

Title: Fluctuations and reheating after Inflation

Date: Apr 25, 2006 11:00 AM

URL: <http://pirsa.org/06040022>

Abstract:

# Fluctuations and Reheating after Inflation

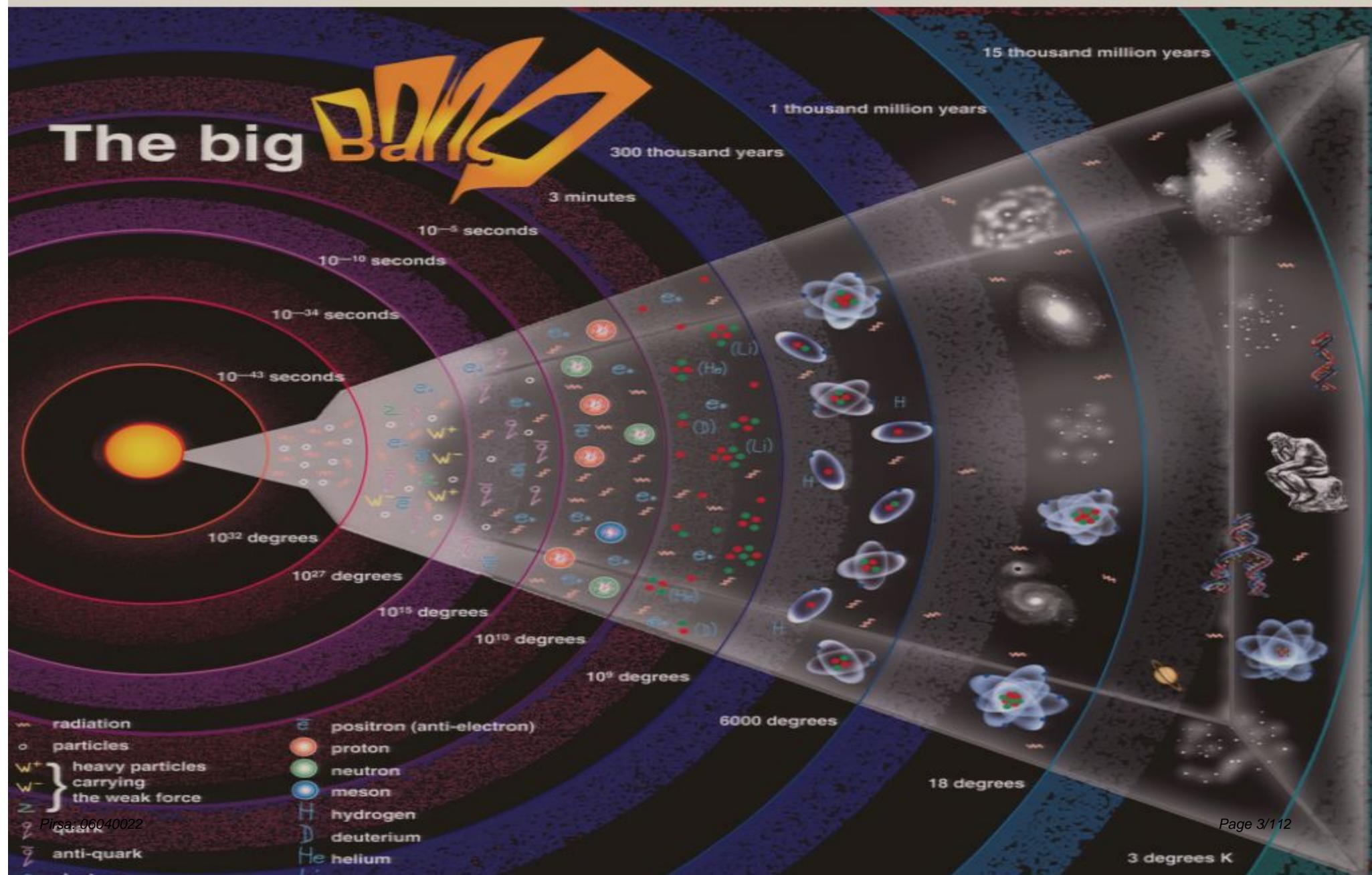
**Top-down approach to inflation:**  
seeks to embed it in fundamental theory

**Bottom-up approach to inflation:**  
reconstruction of acceleration trajectories

Lev Kofman

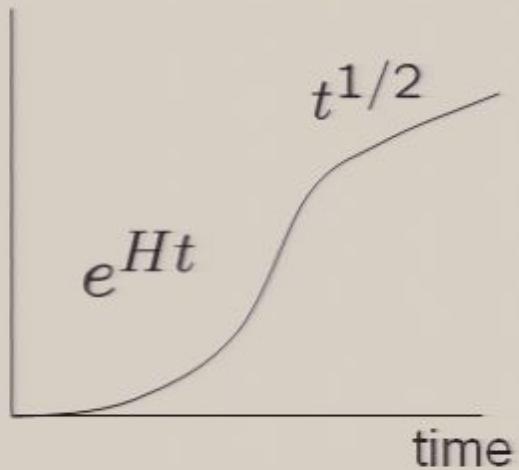


# Early Universe



## Early Universe Inflation

Scale factor  $a(t)$



Equation of State  $t \leq 10^{-35}$  sec

$$p \approx -\epsilon$$

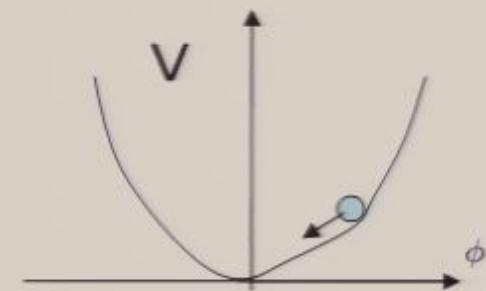
Inflation  $a(t) \approx e^{Ht}$

## Realization of Inflation

Scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

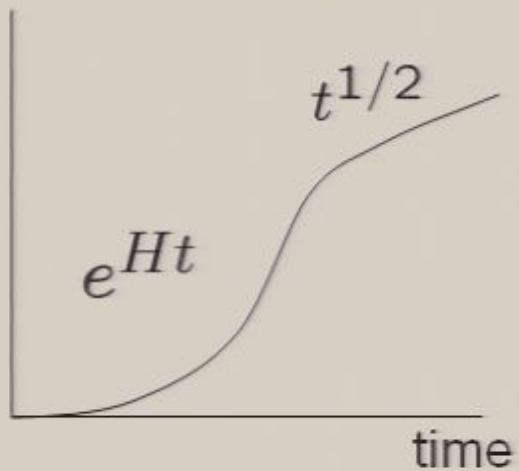
$$p = \frac{1}{2}\dot{\phi}^2 - V$$
$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$



$$R + R^2 + R_{ij}R^{ij}$$

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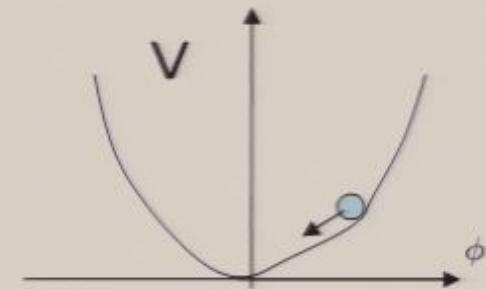
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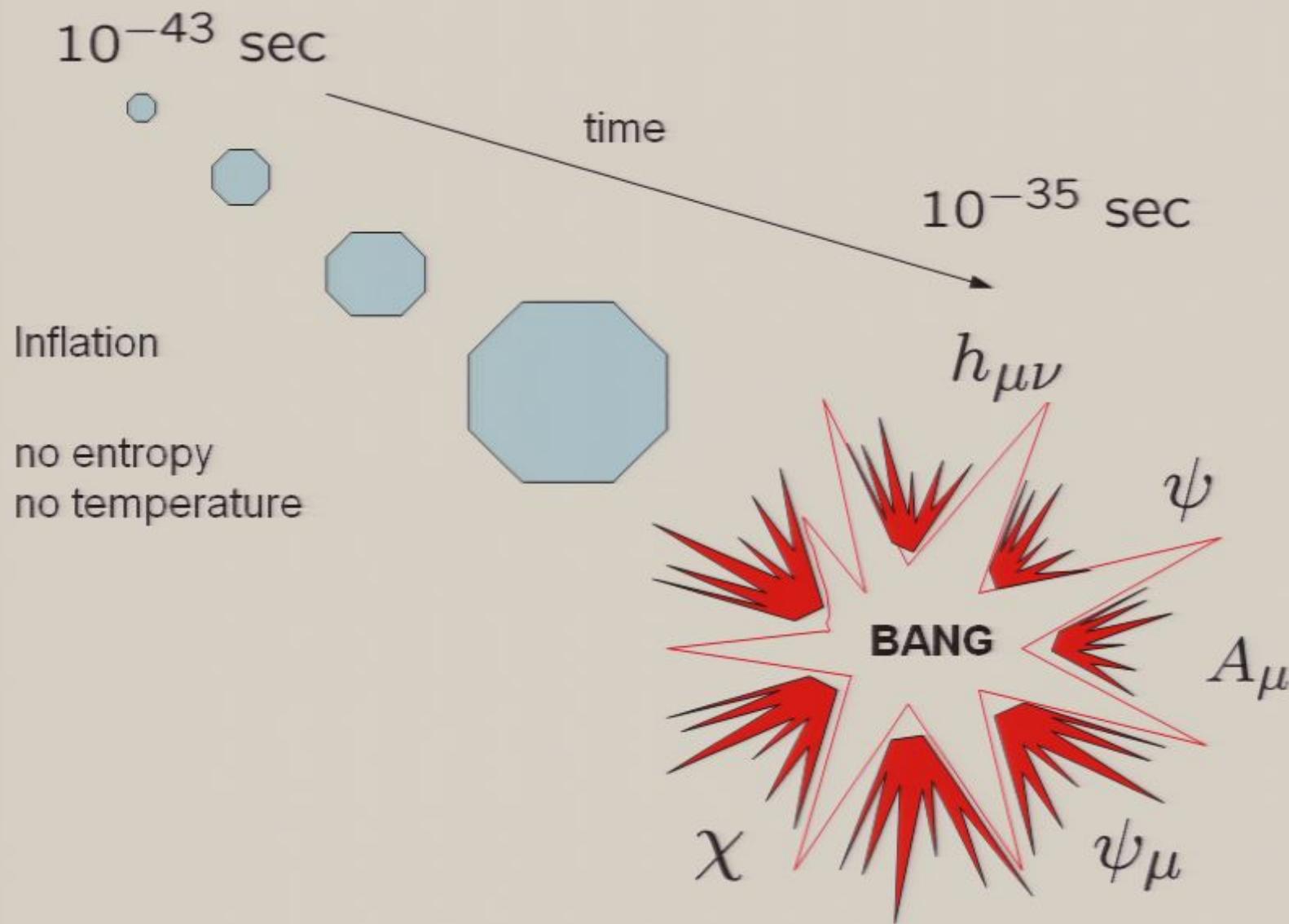
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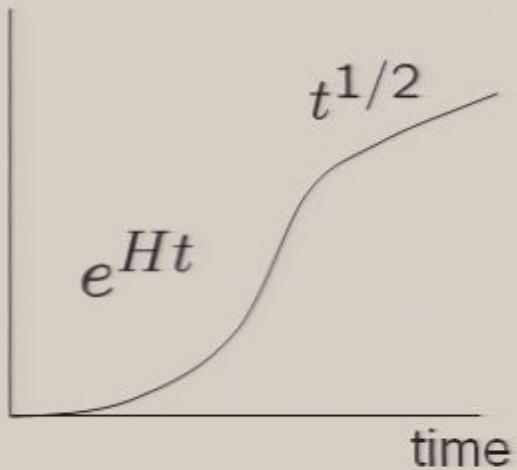


## Particlegenesis



## Early Universe Inflation

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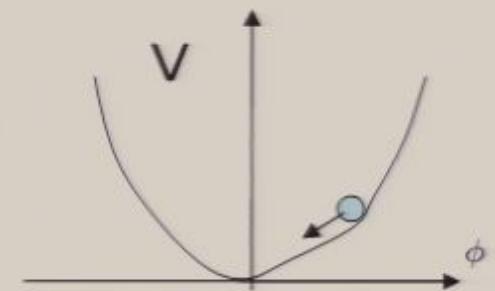
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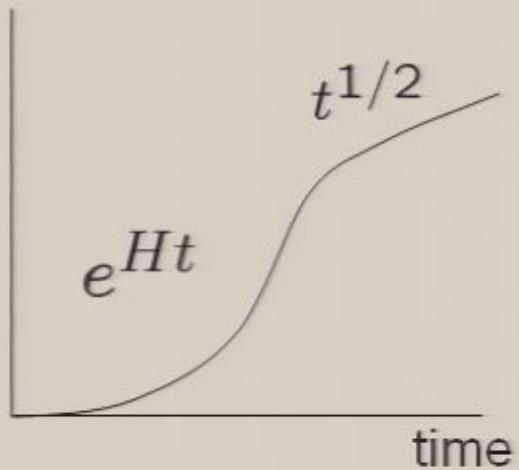
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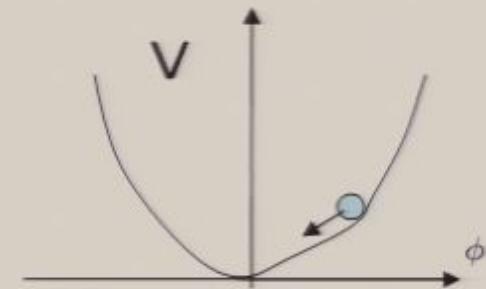
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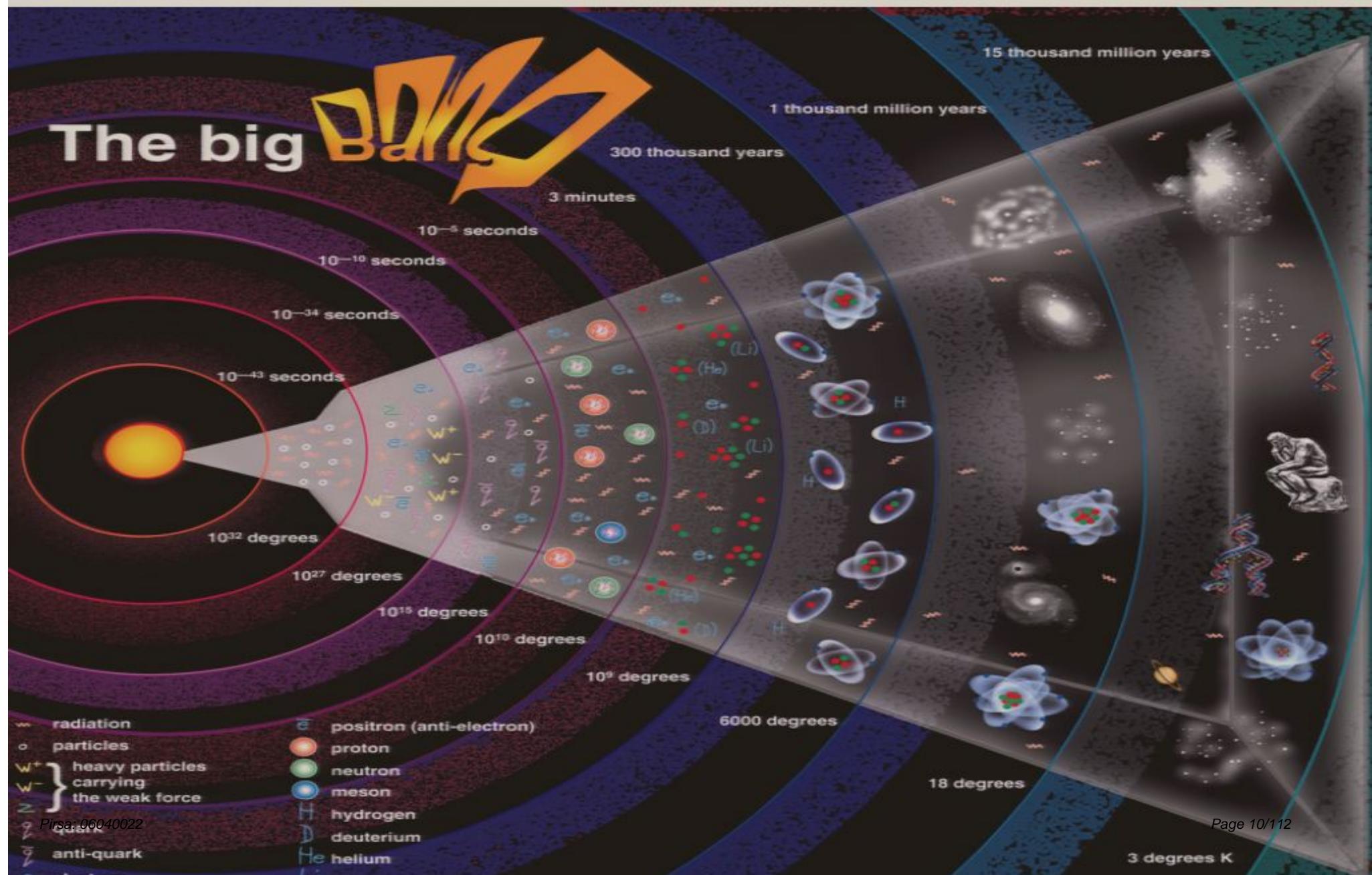
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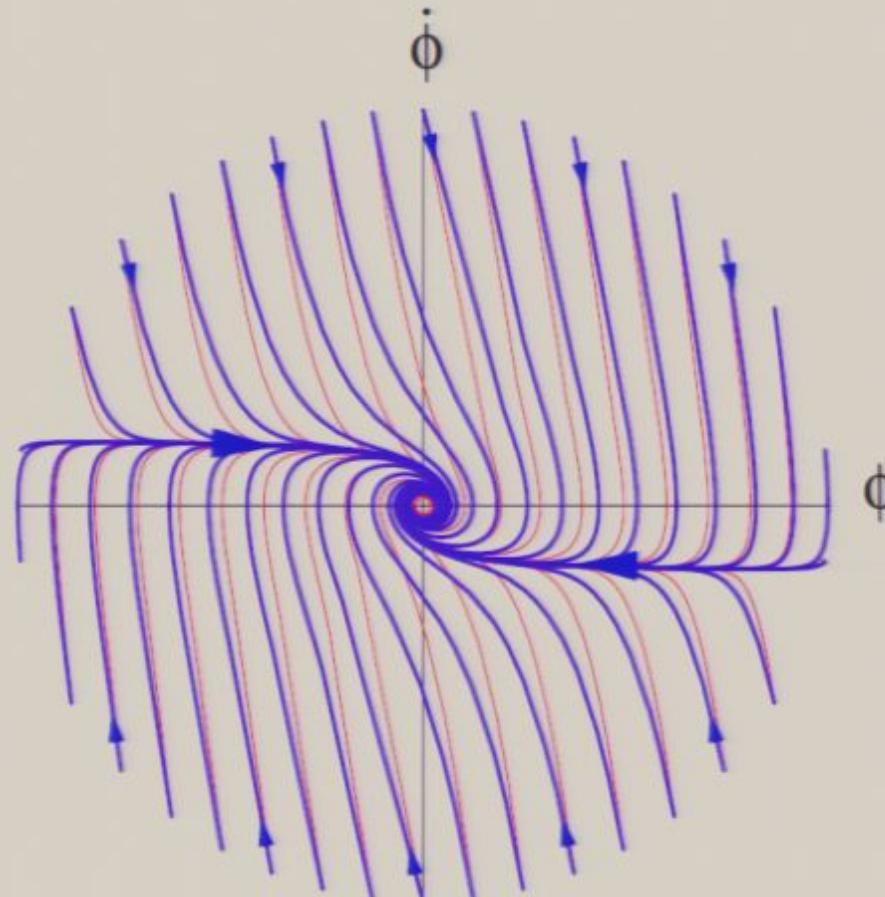
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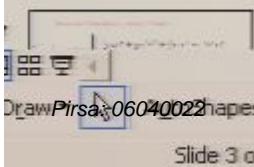
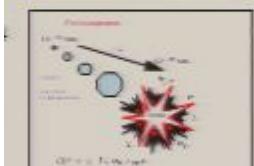
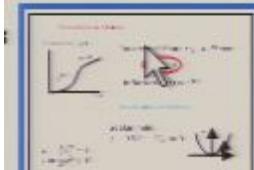
# Early Universe



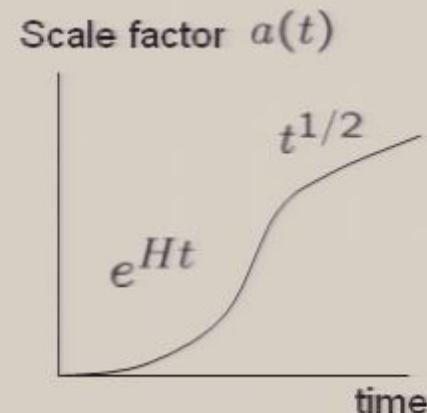
## Phase portrait of inflation



Click to add notes



## Early Universe Inflation



Equation of State  $t \leq 10^{-35}$  sec

$$p \approx -\epsilon$$

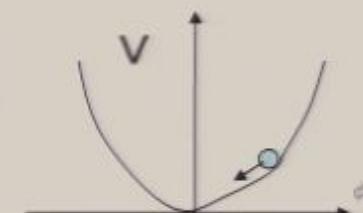
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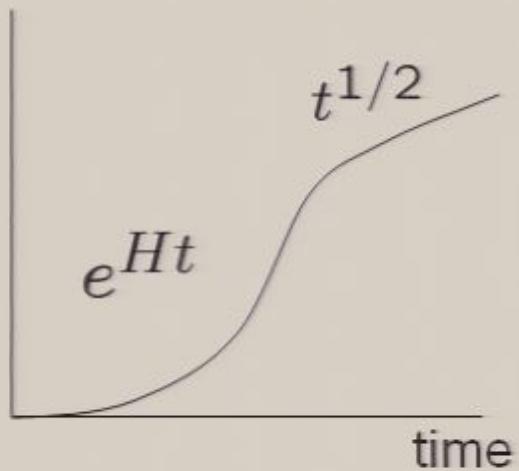
$$p = \frac{1}{2}\dot{\phi}^2 - V$$
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Click to add notes

## Early Universe Inflation

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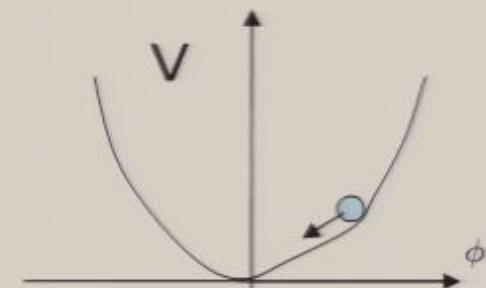
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## Realization of Inflation

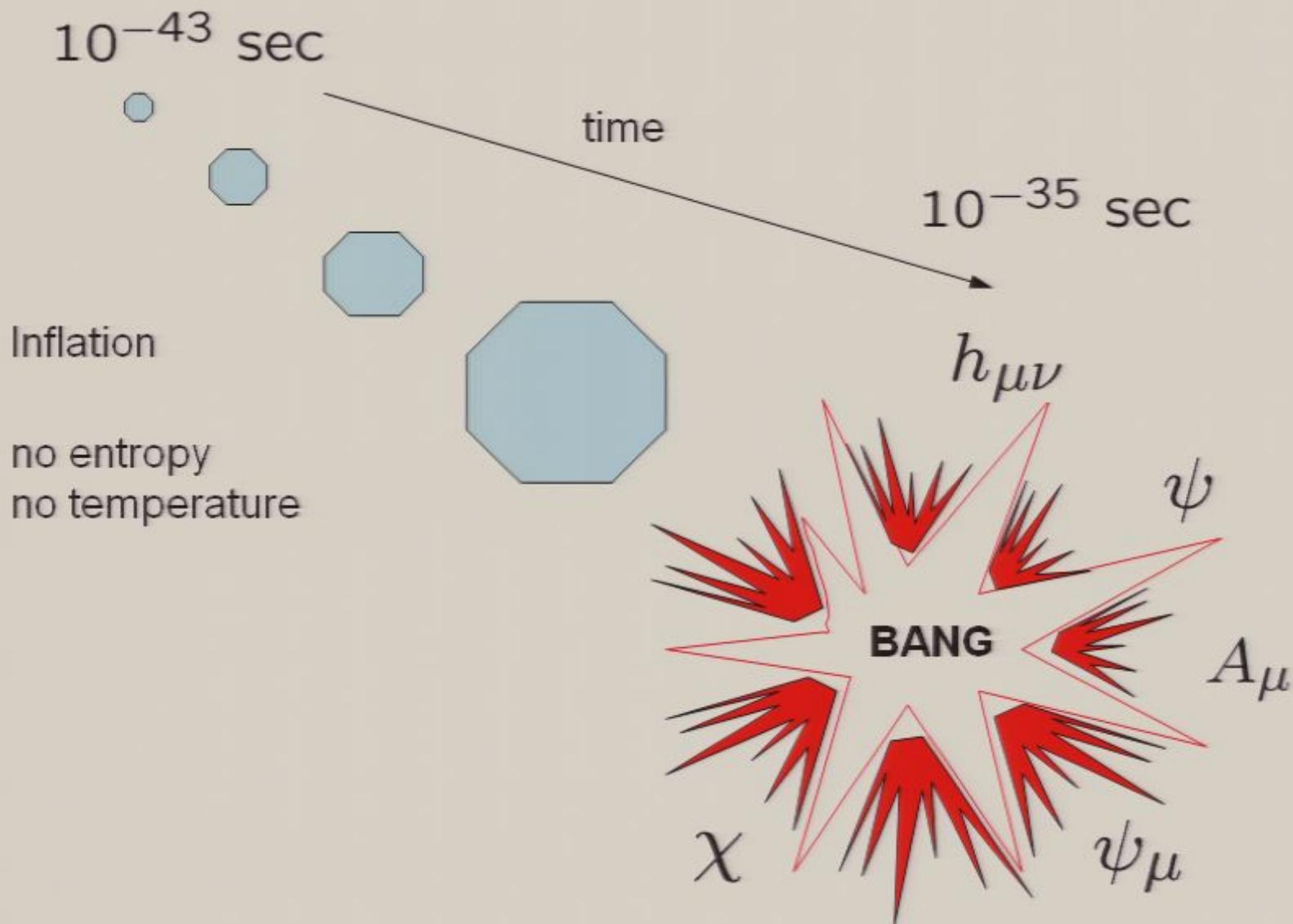
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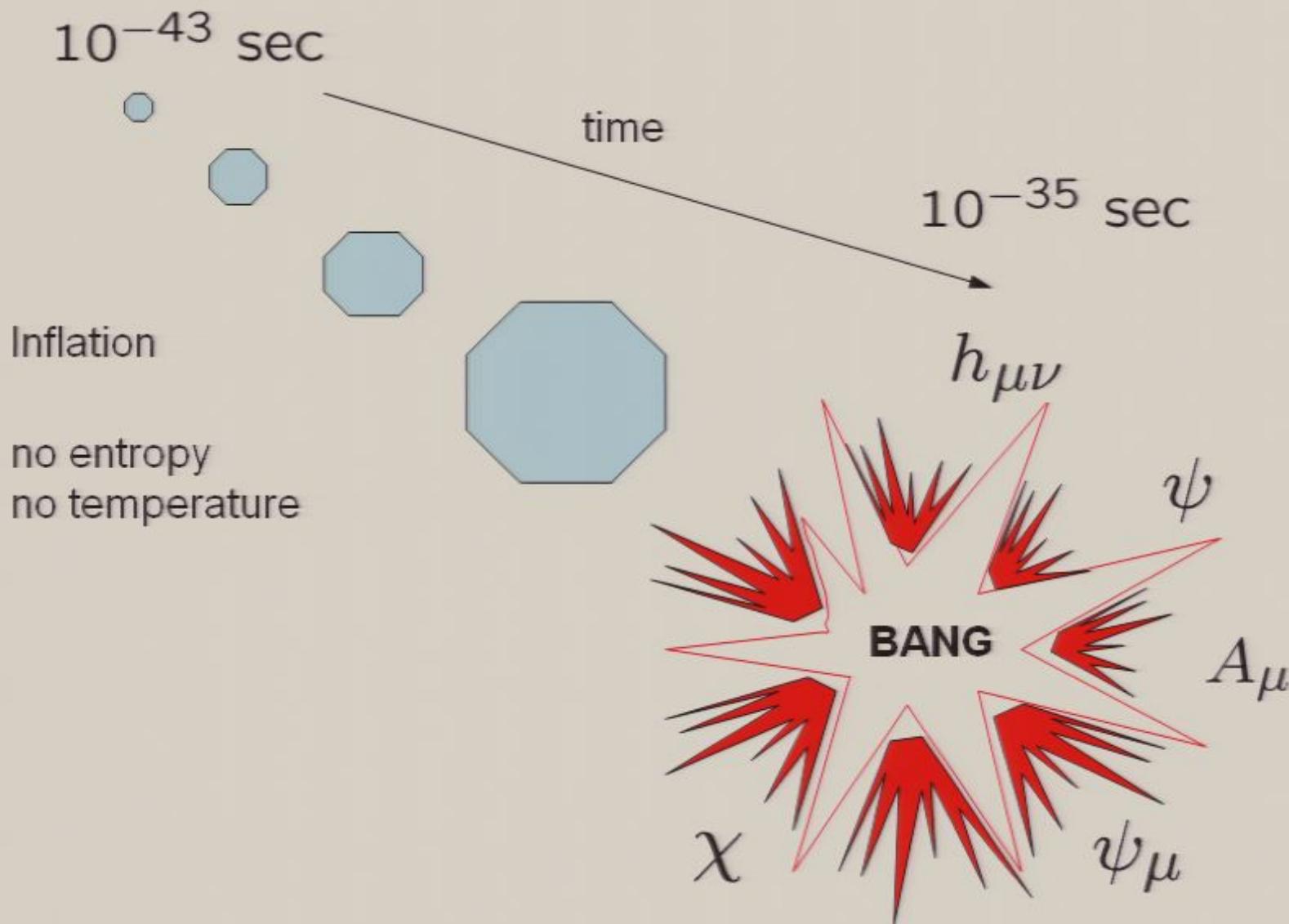
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$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$



# Particlegenesis

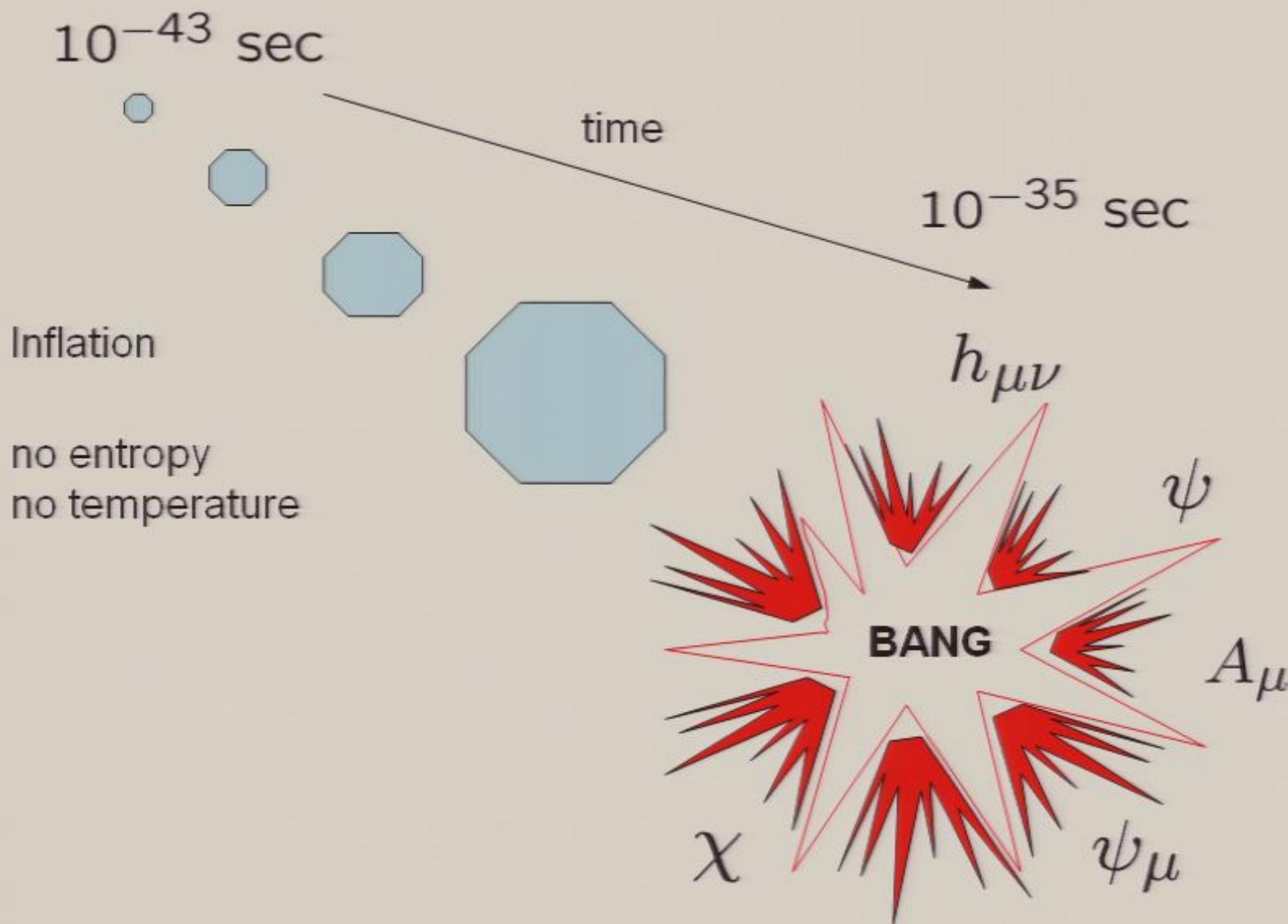


# Particlegenesis



$$\begin{aligned}
e^{-1}\mathcal{L} = & -\tfrac{1}{2}M_P^2 \left[ R + \bar{\psi}_\mu R^\mu + \mathcal{L}_{SG,torsion} \right] - g_i{}^j \left[ M_P^2 (\hat{\partial}_\mu z^i)(\hat{\partial}^\mu z_j) + \bar{\chi}_j \not{\partial} \chi^i + \bar{\chi}^i \not{\partial} \chi_j \right] \\
& + (\text{Re } f_{\alpha\beta}) \left[ -\tfrac{1}{4} F_{\mu\nu}^\alpha F^{\mu\nu\beta} - \tfrac{1}{2} \bar{\lambda}^\alpha \not{\partial} \lambda^\beta \right] + \tfrac{1}{4} i(\text{Im } f_{\alpha\beta}) \left[ F_{\mu\nu}^\alpha \tilde{F}^{\mu\nu\beta} - \partial_\mu (\bar{\lambda}^\alpha \gamma_5 \gamma^\mu \lambda^\beta) \right] \\
& - M_P^{-2} e^K \left[ -3WW^* + (\mathcal{D}^i W) g^{-1}{}_i{}^j (\mathcal{D}_j W^*) \right] - \tfrac{1}{2} (\text{Re } f)^{-1}{}^{\alpha\beta} \mathcal{P}_\alpha \mathcal{P}_\beta \\
& + \tfrac{1}{8} (\text{Re } f_{\alpha\beta}) \bar{\psi}_\mu \gamma^{\nu\rho} (F_{\nu\rho}^\alpha + \hat{F}_{\nu\rho}^\alpha) \gamma^\mu \lambda^\beta \\
& + \left\{ M_P g_j{}^i \bar{\psi}_\mu L (\hat{\partial} z^j) \gamma^\mu \chi_i + \bar{\psi}_R \cdot \gamma \left[ \tfrac{1}{2} i \lambda_L^\alpha \mathcal{P}_\alpha + \chi_i Y^3 M_P^{-4} \mathcal{D}^i W \right] \right. \\
& \quad + \tfrac{1}{2} Y^3 M_P^{-3} W \bar{\psi}_{\mu R} \gamma^{\mu\nu} \psi_{\nu R} - \tfrac{1}{4} M_P^{-1} f_{\alpha\beta}^i \bar{\chi}_i \gamma^{\mu\nu} \hat{F}_{\mu\nu}^{-\alpha} \lambda_L^\beta \\
& \quad - Y^3 M_P^{-5} (\mathcal{D}^i \mathcal{D}^j W) \bar{\chi}_i \chi_j + \tfrac{1}{2} i (\text{Re } f)^{-1}{}^{\alpha\beta} \mathcal{P}_\alpha M_P^{-1} f_{\beta\gamma}^i \bar{\chi}_i \lambda^\gamma - 2 M_P \xi_\alpha{}^i g_i{}^j \bar{\lambda}^\alpha \chi_j \\
& \quad + \tfrac{1}{4} M_P^{-5} Y^3 (\mathcal{D}^j W) g^{-1}{}_j{}^i f_{\alpha\beta i} \bar{\lambda}_R^\alpha \lambda_R^\beta \\
& \quad - \tfrac{1}{4} M_P^{-1} f_{\alpha\beta}^i \bar{\psi}_R \cdot \gamma \chi_i \bar{\lambda}_L^\alpha \lambda_L^\beta + \tfrac{1}{4} M_P^{-2} (\mathcal{D}^i \partial^j f_{\alpha\beta}) \bar{\chi}_i \chi_j \bar{\lambda}_L^\alpha \lambda_L^\beta + \text{h.c.} \Big\} \\
& + g_j{}^i \left( \tfrac{1}{8} e^{-1} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \psi_\rho \bar{\lambda}^j \gamma_\sigma \chi_i - \bar{\psi}_\mu \chi^j \bar{\psi}^\mu \chi_i \right) \\
& + M_P^{-2} \left( R_{ij}^{k\ell} - \tfrac{1}{2} g_i{}^k g_j{}^\ell \right) \bar{\chi}^i \chi^j \bar{\chi}_k \chi_\ell \\
& + \tfrac{3}{64} M_P^{-2} \left( (\text{Re } f_{\alpha\beta}) \bar{\lambda}^\alpha \gamma_\mu \gamma_5 \lambda^\beta \right)^2 - \tfrac{1}{16} M_P^{-2} f_{\alpha\beta}^i \bar{\lambda}_L^\alpha \lambda_{LG}^\beta g^{-1}{}_i{}^j f_{\gamma\delta j} \bar{\lambda}_R^\gamma \lambda_R^\delta \\
& + \tfrac{1}{8} (\text{Re } f)^{-1}{}^{\alpha\beta} M_P^{-2} \left( f_{\alpha\gamma}^i \bar{\chi}_i \lambda^\gamma - f_{\alpha\gamma i} \bar{\chi}^i \lambda^\gamma \right) \left( f_{\beta\delta}^j \bar{\chi}_j \lambda^\delta - f_{\beta\delta j} \bar{\chi}^j \lambda^\delta \right).
\end{aligned}$$

# Particlegenesis



## Schwinger formula for particle creation

Scalar ED     $(D_\mu D^\mu + \mu^2) \chi = 0$

classical

$$\vec{E}$$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$A_\mu = (0, 0, 0, -Et)$$

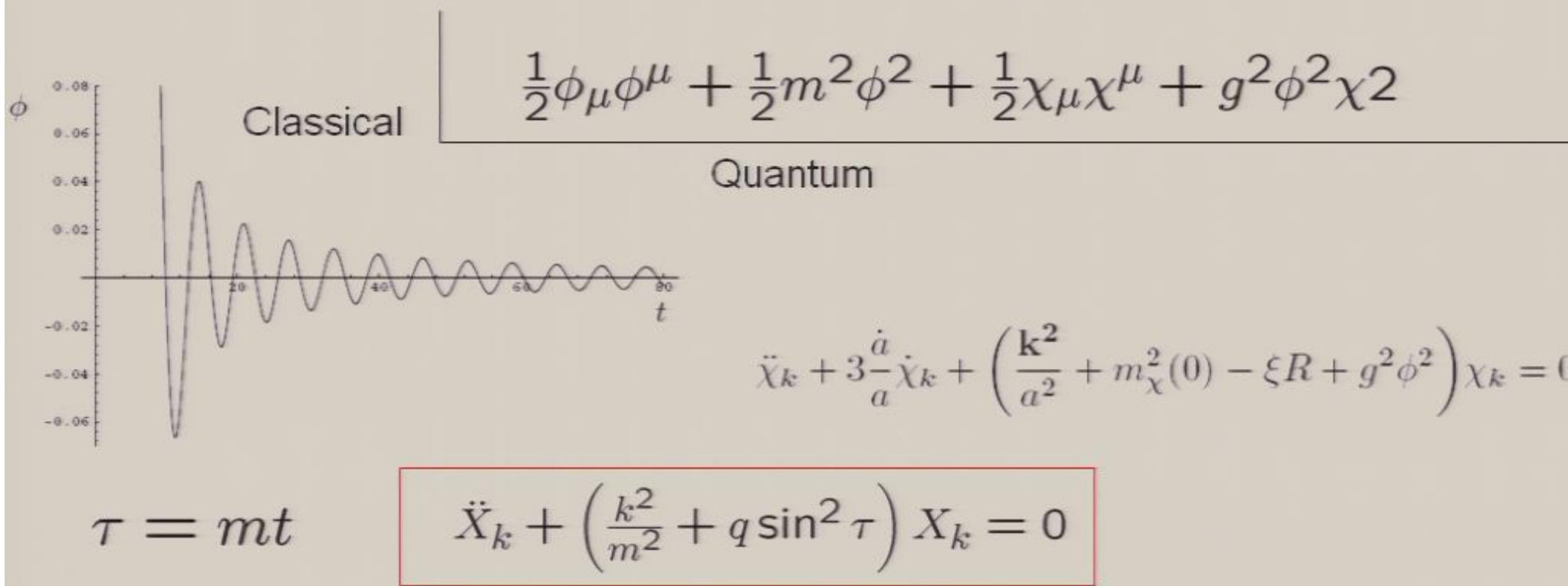
Quantum     $\chi = X_k(t)e^{i\vec{k}\vec{x}}$

$$\tau = eEt$$

$$\boxed{\frac{d^2 X_k}{d\tau^2} + (\kappa^2 + \tau^2) X_k = 0}$$

$$n_k = |\beta_k|^2 = \exp\left(\frac{\pi(k^2 + \mu^2)}{eE}\right)$$

## Resonant Preheating in Chaotic Inflation



parameter  $q = \frac{g^2\phi_0^2}{m^2} \sim g^2 10^{10}$

Occupation number

$$n_k = \frac{\omega_k}{2} \left( \frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}$$

$$n_k \sim e^{\mu t}$$

$$\kappa^2 = \frac{k^2}{gm\phi_0}$$

Method of successive scatterings

$$n_k^{j+1} = e^{-\pi\kappa^2} + \left(1 + 2e^{-\pi\kappa^2}\right) n_k^j - 2e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}} \sqrt{n_k^j(1 + n_k^j)} \sin \theta_{tot}^j$$

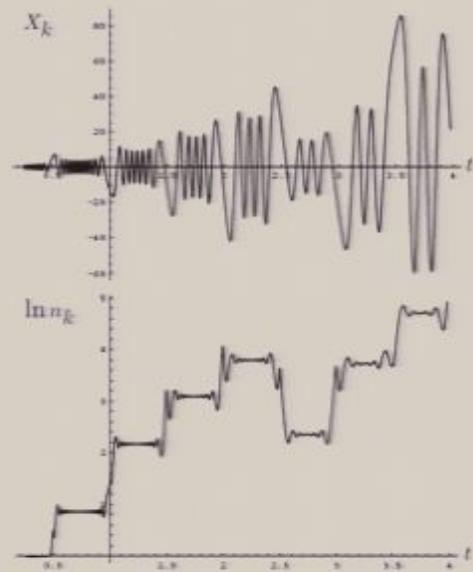
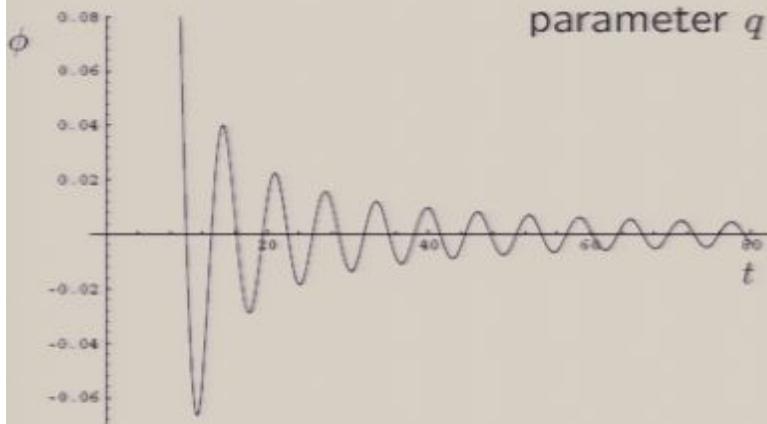
$$n_k^{j+1} \approx \left(1 + 2e^{-\pi\kappa^2} - 2 \sin \theta_{tot}^j e^{-\frac{\pi}{2}\kappa^2} \sqrt{1 + e^{-\pi\kappa^2}}\right) n_k^j$$

$$n_{j+1} \approx e^{4\pi j \mu_k} = e^{2\mu_k m t}$$

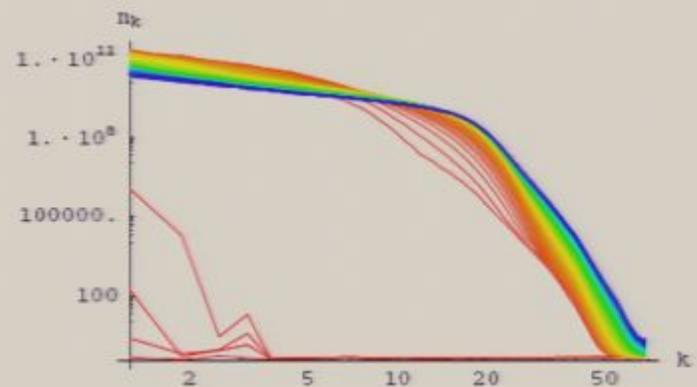
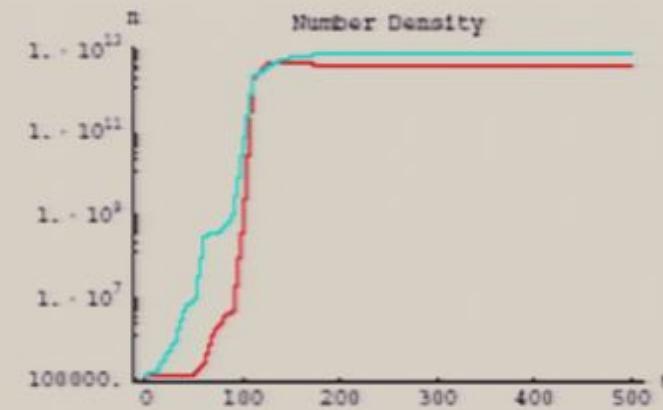
# Resonant Preheating in Chaotic Inflation

$$g^2 \phi^2 \chi^2$$

parameter  $q = \frac{g^2 \phi_0^2}{m^2} \sim g^2 10^{10} \gg 1$



$$\delta\chi$$



$$D(t)^2 \equiv \sum_A (|f'_A - f_A|)^2 + (|\dot{f}'_A - \dot{f}_A|)^2$$

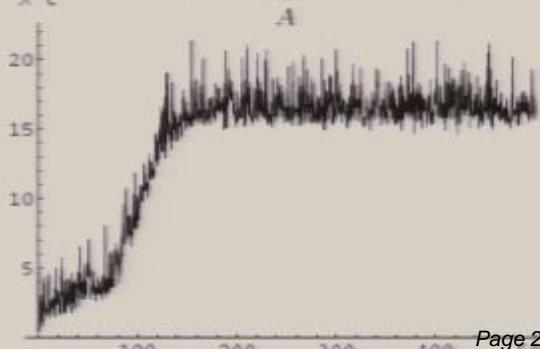
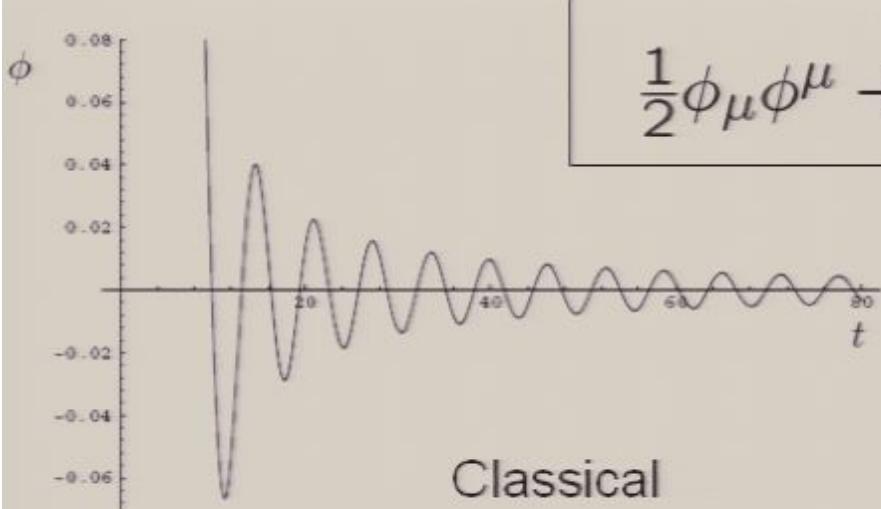


FIG. 8. The Lyapunov exponent  $\lambda'$  for the fields  $\phi$  and  $\chi$  using the normalized distance function  $\Delta$ .



$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

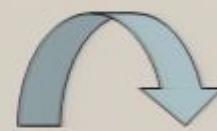
Classical

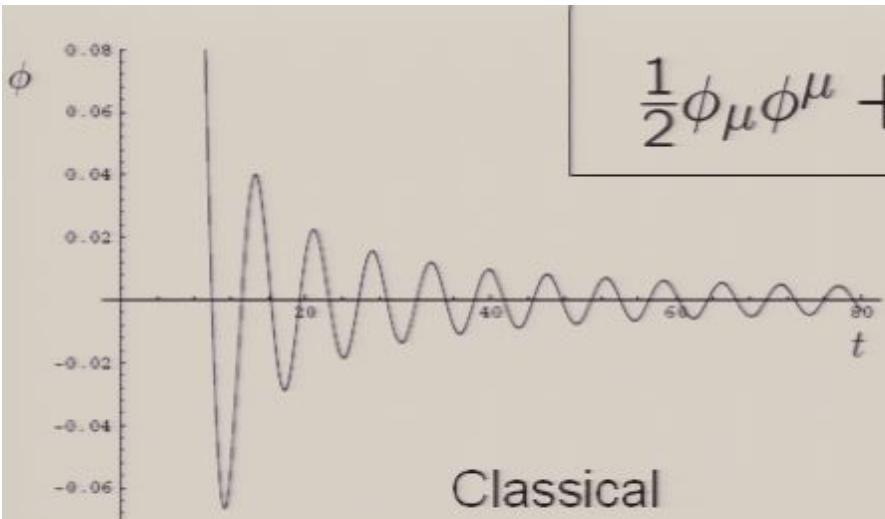
$$\phi_0 + \phi$$

**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$



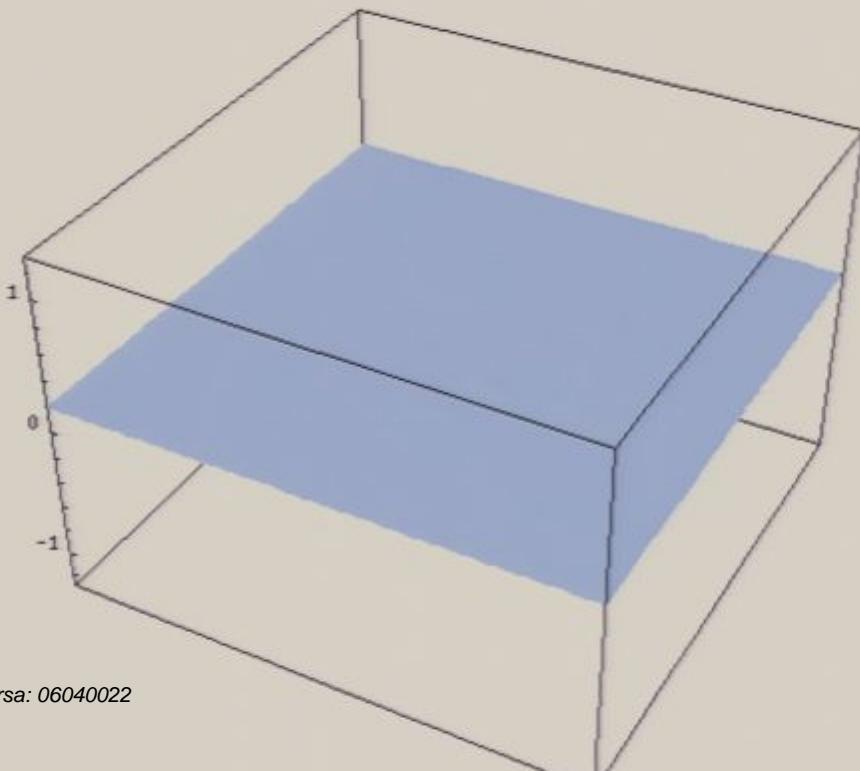


$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

$$\phi_0 + \phi$$

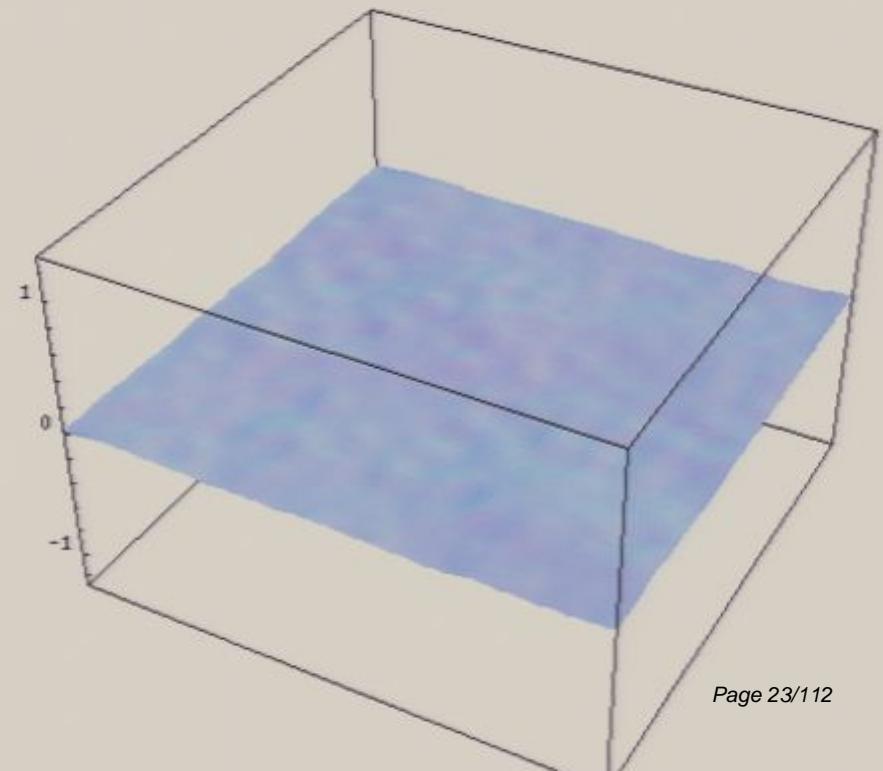
Slices for  $t=95.702$

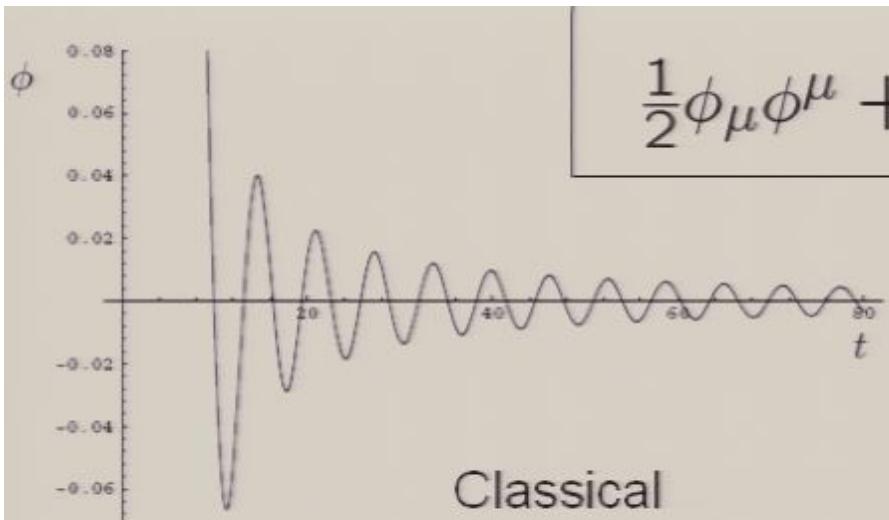


**Decay of inflaton  
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Quantum

$$\chi$$

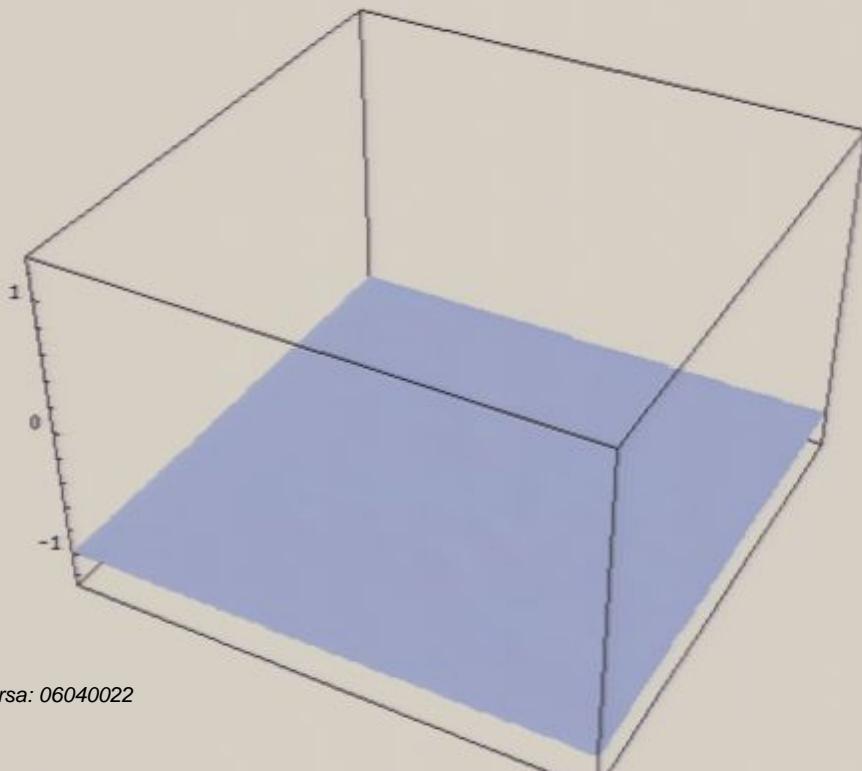




$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

$$\phi_0 + \phi$$

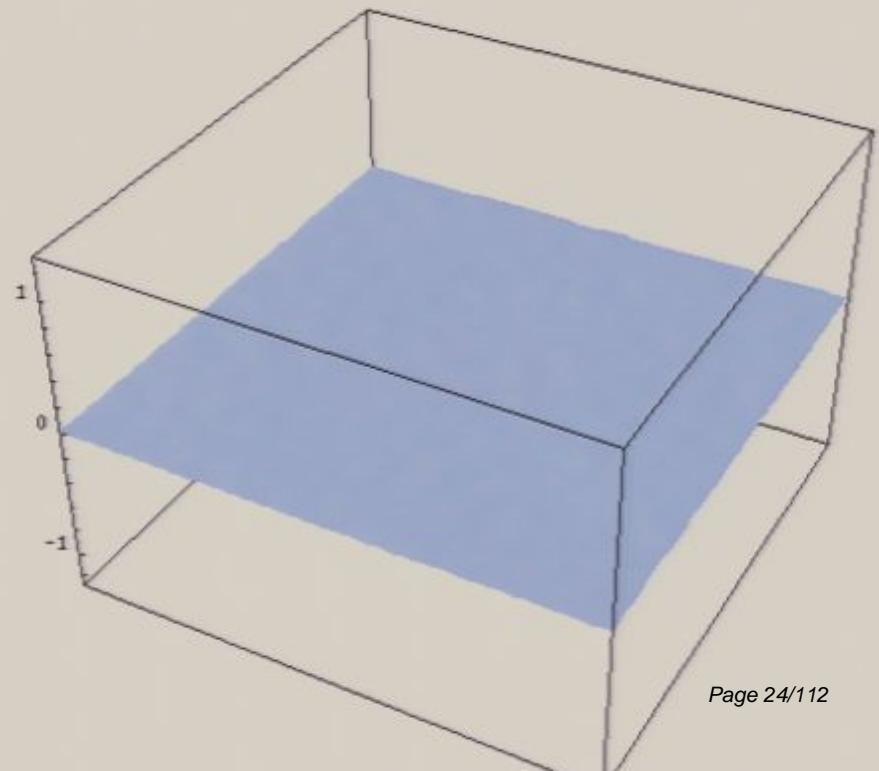


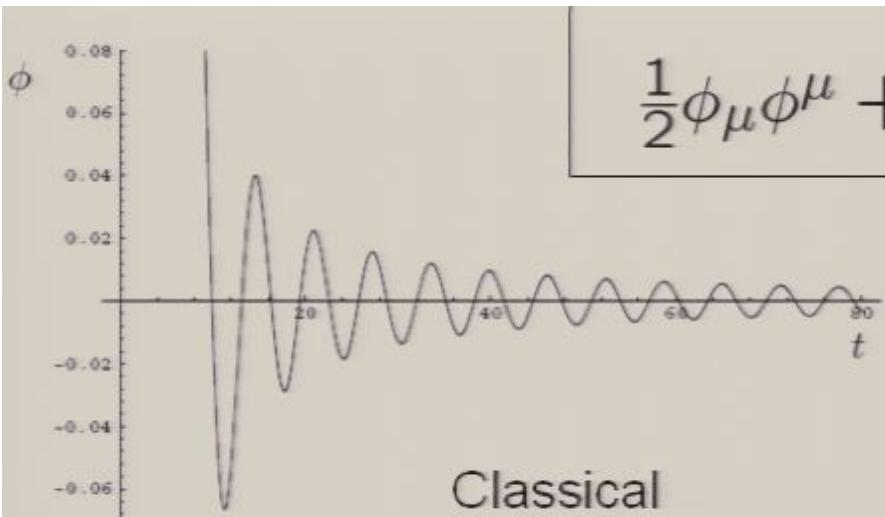
**Decay of inflaton  
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$$\chi$$

Slices for  $t=97.7025$

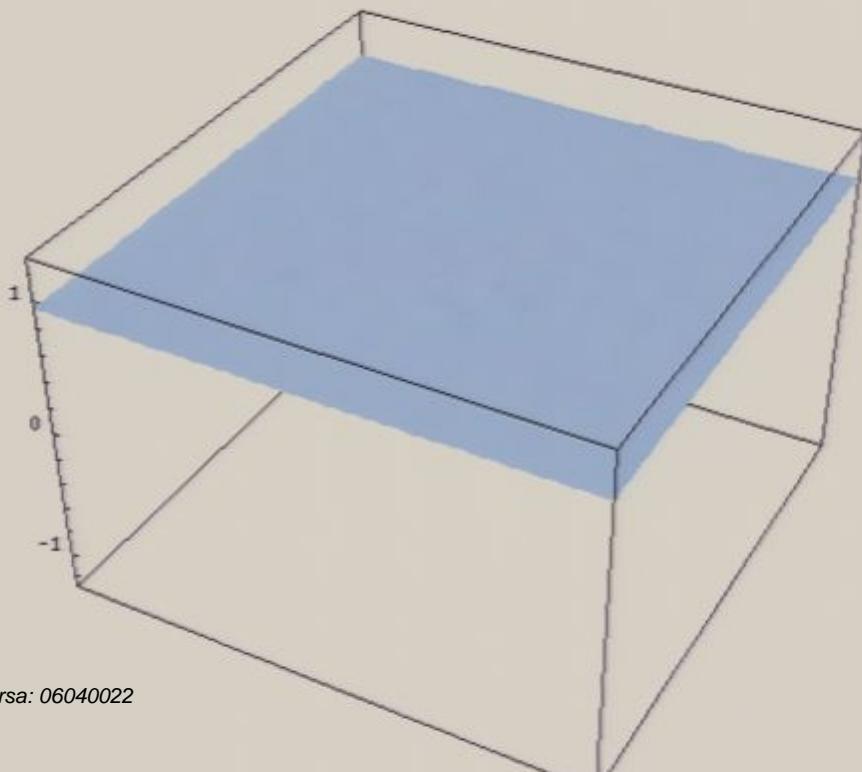




$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

$$\phi_0 + \phi$$

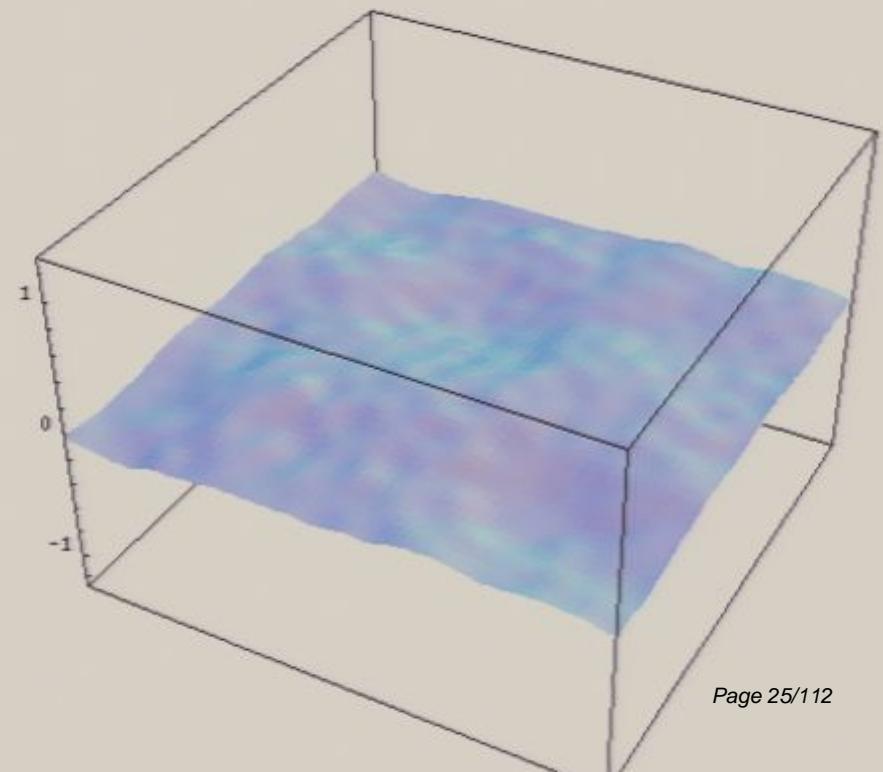


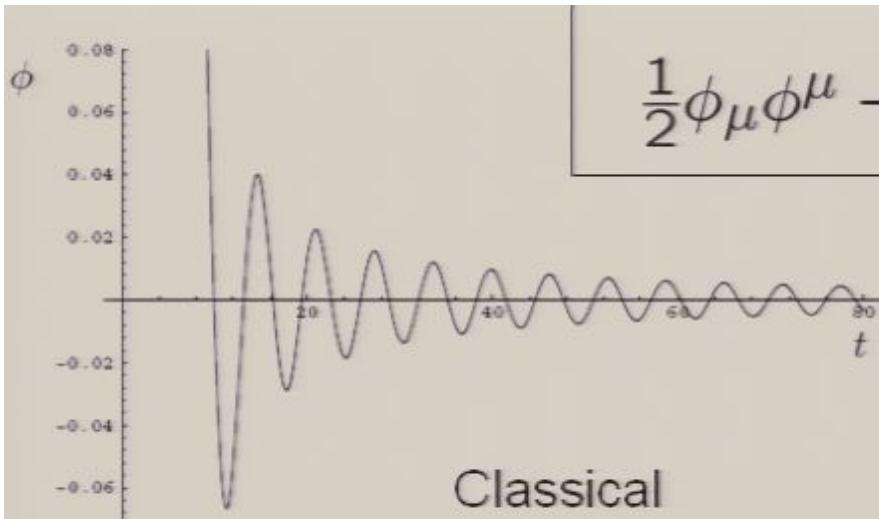
## Decay of inflaton and preheating after inflation

Quantum

$$\chi$$

Slices for  $t=100, 203$





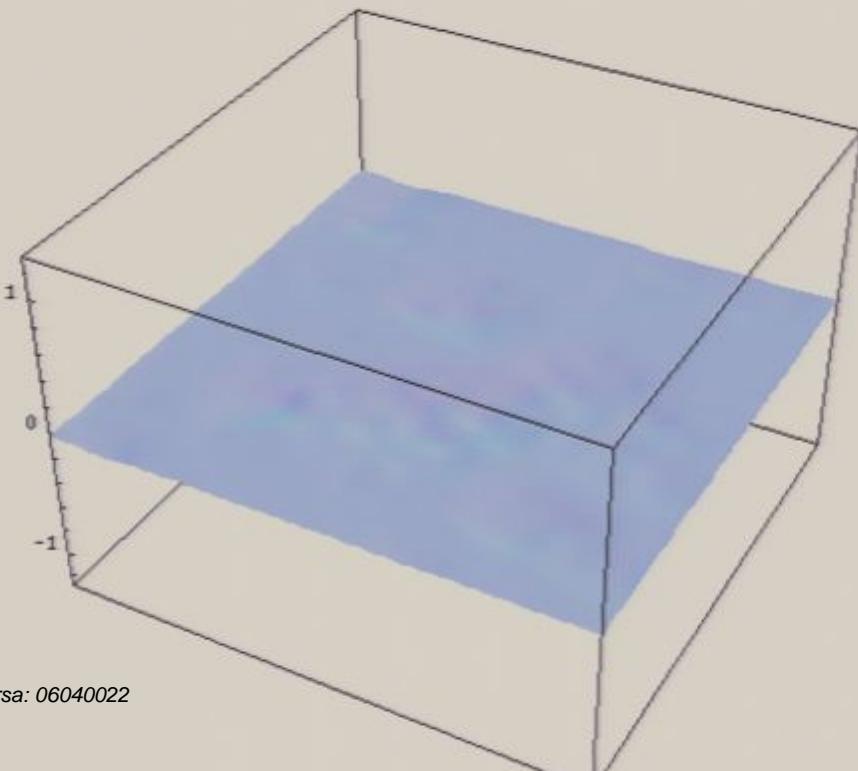
$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

**Decay of inflaton  
and preheating after inflation**

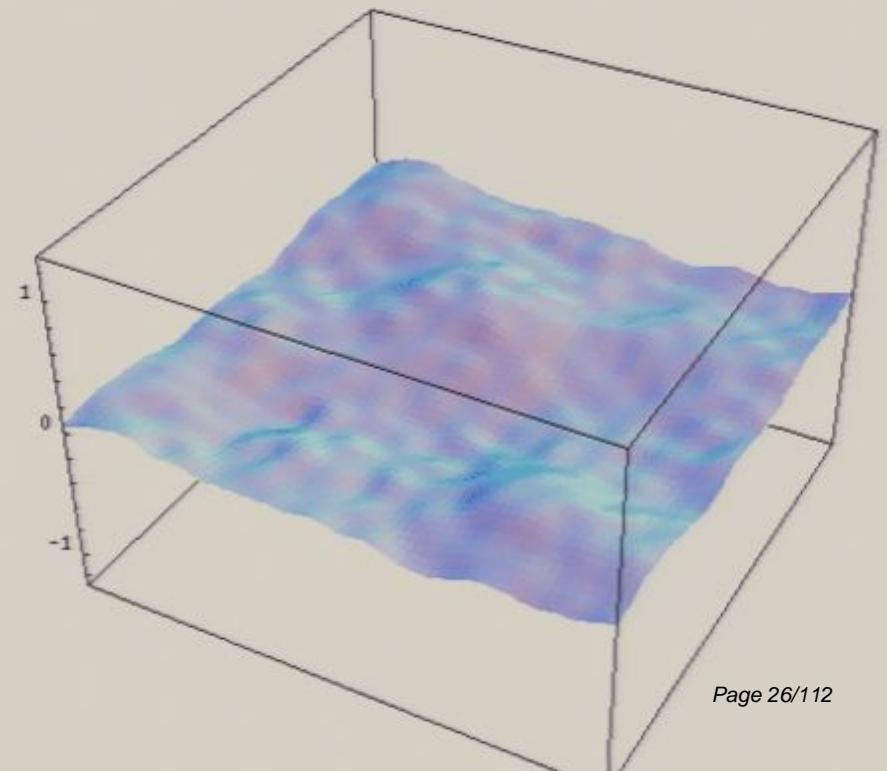
$\phi_0 + \phi$

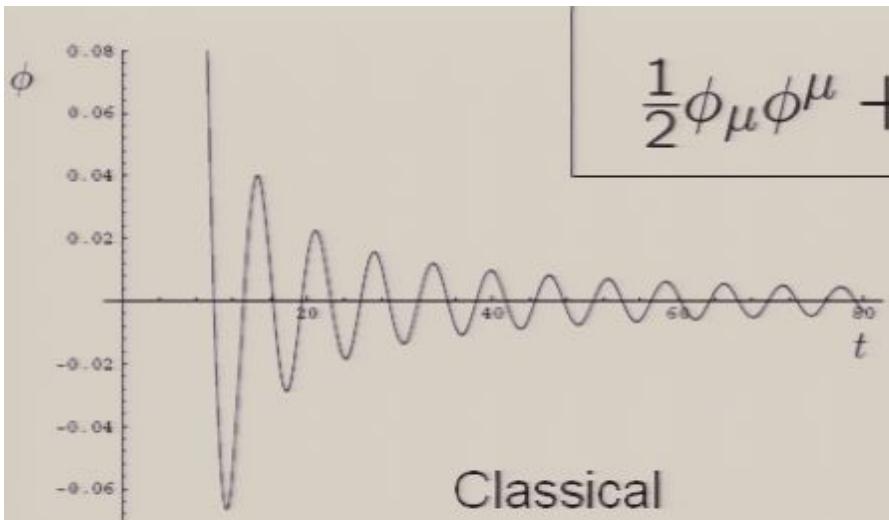
slices for  $t=102, 203$



Quantum

$\chi$

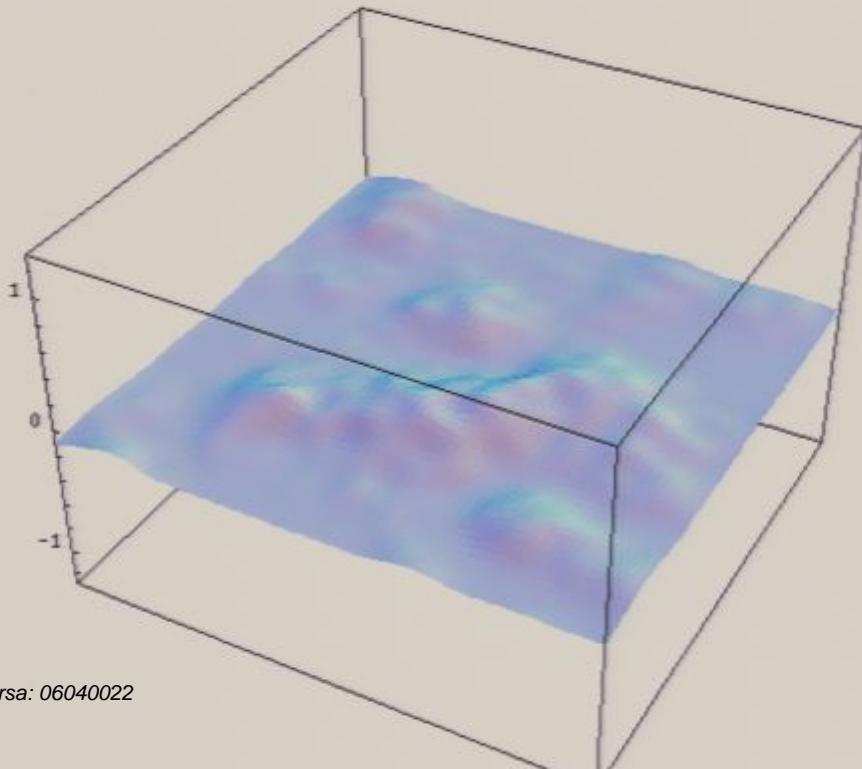




$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

$$\phi_0 + \phi$$

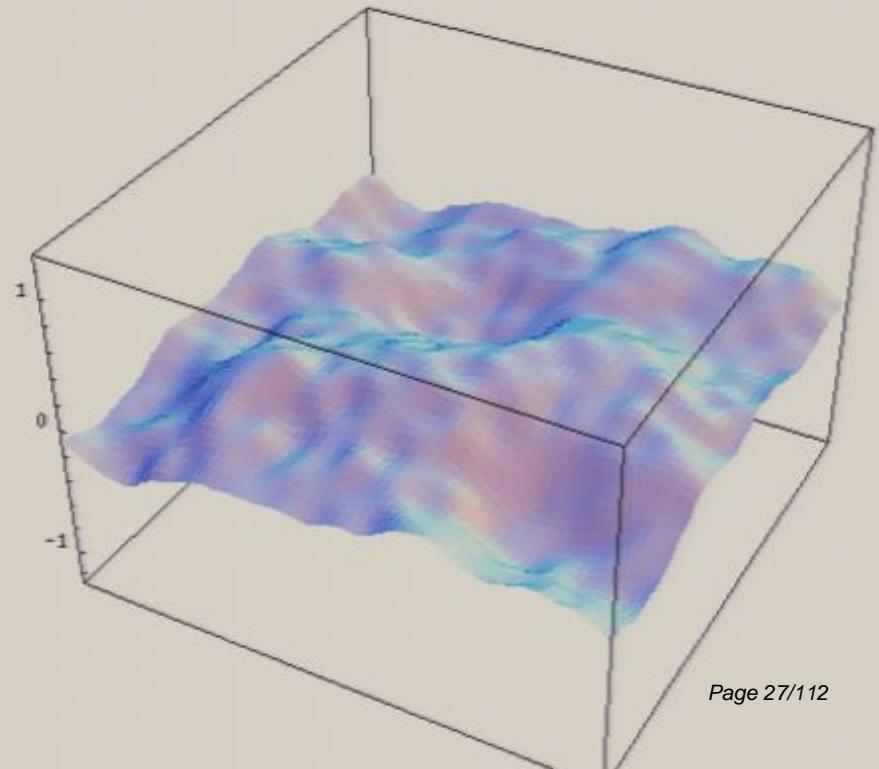


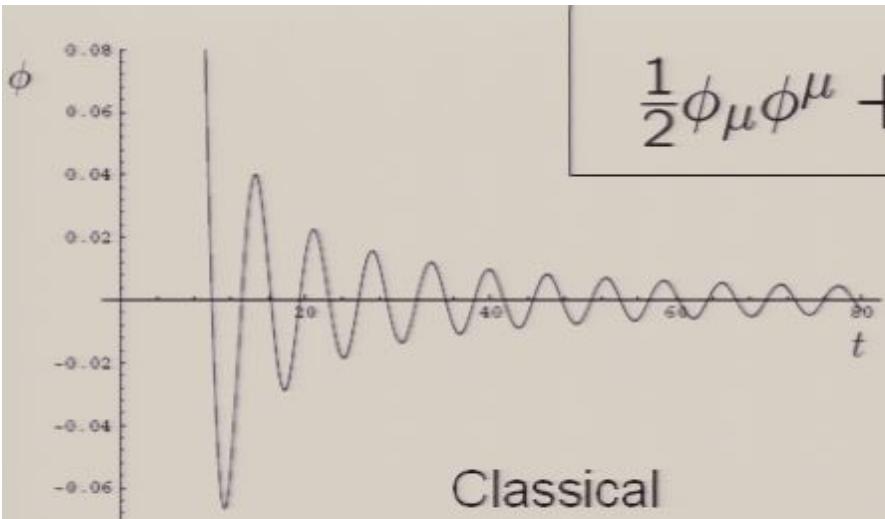
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=104, 704$





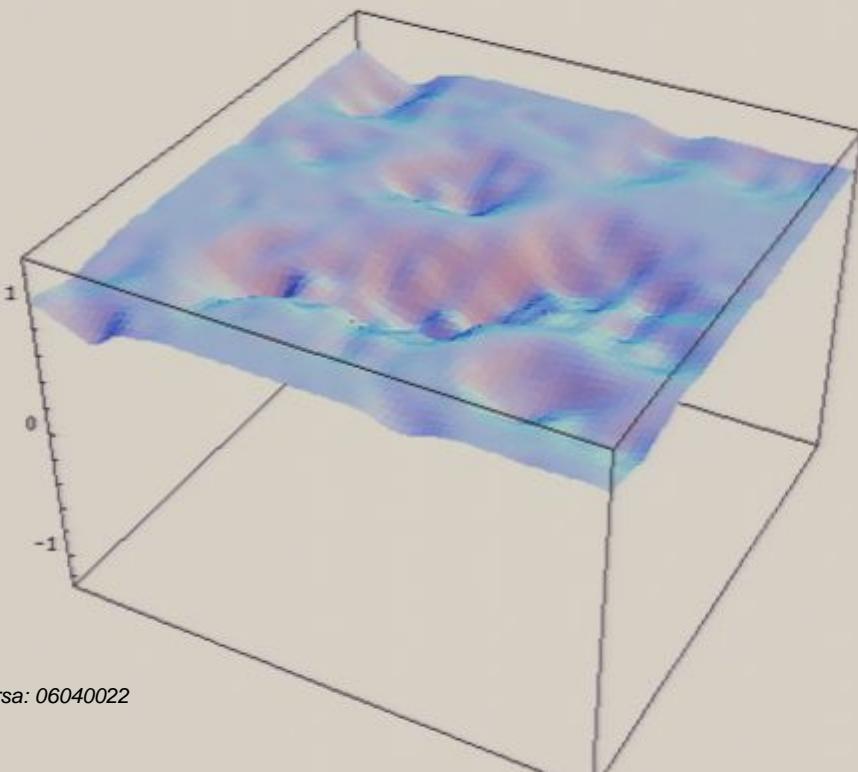
$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

**Decay of inflaton  
and preheating after inflation**

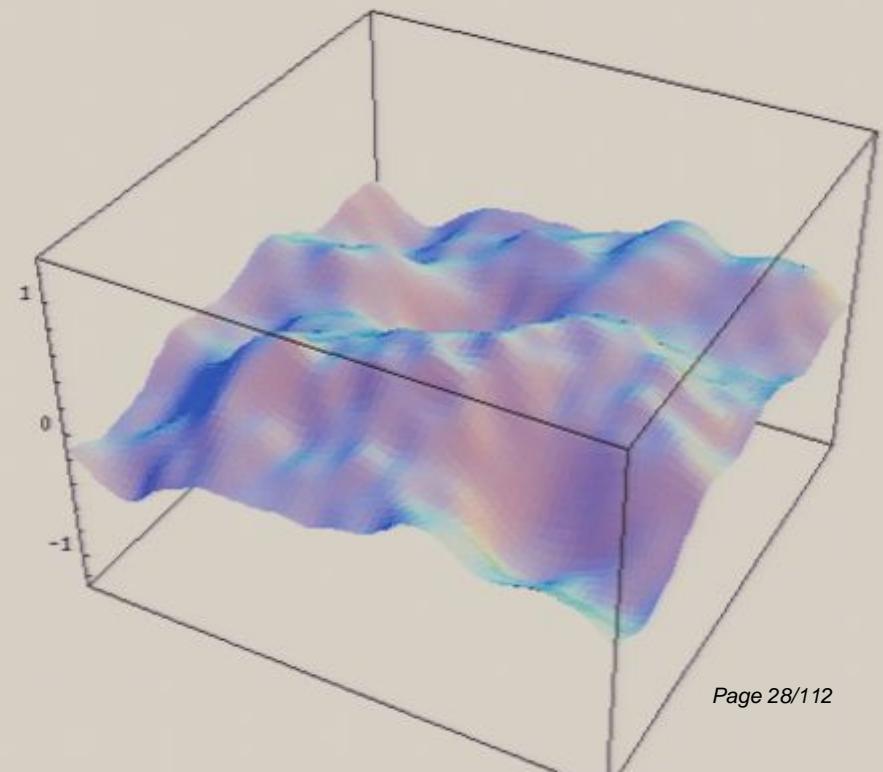
$$\phi_0 + \phi$$

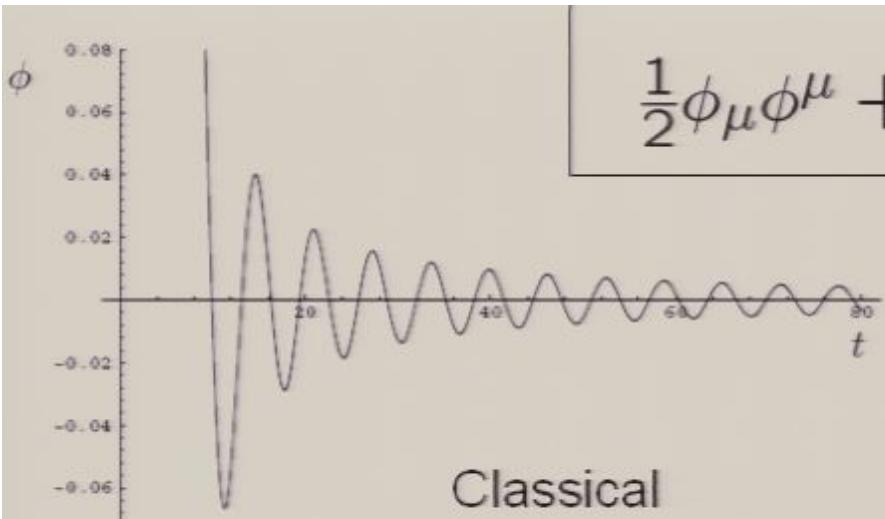
slices for  $t=106.704$



Quantum

$$\chi$$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

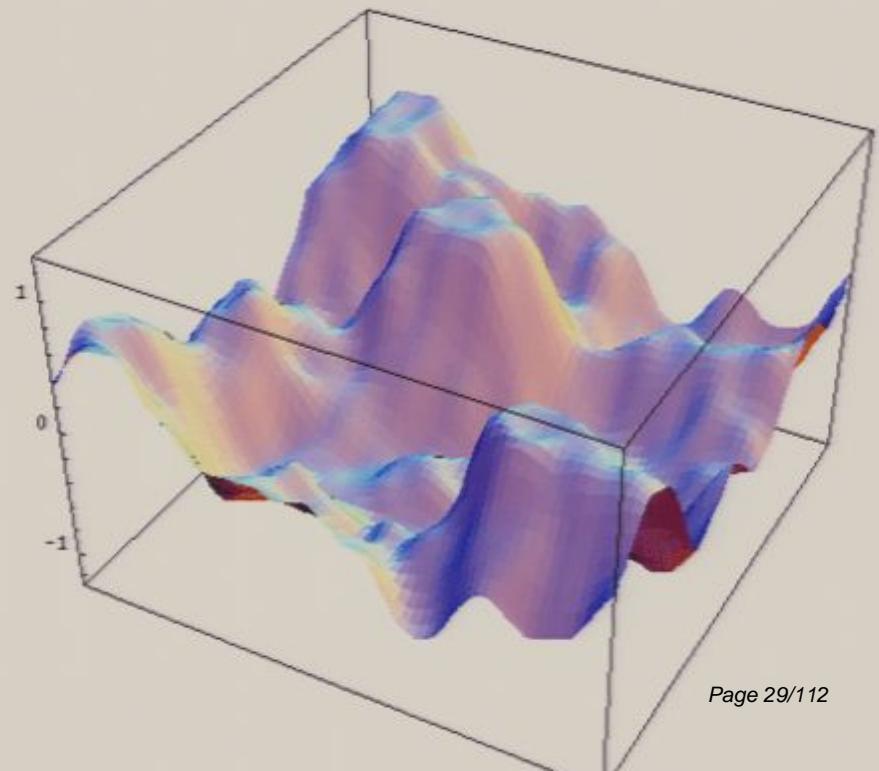
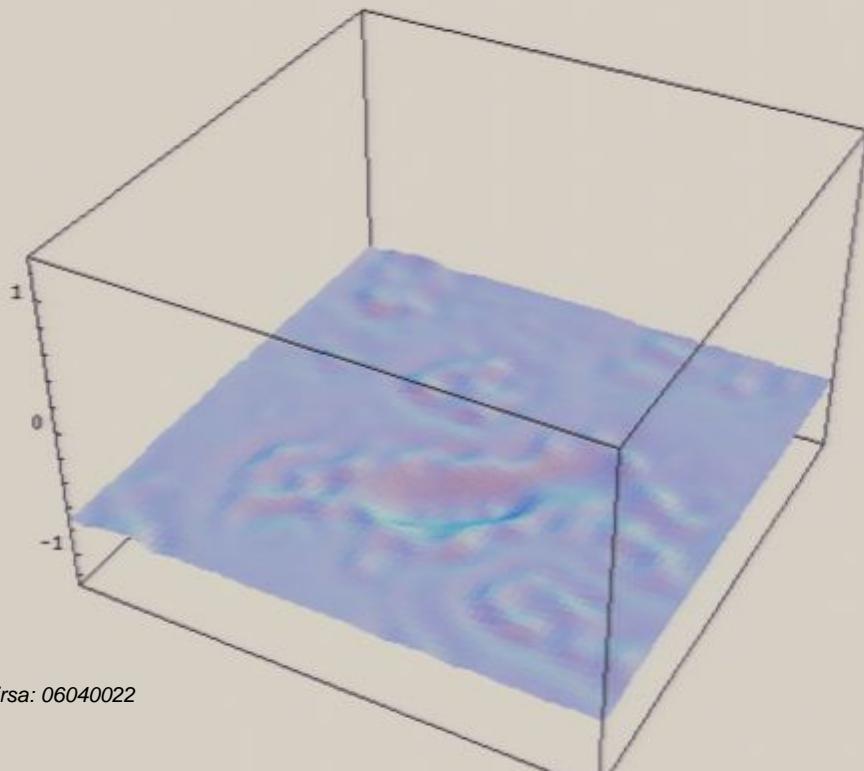
$$\phi_0 + \phi$$

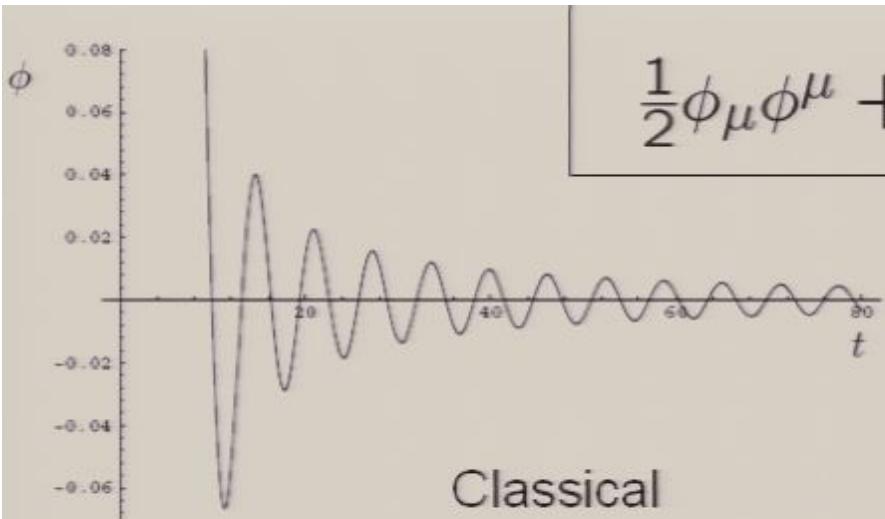
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=109.205$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

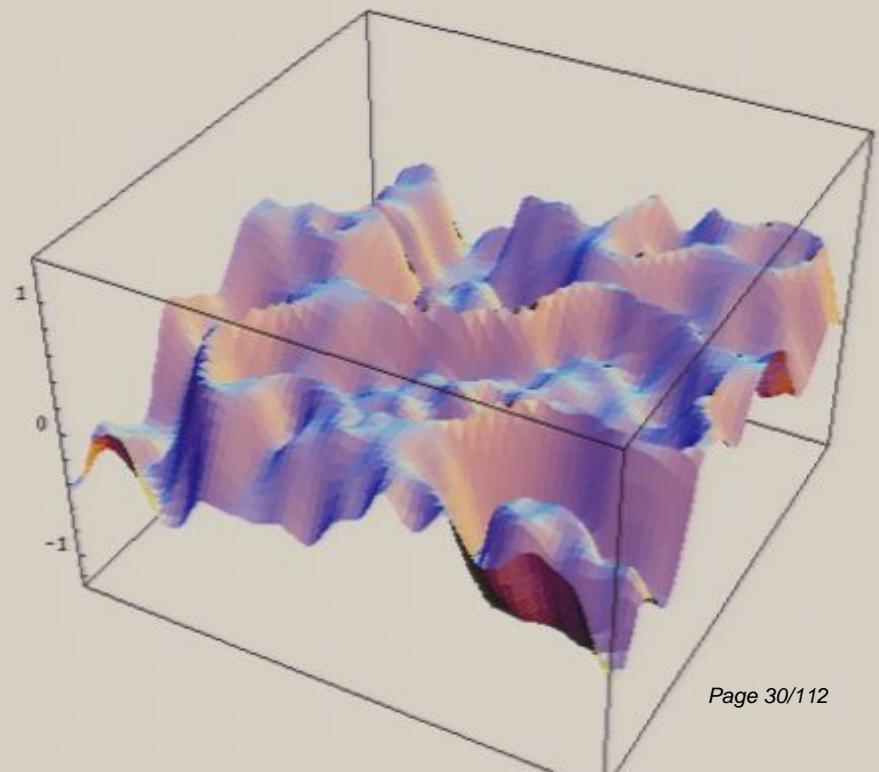
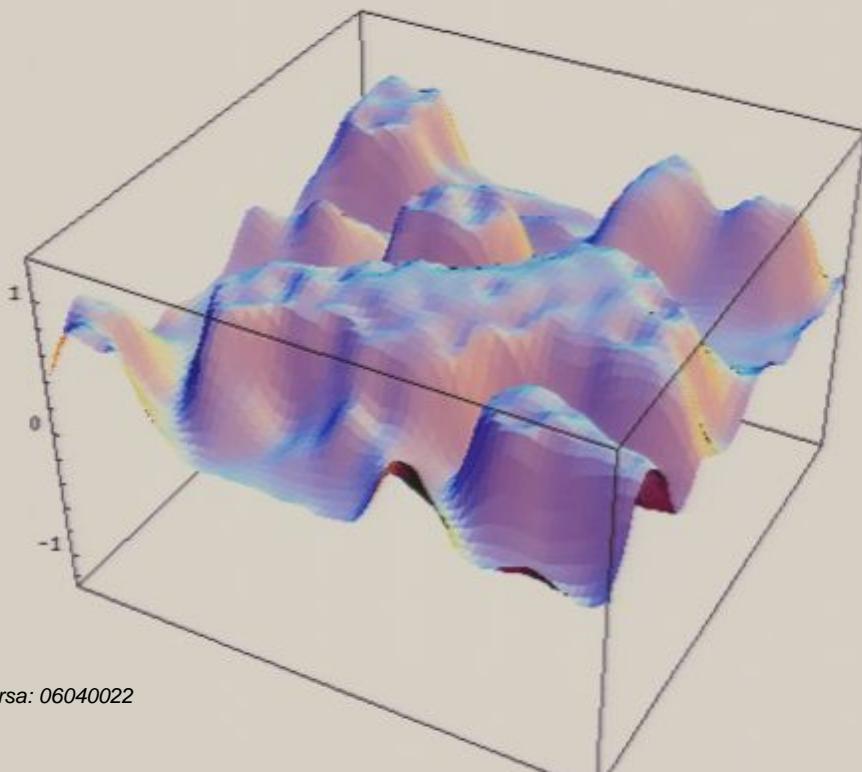
$$\phi_0 + \phi$$

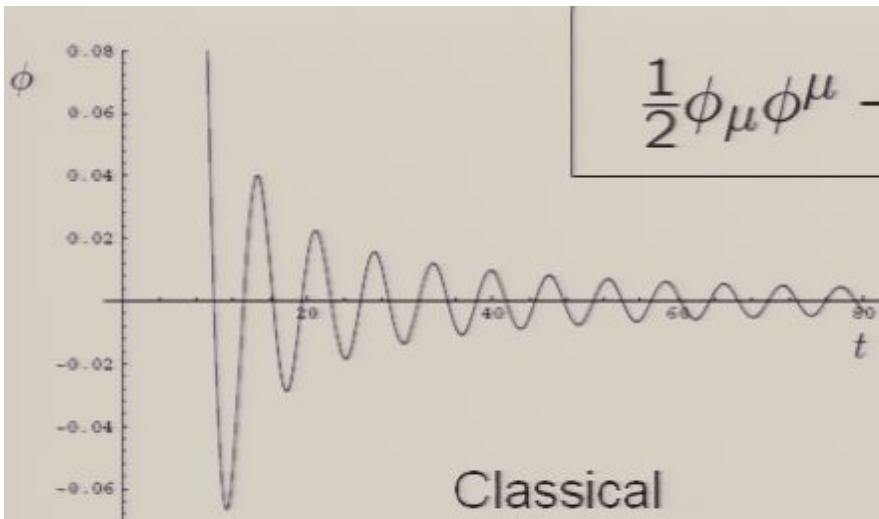
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

Slices for  $t=111.705$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

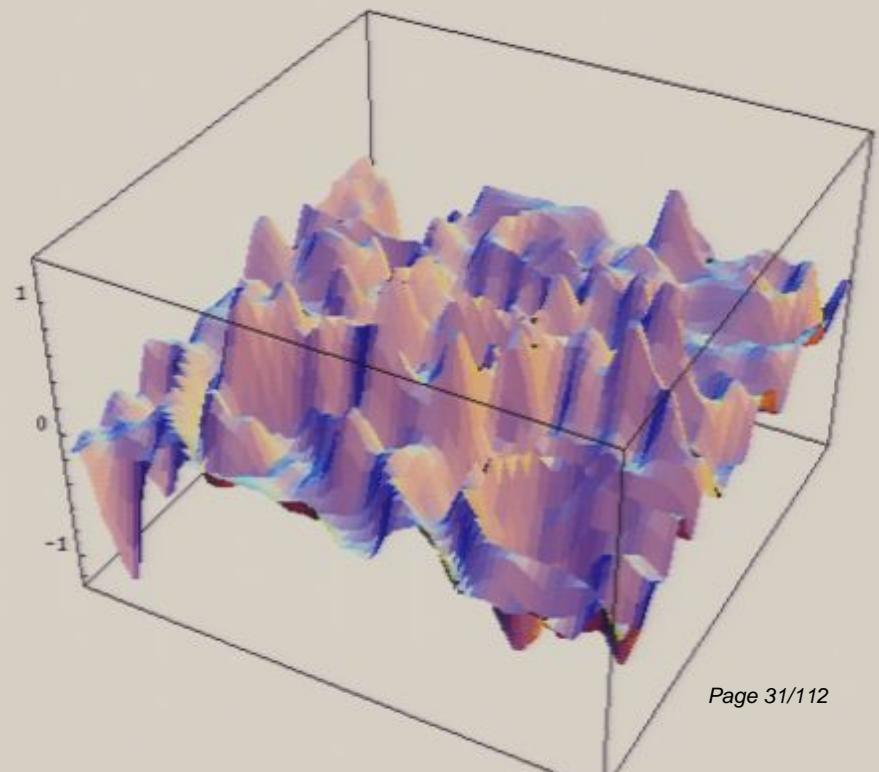
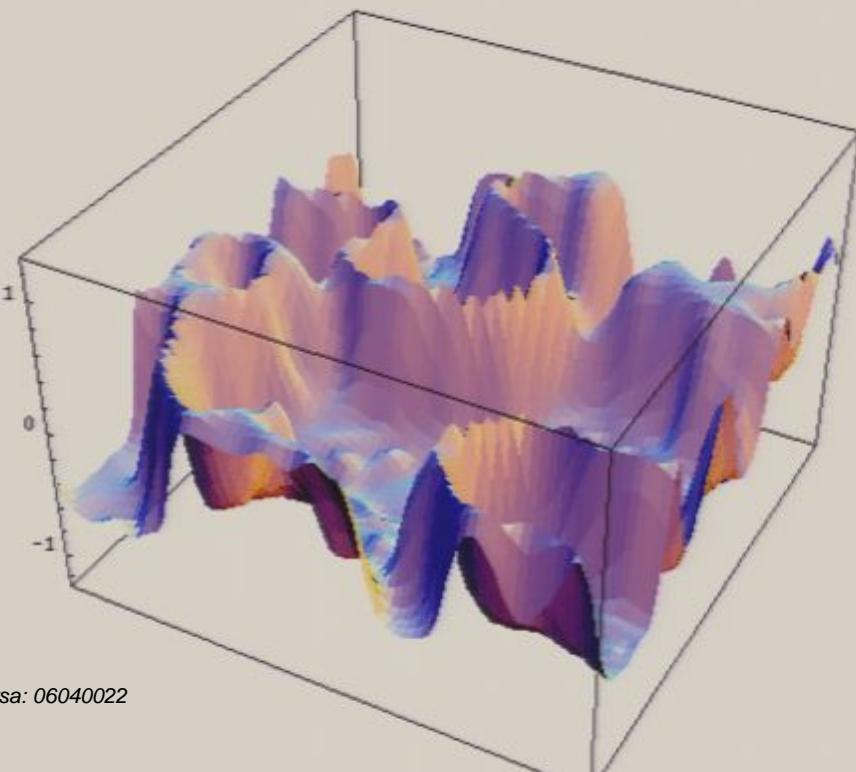
$$\phi_0 + \phi$$

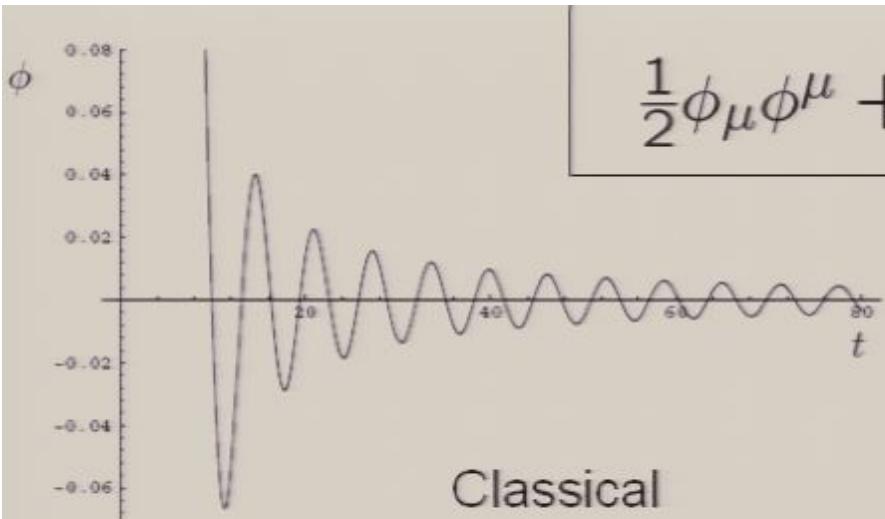
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=114.206$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

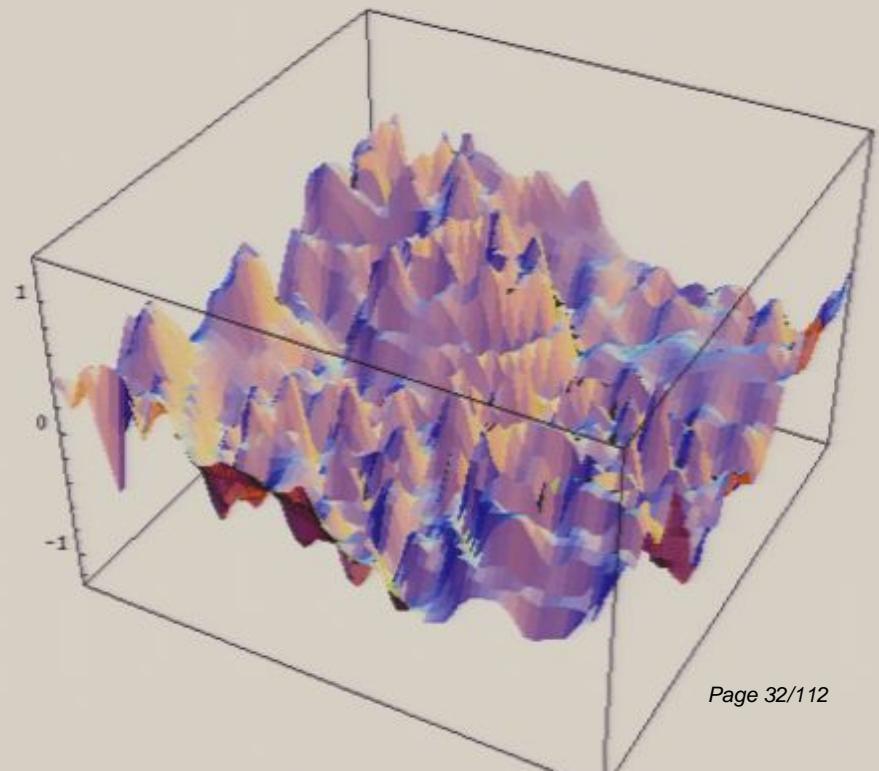
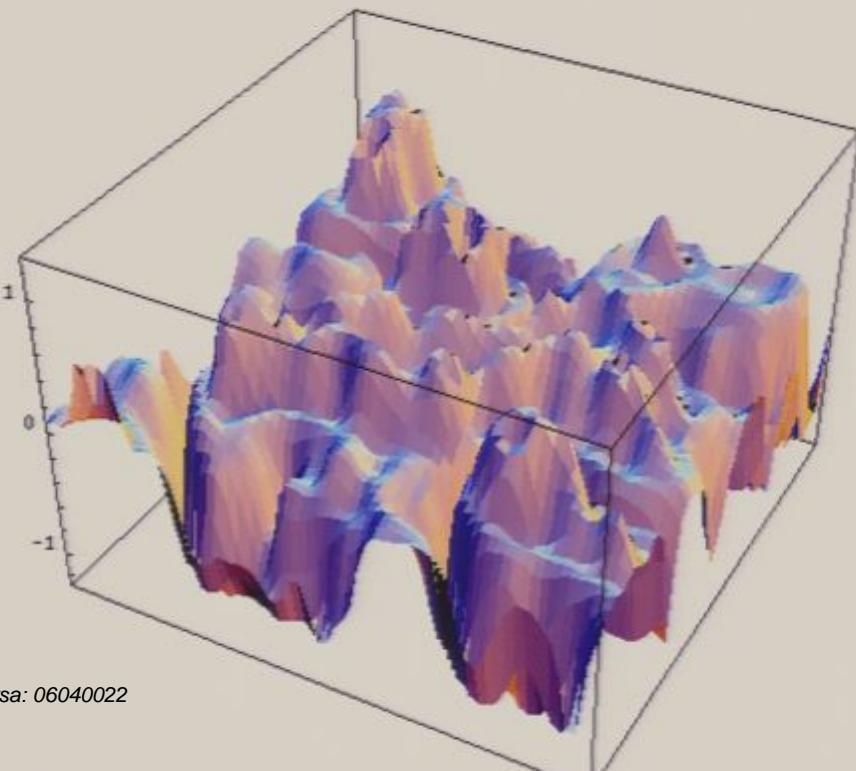
$$\phi_0 + \phi$$

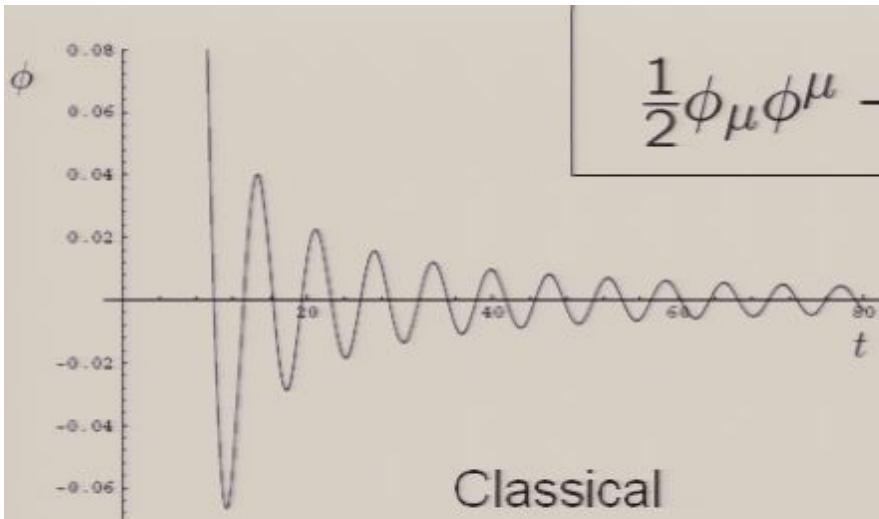
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=116.707$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

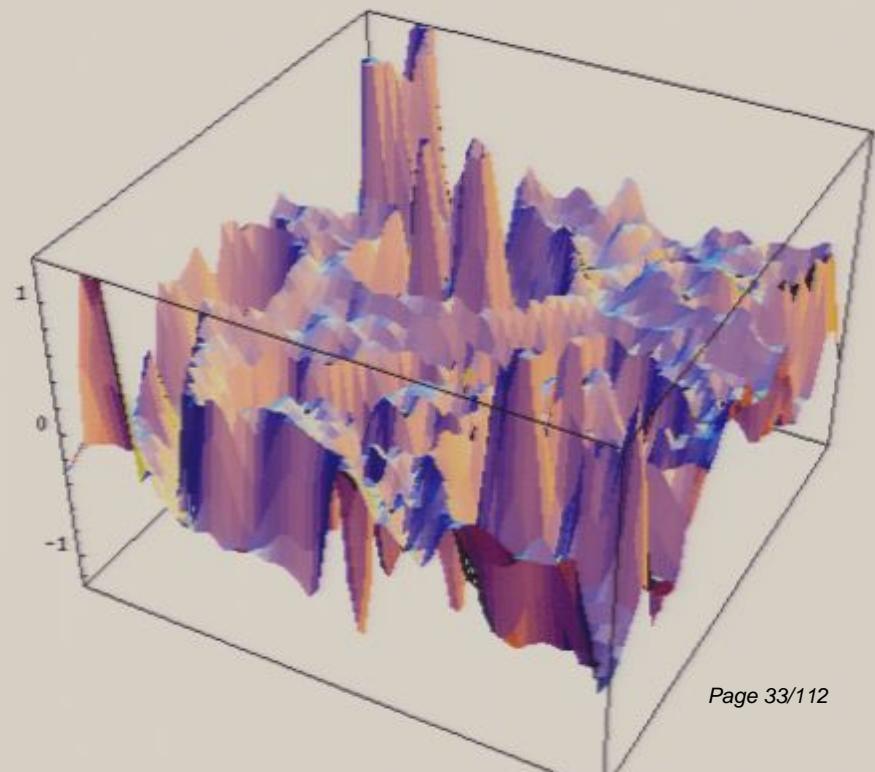
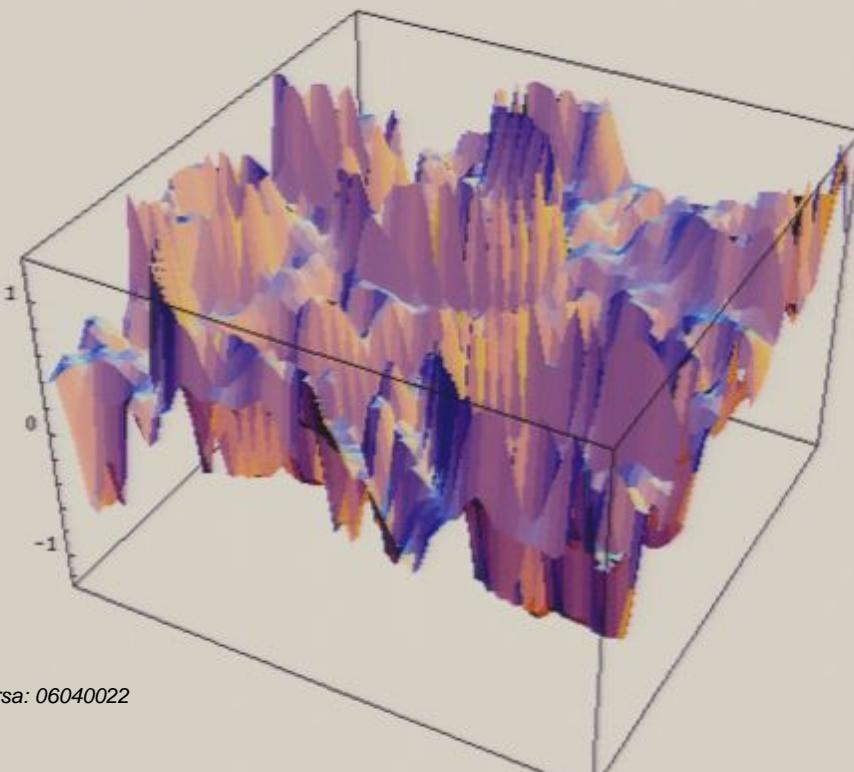
$$\phi_0 + \phi$$

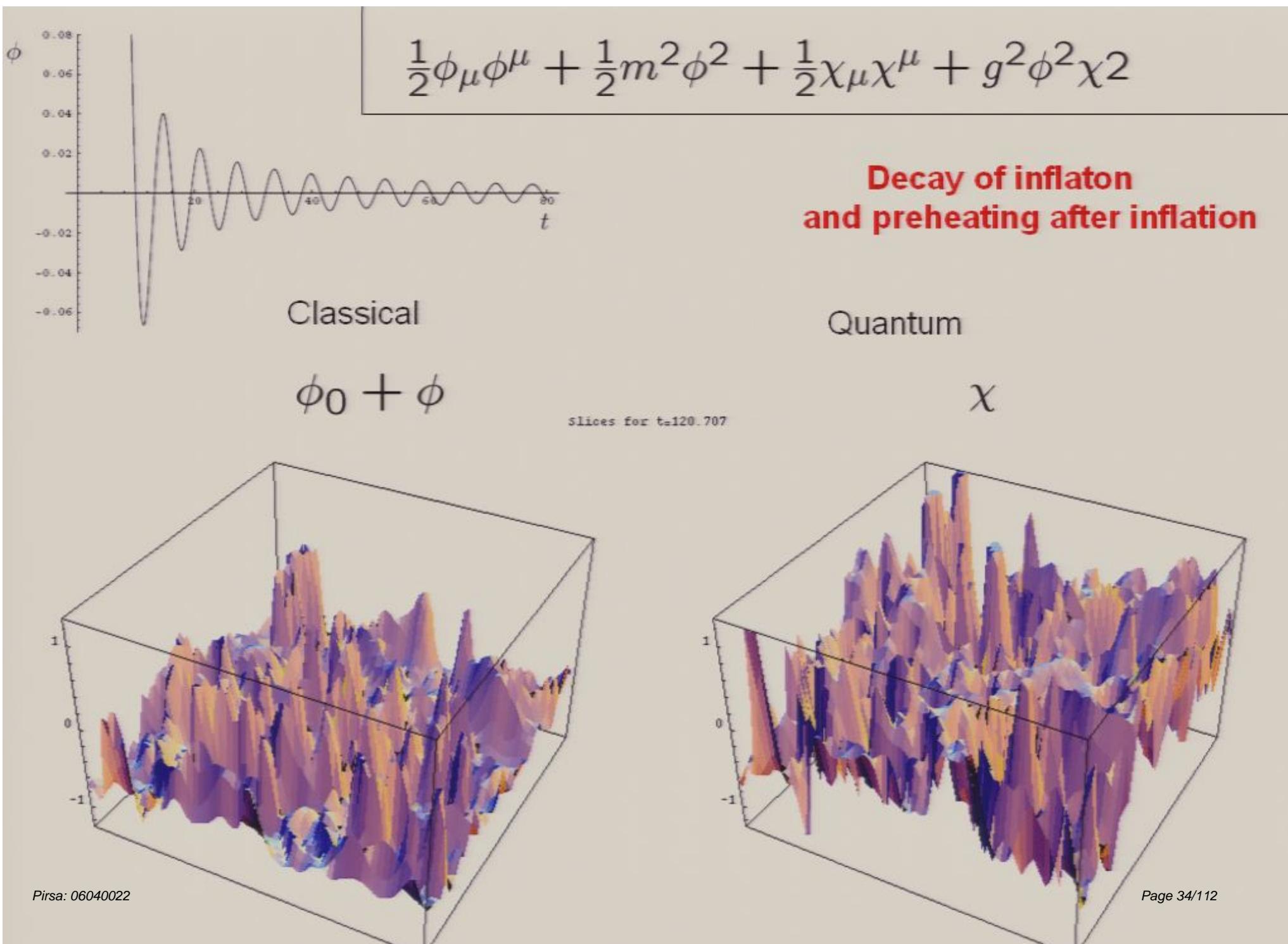
**Decay of inflaton  
and preheating after inflation**

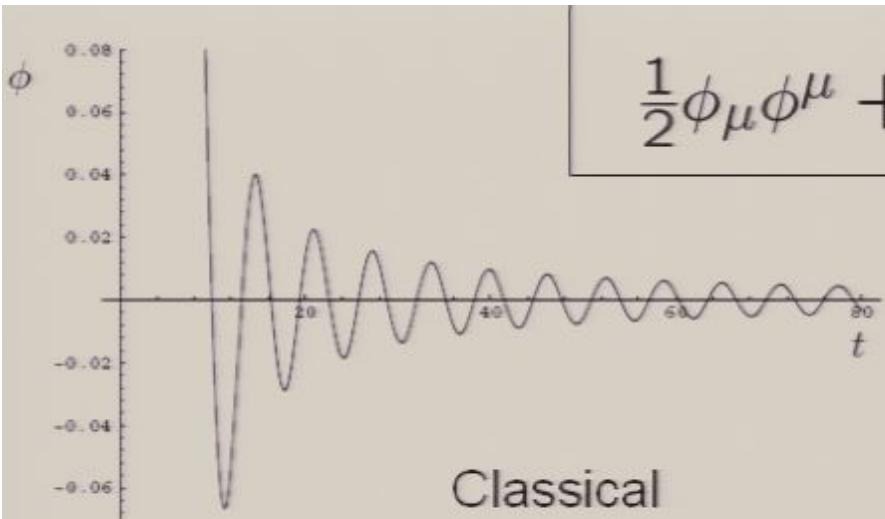
Quantum

$$\chi$$

slices for  $t=119.207$







$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

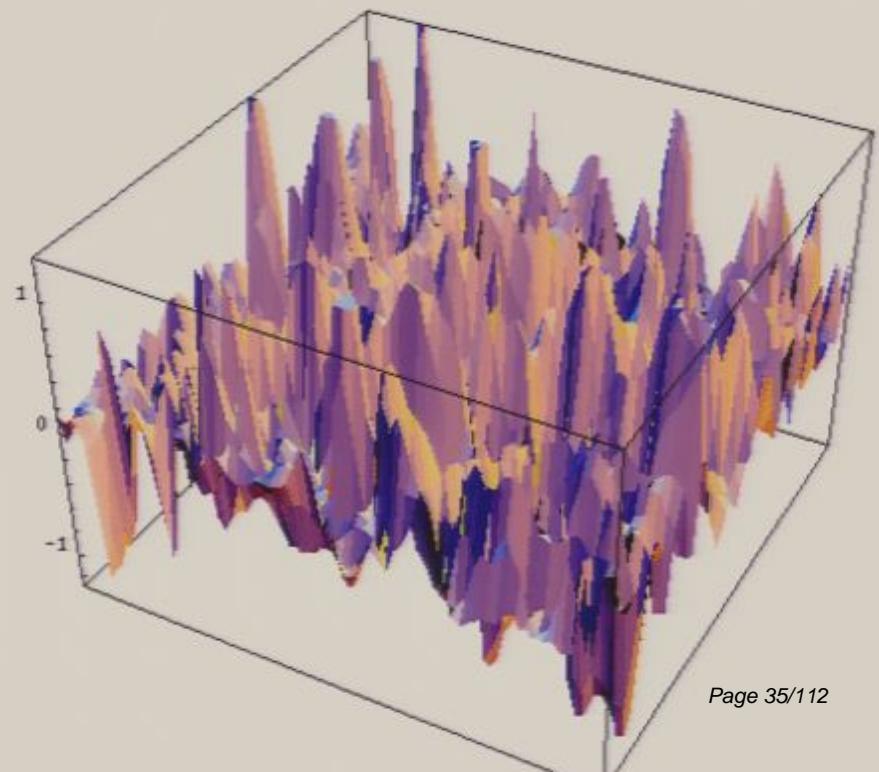
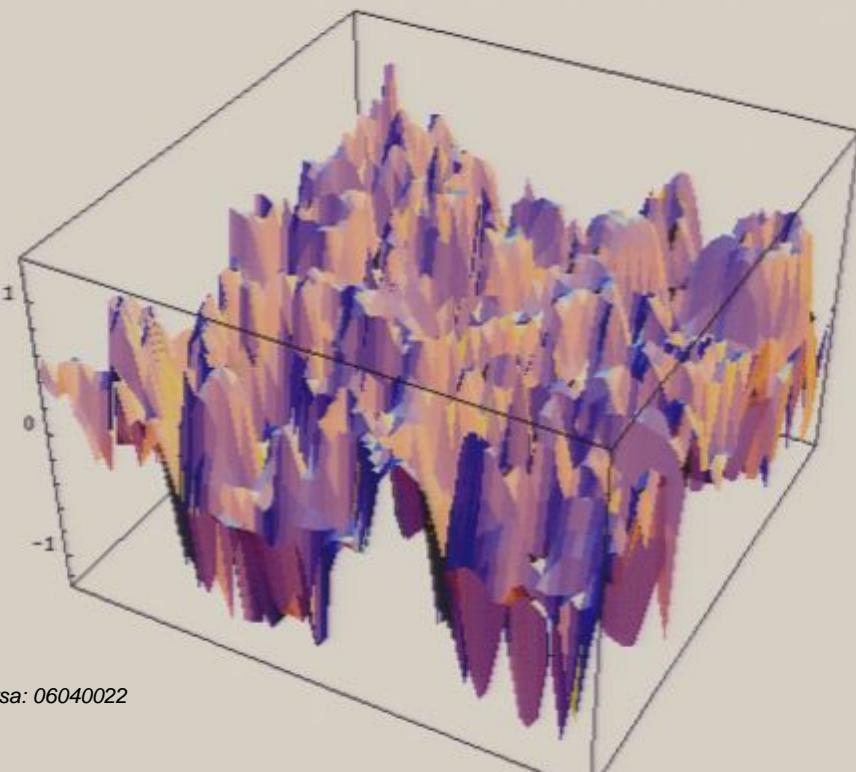
$$\phi_0 + \phi$$

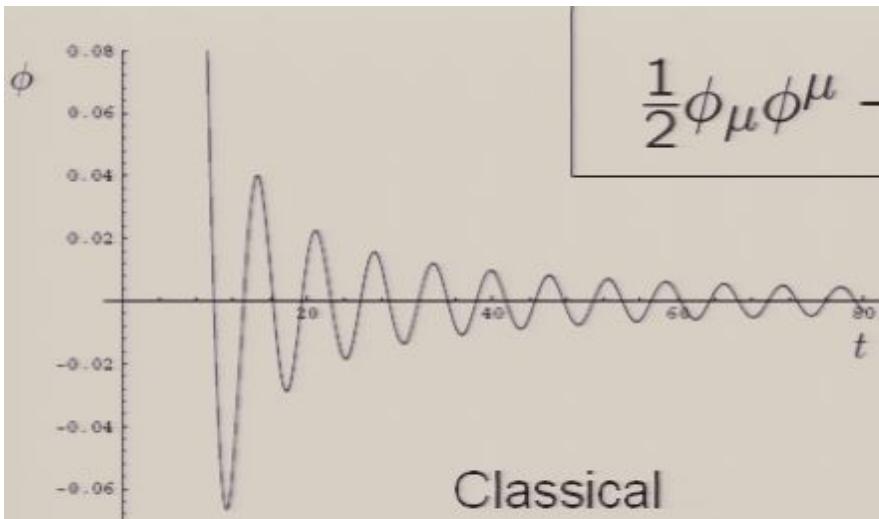
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=122.708$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

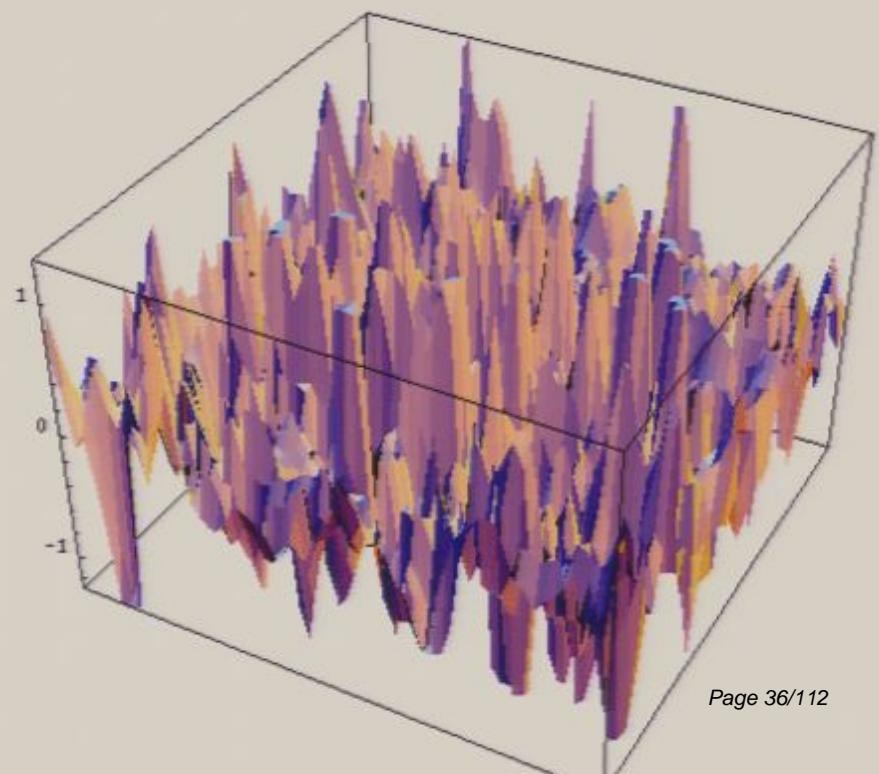
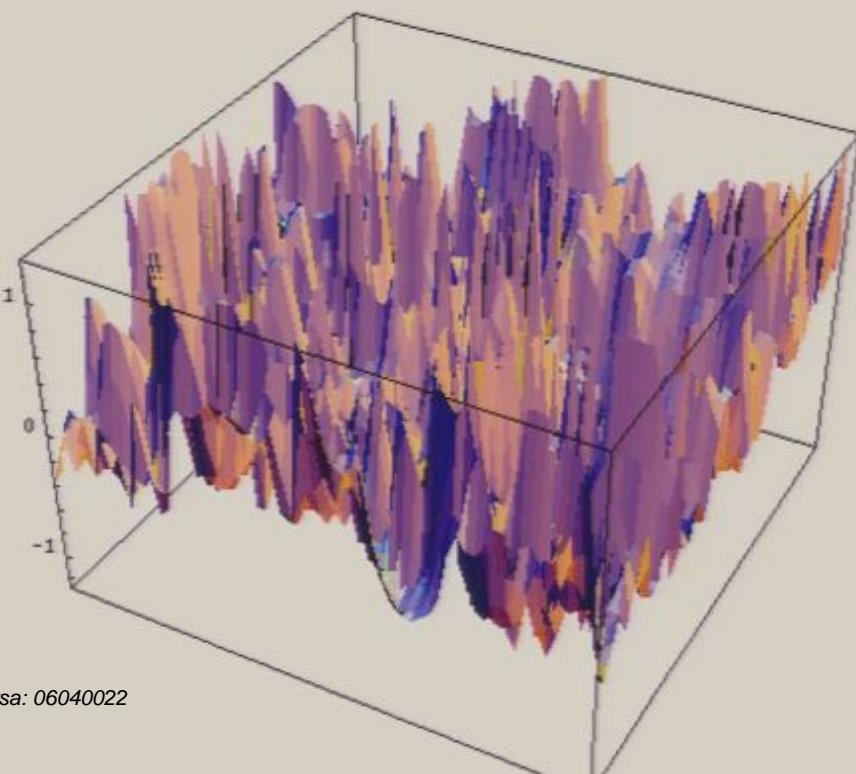
$$\phi_0 + \phi$$

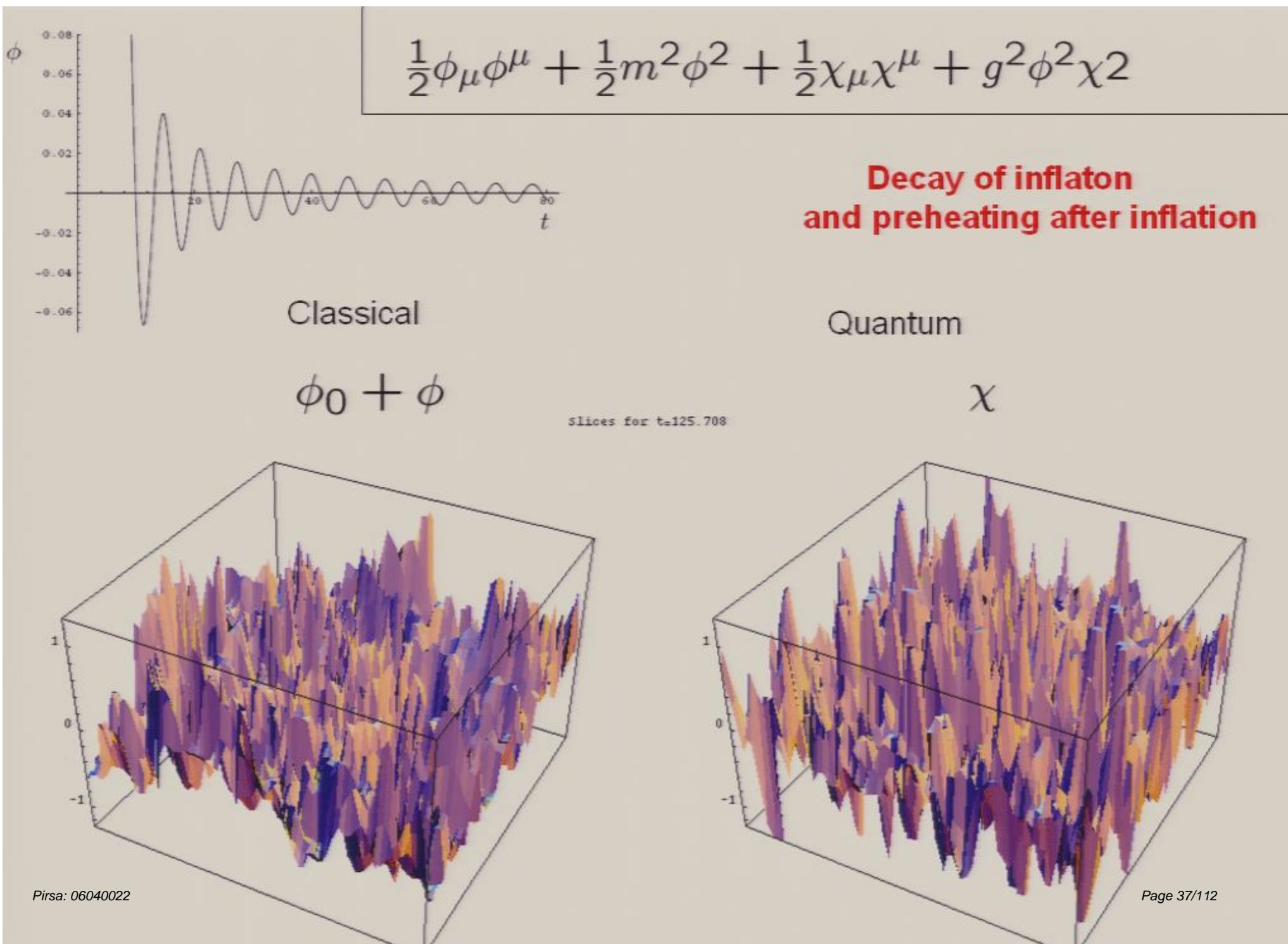
**Decay of inflaton  
and preheating after inflation**

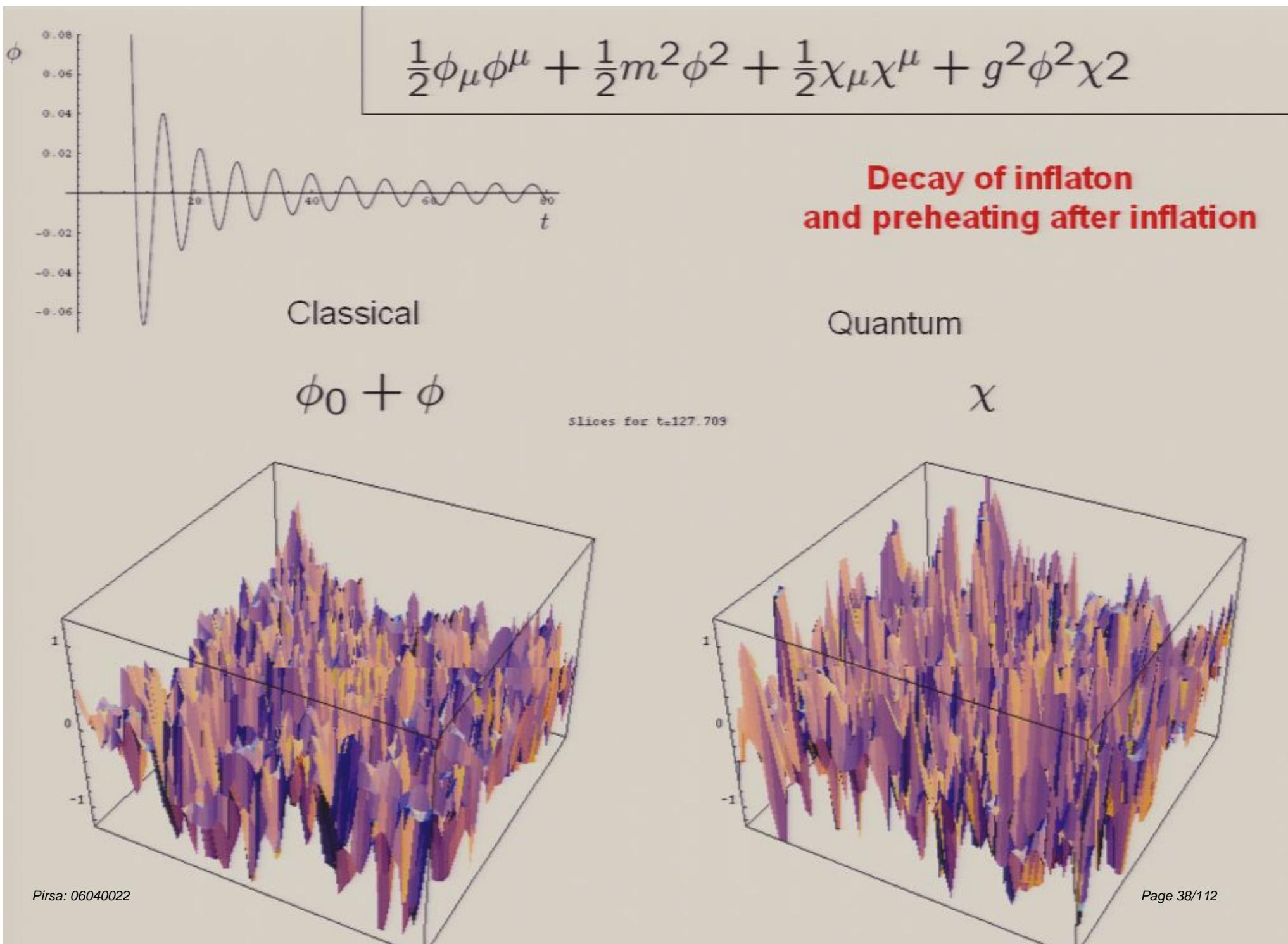
Quantum

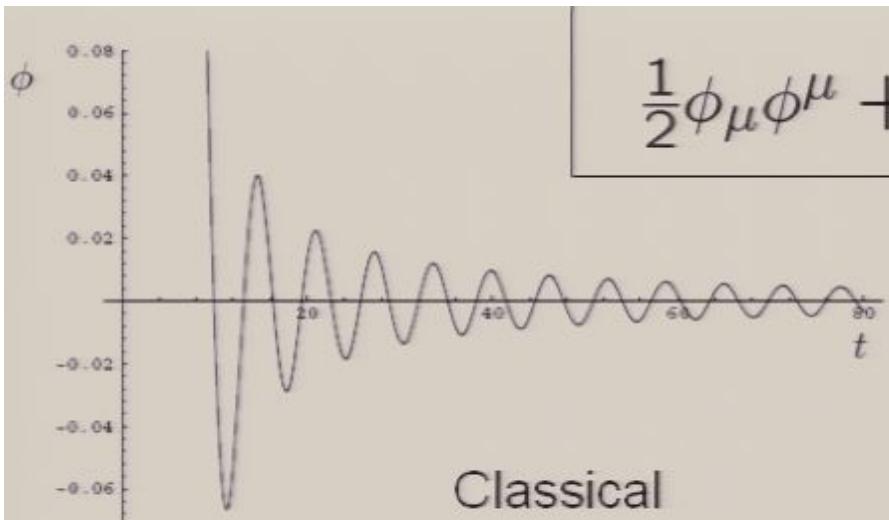
$$\chi$$

slices for  $t=125, 208$







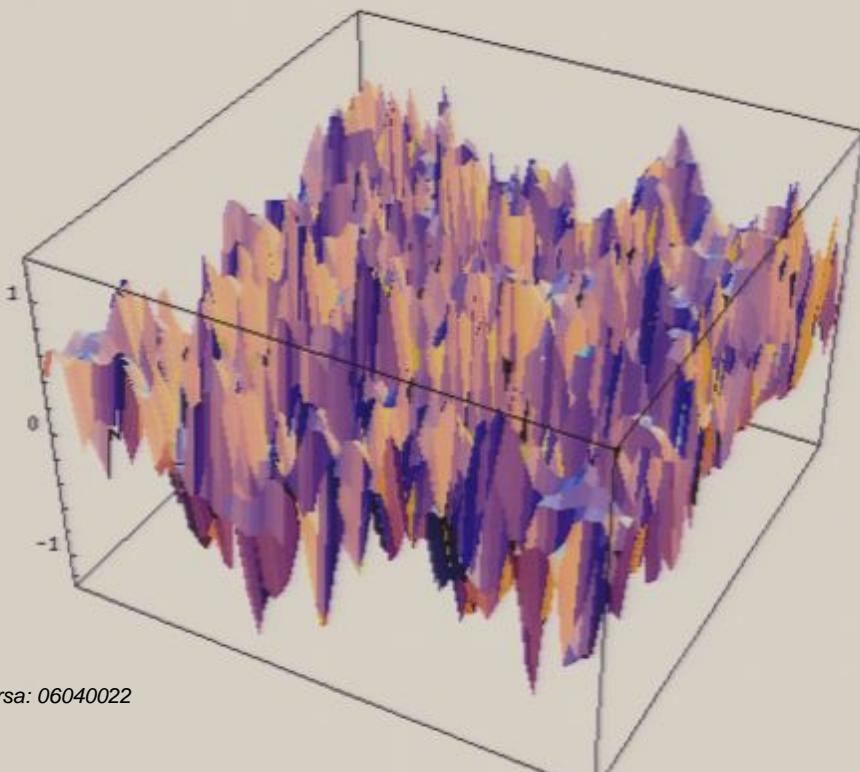


$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

$$\phi_0 + \phi$$

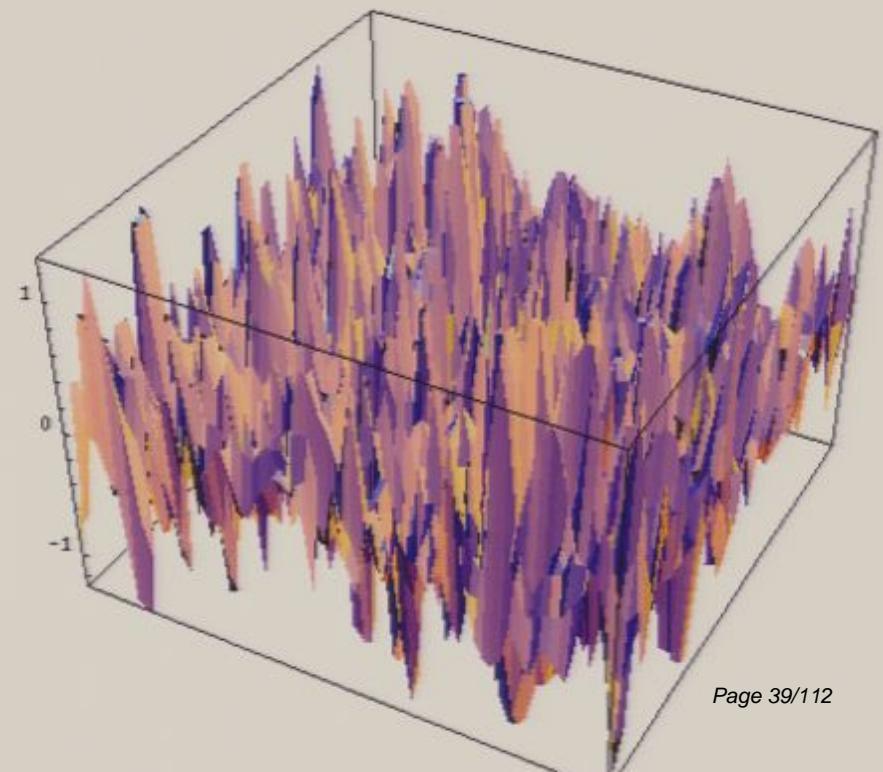
slices for  $t=128.729$

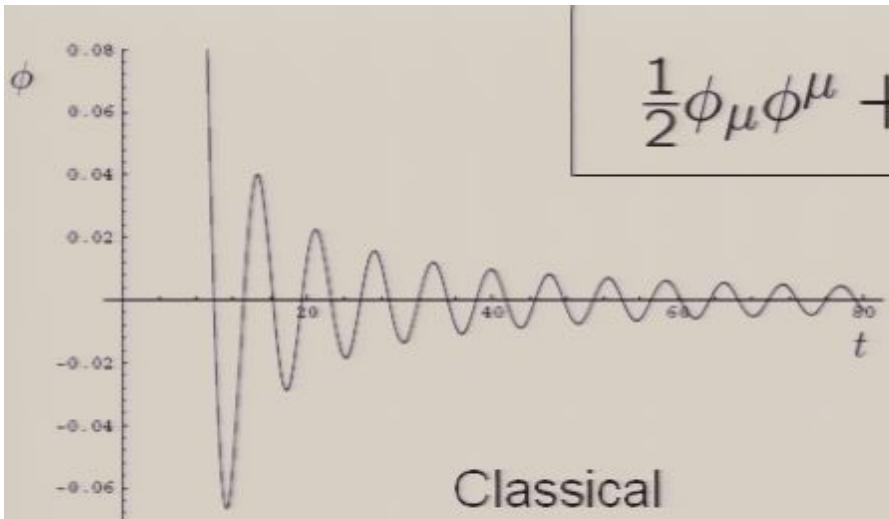


**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

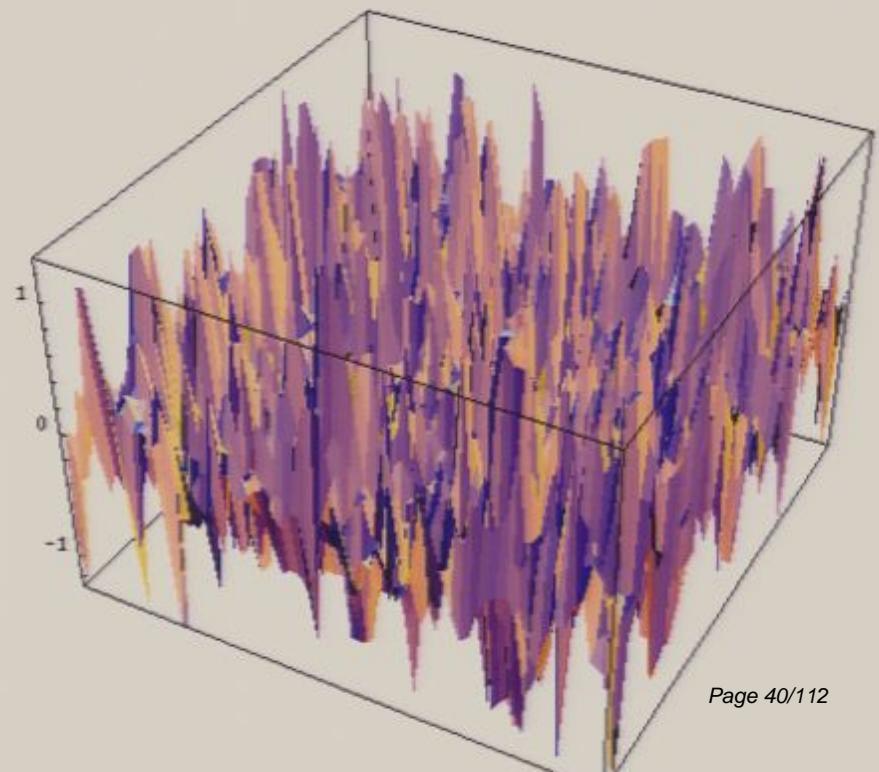
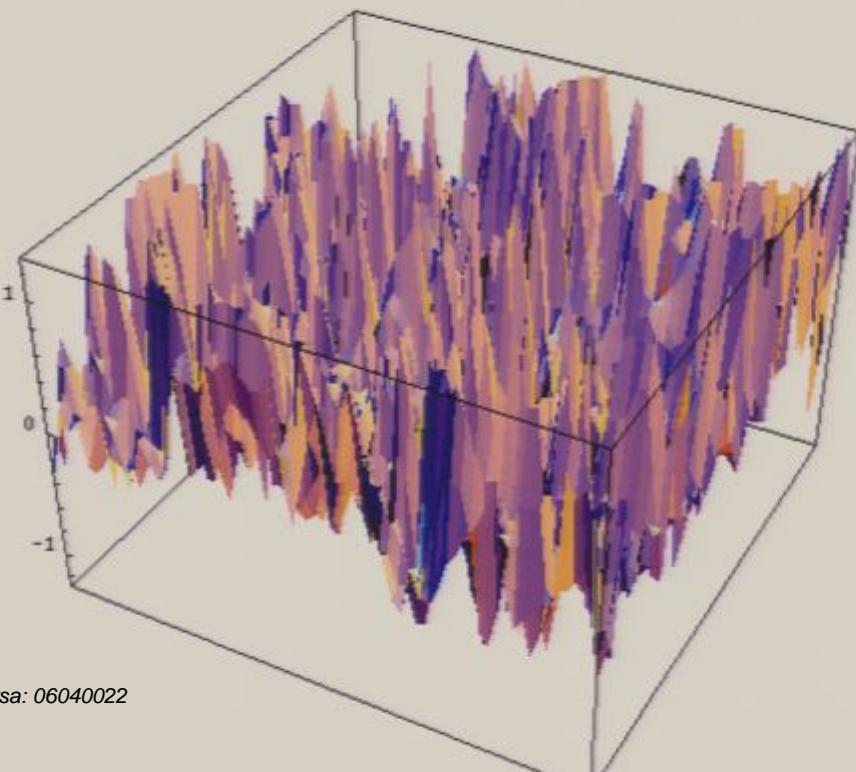
$$\phi_0 + \phi$$

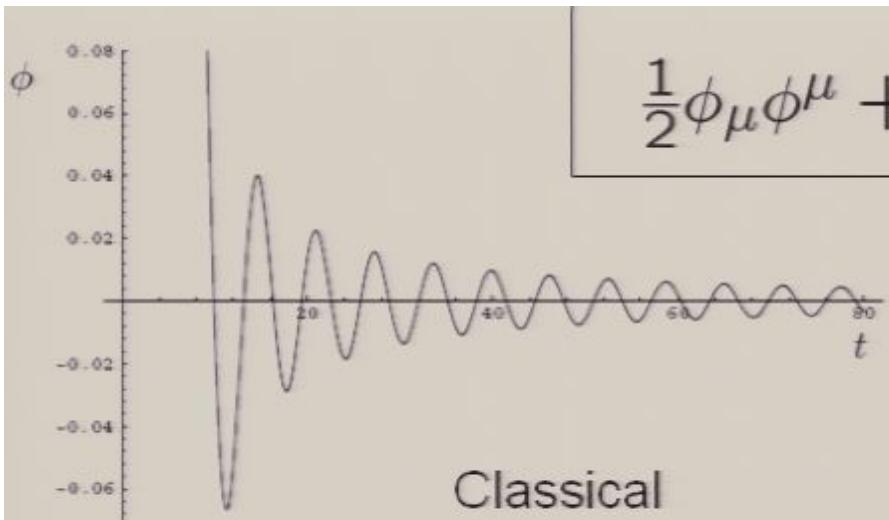
**Decay of inflaton  
and preheating after inflation**

Quantum

$$\chi$$

slices for  $t=130.767$





$$\frac{1}{2}\phi_\mu\phi^\mu + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\chi_\mu\chi^\mu + g^2\phi^2\chi^2$$

Classical

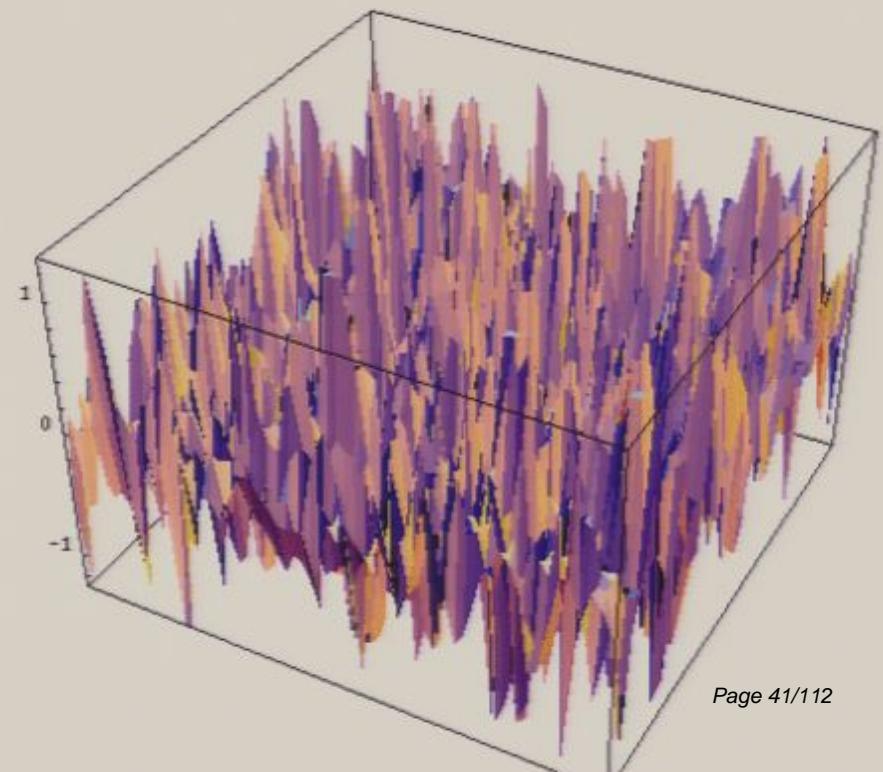
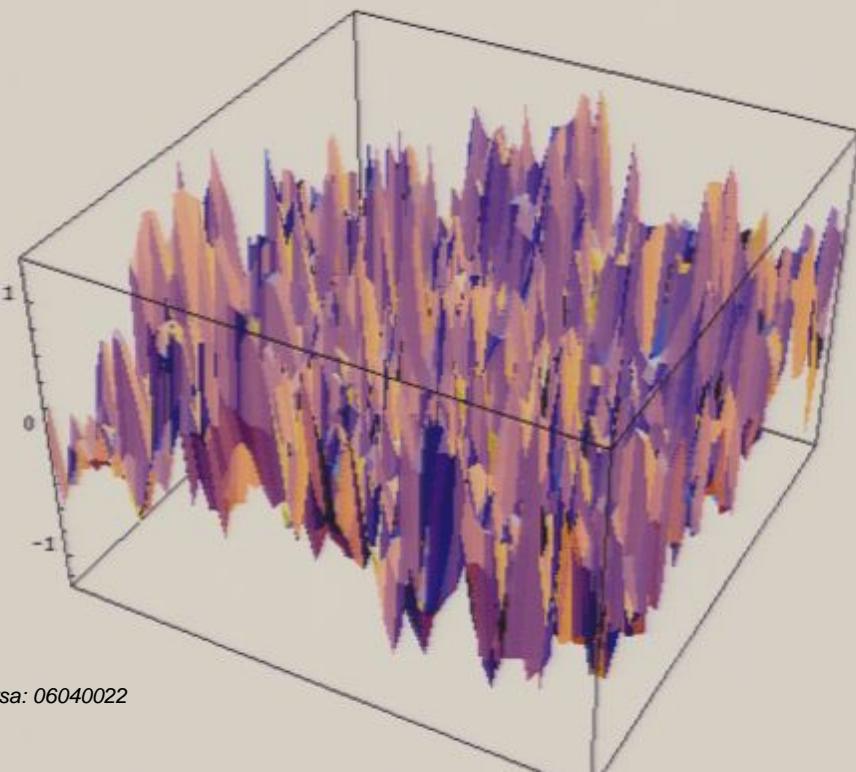
$\phi_0 + \phi$

Decay of inflaton  
and preheating after inflation

Quantum

$\chi$

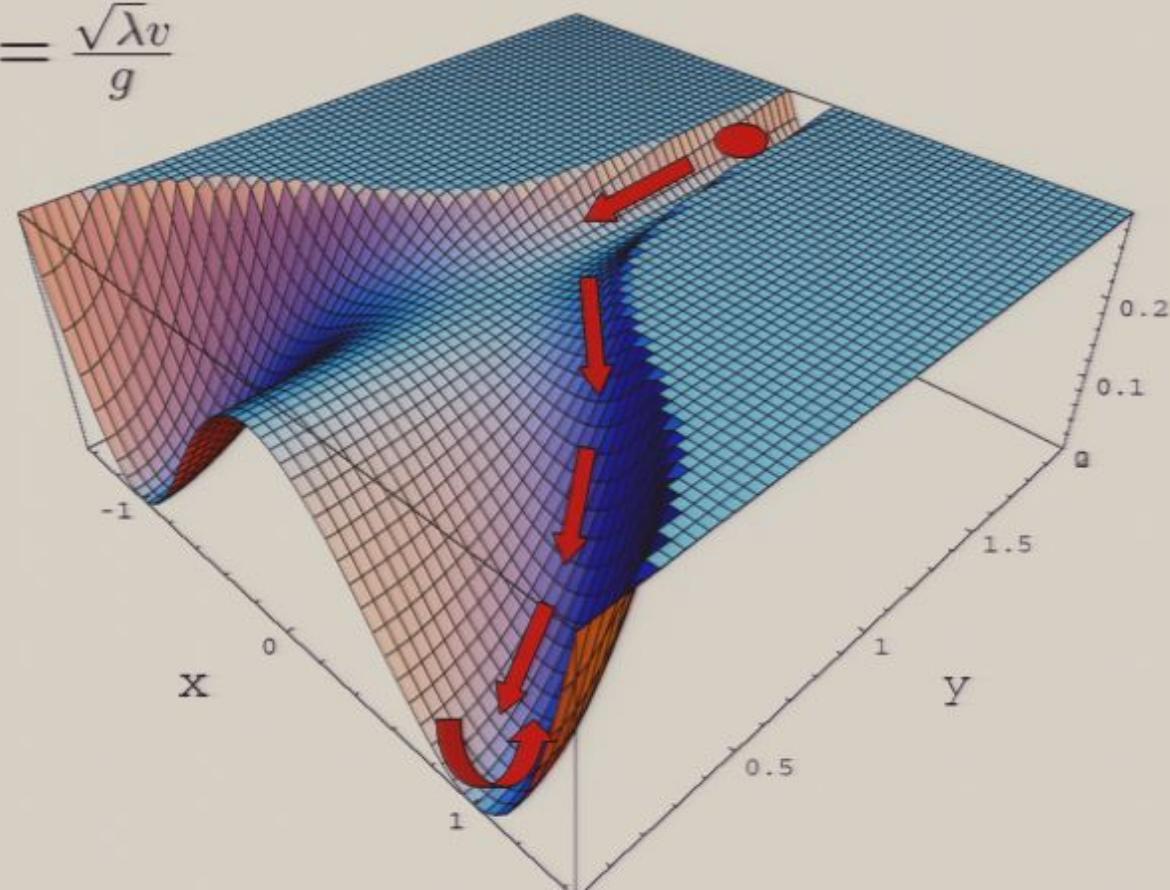
slices for  $t=131.277$



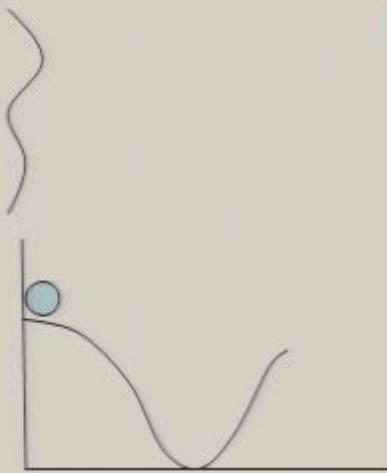
## Tachyonic Preheating in Hybrid Inflation

$$V(\phi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$$

bifurcation point  $\phi_c = \frac{\sqrt{\lambda}v}{g}$



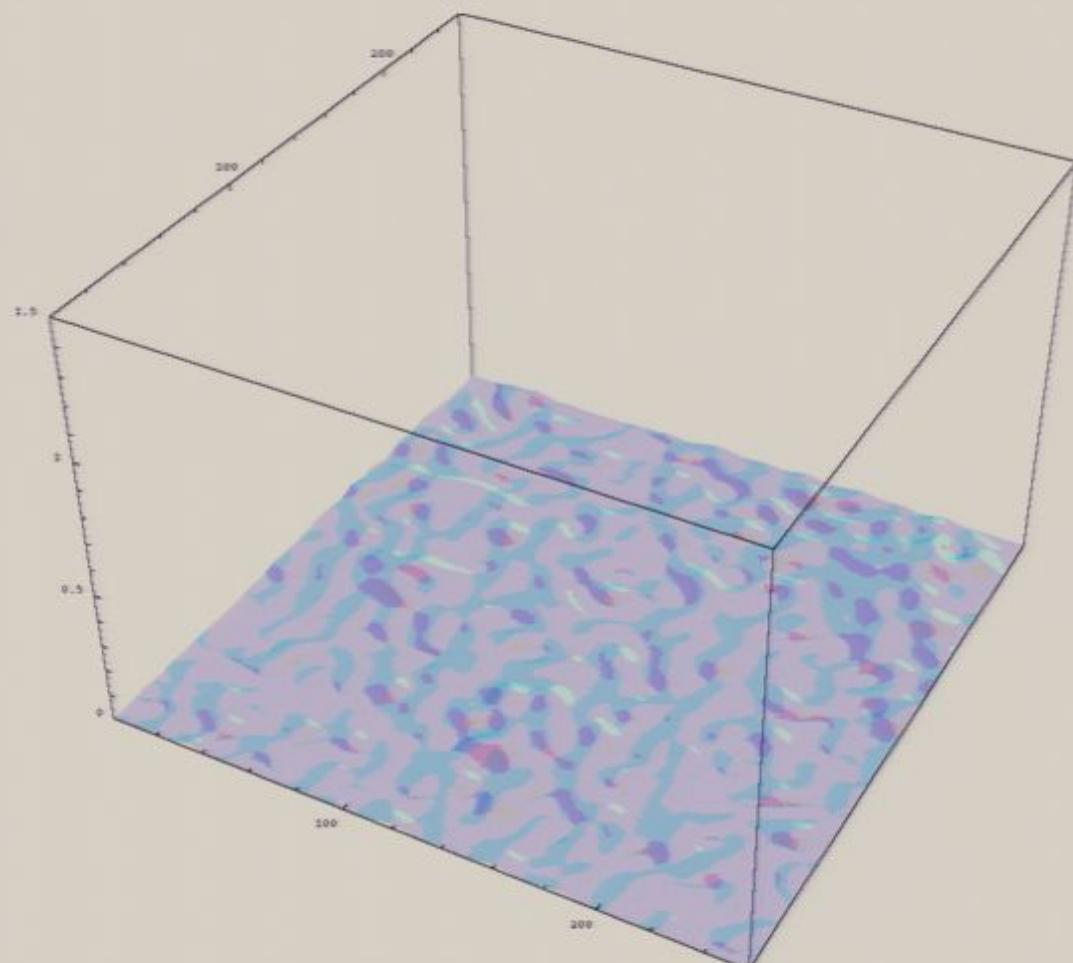
Felder, LK, Linde, 01



## Tachyonic Preheating

$$V_F = V_0 + \frac{\lambda}{4}\sigma^4 - \frac{\lambda^3}{4}\sigma^3 + \lambda\sigma^2$$

$\sigma(t, \vec{x})$

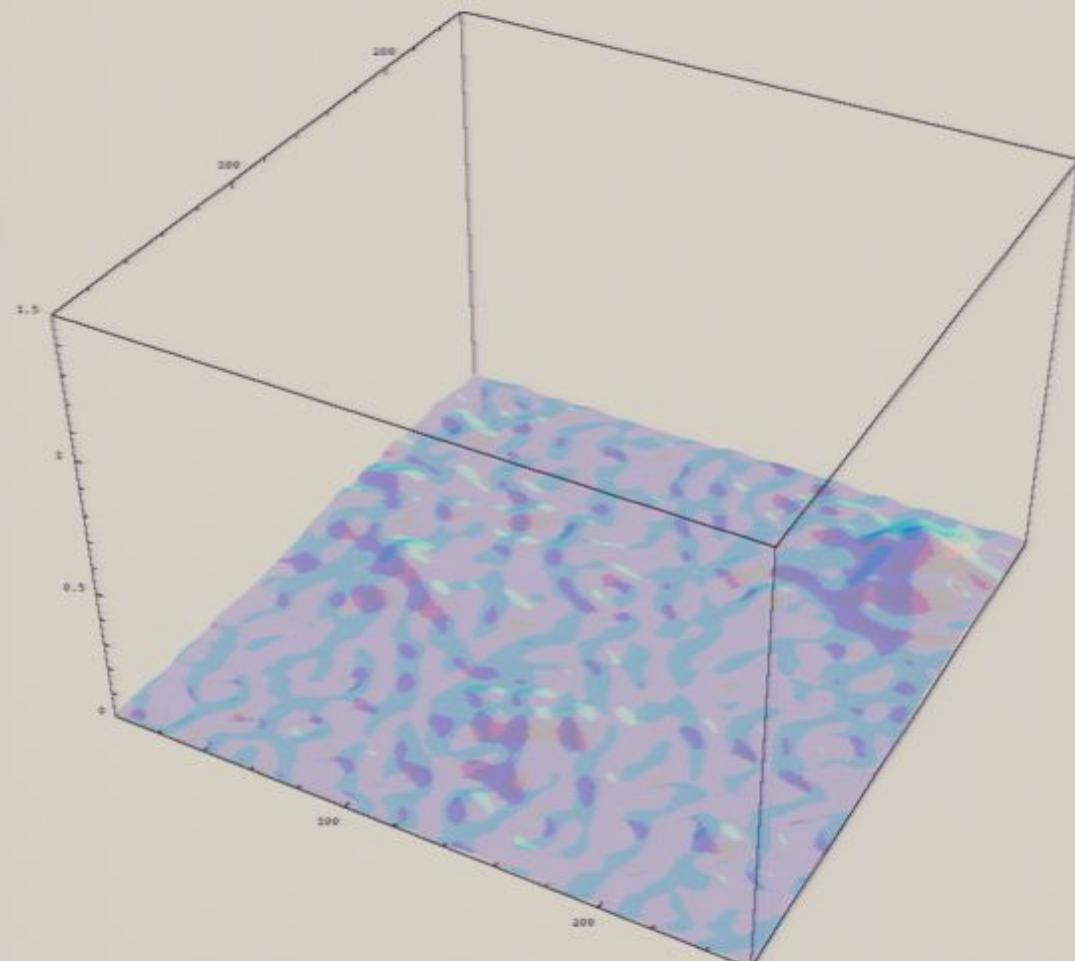


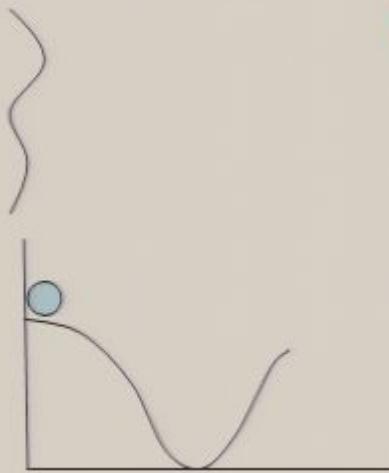


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$\sigma(t, \vec{x})$

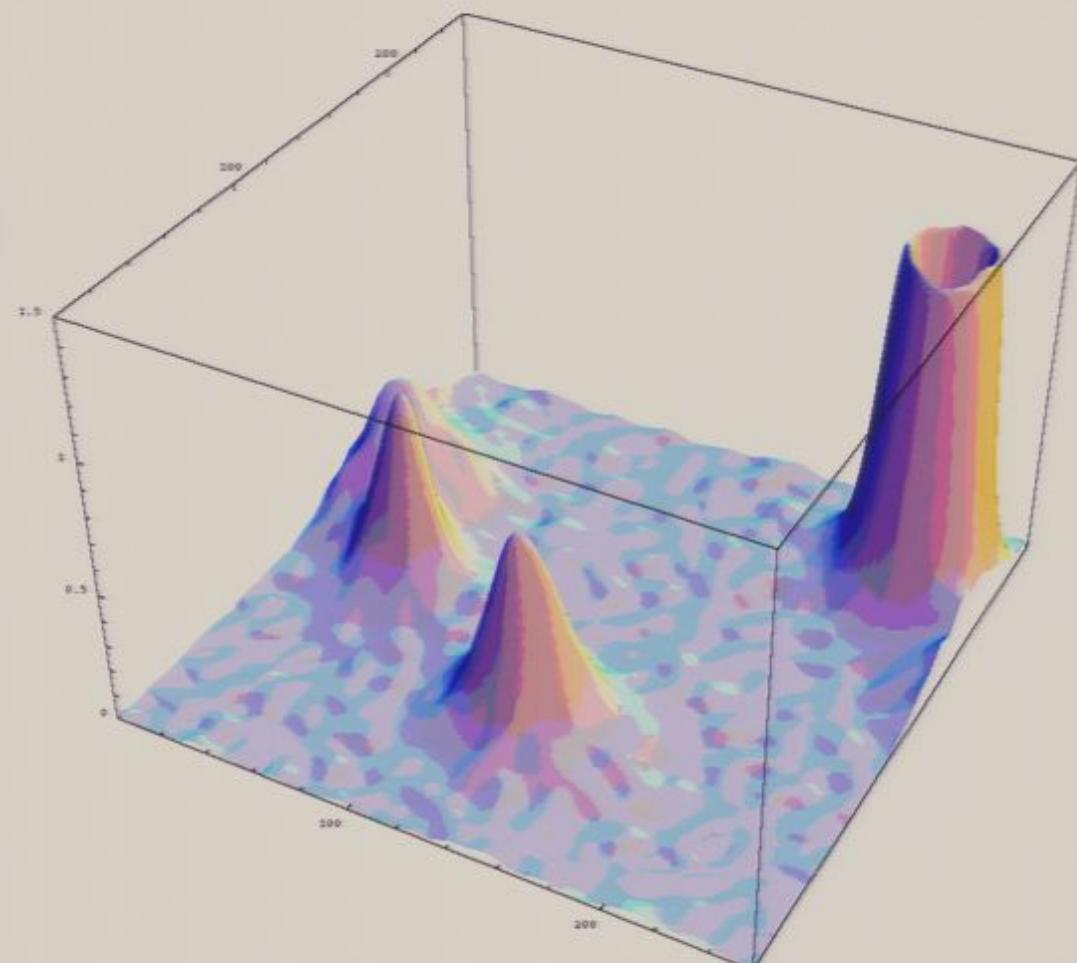


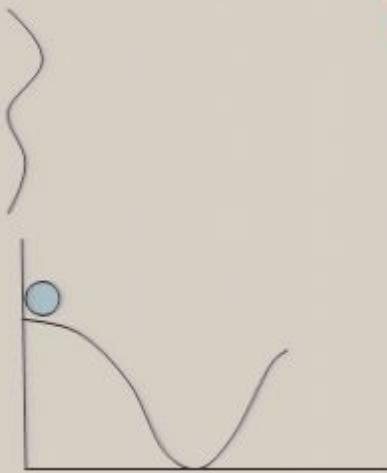


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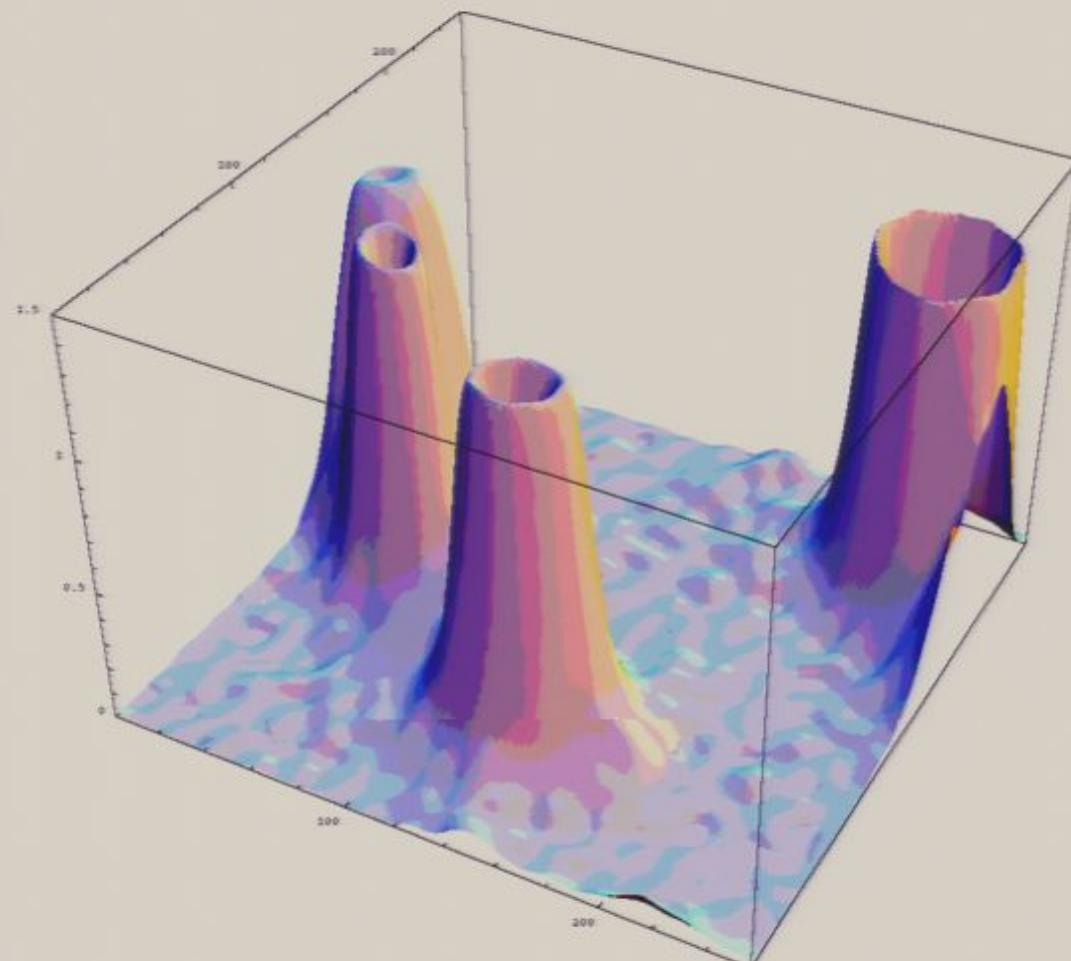




## Tachyonic Preheating

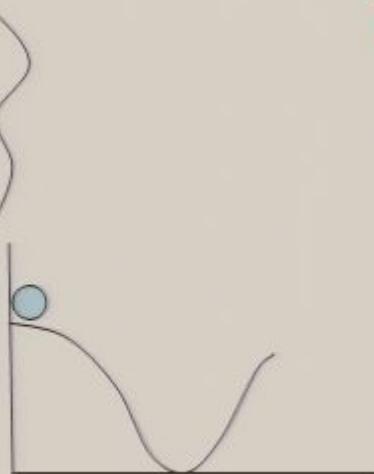
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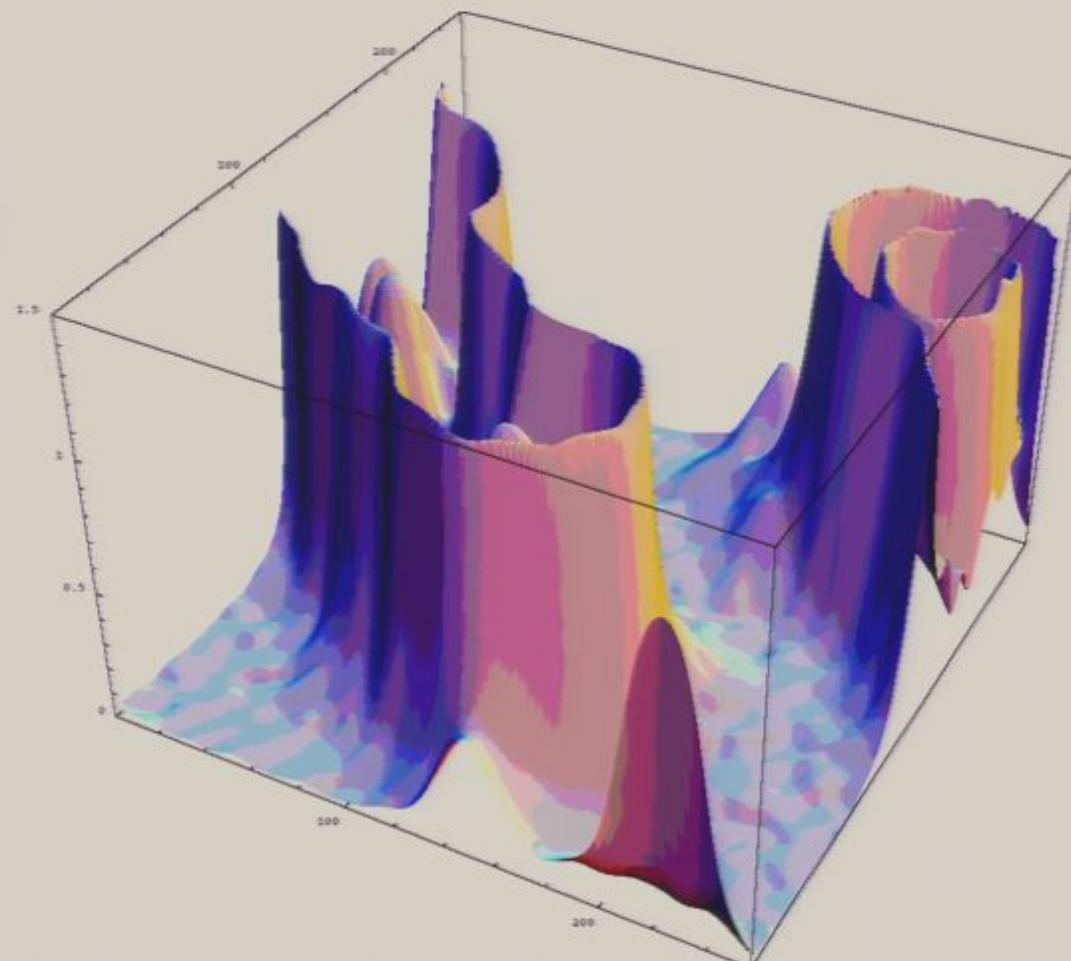


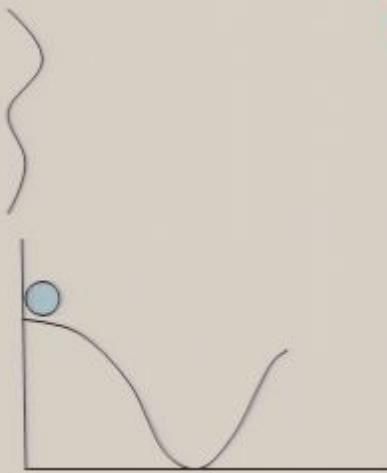
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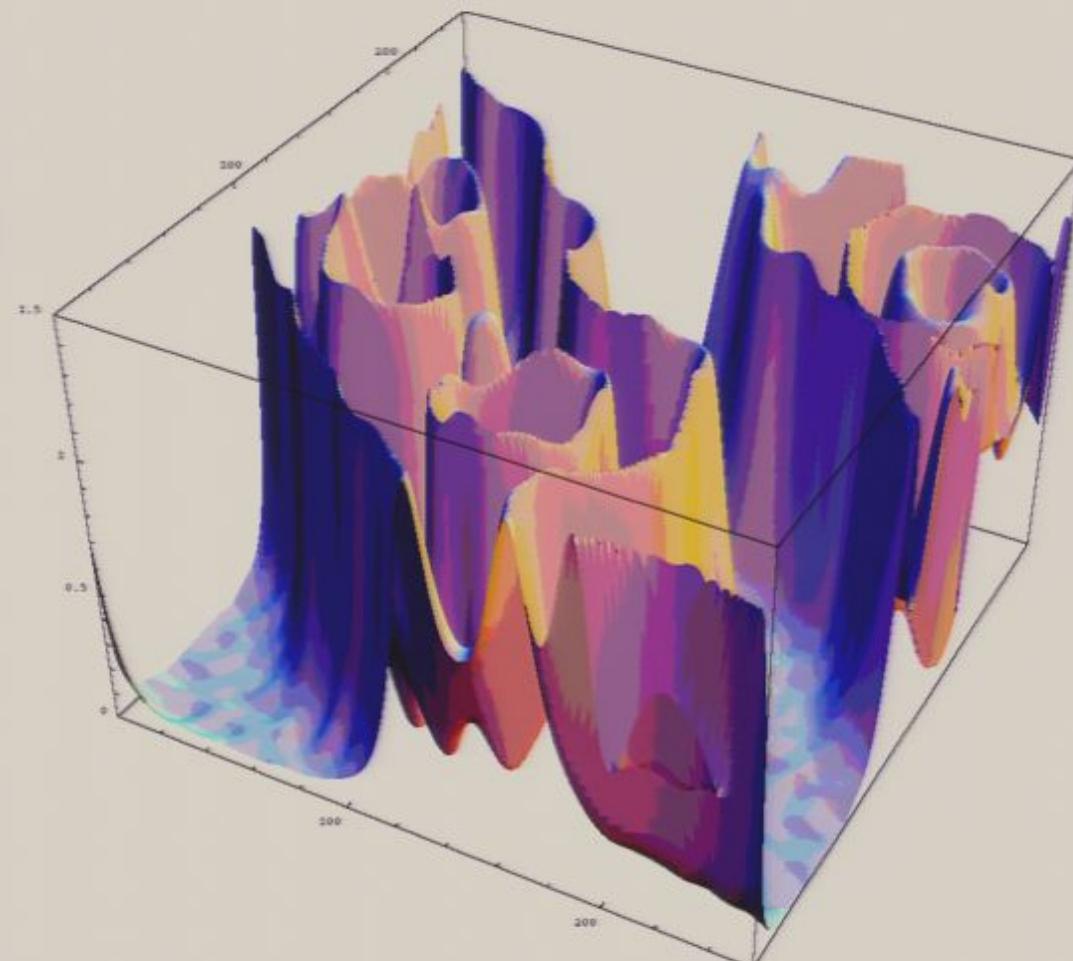


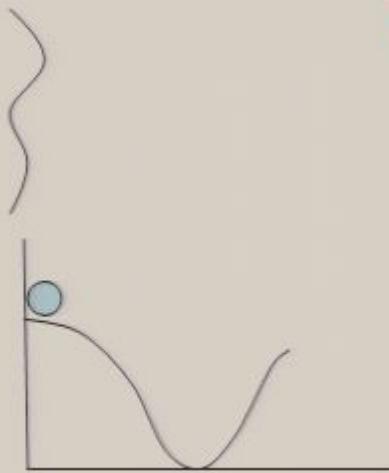


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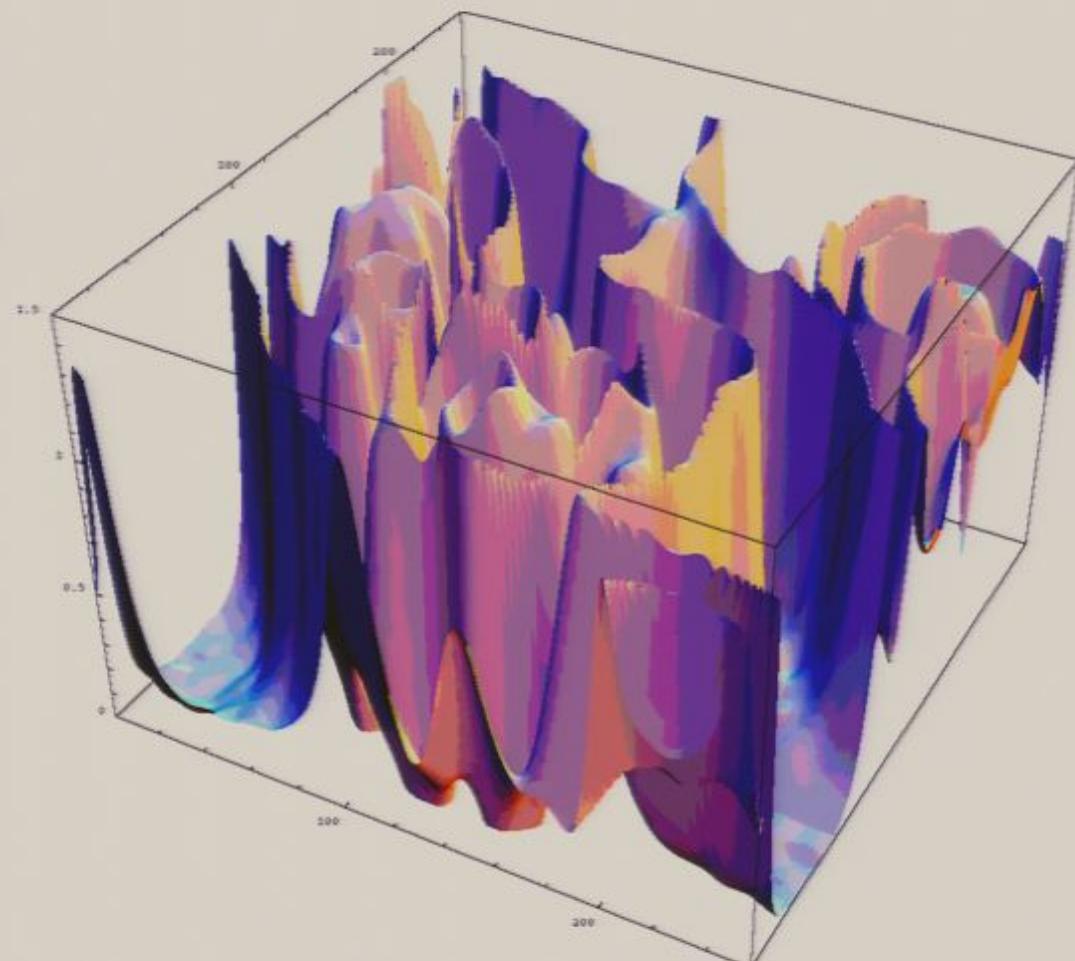


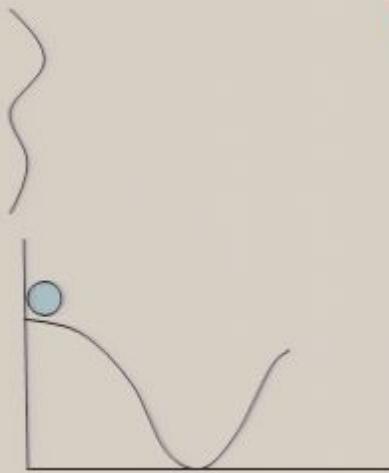


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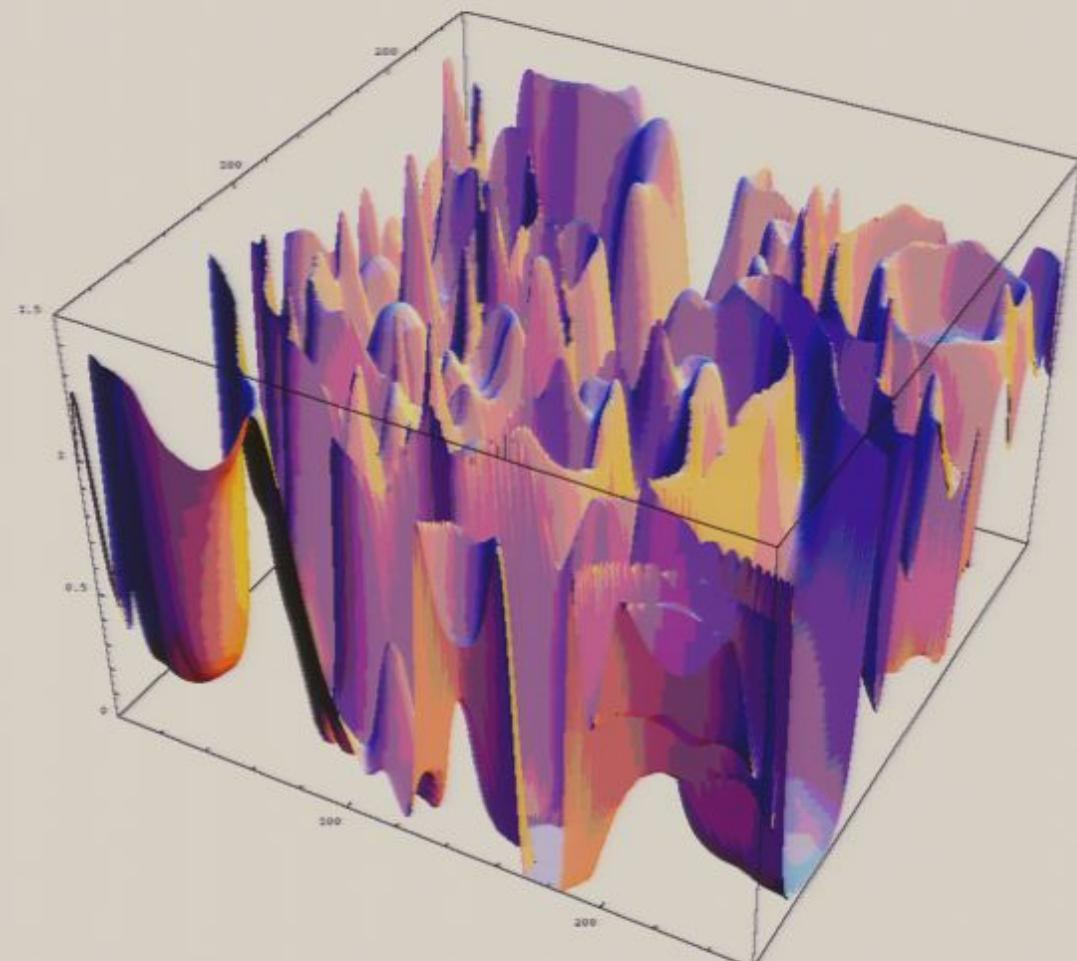


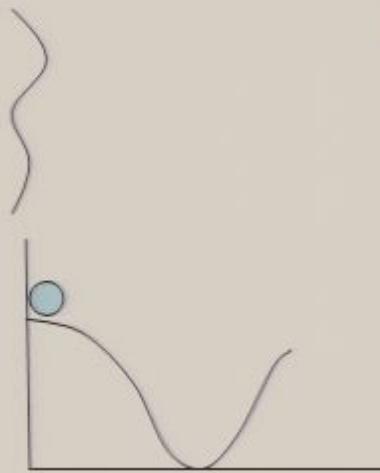


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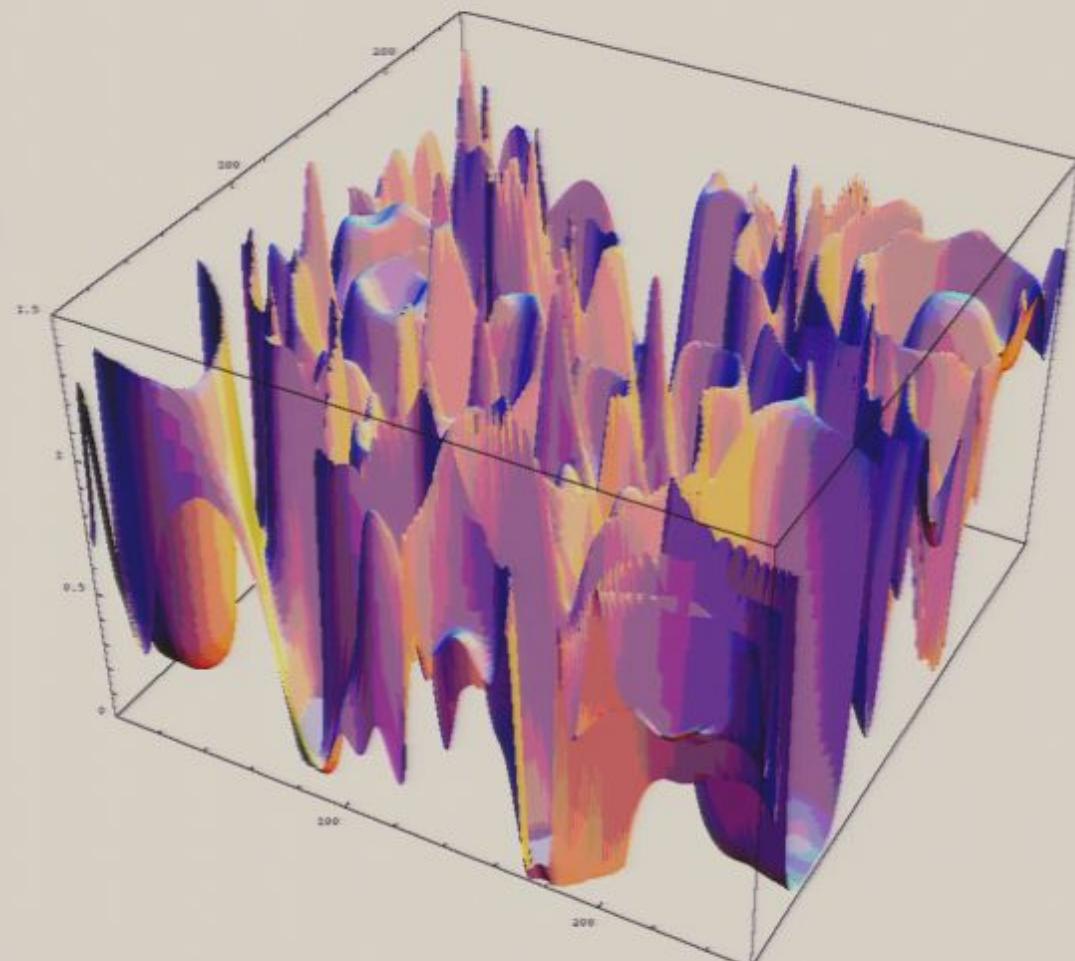




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$$V_F = V_0 + \frac{\lambda}{4}\sigma^4 - \frac{\lambda^3}{4}\sigma^3 + \lambda\sigma^2$$

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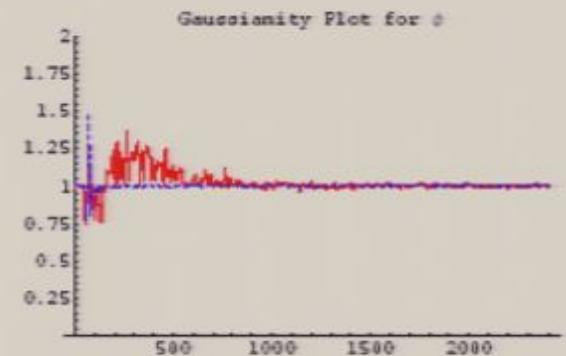
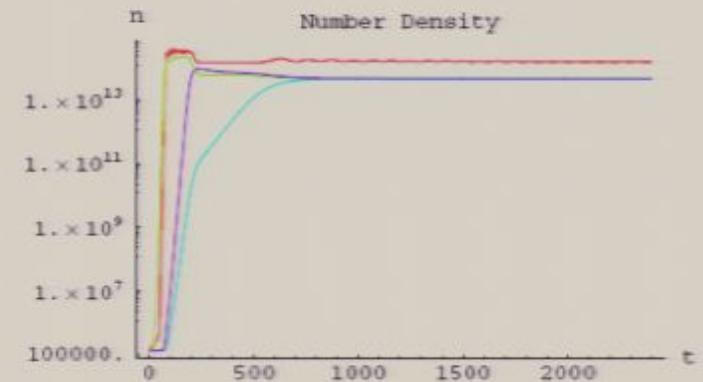
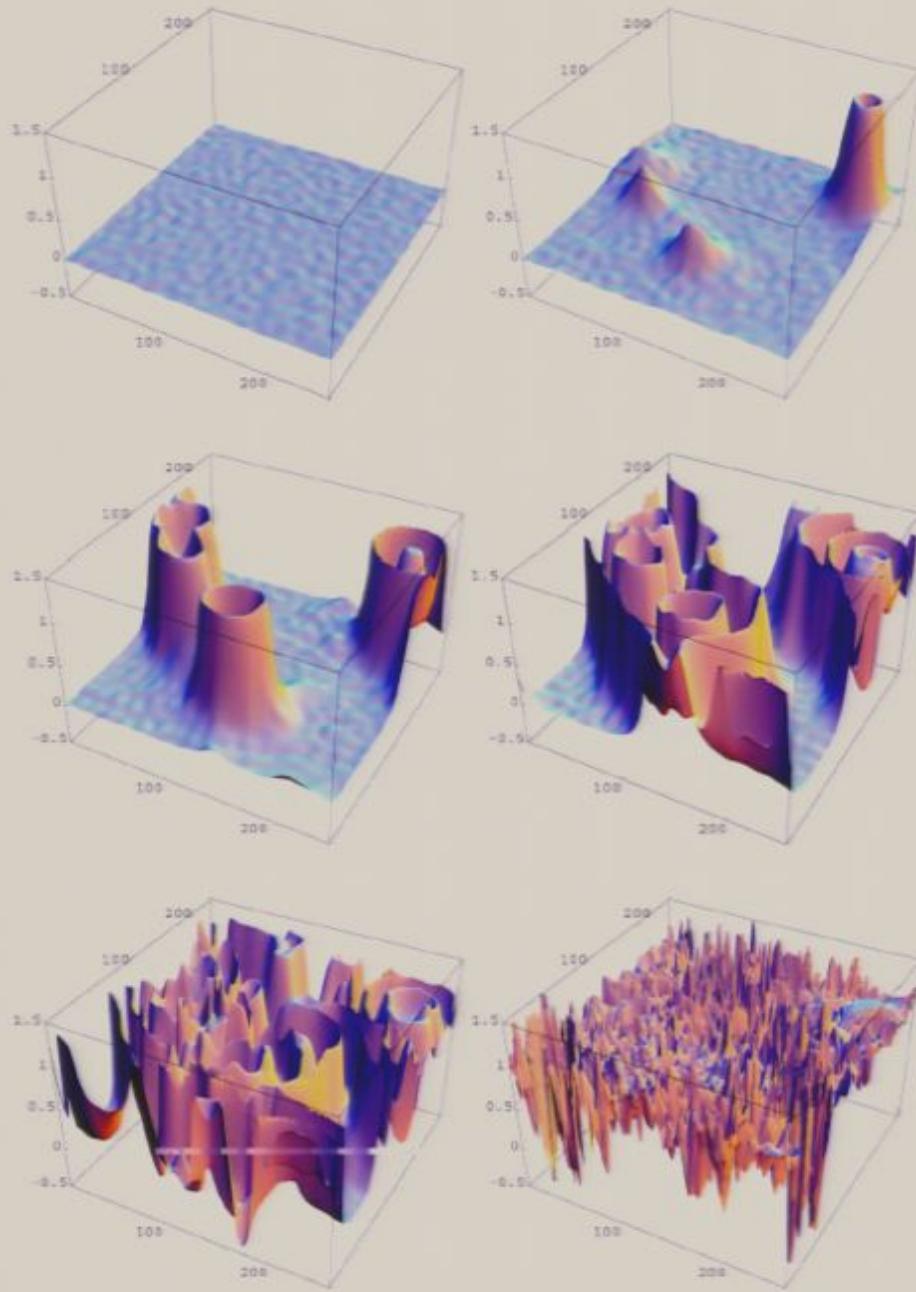


FIG. 10. Deviations from Gaussianity for the field  $\phi$  as a function of time. The solid, red line shows  $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$  and the dashed, blue line shows  $3\langle\delta\phi^2\rangle^2/\langle\delta\phi^4\rangle$ . Page 52/112

## Probing preheating with stochastic background gravitational radiation

$$\Omega_{gw} h^2 = \Omega_\tau h^2 \frac{d\rho_{gw}(a_e)}{d\ln\omega} \left(\frac{g_0}{g_*}\right)^{1/3}$$

### estimation

$$\frac{\rho_{gw}}{\rho_r} \sim (RH)^2$$

$$f \sim \frac{M}{10^{15} GeV} 10^8 \text{ Hz}$$

### numerics

$$\frac{dE}{d\Omega} = 2G\Lambda_{ij,lm}\omega^2 T^{ij*}(\vec{k},\omega)T^{lm}(\vec{k},\omega)d\omega$$



$$\begin{aligned} \Lambda_{ij,lm}(\hat{k}) = & \delta_{ij}\delta_{lm} - 2\hat{k}_j\hat{k}_m\delta_{il} + \frac{1}{2}\hat{k}_i\hat{k}_j\hat{k}_l\hat{k}_m \\ & - \frac{1}{2}\delta_{ij}\delta_{lm} + \frac{1}{2}\delta_{ij}\hat{k}_l\hat{k}_m + \frac{1}{2}\delta_{jl}\hat{k}_i\hat{k}_m \end{aligned}$$

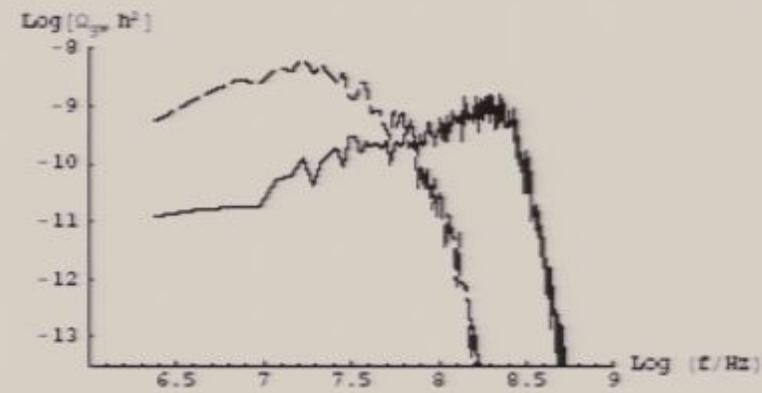
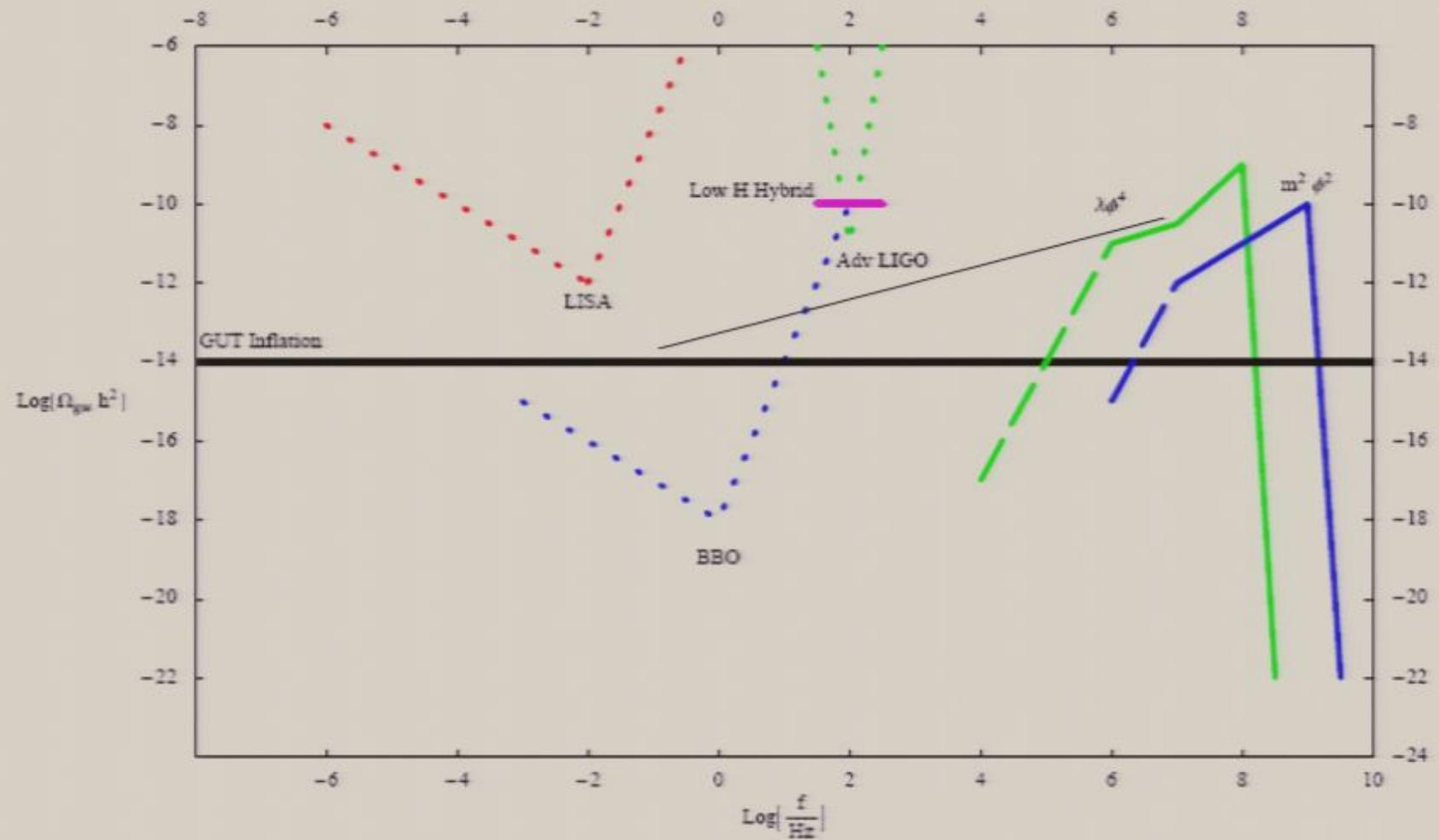


FIG. 1: The gravitational spectrum for the  $\lambda\phi^4$  model with  $\lambda = 10^{-14}$  and  $g^2/\lambda = 1.2$  (dash line) and 120 (full line) respectively. As expected, it is peaked around  $10^7 \sim 10^8$  Hz and spans about 2 decades. The horizon size at the time of preheating imposes the low frequency cut-off, while the high frequency cut-off is due to the fact that high momentum  $\chi$  particles are energetically too expensive to be created. Notice that the peak frequency is higher for the larger value of  $g^2/\lambda$ .



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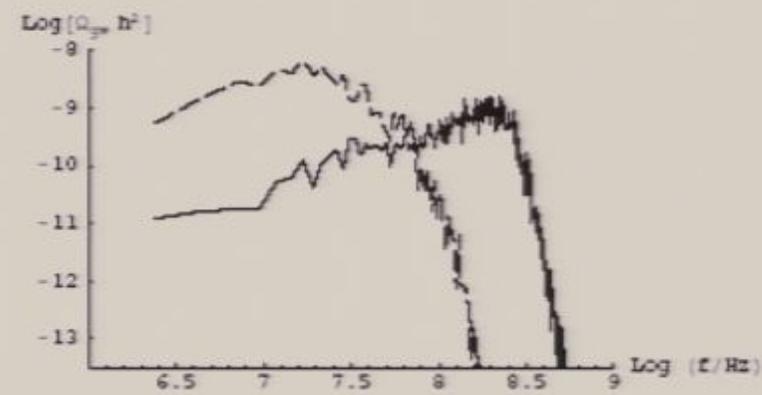


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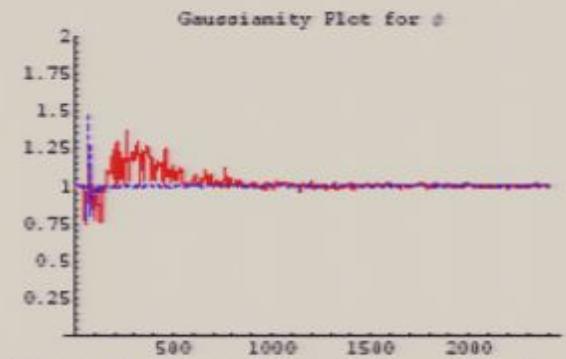
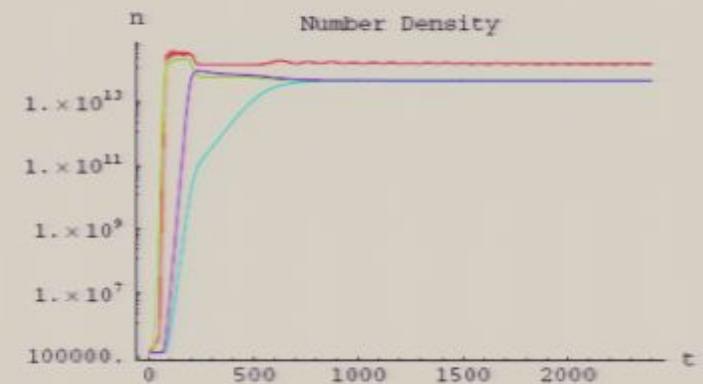
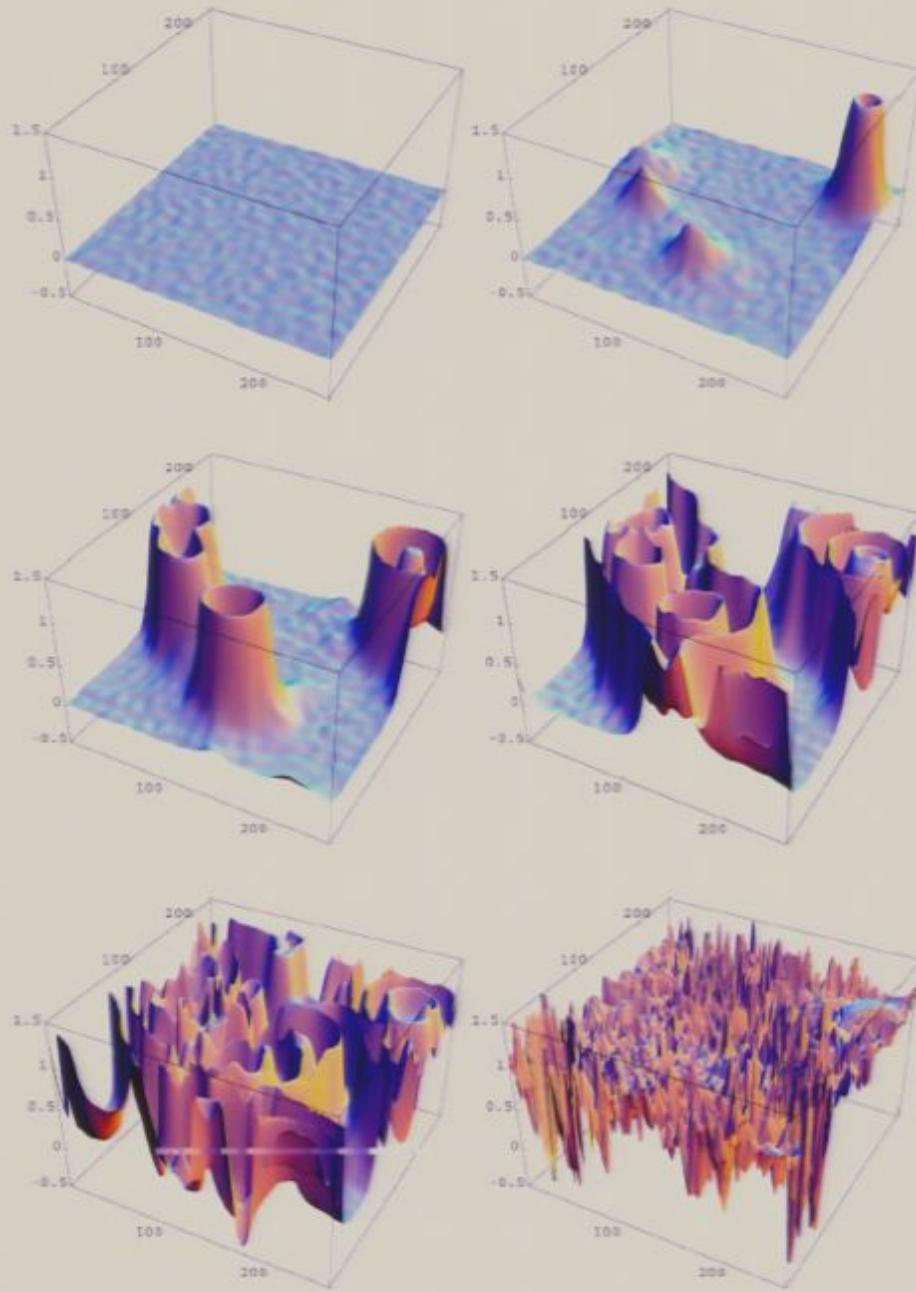
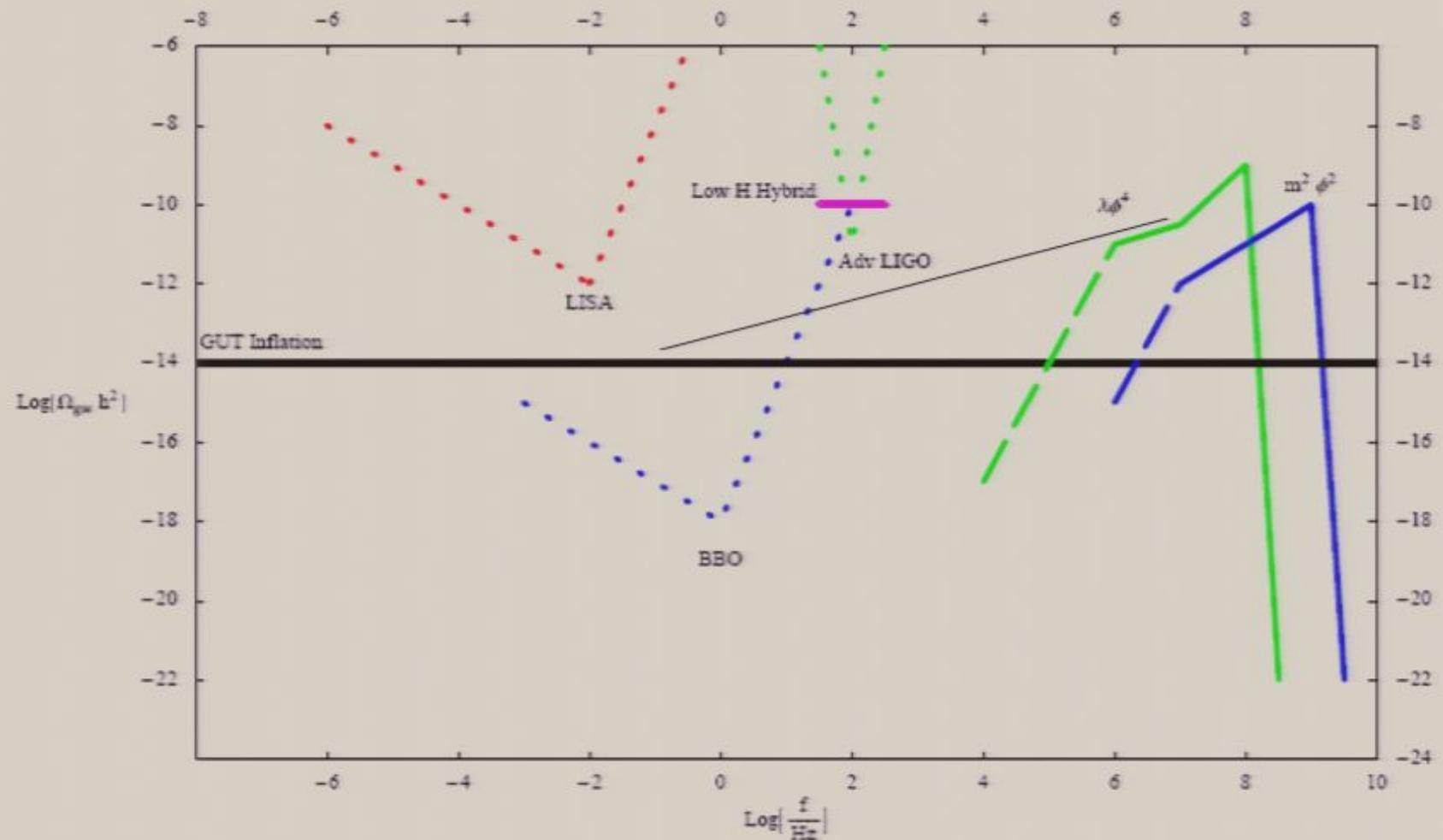


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## Three-linear interaction

In expanding universe complete inflaton decay requires 3-legs interactions

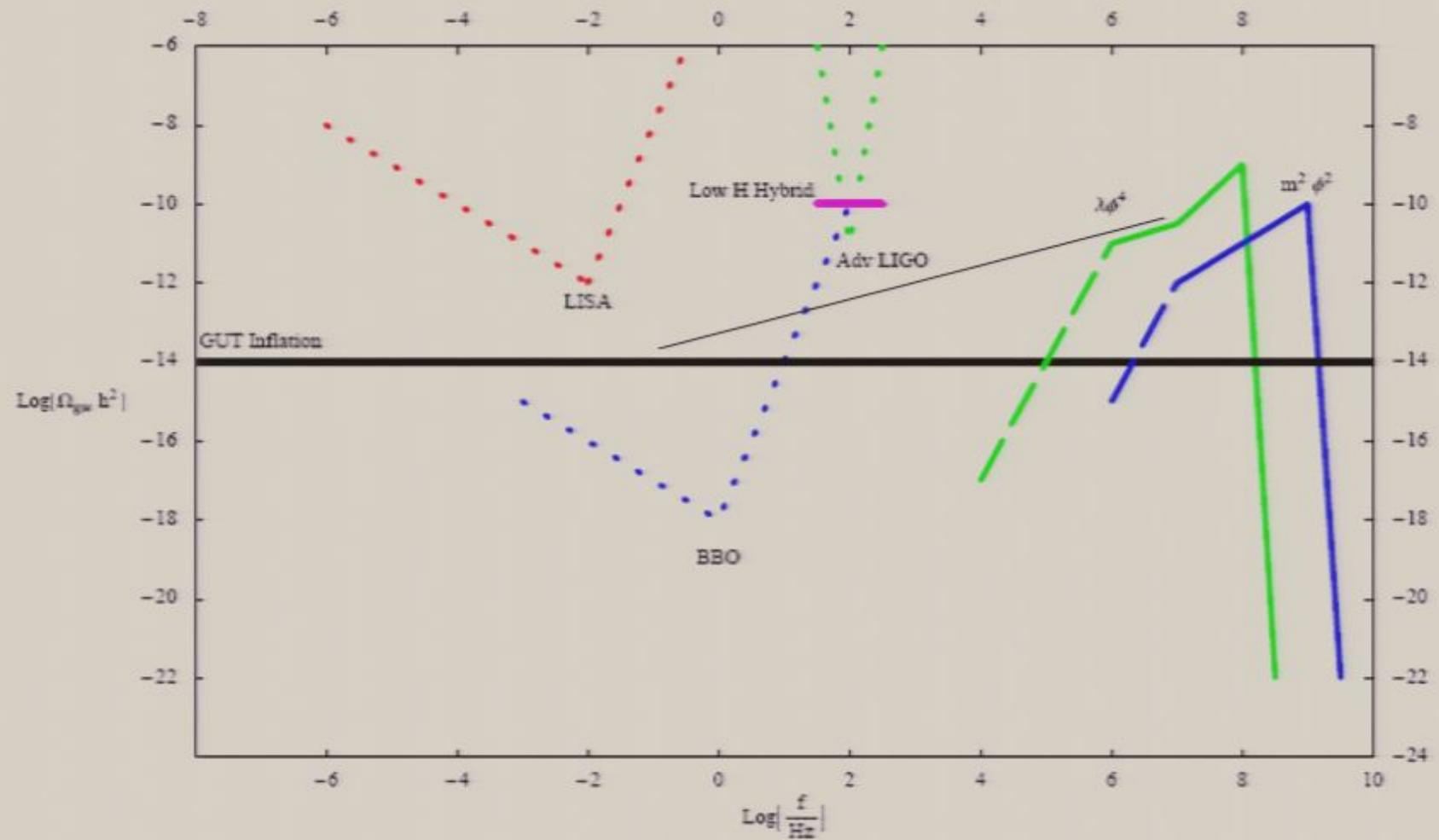
$$V = \frac{m^2}{2} \phi^2 + \frac{\sigma}{2} \phi \chi^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{\lambda}{4} \chi^4 \iff W = \frac{m}{2\sqrt{2}} \phi^2 + \frac{g}{2\sqrt{2}} \phi \chi^2$$
$$\lambda = g^2/2 \text{ and } \sigma = gm$$

$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0$$

$$mt = 2z - \frac{\pi}{2}, A_k = \frac{4k^2}{m^2} \text{ and } q = \frac{2\sigma\Phi}{m^2}$$

$A_k \geq 2q$       **Broad Parametric Resonance**

$0 < A_k < 2q$       **Tachyonic Resonance**      Dufaux et al 06



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In expanding universe complete inflaton decay requires 3-legs interactions

$$V = \frac{m^2}{2} \phi^2 + \frac{\sigma}{2} \phi \chi^2 + \frac{g^2}{2} \phi^2 \chi^2 + \frac{\lambda}{4} \chi^4 \iff W = \frac{m}{2\sqrt{2}} \phi^2 + \frac{g}{2\sqrt{2}} \phi \chi^2$$

$\lambda = g^2/2$  and  $\sigma = gm$

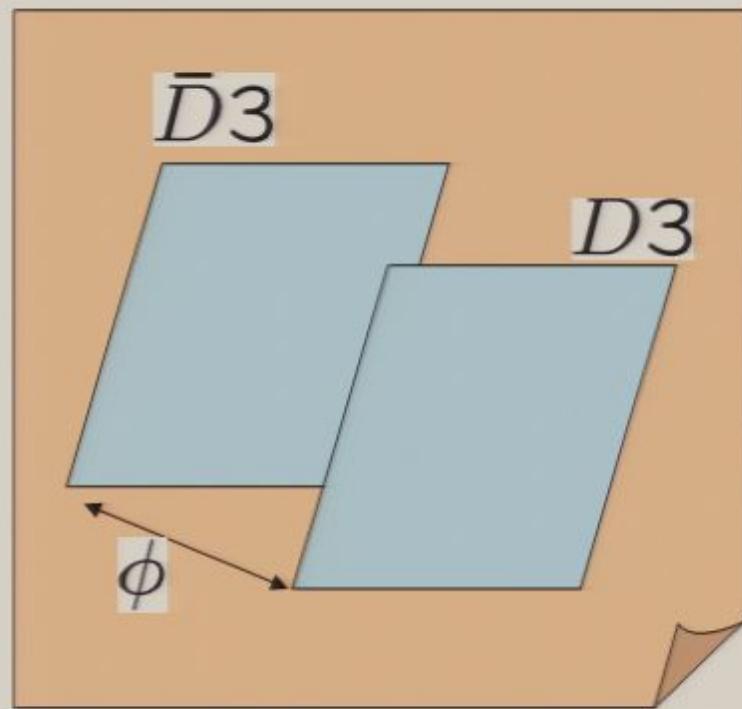
$$\chi_k'' + (A_k - 2q \cos 2z) \chi_k = 0$$

$$mt = 2z - \frac{\pi}{2}, A_k = \frac{4k^2}{m^2} \text{ and } q = \frac{2\sigma\Phi}{m^2}$$

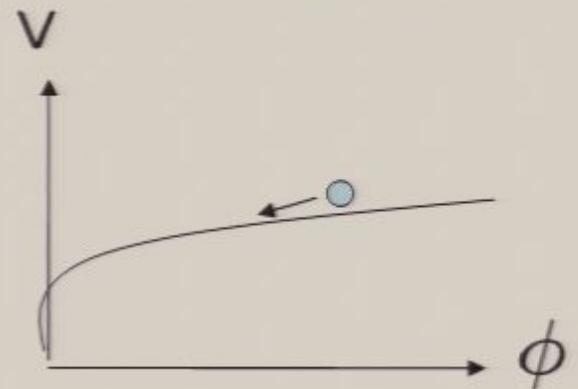
$A_k \geq 2q$       **Broad Parametric Resonance**

$0 < A_k < 2q$       **Tachyonic Resonance**      Dufaux et al 06

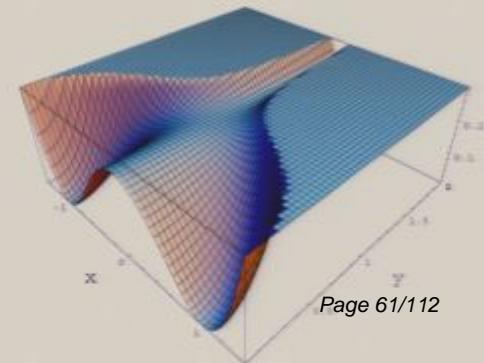
## Inflaton with branes in String Theory



4-dim picture

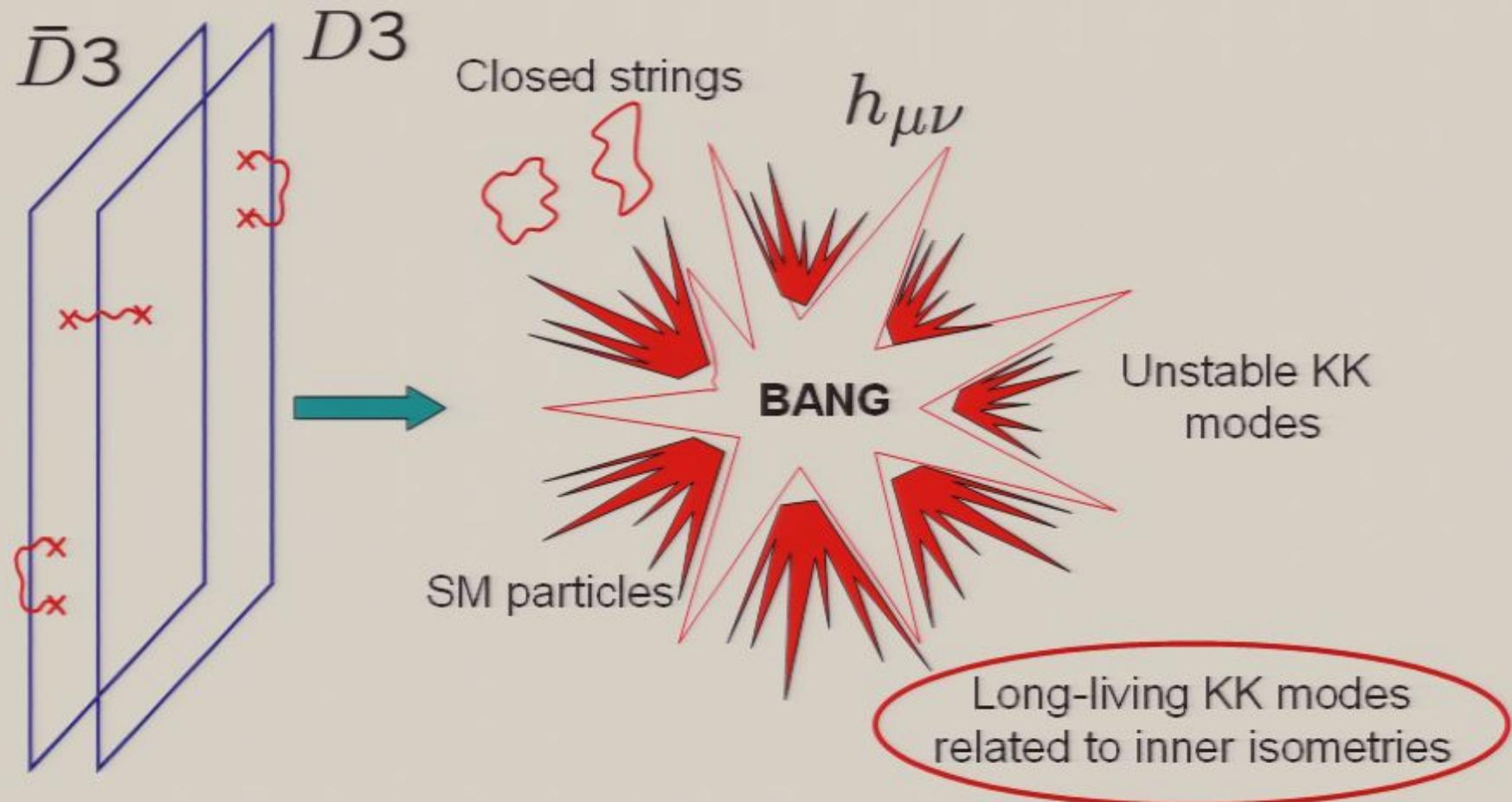


Prototype of  
hybrid inflation



## End point of inflation

LK, Yi 05



Open strings

between branes are unstable

## Cascading Energy from Inflaton to Radiation

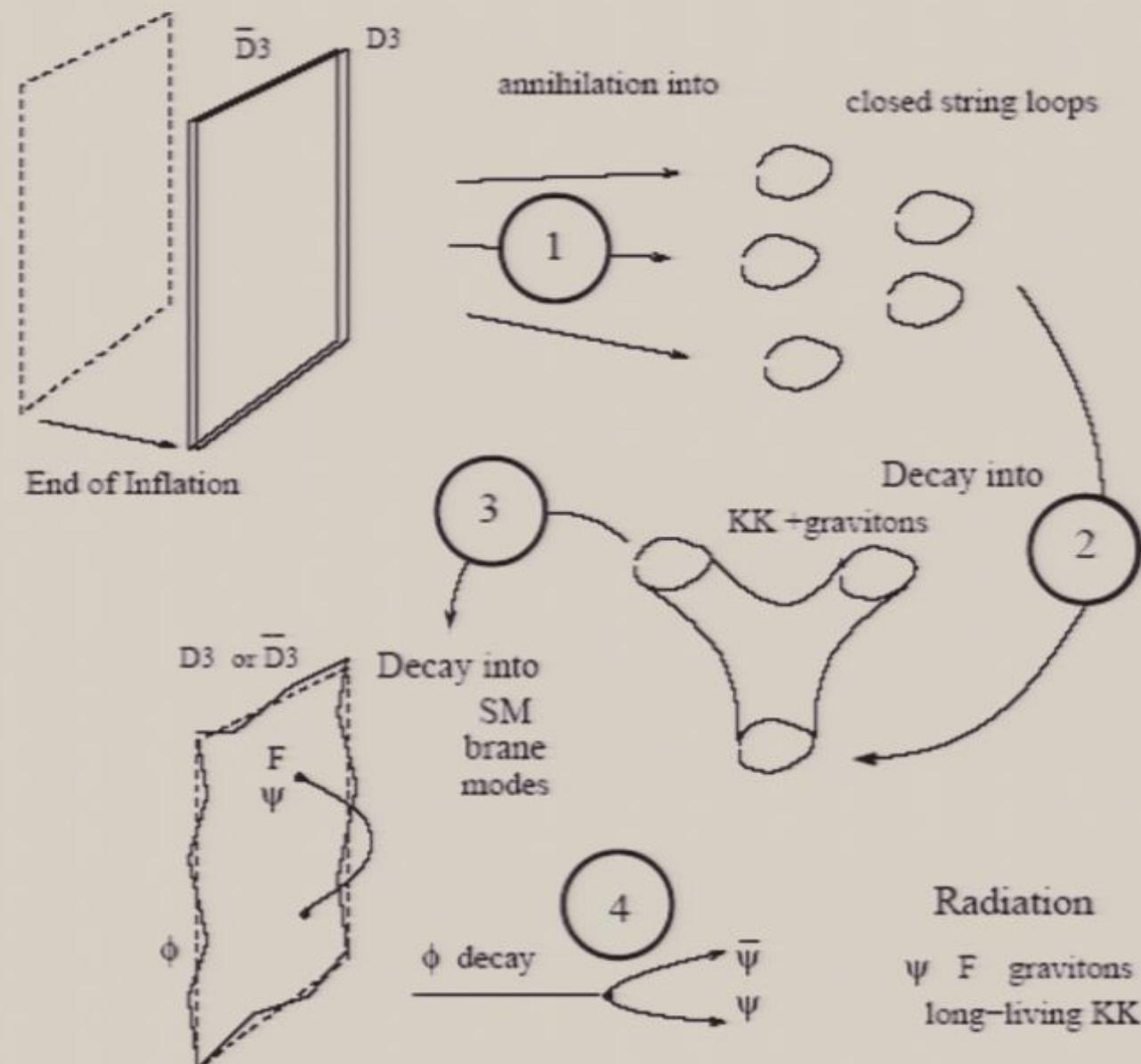
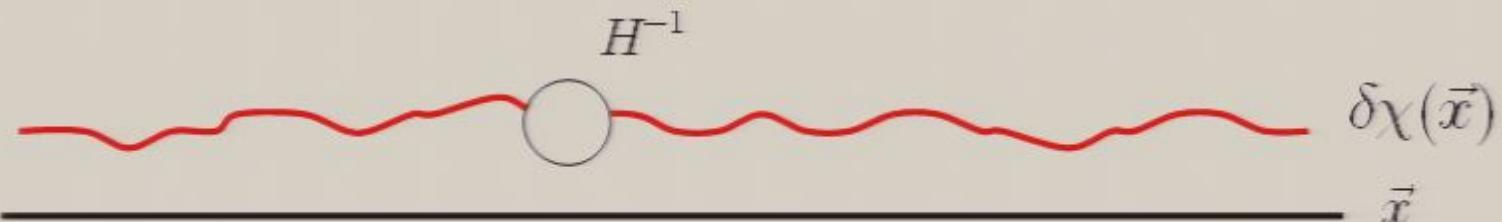


Figure 2: Identifying the channels of D-brane decay



## Light field at inflation

$$\delta\chi = \int d^3k (a_k \chi_k(t) e^{i\vec{k}\vec{x}} + h.c.)$$



Scalar metric Fluctuations from Inflation

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\vec{x}^2$$

**Initial conditions from Inflation**  $\rightarrow$



Random Gaussian Field  $\Phi(\vec{x})$

$$a = e^{Ht}$$

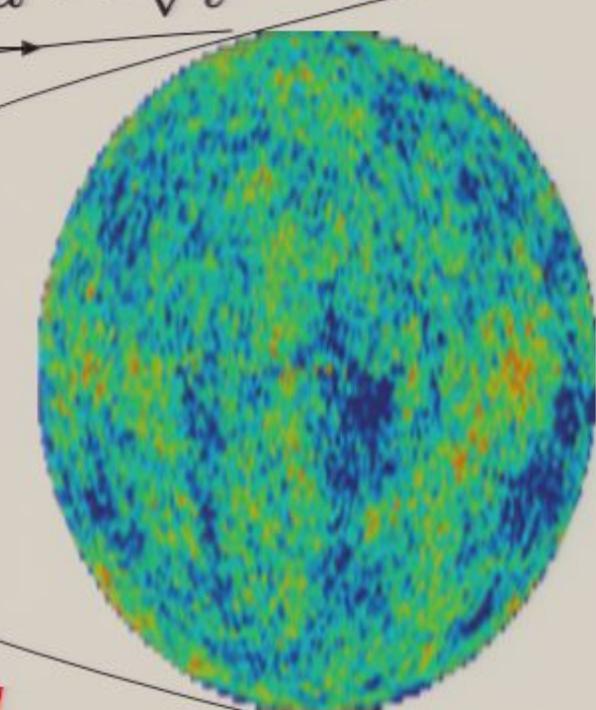
$$\Omega_{tot} = 1$$

$$k^3 \Phi_k^2 \rightarrow P_s = A_s k^{n_s - 1}$$

$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$

$$N = 62 - \ln \frac{10^{16} GeV}{V_{inf}^{1/4}}$$

$$a = \sqrt{t}$$



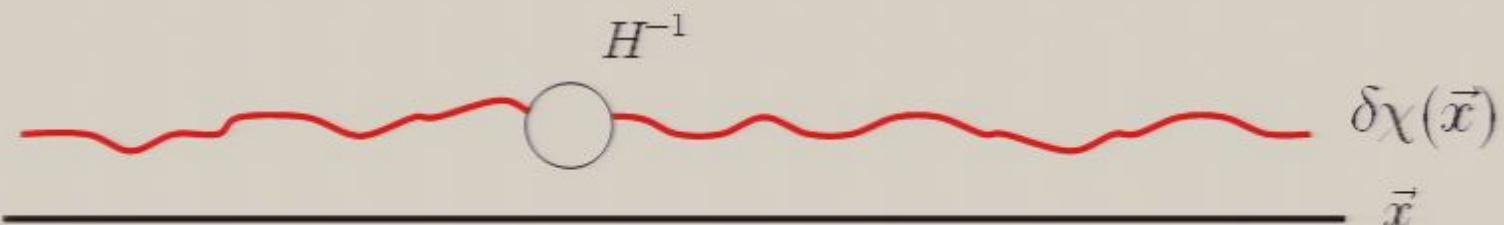
**inflation**



**Hot FRW**

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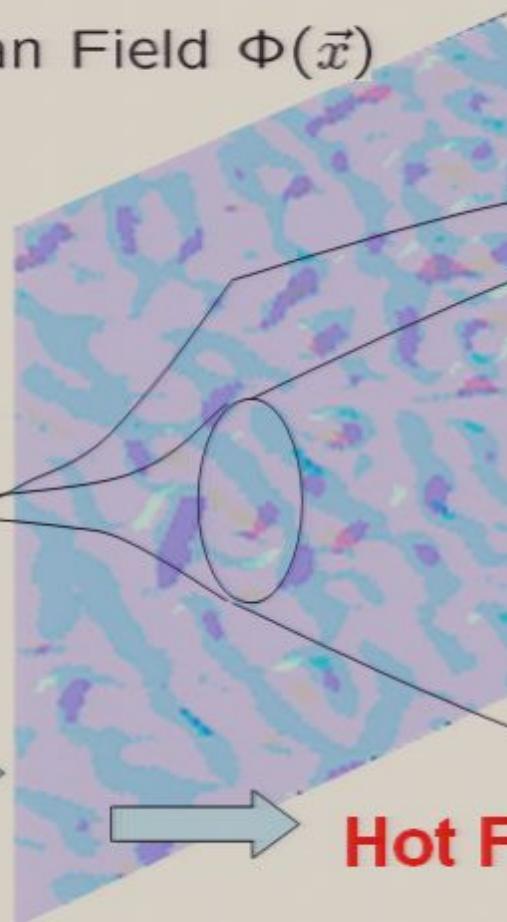
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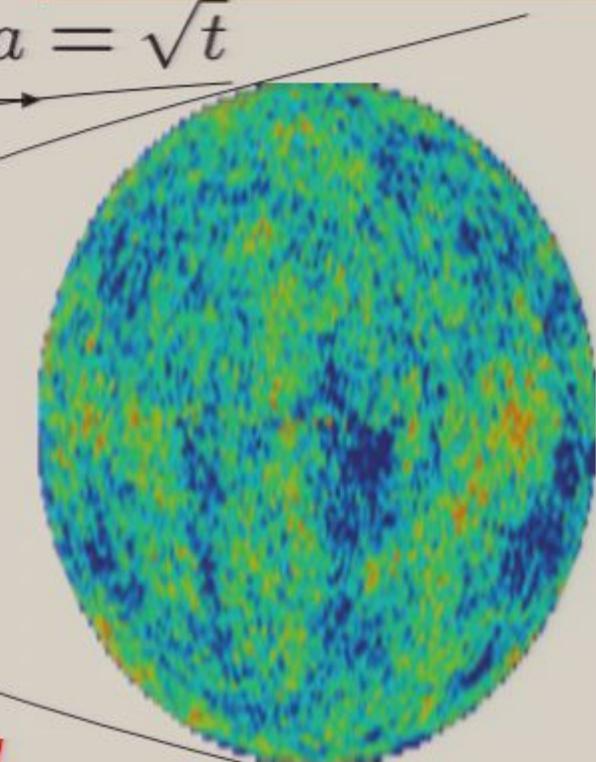
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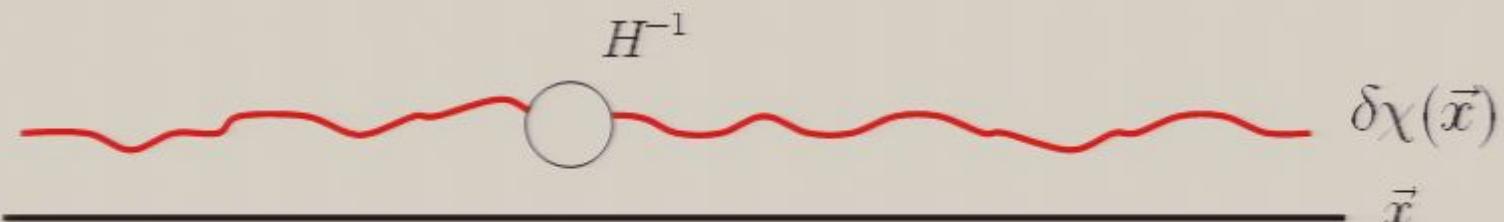


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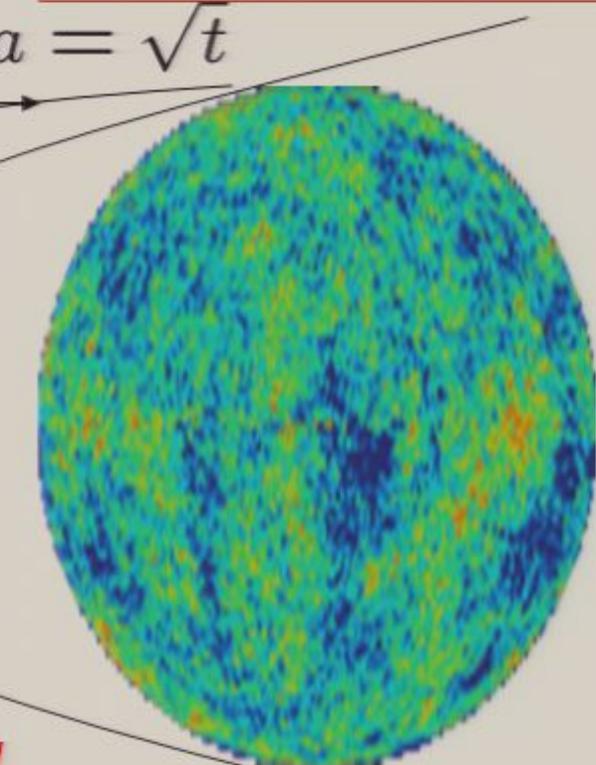
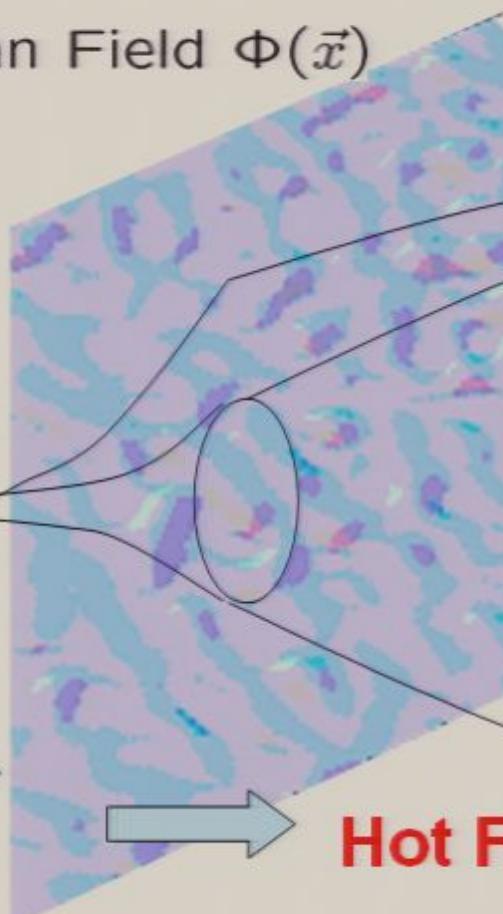
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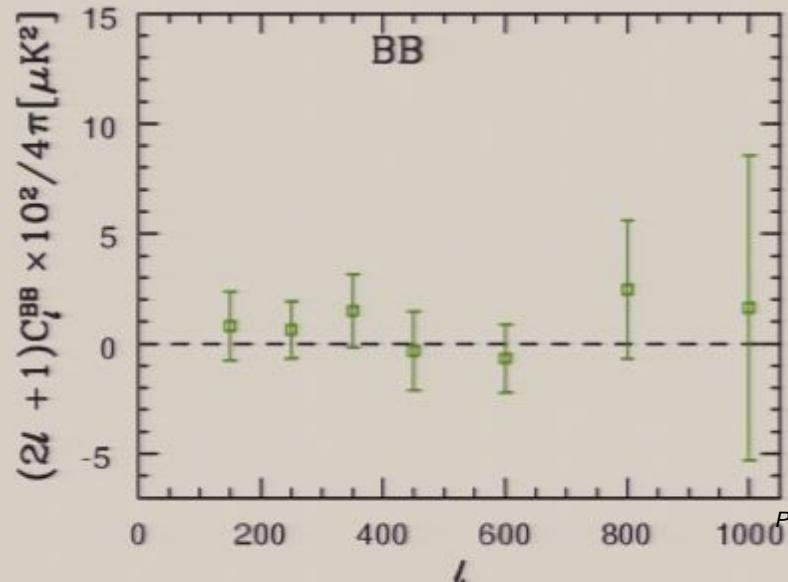
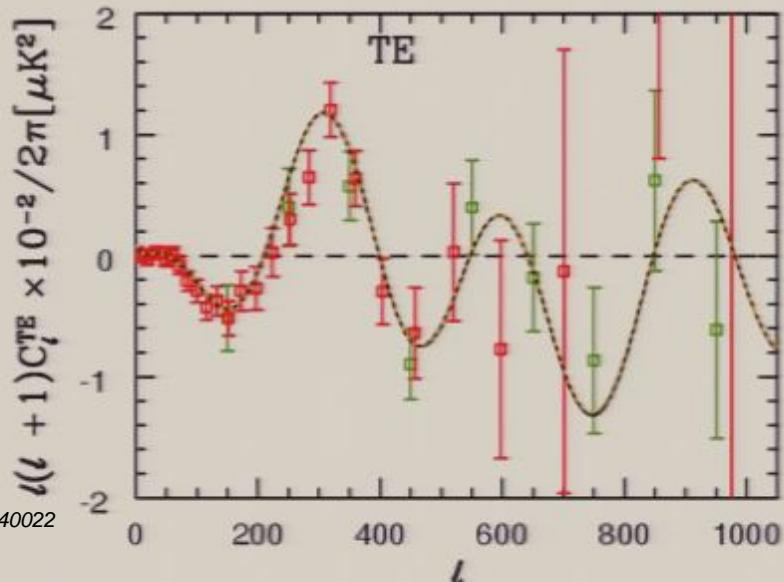
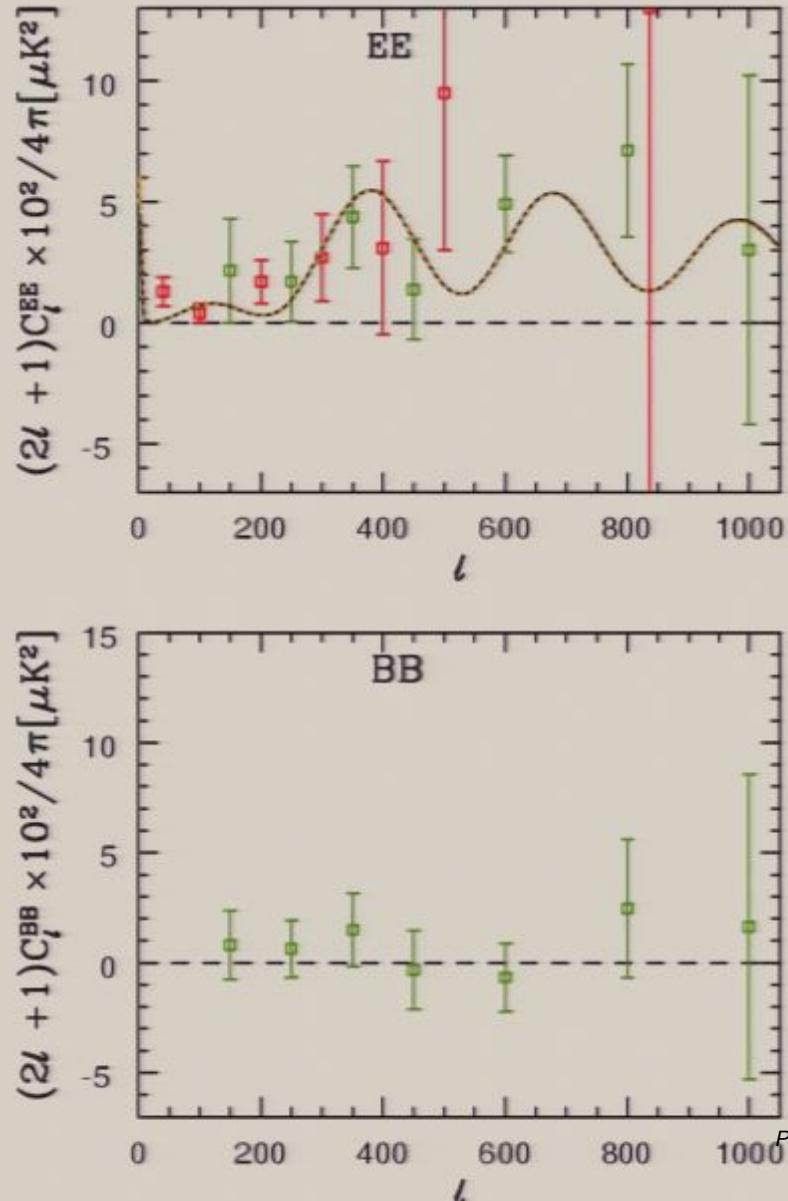
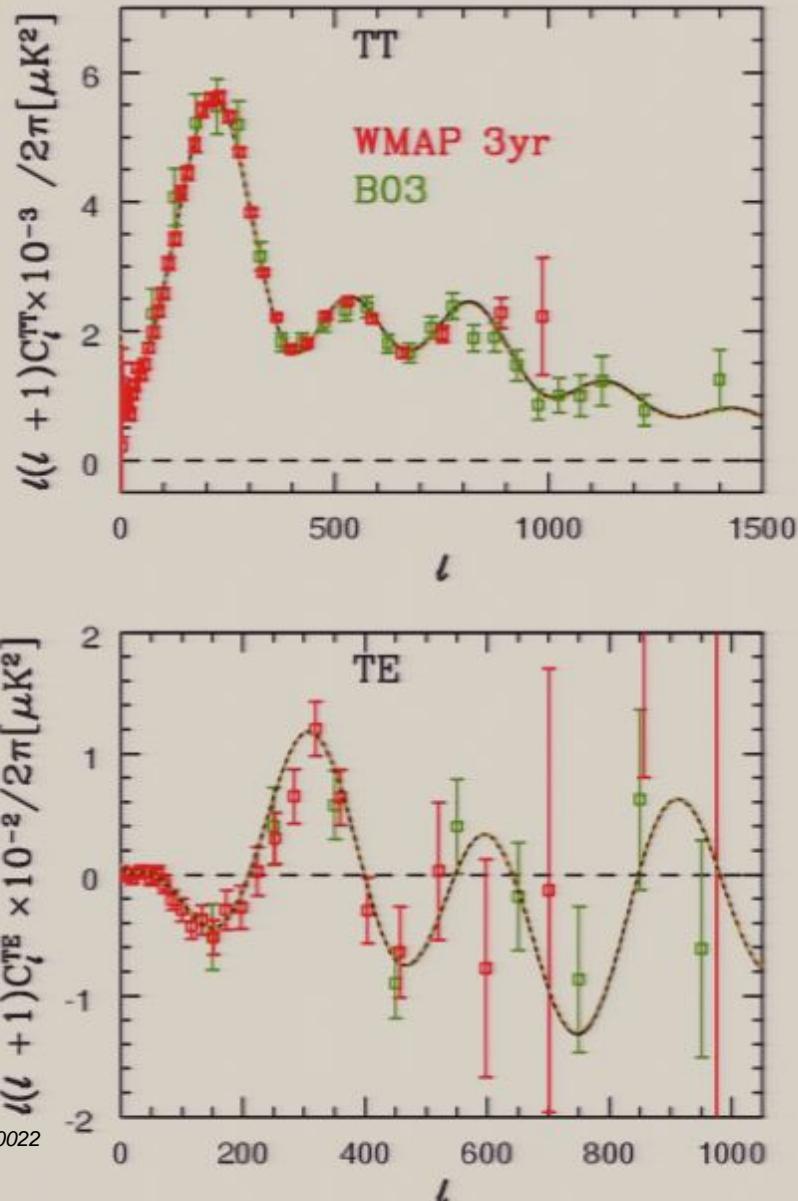
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**Hot FRW**



# WMAP3 sees 3<sup>rd</sup> pk, B03 sees 4<sup>th</sup>



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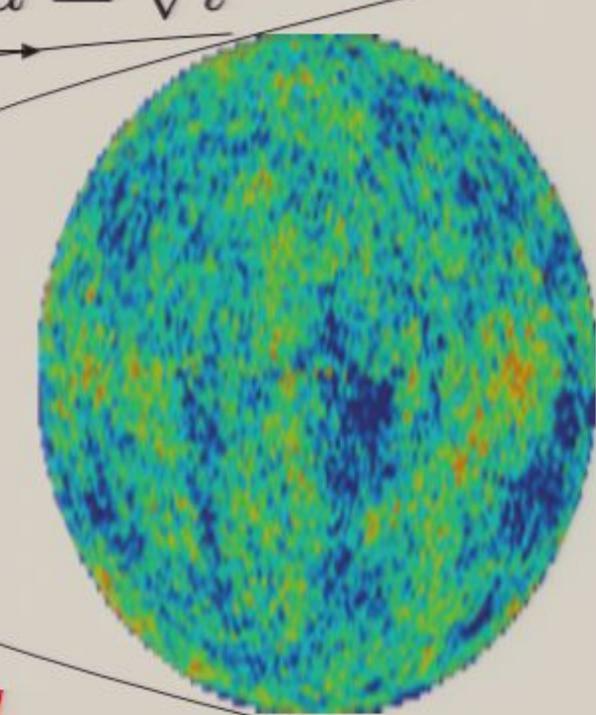
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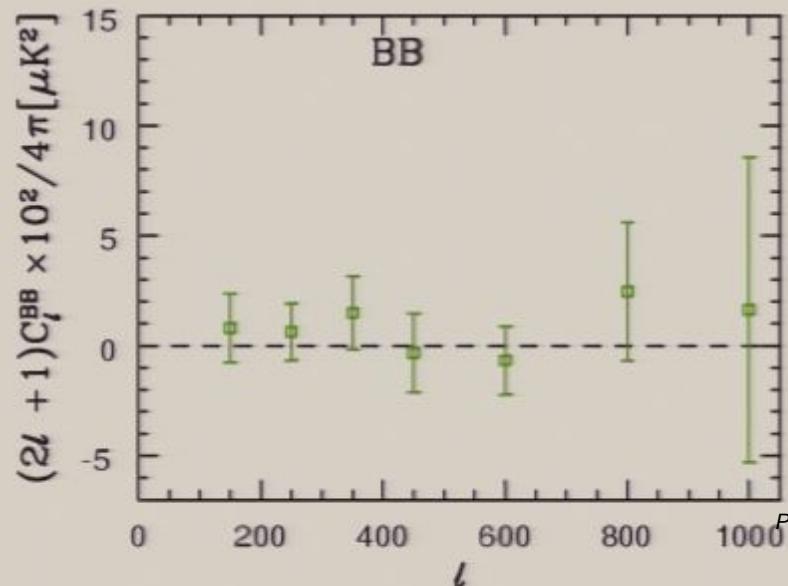
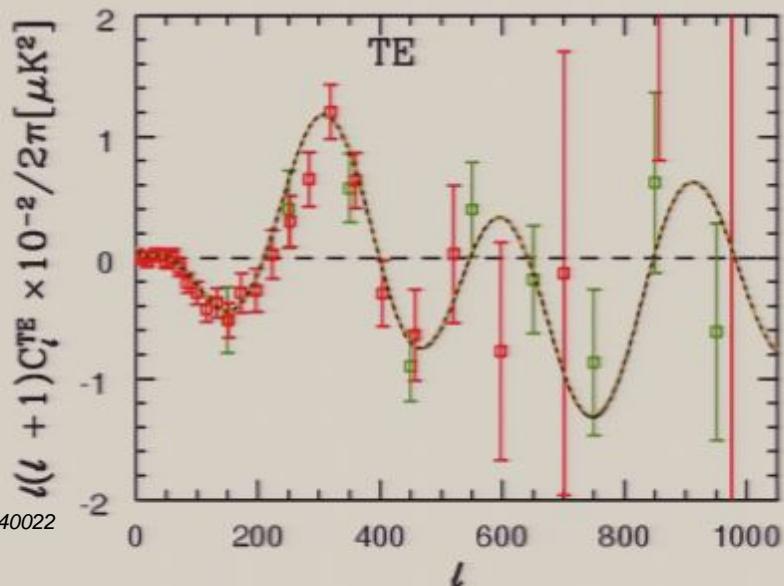
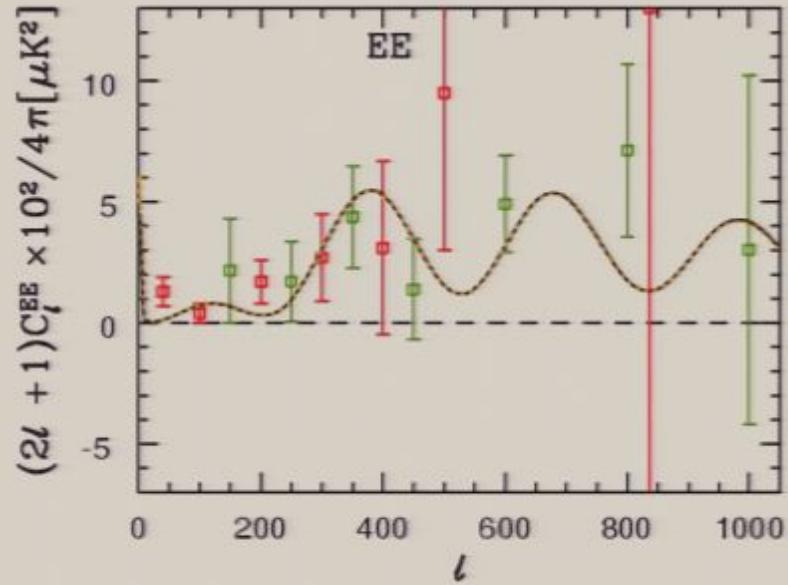
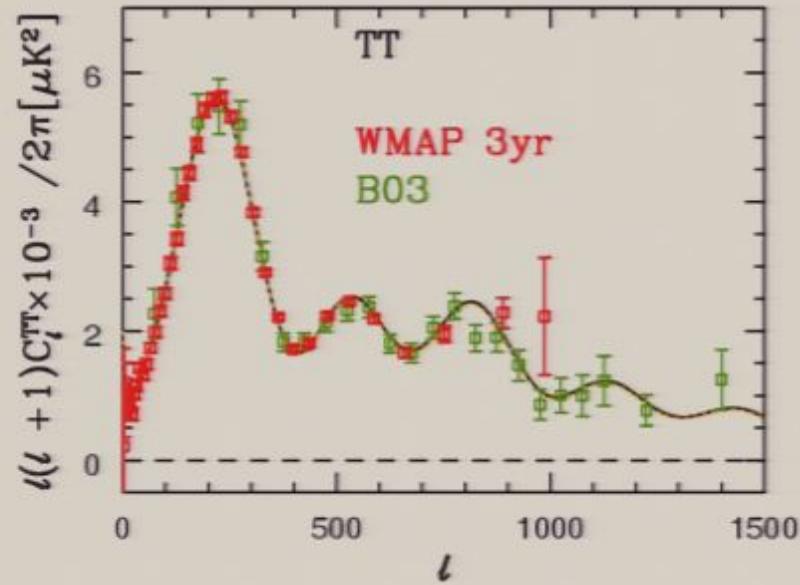


**inflation**

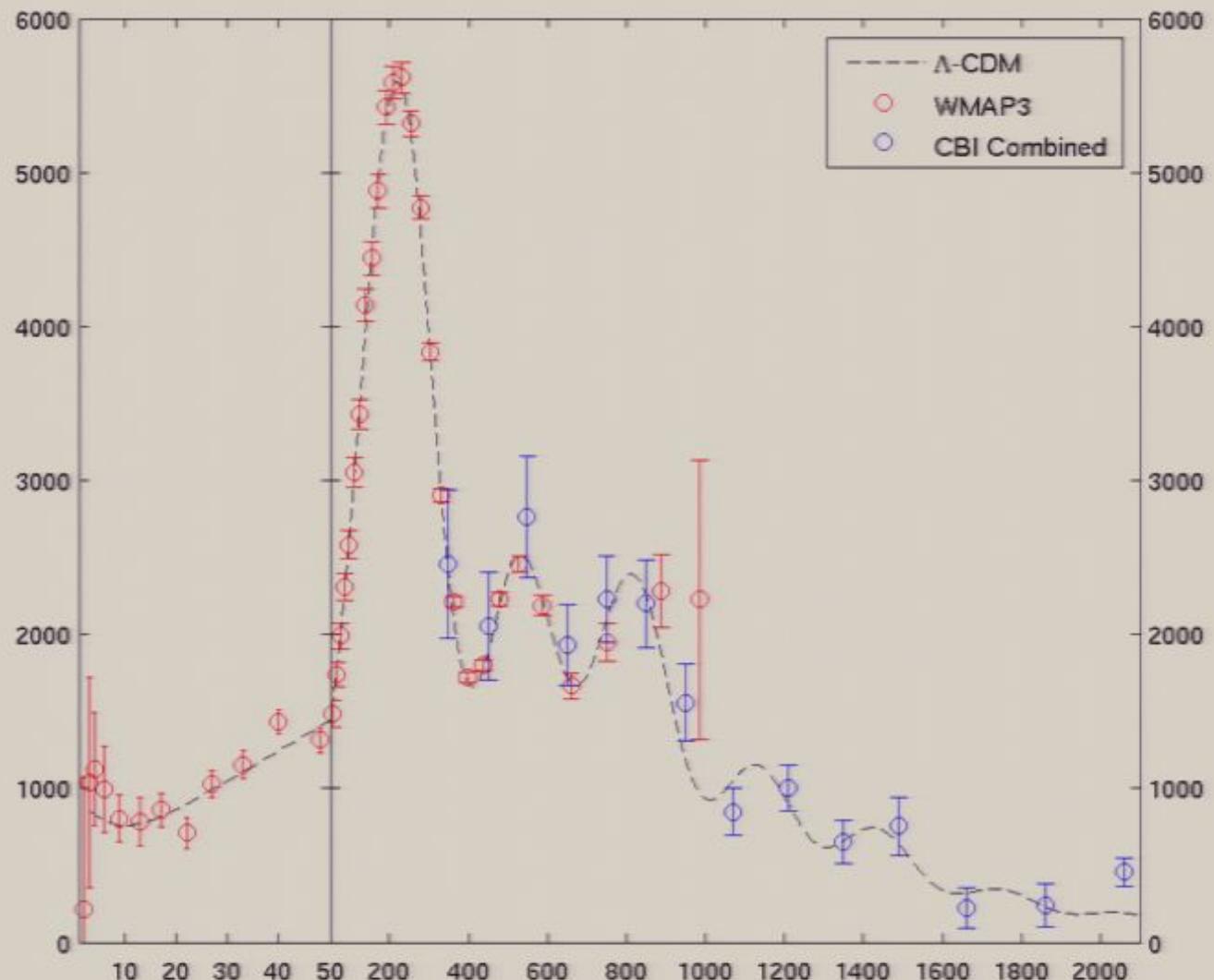


**Hot FRW**

# WMAP3 sees 3<sup>rd</sup> pk, B03 sees 4<sup>th</sup>



# CBI combined TT sees 5<sup>th</sup> pk



## Inflation in the context of ever changing fundamental theory

1980

$R^2$ -inflation

Old Inflation

New Inflation

Chaotic inflation

Double Inflation

Power-law inflation

SUGRA inflation

Extended inflation

1990

Hybrid inflation

SUSY F-term  
inflation

Assisted inflation

SUSY D-term  
inflation

Brane inflation

2000

SUSY P-term  
inflation

Super-natural  
Inflation

K-flation

N-flation

$D3 - D7$  inflation

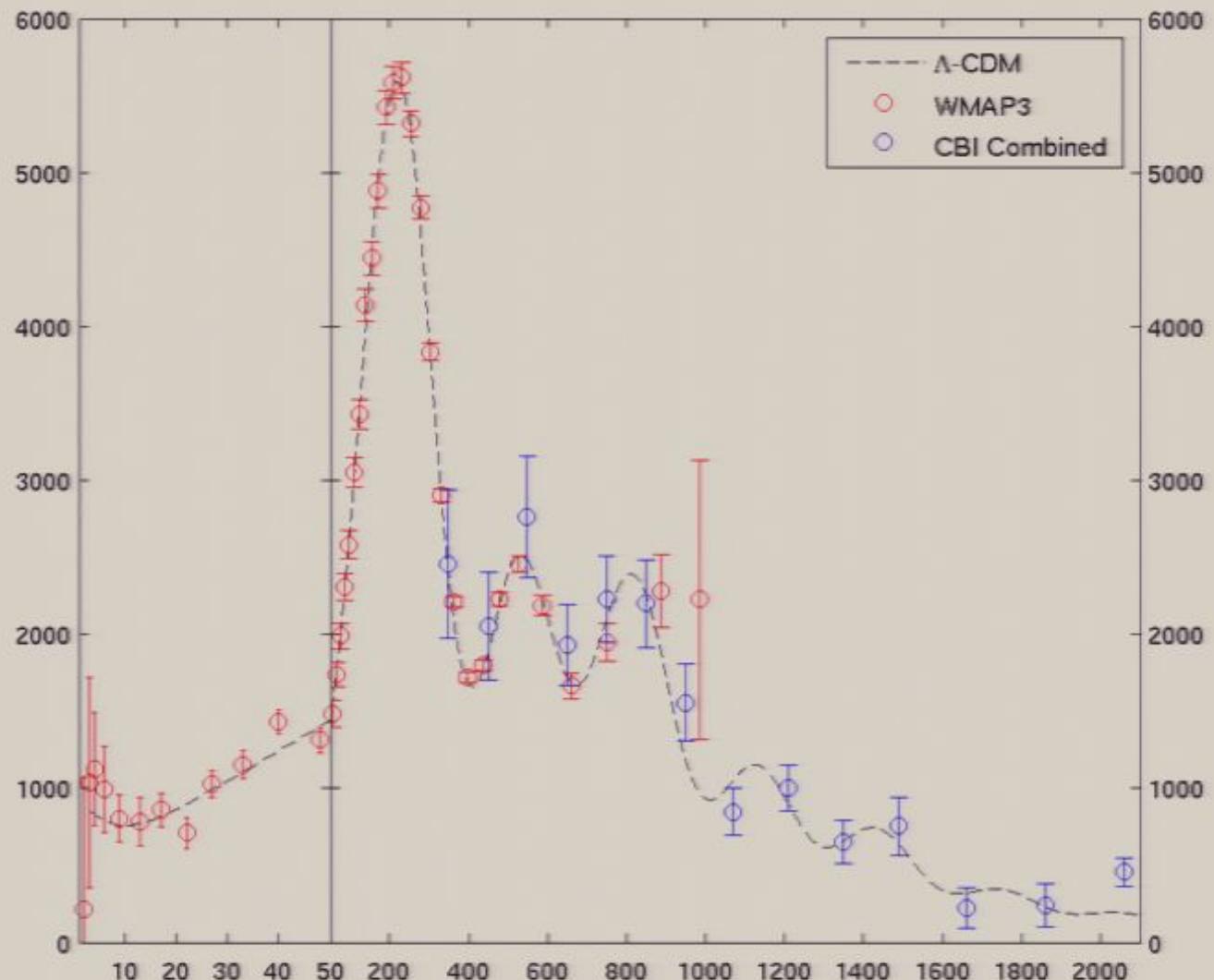
DBI inflation

Racetrack inflation

Tachyon inflation

Warped Brane  
inflation

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$D_3 - D_7$  inflation

DBI inflation

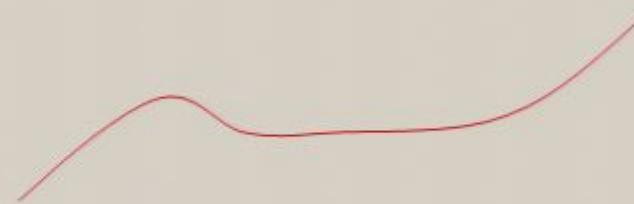
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Tachyon inflation

Warped Brane  
inflation

String theory landscape

$10^{500}$  vacua



$$V(\phi_1, \dots, \phi_{1000}, \dots)$$

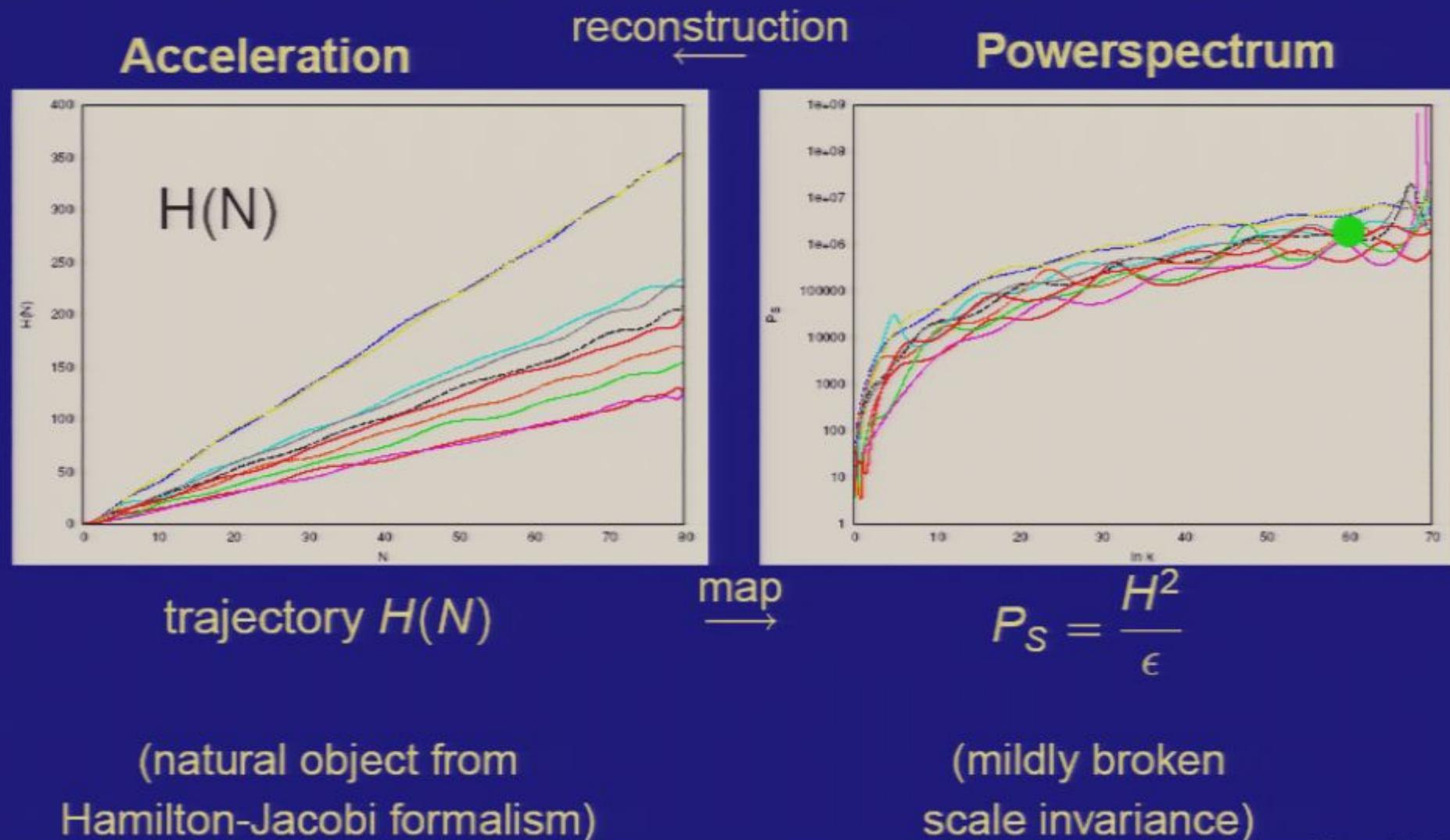
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# The Eye Of The Needle



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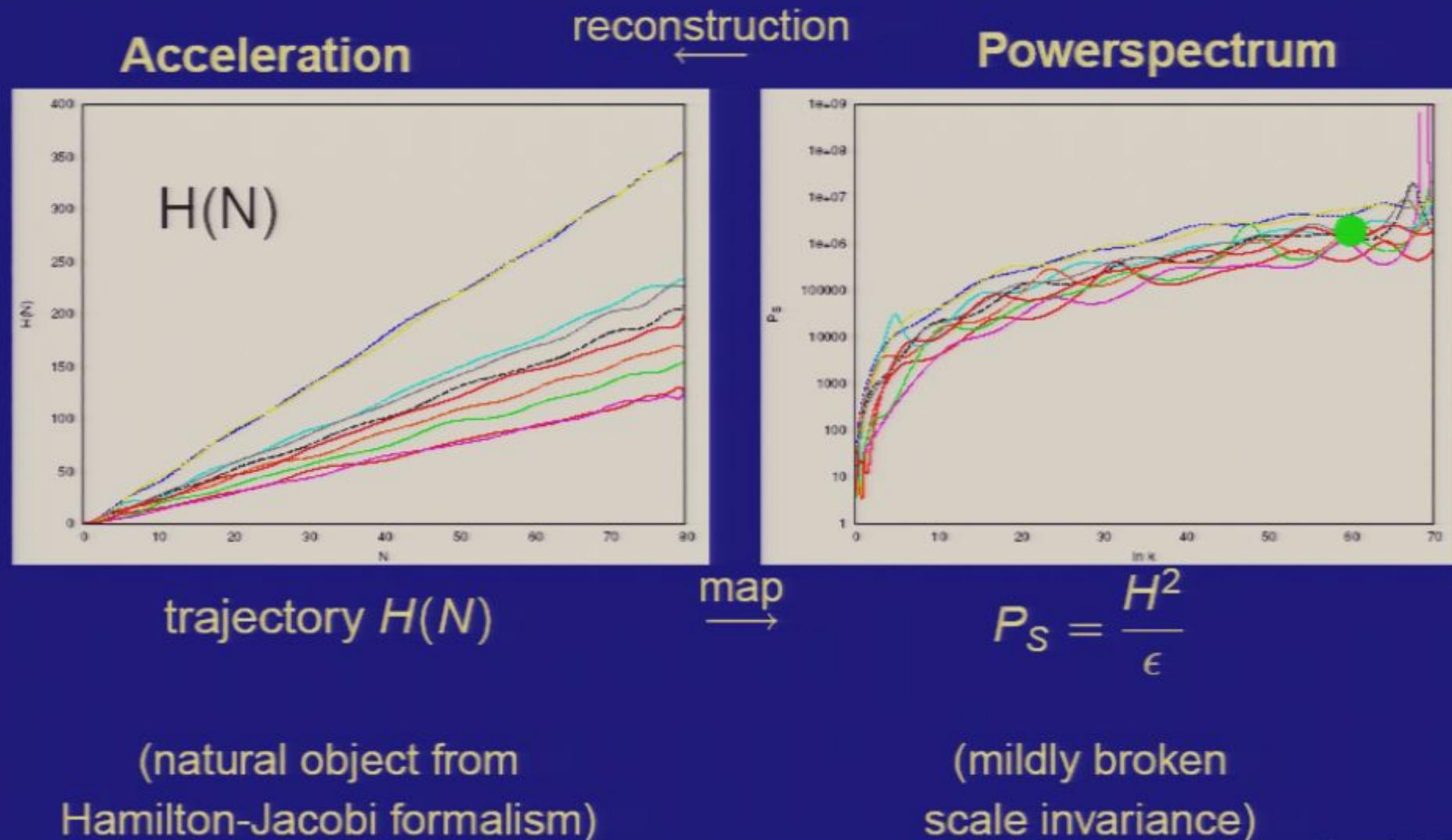
DBI inflation

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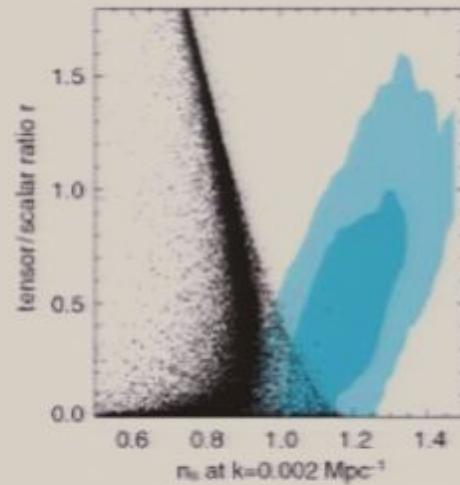
Warped Brane  
inflation

# The Eye Of The Needle



Bottom-up

## Scanning Inflation



$$P_s(k) = A_s k^{n_s - 1}, \quad r = P_{GW}/P_s$$

RG flow method

Small slow roll parameters

$$\epsilon = \frac{M_p^2 H'^2}{4\pi H^2}, \quad \eta = \frac{M_p^2 H''}{4\pi H^2}, \quad \xi \sim H''' \text{ etc.}$$

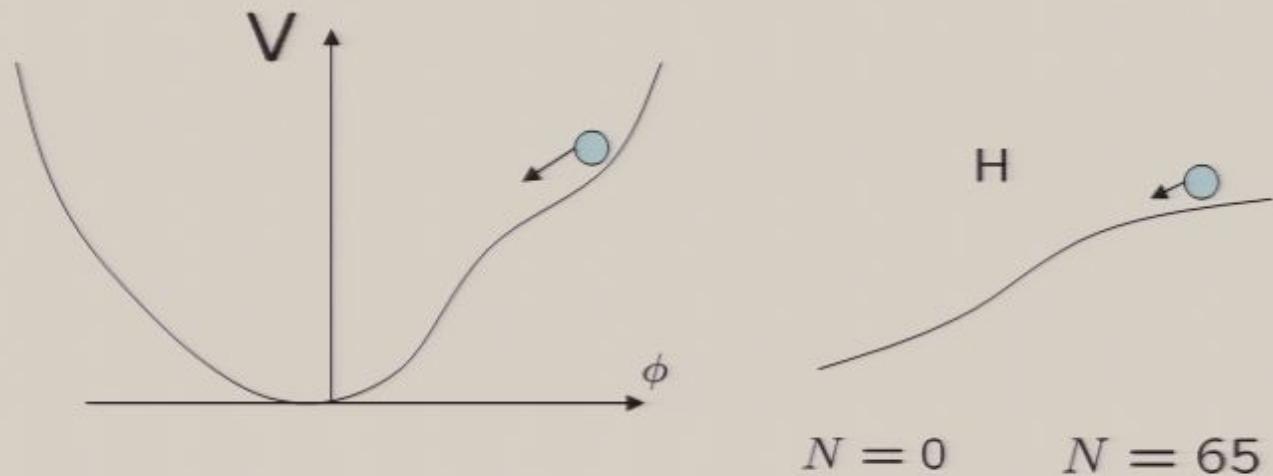
Flow eqs.

$$\frac{d\epsilon}{dN} = \epsilon(\sigma + 2\epsilon)$$

$$\frac{d\sigma}{dN} = -5\sigma\epsilon - 12\epsilon^2 + 2\xi$$

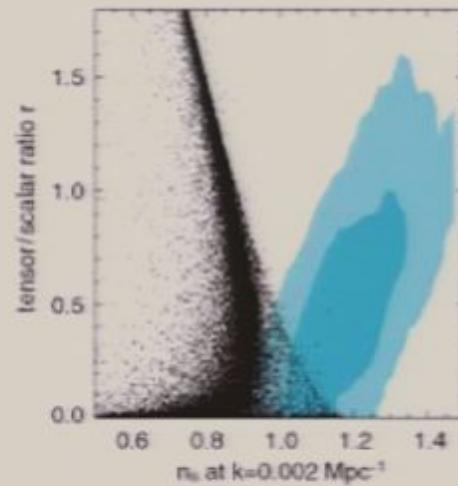
$$\frac{d\xi}{dN} \sim \text{cubic in } \epsilon, \sigma, \xi + \lambda$$

etc



Bottom-up

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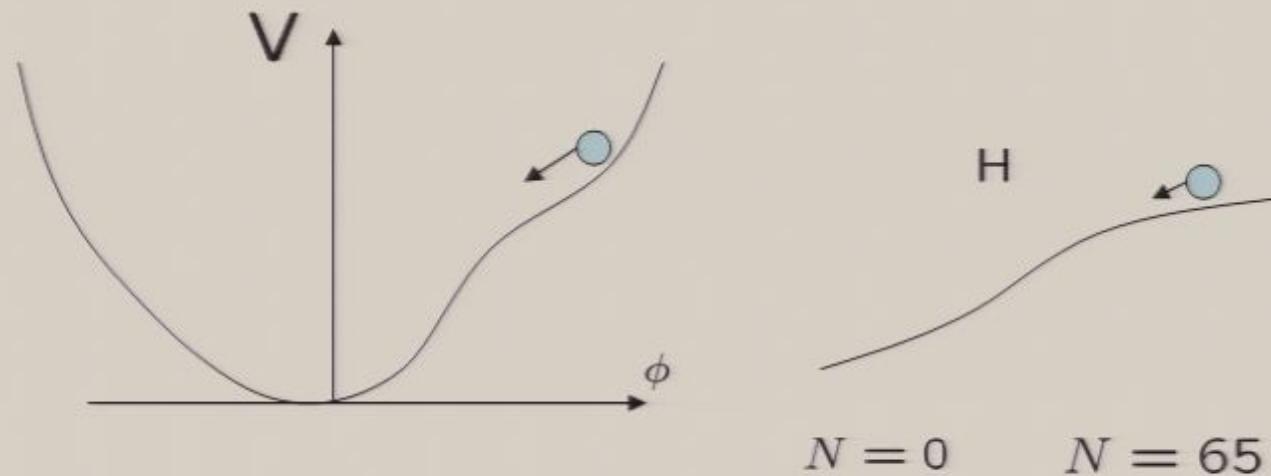
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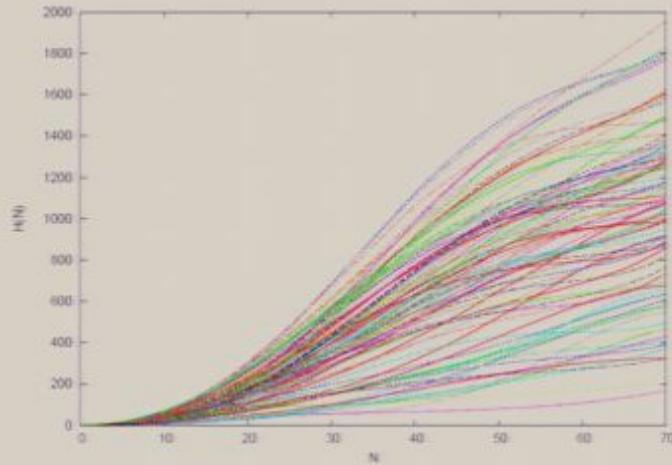
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# Ensemble of Inflationary trajectories



Chebyshev decomposition

$$H(x) = \sum c_n T_n(x), \quad x = \frac{2N - N_{\max}}{N_{\max}}$$

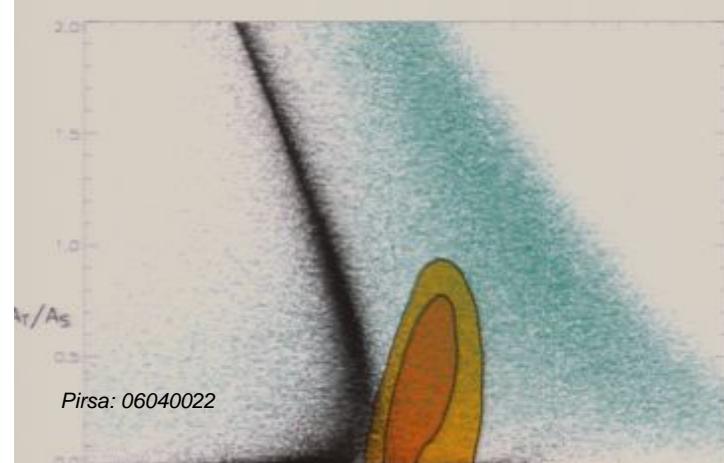
$$\|H(x) - \sum^M c_n T_n(x)\| = \min.$$

$$0 < \frac{dH}{dN} < H \quad \frac{dH}{dN} = H \text{ at } N = 65$$

Related methods of trajectory generation

Space of models opens wide

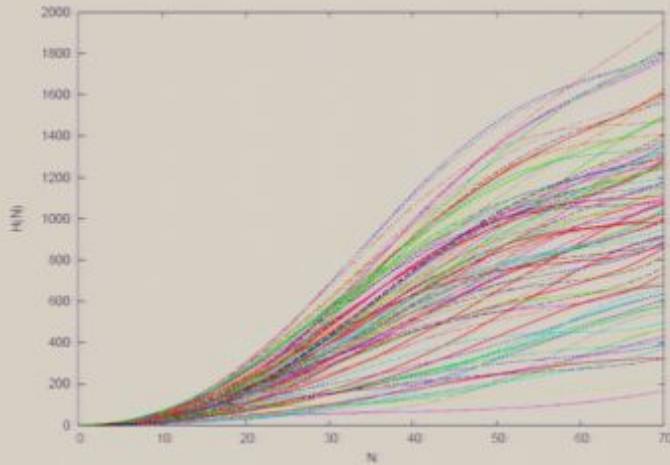
speed  $10^5$  up vs RGF



$$f(x)$$

$$R^2 + \partial R_{ij} R^{ij} = -1 + \dots$$

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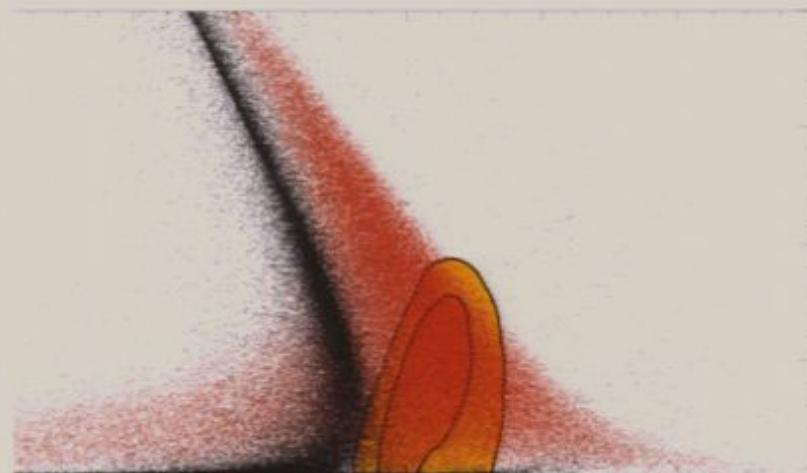
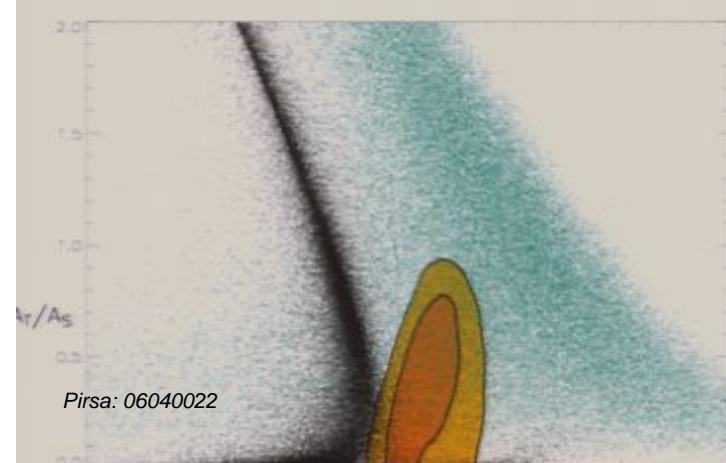
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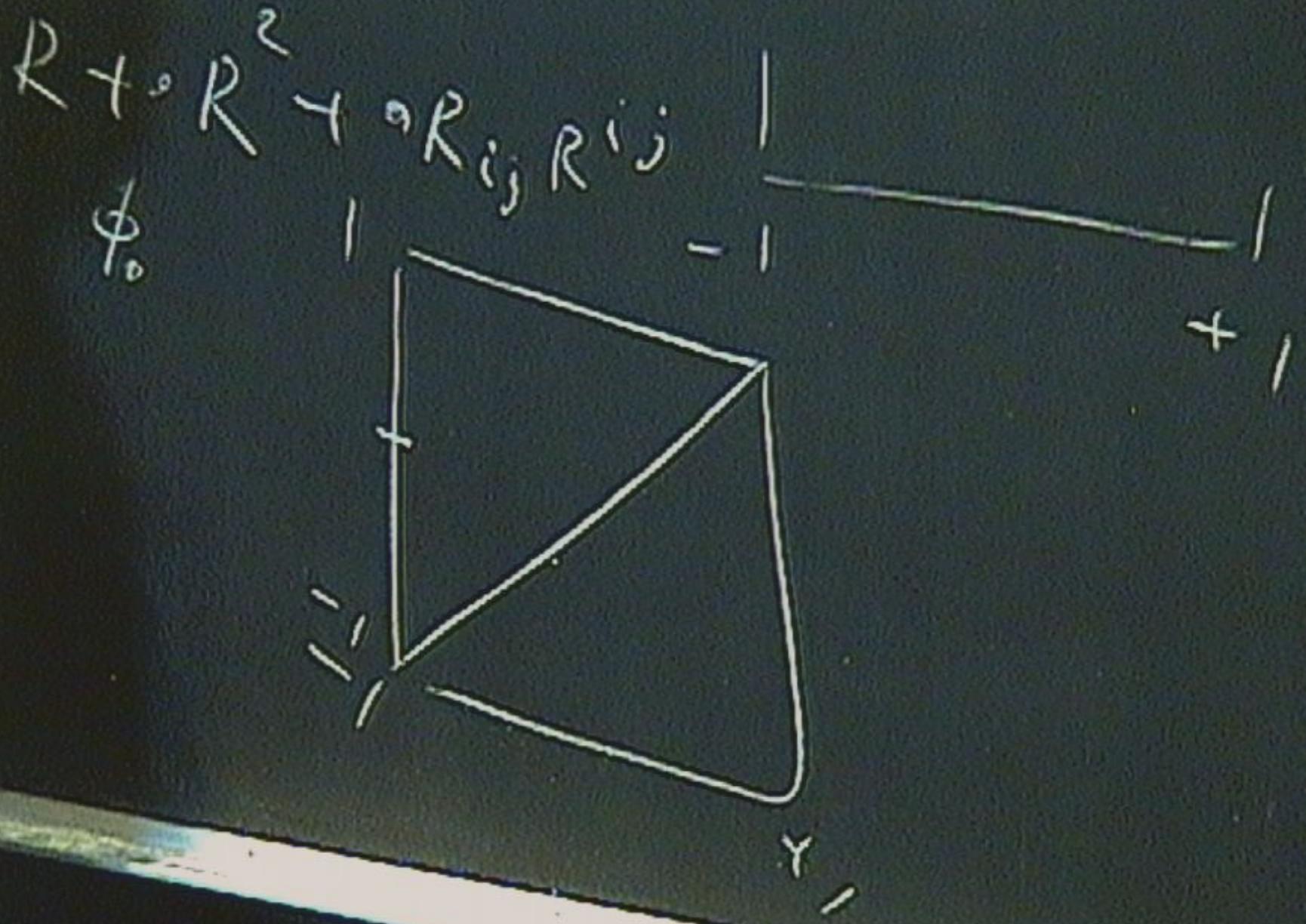
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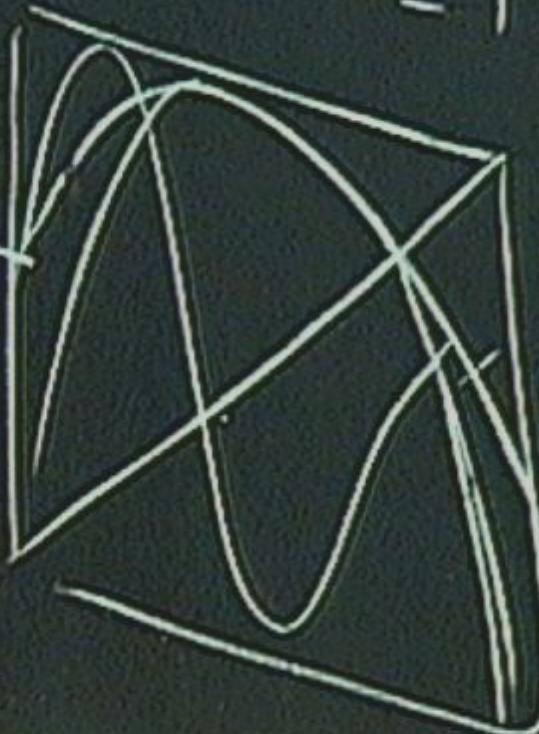




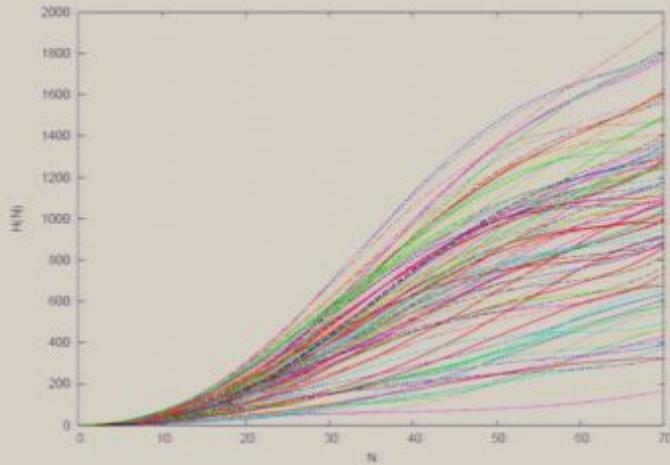
$$R + R^2$$

$$\phi_0$$

$$+ \gamma^a R_{ij} R^{ij}$$


$$\gamma$$
$$+$$

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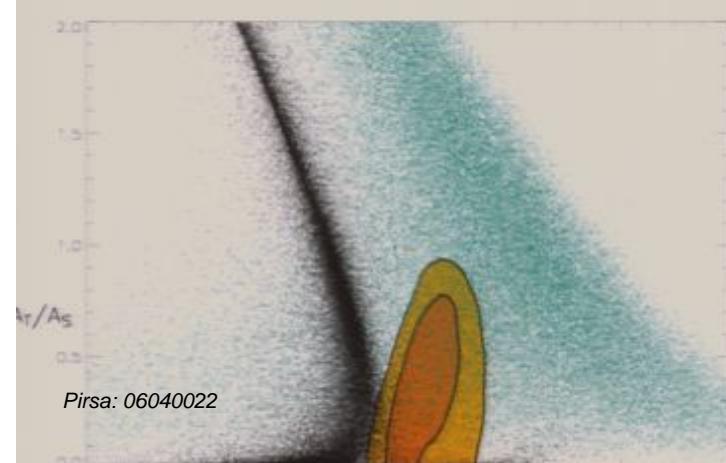
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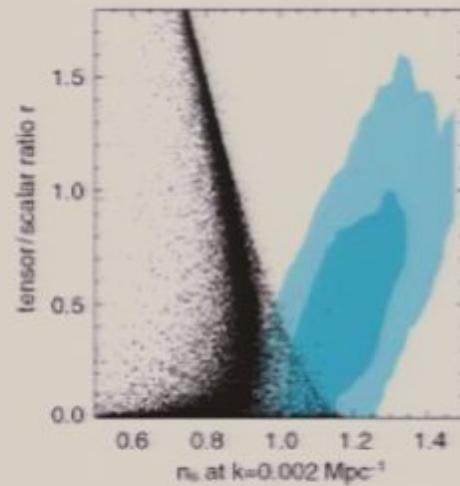
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Bottom-up

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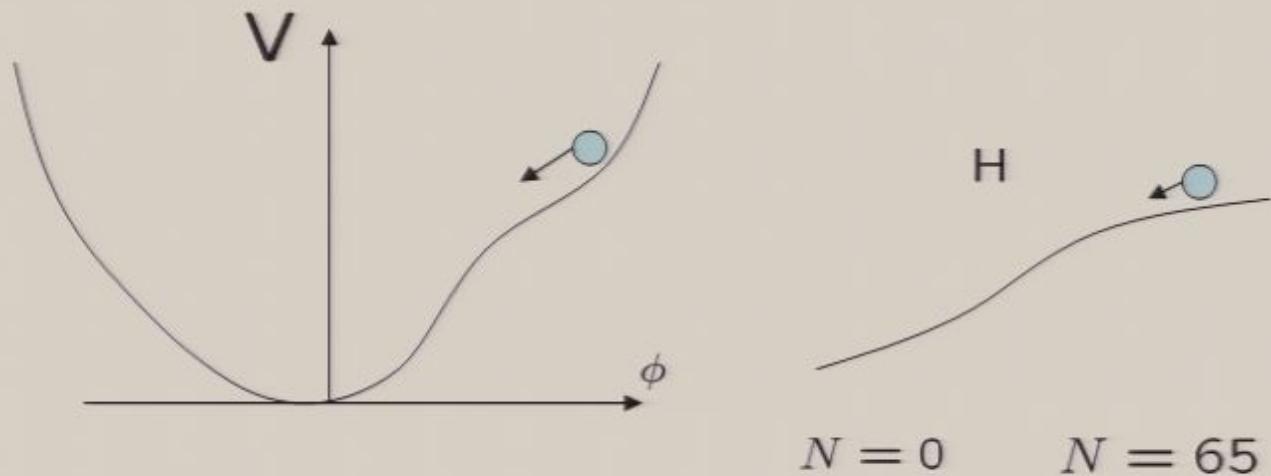
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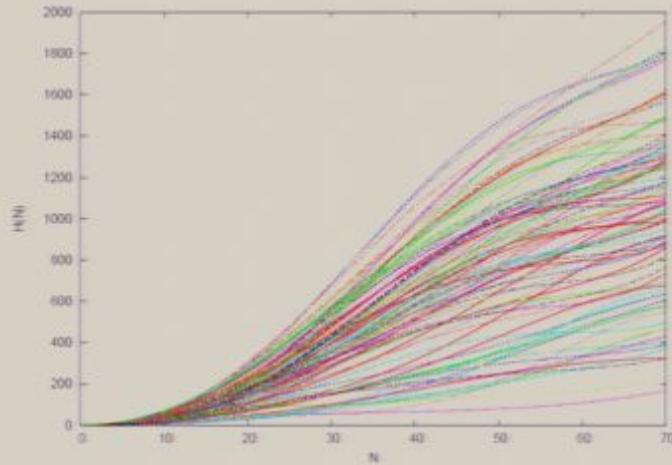
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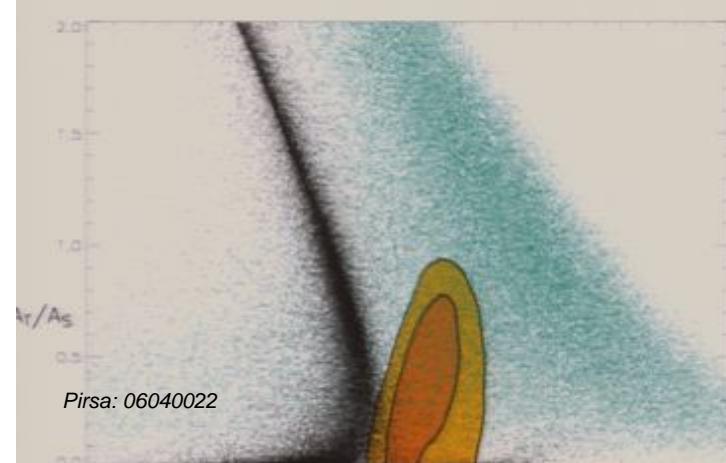
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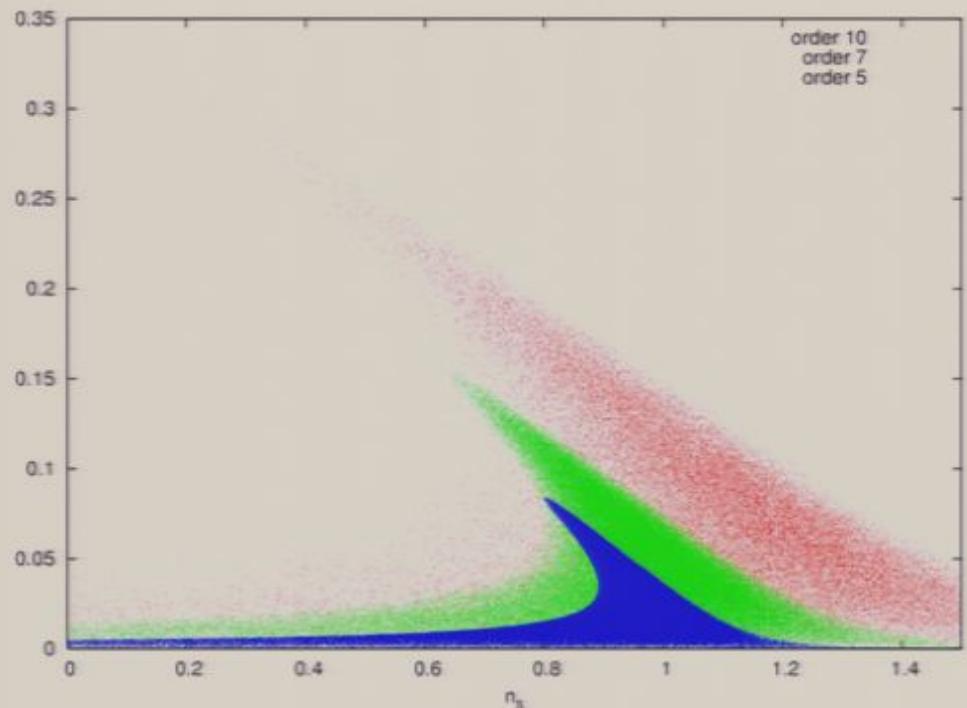
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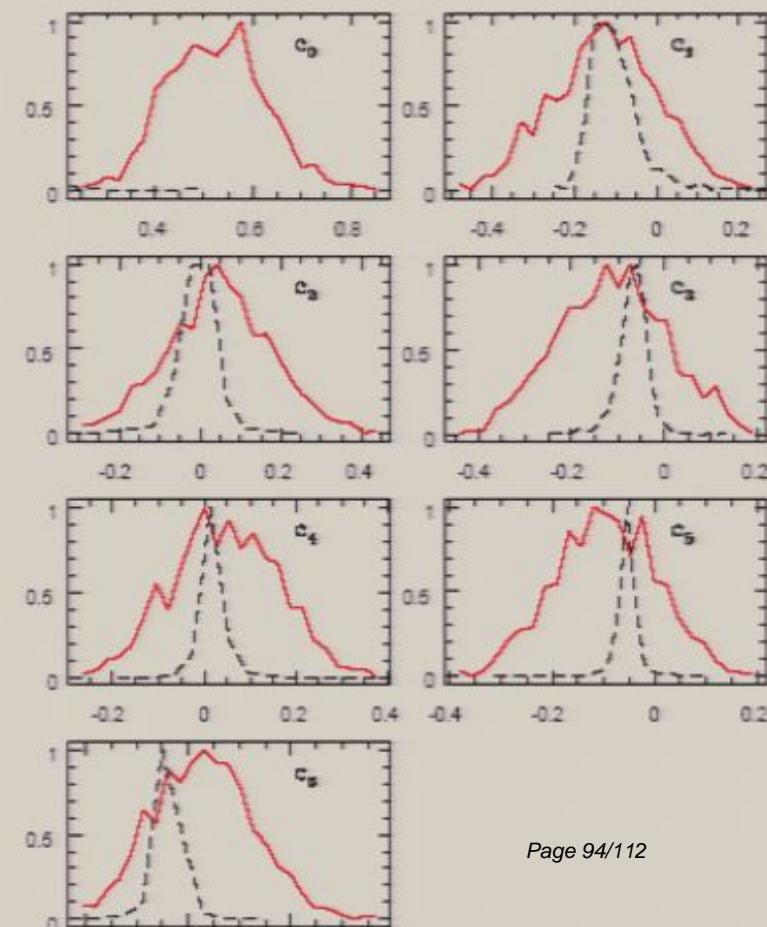
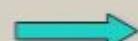
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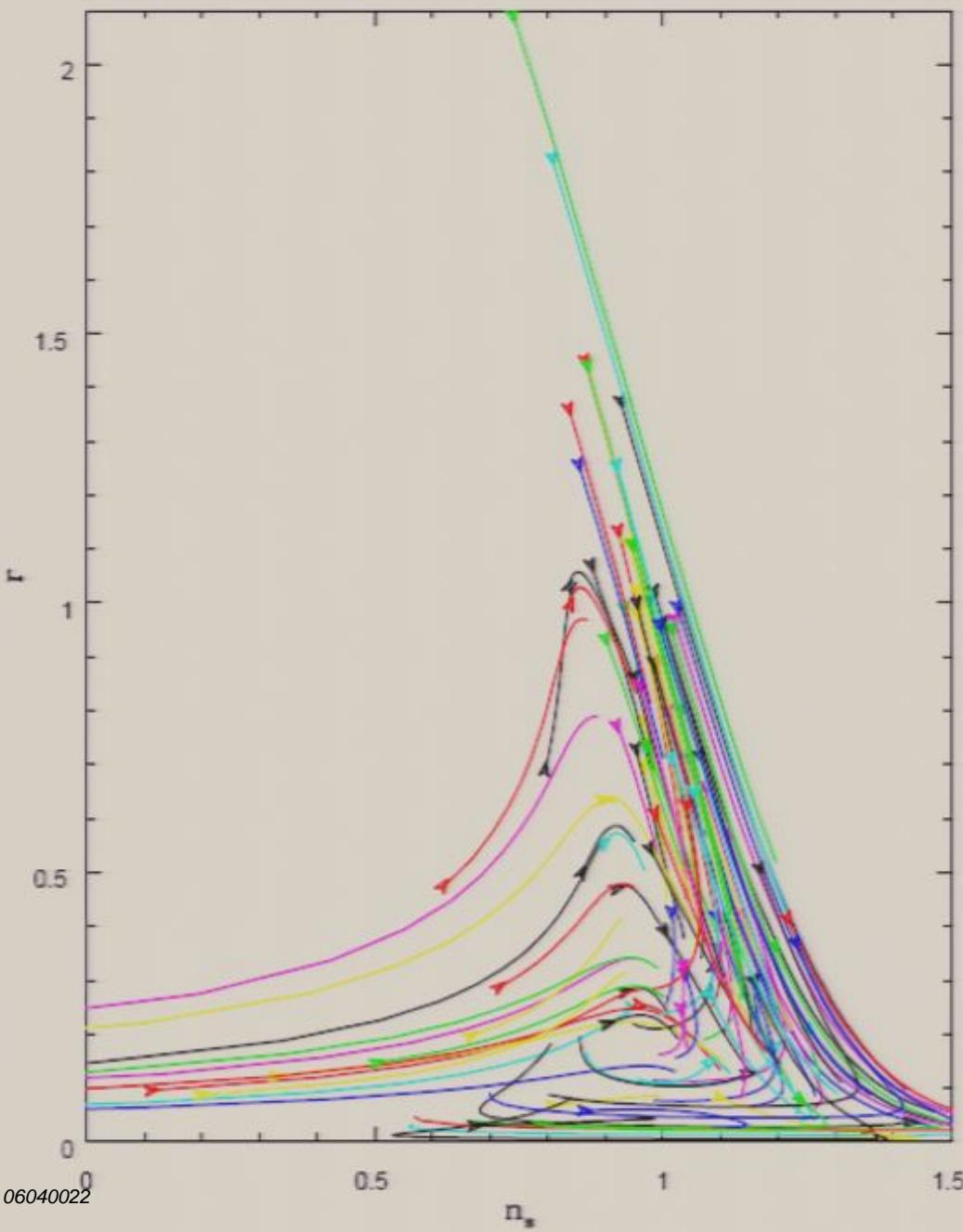




space opens more with higher order polynomials

comparison of  $c_n$  (red) in our method vs  $c_n$  (black) of Chebyshev transform of trajectories generated with RG flow





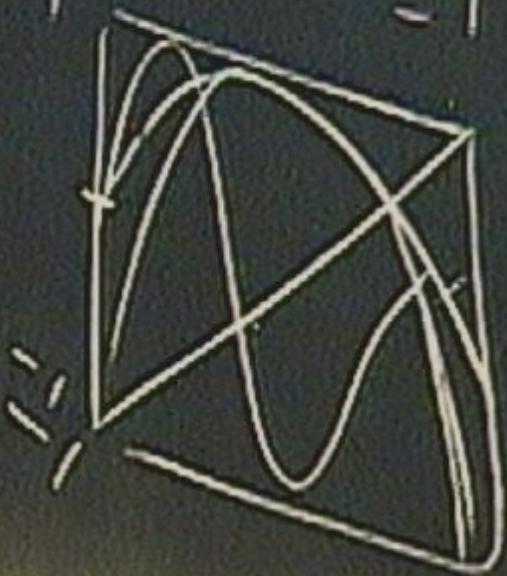
$\epsilon, \eta \dots$

$H(N) \longrightarrow P(k)$

$n_s, n_t, r, dn/d\ln k, A_s, \dots$

$f(x)$ 

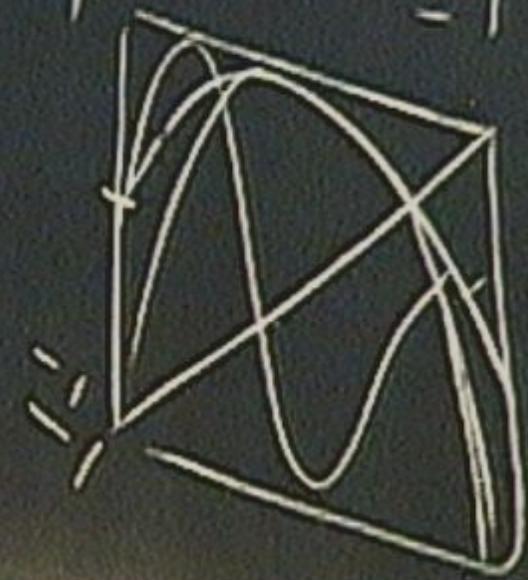
$$R + R^2 + \dots + R_{ij} R^{ij}$$

 $\phi_0$  $y_i$

$f(x)$

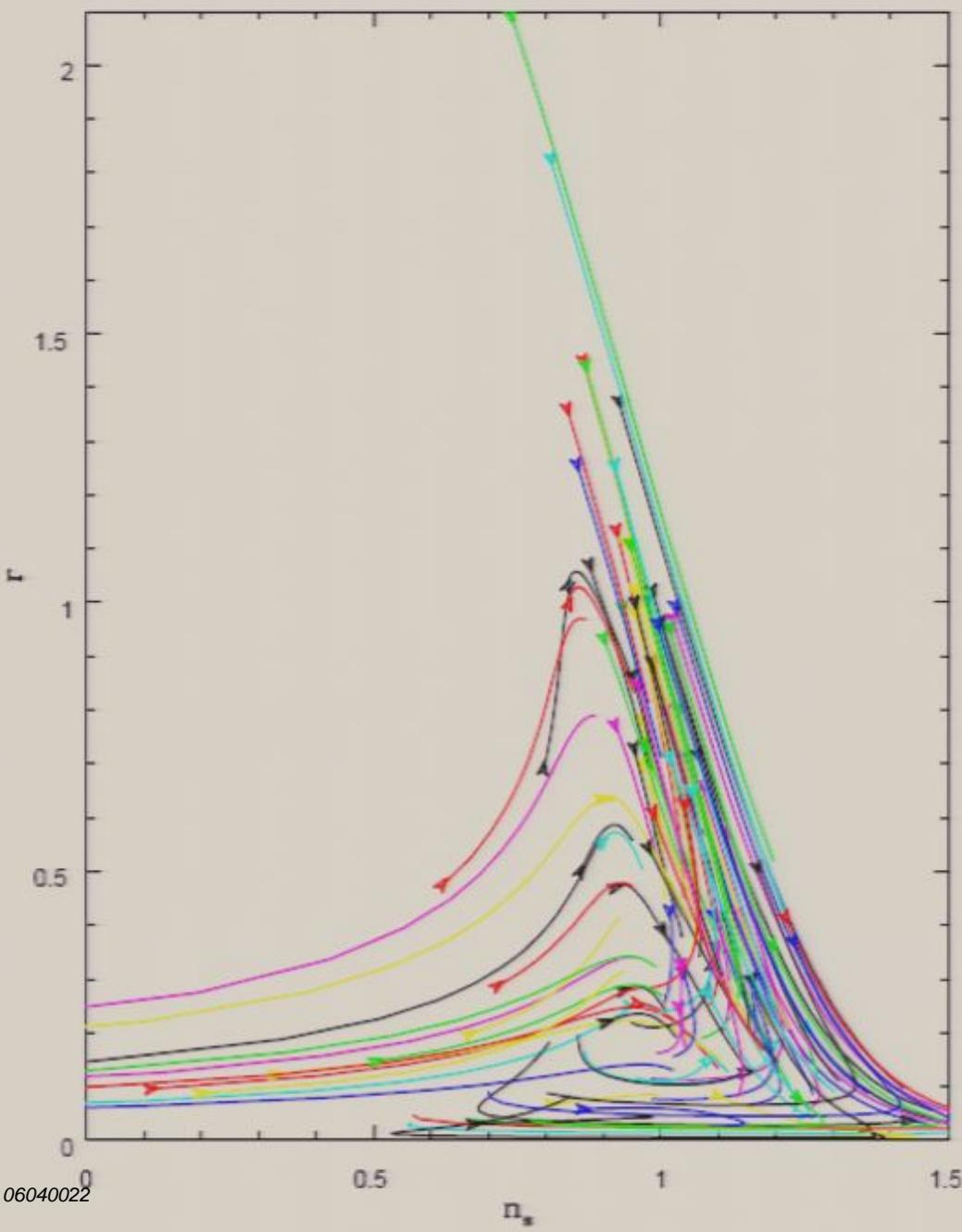
$$R^+ - R^2 + \partial R_{ij} R^{ij}$$

$\phi_0$



$y$



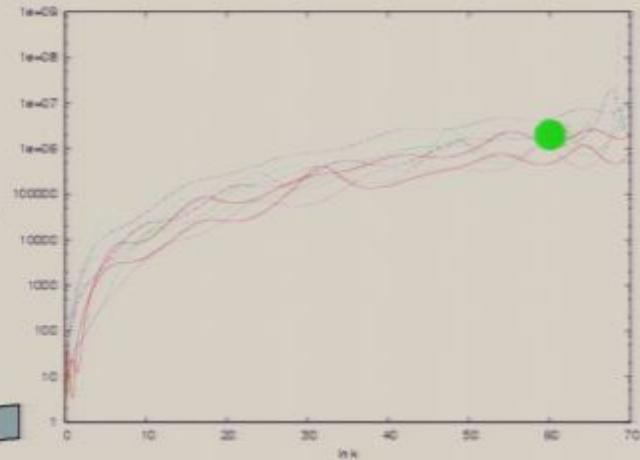


$\epsilon, \eta \dots$

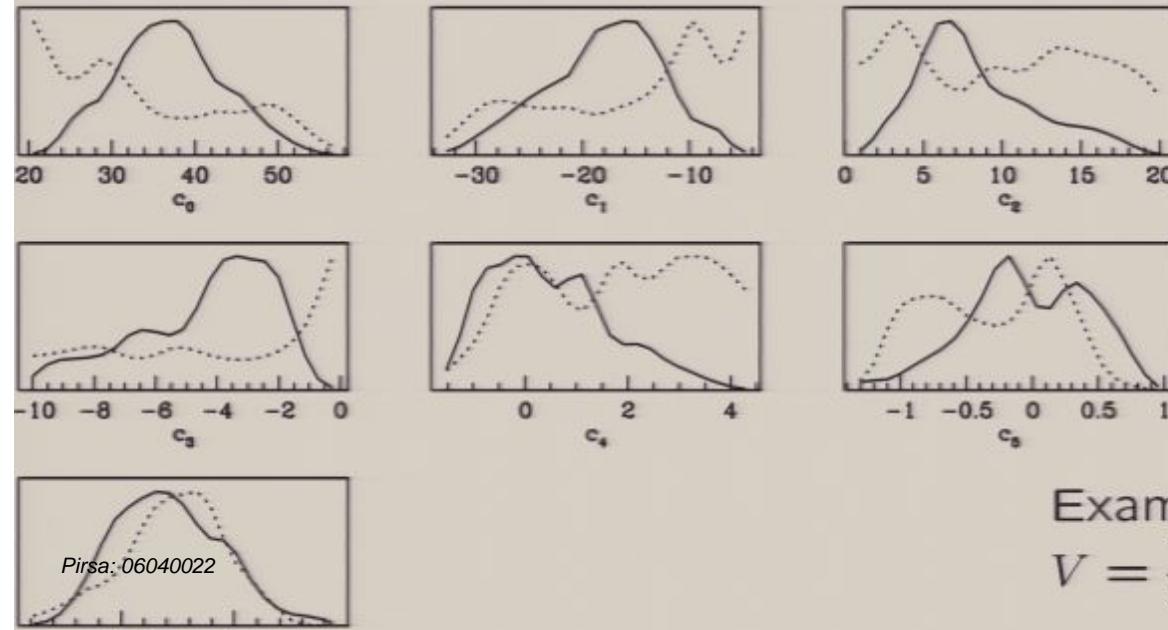
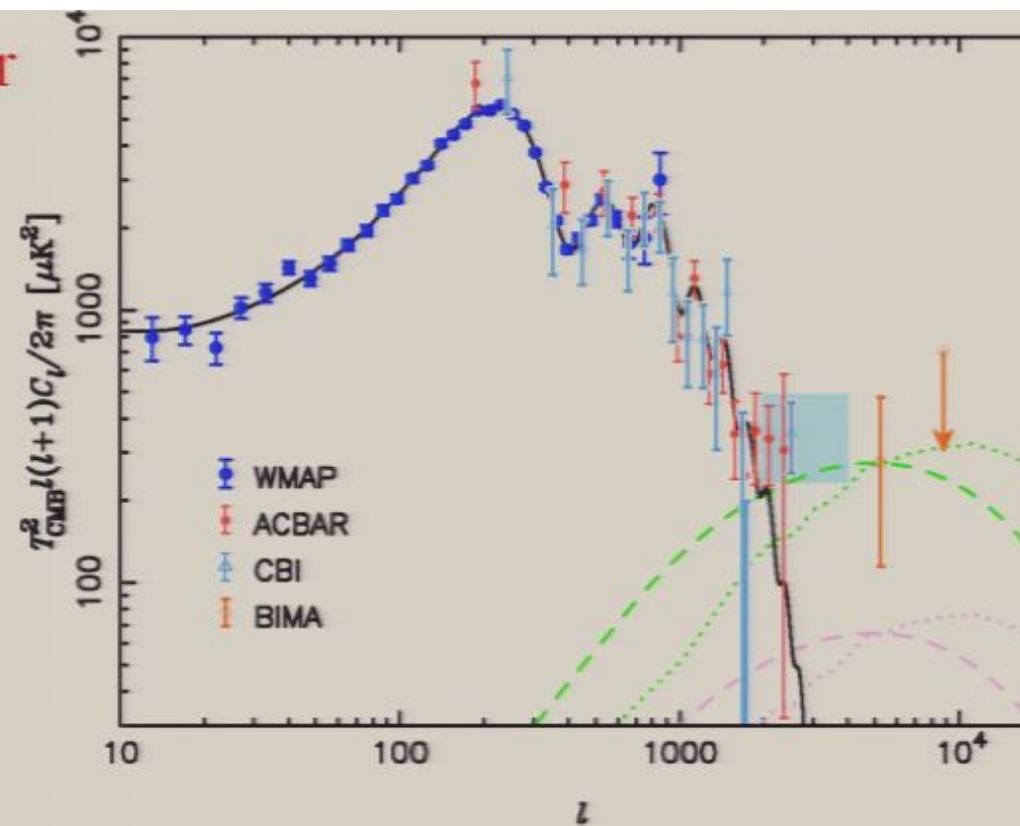
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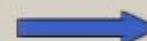
# Observational constraints on trajectory



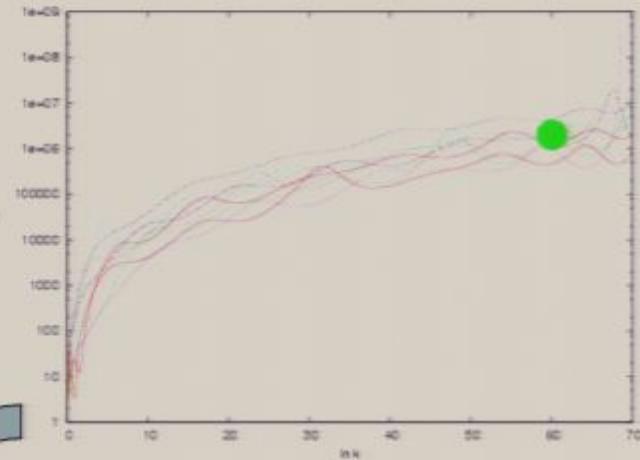
Markov Chain Monte Carlo



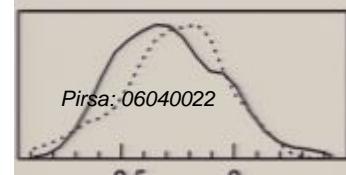
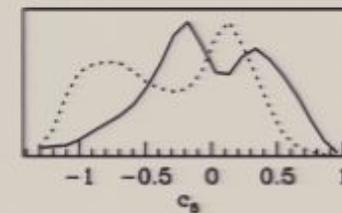
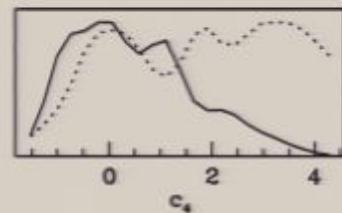
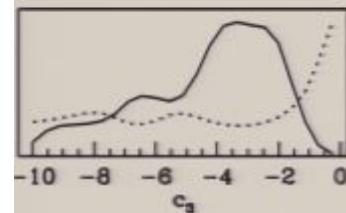
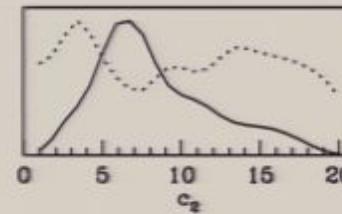
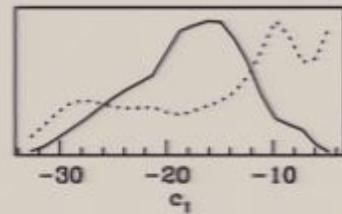
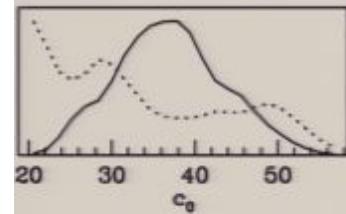
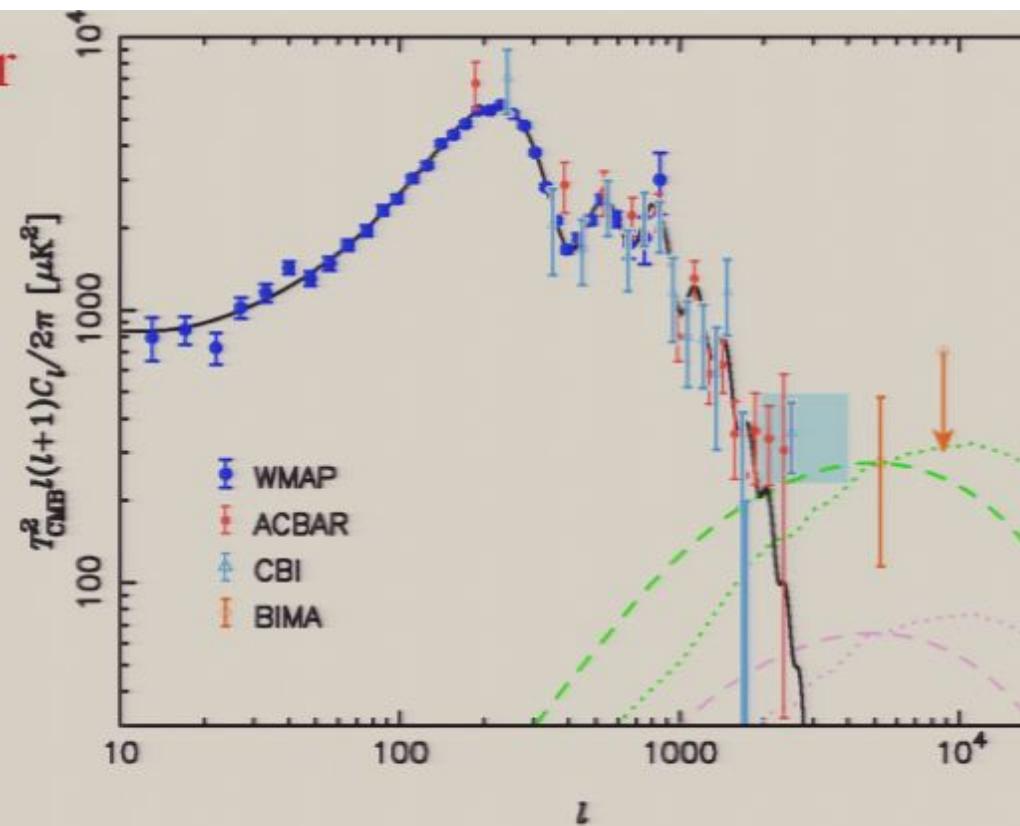
Example of  $c_n$  for  
 $V = \frac{1}{2}m^2\phi^2$



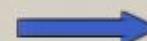
# Observational constraints on trajectory



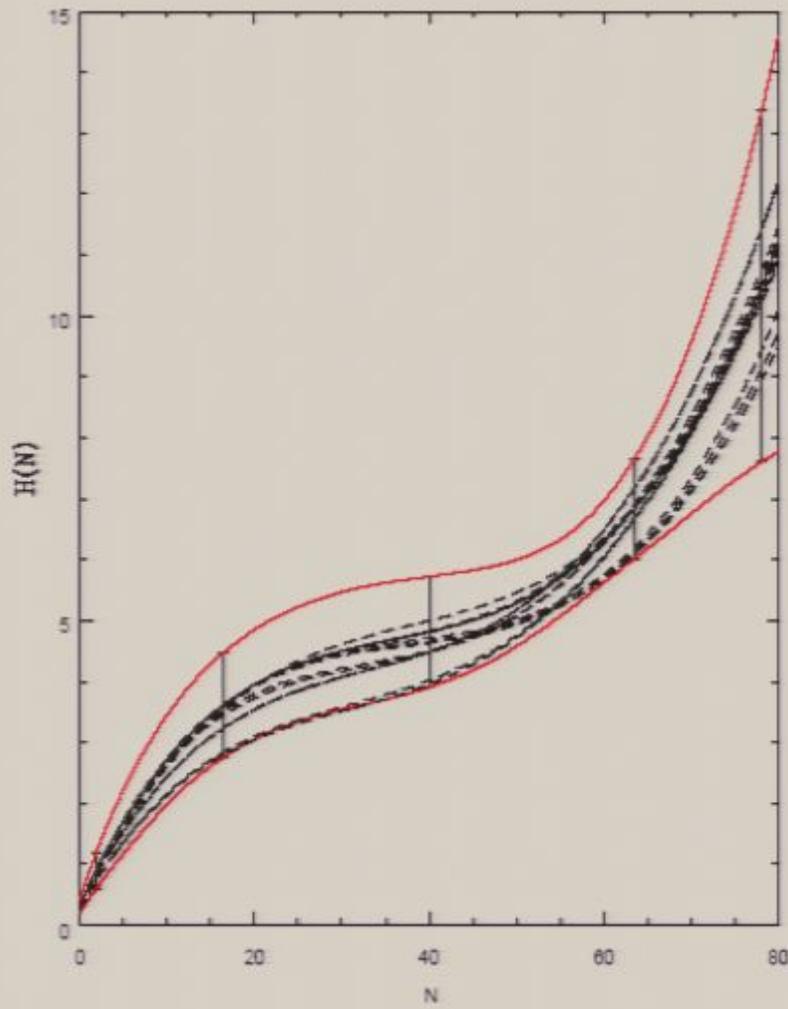
Markov Chain Monte Carlo



Example of  $c_n$  for  
 $V = \frac{1}{2}m^2\phi^2$



## Reconstruction of Inflationary Trajectory



$$f(x) = \sum_{j=0}^n c_j T_j(x)$$

Chebyshev Polynomials Nodal Points Method

$$f(x) = \sum f(x_j) \phi_j^{(n)}(x)$$

 set of nodal points  $x_j^{(n)}$

## Variations

$$f(x) = H, \log H, \epsilon, \log \epsilon, P_s, P_t$$

$$x = N, \log k$$

Trajectories cf.

**WMAP1+BO3+CBI+DASI+VSA+Acbar+Maxima+SDSS+2dF**

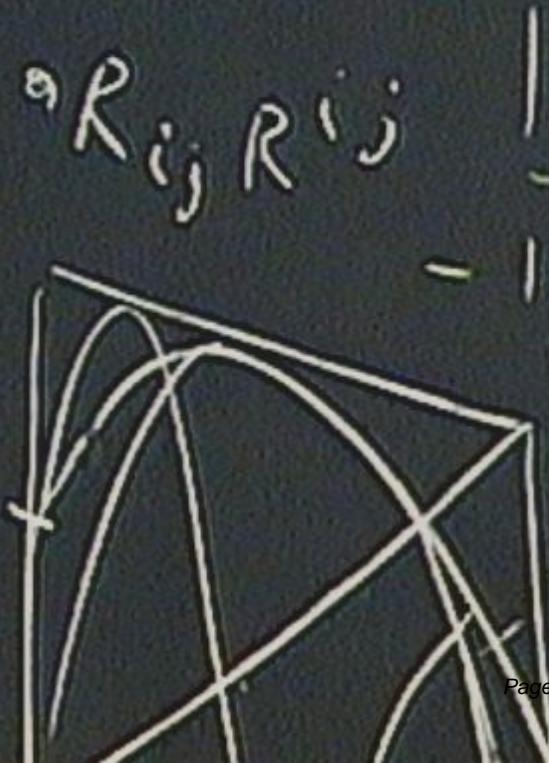
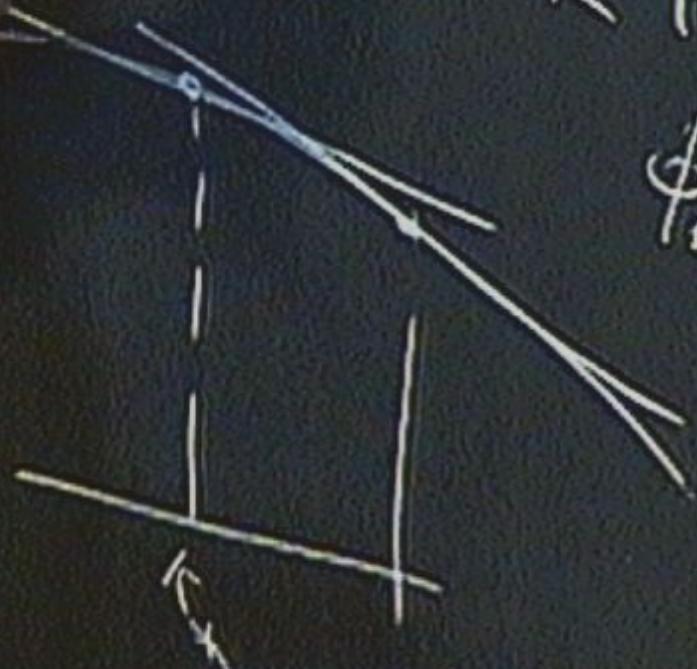
$$w_s = 1 + \frac{1}{2} \frac{d w_s}{d \theta_{\text{min}}}$$

$$P_s = A^k$$

b: R

$$R + R^2 + R_{ij} R^{ij} +$$

$\phi_d$

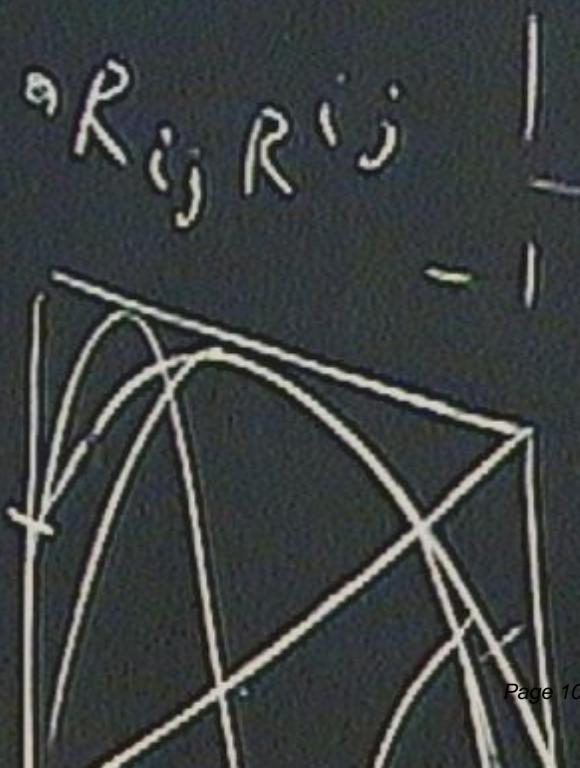
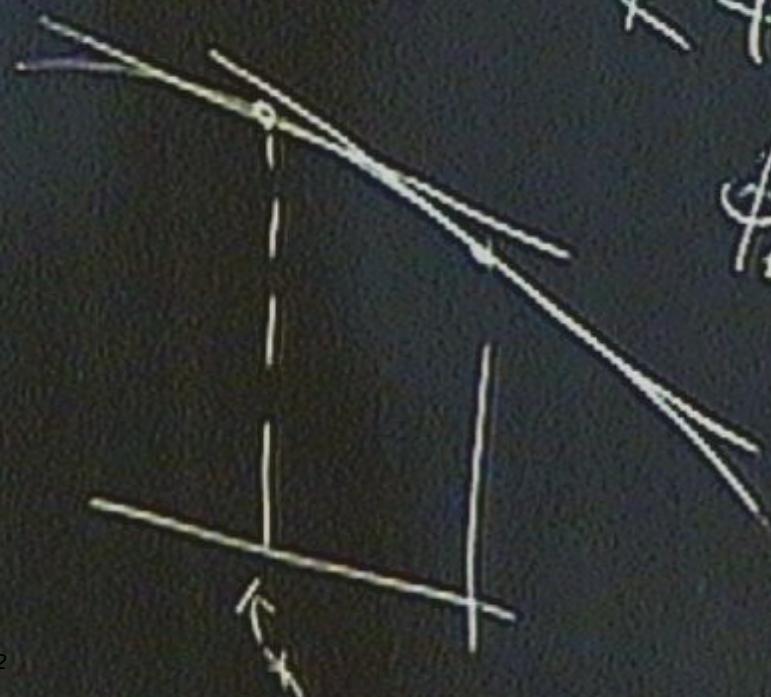


$$w_s = 1 + \frac{1}{2} \frac{d w_s}{d \theta_{\text{min}}}$$

$$P_s = A K$$

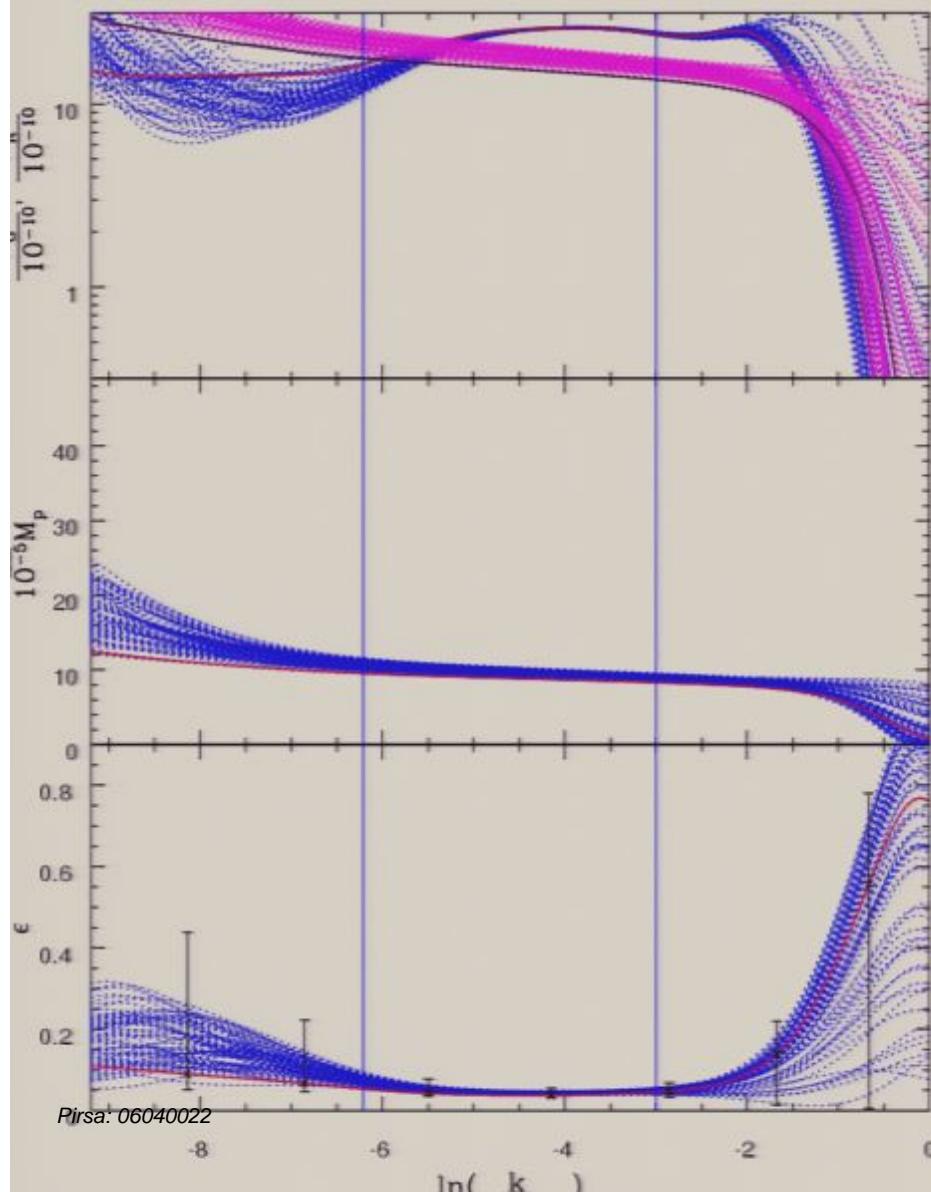
$$\log P_s$$

$$R + R^2 + R_{ij} R^{ij} + \dots$$



$\epsilon(\ln k)$  reconstructed from CMB+LSS data using Chebyshev expansion  
 (order 15 nodal points)  
 and Markov Chain Monte Carlo Method. T/S consistency function is imposed  
 Probe of CMB+LSS window only 1-folds

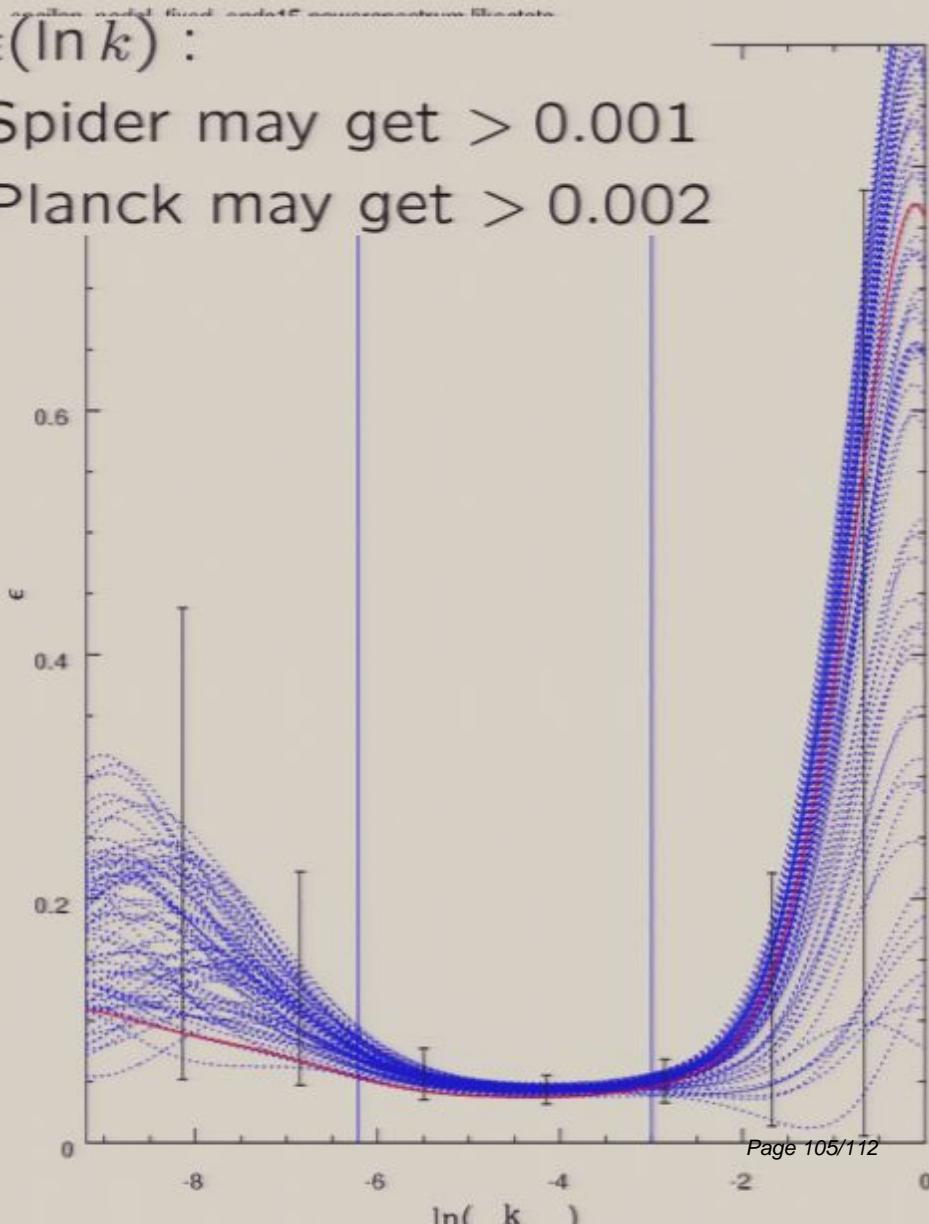
epsilon\_nodal\_fixed\_ends15.powerspectrum.likestats



$\epsilon(\ln k)$ :

Spider may get  $> 0.001$

Planck may get  $> 0.002$



## Top-down approach:

no priors

"Best fit" model is not usual:

features in the potential  
suppression of scalar mode at large scales

large tensor mode

mutually (almost) compensated features in

$$\left(\frac{\Delta T}{T}\right)_{tot}^2 = \left(\frac{\Delta T}{T}\right)_s^2 + \left(\frac{\Delta T}{T}\right)_t^2$$

## Bottom-up:

personal priors

Non-vanishing probability

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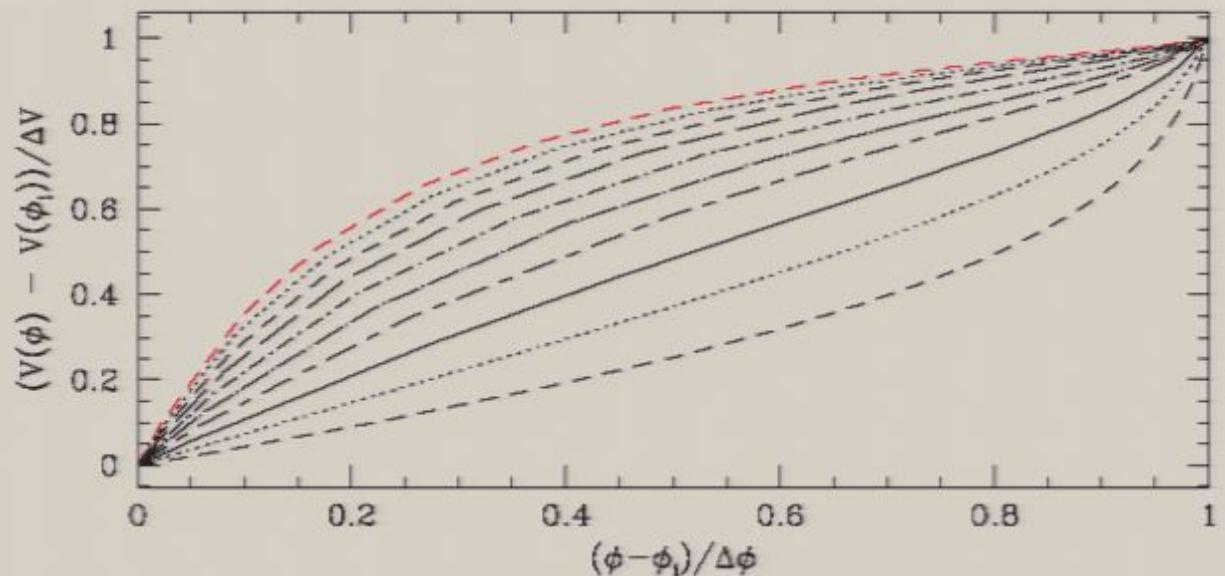
Non-vanishing probability

## Degeneracy of the Potential Reconstruction

known  $P_s(k) \rightarrow$  reconstruct  $V(\phi)$

$$P_s(k) = \frac{8\pi H^4}{M_p^4 H'^2}$$

$$V(\phi) = \frac{M_p^4}{32\pi^2} \left( \frac{12\pi}{M_p^2} H^2 - H'^2 \right)$$



$$\phi - \phi_0 = \frac{M_p^2}{2} \int_{\ln k_0}^{\ln k} d \ln k' \frac{\sqrt{P_s(k')}}{H^2(k')} \frac{dH}{d \ln k'}$$

$$\frac{dH}{d \log k} = \frac{H^3}{H^2 - \pi M_p^2 P_s(k)}$$

Degeneracy is lifted by fixing  $P_{GW} = \frac{H_*^2}{M_p^2}$

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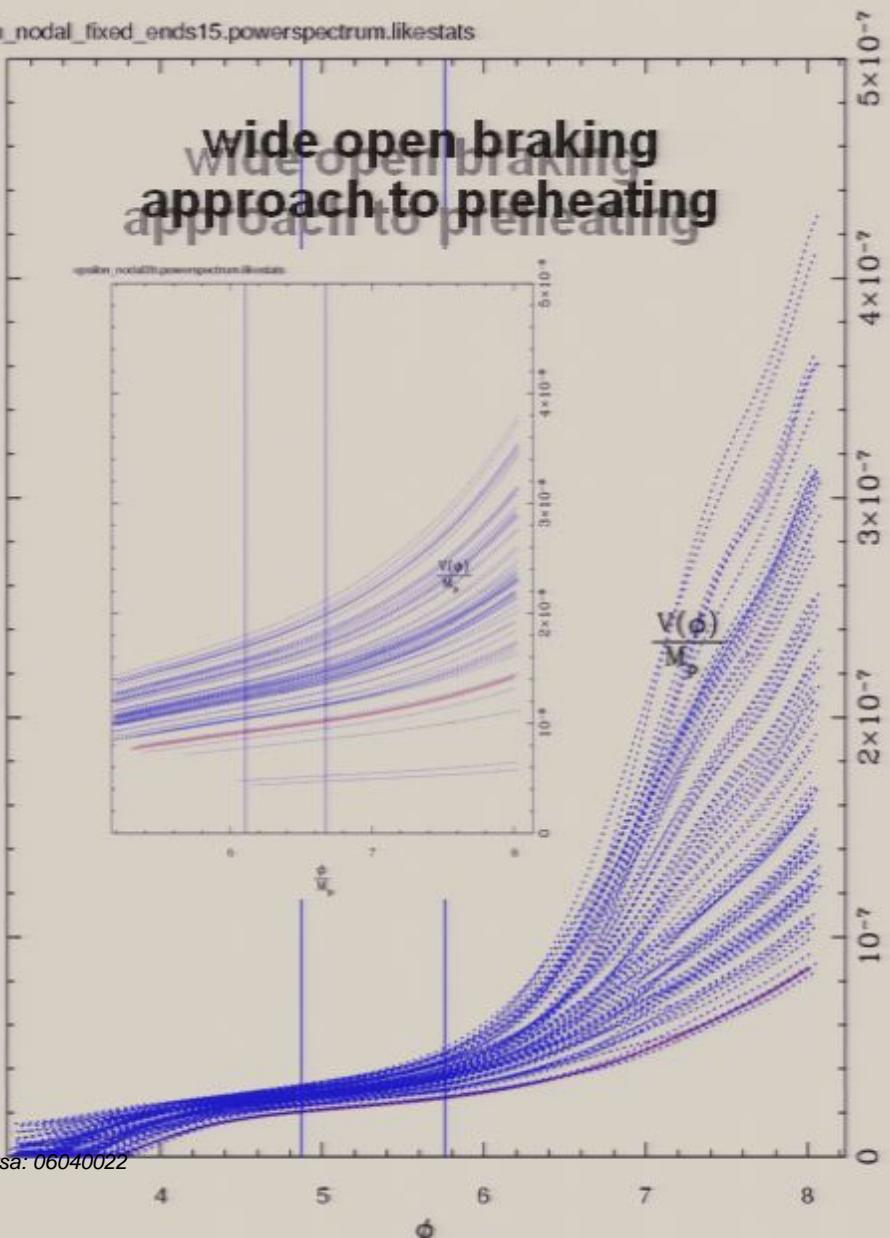
## Bottom-up:

personal priors

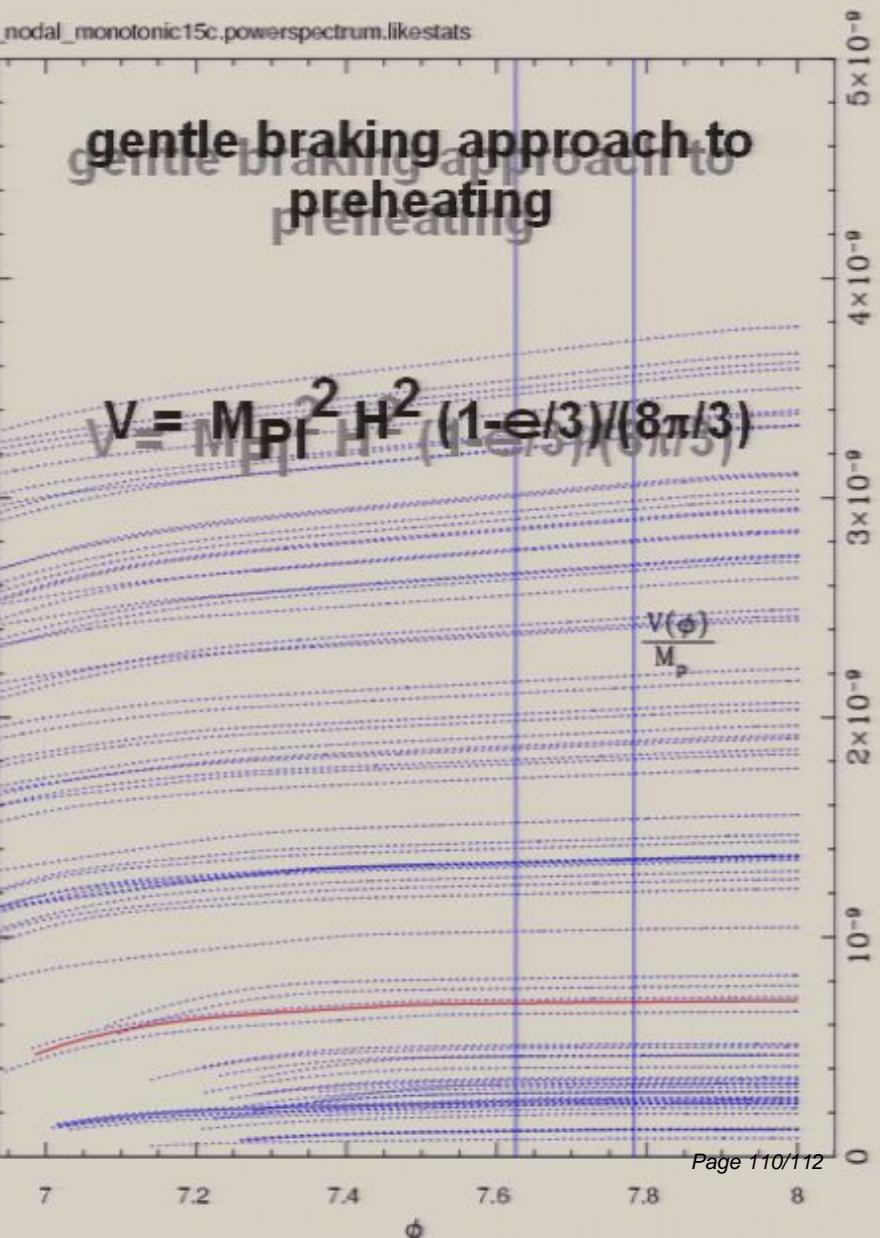
Non-vanishing probability

$V(\phi)$  reconstructed from CMB+LSS data using Chebyshev expansions (uniform order 15 nodal point) of (uniform order 3 nodal point) of (monotonic order 15 nodal point) and Markov Chain Monte Carlo methods...

epsilon\_nodal\_fixed\_ends15.powerspectrum.likestats

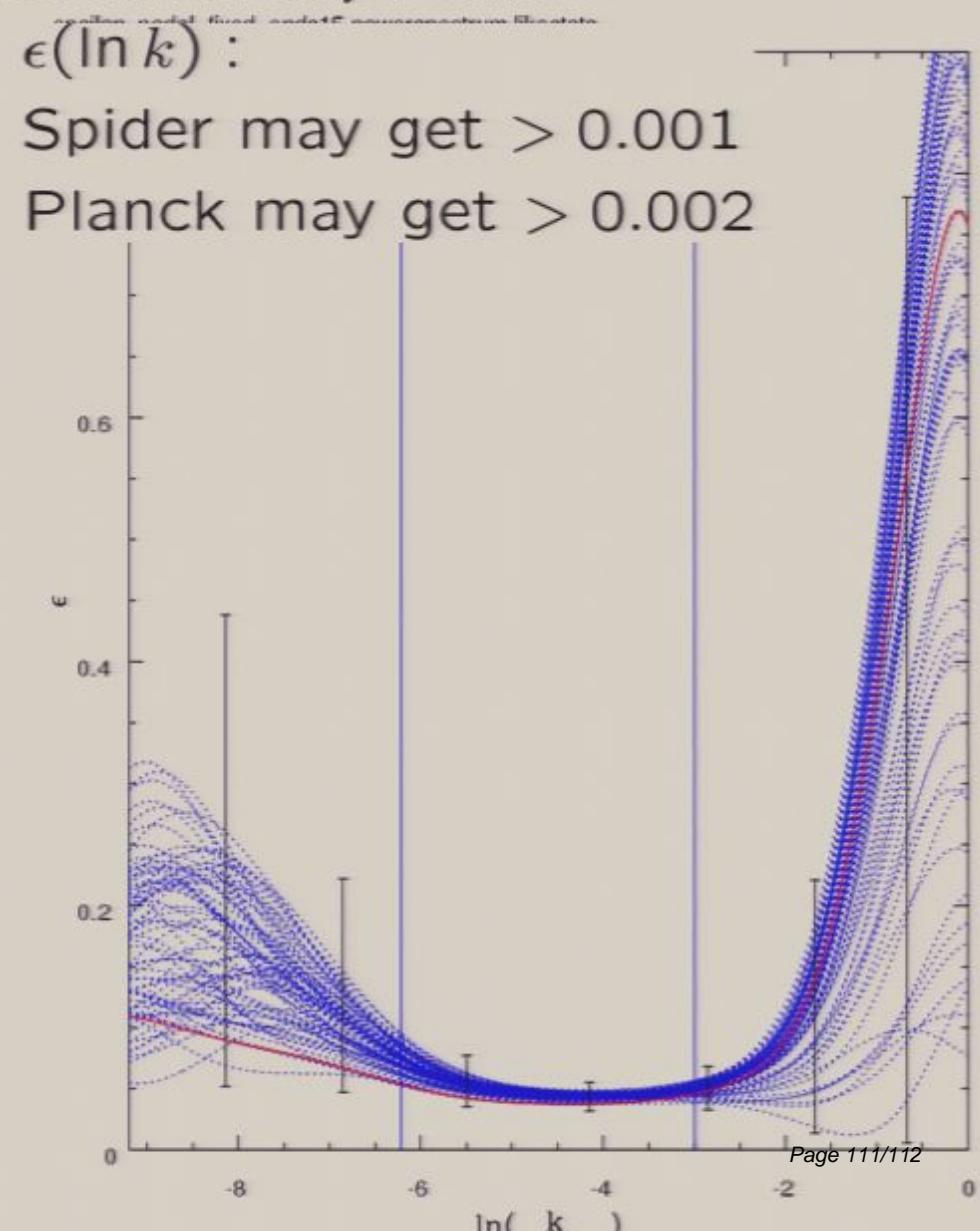
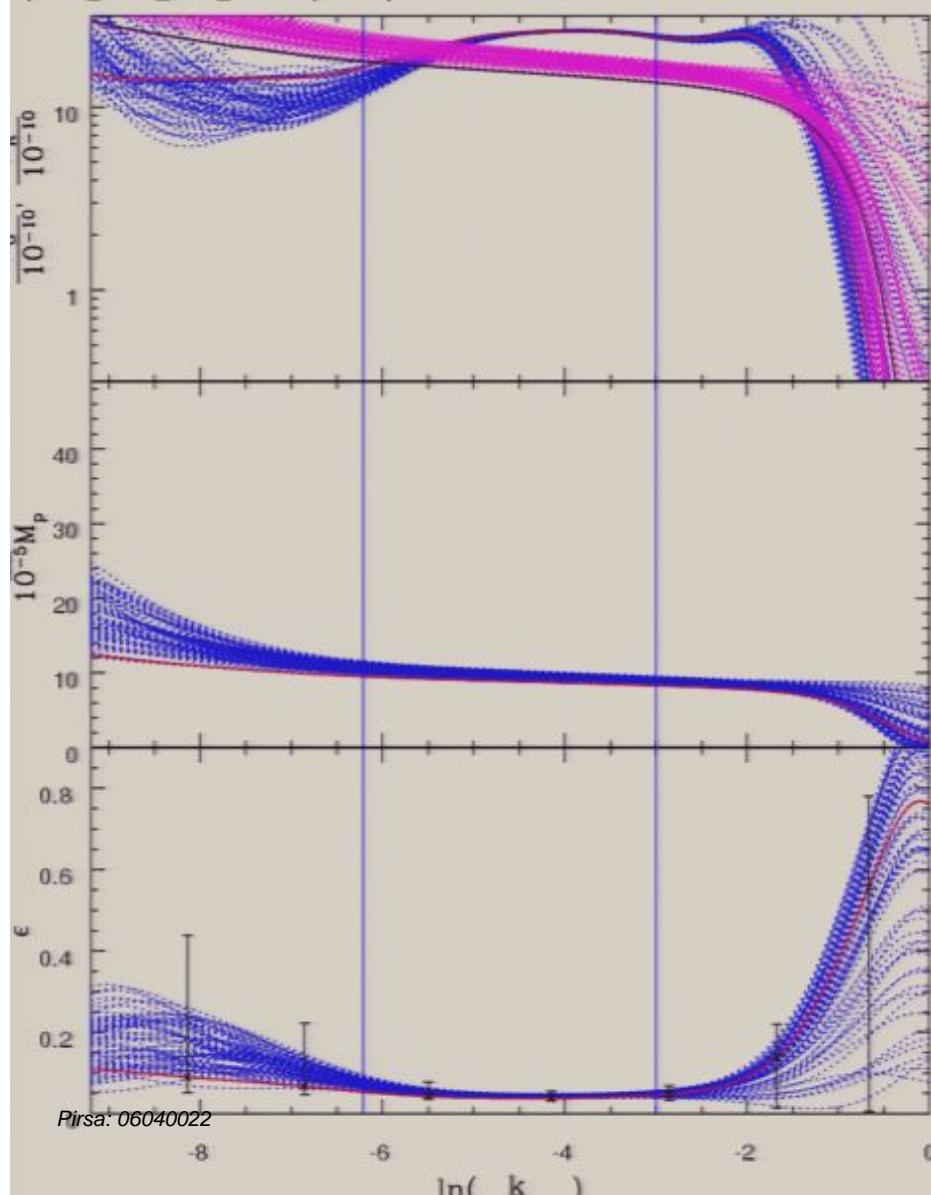


epsilon\_nodal\_monotonic15c.powerspectrum.likestats



$\epsilon(\ln k)$  reconstructed from CMB+LSS data using Chebyshev expansion  
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 and Markov Chain Monte Carlo Method. T/S consistency function is imposed  
 Probe of CMB+LSS window only 1-folds

epsilon\_nodal\_fixed\_ends15.powerspectrum.likestats



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## Bottom-up:

personal priors

Non-vanishing probability