

Title: World of Wonders: Special Relativity

Date: May 25, 2006 08:00 PM

URL: <http://pirsa.org/06050013>

Abstract: <kw> teaching relativity, Galilean relativity, Einstein\'s postulates, Lorentz-Einstein transformations, time, length, Galilean transformation equations, spatial separating, temporal separation, laws of mechanics, Maxwell\'s equations, Michelson-Morley </kw>

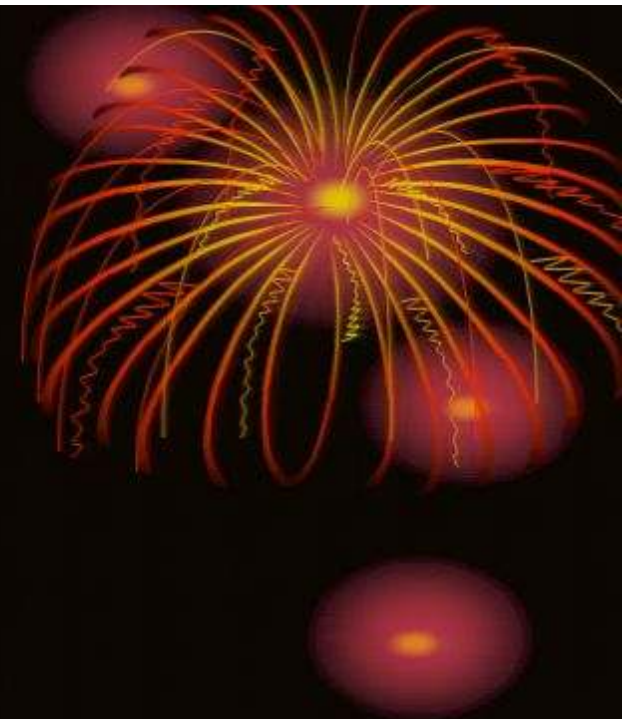
World of Wonders: Special Relativity

(avoiding pitfalls when
teaching special relativity)

Ernie McFarland

UNIVERSITY
of GUELPH

OAPT Conference 2006
Perimeter Institute



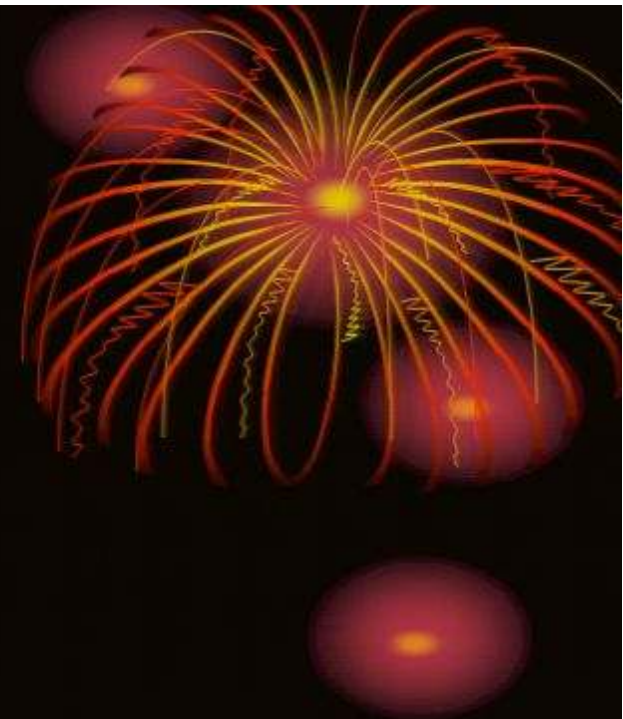
World of Wonders: Special Relativity

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Overview

- Galilean relativity
- Events
- Einstein's postulates
- Lorentz-Einstein transformations
- Relativity of time
- Relativity of length
- What does $E = mc^2$ mean?

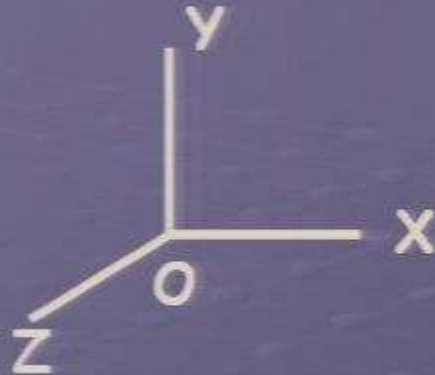


Pitfall-avoidance Tip:

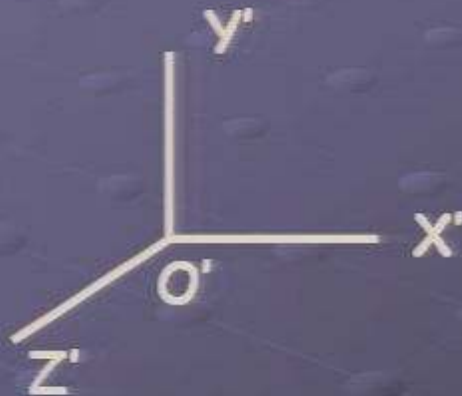
A good introduction to Galilean relativity is very useful before special relativity is started.

Galilean Relativity

Frame S
(inertial
frame)



Frame S'



Frame S' moves with constant speed v in the $+x$ direction relative to frame S .

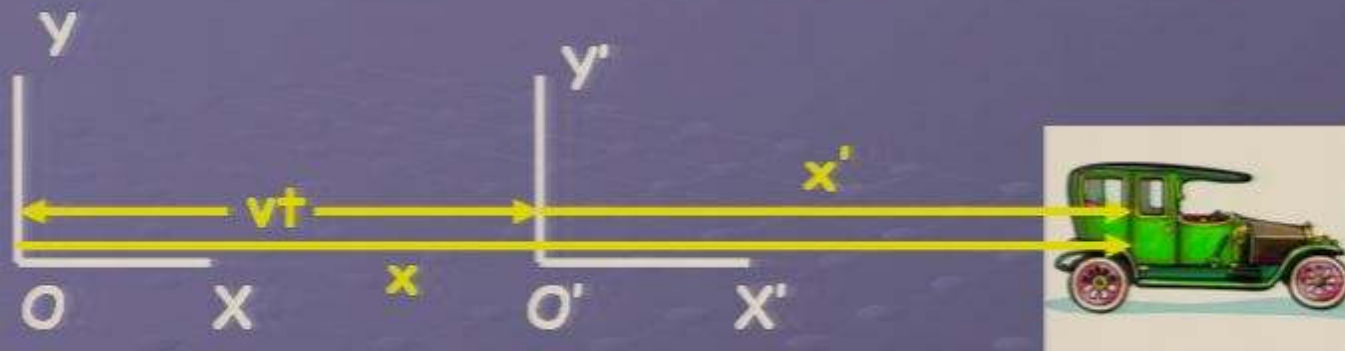
Galilean Relativity

For simplicity, assume that the origins of the two frames coincide at time $t = 0$.

At time t , O' will have moved a distance of vt from O .



Galilean Transformation Equations



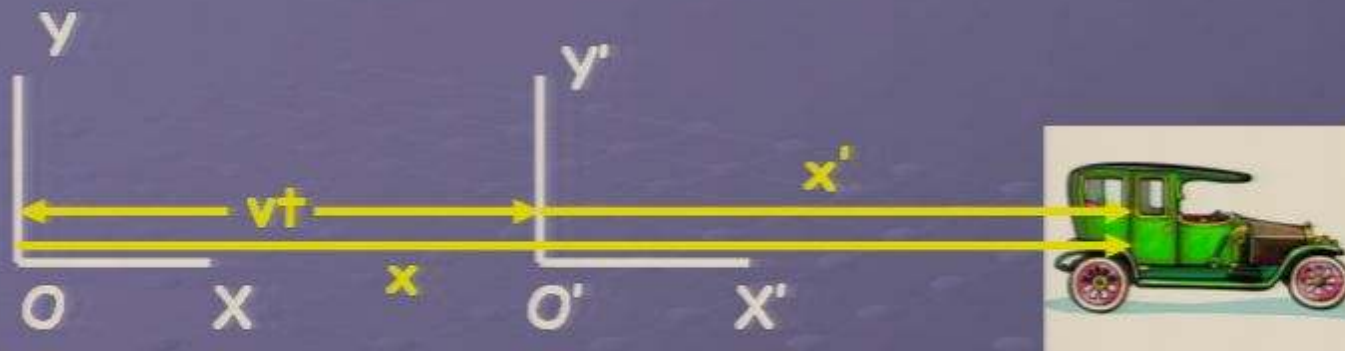
At time t , the **position** of the object (car) is: x relative to (the origin of) frame S , and x' relative to (the origin of) frame S' .

From the drawing, $x = x' + vt$

Taking time derivatives: $dx/dt = dx'/dt + d(vt)/dt$
or: $u = u' + v$

where u and u' are the velocities of the car relative to S and S' , respectively.

Galilean Transformation Equations



So far: $x = x' + vt$
and $u = u' + v$

Take another time derivative:
 $du/dt = du'/dt + dv/dt$

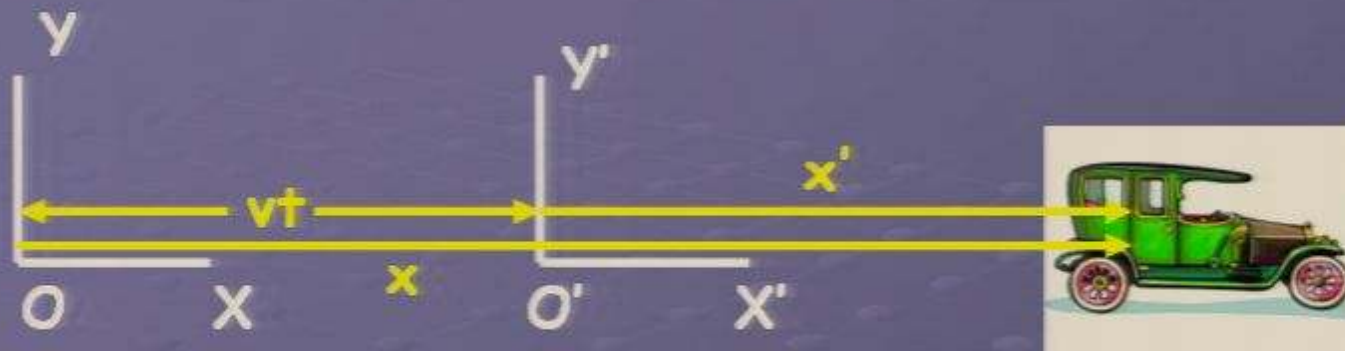
v is constant $\rightarrow dv/dt = 0$.

Therefore: $du/dt = du'/dt$

or: $a = a'$

where a and a' are the accelerations of the car,
relative to S and S' , respectively.

Galilean Transformation Equations



So, for the positions, velocities, and accelerations of the car:

$$x = x' + vt$$

$$u = u' + v$$

$$a = a'$$

These are the Galilean transformation equations (along with $y = y'$, $z = z'$, and $t = t'$).

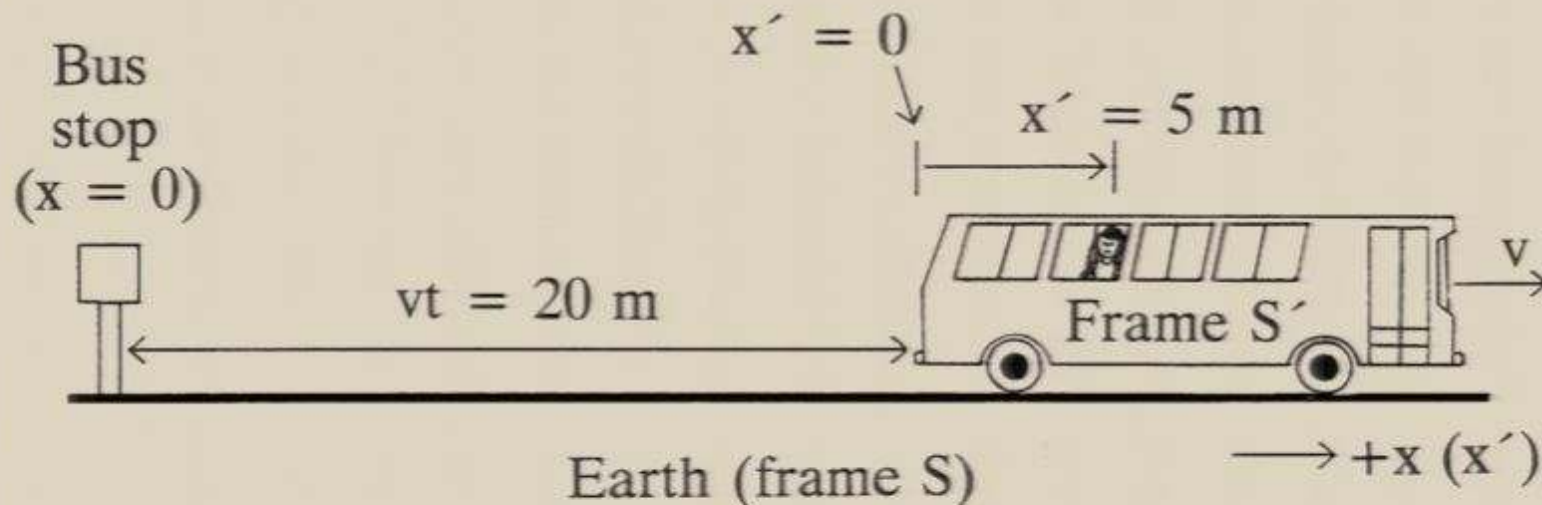
A bus is travelling forward at 25 m/s relative to Earth. The rear of the bus passes a bus stop, and 0.80 s later, a woman passenger sitting 5 m from the rear of the bus stands up. How far is she from the bus stop at this time?

Define frame S (Earth) and S' (bus).

Choose origins of the frames (see drawing). These origins coincide at time $t = 0$.

Choose $+X, +X'$ direction.

(S' moves in $+X$ direction with constant speed v .)



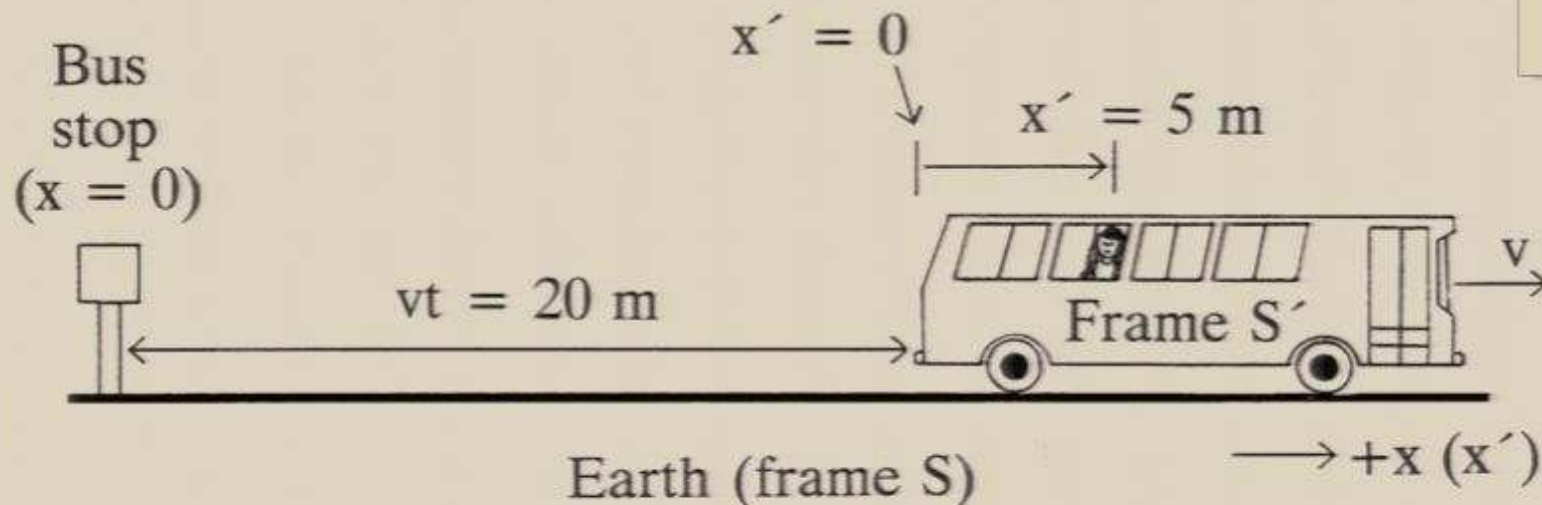
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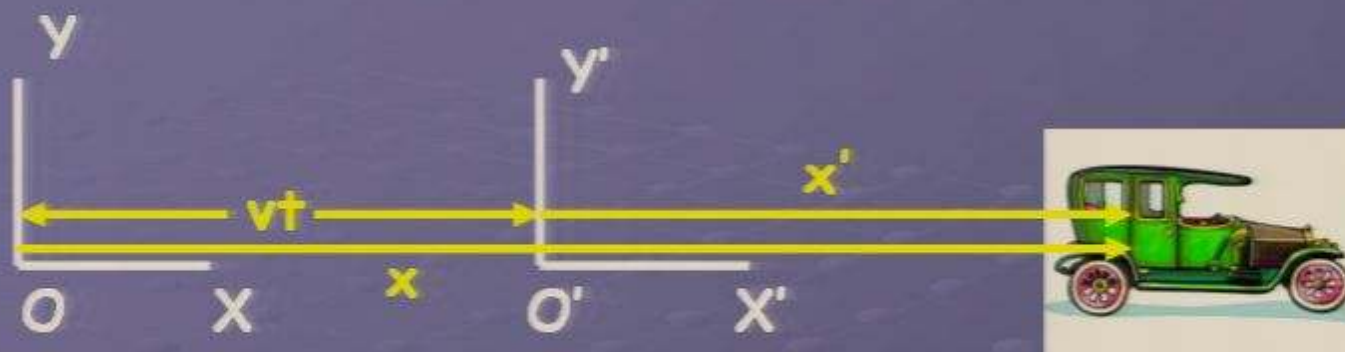
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Galilean Transformation Equations



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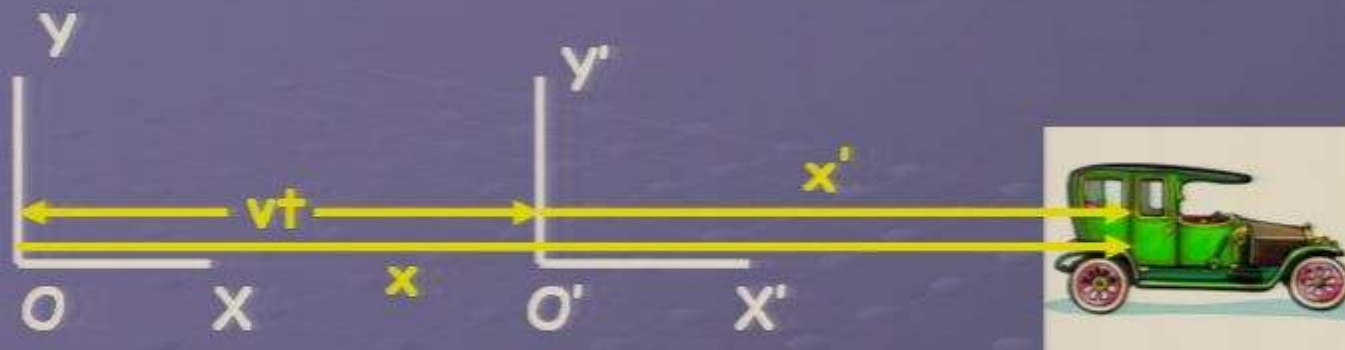
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Galilean Transformation Equations



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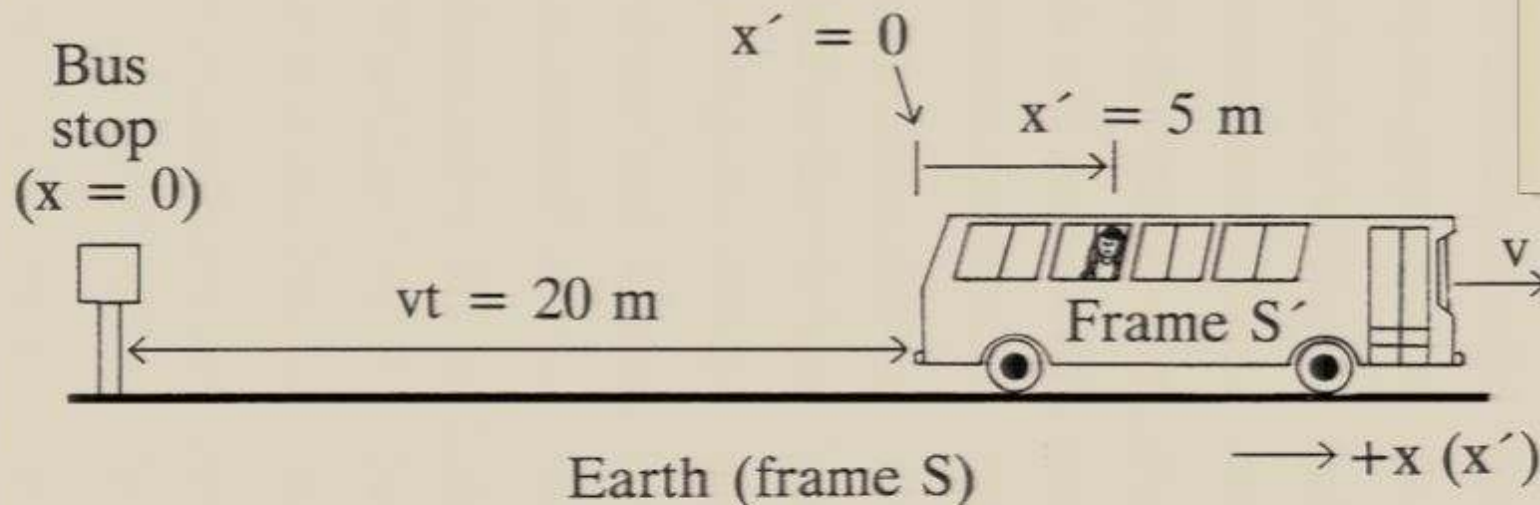
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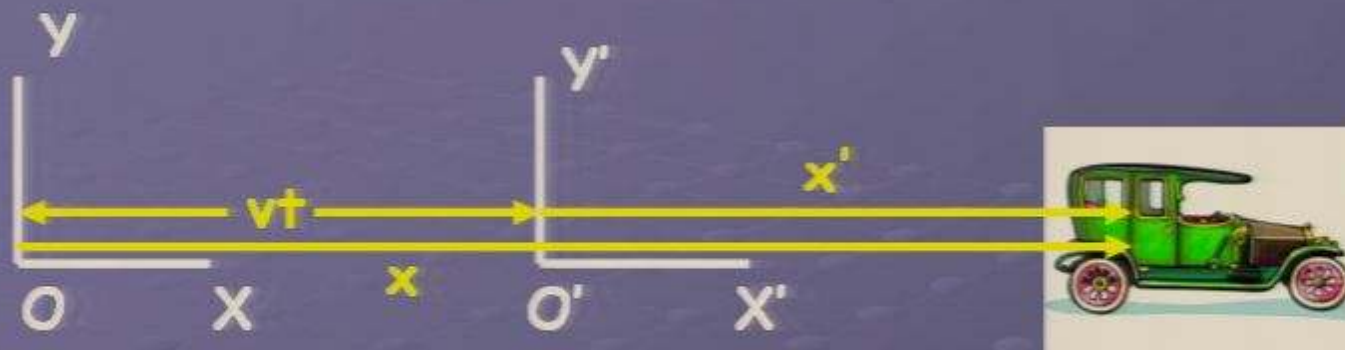
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Galilean Transformation Equations



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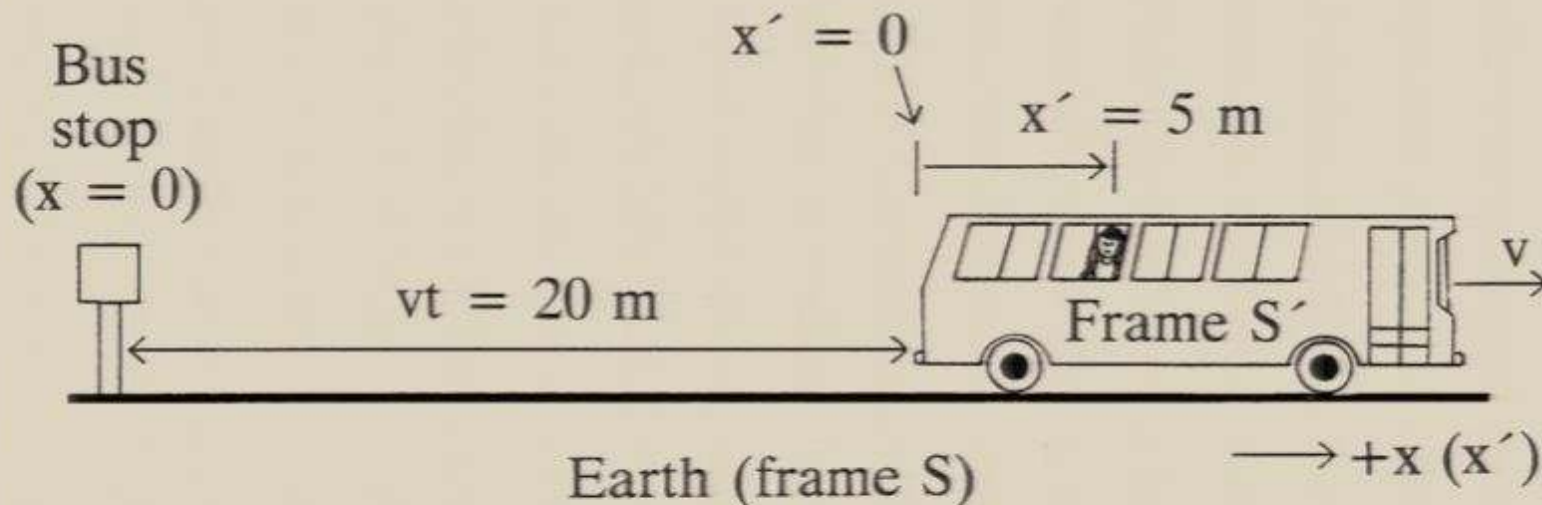
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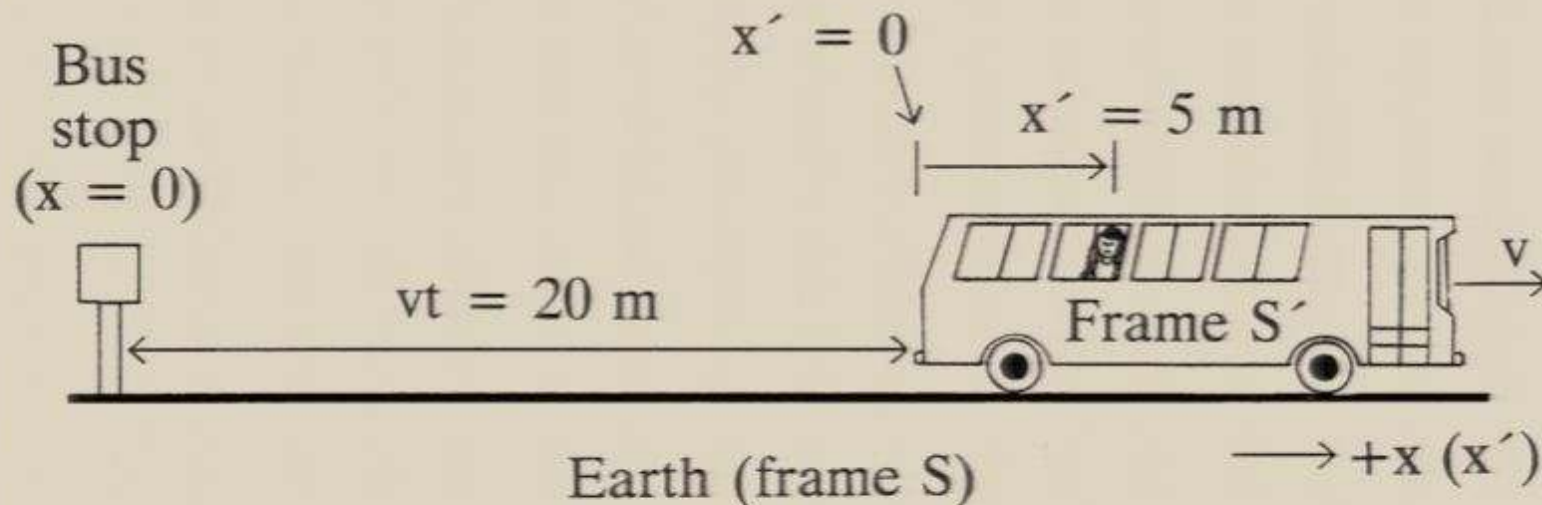
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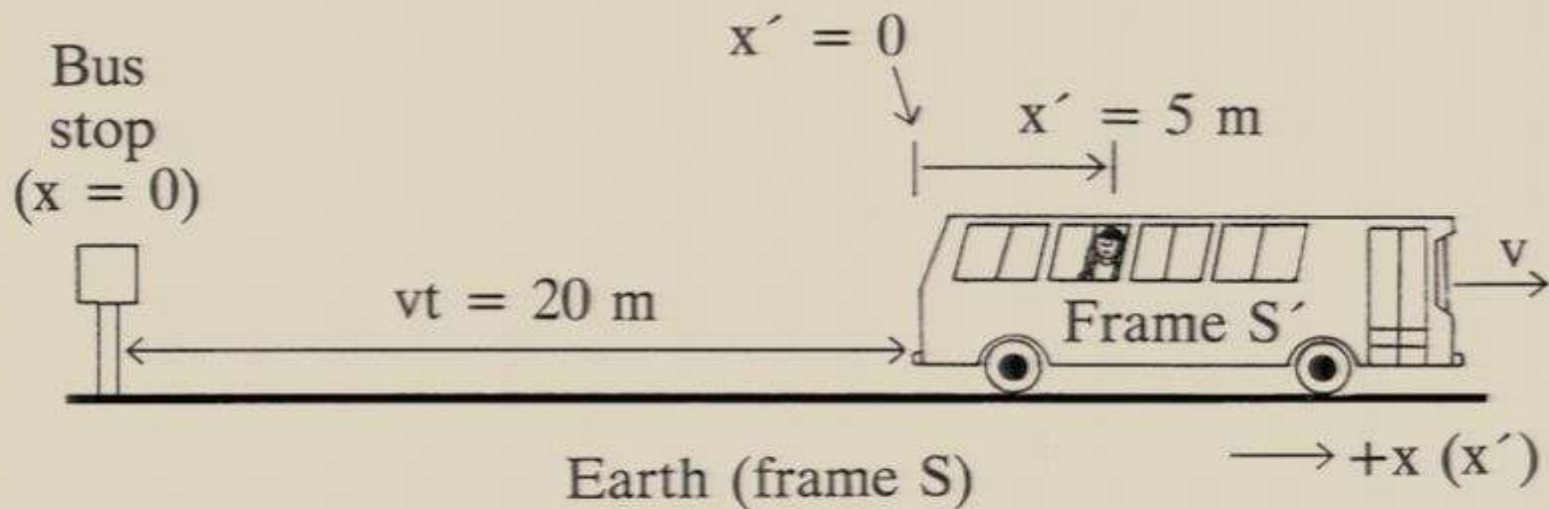
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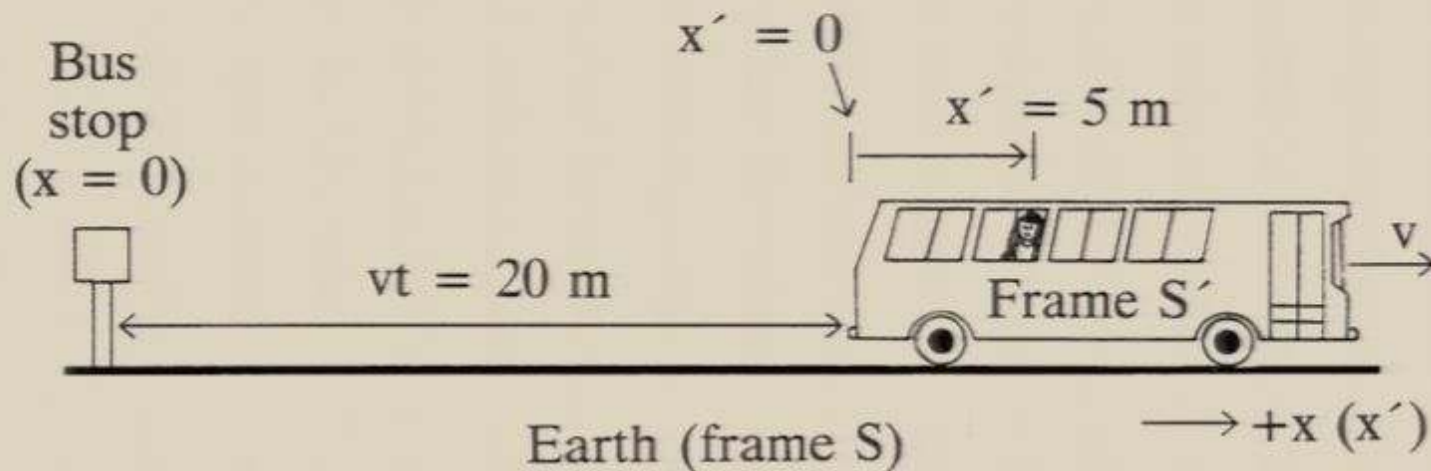
(S' moves in $+X$ direction with constant speed v .)





$$\begin{aligned}
 \text{Use } x &= x' + vt \\
 &= (5 \text{ m}) + (25 \text{ m/s})(0.80 \text{ s}) \\
 &= 25 \text{ m}
 \end{aligned}$$

Therefore, the woman is 25 m from the bus stop.



As the woman is sitting on the bus, which is travelling forward at 25 m/s relative to Earth, what is her velocity

(a) relative to Earth? (b) relative to the bus?

(c) What is the velocity of Earth relative to the bus?

During a time of 0.80 s , how far does the woman move

(d) relative to Earth? (e) relative to the bus?

Overview

- Galilean relativity
- ■ Events
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- Lorentz-Einstein transformations
- Relativity of time
- Relativity of length
- What does $E = mc^2$ mean?



EVENTS



Anything that happens at a particular time and place relative to a given frame is called an **event**.

Event #1 occurs at

position x_1 relative to frame S , and x_1' relative to S' .

It occurs at time t_1 relative to S and S' .

Event #2 occurs at

x_2 relative to S and x_2' relative to S' , at time t_2 .

The **spatial separation** between the events is

$$\Delta x = x_2 - x_1 \text{ relative to } S$$

and $\Delta x' = x_2' - x_1'$ relative to S' .

The **temporal separation** between the events is

$$\Delta t = t_2 - t_1 \text{ relative to } S \text{ and } S'.$$

$x = x' + vt$ can be used for each event:

$$x_2 = x_2' + vt_2$$

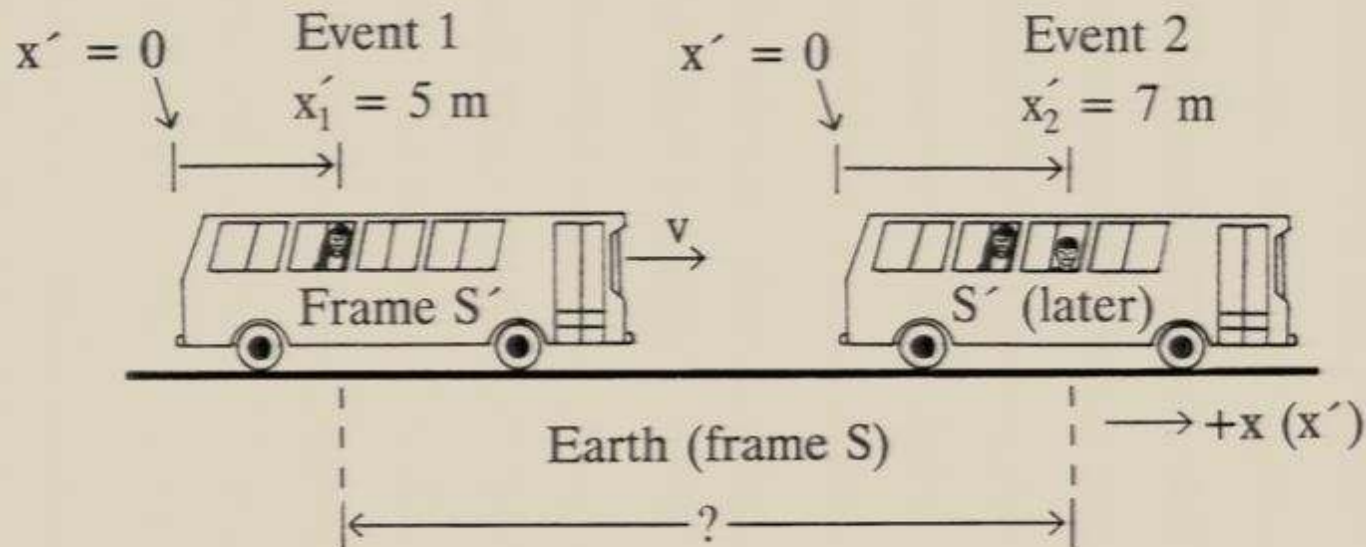
$$x_1 = x_1' + vt_1$$

Subtracting: $(x_2 - x_1) = (x_2' - x_1') + v(t_2 - t_1)$

i.e., $\Delta x = \Delta x' + v\Delta t$

or: $\Delta x' = \Delta x - v\Delta t$

Back to the bus example: At a time of 1.0 s after the woman stands up, a man sitting 2 m in front of her coughs. What is the distance separating these events relative to Earth (frame S)?



$$\begin{aligned}
 \Delta x &= \Delta x' + v \Delta t \\
 &= (x'_2 - x'_1) + v (t_2 - t_1) \\
 &= (7 - 5) \text{ m} + (25 \text{ m/s})(1.0 \text{ s}) \\
 &= 27 \text{ m}
 \end{aligned}$$

Galilean Relativity Principle

The laws of **mechanics** are the same in all inertial frames of reference.

Transforming Laws of Physics

Example: conservation of linear momentum in a one-dimensional collision of two particles

Relative to frame S : $m_1 u_1 + m_2 u_2 = m_1 u_3 + m_2 u_4$
(before coll.) (after coll.)

But $u = u' + v$, and so $u_1 = u_1' + v$, etc.

$$\therefore m_1(u_1' + v) + m_2(u_2' + v) = m_1(u_3' + v) + m_2(u_4' + v)$$

$$m_1 u_1' + m_1 v + m_2 u_2' + m_2 v = m_1 u_3' + m_1 v + m_2 u_4' + m_2 v$$

$$m_1 u_1' + m_2 u_2' = m_1 u_3' + m_2 u_4'$$

\therefore linear momentum is conserved relative to S' as well

Major difficulty

Maxwell's equations re: electricity & magnetism
don't transform into Maxwell's equations when
the Galilean transformation equations are used.

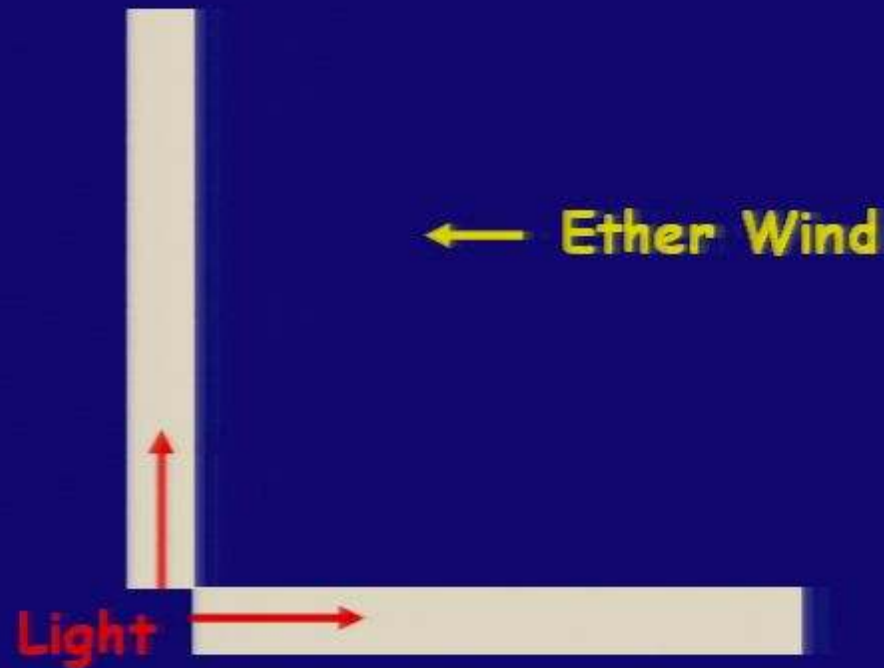
From Maxwell's equations,
the speed of electromagnetic radiation (light) in vacuum is

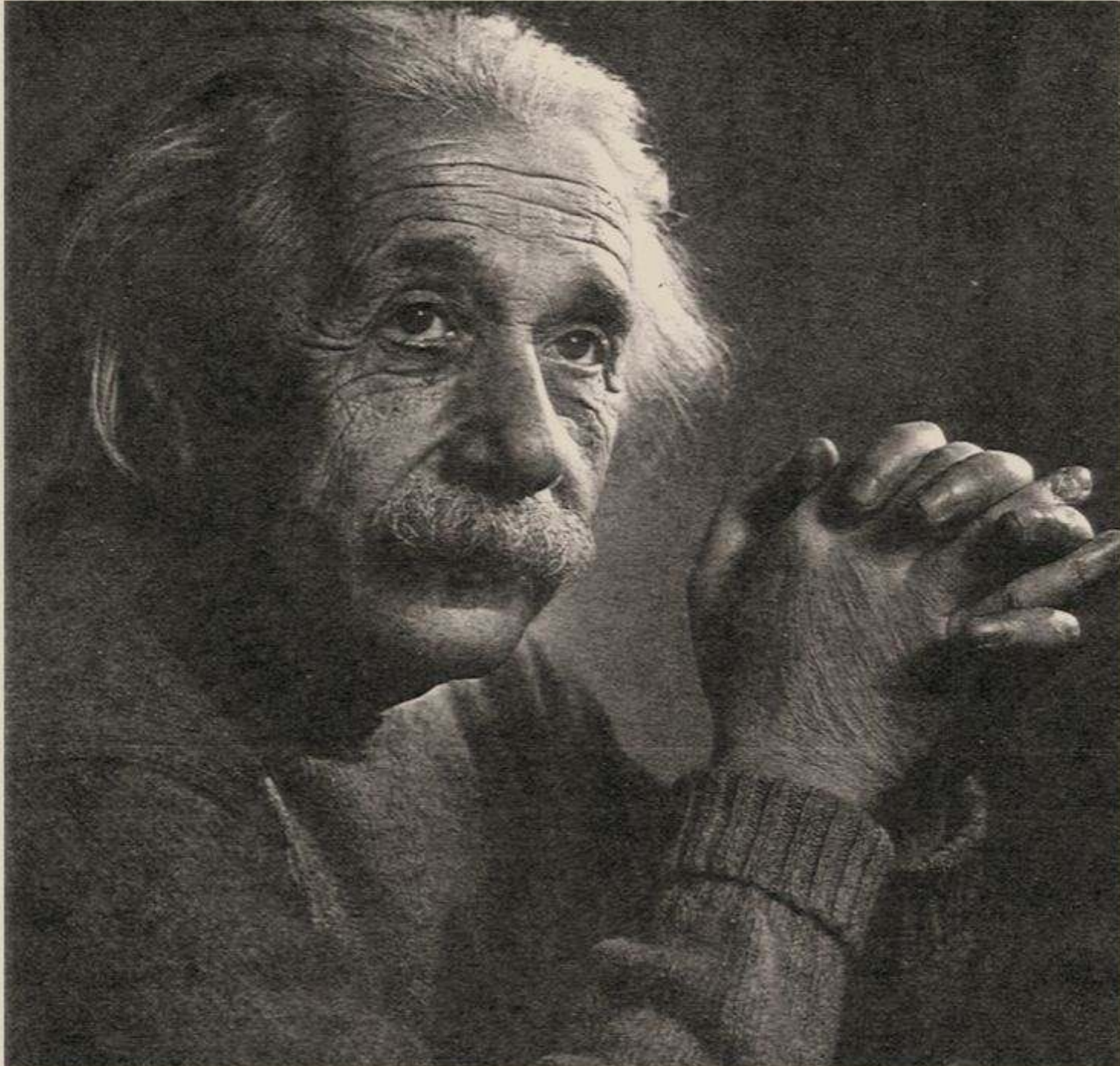
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

relative to ???



Michelson-Morley Experiment (1887)

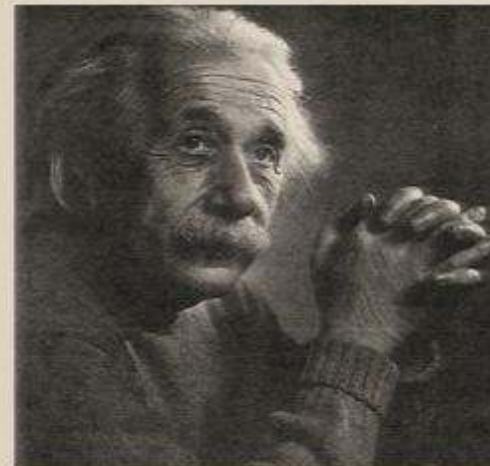




Overview

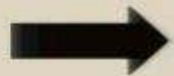
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Einstein's postulates:



1. **All** the laws of physics are the same in all inertial frames of reference.
2. The speed of light in vacuum is the same in all inertial frames of references.

Einstein's postulates



- ① Lorentz-Einstein transformations
(laws of mechanics and E & M transform into themselves relative to any inertial frame)
- ② explanation of Michelson-Morley experiment

MAX BORN

THE
BORN-EINSTEIN
LETTERS
1916-1955

Friendship, Politics and Physics
in Uncertain Times



Introduction by Werner Heisenberg Foreword by Bertrand Russell

New Preface by Diana Buchwald and Kip Thorne

Relative to any one frame, the calculation of positions, velocities, times, etc., is straightforward, even for rapidly moving objects.



Pitfall-avoidance Tip:

Useful practice --

questions involving positions, velocities,

and times, **relative to only one frame of**

reference, for rapidly moving objects.

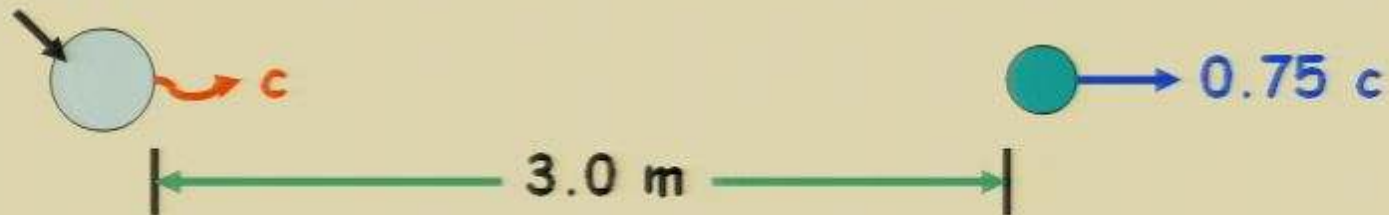
A nucleus, stationary relative to Earth, emits a gamma ray that is later absorbed by a second nucleus moving away from the emitting nucleus with a constant speed of $0.75 c$ relative to Earth. If the nuclei are separated by 3.0 m (in the Earth frame) when the gamma ray is emitted,

- how long (in the Earth frame) is the gamma ray in flight?
- how far (in the Earth frame) does the gamma ray travel?

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- (a) how long (in the Earth frame) is the gamma ray in flight?
- (b) how far (in the Earth frame) does the gamma ray travel?

nucleus
at rest



relative to Earth

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Galilean transformation equations for spatial and temporal separations:

$$\Delta x = \Delta x' + v\Delta t \quad \text{or} \quad \Delta x' = \Delta x - v\Delta t$$

$$\Delta t = \Delta t'$$

Lorentz-Einstein transformations:

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$

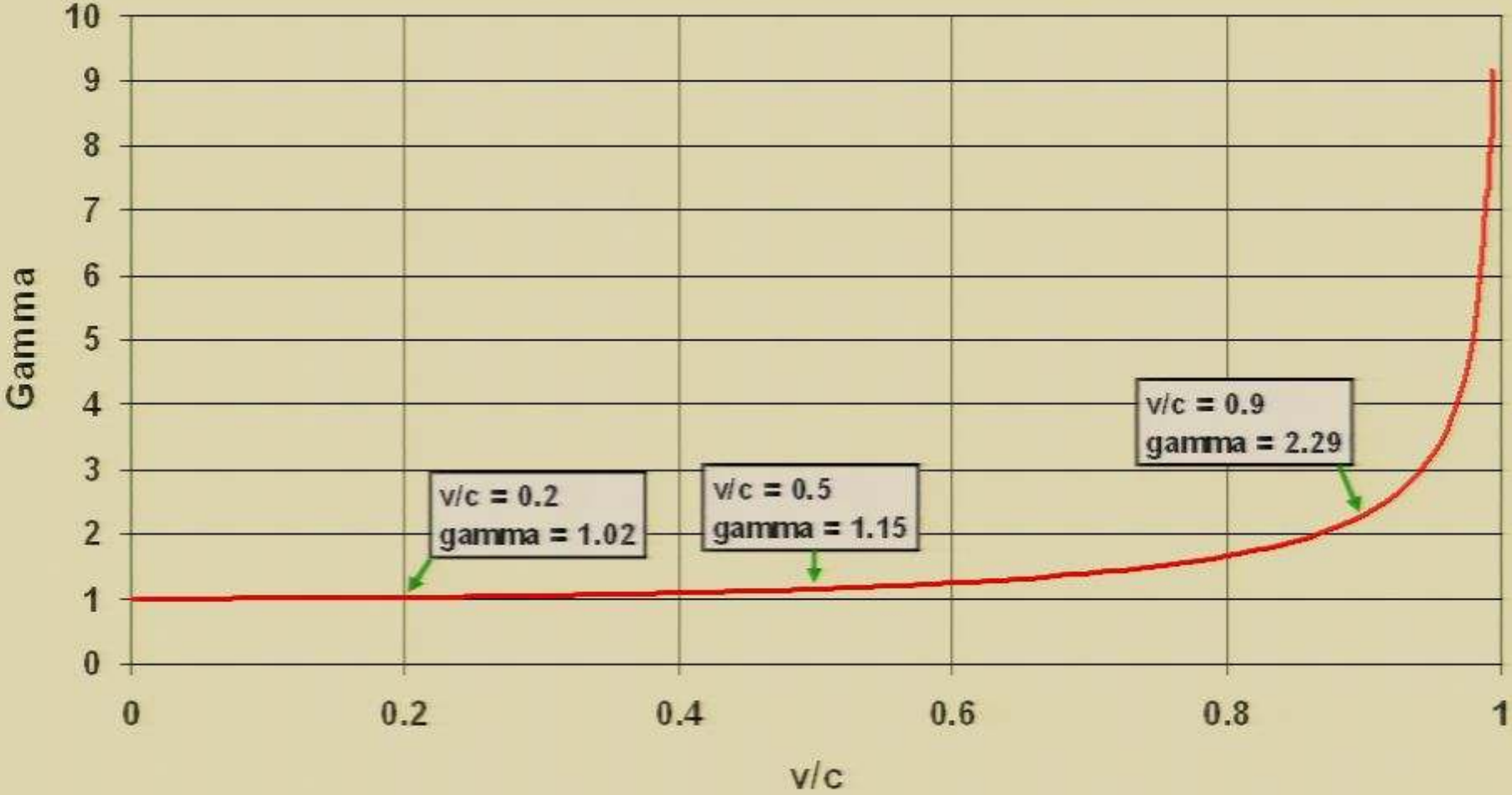
$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$$

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Gamma vs. v/c



Ideas about science from essays, exams and classroom discussions from fifth- and sixth-graders at Jefferson Elementary School in St. Louis, Missouri:

- Vacuums are nothings. We only mention them to let them know we know they're there.

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- You can listen to thunder after lightning and tell how close you got to getting hit. If you don't hear it you got hit, so never mind.
- Some people can tell what time it is by looking at the sun. But I have never been able to make out the numbers.

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Gedanken Experiments



Relativity of Time



Suppose that the spatial and temporal separations (Δx and Δt) between two events are measured relative to S .

Relative to S' , the temporal separation is $\Delta t'$.

Will $\Delta t'$ be larger, smaller, or equal to Δt ??

From $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$,
 $\Delta t'$ could be larger, smaller, or equal to Δt !!

It depends on the sizes of v (and hence γ), Δt , and Δx .



Time Dilation



Suppose that two events occur at the **same position** relative to S , and the temporal separation (relative to S) between the events is Δt .

$$\Delta x = 0 \text{ (same position)}. \quad \text{Therefore, } \Delta t' = \gamma(\Delta t - v\Delta x/c^2) \\ = \gamma\Delta t > \Delta t$$

Suppose that two events occur at the **same position** relative to S' , and the temporal separation (relative to S') between the events is $\Delta t'$.

$$\Delta x' = 0 \text{ (same position)}. \quad \text{Therefore, } \Delta t = \gamma(\Delta t' + v\Delta x'/c^2) \\ = \gamma\Delta t' > \Delta t'$$



Time Dilation



When two events occur at the **same position** relative to one frame, with a time interval ΔT_0 (subscript "0" for zero spatial separation), the time interval between these two events relative to any other frame is $\Delta T = \gamma \Delta T_0 > \Delta T_0$.

This is time dilation.

Easy-to-remember tip:

two-position time (interval) > one-position time (interval)

A useful style of question:

In which of the following situations is the time interval between the two events, as measured relative to one frame, equal to γ multiplied by the time interval measured relative to the other frame?

(a) A high-energy electron is emitted (event #1) by a distant star, which is moving rapidly away from Earth. The electron, however, is moving toward Earth and is detected upon arrival there (event #2). The frames are the star and Earth.

(b) . . .

(c) . . .

A useful style of question:

In which of the following situations is the time interval between the two events, as measured relative to one frame, equal to γ multiplied by the time interval measured relative to the other frame?

- (a) A high-energy electron is emitted (event #1) by a distant star, which is moving rapidly away from Earth. The electron, however, is moving toward Earth and is captured upon arrival there (event #2). The frames are the rest frame of the star and Earth.

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Pitfall-avoidance Tips:

It is worth emphasizing that time dilation
is a special case.

Avoid "moving clocks run slow."

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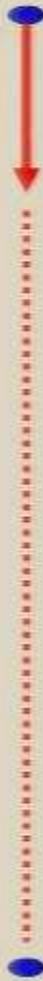
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(b) . . .

(c) . . .

Relative to Earth (frame S)

Muon (frame S') created here (event #1)



$v = 0.995 c$
 $\gamma = 10$

Muon decays here (event #2)

Rel. to S':
 $\Delta t' = 2 \mu s$

Rel. to S:
 $\Delta t = 20 \mu s$

Earth (frame S)

-- a comment about questions involving high-speed
subatomic particles vs. questions involving
high-speed spaceships, sticks, etc.

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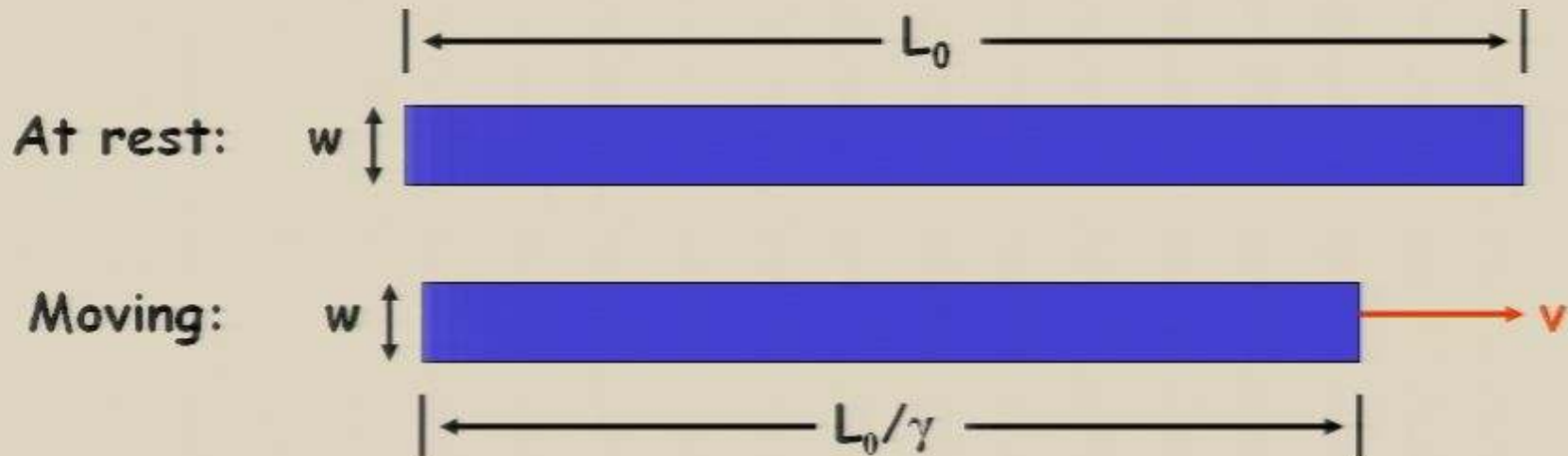


Relativity of Length



If an object at rest has a (rest) length L_0 ,
then its length when moving at speed v is L_0/γ .

This **length contraction** occurs only in the direction of motion.





Pitfall-avoidance Tip:

The spatial separation between two events may or may not correspond to the length of an object.

An electron is moving at a speed of $3c/5$ along the beam line of an accelerator. At this speed,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 5/4 = 1.25$$

As the electron travels 1.00 m as measured in the frame of reference of the beam line (or laboratory), how far does the electron travel relative to the frame of reference of the electron?

(A) 1.00 m

(B) 1.25 m

(C) 0.800 m

(D) 0.600 m

(E) 0 m

Relative to Earth (frame S)

Muon (frame S') created here
(event #1)

$$v = 0.995 c$$

$$\gamma = 10$$

Rel. to Earth (S), $L_0 = 6 \text{ km}$

$$\begin{aligned} L_0 &= v \Delta t \\ &= (0.995 c)(20 \mu\text{s}) \\ &= 6 \text{ km} \end{aligned}$$

Rel. to S':
 $\Delta t' = 2 \mu\text{s}$

Rel. to S:
 $\Delta t = 20 \mu\text{s}$

Muon decays here (event #2)

Earth (frame S)

Relative to muon (frame S')

Muon (frame S') creation and decay

$$\Delta x' = 0$$

Rel. to muon, the Earth and atm. are moving at $v = 0.995 c$, and the height of the atm. is $L = L_0/\gamma = 0.6 \text{ km}$.



$$v = 0.995 c$$
$$\gamma = 10$$

Earth (frame S)

Summary

Rel. to S' :

$$\Delta t' = 2 \mu\text{s}, \text{ and } L = 0.6 \text{ km}$$

Rel. to S :

$$\Delta t = 20 \mu\text{s}, \text{ and } L_0 = 6 \text{ km}$$

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- Galilean relativity
- Events
- Einstein's postulates
- Lorentz-Einstein transformations
- Relativity of time
- Relativity of length
- ➔ ■ What does $E = mc^2$ mean?



LEPTONS

e

$$J = \frac{1}{2}$$

Mass $m = (546.5799045 \pm 0.00000004) \times 10^{-6} \mu$

Mass $m = 0.51099892 \pm 0.00000004 \text{ MeV}$

$|m_e - m_\mu| / m < 8 \times 10^{-9} \text{ CL} = 90\%$

$|g_V - g_A| / g < 4 \times 10^{-8}$

Magnetic moment $\mu = 1.001159652127 \pm 0.000000000004 \mu_B$

$(g_V - g_A) / g_{\text{average}} = (-0.5 \pm 2.1) \times 10^{-13}$

Electric dipole moment $d = (0.07 \pm 0.07) \times 10^{-26} \text{ ecm}$

Mean life $\tau > 4.6 \times 10^{26} \text{ yr. CL} = 90\% [4]$

 μ

$$J = \frac{1}{2}$$

Mass $m = 0.113429264 \pm 0.000000000 \mu$

Mass $m = 105.658369 \pm 0.000009 \text{ MeV}$

Mean life $\tau = (2.19703 \pm 0.00004) \times 10^{-13} \text{ s}$

$\tau_{\mu \rightarrow e \gamma} / \tau = 1.00002 \pm 0.00008$

$\tau_{\mu \rightarrow e \nu \bar{\nu}} / \tau = 658.654$

Magnetic moment $\mu = 1.0011559163 \pm 0.0000000006 \mu_B / 2m_\mu$

$(g_V - g_A) / g_{\text{average}} = (-2.6 \pm 1.6) \times 10^{-8}$

Electric dipole moment $d = (3.7 \pm 3.6) \times 10^{-19} \text{ ecm}$

14 Lepton Summary Table

LEPTONS

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$$J = \frac{1}{2}$$

$$\text{Mass } m = (548.57990945 \pm 0.00000024) \times 10^{-6} \text{ u}$$

$$\text{Mass } m = 0.51099892 \pm 0.00000004 \text{ MeV}$$

$$|m_{e^-} - m_{e^+}|/m < 8 \times 10^{-9}, \text{ CL} = 90\%$$

$$|q_{e^-} + q_{e^+}|/e < 4 \times 10^{-8}$$

$$\text{Magnetic moment } \mu = 1.001159652187 \pm 0.000000000004 \mu_B$$

$$(g_{e^-} - g_{e^+}) / g_{\text{average}} = (-0.5 \pm 2.1) \times 10^{-12}$$

$$\text{Electric dipole moment } d = (0.07 \pm 0.07) \times 10^{-26} \text{ e cm}$$

$$\text{Mean life } \tau > 4.6 \times 10^{26} \text{ yr, CL} = 90\% \text{ [a]}$$

μ

$$J = \frac{1}{2}$$

$$\text{Mass } m = 0.1134289264 \pm 0.0000000030 \text{ u}$$

$$\text{Mass } m = 105.658369 \pm 0.000009 \text{ MeV}$$

$$\text{Mean life } \tau = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\tau_{\mu^-} / \tau_{\mu^+} = 1.00002 \pm 0.00008$$

$$c\tau = 658.654 \text{ m}$$

$$\text{Magnetic moment } \mu = 1.0011659160 \pm 0.0000000006 e\hbar/2m_\mu$$

$$(g_{\mu^-} - g_{\mu^+}) / g_{\text{average}} = (-2.6 \pm 1.6) \times 10^{-8}$$

$$\text{Electric dipole moment } d = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}$$

Particle
Physics
Booklet

[http://
pdg.lbl.gov](http://pdg.lbl.gov)

2005
Booklet
not yet
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For a moving object, 

$$E_{\text{TOTAL}} = mc^2 + \text{Kinetic Energy (KE)}$$

$$\text{or: } KE = E_{\text{TOTAL}} - mc^2$$

It can be shown that: $E_{\text{TOTAL}} = \gamma mc^2$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (v = \text{speed of object})$$

$$\begin{aligned} \text{Therefore, } KE &= \gamma mc^2 - mc^2 \\ &= (\gamma - 1) mc^2 \\ &\neq \frac{1}{2} mv^2 \quad \text{unless } v \ll c \end{aligned}$$

Back to: $E_{\text{TOTAL}} = \gamma mc^2$

Should γm be interpreted as the mass of the moving object, that is, can we write $E_{\text{TOTAL}} = Mc^2$? (where $M = \gamma m$)

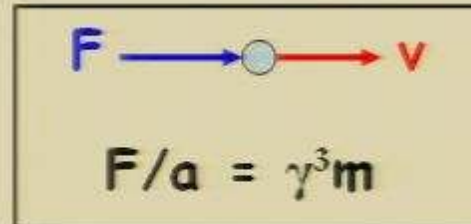
What is mass?

Two kinds:

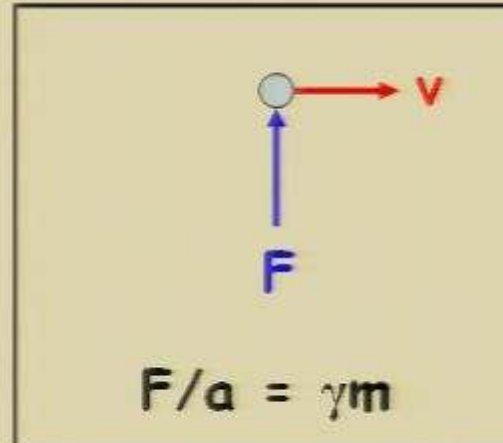
- inertial mass
- gravitational mass

Inertial mass = F/a

For high-speed objects:



But:



The inertial mass of a rapidly moving object cannot be uniquely defined.

Gravitational mass

For low-speed objects: $F = Gm_1m_2/r^2$

For rapidly moving objects, use general relativity.

→ in general it is difficult to define a relativistic gravitational mass.

The only uniquely defined mass is rest mass.

Einstein: 1948 Letter in German

1) Es ist nicht gut von der Masse $\frac{M}{\sqrt{1-\frac{v^2}{c^2}}}$ eines bewegten Körpers zu sprechen, da für M keine klare Definition gegeben werden kann. Man beschränkt sich besser auf die 'Ruhe-Masse' m . Dasselbe kann man ja den Quadraten für momentum und Energie geben, wenn man das Verhältnis erhalten muss bewegter Körper angeben will.

"It is not good to introduce the concept of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' m . Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion."

Energy and Momentum

$$E_{TOTAL}^2 = p^2 c^2 + (mc^2)^2$$

where momentum $p = \gamma m v$
for an object having rest mass m

For photons, $m = 0$

Therefore, $E_{TOTAL} = pc$

(and $p = h/\lambda = hf/c$
 $\rightarrow E_{TOTAL} = hc/\lambda = hf$)

Summary -- Avoid Pitfalls by:



- ✓ having a good introduction to Galilean relativity
- ✓ asking questions involving positions, velocities, and times, relative to only one frame of reference, for rapidly moving objects
- ✓ emphasizing that time dilation is a special case
- ✓ avoiding “moving clocks run slow”
- ✓ remembering that spatial separation might not correspond to an object’s length
- ✓ following Einstein’s advice about mass

References:

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E. McFarland, *Einstein's Special Relativity: Discover it for Yourself*, Trifolium Books, 1998
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