Title: Expressing the equation of state parameter in terms of the three dimensional cosmic shear

Date: Apr 11, 2006 11:00 AM

URL: http://pirsa.org/06040009

Abstract: We express the total equation of state parameter of a spatially flat Friedman-Robertson-Walker universe in terms of derivatives of the red-shift dependent spin-weighted angular moments of the two-point correlation function of the three dimensional cosmic shear. In the talk I will explain all the technical terms in the first sentence, I will explain how such an expression is obtained and highlight its relevance for determining the expansion history of the universe.

The cosmic equation of state parameter in terms of the 3D cosmic shear

+ Daniel Levy,
Edmund Bertschinger (?)
to appear

Ram Brustein

אוניברסיטת בן-גוריון

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- Need an observable to determine the perturbation's time (redshift)-dependence that can be calculated and measured reliably (~ 1%)

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The three dimensional cosmic shear

The Plan: Express the total EOS of a flat FRW universe in terms of derivatives of the red-shift dependent spin-weighted angular moments of the 2-point correlation function of the 3D cosmic shear

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1. Express the total EOS in terms of the growing mode of the metric perturbation Φ in the conformal-Newtonian gauge for the case of adiabatic perturbations with vanishing speed of sound.

The Plan: Express the total EOS of a flat FRW universe in terms of derivatives of the red-shift dependent spin-weighted angular moments of the 2-point correlation function of the 3D cosmic shear

1. Express the total EOS in terms of the growing mode of the metric perturbation Φ in the conformal-Newtonian gauge for the case of adiabatic perturbations with vanishing speed of sound.

2. Express the metric perturbation in terms of derivatives of the angular moments of the shear correlation function. (final explicit expression for the case of a Harrison-Zeldovich spectrum)

Background probes: a reminder

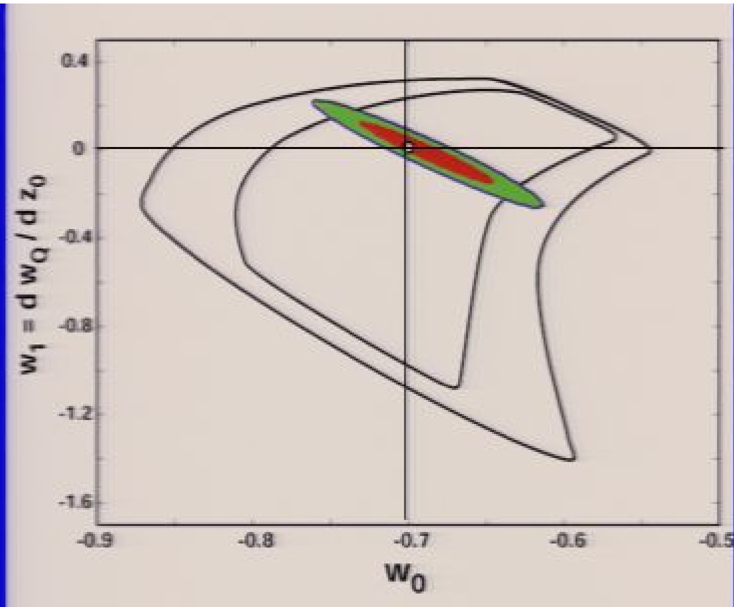
Luminosity distance d_L vs. redshift z

$$d_L = \frac{1+z}{H_0} \int_1^{1+z} dx\, e^{-\frac{3}{2}\int_1^x \frac{dy}{y}(1+w_{tot})}$$

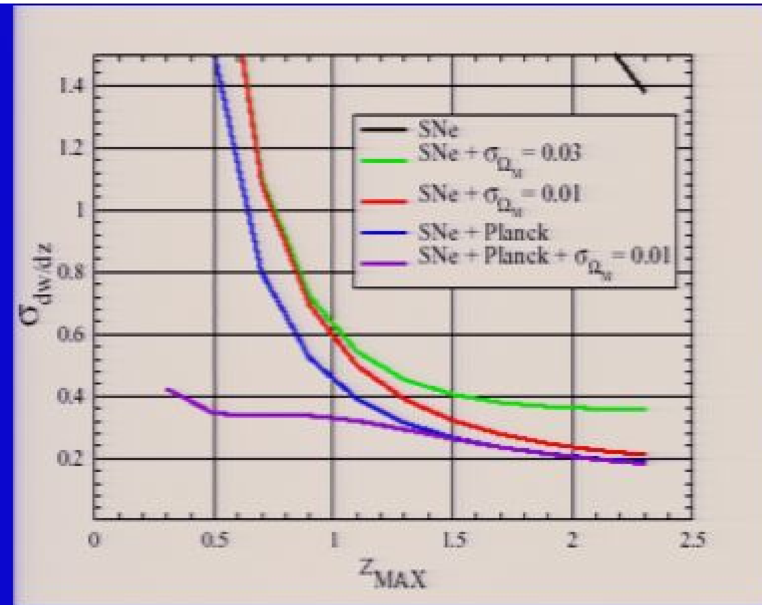
I. Maor, RB,
P. Steinhardt
later : many others

$$w_{tot} = -1 - \frac{2}{3}(1+z)\partial_z\left(\log\left[\partial_z \frac{d_L}{1+z}\right]\right)$$

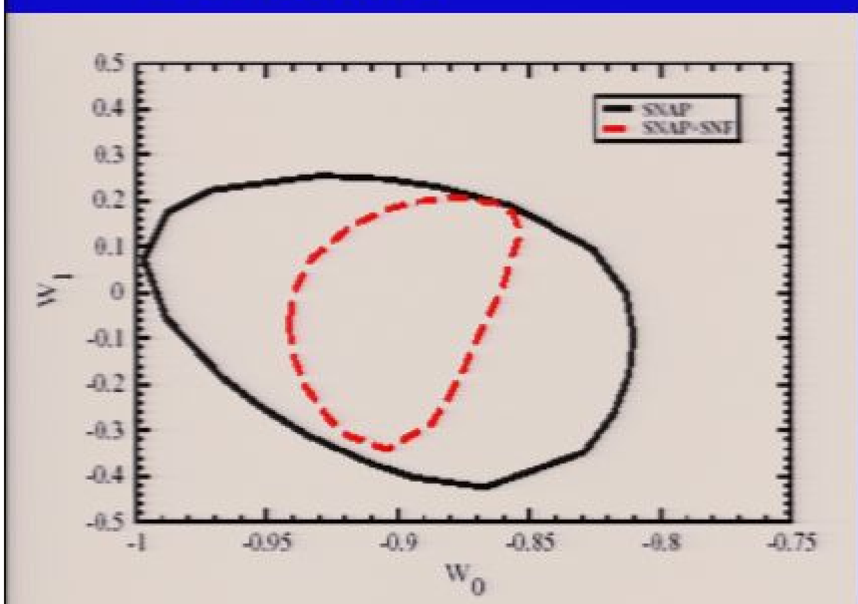
Assumptions on the form of w_{tot} or d_L, combining different observables (dN/dz, d_A, etc., etc.) can improve the quality of inversion



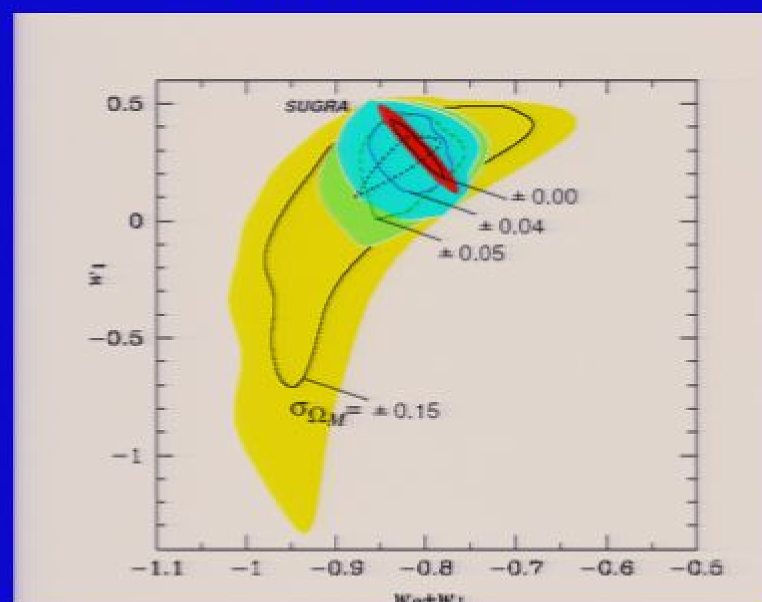
I. Maor et al



J. Frieman et al



P. Antilogus



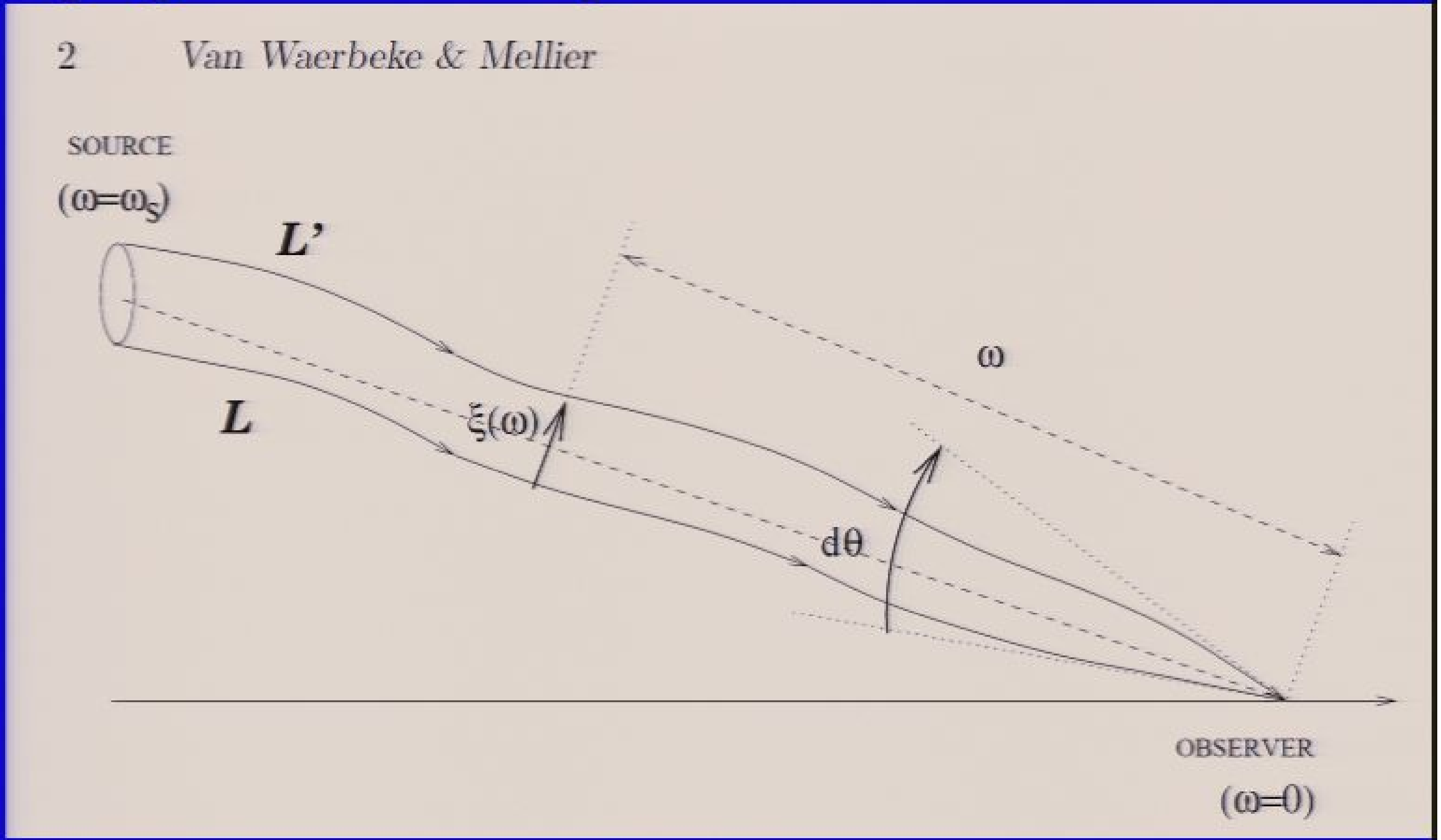
Weller & Albrecht

CANNOT MEASURE w'

Similar conclusions : many more

The cosmic shear

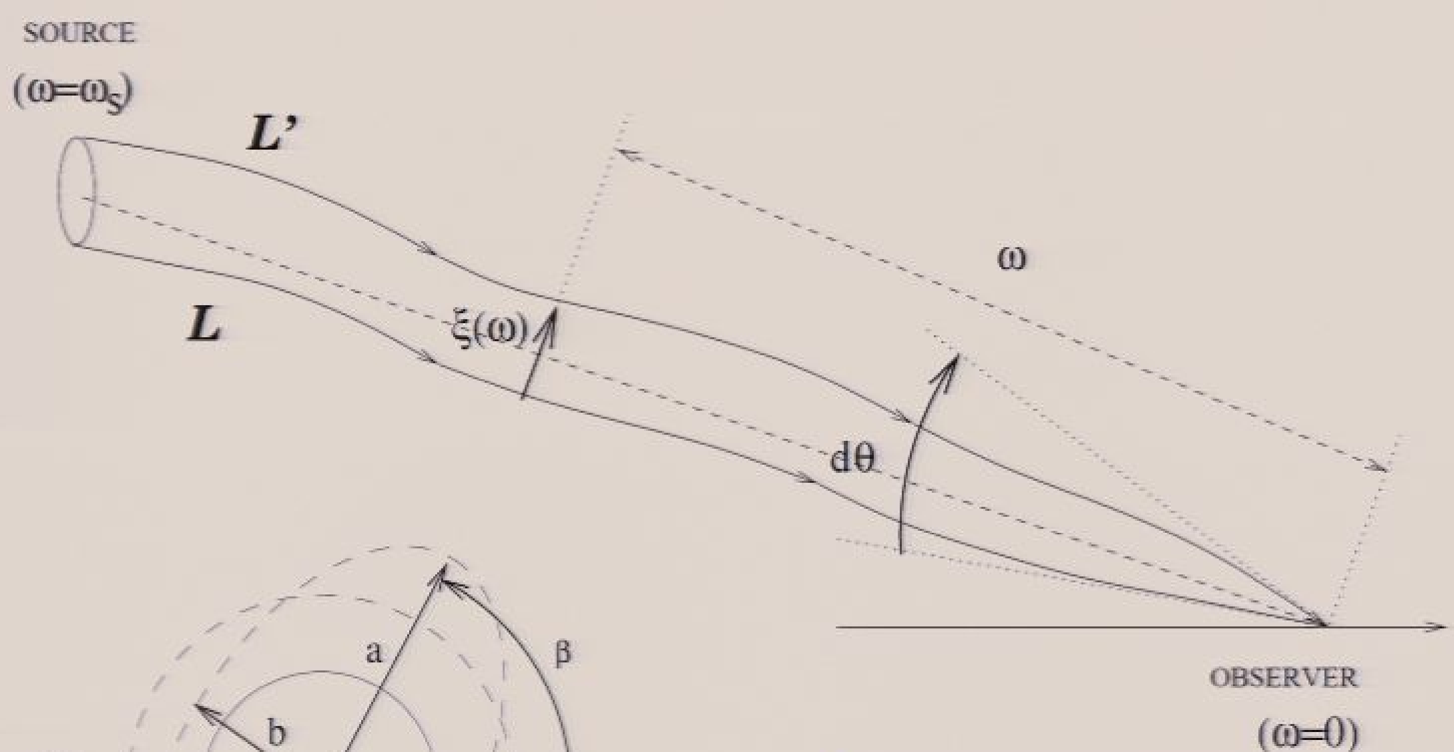
Light propagation in the perturbed universe

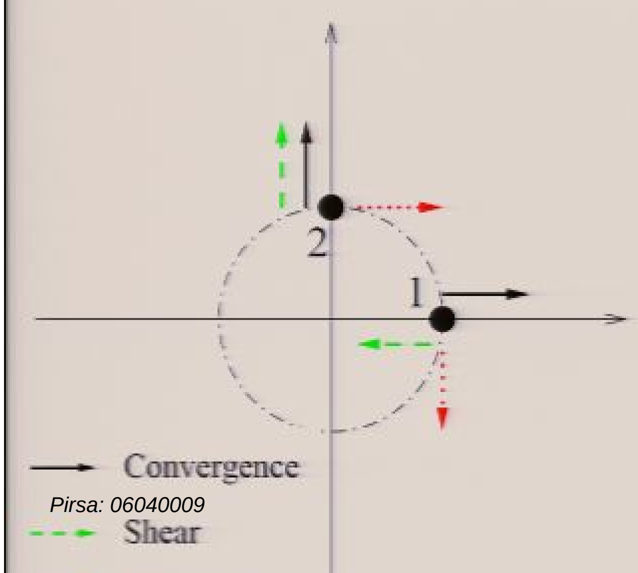


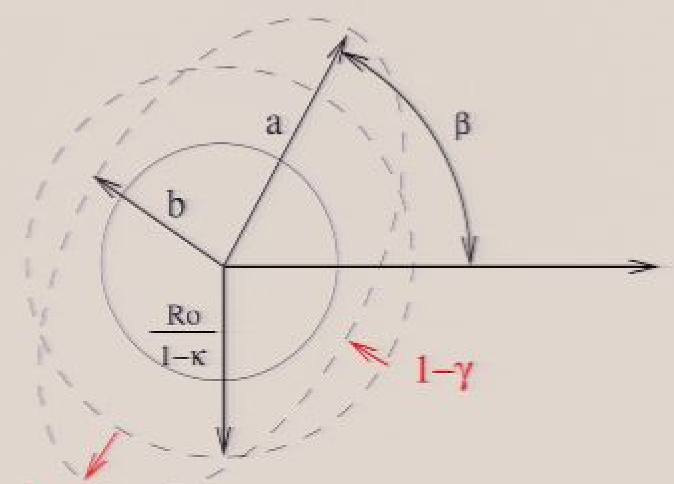
The cosmic shear

Light propagation in the perturbed universe

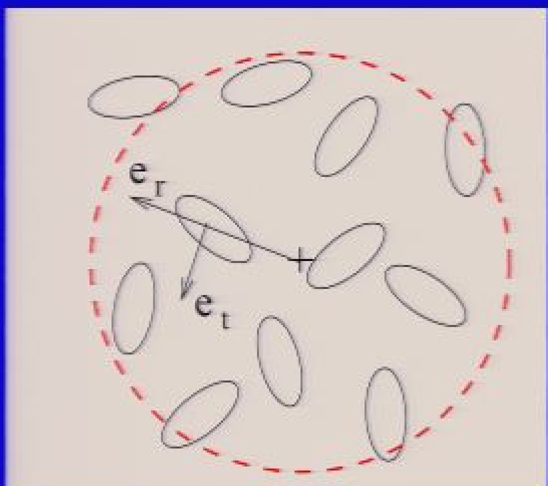
2 Van Waerbeke & Mellier





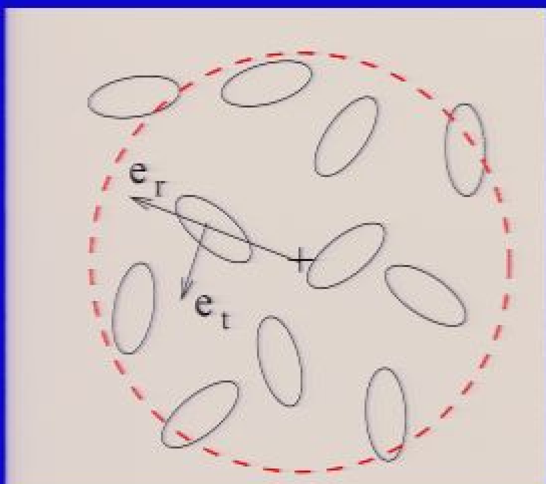


Measuring the cosmic shear



WL induced distortions on individual galaxies are small (0.1 −2%)
Intrinsic scatter in the shapes of the galaxies is large (30%) ➔
WL must be measured statistically.

Measuring the cosmic shear



arXiv:astro-ph/0511090 CFHTLS
arXiv:astro-ph/0511089 → Canada UW
First significant measurements

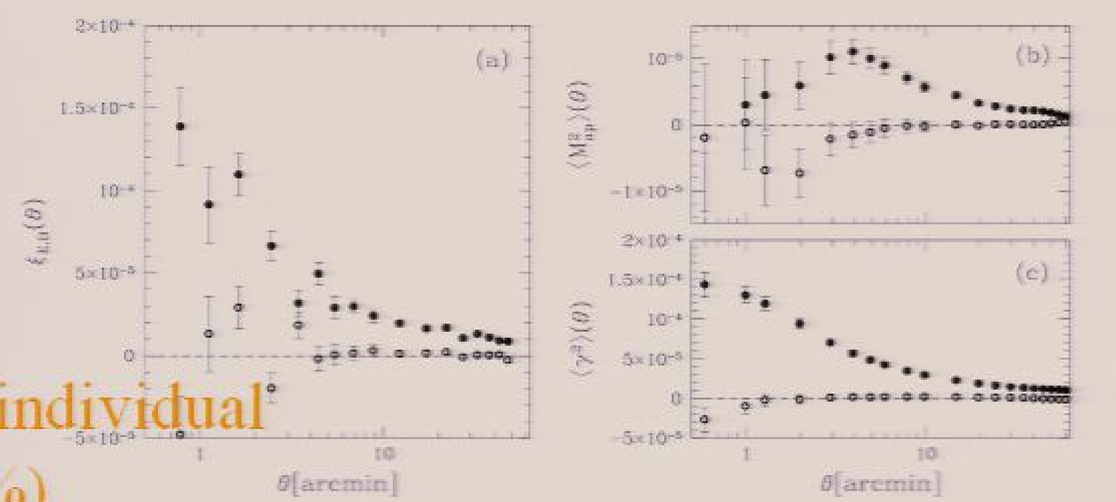


FIG. 4. — Panel a) 'E' and 'B'-mode shear correlation functions (filled and open points, respectively) measured from ~ 22 deg^2 of i' data from the CFHT Legacy Survey. The error bars only indicate the statistical errors determined for the combined signal from the data from W1 and W3 fields. Panel b) 'E' and 'B'-mode measurements for the aperture mass statistic. Note that the 'B'-mode is consistent with zero on all scales. Panel c) as before but now for the top-hat variance.

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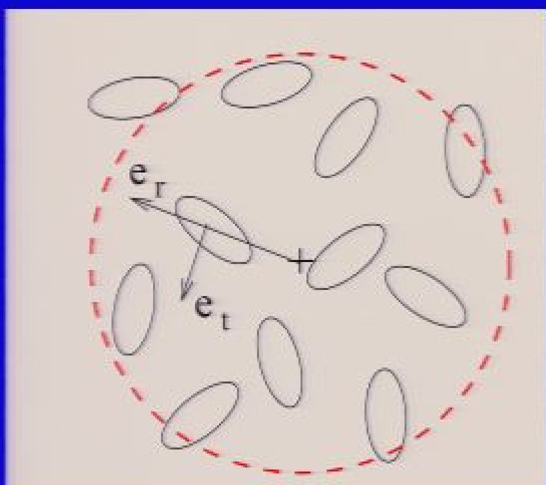

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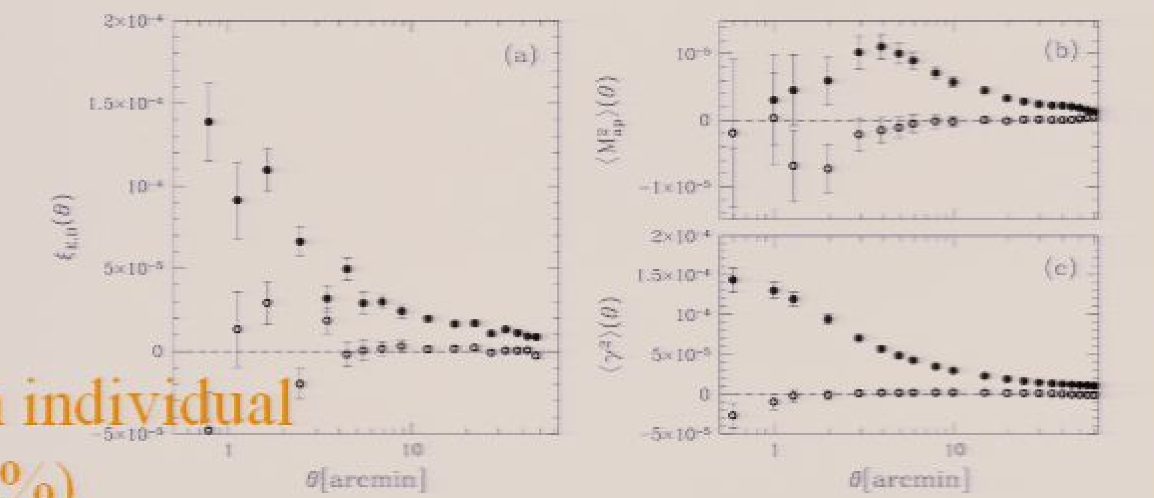

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3D: "double" future

w_{tot} in terms of Φ

The background and the perturbations

$$ds^2 = a^2(\eta)\left(-d\eta^2 + dw^2 + w^2 d\Omega^2\right), \qquad K = 0$$

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho,$$
$$\mathcal{H}' = -4\pi G(\rho + p) a^2,$$
$$\rho' + 3\mathcal{H}(\rho + p) = 0.$$

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$$\begin{aligned} \nabla^2\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Phi &= 4\pi G a^2 \delta\rho, \\ \Phi'' + 3\mathcal{H}\Phi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Phi &= 4\pi G a^2 \delta p. \end{aligned}$$

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$$c_S^2 = \frac{\partial\delta p}{\partial\delta\rho}$$

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$$\Phi'' + 3\mathcal{H}(1 + c_S^2)\Phi' - c_S^2 \nabla^2\Phi + (2\mathcal{H}' + (1 + 3c_S^2)\mathcal{H}^2\Phi = 0$$

Adiabatic perturbations $\delta S = 0$

w_{tot} in terms of Φ

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Adiabatic perturbations $\delta S = 0$

$w_{\rm tot}$ in terms of Φ

Solutions of the perturbation equations

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$$\Phi = 4\pi G\sqrt{\rho + p}\, u \Longrightarrow u'' - c_S^2\nabla^2 u - \frac{\theta''}{\theta}u = 0$$

$$\theta(\eta) = \frac{1}{a(\eta)}\frac{1}{\sqrt{1+w_{\rm tot}(\eta)}}$$

$$w_{\rm tot} = \frac{p}{\rho}$$

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$$(q\eta)^2 c_S^2 \ll 1 \Longrightarrow u(\vec{x},\eta) = C_1(\vec{x})\theta(\eta) + C_2(\vec{x})\theta(\eta)\int d\tilde{\eta}\frac{1}{\theta(\tilde{\eta})^2}$$

"growing solution"

w_{tot} in terms of Φ

The "growing" solution for vanishing c_S^2

$$(q\eta)^2 c_S^2 \ll 1$$

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$$\Phi_+(\eta,\vec{x}) = \sqrt{\frac{2\pi G}{3}}C_2(\vec{x})\frac{\sqrt{\rho}}{a}\int d\rho\frac{a(\rho)}{\rho^{3/2}}$$

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$$\chi = \frac{a/a_0}{\sqrt{\rho/\rho_0}}$$

$$\Phi_+(\chi,\vec{x}) = \sqrt{24\pi G}C_2(\vec{x})\frac{1}{\chi}\int_0^\chi d\widetilde{\chi}\frac{1+w_{\rm tot}(\widetilde{\chi})}{5+3w_{\rm tot}(\widetilde{\chi})}$$

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Factorization

$$\Phi_+(\chi,\vec{x}) = C(\vec{x})\Phi_T(\chi)$$

w_{tot} in terms of Φ

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$$\chi = \frac{a/a_0}{\sqrt{\rho/\rho_0}} = \frac{1}{1+z}\frac{1}{H(z)/H_0}$$

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$$I(z) = \int_z^{\infty} d\tilde{z}\,\frac{5+3w_{\text{tot}}(\tilde{z})}{2(1+\tilde{z})}$$

$$\Phi_T(z) = \Phi_T(z\to\infty) + e^{-I(z)}\int_z^{\infty} d\tilde{z}\; e^{I(\tilde{z})}\,\frac{1+w_{\text{tot}}(\tilde{z})}{2(1+\tilde{z})}$$

$$w_{\text{tot}}(z) = -\frac{2(1+z)\partial_z\Phi_T(z) + 1 - 5\Phi_T(z)}{1-3\Phi_T(z)}$$

$w_{\rm tot}$ in terms of Φ

The "growing" solution for vanishing c_S^2

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w_{tot} in terms of Φ

$$\Phi_T(\chi) = \frac{1}{\chi}\int_0^{\chi} d\widetilde{\chi}\frac{1+w_{\text{tot}}(\widetilde{\chi})}{5+3w_{\text{tot}}(\widetilde{\chi})}$$

$$\chi = \frac{a/a_0}{\sqrt{\rho/\rho_0}} = \frac{1}{1+z}\frac{1}{H(z)/H_0}$$

$$\Phi_T(z) = \frac{1}{\widetilde{\chi}(z)}\int_{\infty}^{z} d\widetilde{z}\,\frac{d\widetilde{\chi}(\widetilde{z})}{d\widetilde{z}}\frac{1+w_{\text{tot}}(\widetilde{z})}{5+3w_{\text{tot}}(\widetilde{z})}$$

$$\frac{\partial_z\chi(z)}{\chi(z)} = -\frac{1}{1+z} - \frac{3}{2}\frac{1+w_{\text{tot}}(z)}{1+z} = -\frac{5+3w_{\text{tot}}(z)}{2(1+z)}$$

w_{tot} in terms of Φ

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$$\Phi_T(z) - \frac{2(1+z)}{5+3w_{\mathrm{tot}}(z)}\partial_z\Phi_T(z) = \frac{1+w_{\mathrm{tot}}(z)}{5+3w_{\mathrm{tot}}(z)}$$

$$I(z) = \int_z^{\infty} d\tilde{z}\,\frac{5+3w_{\mathrm{tot}}(\tilde{z})}{2(1+\tilde{z})}$$

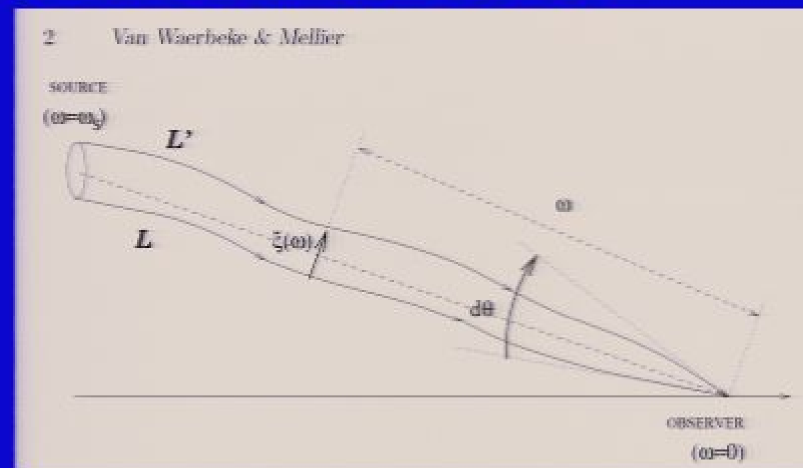
$$\Phi_T(z) = \Phi_T(z\to\infty) + e^{-I(z)}\int_z^{\infty} d\tilde{z}\; e^{I(\tilde{z})}\,\frac{1+w_{\mathrm{tot}}(\tilde{z})}{2(1+\tilde{z})}$$

$$w_{\mathrm{tot}}(z) = -\frac{2(1+z)\partial_z\Phi_T(z)+1-5\Phi_T(z)}{1-3\Phi_T(z)}$$

The cosmic shear

Light propagation in the perturbed universe

Background: $\vec{\xi}(w) = w\vec{\vartheta}$

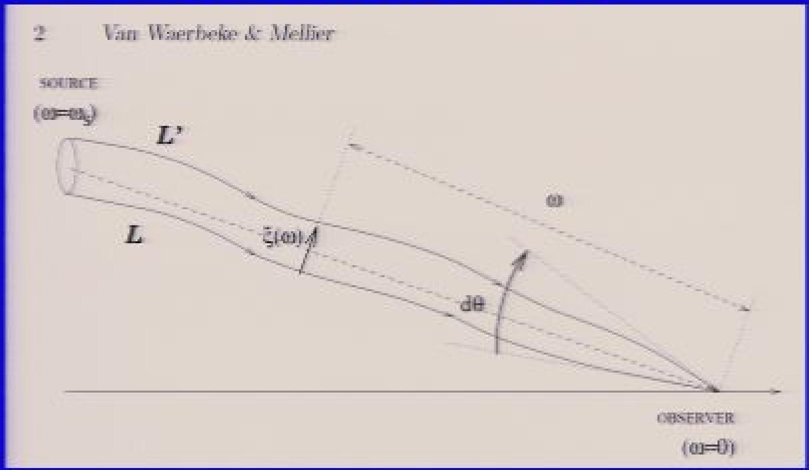


The cosmic shear

Light propagation in the perturbed universe

Background: $\vec{\xi}(w) = w\vec{\vartheta}$

Linear pert.: $\vec{\xi}(w) = w\mathcal{A}(w)\vec{\vartheta}$



$$\mathcal{A} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\gamma = \gamma_1 + i\gamma_2$$

Convergence	κ
Shear	γ

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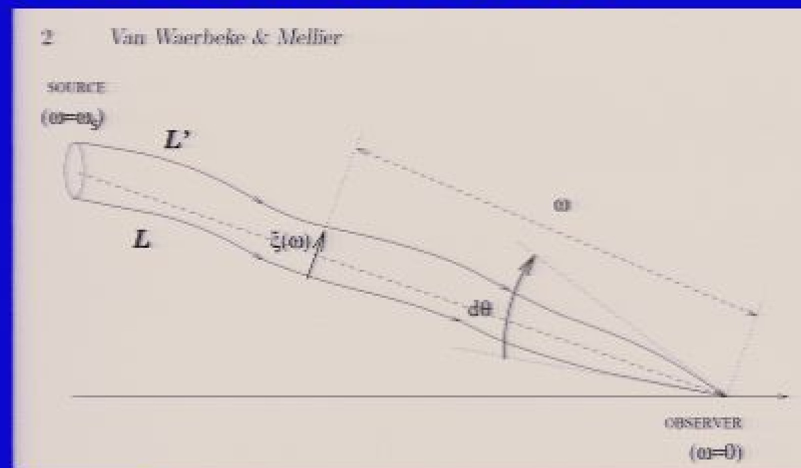

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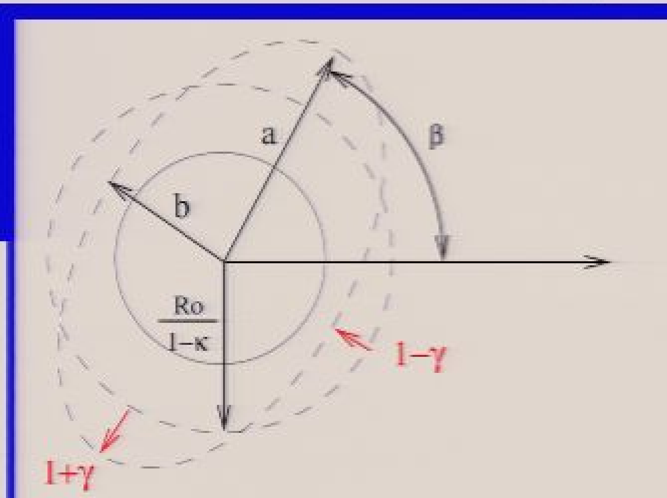

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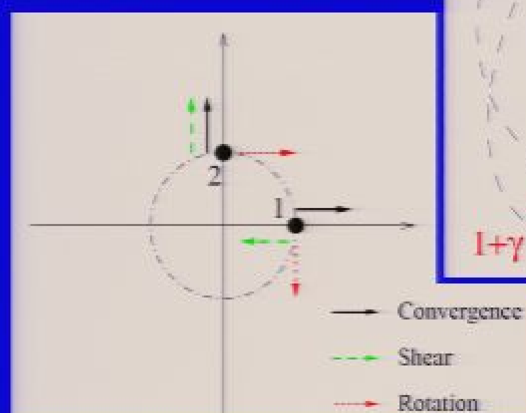

The 3D Cosmic Shear

The geodesic deviation equation & it's solution

$$\frac{d^2\vec{\xi}}{dv^2} = \begin{pmatrix} \mathcal{R} + \Re e(\mathcal{F}) & \Im m(\mathcal{F}) \\ \Im m(\mathcal{F}) & \mathcal{R} - \Re e(\mathcal{F}) \end{pmatrix} \vec{\xi}.$$

$$\vec{\xi}(w) = w\mathcal{A}(w)\vec{\vartheta}$$

$$\mathcal{R} = -\frac{1}{2}R_{\alpha\beta}k^\alpha k^\beta = \mathcal{R}^{(FRW)} - \left[\Phi'' + \nabla^2\Phi - 2\Phi(\mathcal{H}' + 2\mathcal{H}^2) + \frac{2}{a}(a\,\Phi_{,r})'\right] + \cdots$$

$$\mathcal{F} = -\frac{1}{2}C_{\alpha\beta\gamma\delta}\varepsilon^{*\,\alpha}k^\beta\varepsilon^{*\,\gamma}k^\delta = \left[\frac{\Phi_{|\phi\phi}}{w^2\sin^2\theta} - \frac{\Phi_{|\theta\theta}}{w^2}\right] + 2i\frac{\Phi_{|\theta\phi}}{w^2\sin\theta} + \cdots$$

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Iterative solution

$$\mathcal{A}_{ij} = \delta_{ij} + \int_0^w dw'\frac{(w-w')w'}{w}\mathcal{T}^{(1)}_{ij}$$

$$\mathcal{T}^{(1)} = \begin{pmatrix} \mathcal{R}^{(1)} + \Re e(\mathcal{F}^{(1)}) & \Im m(\mathcal{F}^{(1)}) \\ \Im m(\mathcal{F}^{(1)}) & \mathcal{R}^{(1)} - \Re e(\mathcal{F}^{(1)}) \end{pmatrix}$$

The 3D Cosmic Shear

The convergence and shear

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$$\begin{aligned} \kappa &= 1 - \frac{1}{2}\mathrm{Tr}\mathcal{A} = -\int_0^w dw' \frac{(w-w')w'}{w}\mathcal{R}^{(1)} \\ &= \int_0^w dw' \frac{(w-w')w'}{w}\left[\Phi'' + \nabla^2\Phi - 2\Phi(\mathcal{H}' + 2\mathcal{H}^2) + \frac{2}{a}(a\,\Phi_{,r})'\right] \end{aligned}$$

$$\begin{aligned} \gamma = \gamma_1 + i\gamma_2 &= \frac{1}{2}(\mathcal{A}_{11} - \mathcal{A}_{22} + 2i\mathcal{A}_{12}) \\ &= \int_0^w dw' \frac{(w-w')w'}{w}\left[\frac{\Phi_{|\phi\phi}}{w'^2\sin^2\theta} - \frac{\Phi_{|\theta\theta}}{w'^2} + 2i\frac{\Phi_{|\theta\phi}}{w'^2\sin\theta}\right] \end{aligned}$$

The 3D Cosmic Shear

Spin-weighted angular moments of the 2-point correlation function

The shear is a rank 2 tensor and hence needs to be expanded in spin-weighted spherical harmonics ${}_sY_{l,m}$

$$\gamma = \int_0^w dw' \frac{(w-w')w'}{w}\left[\frac{\Phi_{|\phi\phi}}{w'^2\sin^2\theta} - \frac{\Phi_{|\theta\theta}}{w'^2} + 2i\frac{\Phi_{|\theta\phi}}{w'^2\sin\theta}\right]$$

$$\eth\ {}_sf(\theta,\phi) = -\sin^s(\theta)\left[\frac{\partial}{\partial\theta} + i\csc(\theta)\frac{\partial}{\partial\phi}\right]\sin^{-s}(\theta)\ {}_sf(\theta,\phi)$$

$$\bar{\eth}\ {}_sf(\theta,\phi) = -\sin^{-s}(\theta)\left[\frac{\partial}{\partial\theta} - i\csc(\theta)\frac{\partial}{\partial\phi}\right]\sin^{s}(\theta)\ {}_sf(\theta,\phi)$$

$$\eth\ {}_sY_{l,m} = [(l-s)(l+s+1)]^{1/2}\ {}_{s+1}Y_{l,m}$$

The 3D Cosmic Shear

The convergence and shear

$$\vec{\xi}(w) = w\mathcal{A}(w)\vec{\vartheta}$$

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$$\mathcal{T}^{(1)} = \begin{pmatrix} \mathcal{R}^{(1)} + \Re e(\mathcal{F}^{(1)}) & \Im m(\mathcal{F}^{(1)}) \\ \Im m(\mathcal{F}^{(1)}) & \mathcal{R}^{(1)} - \Re e(\mathcal{F}^{(1)}) \end{pmatrix}$$

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The 3D Cosmic Shear

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$$\eth \; {}_sY_{l,m} = [(l-s)(l+s+1)]^{1/2} \; {}_{s+1}Y_{l,m}$$

$$\gamma(w,\theta,\varphi) = \int_0^w dw' \frac{w-w'}{ww'} \eth\eth\Phi(w',\theta,\varphi)$$

$$\Phi_+(\chi,\vec{x}) = C(\vec{x})\Phi_T(\chi)$$

$$\gamma(w,\theta,\varphi) = \int_0^w dw' \frac{w-w'}{ww'} \Phi_T(w')\eth\eth\, C(w',\theta,\varphi)$$

And now to the 2-point function …

The 3D Cosmic Shear

The spin-weighted angular moments of shear 2-point function

$$\gamma(w,\theta,\varphi) = \int_0^w dw' \frac{w-w'}{ww'} \Phi_T(w') \eth\eth\, C(w',\theta,\varphi)$$

The 3D Cosmic Shear

The spin-weighted angular moments of shear 2-point function

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$$\langle C(\vec{q}_1), C^*(\vec{q}_2)\rangle = f(q)\delta(\vec{q}_1 - \vec{q}_2)$$

$$\langle \gamma(w_1,\theta_1,\varphi_1), \gamma^*(w_2,\theta_2,\varphi_2)\rangle =$$

$$= \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1-u_1}{w_1\,u_1} \frac{w_2-u_2}{w_2\,u_2} \Phi_T(u_1)\Phi_T(u_2)\eth_1\eth_1\,\bar{\eth}_2\bar{\eth}_2 \int d^3q f(q) e^{i\vec{q}(\vec{x}_1-\vec{x}_2)}$$

The 3D Cosmic Shear

The spin-weighted angular moments of shear 2-point function

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$$= (4\pi)^2 \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1-u_1}{w_1\, u_1}\frac{w_2-u_2}{w_2\, u_2} \Phi_T(u_1)\Phi_T(u_2) \eth_1\eth_1\, \bar{\eth}_2\bar{\eth}_2 \int dq q^2 f(q)$$
$$\times \sum_{l,m} j_l(qu_1) j_l(qu_2) Y_{lm}(\theta_1,\varphi_1) Y^*_{lm}(\theta_2,\varphi_2).$$

$$\langle \gamma(w_1, \theta_1, \varphi_1), \gamma^*(w_2, \theta_2, \varphi_2) \rangle =$$

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$$\times \sum_{l,m} j_l(qu_1) j_l(qu_2) Y_{lm}(\theta_1, \varphi_1) Y^*_{lm}(\theta_2, \varphi_2).$$

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$$\times \sum_{l,m} \frac{(l+2)!}{(l-2)!} j_l(qu_1) j_l(qu_2)\ {}_2Y_{lm}(\theta_1,\varphi_1)\ {}_{-2}Y^*_{lm}(\theta_2,\varphi_2).$$

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$$\sum_m\ {}_{s_1}Y^*_{l,m}(\theta_1,\varphi_1)\ {}_{s_2}Y_{l,m}(\theta_2,\varphi_2) = \sqrt{\frac{2l+1}{4\pi}}\ {}_{s_2}Y_{l,-s_1}(\beta,\alpha)e^{-is_2\delta}$$

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$$\times \sum_l \sqrt{\frac{2l+1}{4\pi}} \frac{(l+2)!}{(l-2)!} j_l(qu_1) j_l(qu_2)\ {}_2Y_{l,2}(\beta, 0).$$

β – angle between two directions

The 3D Cosmic Shear

The shear spin-weight 2 angular power spectrum

$$\langle\gamma(w_s,\theta_1,\varphi_1),\gamma^*(w_s,\theta_2,\varphi_2)\rangle$$
$$= (4\pi)^2 \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1-u_1}{w_1\,u_1}\frac{w_2-u_2}{w_2\,u_2}\Phi_T(u_1)\Phi_T(u_2)\int dq q^2 f(q)$$
$$\times \sum_l \sqrt{\frac{2l+1}{4\pi}\frac{(l+2)!}{(l-2)!}} j_l(qu_1)j_l(qu_2)\ {}_2Y_{l,2}(\beta,0).$$

$$\langle\gamma,\gamma^*\rangle = \sum_l \sqrt{\frac{2l+1}{4\pi}}\ {}_2C_l\ {}_2Y_{l,-2}$$

$${}_2C_l = (4\pi)^2\frac{(l+2)!}{(l-2)!}\int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1-u_1}{w_1\,u_1}\frac{w_2-u_2}{w_2\,u_2}\Phi_T(u_1)\Phi_T(u_2)\int dq q^2 f(q) j_l(qu_1)j_l(qu_2)$$

$$ {}_2C_l = (4\pi)^2 \frac{(l+2)!}{(l-2)!} \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1 - u_1}{w_1\, u_1} \frac{w_2 - u_2}{w_2\, u_2} \Phi_T(u_1)\Phi_T(u_2) \int dq q^2 f(q) j_l(qu_1) j_l(qu_2) $$

$$ \langle C(\vec{q}_1), C^*(\vec{q}_2) \rangle = f(q)\delta(\vec{q}_1 - \vec{q}_2) $$

Assume power law primordial spectrum

$$ q^3 f(q) = A q^{n-1} $$

$$ {}_2C_l = (4\pi)^2 \frac{(l+2)!}{(l-2)!} \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1 - u_1}{w_1\, u_1} \frac{w_2 - u_2}{w_2\, u_2} \Phi_T(u_1)\Phi_T(u_2) \int dq q^2 f(q) j_l(qu_1) j_l(qu_2) $$

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Assume power law primordial spectrum

$$ q^3 f(q) = A q^{n-1} $$

For n = 1

$$ \int dq \frac{1}{q} j_l(qu_1) j_l(qu_2) = \frac{1}{2l} \begin{cases} \left(\frac{u_1}{u_2}\right)^l & u_2 > u_1 \\ \left(\frac{u_2}{u_1}\right)^l & u_2 < u_1 \end{cases} $$

* Other n's can also be calculated

$$ \begin{cases} \left(\frac{u_1}{u_2}\right)^l & u_2 > u_1 \\ \left(\frac{u_2}{u_1}\right)^l & u_2 < u_1 \end{cases} \simeq \frac{2l}{l^2 - 1} u_1 \delta(u_1 - u_2) $$

The 3D Cosmic Shear

The shear spin-weight 2 angular power spectrum

$${}_2C_l = (4\pi)^2 \frac{(l+2)!}{(l-2)!} \int_0^{w_1} du_1 \int_0^{w_2} du_2 \frac{w_1 - u_1}{w_1\, u_1} \frac{w_2 - u_2}{w_2\, u_2} \Phi_T(u_1)\Phi_T(u_2) \int dq q^2 f(q) j_l(qu_1) j_l(qu_2)$$

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$$ \langle C(\vec{q}_1), C^*(\vec{q}_2) \rangle = f(q)\delta(\vec{q}_1 - \vec{q}_2) $$

Assume power law primordial spectrum $q^3 f(q) = A q^{n-1}$

For n = 1

$$ \int dq \frac{1}{q} j_l(qu_1) j_l(qu_2) = \frac{1}{2l} \begin{cases} \left(\frac{u_1}{u_2}\right)^l & u_2 > u_1 \\ \left(\frac{u_2}{u_1}\right)^l & u_2 < u_1 \end{cases} $$

* Other n's can also be calculated

$$ \begin{cases} \left(\frac{u_1}{u_2}\right)^l & u_2 > u_1 \\ \left(\frac{u_2}{u_1}\right)^l & u_2 < u_1 \end{cases} \simeq \frac{2l}{l^2 - 1} u_1 \delta(u_1 - u_2) $$

The 3D Cosmic Shear

The shear spin-weight 2 angular power spectrum

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Φ in terms of ${}_2C_l$

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The 3D Cosmic Shear

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$$ \partial_{w_1}(w_1^2 \partial_{w_1}\, {}_2C_l(w_1, w_2)) = (4\pi)^2 A l(l+2) \frac{w_2 - w_1}{w_2} \Phi_T^2(w_1) $$

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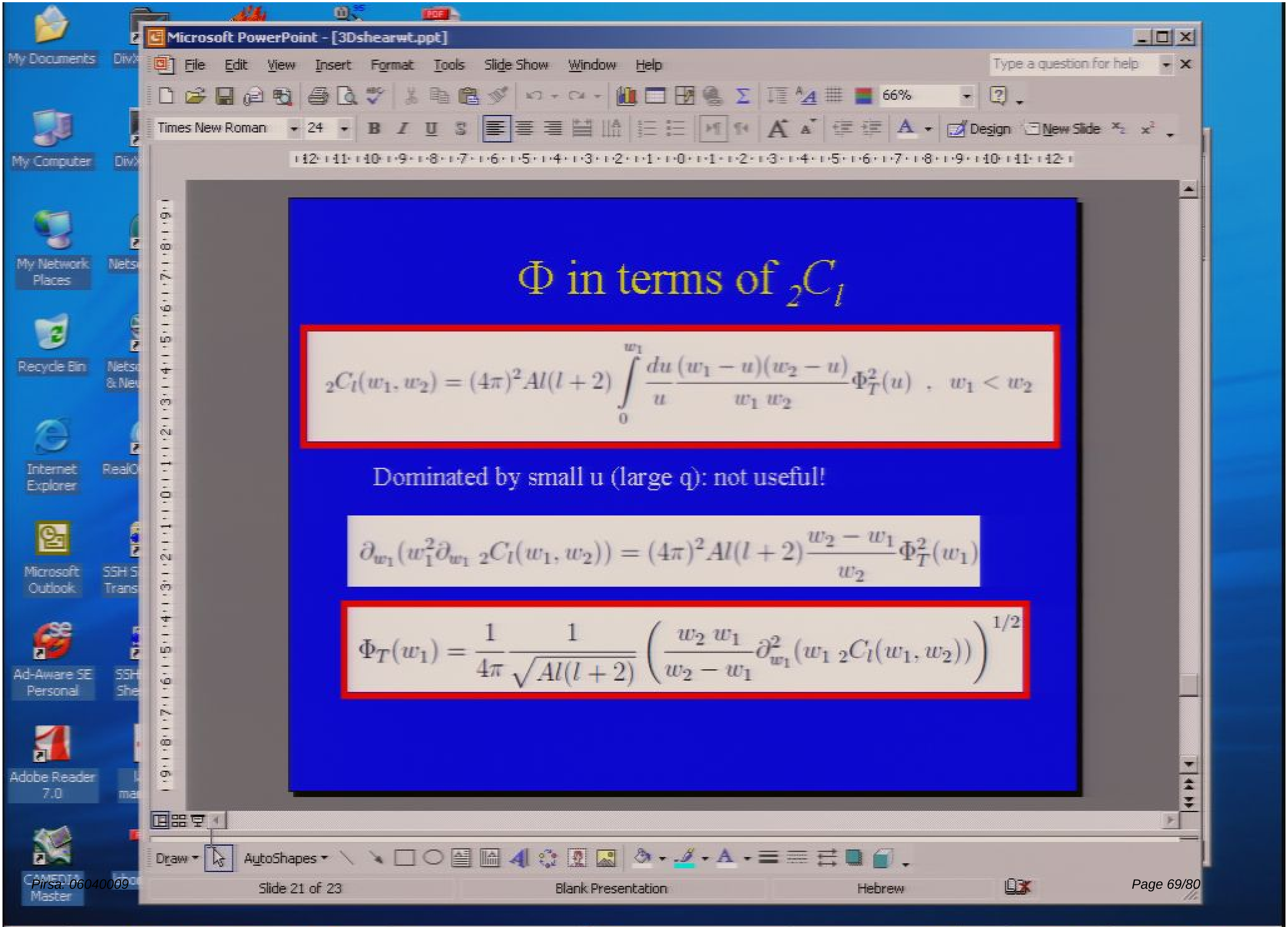
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Φ in terms of ${}_2C_l$
${}_2C_l(w_1, w_2) = (4\pi)^2 Al(l+2)\int_0^{w_1}\frac{du}{u}\frac{(w_1-u)(w_2-u)}{w_1\, w_2}\Phi_T^2(u)\ ,\quad w_1 < w_2$
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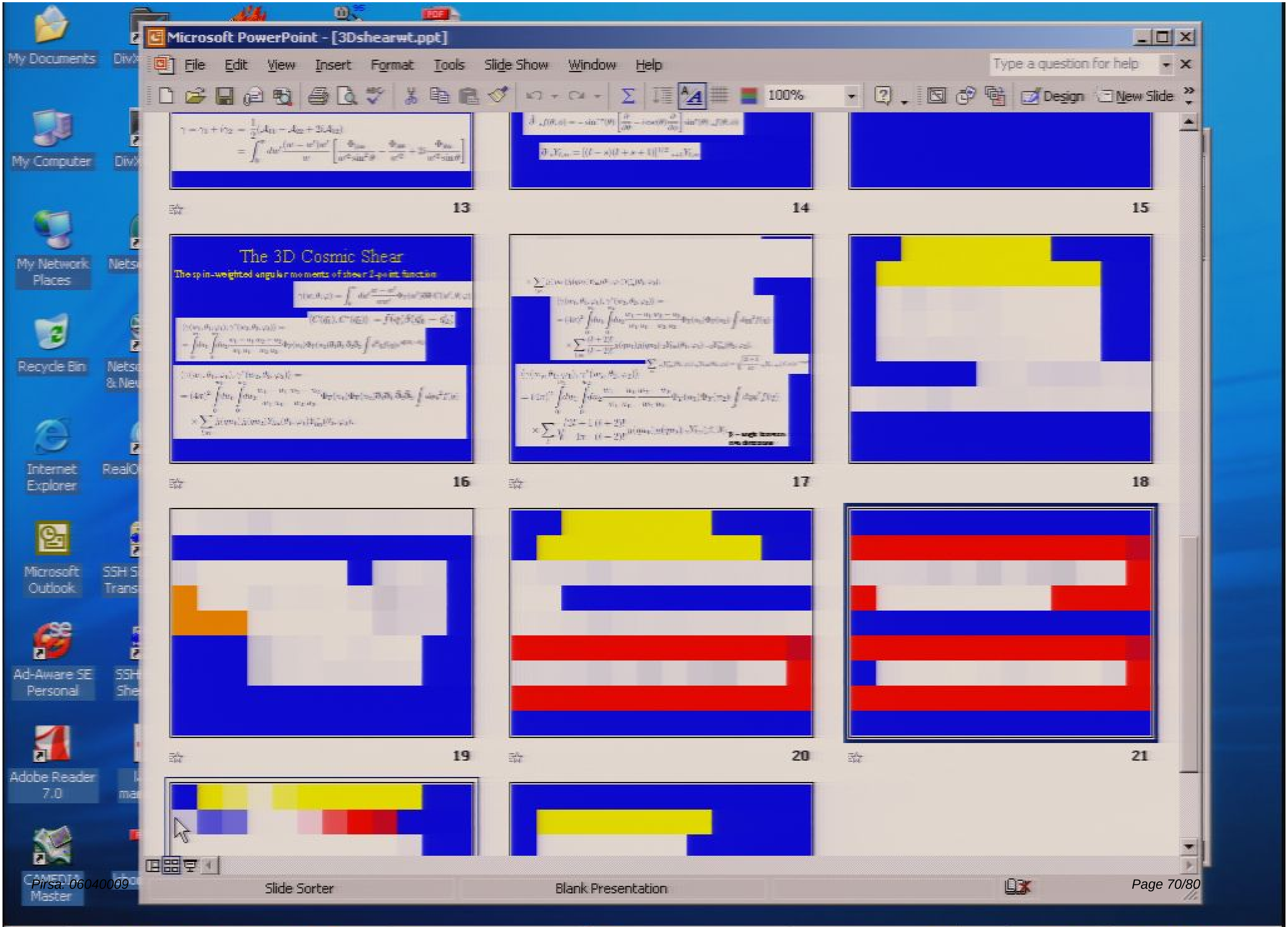

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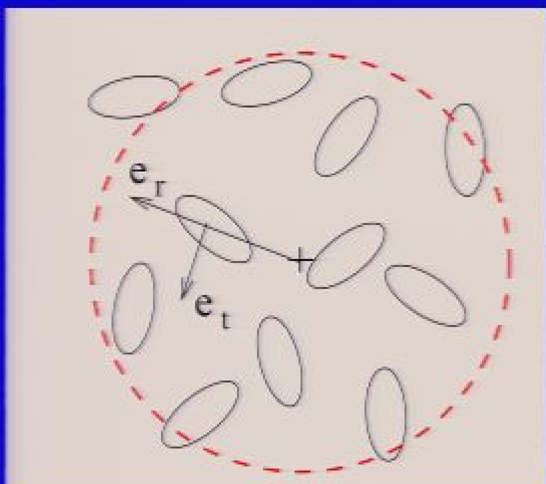
Measuring the cosmic shear



arXiv:astro-ph/0511090 CFHTLS
arXiv:astro-ph/0511089 → Canada UW
First significant measurements

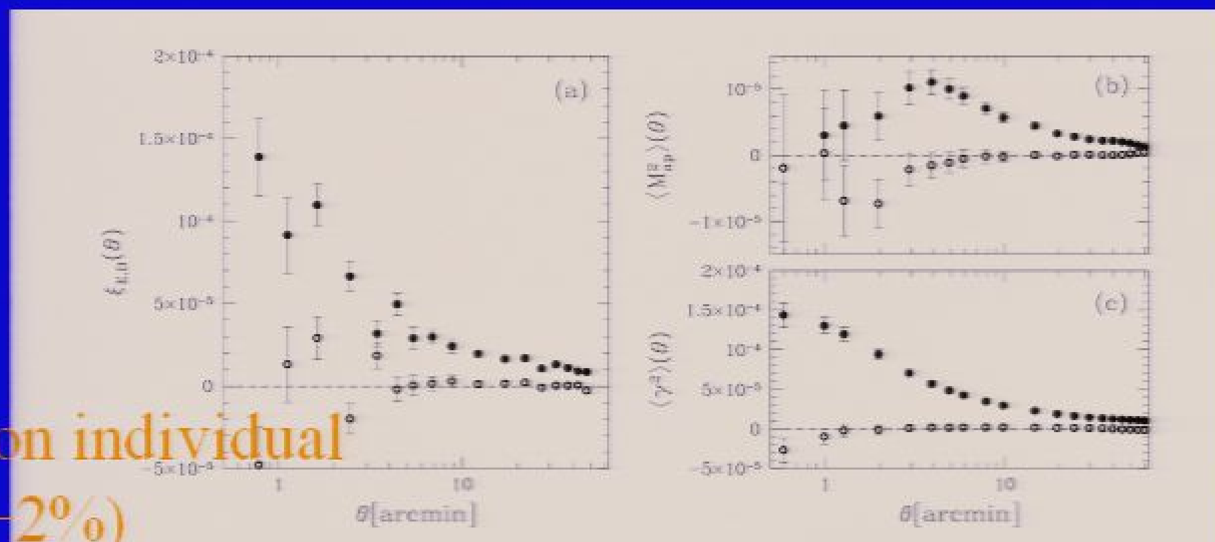


FIG. 4.— Panel a) 'E' and 'B'-mode shear correlation functions (filled and open points, respectively) measured from ~ 22 deg^2 of i' data from the CFHT Legacy Survey. The error bars only indicate the statistical errors determined for the combined signal from the data from W1 and W3 fields. Panel b) 'E' and 'B'-mode measurements for the aperture mass statistic. Note that the 'B'-mode is consistent with zero on all scales. Panel c) as before but now for the top-hat variance.

WL induced distortions on individual galaxies are small (0.1 – 2%)
Intrinsic scatter in the shapes of the galaxies is large (30%) ➔
WL must be measured statistically.

w_{tot} in terms of derivatives of ${}_2C_l$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + dw^2 + w^2 d\Omega^2\right), \qquad K = 0$$

→ For light → $w = \eta$

$d_L = (1+z)\, w$ →

$$w = \frac{d_L(z)}{(1+z)}$$

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$$w_{\text{tot}}(z) = -\frac{2(1+z)\partial_z \Phi_T(z) + 1 - 5\Phi_T(z)}{1 - 3\Phi_T(z)}$$

Conclusions and outlook

- Perturbation probes are useful

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- Measurement is possible, challenging (!)
- Measurement requires combining results from different experiments