Title: Accelerated expansion from structure formation

Date: Apr 11, 2006 04:00 PM

URL: http://pirsa.org/06040011

Abstract: I discuss the backreaction of inhomogeneities on the expansion of the universe. The average behaviour of an inhomogeneous spacetime is not given by the Friedmann-Robertseon-Walker equations. The new terms in the exact equations hold the possibility of explaining the observed acceleration without a cosmological constant or new physics. In particular, the coincidence problem may be solved by a connection with structure formation.

Accelerated expansion from structure formation

Syksy Räsänen

CERN

Perimeter Institute cosmology seminar, April 11, 2006

Dark energy

Dark energy

- Observations (SNIa): $d_L(z)$.

Dark energy

- Observations (SNIa): $d_L(z)$.

Dark energy

- Observations (SNIa): $d_L(z)$.
- Interpreted in the context of the homogeneous and isotropic FRW model:

Dark energy

- Observations (SNIa): d_L(z).
- Interpreted in the context of the homogeneous and isotropic FRW model:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N(\rho + 3p) \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\dfrac{k}{a^2} \end{cases} \qquad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Dark energy

- Observations (SNIa): $d_L(z)$.
- Interpreted in the context of the homogeneous and isotropic FRW model:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N(\rho + 3p) \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N\rho - 3\dfrac{k}{a^2} \end{cases} \qquad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

- For dust, $p = 0 \Rightarrow \rho_{\rm m} \propto a^{-3}$ decreases too fast

  $\Rightarrow$ add a component which decays more slowly than dust

  $\Leftrightarrow p < 0$

Dark energy

- Observations (SNIa): $d_L(z)$.
- Interpreted in the context of the homogeneous and isotropic FRW model:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N(\rho + 3p) \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\dfrac{k}{a^2} \end{cases} \qquad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

- For dust, $p = 0 \Rightarrow \rho_m \propto a^{-3}$ decreases too fast

  $\Rightarrow$ add a component which decays more slowly than dust

  $\Leftrightarrow p < 0$
- The issue is not $p < 0$ but $z \sim 1$: why $\rho_m \sim \rho_{de}$ today.

  This is **the coincidence problem.**

Reasons for the ‘coincidence’

Reasons for the 'coincidence'

1) Coincidence

$(\rho_{m0})^{1/4} \sim 10^{-3}$ eV $\sim (\mathrm{TeV})^2/M_{\mathrm{Pl}}$

Reasons for the 'coincidence'

1) Coincidence

$(\rho_{m0})^{1/4} \sim 10^{-3}$ eV $\sim (\text{TeV})^2/M_{Pl}$

2) Anthropic

We cannot observe domains where $|\rho_\Lambda| \gg \rho_{m0}$.

Reasons for the 'coincidence'

1) Coincidence

$(\rho_{m0})^{1/4} \sim 10^{-3}$ eV $\sim (\text{TeV})^2/M_{Pl}$

2) Anthropic

We cannot observe domains where $|\rho_\Lambda| \gg \rho_{m0}$.

3) Dynamical

Late-time events in the visible universe:

- $z \sim 3500$ matter-radiation equality
- $z \sim 1100$ matter-radiation decoupling
- $z \sim 10\text{-}100$ growth of structure
- $z \sim 0$ maximum size of collapsed objects

Backreaction

Backreaction

- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**

Backreaction

- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**
- The Einstein equation is non-linear in the metric:

  $<G_{\mu\nu}(g_{\alpha\beta})> \neq G_{\mu\nu}(<g_{\alpha\beta}>)$

  $\Leftrightarrow$ average quantities ($<\rho>$, $<\theta>$, …) do not satisfy the Einstein equation.

Backreaction

- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**
- The Einstein equation is non-linear in the metric:
  $<G_{\mu\nu}(g_{\alpha\beta})> \neq G_{\mu\nu}(<g_{\alpha\beta}>)$
  $\Leftrightarrow$ average quantities ($<\rho>$, $<\theta>$, …) do not satisfy the Einstein equation.
- This is **the fitting problem** (Ellis 1983): how do we find the homogeneous model that best fits the inhomogeneous universe?

FRW assumptions

Backreaction

- **The average behaviour of an inhomogeneous spacetime is not the same as the behaviour of the corresponding smooth spacetime.**
- The Einstein equation is non-linear in the metric:
  $<G_{\mu\nu}(g_{\alpha\beta})> \neq G_{\mu\nu}(<g_{\alpha\beta}>)$
  $\Leftrightarrow$ average quantities ($<\rho>$, $<\theta>$, …) do not satisfy the Einstein equation.
- This is **the fitting problem** (Ellis 1983): how do we find the homogeneous model that best fits the inhomogeneous universe?

FRW assumptions

FRW assumptions

1) FRW metric

Observables can be computed from an average scale factor.

FRW assumptions

1) FRW metric

Observables can be computed from an average scale factor.

2) FRW dynamics

The average scale factor evolves according to the FRW equations.

FRW assumptions

1) FRW metric

Observables can be computed from an average scale factor.

2) FRW dynamics

The average scale factor evolves according to the FRW equations.

3) FRW + perturbations

The inhomogeneities evolve according to linear perturbation theory around the average.

Backreaction, exactly

Backreaction, exactly

- Let us take a universe with irrotational dust. The metric is

$$ds^2 = -dt^2 + {}^{(3)}g_{ij}(t, x^i)dx^i dx^j$$

Backreaction, exactly

- Let us take a universe with irrotational dust. The metric is

$$ds^2 = -dt^2 + {}^{(3)}g_{ij}(t,x^i)dx^i dx^j$$

- The Einstein equation gives the following exact, local equations:

$$\begin{cases} \dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G_N \rho - 2\sigma^2 \\ \frac{1}{3}\theta^2 = 8\pi G_N \rho - \frac{1}{2}{}^{(3)}R + \sigma^2 \\ \dot{\rho} + \theta\rho = 0 \end{cases}$$

- Here θ is the expansion rate of the local volume element, $\sigma^2 \geq 0$ is the shear and ${}^{(3)}R$ is the spatial curvature.

- The FRW equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \rho \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\dfrac{k}{a^2} \end{cases}$$

$$\partial_t \rho + 3\frac{\dot{a}}{a}\rho = 0$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \dfrac{1}{2}\langle R \rangle - \dfrac{1}{2} Q \end{cases}$$

$$\partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a}\langle \rho \rangle = 0$$

- The FRW equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \rho \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\dfrac{k}{a^2} \end{cases}$$

$$\partial_t \rho + 3\frac{\dot{a}}{a}\rho = 0$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \dfrac{1}{2}\langle R \rangle - \dfrac{1}{2} Q \end{cases} \qquad \partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a}\langle \rho \rangle = 0$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \dfrac{1}{2}\langle R \rangle - \dfrac{1}{2} Q \end{cases} \qquad \partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a}\langle \rho \rangle = 0$$

- Here $a(t) \propto \left(\int d^3x \sqrt{{}^{(3)}g} \right)^{1/3}$. The backreaction is contained in

$$Q \equiv \frac{2}{3}\left(\langle \theta^2 \rangle - \langle \theta \rangle^2 \right) - 2\langle \sigma^2 \rangle \qquad \langle A \rangle \equiv \frac{\int d^3x \sqrt{{}^{(3)}g}\, A}{\int d^3x \sqrt{{}^{(3)}g}}$$

- The Buchert equations:

$$\begin{cases} 3\frac{\ddot{a}}{a} = -4\pi G_N \langle\rho\rangle + Q \\ 3\frac{\dot{a}^2}{a^2} = 8\pi G_N \langle\rho\rangle - \frac{1}{2}\langle R\rangle - \frac{1}{2}Q \end{cases}$$

$$\partial_t \langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0$$

- Here $a(t) \propto \left(\int d^3x \sqrt{^{(3)}g}\right)^{1/3}$. The backreaction is contained in

$$Q \equiv \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) - 2\langle\sigma^2\rangle \qquad \langle A\rangle \equiv \frac{\int d^3x \sqrt{^{(3)}g}A}{\int d^3x \sqrt{^{(3)}g}}$$

- The average expansion can accelerate, even though the local expansion rate decelerates everywhere.
- What is the physical meaning of this?

Backreaction, exactly

- Let us take a universe with irrotational dust. The metric is

$$ds^2 = -dt^2 + {}^{(3)}g_{ij}(t,x^i)dx^i dx^j$$

- The Einstein equation gives the following exact, local equations:

$$\begin{cases} \dot{\theta} + \frac{1}{3}\theta^2 = -4\pi G_N \rho - 2\sigma^2 & \Rightarrow Ht \leq 1 \\ \frac{1}{3}\theta^2 = 8\pi G_N \rho - \frac{1}{2}{}^{(3)}R + \sigma^2 & \\ \dot{\rho} + \theta\rho = 0 & \end{cases}$$

- Here θ is the expansion rate of the local volume element, $\sigma^2 \geq 0$ is the shear and ${}^{(3)}R$ is the spatial curvature.

- The FRW equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \rho \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \rho - 3\dfrac{k}{a^2} \end{cases}$$

$$\partial_t \rho + 3\frac{\dot{a}}{a}\rho = 0$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \dfrac{1}{2}\langle R \rangle - \dfrac{1}{2} Q \end{cases} \qquad \partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a}\langle \rho \rangle = 0$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle\rho\rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle\rho\rangle - \dfrac{1}{2}\langle R\rangle - \dfrac{1}{2}Q \end{cases} \qquad \partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0$$

- Here $a(t) \propto \left(\int d^3x \sqrt{^{(3)}g}\right)^{1/3}$. The backreaction is contained in

$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle \qquad \langle A\rangle \equiv \frac{\int d^3x \sqrt{^{(3)}g}\,A}{\int d^3x \sqrt{^{(3)}g}}$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \dfrac{1}{2}\langle R \rangle - \dfrac{1}{2} Q \end{cases} \qquad \partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a}\langle \rho \rangle = 0$$

- Here $a(t) \propto \left(\int d^3x \sqrt{^{(3)}g} \right)^{1/3}$. The backreaction is contained in

$$Q \equiv \frac{2}{3}\left(\langle \theta^2 \rangle - \langle \theta \rangle^2\right) - 2\langle \sigma^2 \rangle \qquad \langle A \rangle \equiv \frac{\int d^3x \sqrt{^{(3)}g}\, A}{\int d^3x \sqrt{^{(3)}g}}$$

- The average expansion can accelerate, even though the local expansion rate decelerates everywhere.
- What is the physical meaning of this?

Acceleration from collapse

Acceleration from collapse

- Linear perturbations around the FRW equations break down at $\delta \sim 1$.
- A simple treatment of a forming structure: the spherical collapse model.

Acceleration from collapse

- Linear perturbations around the FRW equations break down at $\delta \sim 1$.
- A simple treatment of a forming structure: the spherical collapse model.

- The FRW equations themselves break down when perturbations with $\delta \sim 1$ occupy a large fraction of space.
- A toy model of structure formation: the union of an underdense and an overdense spherical region.

Acceleration from collapse

- Linear perturbations around the FRW equations break down at $\delta \sim 1$.
- A simple treatment of a forming structure: the spherical collapse model.

- The FRW equations themselves break down when perturbations with $\delta \sim 1$ occupy a large fraction of space.
- A toy model of structure formation: the union of an underdense and an overdense spherical region.

- For an empty void we have $a_1 \propto t$ and for an overdensity we have $a_2 \propto 1\text{-}\cos\varphi$, $t \propto \varphi\text{-}\sin\varphi$.
- The overall scale factor is $a = (a_1{}^3 + a_2{}^3)^{1/3}$.

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_1 + \frac{a_2^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_2 = v_1 H_1 + v_2 H_2$$

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2$$

$$H_1 t = 1$$

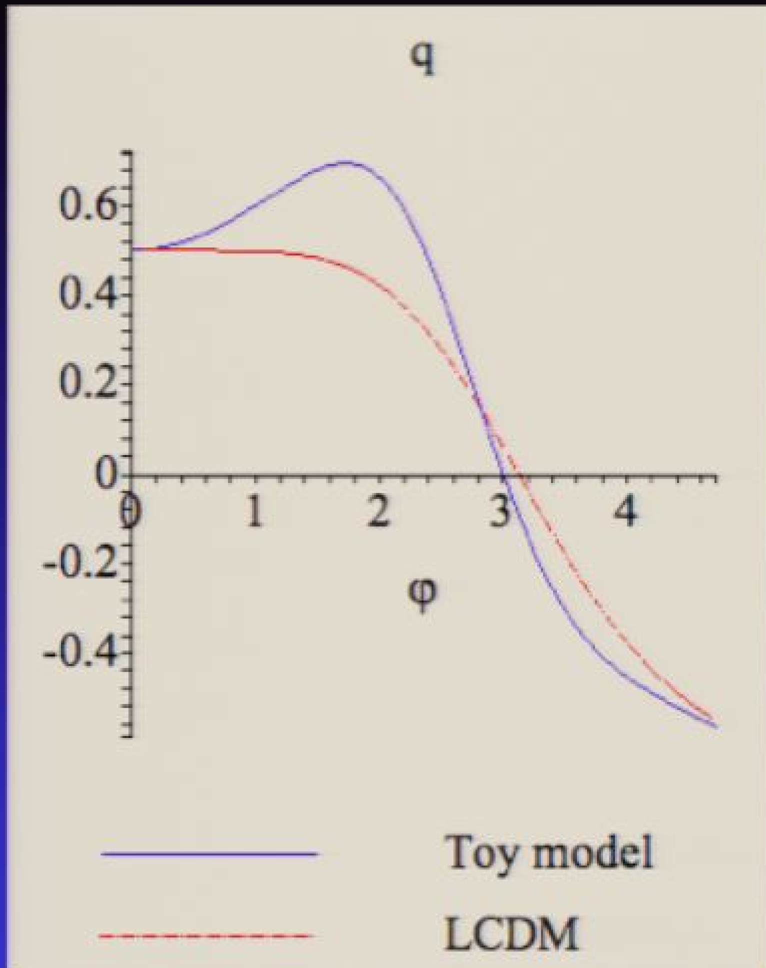
$$H_2 t = \frac{\sin\varphi(\varphi - \sin\varphi)}{(1 - \cos\varphi)^2}$$

$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^{\,3}}{a_1^{\,3}+a_2^{\,3}} H_1 + \frac{a_2^{\,3}}{a_1^{\,3}+a_2^{\,3}} H_2 = v_1 H_1 + v_2 H_2$$

$$H_1 t = 1$$

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{H_1^{\,2}}{H^2} v_1 q_1 + \frac{H_2^{\,2}}{H^2} q_2 v_2 - 2 v_1 v_2 \frac{(H_1 - H_2)^2}{H^2}$$

$$H_2 t = \frac{\sin\varphi(\varphi - \sin\varphi)}{(1-\cos\varphi)^2}$$

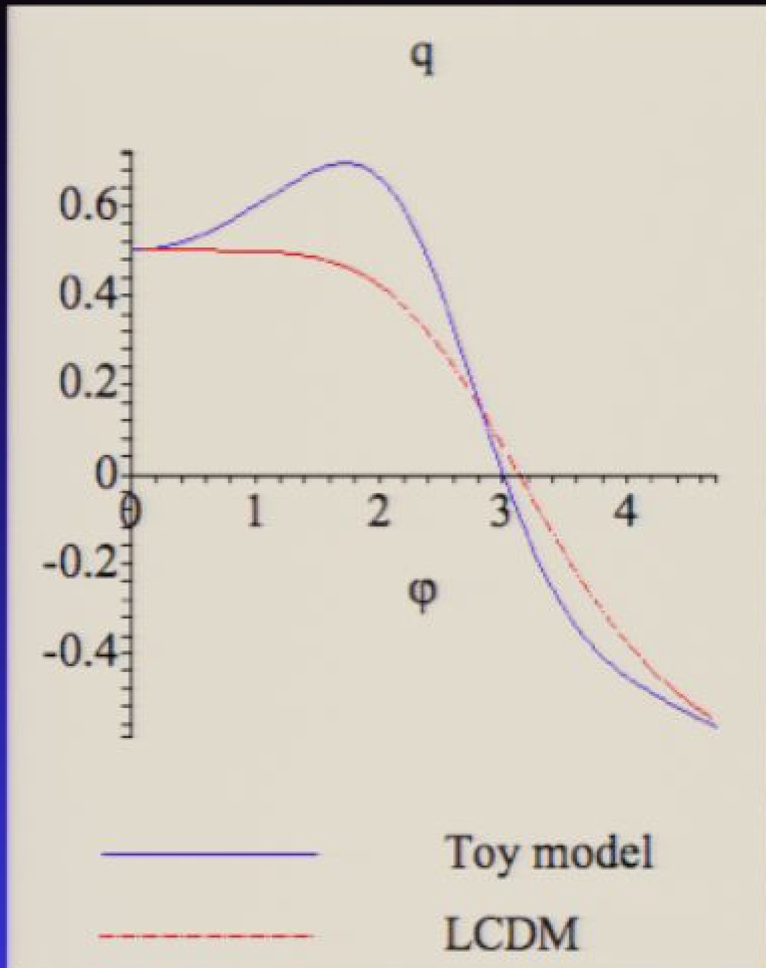


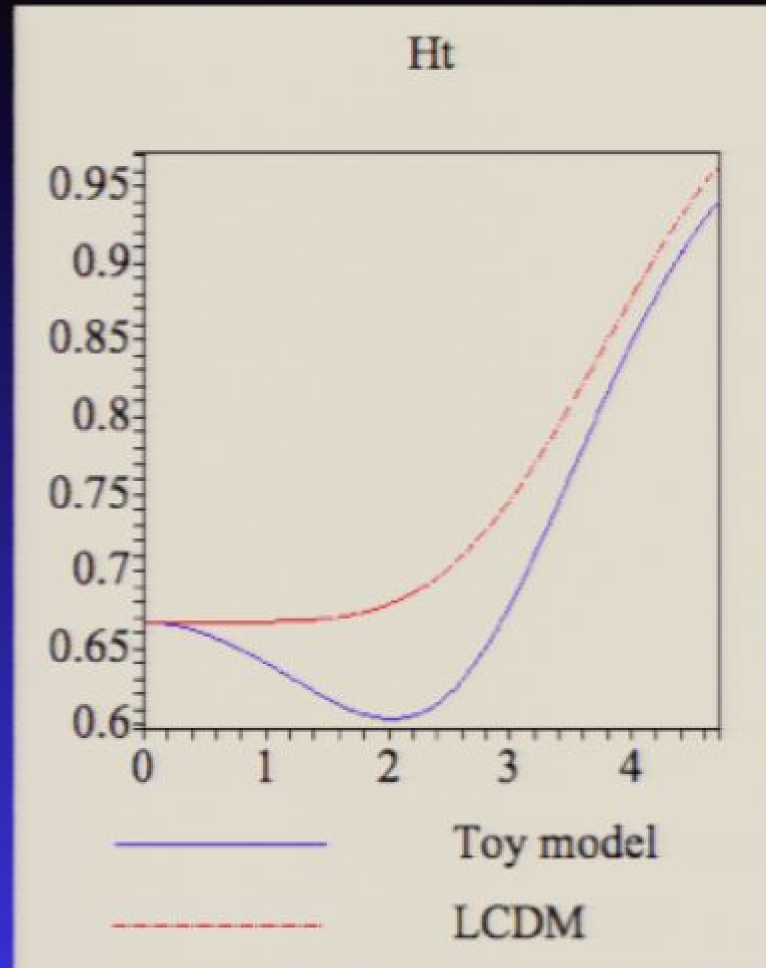
$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_1 + \frac{a_2^{\ 3}}{a_1^{\ 3} + a_2^{\ 3}} H_2 = v_1 H_1 + v_2 H_2$$

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{H_1^{\ 2}}{H^2} v_1 q_1 + \frac{H_2^{\ 2}}{H^2} q_2 v_2 - 2 v_1 v_2 \frac{(H_1 - H_2)^2}{H^2}$$

$$H_1 t = 1$$

$$H_2 t = \frac{\sin\varphi(\varphi - \sin\varphi)}{(1-\cos\varphi)^2}$$





$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2 \qquad H_1 t = 1$$

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{H_1^2}{H^2} v_1 q_1 + \frac{H_2^2}{H^2} q_2 v_2 - 2 v_1 v_2 \frac{(H_1 - H_2)^2}{H^2} \qquad H_2 t = \frac{\sin\varphi(\varphi - \sin\varphi)}{(1-\cos\varphi)^2}$$

Collapse and coincidence

Collapse and coincidence

- In the real universe, perturbations are nested inside each other hierarchically, so part of the universe is always collapsing.

Collapse and coincidence

- In the real universe, perturbations are nested inside each other hierarchically, so part of the universe is always collapsing.
- First structures collapse around $z \sim 50$.

Collapse and coincidence

- In the real universe, perturbations are nested inside each other hierarchically, so part of the universe is always collapsing.
- First structures collapse around **z** ~ 50.
- The size of the structures which are about to collapse relative to the horizon size grows, saturating at $k^2_{NL}/(aH)^2 \approx 10^{-5}$ around 10-100 billion years.

Collapse and coincidence

- In the real universe, perturbations are nested inside each other hierarchically, so part of the universe is always collapsing.
- First structures collapse around **z** ~ 50.
- The size of the structures which are about to collapse relative to the horizon size grows, saturating at $k^2_{NL}/(aH)^2 \approx 10^{-5}$ around 10-100 billion years.
- One would expect the departure from the FRW equations to be largest when the structures reach their maximum size.

Conclusion

Conclusion

- The FRW equations do not describe the expansion of an inhomogeneous space.

Conclusion

- The FRW equations do not describe the expansion of an inhomogeneous space.
- The Buchert equations show that even when the local expansion decelerates everywhere, the average expansion can accelerate.

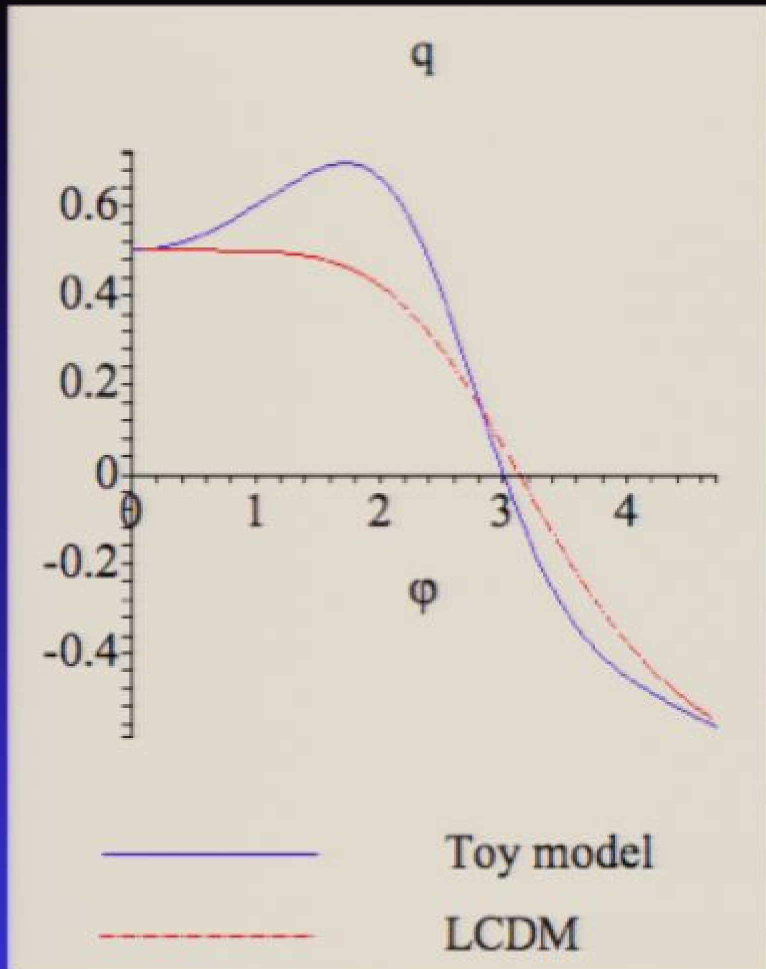
Conclusion

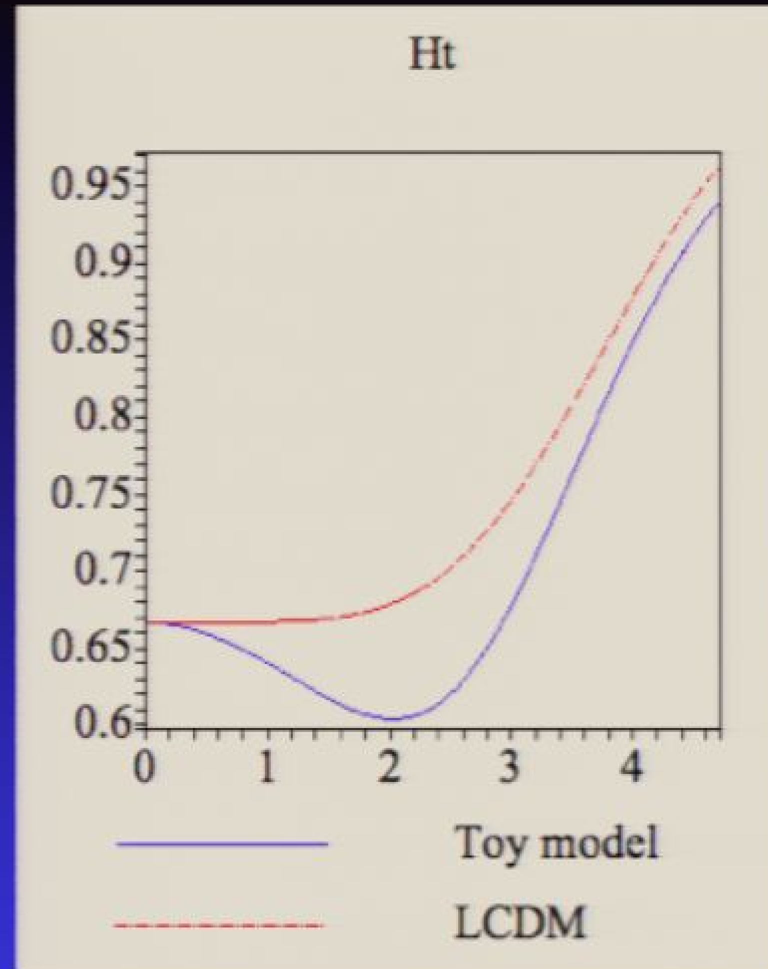
- The FRW equations do not describe the expansion of an inhomogeneous space.
- The Buchert equations show that even when the local expansion decelerates everywhere, the average expansion can accelerate.
- Acceleration is intimately related to collapse, and structure formation has a preferred time around the acceleration era.

Conclusion

- The FRW equations do not describe the expansion of an inhomogeneous space.
- The Buchert equations show that even when the local expansion decelerates everywhere, the average expansion can accelerate.
- Acceleration is intimately related to collapse, and structure formation has a preferred time around the acceleration era.

- The next step is to build a quantitative model.
- The FRW metric assumption will also have to be checked.





$$H \equiv \frac{\dot{a}}{a} = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 = v_1 H_1 + v_2 H_2$$

$$H_1 t = 1$$

$$H_2 t = \frac{\sin\varphi(\varphi - \sin\varphi)}{(1 - \cos\varphi)^2}$$

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{H_1^2}{H^2} v_1 q_1 + \frac{H_2^2}{H^2} q_2 v_2 - 2 v_1 v_2 \frac{(H_1 - H_2)^2}{H^2}$$

- The Buchert equations:

$$\begin{cases} 3\dfrac{\ddot{a}}{a} = -4\pi G_N \langle\rho\rangle + Q \\ 3\dfrac{\dot{a}^2}{a^2} = 8\pi G_N \langle\rho\rangle - \dfrac{1}{2}\langle R\rangle - \dfrac{1}{2}Q \end{cases} \qquad \partial_t\langle\rho\rangle + 3\frac{\dot{a}}{a}\langle\rho\rangle = 0$$

- Here $a(t) \propto \left(\int d^3x\sqrt{{}^{(3)}g}\right)^{1/3}$. The backreaction is contained in

$$Q \equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle \qquad \langle A\rangle \equiv \frac{\int d^3x\sqrt{{}^{(3)}g}A}{\int d^3x\sqrt{{}^{(3)}g}}$$

- The average expansion can accelerate, even though the local expansion rate decelerates everywhere.
- What is the physical meaning of this?