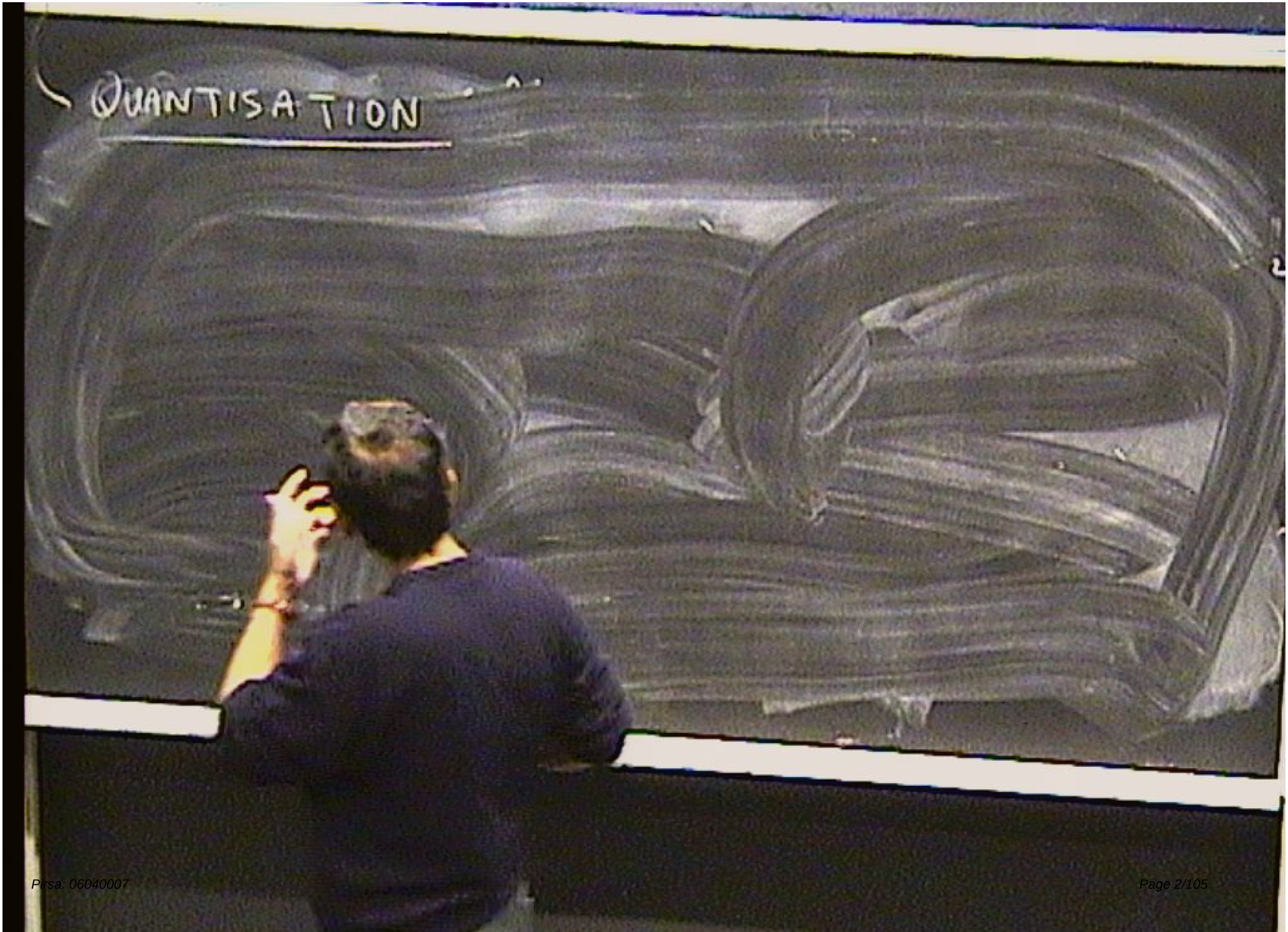
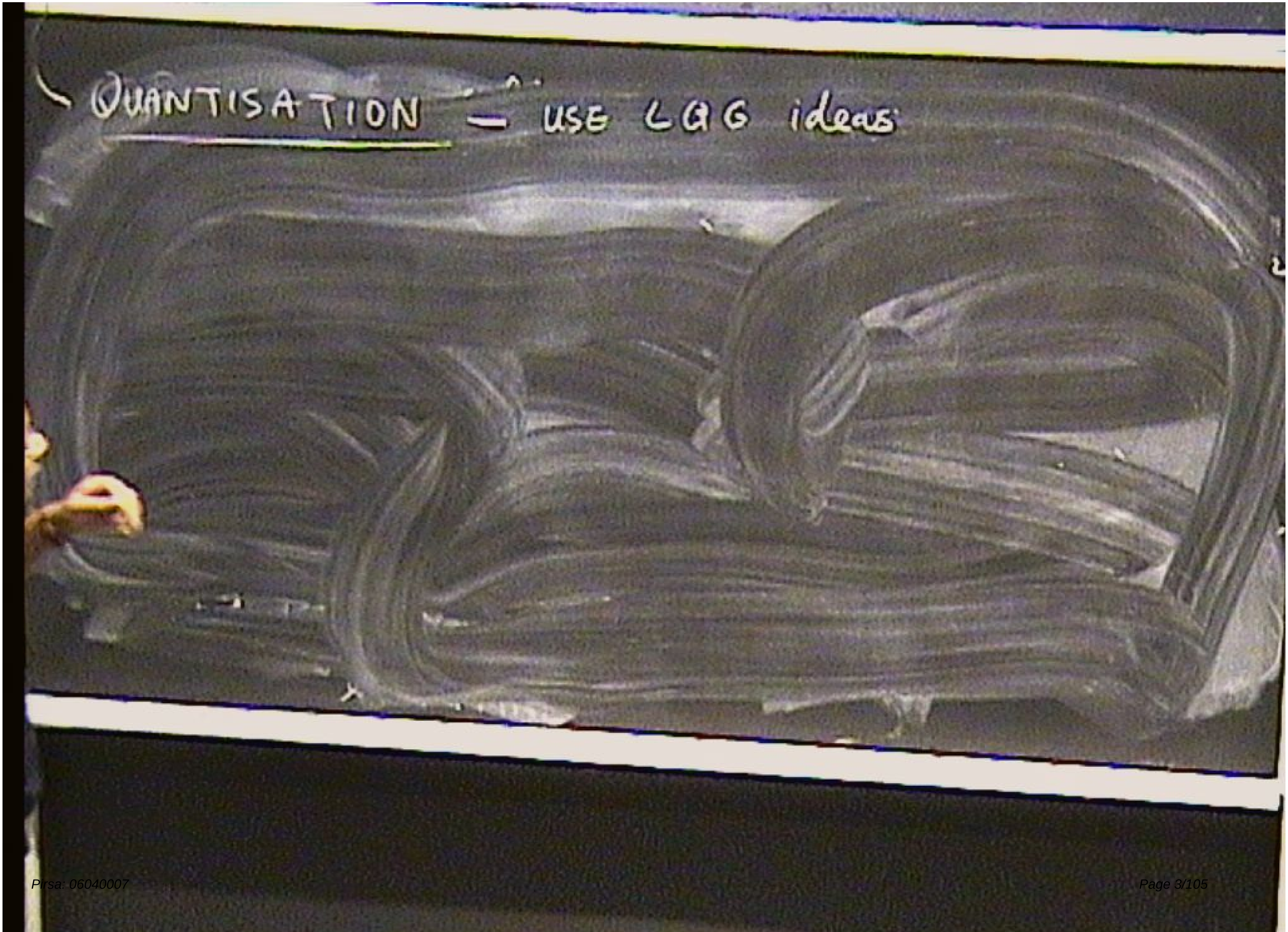
Title: Introduction to quantum gravity - Part 23

Date: Apr 05, 2006 08:00 PM

URL: http://pirsa.org/06040007

Abstract:


QUANTISATION


QUANTISATION — USE LQG ideas:

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$$\{ R(r,t), P_R(r',t) \} = \delta(r-r')$$

$$\{ Q \; \ldots \; \} = \delta(r-r')$$

QUANTISATION — use LQG ideas

$$\{R(r,t),\, P_R(r',t)\} = \delta(r-r')$$

$$\{Q \;\ldots\ldots\} = \delta(r-r')$$

$$\{A^i_a(x),\, E^b_j(x')\} = \delta^i_j\, \delta_a^b\, \vec{\delta}(x-x')$$

$$\downarrow$$

QUANTISATION — USE LQG ideas

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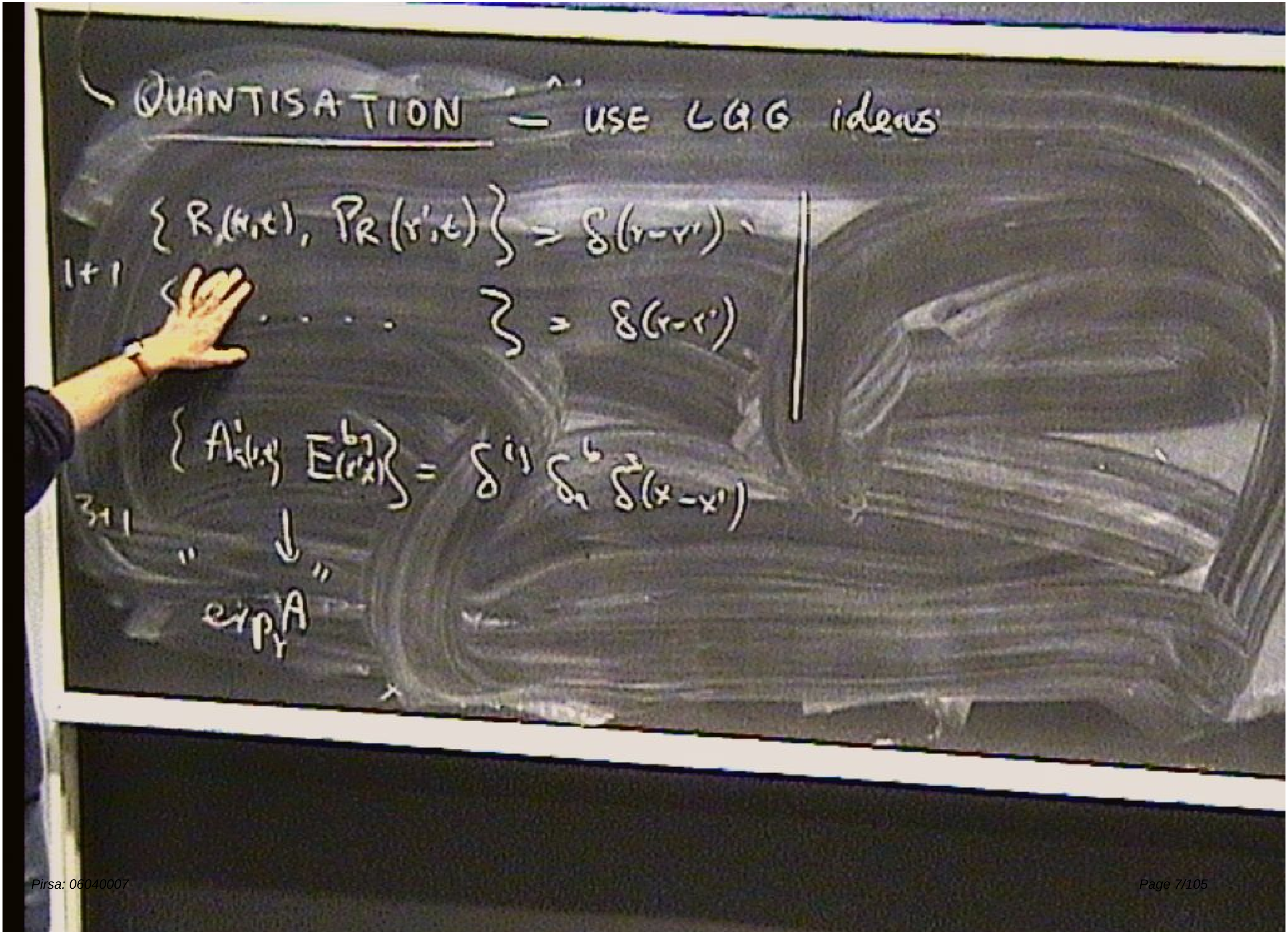
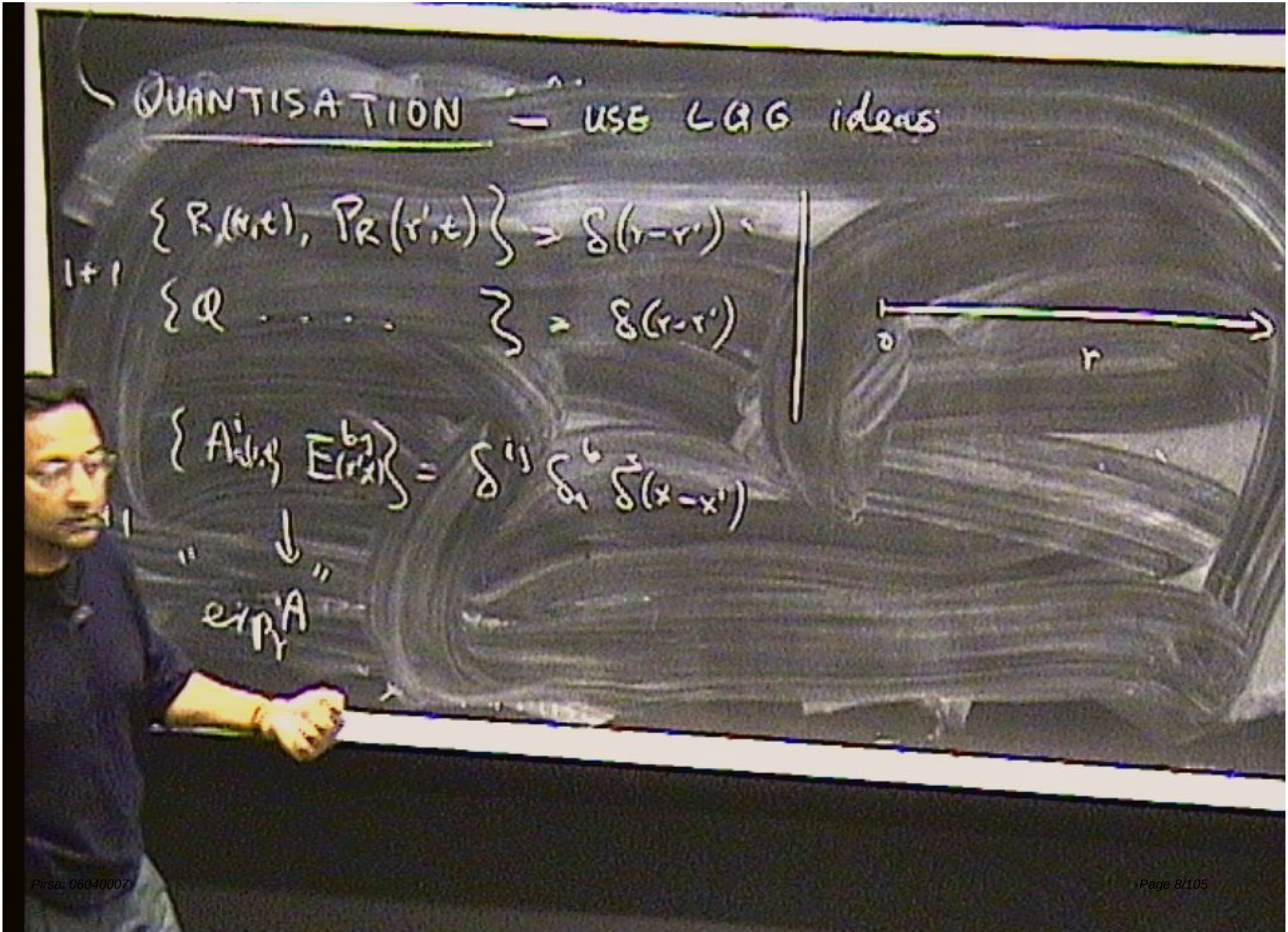
$$\downarrow$$

" $\exp_\gamma A$ "


QUANTISATION — USE LQG ideas
{ R(r,t), P_R(r',t) } = δ(r−r')
1+1
{ . . . . . } = δ(r−r')
{ A(x) E(x') } = δ δ δ(x−x')
3+1
" ↓ "
exp A


QUANTISATION — USE LQG ideas
{ P_r(r,t), P_R(r',t) } = δ(r−r')
1+1
{ Q . . . . . . } = δ(r−r')
0
r
{ A^i_a(x), E^b_j(x') } = δ^i_j δ^b_a δ(x−x')
" ↓ "
e^{i p_γ A}

QUANTISATION — use LQG ideas

1+1

$$\{R(r,t),\ P_R(r',t)\} = \delta(r-r')$$

$$\{Q \ \ldots\ \} = \delta(r-r')$$

3+1

$$\{A^i_a(x),\ E^b_j(x')\} = \delta^i_j\, \delta^b_a\, \delta(\vec{x}-\vec{x}')$$

$$\downarrow$$

$$\text{"}\, e^{i\int_\gamma A} \,\text{"}$$

Graph: $\{r_i\}_{i=1}^N$ of points chosen from $[0,\infty)$.

QUANTISATION — USE LQG ideas

1+1 $\{R_{,}(r,t), P_R(r',t)\} = \delta(r-r')$

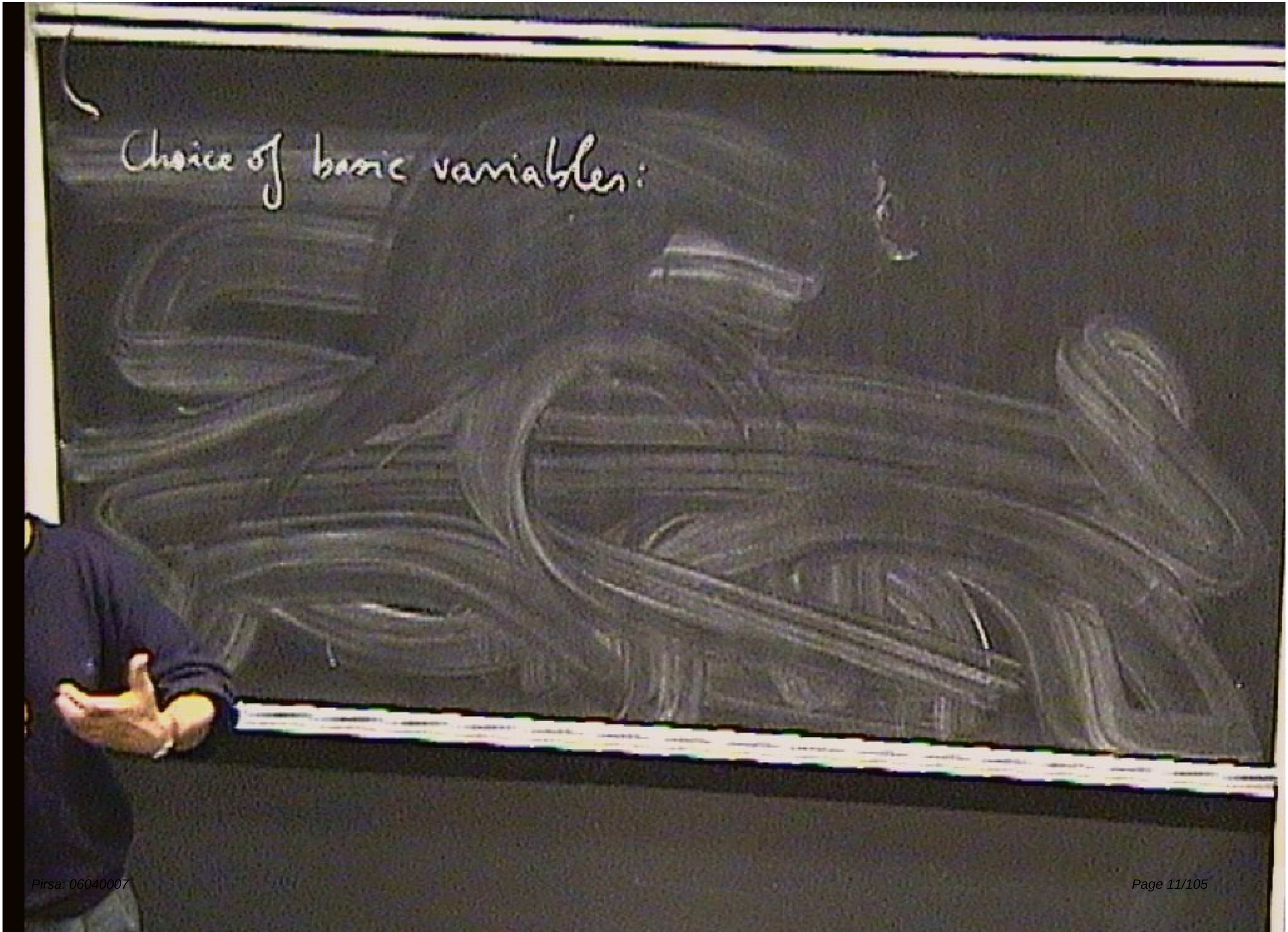
$\{Q \; \ldots \ldots \} = \delta(r-r')$

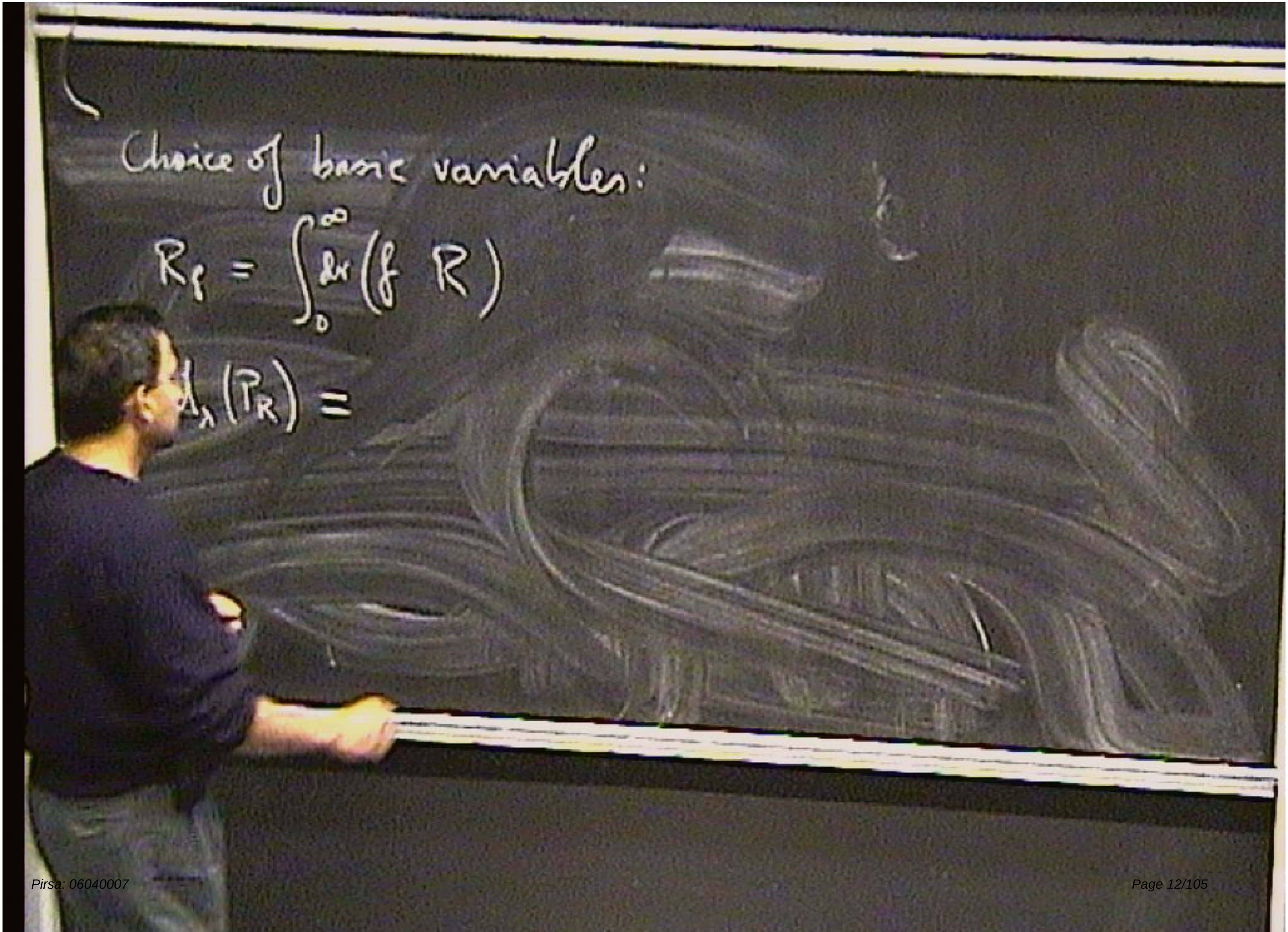
3+1 $\{A^i_a(x), E^b_j(x')\} = \delta^i_j \delta^b_a \delta(x-x')$

$\downarrow$

" $e^{i p_\gamma A}$ "

Graph: $\{r_i\}^N_{i=1}$ of points chosen from $[0,\infty)$.


Choice of basic variables:


Choice of basic variables:
$R_f = \int_0^\infty dr \, (f \, R)$
$(P_R) =$

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Choice of basic variables:

$$R_f = \int_0^\infty dr\,(f\,R) \qquad A_f = \int_0^\infty dr\,(f\,A)$$

$$U_\lambda(P_R) = \exp(i\lambda P_R) \qquad U_\lambda(P_A) = \exp(i\lambda P_A)$$

Choice of basic variables:

$$R_f = \int_0^\infty dr\,(f\,R) \qquad A_f = \int_0^\infty dr\,(f\,A)$$

$$U_\lambda(P_R) = \exp(i\lambda P_R) \qquad U_\lambda(P_A) = \exp(i\lambda P_A)$$

$$\{R_f, U_\lambda(P_R)\} = i\lambda\, f(r)\, U_\lambda$$

Choice of basic variables:

$$R_f = \int_0^\infty dr\,(f\,R) \qquad \mathcal{A}_f = \int_0^\infty dr\,(f\,\mathcal{A})$$

$$U_\lambda(P_R)(r) = \exp(i\lambda P_R(r)) \qquad U_\lambda(P_{\mathcal{A}}) = \exp(i\lambda P_{\mathcal{A}})$$

$$\{R_f, U_\lambda(P_R)\} = i\lambda f(r)\, U_\lambda$$

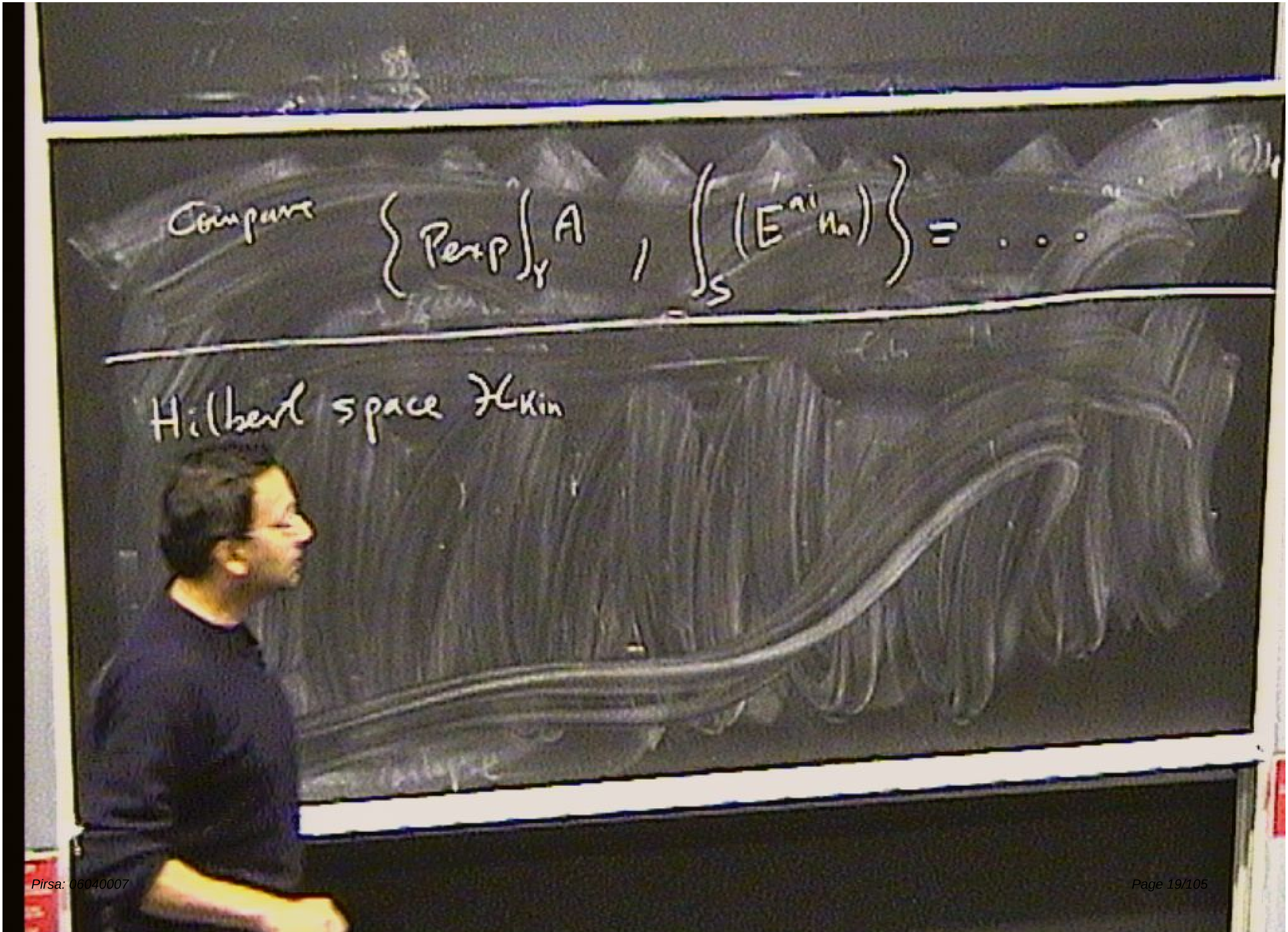
Choice of basic variables:

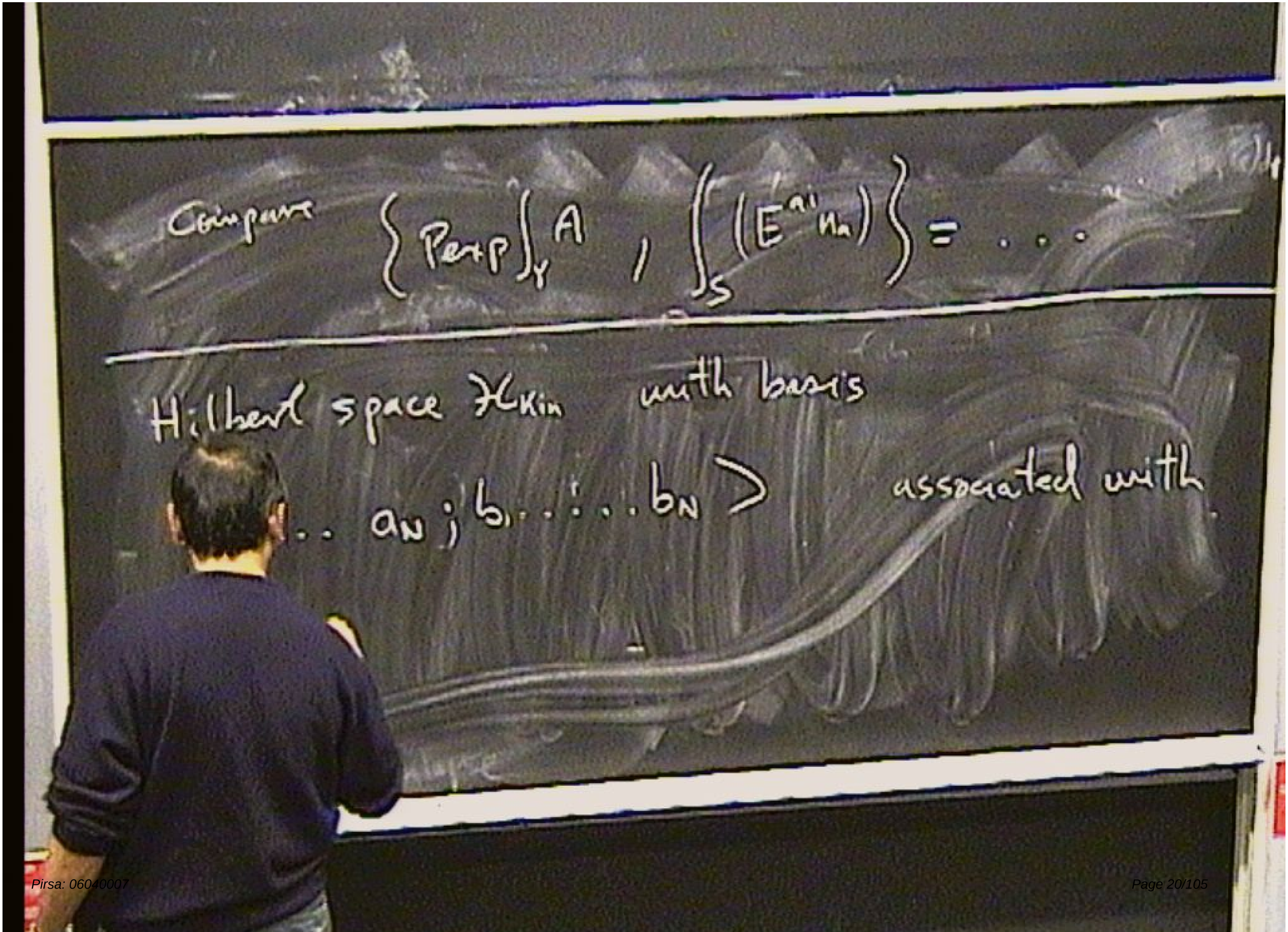
$$R_f = \int_0^\infty dr\,(f\,R) \qquad\qquad Q_f = \int_0^\infty dr\,(f\,Q)$$

$$U_\lambda(P_R)(t) = \exp(i\lambda P_R(t)) \qquad\qquad U_\lambda(P_Q) = \exp(i\lambda P_Q)$$

$$\{R_f, U_\lambda(P_R)\} = i\lambda f(r)\, U_\lambda \qquad\qquad \{Q_f, U_\lambda(P_Q)\} = i\lambda f(r)\, U_\lambda(P_Q)$$

Compare $\left\{ P\exp\int_\gamma A \, , \int_S (E^{ai} n_a) \right\} =$


Compare $\left\{ P\exp\int_\gamma A \, , \int_S (E^{ai} n_a) \right\} = \ldots$
Hilbert space $\mathcal{H}_{Kin}$


Compare $\{ P\exp\int_\gamma A \, , \int_S (E^{ai} n_a) \} = \ldots$
Hilbert space $\mathcal{H}_{kin}$ with basis
$\ldots a_N ; b_1 \ldots \ldots b_N \rangle$ associated with

Compare $\left\{ P\exp\int_\gamma A \;,\; \int_S (E^{ai} n_a) \right\} = \ldots$

Hilbert space $\mathcal{H}_{kin}$ with basis

$|a_1 \ldots a_N ; b_1 \ldots \ldots b_N \rangle$ associated with

a "graph" $\{\gamma_i\}$

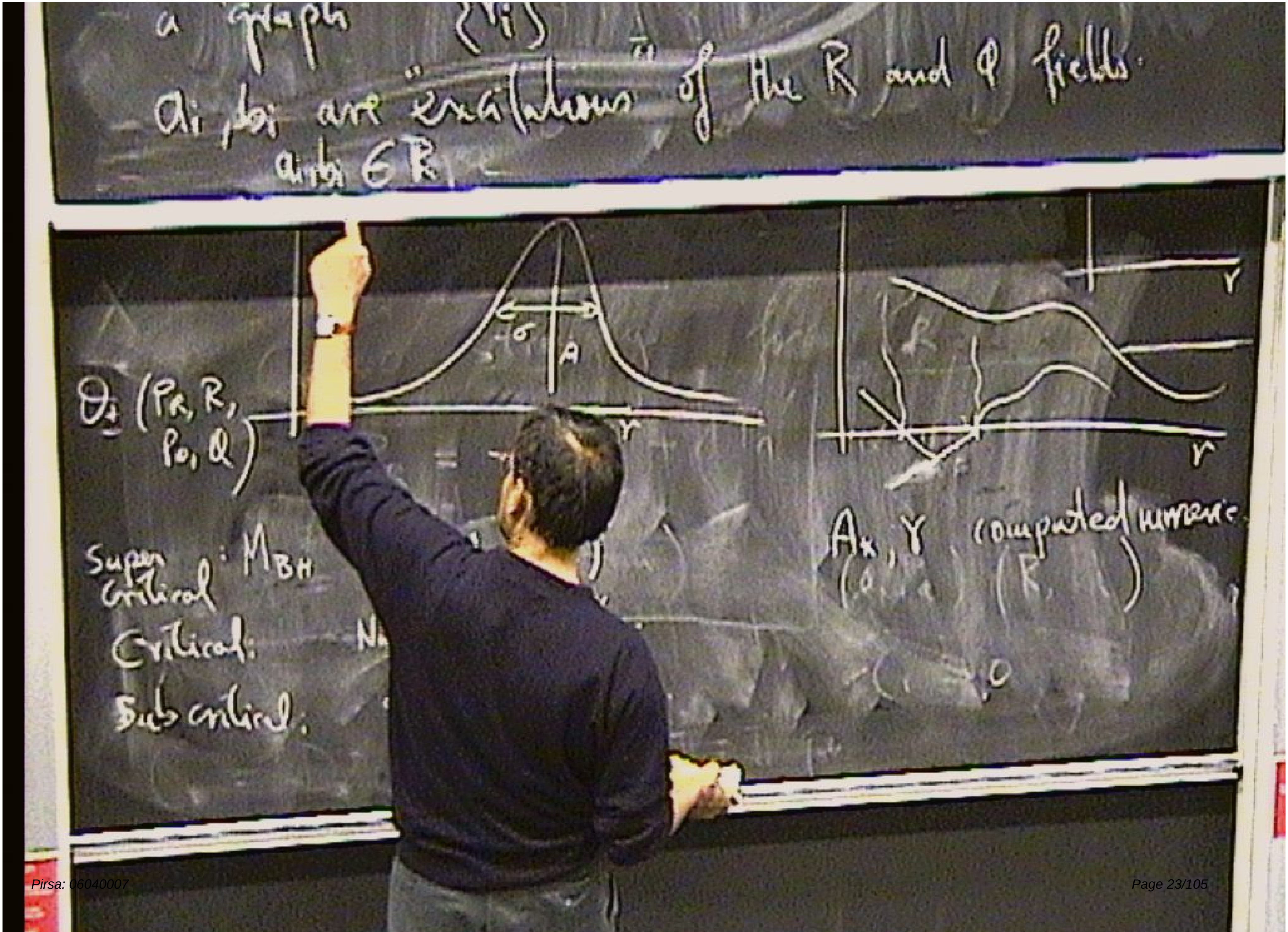
Compare $\left\{ \mathrm{P}\exp\int_\gamma A \, , \int_S (E^{ai} n_a) \right\} = \ldots$

Hilbert space $\mathcal{H}_{kin}$ with basis

$|a_1 \ldots a_N ; b_1 \ldots \ldots b_N \rangle$ associated with

a "graph" $\{v_i\}$

a_i, b_i are "excitations" of the R and ϕ fields.


a graph
ai, bi are excitations of the R and Φ fields.
ai, bi ∈ ℝ
Θ+ (PR, R, PΦ, Φ)
σ
A
r
Super critical: MBH
Critical:
Sub critical:
Ax, Y computed numeric

Compare $\left\{ P\exp \int_\gamma A \ , \int_S (E^{a i} n_a) \right\} = \ldots$

Hilbert space $\mathcal{H}_{kin}$ with basis

$| a_1 \ldots a_N ; b_1 \ldots b_N \rangle$ associated with

a "graph" $\{\gamma_i\}$

a_i , b_i are "excitations" of the R and φ fields.

$a_i, b_i \in \mathbb{R}$

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$$\hat{R}_f \, |a_1 \ldots a_N ; b_1 \ldots b_N \rangle = \sum_{i=1}^{N} \left(f(r_i)\, a_i \right) | \quad ; \quad \rangle$$

$$\hat{U}_n(P_R) | \qquad \rangle =$$

a_i, b_i are "excitations" of the R and Φ fields

$a_i, b_i \in \mathbb{R}$

$$\hat{R}_f \, |a_1 \dots a_N ; b_1 \dots b_N \rangle = \sum_{i=1}^{N} \big(f(r_i)\, a_i\big) \, | \quad ; \quad \rangle$$

$$\hat{U}_\lambda(P_R)(r_2) \, | \quad ; \quad \rangle =$$

a_i, b_i are "excitations" of the R and φ fields.

$a_i, b_i \in \mathbb{R}$

$$\hat{R}_f \,|a_1 \ldots a_N ; b_1 \ldots b_N\rangle = \sum_{i=1}^{N} \big(f(r_i)\, a_i\big)\, | \quad ; \quad \rangle$$

$$\hat{U}_\lambda(P_R)(r_2)\,|a_1\, a_2 \ldots a_N ; b_1 \ldots b_N\rangle = |a_1\, a_2-\lambda \ldots a_N ; b_1 \ldots b_N\rangle$$

Choice of basic variables:

$$R_f = \int_0^\infty dr \, (f \, R) \qquad \mathcal{A}_f = \int_0^\infty dr \, (f \, \mathcal{A})$$

$$U_\lambda(P_R)(r) = \exp(i\lambda P_R(r)) \qquad U_\lambda(P_\mathcal{A}) = \exp(i\lambda P_\mathcal{A})$$

$$\{R_f, U_\lambda(P_R)\} = i\lambda f(r) \, U_\lambda \qquad \{\mathcal{A}_f, U_\lambda(P_\mathcal{A})\} = i\lambda f(r) \, U_\lambda(P_\mathcal{A})$$

$$\lambda \in \mathbb{R}$$

a_i, b_i are "excitations" of the R and φ fields.

$a_i, b_i \in \mathbb{R}$

$$\hat{R}_f \, |a_1 \ldots a_N ; b_1 \ldots b_N\rangle = \sum_{i=1}^{N} \left(f(x_i)\, a_i\right) | \quad i \rangle$$

$$\hat{U}_\lambda(P_R)(x_i)\, |a_1\, a_2 \ldots a_N ; b_1 \ldots b_N\rangle = |a_1 \ldots a_i - \lambda \ldots a_N ; b_1 \ldots b_N\rangle$$

$$\left[\hat{R}_f , \hat{U}_\lambda(P_R)\right] =$$

a_i, b_i are "excitations" of the R and φ fields.

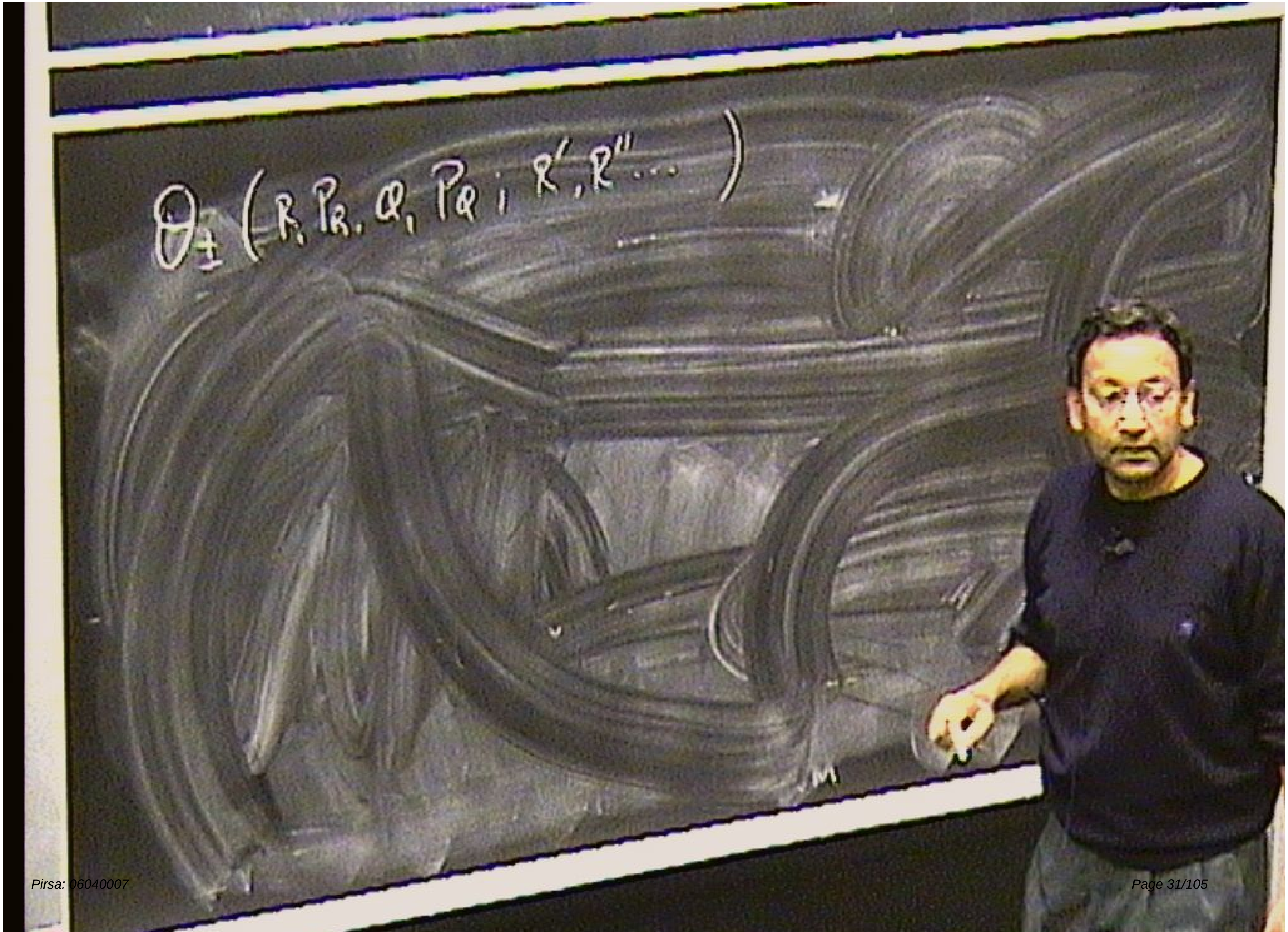
$a_i, b_i \in \mathbb{R}$

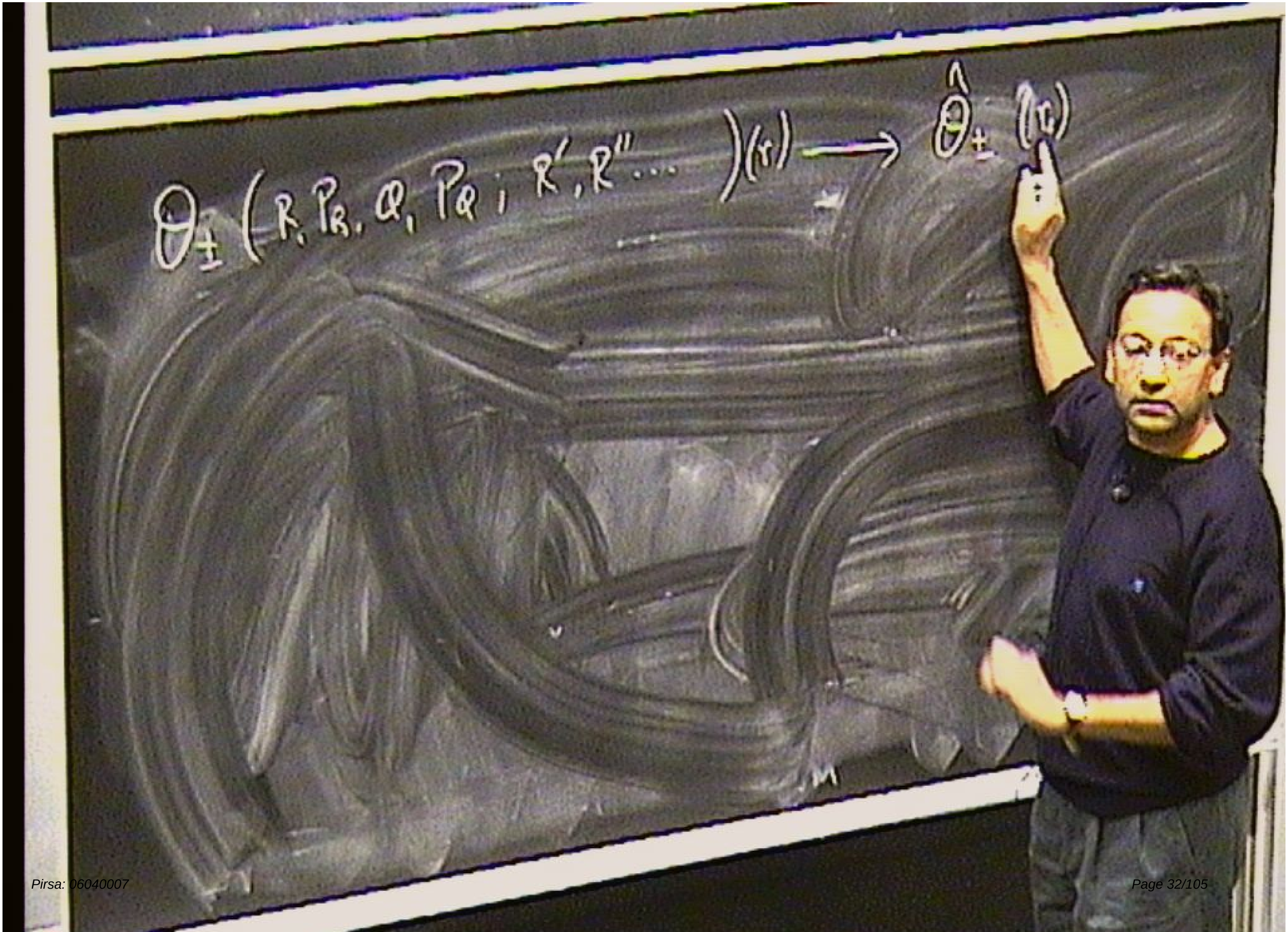
$$\hat{R}_f |a_1 \ldots a_N ; b_1 \ldots b_N\rangle = \sum_{i=1}^{N} \left(f(r_i) a_i\right) | \quad ; \quad \rangle$$

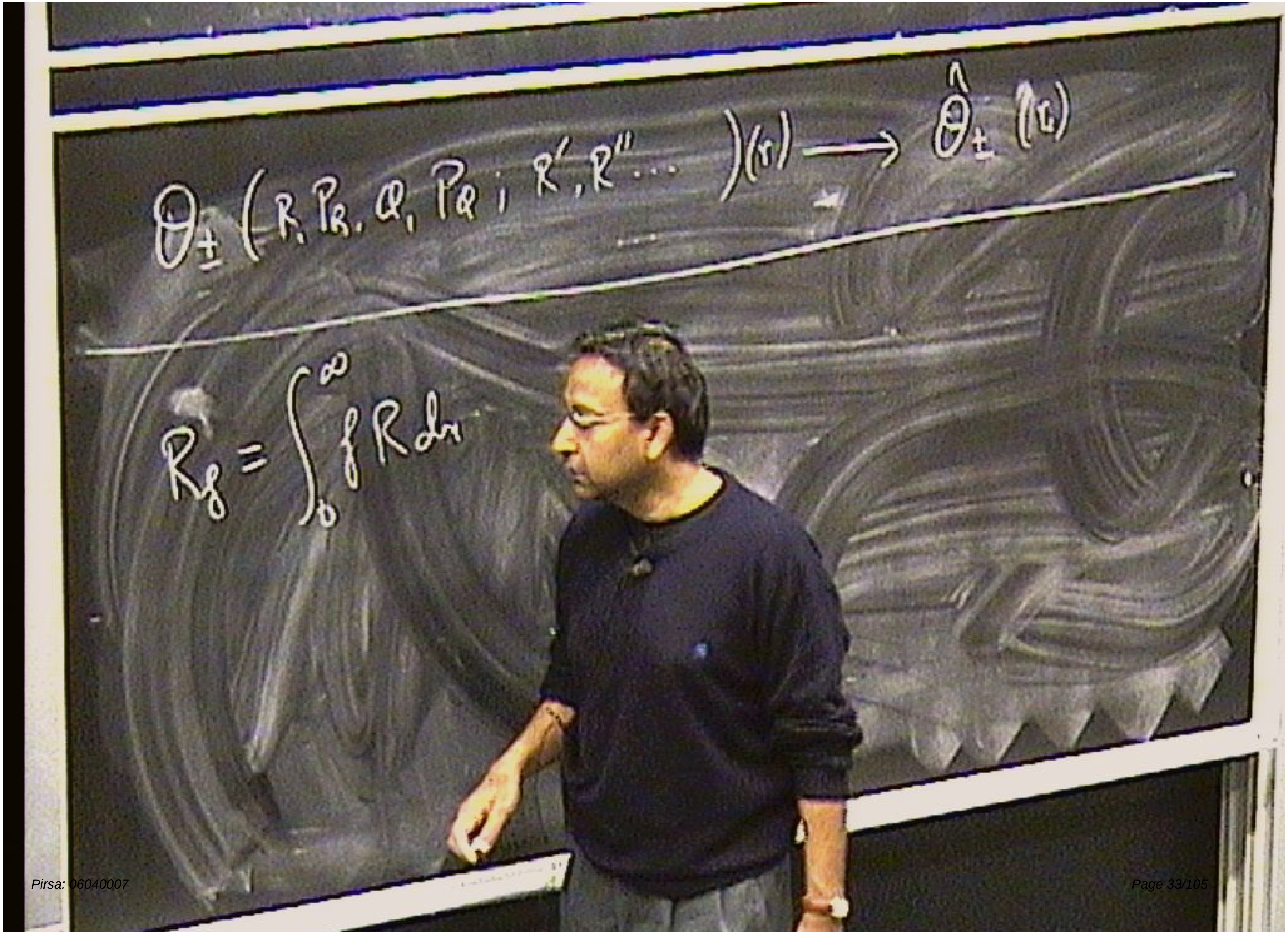
$$\widehat{U_\lambda(P_R)(r_2)} |a_1 a_2 \ldots a_N ; b_1 \ldots b_N\rangle = |a_1 \ldots a_2 - \lambda \ldots a_N ; b_1 \ldots b_N\rangle$$

$$\left[\hat{R}_f , \widehat{U_\lambda(P_R)(r_2)}\right] = \lambda f(r_2) \hat{U}_\lambda(P_R)(r_2)$$

Same for $\hat{\varphi}_f$, $\widehat{U_\lambda(P_\varphi)}$


$\Theta_\pm(R, P_R, Q, P_Q ; R', R'' \ldots)$


$\Theta_{\pm}(R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_{\pm}(r)$


$\Theta_{\pm}(R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_{\pm}(r)$
$R_g = \int_0^\infty f R \, dr$

$$\Theta_{\pm}(R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_{\pm}(r_1)$$

$$f_\epsilon(r, r_1) = A e^{-(r-r_1-\epsilon)^2}$$

$$R_f = \int_0^\infty f R \, dr_1$$

$$\Theta_{\pm}\left(R, P_R, Q, P_Q ; R', R'' \ldots\right)(r) \longrightarrow \hat{\Theta}_{\pm}(r_i)$$

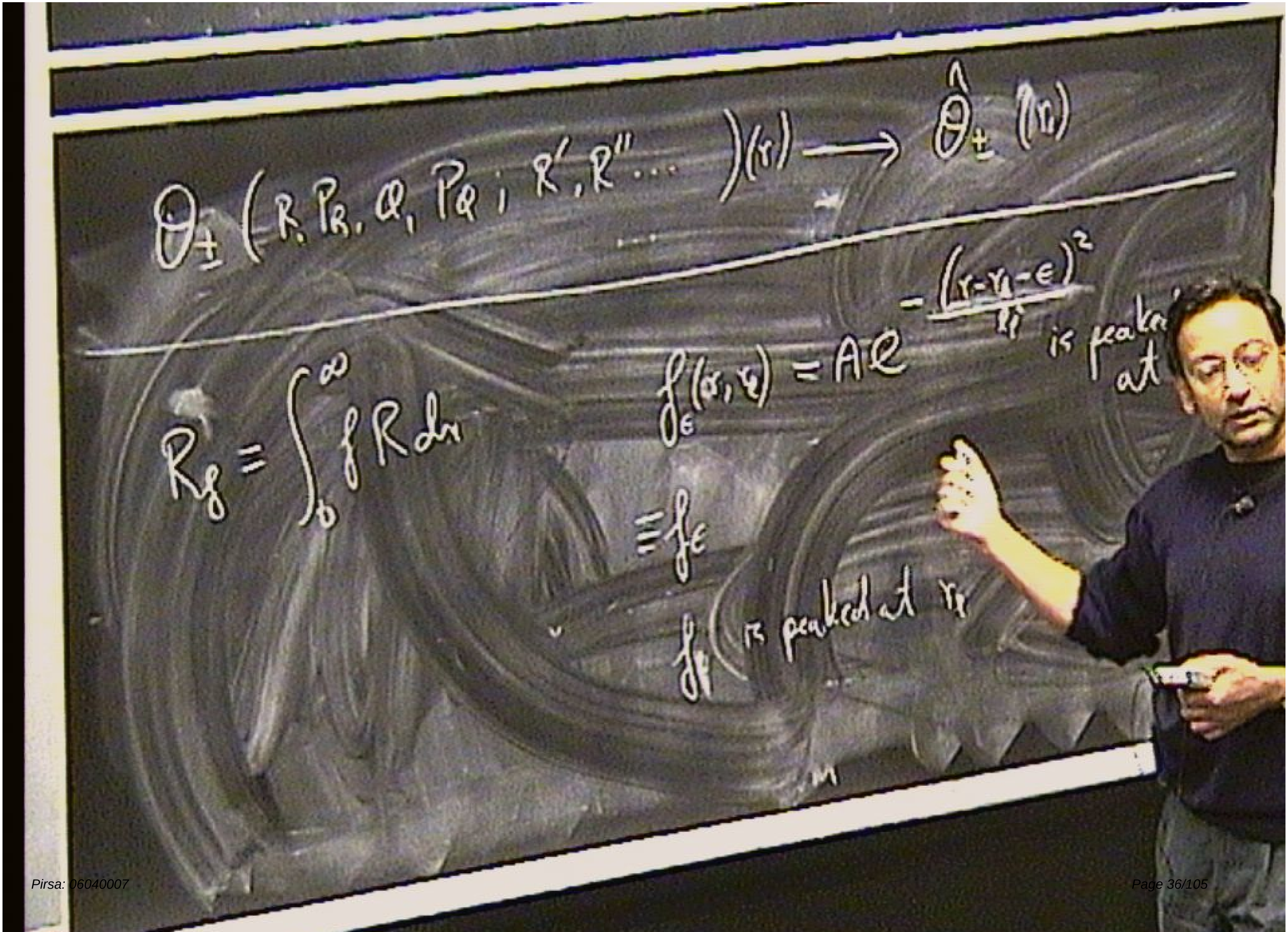
$$f_\epsilon(r, r_i) = A e^{-(r-r_i-\epsilon)^2}$$

is peaked at $r_i+\epsilon$

$$R_g \equiv \int_0^\infty f R \, dr_i$$

$$\equiv f_\epsilon$$

f_g is peaked at r_i


$\Theta_\pm (R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_\pm (r_i)$
$R_g \equiv \int_0^\infty f R \, dr$
$f_\epsilon(r, r_i) = A e^{-\frac{(r - r_i - \epsilon)^2}{\epsilon_i}}$
is peaked at
$\equiv f_\epsilon$
f_ϵ is peaked at r_i

$$\Theta_\pm \left(R, P_R, Q, P_Q ; R', R'' \ldots \right)(r) \longrightarrow \hat{\Theta}_\pm (r_i)$$

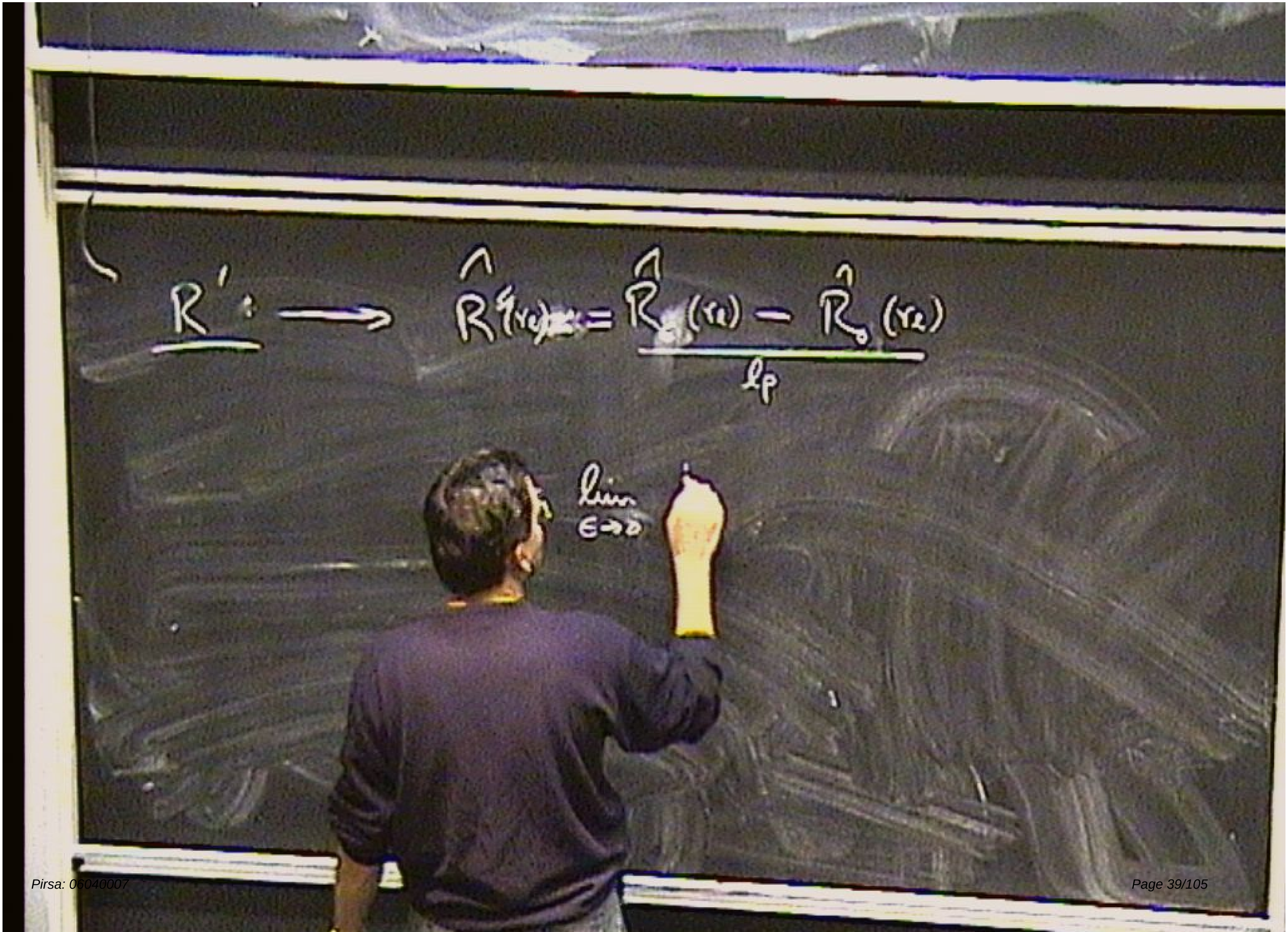
$$f_\epsilon(r, r_i) = A e^{-\frac{(r-r_i-\epsilon)^2}{k_i}}$$ is peaked at $r_i+\epsilon$

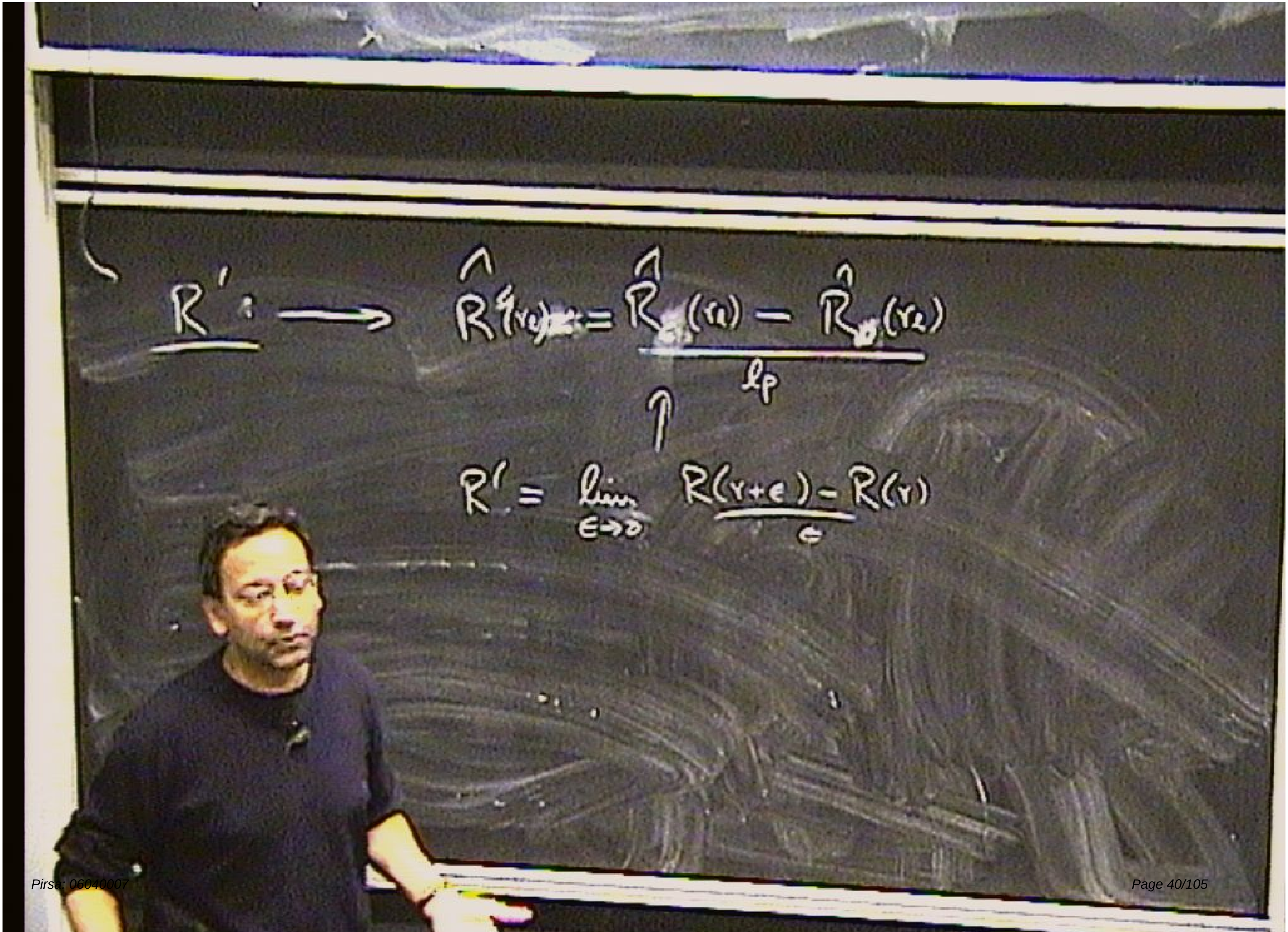
$$R_g \equiv \int_0^\infty f R \, dr$$

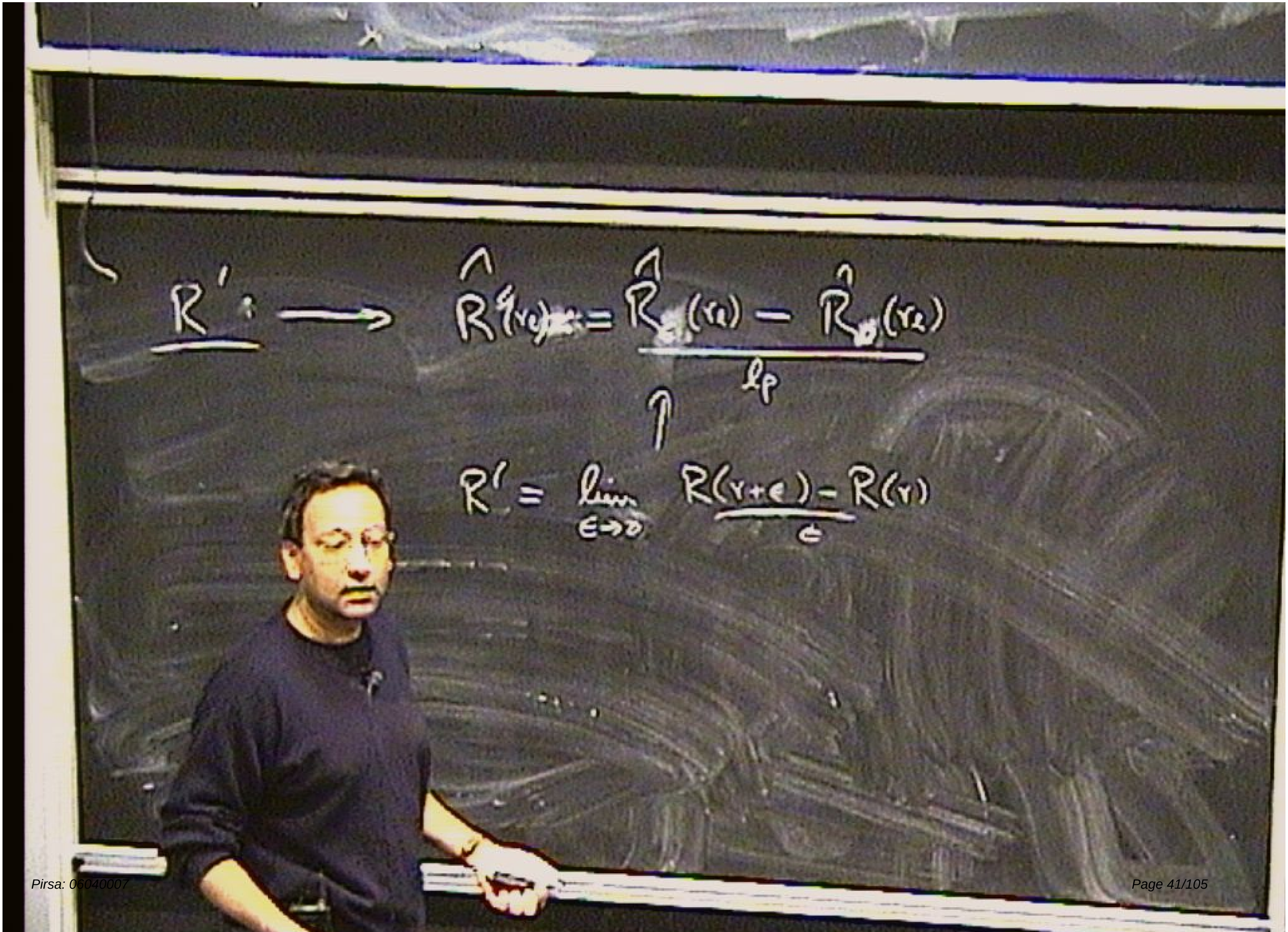
$$\equiv f_\epsilon$$

f_ϵ is peaked at r_i

$\underline{R':}\longrightarrow \hat{R}:$


R' : $\longrightarrow$ $\hat{R}^{\epsilon}(r_2) = \dfrac{\hat{R}_{\epsilon}(r_2) - \hat{R}_0(r_2)}{\ell_p}$
$\lim_{\epsilon \to 0}$


$\underline{R'}$: $\longrightarrow$ $\hat{R}'(r_2) = \dfrac{\hat{R}(r_1) - \hat{R}(r_2)}{l_p}$
$R' = \lim_{\epsilon \to 0} \dfrac{R(r+\epsilon) - R(r)}{\epsilon}$



$R' : \longrightarrow \hat{R}'(r_2) = \frac{\hat{R}(r_2) - \hat{R}(r_2)}{l_p}$
$R' = \lim_{\epsilon \to 0} \frac{R(r+\epsilon) - R(r)}{\epsilon}$

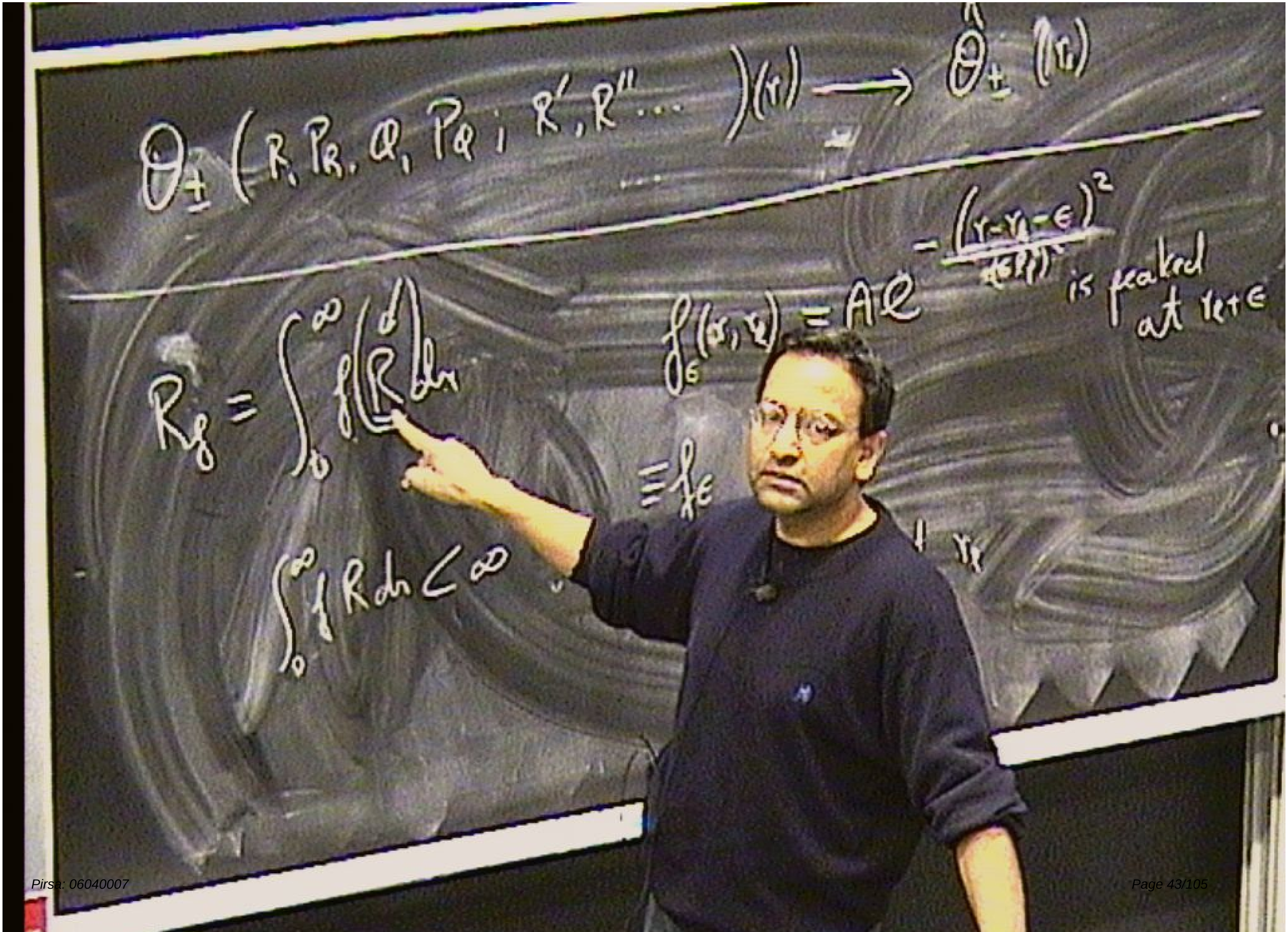
$$\Theta_\pm(R, P_R, Q, P_Q;\ R', R'' \dots)(r) \longrightarrow \hat{\Theta}_\pm(r_i)$$

$$f_\epsilon(r, r_i) = A e^{-\frac{(r-r_i-\epsilon)^2}{\sigma(r_i)}}$$ is peaked at $r_i+\epsilon$

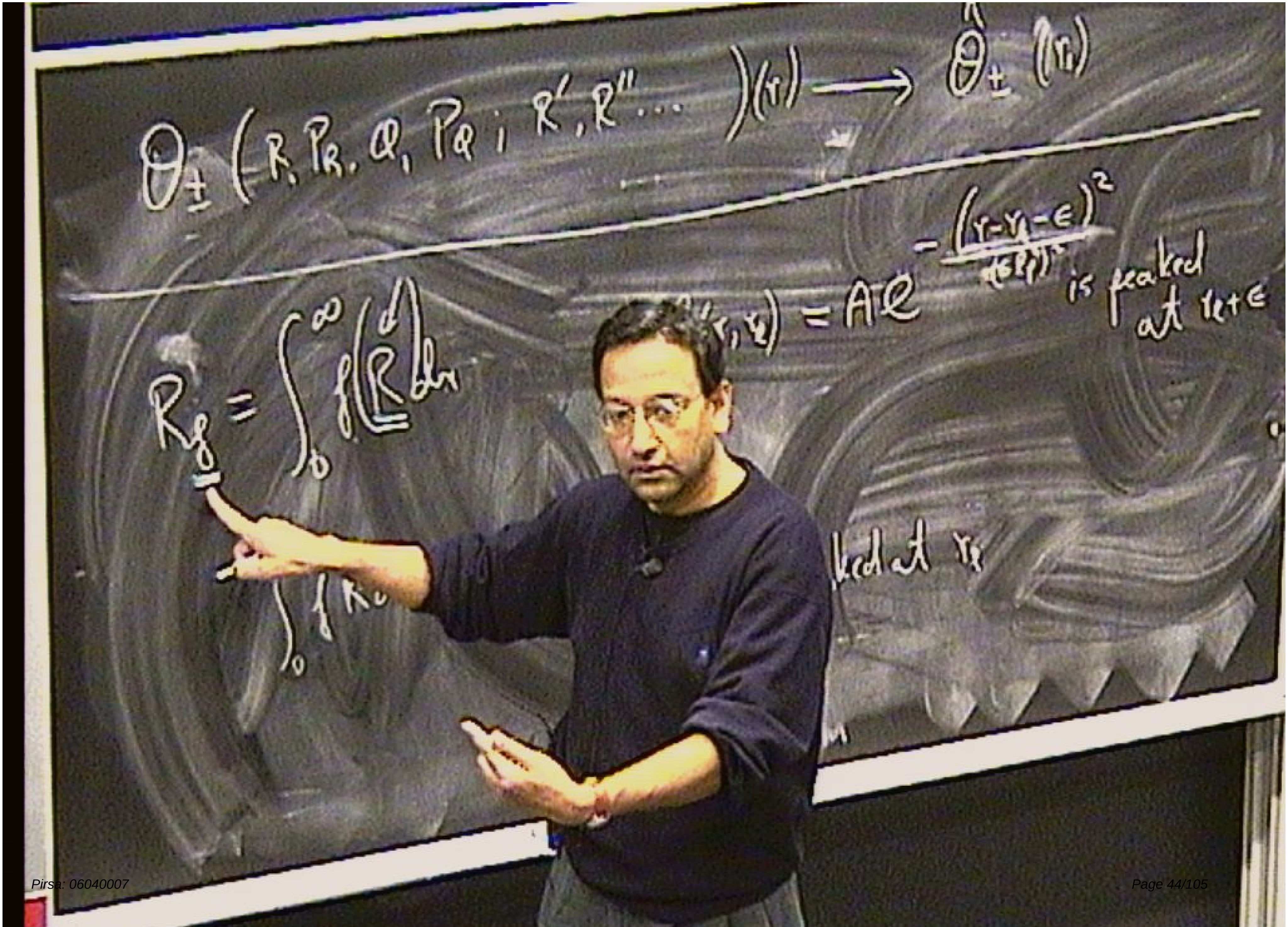
$$R_f = \int_0^\infty f R \, dr$$

$$\equiv f_\epsilon$$

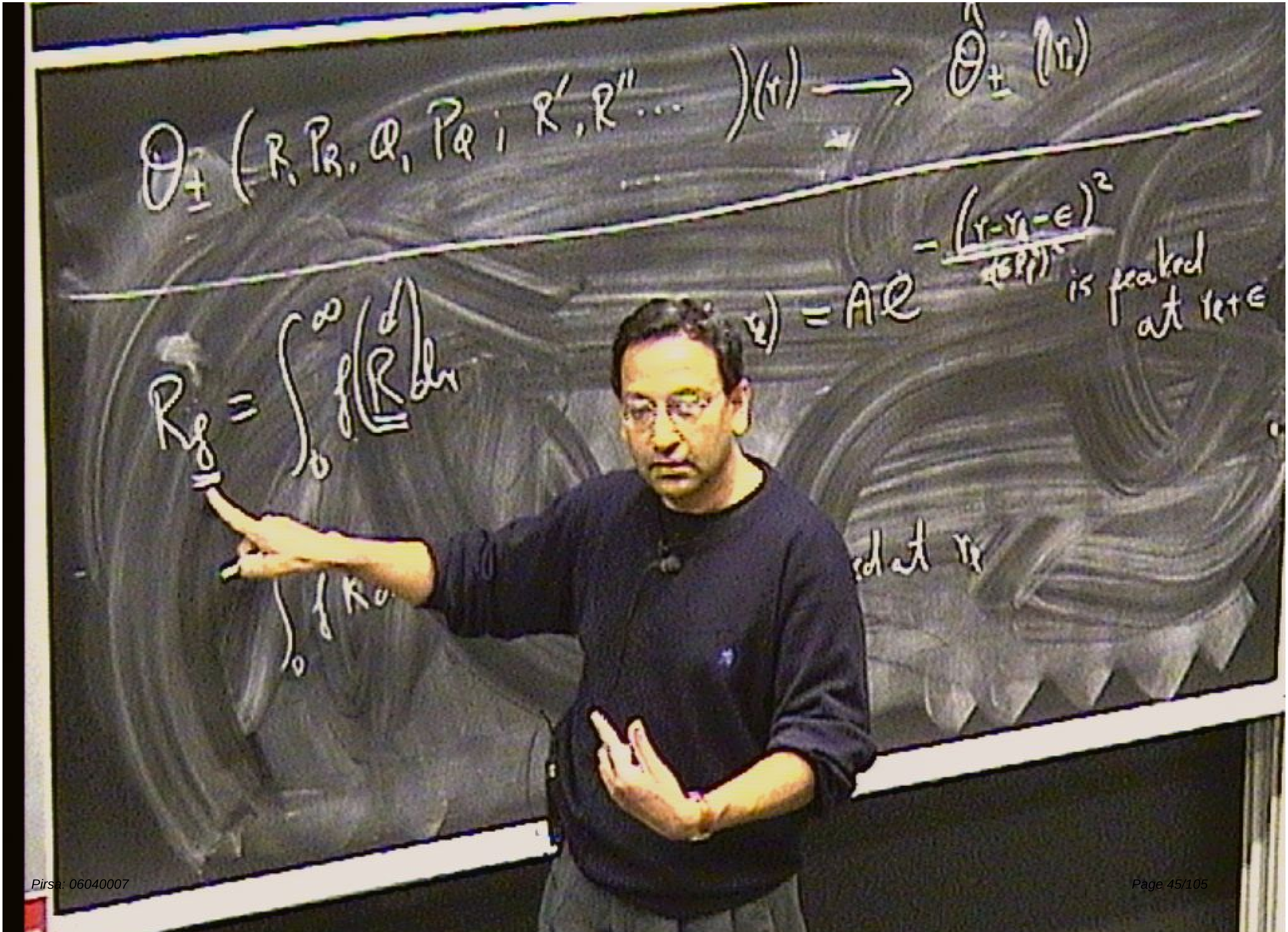
f is peaked at r_i

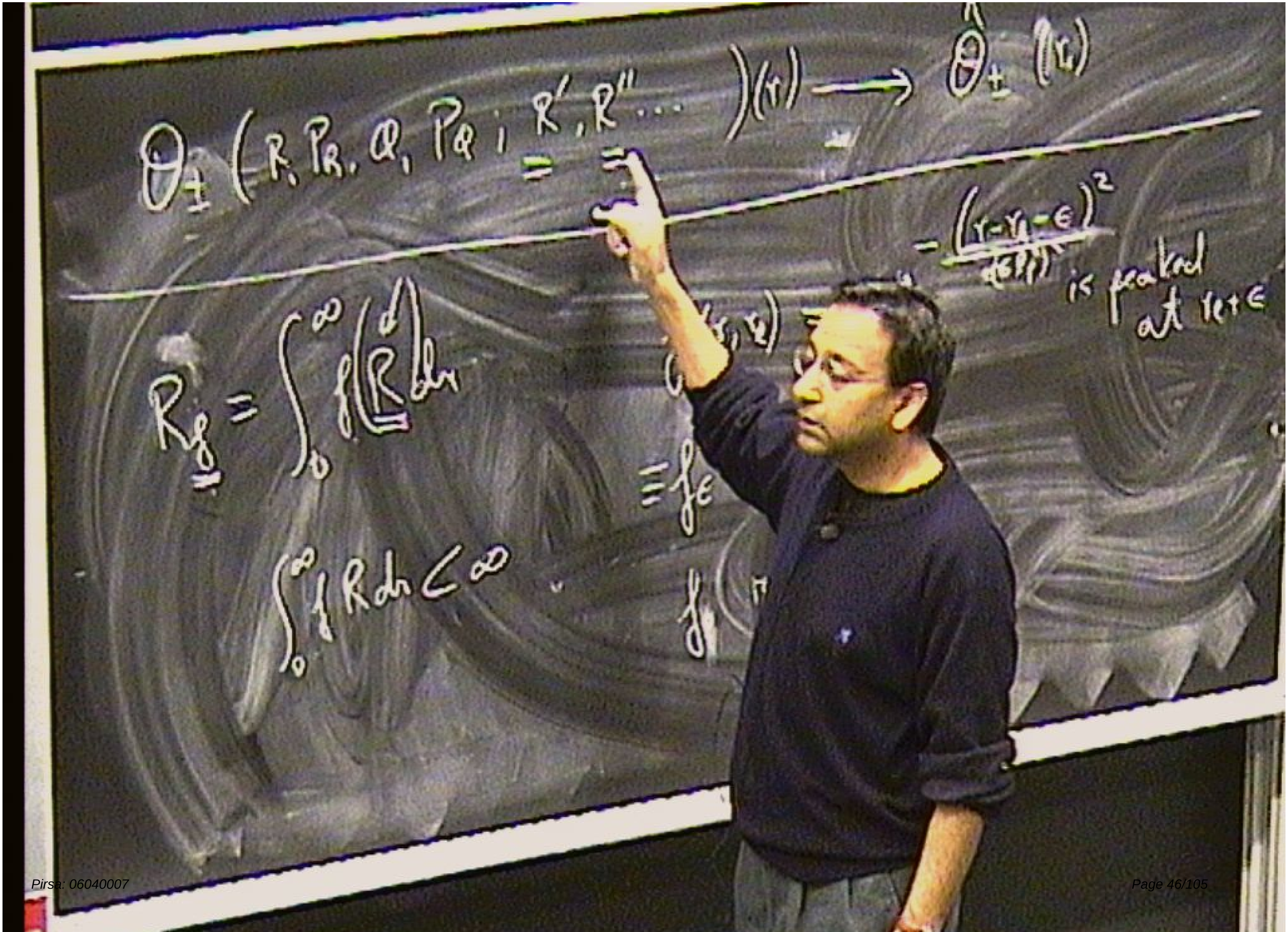


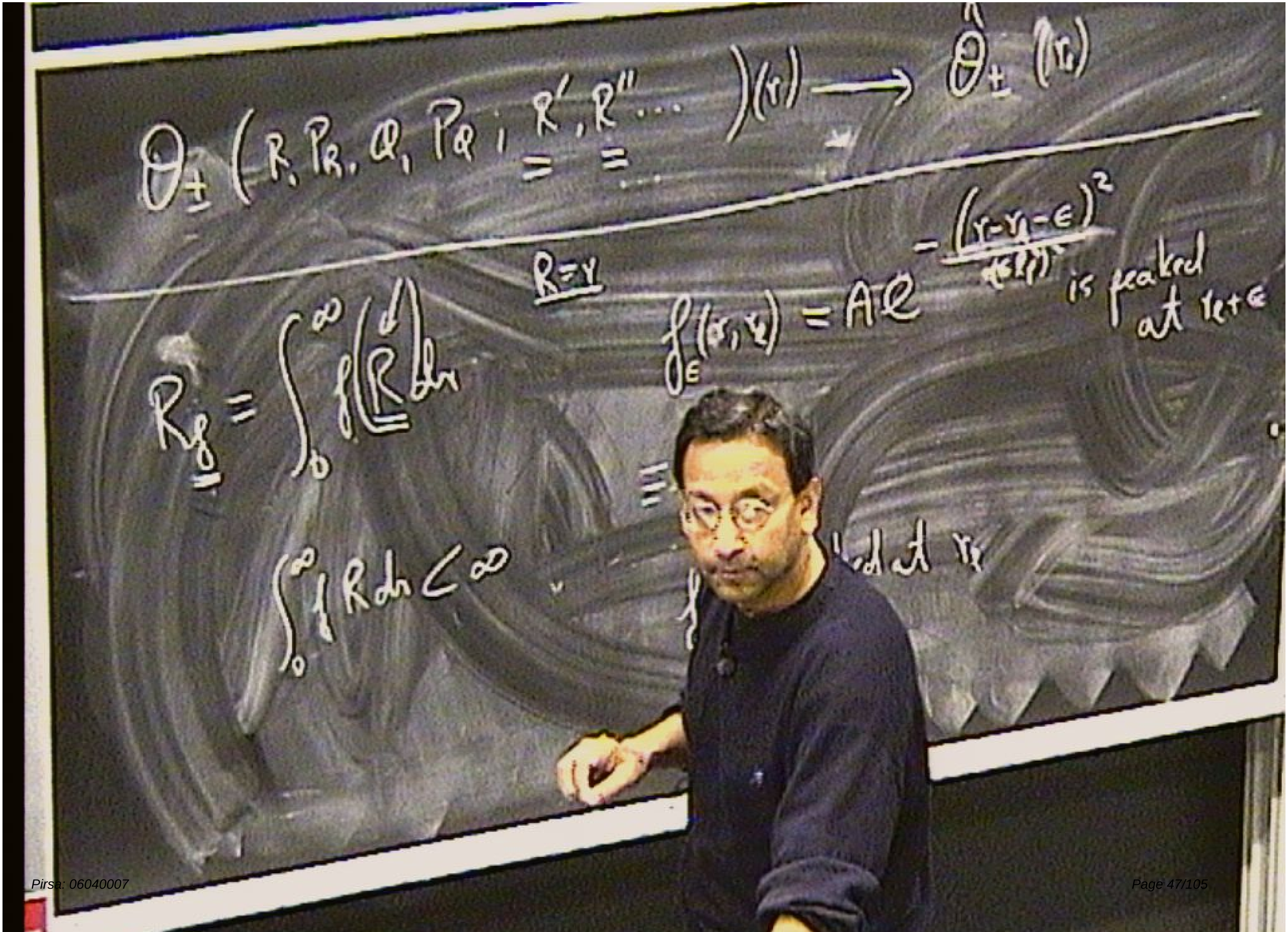
Θ± (R, P_R, Q, P_Q ; R', R'' ...)(r) → Θ̂± (r_1)
f_ε(r, r_1) = A e^{-(r - r_1 - ε)^2}
is peaked at r_1+ε
R_f = ∫_0^∞ f (R) dr
≡ f_ε
∫_0^∞ f R dr < ∞



$\Theta_\pm (R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_\pm (r_1)$
$= A e^{-\frac{(r - r_1 - \epsilon)^2}{(\delta r)^2}}$
is peaked at $r_1+\epsilon$
$R_g = \int_0^\infty f(R) dr$


$\Theta_\pm (R, P_R, Q, P_Q ; R', R'' \ldots)(r) \longrightarrow \hat{\Theta}_\pm (r_1)$
$= A e^{-\frac{(r - r_1 - \epsilon)^2}{(\delta r_1)^2}}$
is peaked at $r_1 + \epsilon$
$R_g = \int_0^\infty f(R) dr$


is peaked
at r₁+ε


Θ± (R, P_R, Q, P_Q ; R′, R″ …)(r) → Θ̂± (r_t)
R=r
f_ε(r, r_t) = A e^{−(r−r_t−ε)^2}
is peaked at r_t+ε
R_g = ∫_0^∞ f R dr
∫_0^∞ f R dr < ∞

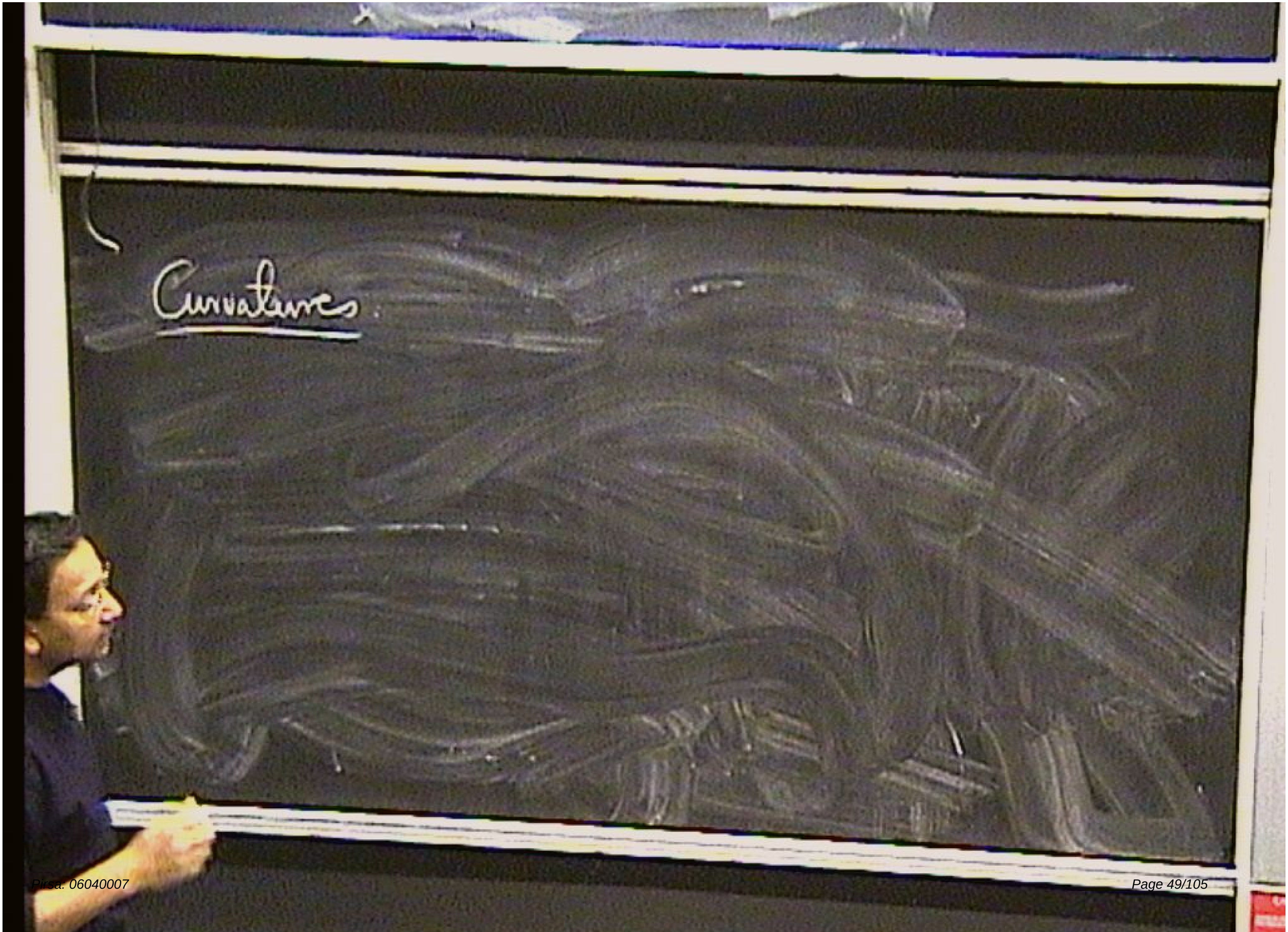
$$\Theta_\pm\left(R, P_R, Q, P_Q; R', R'' \ldots\right)(r) \longrightarrow \hat{\Theta}_\pm(r_i)$$

$$f_\epsilon(r, r_i) = A e^{-\frac{(r-r_i-\epsilon)^2}{\sigma(\epsilon)^2}}$$ is peaked at $r_i+\epsilon$

$$R_f \equiv \int_0^\infty f\,R\,dr \equiv f_\epsilon$$

$$\int_0^\infty f\,R\,dr < \infty$$

f is peaked at r_i


Curvatures.

Curvatures

$$\frac{1}{R^n}$$

$$R^2 \, d\Omega^2$$

Curvatures : $\frac{1}{R^n}$ $R^2 d\Omega^2$

$$\wedge$$

$$\frac{1}{R^n}$$

Curvatures

$$\frac{1}{R^n} \qquad R^2 \, d\Omega^2$$

$$\widehat{\frac{1}{R^n}} = ?$$

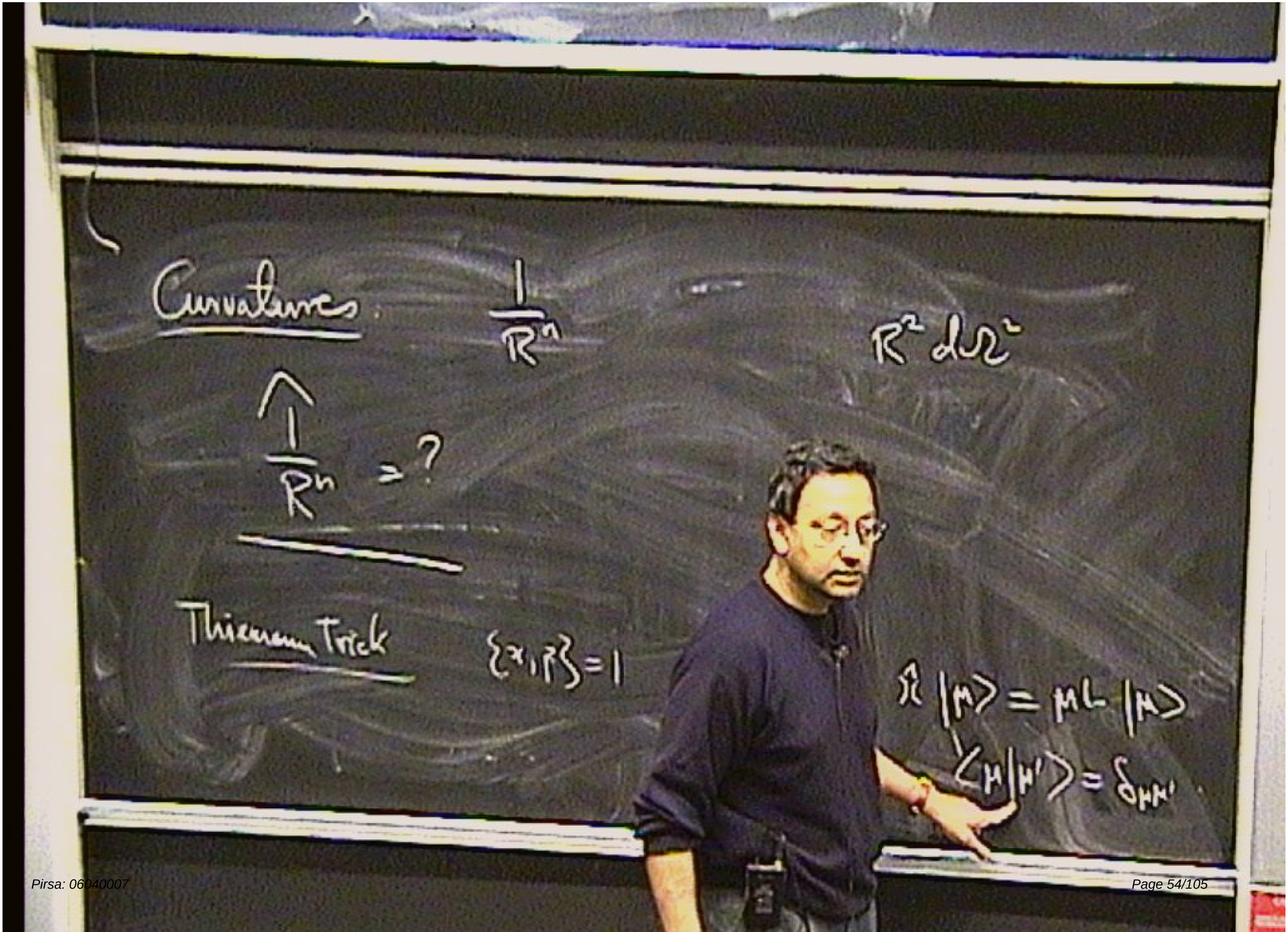
Curvatures : $\frac{1}{R^n}$

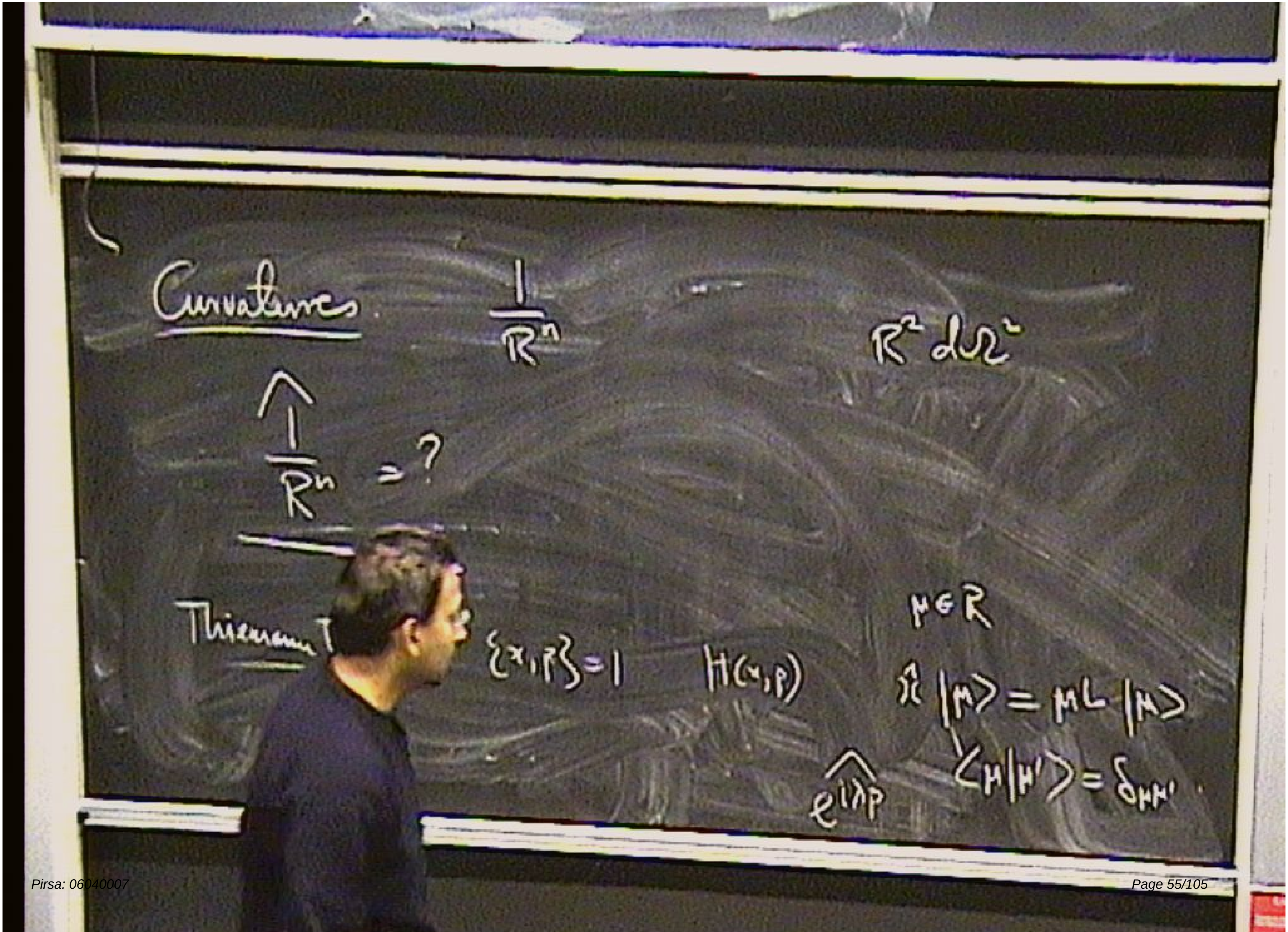
$R^2 \, d\Omega^2$

$\widehat{\frac{1}{R^n}} = ?$

Thiemann Trick

$\{x, p\} = 1 \qquad H(x,p)$


Curvatures: $\frac{1}{R^n}$
$R^2 d\Omega^2$
$\widehat{\frac{1}{R^n}} \Rightarrow ?$
Thiemann Trick
$\{x, R\} = 1$
$\hat{R}\,|\mu\rangle = \mu L\,|\mu\rangle$
$\langle\mu|\mu'\rangle = \delta_{\mu\mu'}$



Curvatures
1/Rⁿ
R² dΩ²
1/Rⁿ = ?
Thiemann
{x, p} = 1
H(x, p)
μ ∈ R
x̂ |μ> = μL |μ>
<μ|μ'> = δμμ'
e^(iλp)

Curvatures $\frac{1}{R^n}$ $R^2 d\Omega^2$

$\widehat{\frac{1}{R^n}} = ?$

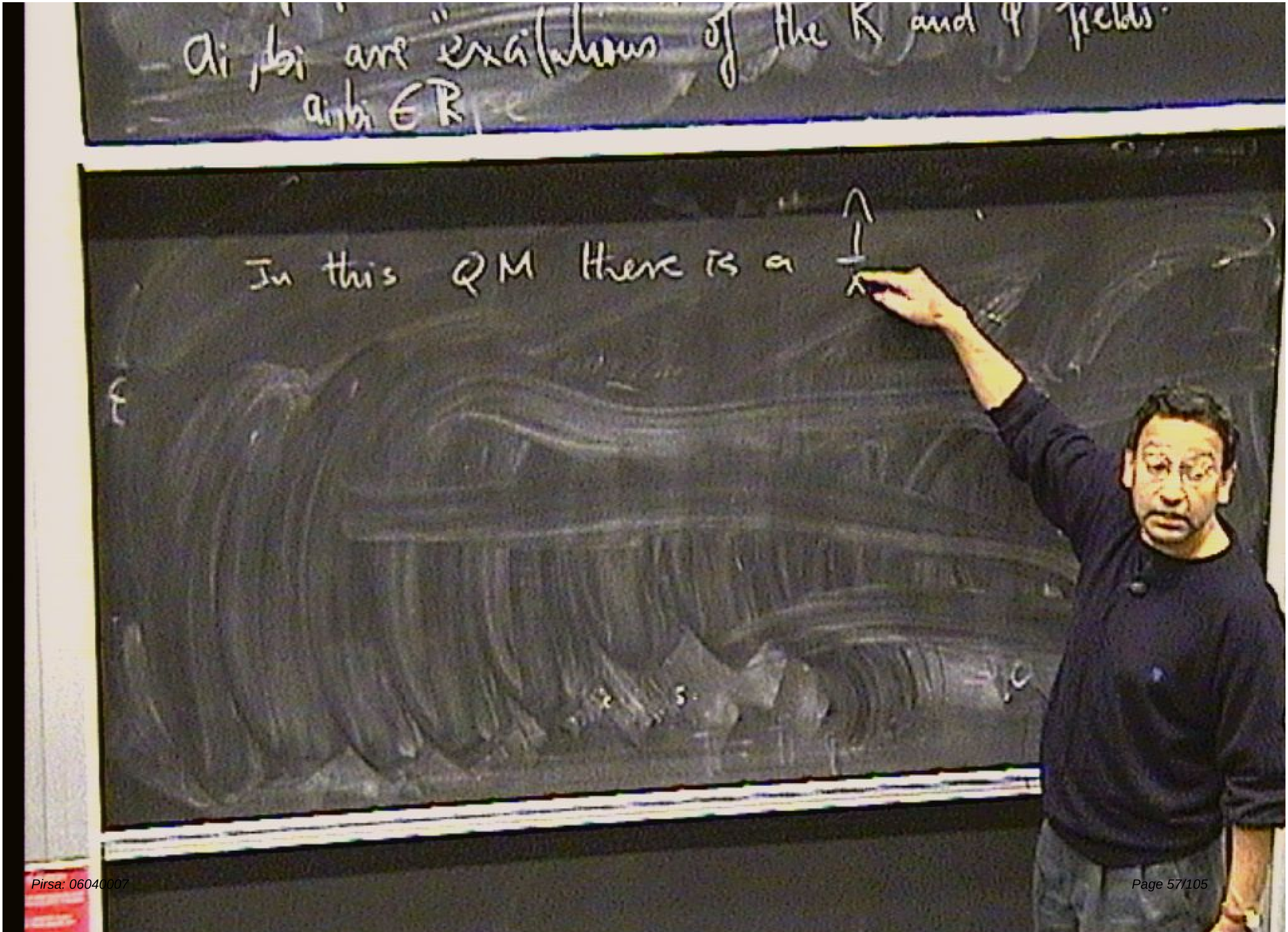
Thiemann Trick $\{x, p\} = 1$ $H(x, p)$

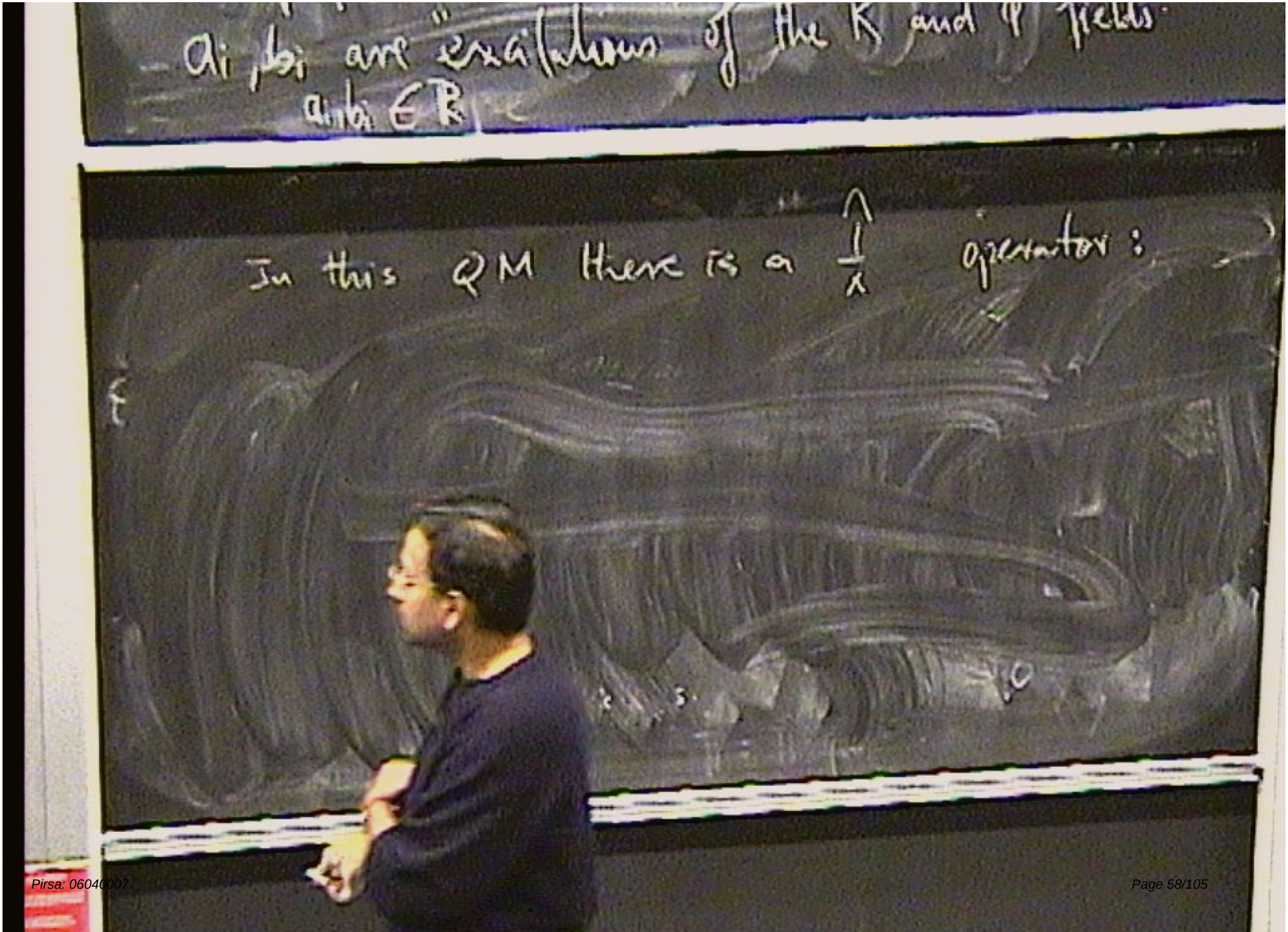
$\mu \in \mathbb{R}$

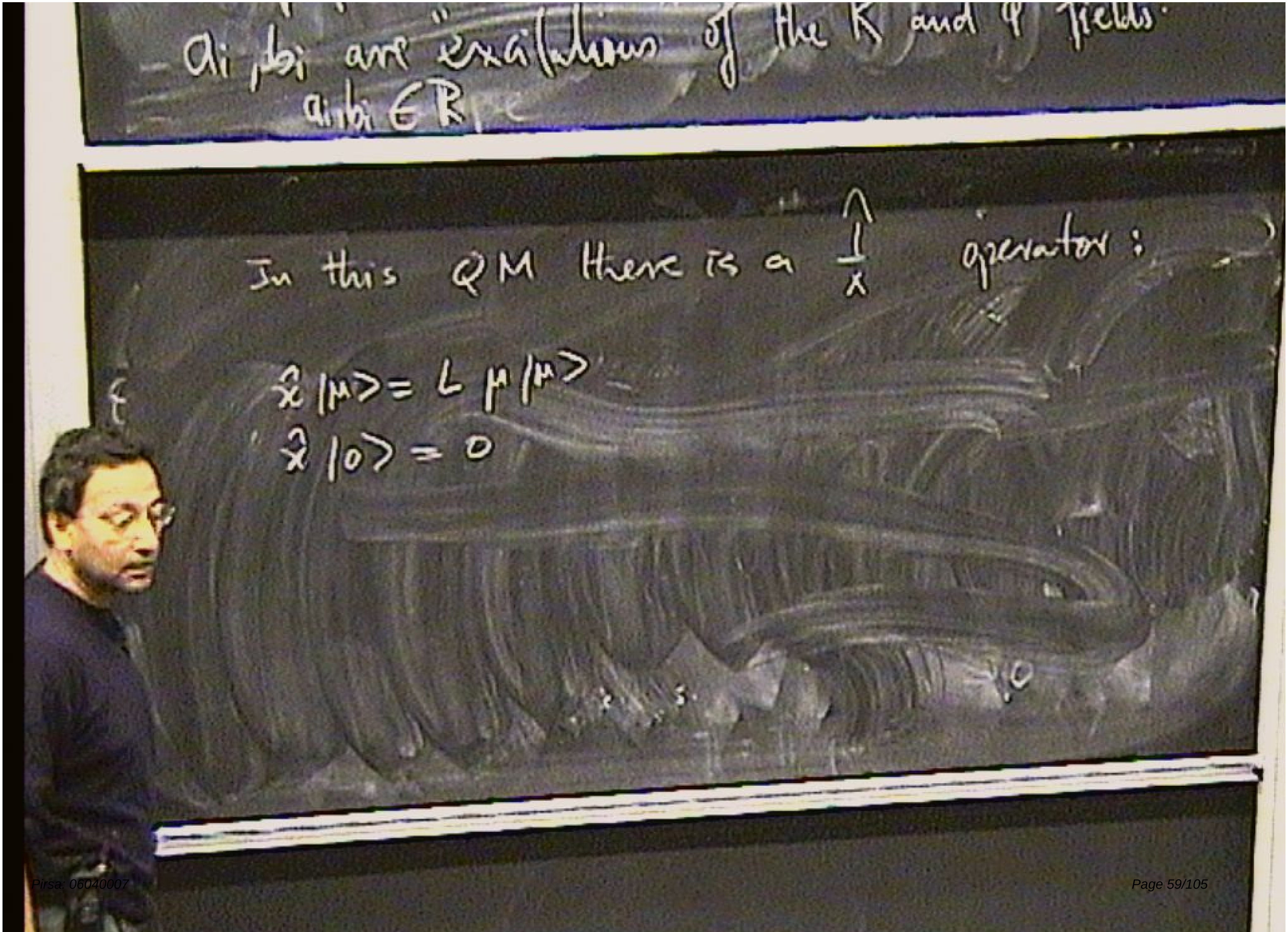
$\hat{x} |\mu\rangle = \mu L |\mu\rangle$

$\langle \mu | \mu' \rangle = \delta_{\mu\mu'}$

$\widehat{e^{i\lambda p}} |\mu\rangle = |\mu - \lambda\rangle$


ai, bi are "excitations" of the R and φ fields.
ai,bi ∈ R
In this QM there is a

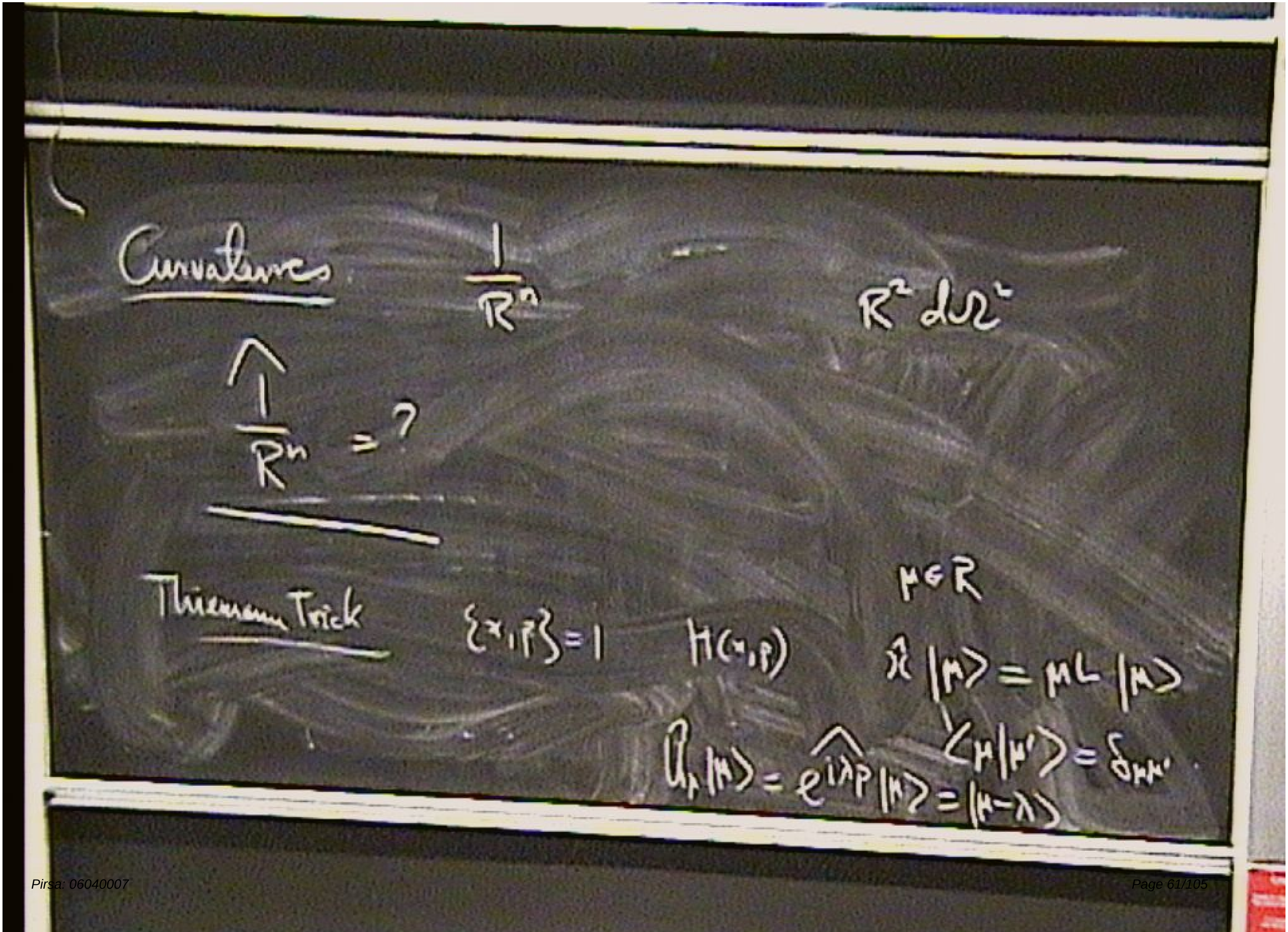

ai, bi are "excitations" of the K and φ fields.
ai, bi ∈ R
In this QM there is a 1/λ operator :


ai, bi are excitations of the K and φ fields.
ai, bi ∈ ℝ
In this QM there is a 1/x operator:
x̂ |μ> = L μ |μ>
x̂ |0> = 0

- a_i, b_i are "excitations" of the K and φ fields.
$a_i, b_i \in \mathbb{R}$

In this QM there is a $\widehat{\frac{1}{x}}$ operator:

$$\hat{x}\,|\mu\rangle = L\,\mu\,|\mu\rangle$$
$$\hat{x}\,|0\rangle = 0\,|0\rangle$$



Curvatures $\frac{1}{R^n}$
$R^2 d\Omega^2$
$\widehat{\frac{1}{R^n}} = ?$
Thiemann Trick
$\{x, p\} = 1$
$H(x,p)$
$\mu \in \mathbb{R}$
$\hat{x} |\mu\rangle = \mu L |\mu\rangle$
$\langle \mu | \mu' \rangle = \delta_{\mu\mu'}$
$U_\lambda |\mu\rangle = e^{i\lambda \hat{p}} |\mu\rangle = |\mu - \lambda\rangle$

- a_i, b_i are "excitations" of the K and φ fields.
 $a_i, b_i \in \mathbb{R}$

In this QM there is a $\widehat{\frac{1}{x}}$ operator:

$$\hat{x}\,|\mu\rangle = L\,\mu\,|\mu\rangle$$
$$\hat{x}\,|0\rangle = 0\,|0\rangle$$

$$\hat{p} := \lim_{\lambda\to 0} \frac{e^{i\lambda\hat{p}} - I}{\lambda}$$

a_i, b_i are "excitations" of the K and φ fields.
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a_i, b_i are "excitations" of the K and φ fields.
$a_i, b_i \in \mathbb{R}$

In this QM there is a $\widehat{\frac{1}{x}}$ operator:

$$\hat{x}|\mu\rangle = L\mu|\mu\rangle$$
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$$\hat{p} := \lim_{\lambda\to 0} \frac{e^{i\lambda\hat{p}} - I}{\lambda} \qquad \frac{\langle\mu| e^{i\lambda\hat{p}} - I |\mu\rangle}{\lambda}$$

$$= \frac{0-1}{\lambda} = -\frac{1}{\lambda}$$

- a_i, b_i are excitations of the K and φ fields.
$a_i, b_i \in \mathbb{R}$

In this QM there is a $\widehat{\frac{1}{x}}$ operator:

$\hat{x}|\mu\rangle = L\mu|\mu\rangle$

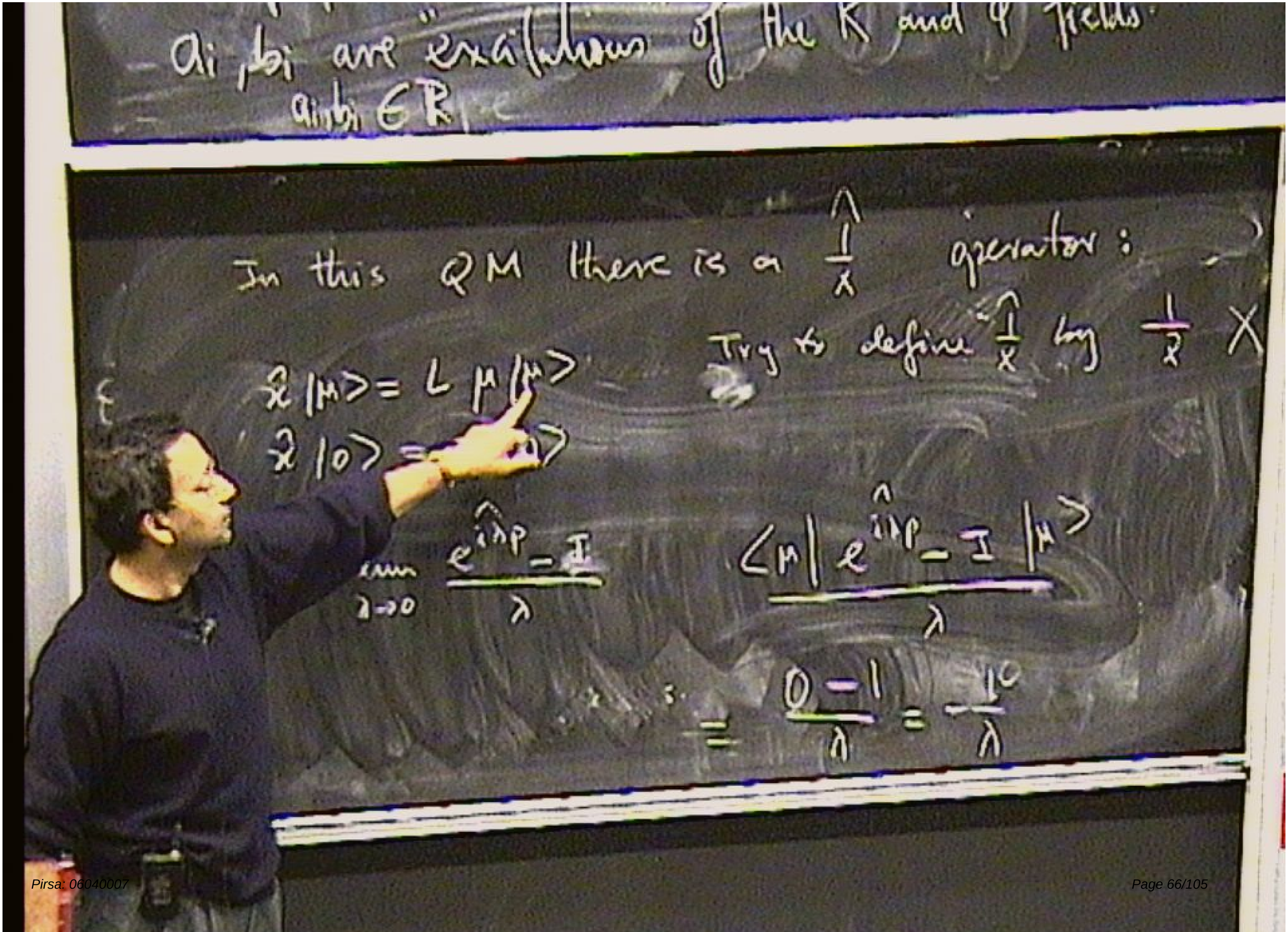
$\hat{x}|0\rangle = 0\,|0\rangle$

Try to define $\widehat{\frac{1}{x}}$ by $\frac{1}{\hat{x}}$

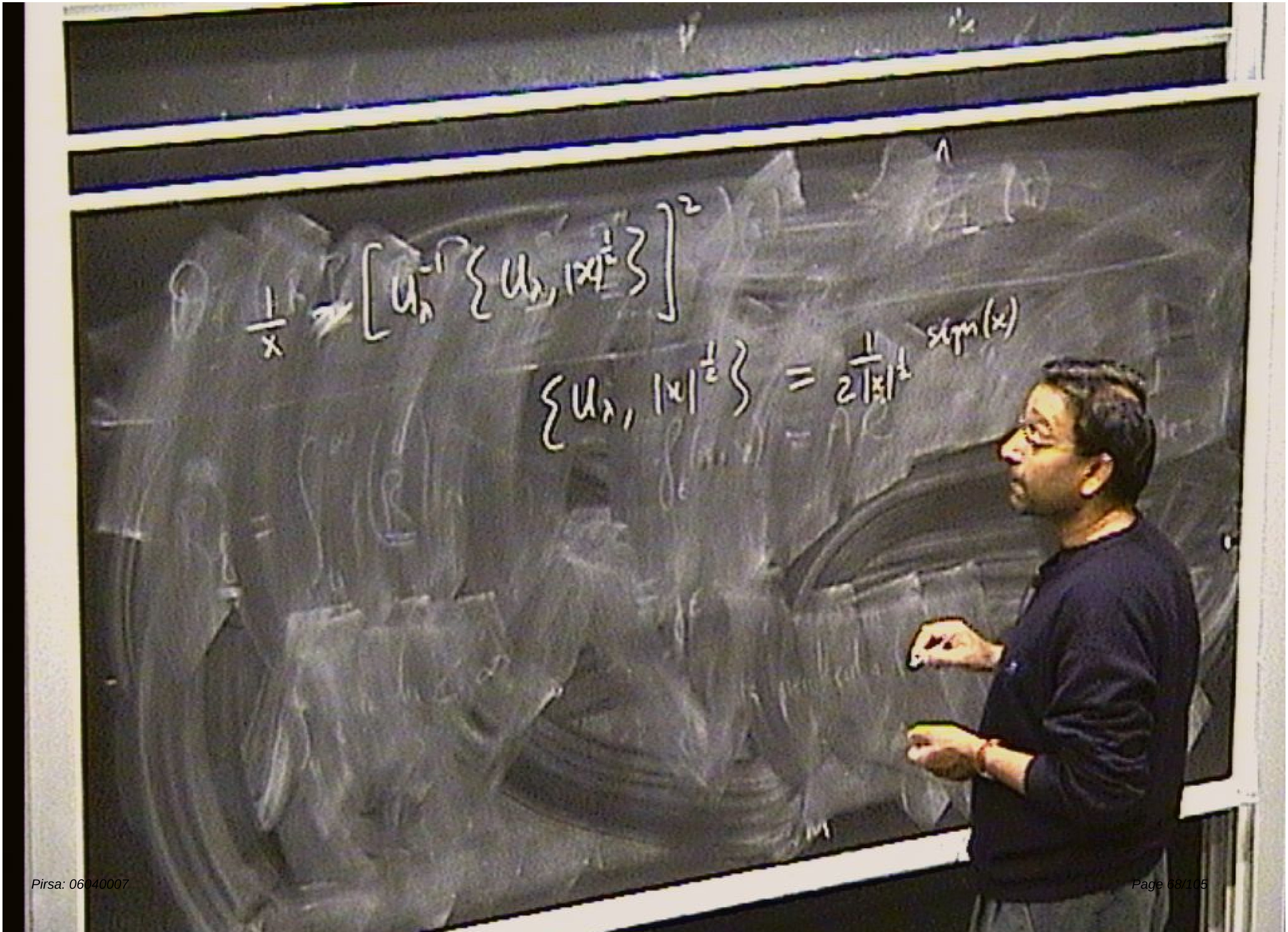
$$\hat{p} := \lim_{\lambda \to 0} \frac{e^{i\lambda \hat{p}} - I}{\lambda}$$

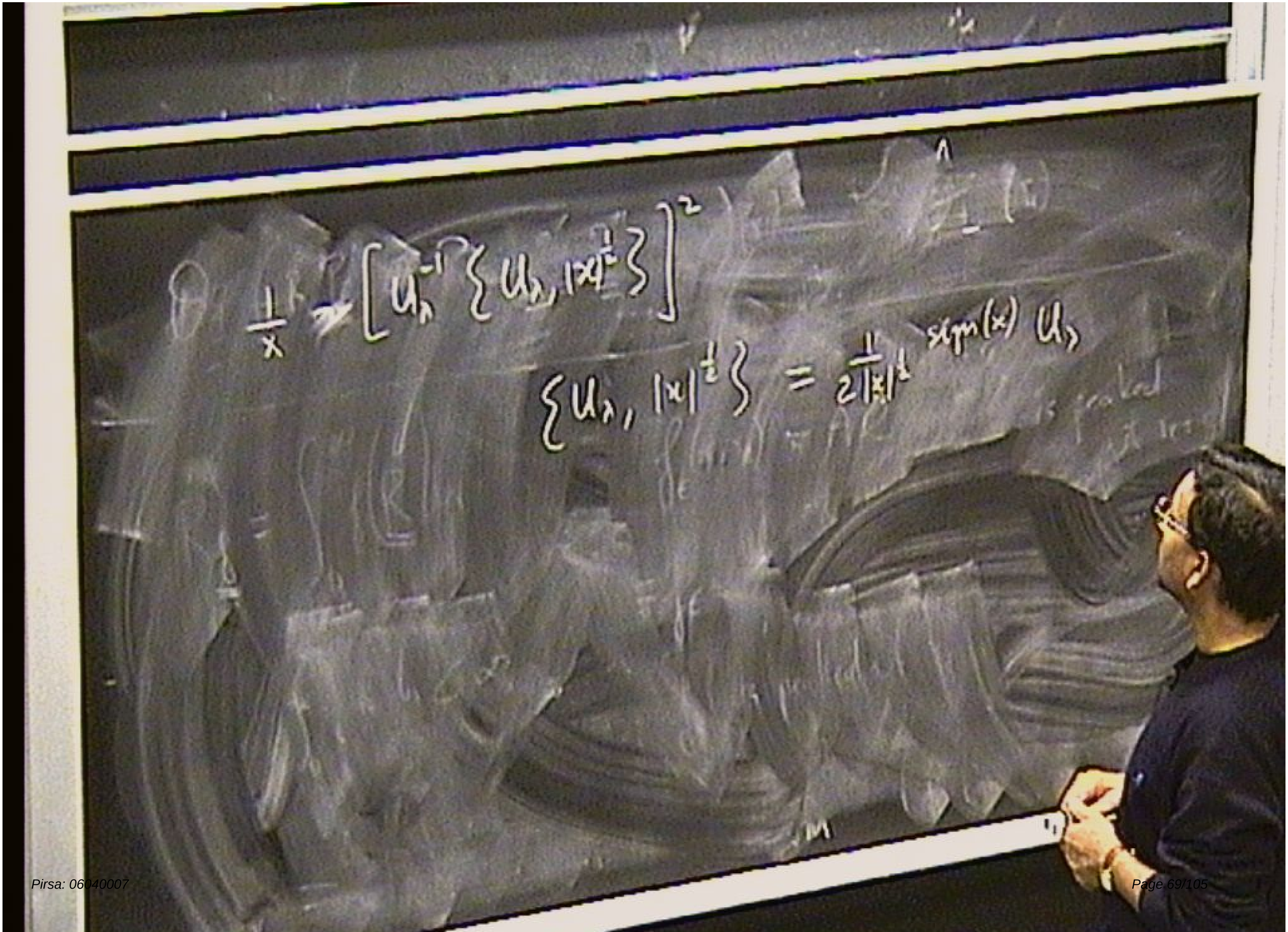
$$\frac{\langle\mu|\, e^{i\lambda\hat{p}} - I \,|\mu\rangle}{\lambda}$$

$$= \frac{0 - 1}{\lambda} = -\frac{1}{\lambda}$$


ai, bi are "excitations" of the K and φ fields.
ai, bi ∈ ℝ
In this QM there is a 1/x operator :
Try to define 1/x by 1/x X
x |μ> = L μ |μ>
x |0> =
lim λ→0 (e^{iλp} − I)/λ
<μ| e^{iλp} − I |μ> / λ
= (0 − 1)/λ = −1/λ

$$\frac{1}{x}\left[u_n^{-1}\{u_n, |x|^{\frac{1}{2}}\}\right]^2$$


$\frac{1}{x} \to \left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2$
$\{U_\lambda, |x|^{\frac{1}{2}}\} = \frac{1}{2|x|^{\frac{1}{2}}}\,\mathrm{sign}(x)$

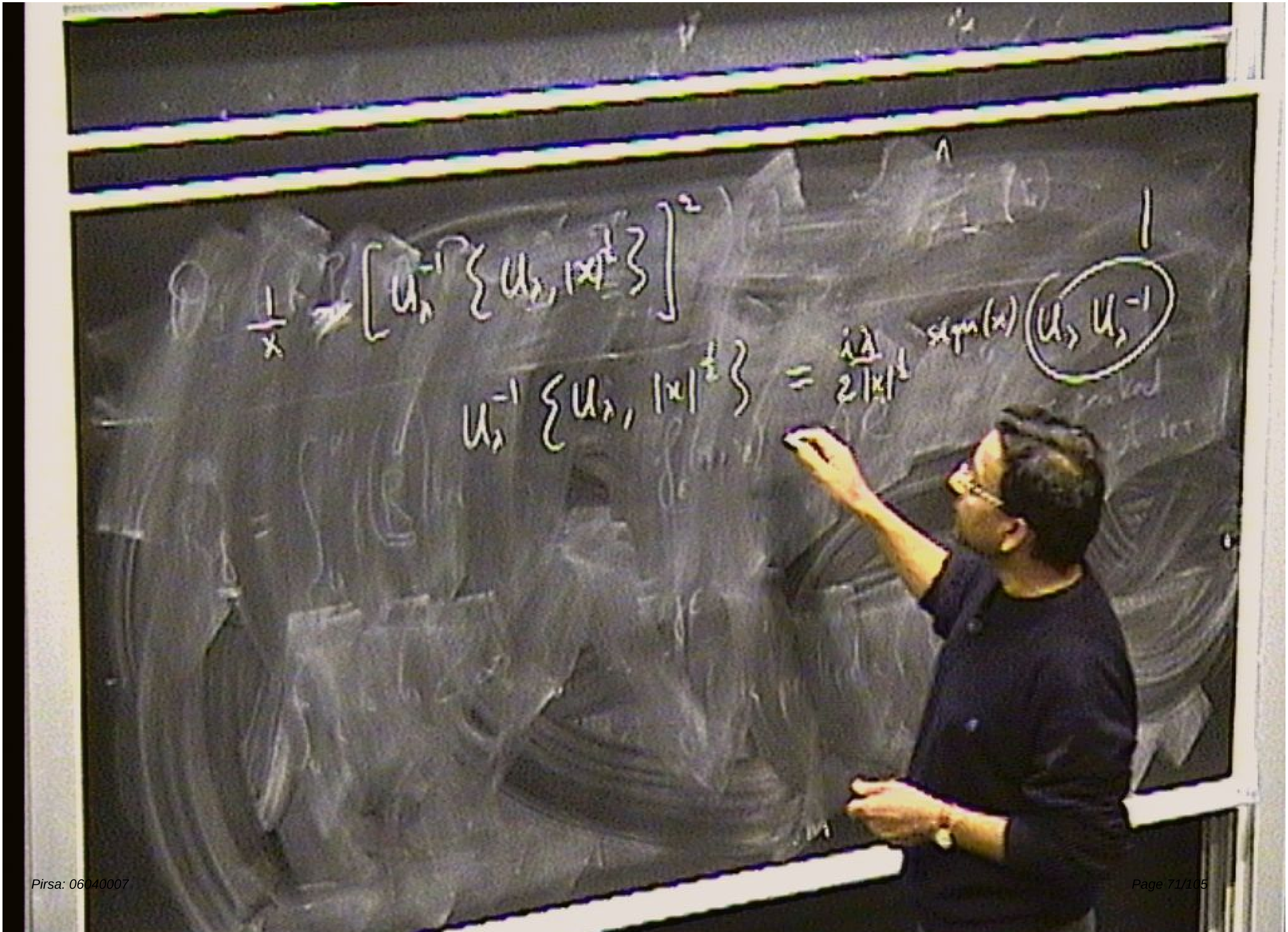


$\frac{1}{x} \to \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$
$\{ U_\lambda, |x|^{\frac{1}{2}} \} = \frac{1}{2|x|^{\frac{1}{2}}} \, \mathrm{sign}(x) \, U_\lambda$

$$\frac{1}{x} = \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$$

$$U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} = \frac{1}{2|x|^{\frac{1}{2}}} \operatorname{sign}(x) \, U_\lambda U_\lambda^{-1}$$

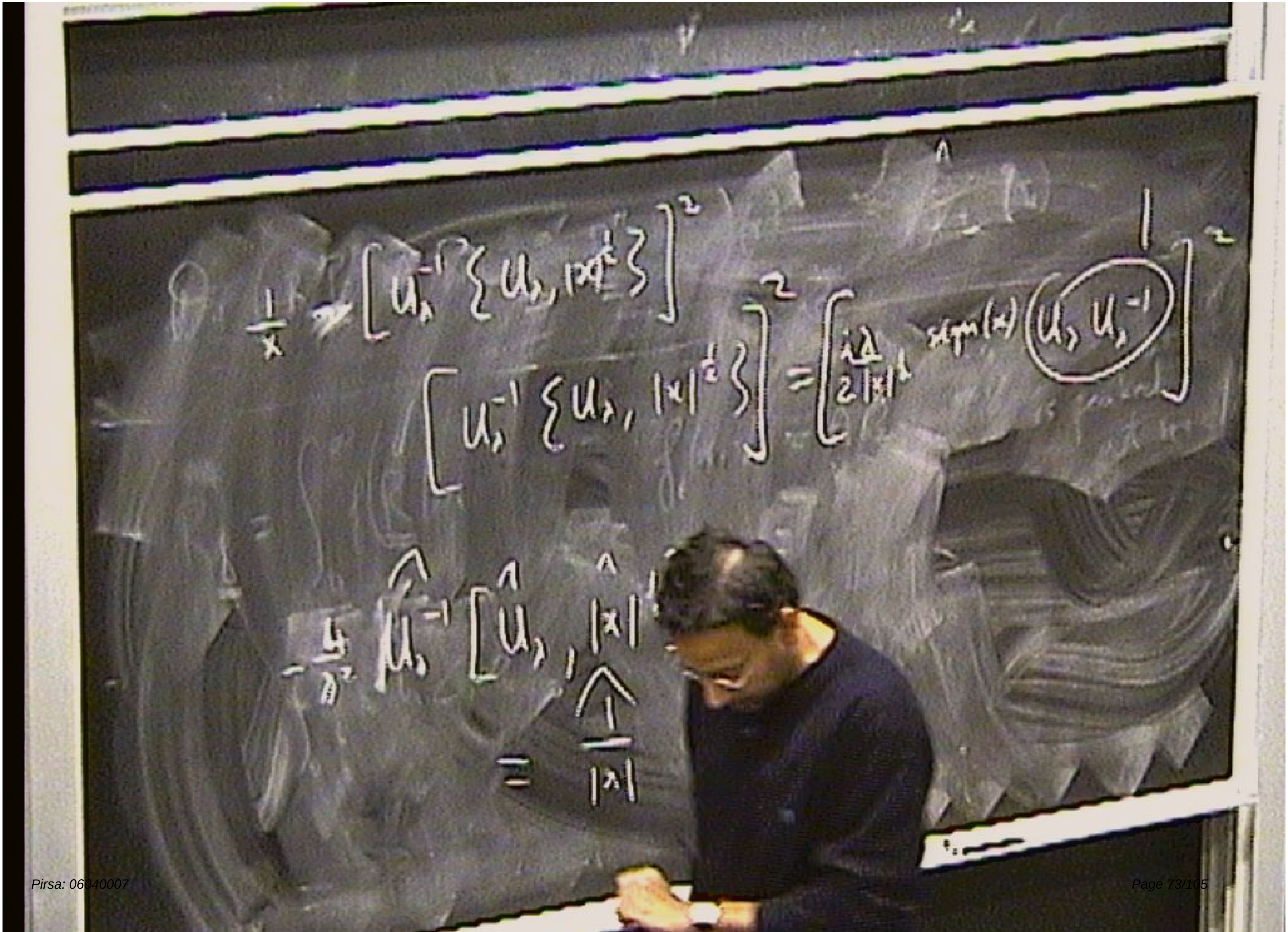
1


$\frac{1}{x}\left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac12}\}\right]^2$
$U_\lambda^{-1}\{U_\lambda, |x|^{\frac12}\} = \frac{i\lambda}{2|x|^{\frac12}}\,\mathrm{sign}(x)\,U_\lambda U_\lambda^{-1}$

$$\frac{1}{x} \quad \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$$

$$\left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}} \operatorname{sign}(x) \left(U_\lambda U_\lambda^{-1} \right) \right]^2$$

$$= \frac{-\lambda^2}{4|x|^2}$$

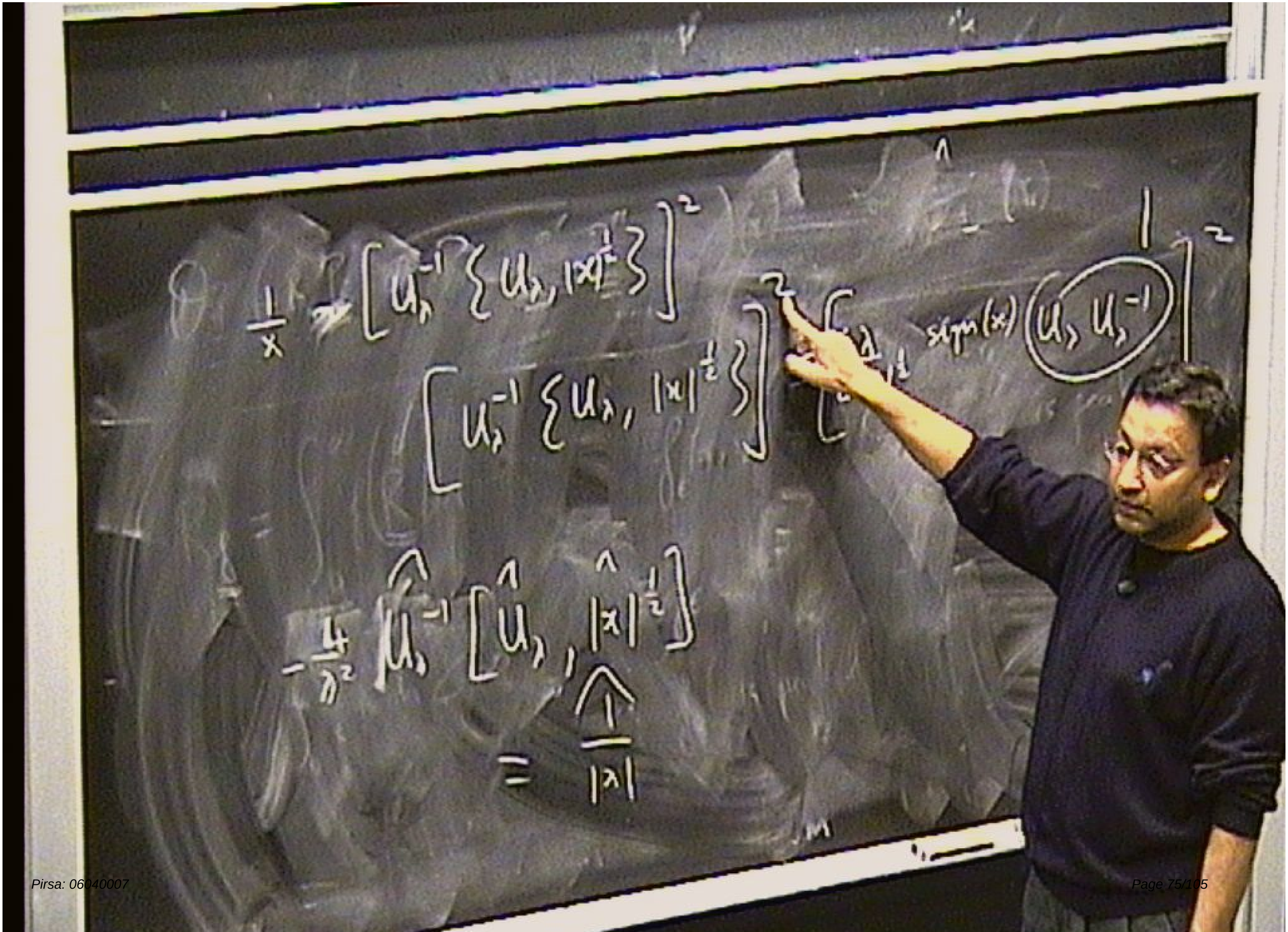


$$\frac{1}{x} = \left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2$$

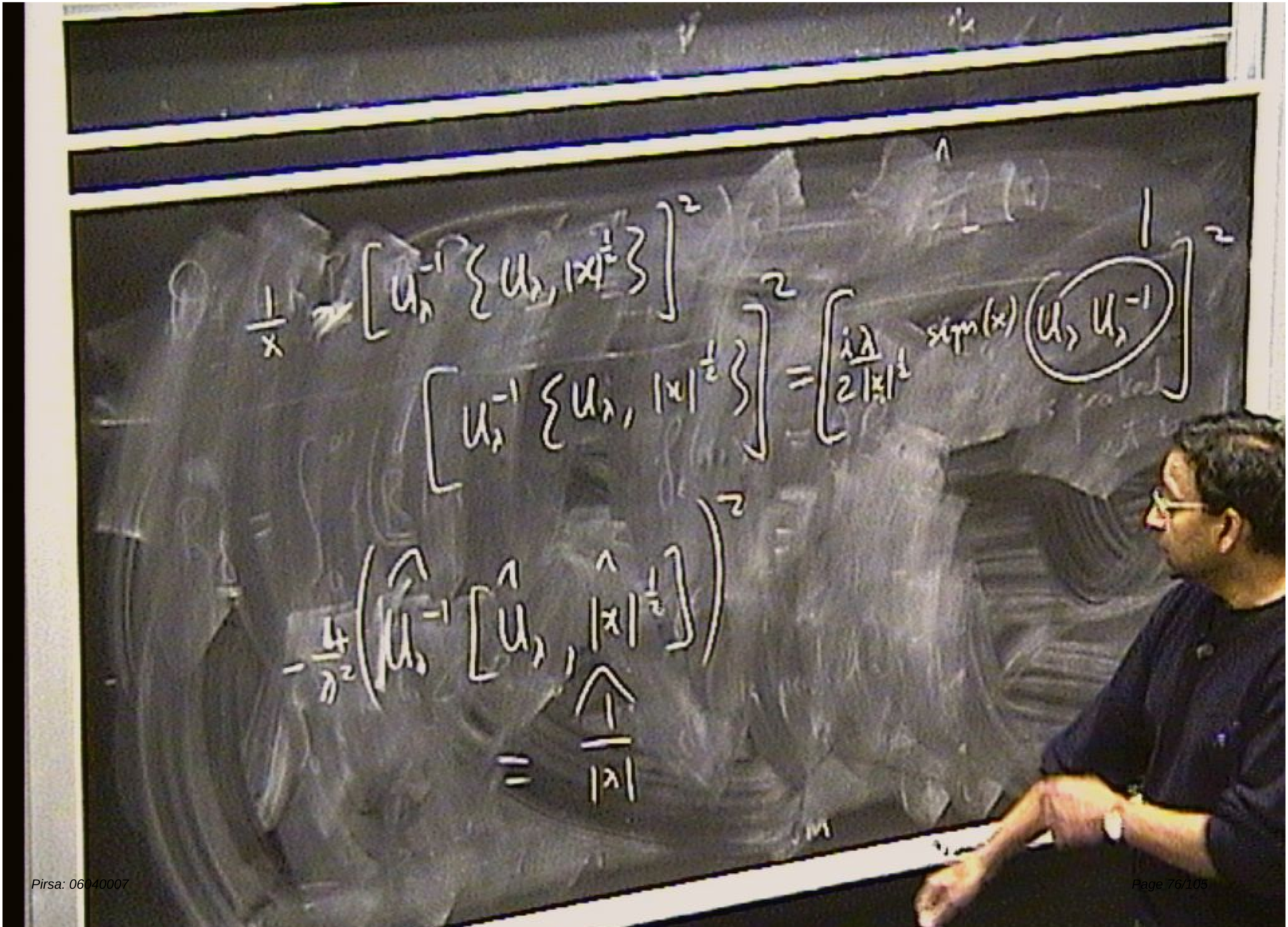
$$\left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}}\,\mathrm{sign}(x)\,\overset{1}{U_\lambda U_\lambda^{-1}}\right]^2$$

$$-\frac{4}{\lambda^2}\,\hat{U}_\lambda^{-1}\left[\hat{U}_\lambda, \widehat{|x|}^{\frac{1}{2}}\right]$$

$$= \widehat{\frac{1}{|x|}}$$



1/x [U_λ^{-1} {U_λ, |x|^{1/2}}]^2
[U_λ^{-1} {U_λ, |x|^{1/2}}]^2
sign(x) U_λ U_λ^{-1}
-4/ħ^2 U_λ^{-1} [U_λ, |x|^{1/2}]
= 1/|x|


$\frac{1}{x} = \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$
$\left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}} \operatorname{sign}(x) \, U_\lambda U_\lambda^{-1} \right]^2$
$-\frac{4}{\lambda^2} \left(\hat{U}_\lambda^{-1} \left[\hat{U}_\lambda, \widehat{|x|}^{\frac{1}{2}} \right] \right)^2 = \widehat{\frac{1}{|x|}}$

$$\frac{1}{x} \to \left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2$$

$$\left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}} \mathrm{sign}(x)\, U_\lambda U_\lambda^{-1}\right]^2$$

$$-\frac{4}{\lambda^2}\left(\hat{U}_\lambda^{-1}\left[\hat{U}_\lambda, \widehat{|x|}^{\frac{1}{2}}\right]\right)^2 = \widehat{\frac{1}{|x|}}$$

$$\widehat{\frac{1}{|x|}}\,|\mu\rangle = f(\mu,\lambda)$$

$$\frac{1}{x} \to \left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2$$

$$\left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}}\, \mathrm{sign}(x) \left(U_\lambda U_\lambda^{-1}\right)\right]^2$$

$$-\frac{4}{\lambda^2}\left(\widehat{U_\lambda^{-1}}\left[\hat{U}_\lambda, \widehat{|x|^{\frac{1}{2}}}\right]\right)^2 = \widehat{\frac{1}{|x|}}$$

$$\widehat{\frac{1}{|x|}}\,|\mu\rangle = f(\mu, \lambda)\,|\mu\rangle.$$

$$\frac{1}{x} \to \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$$

$$\left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}} \operatorname{sign}(x) \left(U_\lambda U_\lambda^{-1} \right) \right]^2$$

$$-\frac{4}{\hbar^2} \left(\hat{U}_\lambda^{-1} \left[\hat{U}_\lambda, \widehat{|x|}^{\frac{1}{2}} \right] \right)^2 = \widehat{\frac{1}{|x|}}$$

$$\widehat{\frac{1}{|x|}} |\mu\rangle = f(\mu, \lambda) |\mu\rangle .$$

$$f(\mu, \lambda) \xrightarrow{\mu \to \infty} \frac{1}{|\mu|}$$

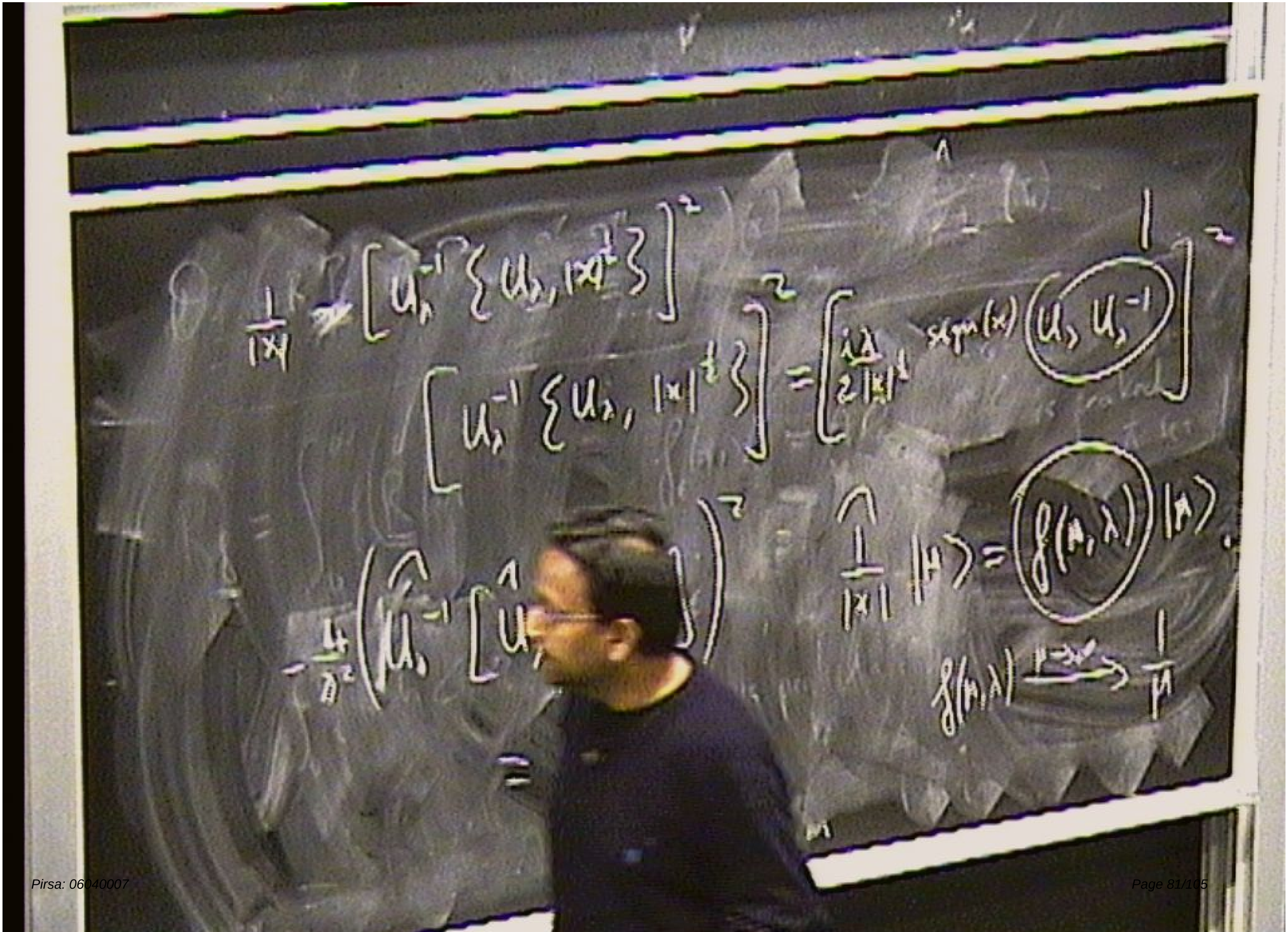
$$\frac{1}{x} \to \left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2$$

$$\left[U_\lambda^{-1}\{U_\lambda, |x|^{\frac{1}{2}}\}\right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}}\,\mathrm{sgn}(x)\,(U_\lambda U_\lambda^{-1})\right]^2$$

$$-\frac{4}{\lambda^2}\left(\hat{U}_\lambda^{-1}\left[\hat{U}_\lambda, \widehat{|x|^{\frac{1}{2}}}\right]\right)^2 = \widehat{\frac{1}{|x|}}$$

$$\widehat{\frac{1}{|x|}}\,|\mu\rangle = f(\mu,\lambda)\,|\mu\rangle$$

$$f(\mu,\lambda) \xrightarrow{\mu\to\infty} \frac{1}{\mu}$$



$\frac{1}{|x|} = [U_\lambda^{-1} \{U_\lambda, |x|^{1/2}\}]^2$
$[U_\lambda^{-1} \{U_\lambda, |x|^{1/2}\}] = [\frac{i\lambda}{2|x|^{1/2}} \mathrm{sgn}(x) (U_\lambda U_\lambda^{-1})]$
$\widehat{\frac{1}{|x|}} |\mu\rangle = f(\mu, \lambda) |\mu\rangle$
$f(\mu, \lambda) \xrightarrow{\mu \gg \lambda} \frac{1}{\mu}$

$$\frac{1}{|x|} \approx \left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2$$

$$\left[U_\lambda^{-1} \{ U_\lambda, |x|^{\frac{1}{2}} \} \right]^2 = \left[\frac{i\lambda}{2|x|^{\frac{1}{2}}} \, x\,\mathrm{sgn}(x) \left(U_\lambda U_\lambda^{-1} \right) \right]^2$$

$$-\frac{4}{\lambda^2} \left(\hat{U}_\lambda^{-1} \left[\hat{U}_\lambda , \widehat{|x|}^{\frac{1}{2}} \right] \right)^2 = \widehat{\frac{1}{|x|}}$$

$$\widehat{\frac{1}{|x|}} |\mu\rangle = f(\mu, \lambda) |\mu\rangle .$$

$$f(\mu,\lambda) \xrightarrow{\mu \to \infty} \frac{1}{\mu}$$

Curvatures: $\frac{1}{R^n}$

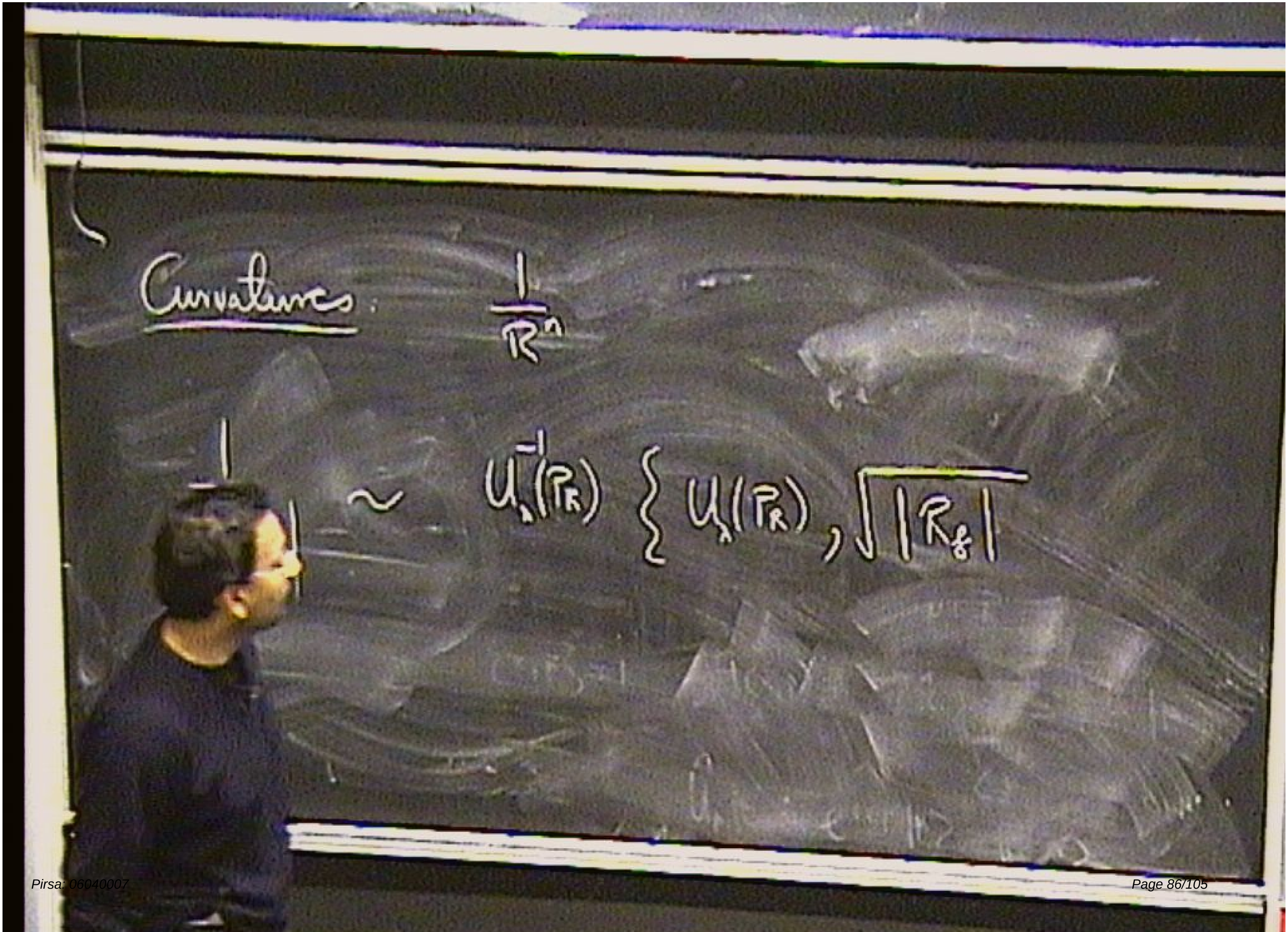
$\frac{1}{R}$

Curvatures. $\frac{1}{R^n}$

$\frac{1}{|R_f|}$

Curvatures : $\frac{1}{R^n}$

$$\frac{1}{|R_f|} \sim U_\lambda^{-1}(P_R) \left\{ U_\lambda(P_R) \right.$$


Curvatures:
1/R^n
U_λ^{-1}(P_R) { U_λ(P_R), √|R_g| }

Curvatures: $\frac{1}{R^n}$

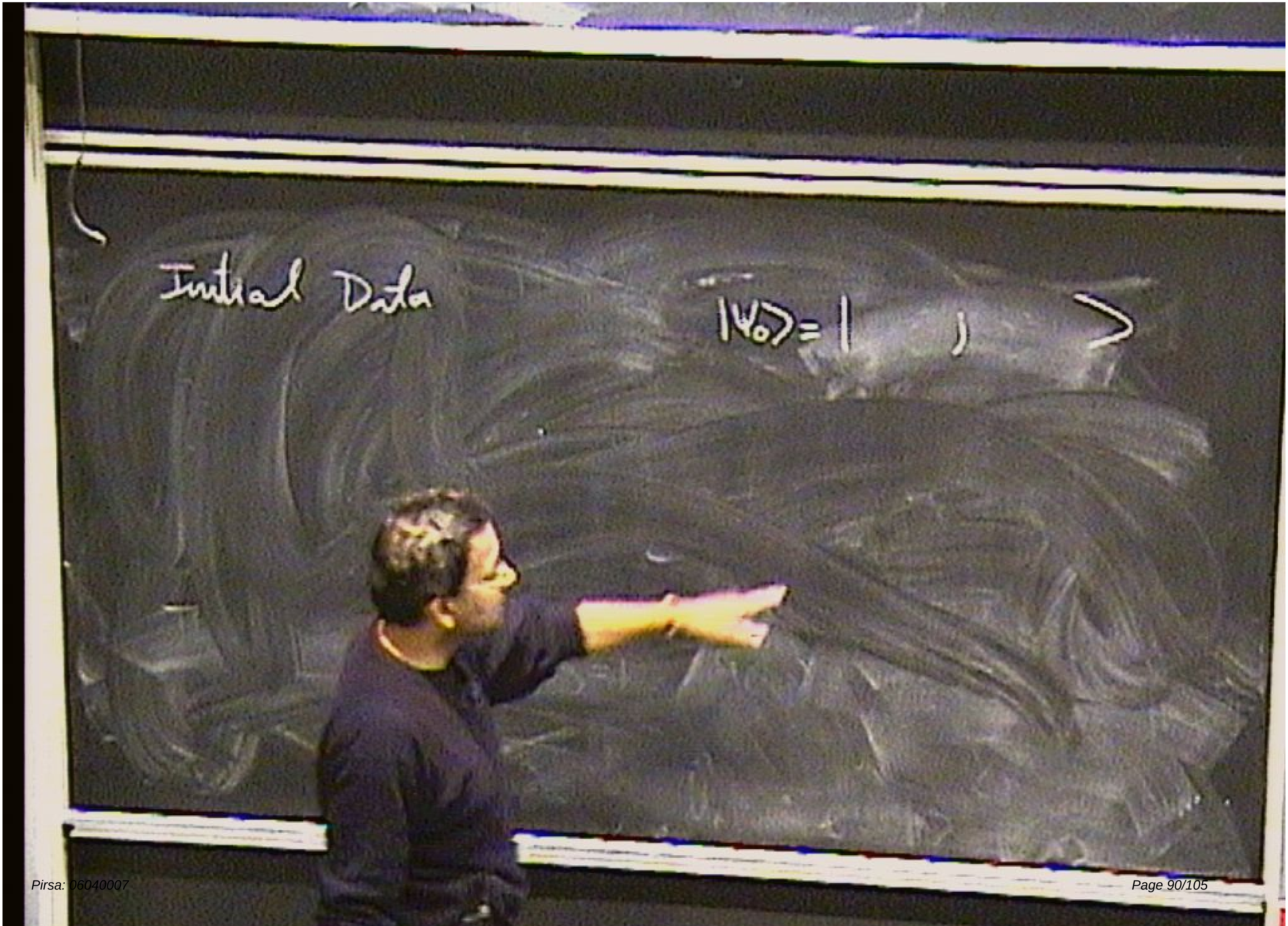
$$\frac{1}{|R_f|} \sim U_\lambda^{-1}(P_R)\,\left\{ U_\lambda(P_R), \sqrt{|R_g|} \right\}$$

Curvatures: $\frac{1}{R^{\Lambda}}$

$$\frac{1}{|R_f|} \sim \left(U_\lambda^{-1}(P_R) \left\{ U_\lambda(P_R), \sqrt{|R_f|} \right\} \right)^2$$

Curvatures: $\frac{1}{R^n}$

$$\frac{1}{|R_f|} \sim \left(U_\lambda^{-1}(p_R) \left\{ U_\lambda(p_R), \sqrt{|R_g|} \right\} \right)^2$$

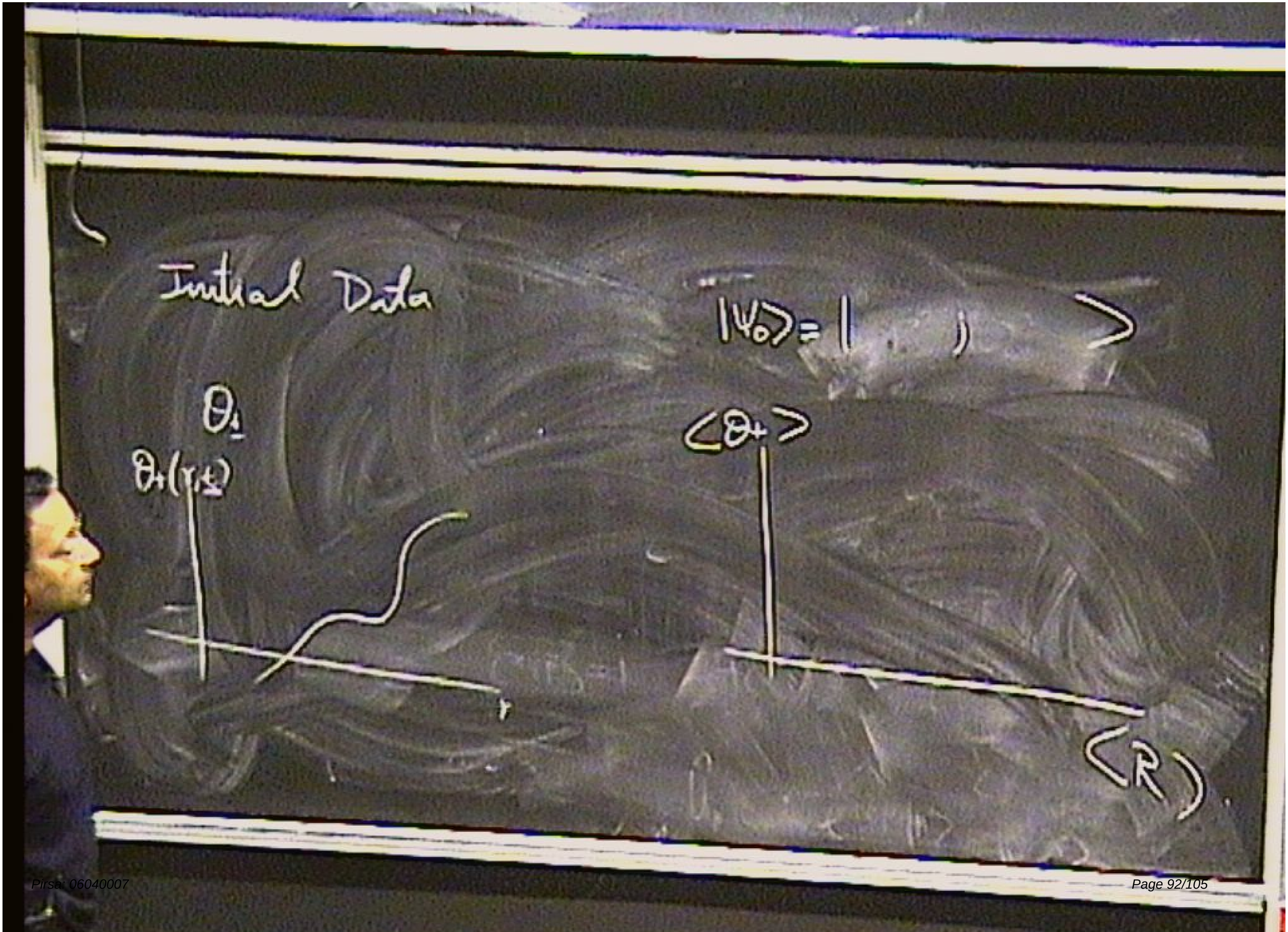

Initial Data
|Ψ₀⟩ = | , ⟩

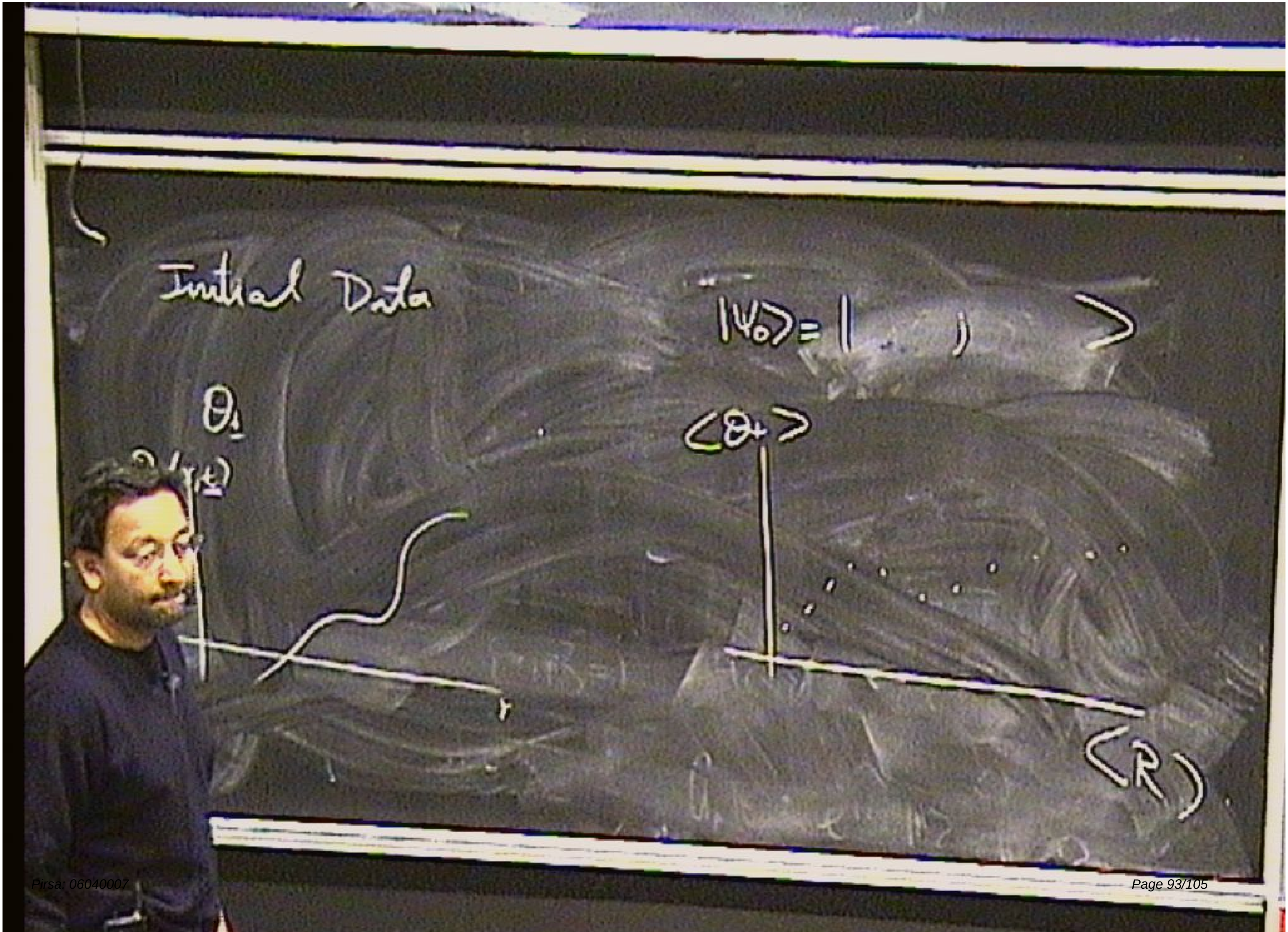
Initial Data

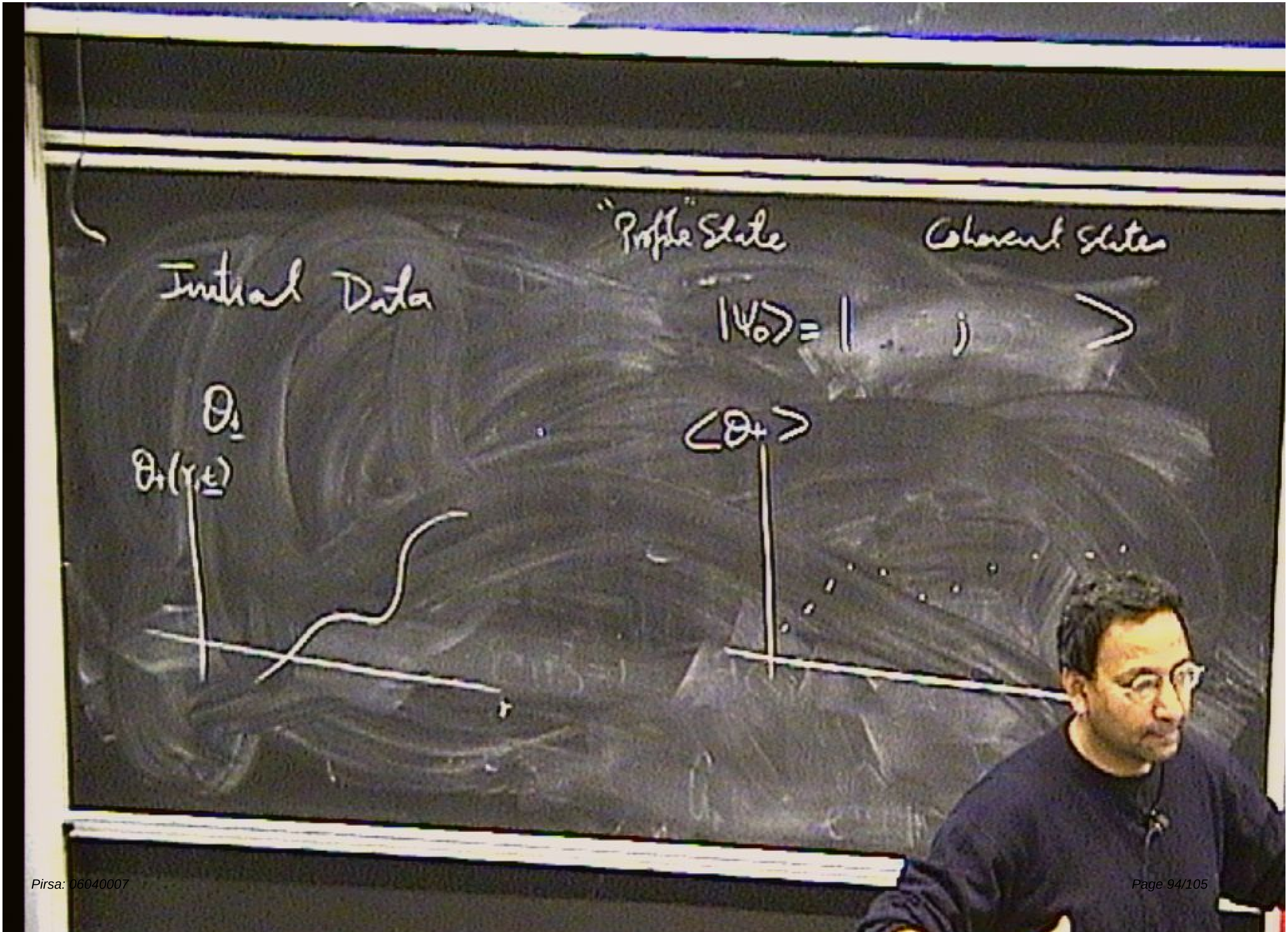
$|\Psi_0\rangle = | \; \ldots \; ; \qquad \rangle$

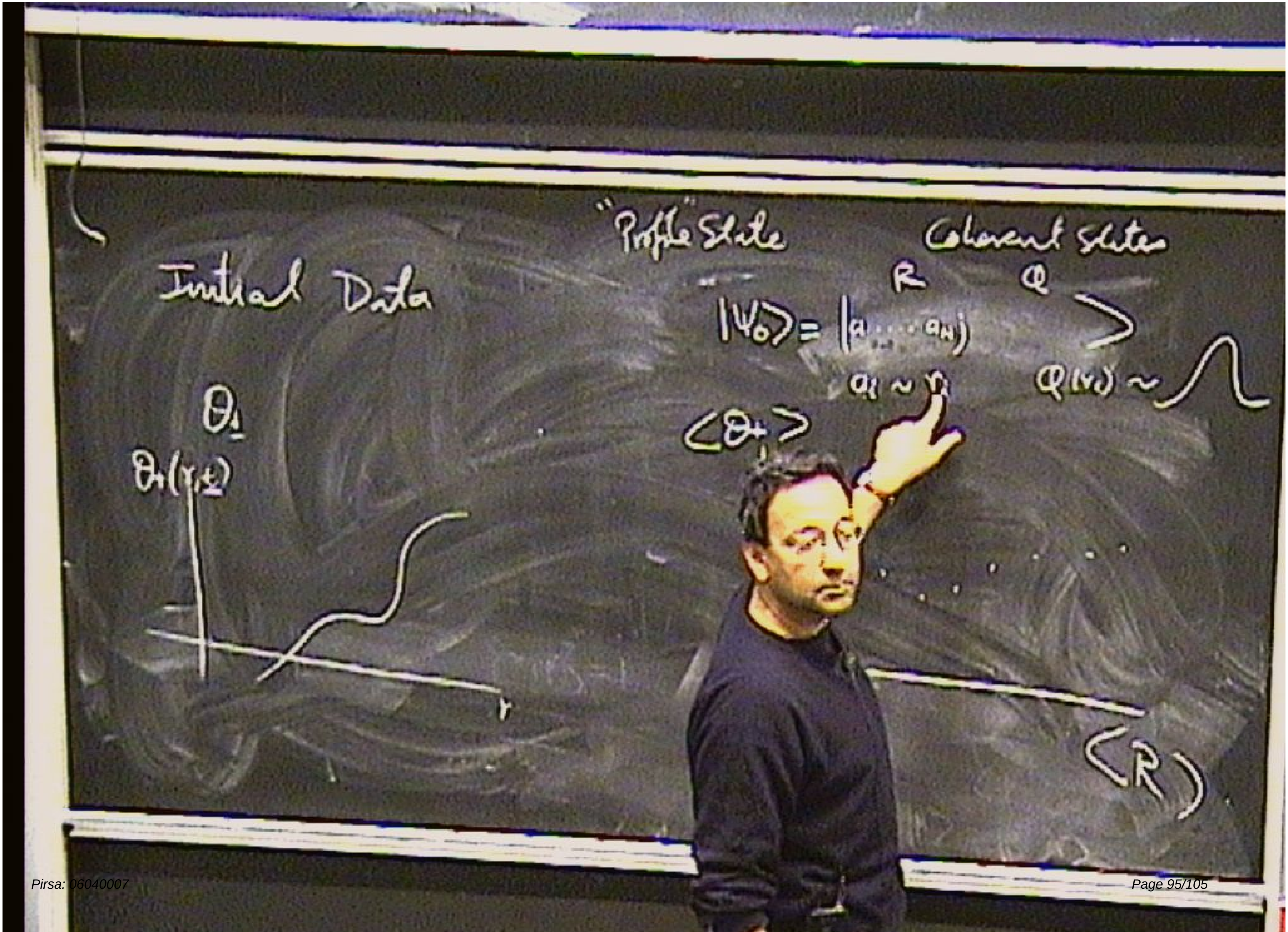
θ_i

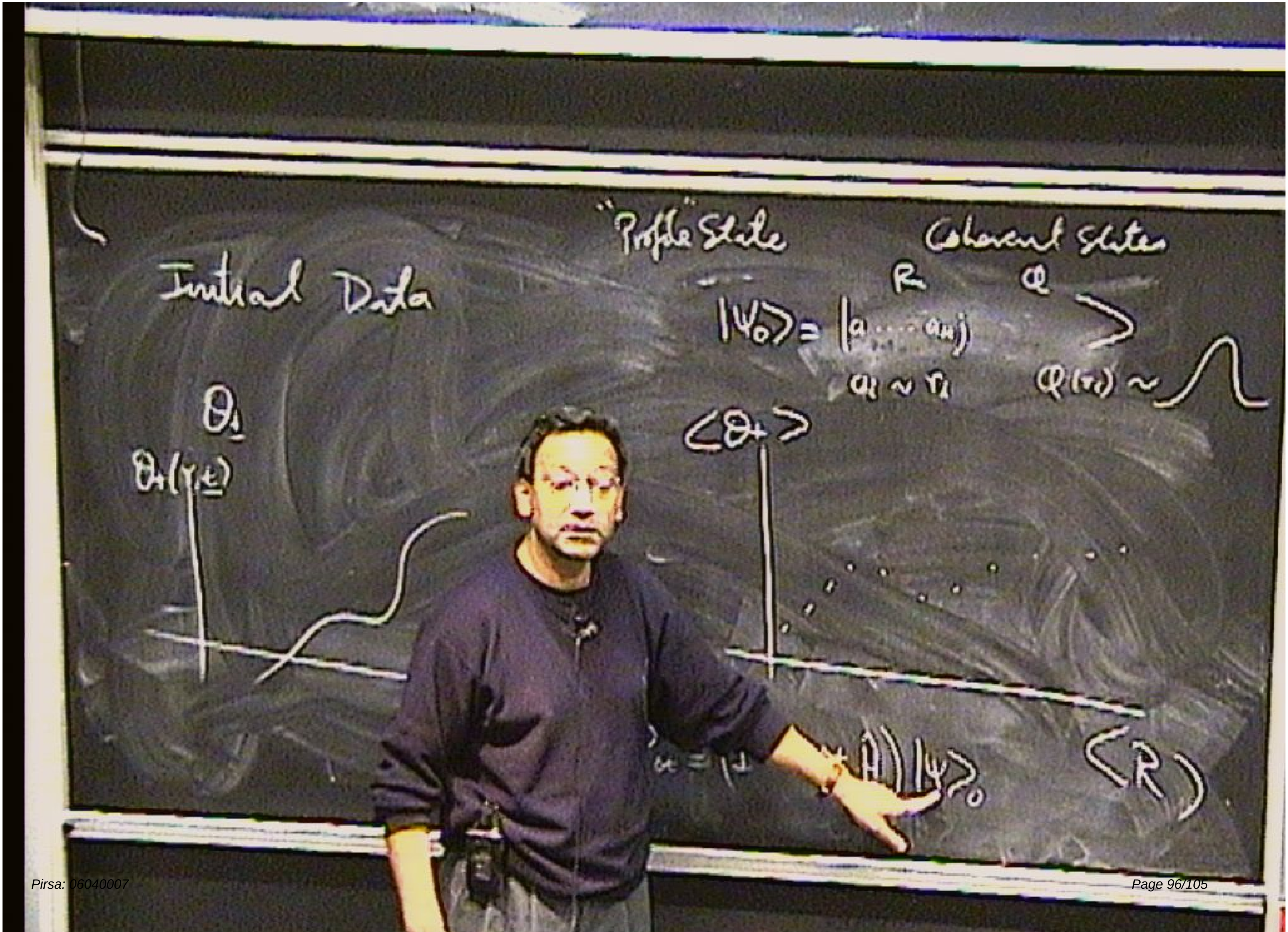
$\theta_i(r, t)$

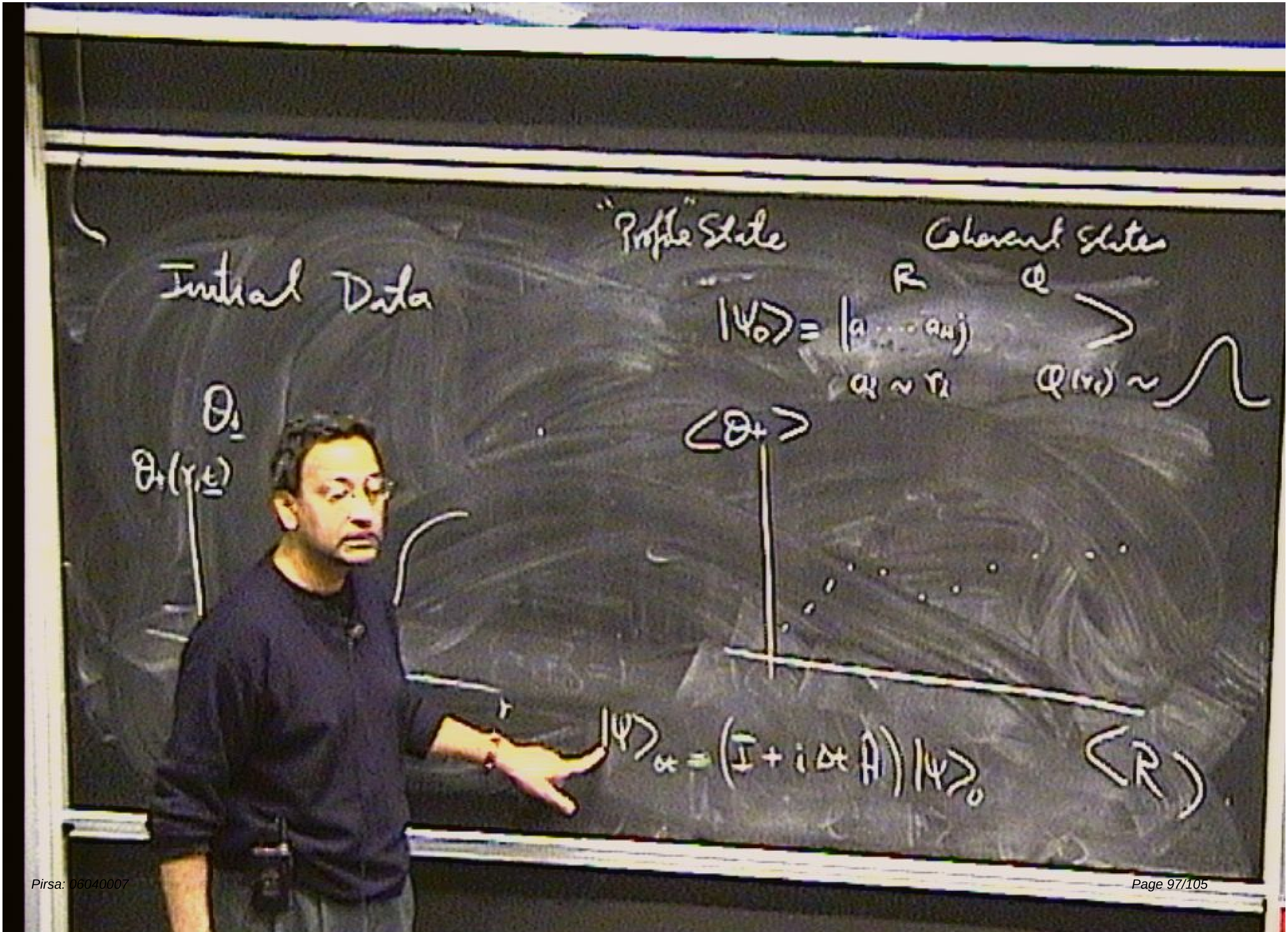

Initial Data
|ψ₀⟩ = |) ⟩
θ+
θ+(r,t)
⟨θ+⟩
r
⟨R⟩

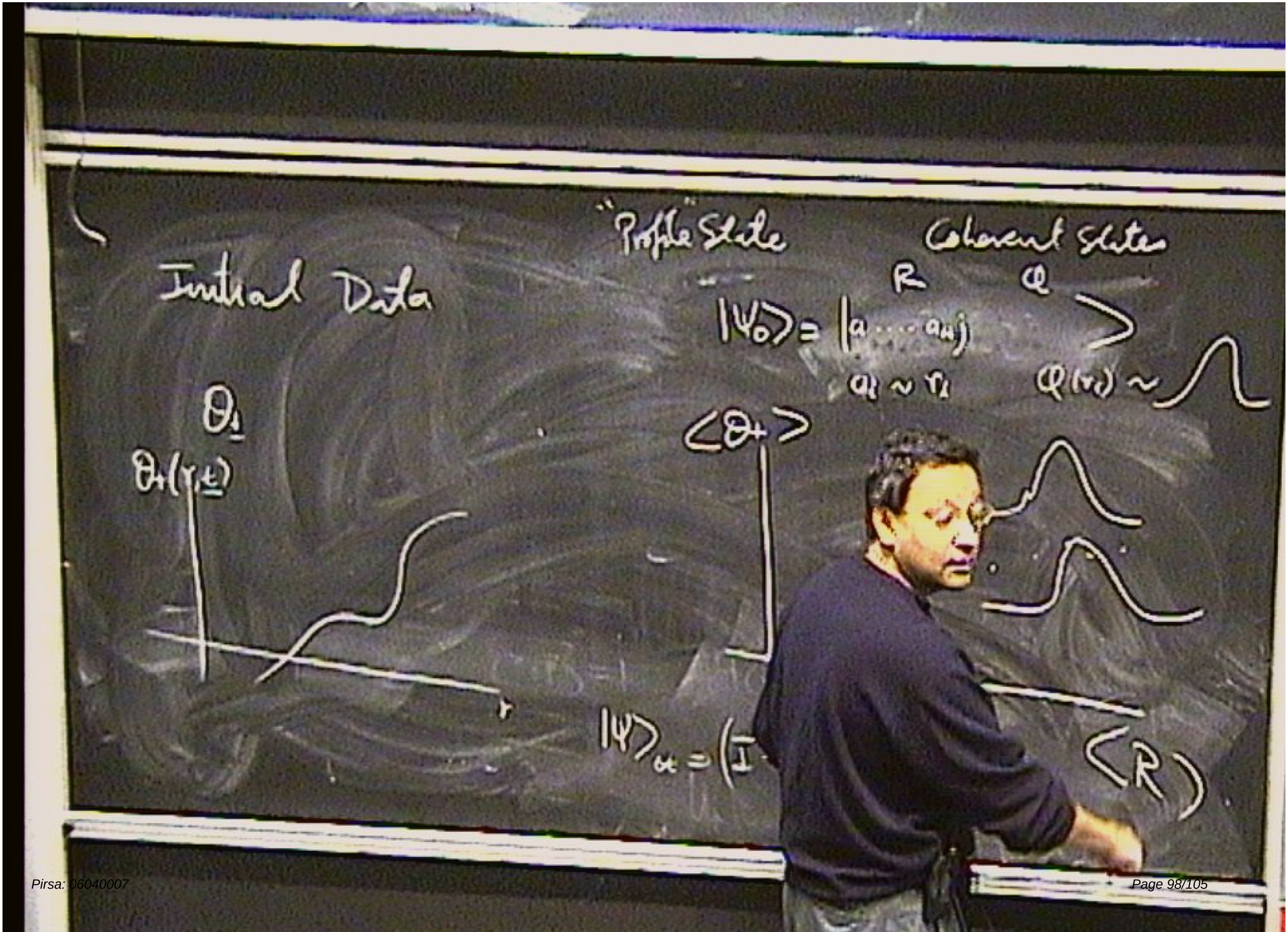

Initial Data

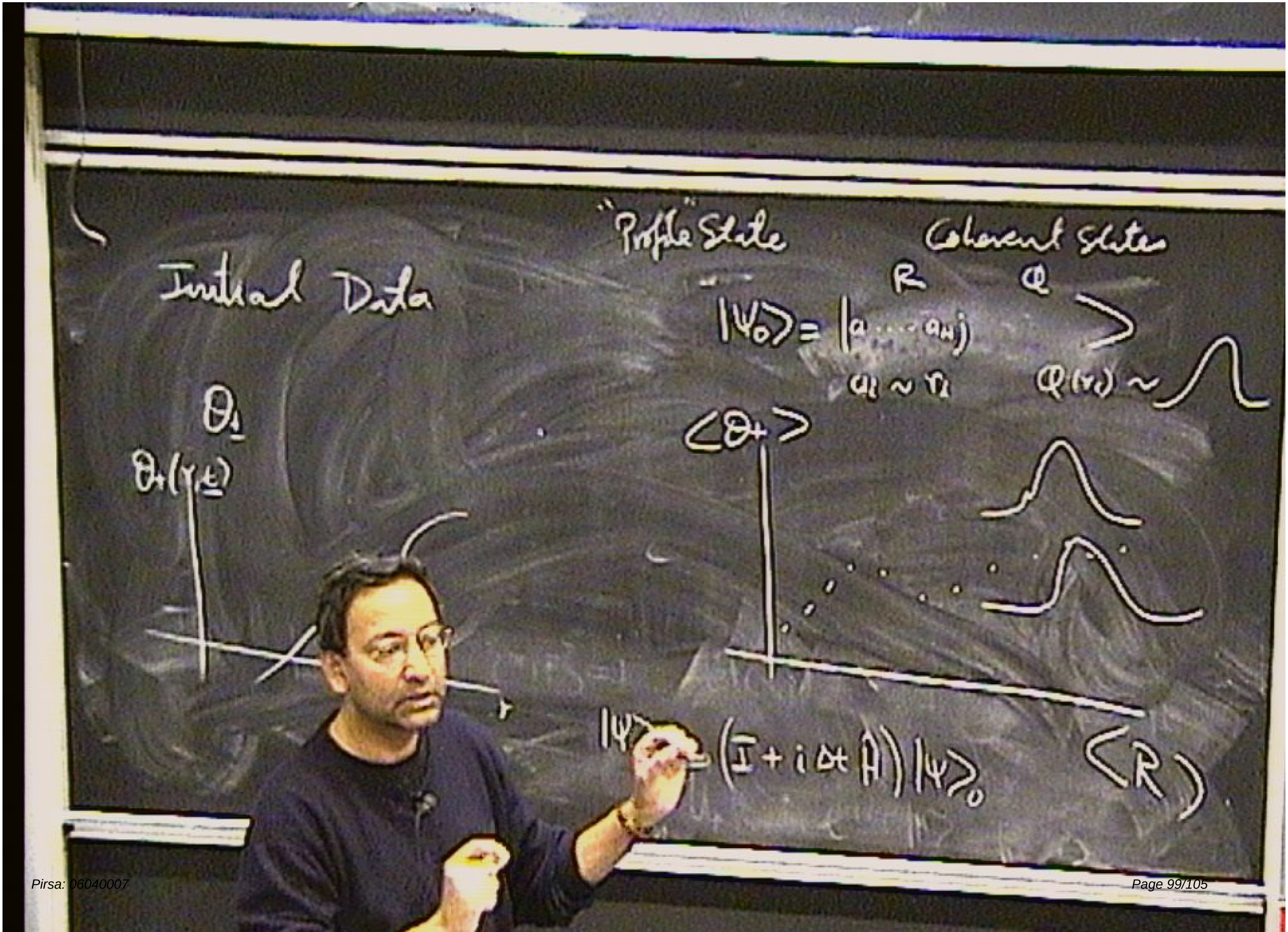

"Profile" State
Coherent States
Initial Data
|Ψ₀⟩ = | j ⟩
⟨θ+⟩
θ+(r,t)

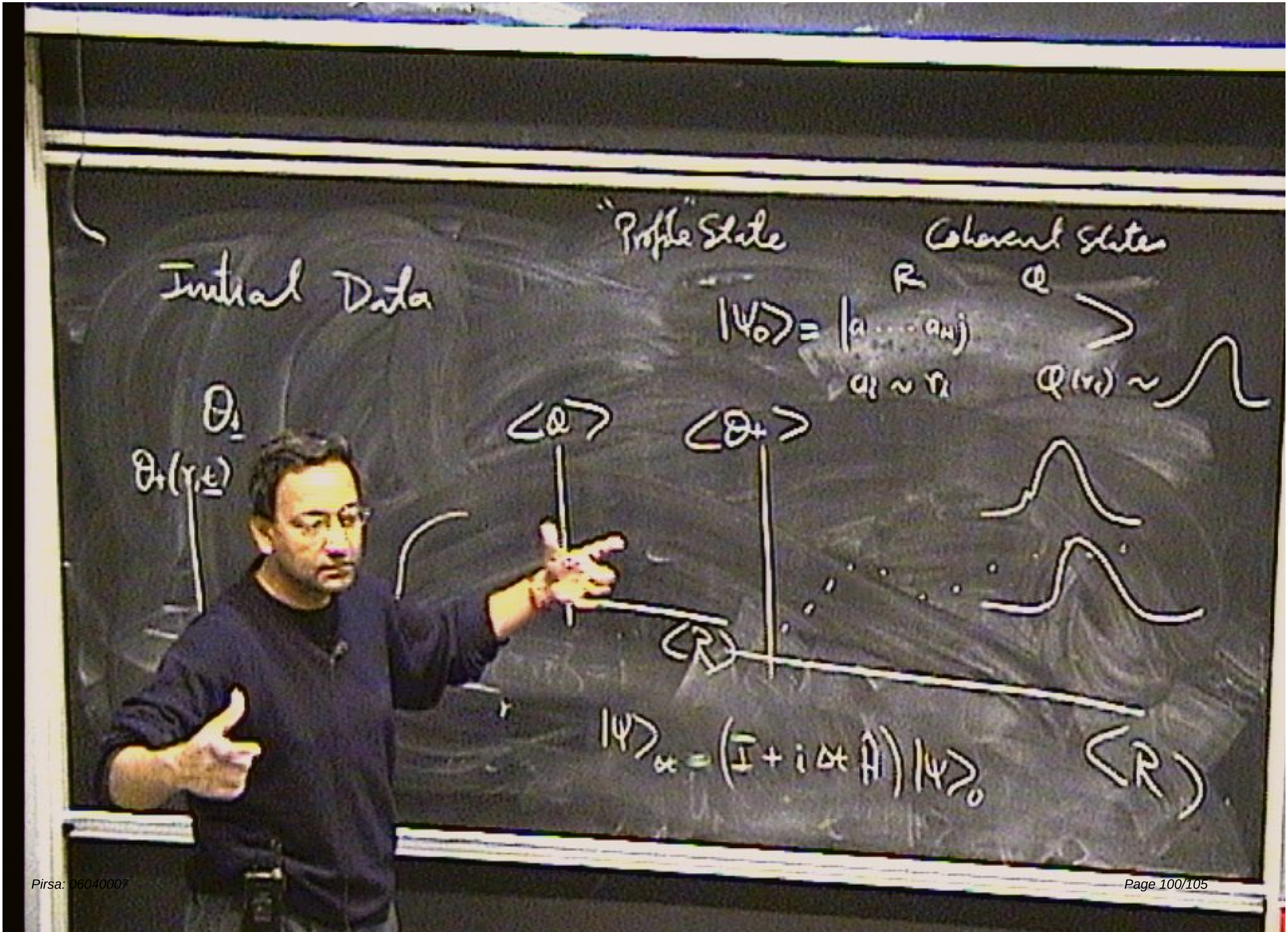

"Profile" State
Coherent states
Initial Data
⟨R⟩

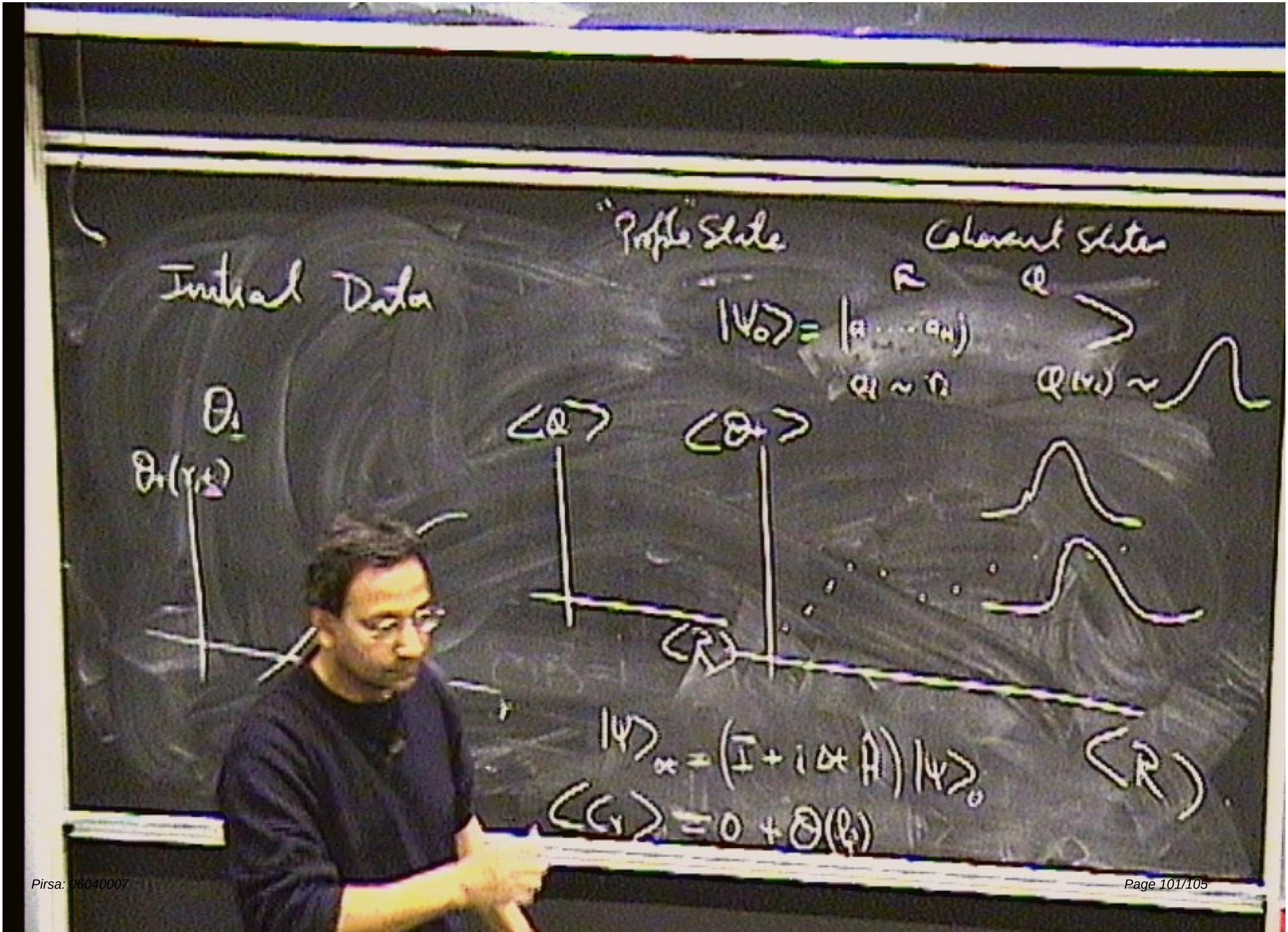

Initial Data
"Profile State"
Coherent States


"Profile" State
Coherent States
Initial Data
R
Q
⟨R⟩


"Profile" State
Coherent States
Initial Data


"Profile" State
Coherent States
Initial Data
R
Q
|ψ0> = |a ... aN)
ai ~ ri
Q(ri) ~
θt
<θt>
θt(r,t)
|ψ>t = (I + i Δt Ĥ) |ψ>0
<R>


"Profile State"
Coherent states
Initial Data
R
Q
|Ψ0> = |a ... aN)
ai ~ ri
Q(ri) ~
<Q>
<θ+>
θ+(r,t)
<R>
|Ψ>δt = (I + iδt Ĥ)|Ψ>0


"Profile" State
Coherent States
Initial Data
|ψ₀> = |a₁ ... a_N)
⟨R⟩
|ψ⟩_α = (I + iα A) |ψ⟩₀
⟨C_r⟩ = 0 + O(ℓ)

"Profile" State

Coherent states

Initial Data

$$|\psi_0\rangle = \Big|\overset{R}{a_1 \dots a_N)}\ \overset{Q}{\ }\Big\rangle$$

$a_i \sim r_i$

$Q(r_i) \sim$

θ_t

$\theta_t(r,t)$

$\langle Q\rangle$

$\langle \theta_t\rangle$

$\langle R)$

r

$\langle R)$

$$|\psi\rangle_{\Delta t} = \left(I + i\,\Delta t\,\hat{H}\right)|\psi\rangle_0$$

$$\langle C_r\rangle = 0 + \mathcal{O}(\hbar)$$

a_i, b_i are "excitations" of the R and Φ fields.

$a_i, b_i \in \mathbb{R}$

In this QM there is a $\widehat{\frac{1}{x}}$ operator:

Try to define $\widehat{\frac{1}{x}}$ by $\frac{1}{\hat{x}}$ X

$$\hat{x}\,|\mu\rangle = L\,\mu\,|\mu\rangle$$

$$\hat{x}\,|0\rangle = 0\,|0\rangle$$

$$\hat{p} := \lim_{\lambda \to 0} \frac{e^{i\lambda \hat{p}} - I}{\lambda} \qquad \frac{\langle \mu | e^{i\lambda \hat{p}} - I | \mu \rangle}{\lambda}$$

$$C_r = P_\Phi \Phi' + P_R R' - P_\Lambda'(\ \ )^k \qquad = \frac{0-1}{\lambda} = -\frac{1}{\lambda}$$

a_i, b_i are "excitations" of the R and Φ fields
$a_i, b_i \in \mathbb{R}$

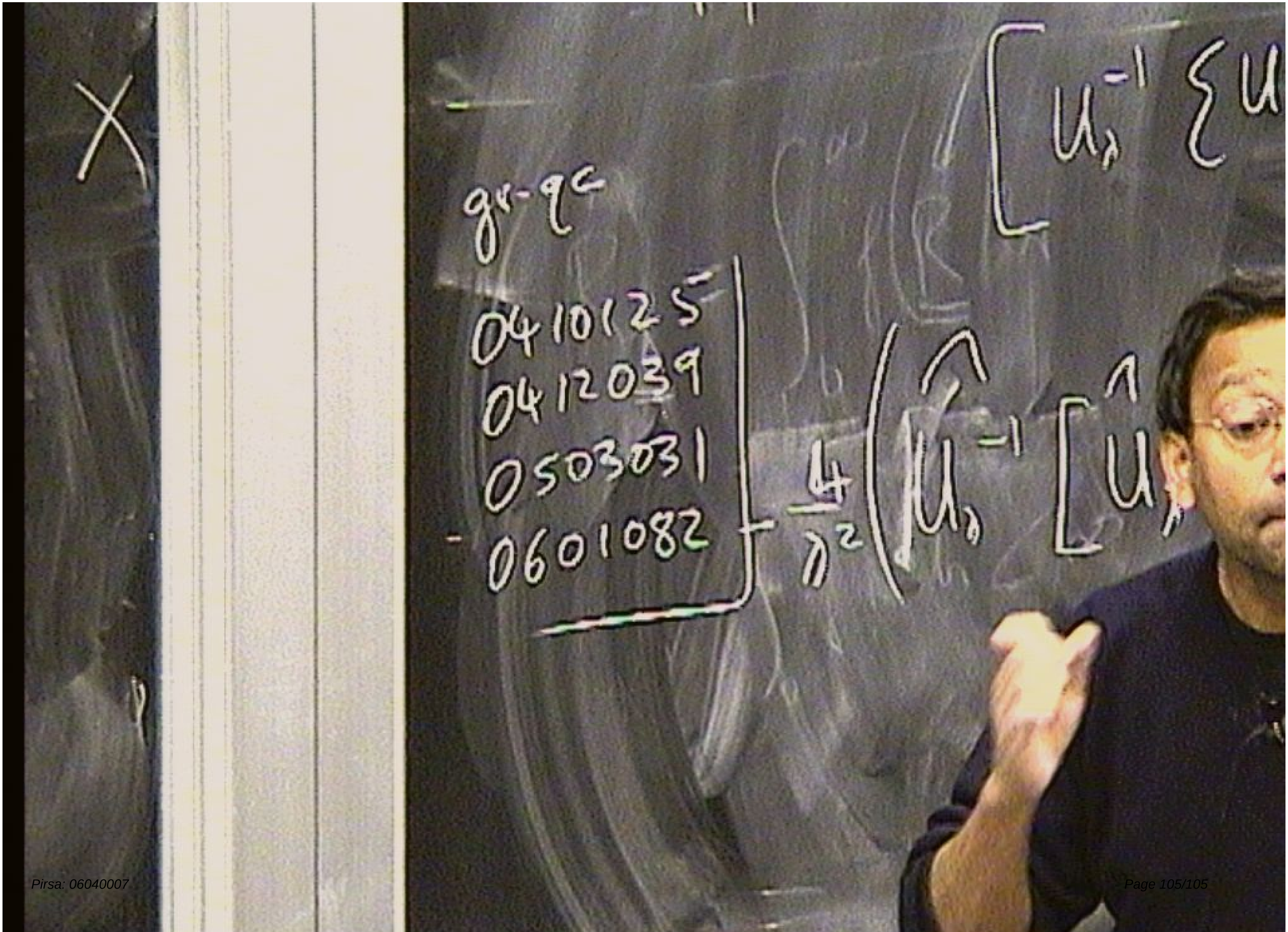
In this QM there is a $\widehat{\frac{1}{x}}$ operator:

$\hat{x}|\mu\rangle = L\mu|\mu\rangle$ Try to define $\widehat{\frac{1}{x}}$ by $\frac{1}{\hat{x}}$ X

$\hat{x}|0\rangle = 0|0\rangle$

$$\hat{p} := \lim_{\lambda \to 0} \frac{e^{i\lambda \hat{p}} - I}{\lambda} \qquad \frac{\langle \mu | e^{i\lambda \hat{p}} - I | \mu \rangle}{\lambda}$$

$$C_r = P_\Phi \Phi' + P_R R' - P_\Lambda'(\ \ )^2 \qquad = \frac{0-1}{\lambda} = -\frac{1}{\lambda}$$


gr-qc
0410125
0412039
0503031
0601082