Title: From Trees to Loops and Back

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Abstract:

From Trees to Loops and Back

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based on hep-th/0510253 AB-Spence-Travaglini

and also
hep-th/0412108 Bedford-AB-Spence-Travaglini
hep-th/0410280 Bedford-AB-Spence-Travaglini
hep-th/0407214 AB-Spence-Travaglini

Outline

- **Motivation & Aims**
- **Scattering Amplitudes in Gauge Theory**
  - Colour Decomposition & Spinor Helicity Formalism
  - Twistor Space
  - MHV Diagrams
- **1-Loop Amplitudes from MHV Vertices**
- **Proof of Equivalence of MHV & Feynman Diagrams**
  - Covariance of MHV Loop-Diagrams - Feynman Tree Theorem
  - Discontinuities
  - Factorisation: Collinear Limits & Soft Limits
- **Conclusions**

Witten 2003

weak/weak

Perturbative N=4 SYM = Topological String on Twistor Space

Why is that interesting ?

- Explains unexpected simplicity of scattering amplitudes in Yang Mills & gravity

→ Simple Geometric Structure in Twistor Space

→ New Differential Equations for Amplitudes

- New tools to calculate amplitudes

→ MHV Diagrams for trees and loops

Generalized Unitarity

New Recursion Relations

Motivation

- LHC is coming
  - Precision pert. QCD calculations
  - Long wishlist of processes to be computed
- New techniques are needed
  - Textbook methods hide simplicity of amplitudes
  - Intermediate expressions are large
  - Factorial growth of nr. of diagrams, e.g. gluon scattering

g g => m g	m=5	m=6	m=8
	559405	10525900	224449225

Motivation cont'd

- Luckily we do not have to use textbook techniques
  - color decomposition
  - spinor helicity
  - unitarity
  - supersymmetry
  - string theory
  - .....
- and since 2004
  - twistor string (inspired) techniques

Color Decomposition

$$\mathcal{A}_n(1,2,3,\ldots,n) = \sum_{\sigma\in S_n/Z_n} \mathrm{Tr}(T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(n)}})A_n(\sigma(1),\ldots,\sigma(n))$$

- at tree level Yang-Mills is planar
- only diagrams with fixed cyclic ordering contribute to the "color stripped amplitudes" A_n
  - analytic structure simpler
- At loop level, also multi-traces; subleading in $1/N$
  - At one-loop simple relation between planar & non-planar terms

Spinor helicity formalism

- Responsible for the existence of compact formulas of tree and loop amplitudes in massless theories

- The 4D Lorentz Group (complexified) $SL(2,\mathbb{C}) \times SL(2,\mathbb{C})$

$$p_\mu \Longleftrightarrow p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}}\,;\;\; a,\dot{a} = 1,2$$

massless on-shell $\quad p_\mu p^\mu = \det p_{a\dot{a}} = 0 \Rightarrow p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$

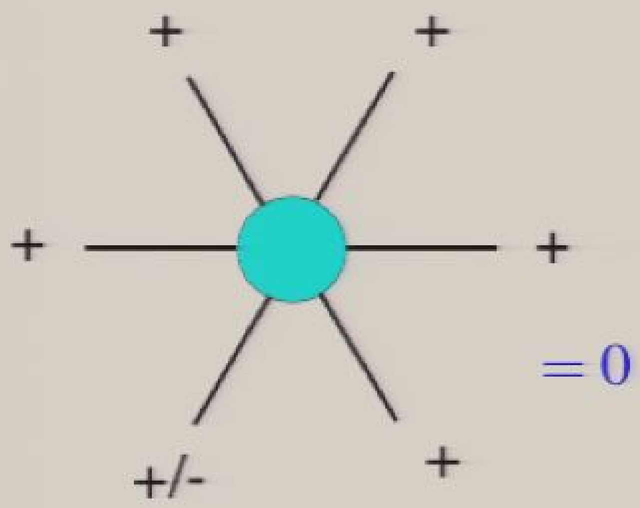
Note: In real Mink $\tilde{\lambda} = \bar{\lambda}$

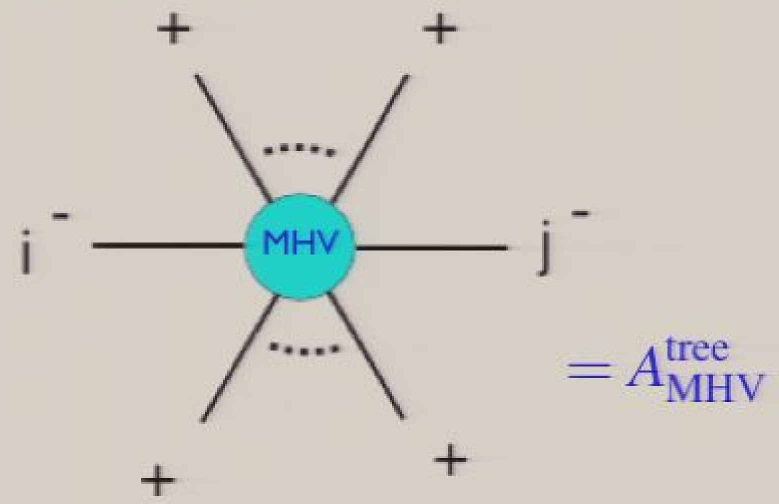
- Spinor Products

$$\langle i\,j\rangle \equiv \lambda^i_a \lambda^j_b \varepsilon^{ab}\,,\;\; [i\,j] \equiv \tilde{\lambda}^i_{\dot{a}} \tilde{\lambda}^j_{\dot{b}} \varepsilon^{\dot{a}\dot{b}} \qquad \Rightarrow\;\; 2p_i \cdot p_j = \langle i\,j\rangle\,[ji]$$

- $\{p^\mu_i, \varepsilon^\mu_i\}$ are redundant; the spinor variables $\left\{\lambda^a_i, \tilde{\lambda}^{\dot{a}}_i\right\}$ contain just the right d.o.f. to describe momentum & wavefnct./polarization of massless particles of arbitrary helicity h

n-Gluon Tree MHV-Amplitudes





$$A^{\text{tree}}_{\text{MHV}} = i g^{n-2} (2\pi)^4 \delta^{(4)} \left(\sum p_i\right) \frac{\langle i j \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

MHV Amplitude
Parke-Taylor;
Berends-Giele

- Very Simple!
- Holomorphic, depends only on λ_i , not on $\tilde{\lambda}_i$
- Correct for Super Yang-Mills, pure glue & QCD
- In N=4 SYM similar formulas for amplitudes with two gluons replaced by fermions/scalars

Twistor Space

... is a "1/2 Fourier transform" of spinor space:

$$(\lambda_a, \tilde{\lambda}_{\dot{a}}) \quad \Rightarrow \quad (\lambda_a, \mu_{\dot{a}})$$

- Twistor Space is complex 4 dim'l $\quad (\lambda_1, \lambda_2, \mu^{\dot{1}}, \mu^{\dot{2}})$
- Amplitudes are homogeneous functions on twistor space

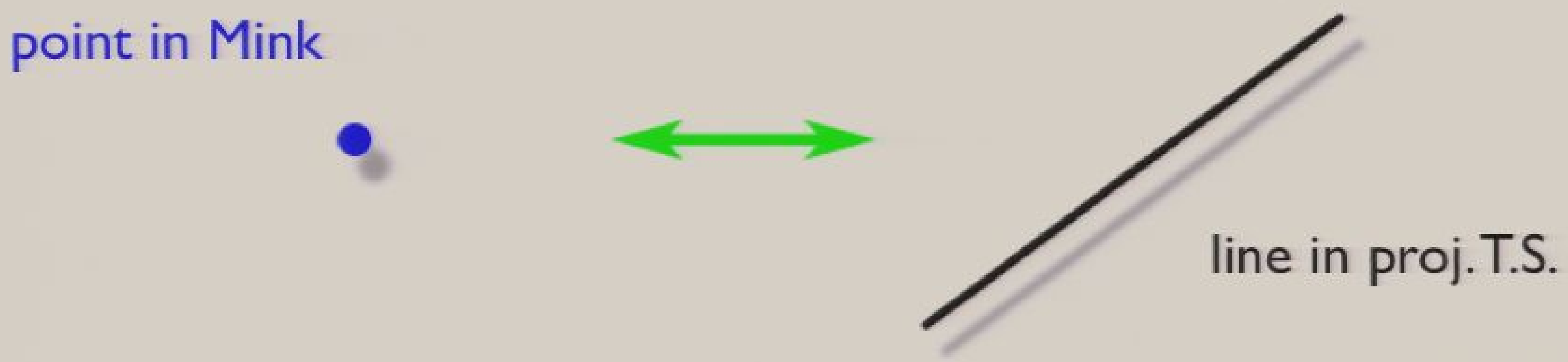
➡ Projective Twistor Space $\quad \mathbb{CP}^3$

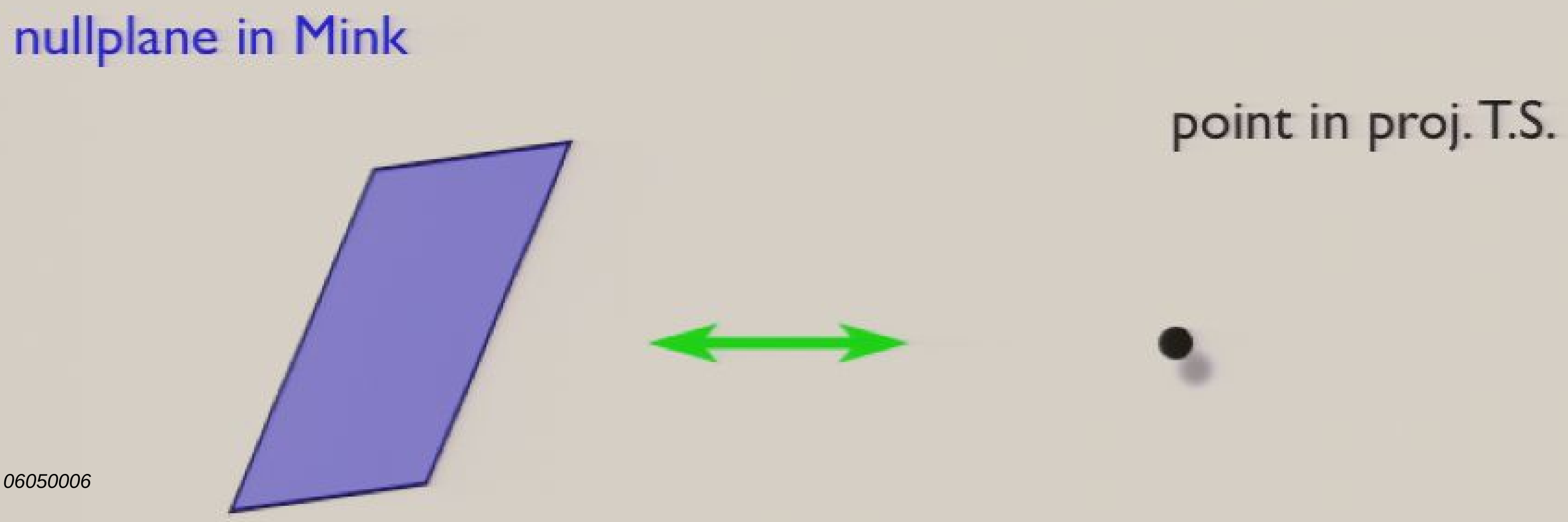
$$(\lambda, \mu) \sim (t\lambda, t\mu)$$

Twistor Space cont'd

- Relations between Minkowski space and projective T. S.

  Incidence Relation: $\mu^{\dot{a}} + x^{a\dot{a}}\lambda_a = 0$



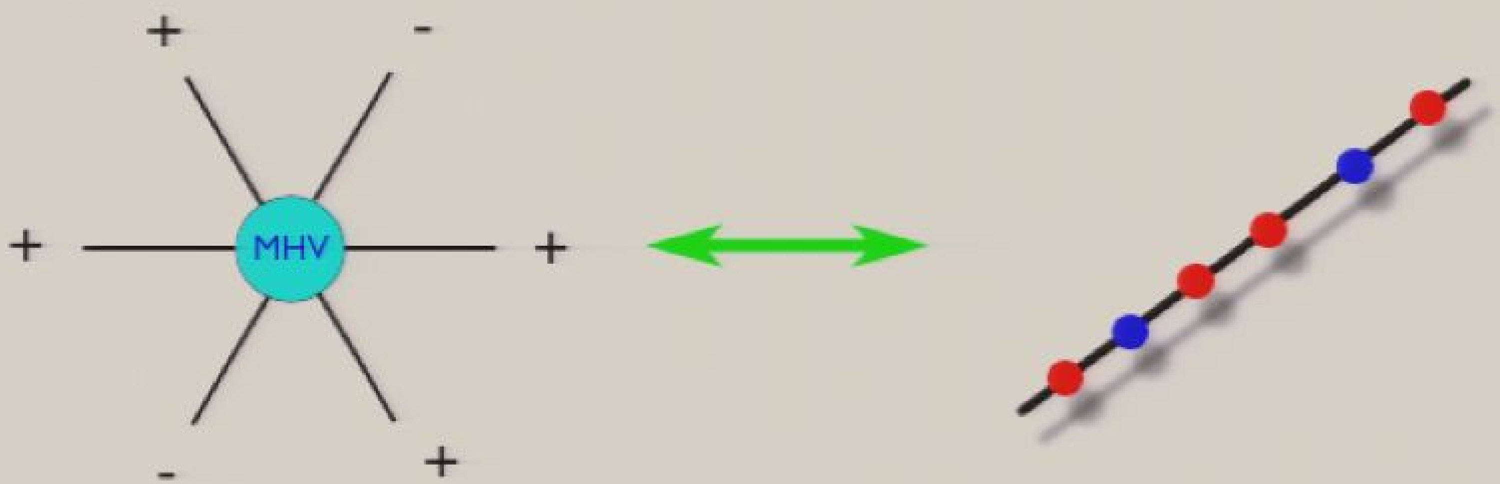


Amplitudes in Twistor Space

- MHV amplitudes are holomorphic (except for momentum conservation); perform 1/2 Fourier transform

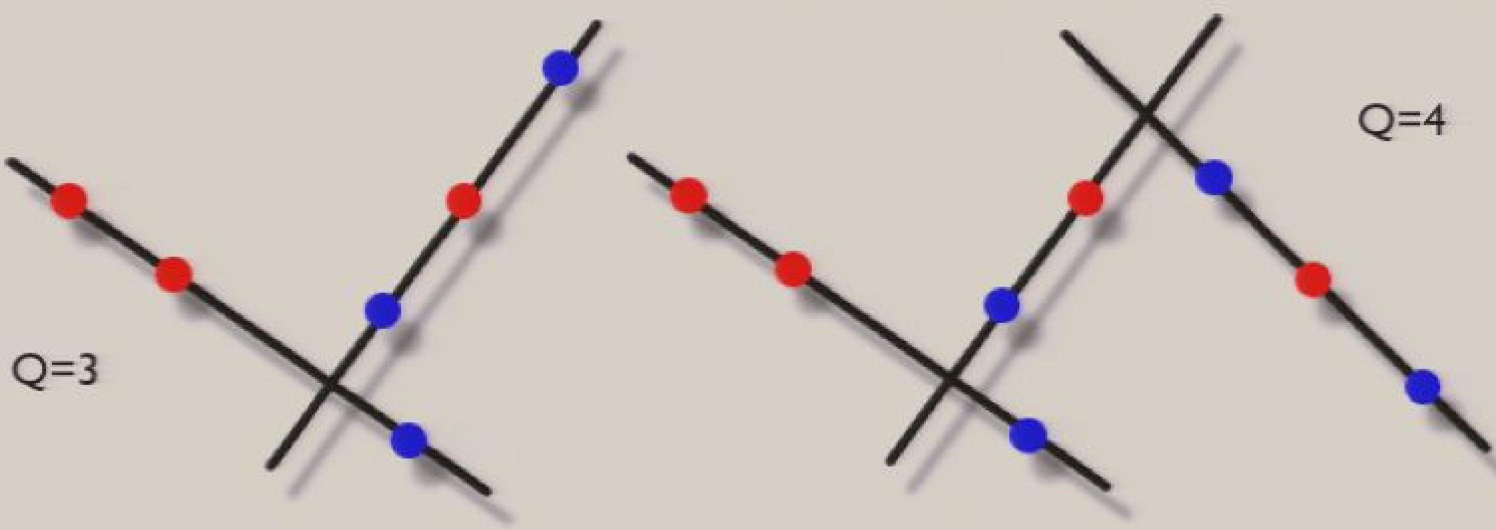
$$\Rightarrow A_{\mathrm{MHV}} \int dx \int \prod_i d\tilde{\lambda}_i e^{i\mu_i\tilde{\lambda}_i} e^{ix\lambda_i\tilde{\lambda}_i} \sim \prod_i \delta^{(2)}(\mu_i + x\lambda_i)$$

Hence: For MHV amplitudes all points (=ext. gluons) lie on a line in projective Twistor Space



Amplitudes in Twistor Space cont'd

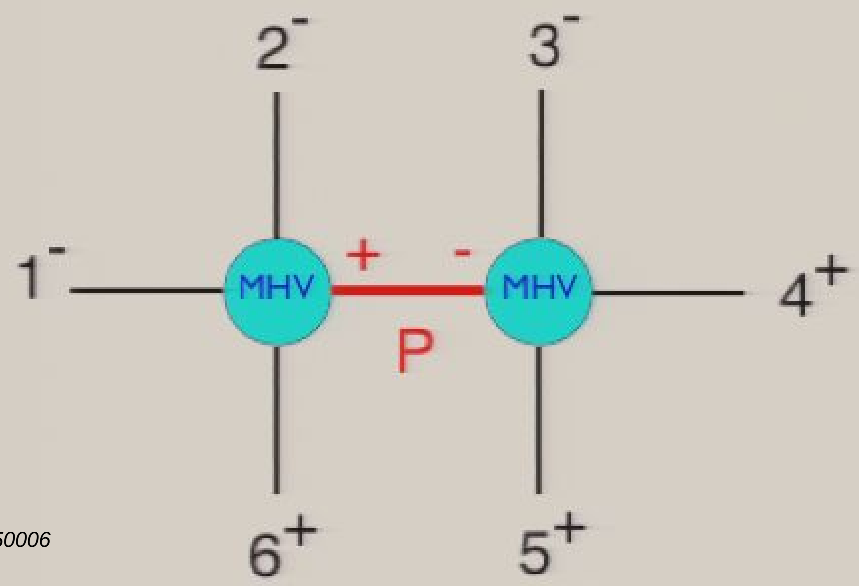
- Witten's conjecture (2003): L-loop amplitudes with Q negative helicity gluons localise on curves of degree=Q-1+L and genus<=L
- Localisation properties of amplitudes in proj. T.S. translate into differential operators obeyed by the amplitudes in momentum space: $\mu \rightarrow i\partial/\partial\tilde{\lambda}$
- For non-MHV tree amplitudes "experiments" with diff. operators reveal (curves are actually degenerate):



MHV Diagrams

- MHV amplitude = Line in T.S. = local interaction in Mink
- CSW Rules (Cachazo-Svrcek-Witten)
  - MHV amplitudes continued off-shell as local vertices
  - Connect MHV vertices with scalar propagators: $\frac{1}{P^2}$
  - Sum diagrams with fixed cyclic ordering of ext. lines

Ex: $\langle 1^- 2^- 3^- 4^+ 5^+ 6^+ \rangle$



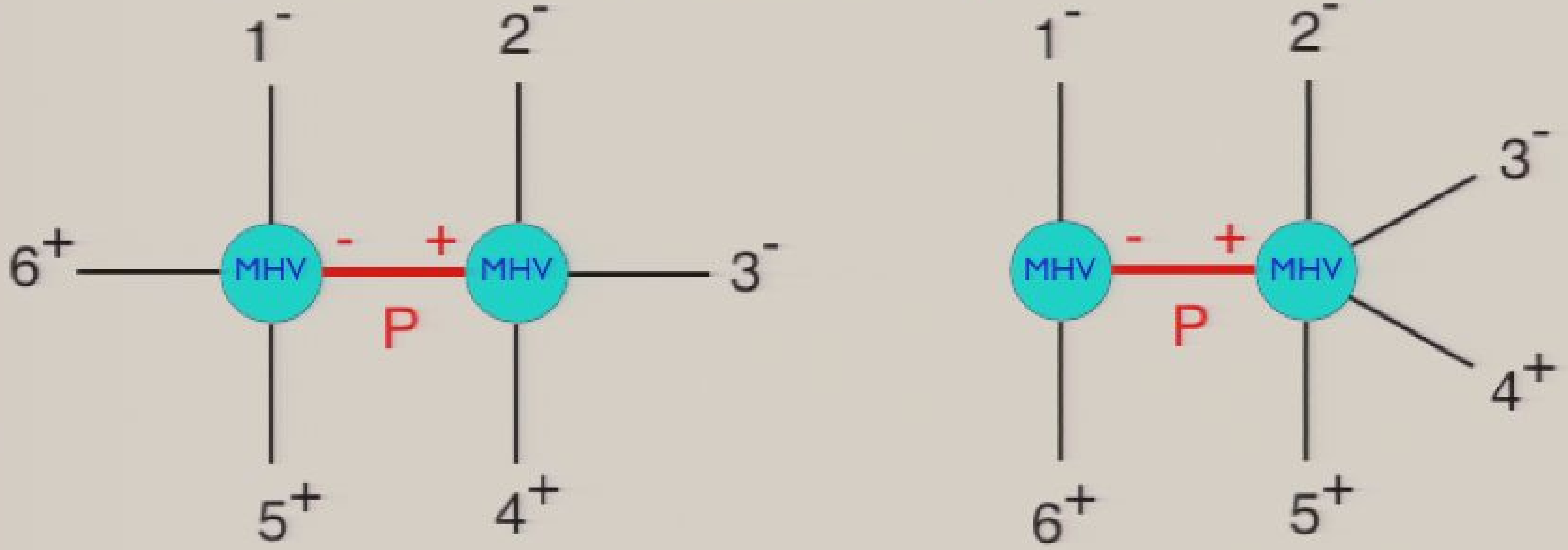
Off-shell continuation of spinor:

$$\lambda_{Pa} = P_{a\dot{a}} \eta^{\dot{a}}$$

$\eta^{\dot{a}} \ldots$ reference spinor

MHV diagrams cont'd

some of the 5 missing diagrams of $\langle 1^- 2^- 3^- 4^+ 5^+ 6^+ \rangle$



- Reproduce known and obtain new scattering amplitudes in any massless gauge theory ➡ dramatic simplifications
- Correct factorisation:
  multiparticle poles & collinear/soft limits
- η dependence disappears in sum over diagrams

MHV diagrams - applications

- Amplitudes of gluons with fermions/scalars Georgiou-Khoze, Wu-Zhu
- Amplitudes with quarks Georgiou-Khoze, Su-Wu
- Higgs plus partons Dixon-Glover-Khoze, Badger-Glover-Khoze
- Electroweak vector boson currents Bern-Forde-Kosower-Mastrolia
- Lagrangian Derivation? Initial Steps have been made using light cone formalism (Mansfield hep-th/0511264, Gorsky-Rosly hep-th/0510111)

From Trees to Loops

(AB-Spence-Travaglini)

- Original prognosis from twistor string theory was negative (Berkovits-Witten), "pollution" with Conformal SUGRA modes

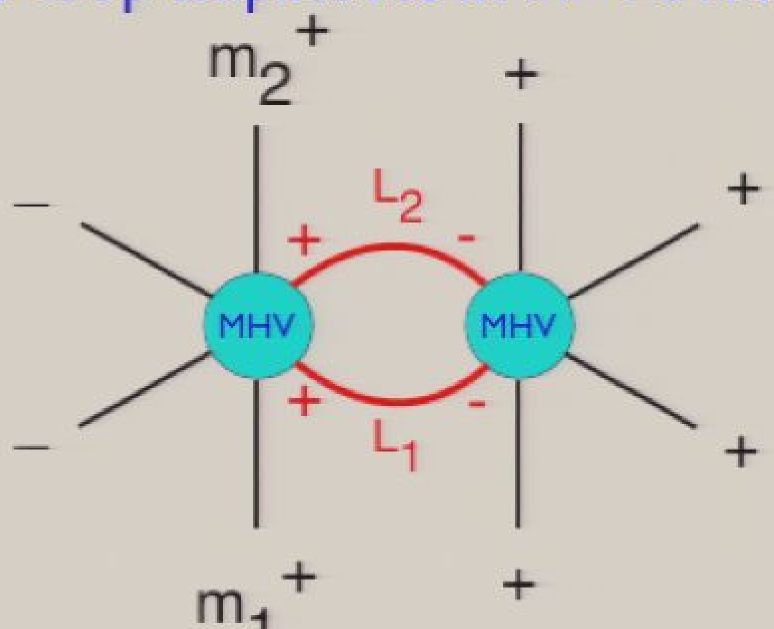
- Try anyway:

  - Connect V=Q-1+L MHV vertices, using the same off-shell continuation as for trees

  - Chose measure, perform loop integration (Dim. Reg.)

- Simplest Ex.: MHV 1-loop amplitudes in N=4 SYM

$$\int d\mathcal{M} \sum_{m_1, m_2, h}$$

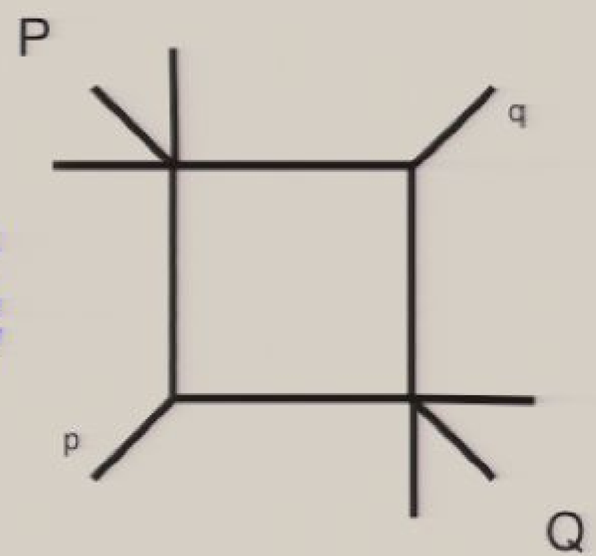


MHV one-loop amplitudes in N=4 SYM

- Computed by Bern-Dixon-Dunbar-Kosower (1994) using four-dim'l cut-constructibility (works for SUSY, massless theories) = Unitarity
- Result is expressed in terms of "2-mass easy box functions"

$$I^{2me}(s,t,P^2,Q^2) = \int d^{4-2\varepsilon}L \frac{1}{L^2(L-p)^2(L-P-p)^2(L+Q)^2}$$

$$A_{MHV}^{1-loop} = A_{MHV}^{tree} \times \sum_{p,q}$$



MHV vertices at one-loop

Loop integration (schematically):

$$A_{\mathrm{MHV}}^{1-loop} = \sum_{m_1,m_2,h} \int d\mathcal{M}\, A_L^{\mathrm{tree}}(-L_1, m1, \ldots, m_2, L_2)$$
$$\times A_R^{\mathrm{tree}}(-L_2, m2+1, \ldots, m_1-1, L_1)$$

Loop measure:

$$d\mathcal{M} = \frac{d^4 L_1}{L_1^2 + i\varepsilon} \frac{d^4 L_2}{L_2^2 + i\varepsilon} \delta^{(4)}(L_2 - L_1 + P_L)$$

Off-shell continutation (as before) $L_\mu = l_\mu + z\eta_\mu$

reference null-vector

Hence

$$\frac{d^4 L}{L^2 + i\varepsilon} = \frac{dz}{z} \quad \times \quad d^4 l\, \delta^{(+)}(l^2)$$

dipersive measure

phase space measure

N=4 SYM one-loop cont'd

Putting everything together and integrating over $z' = z_1 + z_2$
we find, using $z = z_1 - z_2$

$$dM = \frac{dz}{z} \times dLIPS(l_2, -l_1; P_{L;z})$$

$$P_{L;z} = P_L - z\eta$$

dLIPS is the 2-particle Lorentz inv. phase space measure and the corresponding integral calculates the branchcut or discontinuity of the amplitude! Note however the shift in $P_{L;z} = P_L - z\eta$

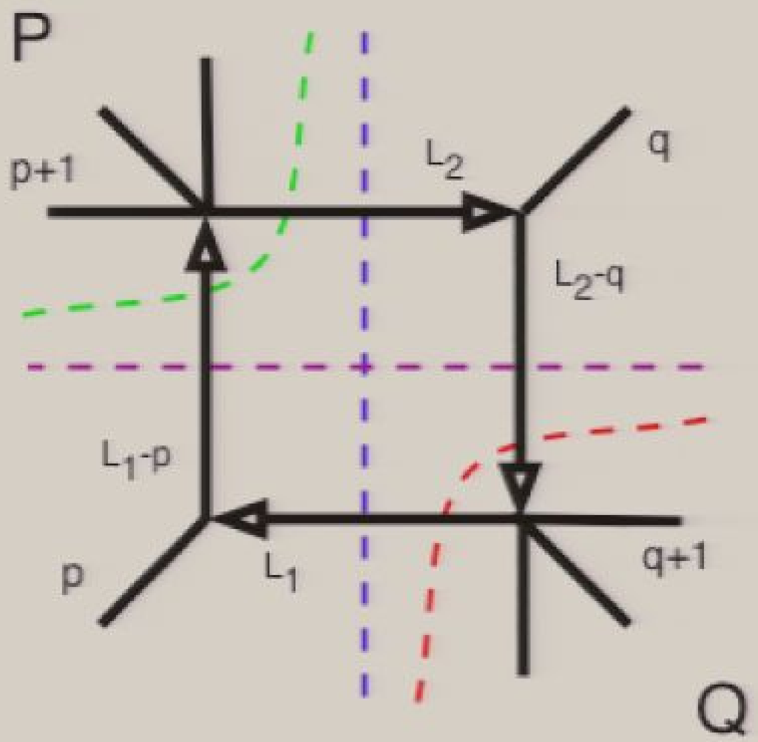
The remaining integration over z is a dispersion (type) integral, which reproduces the full amplitude!

➡ The Return of the Analytic S-Matrix

N=4 SYM one-loop cont'd

After some manipulations we find the result to be the sum over contributions from all possible cuts of all possible 2-mass easy box functions (to all orders in DR parameter ε)

$$A^{\text{tree}}_{\text{MHV}} \times$$



Note: only after summing over the four cuts dependence on η disappears!

Summary of N=4 SYM at 1-loop

- Agrees with result of (Bern-Dixon-Dunbar-Kosower)
- Incorporates large numbers of conventional Feynman diags
- Naturally leads to "dispersion integrals"
- Non-trivial check of MHV diagrammatic method
  - covariance (no dependence on η)
  - non-MHV amplitudes (later in the talk)
- Simpler form of "2-mass easy box function"

$$I^{2me}(s,t,P^2,Q^2) = -\frac{1}{\varepsilon^2}\Big[(-s)^{-\varepsilon} + (-t)^{-\varepsilon} - (-P^2)^{-\varepsilon} - (-Q^2)^{-\varepsilon}\Big]$$
$$+\mathrm{L}i_2(1-aP^2) + \mathrm{L}i_2(1-aQ^2) - \mathrm{L}i_2(1-as) - \mathrm{L}i_2(1-at)\,,$$
$$a = \frac{P^2+Q^2-s-t}{P^2Q^2-st} = \frac{u}{P^2Q^2-st}$$

Generalisations

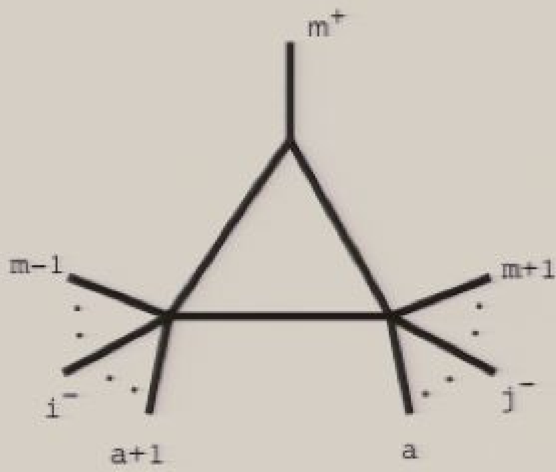
- In principle our approach can readily be applied to non-MHV amplitudes and theories with less supersymmety
- MHV, one-loop amplitudes in N=1 SYM (Bedford-AB-Spence-Travaglini)
  - Contribution of a chiral multiplet (susy decomposition)

    $$A^{\mathcal{N}=1,\text{vector}} = A^{\mathcal{N}=4} - 3A^{\mathcal{N}=1,\text{chiral}}$$
  - Result involves scalar box & triangle functions
  - MHV diagram method agrees with BDDK
  - Works despite the absence of Twistor String Dual of N=1 SYM

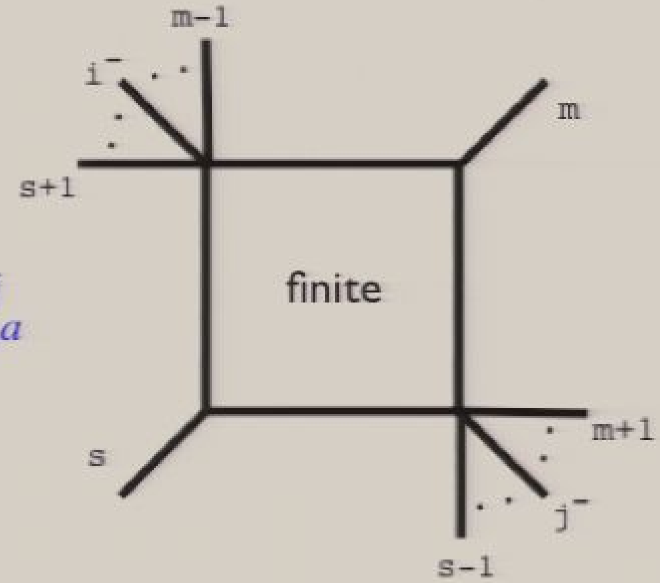
MHV, one-loop in N=1 SYM

$$A_{chiral}^{1-loop,MHV} = A^{\text{tree},MHV} \times I$$

$$I = \sum_{m,s} b_{m,s}^{i,j}$$



$$+ \sum_{m,a} c_{m,a}^{i,j}$$



MHV, 1-loop amplitudes in Yang-Mills

- non-supersymmetric theories are not "4D cut-constructible"
  - Amplitudes contain rational terms that are not linked to terms containing cuts (can be obtained from new on-shell recursion relations (Bern, Forde, Dixon, Kosower))
- From MHV vertices we obtain cut-containing terms
- SUSY decomposition

$$A^g = (A^g + 4A^f + 3A^s) - 4(A^f + A^s) + A^s$$

To be computed

Yang-Mills, 1-loop cont'd

- Result is expressed in terms of
  - finite box functions: $I^{2\text{me}}_{finite} = B(s,t,P^2,Q^2)$
  - triangle functions: $T^{(r)}(p,P,Q) = \dfrac{\log(Q^2/P^2)}{(Q^2-P^2)^r}$
- Coefficient of B is: $\left(b^{ij}_{m_1 m_2}\right)^2$
- Agrees with 5-point result and the case of adjacent negative helicity gluons of (BDDK)
- New Result for negative helicity gluons in arbitrary position
  - First new result for QCD from MHV diagrams !

Evidence for MHV diagrams so far

- Tree Level Amplitudes ⟹ several proofs (CSW, Britto-Cachazo-Feng-Witten, Risager)
- One-Loop Amplitudes in (S)YM
  - MHV 1-loop Amplitudes in N=4 SYM (AB-Spence-Travaglini)
  - MHV 1-loop amplitudes in N=1 SYM (Bedford-AB-Spence-Travaglini, Quigley-Rozali)
  - Cut-Constructible Parts of MHV 1-loop Amplitudes in pure Yang-Mills (Bedford-AB-Spence-Travaglini)

Q: Do MHV Diagrams provide a new, complete, perturbative expansion of SUSY Yang-Mills ?

From Loops Back to Trees

via the Feynman Tree Theorem (FTT)

- We want to show that MHV diagrams are equivalent to Feynman diagrams for generic one-loop amplitudes in SYM
    - Step 1: Proof of Covariance
    - Step 2: Discontinuities
    - Step 3: Kinematic Limits
- Step1: proof of covariance using FTT
- FTT is based on the decomposition of the usual Feynman propagator:

$$\Delta_F(P) = \Delta_R(P) + 2\pi\delta^{(-)}(P^2 - m^2)$$

$$\delta^{(-)}(P^2 - m^2) \equiv \delta(P^2 - m^2)\theta(-P_0)$$

FTT cont'd

- Assume we use Feynman rules with $\Delta_R(P)$ instead of $\Delta_F(P)$
- Since $\Delta_R(P)$ is a causal propagator (contrary to $\Delta_F(P)$) any loop integral with local vertices has support for:

$$t_1 > t_2 > \cdots > t_n > t_1$$

Since there are no closed time-like curves in Minkowski space this integral vanishes!

$$I_R = \int \prod_i d^4x_i \, \Delta_R(x_1 - x_2) V(x_2) \Delta_R(x_2 - x_3) V(x_3) \cdots \Delta_R(x_n - x_1) V(x_1) = 0$$

FTT cont'd

- Now use the decomposition of $\Delta_R(P)$ into $\Delta_F(P)$ and an on-shell delta-function

$$I_R := \int \frac{d^4L}{(2\pi)^4} f(L,\{K_i\}) \prod_i \left[\Delta_F(L+K_i) - 2\pi\delta^{(-)}\left((L+K_i)^2\right)\right] = 0$$

to find the FTT

$$I_F = -\int \frac{d^4L}{(2\pi)^4} f(L,\{K_i\}) \prod_i{}' \left[\Delta_F(L+K_i) - 2\pi\delta^{(-)}\left((L+K_i)^2\right)\right]$$

In a nutshell: the FTT reduces Loops to Trees! Or more precisely to the sum of all possible cuts.

$$I_F = I_{1-cut} + I_{2-cut} + I_{3-cut} + I_{4-cut}$$

FTT and MHV Diagrams

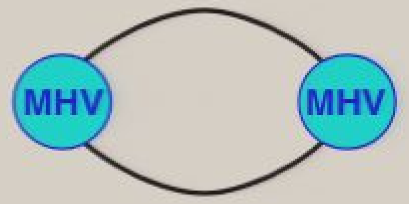
- Because of the local character in Minkowski space of MHV vertices we can apply the FTT directly to MHV diagrams.
- This will allow us to find a simple proof of covariance for the sum of MHV diagrams contributing to generic (one-)loop amplitudes
- The amplitude is given by a sum of terms in which at least one loop leg is cut

$$\mathcal{A} = \mathcal{A}_{1-cut} + \mathcal{A}_{2-cut} + \mathcal{A}_{3-cut} + \mathcal{A}_{4-cut}$$

The key point is that each set of p-particle cut diagrams sums to a covariant expression!

FTT and MHV Diagrams cont'd

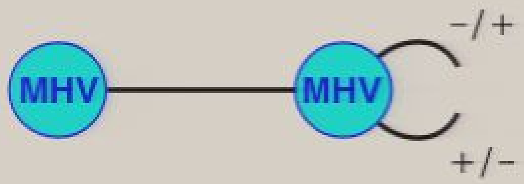
- MHV one-loop amplitudes



The MHV diagrams have the following 1-particle and 2-particle cuts

The 2-particle cuts give a phase space integral of a product of on-shell tree amplitudes and hence are covariant

FTT and MHV Diagrams cont'd



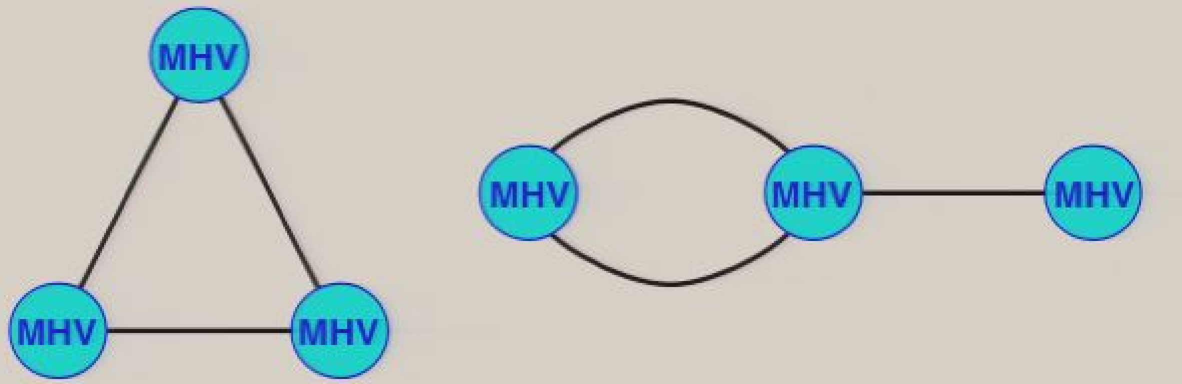
"missing diagrams"

- More care needed for 1-particle cuts: sum only over diagrams with cut legs on different MHV vertices.
- Two alternative justifications to exclude these diagrams

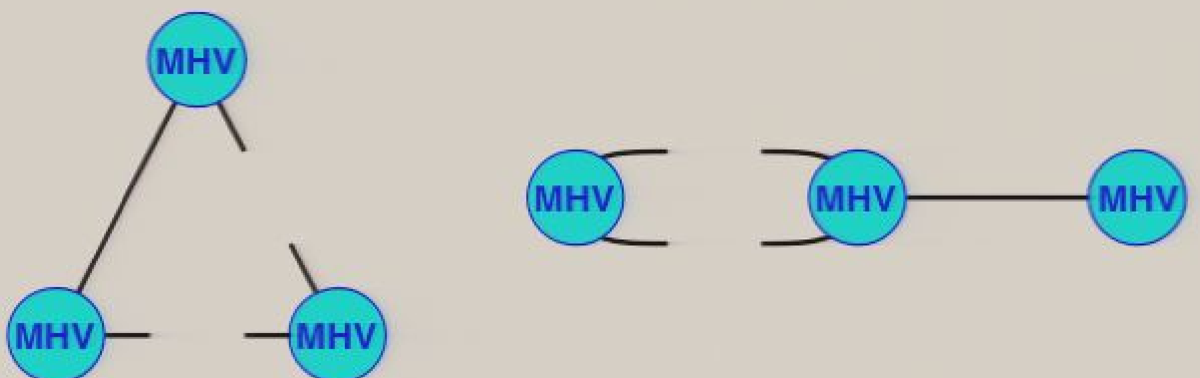
  1. In supersymmetric theories the missing diagrams give a vanishing integrand, after summing over internal particle species.
  2. cut legs are (anti-)collinear ⇒
     "missing diagram" = (splitting function) x (tree diagram)
     These tree diagrams sum to an amplitude.

FTT and MHV Diagrams cont'd

- more complicated examples can be treated in complete analogy
- NMHV Amplitudes

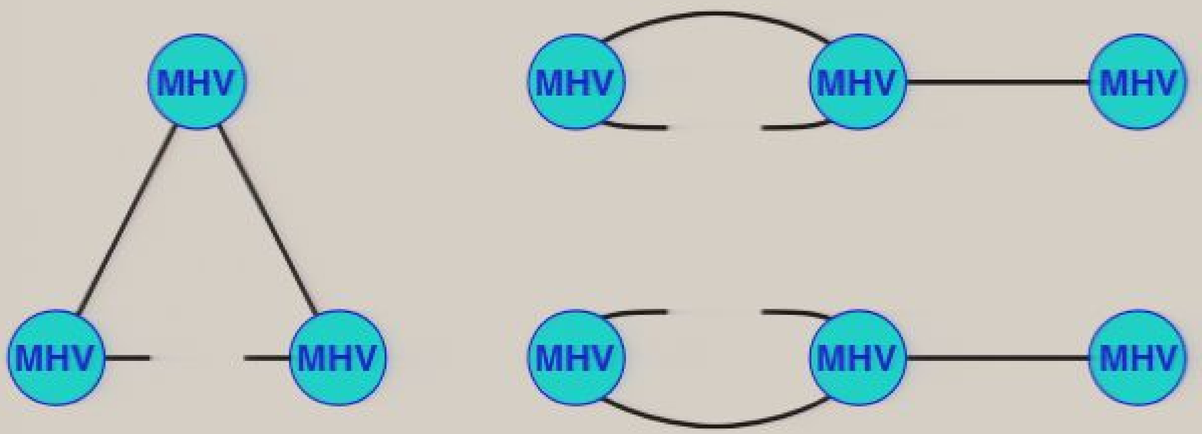


MHV diagrams

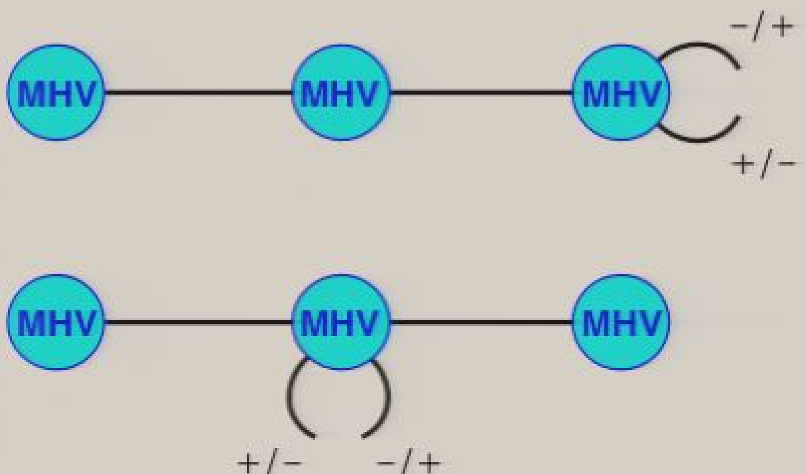


2-particle cut diagrams

FTT and MHV Diagrams cont'd



one-particle cut diagrams



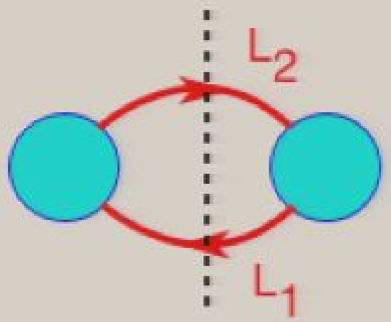
"missing" diagrams

Discontinuities

- One-loop MHV diagrams give covariant expressions
- Step 2: check that these expressions have the correct discontinuities or unitarity cuts in all channels.
- Straightforward; the diagrammatics is the same for Feynman 2-particle cuts in the FTT and a unitarity 2-particle cuts.
- In a particular channel one fixes two propagators and replaces them by two on-shell delta functions. Summing all MHV diagrams sharing the same 2-particle cut, one obtains the full tree amplitudes on both sides of the cut. LIPS integration produces then the expected discontinuity.

Discontinuities cont'd

- This argument applies also to generalised unitarity cuts.
- Note: although the diagrammatics look the same, a Feynman 2-particle cut is different from a unitarity 2-particle cut. In particular a Feynman/Unitarity 2-particle cut vanishes above/below the 2-particle threshold!



In a Feynman cut:

$L_{10} < 0$ and $L_{20} < 0$

In a Unitarity cut:

$L_{10} < 0$ and $L_{20} > 0$

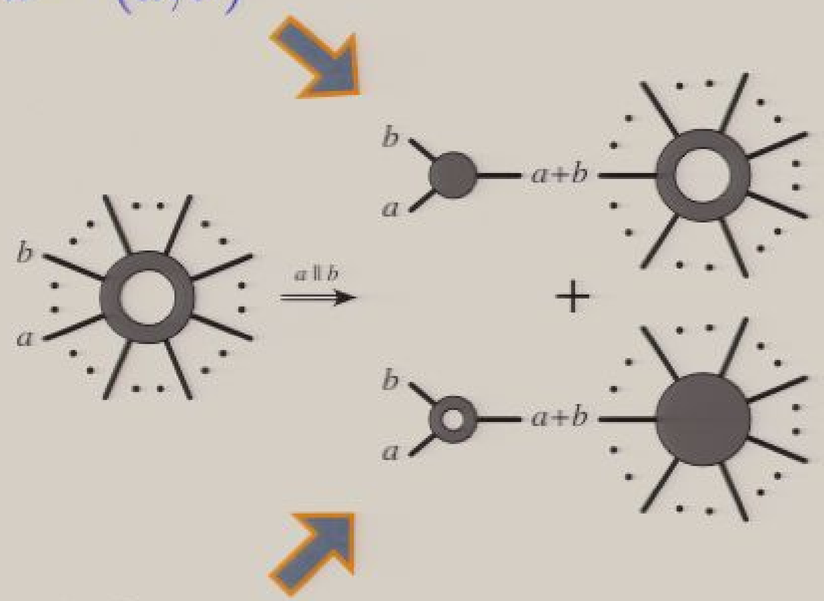
Factorisation

- MHV one-loop diagrams give covariant formulas with all the correct (generalised) cuts
- Step 3: Check that all physical poles in possible kinematic limits are correct. In particular we will check the universal collinear and some of the soft limits.
- Unphysical, η–dependent singularities (and cuts) can be excluded by our proof of covariance
- The remaining ambiguity must be a polynomial term, which can be ruled out on dimensional grounds (as in BCFW)

Universal collinear factorisation

- Consider a one-loop amplitude $\mathcal{A}_n^{1-loop}$ in the limit when momenta a and b become collinear (parallel)

$Split^{tree}(a,b)$



$Split^{1-loop}(a,b)$

Universal Collinear Factorisation

Involves tree and one-loop splitting functions:

$$Split_{-}^{tree}(a^{+},b^{+}) = \frac{1}{\sqrt{z(1-z)}}\frac{1}{\langle ab\rangle},\ \ Split_{+}^{tree}(a^{-},b^{-}) = -\frac{1}{\sqrt{z(1-z)}}\frac{1}{[ab]}$$

With $k_a := zk_P,\ k_b := (1-z)k_P,\ k_P^2 \to 0$

The one-loop splitting function is

$$Split^{1-loop}(a,b) = Split^{tree}(a,b) \times r(z)$$

- All order in ϵ expression for $r(z)$ was calculated by (Kosower-Uwer, Bern-Del Duca-Kilgore-Schmidt)

$$r(z) := \frac{c_\Gamma}{\varepsilon^2}\left(\frac{-s_{ab}}{\mu^2}\right)^{-\varepsilon}\left[1 - {}_2F_1\left(1,-\varepsilon,1-\varepsilon,\frac{z-1}{z}\right) - {}_2F_1\left(1,-\varepsilon,1-\varepsilon,\frac{z}{z-1}\right)\right]$$

Collinear Limits from MHV Diagrams

- At tree level, collinear limits come out as expected (CSW) as well as soft limits

- Two types of collinear limits in MHV diagram method

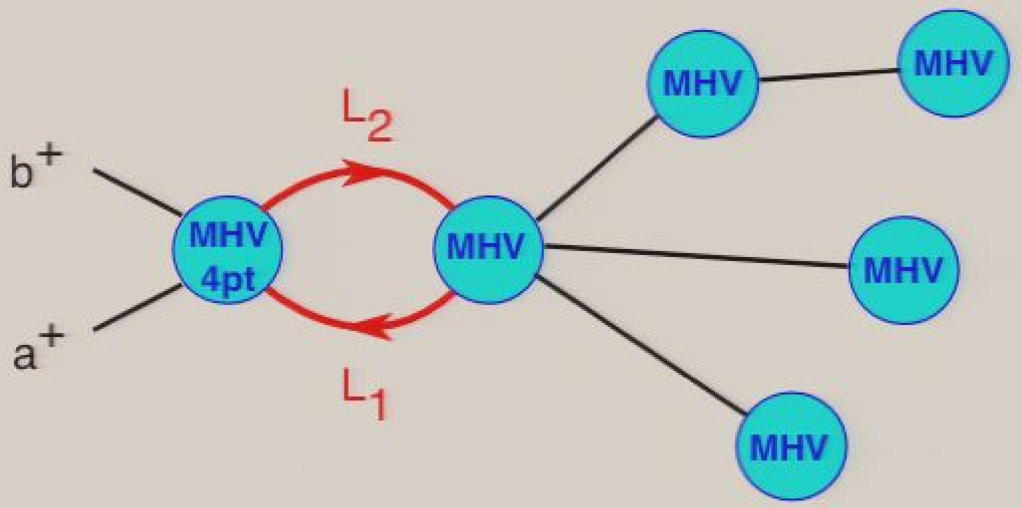
  - A-type: $++ \Rightarrow +$ and $+- \Rightarrow -$

  - B-type: $+- \Rightarrow +$ and $-- \Rightarrow -$

- Both legs belong to the same MHV vertex, for B-type this must be a 3-point vertex

Collinear Limits from MHV Diagrams

- Also at 1-loop level we have to distinguish A/B-type
- "singular channel" (Kosower) and "non-singular channel" MHV diagrams
  - "non-singular channel": Tree splitting function
  - "singular channel": 1-loop splitting function



"singular channel" MHV diagram

1-loop splitting functions from MHV diagrams

- If legs a and b end on different MHV vertices
  ➡ no contribution to collinear limit

- A-type collinear limits

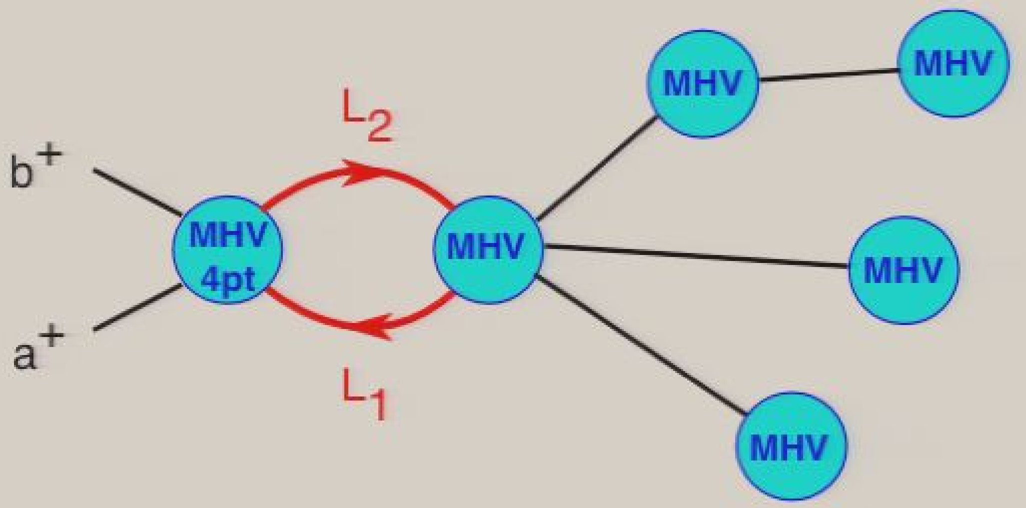
  - all order 1-loop splitting function from generic "singular channel" diagram shown before

  - requires all order in ε form of the 2-mass easy box function (slight generalisation of calculation in (AB-Spence-Travaglini))

  - tree-level splitting function from "non-singular channel" diagrams, where legs a and b are a proper subset of legs attached to MHV vertex

Collinear Limits from MHV Diagrams

- Also at 1-loop level we have to distinguish A/B-type
- "singular channel" (Kosower) and "non-singular channel" MHV diagrams
  - "non-singular channel": Tree splitting function
  - "singular channel": 1-loop splitting function



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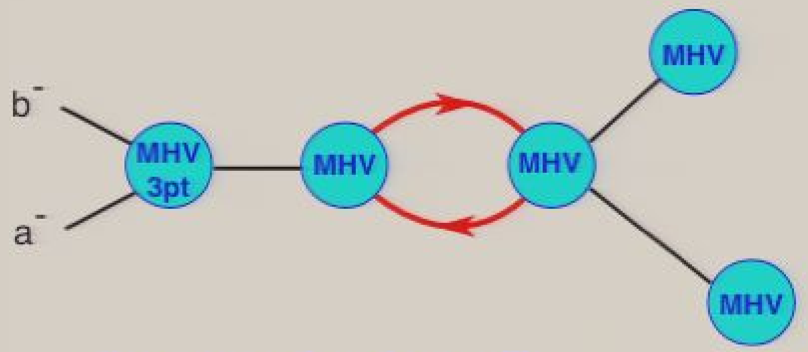
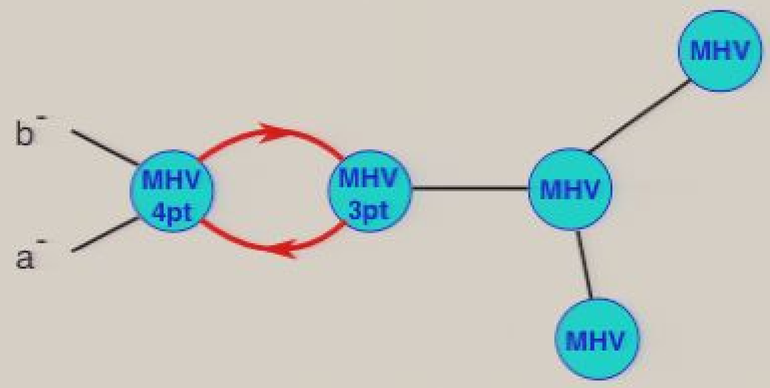
One-loop splitting functions cont'd

- B-type collinear limits need special attention
  - all order one-loop splitting function from "singular channel" diagram
  - agrees with known results



"non-singular channel" MHV diagram, contributes to tree level splitting function



"singular channel" MHV diagram

Soft Limits

- Behaviour of one-loop amplitudes when one leg s becomes soft is given by:

$$\mathcal{A}_n^{1-loop}(1,\ldots,a,s,b,\ldots,n) \xrightarrow{k_s \to 0}$$

$$Soft^{tree}(a,s,b)\, \mathcal{A}_{n-1}^{1-loop}(1,\ldots,a,b,\ldots,n)$$

$$+ Soft^{1-loop}(a,s,b)\, \mathcal{A}_{n-1}^{tree}(1,\ldots,a,b\ldots,n)$$

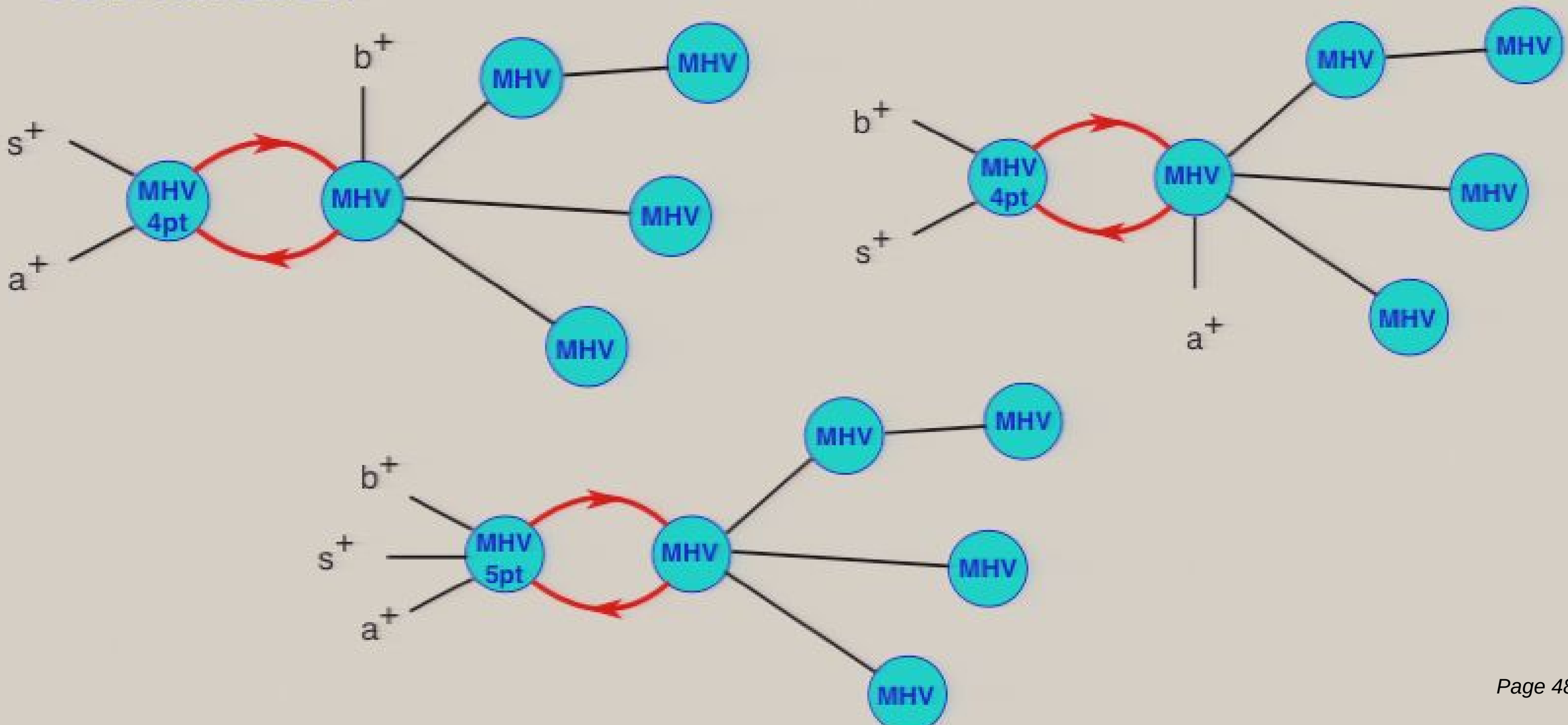
with

$$Soft^{tree}(a,s^+,b) = \frac{\langle a\, b\rangle}{\langle a\, s\rangle\, \langle s\, b\rangle}$$

$$Soft^{1-loop}(a,s,b) = Soft^{tree}(a,s,b)\left(-\frac{c_\Gamma}{\varepsilon^2}\frac{\pi\varepsilon}{\sin(\pi\varepsilon)}\right)\left(-\frac{s_{ab}}{s_{as}s_{sb}}\mu^2\right)^{\varepsilon}$$

Soft Limits from MHV Diagrams

- For concreteness we discuss the soft limit: $a^+ s^+ b^+ \longrightarrow a^+ b^+$
- Three MHV diagrams contribute in this case, the first two being familiar from the collinear limits
- Again the MHV diagrams reproduce the all order, one-loop soft function



Summary

- MHV diagrams are an efficient tool to calculate tree and 1-loop amplitudes in (S)YM
- In SYM at 1-loop we have shown
  - covariance (from FTT)
  - correct cuts (by construction)
  - correct soft and collinear limits, (to all orders in ϵ)
    - Multiparticle Poles?
- Further applications of FTT
  - rederivation of MHV 1-loop measure
  - FTT applies also to massive/non-susy theories

Outlook

- MHV diagrams work better than expected
  - all order in ϵ one-loop splitting and soft functions and 4-point one-loop amplitude in N=4 SYM
- Should work for higher loops (work in progress)
  - Connections with integrability (Minahan-Zarembo ...) and higher loop recursion relations (Anastasiou-Bern-Dixon-Kosower, Bern-Dixon-Smirnov, Cachazo-Spradlin-Volovich, Bern-Czakon-Kosower-Roiban-Smirnov) ?