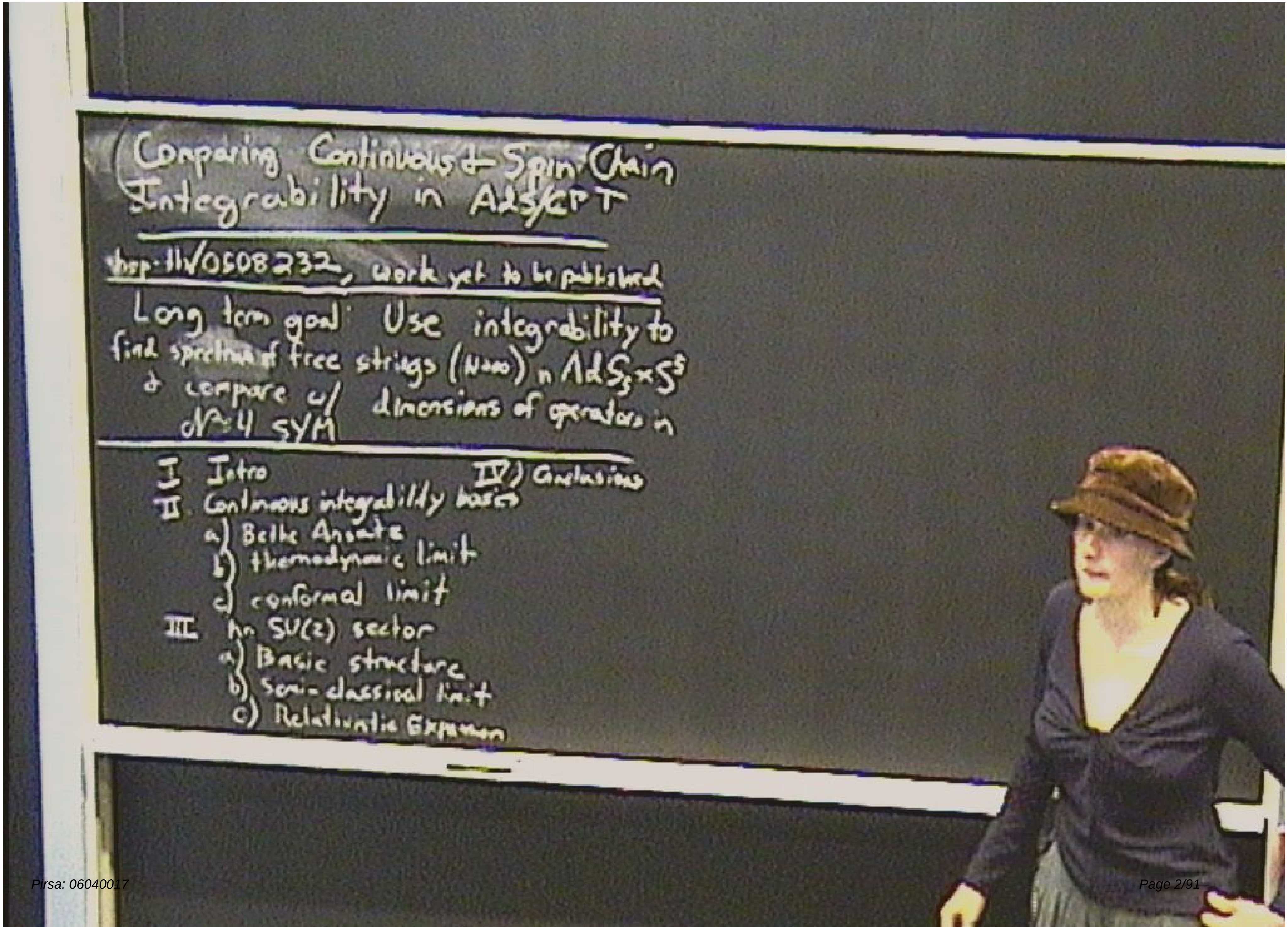
Title: Connections between Spin-Chain and continuous Integrability in AdS/CFT

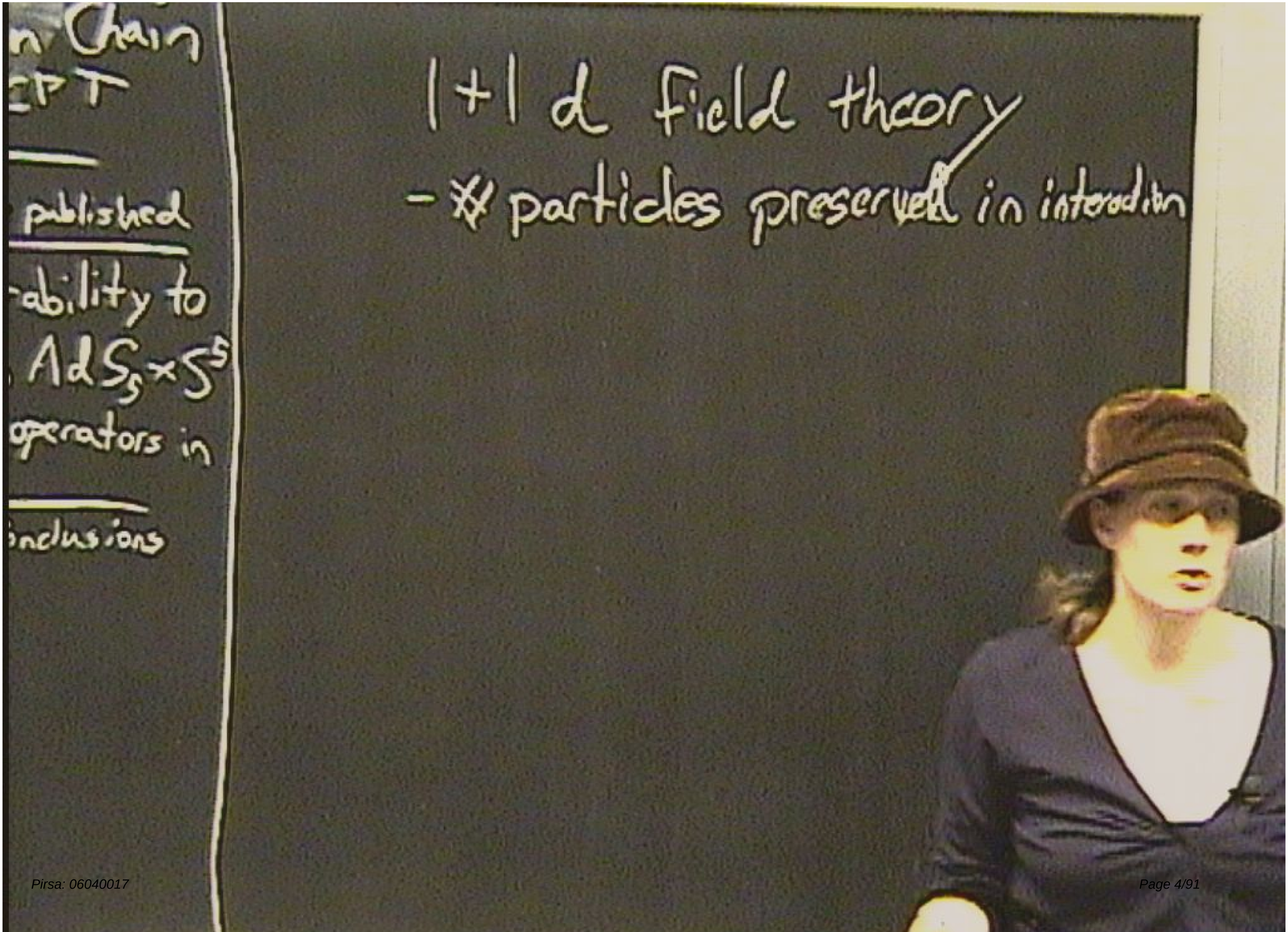
Date: Apr 18, 2006 02:00 PM

URL: http://pirsa.org/06040017

Abstract:


Comparing Continuous & Spin Chain
Integrability in AdS/CFT
hep-th/0508232, work yet to be published
Long term goal: Use integrability to find spectrum of free strings (N→∞) in AdS5×S5
& compare w/ dimensions of operators in N=4 SYM
I Intro
II Continuous integrability basics
a) Bethe Ansatz
b) thermodynamic limit
c) conformal limit
III In SU(2) sector
a) Basic structure
b) Semi-classical limit
c) Relativistic Expansion
IV) Conclusions


n Chain
CFT
published
ability to
AdS5×S5
operators in
nclusions
1+1 d field theory


n Chain
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ability to
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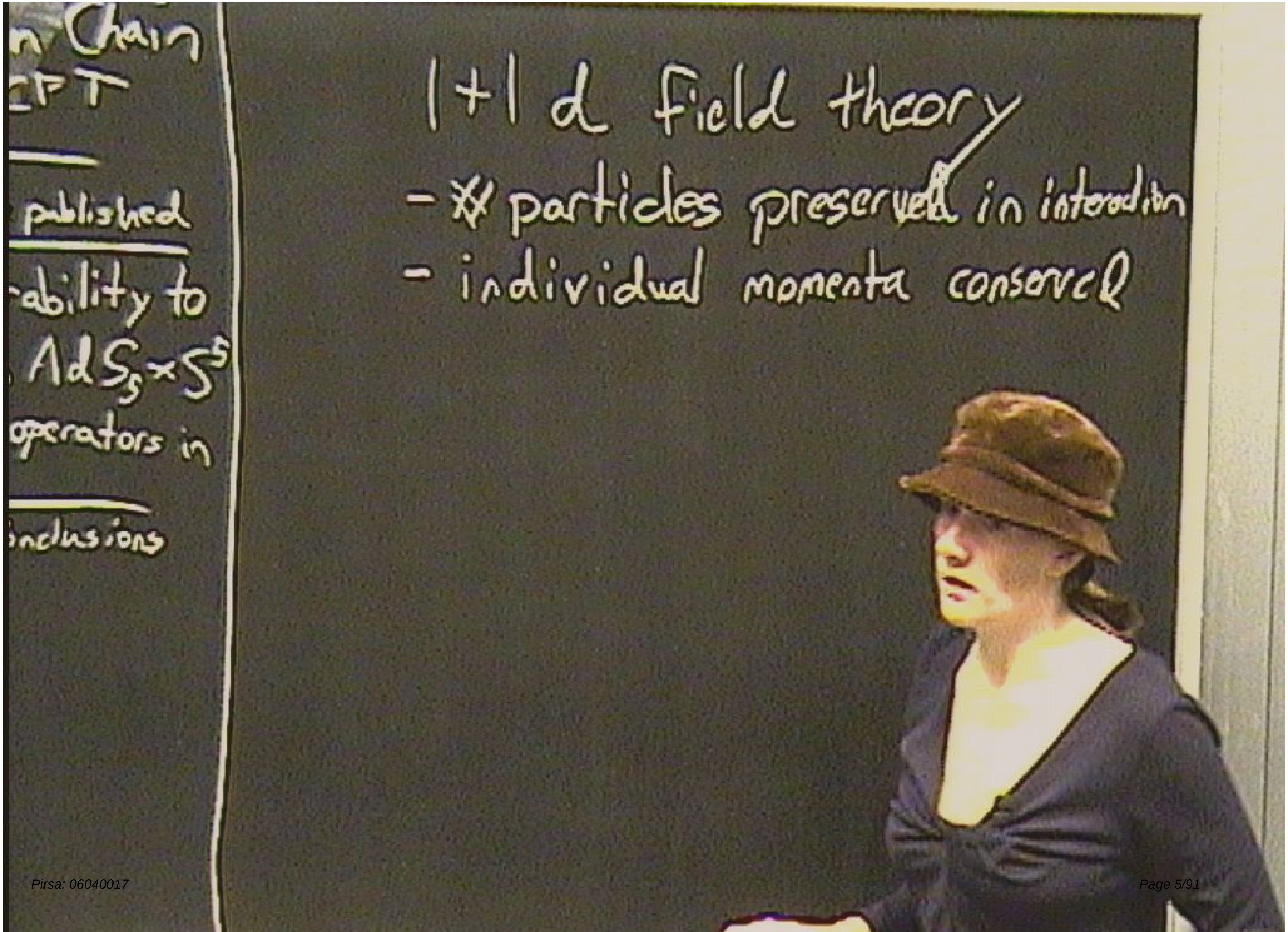
operators in
onclusions
1+1 d field theory
- # particles preserved in interaction


n Chain
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- # particles preserved in interaction
- individual momenta conserved

n Chain
CFT

published

ability to
$AdS_5 \times S^5$
operators in

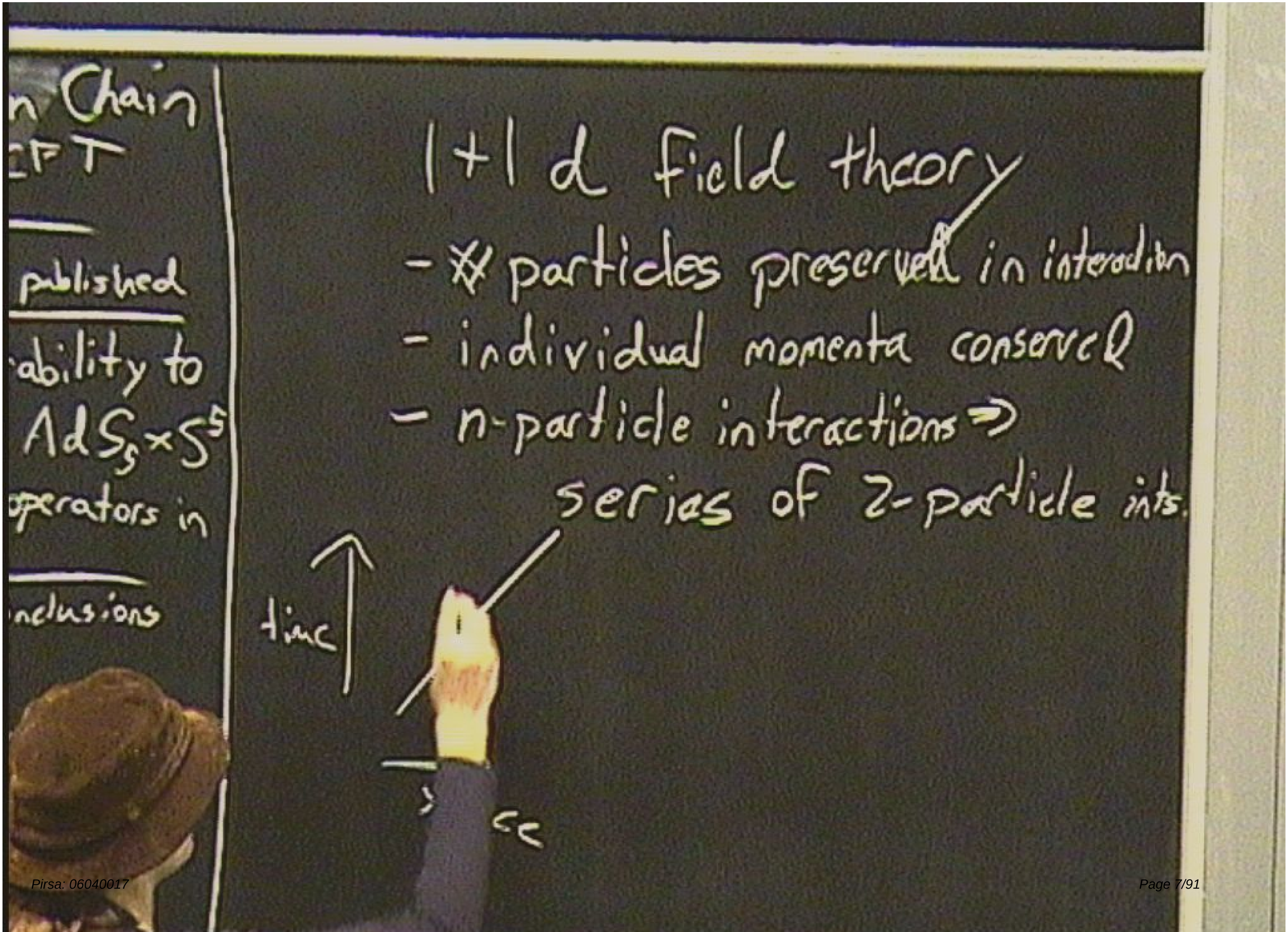
nclusions

1+1 d field theory

- ~~#~~ particles preserved in interaction
- individual momenta conserved
- n-particle interactions $\Rightarrow$
series of 2-particle ints.

time

space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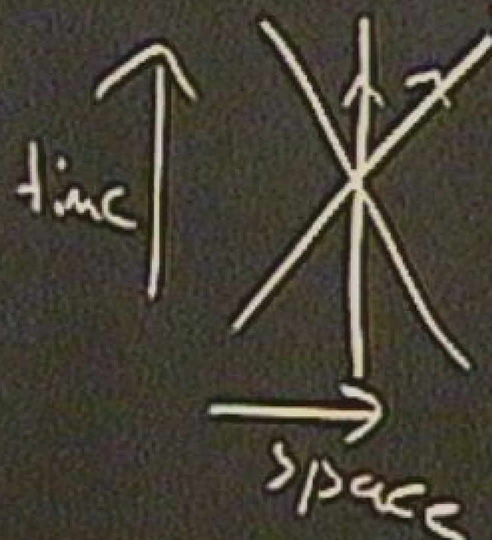

n Chain
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published

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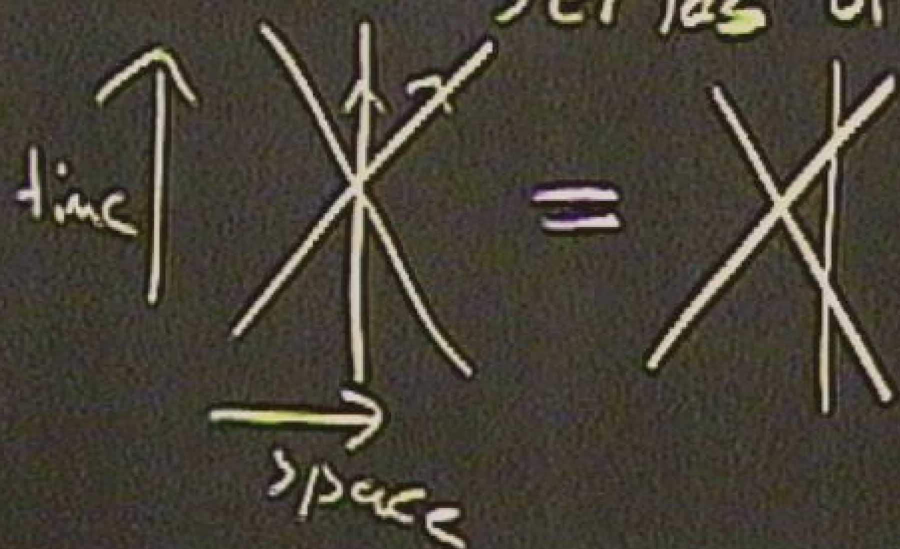

n Chain
EPT

published

ability to
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1+1 d field theory

- # particles preserved in interaction
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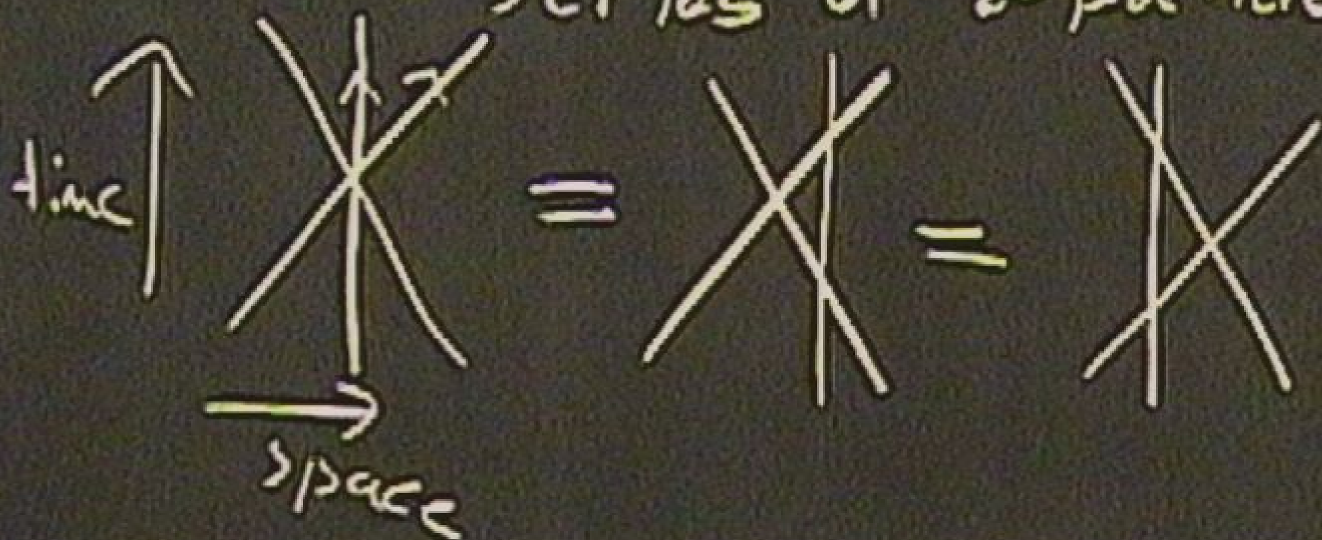
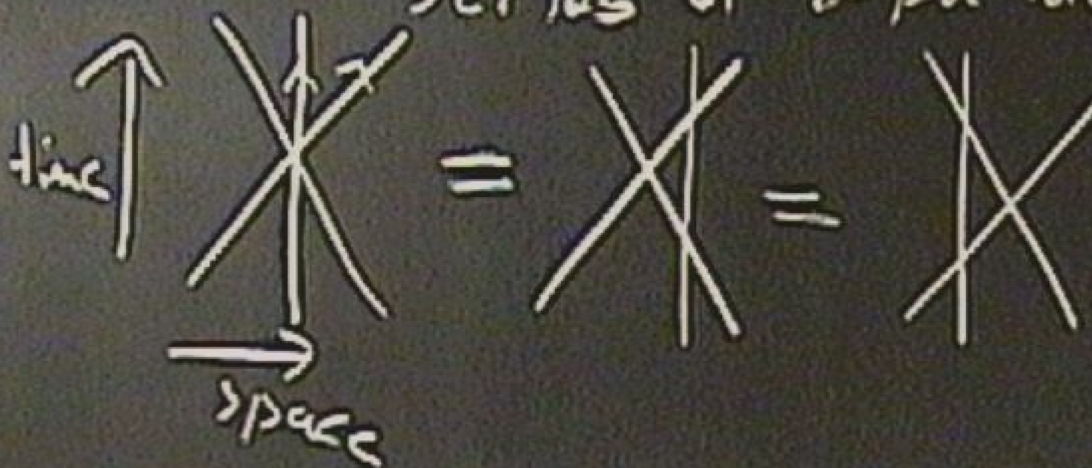


integrability to
N→∞) in $AdS_5 \times S^5$
ons of operators in

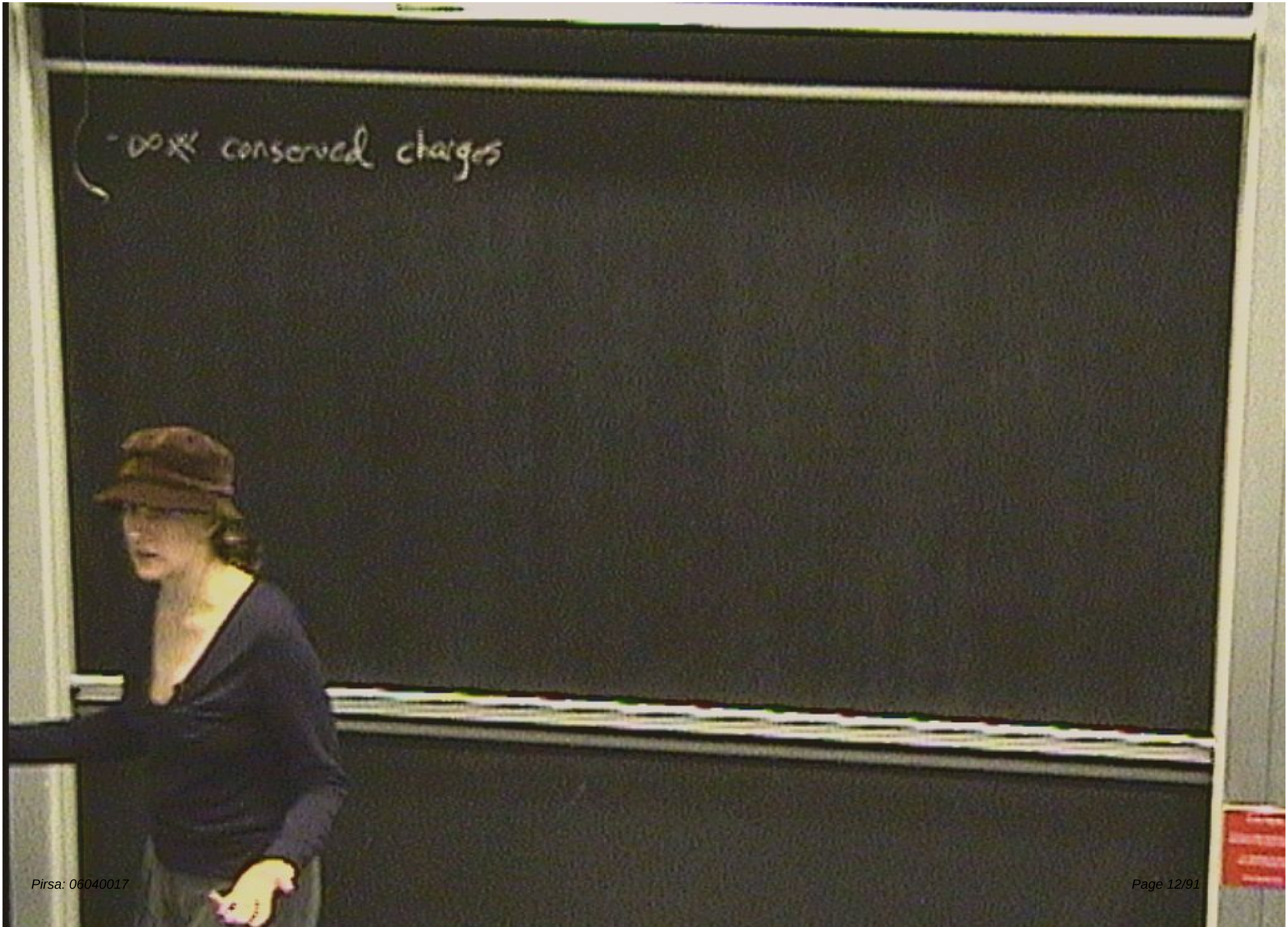
IV) Conclusions
ases

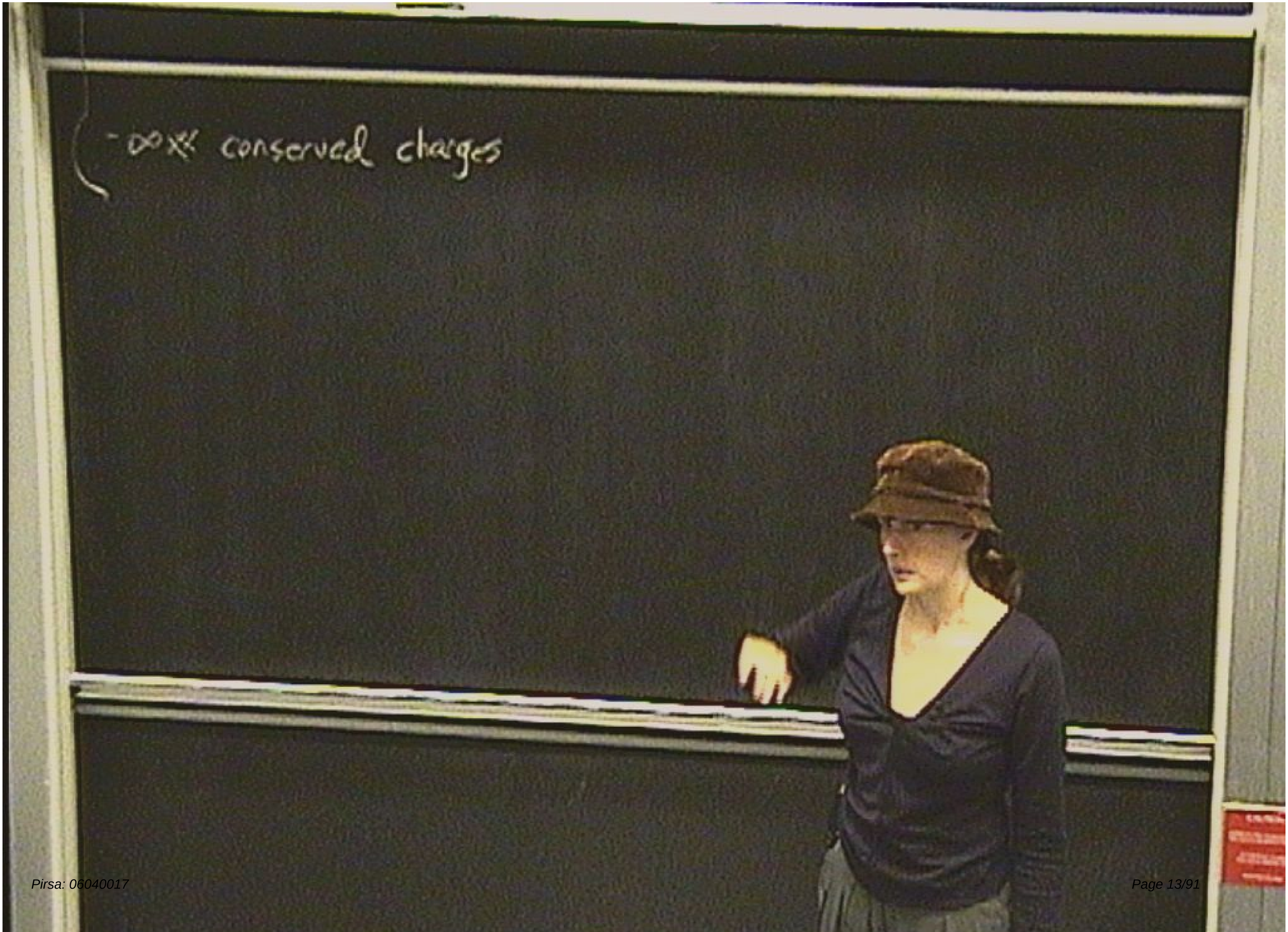
1+1 d field theory

- # particles preserved in interaction
- individual momenta conserved
- n-particle interactions ⇒ series of 2-particle ints



- S-matrix is "factorizeable"


- ∞ conserved charges



- ∞ly conserved charges

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- periodic dimension length L

set of identical bosons

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set of n identical bosons

$$e^{ip_i L} = \prod_{j \neq i} S(\theta_i - \theta_j) + \left(\substack{\text{corrections} \\ \mathcal{O}(1/L)} \right)$$

- ∞ # conserved charges
- periodic dimension length L

set of n identical bosons

$$e^{ip_i L} = \prod_{j \neq i} S(\theta_i - \theta_j) + \left(\begin{matrix} \text{corrections} \\ \mathcal{O}(1/L) \end{matrix} \right)$$

$p_i = m \sinh \theta_i$ = momentum of ith particle

- ... conserved charges
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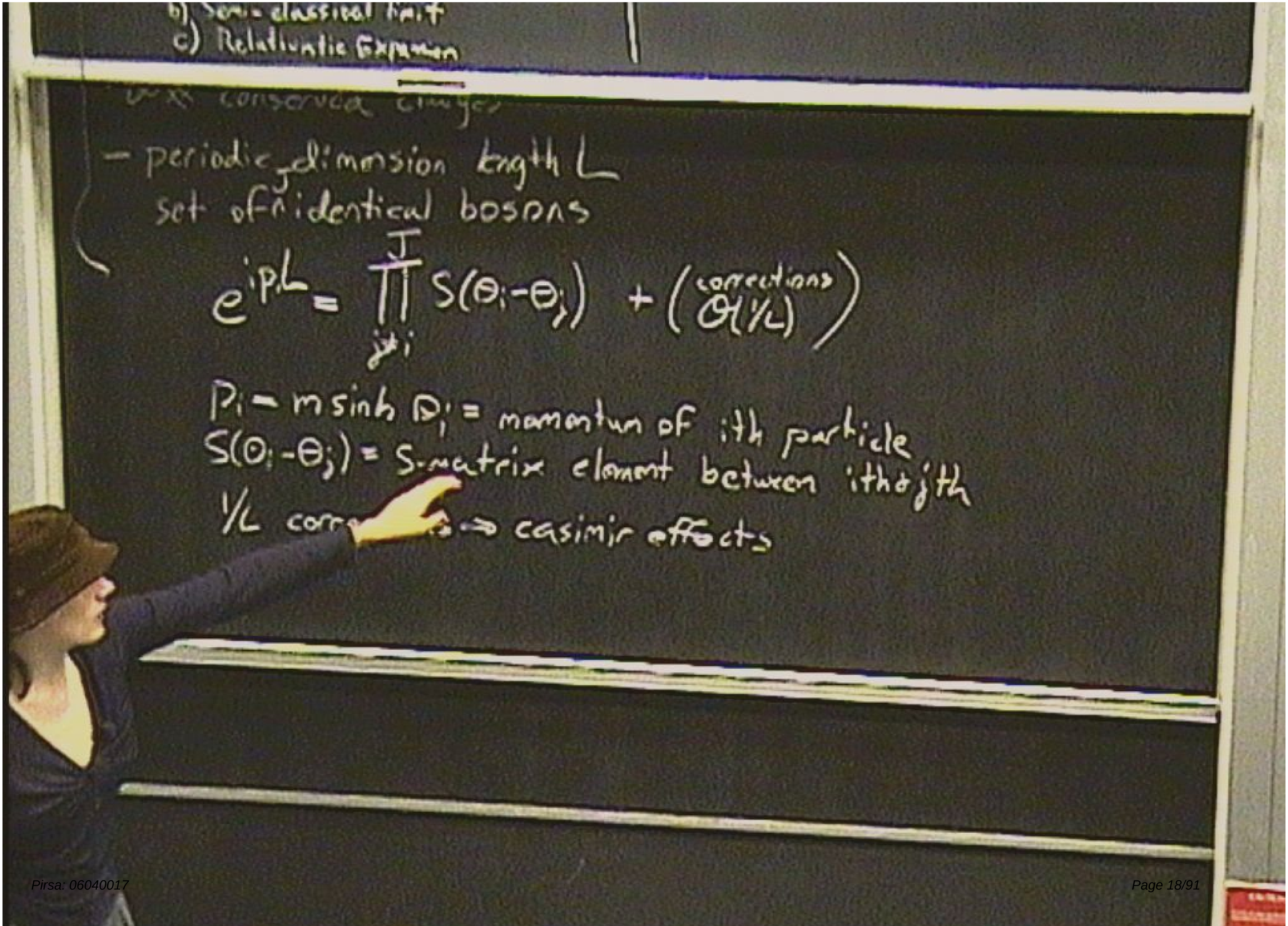
b) Semi-classical limit
c) Relativistic Expansion
conserved charges
- periodic dimension length L
set of n identical bosons
e^{ip_i L} = ∏_{j≠i} S(θ_i - θ_j) + (corrections O(1/L))
p_i = m sinh θ_i = momentum of ith particle
S(θ_i - θ_j) = S-matrix element between ith & jth
1/L corr ⇒ casimir effects

b) Semi-classical limit
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- ... conserved charges

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set of n identical bosons

$$e^{ip_i L} = \prod_{j \neq i}^{J} S(\theta_i - \theta_j) + \left(\begin{matrix} \text{corrections} \\ \mathcal{O}(1/L) \end{matrix} \right)$$

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$1/L$ corrections $\Rightarrow$ casimir effects

want $L \to \infty$

$$i p_i L = \sum_{j \neq i} \ln S(\theta_i - \theta_j) + 2\pi i n_i + \cdots$$

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$J \to \infty$

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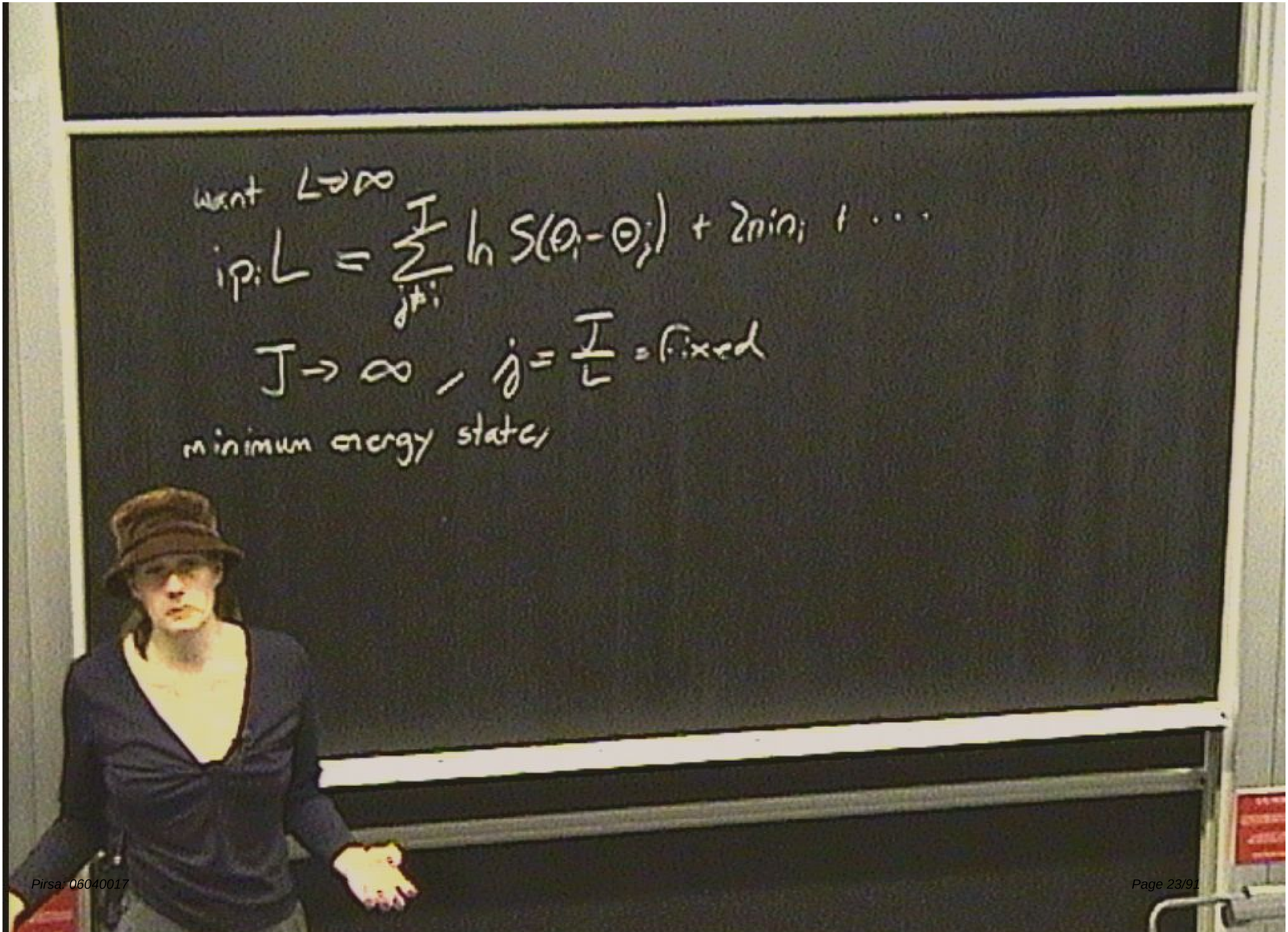
$$i p_i L = \sum_{j \neq i}^{I} \ln S(\theta_i - \theta_j) + 2\pi i n_i + \cdots$$

$$J \to \infty, \quad j = \frac{I}{L} = \text{fixed}$$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minimum energy states

want $L \to \infty$

$$i p_i L = \sum_{j \neq i}^{I} \ln S(\theta_i - \theta_j) + 2\pi i n_i + \dots$$

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minimum energy state, total momentum $= 0$

$$\Rightarrow n_i = n$$

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minimum energy states, total momentum $= 0$

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$$\frac{1}{L(\theta_{i+1} - \theta_i)} = \rho(\theta_i) = \text{density in rapidity space}$$

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minimum energy states, total momentum $= \emptyset$

$$\Rightarrow n_i = n$$

$$\frac{1}{L(\theta_{i+1} - \theta_i)} = \rho(\theta_i) = \text{density in rapidity space}$$

$$\rho(\theta) - \int_{-B}^{B} K(\theta - \theta')\rho(\theta')\,d\theta' = m\cosh\theta$$

$$K(\theta) = \frac{1}{2\pi i}\,\partial_\theta \ln S(\theta)$$

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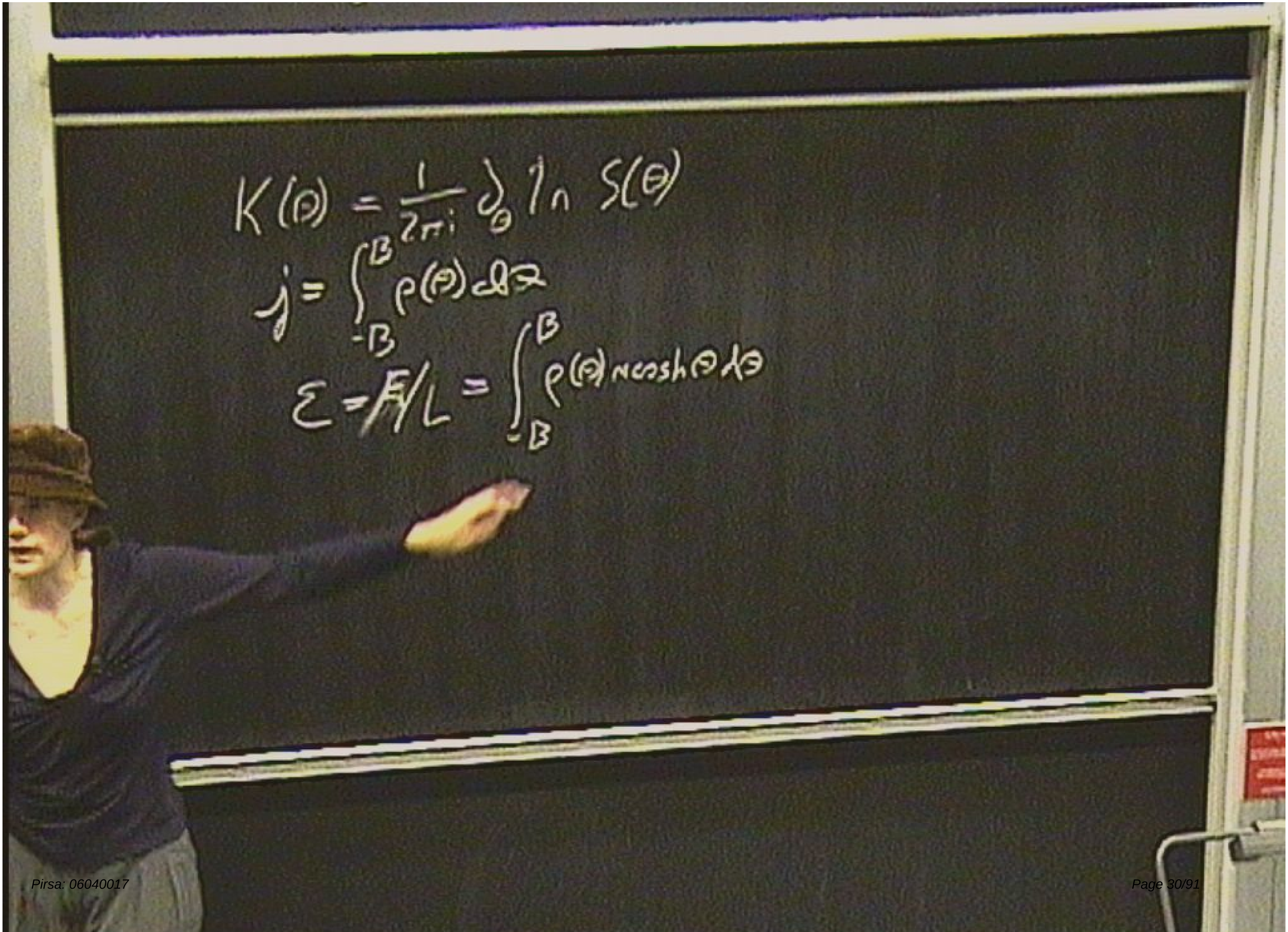
$$j = \int_{-B}^{B} \rho(\theta)\, d\theta$$


$K(\theta) = \frac{1}{2\pi i}\partial_\theta \ln S(\theta)$
$j = \int_{-B}^{B} \rho(\theta)\, d\theta$
$\mathcal{E} = \cancel{E}/L = \int_{-B}^{B} \rho(\theta)\, m\cosh\theta\, d\theta$

$$K(\theta) = \frac{1}{2\pi i}\partial_\theta \ln S(\theta)$$

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$-B < \theta < B$ range of allowed rapidities

"filled Fermi Sea"

$$K(\theta) = \frac{1}{2\pi i}\,\partial_\theta \ln S(\theta)$$

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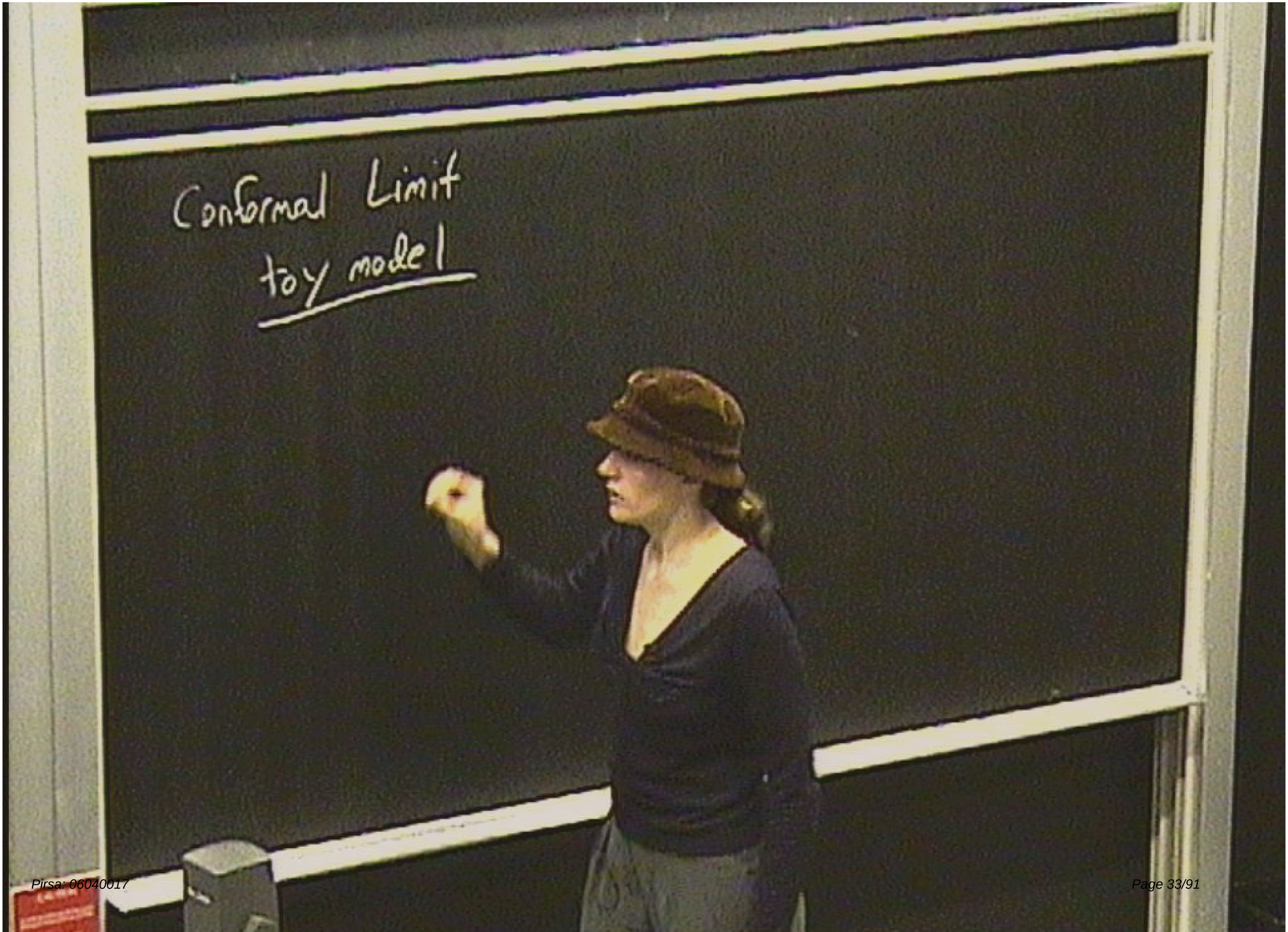
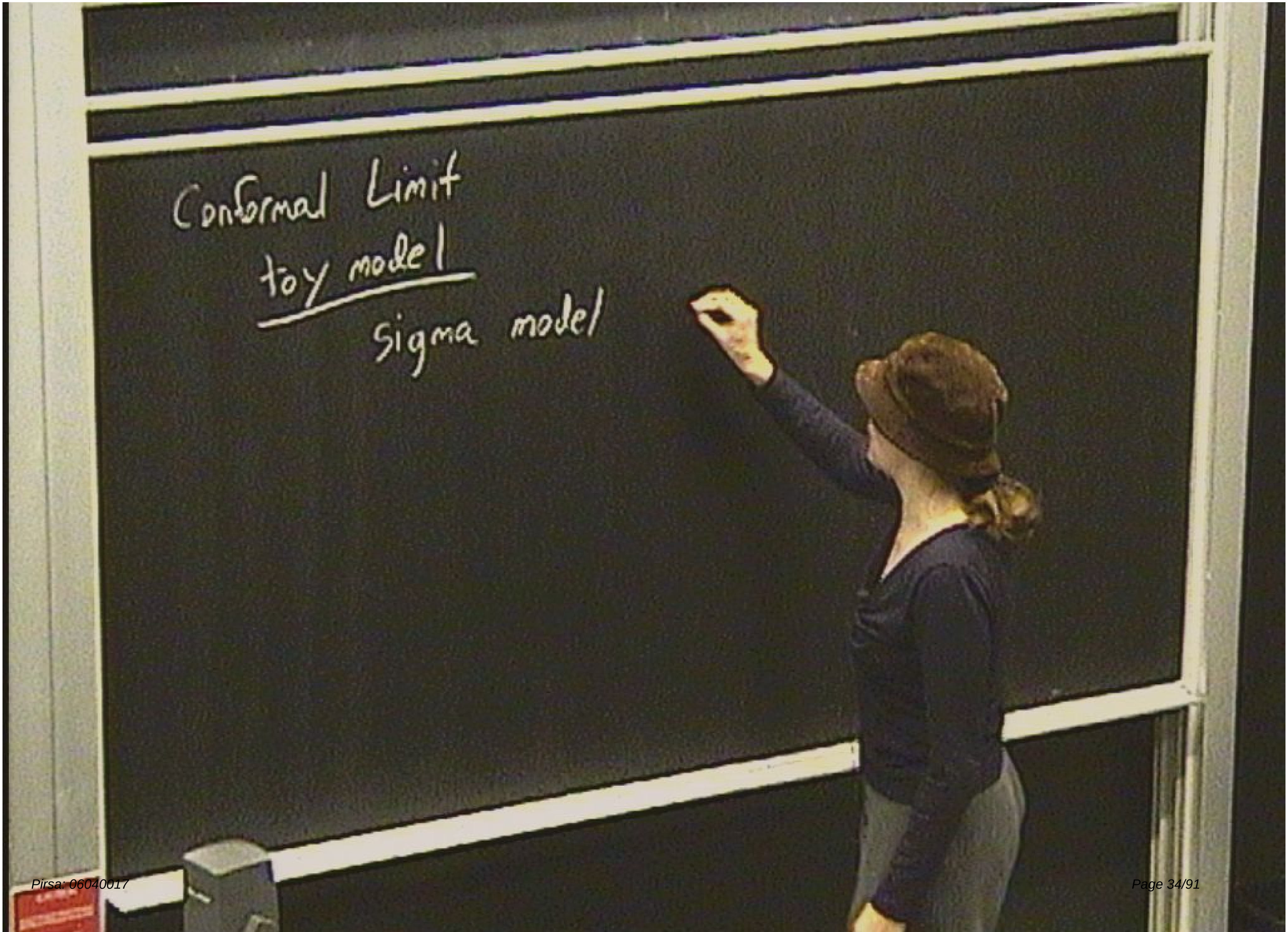
$$\mathcal{E} = E/L = \int_{-B}^{B} \rho(\theta)\, m\cosh\theta\, d\theta$$

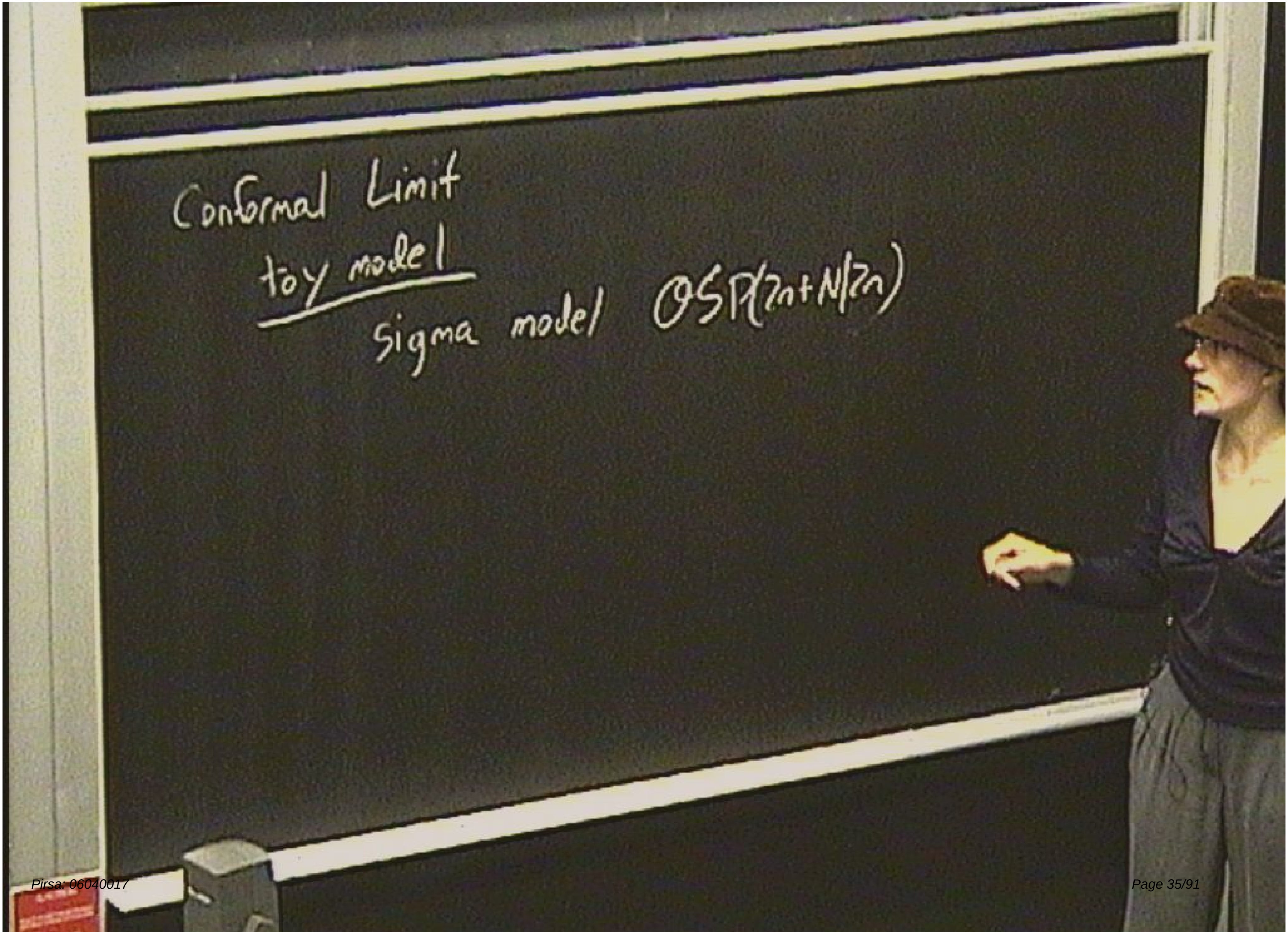
$-B < \theta < B$ range of allowed rapidities

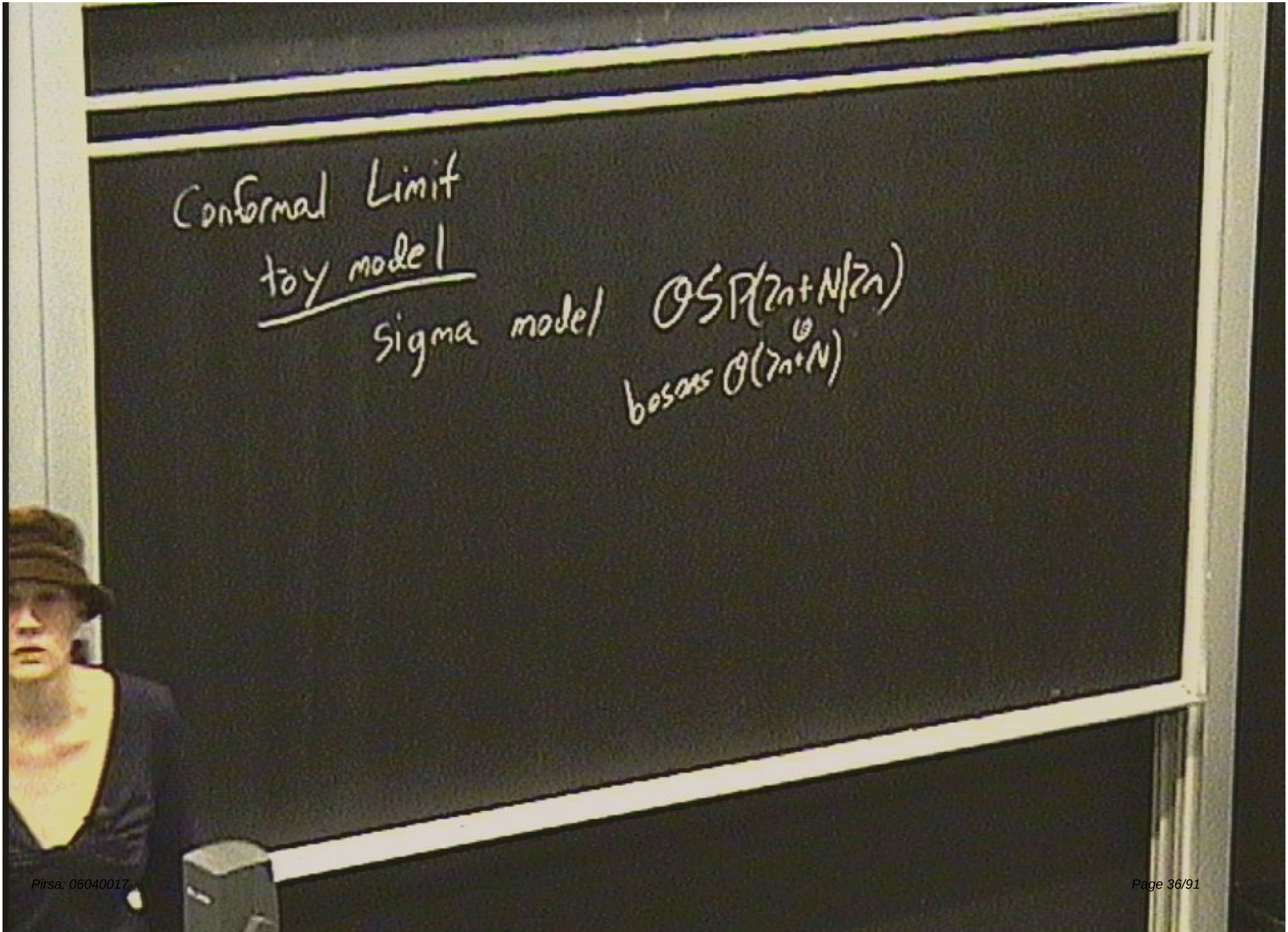
"filled Fermi Sea"

more excited state $\Rightarrow$ more interesting range


Conformal Limit
toy model


Conformal Limit
toy model
sigma model


Conformal Limit
toy model
sigma model OSP(2n+N|2n)


Conformal Limit
toy model
sigma model OSP(2n+N|2n)
bosons O(2n+N)

Conformal Limit

toy model

sigma model $OSP(2n+N|2n) \xrightarrow{N \to 2} OSP(2n+2|2n)$

bosons $O(2n+N)$

is conformal thy

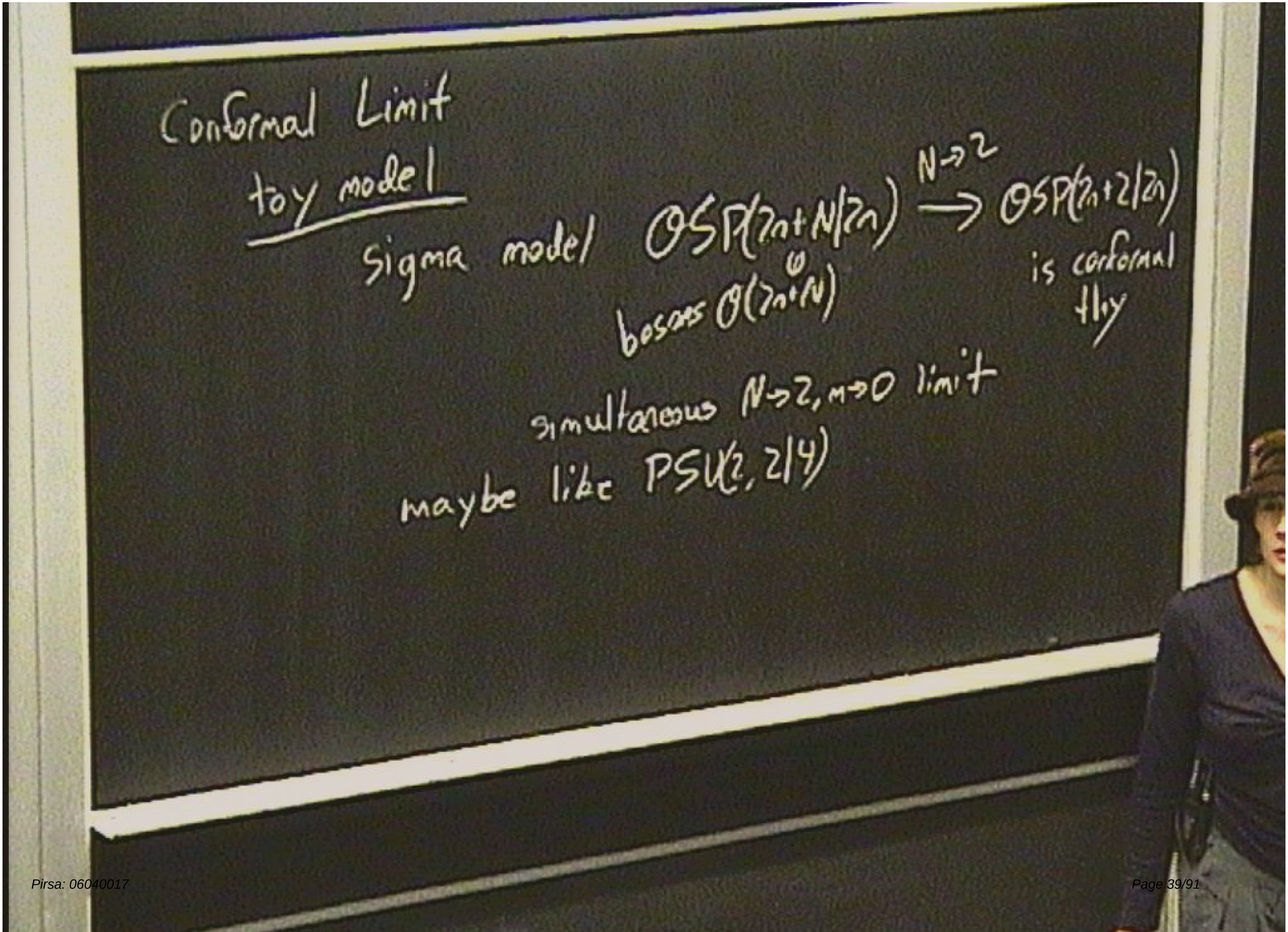
Conformal Limit

toy model

sigma model $OSP(2n+N|2n) \xrightarrow{N\to 2} OSP(2n+2|2n)$ is conformal thy

bosons $O(2n+N)$

simultaneous $N\to 2, n\to 0$ limit


Conformal Limit
toy model
Sigma model OSP(2n+N|2n) N→2 → OSP(2n+2|2n)
is conformal thy
bosons O(2n+N)
simultaneous N→2, n→0 limit
maybe like PSU(2,2|4)

Conformal Limit

toy model

sigma model $OSP(2n+N|2n) \xrightarrow{N\to 2} OSP(2n+2|2n)$ is conformal thy

bosons $O(2n+N)$

simultaneous $N\to 2, m\to 0$ limit

maybe like $PSU(2,2|4)$

semi-classical, $O(4) \in OSP$

$S^3 \in AdS_5 \times S^5$

$$\rho(\theta) - \int_{-B} K(\theta-\theta')\rho(\theta')d\theta' = m\cosh\theta$$

$$K(\theta) = \frac{1}{2\pi i}\partial_\theta \ln S(\theta)$$

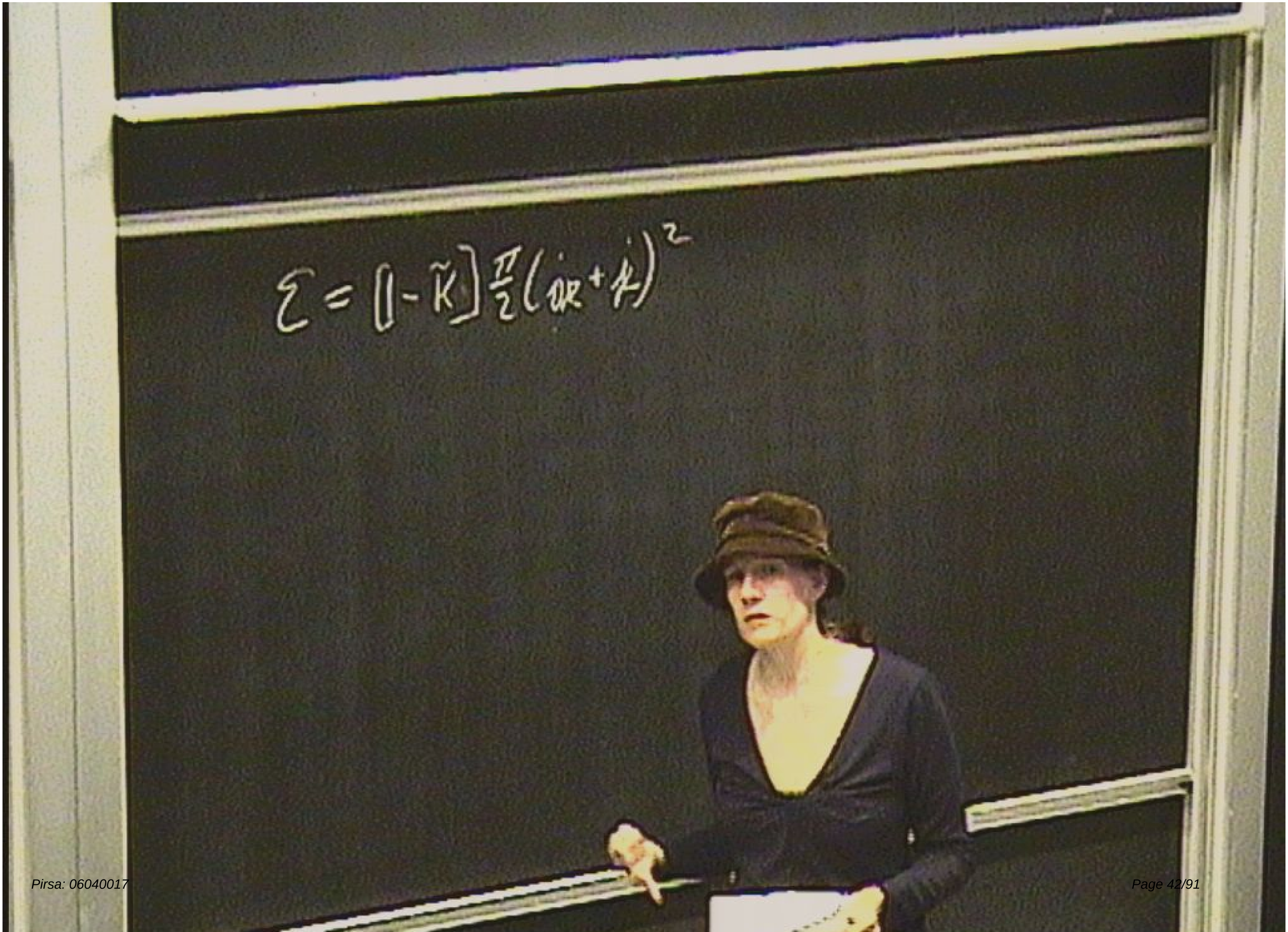
$$j = \int_{-B}^{B} \rho(\theta)\,d\theta$$

$$\mathcal{E} = E/L = \int_{-B}^{B} \rho(\theta)\, m\cosh\theta\, d\theta$$

$-B < \theta < B$ range of allowed rapidities

"filled Fermi Sea"

more excited state ⇒ more interesting range


$\xi = [1-\tilde{K}]\frac{\pi}{2}(\dot{a}R+\dot{r})^2$

$$\mathcal{E} = [1-\tilde{K}]\frac{\pi}{2}(j_R + j_L)^2$$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = j_R\,K(x-\phi) + j_L\,K(x+\phi)$$

$$x = (N-2)\ln\frac{\Lambda}{m}$$

$$\mathcal{E} = [1 - \tilde{K}] \frac{\pi}{2} (\dot{j}_R + \dot{j}_L)^2$$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi - \phi') \rho(\phi') d\phi' = \dot{j}_R K(x - \phi) + \dot{j}_L K(x + \phi)$$

$$\dot{j} = \dot{j}_R + \dot{j}_L + \int_{-x}^{x} \rho(\phi) d\phi$$

$$x = (N-2) \ln \frac{\Lambda}{m}$$

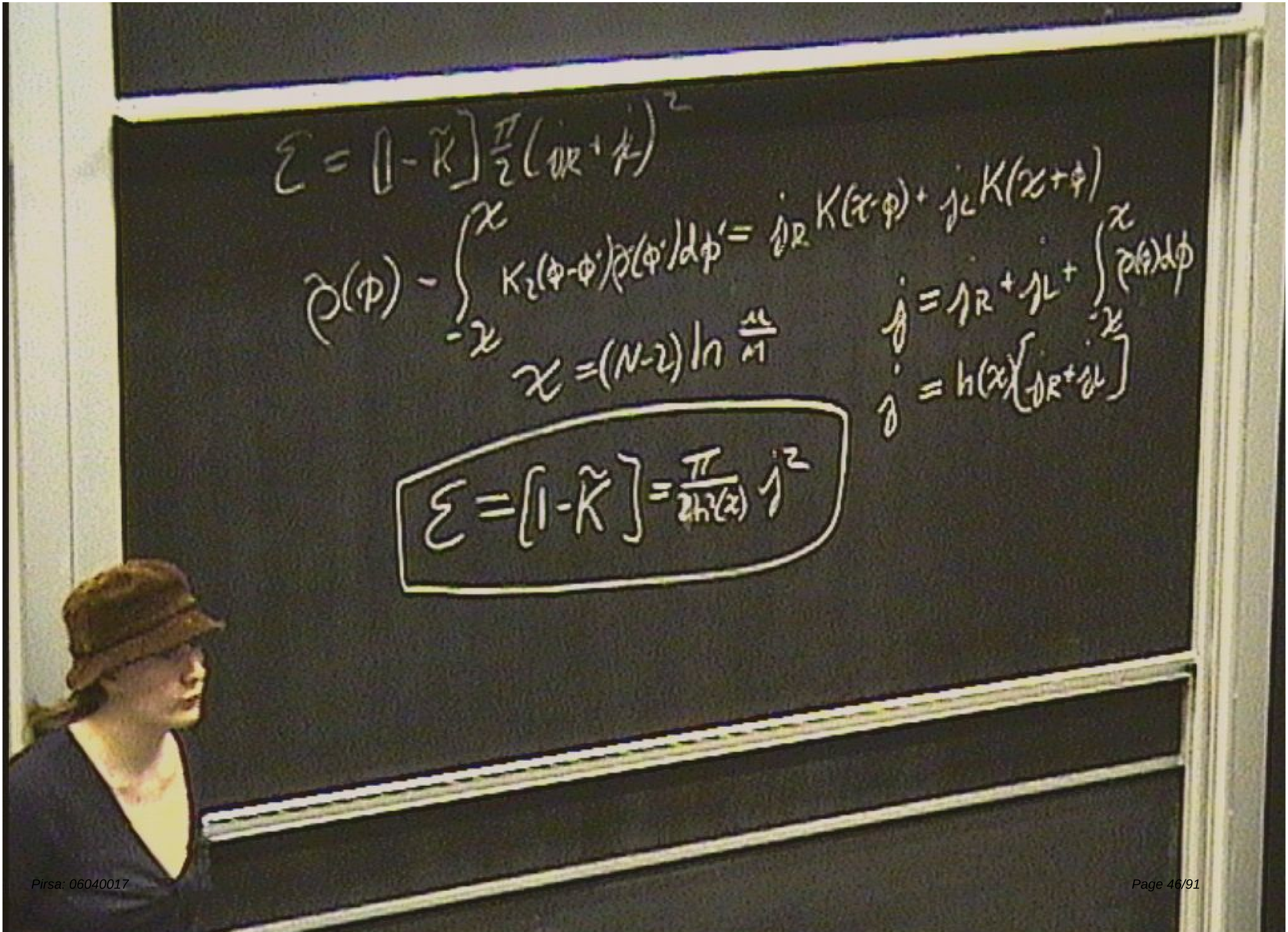
$$\mathcal{E} = [1-\tilde{K}]\,\frac{\pi}{2}\left(\dot{j}_R + \dot{j}_L\right)^2$$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = \dot{j}_R\, K(x-\phi) + \dot{j}_L\, K(x+\phi)$$

$$x = (N-2)\ln\frac{\mu}{m}$$

$$\dot{j} = \dot{j}_R + \dot{j}_L + \int_{-x}^{x}\rho(\phi)\,d\phi$$

$$= h(x)\left(\dot{j}_R + \dot{j}_L\right)$$



E = [1 - K̃] π/2 (j_R + j_L)^2
ρ(φ) - ∫_{-x}^{x} K_2(φ-φ')ρ(φ')dφ' = j_R K(x-φ) + j_L K(x+φ)
j = j_R + j_L + ∫_{-x}^{x} ρ(φ)dφ
x = (N-2) ln
j = h(x)[j_R + j_L]
E = [1 - K̃] = π/(2h^2(x)) j^2

$$\mathcal{E} = [1-\tilde{K}]\frac{\pi}{2}(\dot{j}_R + \dot{j}_L)^2$$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = \dot{j}_R K(x-\phi) + \dot{j}_L K(x+\phi)$$

$$\dot{j} = \dot{j}_R + \dot{j}_L + \int_{-x}^{x}\rho(\phi)\,d\phi$$

$$x = (N-2)\ln\frac{\mu}{m}$$

$$\dot{j} = h(x)[\dot{j}_R + \dot{j}_L]$$

$$\boxed{\mathcal{E} = [1-\tilde{K}] = \frac{\pi}{2h^2(x)}\dot{j}^2}$$

find relationship b/w x + λ

$$\mathcal{E} = [1-\tilde{K}]\,\frac{\pi}{2}\left(\dot{j}_R + \dot{j}_L\right)^2$$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = \dot{j}_R K(x-\phi) + \dot{j}_L K(x+\phi)$$

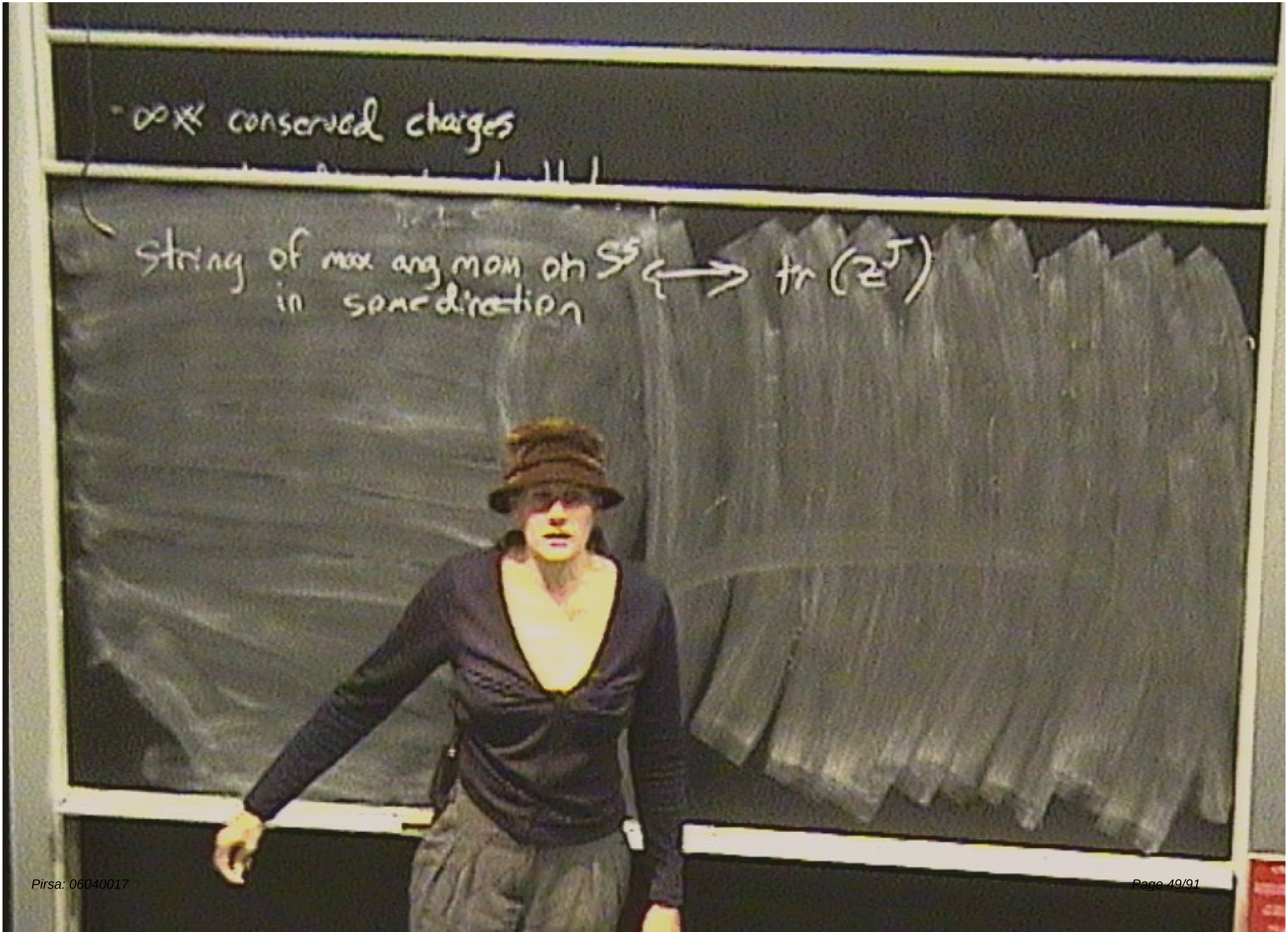
$$\dot{j} = \dot{j}_R + \dot{j}_L + \int_{-x}^{x} \rho(\phi)\,d\phi$$

$$x = (N-2)\ln\frac{\mu}{M}$$

$$\dot{j} = h(x)\left[\dot{j}_R + \dot{j}_L\right]$$

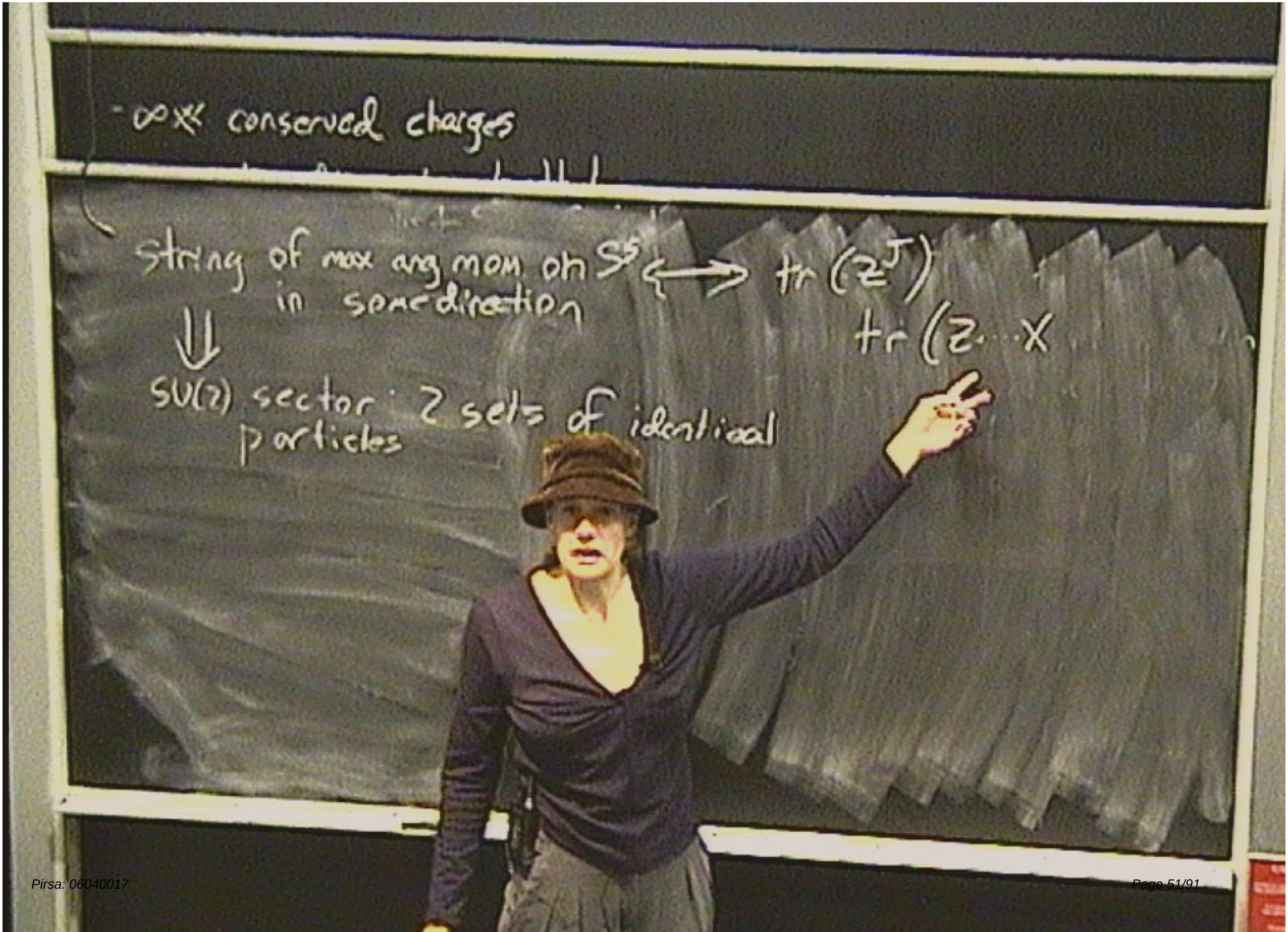
$$\boxed{\mathcal{E} = [1-\tilde{K}] = \frac{\pi}{2h^2(x)}\,\dot{j}^2}$$

find relationship b/w x & λ


∞ conserved charges
string of max ang mom on S5 ⟷ tr(Z^J)
in some direction

- ∞ conserved charges

String of max ang mom on S^5 $\longleftrightarrow$ $\mathrm{tr}(Z^J)$
in some direction



∞ conserved charges
string of max ang mom on S5 in some direction
tr(ZJ)
tr(Z ··· X
SU(2) sector: 2 sets of identical particles

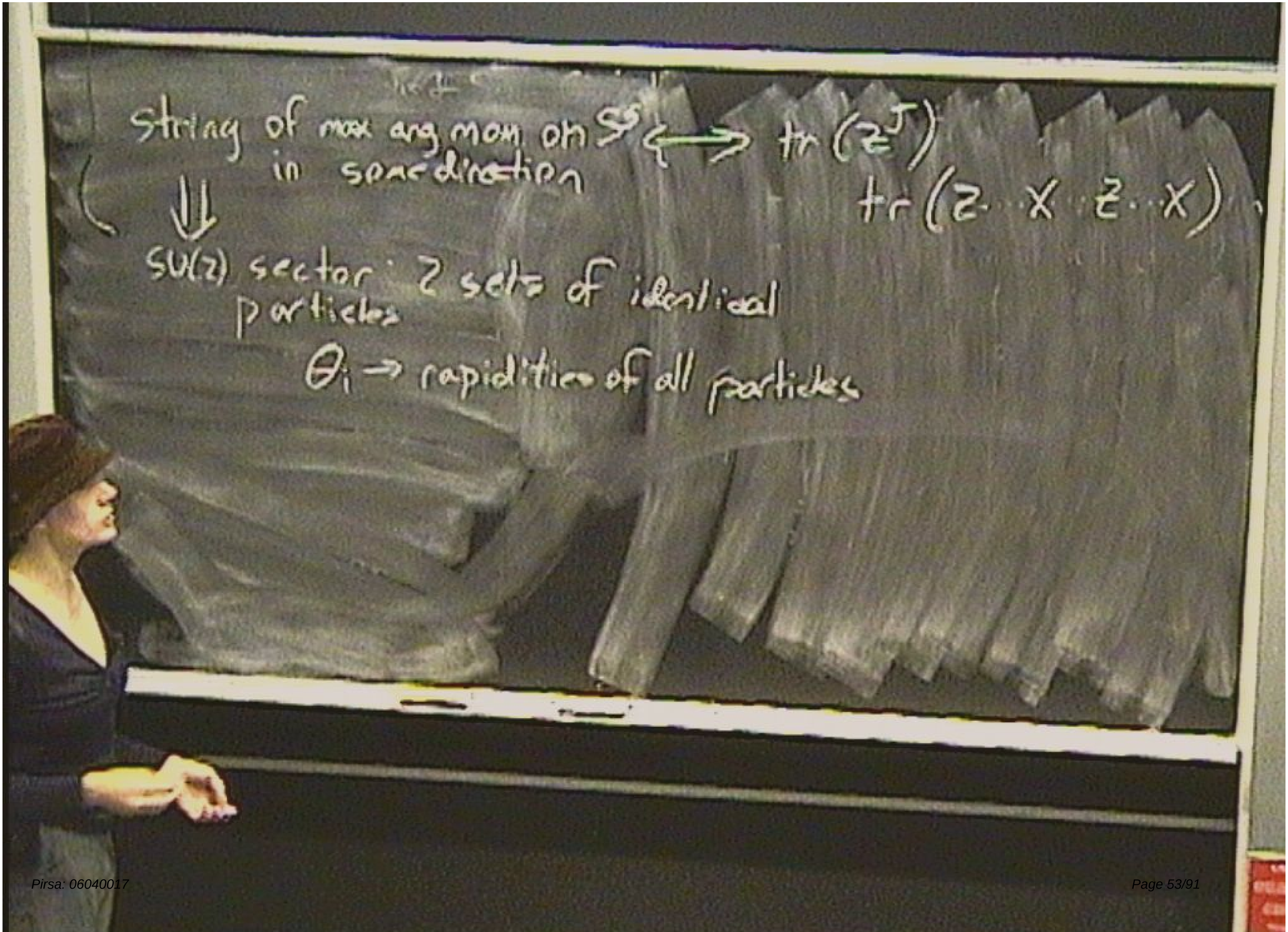
- ∞ #s conserved charges

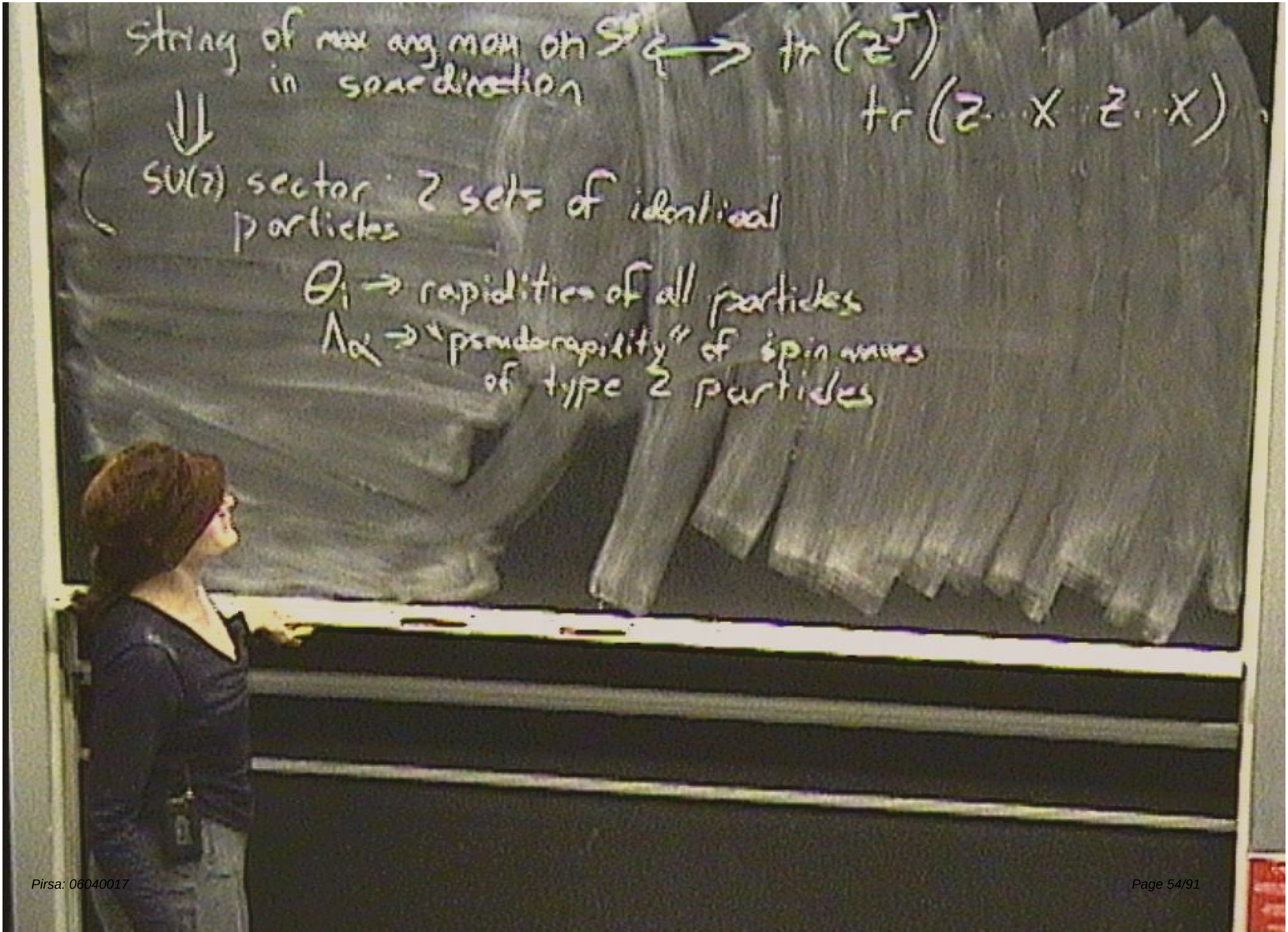
string of max ang mom on S^5 $\longleftrightarrow$ $\mathrm{tr}(Z^J)$
in some direction

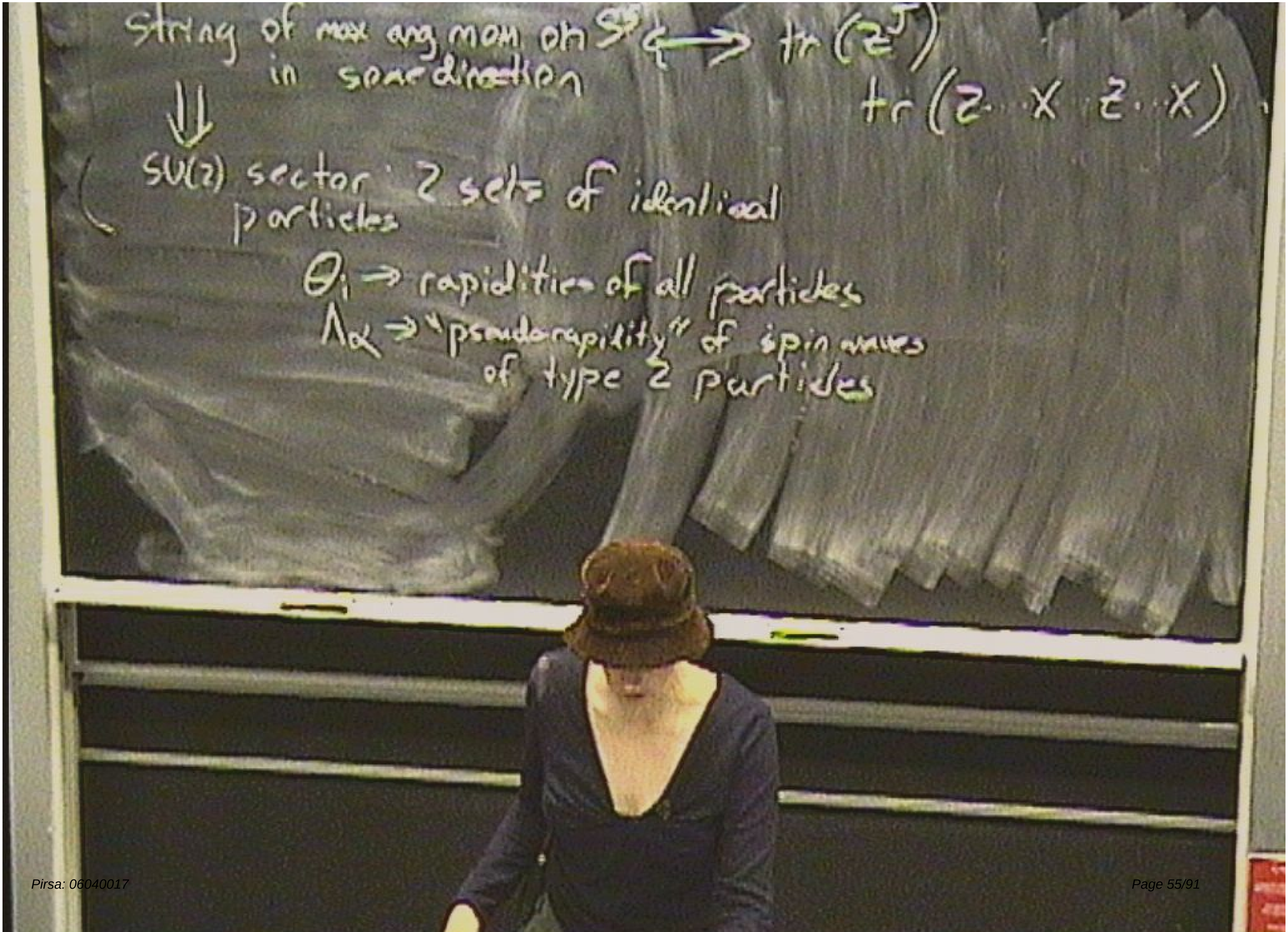
$\Downarrow$

$SU(2)$ sector: 2 sets of identical particles

$\mathrm{tr}(Z \cdots X \cdots Z \cdots X)$


String of max ang mom on S5 ⟷ tr (Z^J)
in space direction
tr (Z··X··Z··X)
⇓
SU(2) sector: 2 sets of identical particles
θi → rapidities of all particles


string of max ang mom on S^5 $\Longleftrightarrow$ $tr(Z^J)$
in some direction
$tr(Z \cdots X \cdots Z \cdots X)$
$\Downarrow$
SU(2) sector: 2 sets of identical particles
$\theta_i \Rightarrow$ rapidities of all particles
$\Lambda_\alpha \Rightarrow$ "pseudorapidity" of spin waves of type 2 particles


string of max ang mom on S^5 $\Longleftrightarrow$ $tr(Z^J)$
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$tr(Z \cdots X \cdots Z \cdots X)$
$\Downarrow$
SU(2) sector: 2 sets of identical particles
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string of max ang mom on S^5 $\longleftrightarrow$ $tr(Z^J)$
in some direction

$tr(Z \cdots X \cdots Z \cdots X)$

$\Downarrow$

SU(2) sector: 2 sets of identical particles

$\theta_i \to$ rapidities of all particles

$\Lambda_\alpha \to$ "pseudorapidity" of spin waves of type 2 particles

$$e^{ip_i L} = \left\{ \prod_{j=1}^{J} S(\theta_i - \theta_j) \right\} \prod_{\alpha=1}^{J_2} \frac{i\Lambda_\alpha - i\theta_i + \frac{\pi}{2}}{i\Lambda_\alpha - i\theta_i - \frac{\pi}{2}}$$

$$\prod^{J} \frac{i\Lambda_\alpha - i\theta_i + \frac{\pi}{2}}{\quad}$$

collisions casimir effects

string of max ang mom on S^5 $\longleftrightarrow$ $\mathrm{tr}(Z^J)$
in some direction

$\mathrm{tr}(Z \cdots X \cdots Z \cdots X)$

$\Downarrow$

SU(2) sector: 2 sets of identical particles

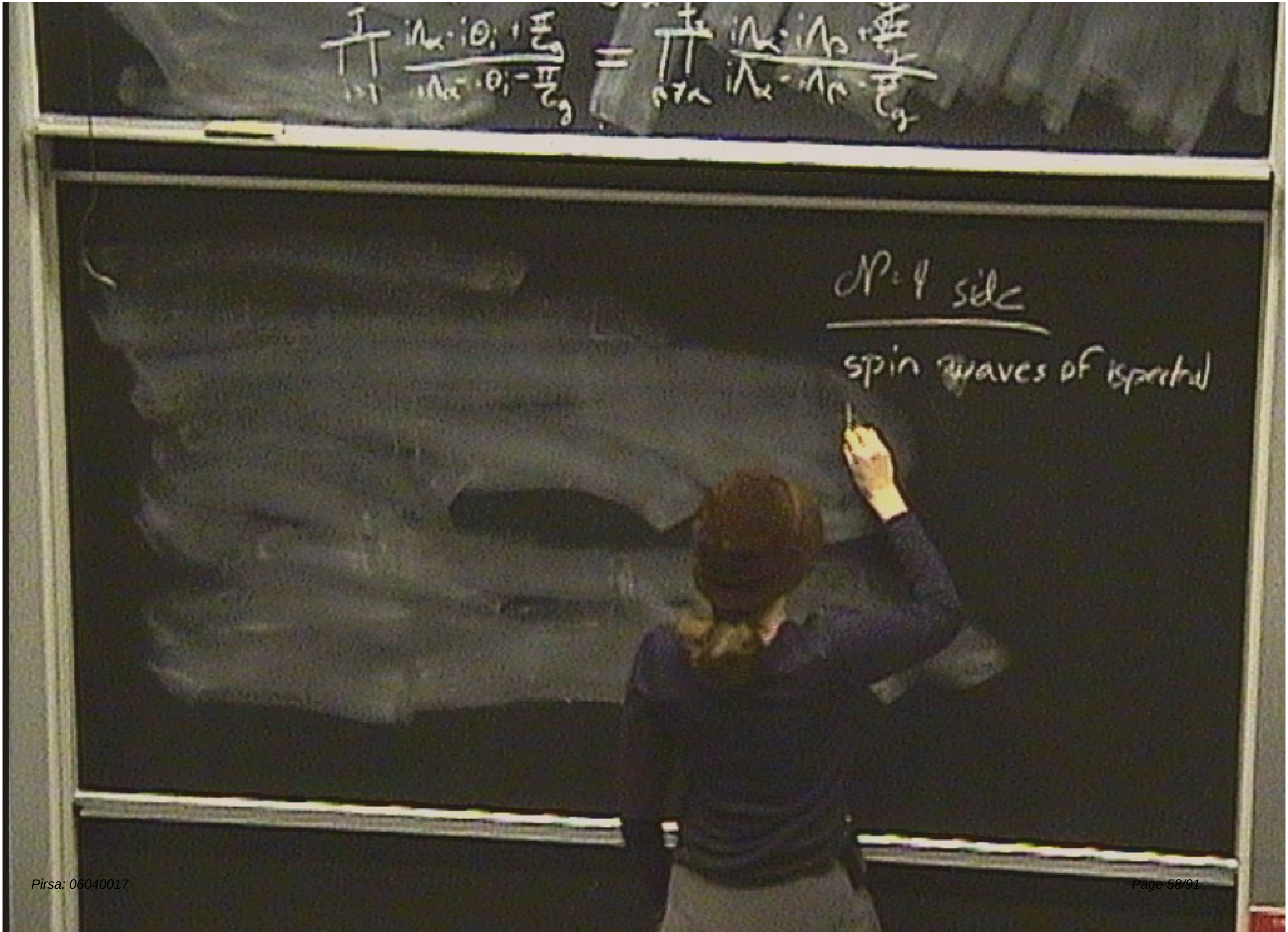
$\theta_i \to$ rapidities of all particles

$\Lambda_\alpha \to$ "pseudo rapidity" of spin waves of type 2 particles

$$e^{ip_i L} = \left\{ \prod_{i=1}^{J} S(\theta_i - \theta_j) \right\} \prod_{\alpha=1} \frac{i\Lambda_\alpha - i\theta_i + \frac{\pi}{2g}}{i\Lambda_\alpha - i\theta_i - \frac{\pi}{2g}}$$

$$\prod_{i=1}^{J} \frac{i\Lambda_\alpha - i\theta_i + \frac{\pi}{2g}}{i\Lambda_\alpha - i\theta_i - \frac{\pi}{2g}} = \prod_{\beta \neq \alpha} \frac{i\Lambda_\alpha - i\Lambda_\beta + \frac{\pi}{g}}{i\Lambda_\alpha - i\Lambda_\beta - \frac{\pi}{g}}$$

1/L corrections $\to$ casimir effects


$\mathcal{N}=4$ side
spin waves of isospectral

$$\prod_{j=1} \frac{i\lambda_\alpha - i\theta_j + \frac{\xi}{2}}{i\lambda_\alpha - i\theta_j - \frac{\xi}{2}} = \prod_{\beta\neq\alpha} \frac{i\lambda_\alpha - i\lambda_\beta + \frac{\xi}{2}}{i\lambda_\alpha - i\lambda_\beta - \frac{\xi}{2}}$$

$\mathcal{N}=4$ side

spin waves of spectral parameters u_α

$\alpha = 1, \ldots, J_2$

$$\prod_{i=1}^{J} \frac{i\Lambda_\alpha - i\Theta_i + \frac{\pi}{2g}}{i\Lambda_\alpha - i\Theta_i - \frac{\pi}{2g}} = \prod_{\beta\neq\alpha} \frac{i\Lambda_\alpha - i\Lambda_\beta + \frac{\pi}{g}}{i\Lambda_\alpha - i\Lambda_\beta - \frac{\pi}{g}}$$

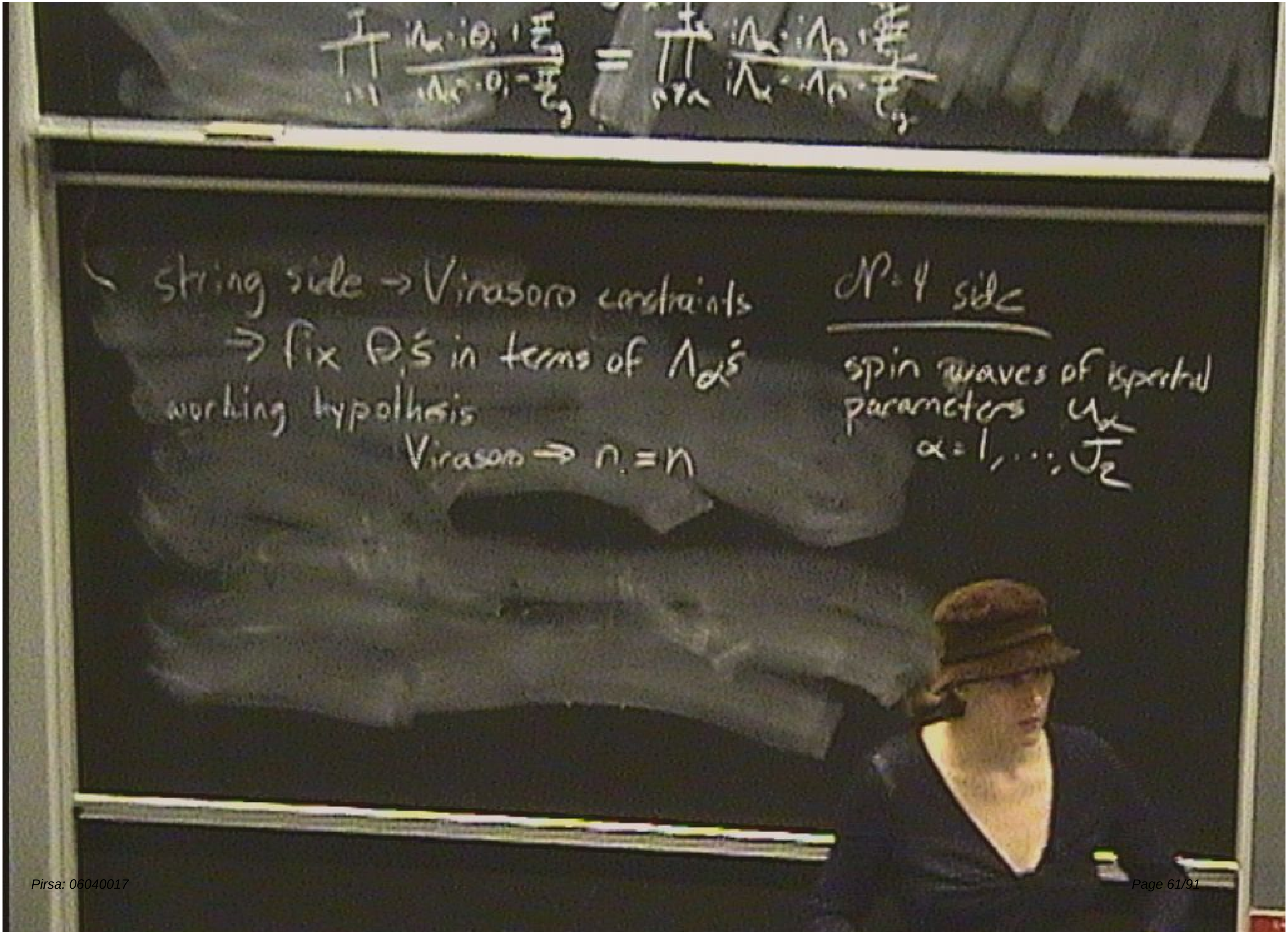
string side → Virasoro constraints

⇒ fix Θ_i's in terms of Λ_α's

$\mathcal{N}=4$ side

spin waves of spectral parameters u_α

$\alpha = 1, \ldots, J_2$


string side → Virasoro constraints
⇒ fix Θ_i's in terms of Λ_α's
working hypothesis
Virasoro ⇒ $n_i = n$
$\mathcal{N}=4$ side
spin waves of spectral parameters u_α
$\alpha = 1, \ldots, J_2$

string side $\Rightarrow$ Virasoro constraints

$\Rightarrow$ fix Θ's in terms of Λ_α's

working hypothesis

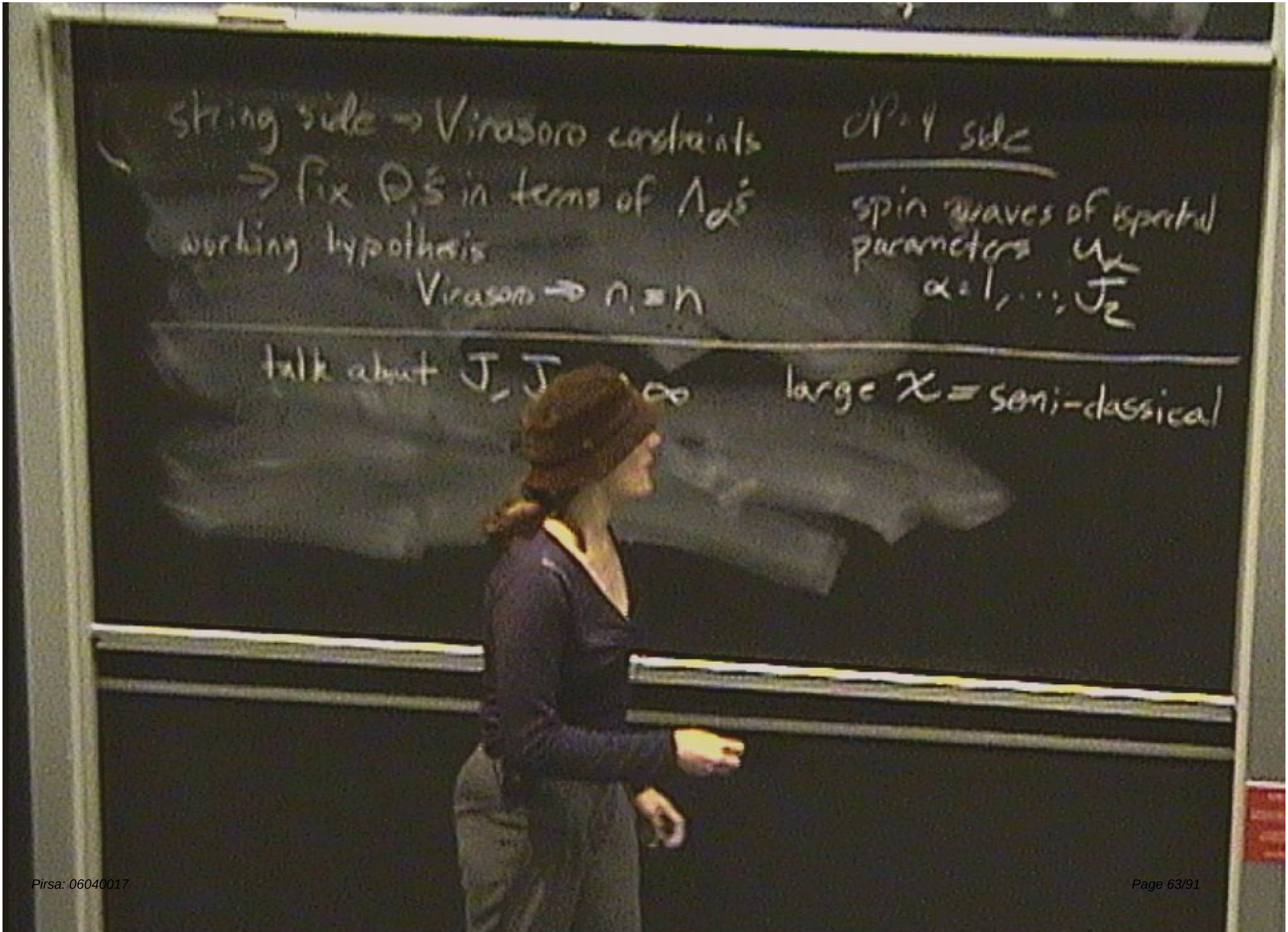
Virasoro $\Rightarrow$ $n_i = n$

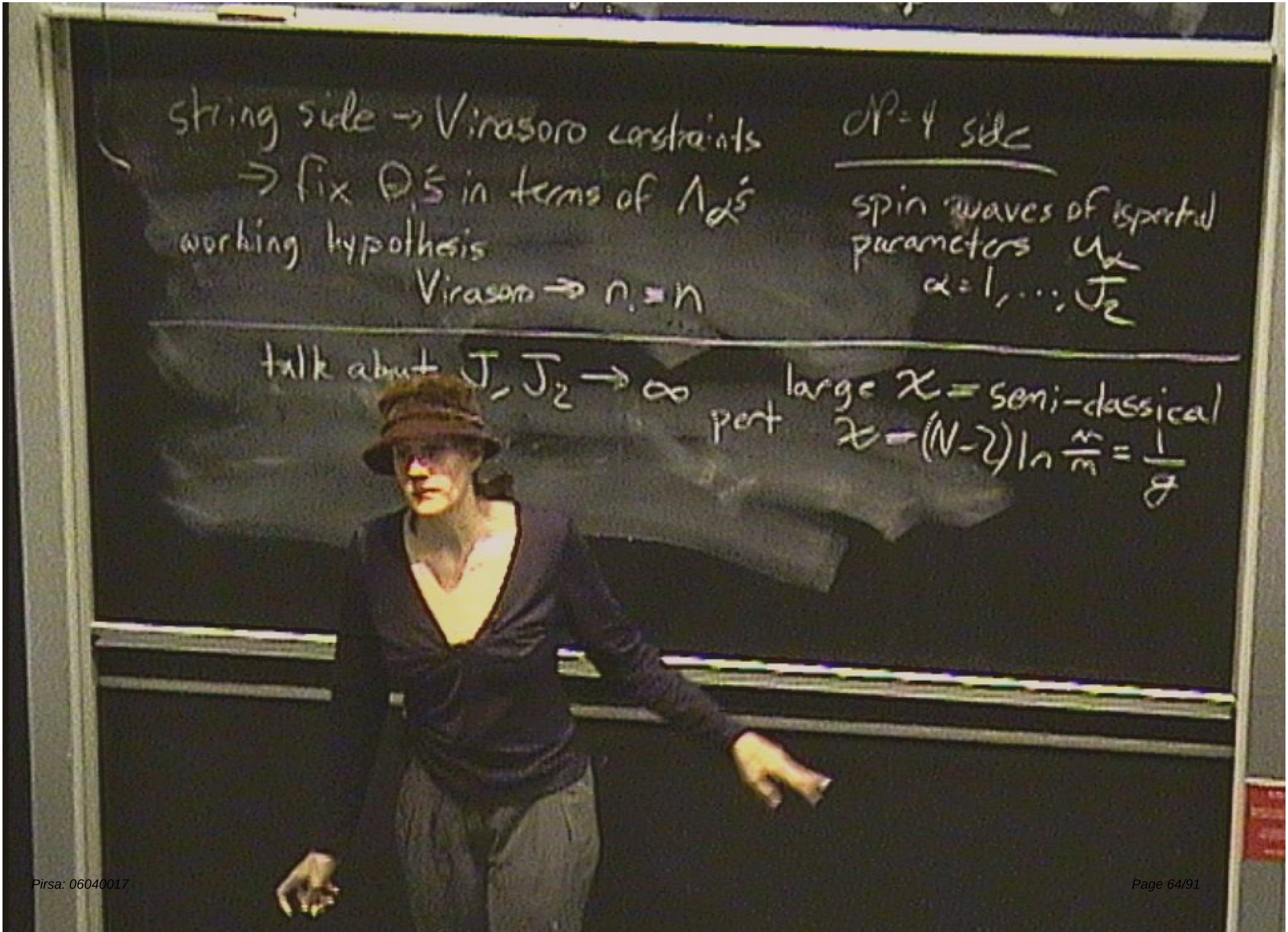
$\mathcal{N}=4$ side

spin waves of spectral parameters u_α

$\alpha = 1, \ldots, J_2$

talk about $J, J_2 \rightarrow \infty$


string side ⇒ Virasoro constraints
⇒ fix θ's in terms of Λ_α's
working hypothesis
Virasoro ⇒ n_1 = n
talk about J, J_2 → ∞
N=4 side
spin waves of spectral
parameters u_α
α = 1, …, J_2
large x ⇒ semi-classical


string side → Virasoro constraints
⇒ fix Θ_i's in terms of Λ_α's
working hypothesis
Virasoro ⇒ n_i = n
N=4 side
spin waves of spectral parameters u_α
α = 1, ..., J_2
talk about J_1, J_2 → ∞
large 𝒳 = semi-classical
pert
𝒳 = (N-2) ln n/m = 1/g

string side → Virasoro constraints
⇒ fix θ_i's in terms of Λ_α's
working hypothesis
Virasoro ⇒ $n_i = n$

$\mathcal{N}=4$ side

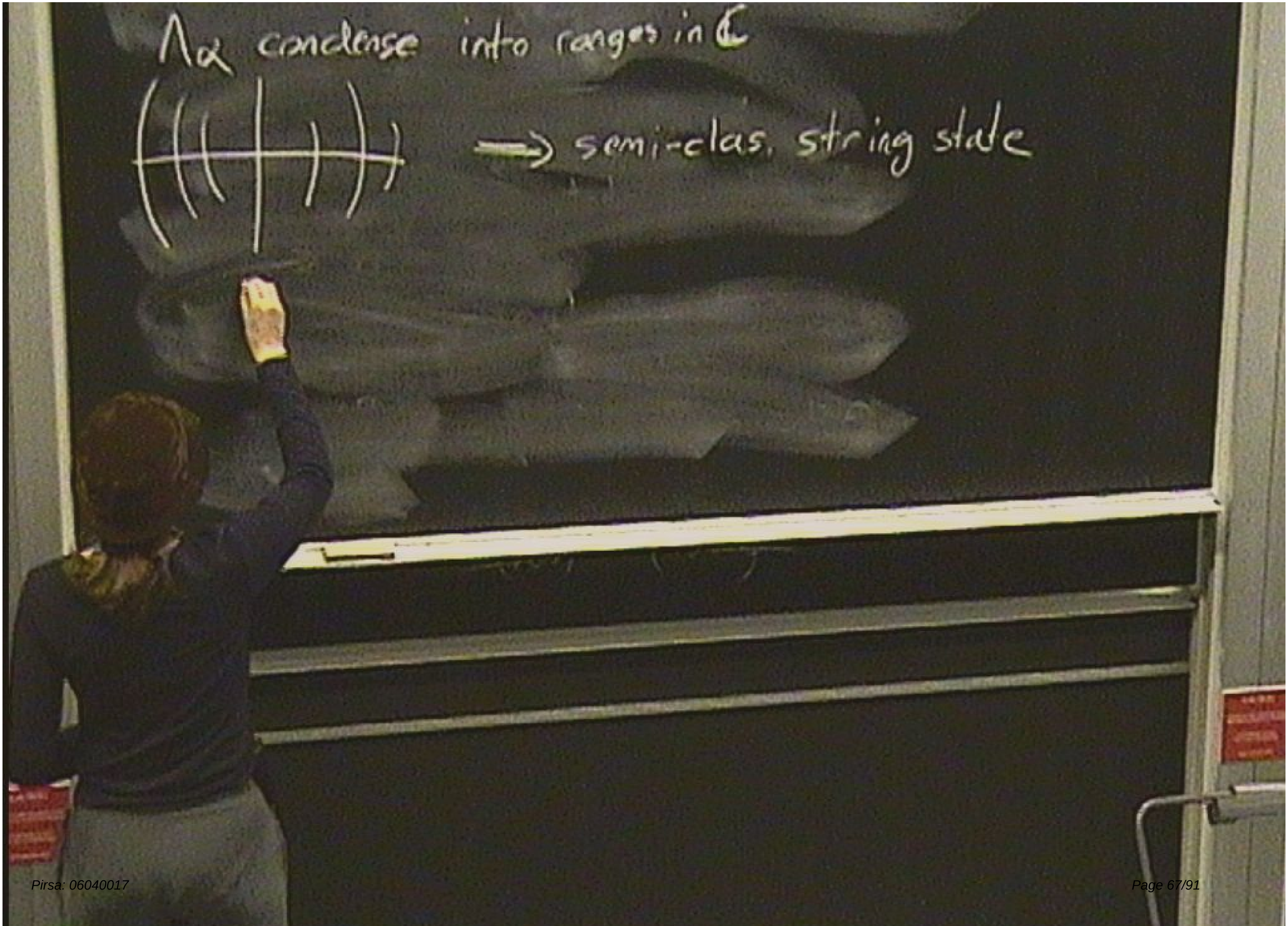
spin waves of spectral parameters u_α
$\alpha = 1, \ldots, J_2$

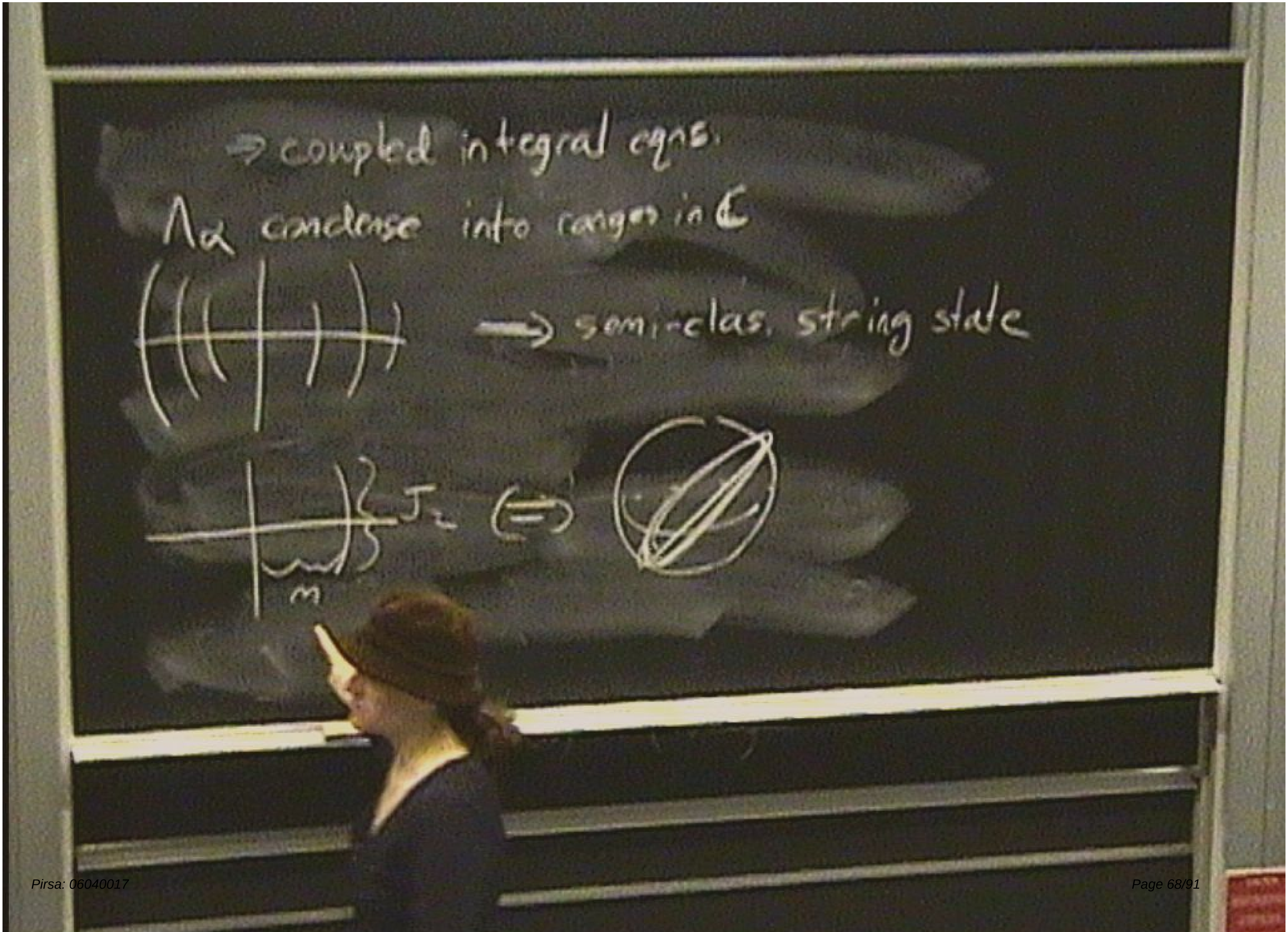
talk about $J, J_2 \to \infty$

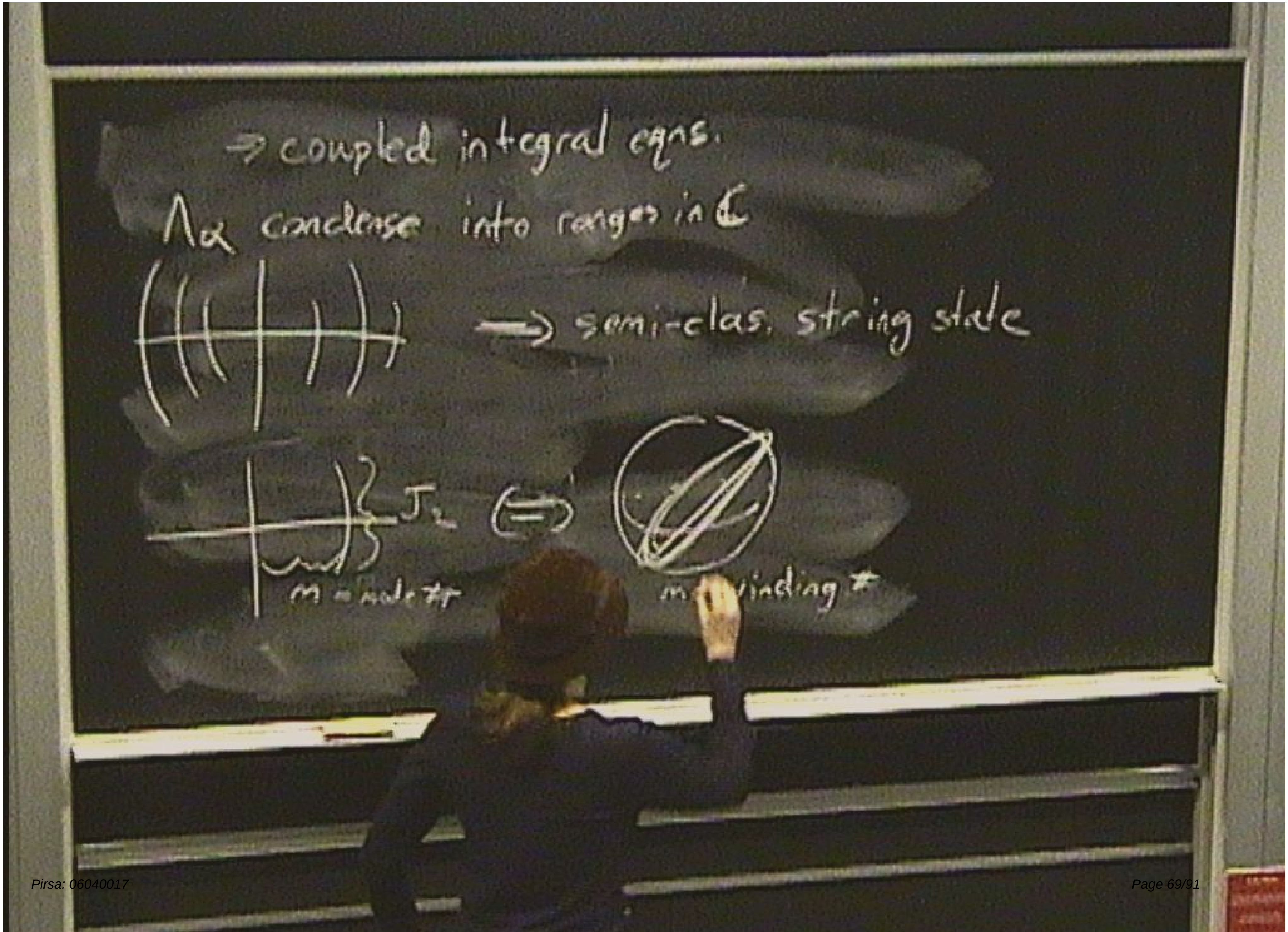
large $\mathcal{X}$ = semi-classical
pert. $\mathcal{X} = (N-2)\ln\frac{n}{m} = \frac{1}{g^2}$
$\Rightarrow \mathcal{X} = \sqrt{\lambda}$
semi-clas.

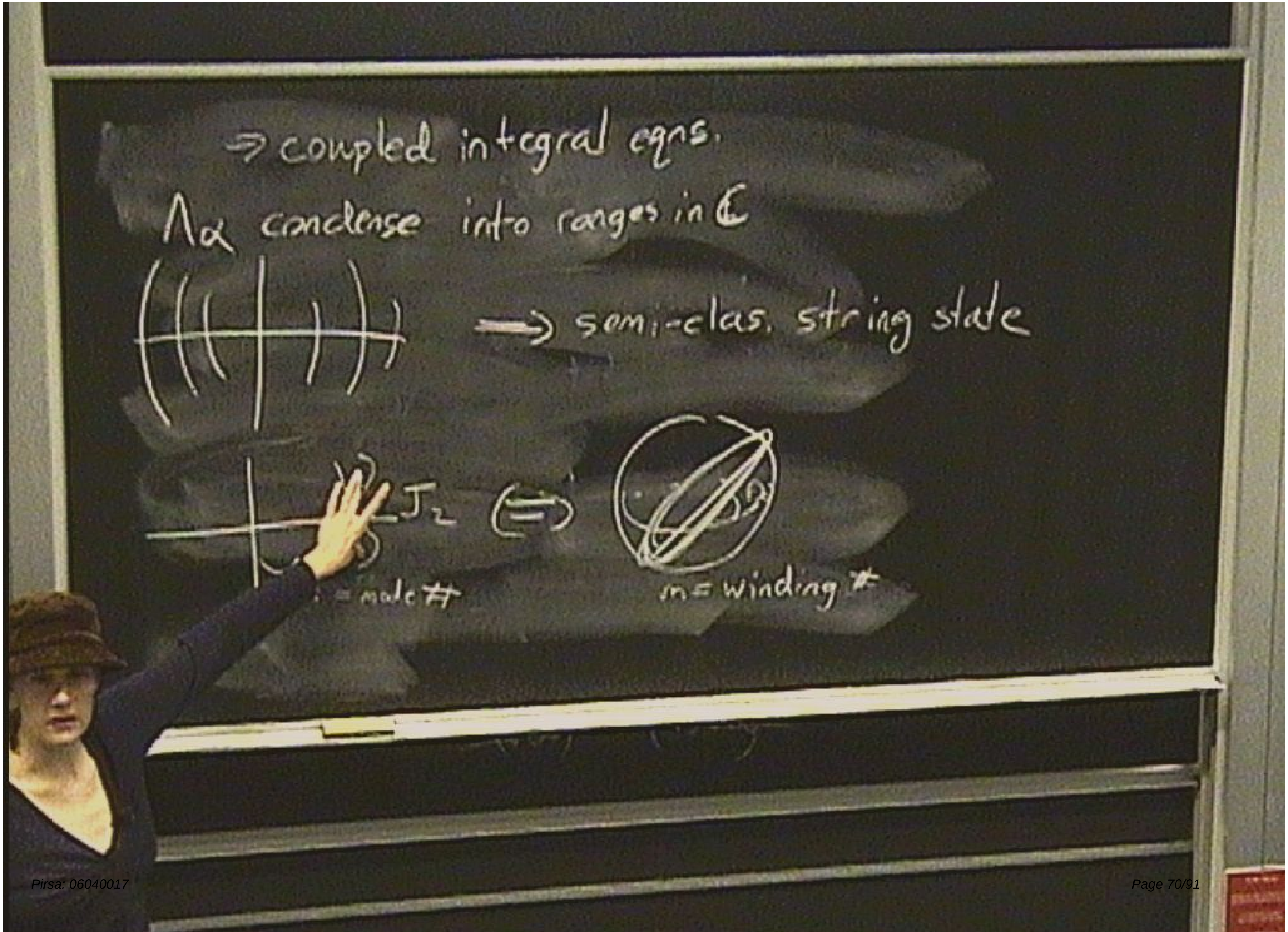
$$K(\theta) = \frac{1}{2\pi i}\,\partial_\theta \ln S(\theta)$$

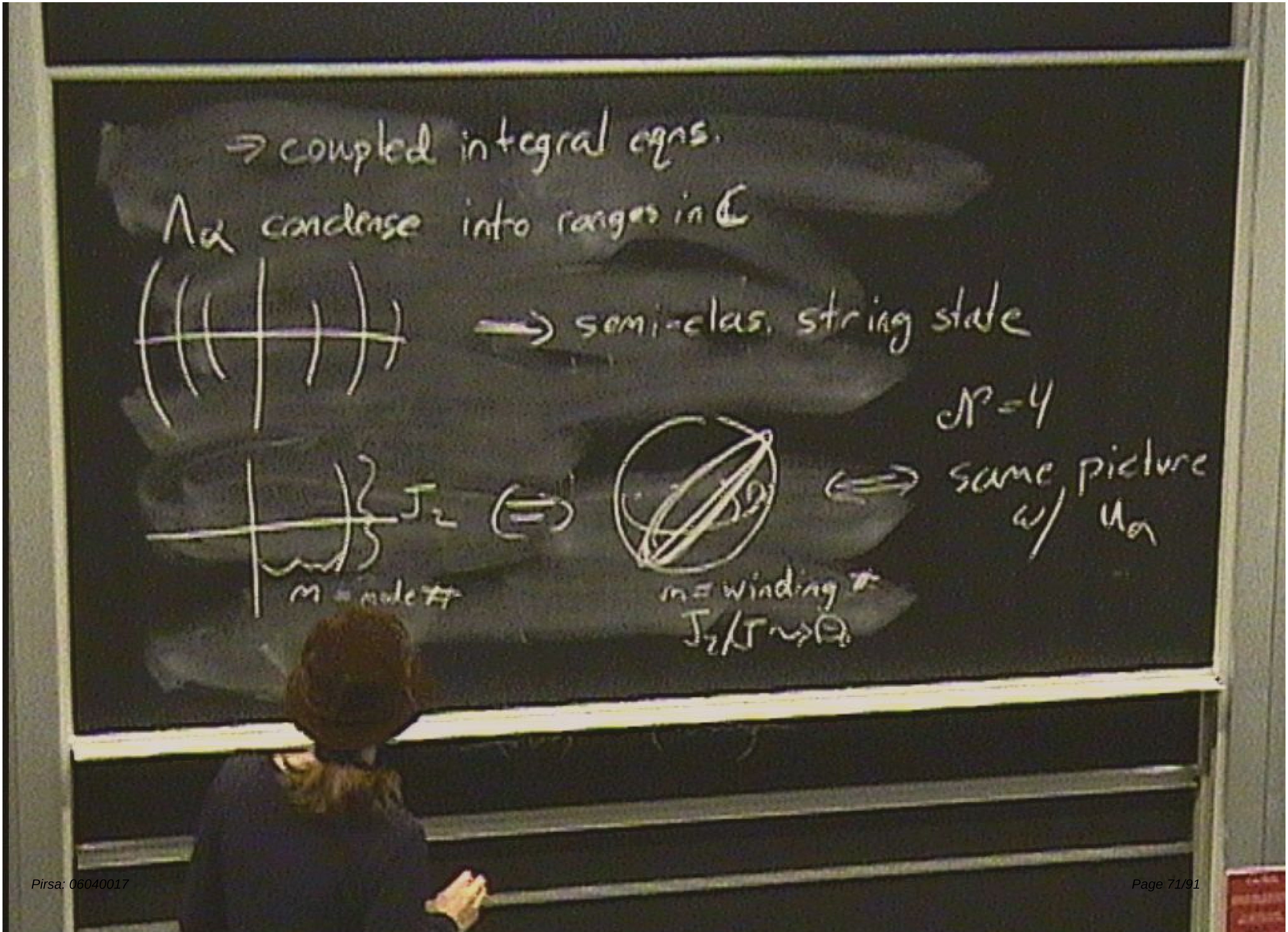
$\Rightarrow$ coupled integral eqns.

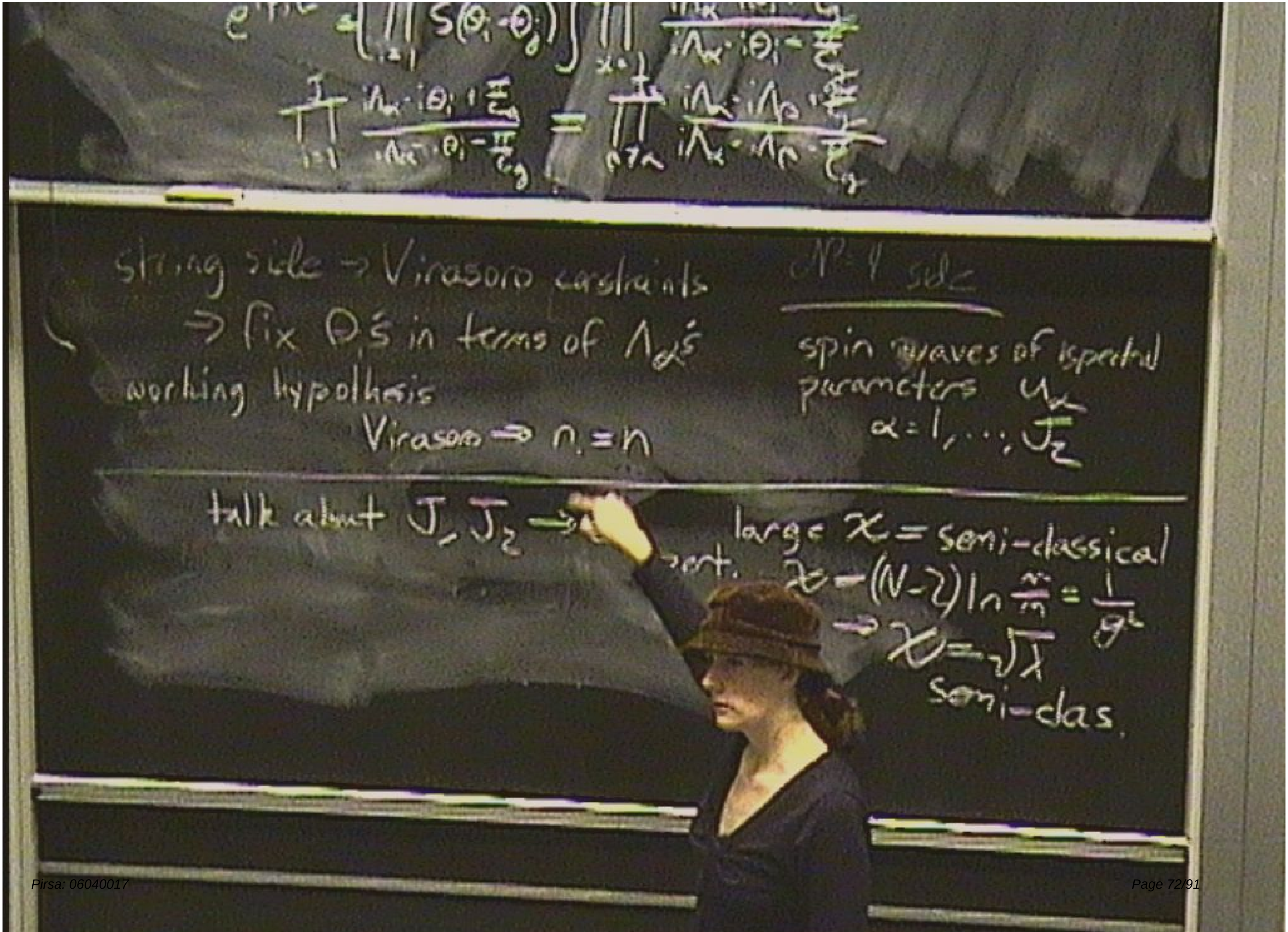

Λα condense into ranges in ℂ
⟹ semi-clas. string state

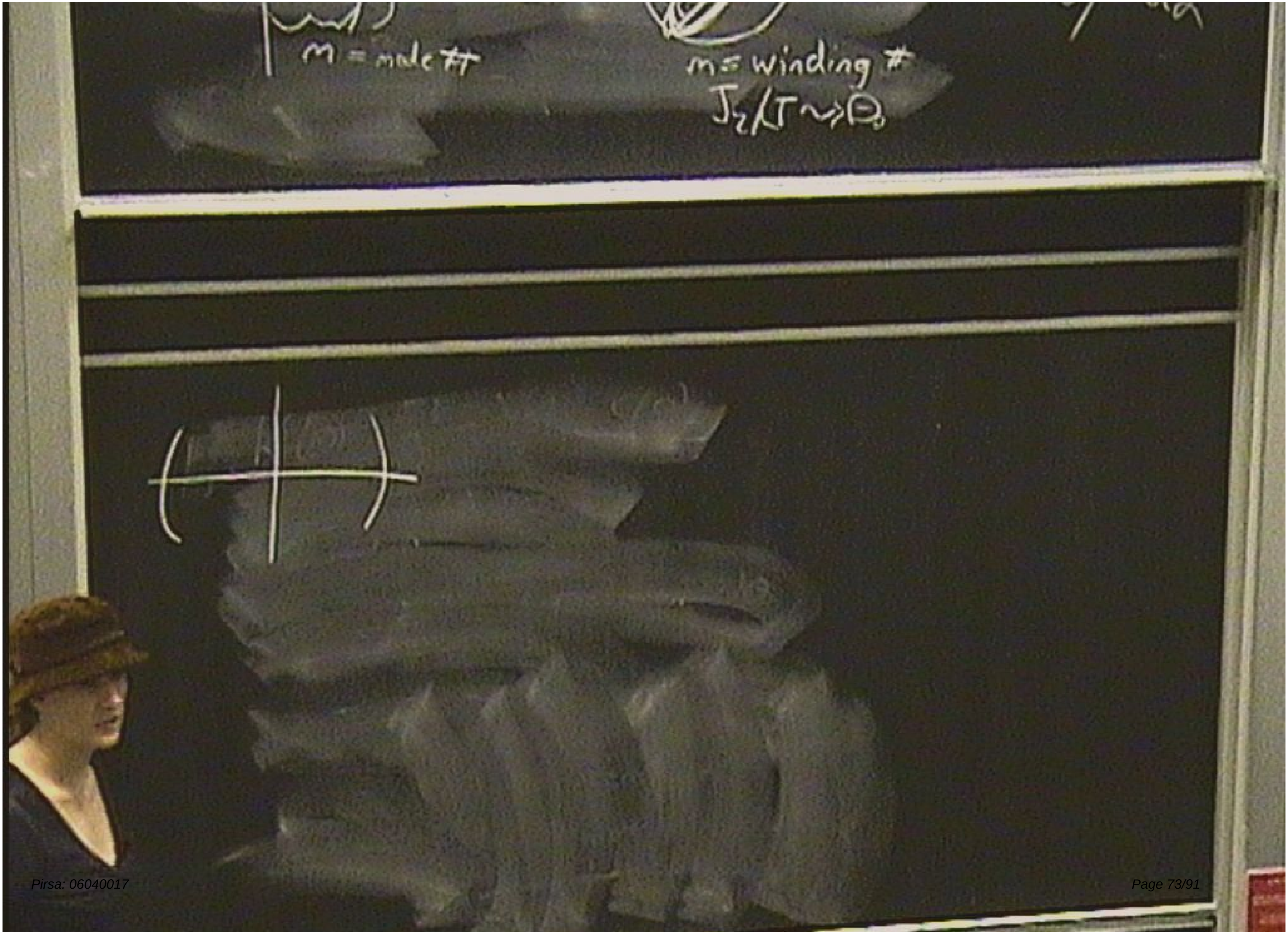

⇒ coupled integral eqns.
Λ_α condense into ranges in $\mathbb{C}$
⟹ semi-clas. string state
J_2
⟺
m

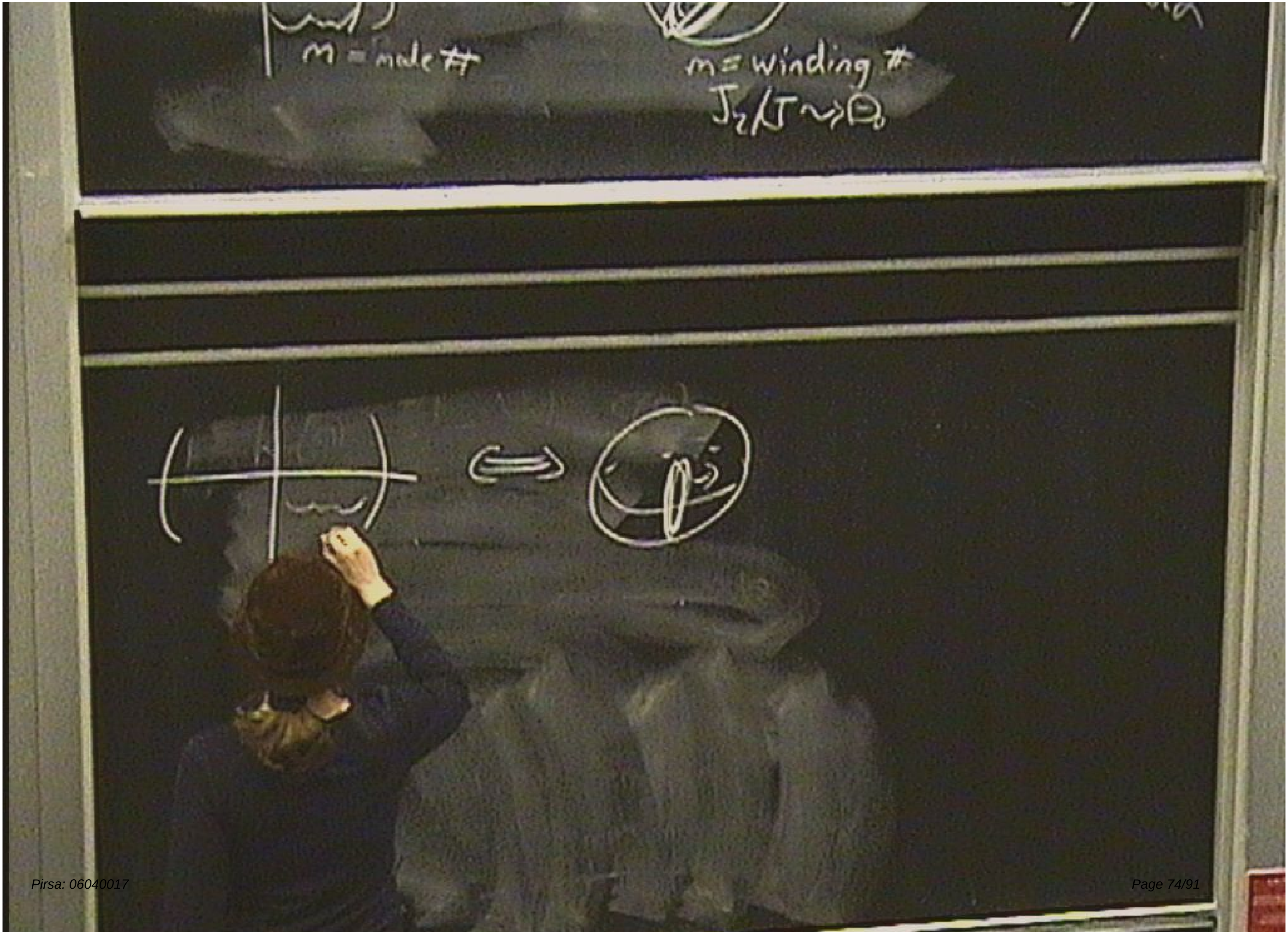

→ coupled integral eqns.
Λα condense into ranges in ℂ
⟹ semi-clas. string state
J₂
m = mode #
winding #

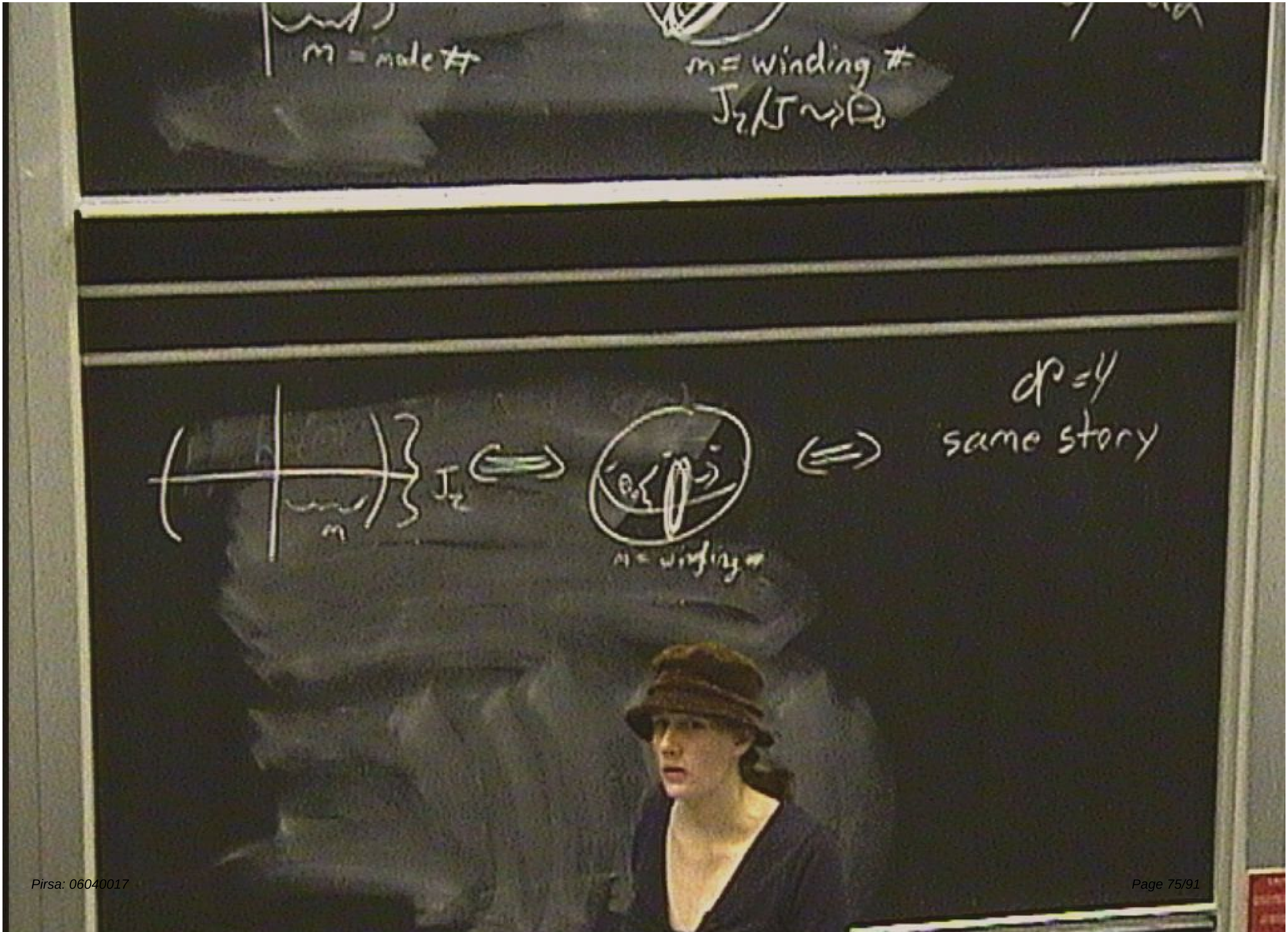

⇒ coupled integral eqns.
Λα condense into ranges in ℂ
⟹ semi-clas. string state
J2
⟺
= mode #
m = winding #


⇒ coupled integral eqns.
Λα condense into ranges in ℂ
→ semi-clas. string state
N=4
J2
⟺
⟺ same picture w/ uα
m = mode #
m = winding #


string side → Virasoro constraints
⇒ fix Θ's in terms of Λ's
working hypothesis
Virasoro ⇒ n = n
N=4 side
spin waves of spectral parameters
α = 1, ..., J2
talk about J, J2 ⇒
large = semi-classical
semi-clas.


m = mode #
m = winding #


m = mode #
m = winding #


m = mode #
m = winding #
Jz/J ~ Ω
Jz
m
m = winding #
d = 4
same story

$$\mathcal{E} = [1-\tilde{K}]\,\frac{\pi}{2}\left(\dot{x}+\dot{\phi}\right)^2$$

$$\int^{x} \ldots \; K(x-\phi) + \dot{\jmath}_L K(x+\phi)$$

Could also fix J_2 (BMN story)

impurities (Λ_α) $\rightarrow$ effective mass

$$\mathcal{E} = [1-\tilde{K}]\frac{\pi}{2}(\dot{x}+\dot{\phi})^2$$

$$\int^{x} \qquad \ldots = i_1 K(x-\phi) + i_2 K(x+\phi)$$

Could also fix J_2 (BMN story)

<u>impurities</u> (Λ_α) $\rightarrow$ effective mass $M = \frac{2\pi j}{\mathcal{K}}$

$$\mathcal{E} = [1-\tilde{K}]\frac{\pi}{2}(\dot{x}+\dot{\phi})^2$$

$$\dot{\imath}_R K(x-\phi) + \dot{\imath}_L K(x+\phi)$$

Could also fix J_2 (BMN story)

impurities (Λ_α) → effective mass $M = \frac{2\pi j}{\chi}$

$$(\Delta E + M)^2 = \Delta P^2 + M^2$$

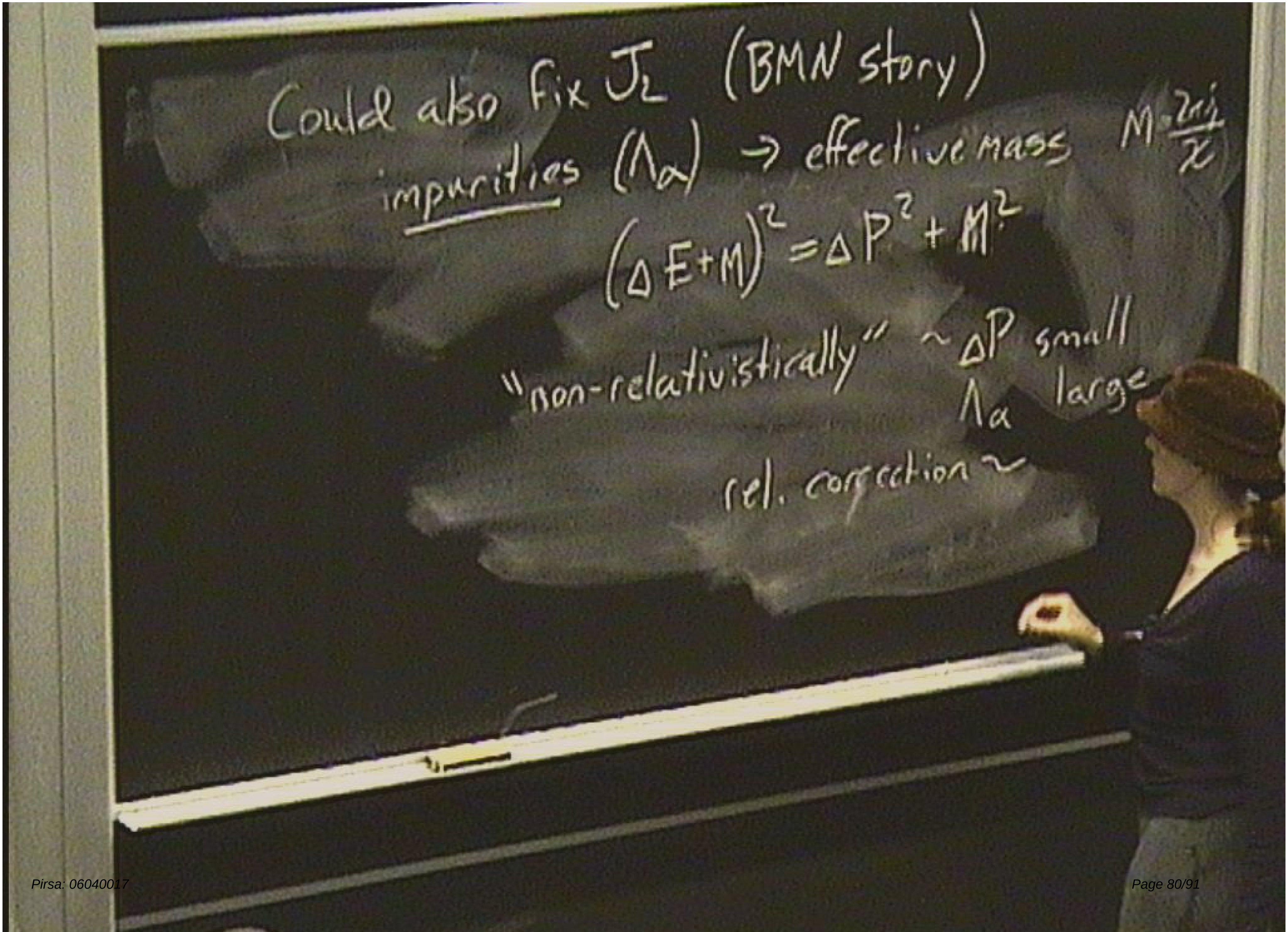
$$\mathcal{E} = [1-\tilde{K}]\,\frac{\pi}{2}\left(\dot{x}+\dot{\phi}\right)^2$$

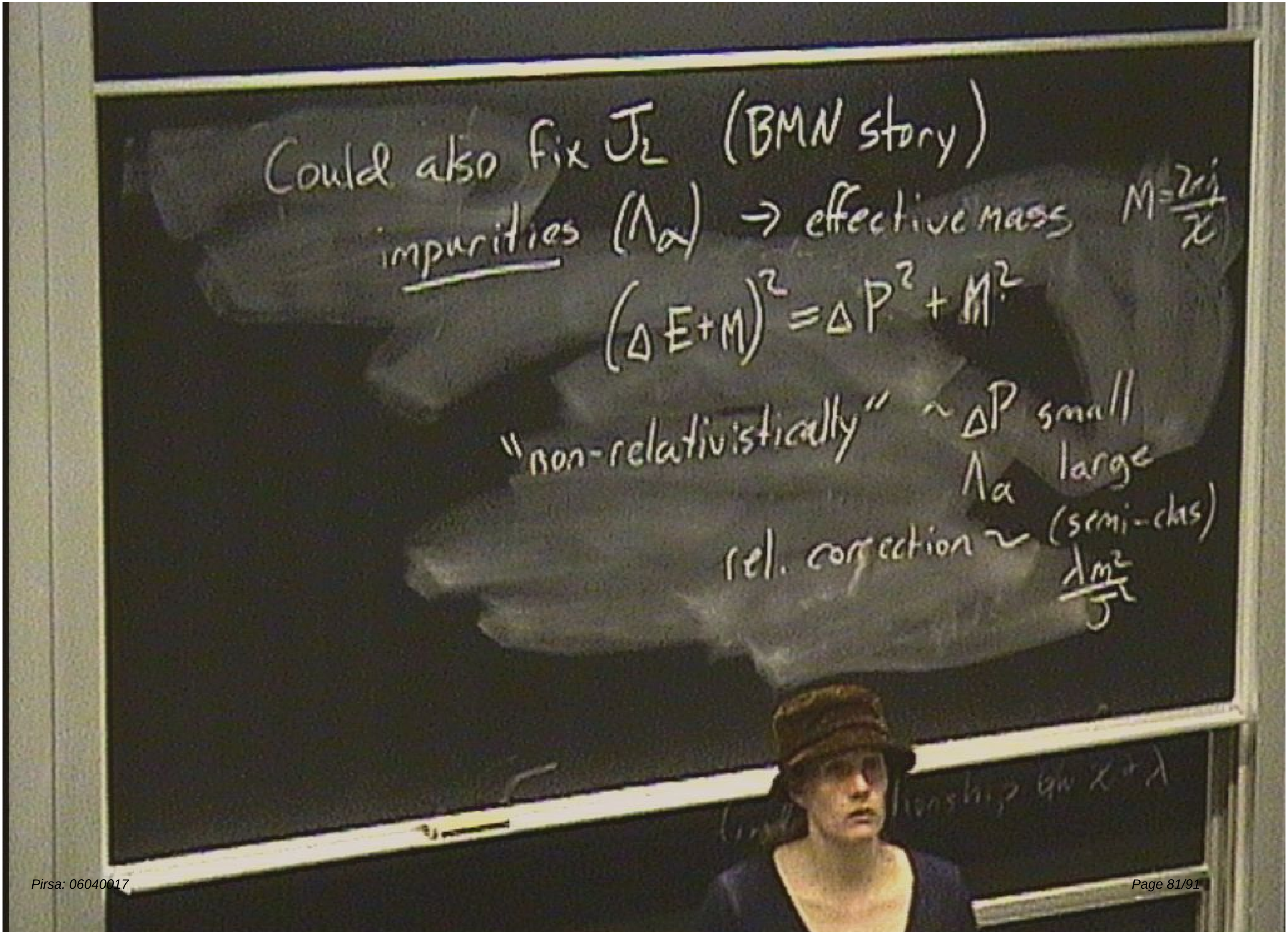
$$\int^{x} \ldots = j_1 K(x-\phi) + j_2 K(x+\phi)$$

Could also fix J_2 (BMN story)

impurities (Λ_α) $\rightarrow$ effective mass $M = \frac{2\pi j}{x}$

$$(\Delta E + M)^2 = \Delta P^2 + M^2$$


Could also fix J_2 (BMN story)
impurities (Λ_α) → effective mass $M = \frac{2\pi j}{\chi}$
$(\Delta E + M)^2 = \Delta P^2 + M^2$
"non-relativistically" ~ ΔP small
Λ_α large
rel. correction ~


Could also fix J_2 (BMN story)
impurities (Λ_α) → effective mass $M = \frac{2\pi j}{\kappa}$
$(\Delta E + M)^2 = \Delta P^2 + M^2$
"non-relativistically" ~ ΔP small
Λ_α large
rel. correction ~ (semi-clas)
$\frac{\lambda m^2}{J^2}$

rel. correction ~ (semi-clas) $\frac{\lambda m^2}{\beta^2}$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = j_R\, K(x-\phi) + j_L\, K(x+\phi)$$

$+$

rel. correction ~ (semi-class) $\frac{\lambda m^2}{J^2}$

$$\rho(\phi) - \int_{-\chi}^{\chi} K_2(\phi-\phi')\rho(\phi')\,d\phi' = j_R K(\chi-\phi) + j_L K(\chi+\phi) + \int \frac{\sigma(v)\,dv}{\frac{1}{4}+(v-\phi)^2}$$

v_σ … scaling of

rel. correction ~ (semi-class) $\frac{\lambda m^2}{g^2}$

$$\rho(\phi) - \int_{-x}^{x} K_2(\phi-\phi')\rho(\phi')\,d\phi' = j_R K(x-\phi) + j_L K(x+\phi) + \int \frac{\sigma(v)\,dv}{\frac{1}{4}+(v-\phi)^2}$$

$v_\alpha \Leftrightarrow$ rescaling of λ_α

rel exp. $\left(\frac{\phi}{v}\right)^2$

rel. correction $\sim$ (s...)

$\frac{\Delta m^2}{\dots}$

$$\rho(\phi) - \int_{-\chi}^{\chi} K_2(\phi-\phi')\rho(\phi')\,d\phi' = j_R K(\chi-\phi) + j_L K(\chi+\phi)$$

$$+ \sum_{\alpha=1} \frac{\pi}{2} \dots \frac{\dots}{\frac{1}{4}+(v_\alpha-\phi)^2}$$

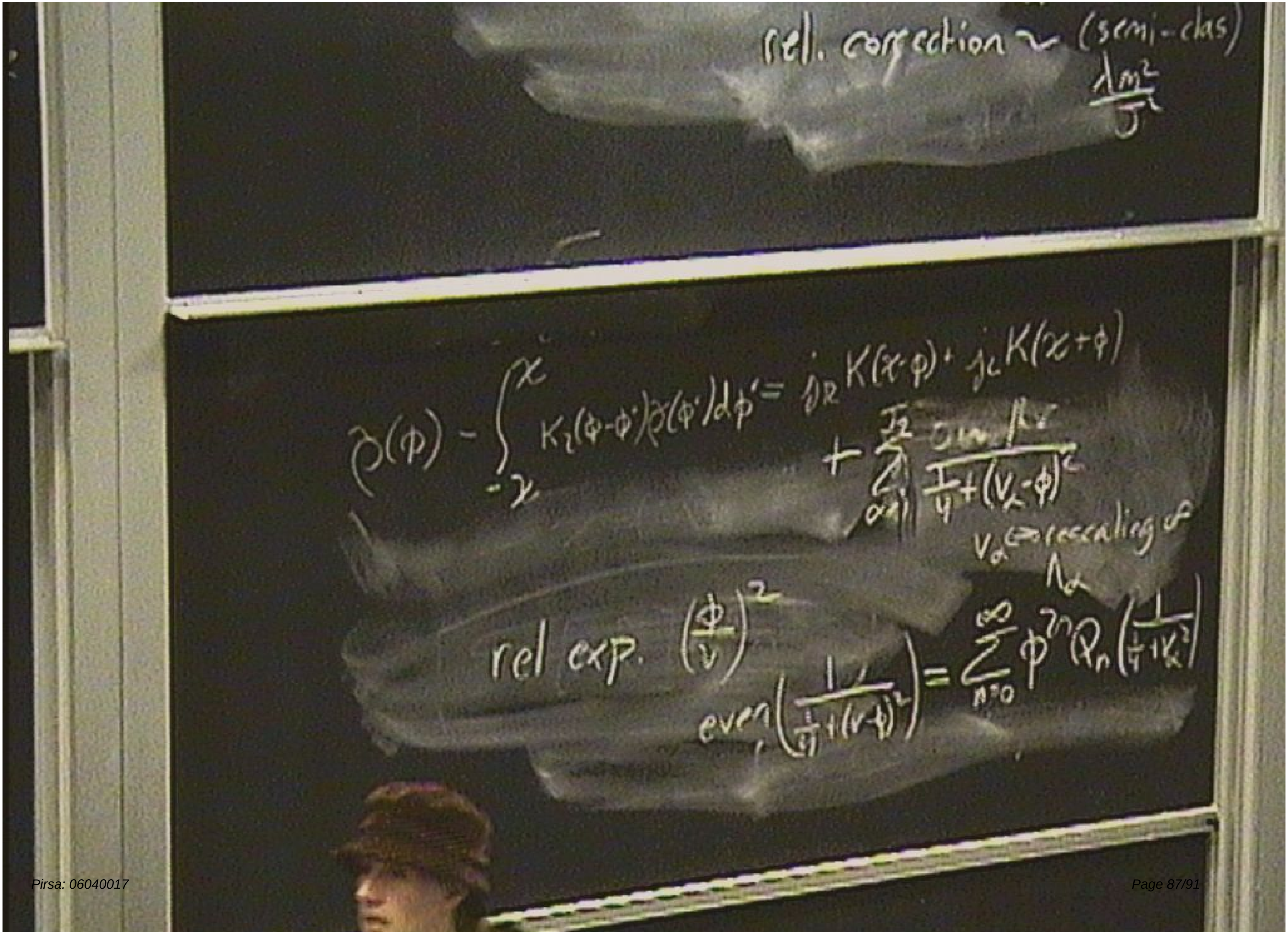
$v_\alpha \Leftrightarrow$ rescaling of N_α

rel exp. $\left(\frac{\phi}{v}\right)^2$

$$\text{even}\left(\frac{1}{\frac{1}{4}+(v-\phi)^2}\right) = \sum_{n=0}^{\infty} \phi^{2n} R_n\left(\frac{1}{\frac{1}{4}+v_\alpha^2}\right)$$

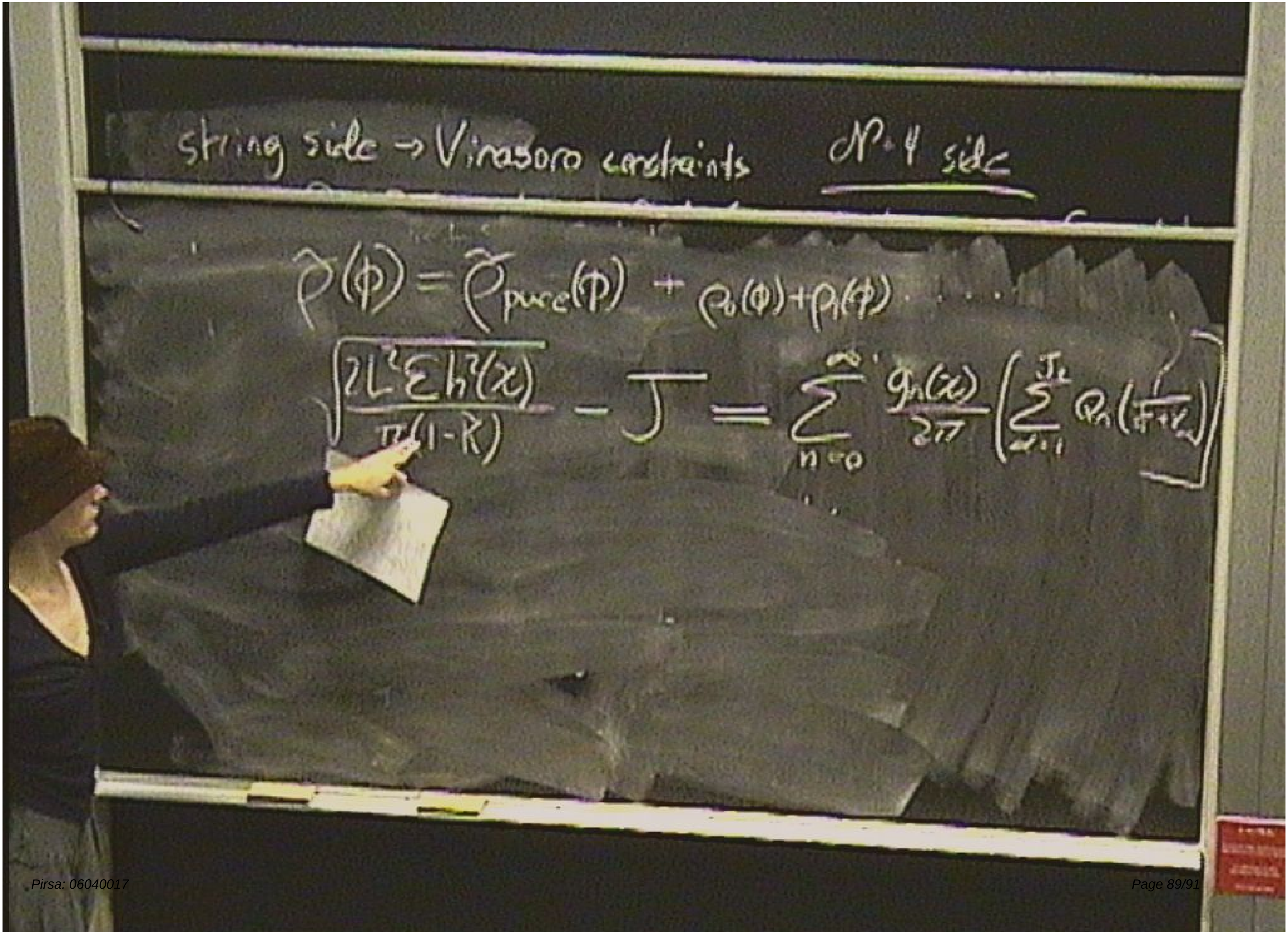
string side → Virasoro constraints $\quad \underline{\mathcal{N}=4 \text{ side}}$

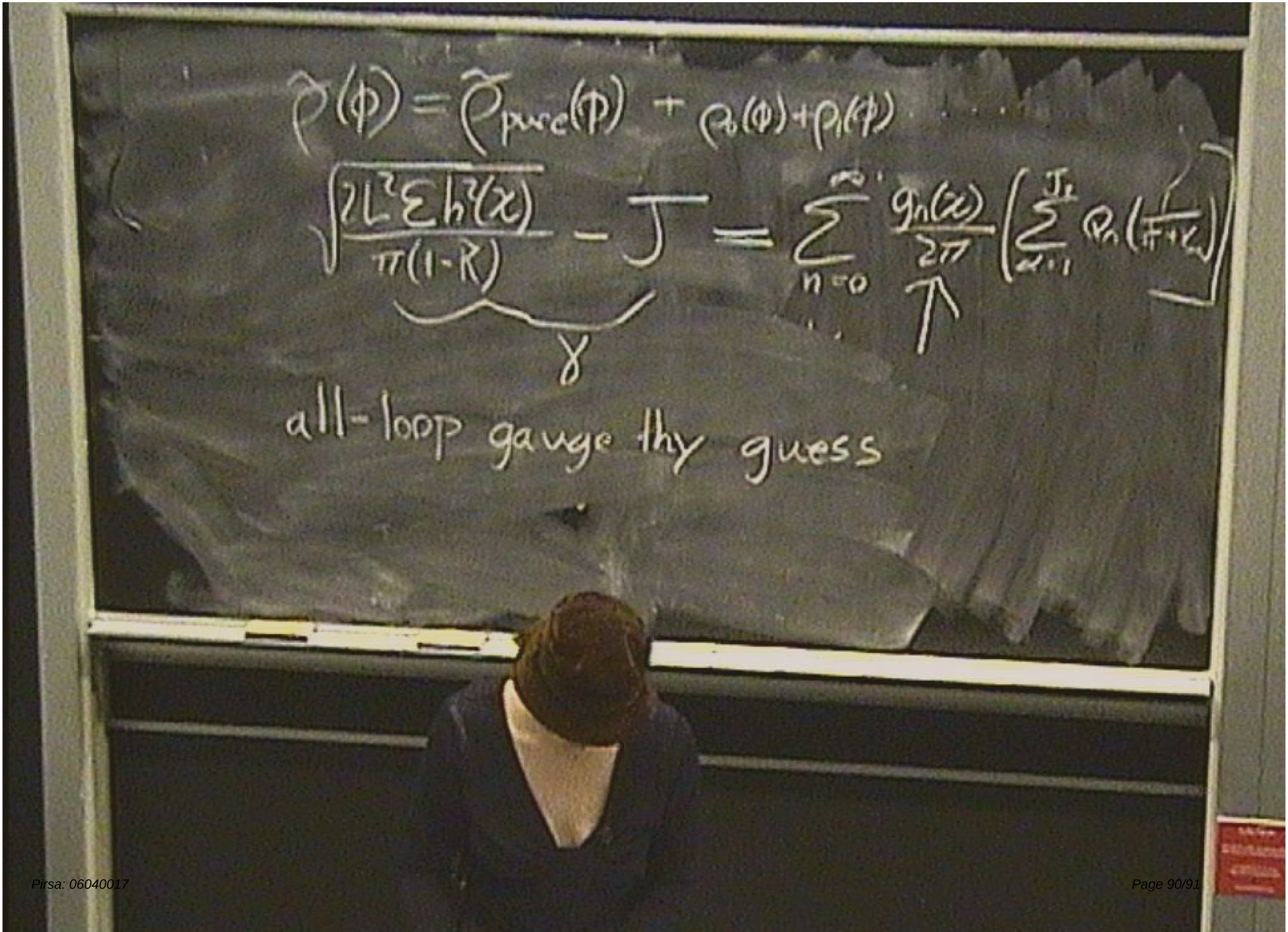
$$\tilde{\rho}(\phi) = \tilde{\rho}_{pure}(\phi)$$


rel. correction ~ (semi-clas)
Λm²/J
ρ(φ) − ∫ K₂(φ−φ')ρ(φ')dφ' = j_R K(x−φ) + j_L K(x+φ)
1/4 + (v_α − φ)²
v_α ⟺ rescaling of Λ_α
rel exp. (φ/v)²
even(1/(1/4 + (v−φ)²)) = Σ φ^{2n} Q_n(1/(1/4 + v_α²))

string side → Virasoro constraints $\underline{\mathcal{N}=4 \text{ side}}$

$$\tilde{\rho}(\phi) = \tilde{\rho}_{pure}(\phi) + \rho_0(\phi) + \rho_1(\phi) \ldots$$


string side → Virasoro constraints
$\mathcal{N}=4$ side
$\tilde{\rho}(\phi) = \tilde{\rho}_{pure}(\phi) + \rho_0(\phi) + \rho_1(\phi) \ldots$
$\sqrt{\frac{2L^2 \Sigma h^2(x)}{\pi(1-R)}} - J = \sum_{n=0}^{\infty}{}' \frac{g_n(x)}{2\pi} \left(\sum_{k=1}^{J_k} \rho_n \right)$



$\tilde{\rho}(\phi) = \tilde{\rho}_{pure}(\phi) + \rho_0(\phi) + \rho_1(\phi) \ldots$
$\underbrace{\sqrt{\frac{2L^2 \Sigma h^2(x)}{\pi(1-R)}} - J}_{\gamma} = \sum_{n=0}^{\infty} \frac{g_n(x)}{2\pi} \left(\sum_{m=1}^{J_n} \rho_n(\ldots) \right)$
all-loop gauge thy guess

$$\tilde{\rho}(\phi) = \tilde{\rho}_{pure}(\phi) + \rho_0(\phi) + \rho_1(\phi) \ldots$$

$$\underbrace{\sqrt{\frac{2L^2 \mathcal{E} h^2(x)}{\pi(1-R)}} - J}_{\gamma} = \sum_{n=0}^{\infty} \frac{g_n(x)}{2\pi} \left(\sum_{\alpha=1}^{J_2} Q_n\left(\frac{1}{t_\alpha + v_\alpha}\right) \right)$$

all-loop gauge thy guess

$$\gamma = \sum_{n=0}^{\infty} \frac{2(2n)!}{(n+1)!\,n!} \left(\frac{\lambda}{16\pi^2}\right)^{n+1} \left[\sum_{\alpha=1}^{J_2} Q_n\left(\frac{1}{t_\alpha + v_\alpha}\right) \right]$$