

Title: Introduction to quantum gravity - Part 25

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

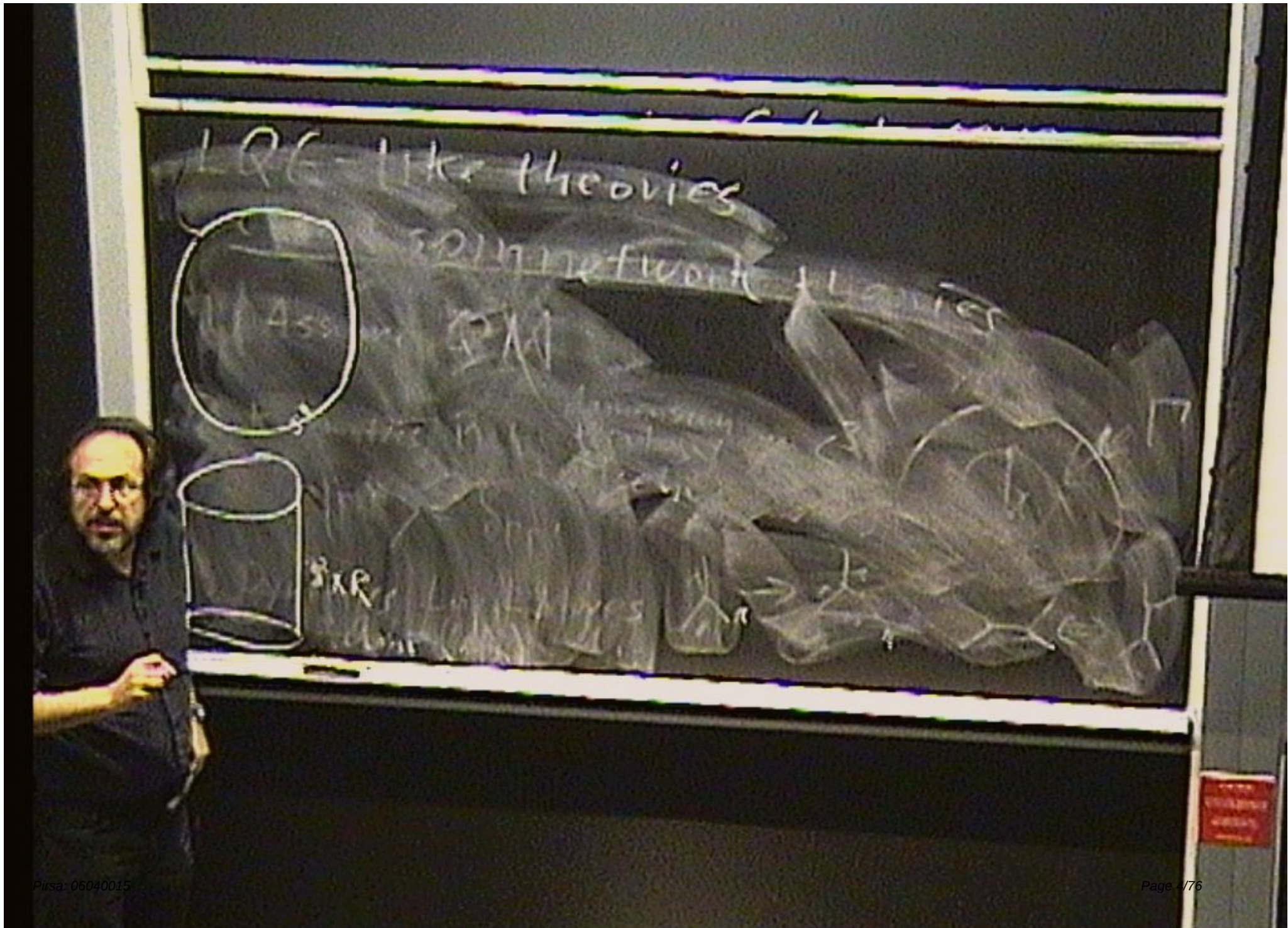
- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras

The key observational question is:

*What is the symmetry of the ground state?*

The key observational question is:

*What is the symmetry of the ground state?*



LQC-like theories



$$H = \int \frac{NC}{2} + \int \frac{NH}{\partial \xi}$$



# LQG-like theories

$$H = \int \frac{N^2 C}{2} + \int N H_{\text{th}} + \text{Lagrangian}$$

In classical GR  $A(x) \rightarrow \infty$   
define Asymp flat

$$H = H_{\text{ADM}}$$

# LQG-like theories



$$H = \int_{\Sigma} NC + \int_{\partial \Sigma} NH + \text{boundary terms}$$

In classical GR  $A(\partial \Sigma) \rightarrow \infty$   
 define Asymp flat

show for  
 Witten

$$H = H_{ADM} \geq 0 \quad \& \quad H_{ADM} = 0 \iff \text{Flat spacetime}$$

$\partial \Sigma$

# LQG-like theories



$$H = \int \frac{N^2}{2} + \int \frac{N^2}{2} + \dots$$

In classical GR  $A(\partial \Sigma) \rightarrow \infty$   
 define Asymp Flat

Structure  
 with

$$H = H_{ADM} \geq 0 \quad \& \quad H_{ADM} = 0 \Leftrightarrow \text{Flat spacetime}$$

QM conjecture:

$$\exists \hat{H}_{ADM}$$

# LQG-like theories



$$H = \int \frac{N^2 C^2}{2} + \int \frac{N^2 h^2}{2\epsilon} + L_{\text{matter}}$$

In classical GR  $A(\partial\mathcal{R}) \rightarrow \infty$   
 define Asymp Flat



Witten  $H = H_{\text{ADM}} \geq 0$  +  $H_{\text{ADM}} = 0 \Leftrightarrow$  Flat spacetime

QM conjecture:

$$\exists \uparrow H_{\text{ADM}} \quad H_{\text{ADM}} \geq 0 \text{ on physical states} \quad H_{\text{ADM}} |G\rangle = 0$$

# LQC-like theories



$$H = \int NC + \int NH + L_{\text{matter}}$$

In classical GR  $A(\partial\Sigma) \rightarrow \infty$   
 define Asymp Flat



classical GR  
 with

$$H_{ADM} \geq 0 \quad \& \quad H_{ADM} = 0 \Leftrightarrow \text{Flat spacetime}$$

$H_{ADM} \geq 0$  physical states

$$H_{ADM}|G\rangle = 0$$

# LQG-like theories

$$H = \int_{\Sigma} NC + \int_{\partial\Sigma} NKh + \dots$$

In classical GR  $A(\partial\Sigma) \rightarrow \infty$   
define Asymp Flat

With  $H = H_{ADM} \geq 0$  +  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime

QM conjecture:

$\exists \hat{H}_{ADM}$   $\hat{H}_{ADM} \geq$  physical shifts

$$\hat{H}_{ADM} |G\rangle = 0$$

In classical GR  $A(\partial \Sigma) \rightarrow \infty$   
 $A(\Sigma)$  is finite Asymp flat  
 show that  $H = H_{ADM} \geq 0$  &  $H_{ADM} = 0 \iff$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM}$   $\hat{H}_{ADM} \geq 0$  on physical states  $\boxed{\hat{H}_{ADM} | \psi \rangle = 0}$

Configuration space  
G-connections on  $M$   
 $G \ltimes \text{Diff}(M)$   
 $G = \text{SU}(2) \times \mathbb{H}$   
 (the edges)

In classical GR  $A(\partial\Sigma) \rightarrow \infty$   
 follow Asymptotic flat  
 structure  
 Witten  $H = H_{ADM} \geq 0$  &  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \uparrow H_{ADM} \geq 0$  on physical states  
 $H_{ADM} |G\rangle = 0$



solutions to Asymptotic constraints  
 $\det \tilde{E}^a_b = 0 \quad q q^b = \tilde{E}^a_c \tilde{E}^c_b$



In classical GR  $A(\partial\mathcal{E}) \rightarrow \infty$   
 AdS asymptotic flat  
 structure  $H = H_{\text{ADM}} \geq 0 + H_{\text{ADM}} = 0 \Leftrightarrow$  Flat spacetime  
 Witten  
 QM conjecture:  
 $\exists \hat{H}_{\text{ADM}} \quad H_{\text{ADM}} \geq 0$  on physical states  
 $\boxed{\hat{H}_{\text{ADM}} |0\rangle = 0}$

solutions to Ash. constraints  
 $\det \tilde{E}^a_i = 0 \quad \uparrow \downarrow = \tilde{E}^a_i \tilde{E}^b_j$   
 Deser



# LQG-like theories



$$H = \int_{\Sigma} NC + \int_{\Sigma} NH$$

In (classical) GR  $A(\partial\Sigma) \rightarrow \infty$   
 define Asymp flat, det  $\mathcal{L}_{\text{ADM}} \neq 0$



Starts to  
 with

$$H_{\text{ADM}} \geq 0 \quad \& \quad H_{\text{ADM}} = 0 \Leftrightarrow \text{Flat spacetime}$$

$H_{\text{ADM}} \geq 0$  is physical  
 shift

$$H_{\text{ADM}}|G\rangle = 0$$

$A(x)$  in classical GR:  $H(t) \neq 0$   
 define Asympt. flat,  $\det g_{\mu\nu} \neq 0$   
 strong form:  $H = H_{ADM} \geq 0$  &  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \quad H_{ADM} \geq 0$  on physical states  $\hat{H}_{ADM} |E\rangle = 0$

solutions to Ash. constraints  
 $\det g = 0 \quad g g^{\mu\nu} = \vec{E}^a \cdot \vec{E}_a$ 
Dosen



$$H = \int d^3x \mathcal{H} + \int d^3x \mathcal{H}_{int}$$

In (classical) GR  $A(z) \rightarrow \infty$   
define Asymp Flat, det  $g_{\mu\nu} \neq 0$

structure  
Wigner  $H = H_{ADM} \geq 0 + H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
QM conjecture:  
 $\exists \hat{H}_{ADM} \quad H_{ADM} \geq 0$  physical states  $\hat{H}_{ADM} |E\rangle = 0$

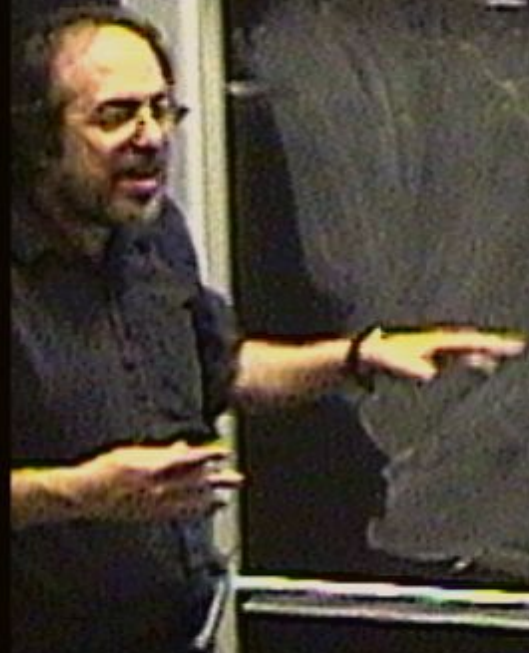


$A(x)$  Belong Asymp. Flat,  $\det L \neq 0$   
 character  $H = H_{ADM} \geq 0 + H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 Witten QM conjecture:  
 $\exists \psi \in H_{ADM} \geq 0$  such that  $H_{ADM} \psi = 0$   
 $\boxed{H_{ADM} \psi = 0}$

solutions to Asch. constraints  
 $\det E = 0 \quad q q^{\phi} = \tilde{E}^{\phi\alpha} \tilde{E}_{\alpha}^{\phi}$   
 $\boxed{D_{\alpha} \pi^{\alpha} = 0}$

$A(x)$  define Asympt. Flat,  $\det \mathcal{L}_n \neq 0$   
 Wilson  $H = H_{ADM} \geq 0 + H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \quad H_{ADM} \geq 0 \text{ on physical states} \quad \boxed{\hat{H}_{ADM} |E\rangle = 0}$

solutions to Asht. constraints  
 $\det \tilde{E} = 0 \quad q q^b = \tilde{E}^{a i} \tilde{E}^{b j} \delta_{ij}$   
 [Deser]



$A(\Sigma)$   $\rightarrow$  Action Asymp. Flat, det  $g_{\mu\nu} \neq 0$   
 Stromer-Witten  $H = H_{ADM} \geq 0$  +  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \quad H_{ADM} \geq 0$  on physical states  $\hat{H}_{ADM} |\psi\rangle = 0$

$l_{90}$  is a minimal length



$A(\rho)$  is John Asymp Flt, det  $Z_n \neq 0$   
 show that  $H = H_{ADM} \geq 0 + H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \hat{H}_{ADM} \geq 0$  on physical states  $\hat{H}_{ADM} | \epsilon \rangle = 0$

$l_{90}$  is a minimal length

$A(\mathbb{R})$  is defined as a Hilbert space, where  $\mathbb{Z}_2 \neq 0$   
 strong form  $H = H_{ADM} \geq 0$  &  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 Witten  
 QM conjecture:  
 $\exists \hat{A}_{ADM} \quad H_{ADM} \geq 0$  on physical states  
 $\boxed{H_{ADM}|\psi\rangle = 0}$

$l_{90}$  is a minimal length & observer independent

$A(\vec{x})$  is a wave function,  $\det \mathcal{L}_m \neq 0$   
 structure  $H = H_{\text{non}} \geq 0 + H_{\text{lin}} > 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{\text{lin}} \quad H_{\text{lin}} \geq 0$  physical states  $\boxed{H_{\text{lin}}|0\rangle = 0}$

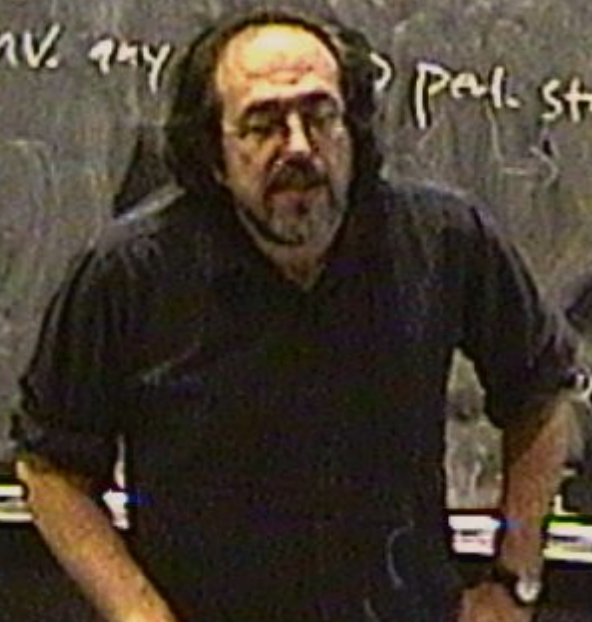
$l_{\text{pl}}$  is a minimal length independent  
 Lorentz invariance  $\mathcal{P}$

$A(\infty)$  Defining Asymptotic Flat,  $\det Z_{ab} \neq 0$   
 structure  $H = H_{ADM} \geq 0 + H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 Witten  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \geq 0$  on physical states  $\hat{H}_{ADM} | \psi \rangle = 0$

$l_{pl}$  is a minimal length & observer independent

Lorentz invariance  $\mathcal{P}$

• Poincare Inv. any  $\rightarrow$  part. strings theory



$A(x)$  Define Asympt. Field, det  $Z_m \neq 0$   
 with  $H = H_{ADM} \geq 0$  +  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{ADM} \quad H_{ADM} \geq 0 \Rightarrow \hat{H}_{ADM} |0\rangle = 0$

$l_{pl}$  is a minimal length & observer independent

Lorentz invariance  $P$

• Polymorphic Inv. anyway  $\Rightarrow$  pol. string theory



$A(\sigma)$  define Hopping Field, det  $Z_n \neq 0$   
 structure  
 Witten  $H = H_{\text{ADM}} \geq 0 + H_{\text{ADM}} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjecture:  
 $\exists \hat{H}_{\text{ADM}} \quad H_{\text{ADM}} \geq 0$  on physical states  $\boxed{\hat{H}_{\text{ADM}} |0\rangle = 0}$

$l_{\text{pl}}$  is a minimal length & observer independent

Lorentz invariance  $P$

- Polymers Inv. anyway  $\Rightarrow$  pol. strings theory
- $l_{\text{pl}}$  is broken  $\Rightarrow$  preferred frame



$A(\infty)$  is defined as a limit, where  $2n \neq 0$   
 structure  $H = H_{\text{non}} \geq 0 + H_{\text{non}} = 0 \Leftrightarrow$  Flat spacetime  
 Witten QM conjectures:  
 $\exists \hat{H}_{\text{non}} \quad H_{\text{non}} \geq 0 \quad \text{physically stable} \quad \boxed{H_{\text{non}}|6\rangle = 0}$

$l_{\text{pl}}$  is a minimal length in observer independent

Lorentz  $\rightarrow$   $\mathcal{P}$

- Poincaré
- Lorentz  $\rightarrow$  way  $\Rightarrow$  post. stage theory
- preferred frame

$\Lambda(\vec{p})$  Define Hopping field, let  $2m \neq 0$   
 character  $H = H_{\text{kin}} \geq 0 + H_{\text{pot}} = 0 \Leftrightarrow$  Flat spacetime  
 Witten QM conjecture:  
 $\exists \hat{H}_{\text{kin}} \hat{H}_{\text{pot}} \geq 0$  physical states  $\boxed{H_{\text{kin}} | \epsilon \rangle = 0}$

$l_{\text{pl}}$  is a minimal length & observer independent  
 Lorentz invariance  $P$

- Poincaré Inv. anyway  $\Rightarrow$  path strings through  
 type inv is broken  $\Rightarrow$  preferred frame  
 Poincaré  $P =$  little group for  $l_{\text{pl}}$  and  $l_{\text{ph}}$



$A(x)$  is a vector field,  $\text{div } A \neq 0$   
 then for  $H = H_{\text{ADM}} \geq 0$  &  $H_{\text{ADM}} = 0 \Leftrightarrow$  Flat spacetime  
 Witten QM conjecture:  
 $\exists \hat{H}_{\text{ADM}} \quad H_{\text{ADM}} \geq 0$  on physical states  $\hat{H}_{\text{ADM}} |6\rangle = 0$

$\ell_{\text{Pl}}$  is a minimal length & observer independent

Lorentz invariance  $P$

- Polymers Inv. anyway  $\Rightarrow$  path strings theory
- Lorentz inv is broken  $\Rightarrow$  preferred frame  
preferred  $P \sim$  lattice spacing  $\Delta x$  and  $\Delta t$

$A(x)$  is defined as asymptotic field, where  $\lambda_n \neq 0$   
 Witten  $H = H_{ADM} \geq 0$  +  $H_{ADM} = 0 \iff$  Flat spacetime  
 QM conjecture:  
 $\exists H_{ADM} \geq 0$  on physical states  $H_{ADM}|6\rangle = 0$

$l_{pl}$  is a minimal length independent

Lorentz invariance  $P$



Inv. symmetry  $\Rightarrow$  preferred frame  
 $\Rightarrow$  preferred frame  
 $\Rightarrow$  little group for  $l_{pl}$

which is  $\langle H_{ADM} \rangle = 0 \Leftrightarrow$  Flat spacetime  
 QM conjectures  
 $\langle H_{ADM} \rangle = 0$   
 $\langle H_{ADM} \rangle = 0$   
 $\langle H_{ADM} \rangle = 0$   
 $\langle H_{ADM} \rangle = 0$

$l_{pl}$  is a minimal length & distance independent

Lorentz invariance  $P$



- Poincaré Inv.  $\Rightarrow$  part. sym. at part
- Lorentz inv.  $\Rightarrow$  preferred frame
- $P =$  little group of  $l_{pl}$  with  $l_{pl}$

which is  $H_{ADM} \approx 0$  &  $H_{ADM} = 0 \Leftrightarrow$  Flat spacetime  
 QM conjectures  
 $\exists H_{ADM} \approx 0$   $H_{ADM} \approx 0$   $H_{ADM} \approx 0$   $H_{ADM} \approx 0$   
 $H_{ADM} | \epsilon \rangle = 0$

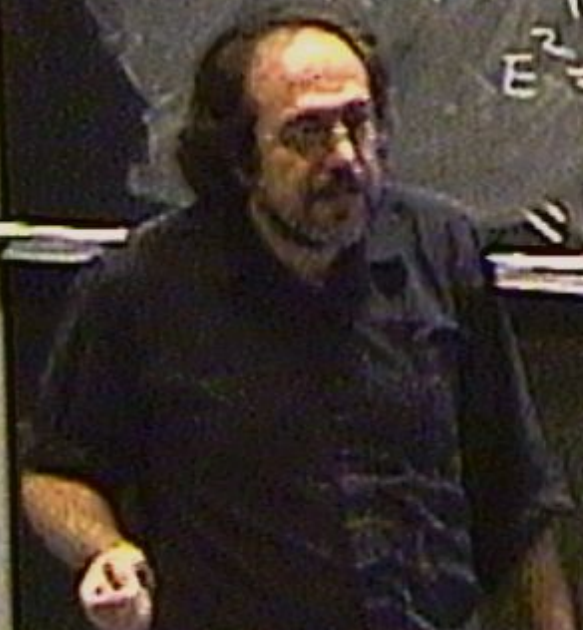
$\ell_{pl}$  is a minimal length in discrete independent

Lorentz invariance  $P$



- Poincare Inv. anyway  $\Rightarrow$  part. strings & part.
- Lorentz inv is broken  $\Rightarrow$  preferred frame

$\vec{p} = (p^1, p^2, p^3)$   
 $E^2 = p^2 + 2\epsilon p^3 + p^4 E^4 + \dots$







Lovely language P

- Poincaré Inv. anyway  $\Rightarrow$  path stays N/away
- Lorentz Inv is broken  $\Rightarrow$  preferred frame

$$1 = \frac{\partial E}{\partial P} = 1 + 2\alpha_P E^2 + \dots$$

$$E^2 = P^2 + 2\alpha_P E^3 + \beta_P P^2 E^4 + \dots$$

$\mathcal{H} \subset \mathcal{H}_{\text{phys}} \Rightarrow \mathcal{H} \cap \mathcal{H}_{\text{phys}} = \mathcal{H} \Leftrightarrow \text{Flat spacetime}$   
 QM conjectures  
 $\exists \mathcal{H}_{\text{phys}} \mathcal{H}_{\text{phys}} \supset \mathcal{H}$  on physical states  
 $\mathcal{H}_{\text{phys}} |0\rangle = 0$

$$H = \frac{1}{2}(E^2 + B^2) +$$

which is  $H_{ADM} \approx 0$  &  $H_{ADM} = 0 \Leftrightarrow$  flat spacetime  
 QM conjecture:  
 $\exists \psi_0$  such that  $H_{ADM} \psi_0 = 0$   $H_{ADM} |\psi\rangle = 0$

$$H = \frac{1}{2}(\vec{E}^2 + \vec{B}^2) + \frac{1}{M_{Pl}} \epsilon^{\mu\nu\rho\sigma} E_\mu \partial_\nu B_\rho$$



$$\langle \psi | \hat{H} | \psi \rangle = \int d^3x \psi^* \hat{H} \psi$$

Canonical P.M.

$$V = \frac{2E}{\partial p} = 1 \times \frac{2E}{\partial p} \quad \dots \quad E^2 = p^2 + \frac{1}{2} p^4 + \frac{1}{2} p^4 E^2$$

$$H = \frac{1}{2} (E^2 + B^2) \quad \frac{1}{M_{Pl}^2} \epsilon^{\mu\nu\rho\sigma} E_\mu \partial_\nu B_\rho$$

$$\text{Gauge Transformations} \quad A_\mu \rightarrow A_\mu + \partial_\mu \chi \quad \psi \rightarrow e^{i q \chi} \psi$$

Canonical Momentum

$$V = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = 1 + \frac{1}{2} \kappa_F E^2 \quad \dots \quad E^2 = P^2 + \kappa_F E^3 + P \kappa_F^2 E^4 \quad \dots$$

$$H = \frac{1}{2} (E^2 + B^2) + \frac{1}{M_{Pl}} \epsilon^{\mu\nu\lambda} E_\mu \partial_\nu B_\lambda$$



$\nabla \cdot \mathbf{E} = \rho$      $\nabla \cdot \mathbf{B} = 0$      $\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$      $\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}$

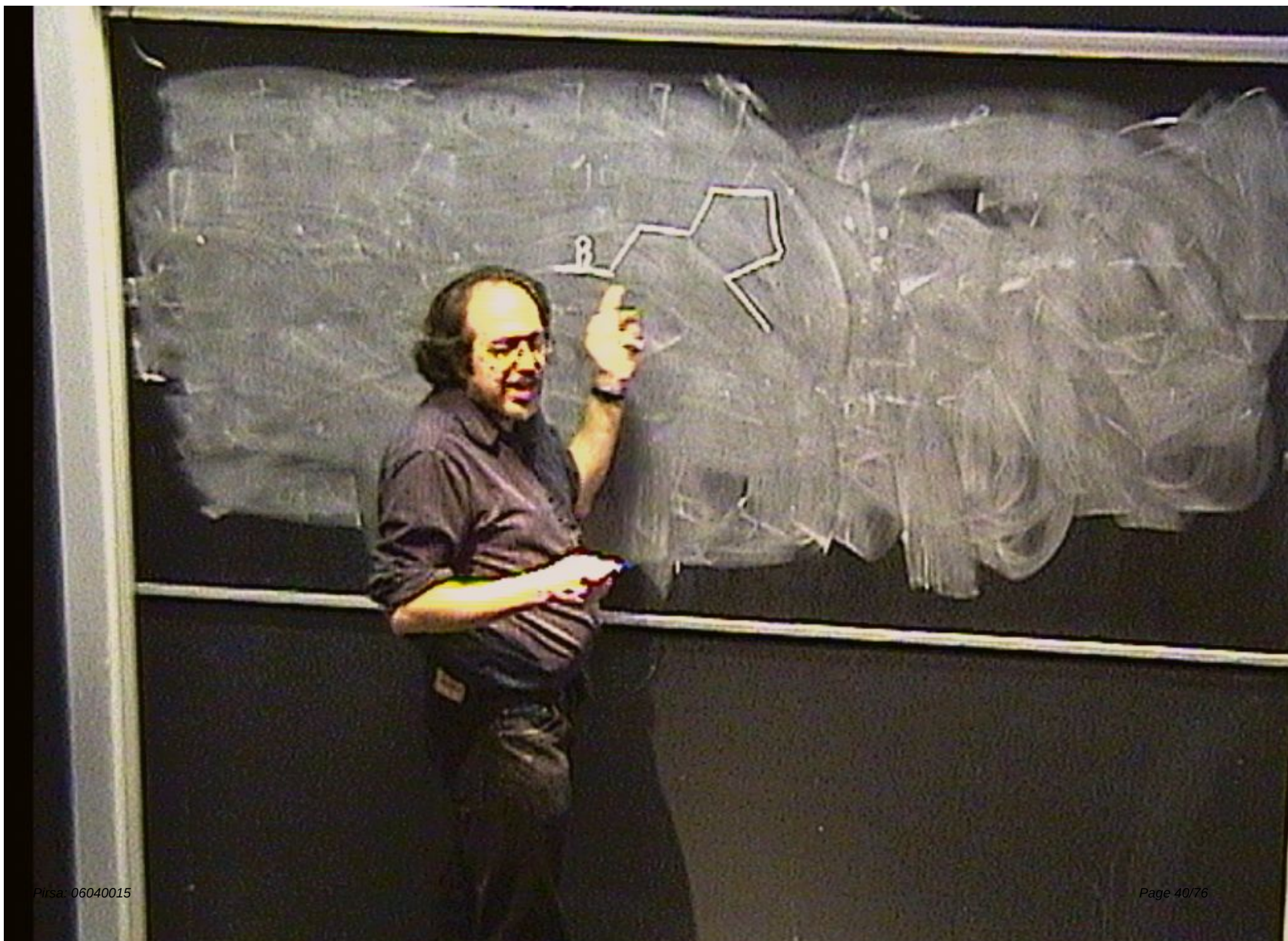
Continuum Limit

$$V = \frac{\partial E}{\partial p} = 1 + \dots$$

$$E^2 = p^2 + \alpha_p E^3 + \beta \alpha_p^2 E^4$$

$$H = \frac{1}{2} (E^2 + B^2) + \frac{10^{-16}}{M_{Pl}^2} \epsilon^{\mu\nu} E_\mu \partial_\nu B_\mu$$





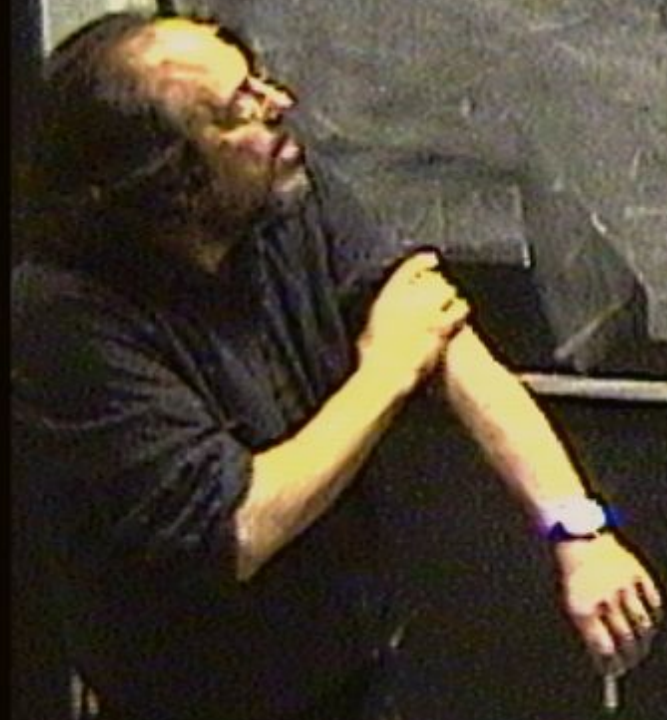
3d option DSR

3d optm DSR

$$E_p = \frac{h\nu}{T_p}$$

3d option DSR

$$E_p = \frac{h\nu}{E_p}$$



3d option DSR

$$E_P = \frac{h^2 c^2}{4p}$$

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$



## Principles of deformed special relativity (DSR):

- 1) Relativity of inertial frames
- 2) The constancy of  $c$ , a velocity
- 3) The constancy of an energy  $E_p$
- 4)  $c$  is the universal speed of photons for  $E \ll E_p$ .

### 3.1 option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk says  
Snyder says

$$E_P = \frac{hc}{\ell_P}$$

# 3d option DSR

$$E_p = \frac{h^2 c}{\lambda_p}$$

$$V(E) = C + O\left(\frac{E}{E_p}\right)$$

Folk 50s

Snyder 50s

Lubliner

Maj

90's  $\chi$ -Minkowski  
 $\chi$ -Folner's

### 3d option DSR

$$E_p = \frac{h^2 c}{\lambda_p}$$

$$V(E) = C + O\left(\frac{E}{E_p}\right)$$

Folk 50s

Snyder 60s

Lukersmith 70s  
Majid

90's

X - Minkowski

X - Poincaré group

# 3d option DSR

$$E_p = \frac{h^2 c^2}{2m_p}$$

$$V(E) = C + O\left(\frac{E}{E_p}\right)$$

Folk sos

Snyder sos

Lukierski et al (2003) 90%  $\gamma$ -Mistake

Majid

Amelino-Canella  $\infty$



# 3d option DSR

$$E_p = \frac{1}{2} \frac{c^2}{v_p}$$

$$V(E) = C + O\left(\frac{E}{E_p}\right)$$

Folk 50s

Snyder 60s

Lukierski et al (90s) 90's  
Majid

Amelino-Canella 00  
Jond & Ia

K-Minkowski  
K-Poincaré sym  
Physikal

# 3d option DSR

$$E_p = \frac{h^2 c^2}{4p}$$

$$V(E) = C + O\left(\frac{E}{E_p}\right)$$

50's  
60's

(1940's) 90's K-Minkowski  
K-Polymerization  
Polymerization

### 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk 50s

Snyder 50s

Lutenski et al (90s) 90s

Majid

Amelina-Camelia

Jonah d. Ia

SO(2,3)

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk sus

Snyder

Lukic

Majid

Amr

10's K - Mistake  
K - Polymers  
Physicists

$SO(4,1)_q$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk ses

Snyder ses

Lukenski + (1981) 90's K-Minkunster

Majid

K - Polycarbonate

Amelino-Gemelli 00 Physikalisch

1000 d. la

SO(4,1)g

$$1 \rightarrow e^{\frac{2\pi i}{f_2}}$$

$$k = \frac{c\eta}{c\lambda}$$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk sos

Snyder sos

Lukienkier  
Majid

Amelun

Jon

K - Miskunster  
K - Polnerer  
Physikler

SO(4,1)g

$$q = e^{\frac{2\pi i}{r_{12}}}$$

M<sub>AB</sub>

$$k = \frac{c\eta}{G\Lambda}$$

Contraction

$$A = 9,5$$

$$q = 4,187$$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk sus

Snyder sus

Lukierski et al 90's  
Majid

Amelino-Canella et al  
2000 & la.

$$SO(4,1)$$

$$l = e^{\frac{2\pi i}{\alpha}}$$

$$M_{AB}$$

$$K = \frac{cA}{\epsilon A}$$

Contraction

$$A = 4,5$$

$$q = 4183$$

$$\sqrt{\Lambda} M_{Pl} = P_c$$

$$\Lambda = \frac{1}{R^2}$$

$$A \rightarrow 0, R \rightarrow \infty$$

# 3d option DSR

$$V(E) = C + O(\frac{E}{E_P})$$

Folk SOS

Snyder SOS

Lukierski et al (2003)  
Majid

Amelino-Canelli  
Jond & L.

$$SO(4,1)$$

$$M_{4,1}$$

Contraction

$$\sqrt{\Lambda} M_{4,1} = P_4$$

$$SO(11,4)$$

$$\rightarrow P_{11,4}$$

$$l \rightarrow e^{\frac{2012}{r_{12}}}$$

$$k = \frac{c\eta}{c\Lambda}$$

$$A = 9,5$$

$$q = 4123$$

$$\Lambda = \frac{1}{R^2}$$

# 3.1 option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk 50s

Snyder 50s

Lukierski et al (70s) 90's  
Majid

Amelino-Canella  
Jond & Ia.

$$SO(2,1)$$

$$M_{AB}$$

Contraction

$$\sqrt{\Lambda} M_{SC} = P_2$$

$$\Lambda \rightarrow 0, R \rightarrow \infty$$

$$SO(1,4)$$

$$\rightarrow Poinc$$

$$l = e^{\frac{2012}{k12}}$$

$$k = \frac{c\eta}{G\Lambda}$$

$$A = 4,5$$

$$4 = 4,123$$

$$\Lambda = \frac{1}{R^2}$$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk sus  
vden cos

stevski et al 90's } K-Minkowski  
1994 } K-Poincare  
(metric  $\infty$  ) } Physikalisch

$$SO(2,1)$$

$$M_{AB}$$

Contraction

$$\sqrt{\Lambda} M_{SC} = P_\mu$$

$$\Lambda \rightarrow 0, R \rightarrow \infty$$

$$SO(1,4)$$

$$\{M_{5\mu}, M_{5\nu}\} = M_{\mu\nu}$$

$$\{P_\mu, P_\nu\} = \frac{1}{R} M_{\mu\nu} \rightarrow 0$$

$$l = R \frac{2\pi\hbar}{m c}$$

$$k = \frac{c\eta}{G\Lambda}$$

$$A = 9,5$$

$$q = 4183$$

$$\Lambda = \frac{1}{R^2}$$

Poincaré

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk 50s

Snyder 60s

Luttenstrich (190s) 90s  
Majid

Amelina-Camelie 00  
Jono & Co.

K-Mistumter  
K-Polheresein  
Phyralden

SO(4,1)g

M<sub>AB</sub>

Contraction

$$q = e^{\frac{2\pi i c}{R^2}}$$

$$k = \frac{60 R^2}{\sqrt{10}}$$

$$n = 4,5$$

$$q = 4,123$$

$$A = \frac{1}{R^2}$$

→ Polite

$$\} = M_{16}$$

$$= \frac{1}{R} M_{16} \rightarrow 0$$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk SOS

Snyder SOS

Lukierski  
Majid

Amelino

Jonckheere

Minkowski

Polynomial

$$SO(4,1)_q$$

$$q = e^{\frac{20\pi}{112}}$$

$$M_{AB}$$

$$k = \frac{c \hbar R^2}{\Delta E \Delta t}$$

Contraction

$$A = 9,5$$

$$q = 4,183$$

$$\Lambda = \frac{1}{R^2}$$

$$\sqrt{\Lambda} M_{SC} = P_2$$

$$\Lambda \rightarrow 0, R \rightarrow \infty$$

$$SO(1,4)_q$$

Polynomial

$$[M_{SC}, M_{SC}] = M_{SC}$$

$$[P_a, P_b] = \frac{1}{R} M_{ab} \rightarrow 0$$

# 3d option DSR

$$V(E) = C + O\left(\frac{E}{E_P}\right)$$

Folk 50s

Snyder 50s

Lukierski et al (90s) 90's  $\kappa$ -Minkowski  
Majid  $\kappa$ -Poincaré

Amelino-Camelia  $\infty$  Physical  
Jouko et al.

$$SO(4,1)_\kappa$$

$$M_{AB}$$

Contraction

$$\sqrt{\Lambda} M_{56} = P_4$$

$$\Lambda \rightarrow 0, R \rightarrow \infty$$

$$SO(11,4)_\kappa$$

$$\{M_{56}, M_{56}\} = M_{56}$$

$$\{P_4, P_4\} = \frac{1}{\kappa} M_{56} \rightarrow 0$$

$$q = e^{\frac{2\pi i}{\kappa R}}$$

$$k = \frac{c\pi R^2}{\Lambda^2}$$

$$A = 9,5$$

$$q = 4103$$

$$\Lambda = \frac{1}{R^2}$$

Poincaré

BIG Q!

$$q_{ab}(\omega)$$

BIG Q:

$$I_{ab}(\omega)$$

$$H(\omega) = \frac{1}{\omega} \epsilon^a \epsilon^b$$

BIG Q!

$$q_{ab}(\omega)$$

$$H(\omega) = \chi(\omega) e^{i\phi}$$

BIG Q!

"Rainbow geometry"  
 $q_{ab}(\omega)$

$$H(\omega) = \frac{1}{2} \epsilon^a \epsilon^b$$

BIG Q:

$$\left| \frac{\text{"Rainbow geometry"}}{q_{ab}(\omega)} \right| = \text{Mik-convol sum} \left| H(\omega) = \frac{2(\omega)E^1 E^L}{\omega} \right|$$

BIG Q:

$$\left| \begin{array}{l} \text{"Rainbow geometry"} \\ \hline \end{array} \right| = \text{Microscopical source} \quad \left| \begin{array}{l} H(\omega) = \frac{1}{2} \epsilon^+ \epsilon^- \\ \hline \end{array} \right|$$

+ matter  
2+1 Grav. & DSR

BIG Q:

$$\left| \text{"Rainbow geometry"} \right| = \text{Anti-Commut geometry} \quad \left| H(\omega) = \frac{2\omega}{\omega^2 + \omega_0^2} E^1 E^2 \right|$$

$$q_{ab}(\omega)$$

+ matter

2+1 Grav'n DSR  
deformed E-H-relati

BIG Q:

$$\left| \text{"Rainbow geometry"} \right| = \text{Mik-Conrad geometry} \quad \left| H(\omega) = \frac{2\omega(\omega)}{E^2 E^2} \right|$$

$$2ab(\omega)$$

+ matter

2+1 GUT & DSR

- deformed E & B-relate
- deformed conservation laws



BIG Q:

$$\left| \text{"Rainbow geometry"} \right| = \text{Minkowski geometry} \quad \left| H(\omega) = \frac{q(\omega) E^4 E^5}{L_{ab}(\omega)} \right|$$

+ matter

2+1 Grav & DSR

- deformed E-Relativity
- deformed conservation laws
- $\eta$ -deformed symmetry  $\Lambda \neq 0$

BIG Q:

$$\left| \text{"Rainbow geometry"} \right| = \text{Minkowski sum} \left| H(\omega) = \gamma(\omega) E^+ E^- \right|$$

$$\gamma_{ab}(\omega)$$

+ matter

2+1 GR & DSR

- deformed E-Relativity
- deformed conservation laws
- $\eta$ -deformed symmetry  $\Lambda \geq 0$

$\rightarrow$  E-thermodynamics



BIG Q:

$$\left| \text{"Rainbow geometry"} \right| = \text{Minkowski space} \left| \begin{array}{l} \mathcal{L}_{ab}(\omega) \\ + \text{matter} \end{array} \right| \quad \left| H(\omega) = \mathcal{L}_{ab}(\omega) E^a E^b \right|$$

2+1 Grav. & DSR  
 • deformed E & B-relate  
 • deformed conservation laws  
 •  $\eta$ -deformed symmetry  $\Lambda \neq 0$   
 • E & B-relate

3+1 semiclassical argument?

BIG Q:

"Rainbow geometry" = MacCallum's geometry  
 $g_{ab}(\omega)$

+ matter

2+1 Gruen & DSR

• deformed E- $\vec{p}$ -relate

• deformed conservation laws

•  $\eta$ -deformed sym

• E- $\vec{p}$  & Lorentz

3+1 Semiclassical

$$H(\omega) = \frac{2\omega}{\Lambda} E^+ E^-$$

$$LED \sim M_{Pl} \sim 10 \text{ TeV}$$

# BIG Q!

"Rainbow geometry" = Multi-Comit geometry  
 $\mathcal{L}_{ab}(\omega)$

+ matter

2+1 Gravitational DSR

- deformed E-PB-relate
- deformed conservation laws
- $\eta$ -deformed symmetry

→ E-extended Lorentz

3+1 semiclassical

$$H(\omega) = \mathcal{L}(\omega) E^+ E^-$$

LED  $\sim M_{Pl} \sim 10 \text{ TeV}$

⇒ LHC experiments?

# BIG Q!

$$\left| \frac{\text{"Rainbow symmetry"}}{q_{ab}(\omega)} \right| = \text{Mandelstam symmetry} \quad \left| H(\omega) = \frac{q(\omega) E^4 E^4}{\dots} \right|$$

+ matter

2+1 Grav & DSR

- deformed E- $\vec{p}$ -relate
- deformed conservation laws
- $\eta$ -deformed symmetry  $\Lambda \neq 0$
- E- $\vec{p}$  laws

3+1 semiclassical argument?

DSR, QFT

LEP,  $\sim 10$  TeV

arguments?

