Title: Dimer models and Mirror Symmetry

Date: Apr 11, 2006 02:00 PM

URL: http://pirsa.org/06040010

Abstract:

Dimer Models and Mirror Symmetry

Based on 0511287 B. Feng, Y.-H. He, K.K., C. Vafa
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(0504110 S. Franco, A. Hanany, K.K, D. Vegh, B. Wecht)

Claim

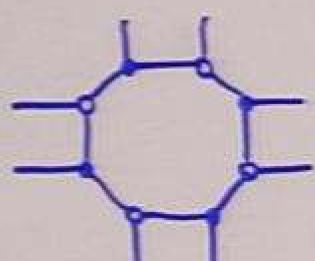
Combinatorics of certain planar graphs on T^2 encodes information about

- geometry of local toric CY mfds
- local mirror symmetry
- topological B/A-branes on toric CY/mirror

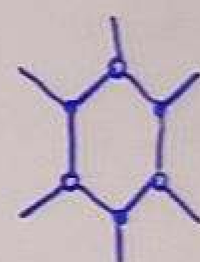
Application:

Holomorphic sector of $\mathcal{N}=1$ SUSY gauge theory in $d=3+1$ living on world-volume of D-branes on CY

e.g.



$\mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^1}(-K)$



$\mathcal{O}_{\mathbb{P}^2}(-3)$

Type IIB Superstring Perspective

• $\mathcal{M}$ a non-compact, toric CY_3

global $U(1)^3 \subset (\mathbb{C}^*)^3$ action

→ complex cone over toric 4-mfd

(→ real cone over toric Sasaki-Einstein 5 mfd; X_5

AdS/CFT → $AdS_5 \times X_5$)

e.g. $\mathcal{O}_{\mathbb{P}^2}(-3)$

SUSY D-branes filling (3+1)-d wrapping hol. cycles on CY

→ world-volume theory is $\mathcal{N}=1$, $d=3+1$ gauge theory

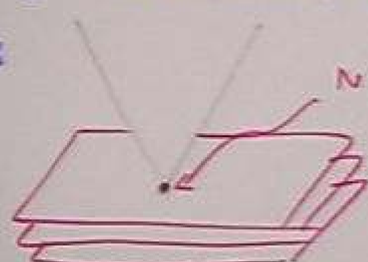
$\mathcal{N}=1$ SUSY vacua determined by

– F-flatness → holomorphic data • depends on C.S. moduli

– D-flatness → stability ; D-branes mutually BPS

• depends on Kähler moduli

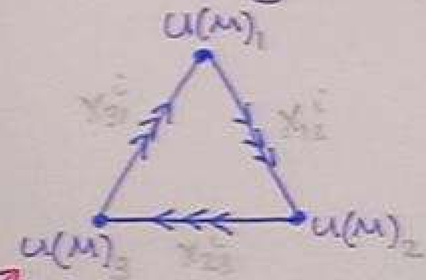
e.g. vary Kähler modulus of $\mathbb{P}^2$ to obtain $\mathbb{C}^3/\mathbb{Z}_3$



N D3-branes at orbifold point.

2

World-volume theory is a quiver gauge theory



$$W = \epsilon_{ijk} \, \mathrm{Tr}\, X_{12}^i X_{23}^j X_{31}^k$$

Geometrically, these correspond to 3 fractional branes localized to orbifold fixed pt. / wrapped on collapsed $\mathbb{P}_2$

F-flatness : $\dfrac{\partial W}{\partial X_{ij}^k} = 0$

D-flatness : $\sum_{k=1}^{3} \left(|X_{i,i+1}^k|^2 - |X_{i+1,i+2}^k|^2 \right) = \zeta_k \; ; \; \sum_i \zeta_k = 0$

2 real D.O.F. ~ $B + iJ$

Holomorphic sector (matter content + W) is accessible to topological B-model on $\mathcal{M}$ → spectrum of topological B-branes does not depend on Kähler moduli

$\parallel^{ly}$, topological A-branes on $\mathcal{W}$ do not depend on C.S.

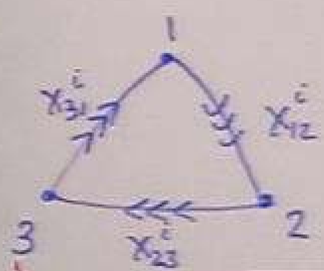
→ important later

• Introduce dimer models & show how they encode hol. data

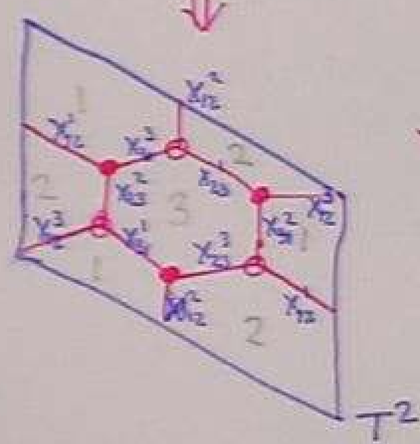
3

Dimer Models on T^2

e.g.



$+ \quad W = \epsilon_{ijk} \, \mathrm{Tr} \, X_{12}^i X_{23}^j X_{31}^k$



"Dimer graph"
Γ

Dictionary

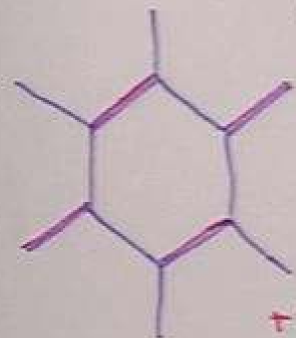
Faces $\longleftrightarrow$ Gauge groups $U(N_i)$

Edges $\longleftrightarrow$ Bifundamental $(N_i, \bar{N}_j)$ chiral multiplets

Vertices $\longleftrightarrow$ W terms (with sign)

Useful tool for studying gauge theory

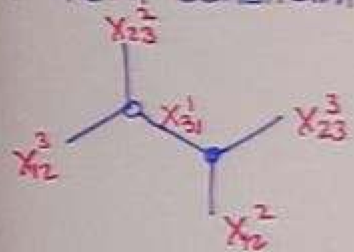
Dimer configurations satisfy F-flatness



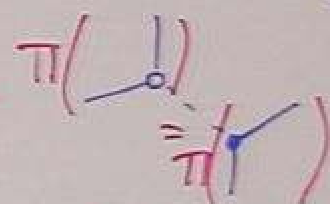
each vertex covered exactly once by a dimer

+5 more

F-term constraints $\frac{\partial W}{\partial X_{ij}^{k}} = 0$ represented graphically



X_{31}^{1} :



$$X_{12}^{3} X_{23}^{2} = X_{12}^{2} X_{23}^{3}$$

Geometrically, the quiver fields X_{ij}^{k} correspond to sections of a line bundle on $\mathcal{M}$

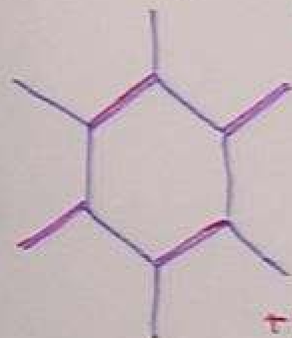
$$X_{ij}^{k} \longleftrightarrow \prod_{\substack{\text{edges} \\ e_i}} p_\alpha^{\langle e_i, p_\alpha \rangle} \qquad \langle e_i, p_\alpha \rangle = \begin{cases} 1 & e_i \in \text{matching } \alpha \\ 0 & \text{otherwise} \end{cases}$$

Matching $\alpha \longleftrightarrow$ coordinate p_α on $\mathcal{M}$ (GLSM field)

$p_\alpha = 0$ defines a toric divisor D_α

X_{ij}^{k} is a section of $\mathcal{O}\left(\sum_{e_i} \langle e_i, p_\alpha \rangle D_\alpha\right)$

Dimer configurations satisfy F-flatness



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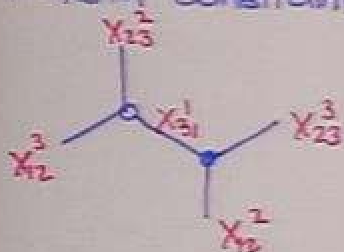
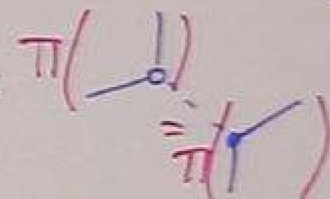


X^1_{31} :



$$X^3_{12} X^2_{23} = X^2_{12} X^3_{23}$$

Geometrically, the quiver fields X^k_{ij} correspond to sections of a line bundle on $\mathcal{M}$

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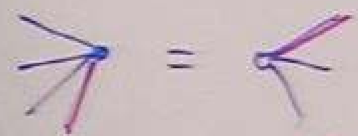
For every F-term constraint, the matchings enumerate all combinations of LHS and RHS edges



dimer on LHS

must pair with RHS

Written in terms of dimer configurations, the F-flatness conditions are trivially satisfied

$$\prod_{c\in LHS}\prod_{\alpha} p_\alpha^{\langle e_c, p_\alpha\rangle} = \prod_{c\in RHS}\prod_{\alpha} p_\alpha^{\langle e_c, p_\alpha\rangle}$$

every p_α appearing here

also appears here

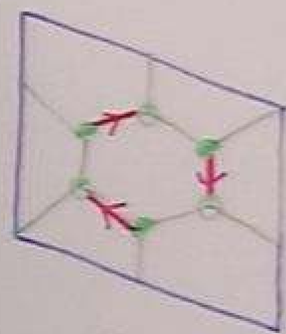
In fact each matching α maps to a lattice point in toric fan generating $\mathcal{M}$ $\rightarrow$ encodes the toric geometry of the CY

Where do the dimer models come from in string theory?

① Map matchings → lattice vectors $\in \mathbb{Z}^3$

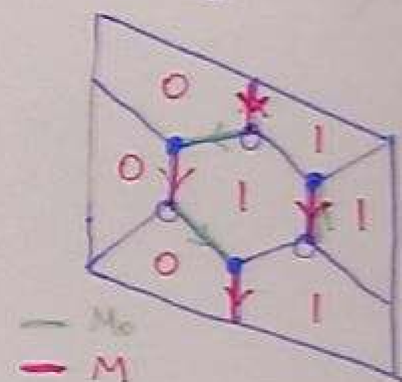
Fix reference matching M_0

e.g.

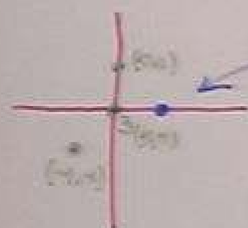


M_0

Differences $M - M_0$ define height function



Δheight $= (1,0)$



Extend $\mathbb{Z}^2$ vectors $\to \mathbb{Z}^3$

$$\vec{m}_i = \begin{cases} (1,0,1) \\ (0,1,1) \\ (0,0,1) \\ (-1,-1,1) \end{cases}$$

Generators of toric fan of $\mathcal{O}_{\mathbb{P}^2}(-3)$

Alternatively, define a GLSM

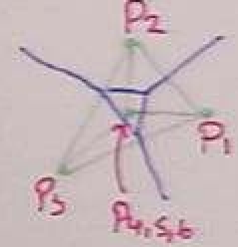
Fields $p_\alpha \longleftrightarrow$ matching α

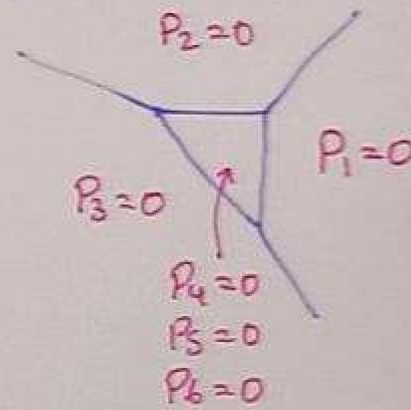
Charges given by linear relations in $\vec{m}_i$

$$\begin{array}{rl} 1 & (1,0,1) \\ 1 & (0,1,1) \\ 1 & (-1,-1,1) \\ -3 & (0,0,1) \\ \hline & (0,0,0) \end{array}$$

$Q = (1,1,1,-3)$

Exhibit space as Lag. T^3 fibration:





$p_\alpha = 0$ defines a toric divisor D_α

$p_\alpha = 0$ is a section of $\mathcal{O}(D_\alpha)$

Introduce intersection pairing

$$\langle e_i, p_\alpha \rangle = \begin{cases} 1 & e_i \in \text{matching } \alpha \\ 0 & \text{otherwise} \end{cases}$$

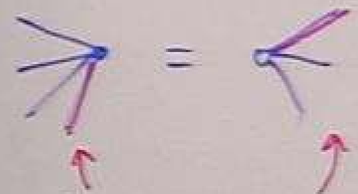
edge $\in \Gamma$ matching α on Γ

Then

$$X_{ij}^{k} \longleftrightarrow \underbrace{\prod_{\substack{e_i \in \Gamma \\ \alpha \in \text{matchings}}} p_\alpha^{\langle e_i, p_\alpha \rangle}}_{\text{section of } \mathcal{O}\left(\sum_{e_i} \langle e_i, p_\alpha \rangle D_\alpha\right)}$$

quiver field

For every F-term constraint, the matchings enumerate all combinations of LHS and RHS edges



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In fact each matching α maps to a lattice point in toric fan generating $\mathcal{M}$ → encodes the toric geometry of the CY

Where do the dimer models come from in string theory?

6

Hint: dimer configurations on Γ are enumerated by the spectral curve of Γ

$$\det K(z,w)=0 \qquad K=\begin{pmatrix} a_{31}^{3}\frac{1}{z} & a_{12}^{1} & a_{23}^{2} \\ a_{23}^{1} & a_{31}^{2}\frac{1}{w} & a_{12}^{3} \\ a_{12}^{2} & a_{23}^{3} & a_{31}^{1}zw \end{pmatrix}$$

adjacency matrix of Γ decorated with $z^{\pm 1}, w^{\pm 1}$

$$\det K = (a_{12}^1 a_{12}^2 a_{12}^3 + a_{23}^1 a_{23}^2 a_{23}^3 + a_{31}^1 a_{31}^2 a_{31}^3) - (a_{12}^2 a_{23}^2 a_{31}^2)\frac{1}{w} - a_{12}^3 a_{23}^3 a_{31}^3 \frac{1}{z} - a_{12}^1 a_{23}^1 a_{31}^1 wz$$

$$\overset{\text{rescale}}{=} 1 + z + w + e^t/zw \qquad e^t = f(a_{ij}^k)$$

which is related to the equation of the mirror CY

$$\det K(z,w) + uv = 0$$

This suggests that the dimer models on T^2 have something to do with mirror symmetry...

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rescale

$$\equiv 1 + z + w + e^t/zw \qquad e^t = f(a_{ij}^k)$$

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Dimer Models on T^2

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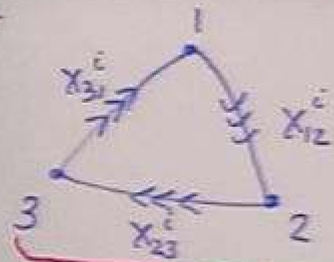

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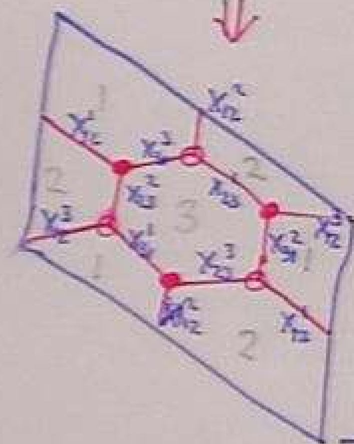

T^2

"Dimer graph" Γ

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$$\det K = (a^1_{12} a^2_{12} a^3_{12} + a^1_{23} a^2_{23} a^3_{23} + a^1_{31} a^2_{31} a^3_{31}) - (a^2_{12} a^2_{23} a^2_{31})\frac{1}{w} - a^3_{12} a^3_{23} a^3_{31}\frac{1}{z} - a^1_{12} a^1_{23} a^1_{31} wz$$

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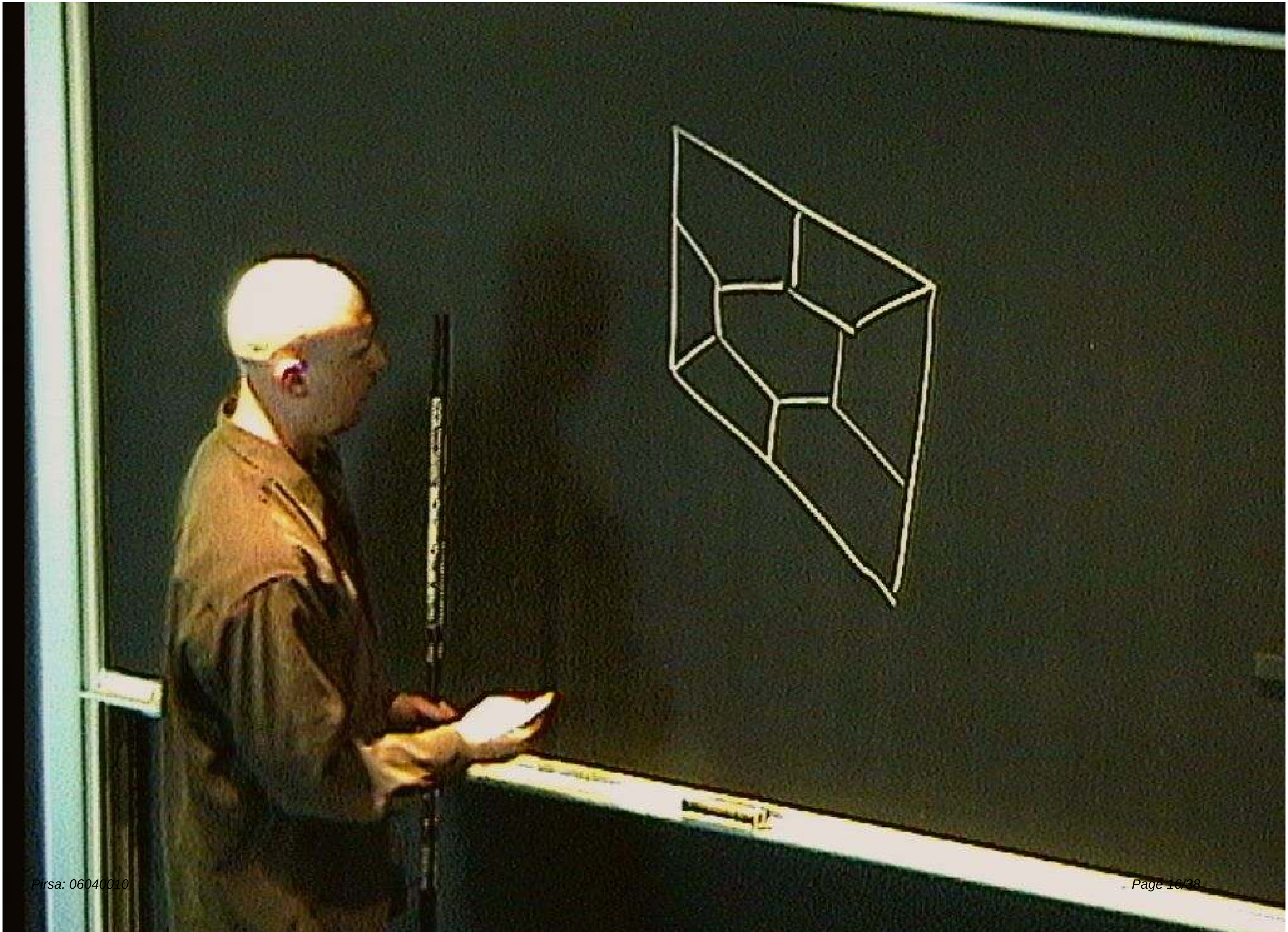

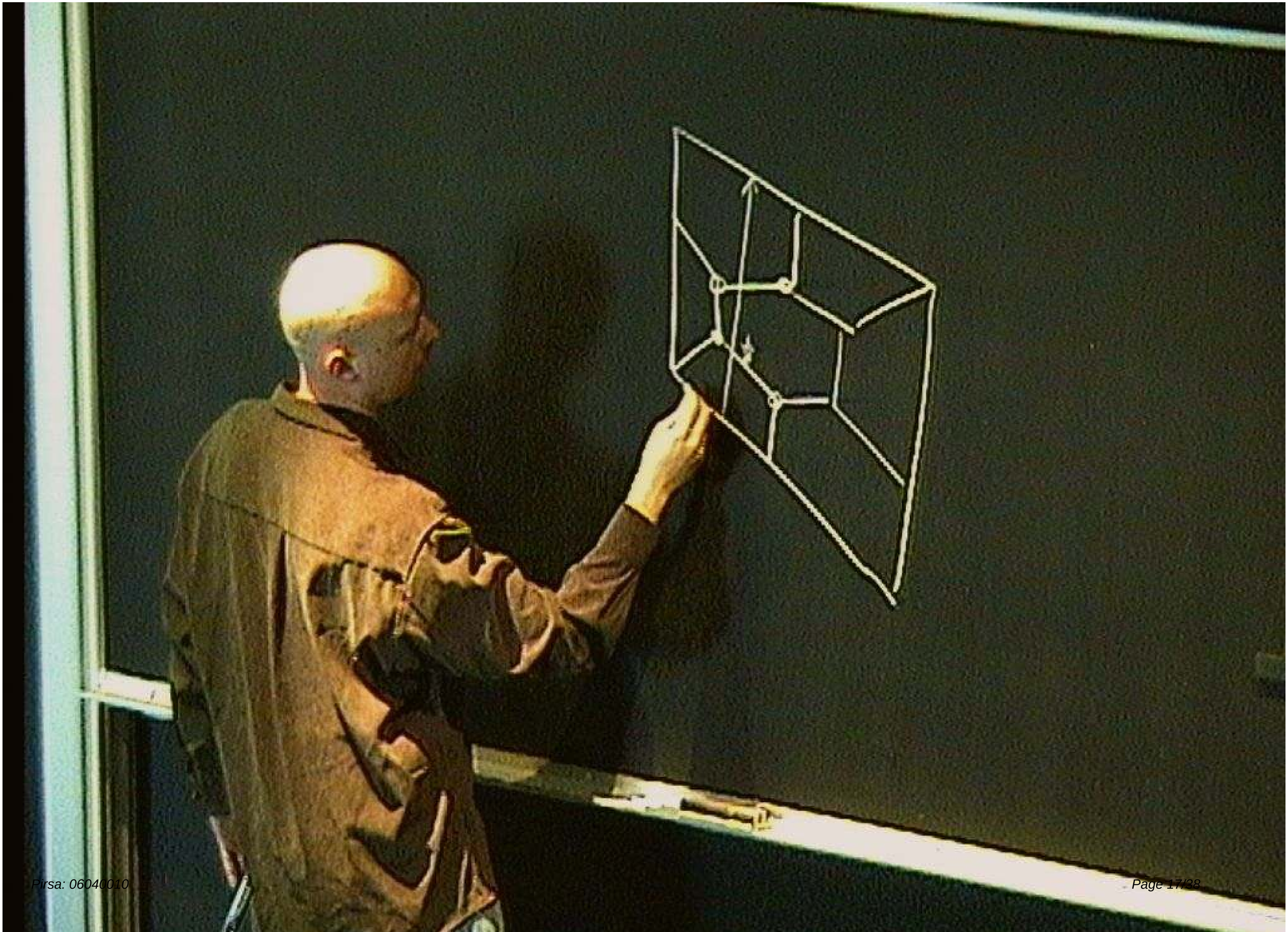

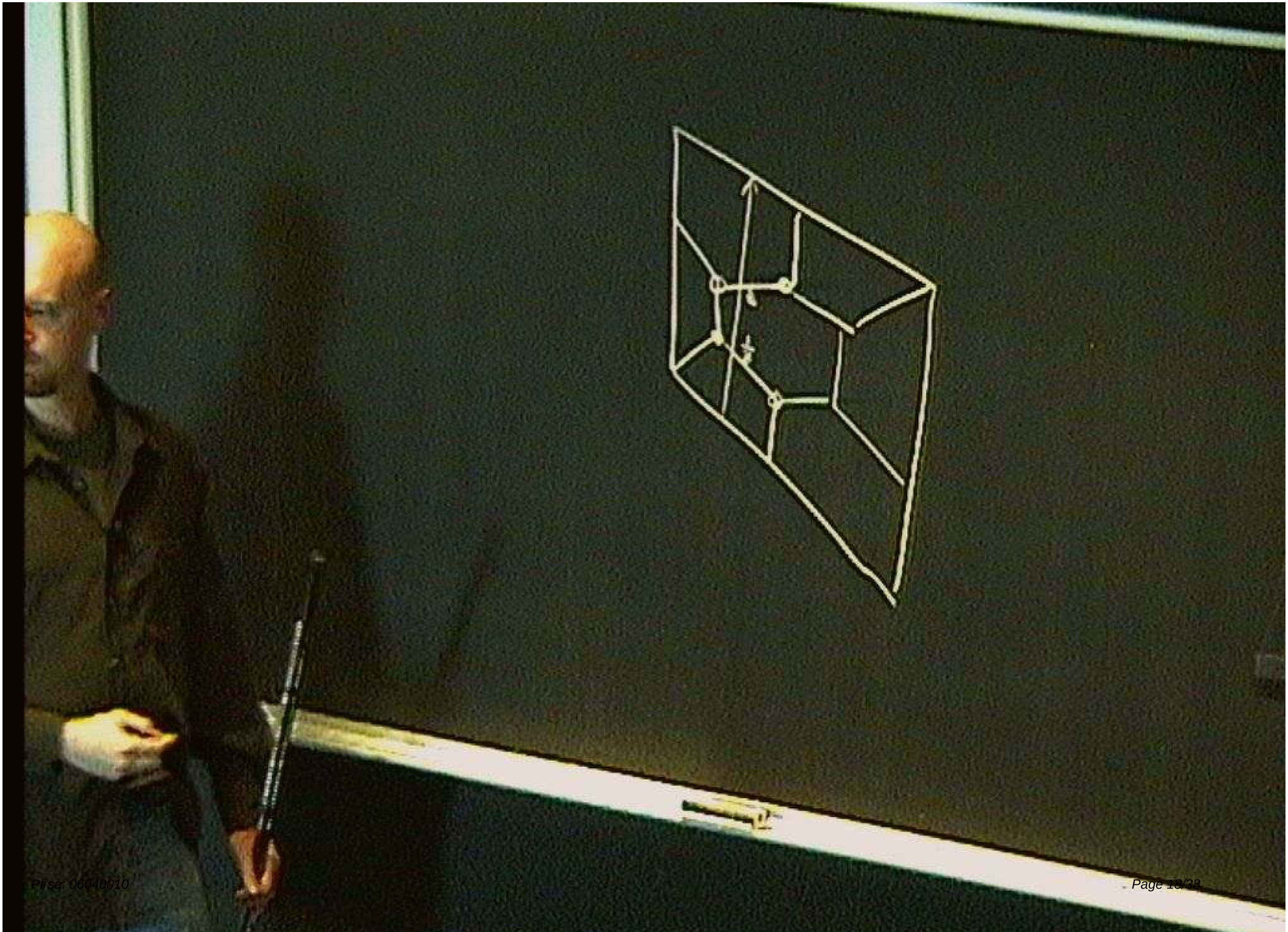

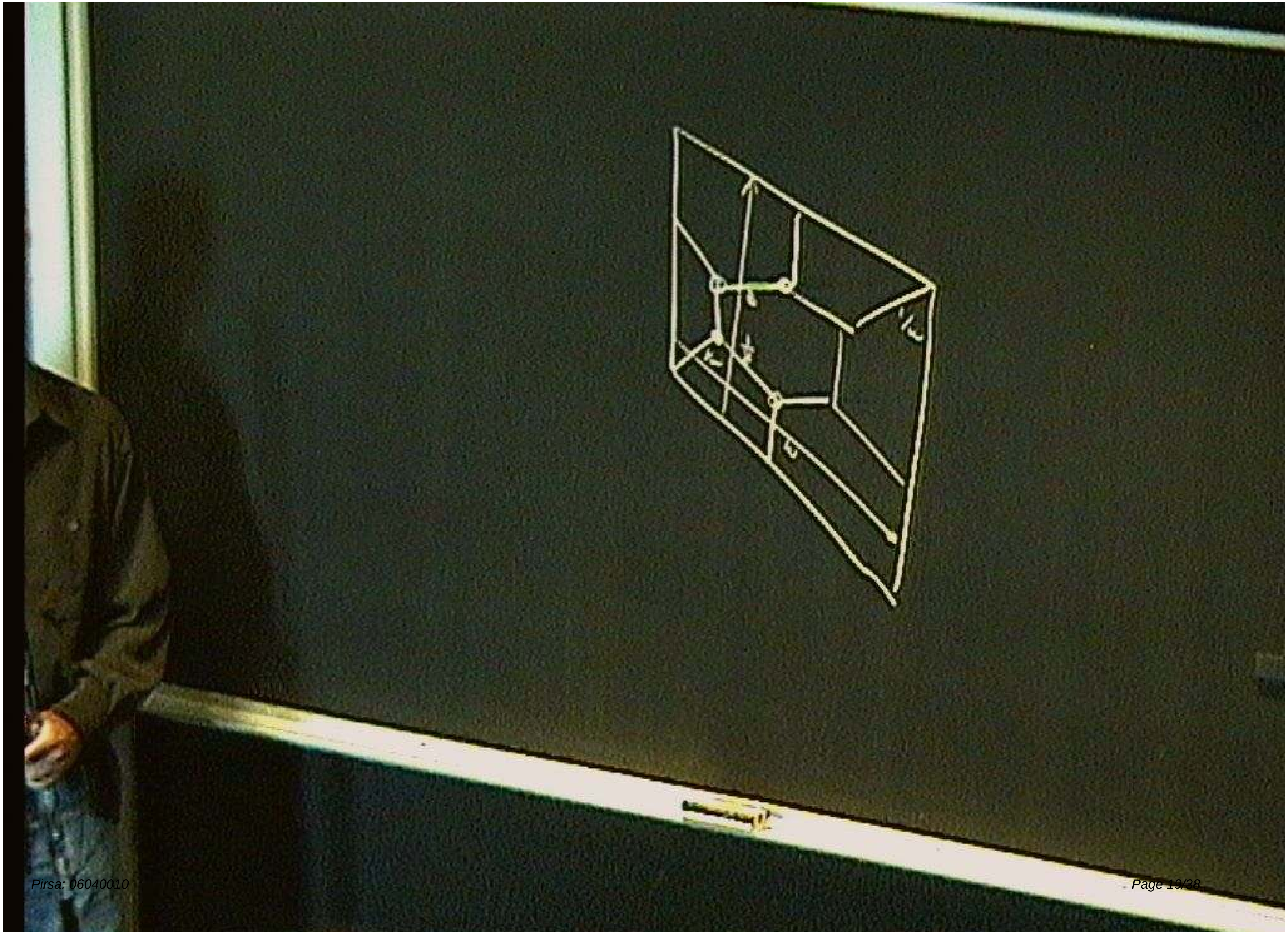

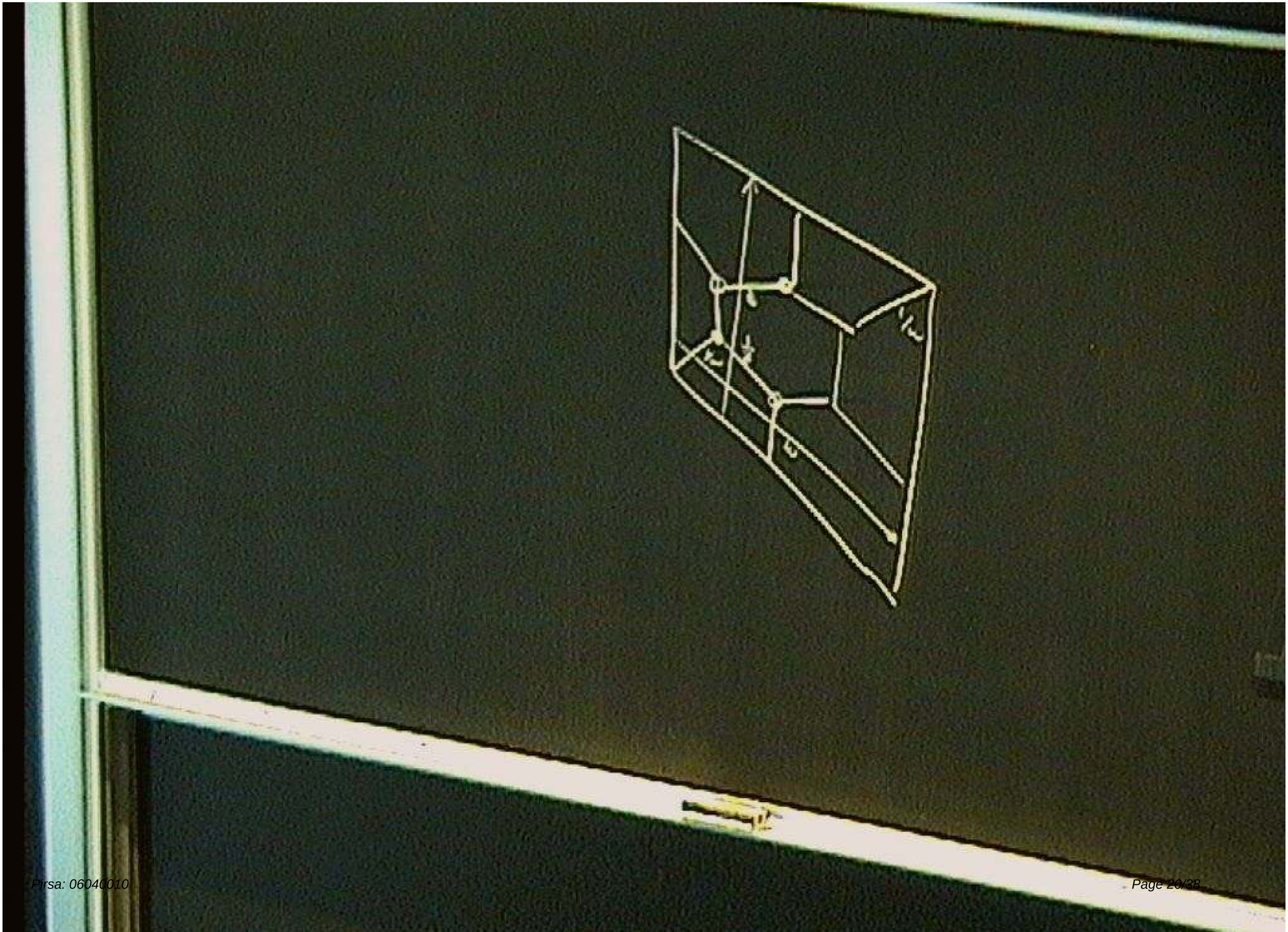

Hint: dimer configurations on Γ are enumerated by the spectral curve of Γ

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$$- (a^2_{12}\,a^2_{23}\,a^2_{31})\frac{1}{w} - a^3_{12}\,a^3_{23}\,a^3_{31}\frac{1}{z} - a^1_{12}\,a^1_{23}\,a^1_{31}\,wz$$

rescale
$$\cong 1 + z + w + e^t/zw \qquad e^t = f(a^k_{ij})$$

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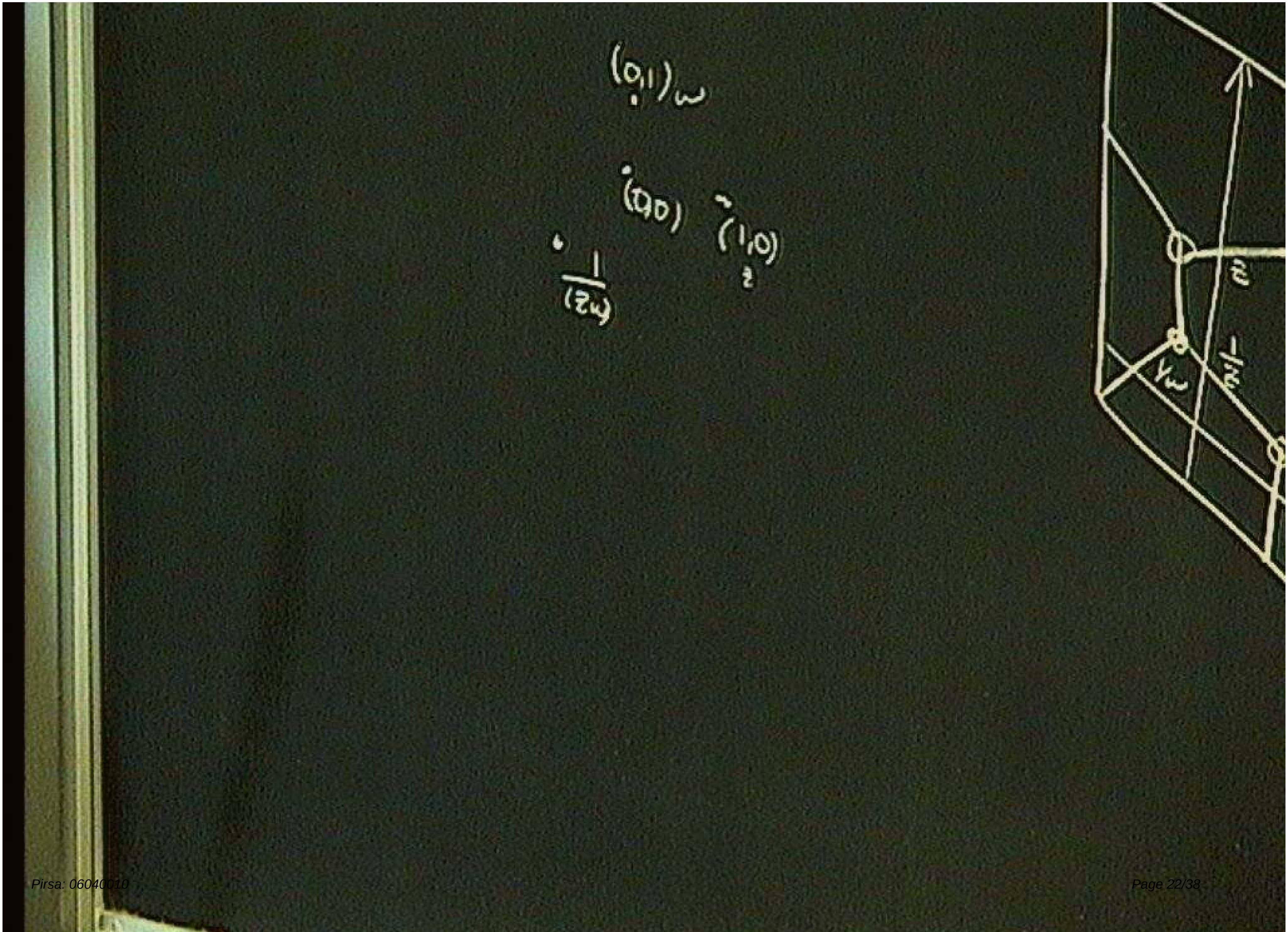
(0,1)w
(0,0)
(1,0)z
1/(zw)
1/w
1/z
z

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7

The Proposal

Recall!

- The fractional branes S_i at orbifold point are fixed but sum to give D0-brane which is free to move
- //^{ly}, at large volume the S_i^{lv} may be defined as sheaves supported on exceptional cycle $E(\mathbb{P}^2)$
- The mirror to a point on $\mathcal{M}$ (D0) has topology T^3

Claim: The mirror to a D0-brane on E has topology

$$\begin{array}{c} S^1 \\ \downarrow \\ T^2 \end{array}$$

where the fibre vanishes along a locus equivalent to $\Gamma \subset T^2$, the graph of the dimer model.

Moreover, the faces of $\Gamma \subset T^2$ are equal (in homology) to the mirror to S_i

Dimer model	Gauge theory	String theory
T^2		$T^2 \subset T^3$ Mirror to point $\in E$
$\Gamma \subset T^2$		Vanishing $S^1 \to T^2$
Face	Gauge group	Mirror to S_i
Edge	Bifundamental	$S_i \cap S_j$; massless open string
Vertex	W term	Disc instanton

8

Rewrite $\quad P(z,w) + uv = 0 \qquad u,v \in \mathbb{C}$

$\qquad\qquad \overset{\text{III}}{\det K(z,w)} \qquad z,w \in \mathbb{C}^*$

as $\quad \left.\begin{array}{l} P(z,w) = \varpi \\ uv = \varpi \end{array}\right\} \quad \begin{array}{c} \Sigma \times \mathbb{C}^* \\ \downarrow \\ \mathbb{C} \end{array}$ Double fibration over w-plane

At every $\varpi \in \mathbb{C}$ $\quad P(z,w) - \varpi = 0$ defines non-compact curve $\subset (\mathbb{C}^*)^2$

$\mathbb{C}^*$ action on $(u,v) \mapsto (\lambda u, \lambda^{-1} v) \quad \lambda \in \mathbb{C}^*$

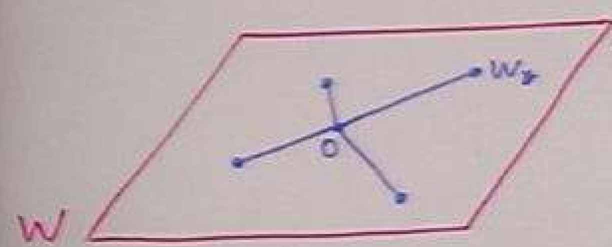
Where does this fibration degenerate?

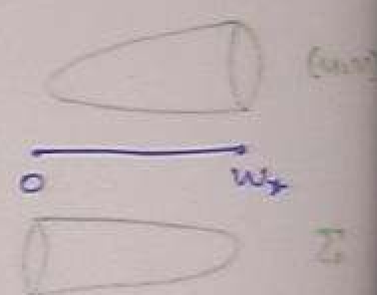
$\mathbb{C}^*$ action degenerates at $W=0 \to$ vanishing S^1 in fibre

$P(z,w) = W$ degenerates when $\dfrac{\partial P}{\partial z} = \dfrac{\partial P}{\partial w} = P - W = 0$

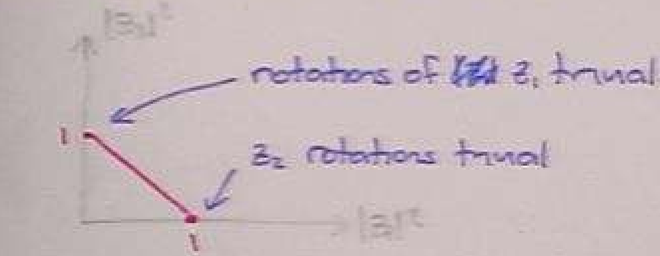
ie. at critical points of $P(z,w)$, $W = W_*$

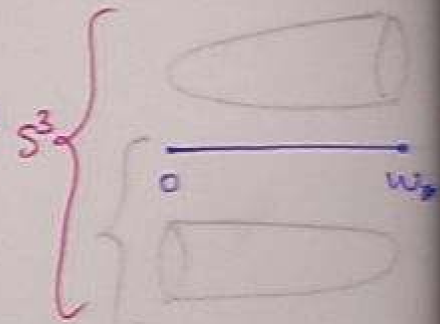
$\to S^1 \subset \Sigma$ vanishing cycle



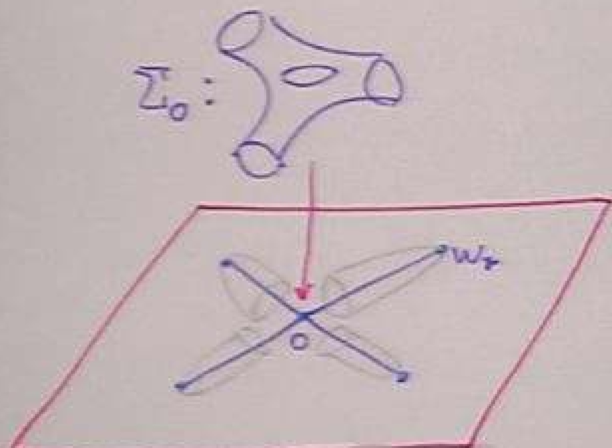


cf $|z_1|^2 + |z_2|^2 = 1 \rightarrow S^3$





Disc with bdy at S^3



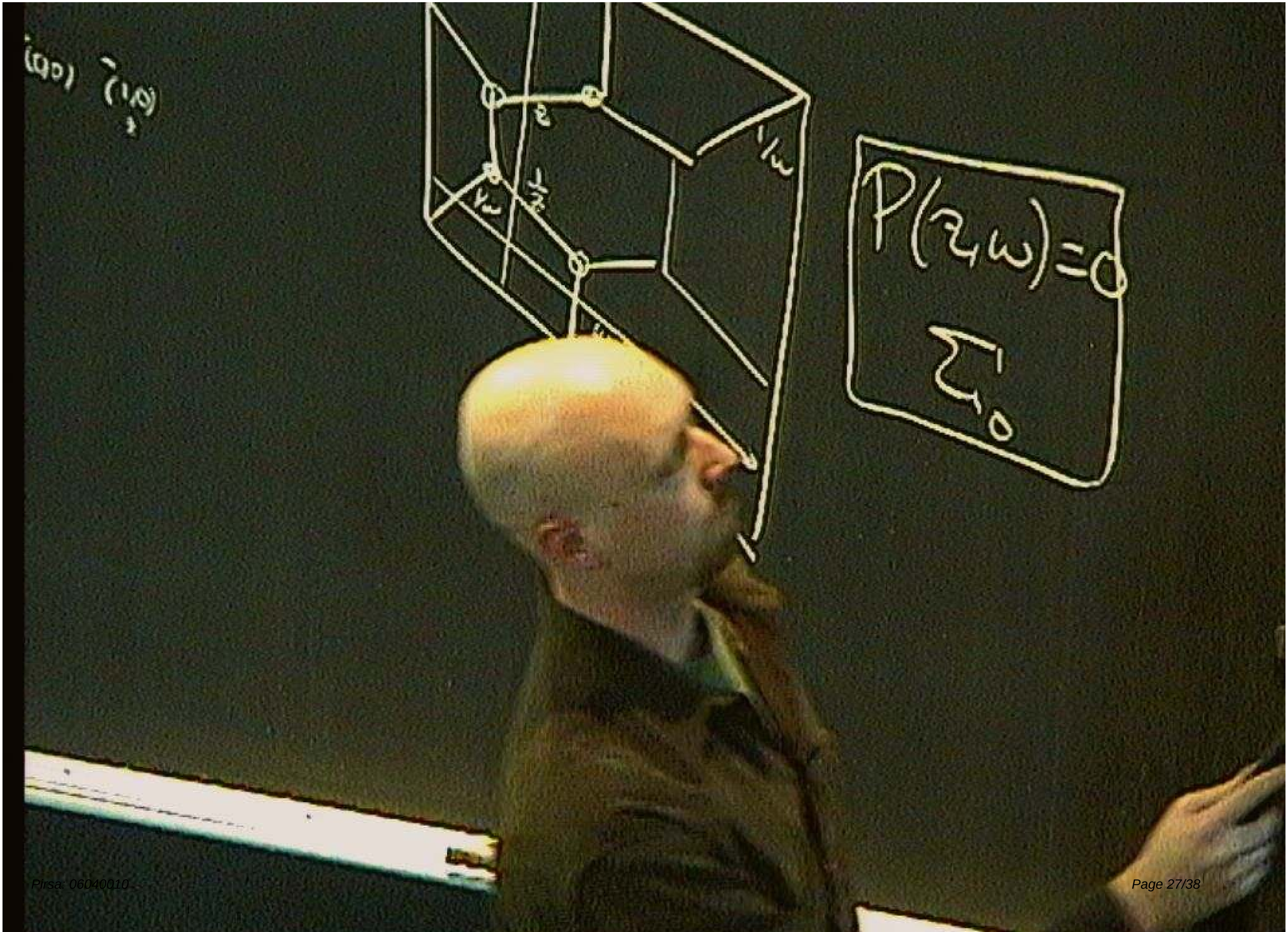
Each interval defines
on $S^3 \subset W$
$\parallel$
$S^1 \rightarrow D_2$

The discs all meet the fibre Σ_0 along 1-cycles

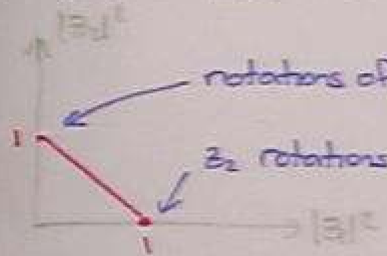
The S^3's are the mirror to the fractional branes [HIV, HI]

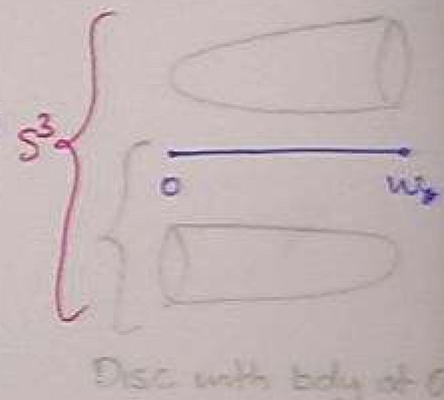
- intersection number gives chiral matter content
- Previously not known how to recover W, non-chiral matter

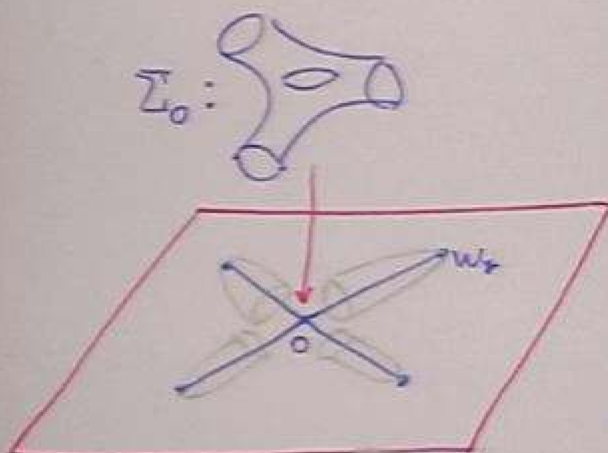
There exists an explicit map from $\Gamma \subset T^2$ to $\Gamma \subset \Sigma_0$ that defines the topology of the fractional branes & gives full data of quiver theory


$P(z,\omega)=0$
$\Sigma_{1,0}$

cf $|z_1|^2 + |z_2|^2 = 1 \to S^3$







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The S^3's are the mirror to the fractional branes [HIV, HI]

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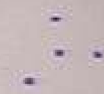
There exists an explicit map from $\Gamma \subset T^2$ to $\Gamma \hookrightarrow \Sigma_0$ that defines the topology of the fractional branes & gives full data of quiver theory e.g. $P(z,w) = 1 + z + w + e^t/zw$

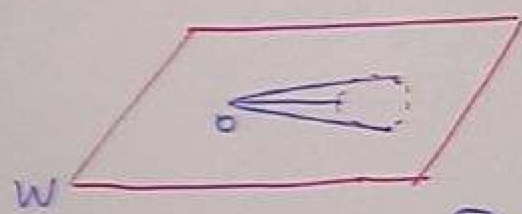


$$\frac{\partial P}{\partial z} = 1 - \frac{e^t}{z^2 w} = 0 = \frac{\partial P}{\partial w} = 1 - \frac{e^t}{z w^2}$$

$$\underbrace{z^2 w = z w^2}_{} = e^t$$

$$zw(z-w)=0 \Rightarrow z = w \quad = e^{t/3}\cdot\lambda \qquad \lambda^3 = 1$$

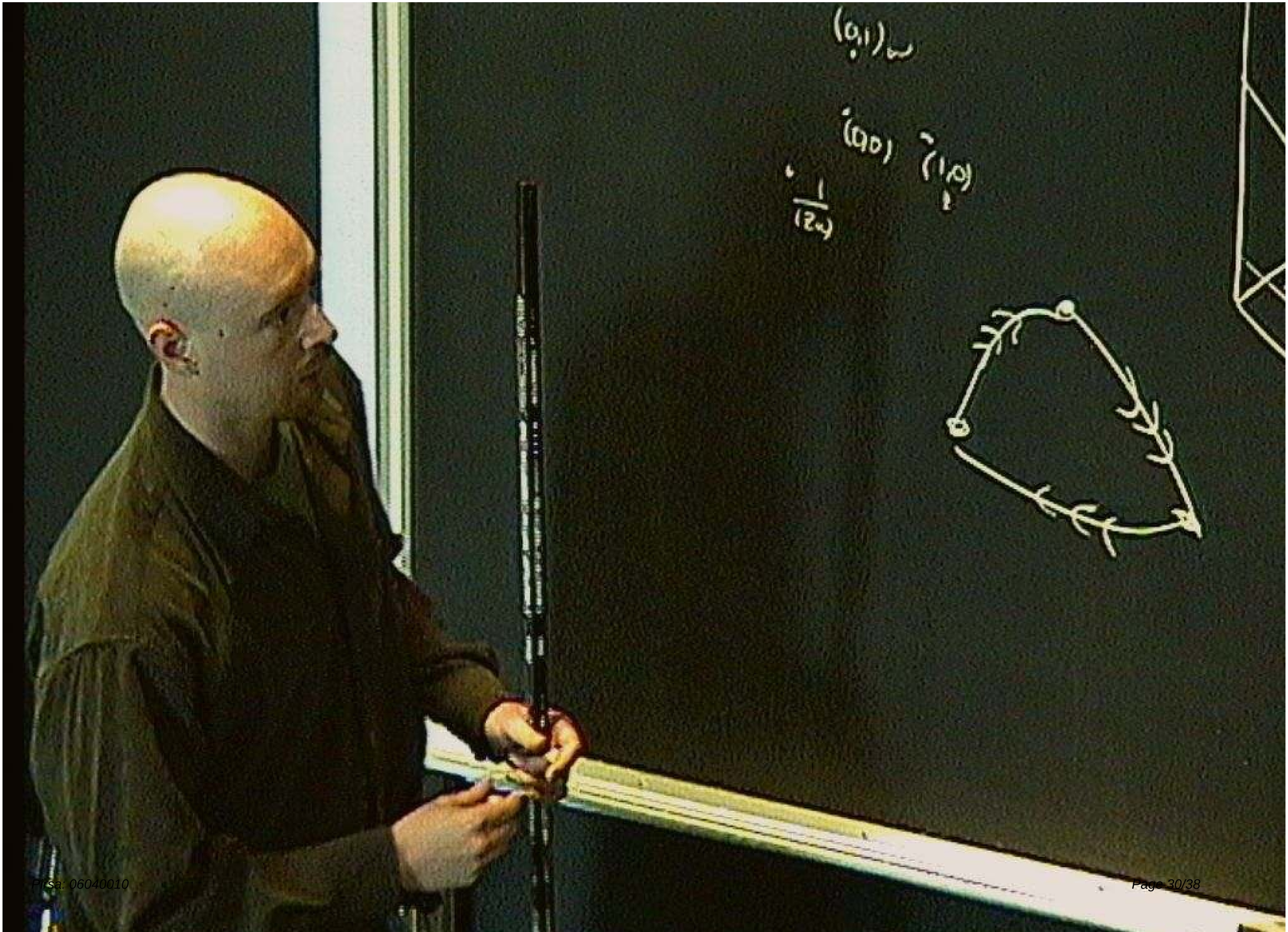
So the critical values are when $P(z,w) = 1 - 3\lambda e^{t/3}$



In the fibre Σ_w near the critical pts a 1-cycle pinches:

In general as long as $P(z,w)=0$ has genus ≥ 1 there are enough critical points to identify the S^3's with the fractional branes

ICo


(0,1)
(0,0)
(1,0)

e.g. $P(z,w) = 1 + z + w + e^t/zw$

$$\frac{\partial P}{\partial z} = 1 - \frac{e^t}{z^2 w} = 0 = \frac{\partial P}{\partial w} = 1 - \frac{e^t}{z w^2}$$

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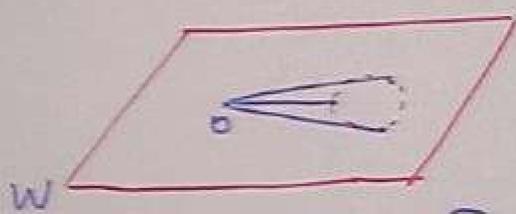


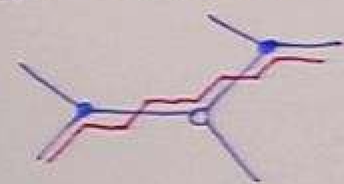
In the fibre Σ_w near the critical pts a 1-cycle pinches:

In general as long as $P(z,w)=0$ has genus ≥ 1 there are enough critical points to identify the S^3's with the fractional branes

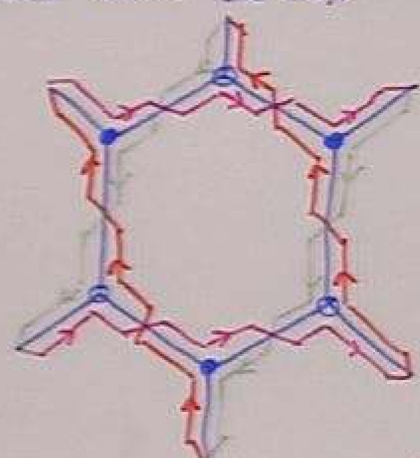
Zig-zag paths

Given a graph $\Gamma \hookrightarrow T^2$, a zig-zag path on T^2 crosses Γ along each edge exactly once, and follows parallel to Γ elsewhere

e.g.



For our dimer graphs in T^2 each edge has two zig-zag paths crossing it (non-trivial!) and moreover they bound closed loops around each vertex



Orientation of loops = colour of vertex

Observation: zig-zag paths on $T^2 \overset{1-1}{\longleftrightarrow}$ punctures of Σ_1

(p,q) winding
$\updownarrow$
(p,q) slope in web

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The Antimap

- Graph Γ defined key properties of theory
  Map $\Gamma \hookrightarrow T^2$ to isomorphic graph $\Gamma \hookrightarrow \Sigma_0$
- Local map, preserves all properties, but

$\Gamma \hookrightarrow \Sigma_0$ describes intersection of fractional branes in mirror CY

$\Gamma \hookrightarrow T^2$ exhibits that the fractional branes combine to form $T^2 \subset T^3$, mirror to D0-brane on $\mathcal{M}$

NB $\Sigma_0 \cap T^2 \neq 0$
i.e. $T^2 \neq \Sigma_0$!

- Antimap : exchanges faces of $\Gamma \hookrightarrow T^2$ w/ zigzags

Do this along every edge

changes genus of embedding

Does it agree with

$\Sigma_0 : P(z,w)=0$?

zig-zag paths now bound part of face in new embedding

$\Gamma \hookrightarrow \Sigma_1$

12

Simple check: $\chi = V + F - E$

T^2: $V + F - E = 0$

 $\mathrm{Newt}(P(z,w))$

$$F \leftarrow N_{gauge} = 2\,\mathrm{Area}(\mathrm{Newt}(P(z,w)))$$

$$= 2I + N_p - 2 \quad \text{(Pick's thm)}$$

I = # int. points

N_p = # edges = # punctures on Σ_0

Antimap exchanges faces $\leftrightarrow$ zigzags $\overset{H}{\updownarrow}$ punctures on Σ_0

Σ : $V + N_p - N_f = 2 - 2I$ $\rightarrow$ correct genus in general

So our new embedding of <u>some</u> graph Γ gives a model for the mirror curve $\Sigma_0 : P(z,w) = 0$

- each face contains puncture
- graph Γ is isomorphic; same $\begin{cases}\text{matchings}\\ \text{edges}\\ \text{vertices}\end{cases}$
- By defining $K(z,w)$ as before ($\sim$ adj. mat) we can promote $\Gamma \hookrightarrow \Sigma_0$ to the algebraic curve $\det K(z,w) = 0$
- Γ knows explicit form of the embedding geometry

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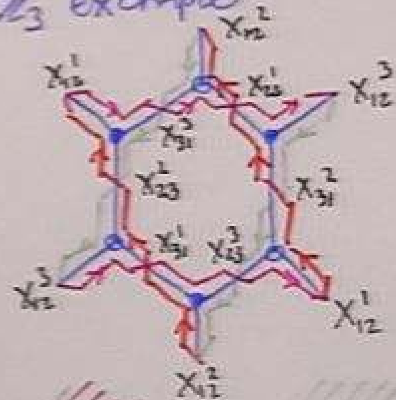
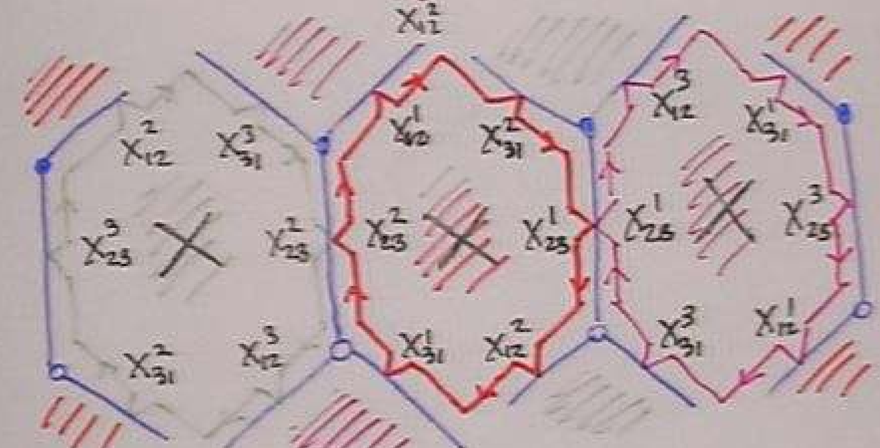
For our $\mathbb{C}^3/\mathbb{Z}_3$ example

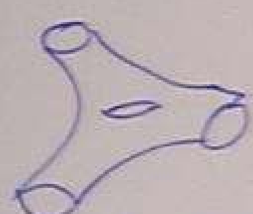
T^2



Σ



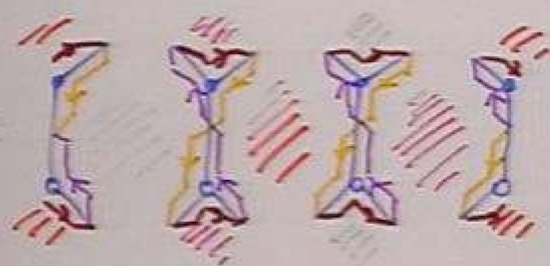
Some graph $\Gamma \subset \Sigma$ defines decomposition of Σ into half-cylinders



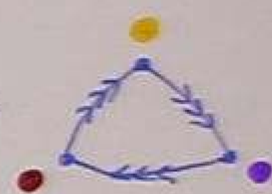
$P(z,w)=0 \to$ genus 1
3 punctures

Finally, we can read off how the fractional branes intersect the curve $P(z,w)=0$ by zig-zag paths on new embedding

Σ_0:



Check: - intersections still encode



- vertices still enclosed by $\pm$ orientation

eg.

So the vertices of the graph $\Gamma \subset \Sigma_0$ are places where an open string worldsheet (disc instanton) can attach to multiple fractional branes

→ gives contribution to W from non-vanishing open disc correlator with n boundary insertions

$\sim e^{A}\,\mathrm{Tr}\, X^1_{31} X^2_{23} X^3_{12}$

Summary

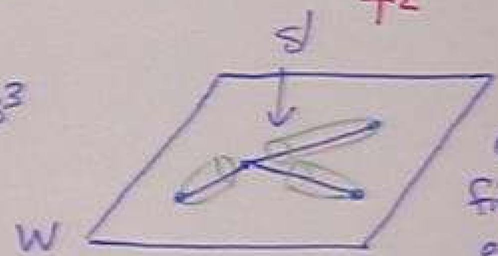
- Dimer model Γ on T^2 is realized in string theory via mirror symmetry

D3-brane at point on $\mathcal{M}$ $\longleftrightarrow$ D6-brane; T^3 on $\mathcal{W}$

" on E exc. cycle $\longleftrightarrow$ $T^3 = \begin{array}{c} S^1 \\ \downarrow \\ T^2 \end{array}$

S_i : $\begin{array}{c} S^1 \\ \downarrow \end{array} \simeq S^3$

frac. brane



S^1 vanishes at 0, finite elsewhere

- $S_i \cap \Sigma_0$ are described by zig-zag paths on embedding of same $\Gamma \hookrightarrow \Sigma_0$

S^1 fibre vanishes along these paths

- We clearly see that the S_i span a T^3 by performing the antimap; the frac branes are indeed the faces of $\Gamma \hookrightarrow T^2$ and the S^1 fibre vanishes along their body.

15a

Advantages:

- explicitly see every intersection of frac. branes
  - → works for theories with non-chiral matter too
- works for genus 0 case (e.g. conifold) when mirror S^3 geometry unclear
- clear picture of superpotential generation
- explains origins of dimer model on T^2; embedded in $\mathcal{W}$ in complicated way but untangled by antimap
- geometry of $\mathcal{W}$ is classical; can define frac. branes $\forall$ C.S. moduli, not just in limit points (orbifold/L.V.)

Open Questions

- Algebraic description of contours $S_i \cap \Sigma_{10}$?
  - using linear sigma model? WIP K.K. K.Hori
- Precise nature of mirror map
  - relation to B-brane construction of HHV 0602041
- Underlying topological string construction
  - connection to ORV — microscopic use of same dimer models to describe closed A-model 0309208
- Extension to more general CYs?
- Beyond topological sector?