Title: World of Wonders: Special Relativity

Date: May 25, 2006 08:00 PM

URL: http://pirsa.org/06050013

Abstract: <kw> teaching relativity, Galilean relativity, Einstein\'s postulates, Lorentz-Einstein transformations, time, length, Galilean transformation equations, spatial separating, temporal separation, laws of mechanics, Maxwell\'s equations, Michelson-Morley </kw>

World of Wonders: Special Relativity

(avoiding pitfalls when teaching special relativity)

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UNIVERSITY of GUELPH

OAPT Conference 2006
Perimeter Institute

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Overview

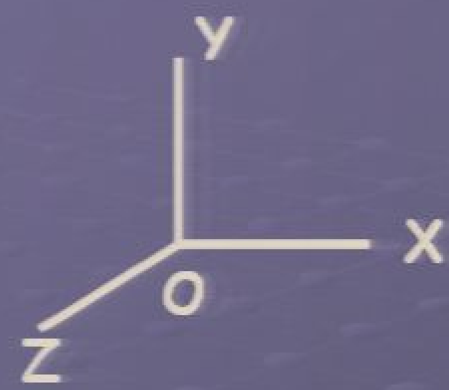
- Galilean relativity
- Events
- Einstein's postulates
- Lorentz-Einstein transformations
- Relativity of time
- Relativity of length
- What does $E = mc^2$ mean?

Pitfall-avoidance Tip:

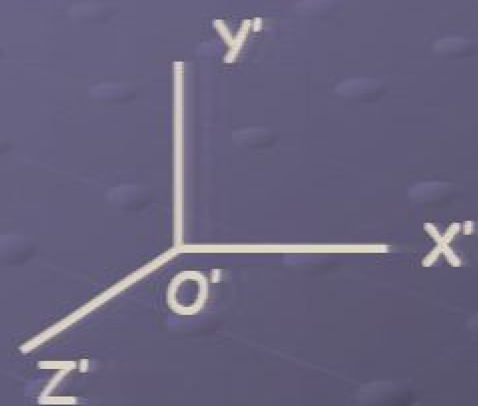
A good introduction to Galilean relativity is very useful before special relativity is started.

Galilean Relativity

Frame S (inertial frame)



Frame S'

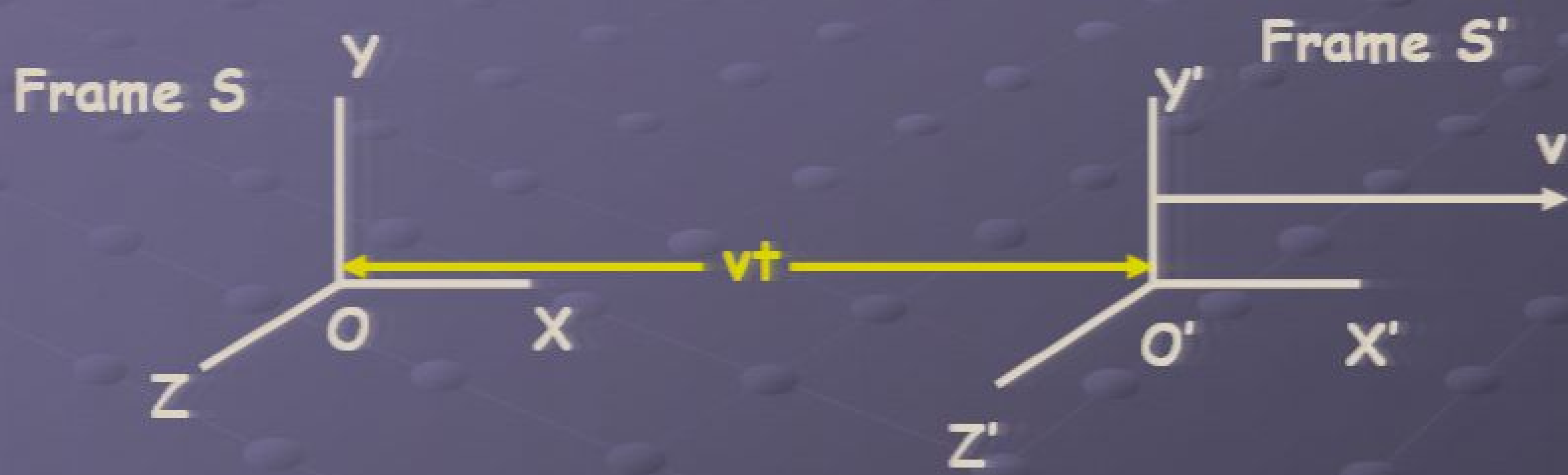


Frame S' moves with constant speed v in the +X direction relative to frame S.

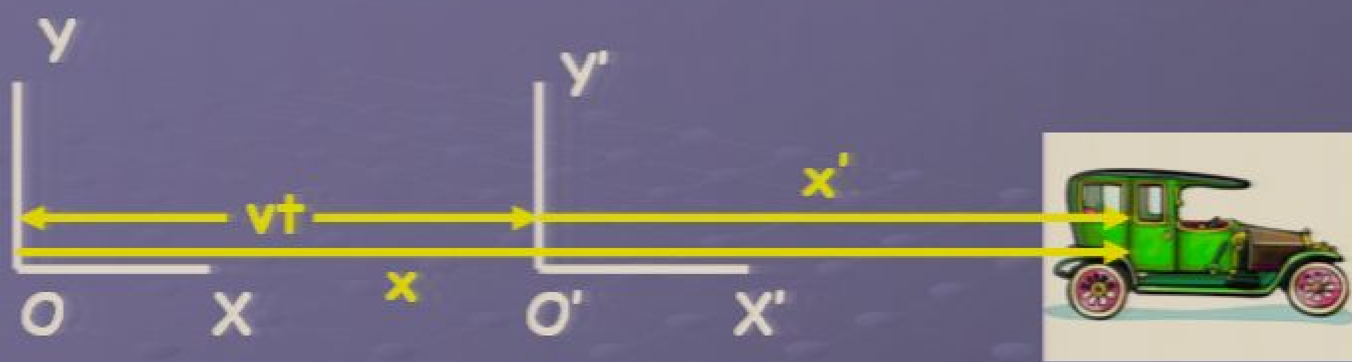
Galilean Relativity

For simplicity, assume that the origins of the two frames coincide at time t = 0.

At time t, *O'* will have moved a distance of vt from *O*.



Galilean Transformation Equations



At time t, the position of the object (car) is:
x relative to (the origin of) frame S, and x' relative to (the origin of) frame S'.

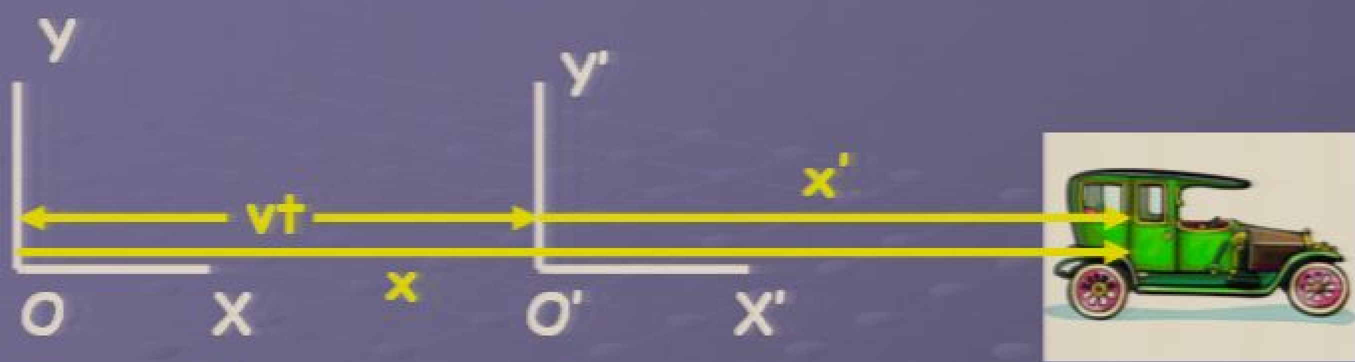
From the drawing, $x = x' + vt$

Taking time derivatives: $dx/dt = dx'/dt + d(vt)/dt$

or: $u = u' + v$

where u and u' are the velocities of the car relative to S and S', respectively.

Galilean Transformation Equations



So far: $x = x' + vt$

and $u = u' + v$

Take another time derivative:

$du/dt = du'/dt + dv/dt$

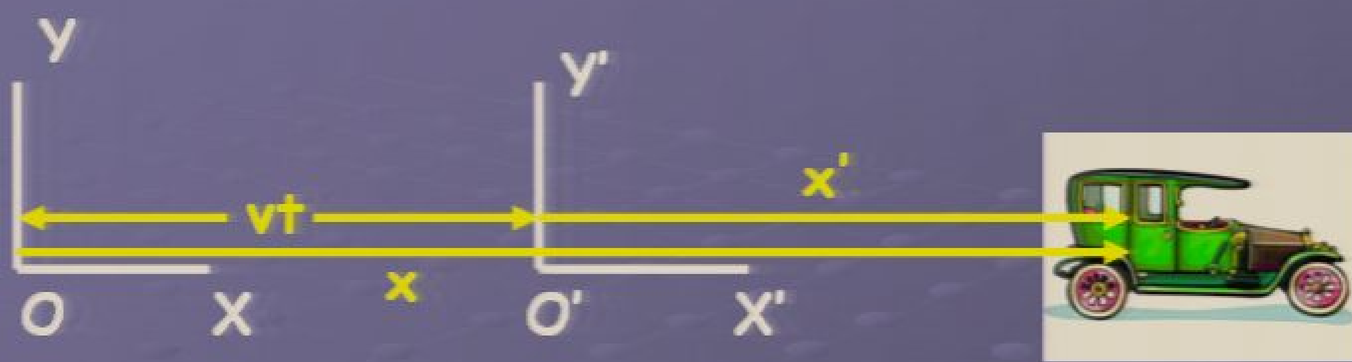
v is constant $\rightarrow$ $dv/dt = 0$.

Therefore: $du/dt = du'/dt$

or: $a = a'$

where a and a' are the accelerations of the car, relative to S and S', respectively.

Galilean Transformation Equations



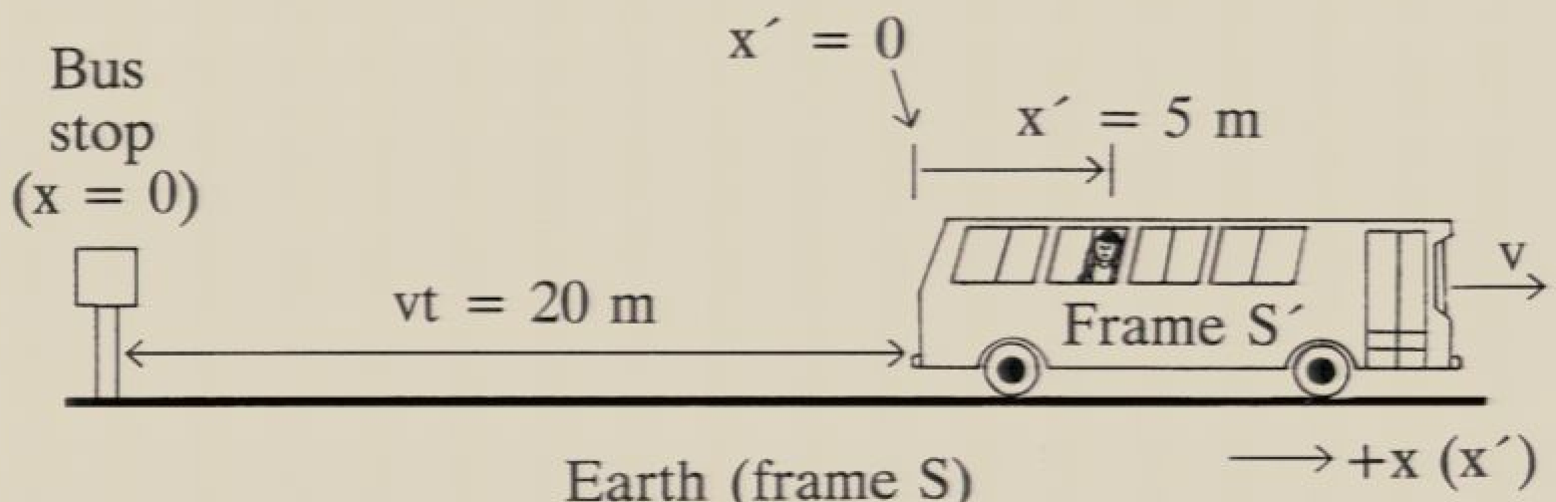
So, for the positions, velocities, and accelerations of the car:

$$x = x' + vt$$
$$u = u' + v$$
$$a = a'$$

These are the Galilean transformation equations (along with $y = y'$, $z = z'$, and $t = t'$).

A bus is travelling forward at 25 m/s relative to Earth. The rear of the bus passes a bus stop, and 0.80 s later, a woman passenger sitting 5 m from the rear of the bus stands up. How far is she from the bus stop at this time?

Define frame S (Earth) and S′ (bus).
Choose origins of the frames (see drawing). These origins coincide at time t = 0.
Choose +X,+X′ direction.
(S′ moves in +X direction with constant speed v.)



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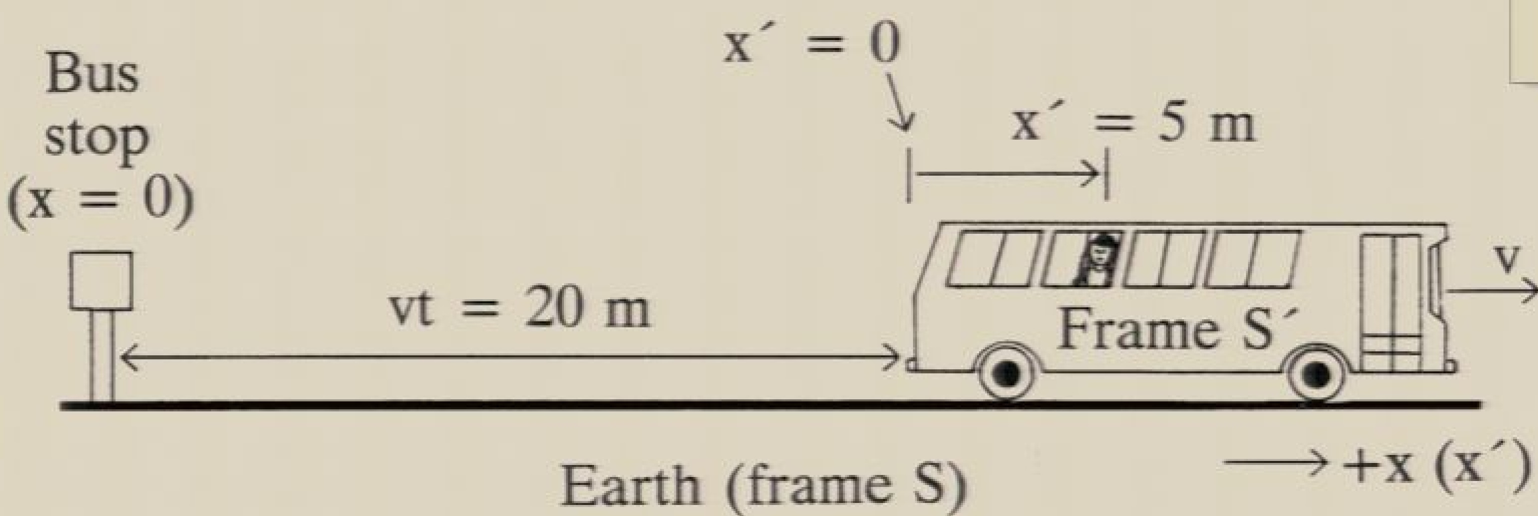

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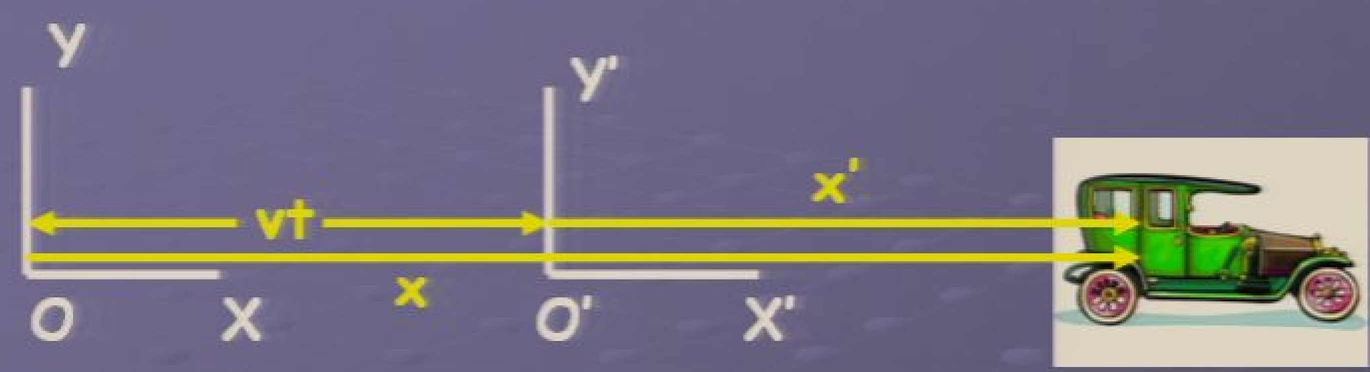

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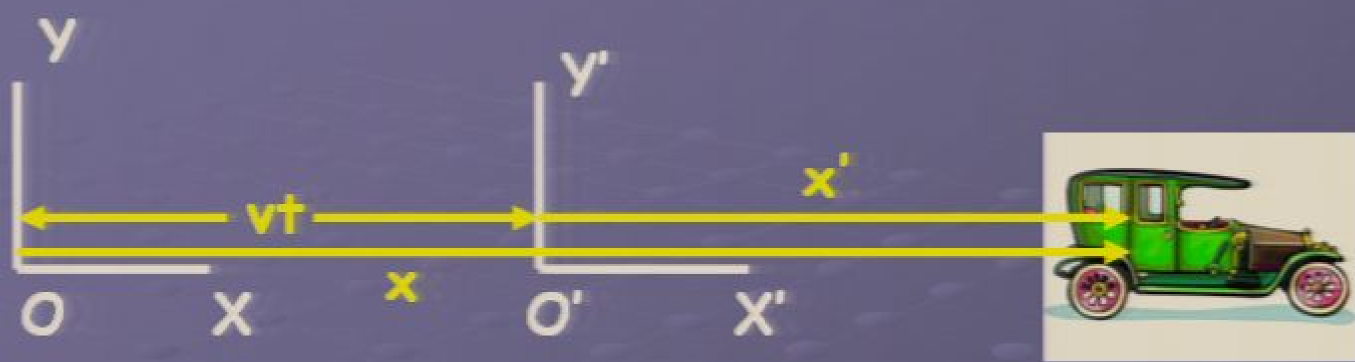

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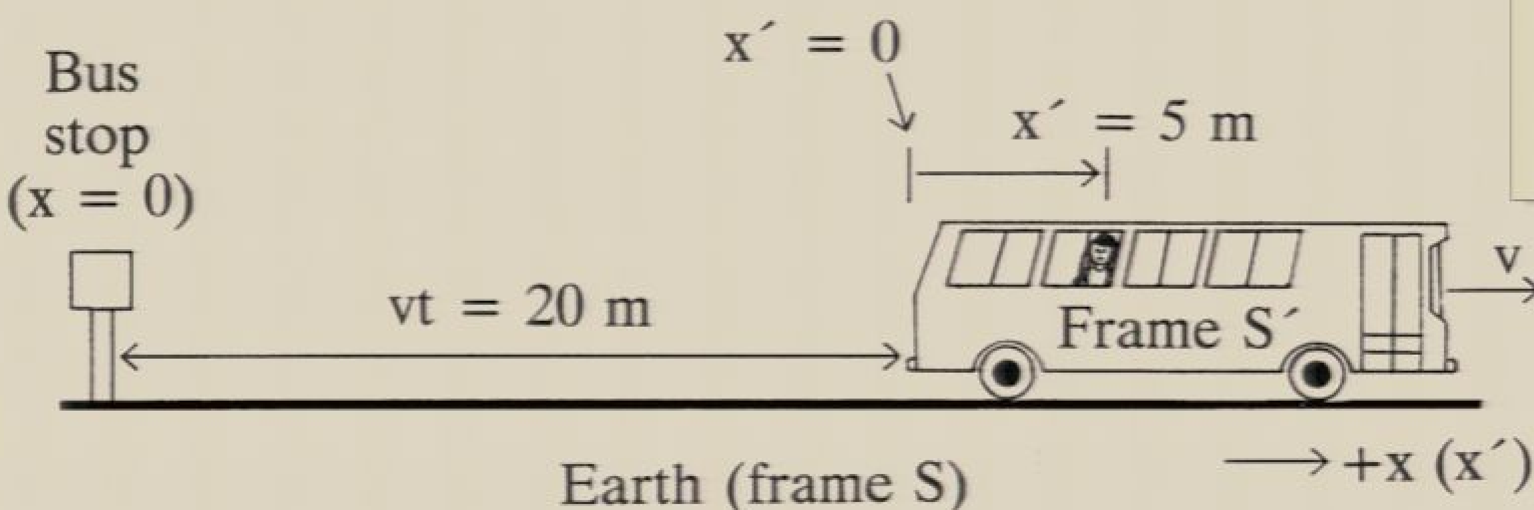

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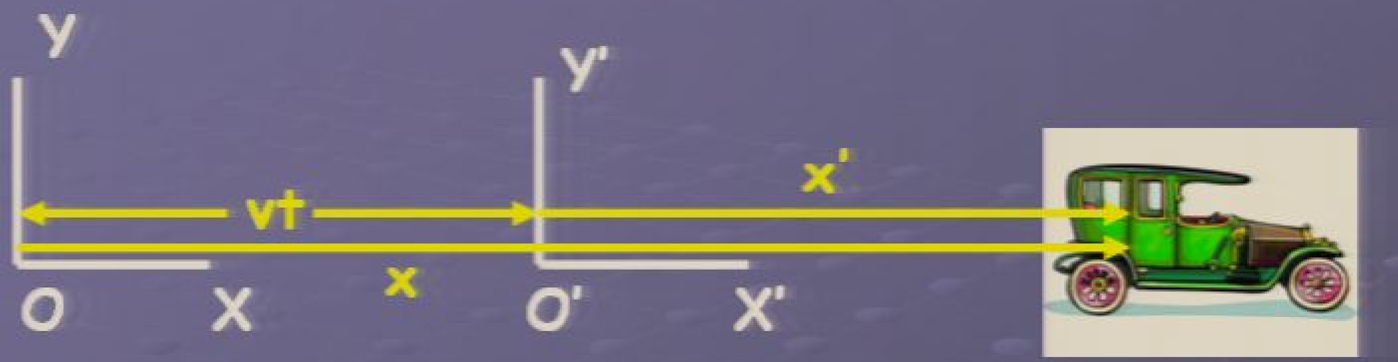

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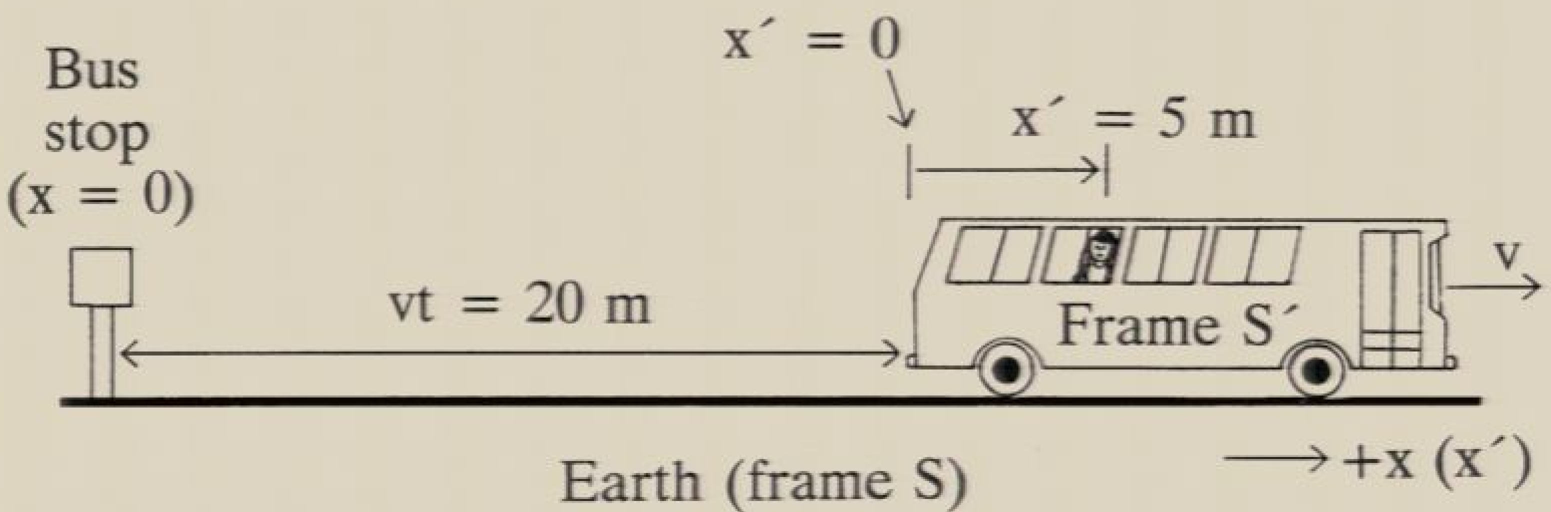

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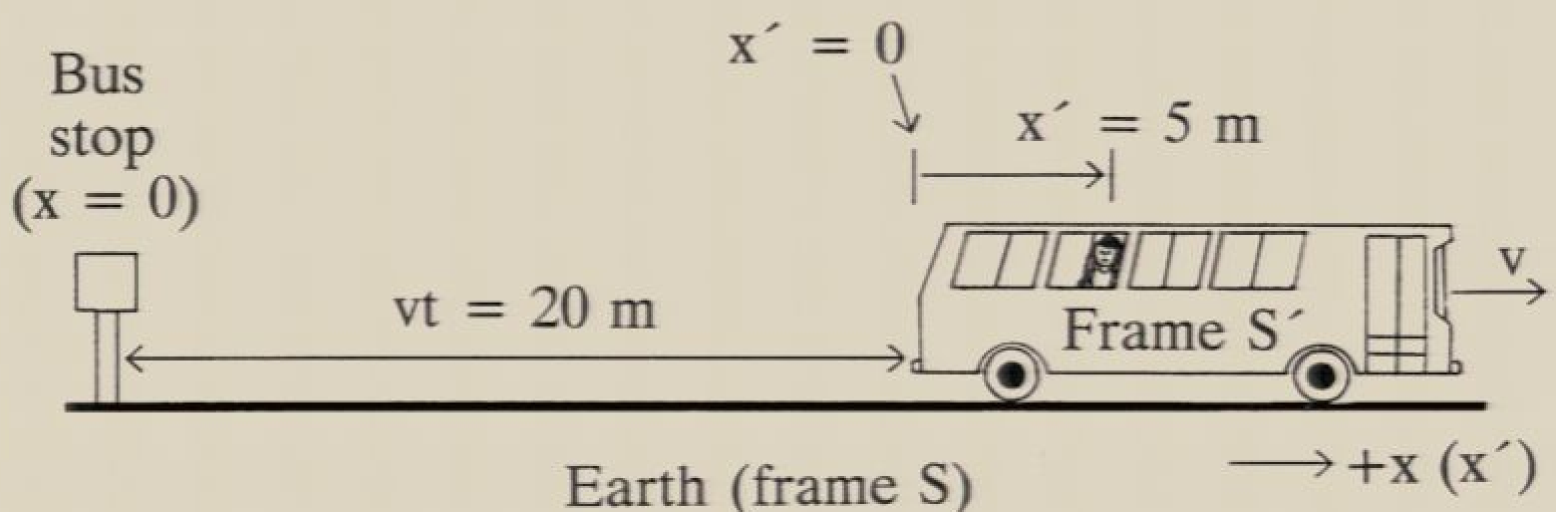
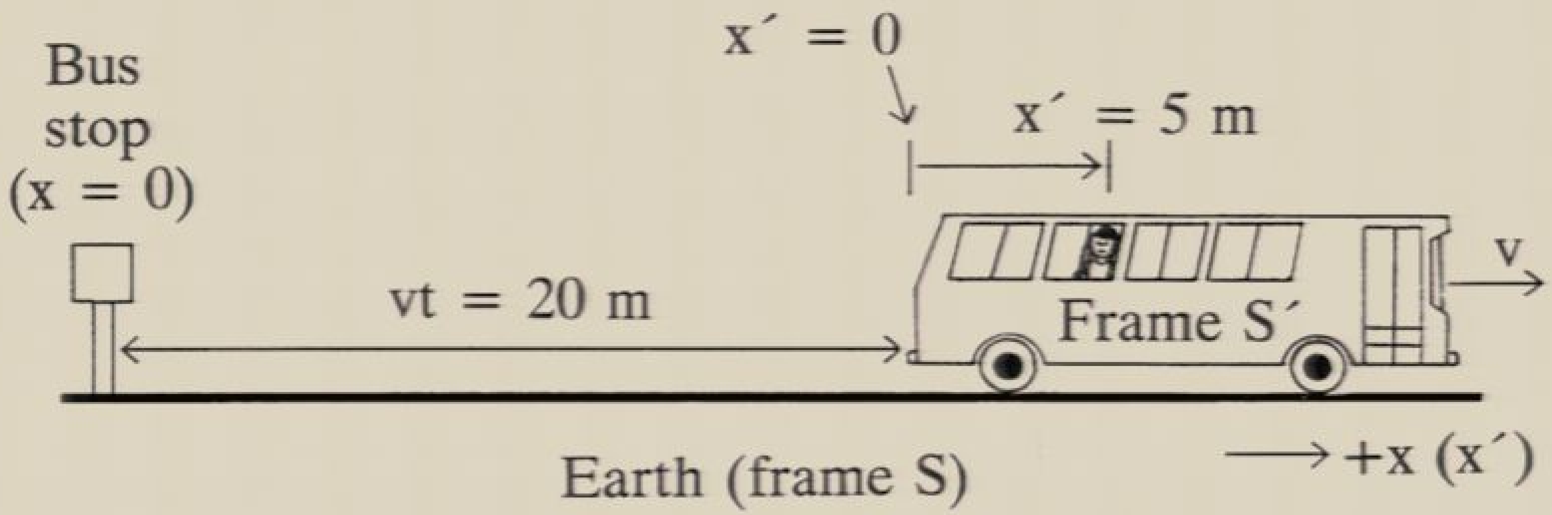


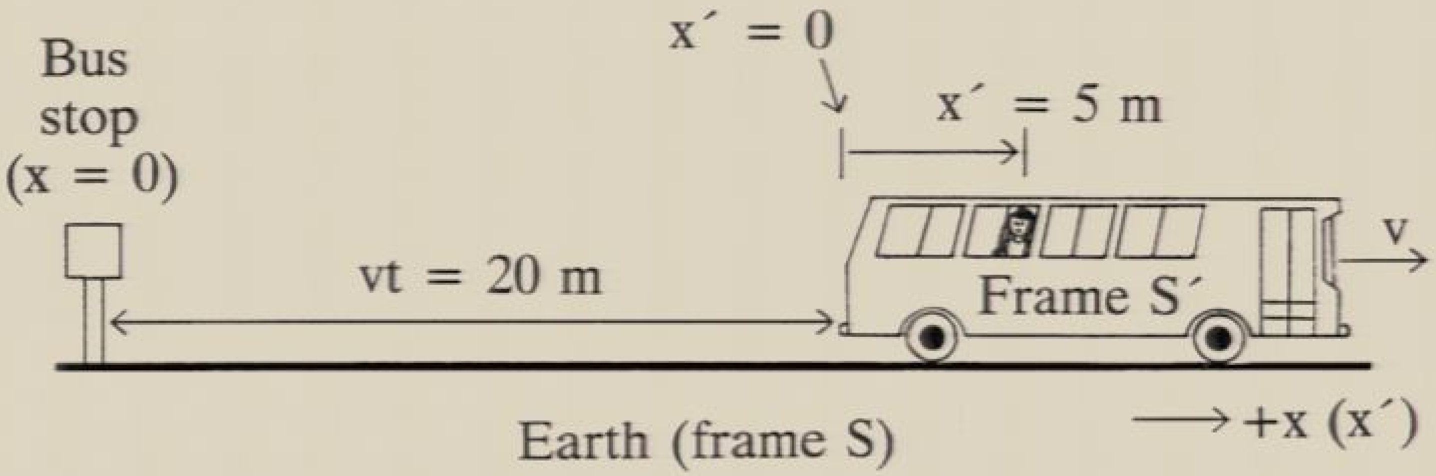


Use $x = x' + vt$

$= (5\ \text{m}) + (25\ \text{m/s})(0.80\ \text{s})$

$= 25\ \text{m}$

Therefore, the woman is 25 m from the bus stop.



As the woman is sitting on the bus, which is travelling forward at 25 m/s relative to Earth, what is her velocity

(a) relative to Earth? (b) relative to the bus?

(c) What is the velocity of Earth relative to the bus?

During a time of 0.80 s, how far does the woman move

(d) relative to Earth? (e) relative to the bus?

Overview

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EVENTS

Anything that happens at a particular time and place relative to a given frame is called an event.

Event #1 occurs at

position x_1 relative to frame S, and x_1' relative to S'.

It occurs at time t_1 relative to S and S'.

Event #2 occurs at

x_2 relative to S and x_2' relative to S', at time t_2 .

The spatial separation between the events is

$\Delta x = x_2 - x_1$ relative to S

and $\Delta x' = x_2' - x_1'$ relative to S'.

The temporal separation between the events is

$\Delta t = t_2 - t_1$ relative to S and S'.

$x = x' + vt$ can be used for each event:

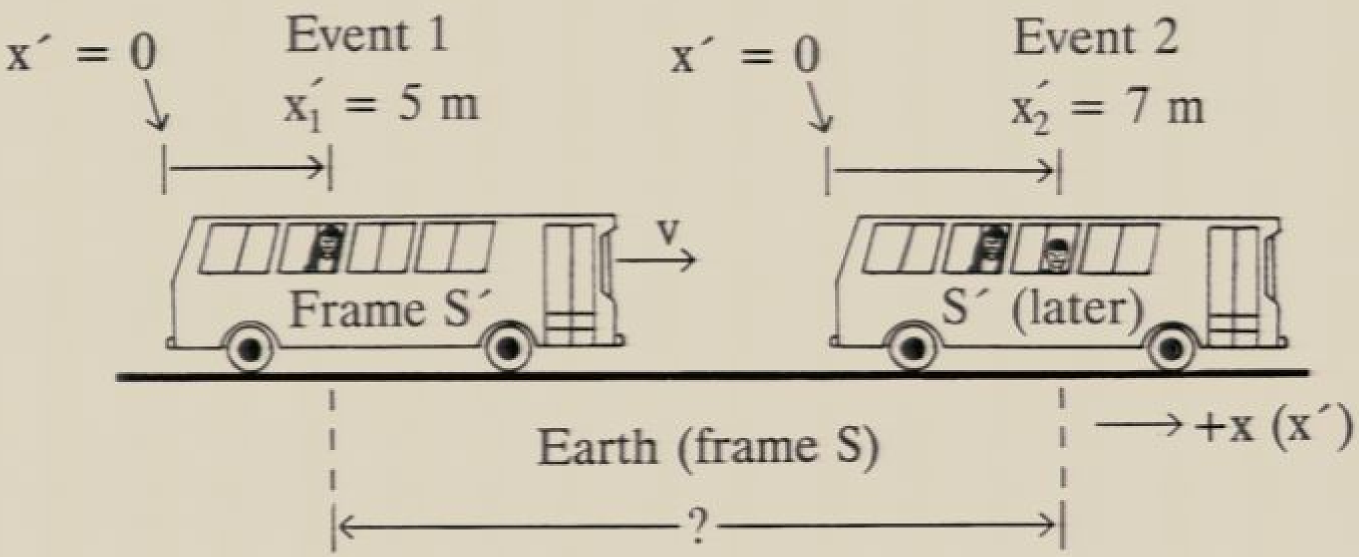
$$x_2 = x_2' + vt_2$$
$$x_1 = x_1' + vt_1$$

Subtracting: $(x_2 - x_1) = (x_2' - x_1') + v(t_2 - t_1)$

i.e., $\Delta x = \Delta x' + v\Delta t$

or: $\Delta x' = \Delta x - v\Delta t$

Back to the bus example: At a time of 1.0 s after the woman stands up, a man sitting 2 m in front of her coughs. What is the distance separating these events relative to Earth (frame S)?



$$\Delta x = \Delta x' + v\Delta t$$
$$= (x_2' - x_1') + v\,(t_2 - t_1)$$
$$= (7 - 5)\text{ m} + (25\text{ m/s})(1.0\text{ s})$$
$$= 27\text{ m}$$

Galilean Relativity Principle

The laws of mechanics are the same in all inertial frames of reference.

Transforming Laws of Physics

Example: conservation of linear momentum in a one-dimensional collision of two particles

Relative to frame S: $m_1u_1 + m_2u_2 = m_1u_3 + m_2u_4$
(before coll.) (after coll.)

But $u = u' + v$, and so $u_1 = u_1' + v$, etc.

$$\therefore m_1(u_1' + v) + m_2(u_2' + v) = m_1(u_3' + v) + m_2(u_4' + v)$$

$$m_1u_1' + m_1v + m_2u_2' + m_2v = m_1u_3' + m_1v + m_2u_4' + m_2v$$

$$m_1u_1' + m_2u_2' = m_1u_3' + m_2u_4'$$

$\therefore$ linear momentum is conserved relative to S' as well

Major difficulty

Maxwell's equations re: electricity & magnetism don't transform into Maxwell's equations when the Galilean transformation equations are used.

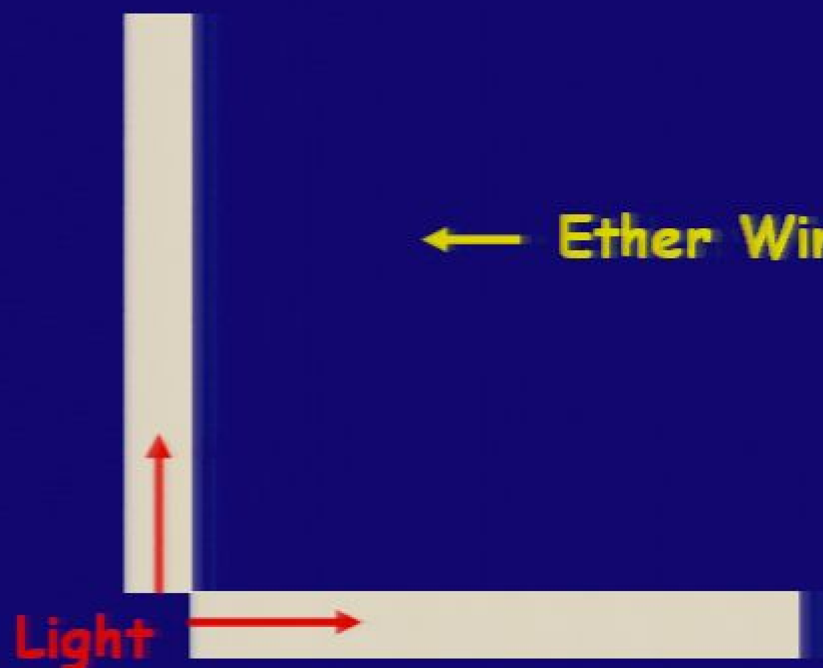
From Maxwell's equations,

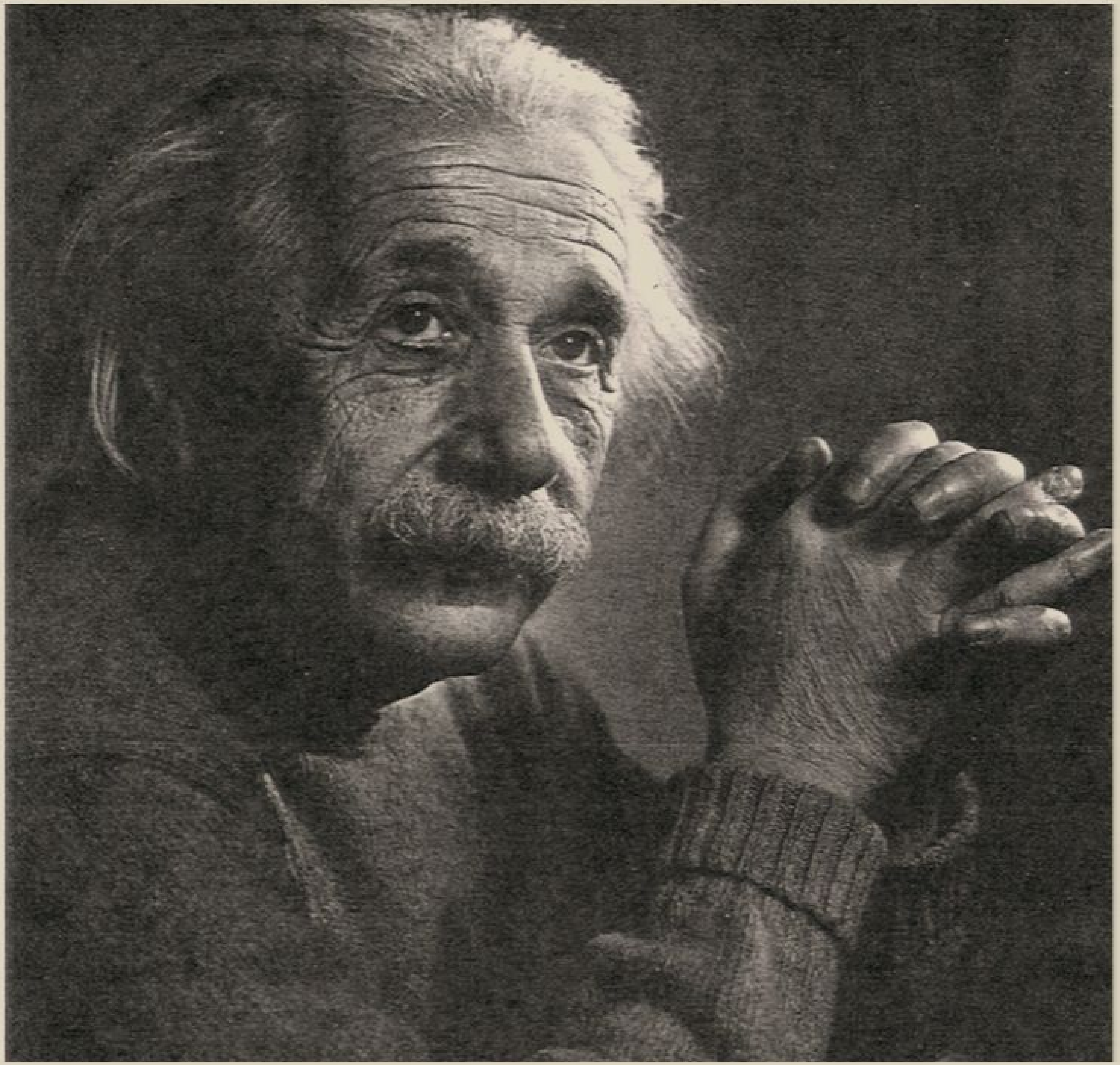
the speed of electromagnetic radiation (light) in vacuum is

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

relative to ???

Michelson-Morley Experiment (1887)





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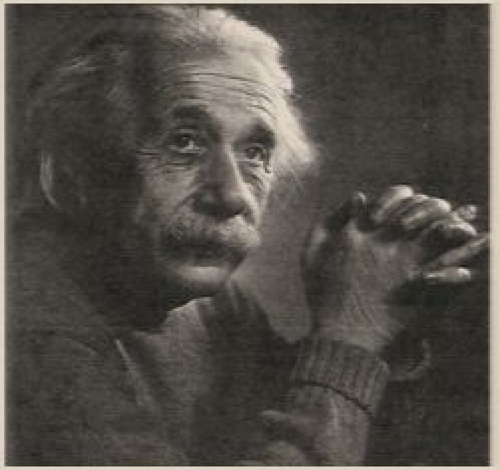
Einstein's postulates:



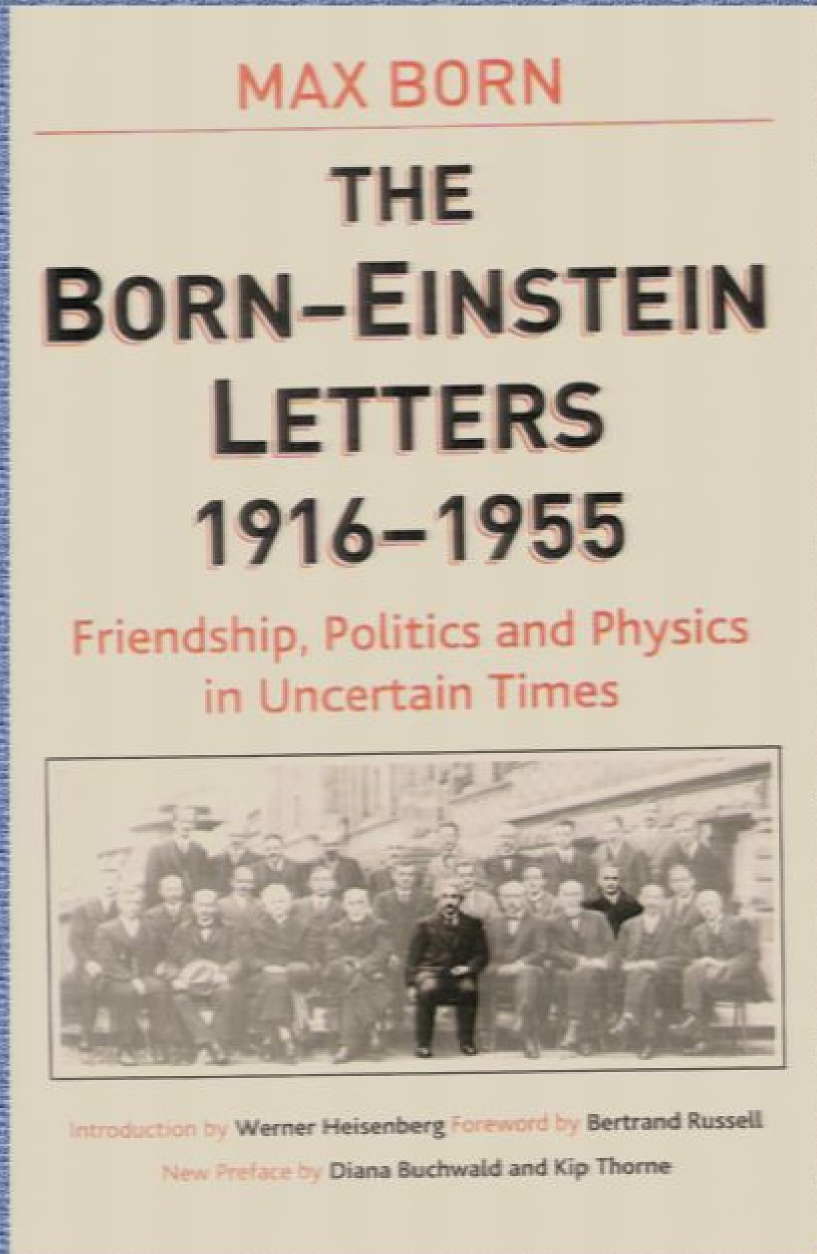
1. All the laws of physics are the same in all inertial frames of reference.

2. The speed of light in vacuum is the same in all inertial frames of references.

Einstein's postulates

➡ ❶ Lorentz-Einstein transformations
(laws of mechanics and E & M transform into themselves relative to any inertial frame)

❷ explanation of Michelson-Morley experiment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LETTERS
1916–1955
Friendship, Politics and Physics
in Uncertain Times
Introduction by Werner Heisenberg Foreword by Bertrand Russell
New Preface by Diana Buchwald and Kip Thorne

Relative to any one frame, the calculation of positions, velocities, times, etc., is straightforward, even for rapidly moving objects.

Pitfall-avoidance Tip:

Useful practice --

questions involving positions, velocities, and times, relative to only one frame of reference, for rapidly moving objects.

A nucleus, stationary relative to Earth, emits a gamma ray that is later absorbed by a second nucleus moving away from the emitting nucleus with a constant speed of 0.75 c relative to Earth. If the nuclei are separated by 3.0 m (in the Earth frame) when the gamma ray is emitted,

(a) how long (in the Earth frame) is the gamma ray in flight?

(b) how far (in the Earth frame) does the gamma ray travel?

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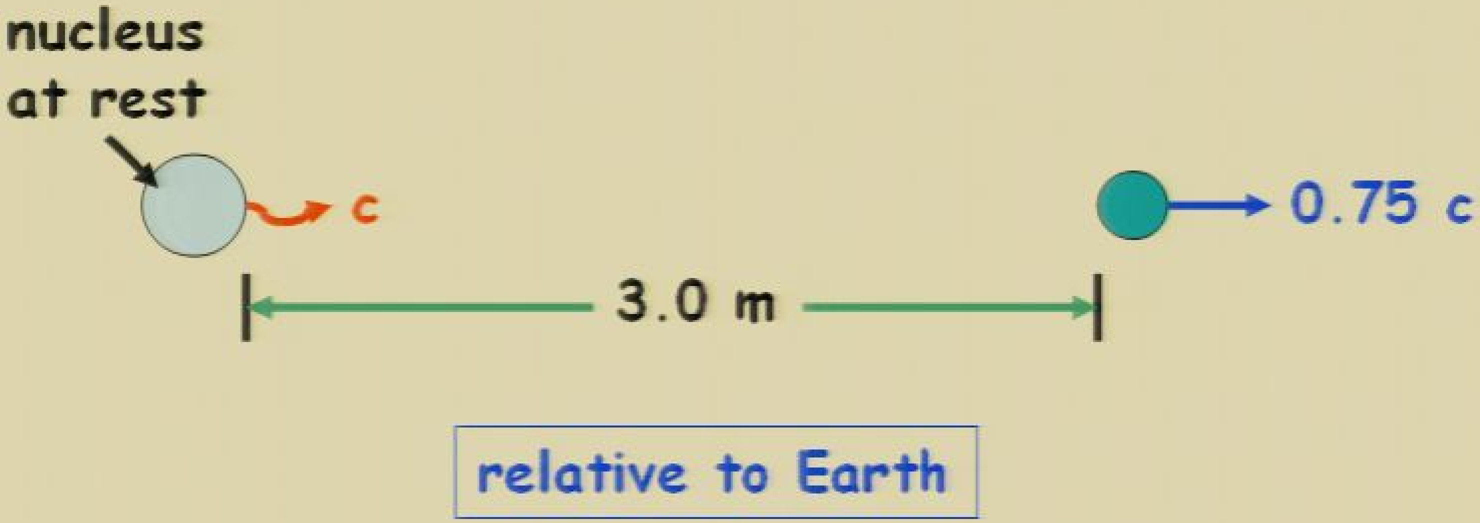

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Galilean transformation equations for spatial and temporal separations:

$$\Delta x = \Delta x' + v\Delta t \quad \text{or} \quad \Delta x' = \Delta x - v\Delta t$$

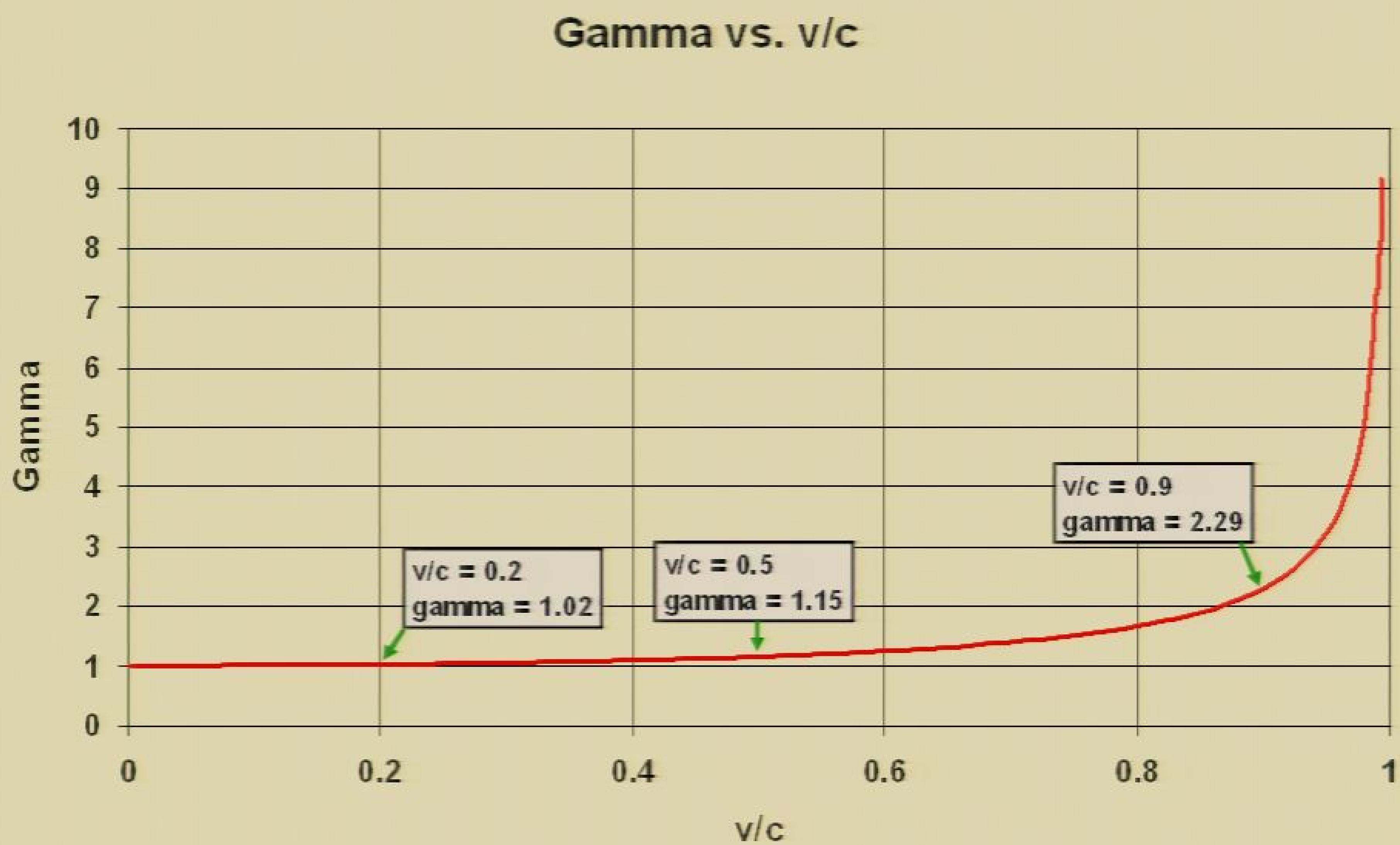
$$\Delta t = \Delta t'$$

Lorentz-Einstein transformations:

$$\Delta x = \gamma(\Delta x' + v\Delta t')$$
$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$$
$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$$

where $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$


Gamma vs. v/c
Gamma
10
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
v/c
v/c = 0.2
gamma = 1.02
v/c = 0.5
gamma = 1.15
v/c = 0.9
gamma = 2.29

Ideas about science from essays, exams and classroom discussions from fifth- and sixth-graders at Jefferson Elementary School in St. Louis, Missouri:

- Vacuums are nothings. We only mention them to let them know we know they're there.

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- I am not sure how clouds get formed, but the clouds know how to do it, and that is the important thing.

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- You can listen to thunder after lightning and tell how close you got to getting hit. If you don't hear it you got hit, so never mind.
- Some people can tell what time it is by looking at the sun. But I have never been able to make out the numbers.

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Gedanken Experiments

Relativity of Time

Suppose that the spatial and temporal separations (Δx and Δt) between two events are measured relative to S.

Relative to S′, the temporal separation is $\Delta t'$.

Will $\Delta t'$ be larger, smaller, or equal to Δt??

From $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$,
$\Delta t'$ could be larger, smaller, or equal to Δt !!

It depends on the sizes of v (and hence γ), Δt, and Δx.

Time Dilation

Suppose that two events occur at the same position relative to S, and the temporal separation (relative to S) between the events is Δt.

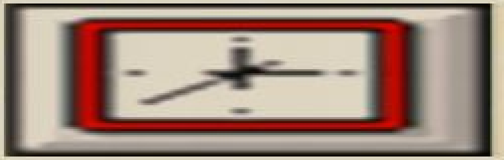
$\Delta x = 0$ (same position). Therefore, $\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$

$$= \gamma\Delta t > \Delta t$$

Suppose that two events occur at the same position relative to S', and the temporal separation (relative to S') between the events is $\Delta t'$.

$\Delta x' = 0$ (same position). Therefore, $\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$

$$= \gamma\Delta t' > \Delta t'$$



Time Dilation

When two events occur at the same position relative to one frame, with a time interval ΔT_0 (subscript "0" for zero spatial separation), the time interval between these two events relative to any other frame is $\Delta T = \gamma \Delta T_0 > \Delta T_0$.

This is time dilation.

Easy-to-remember tip:

two-position time (interval) > one-position time (interval)

A useful style of question:

In which of the following situations is the time interval between the two events, as measured relative to one frame, equal to γ multiplied by the time interval measured relative to the other frame?

(a) A high-energy electron is emitted (event #1) by a distant star, which is moving rapidly away from Earth. The electron, however, is moving toward Earth and is detected upon arrival there (event #2). The frames are the star and Earth.

(b) . . .

(c) . . .

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Pitfall-avoidance Tips:

It is worth emphasizing that time dilation

is a special case.

Avoid "moving clocks run slow."

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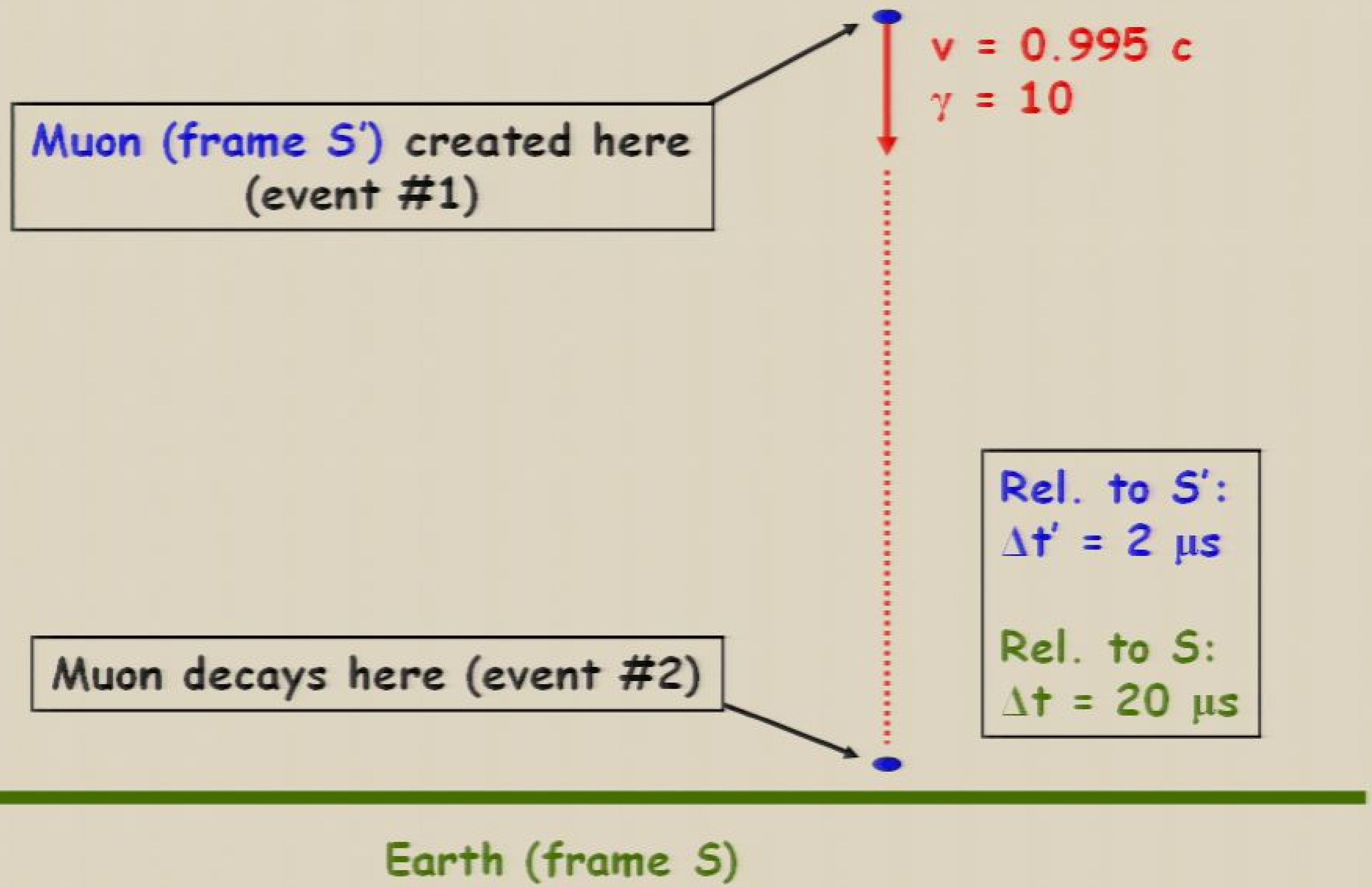
Relative to Earth (frame S)



-- a comment about questions involving high-speed subatomic particles vs. questions involving high-speed spaceships, sticks, etc.

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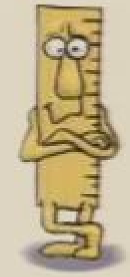
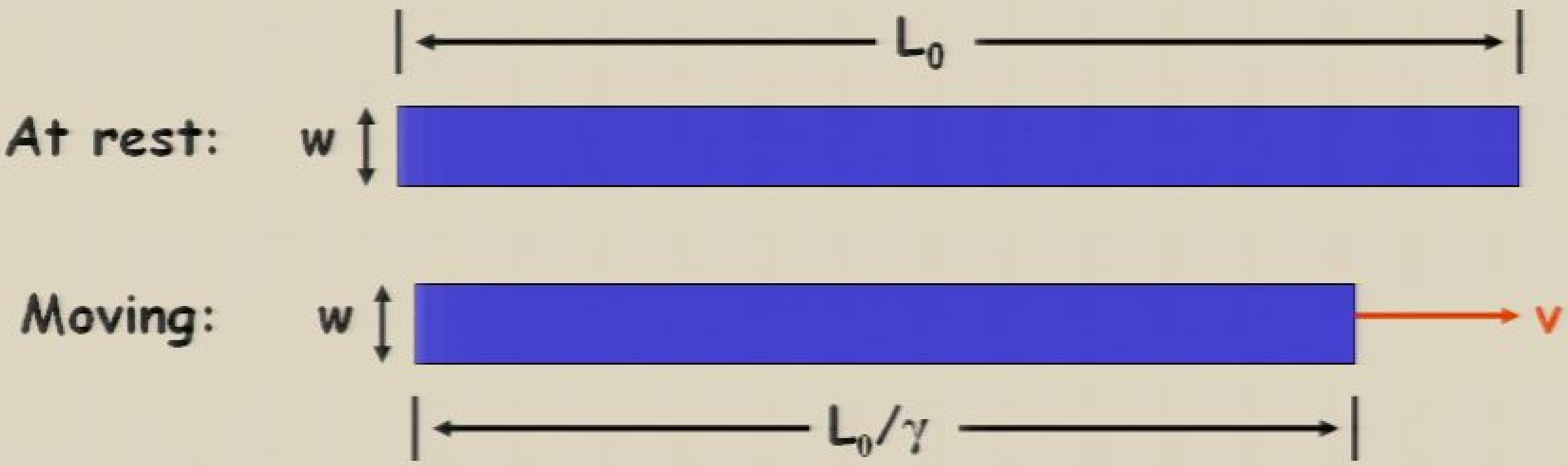
Relativity of Length



If an object at rest has a (rest) length L_0 ,

then its length when moving at speed v is L_0/γ .

This length contraction occurs only in the direction of motion.



Pitfall-avoidance Tip:

The spatial separation between two events may or may not correspond to the length of an object.

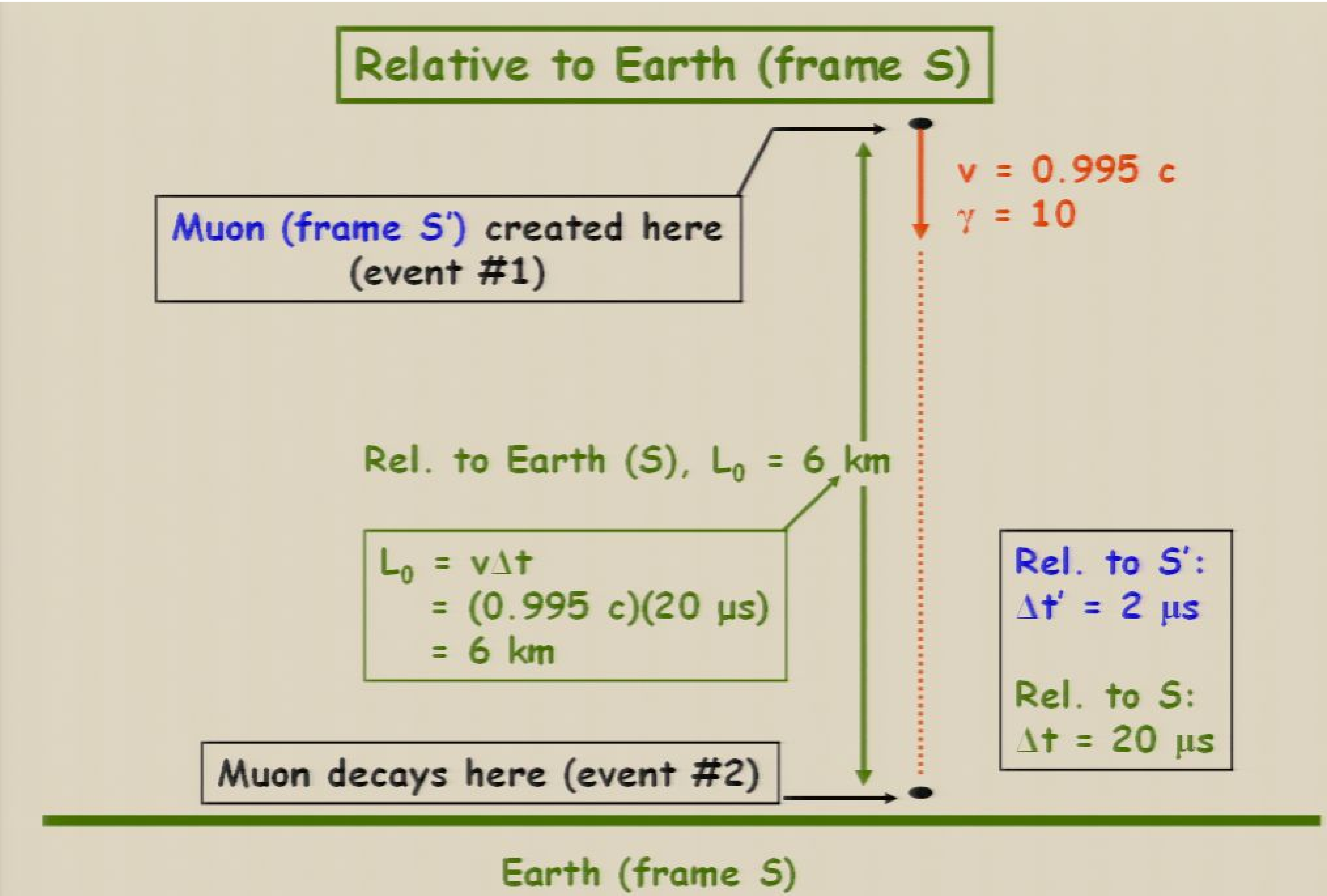
An electron is moving at a speed of 3c/5 along the beam line of an accelerator. At this speed,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 5/4 = 1.25$$

As the electron travels 1.00 m as measured in the frame of reference of the beam line (or laboratory), how far does the electron travel relative to the frame of reference of the electron?

(A) 1.00 m (B) 1.25 m (C) 0.800 m

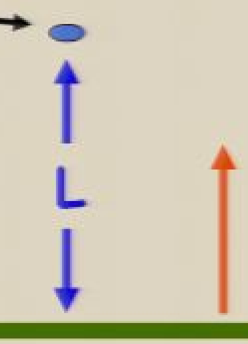
(D) 0.600 m (E) 0 m


Relative to Earth (frame S)
Muon (frame S') created here (event #1)
v = 0.995 c
γ = 10
Rel. to Earth (S), L0 = 6 km
L0 = vΔt
= (0.995 c)(20 μs)
= 6 km
Rel. to S':
Δt' = 2 μs
Rel. to S:
Δt = 20 μs
Muon decays here (event #2)
Earth (frame S)

Relative to muon (frame S')

Muon (frame S') creation and decay
$\Delta x' = 0$

Rel. to muon, the Earth and atm. are moving at $v = 0.995\,c$, and the height of the atm. is $L = L_0/\gamma = 0.6$ km.



$v = 0.995\,c$
$\gamma = 10$

Earth (frame S)

Summary

Rel. to S':
$\Delta t' = 2\ \mu s$, and $L = 0.6$ km

Rel. to S:
$\Delta t = 20\ \mu s$, and $L_0 = 6$ km

Overview

- Galilean relativity
- Events
- Einstein's postulates
- Lorentz-Einstein transformations
- Relativity of time
- Relativity of length
- What does $E = mc^2$ mean?


E=ma²
E=mb²
S. Harris

LEPTONS

e

$J = \frac{1}{2}$

Mass $m = (548.57990945 \pm 0.00000024) \times 10^{-6}$ u
Mass $m = 0.51099892 \pm 0.00000004$ MeV
$|m_{e^+} - m_{e^-}| / m < 8 \times 10^{-9}$, CL = 90%
$|q_{e^+} + q_{e^-}| / e < 4 \times 10^{-8}$
Magnetic moment $\mu = 1.001159652187 \pm 0.000000000004\ \mu_B$
$(g_{e^+} - g_{e^-}) / g_{average} = (-0.5 \pm 2.1) \times 10^{-12}$
Electric dipole moment $d = (0.07 \pm 0.07) \times 10^{-26}$ e cm
Mean life $\tau > 4.6 \times 10^{26}$ yr, CL = 90% [d]

μ

$J = \frac{1}{2}$

Mass $m = 0.1134289264 \pm 0.0000000030$ u
Mass $m = 105.658369 \pm 0.000009$ MeV
Mean life $\tau = (2.19703 \pm 0.00004) \times 10^{-6}$ s
$\tau_{\mu^+} / \tau_{\mu^-} = 1.00002 \pm 0.00008$
$c\tau = 658.654$ m
Magnetic moment $\mu = 1.0011659160 \pm 0.0000000006\ e\hbar/2m_\mu$
$(g_{\mu^+} - g_{\mu^-}) / g_{average} = (-2.6 \pm 1.6) \times 10^{-8}$
Electric dipole moment $d = (3.7 \pm 3.4) \times 10^{-19}$ e cm

LEPTONS

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Particle Physics Booklet

http://pdg.lbl.gov

2005 Booklet not yet available (try in Sept.)

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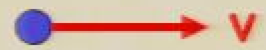
For a moving object,



E_{TOTAL} = mc^2 + Kinetic Energy (KE)

or: KE = E_{TOTAL} - mc2

It can be shown that: $E_{TOTAL} = \gamma mc^2$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (v = \text{speed of object})$$

$$\begin{aligned} \text{Therefore, KE} &= \gamma mc^2 - mc^2 \\ &= (\gamma - 1)\, mc^2 \\ &\neq \tfrac{1}{2} mv^2 \quad \text{unless } v \ll c \end{aligned}$$

Back to: $E_{TOTAL} = \gamma mc^2$

Should γm be interpreted as the mass of the moving object, that is, can we write $E_{TOTAL} = Mc^2$? (where $M = \gamma m$)

What is mass?

Two kinds:

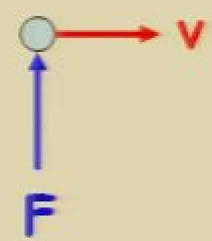
- *inertial* mass
- *gravitational* mass

Inertial mass = F/a

For high-speed objects:

$$F/a = \gamma^3 m$$

But:



$$F/a = \gamma m$$

The inertial mass of a rapidly moving object cannot be uniquely defined.

Gravitational mass

For low-speed objects: $F = Gm_1m_2/r^2$

For rapidly moving objects, use general relativity.

→ in general it is difficult to define a relativistic gravitational mass.

The only uniquely defined mass is rest mass.

Einstein: 1948 Letter in German

1) Es ist nicht gut, von der Masse $M = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ eines bewegten Körpers zu sprechen, da für M keine klare Definition gegeben werden kann. Man beschränkt sich besser auf die "Ruhe-Masse" m. Daneben kann man ja den Ausdruck für momentum und Energie geben, wenn man das Trägheitsverhalten rasch bewegter Körper angeben will.

"It is not good to introduce the concept of the mass $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion."

Energy and Momentum

$$E_{TOTAL}^2 = p^2c^2 + (mc^2)^2$$

where momentum $p = \gamma m v$

for an object having rest mass m

For photons, m = 0

Therefore, $E_{TOTAL} = pc$

(and $p = h/\lambda = hf/c$

$\rightarrow E_{TOTAL} = hc/\lambda = hf$)

Summary -- Avoid Pitfalls by:

- ☑ having a good introduction to Galilean relativity
- ☑ asking questions involving positions, velocities, and times, relative to only one frame of reference, for rapidly moving objects
- ☑ emphasizing that time dilation is a special case
- ☑ avoiding "moving clocks run slow"
- ☑ remembering that spatial separation might not correspond to an object's length
- ☑ following Einstein's advice about mass

References:

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E. McFarland, Einstein's Special Relativity: Discover it for Yourself, Trifolium Books, 1998
ISBN: 1895579236