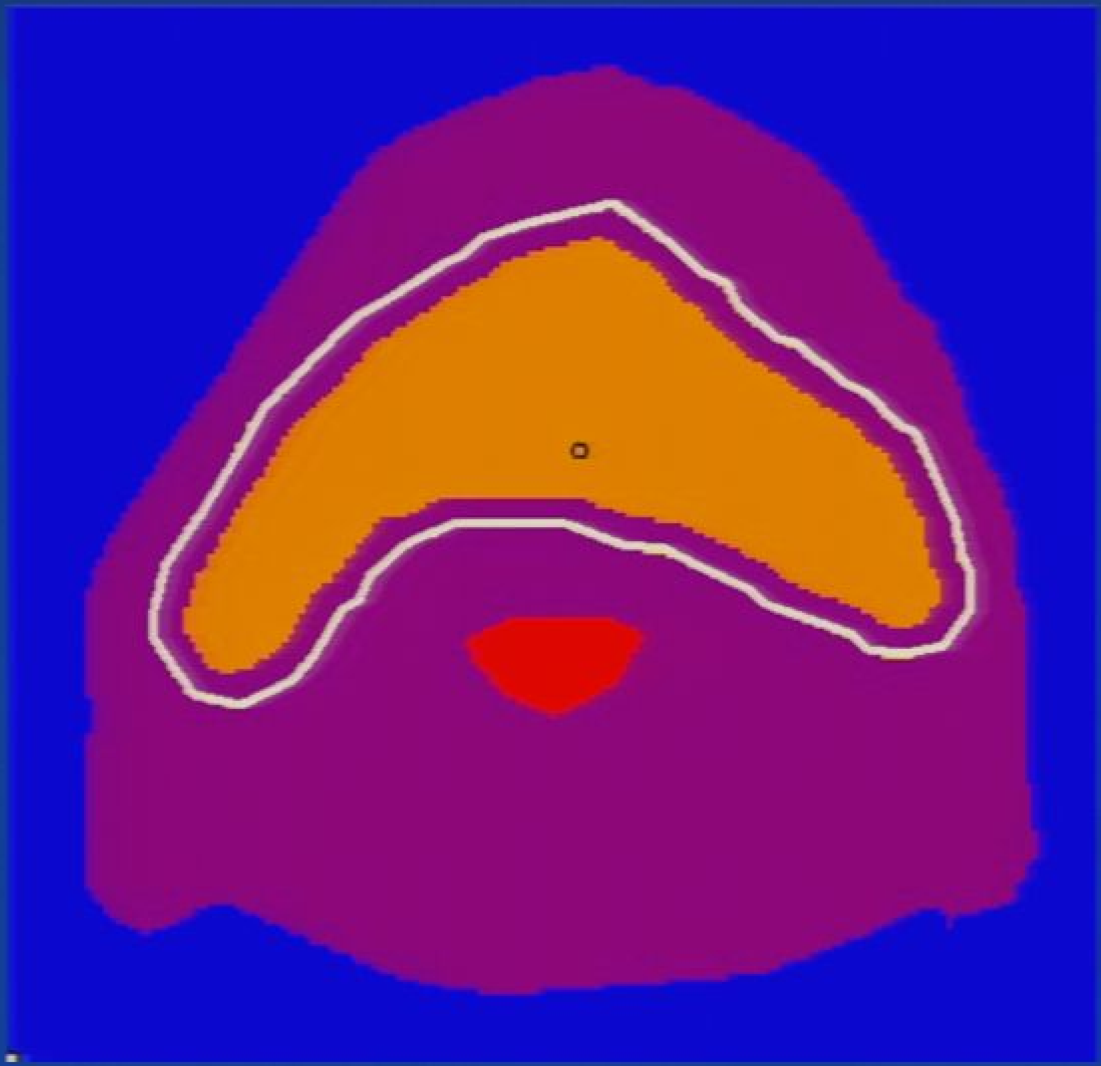


Radiation Delivery

fifty beams, same intensity

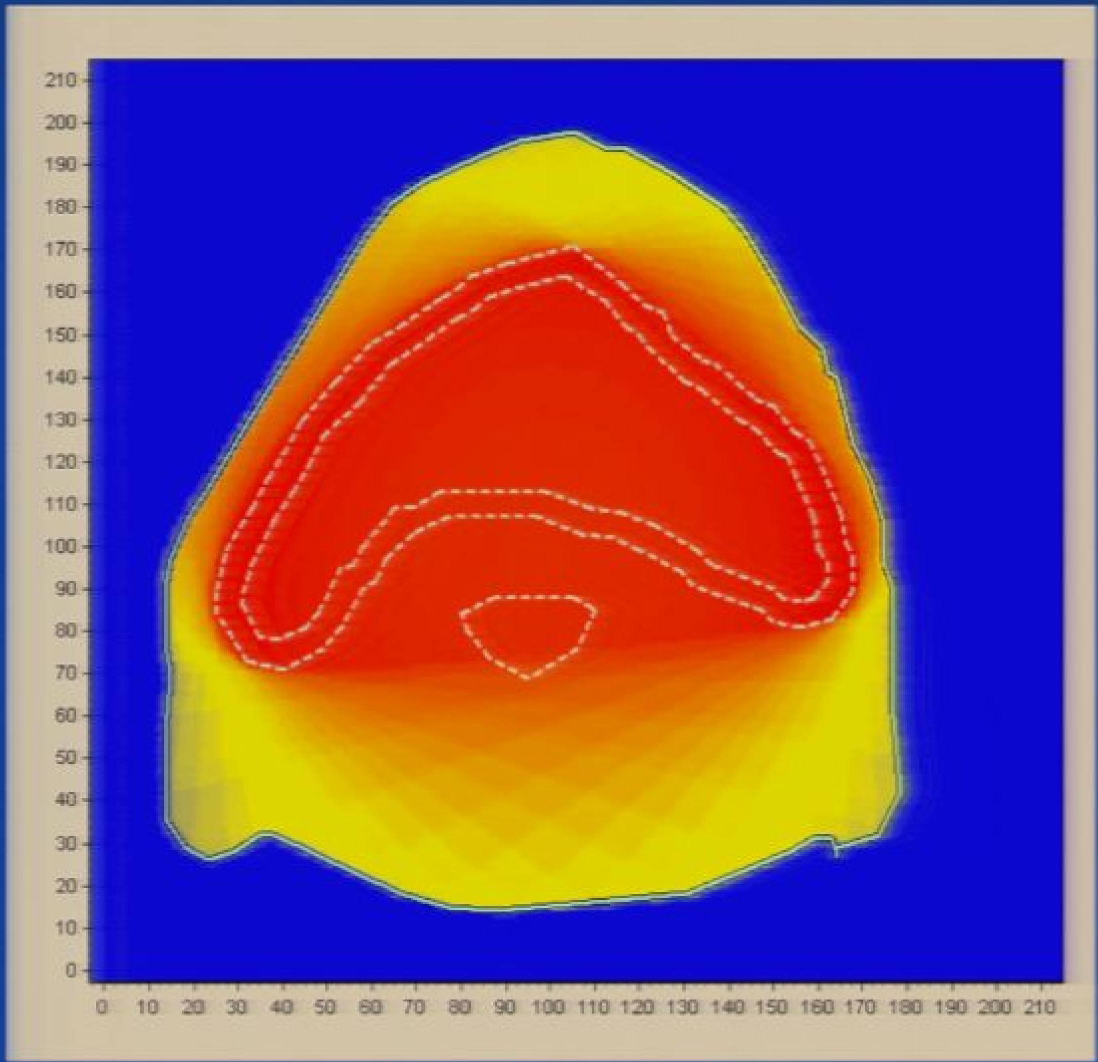


Same case
different slice



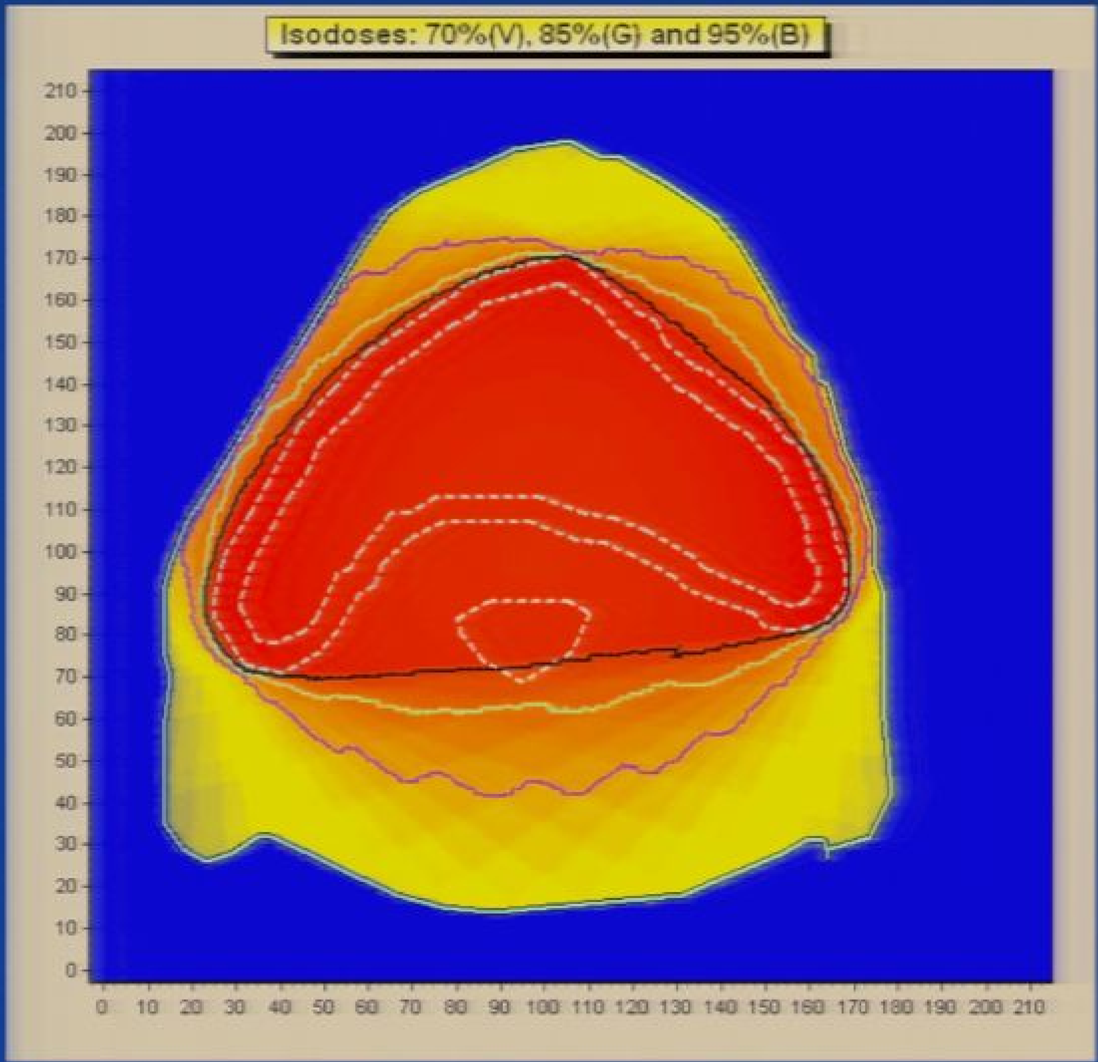
Radiation Delivery

fifty beams, same intensity

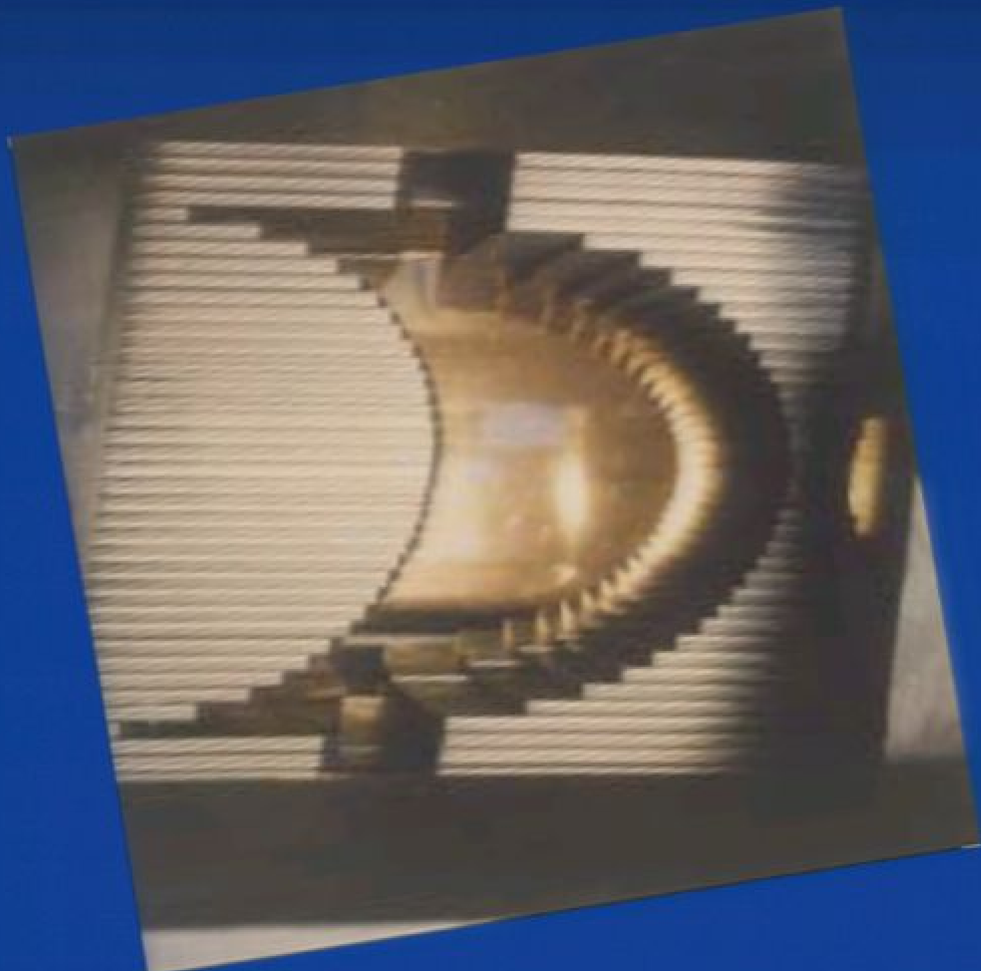


Radiation Delivery

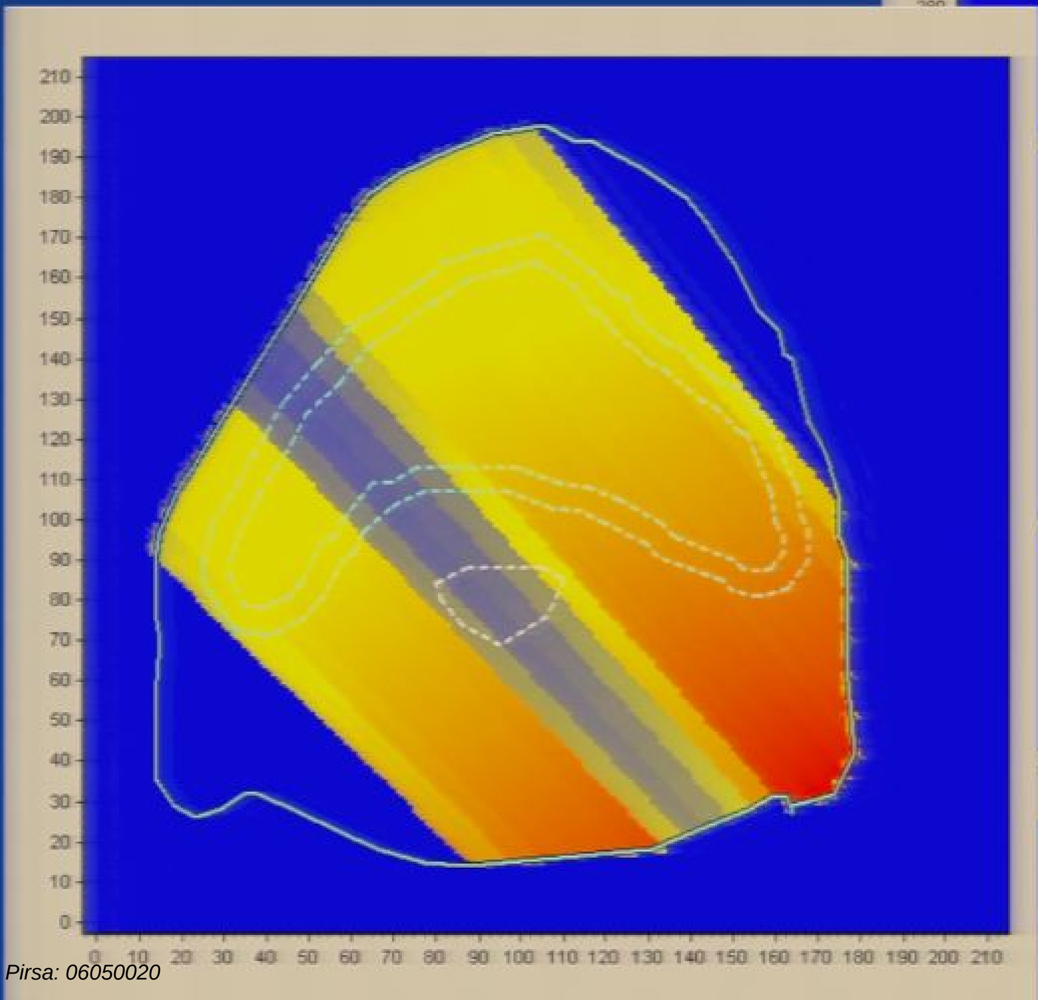
fifty beams, same intensity

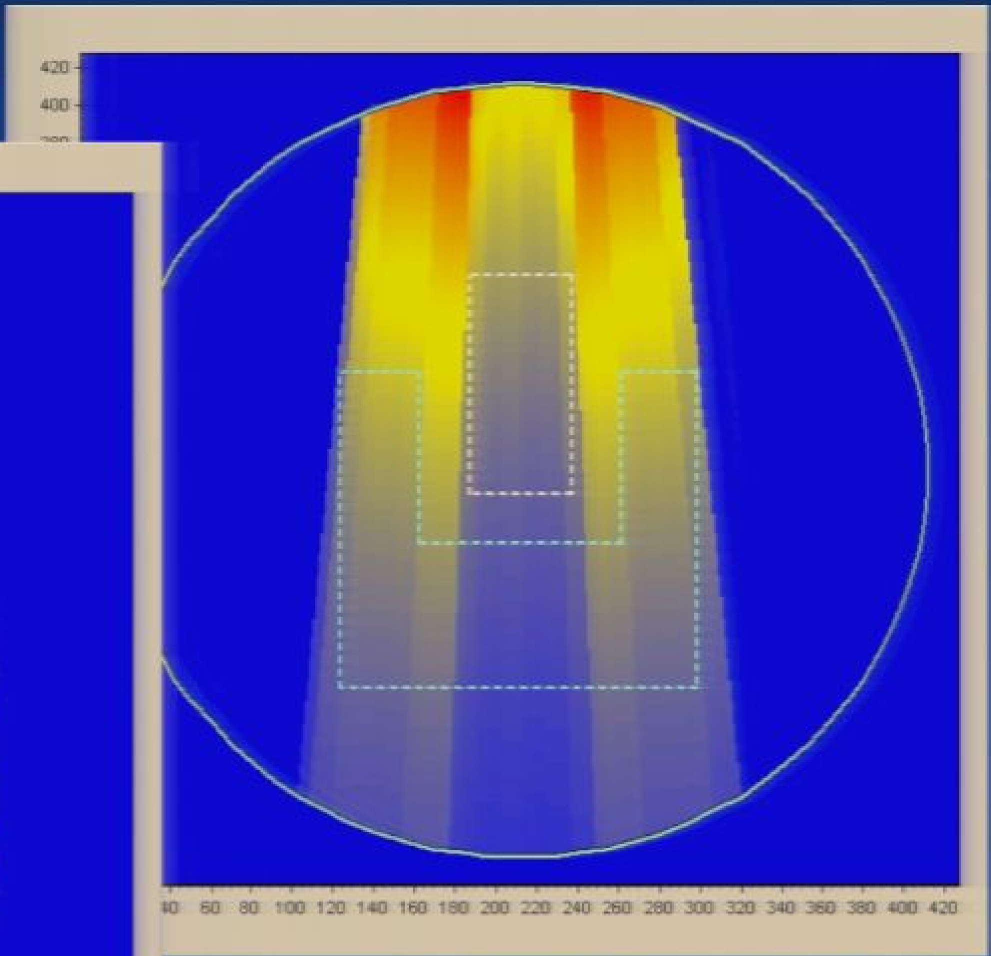


Intensity Modulation



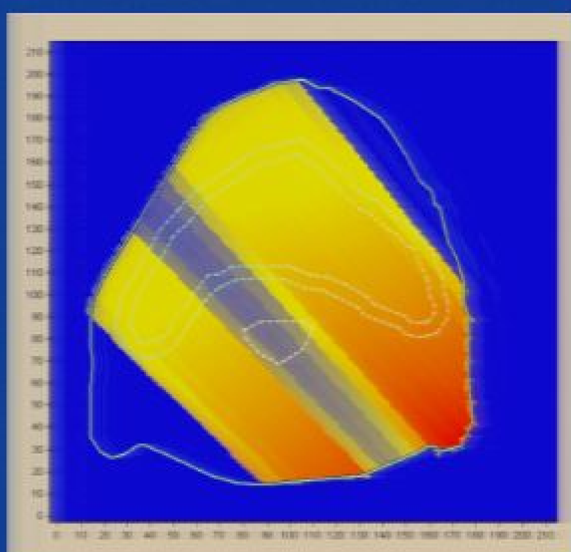
Radiation Delivery - Beamlets

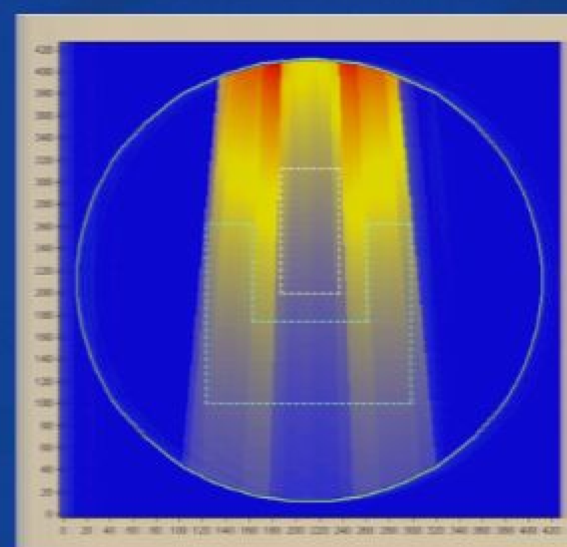




Radiation delivery *(IMRT, Tomotherapy,…)*

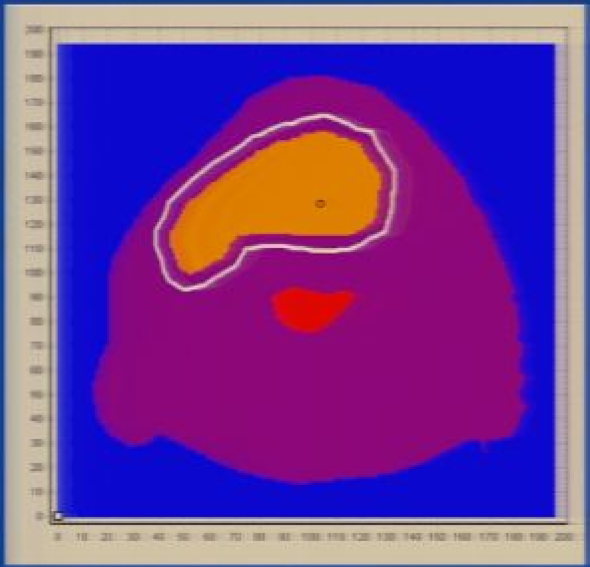
- Irradiate from several (many) gantry angles
- Beams are divided in narrow beamlets
- Each beamlet may have a different weight





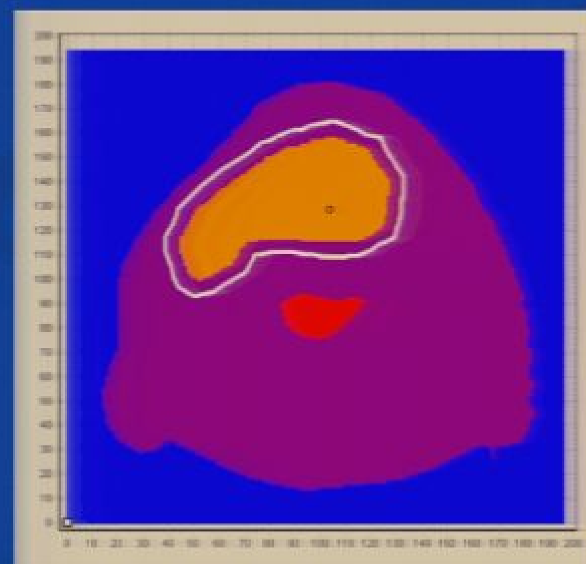
Optimal radiation treatment

- Assign to each beamlet the correct weight in order to obtain:



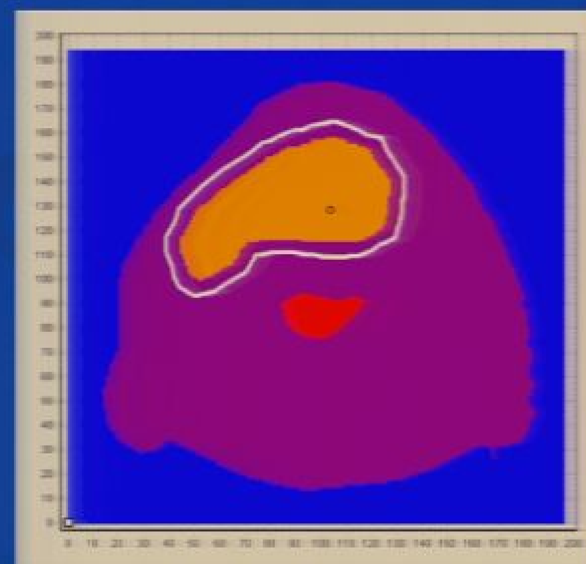
Optimal radiation treatment

- Assign to each beamlet the correct weight in order to obtain:
  - Homogeneous energy deposition inside the Planned Target Volume (PTV)
  - Low (or no) energy deposition inside the Organs at Risk (OAR)
  - Low energy deposition everywhere else inside the outside contours (ATR)



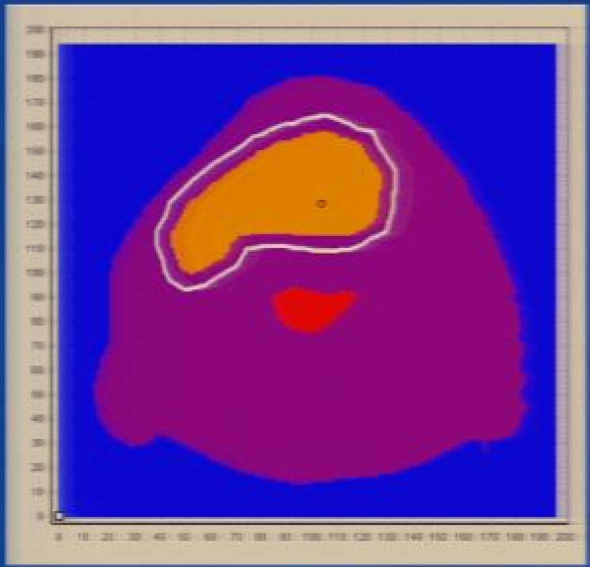
Optimal radiation treatment

- Assign to each beamlet the correct weight in order to obtain:
  - Homogeneous energy deposition inside the Planned Target Volume (PTV)
  - Low (or no) energy deposition inside the Organs at Risk (OAR)
  - Low energy deposition everywhere else inside the outside contours (ATR)



Optimal radiation treatment

- Assign to each beamlet the correct weight in order to obtain:
  - Homogeneous energy deposition inside the Planned Target Volume (PTV)
  - Low (or no) energy deposition inside the Organs at Risk (OAR)
  - Low energy deposition everywhere else inside the outside contours (ATR)
- **BUT HOW?**



Optimal radiation treatment

Inverse Optimization Problem

- We know the dose distribution we need (the final result)

Optimal radiation treatment

Inverse Optimization Problem

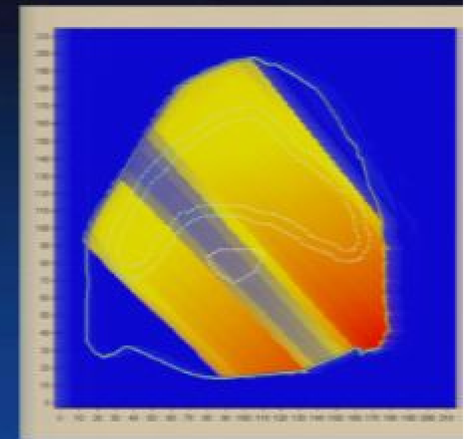
- We know the dose distribution we need (the final result)
- **We do not know what beamlet intensities will yield the desired result**

Optimal radiation treatment

Inverse Optimization Problem

- We know the dose distribution we need (the final result)
- **We do not know what beamlet intensities will yield the desired result**
  - ***i.e.*** **the weight of each beamlet for each beam at each gantry angle --- hundreds or thousands of beamlets!**

Traditional Optimizations



- **SEARCH**

  i.e. By <u>trial and error</u> find the weights of each of the thousand of beamlets such that:

  - The addition of the dose deposited by all beamlets at each point in the Planned Target Volume (PTV) will add up to the prescribed dose for the tumour.
  - The addition of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

Traditional Optimizations

- Rely on a numerical search
  - **Trial and error:** try different values for each of the hundreds (thousands) of beamlets
  - Long computation times
  - Search may 'get trapped' into sub-optimal results

Traditional Optimizations

- Rely on a numerical search
  - **Trial and error:** try different values for each of the hundreds (thousands) of beamlets
  - Long computation times
  - Search may 'get trapped' into sub-optimal results

Traditional Optimizations

- Rely on a numerical search
  - **Trial and error:** try different values for each of the hundreds (thousands) of beamlets
  - Long computation times
  - Search may 'get trapped' into sub-optimal results
- Alternative: optimize by matrix inversion
  - Very (Very) fast, single and best solution

Traditional Optimizations

- Rely on a numerical search
  - **Trial and error:** try different values for each of the hundreds (thousands) of beamlets
  - Long computation times
  - Search may 'get trapped' into sub-optimal results

- Alternative: optimize by matrix inversion
  - Very (Very) fast, single and best solution

Traditional Optimizations

- Rely on a numerical search
  - **Trial and error:** try different values for each of the hundreds (thousands) of beamlets
  - Long computation times
  - Search may 'get trapped' into sub-optimal results
- Alternative: optimize by matrix inversion
  - **Very (Very) fast, single and best solution**
  - Cannot be used (!!!) Why?

Introduction

Traditional Optimizations

- An Object Function O

$$\boxed{O = p_{PTV} \sum_{x \in PTV} \left(D(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(D(x) \right)^2}$$

- ε^{PTV}: the dose prescribed for the PTV (the target volume)
- $D(x)$: the total dose deposited at point x by all the beamlets passing through x.

$$\boxed{D(x) = \sum^{\text{all beamlets}} D_{\text{each beamlet}}(x)}$$

Introduction

Traditional Optimizations

- An Object Function O

$$O = p_{PTV} \sum_{x \in PTV} \left(D(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(D(x) \right)^2$$

  - ***ε^{PTV}:*** the dose prescribed for the PTV (the target volume)
  - ***$D(x)$:*** the total dose deposited at point x by all the beamlets passing through x.

$$D(x) = \sum^{\text{all beamlets}} D_{\text{each beamlet}}(x)$$

Introduction

Traditional Optimizations

Weights:

- Express the dose deposited by each beamlet in terms of a beamlet *weight*

$D_i(x) = w_i \, d_i(x)$

  - w_i - “weight” of beamlet i
  - $d_i(x)$ - Dose deposited at point x by beamlet i with unit weight

Introduction

Traditional Optimizations

- An Object Function O

$$O = p_{PTV} \sum_{x \in PTV} \left(D(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(D(x) \right)^2$$

  - Rewrite the total dose deposited at point x in terms of the dose deposited at x by beamlets of weight w_i:

$$D(x) = \sum_{i}^{\text{all beamlets}} w_i d_i(x)$$

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

- OPTIMIZE THE WEIGHTS!

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

- **OPTIMIZE THE WEIGHTS!**

- Minimize O with respect to all the weights w_i

$$\frac{\partial O}{\partial w_i} = 0 \qquad \text{for all } w_i$$

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$\frac{\partial O}{\partial w_i} = 0 \qquad \text{for all } w_i$$

$$0 = p_{PTV} \sum_{x \in PTV} \left(\sum_{j}^{\text{all beamlets}} w_j d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \sum_{x \in PTV} d_i(x) + p_{OAR} \sum_{x \in OAR} \left(\sum_{j}^{\text{all beamlets}} w_j d_i(x) d_j(x) \right)$$

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$\frac{\partial O}{\partial w_i} = 0 \qquad \text{for all } w_i$$

$$0 = p_{PTV} \sum_{x \in PTV} \left(\sum_{j}^{\text{all beamlets}} w_j d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \sum_{x \in PTV} d_i(x) + p_{OAR} \sum_{x \in OAR} \left(\sum_{j}^{\text{all beamlets}} w_j d_i(x) d_j(x) \right)$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right)$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right)$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right.$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \, \alpha_{ij}^{PTV} - p_{PTV} \varepsilon^{PTV} \beta_i^{PTV} + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \, \alpha_{ij}^{OAR}$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right.$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \, \alpha_{ij}^{PTV} - p_{PTV} \varepsilon^{PTV} \, \beta_i^{PTV} + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \, \alpha_{ij}^{OAR}$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right)$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{PTV} - p_{PTV} \varepsilon^{PTV} \beta_i^{PTV} + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{OAR}$$

$$\alpha_{ij} = p_{PTV} \alpha_{ij}^{PTV} + p_{OAR} \alpha_{ij}^{OAR} \qquad \beta_i = p_{PTV} \beta_i^{PTV}$$

$$\boxed{\sum_j \alpha_{ij} w_j = \beta_i}$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right)$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{PTV} - p_{PTV} \varepsilon^{PTV} \beta_i^{PTV} + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{OAR}$$

$$\alpha_{ij} = p_{PTV} \alpha_{ij}^{PTV} + p_{OAR} \alpha_{ij}^{OAR} \qquad \beta_i = p_{PTV} \beta_i^{PTV}$$

$$\sum_j \alpha_{ij} w_j = \beta_i \qquad w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

Introduction

Traditional Optimizations

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in PTV} d_i(x) d_j(x) \right) - p_{PTV} \varepsilon^{PTV} \left(\sum_{x \in PTV} d_i(x) \right) + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \left(\sum_{x \in OAR} d_i(x) d_j(x) \right)$$

$$0 = p_{PTV} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{PTV} \cdots + p_{OAR} \sum_{j}^{\text{all beamlets}} w_j \alpha_{ij}^{OAR}$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

$$\alpha_{ij} = p_{PTV} \alpha_{ij}^{PTV} + p_{OAR} \alpha_{ij}^{OAR}$$

$$\beta_i = p_{PTV} \beta_i^{PTV}$$

$$\sum_j \alpha_{ij} w_j = \beta_i$$

Matrix Inversion Optimization

- Call $\mathbf{w_j}$ the weight of the beamlet with 'ID number' j
- The optimal set of weights can be found by solving a set of linear algebraic equations:

$$2w_1 + 3w_2 = 23$$
$$4w_1 - 2w_2 = 6$$

Matrix Inversion Optimization

- Call $\mathbf{w_j}$ the weight of the beamlet with 'ID number' j
- The optimal set of weights can be found by solving a set of linear algebraic equations:

$$
\begin{array}{l}
\alpha_{11}w_1 + \alpha_{12}w_2 + \alpha_{13}w_3 + \alpha_{14}w_4 + \ldots = \beta_1 \\
\alpha_{21}w_1 + \alpha_{22}w_2 + \alpha_{23}w_3 + \alpha_{24}w_4 + \ldots = \beta_2 \\
\alpha_{31}w_1 + \alpha_{32}w_2 + \alpha_{33}w_3 + \alpha_{34}w_4 + \ldots = \beta_3 \\
\quad\vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots
\end{array}
$$

- The weights $\mathbf{w_j}$ are unknown
- The other coefficients are known

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

DISASTER!!!

- A number of weights w_i come out **negative**

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

- Constrain all the weights w_i to be positive

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

- Constrain all the weights w_i to be positive

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i > 0$$

$$w_i = \sum_j \alpha_{ij}^{-1} \beta_j$$

- Constrain all the weights w_i to be positive
- **Optimal w_i must be found through a systematic numerical search.**

Introduction

Traditional Optimizations

$$O = p_{PTV} \sum_{x \in PTV} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) - \varepsilon^{PTV} \right)^2 + p_{OAR} \sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} w_i d_i(x) \right)^2$$

$$w_i > 0$$

- Constrain all the weights w_i to be positive
- **Optimal w_i must be found through a systematic numerical search.**

Inverse Planning Optimization

Medical Science Series

THE PHYSICS OF THREE-DIMENSIONAL RADIATION THERAPY

Conformal Radiotherapy, Radiosurgery and Treatment Planning

Steve Webb

Joint Department of Physics,
Institute of Cancer Research and
Royal Marsden Hospital, Sutton, Surrey, UK

Institute of Physics Publishing
Bristol and Philadelphia

Inverse Planning Optimization

CONTENTS

v

Inverse Planning Optimization

Inverse Planning Optimization

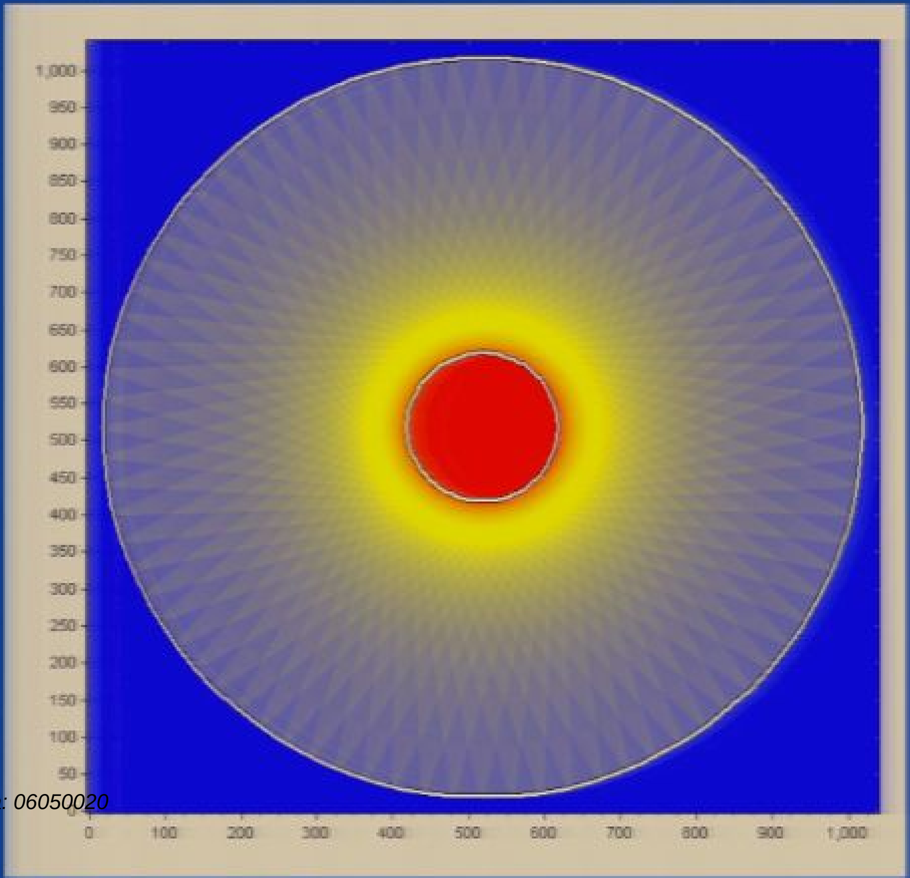
"… All so well and good except for the problem of needing negative beam-weights… …As we have seen, it is apparent that, without unphysical negative intensities, perfect dose distributions can never be achieved…"

"…It is these limitations which iterative, and admittedly computationally expensive, dose-planning algorithms can remove… …Methods of solving the inverse problem which constrain the beam intensities to be positive avoid this difficulty altogether by seeking practical solutions…"

S. Webb (1993)

Is speed the only issue?

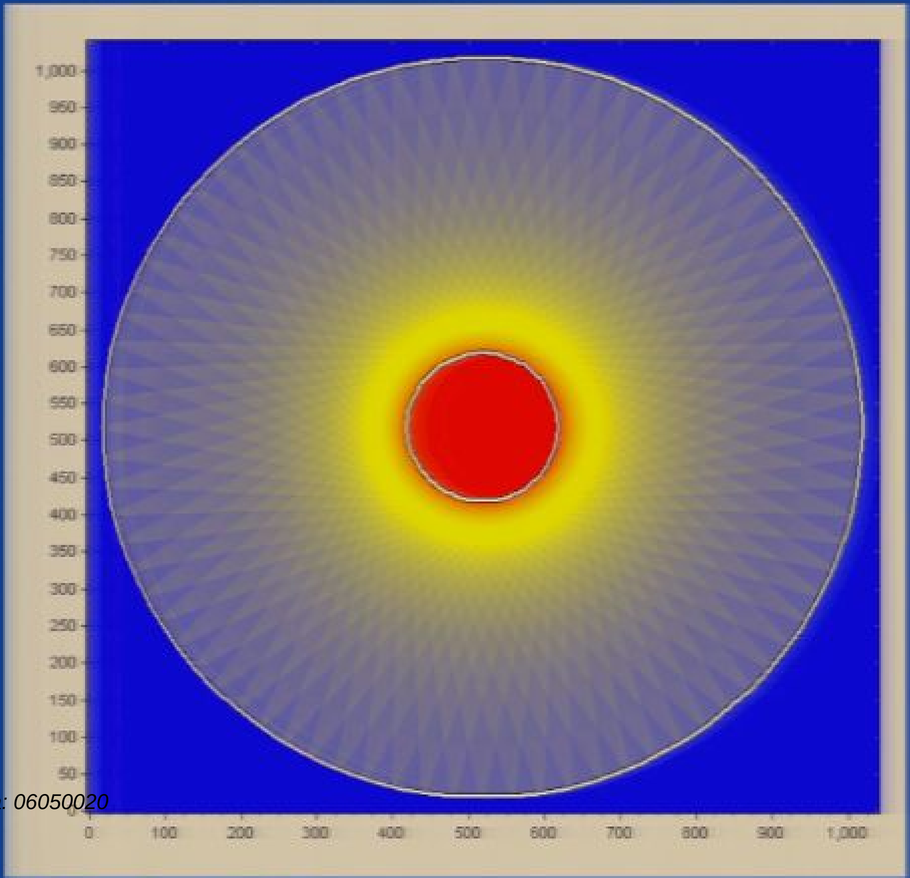
- Consider a system symmetric under rotations:
  Circular PTV and outside contour, centered at the isocentre



$r_{\text{PTV}} = 4$ cm
Grid cell-length = 0.4 mm
Number of beams = 80

Is speed the only issue?

- Consider a system symmetric under rotations:
  Circular PTV and outside contour, centered at the isocentre

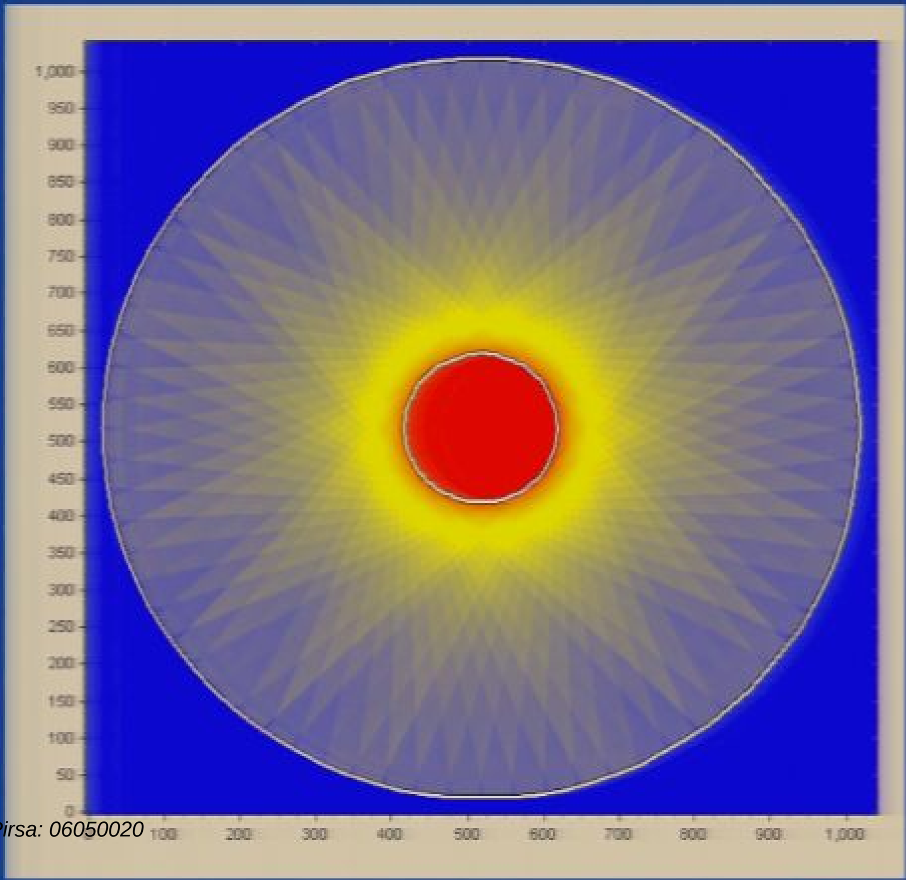


- All beam weights must be the same

r_{PTV} = 4 cm
Grid cell-length = 0.4 mm
Number of beams = 80

Is speed the only issue?

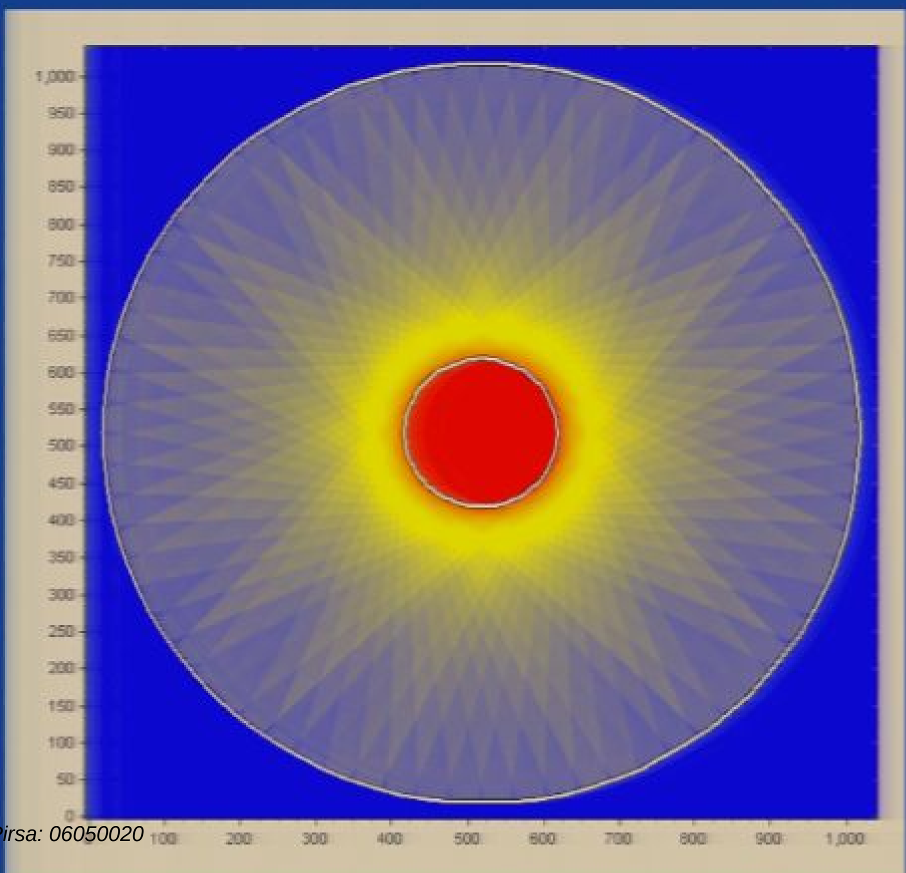
- The symmetry is lost!



$r_{\mathrm{PTV}} = 4$ cm
Grid cell-length = 0.4 mm
Number of beams = 80
Number of negative weights: 8

Is speed the only issue?

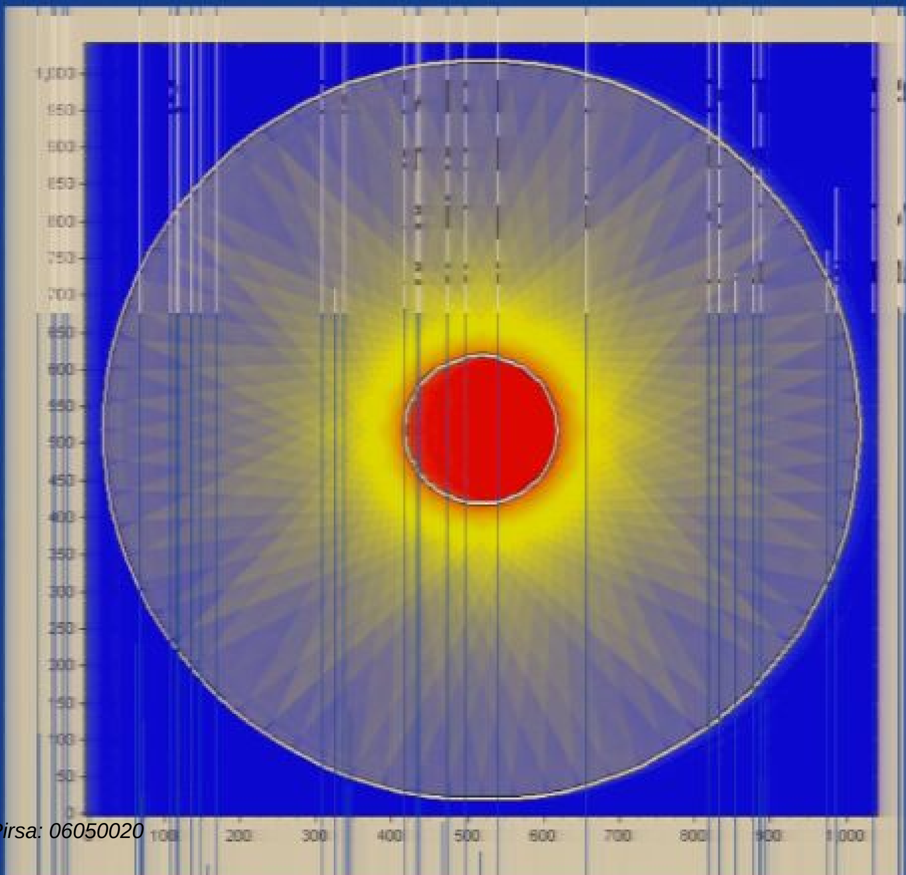
- The symmetry is lost!



$r_{PTV} = 4$ cm
Grid cell-length = 0.4 mm
Number of beams = 80
Number of negative weights: 8

Is speed the only issue?

- The symmetry is lost!



- All beam weights should be the same!

$r_{PTV} = 4$ cm
Grid cell-length = 0.4 mm
Number of beams = 80
Number of negative weights: 8

Inverse Planning Optimization

Inverse Planning Optimization

???

R

Inverse Planning Optimization

Inverse Planning Optimization

The impossible
IS possible

Introducing…

Introducing…

FIDO
Fast Inverse Dose Optimization

- An exact solution of a system of linear algebraic equations with positive weights
- No numerical search!!!

Introducing…

FIDO
Fast Inverse Dose Optimization

- An exact solution of a system of linear algebraic equations with positive weights
- No numerical search!!!

FIDO – The Method

Avoiding negative weights in regions in which dose deposition is undesirable:

FIDO – The Method

Avoiding negative weights in regions in which dose deposition is undesirable:

FIDO – The Method

Avoiding negative weights in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding negative weights in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

- The optimization yields some beamlets that are positive, some that are negative (!!!) and the sum cancels to zero.

e.g. $a + b = 0 \;\rightarrow\; b = -a$

FIDO – The Method

Avoiding negative weights in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.
- The optimization yields some beamlets that are positive, some that are negative (!!!) and the sum cancels to zero.

e.g. $a+b=0 \rightarrow b=-a$

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by <u>**each**</u> beamlet
  at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by **each** beamlet
  at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by **each** beamlet
  at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by **<u>each</u>** beamlet at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

Presently

- The **addition** of the dose deposited by all beamlets at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by <u>each</u> beamlet
  at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

FIDO – The Method

Avoiding interference effects in regions in which dose deposition is undesirable:

New in FIDO:

- The dose deposited by <u>each</u> beamlet
  at each point in the organs at risk (OAR) will be as small as possible or, ideally, zero.

- The only way to have beamlets whose sum cancels to zero is if each beamlet has zero intensity.

e.g. $a^2 + b^2 = 0 \rightarrow b = a = 0$

FIDO – The Method

First Change in the Objective function

$$\sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} D_i(x) \right)^2 \to 0 \quad \text{with} \quad D_i(x) \geq 0 \quad \text{for all beamlets}$$

FIDO – The Method

First Change in the Objective function

- If $$\sum_{x\in OAR}\left(\sum_{i}^{\text{all beamlets}} D_i(x)\right)^2 \rightarrow 0 \quad \text{with} \quad D_i(x)\geq 0 \quad \text{for all beamlets}$$
- then $$\sum_{x\in OAR}\sum_{i}^{\text{all beamlets}} D_i^2(x) \rightarrow 0 \quad \text{with} \quad D_i(x)\geq 0 \quad \text{for all beamlets}$$

FIDO – The Method

First Change in the Objective function

- If $$\sum_{x\in OAR}\left(\sum_{i}^{\text{all beamlets}} D_i(x)\right)^2 \to 0 \quad \text{with} \quad D_i(x)\geq 0 \quad \text{for all beamlets}$$
- then $$\sum_{x\in OAR}\sum_{i}^{\text{all beamlets}} D_i^2(x) \to 0 \quad \text{with} \quad D_i(x)\geq 0 \quad \text{for all beamlets}$$

FIDO – The Method

First Change in the Objective function

- If $\sum_{x \in OAR} \left(\sum_{i}^{\text{all beamlets}} D_i(x) \right)^2 \to 0$ with $D_i(x) \geq 0$ for all beamlets
- then $\sum_{x \in OAR} \sum_{i}^{\text{all beamlets}} D_i^2(x) \to 0$ with $D_i(x) \geq 0$ for all beamlets

- We eliminate the *ad-hoc* constraint $(w_i > 0)$ on *physical* grounds:
  - No interference between beamlets
  - Minimize the dose deposited by **each** beamlet!

FIDO – Summary

- Beams with negative intensities can be avoided

- Fast optimization
  - **Matrix inversion** instead of a numerical search

FIDO – Summary

- Beams with negative intensities can be avoided

$$w = \alpha^{-1} \times \beta$$

- **Fast optimization**
  - **Matrix inversion** instead of a numerical search
- Single best solution:
  Absolute minimum

FIDO – Summary

- Beams with negative intensities can be avoided

$$\boxed{w = \alpha^{-1} \times \beta}$$

- **Fast optimization**
  - **Matrix inversion** instead of a numerical search
- Single best solution:
  Absolute minimum

FIDO – Summary

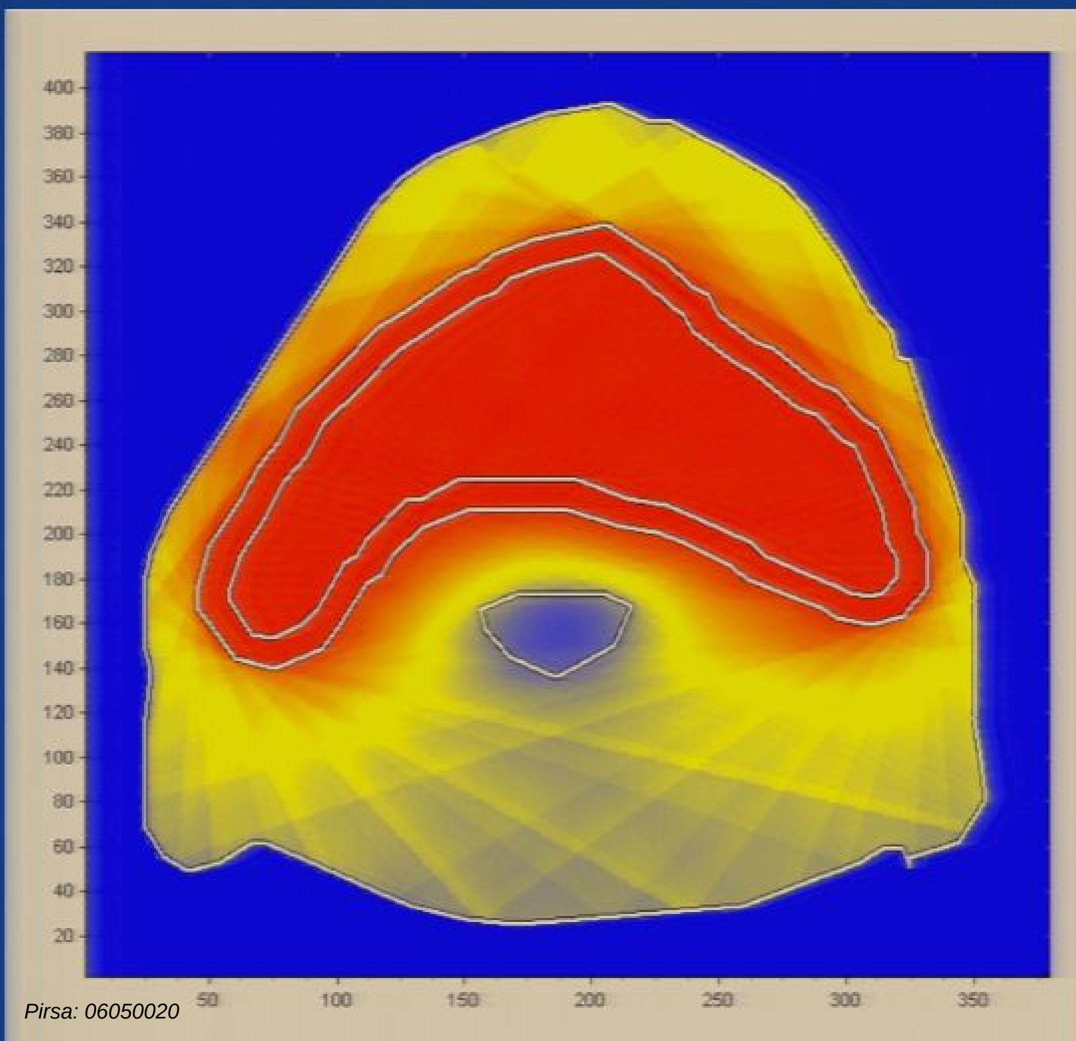
$$w = \alpha^{-1} \times \beta$$

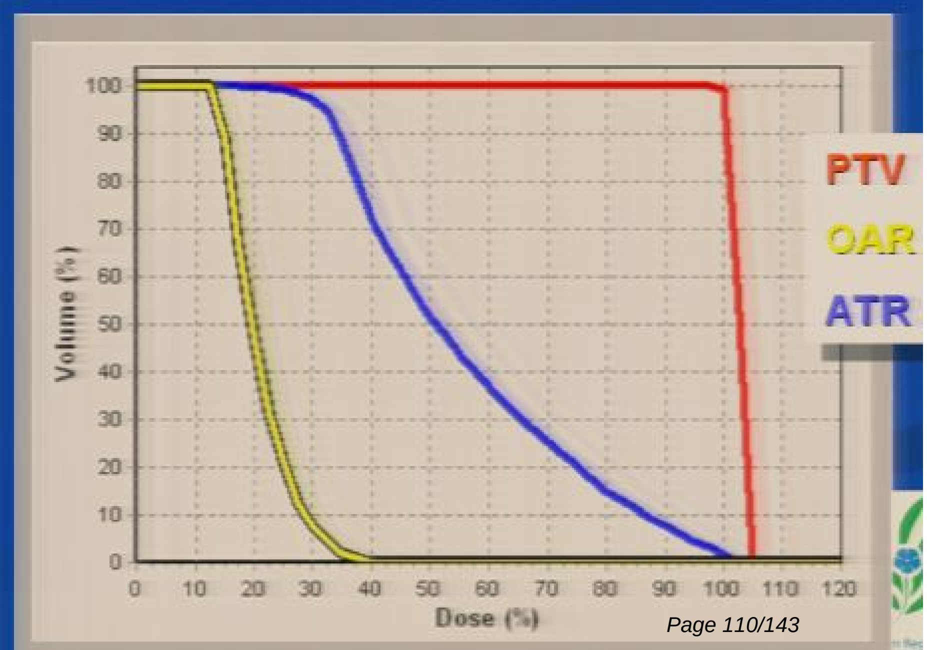
IT WORKS!!!

$$w_i = \alpha_{ij}^{-1}\beta_j$$

Results

20 Gantry Angles: 0^0 to 360^0
total number of beamlets: 988
beamlet width: 2 mm
matrix calculations time: 2”
optimization time: 0.25”

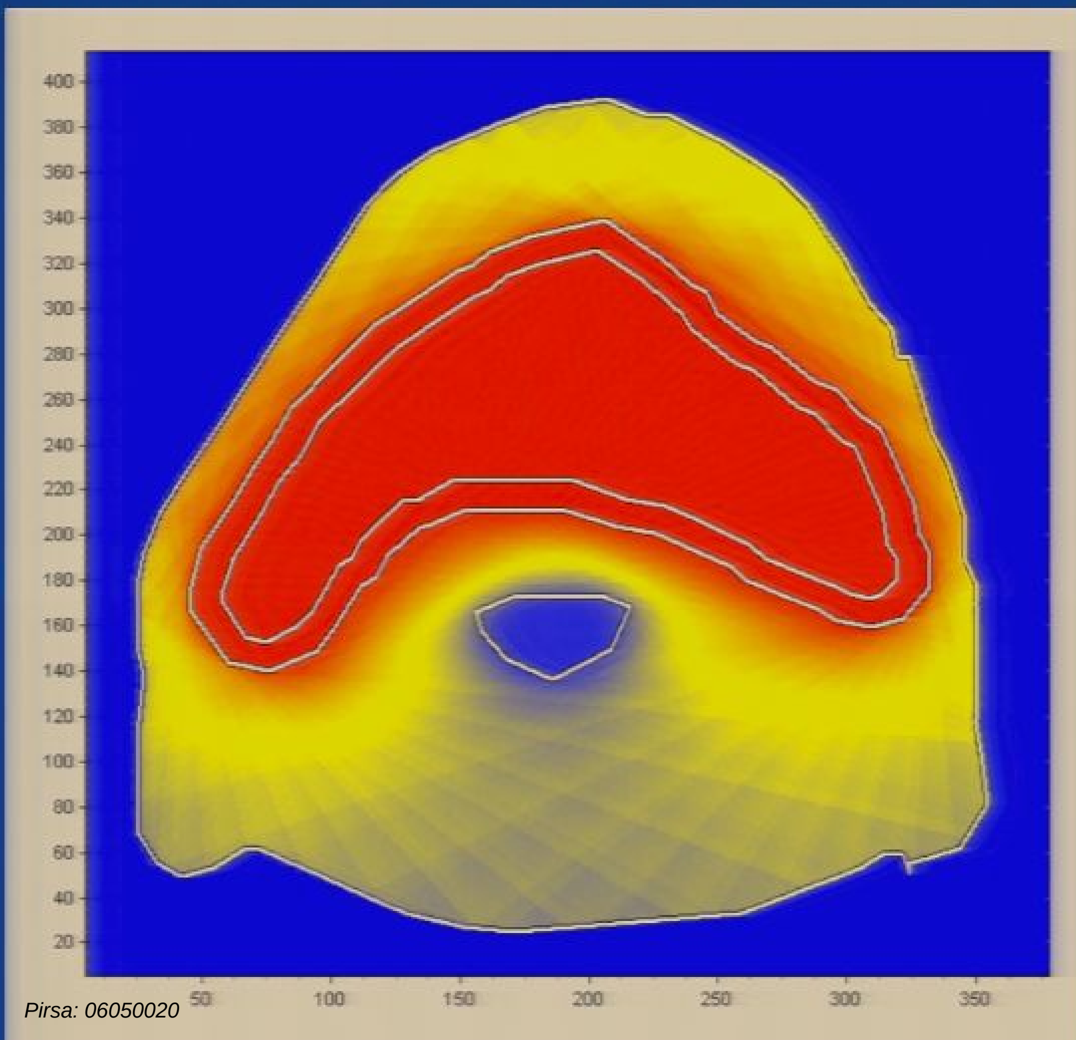


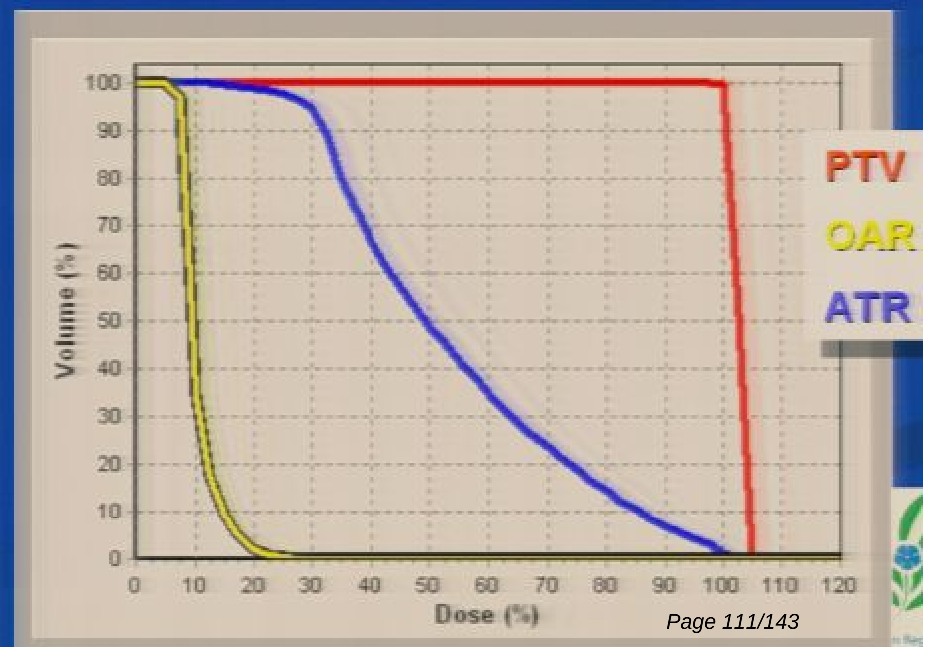


$$w_i = \alpha_{ij}^{-1}\beta_j$$

Results

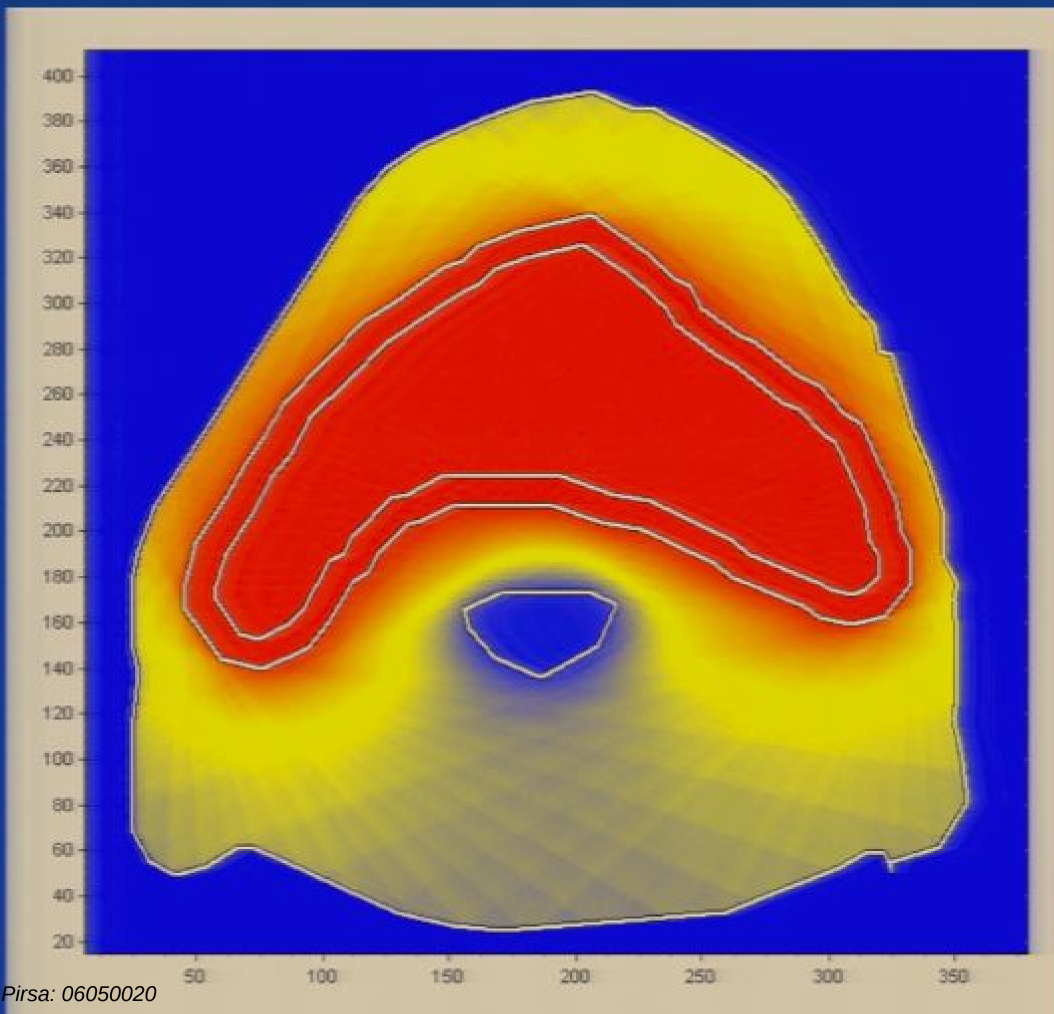
40 Gantry Angles: 0^0 to 360^0
total number of beamlets: 1976
beamlet width: 2 mm
matrix calculations time: 7”
optimization time: 3”



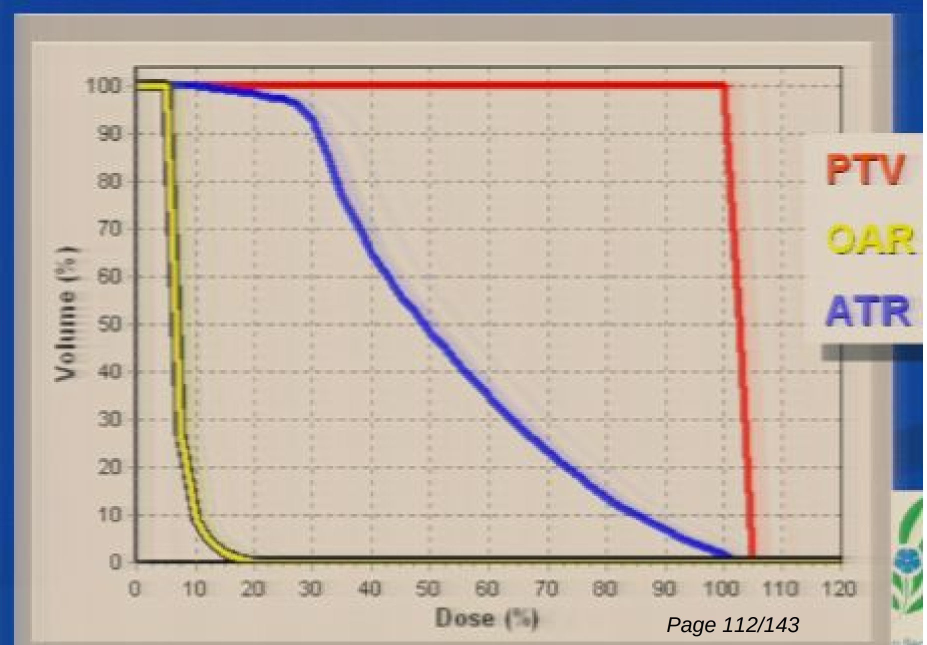


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

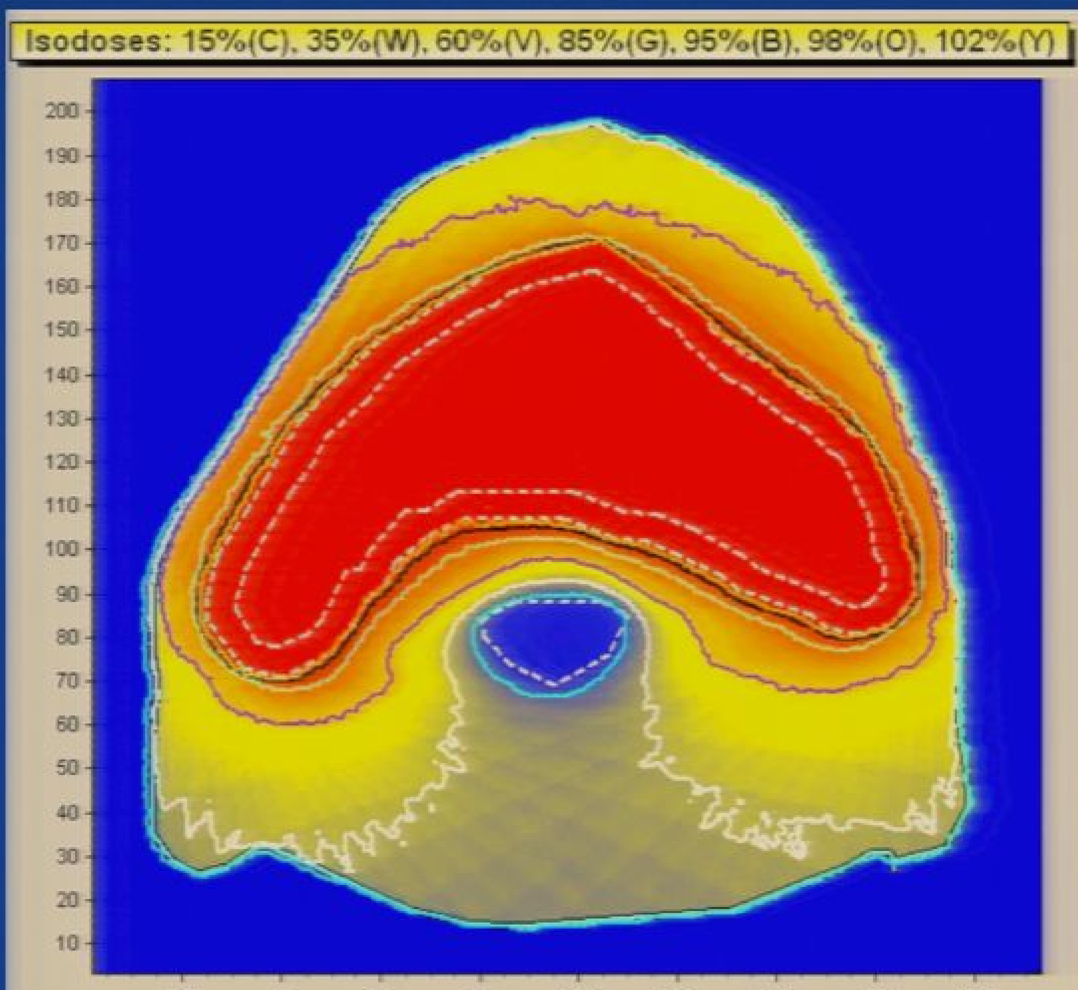


60 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2968
beamlet width: 2 mm
matrix calculations time: 16”
optimization time: 12”



$$w_i = \alpha_{ij}^{-1} \beta_j$$

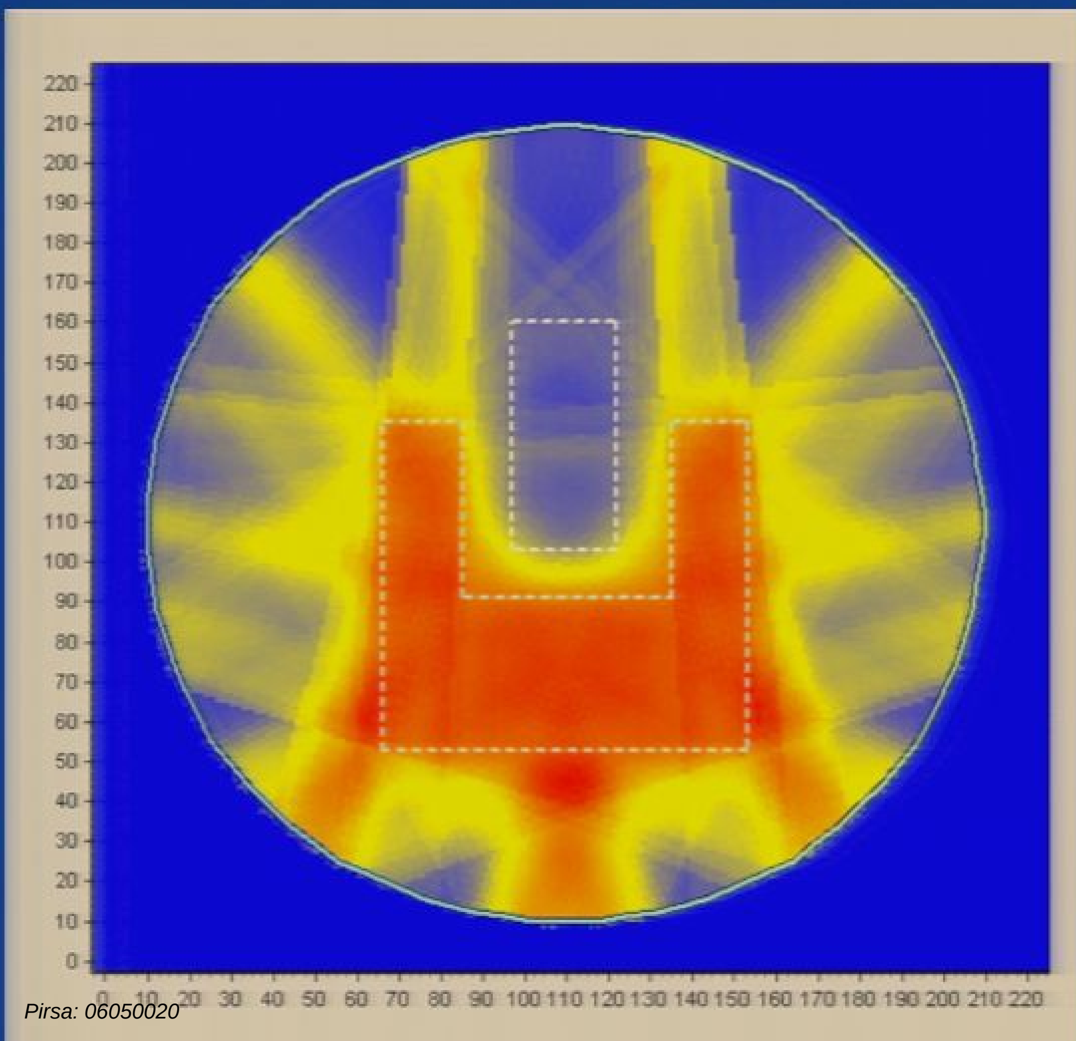
Results



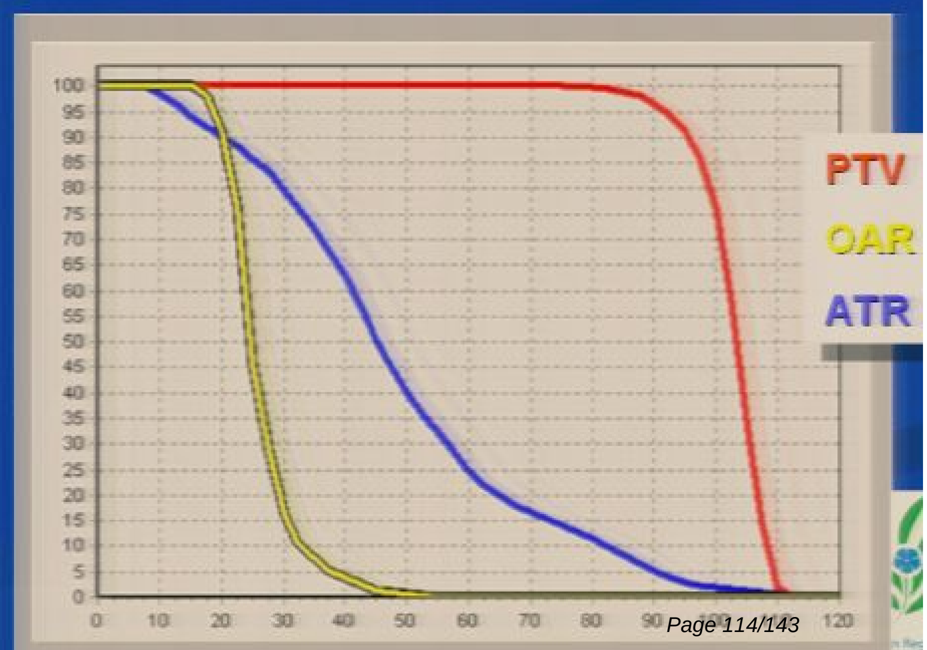
60 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2968
beamlet width: 2 mm
matrix calculations time: 16”
optimization time: 12”

$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

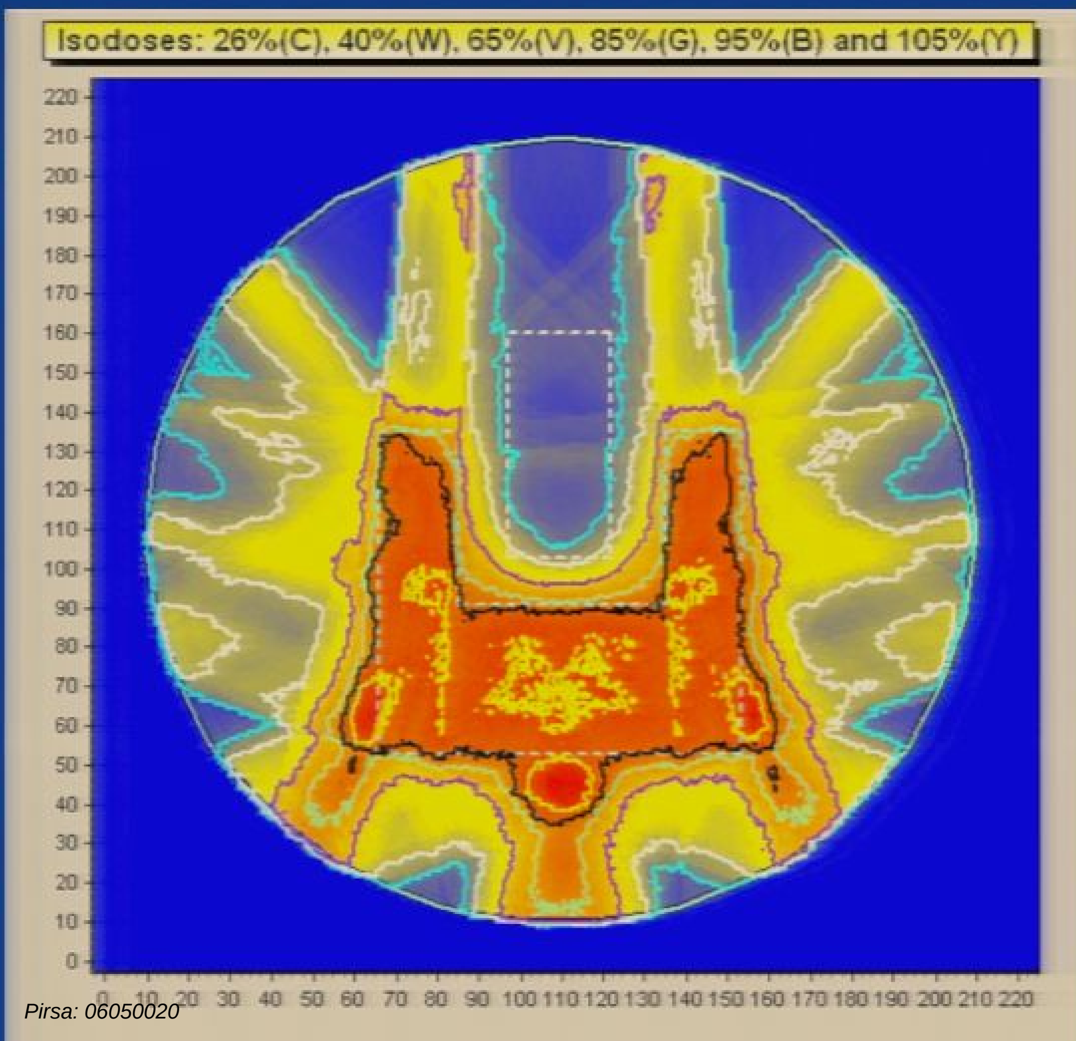


9 Gantry Angles: 0^0 to 360^0
total number of beamlets: 393
beamlet width: 5 mm
matrix calculations time: 0.83”
optimization time: 0.02”

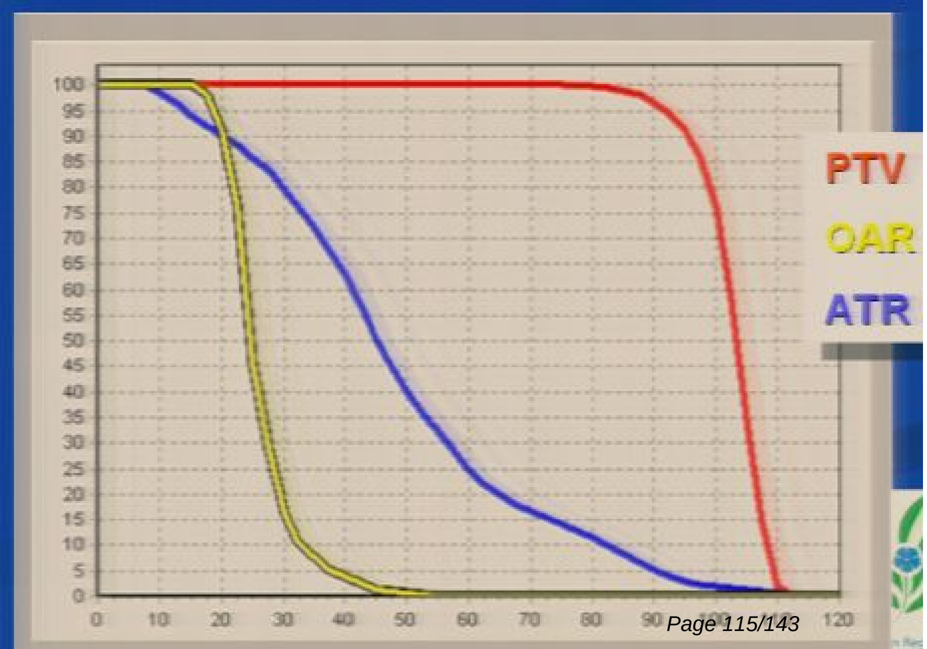


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

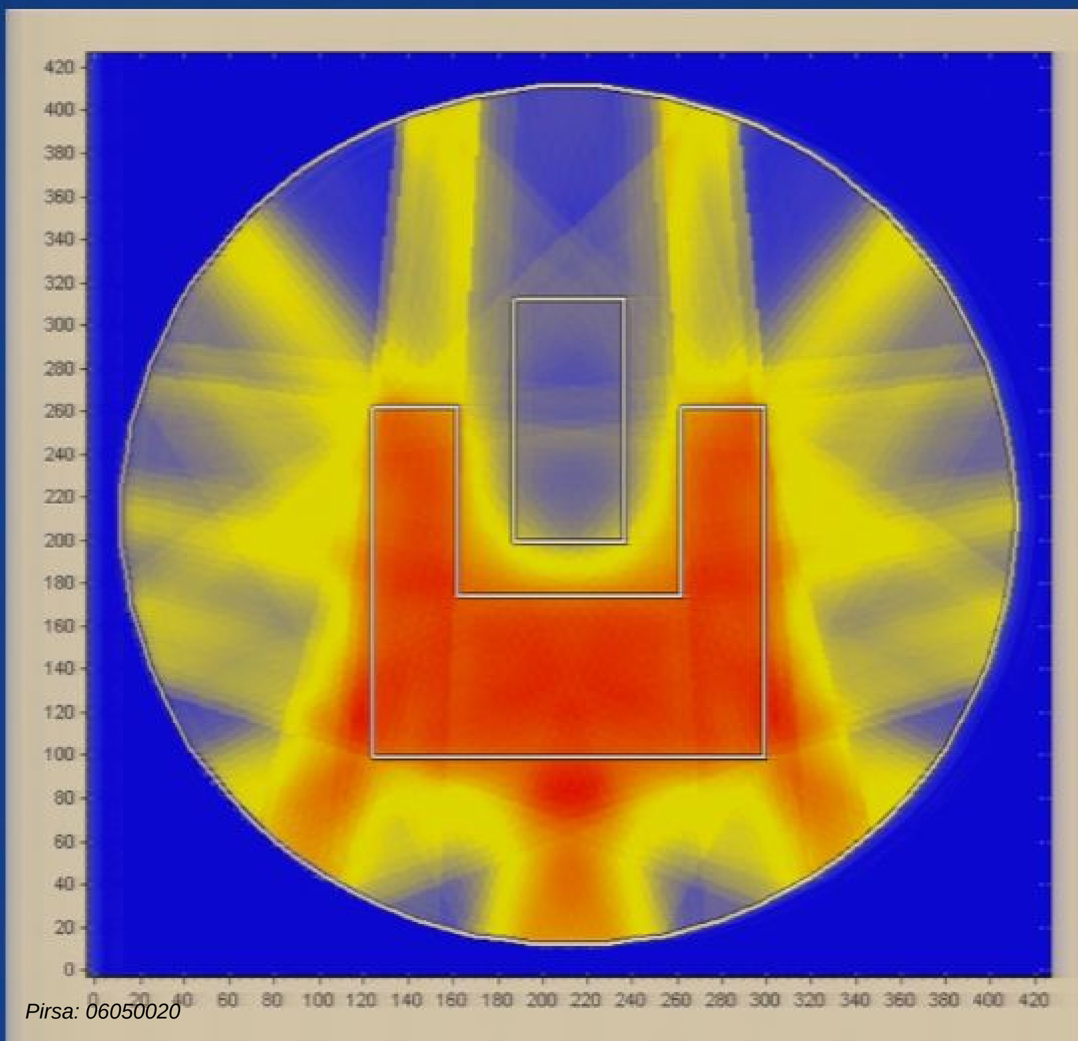


9 Gantry Angles: 0^0 to 360^0
total number of beamlets: 393
beamlet width: 5 mm
matrix calculations time: 0.83”
optimization time: 0.02”

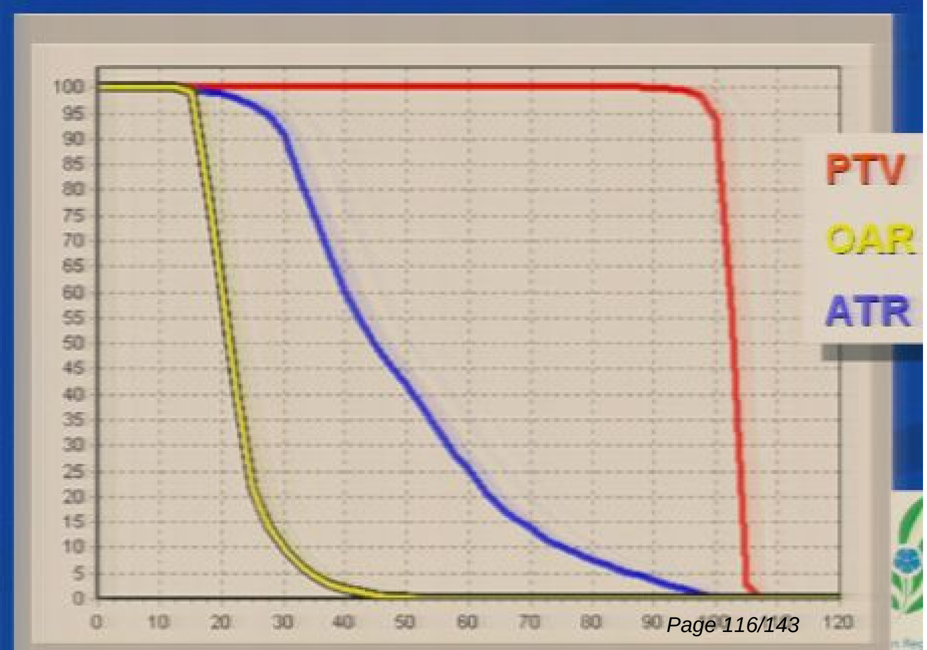


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results



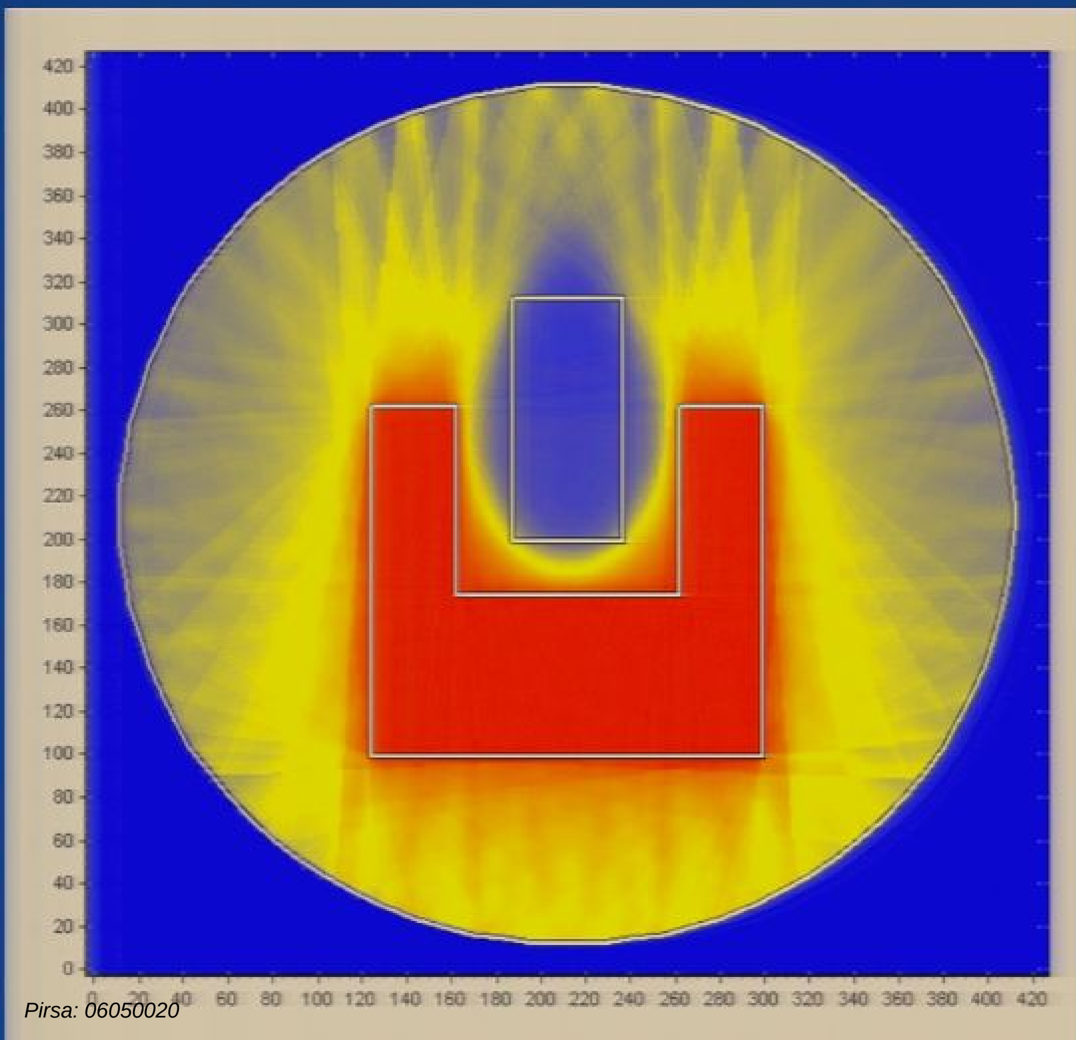
20 Gantry Angles: 0^0 to 360^0
total number of beamlets: 1162
beamlet width: 3.75 mm
matrix calculations time: 14”
optimization time: 0.5”

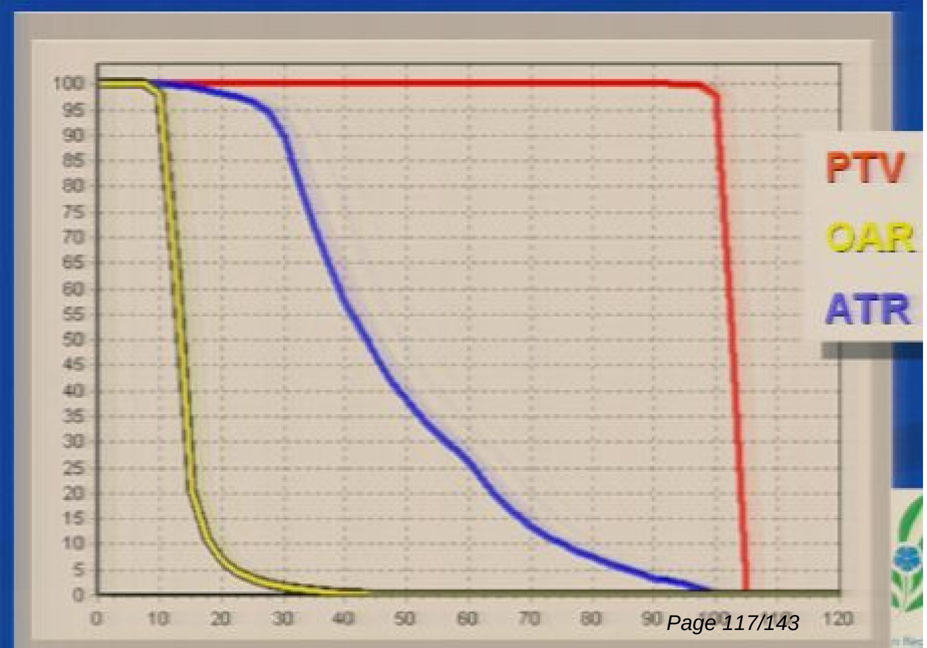


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

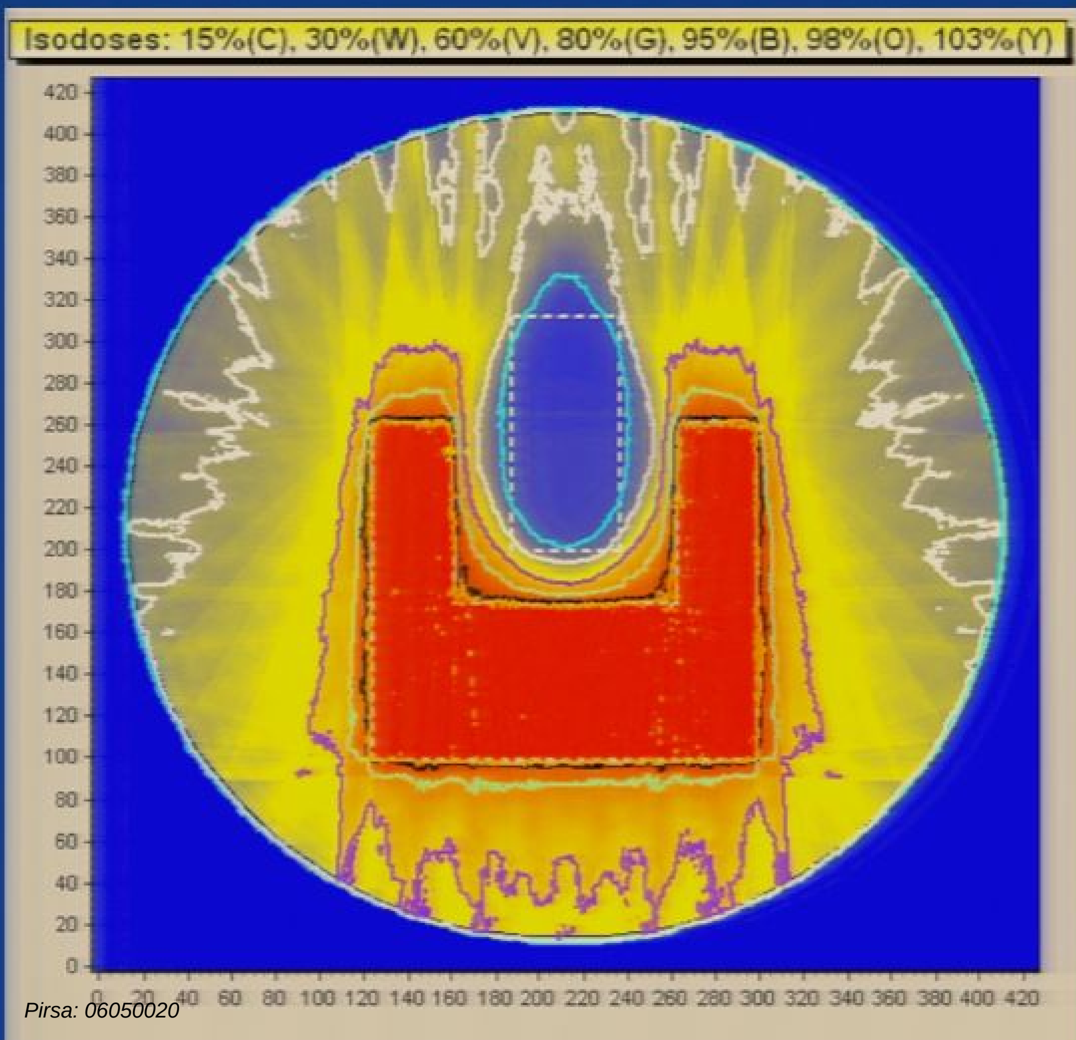
40 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2326
beamlet width: 3.75 mm
matrix calculations time: 48”
optimization time: 5.5”





$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

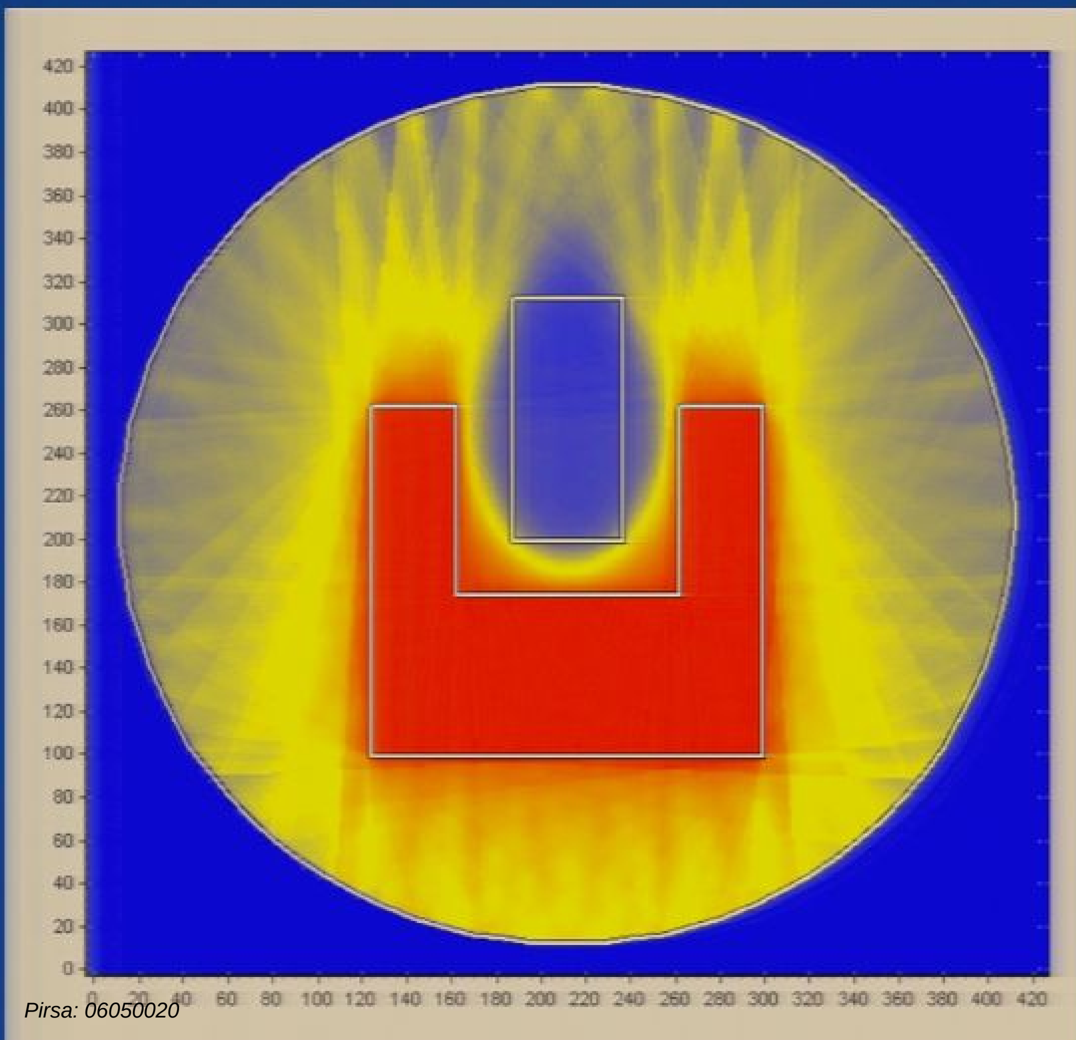


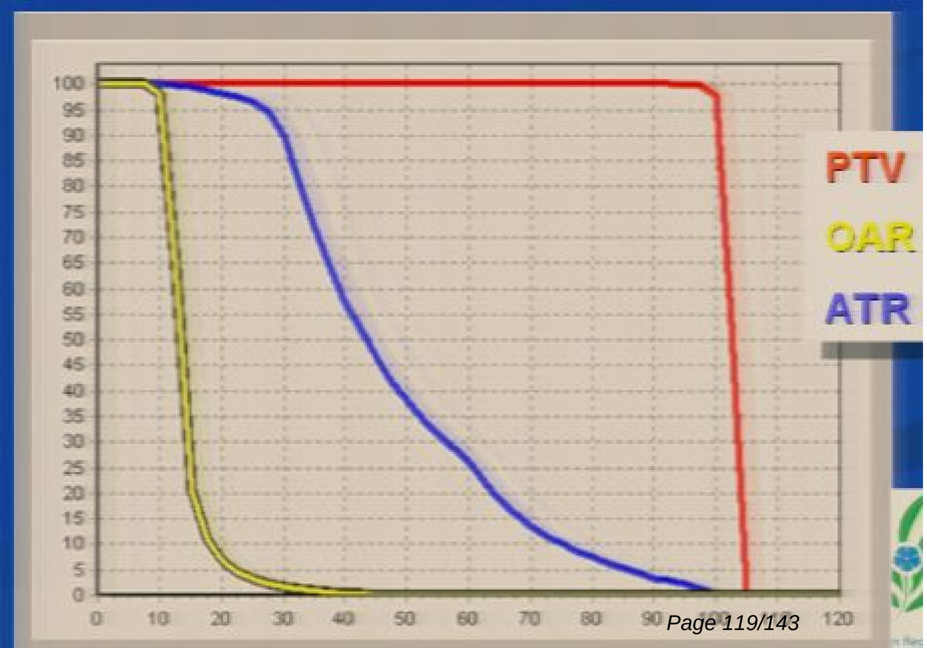
40 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2326
beamlet width: 3.75 mm
matrix calculations time: 48”
optimization time: 5.5”

$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

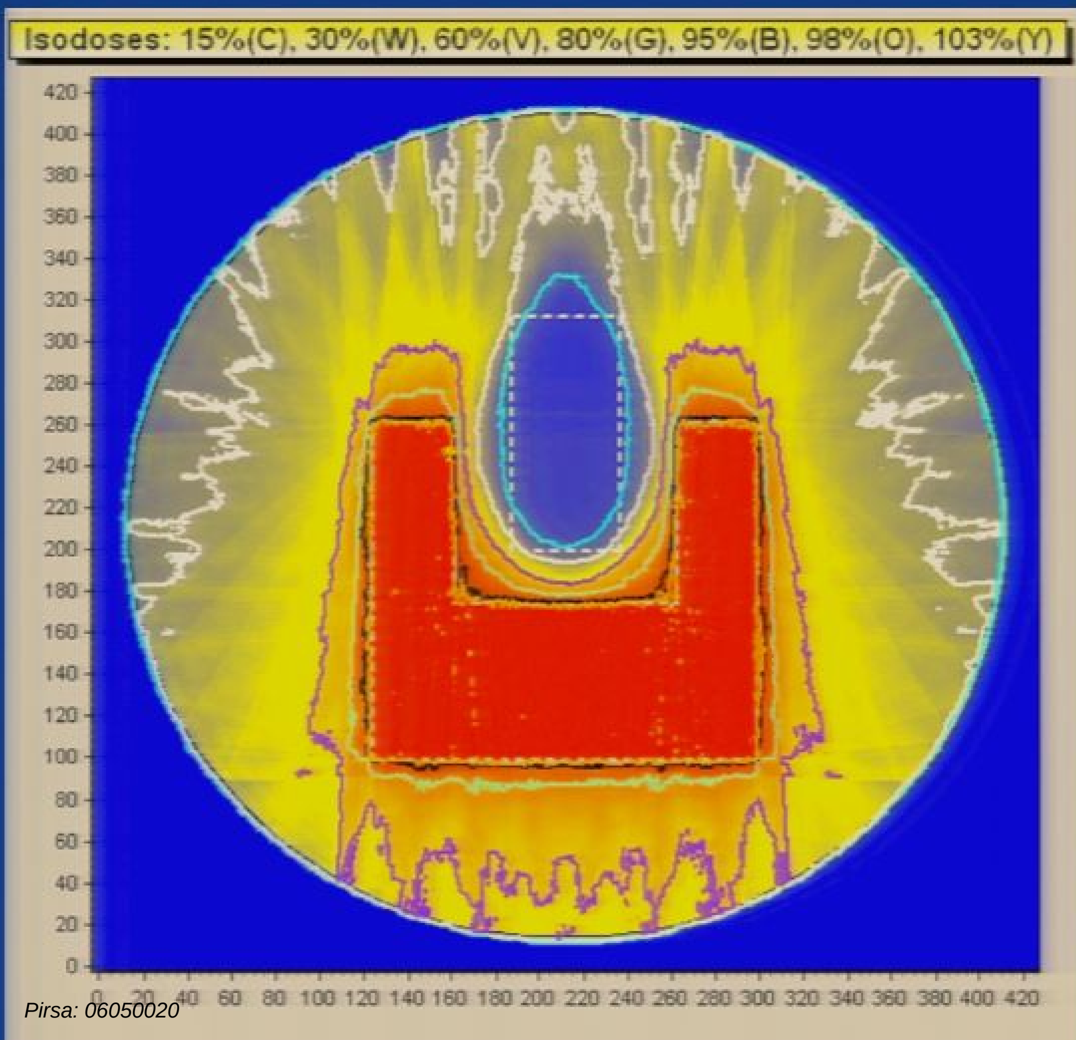
40 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2326
beamlet width: 3.75 mm
matrix calculations time: 48”
optimization time: 5.5”



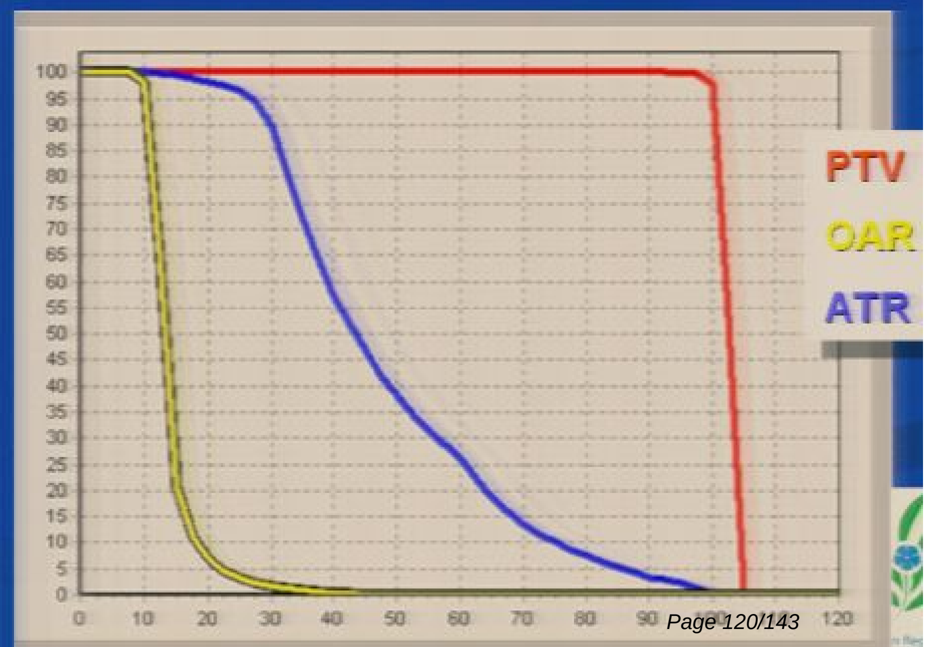


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results



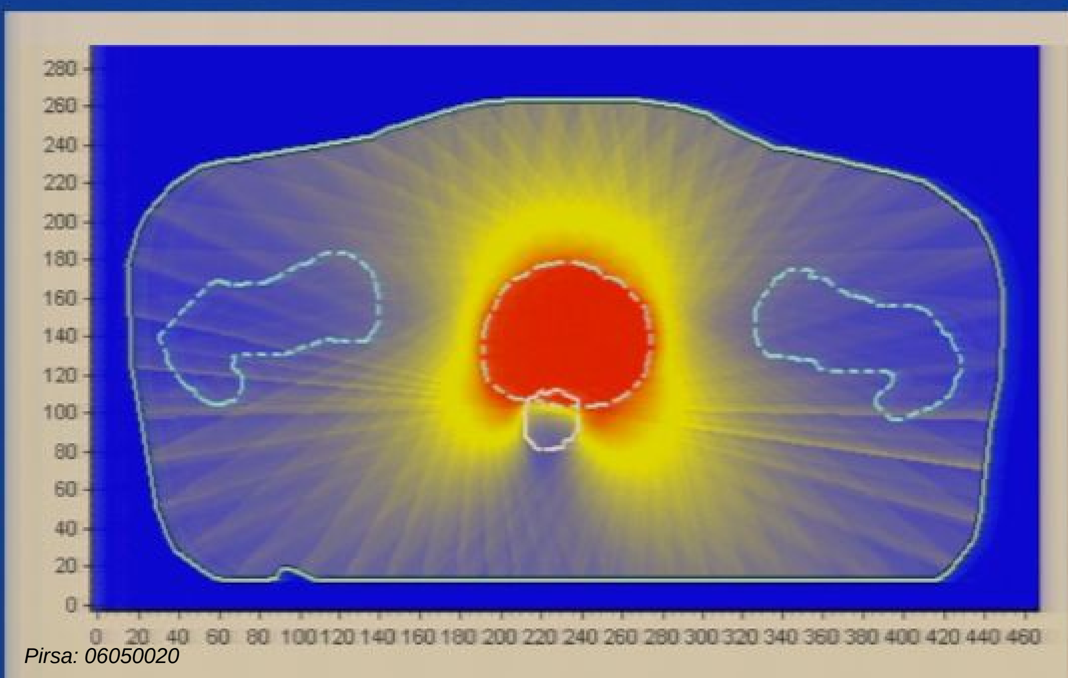
40 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2326
beamlet width: 3.75 mm
matrix calculations time: 48”
optimization time: 5.5”

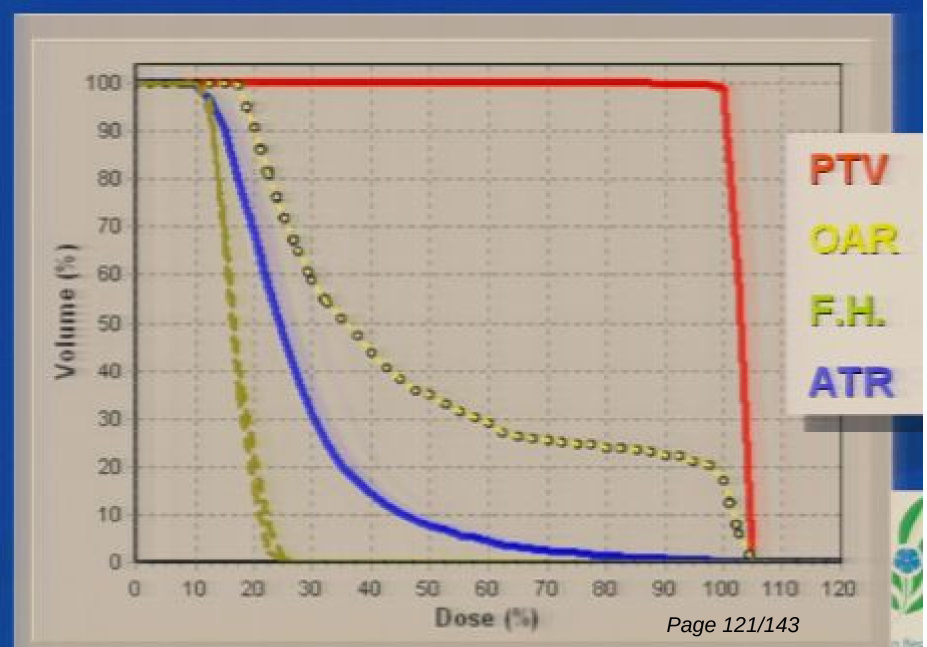


$$w_i = \alpha_{ij}^{-1}\beta_j$$

Results

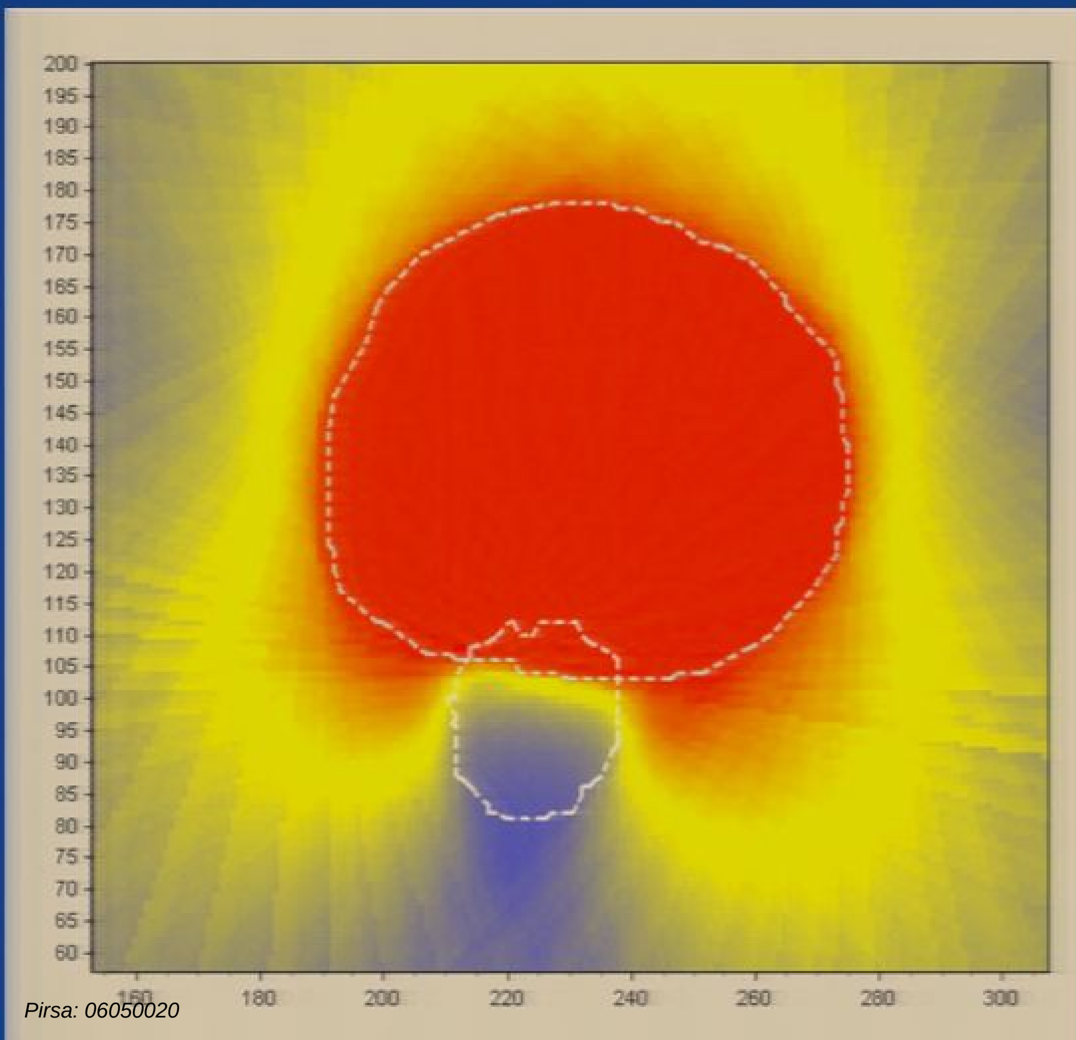
50 Gantry Angles: 0^0 to 360^0
total number of beamlets: 1642
beamlet width: 2.00 mm
matrix calculations time: 6”
optimization time: 1.02”





$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results



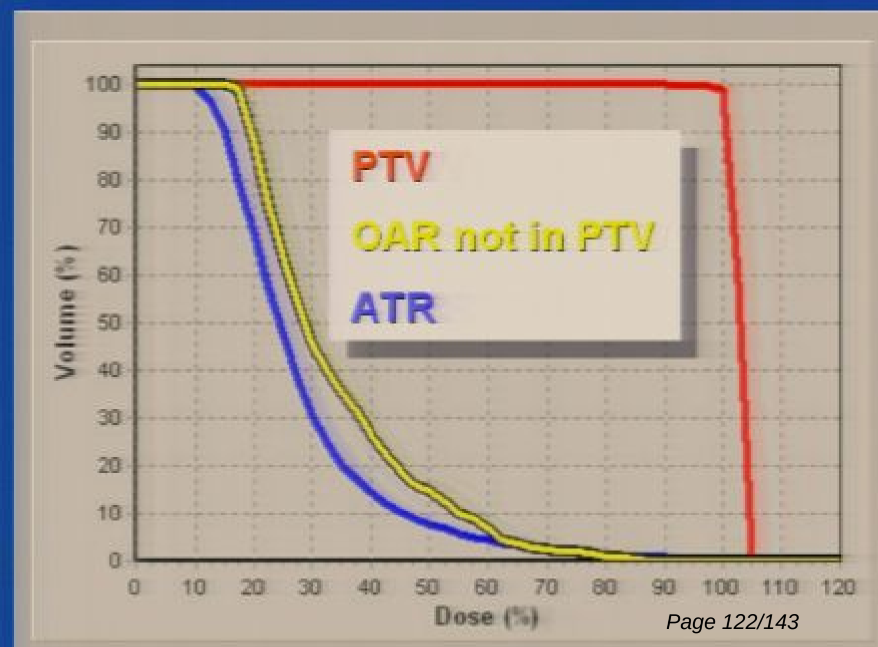
50 Gantry Angles: 0^0 to 360^0

total number of beamlets: 1642

beamlet width: 2.00 mm

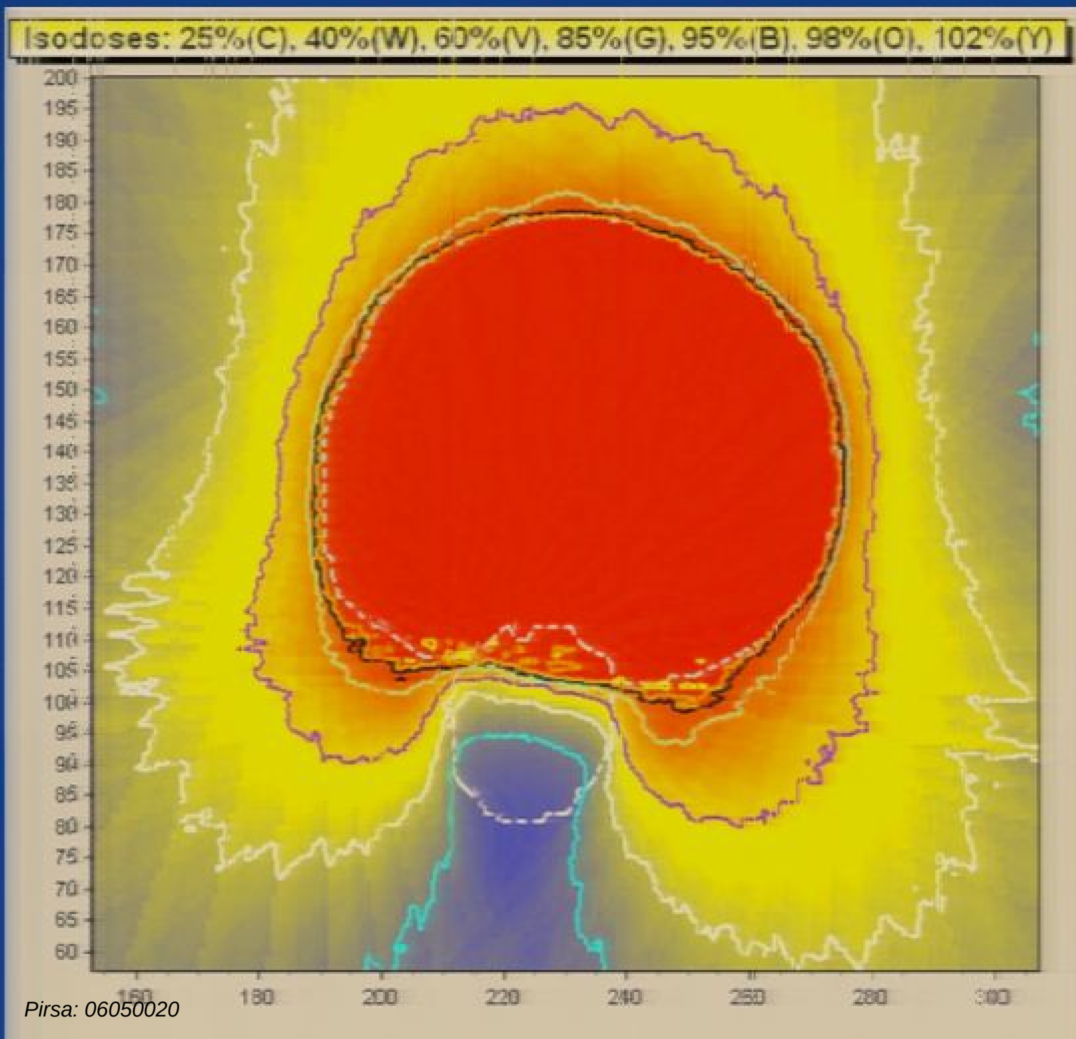
matrix calculations time: 6"

optimization time: 1.02"



$$w_i = \alpha_{ij}^{-1}\beta_j$$

Results



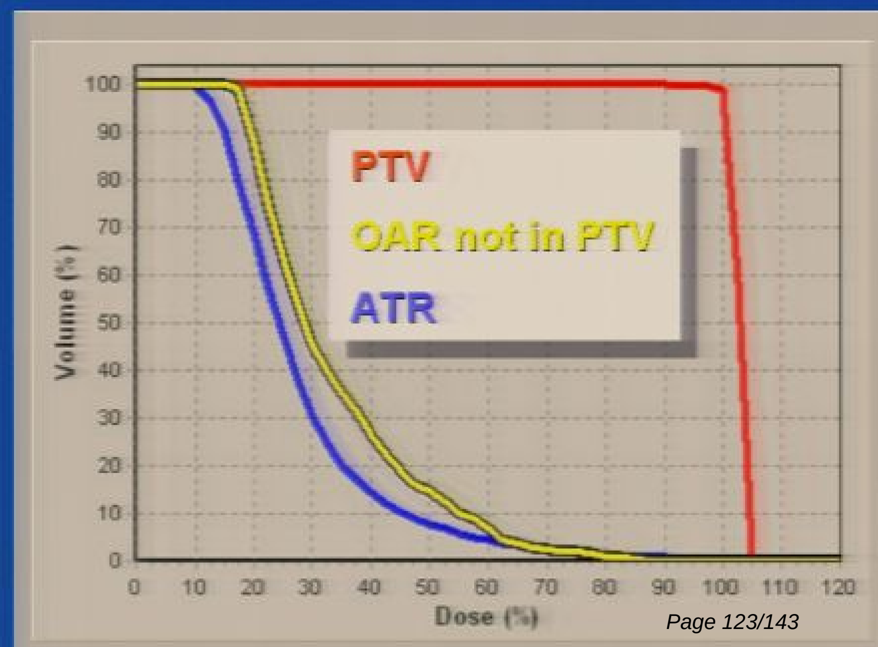
50 Gantry Angles: 0^0 to 360^0

total number of beamlets: 1642

beamlet width: 2.00 mm

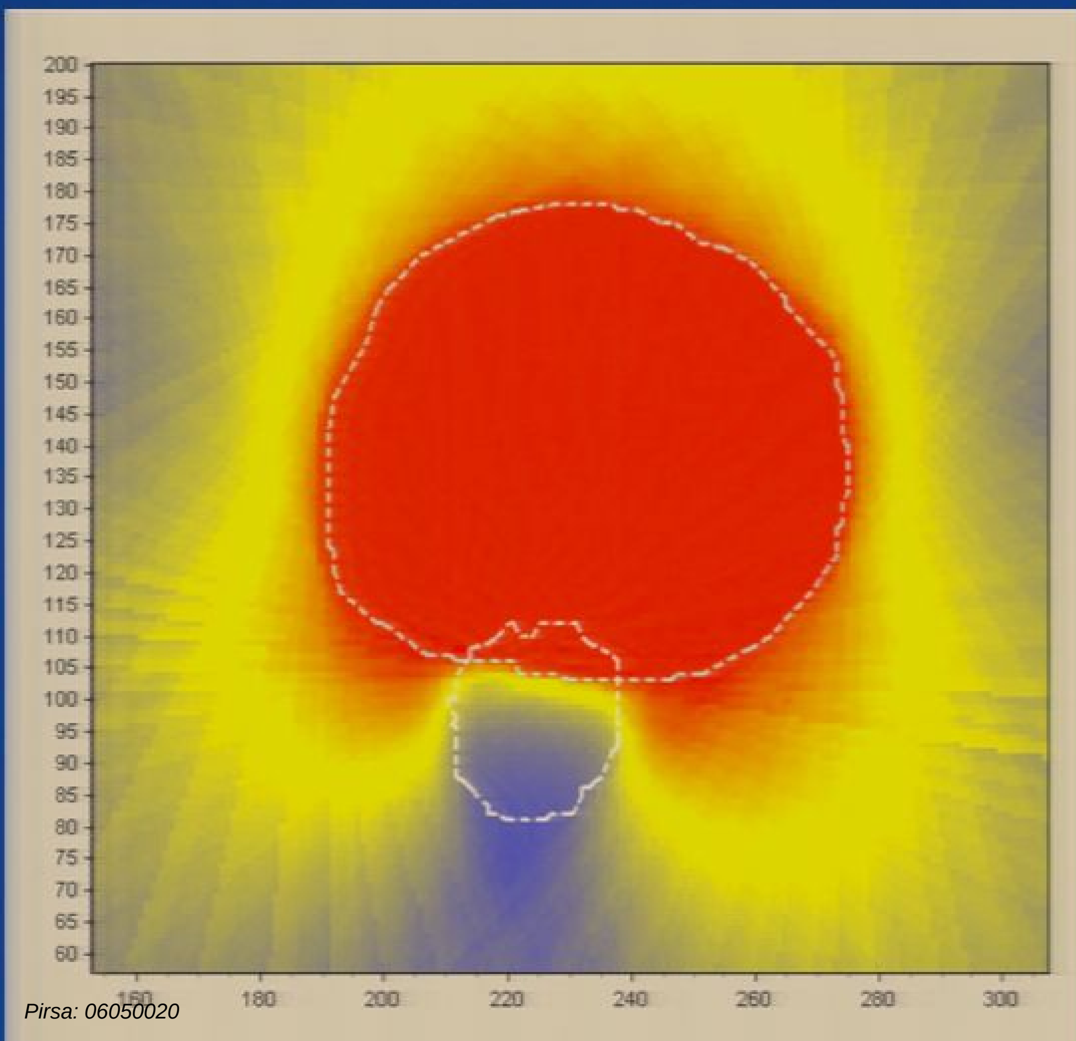
matrix calculations time: 6"

optimization time: 1.02"

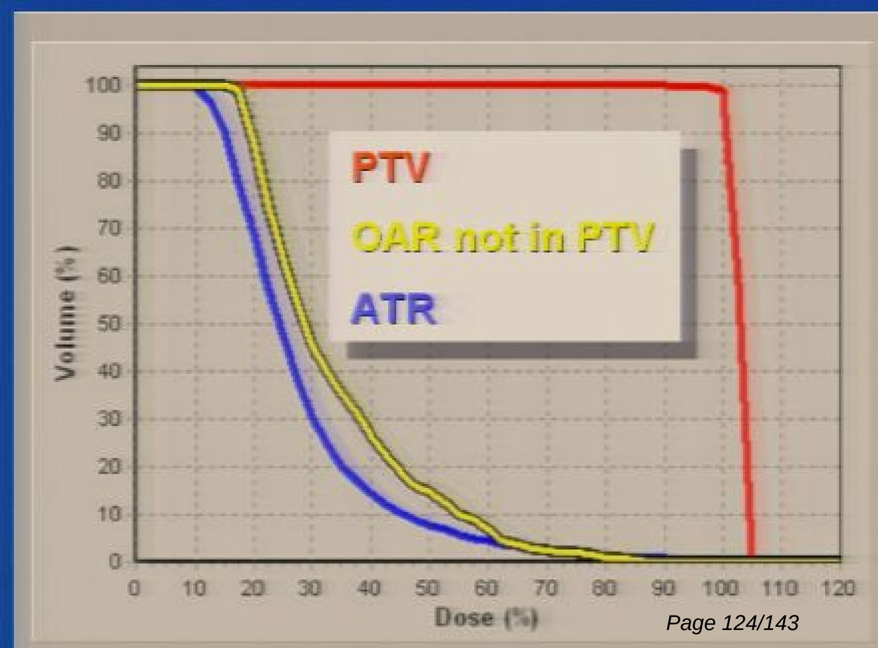


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results



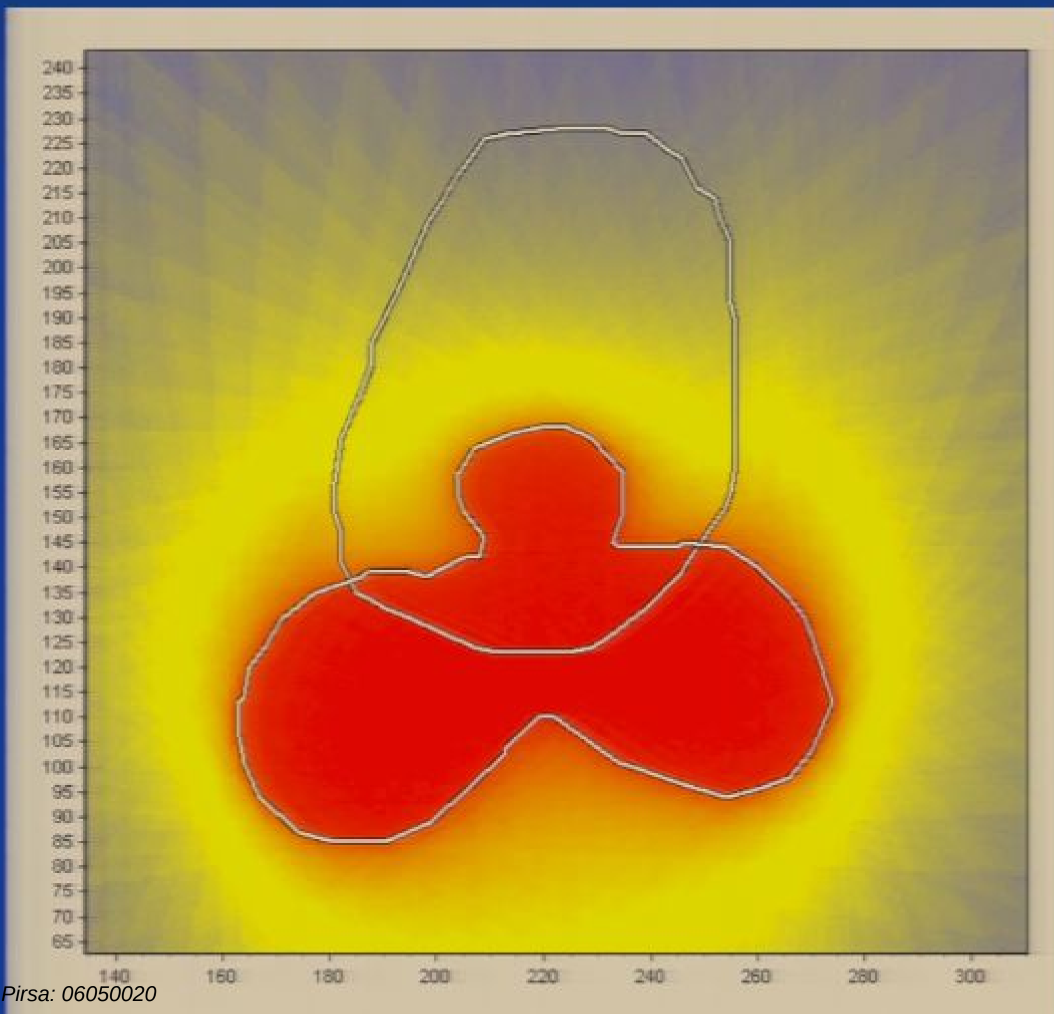
50 Gantry Angles: 0^0 to 360^0
total number of beamlets: 1642
beamlet width: 2.00 mm
matrix calculations time: 6”
optimization time: 1.02”

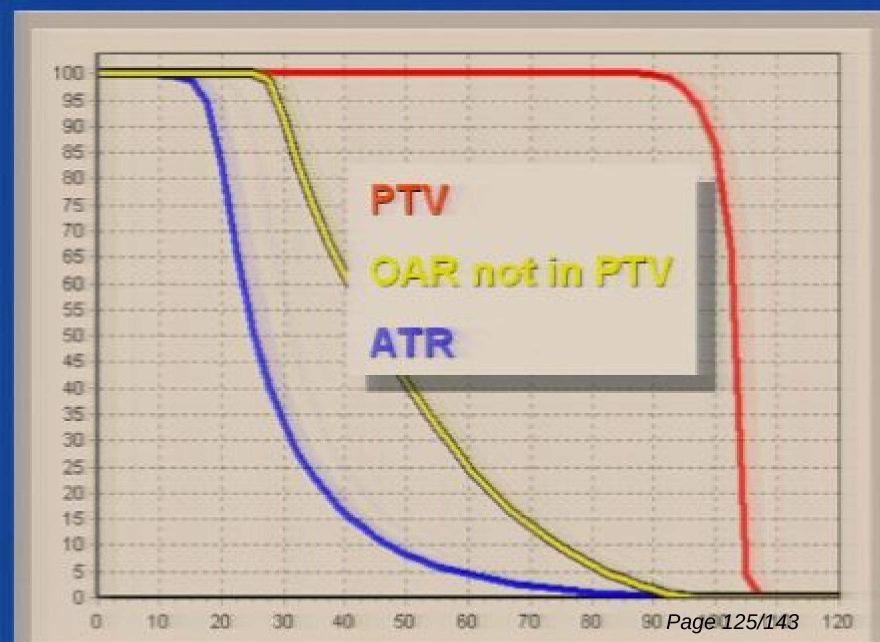


$$w_i = \alpha_{ij}^{-1} \beta_j$$

Results

60 Gantry Angles: 0^0 to 360^0
total number of beamlets: 2314
beamlet width: 2.00 mm
matrix calculations time: 47”
optimization time: 3”





Concluding Remarks

Using the FIDO algorithm we obtain **very conformal dose distributions** in **very short optimization times**

- **Conformal dose distributions:** Allow to reduce the safety margin around the target volume
- **Very short optimization times:** Open the door to adaptive radiation therapy (re-optimize before each fraction is administered)

Why Medical Physics?

Why Medical Physics?

- Immediate benefit to society

Why Medical Physics?

- Immediate benefit to society
- High demand for medical physicists

Why Medical Physics?

- Immediate benefit to society
- High demand for medical physicists
- Exciting field with tremendous future

Why Medical Physics?

- Immediate benefit to society
- High demand for medical physicists
- Exciting field with tremendous future
- Good salaries

Why Medical Physics?

- Immediate benefit to society
- High demand for medical physicists
- Exciting field with tremendous future
- Good salaries

Cutting-edge Research in Medical Physics at Ryerson

http://www.physics.ryerson.ca

- Radiation therapy
- Magnetocarcinotherapy (*magnetic nanoparticles*)
- Biomedical optics (*fiberoptics* + *lasers*)
- Photoacoustic and magnetoacoustic tomography
- Ultrasound imaging and therapeutics
- Thermal Therapy
- X-ray fluorescence (*analysis of trace elements in humans*)

Superb Teaching & Training in Medical Physics at Ryerson

http://www.physics.ryerson.ca

- Princess Margaret
- Sunnybrook
- Heidelberg

Superb Teaching & Training in Medical Physics at Ryerson

http://www.physics.ryerson.ca

- Princess Margaret
- Sunnybrook
- Heidelberg

- Undergraduate projects
- Clinical courses
- Graduate supervision

goldman@ryerson.ca

Superb Teaching & Training in Medical Physics at Ryerson

http://www.physics.ryerson.ca

- Princess Margaret
- Sunnybrook
- Heidelberg

- Undergraduate projects
- Clinical courses
- Graduate supervision

goldman@ryerson.ca

Present Work

- 3-D IMRT
- Scattering effects
- Dose Volume constraints
- Gantry angle optimization
- Intensity Modulated Arc Therapy (*future*)
- Applications to CT image reconstruction (Queen's University)

cap06@brocku.ca

CAP CONGRESS
BROCK UNIVERSITY

Canadian Association of Physicists, 6th Annual

Teacher's Workshop

Monday, June 12th, 2006
8 AM - 4:45 PM

Brock University
Thistle Complex, Room 325

Registration: Free - before June 1st
($25 after June 1st)

Lunch Provided

★ Sponsored by the Canadian Institute for Photonics Innovations

Session Topics:

- Classroom Activities to Engage Students
- Teaching Strategies for Physics in Canada
- Einstein's Impact on Science Culture
- Breakthrough Radiation Therapy of Tumors
- Strategies for Advancing Biophotonics

Special DVD Showing & Presentation: Open to Teachers AND Students!

"The Power of the SUN"

Walter Kohn, *Nobel Laureate*, *UC Santa Barbara*

Poster Sessions to update yourself on a wide variety of modern Physics research!

★ Sponsored by the Canadian Institute for Photonics Innovations

Biophotonics

Special DVD Showing & Presentation: **Open to Teachers AND Students!**

"The Power of the SUN" Walter Kohn, *Nobel Laureate*, *UC Santa Barbara*

Poster Sessions to update yourself on a wide variety of modern Physics research!

Herzberg Memorial Public Lecture

"Energy: Where on Earth are We Going?"

Sunday June 11th, 2006 7pm
Sean O'Sullivan Theatre, Brock University

Ernie McFarland, University of Guelph

This event is tailored to the General Public.
No mathematical or scientific knowledge is necessary.

For more information contact:

Heather Theijsmeijer at heather_theijsmeijer@ridley.on.ca
Or visit: http://cap06.brocku.ca/english/teachers.html

CAP CONGRESS
BROCK UNIVERSITY

cap06@brocku.ca

Canadian Association of Physicists, 6th Annual

Teacher's Workshop

Monday, June 12th, 2006
8 AM - 4:45 PM

Brock University
Thistle Complex, Room 325

Registration: <u>Free</u> - before June 1st
($25 after June 1st)

Lunch Provided

★ Sponsored by the Canadian Institute for Photonics Innovations

Session Topics:

- Classroom Activities to Engage Students
- Teaching Strategies for Physics in Canada
- Einstein's Impact on Science Culture
- Breakthrough Radiation Therapy of Tumors
- Strategies for Advancing Biophotonics

Special DVD Showing & Presentation:

Open to Teachers AND Students!

"The Power of the SUN"

Walter Kohn, *<u>Nobel Laureate</u>, UC Santa Barbara*

<u>Poster Sessions</u> to update yourself on a wide variety of modern Physics research!

goldman@ryerson.ca