Title: Perturbative Gauge Theory as a Gauge Theory in Twistor Space

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Abstract:

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PERTURBATIVE GAUGE THEORY AS A GAUGE THEORY IN TWISTOR SPACE

RUTGER BOELS

(OXFORD)

BASED ON:

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- " , " , "
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(CONTINUED FROM:
- LIONEL MASON, HEP-TH/0507269)

Outline

- Introduction
- Technical intermezzo
- Construct action
- Standard Y.M.
  C.S.W.
- Conclusions / Speculations / Open Problems

INTRODUCTION

- (PERTURBATIVE) GAUGE THEORY IS HARD

- ..... BUT SOME RESULTS ARE (VERY) SIMPLE WHEN EXPRESSED IN SPINOR / HELICITY VARIABLES:

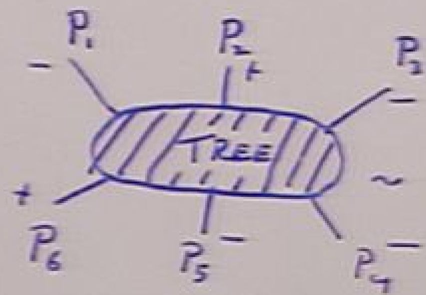
- $a^\mu \to a^{AA'} \left(= \sigma_\mu{}^{AA'} a^\mu \right)$

- $$\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} x^0 + x^1 & x^2 + i x^3 \\ x^2 - i x^3 & x^0 - x^1 \end{pmatrix}$$

- $\mathrm{DET}(a^{AA'}) = g(a,a) \Rightarrow \mathrm{DET}(a) = 0$
$$\Updownarrow$$
$$a^{AA'} = a^A a^{A'}$$

- $p_1{}^A p_{2A} = [12]$
$p_{1A'} p_2{}^{A'} = \langle 12 \rangle$

– CONSIDER 'HELICITY AMPLITUDES'

– 

$$\sim \frac{\langle 26\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$$

MHV- AMPLITUDE

"(2+ , REST–)"

– COLOR ORDERING

– EVEN SIMPLER:

	TREE	LOOP	LOOP+ SUSY
< ALL + >	$=0$	$\neq 0$	$=0$
< + – + + >	$=0$	$\neq 0$	$=0$

– WHY IS THIS SIMPLE?

Twistor Strings

- Witten: Simplicity amplitudes

$\Updownarrow$

(Nair) → - Twistor space ($\mathbb{CP}^3$)

- Topological field theory

- Twistor correspondence:

"on-shell massless spacetime fields"

$\Updownarrow$

"(Dolbeault) cohomology classes on $\mathbb{CP}^3$"

BF? - Study holomorphic Chern-Simons

$$S = \int \underset{(3,0)}{\Omega} \wedge \left(A \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A \right), \quad A: (0,1)$$

$$\Rightarrow \left(\begin{array}{l} \bar{\partial} A = 0 \\ A \to A + \bar{\partial} \chi \end{array} \right.$$

- Open B-model strings?

- NEED CALABI-YAU $\Rightarrow$ STUDY $\mathbb{CP}^{3|4}$

- $\mathcal{A}$ LOOKS LIKE $\mathcal{N}=4$ multiplet ON-SHELL

$$\overset{0}{\mathcal{A}}_{(0,1)} = \overset{0}{\underline{a}} + \psi^i \overset{-1}{\underline{\lambda_i}} + \frac{1}{2!}\psi^i\psi^j \overset{-2}{\underline{\varphi_{ij}}} + \frac{\varepsilon_{ijkl}}{3!}\psi^i\psi^j\psi^k \overset{-3}{\underline{\chi^l}}$$

$$+ \frac{\varepsilon_{ijkl}}{4!}\psi^i\psi^j\psi^k\psi^l \overset{-4}{\underline{b}}$$

- VOLUME ELEMENT:

$$\Omega = \frac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta} Z^\alpha dZ^\beta \wedge dZ^\gamma \wedge dZ^\delta$$

$\boxed{4}$ $(Z \to \lambda Z)$

HOMOGENEOUS COORDINATES

$d^4\psi \leftarrow \boxed{-4}$

$\Rightarrow$ super CY

- IS THIS IT? $\Longleftarrow$ would BE EXTREMELY nice!

$\Rightarrow$ No!

$\Rightarrow$ NEED MORE INTERACTIONS

- WITTEN: ADD (D1 BRANES)

D-INSTANTONS

- $S_{D1} = \int \beta (\bar{\partial} + A) \alpha$

$C \hookrightarrow \mathbb{CP}^{3|4}$

- "INTEGRATE OVER MODULI SPACES OF DIFFERENT EMBEDDINGS"

- ? SCATTERING AMPLITUDES

$d = q - 1 + \ell$

DEGREE CURVE

- GLUONS

LOOPS

- e.g. $g = 0, \; d = 1 \quad (q = 2 \Rightarrow \text{MHV})$

$g = \ell$

$$Z_{(g=0\, d=1)} = \int d^4x \, d^8\vartheta \; \text{DET}(\bar{\partial}_{A(x,\vartheta)})$$

$$= \int d^4x \, d^8\vartheta \int d\alpha \, d\beta \; e^{-\int_{\mathbb{CP}^1} \alpha(\bar{\partial}+A)\beta}$$

$$= \int d^4x \, d^8\vartheta \sum_m C_m A^m$$

CORRELATION FUNCTIONS IN 2D!

- 4pt AMPLITUDE:

$$\langle \alpha_1 A_1 \beta_1 \, \alpha_2 A_2 \beta_2 \, \alpha_3 A_3 \beta_3 \, \alpha_4 A_4 \beta_4 \rangle$$

$\Rightarrow$ MHV

?

$$= (2\pi)^4 \, \delta^4(\textstyle\sum_i p_i) \, \mathrm{Tr}(T_1 T_2) \, \mathrm{Tr}(T_3 T_4) \cdot \frac{(\lambda_3 \lambda_4)^4}{\langle \lambda_1 \lambda_2 \rangle^2 \langle \lambda_3 \lambda_4 \rangle^2}$$

- NOT IN GAUGE THEORY
- $\sim \left(\frac{1}{k^2}\right)^2$
- CONFORMAL SUGRA? ← TREE VS. LOOP LEVEL?

- CHIRAL DETERMINANTS ARE ANOMALOUS

- DOES NOT STOP PEOPLE FROM CALCULATING!

Related Developments

- Different twistor string formulations
- Calculational prescriptions (tree)

CONNECTED — Cachazo/Svrček/Witten

Roiban/Spradlin/Volovich — DISCONNECTED

- Recursion relations

Britto/Cachazo/Feng(/Witten)

- Loop level

Brandhuber/Spence/Travaglini

- Lagrangian framework?

- Mansfield
- Gorsky/Rosly

Twistor actions

(Mason 0507269
Mason/Skinner

- Euclidean signature:
  - Projective Twistor space: $\mathbb{C}P^3$
  - Study space of lines in $\mathbb{C}P^3$

"Dotted/
Undotted"

$$\omega^A = x^{AA'} \pi_{A'}$$

$Z^\alpha = (\omega^A, \pi_{A'})$: Hom. coords on $\mathbb{C}P^3$

- Given (ω, π), what is $x^{AA'}$?
  $\Rightarrow$ need more equations
  $\Rightarrow$ need conjugation
  (determines signature)

$$\hat{\omega}^A = \begin{pmatrix} -\bar{\omega}^1 \\ \bar{\omega}^0 \end{pmatrix} \Leftarrow \omega^A = \begin{pmatrix} \omega^0 \\ \omega^1 \end{pmatrix}$$

- $(\omega^A, \pi_{A'}) \Longrightarrow \left\{ \begin{array}{l} x^{AA'} \\ \pi_{A'} \end{array} \right\}$

$\mathbb{C}P^1$ fibres

- Now for $\mathbb{CP}^{3|4}$

  - Lines $\mathbb{CP}^{1|0} \hookrightarrow \mathbb{CP}^{3|4}$
  - Superlines $\mathbb{CP}^{1|4} \hookrightarrow \mathbb{CP}^{3|4}$

    → CHIRAL

    → ANTI-CHIRAL

$$\left(x_{-}^{AA'},\ [\pi_{A'}],\ \bar{\vartheta} \right)$$

- (0,1) forms
  - 2 spacetime components $\bar{e}^{A}$
  - 1 fibre component $\bar{e}^{0}$

- Holomorphic volume-form

- Combine all this

$\Longrightarrow$

$$\omega^{A} = x^{AA'} \pi_{A'}$$

$$\omega^{A} = x^{AA'} \pi_{A'}$$

$$\psi^{i} =$$

$$\omega^{A} = x^{AA'} \pi_{A'}$$

$$\psi^{i} = \qquad \pi_{A'}$$

$$\omega^A = x^{AA'} \pi_{A'}$$

$$\psi^i = \sigma^{iA'} \pi_{A'}$$

$$\omega^{A} = x^{AA'}\pi_{A'}$$

$$\underset{\overset{\uparrow}{4}}{\psi^{i}} = \underbrace{\vartheta^{iA'}}_{\uparrow\, 8}\pi_{A'}$$

- Now for $\mathbb{C}P^{3|4}$
  - Lines $\mathbb{C}P^{1|0} \hookrightarrow \mathbb{C}P^{3|4}$
  - Superlines $\mathbb{C}P^{1|4} \hookrightarrow \mathbb{C}P^{3|4}$
    - CHIRAL
    - ANTI-CHIRAL

$$\left(x_{-}^{AA'}, [\pi_{A'}], \bar{\vartheta} \right)$$

$\bar{e}^{A}$

- (0,1) FORMS
  - 2 SPACETIME COMPONENTS
  - 1 FIBRE COMPONENT

$\bar{e}^{0}$

- Holomorphic volume-form

- Combine all this $\Longrightarrow$

- Take 'CSW' at face value:

NEEDED: - Chern-Simons propagators

- MHV-vertices

- 'Guess':

$$S = \frac{i}{2\pi}\int \Omega \wedge d^4\psi \wedge \mathrm{Tr}\left(A \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A\right)$$

$$- K \int d\mathfrak{J}\ \boxed{\mathrm{LOG}}\ \mathrm{DET}(\bar{\partial} + A)\Big|_{\underbrace{L(x,\theta)}_{\mathbb{CP}^1\text{-FIBRE}}}$$

$d^4x\, d^8\theta \Leftarrow$ moduli space of degree 1 curves

- This action is gauge-invariant

(contains no anomalous contribution from chiral determinant)

$$A \to A + \bar{\partial}_A \chi$$

function on $\mathbb{CP}^{3|4}$

- Form couplings are peculiar

However $\Rightarrow$

$\Rightarrow$ SPACETIME YANG MILLS?

!

- WE HAVE MORE GAUGE SYMMETRY THAN $D=4$ YM $\Rightarrow$ FIX THIS

$$\bar{\partial}^{*}_{L(x,\vartheta)} A_0 = 0$$

$$A = A_0 \bar{e}^0 + A_A \bar{e}^A$$

$\Rightarrow$ COMPONENTS OF A ARE HARMONIC (W/ SPECIFIC WEIGHT)

$\Rightarrow$ COHOMOLOGY

e.g. WEIGHT $0 \Rightarrow a = \bar{e}^A a_A(\,) \boxed{+0}$

$-4 \Rightarrow b = 3\bar{e}^0 \dfrac{\boxed{B_{\dot{A}'\dot{B}'}(x)}\,\hat{\pi}^{\dot{A}'}\hat{\pi}^{\dot{B}'}}{(\bar{\pi}\cdot\hat{\pi})^2} + \bar{e}^A b_A(\,)$

- RESIDUAL GAUGE FREEDOM:

$$A \to A + \bar{\partial}_A h \;\Big|\; \bar{\partial}^{*}_L \bar{\partial}_L h = 0$$

$$\Rightarrow h = h(x)$$

– GHOSTS DECOUPLE:

$$\int \Omega^{\mathcal{Y}} d\psi \wedge \mathrm{Tr}\left[\underset{\substack{\uparrow \\ \text{BRST}}}{Q}, \bar{c}\, \bar{\partial}_L^{*} A_0\right]$$

$$= \int \Omega^{\mathcal{Y}} d\psi \wedge \mathrm{Tr}\left\{ m\, \bar{\partial}_L^{*} A_0 + \bar{c}\, \bar{\partial}_L^{*}(\bar{\partial}_0 + A_0)\, c \right\}$$

$\Rightarrow$

$\bar{c}$ \ c	-4	-3	-2	-1	0
0			X	X	X
-1				X	X
-2			$\emptyset$		X
-3		$\emptyset$			
-4					

(OR STUDY DIAGRAMS)

- INSERT GAUGE-FIXED FIELDS BACK INTO THE ACTION AND INTEGRATE OUT FERMIONIC COORDINATES

- $S_{CS} = \int \Omega \wedge \bar{\Omega}\, ()\Big(B_{A'B'}\, \hat{\pi}^{A'} \hat{\pi}^{B'} \cdot$

$$\Big(\pi^{C'} \frac{\delta a^{A}}{\delta x^{AC'}} + \frac{1}{2}[a^{A}, a_{A}] \Big) + \ldots\ldots$$

- CONTAINS AUXILLIARY FIELDS $\Rightarrow$ INTEGRATE THOSE OUT

- DO THE INTEGRAL OVER $\mathbb{CP}^1$-FIBRE

- $S_{CS} = \int d^4x\, \frac{1}{2} B_{A'B'} F^{A'B'} \leftarrow$ 'BF'!

- CHALMERS & SIEGEL LAGRANGIAN

SECOND TERM $\Longrightarrow$

$- \int d\vartheta \; \mathrm{LOG} \; \mathrm{DET}(\bar\partial + A)\big|_{L(x^-,\vartheta)}$

A EXPANSION STARTS AT ϑ^2!

IN THIS GAUGE

$\Rightarrow$ FINITE NUMBER OF TERMS

- INTEGRATE OVER ALL FIBRES

- $S_{(2)} = -\frac{K}{2} \int d^4x \; B_{A'B'} B^{A'B'} + \ldots$

- $S_{CS} + S_{(2)}$ IS PERTURBATIVELY EQUIVALENT TO $N=4$ SYM

- INTEGRATING OVER FIBRES IS SPECIAL TO THIS GAUGE

OTHER GAUGES? $\Longrightarrow$

$$\omega^{A} = x^{AA'}\pi_{A'}$$

$$BF^{+}_{n}$$

$$\underset{4}{\psi^{i}} = \underbrace{\bar{\vartheta}^{iA'}}_{8}\pi_{A'}$$

$$\omega^{A} = x^{AA'}\pi_{A'} \qquad BF_{A}^{+} + BB$$

$$\underbrace{\psi^{i}}_{4} = \underbrace{\bar{\theta}^{iA'}}_{8}\pi_{A'}$$

$$\omega^{A} = x^{AA'}\pi_{A'}$$

$$BF^{+}_{n} + BB$$

$$(F^{+})^{2}$$

$$\underset{4}{\psi^{i}} = \underbrace{\bar{\theta}^{iA'}}_{8}\pi_{A'}$$

$$\omega^{A} = x^{AA'}\pi_{A'}$$

$$\underbrace{B F^{+}_{\wedge}} + \underbrace{B B}$$

$$(F^{+})^{2}$$

$$\underset{\uparrow\,4}{\psi^{i}} = \underbrace{\bar{\theta}^{iA'}}_{\uparrow\,8}\pi_{A'}$$

$$- \int d\bar{y} \; \text{LOG DET}(\bar{\partial} + A)\Big|_{L(x, \vartheta)}$$

A EXPANSION STARTS AT ϑ^2!

IN THIS GAUGE

$\Rightarrow$ FINITE NUMBER OF TERMS

- INTEGRATE OVER ALL FIBRES

- $$S_{(2)} = -\frac{\kappa}{2} \int d^4x \; B_{A'B'} B^{A'B'} + \ldots$$

- $S_{CS} + S_{(2)}$ IS PERTURBATIVELY EQUIVALENT TO $\mathcal{N}=4$ SYM

- INTEGRATING OVER FIBRES IS SPECIAL TO THIS GAUGE

OTHER GAUGES? $\Longrightarrow$

CSW - GAUGE

- PICK AXIAL GAUGE (LIGHT CONE / SPACECONE LIKE)

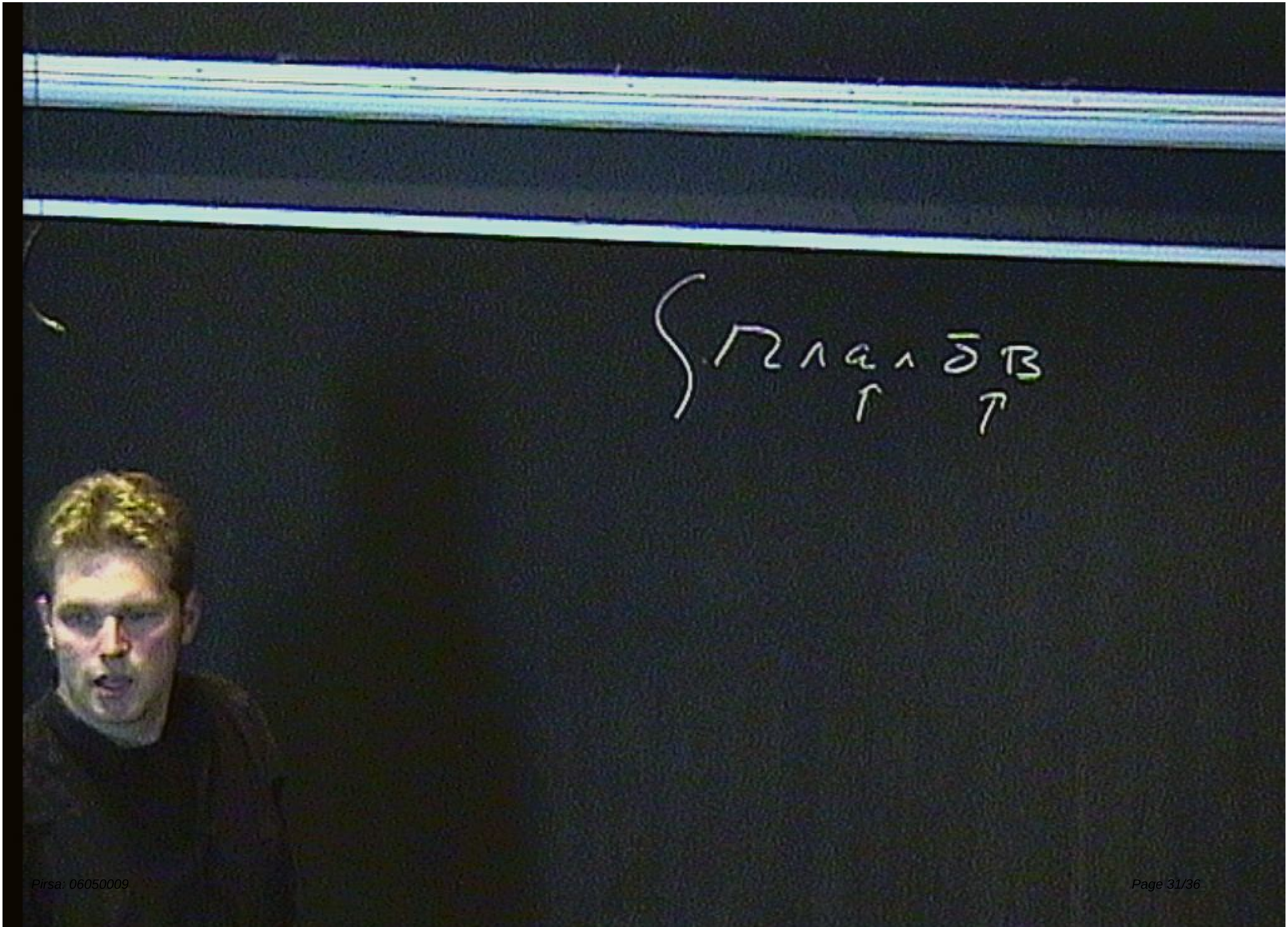
$$\eta^{A} \pi^{A'} \frac{\partial}{\partial x^{AA'}} \lrcorner A = 0$$

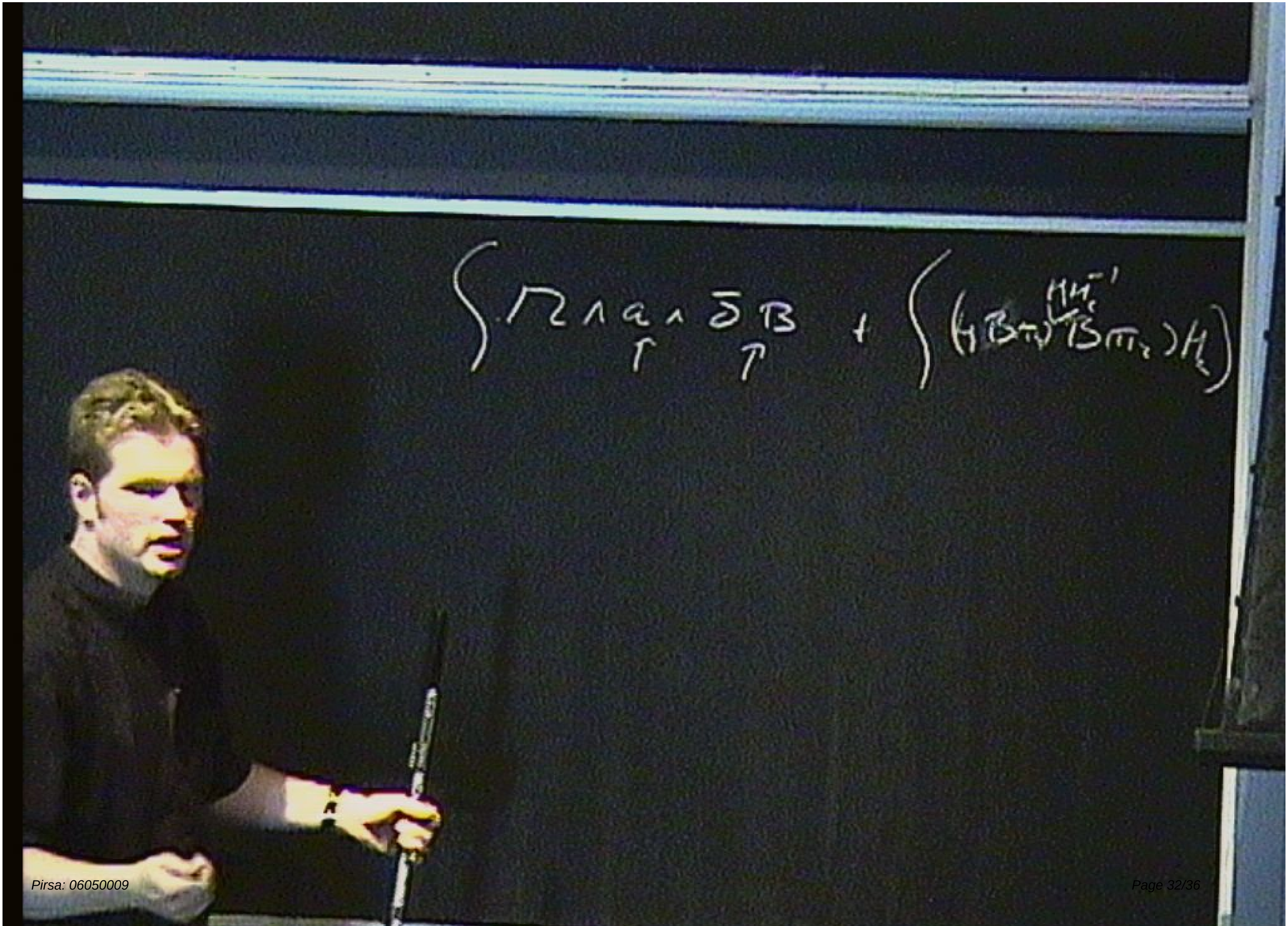
- CS - VERTEX VANISHES
- CALCULATE PROPAGATORS AND ON-SHELL FIELDS $(a_0, b_0, \langle a_0 b_0 \rangle)$
- $S_{(2)} = \sum$ MHV - VERTICES

$$\left(\text{LOG DET} (\bar\partial + A) = \operatorname{Tr} \log \bar\partial + \sum_k \frac{1}{k} \operatorname{Tr} \left(\frac{A}{\bar\partial}\right)^k \qquad \bar\partial^{-1} = \frac{1}{(\pi_1 \cdot \pi_2)} \right)$$

- SLIGHT PROBLEM IN $N=0$

"GOOD"


$\int \Omega \wedge a \wedge \bar{\partial} B$


$\int \Omega \wedge a \wedge \bar{\partial} B$

$$\int \Omega \wedge a \wedge \bar{\partial} B + \int \left(H_1 B(\pi_1) \overset{H H_2^{-1}}{\vee} B(\pi_2) H_2 \right)$$

$$H(a)$$

- OTHER GAUGE THEORIES
  - $\mathcal{N} = 0, 1, 2\,(3)$! ✓
  - MATTER (MASSLESS) ✓
  - MASSIVE? ?

→ REDUCE R-SYMMETRY

→ - TAKE CUE FROM SUSY
  - FIND NEEDED SYMMETRY FOR SPACETIME GAUGE

- E.G. SU(3) WITH MATTER IN THE FUNDAMENTAL AND NO SUSY.....

Conclusions

- $\exists$ FORMULATION OF 'pure' $\mathcal{N}=0,1,2,4$ YANG-MILLS ON TWISTOR SPACE
- QUANTIZATION IS OK
- NO SIGN OF CONFORMAL SUGRA
- PROVIDES AN ALTERNATIVE OFF-SHELL SUPERFIELD FORMULATION

"BUT" $\Rightarrow$

Questions:

- Unitarity? Renormalizability?
- Loops?
- Any new insight into

  < FAVORITE GAUGE THEORY QUESTION >

  - Perturbation series
    - BCFW?
  - Instanton calculations
  - AdS/CFT (integrability)
  - Twisting?
- Čech cohomology?
- Twistor diagrams?
- String Theory?
  - Holomorphic Linking?
  - $B + \bar{B}$ - model?