Title: How should any quantum measuring instrument (including a quantum computer) work?

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Abstract: We will look at the axioms of quantum mechanics as expressed, for example, in the book by M. A. Nielsen and I. L. Chung ("Quantum Computation and Quantum Information"). We then take a critical look at these axioms, raising several questions as we go. In particular, we will look at the possible informational completeness property of the family of operators that we measure. We will propose physical solutions based on the results of quantum mechanics on phase space and the measurement of quantum particles by quantum mechanical means. We illustrate this with both momentum-position measurements and spin measurements.

How Should Any Quantum Measurement Work (Including a Quantum Computer)

Dr. Franklin E. Schroeck, Jr.
University of Denver

Outline:

1) Axioms of Quantum Mechanics (from "Quantum Computation and Quantum Information" by M. A. Nielsen and I. L. Chuang).

2) A critical look at the axioms.

3) A partial solution to the questions that arose.

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Axiom 1) The objective is to categorize a vector in a Hilbert space. Frequently we choose the Hilbert space to be C^2 or R^n. The Hilbert space is of finite dimension.

Axiom 2) Evolution of a closed quantum system is described by a unitary transformation. In the case of a single qubit, it is assumed that any unitary operator can be realized in realistic systems.

Axiom 3) Quantum measurements are described by a countable collection $\{M_m \mid \sum M_m^* M_m = 1\}$ of measurement operators, and

(a) the probability that m occurs in state ψ is

$$p(m) = \langle \psi \mid M_m^* M_m \psi \rangle = \| M_m \psi \|^2,$$

(b) the state of the system after measurement is $\| M_m \psi \|^{-1} M_m \psi$.

Axiom 4) A composite system of states

$\rho_1, \rho_2, \cdots, \rho_n$ is given by $\rho_1 \otimes \cdots \otimes \rho_n$

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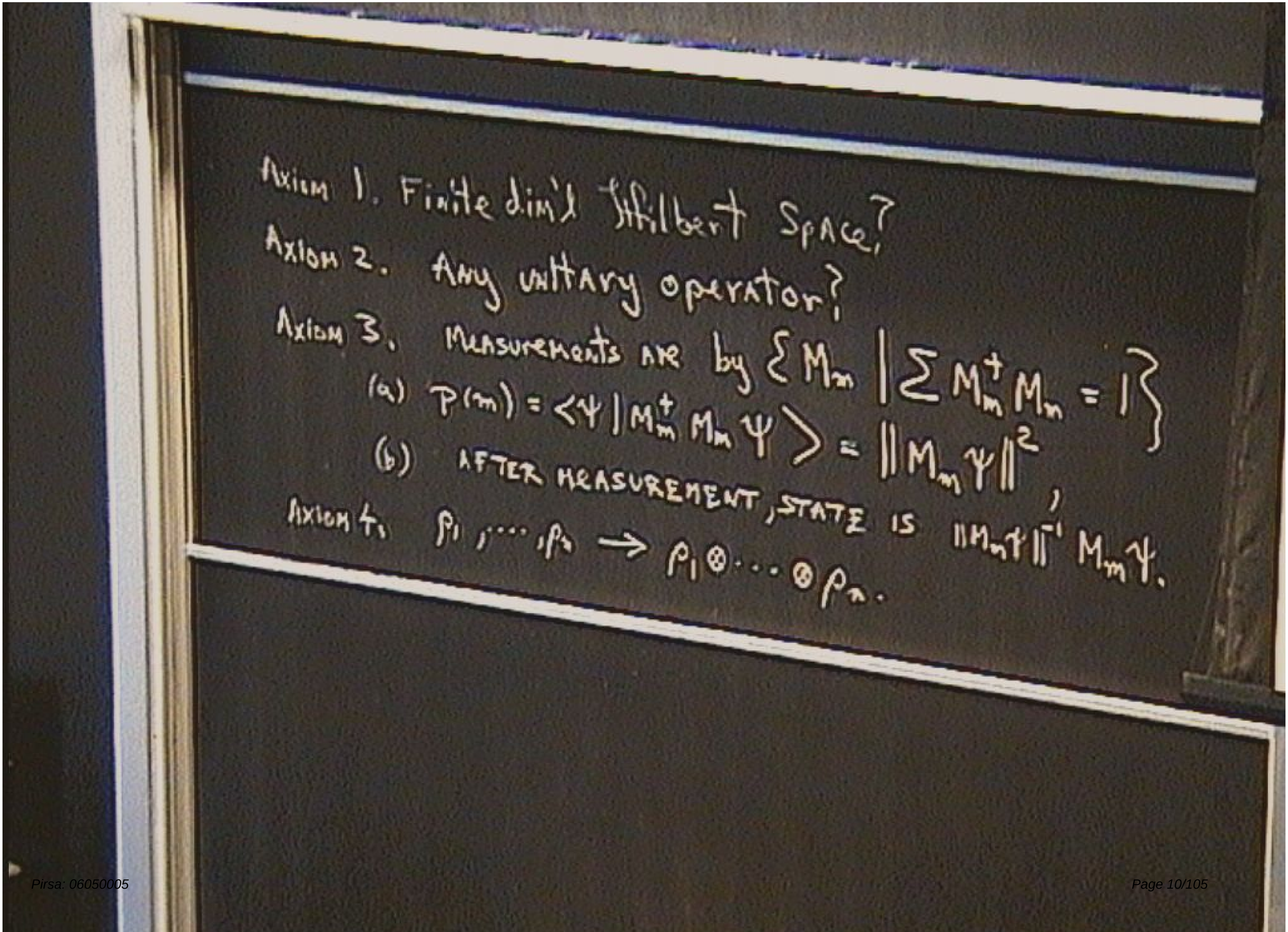
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On Axiom 1) Finite dim^l Hilbert space? dim(spin space) = 2. What about the position q and momentum p of the particle?

In Hilbert space H, take a countable basis expand any wave function. Truncate. p and q take you out of this basis.

There is no finite basis providing a representation of the c.c.r.'s.

Question 1) What is a basis on which to choose to truncate?

On Axiom 2) The dynamics of a particle in a closed quantum system is given by a unitary operator on phase space. Given any state "localized within a certain region of phase space" it may have a slow wave packet spreading, and then we may concentrate on the spin by taking the partial trace. We may not get "any unitary operator" this way.

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On Axiom 3) You don't have von Neumann's collapse scheme; so, the M_ms are not projections. They may be positive operators.

Question 4) Given that you don't have projections in the game, are some or all of the "results" of the Stern-Gerlach experiment, or of quantum computing, or · · · valid? How will we proceed with positive operators?

On Axiom 3 (contin.)) If M_m is a positive operator, then 3(a) remains the same. 3(b) is occasionally wrong. What if the experiment destroys the state, for example?

Question 5) Can we compute the "results" based on a) probabilities, b) dynamics alone?

Definition *We will take a set of operators $\{A_\alpha\}$ to be informationally complete if, whenever ρ and ρ' are density operators, then $Tr(\rho A_\alpha) = Tr(\rho' A_\alpha)$ for all α implies $\rho = \rho'$.*

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Question 6) What set of operators is informationally complete? Take a set with this property and then we may take a finite set of these to approximate the equality of states.

On Axiom 4) Simply taking the tensor product of states to obtain multi-particle states is contrary to entanglement. It is a certain subspace of the tensor product of the Hilbert spaces in which we work.

Question 7) Can we take entangled states and do a similar analysis in the case that the states are unentangled? (Answer - Yes.)

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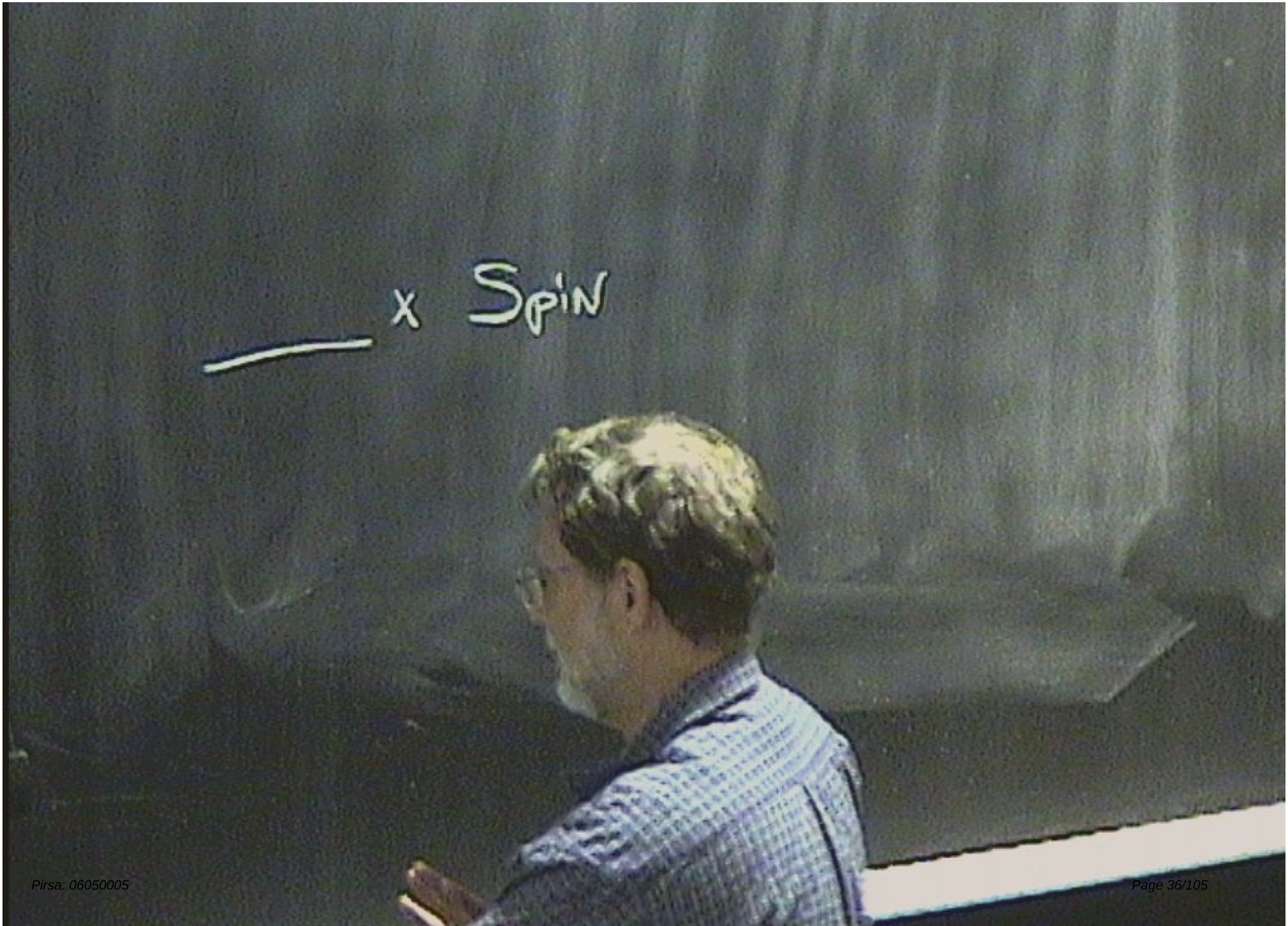
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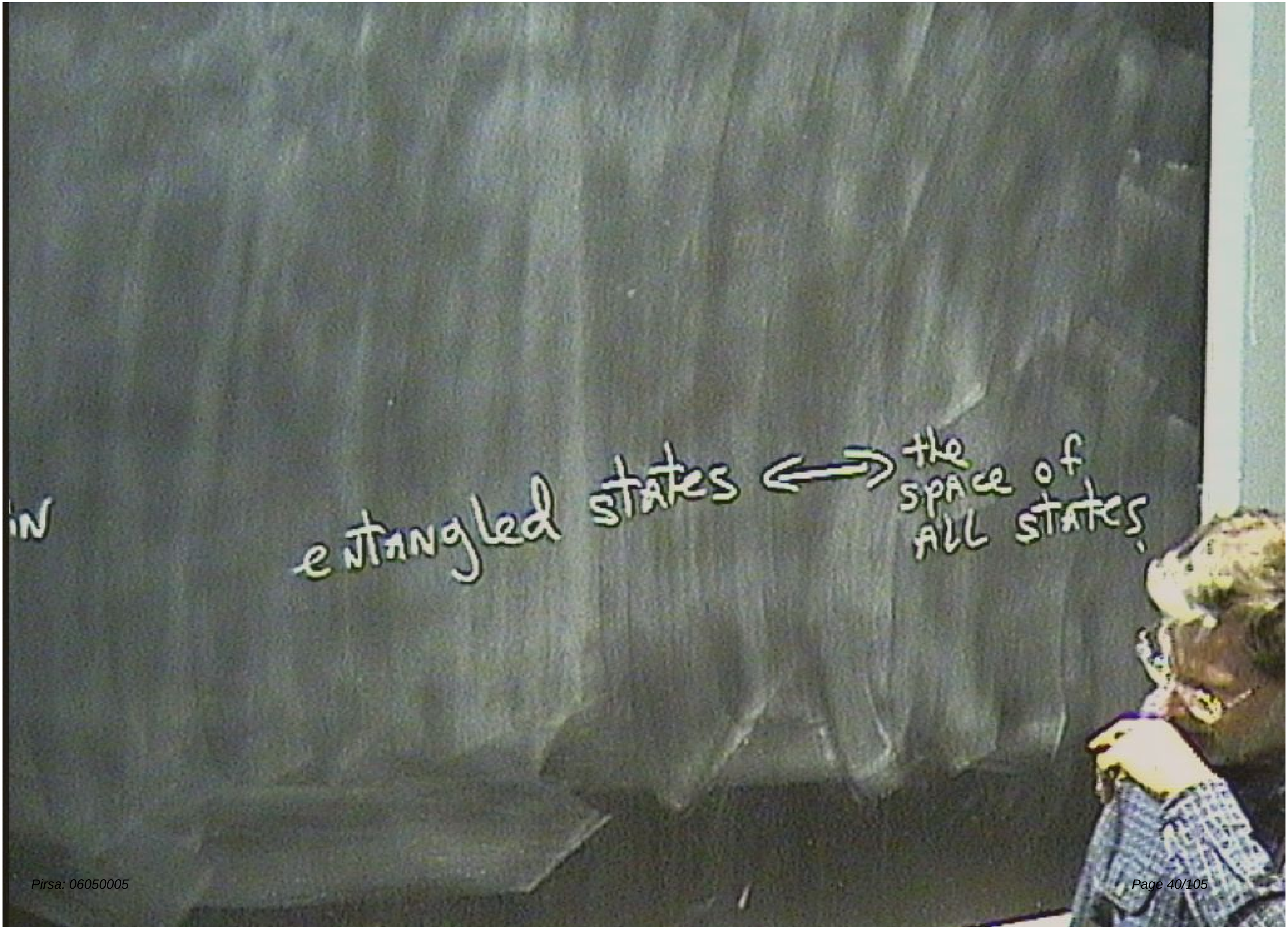
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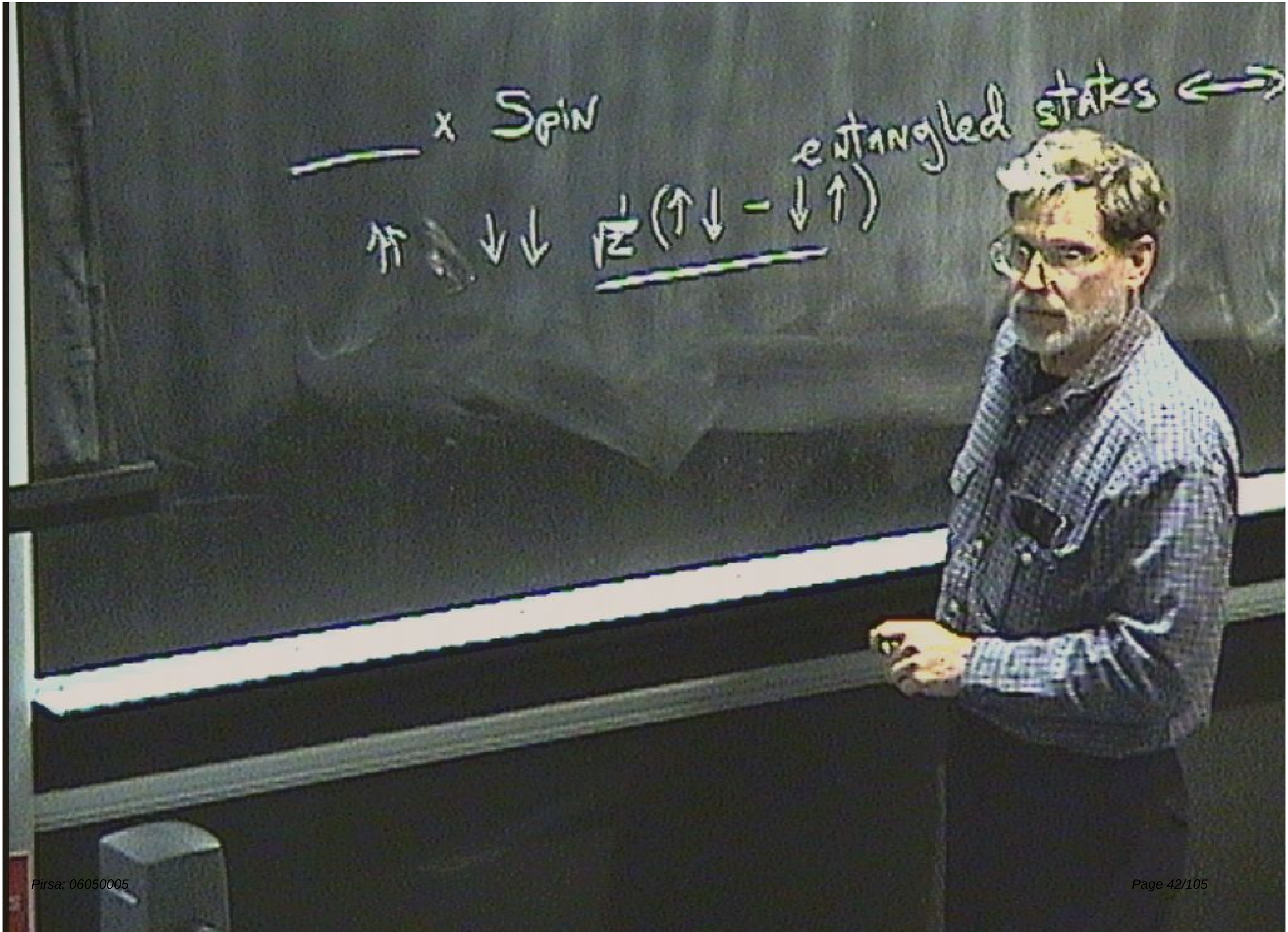
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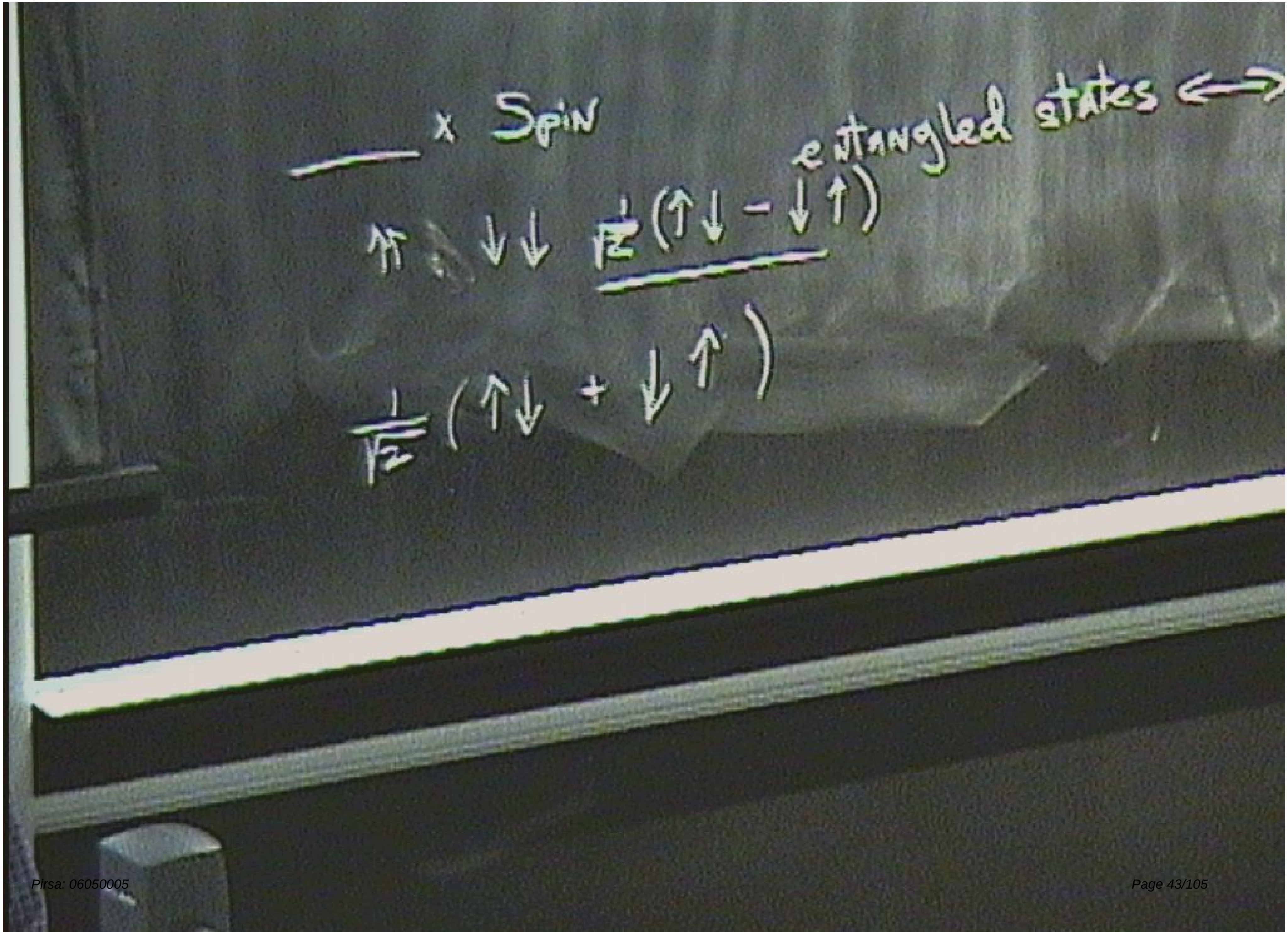
↑↑ ↓↓ $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$


x Spin
entangled states ⟷
↑↑ ↓↓ $\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
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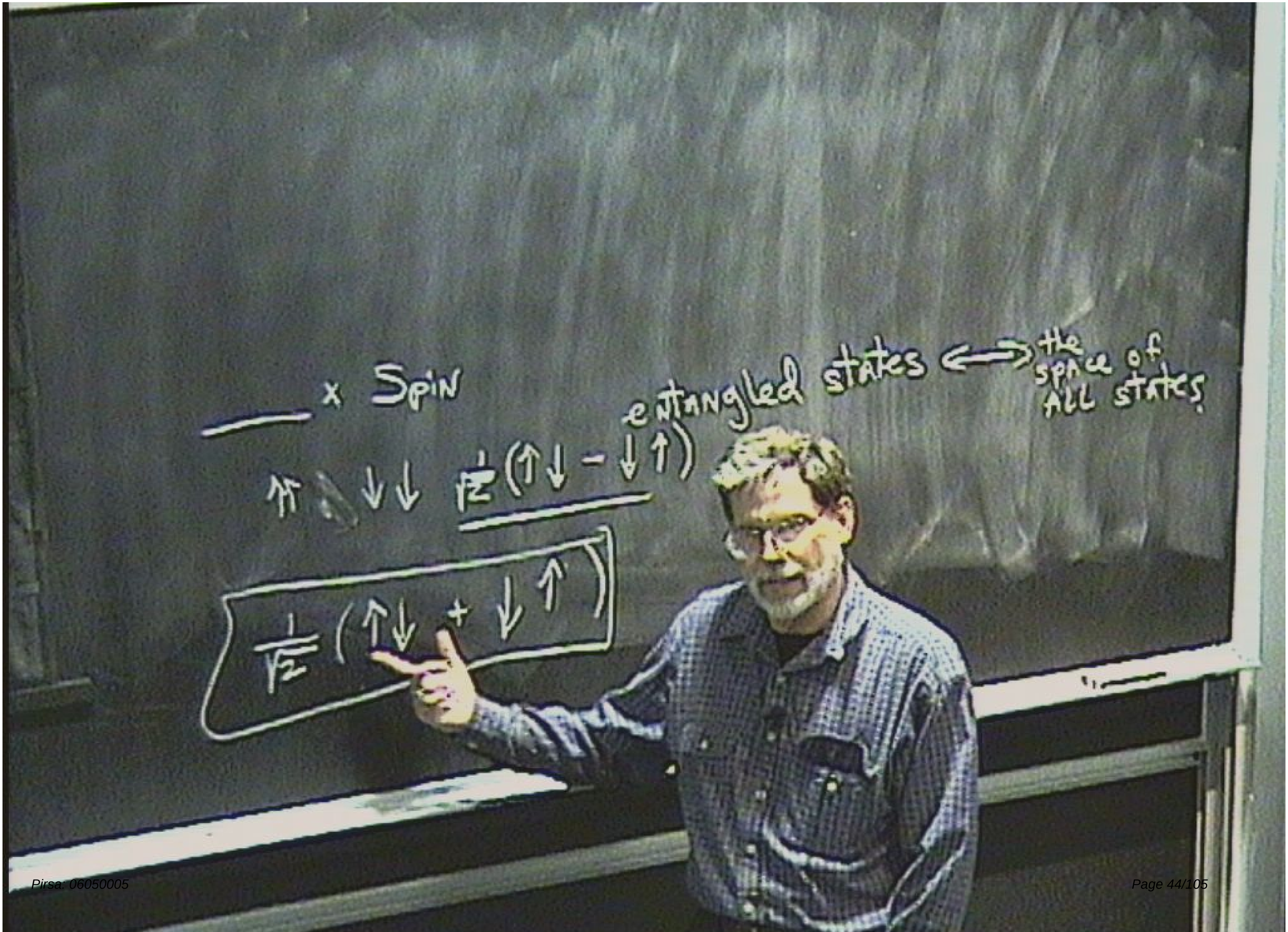
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entangled states ⟷ the space of ALL states
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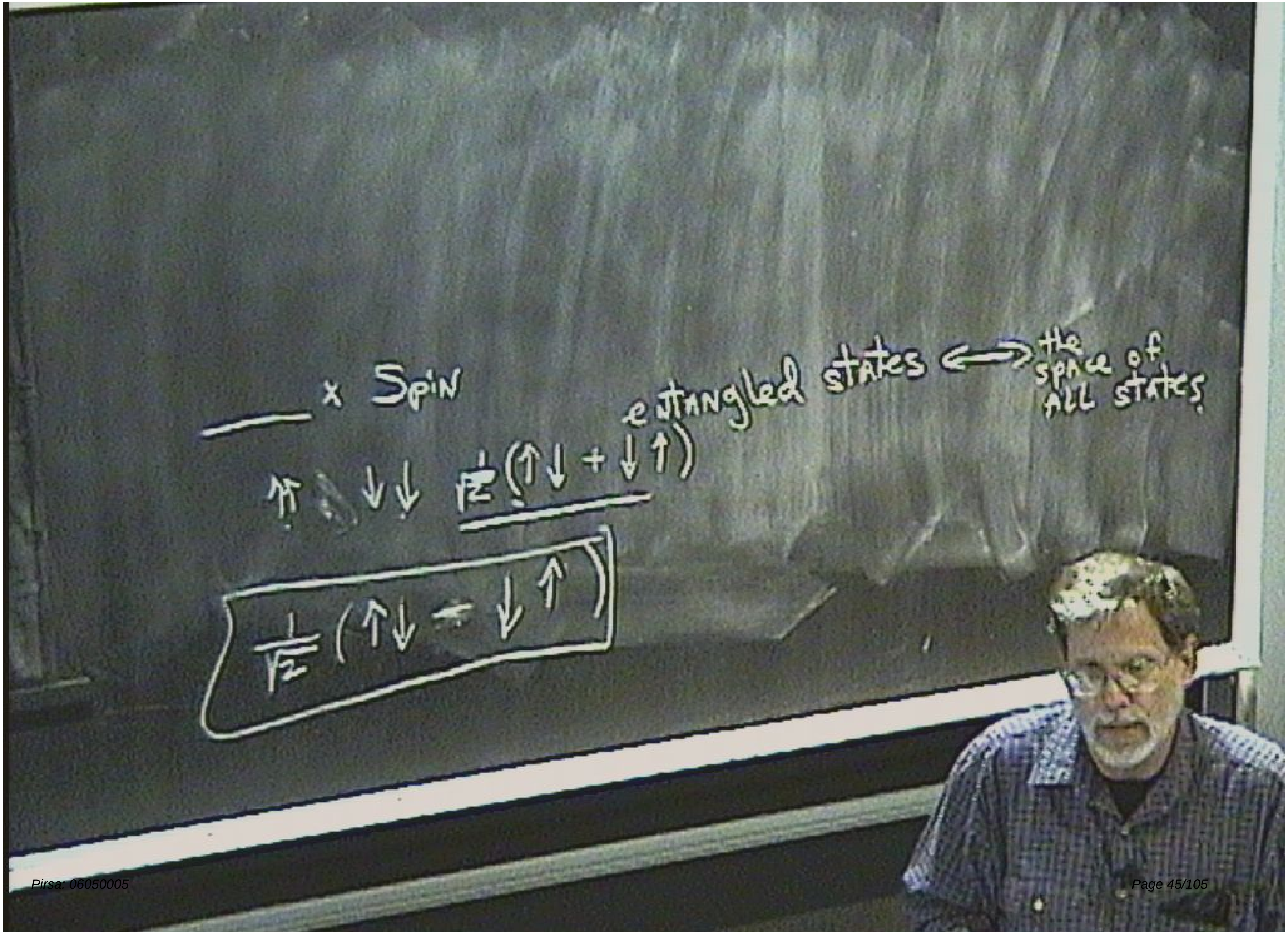
x Spin
entangled states ⟷ the space of ALL states
↑↑ ↓↓ 1/√2 (↑↓ + ↓↑)
1/√2 (↑↓ − ↓↑)

On Axiom 3 (contin.)) The spectral projectors

for momentum and/or position are not informationally complete.

Question 6) What set of operators is informationally complete? Take a set with this property and <u>then</u> we may take a finite set of these to approximate the equality of states.

On Axiom 4) Simply taking the tensor product of states to obtain multi-particle states is contrary to entanglement. It is a certain subspace of the tensor product of the Hilbert spaces in which we work.

Question 7) Can we take entangled states and do a similar analysis in the case that the states are unentangled? (Answer - Yes.)

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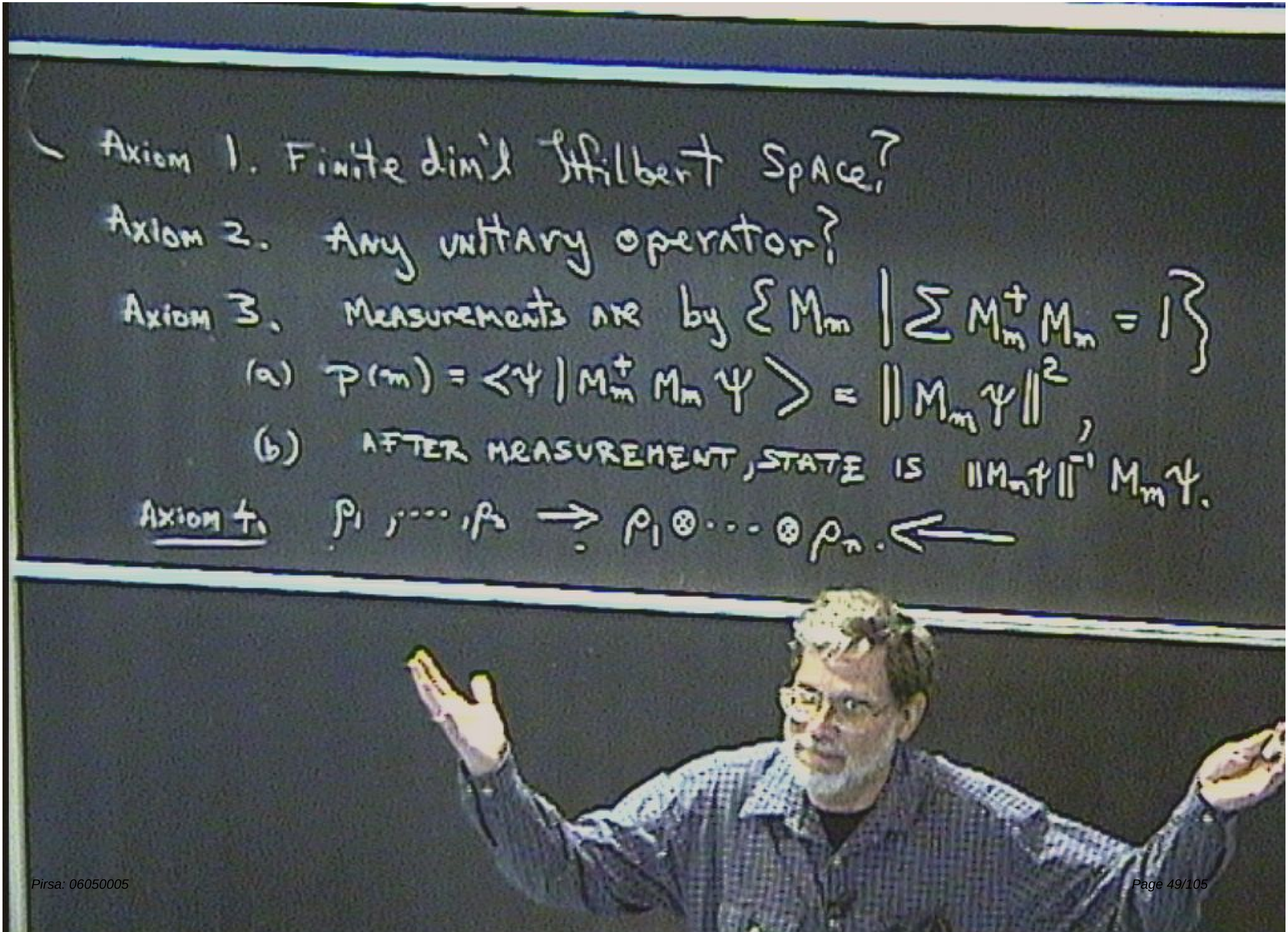
A Possible Solution coming from Quantum Mechanics on Phase Space

Via Wigner, we treat quantum mechanics on phase space as coming from the Poincaré (or Galilei) group and then derive the phase spaces, irreducible representations, etc. as coming from the group itself. The following definition is one non-surprizing result. See F.E. Schroeck, Jr., Quantum Mechanics on


Axiom 1. Finite dim'l Hilbert Space?
Axiom 2. Any unitary operator?
Axiom 3. Measurements are by {M_m | Σ M_m^† M_m = 1}
(a) p(m) = <ψ | M_m^† M_m ψ> = ||M_m ψ||^2,
(b) AFTER MEASUREMENT, STATE IS ||M_m ψ||^-1 M_m ψ.
Axiom 4. ρ_1, ..., ρ_n ⇒? ρ_1 ⊗ ... ⊗ ρ_n.

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Definition *(or Theorem) In relativistic quantum mechnics, the phase space for massive spinning particle is* $\mathrm{R}^3 \oslash \mathrm{R}^3 \oslash S(2)$*. For massive, spin zero particle =* $\mathrm{R}^3 \oslash \mathrm{R}^3$*.* $\mathrm{R}^3 \oslash \mathrm{R}^3$ *denotes the momentum space times position space;* $S(2)$ *is the spin space.*

Spin zero case: We may want to have particles with momentum in a box Δ_1 (say around zero) and position in a box Δ_2. Take a particle with wave function η that has

$$< \eta, P\eta > = 0, < \eta, Q\eta > = 0.$$

Translate it with $U(p,q)$ to obtain

$$< U(p,q)\eta, PU(p,q)\eta > = p,$$

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Take the probability for the localization operator for a general vector ψ to be in $\Delta_1 \times \Delta_2$ to be

$$L^2_{\Delta_1 \times \Delta_2}(\psi)$$

$$= \int_{\Delta_1 \times \Delta_2} |< U(p,q)\eta, \psi >|^2 \, d^3p \, d^3q$$

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Note that we have written just the [illegible] probability for ψ with $U(p,q)\eta$ integrated over the confines of the box $\Delta_1 \times \Delta_2$. (For the interpretation of η, see my book, for example.)

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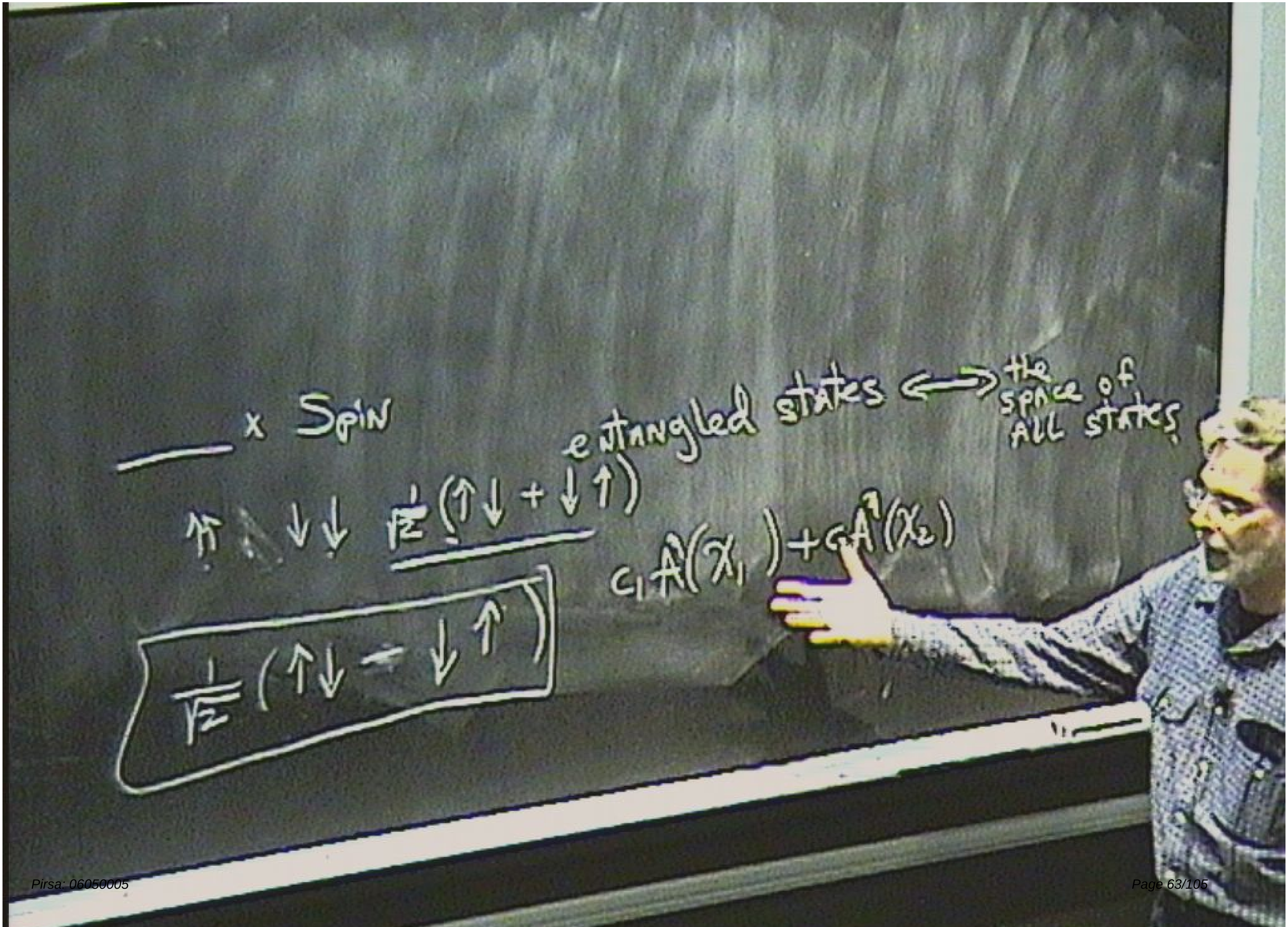
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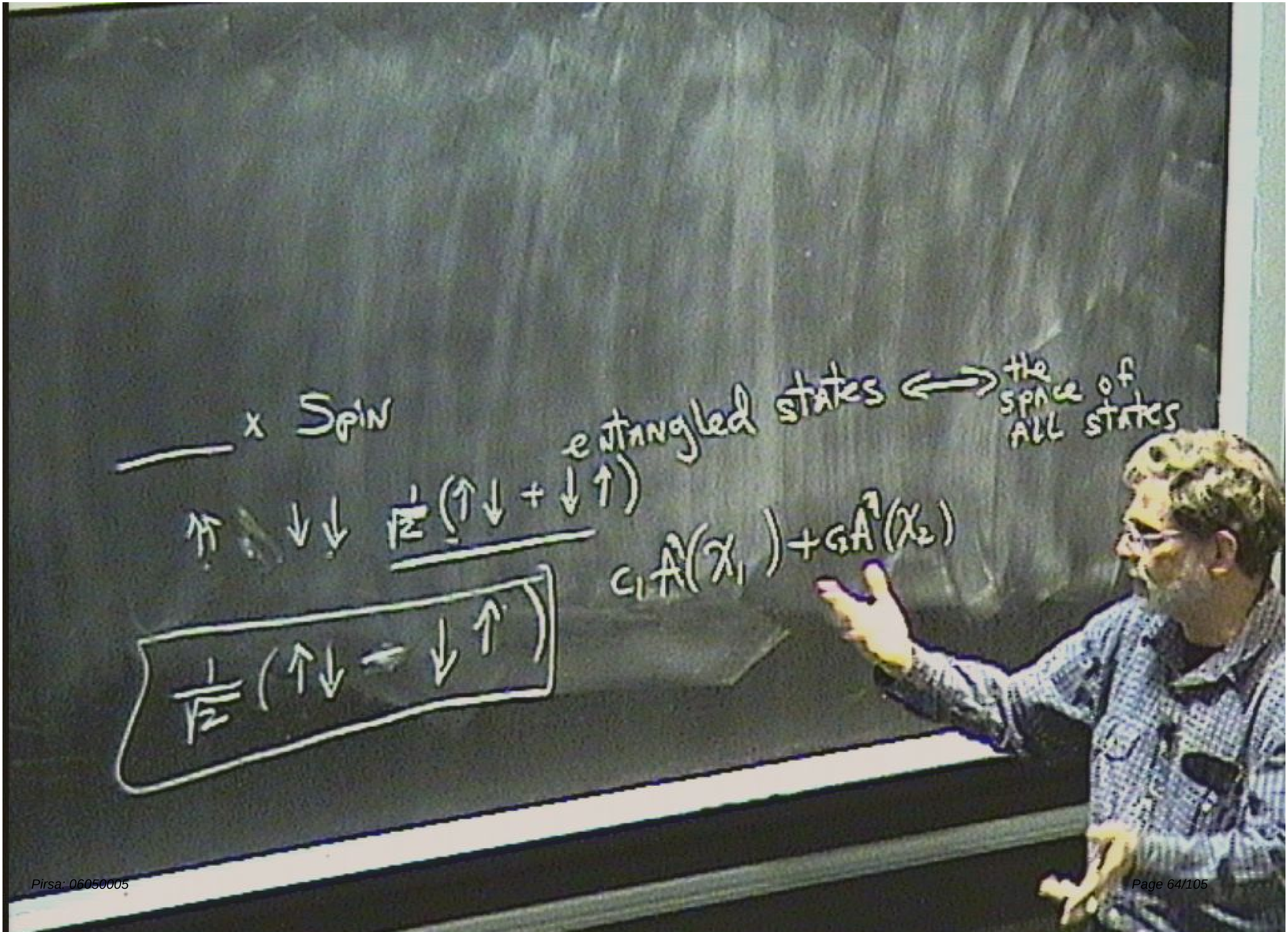
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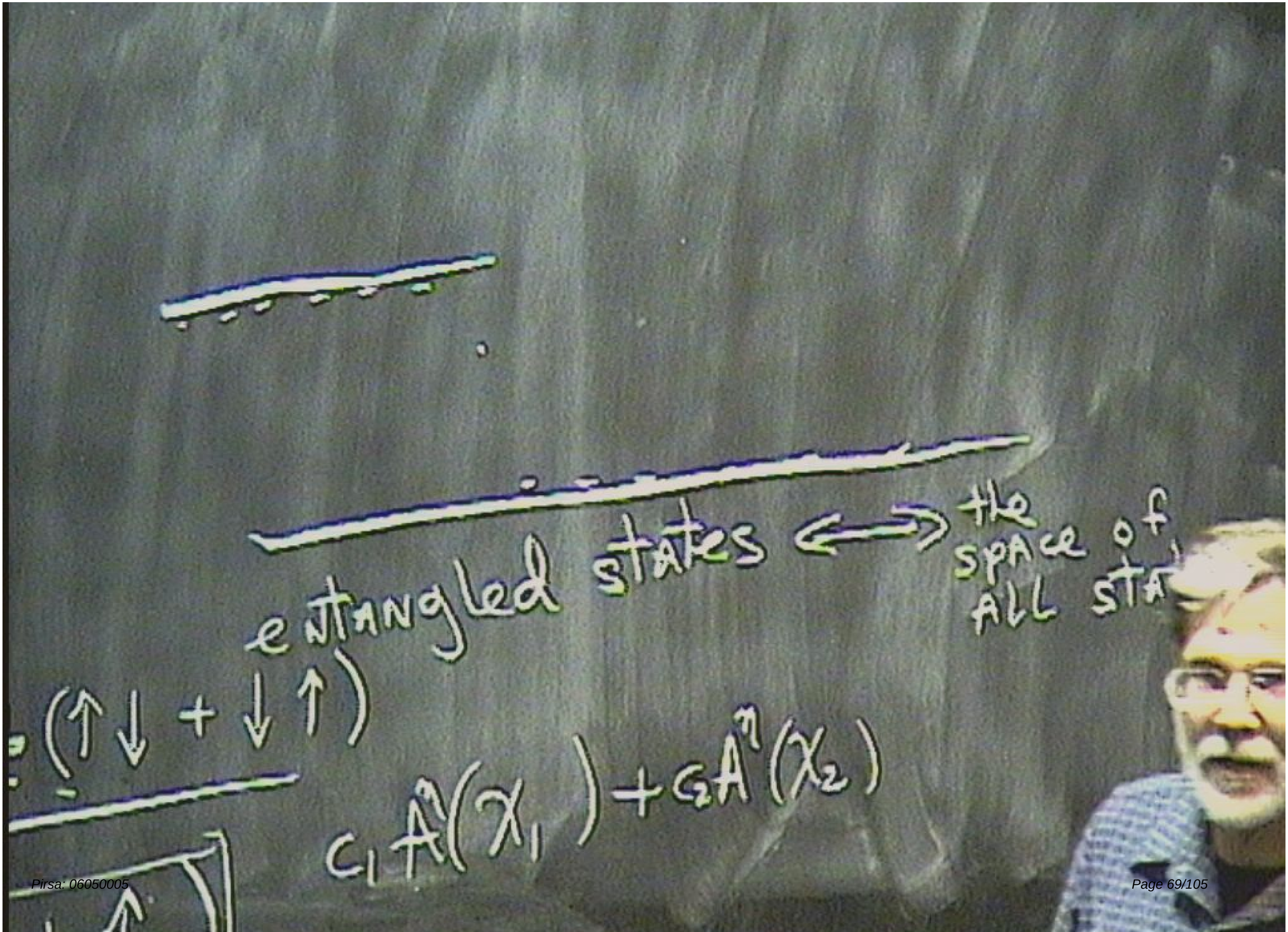
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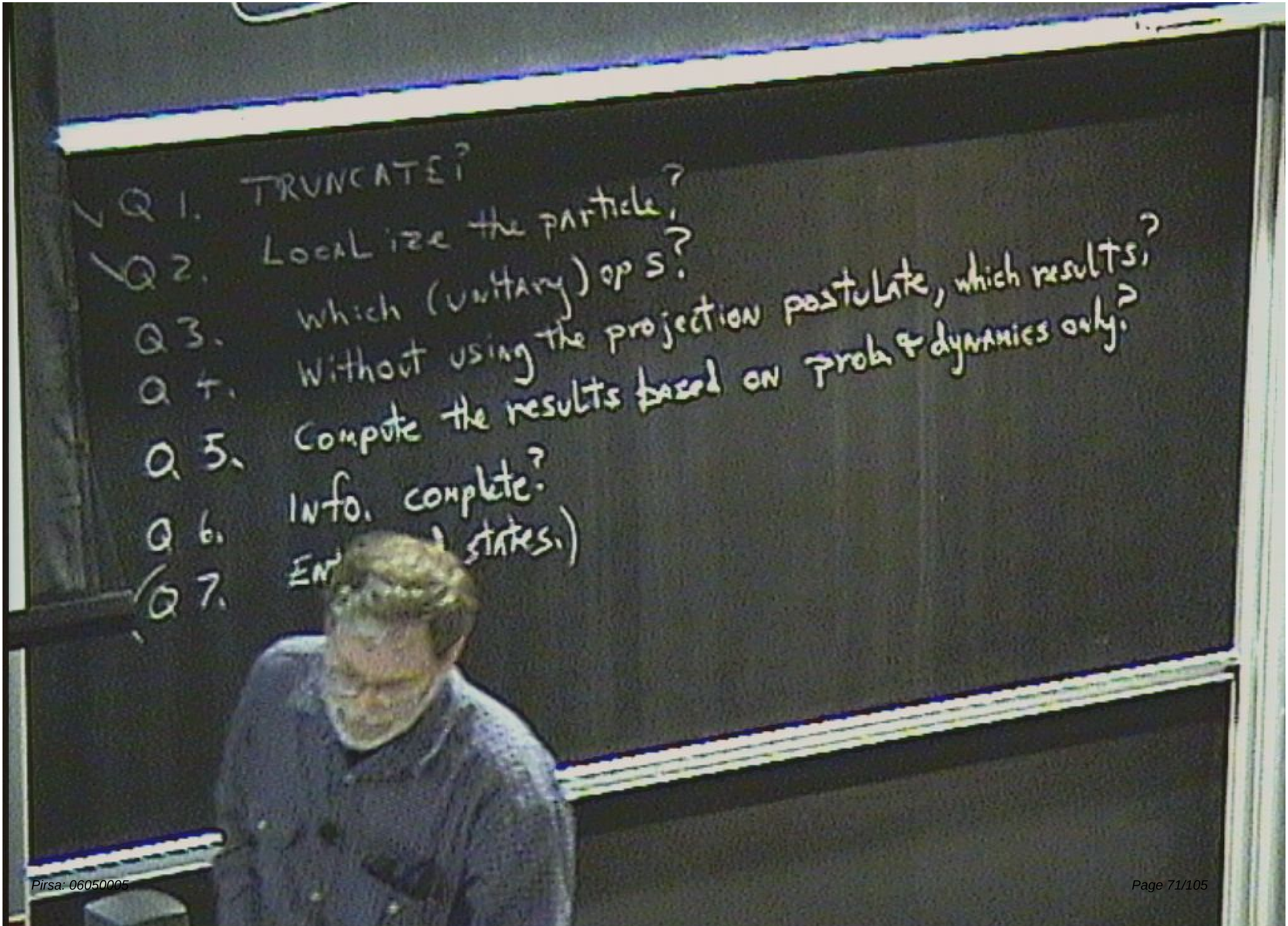
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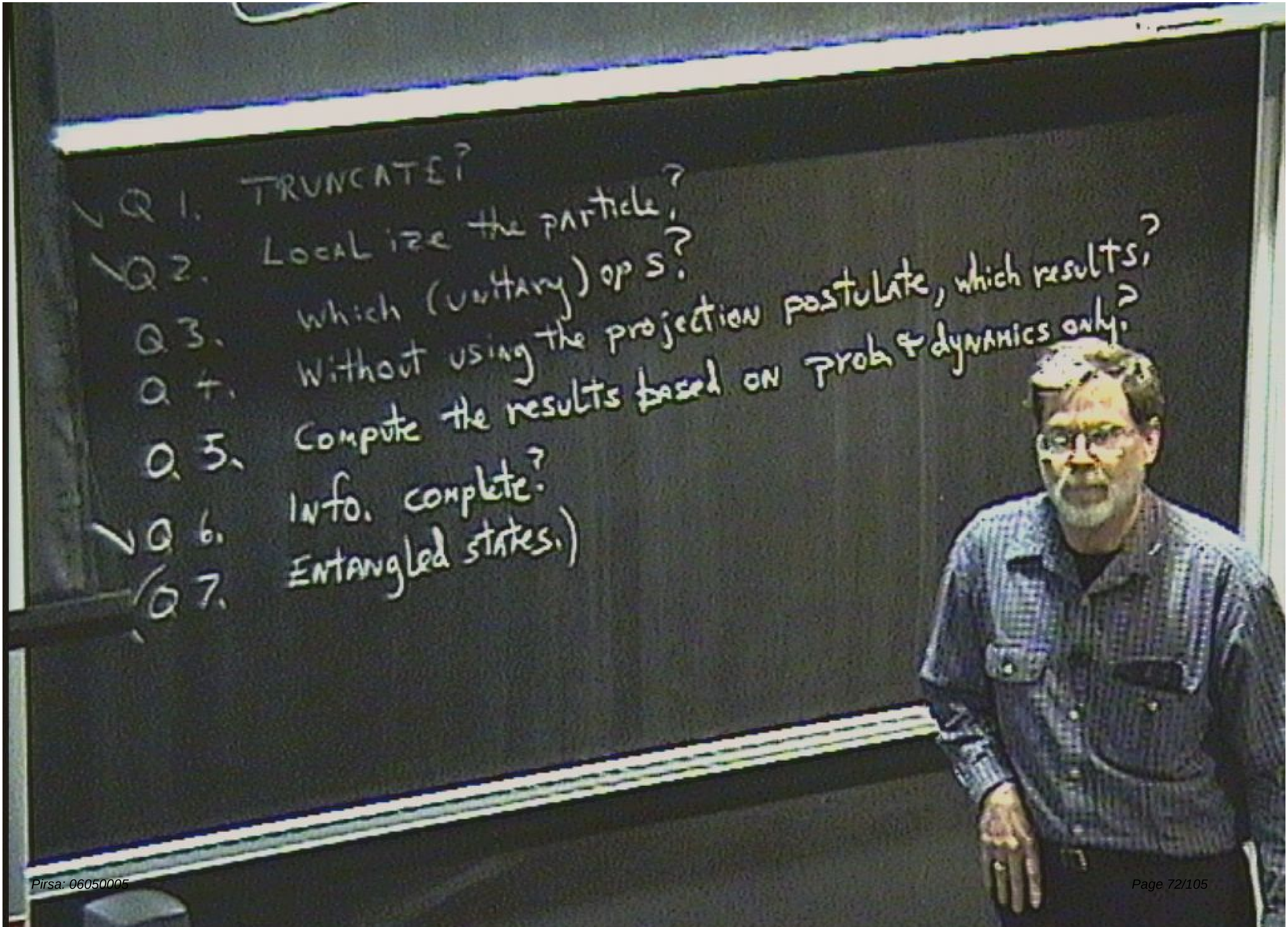
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To answer questions 3 and 4, we first look at the full phase space (momentum, position, and spin) on which we have some Hamiltonian dynamics. Then we will take the projection of that unitary operator onto the part of phase space being considered, and then take the partial trace to get what we have to discuss for a theory based on just momentum, or just position, or just spin. We don't get a unitary dynamics by this means.

Alternatively, we may expand the Hamiltonian dynamics in terms of the eigenbasis we obtained from localizing; then truncate the basis; and then look at the relevant variables for the experiment by marginality. Again we won't get a unitary operator.

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Two examples.

Stern-Gerlach devise: If you take the beam of (bound) electrons as collumated (which they are), then place the Stern-Gerlach device in the beam, and then treat the beam particles as functions of momentum, position, and spin, you find that the "upper" transmitted beam has a great deal of spin "up", but not all. It may be as much as 1 in 25 spin down. (P. Busch and F. Schroeck, Found. Physics 19 (1989), 807-872.) Spin is a quantity which is physically not an eigenfunction of any projection in this experiment, as you can not allign the direction of measurement of the spin perfectly. We may have an eigenfunction of a positive operator with eigenvalue near but not equal to one, obtained in a manner similar to our formulation of $A^{\eta}(\chi_{\Delta_1 \times \Delta_2})$.

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Definition *A qubit is a vector in a two dimensional Hilbert space,* H_0. *Choose an orthonormal basis* $\{|0>, |1>\}$.

Quantum computing: A CNOT gate is supposed to take a control qubit and a target qubit, and if the control is in state |0>, then the target is left alone; if the control is in state |1>, the target is flipped.

This transition may be described by a unitary operator in H_0. But, having a system with only two states is just an approximation, and the unitary operator you have on that two state system doesn't come from any Hamiltonian dynamics!

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Pirsa: 06050005

As states evolve, they undergo wave-function spreading. This is a small spreading in terms of q, but it will change the direction of p slightly, and thus change the spin which will have a drastic effect if the angle of change is appreciable. Said another way, as the evolution takes place, the state goes from one eigenvector of localization with eigenvalue near 1 to a state that is a mixture of eigenvectors; you don't expect it to go to just the orthogonal vector in H_Δ.

The transition is

$$\psi \to A^\eta(\chi_\Delta)\psi \to U(\Delta t)A^\eta(\chi_\Delta)\psi$$

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where $U(\Delta t)$ is the unitary time evolution, Δ is the set for localizing the particle in the full phase space before the evolution and Δ' is for localizing after the evolution, which may be different. Then $M_\Delta = A^\eta(\chi_{\Delta'})U(\Delta t)A^\eta(\chi_\Delta)$, which is not a unitary operator on the original Hilbert space.

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Conclusion

In order to get a truely quantum mechanical theory of measurement we must look a little deeper into the theory of measurement. We may take the previous "results" as just mathematical, and not necessarily physical. For example, we take the unitary operators of time propagation and look at the resultant when we "project" by means of a positive operator. What then will be the analog of Shor's theorem, for example?