Title: Mirage Inflation

Date: May 02, 2006 11:00 AM

URL: http://pirsa.org/06050000

Abstract: A cosmological model based on an inhomogeneous D3-brane moving in an AdS_5 X S_5 bulk is introduced. Although there is no special points in the bulk, the brane Universe has a center and is isotropic around it. The model has an accelerating expansion and its effective cosmological constant is inversely proportional to the distance from the center, giving a possible geometrical origin for the smallness of a present-day cosmological constant. Besides, if our model is considered as an alternative of early time acceleration, it is shown that the early stage accelerating phase ends in a dust dominated FRW homogeneous Universe. Mirage-driven acceleration thus provides a dark matter component for the brane Universe final state. We finally show that the model fulfills the current constraints on inhomogeneities.

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MIRAGE INFLATION

Cristiano Germani
(DAMTP)

in collaboration with

Cristophe Galfard (DAMTP)

Alex Kehagias (Athens)

hep-th/0509136

Class. Quant. Grav. 23, 1999 (2006)

Perimeter Institute, 2 May 2006

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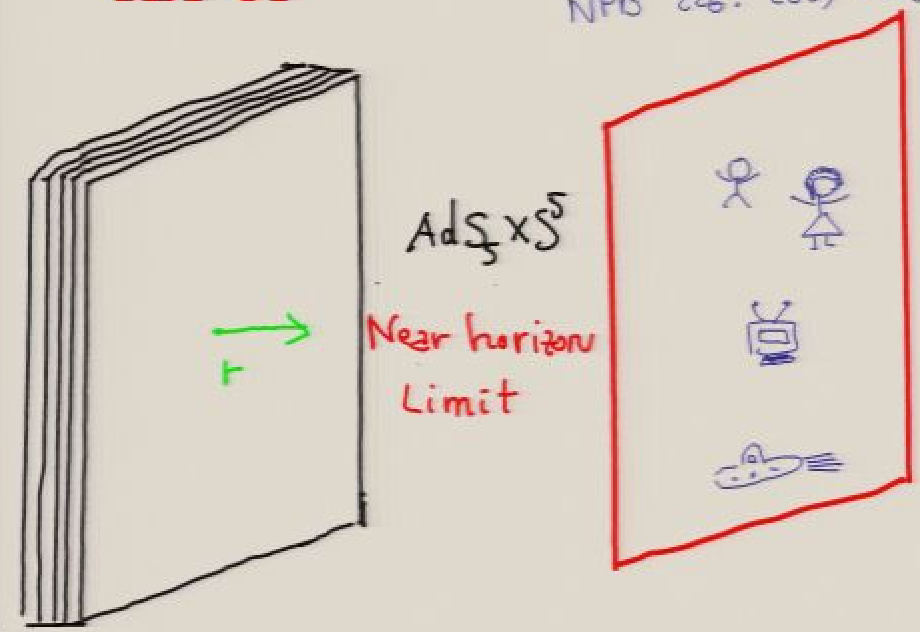
OVERVIEW

- Set up, D-brane Stuff
- Inhomogeneous trajectories
- Cosmic Acceleration
- Conclusions
- Work in Progress

SET-UP

Schwarz
NPB 226: 269, 1983



N D3-branes
($N \gg 1$)

D3 brane, our Universe

$$ds^2 = \frac{r^2}{L^2}\left(-dt^2 + d\rho^2 + \rho^2 d\Omega_2^2\right) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

$L \equiv AdS_5$ length

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Our Universe

$$S_{DBI} = T_3 \int d^4x \, e^{-\phi} \sqrt{\det\left(G^{ind}_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} - B_{\alpha\beta}\right)} +$$

$$+ T_3 \underbrace{\int d^4x \, \hat{C}_4}_{\text{RR Field}} + \text{anomaly terms}$$

Near horizon Limit + Spherical coords

$$\Downarrow$$

$$\phi = \text{const} \qquad \hat{C}_{0..3} = \frac{r^4}{L^4}$$

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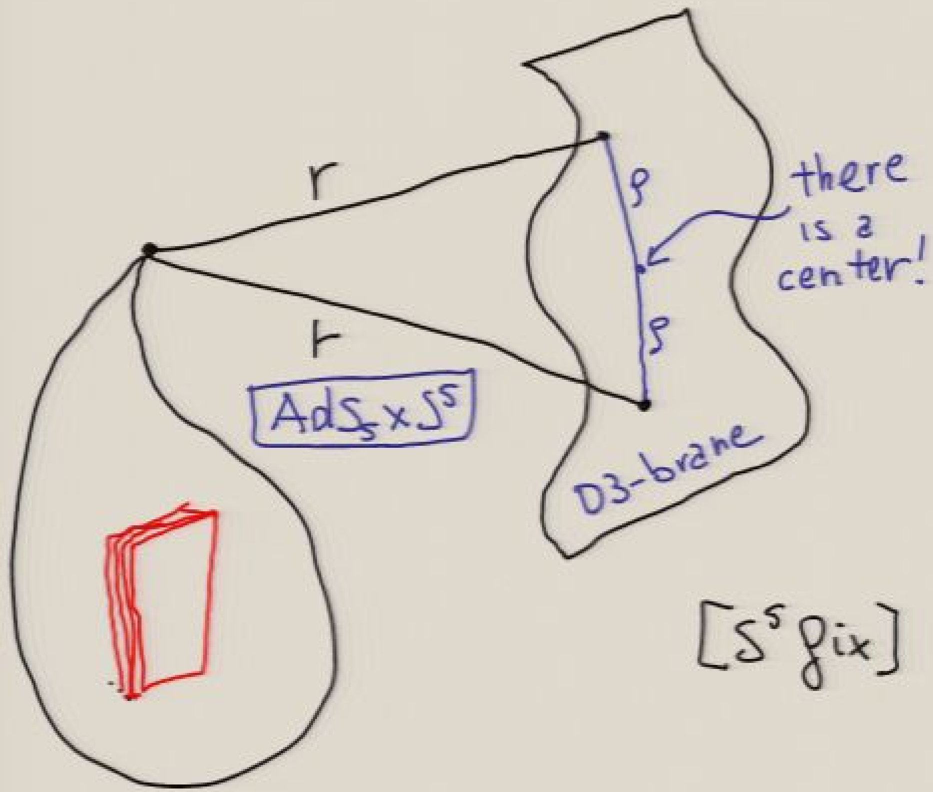
MIRAGE COSMOLOGY



We consider a vacuum Brane ⇒ $B = 0 = F$

Non-homogeneous initial conditions

$\Downarrow$

$r = r(t, \rho)$ on the Brane

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Brane Motion

$$ds^2_{ind} = -\left(\frac{r^2}{L^2} - \dot{r}^2 \frac{L^2}{r^2}\right)dt^2 + \left(\frac{r^2}{L^2} + \frac{r'^2 L^2}{r^2}\right)d\rho^2 + \frac{2\dot{r}r'}{r^2} L^2 \, d\rho \, dt + \frac{r^2}{L^2} \rho^2 d\Omega_2^2$$

$(\dot{} = \partial_t \quad ' = \partial_\rho)$

Lagrangian

$$\frac{\mathcal{L}}{4\pi T_3} = \underbrace{\frac{r^4}{L^4} \rho^2 \sqrt{1 + \frac{r'^2 L^2}{r^4} - \frac{\dot{r}^2 L^4}{r^4}}}_{\sqrt{-\det g^{ind}_{d\rho}}} - \underbrace{\rho^2 \frac{r^4}{L^4}}_{RR}$$

Eq. of motion $\Rightarrow$ $\boxed{\delta_r \mathcal{L} = 0}$

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Non relativistic approximation

$$\boxed{\frac{\dot{r} L^4}{r^4} \ll 1} \qquad (1)$$

$$+$$

Quasi-Homogeneity

$$\boxed{\frac{r' L^4}{r^4} \ll 1} \qquad (2)$$

$$\Downarrow$$

$$\delta_r \mathcal{L} = 0 \Rightarrow \boxed{\ddot{r} - \frac{1}{\rho^2} \partial_\rho (\rho^2 r') = 0}$$

P.S. in the Following we will call

(1) and (2) non-Relativistic

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No singular solution

$$\boxed{r = \frac{\mu}{2}(t-t_0)^2 + \frac{\mu}{6}\rho^2 + r_0}$$

No space of parameter connecting with the homogeneous solutions !

$\Downarrow$

New branch !

Approximations satisfied for

$$\boxed{\mu \ll \frac{r_0^3}{L^4}}$$

No singularity

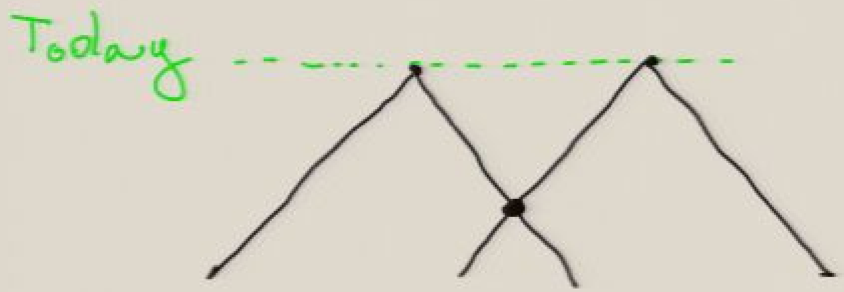
$\Downarrow$

No "big bang"

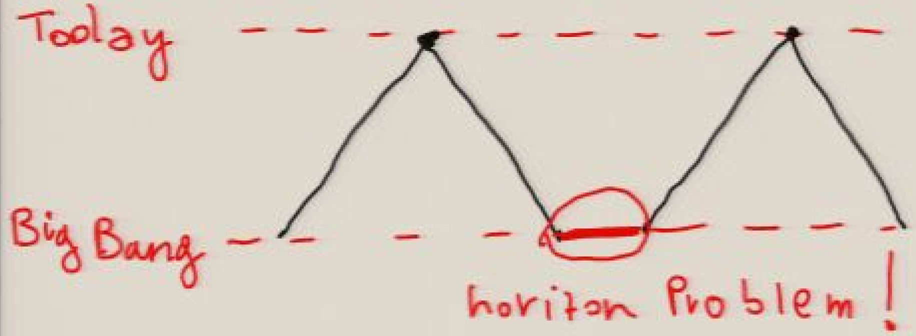
$\Downarrow$

No horizon problem

our model



FRW



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Is the Universe accelerated?

Induced metric on the Brane

$$ds^2 = -\left(\frac{r^2}{L^2} - \frac{\dot{r}^2 L^2}{r^2}\right) dt^2 + \left(\frac{r^2}{L^2} + \frac{r'^2 L^2}{r^4}\right) d\rho^2 + 2\frac{\dot{r} r'}{r^2} L^2 d\rho\, dt + \frac{r^2}{L^2}\rho^2 d\Omega_2^2$$

Non relativistic approximations are valid up to second order variation ($\Gamma^{\alpha}_{\mu\nu}, \dot{\theta}, \ldots$) if we use

$$\boxed{ds^2 \simeq \frac{r^2}{L^2} d\eta^2}$$

$d\eta^2 \equiv$ Minkowski line element

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Light-cone acceleration

$K^\alpha K_\alpha = 0$ Normalization

$K^\alpha \nabla_\alpha K^\beta = 0$ Geodesic

$$\Downarrow$$

$$K^\alpha = F(s+t)\left[-\frac{1}{r^2}, \frac{1}{r^2}, 0, 0\right]$$

where $F(s+t)$ parameterize the curve

Defining the light-cone expansion

$$\theta = \nabla_\alpha K^\alpha$$

and the acceleration parameter

$$Q_N = K^\alpha \nabla_\alpha \theta + \frac{1}{2}\theta^2$$

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$$Q_N = \frac{96\mu^2 F^2}{r^6}(t-t_0)\left[\varrho-(t-t_0)+\frac{r_0}{\mu(t-t_0)}\right]$$

so for $t > t_0$
and

$$\varrho + \frac{r_0}{\mu(t-t_0)} > t-t_0$$

$$\Downarrow$$

Large ϱ and/or early time

$$\boxed{Q_N > 0}$$

Acceleration!

Well .... lets see the time-like

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Fine Chisel
Medium Point
Very Fine Point
Medium Point
Marker (2 mm)
Pen Settings...

$$Q_N = \frac{96\mu^2 F^2}{r^6}(t-t_0)\left[\xi-(t-t_0)+\frac{r_0}{\mu(t-t_0)}\right]$$

so for $t > t_0$ and

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$$Q_N = \frac{96\mu^2F^2}{r^6}(t-t_0)\left[3-(t-t_0)+\frac{r_0}{\mu(t-t_0)}\right]$$

so for $t > t_0$

and

g

La ... ime

Pen and Highlighter Settings

Pen Settings | Highlighter Settings

Current pens:

- Fine Chisel
- Medium Point
- Very Fine Point
- Medium Point
- Marker (2 mm)

Color: | Thickness: | Tip style: Point

Pressure sensitive

OK | Cancel | Help

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Point

More Colors...

OK Cancel Help

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and

$$\varrho + \frac{r_0}{\mu(t-t_0)} > t - t_0$$

$$\Downarrow$$

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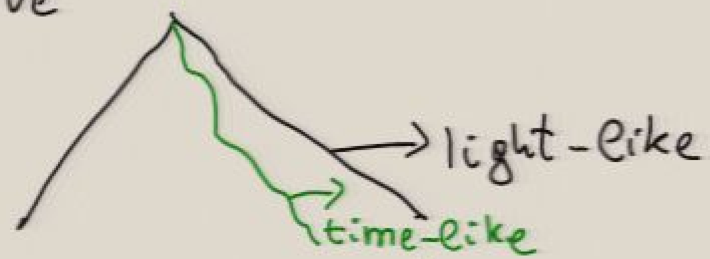
Acceleration!

Well .... Lets see the time-Like

Time-like

Naive



Concrete: we need more approx

Normalization: $u^\alpha u_\alpha = -1 \Rightarrow u^t = \left[\frac{L^2}{r^2} + (u^\rho)^2\right]^{1/2}$

quasi-homogeneous observer:

$$u^\rho \equiv v \ll \frac{L}{r}$$

Geodesic: $\dot{v} = \frac{r' L}{r^2}$ $\quad [v(t=t_0, \rho) = 0]$

$$V = \frac{\alpha(t-t_0)}{2\beta[(t-t_0)^2+\beta]} + \frac{\alpha}{2\beta^{3/2}} \arctan \frac{t-t_0}{\beta^{1/2}}$$

$$\alpha = \frac{4\rho L}{3\mu} \qquad \beta = \frac{\rho^2}{3} + 2\frac{r_0}{\mu}$$

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Suppose we are at large distance from the center ($\rho = 0$)

$\Downarrow$

(*) β large $\Rightarrow$ $\beta \gg (t-t_0)^2$

$\Downarrow$

$$\boxed{V \simeq \frac{\alpha(t-t_0)}{\beta}}$$

Acceleration Parameter

$$Q_t = n^{\lambda} \nabla_{\lambda} \theta + \frac{1}{3} \theta^2 \simeq \frac{288 L^2}{3\mu^2 \rho^6}$$

$Q_t > 0$ in the approximation in particular at early time. *

Late time ???

Late time

Large distance from the center

$\frac{\rho}{t-t_0} \gg 1$ we found

$$Q_t \simeq \frac{288 L^2}{3\mu^2 \rho^6}$$

Suppose we measure

$\rho = \rho_0 + \varepsilon$ with $\varepsilon \ll \rho_0$

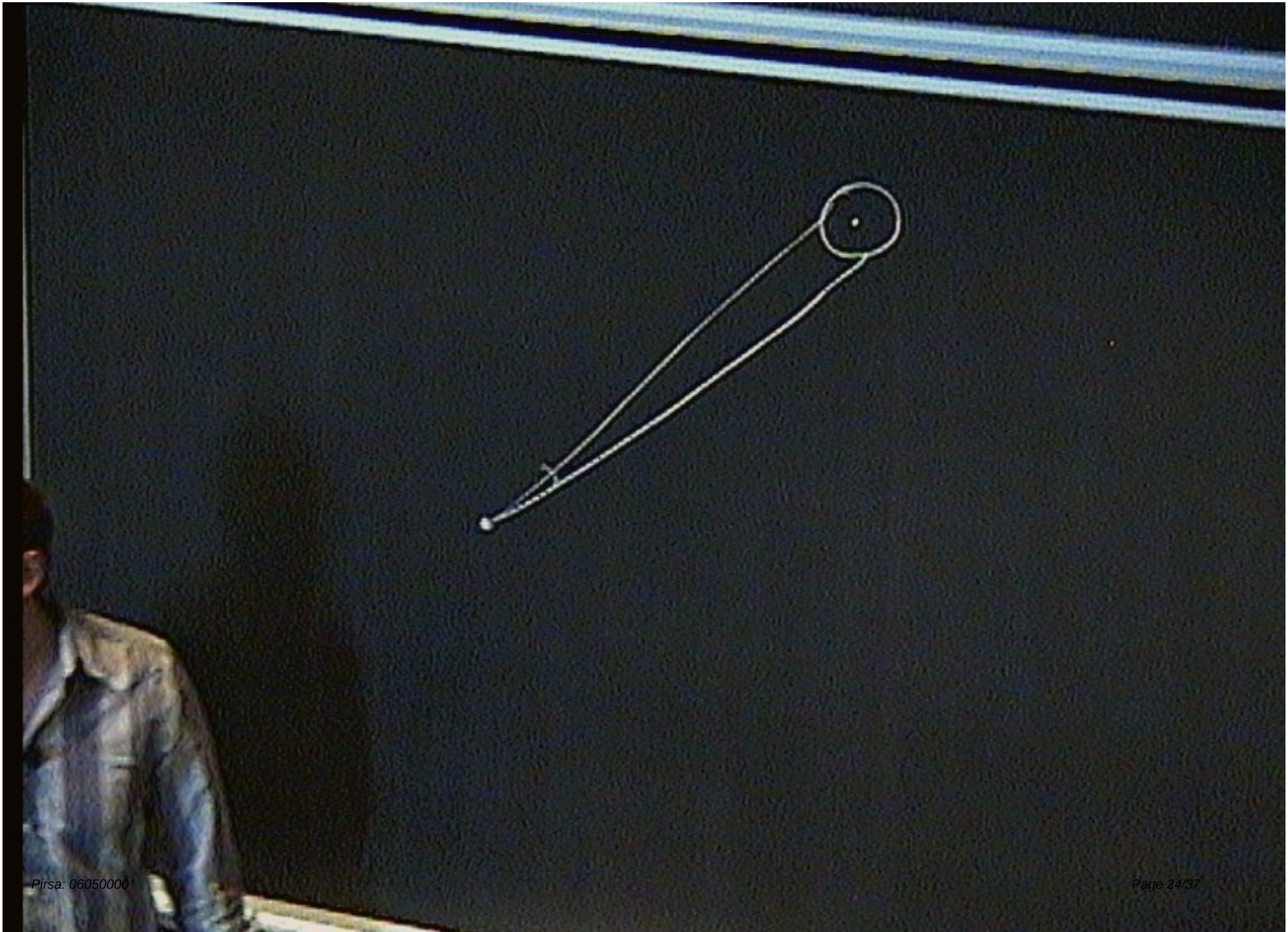
then

$$Q_t \simeq \frac{288 L^2}{3\mu^2 \rho_0^6} = \text{const}$$

Lets interpret Q_t as the cosmological constant

$$\boxed{Q_t = \Lambda}$$

we will comment later!



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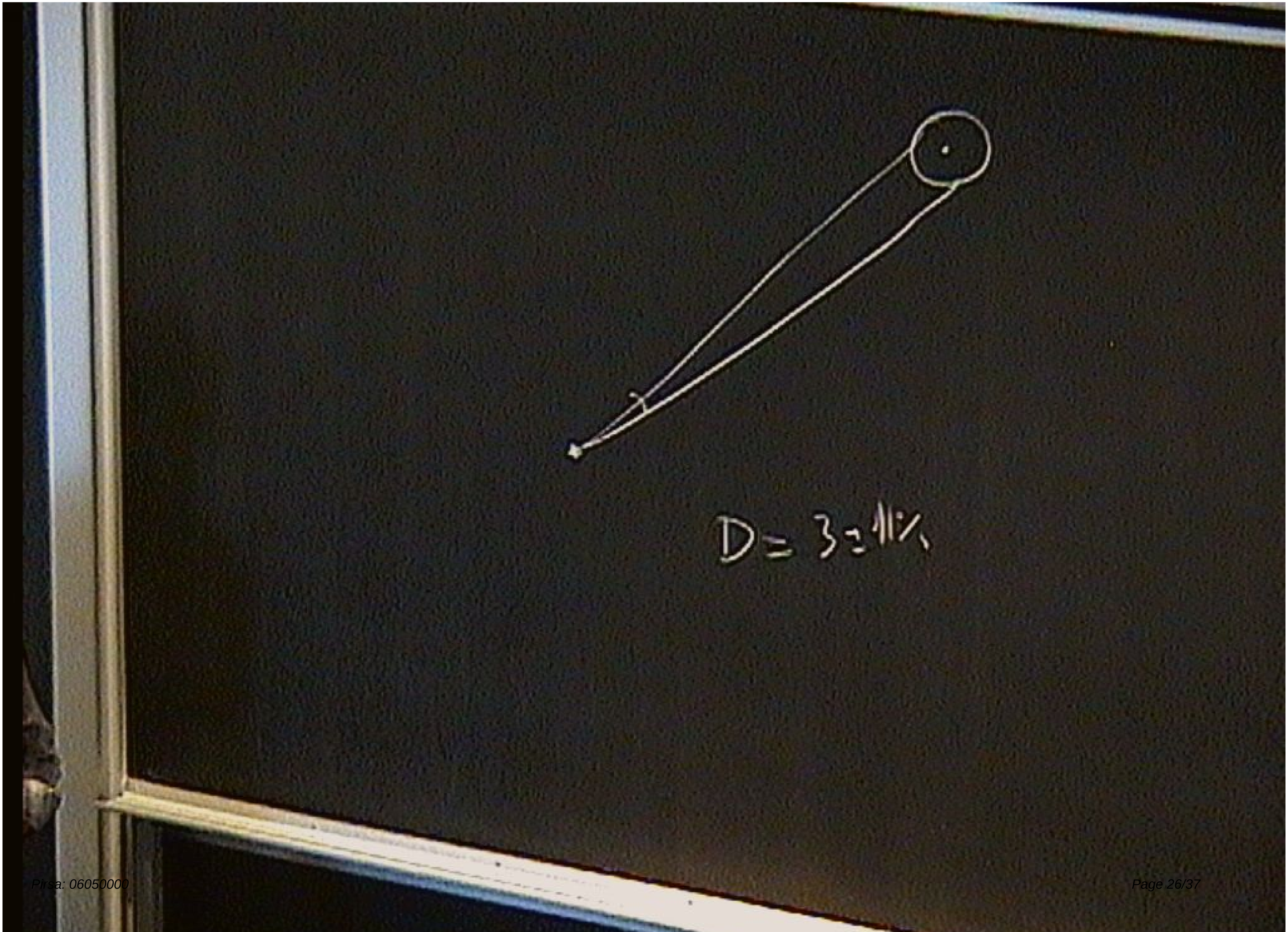
D = 3 ± 4%

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Late time

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Where the acceleration is coming from?

Time-Like geodesics

Shear

$$\sigma^2 = \left(h^{\mu}{}_{\alpha} h^{\nu}{}_{\beta)} \nabla_\mu u_\nu - \frac{1}{3}\theta^2 h_{\alpha\beta}\right)^2 \sim 0$$

Vorticity

$$\omega^2 = \left(\nabla_{[\alpha} u_{\beta]}\right)^2 \sim 0$$

$\Downarrow$

From Raychoudhuri

$$R_{\alpha\beta} u^\alpha u^\beta = -Q < 0$$

Energy conditions are effectively violated!

But is a Mirage matter (geometry!)

$\Downarrow$

No Problem!

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Shear

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FRW effective description

Late time and $\rho = \rho_0 + \varepsilon$

$$ds^2_{ind} \simeq \left[\frac{(t-t_0)^2}{2} + \frac{\rho_0^2}{6} + r_0^2 \right]^2 dz^2$$

Consider t the conformal time and define $\left[\frac{(t-t_0)^2}{2} + \frac{\rho_0^2}{6} + r_0^2 \right] dt = d\tau$

$$\Downarrow$$

$$ds^2_{ind} \simeq -d\tau^2 + a^2(\tau)\, d\vec{x} \cdot d\vec{x}$$

$$Q = \frac{\ddot{a}}{a}$$

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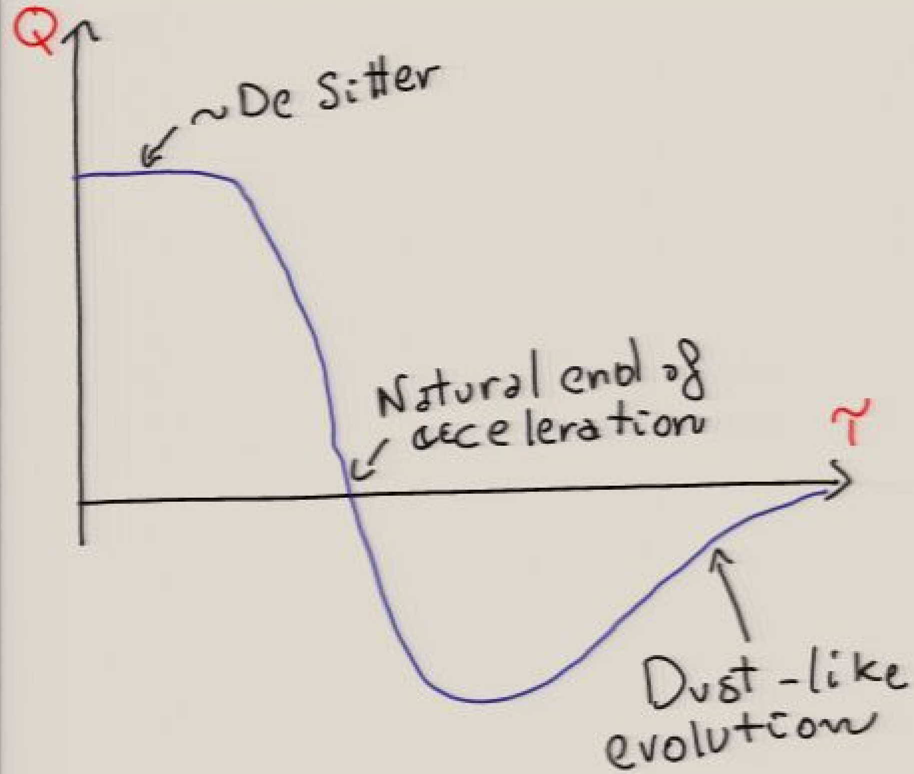

Large time $a \sim \tau^{2/3}$ (Dust)

$\Downarrow$

Dark matter ?

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But the Universe looks homogeneous ?¡?¡

"The only constraint on homogeneity (3D) is the counting of number of objects between us and 150 Mpc"

Hoggs et. al. [Ast. J. 624, 54 (2005)]

The Universe is "homogeneous" in the range 100 ÷ 150 Mpc

or

$$\boxed{D = \frac{\ln V/V_0}{\ln \frac{150}{100}} = 3 \pm 1\%}$$

Fractal dimension

comoving coords

$$(t, \rho, \theta, \varphi) \longrightarrow (\tau, R, \theta, \varphi)$$

where

$$1 = u^\tau = \dot{\tau}\, u^t + \tau' u^\rho$$

$$0 = u^R = \dot{R}\, u^t + R' u^\rho$$

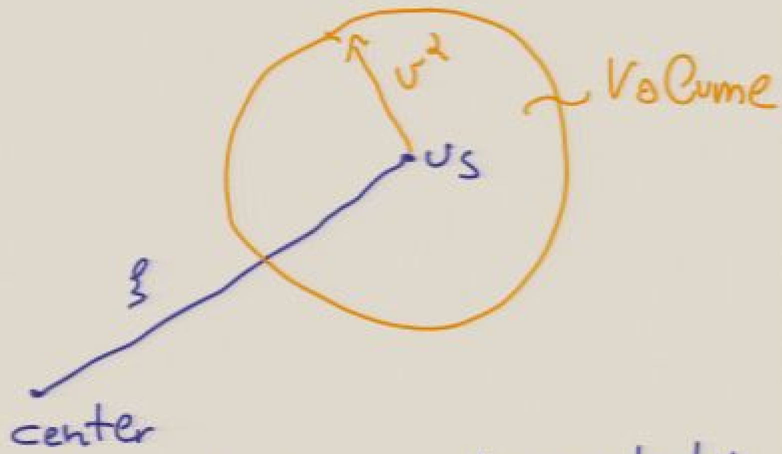
Again for large ρ and the quasi-homogeneous observer

$$ds^2 = -d\tau^2 + \frac{1}{2}\frac{R}{\mu L^2}\, dR^2 + \frac{2\tau}{R}\, dR\, d\tau + \frac{6R^3}{\mu L^2}\, d\Omega_2^2$$

define the radial distance

$$\rho = \sqrt{\frac{6}{\mu L^2}}\; R^{3/2}$$

$$ds^2 = -\, d\tau^2 + \frac{1}{9}\, d\rho^2 + \frac{4\tau}{3\rho}\, d\rho\, d\tau + \rho^2\, d\Omega_2^2$$



we need cartesian coords to calculate the Volume

$$d\xi^2 + \xi^2 d\Omega_2^2 = dx^2 + dy^2 + dz^2$$

defining v^α (s.t. $v^\alpha v_\alpha = 1$)

$$\ell = \int v_\alpha dx^\alpha \,, \quad V = \int \sqrt{h}\, dx\, dy\, dz$$

$$[h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta]$$

Using $Q_t = \Lambda = \dfrac{288 L^2}{\mu^2 \xi_0^6}$

$$V \propto \ell^3 \left(1 - \frac{4\ell}{\sqrt{22}} \sqrt{\Lambda}\right)$$

The only constant playing is Λ !

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Using the best WMAP fit

$$\Lambda^{-\frac{1}{2}} \sim 2.6\ 10^{3}\ \mathrm{Mpc}$$

we get

$$D = 3 \pm 1\%$$

The Universe looks homogeneous!

The inhomogeneity scales can be observed at $\Lambda^{-\frac{1}{2}}$

$\Downarrow$

Λ is the "inhomogeneity scale"

$\Downarrow$

cosmological constant is a geometrical effect!

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Conclusions

- Inhomogeneities on a D3-brane in AdS bulk make the Universe accelerated at early time and/or at large distances
- The inhomogeneity scale at late time is the measured Cosmological constant
- The acceleration naturally end in a "dark Matter" dominated Universe
- Homogeneity tests are fulfilled.

Work to do

(or some free time exercises for you)

- What happen for a more general geometry? (i.e. non spherically symmetric)
- Calculate CMB spectra (i.e. introducing matter)
- Local gravity?
- Effects of matter?