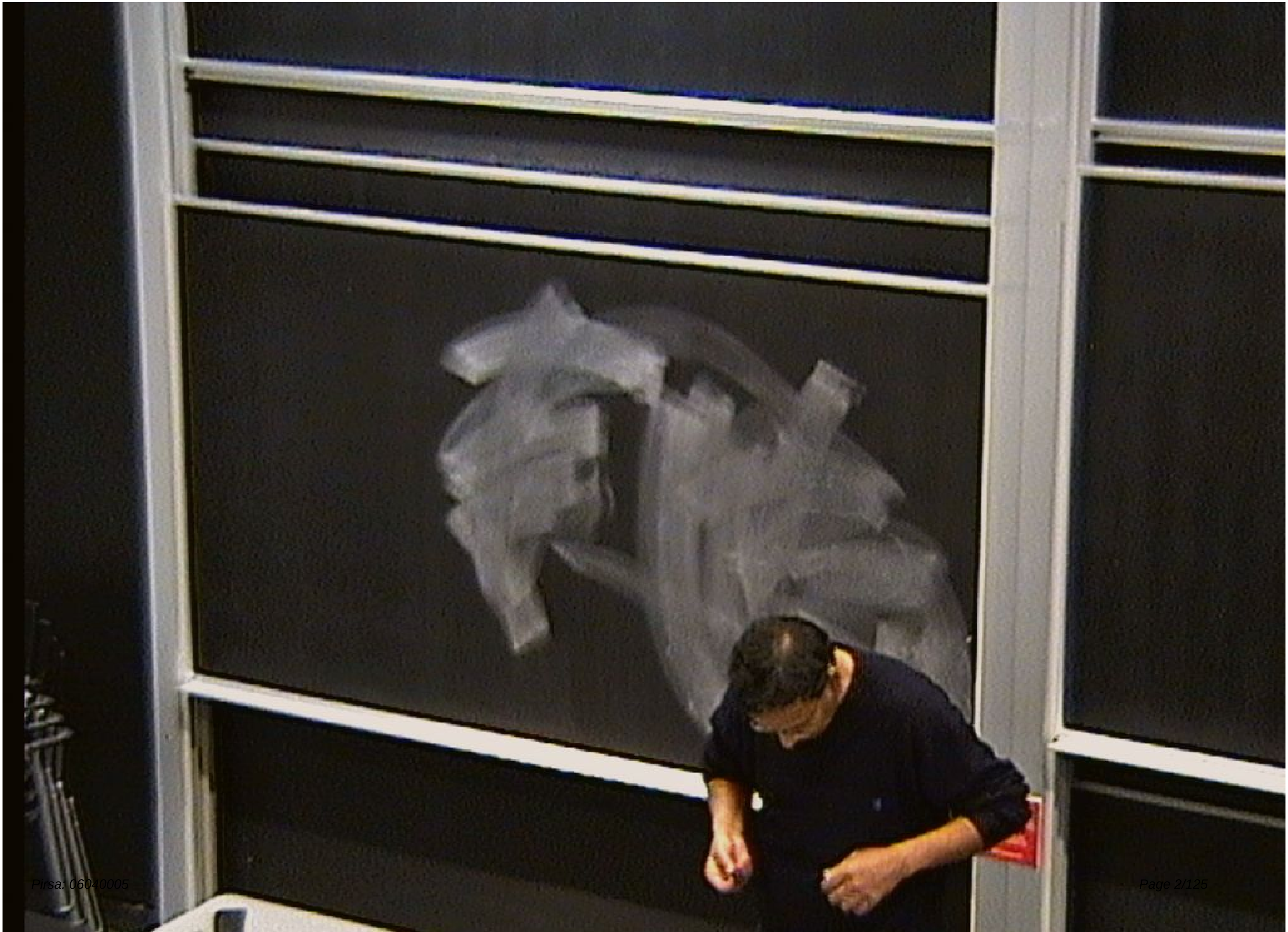
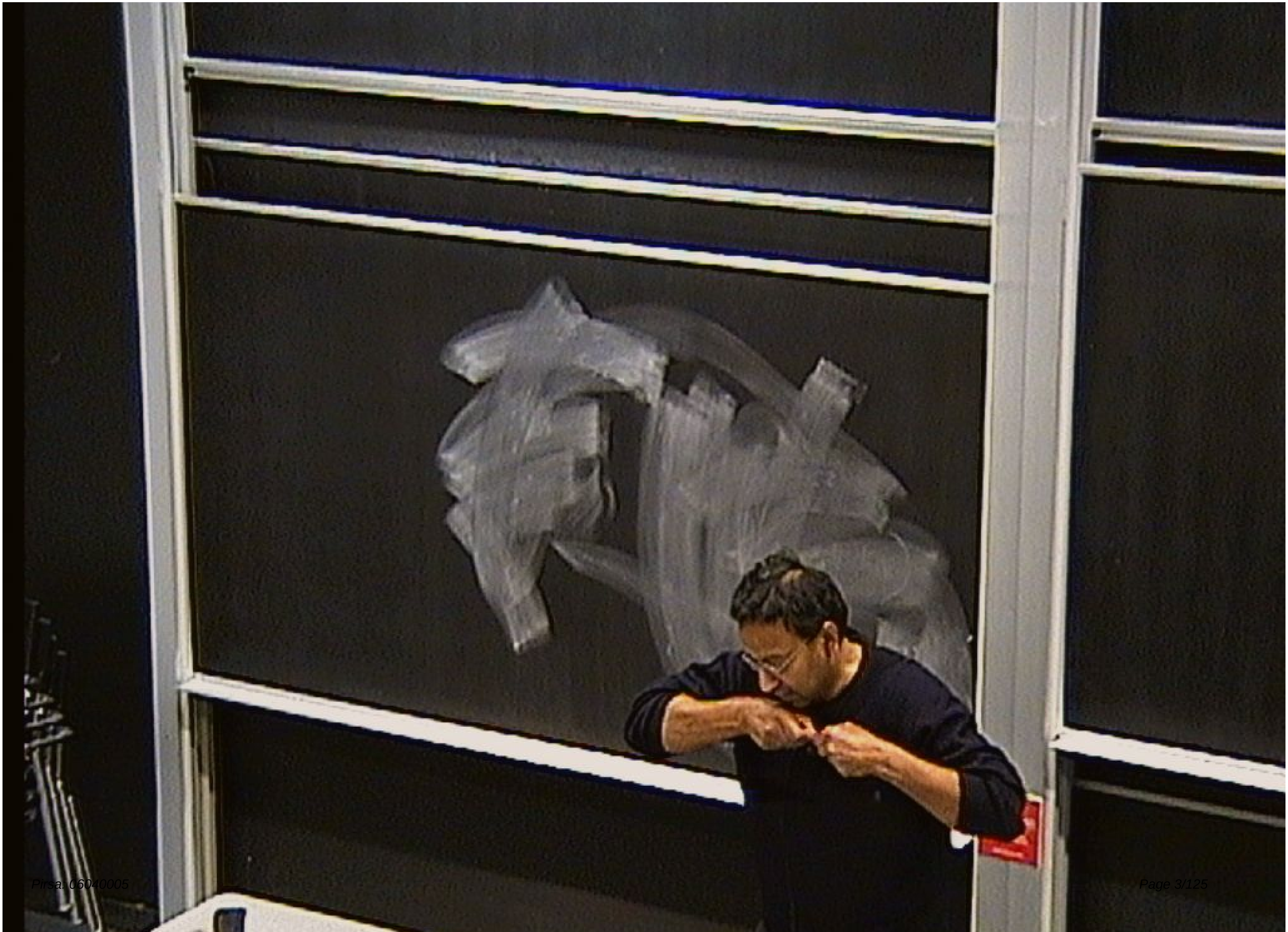
Title: Introduction to quantum gravity - Part 22

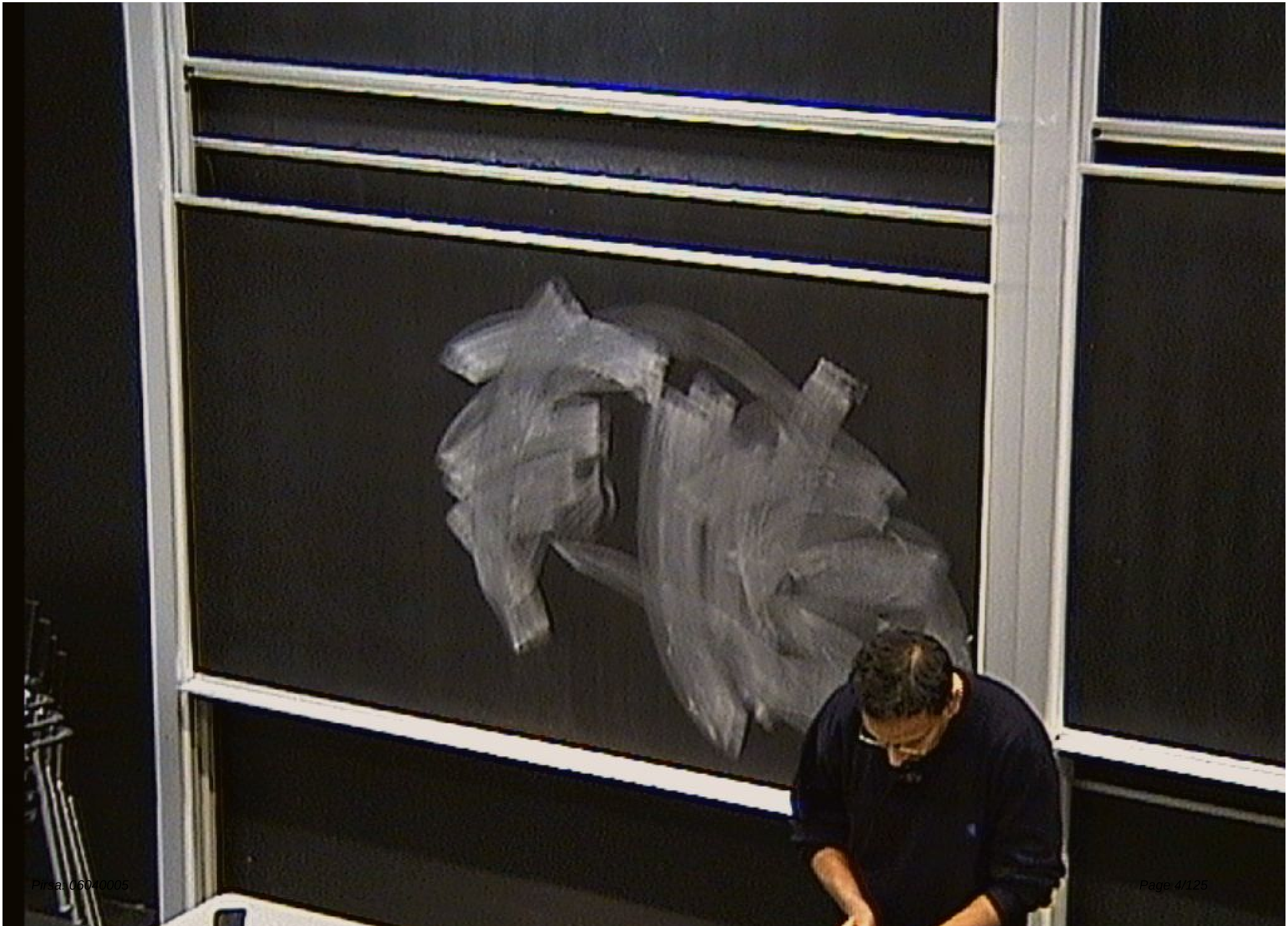
Date: Apr 05, 2006 06:30 PM

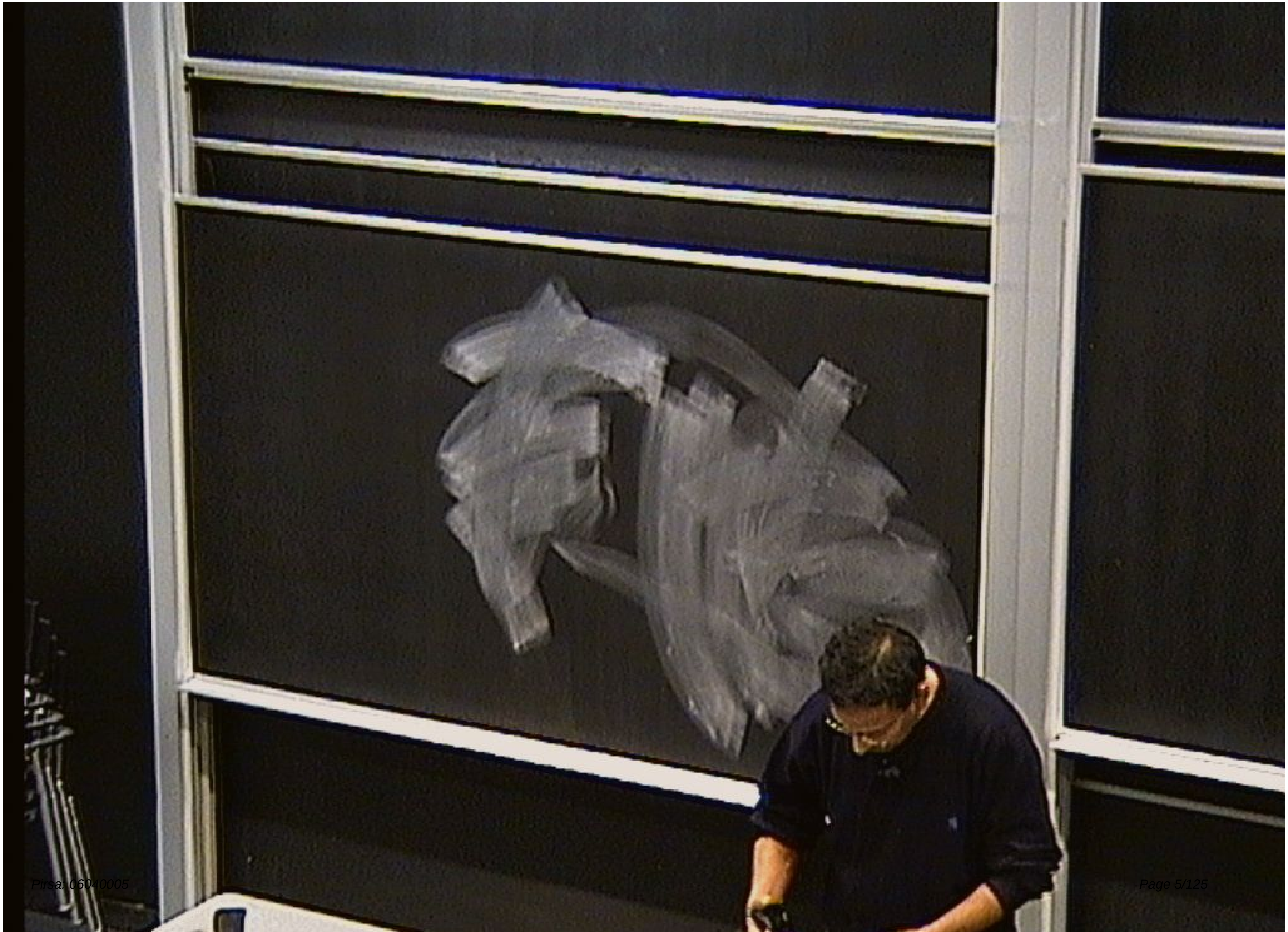
URL: http://pirsa.org/06040005

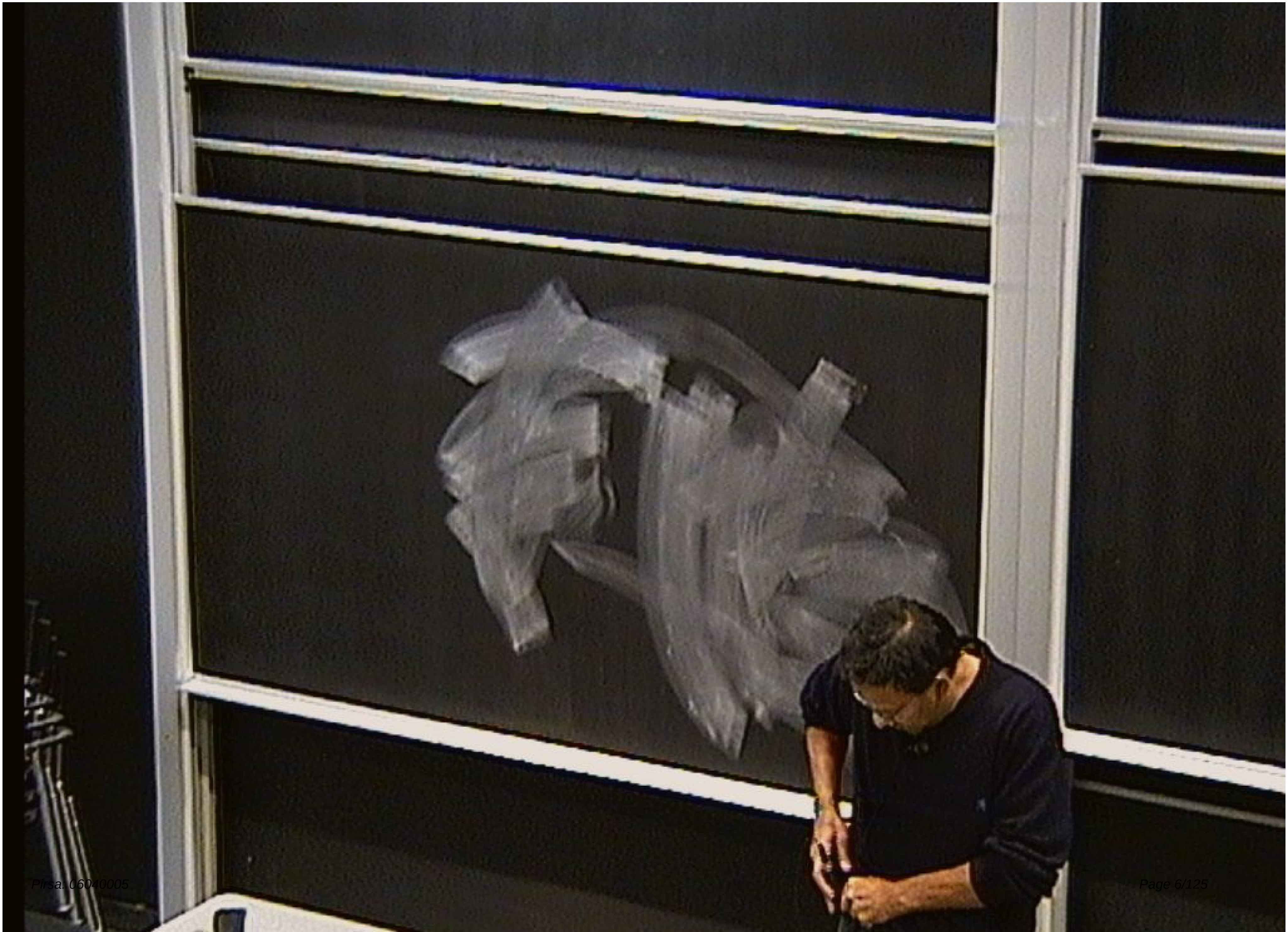
Abstract:

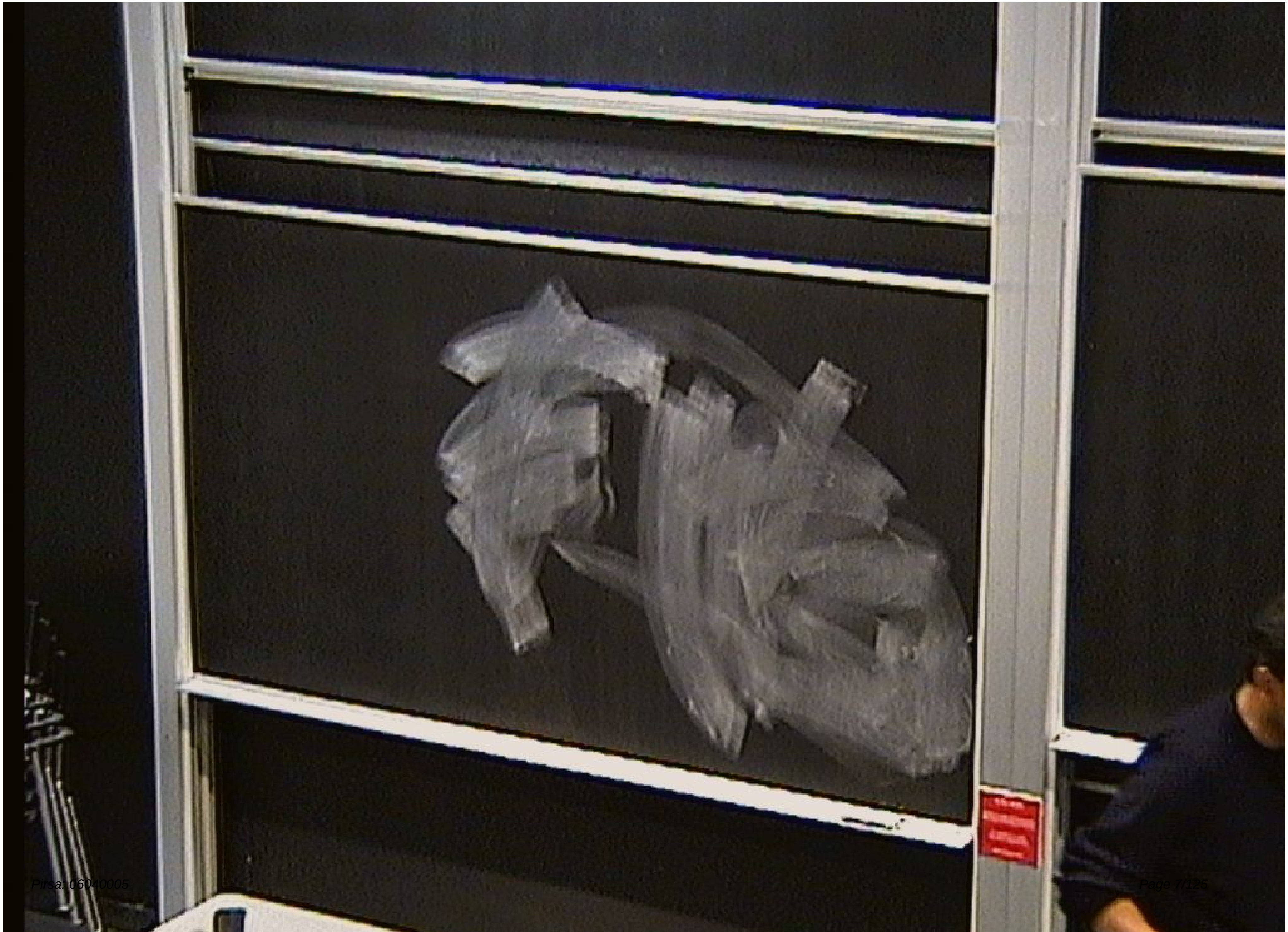


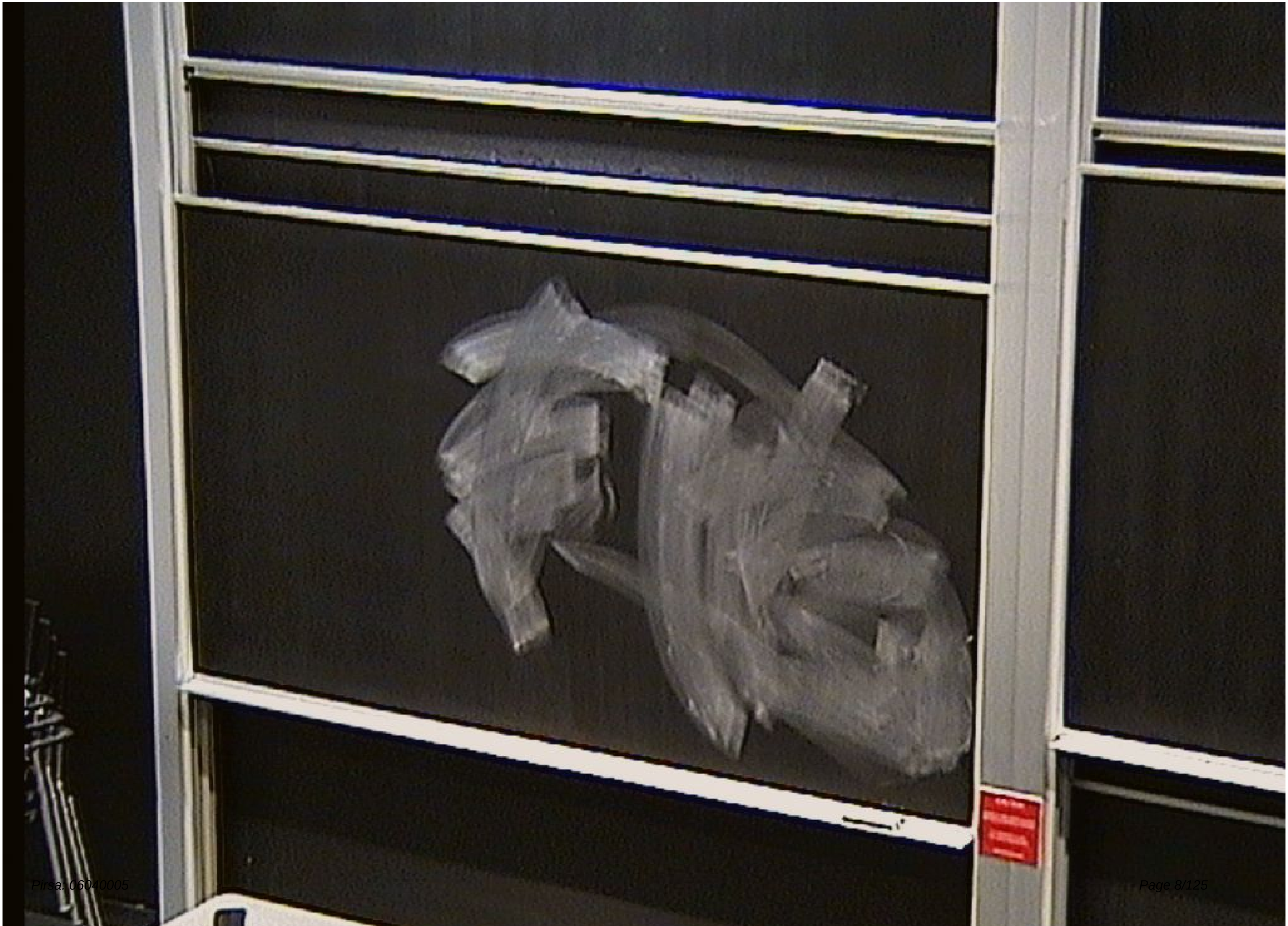


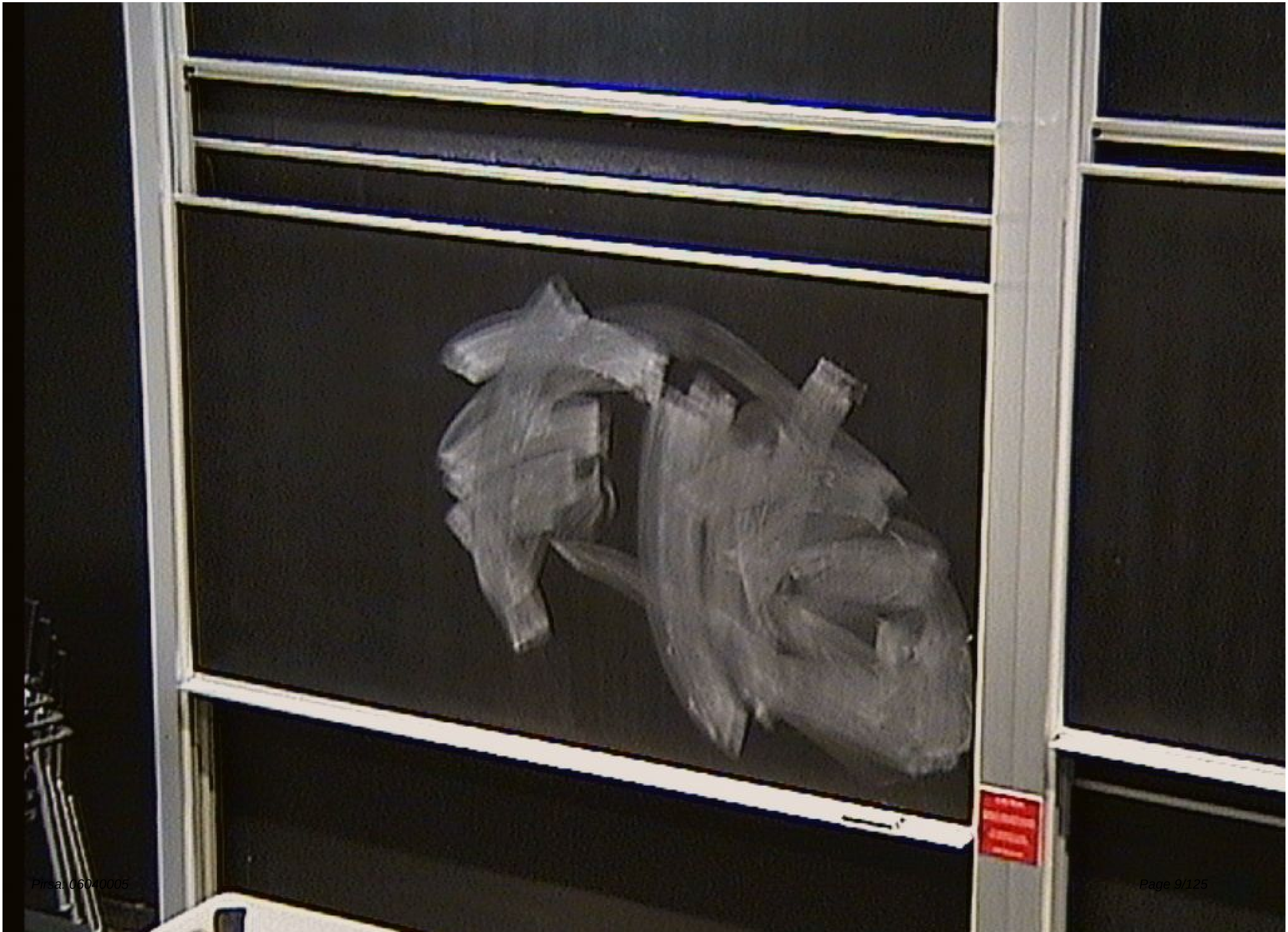


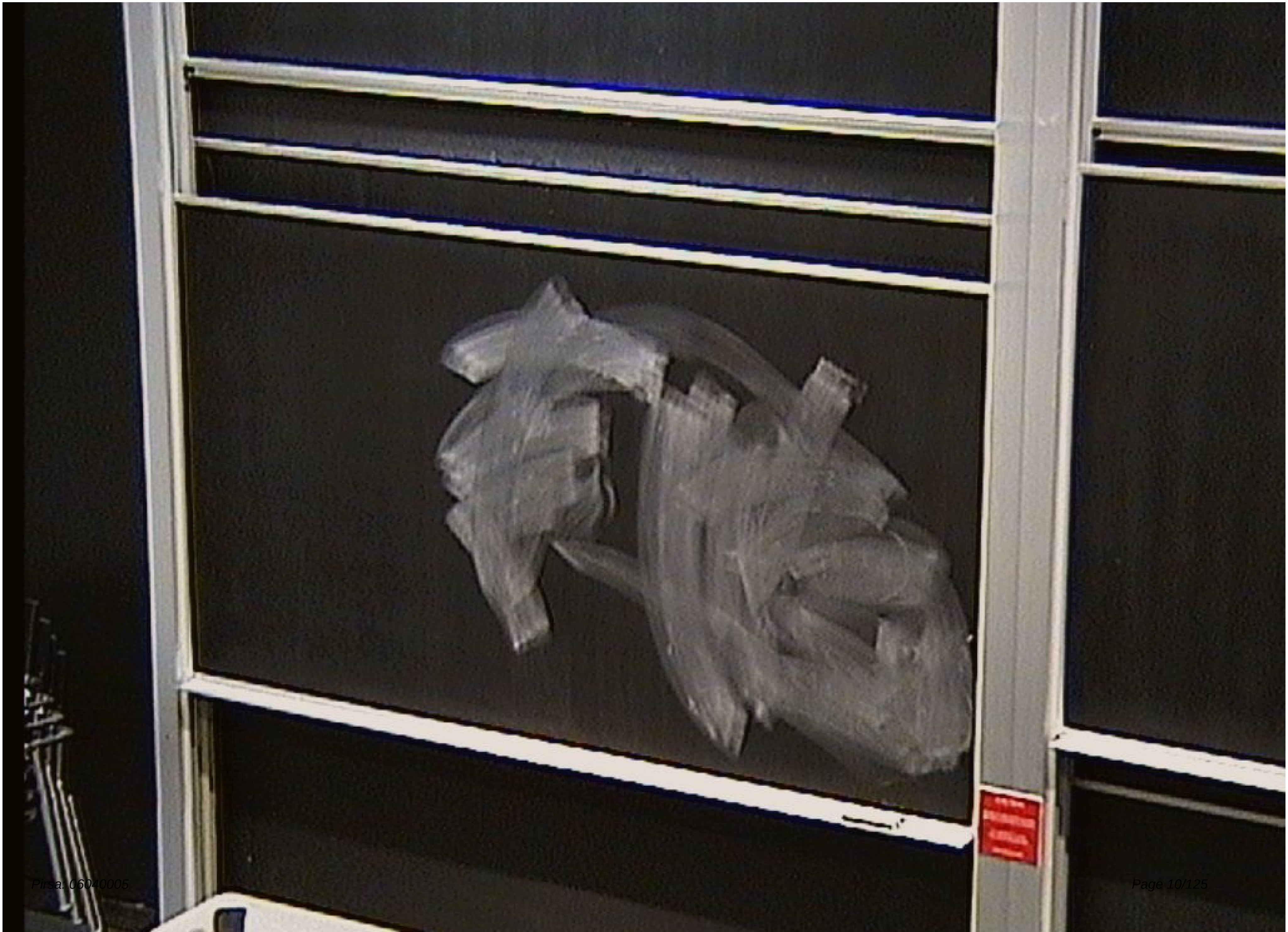


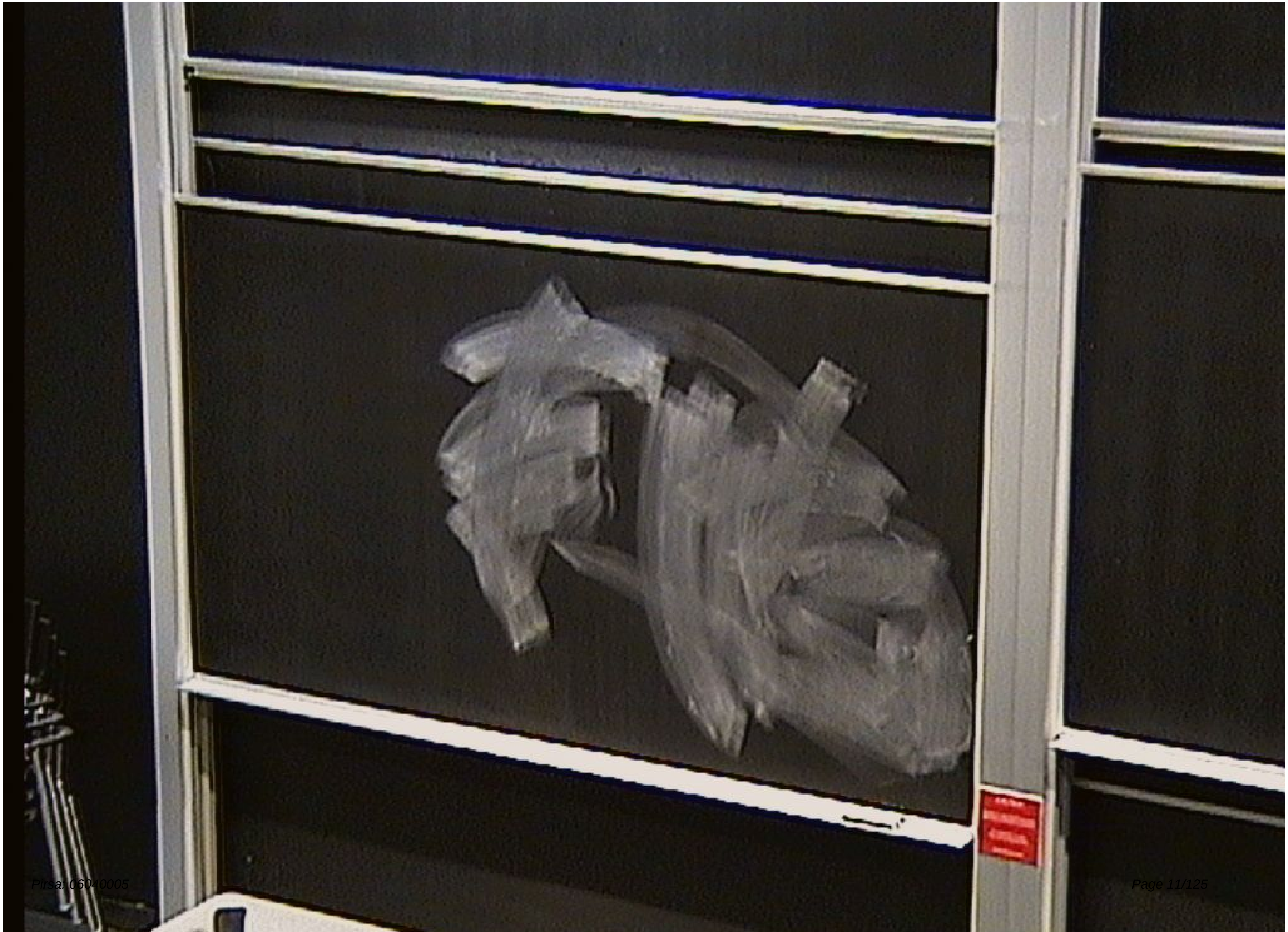


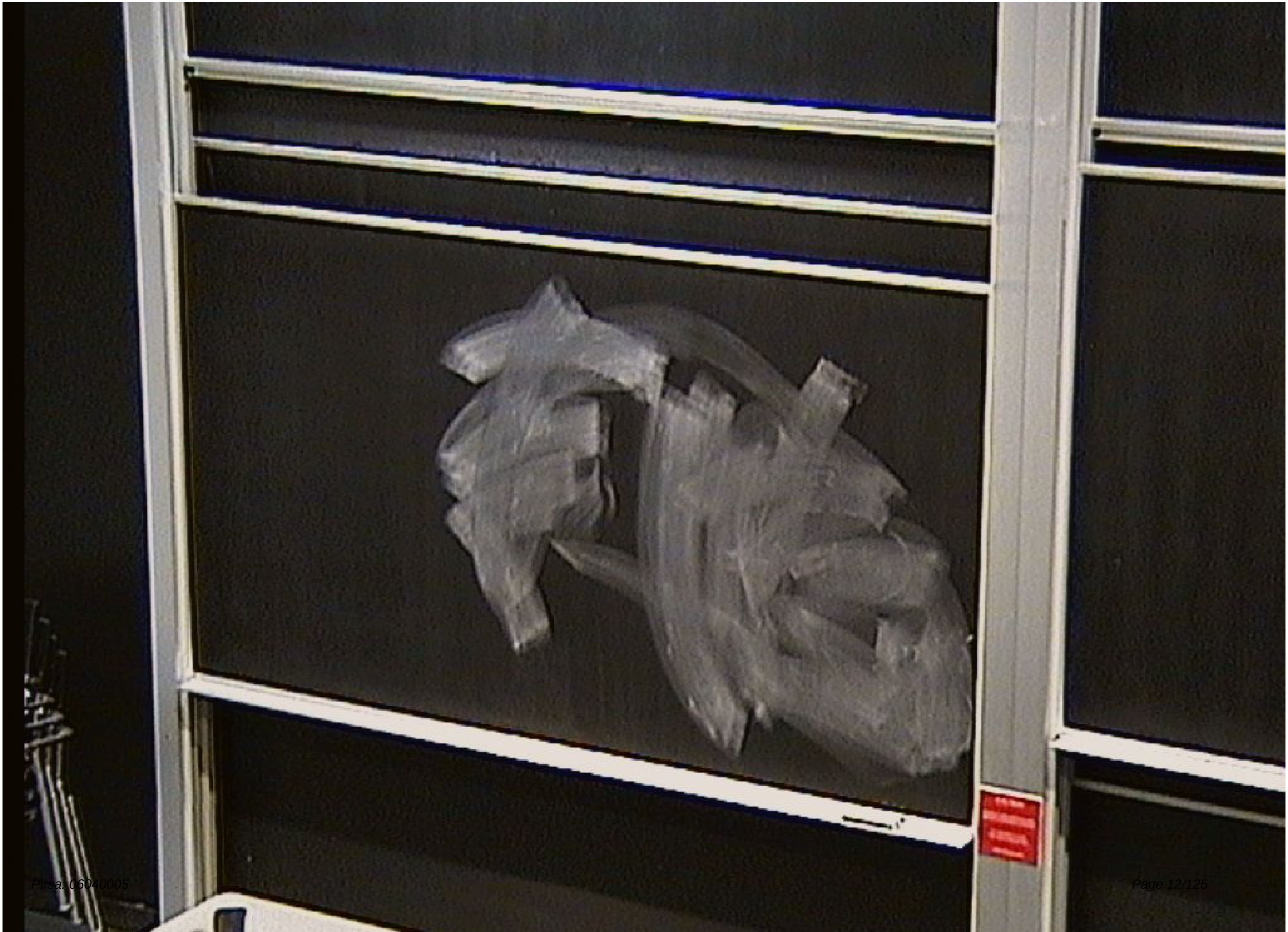


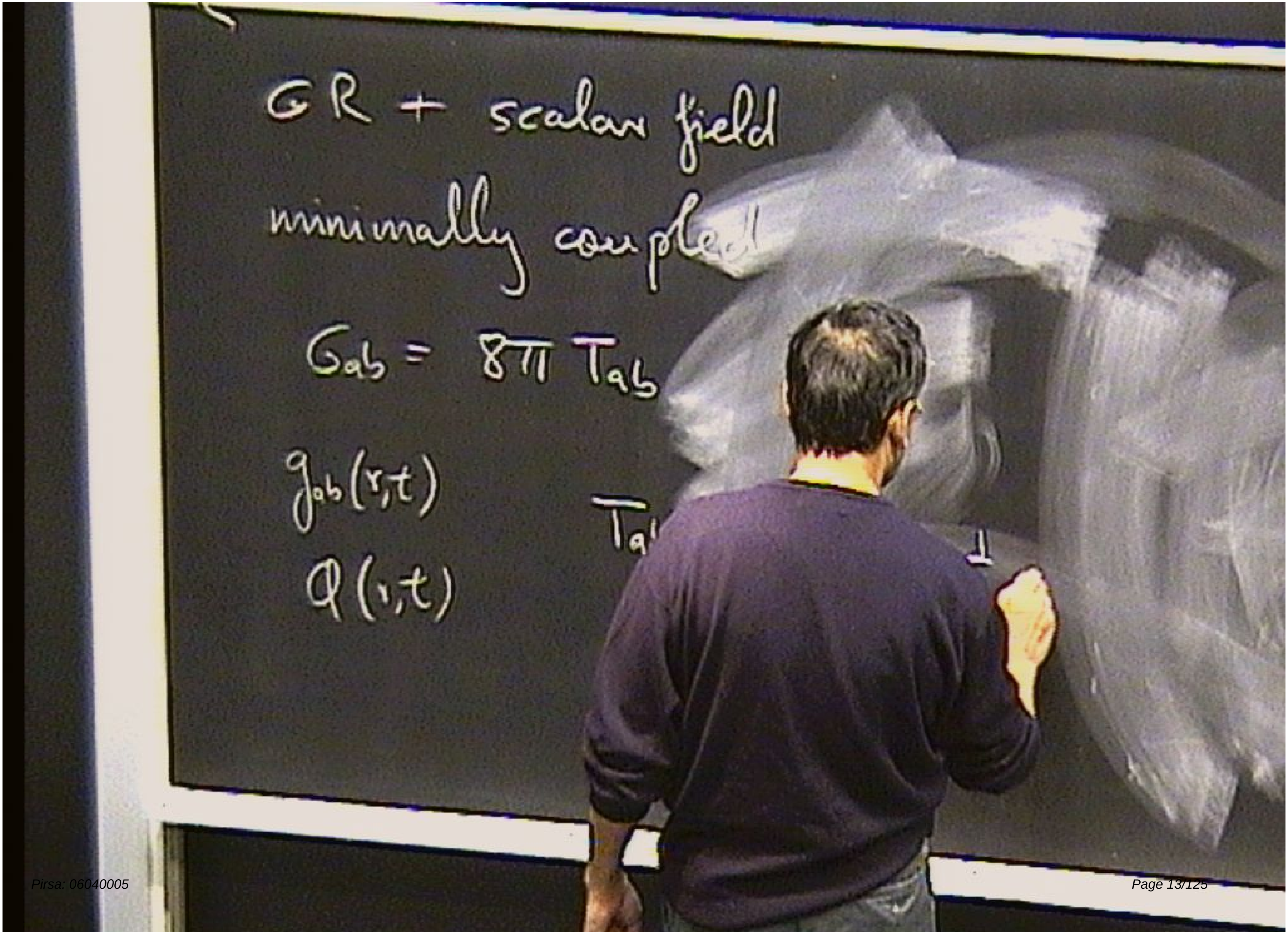











GR + scalar field
minimally coupled
$G_{ab} = 8\pi T_{ab}$
$g_{ab}(r,t)$
$\phi(r,t)$

GR + scalar field

minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$$g_{ab}(r,t)$$

$$\varphi(r,t)$$

$$T_{ab} = \partial_a \varphi \, \partial_b \varphi - \frac{1}{2} g_{ab} (\partial \varphi)^2$$

GR + scalar field

minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$g_{ab}(r,t)$

$\varphi(r,t)$

$$T_{ab} = \partial_a \varphi \, \partial_b \varphi - \frac{1}{2} g_{ab} (\partial \varphi)^2$$

Unruh

$$ds^2 = -dt^2 f(r,t)^2 + dr^2 g^2 + r^2 d\Omega^2$$

GR + scalar field

minimally coupled

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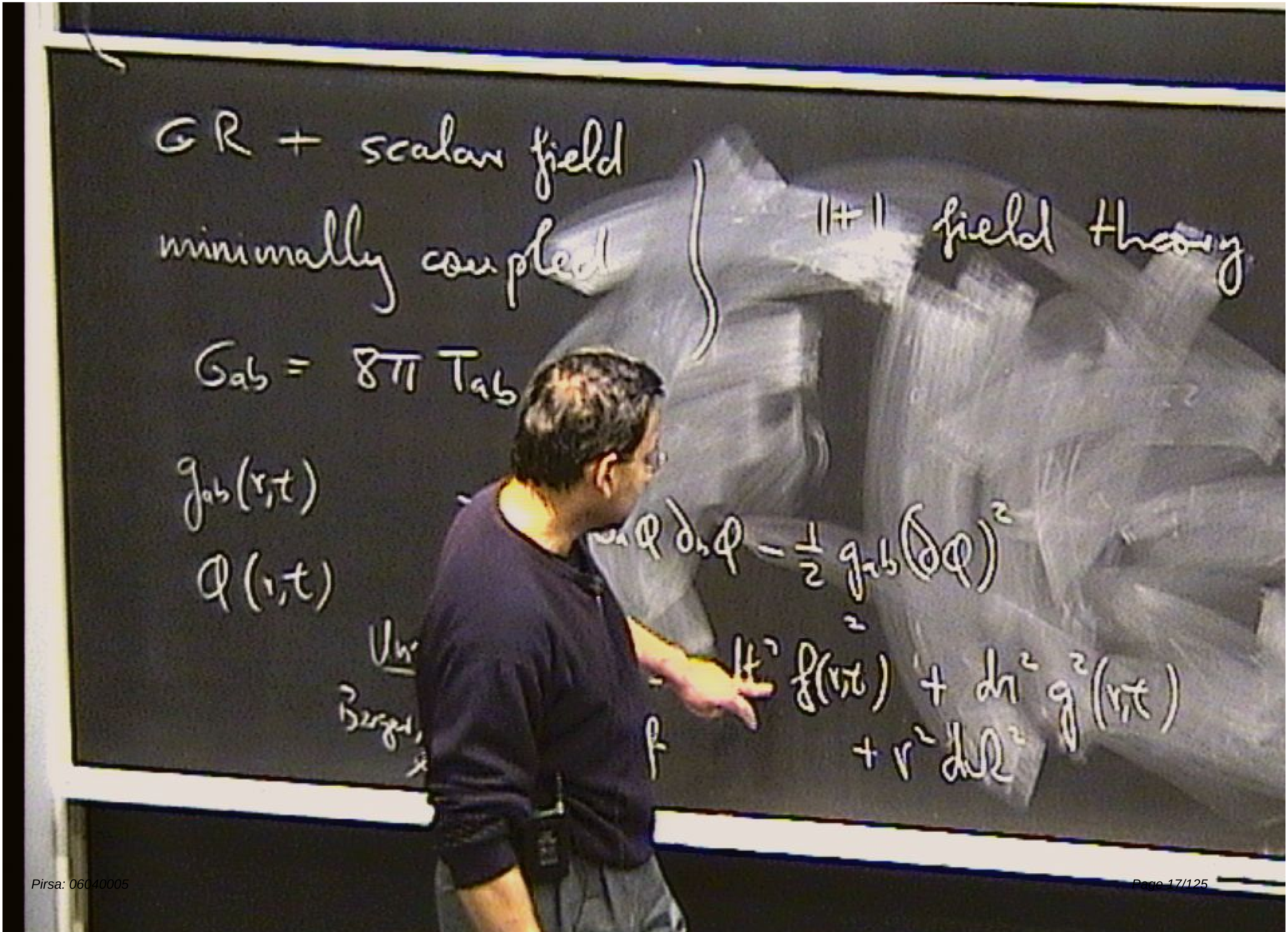
Unruh (1976) $ds^2 = -dt^2 f^2(r,t) + dr^2 g^2(r,t) + r^2 d\Omega^2$

Berger, Chitre, Moncrief & Nutku (1973)

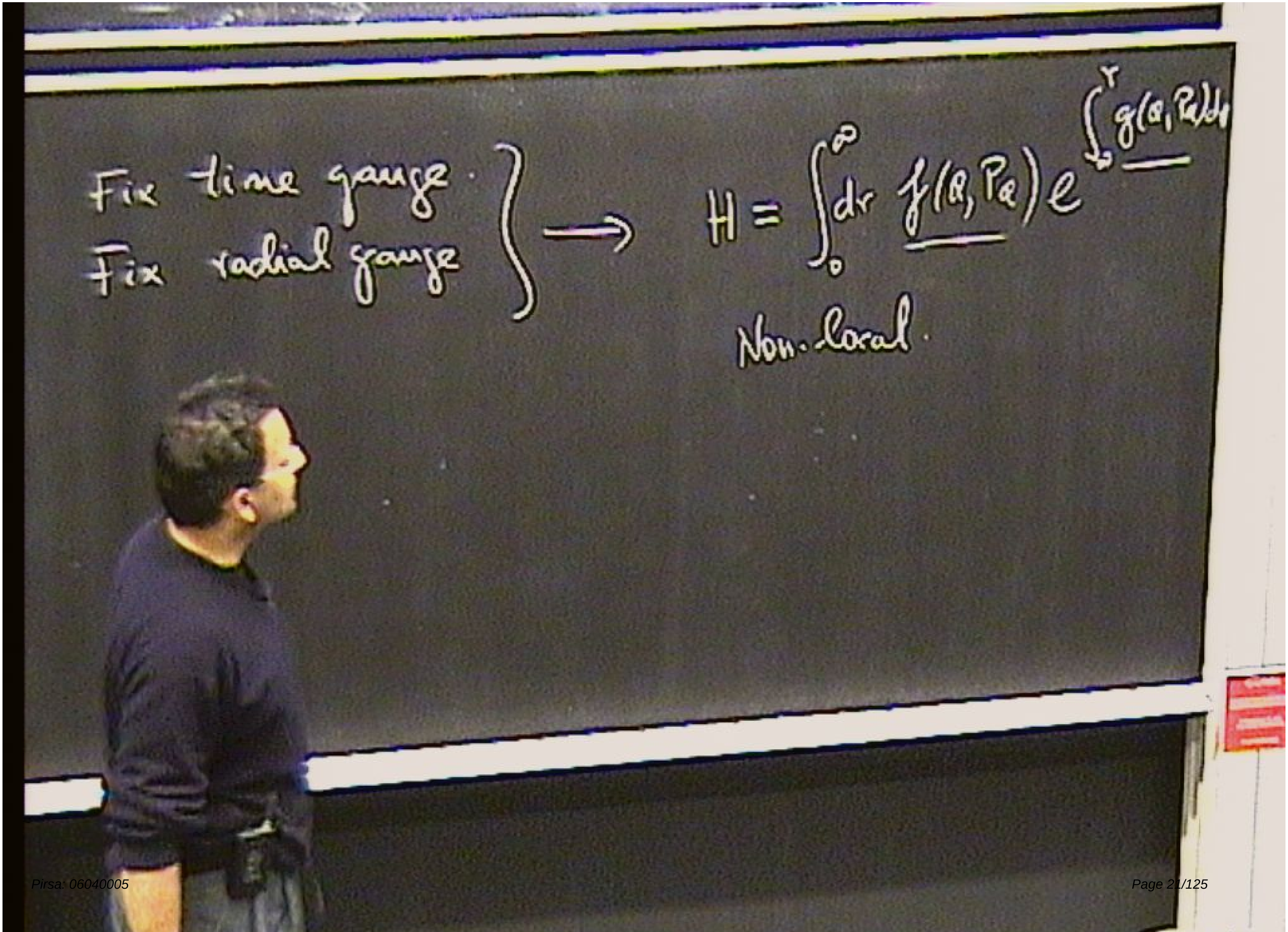

GR + scalar field
minimally coupled
1+1 field theory
Gab = 8π Tab
gab(r,t)
φ(r,t)

Fix time gauge.
Fix radial gauge $\Bigg\} \longrightarrow$

$$\left.\begin{array}{l}\text{Fix time gauge.}\\ \text{Fix radial gauge}\end{array}\right\} \longrightarrow \quad H = \int_0^\infty dr\, f(R, P_R)$$

Fix time gauge
Fix radial gauge

$$\left.\begin{array}{l}\text{Fix time gauge}\\ \text{Fix radial gauge}\end{array}\right\} \longrightarrow \quad H = \int_0^\infty dr\, f(R, P_R)\, e^{\int_0^r g(R, P_R)\, dr}$$


Fix time gauge.
Fix radial gauge
$H = \int_0^\infty dr\, f(R, P_R)\, e^{\int_0^r g(R, P_R) dr}$
Non-local.

Fix time gauge.

Fix radial gauge

$$\left.\begin{array}{l}\text{Fix time gauge.}\\ \text{Fix radial gauge}\end{array}\right\} \longrightarrow \quad H = \int_0^\infty dr\, \underline{f(R, P_R)}\, e^{\int^r \underline{g(R, P_R)}\, dr}$$

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Fix radial gauge

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Non-local.

- Classical ADM formalism.
-

Fix time gauge.
Fix radial gauge

$$\left.\begin{array}{l}\text{Fix time gauge.}\\ \text{Fix radial gauge}\end{array}\right\} \longrightarrow \quad H = \int_0^\infty dr\, \underline{f(R, P_R)}\, e^{\int_0^r \underline{g(R,P_R)}\,dr}$$

Non-local.

Classical ADM formalism – adapted to flat slice coordinates.

Fix time gauge.
Fix radial gauge $\Bigg\} \longrightarrow \quad H = \int_0^\infty dr\, \underline{f(R, P_R)}\, e^{\int_0^r \underline{g(R,P_R)} dr}$

Non-local.

- Classical ADM formalism — adapted to flat slice coordinates.
- Classical results on collapse.
- Quantization (LQG ideas)
- Quantum collapse

Fix time gauge.

Fix radial gauge

$$\longrightarrow \quad H = \int_0^\infty dr\, \underline{f(R, P_R)}\, e^{\int_0^r \underline{g(R,P_R)}\,dr}$$

Non-local.

- Classical ADM formalism — adapted to flat slice coordinates.
- Classical results on collapse.
- Quantization (LQG ideas)
- Quantum collapse
-

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}}\,dt\right)^2 + dr^2 + r^2 d\Omega^2$$

Painleve & Gullstrand.

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Painlevé & Gullstrand. <u>PG</u>

$t = \text{const}$

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Painlevé & Gullstrand. PG

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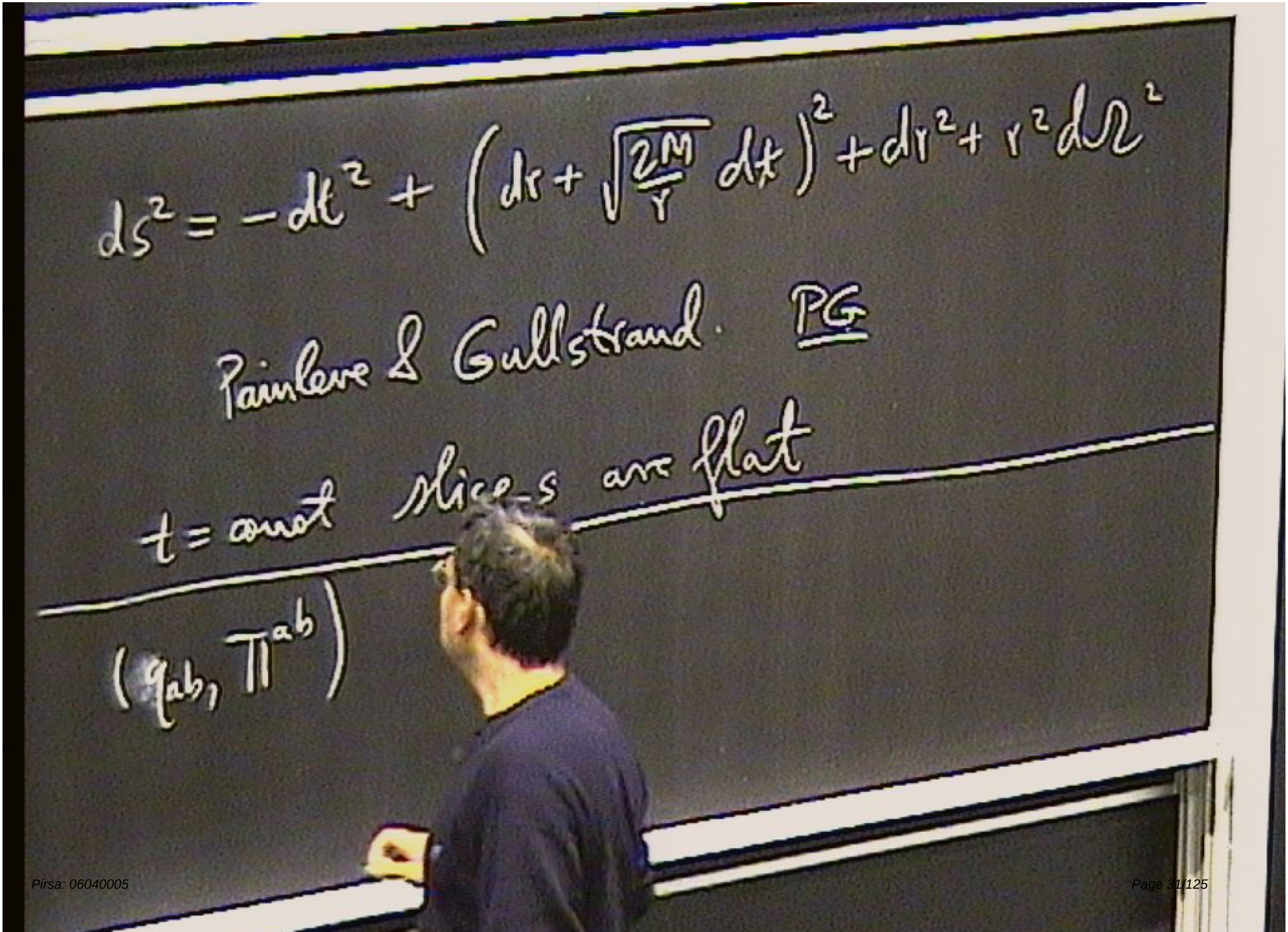
Painleve & Gullstrand. PG

$t = \text{const}$ slices are flat


$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}}\, dt\right)^2 + dr^2 + r^2 d\Omega^2$
Painleve & Gullstrand. PG
t = const slices are flat
(q_{ab}, Π^{ab})

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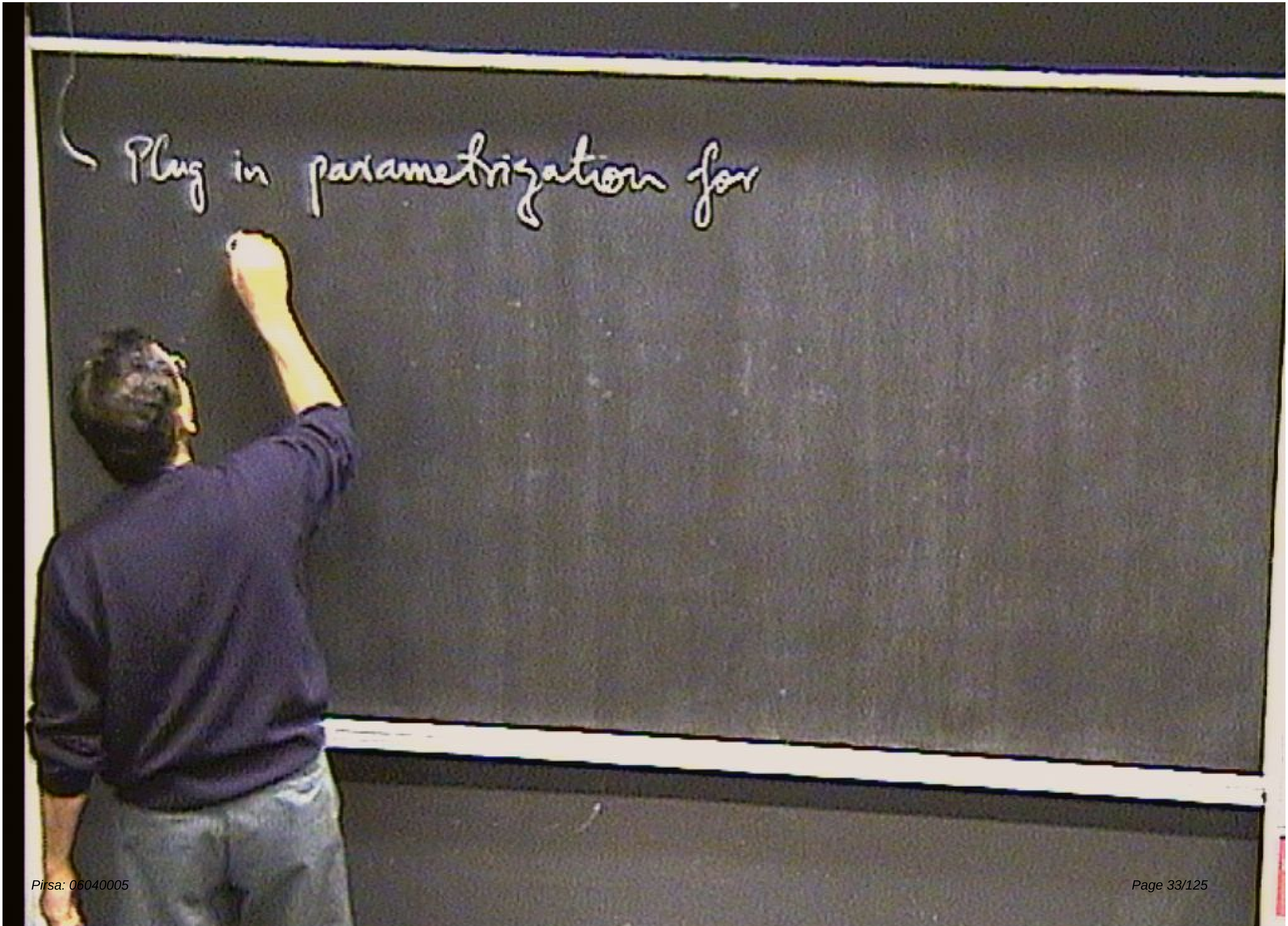
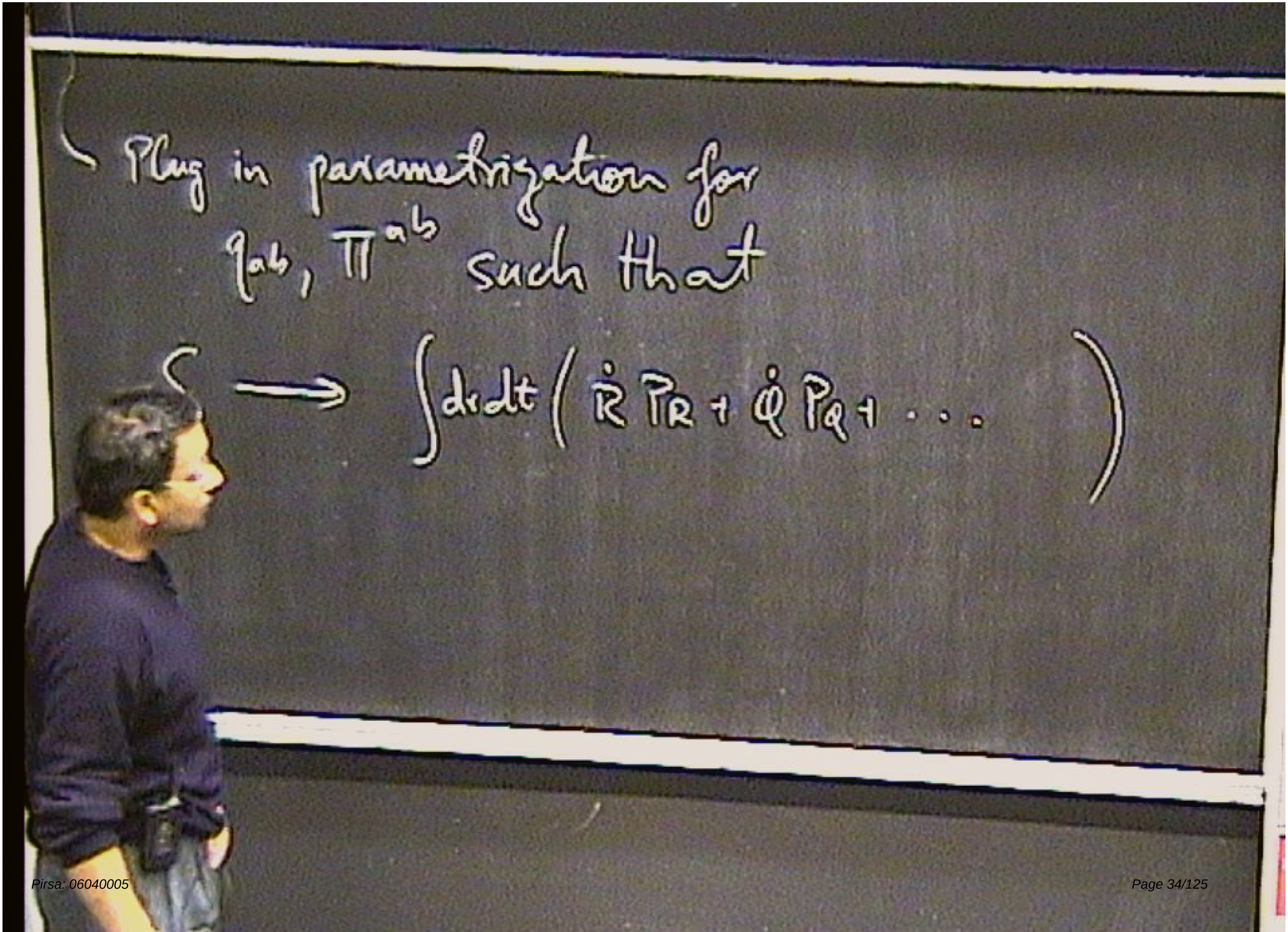
Painleve & Gullstrand. PG

$t = \text{const}$ slices are flat

(q_{ab}, Π^{ab})

$$S_{GR} = \int_M \left\{ \Pi^{ab} \dot{q}_{ab} - N\mathcal{H} - N^a C_a \right\} d^3x\,dt + \int_{\partial M} (\ldots)$$


Plug in parametrization for


Plug in parametrization for
q_{ab}, π^{ab} such that
$S \longrightarrow \int dr\,dt \left(\dot{R} P_R + \dot{Q} P_Q + \dots \right)$

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Plug in parametrization for q_{ab}, π^{ab} such that

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Fall off conditions on phase space variables.

Plug in parametrization for

q_{ab}, π^{ab} such that

$$S \longrightarrow \int dr\,dt \left(\dot{R} P_R + \dot{Q} P_Q + \cdots \right)$$

Fall off conditions on phase space variables.

<u>Schw</u> $\quad q_{ab} = e_{ab} + \frac{f_{ab}(\theta, Q)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$

$$t^2 + \left(dr + \sqrt{\frac{2M}{r}}\, dt\right)^2 + \cancel{dr^2} + r^2 d\Omega^2 \Big)$$

ve & Gullstrand. PG

t slices are flat

$$\Big) \qquad S_{GR} = \int_M \left\{ \pi^{ab} \dot{q}_{ab} - N\mathcal{H} - N^a C_a \right\} d^3x\, dt$$

$$+ \int (\ldots)$$

Fall off conditions on phase space variables.

Schw $\quad q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$

$$\pi^{ab} = \frac{g^{ab}(\theta,\varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

Fall off conditions on phase space variables.

<u>Schw</u>

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

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<u>PG</u>

$$q_{ab} = e_{ab}$$

Fall off conditions on phase space variables.

Schw $\quad q_{ab} = e_{ab} + \dfrac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\dfrac{1}{r^2}\right)$

e_{ab} – flat 3 metric

PG

$$\Pi^{ab} = \frac{g^{ab}(\theta,\varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

$q_{ab} = e_{ab}$

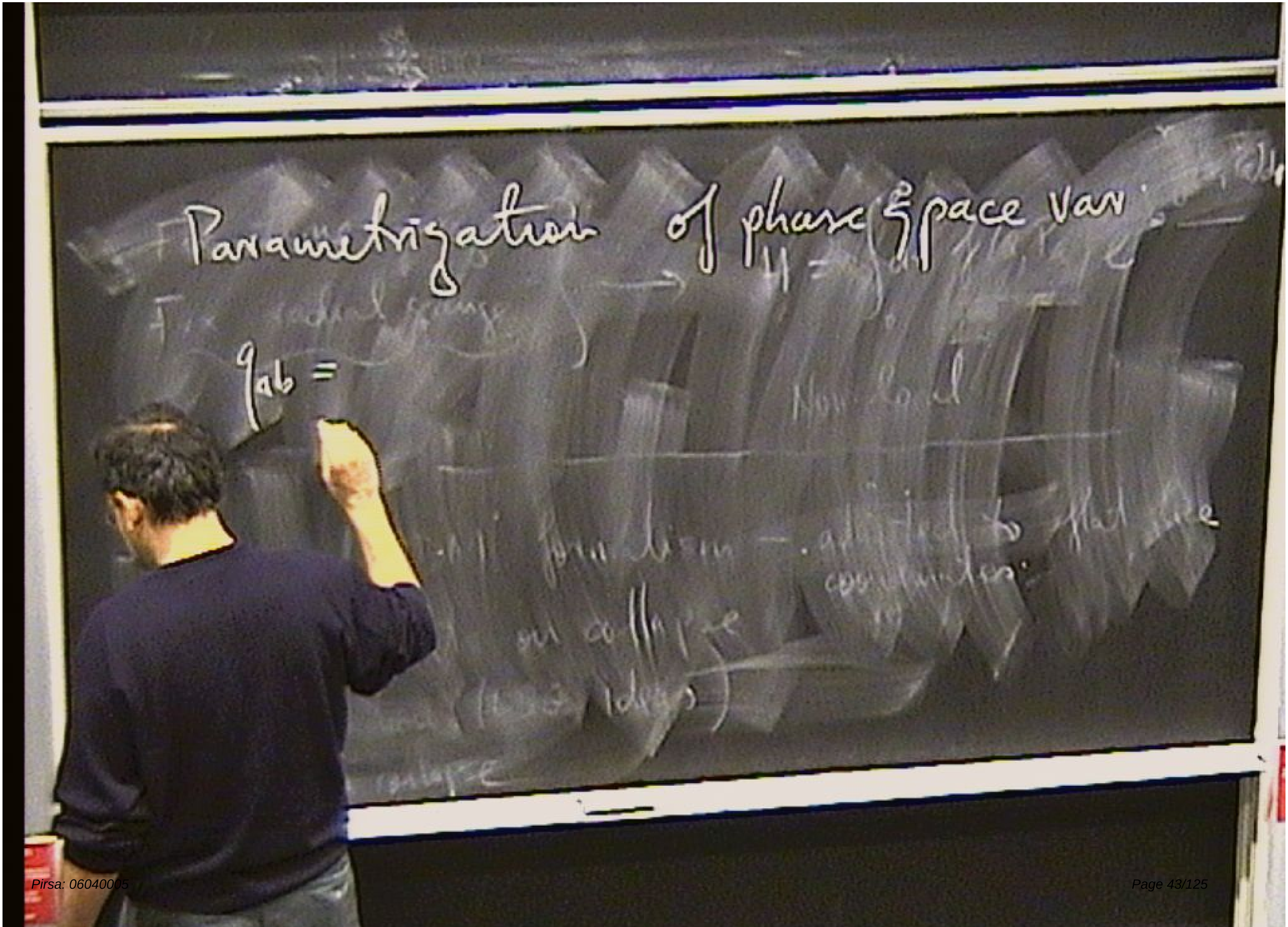
Fall off conditions on phase space variables.

Schw
$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

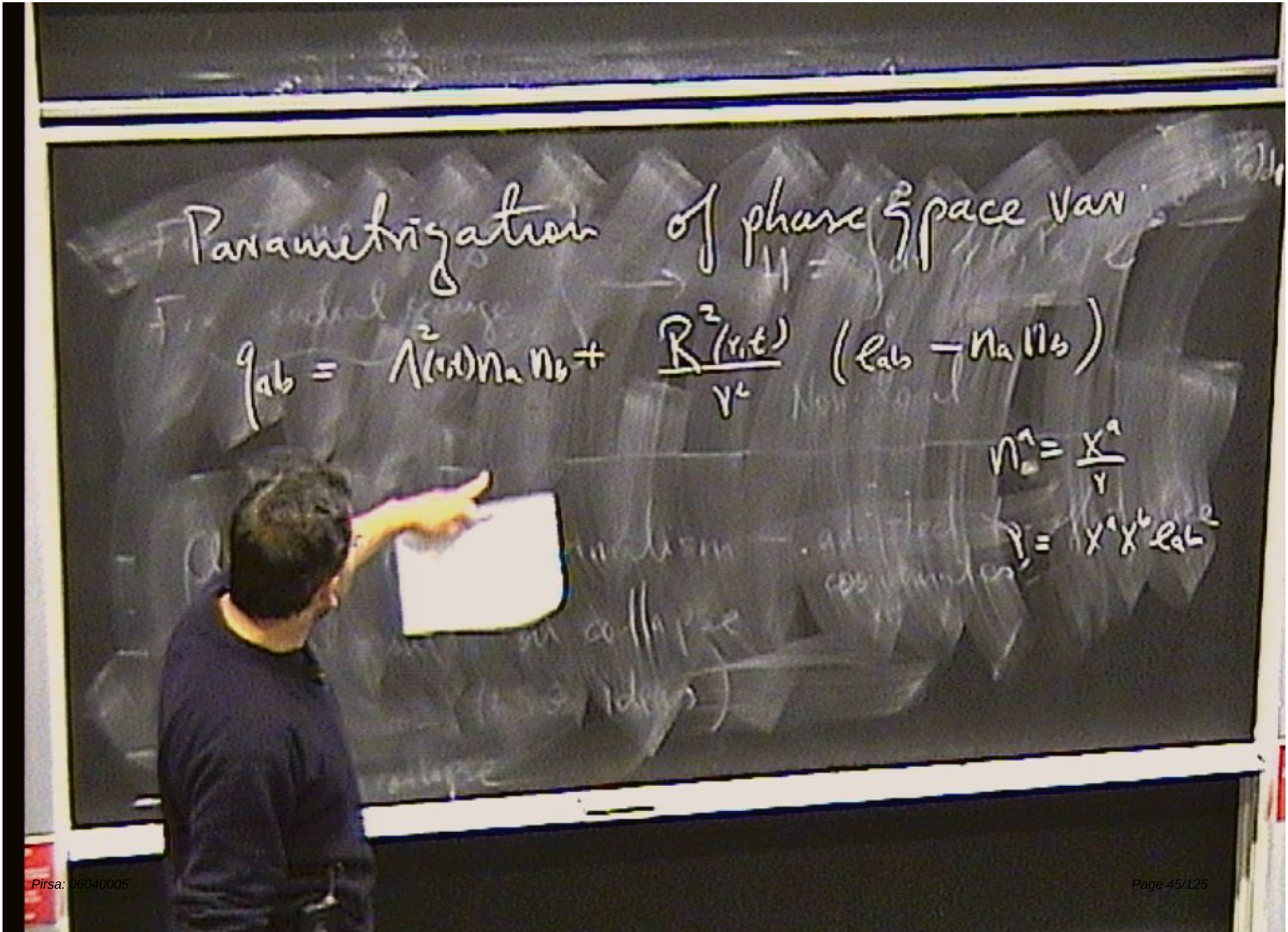
$$\Pi^{ab} = \frac{g^{ab}(\theta,\varphi)}{r^2} + O\left(\frac{1}{r^3}\right)$$

PG ($\varrho=0$)
$$q_{ab} = e_{ab}$$
$$\Pi^{ab} = \frac{c}{r^{3/2}}$$


Parametrization of phase space var
q_{ab} =

Parametrization of phase space var:

$$q_{ab} = \Lambda^2(r,t)\, n_a n_b + \frac{R^2(r,t)}{r^2}\left(e_{ab} - n_a n_b\right)$$


Parametrization of phase space var
$g_{ab} = \Lambda^2(r,t) n_a n_b + \frac{R^2(r,t)}{r^2} (e_{ab} - n_a n_b)$
$n^a = \frac{x^a}{r}$
$r = |x^a x^b e_{ab}|^{\frac{1}{2}}$

Parametrization of phase space var:

$$q_{ab} = \Lambda^2(r,t)\, n_a n_b + \frac{R^2(r,t)}{r^2}\left(e_{ab} - n_a n_b\right)$$

$$n^a = \frac{x^a}{r}$$

$$ds^2 = \Lambda^2(r,t)\, dr^2 + R^2\, d\Omega^2$$

$$r = \sqrt{x^a x^b e_{ab}}$$

Parametrization of phase space var:

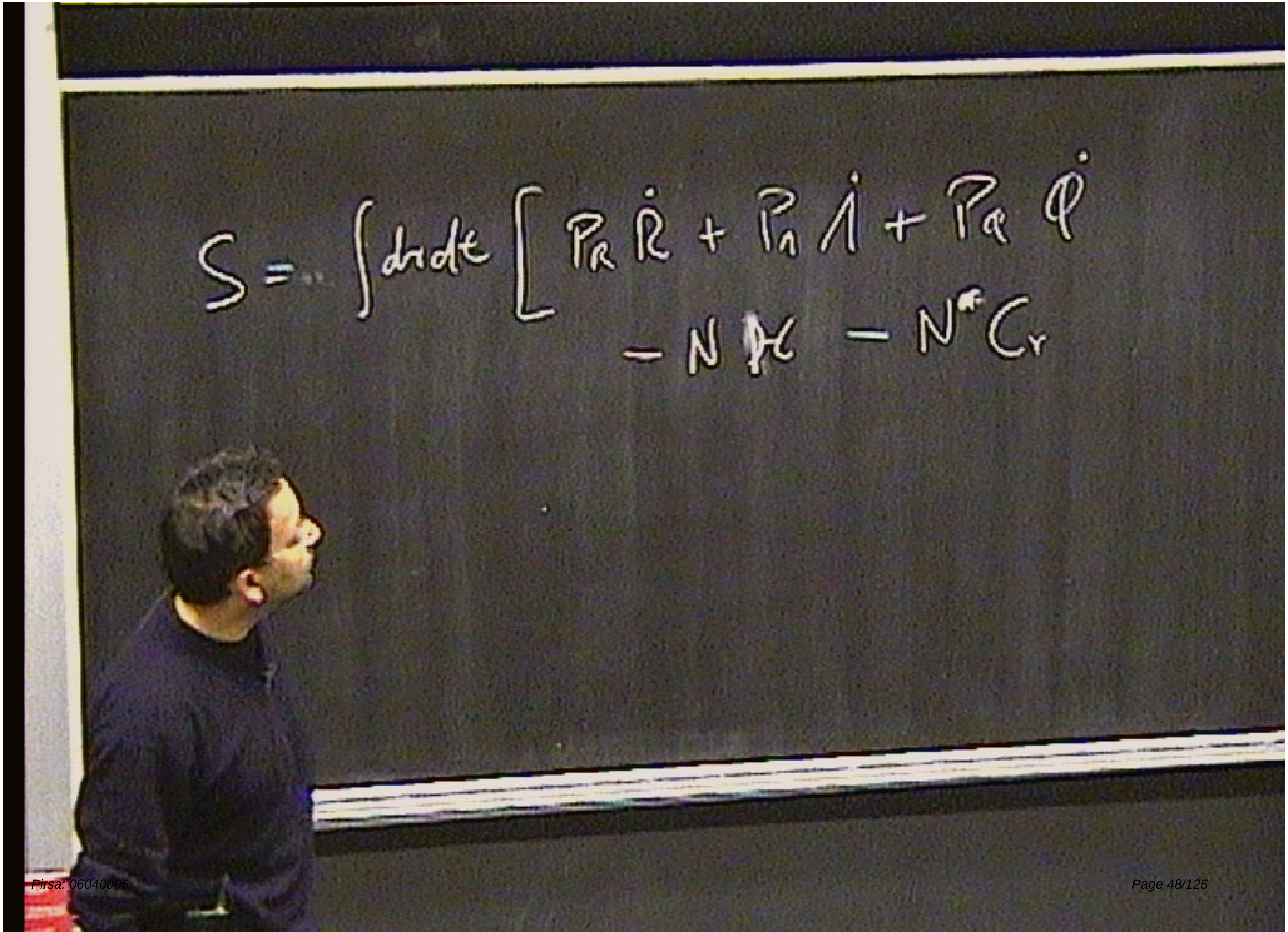
$$g_{ab} = \Lambda^2(r,t)\, n_a n_b + \frac{R^2(r,t)}{r^2}\left(e_{ab} - n_a n_b\right)$$

$$n^a = \frac{x^a}{r}$$

$$ds^2 = \Lambda^2(r,t)\, dr^2 + R^2\, d\Omega^2$$

$$r = \sqrt{x^a x^b e_{ab}}$$

$$\tilde{\Pi}^{ab} = \frac{P_\Lambda}{2\Lambda} n^a n^b + \frac{r^2 P_R}{4R}\left(e^{ab} - n^a n^b\right)$$


$S = \int dr\,dt \left[p_R \dot{R} + p_\Lambda \dot{\Lambda} + p_\varphi \dot{\varphi} \right.$
$\left. - N \mathcal{H} - N^r C_r \right.$

$$
S = \int_M dr\,dt \left[P_R \dot{R} + P_\Lambda \dot{\Lambda} + P_\varphi \dot{\varphi} - N \mathcal{H} - N^r C_r \right] + \int_{\partial M} (\ldots)
$$

$$S = \int_M dr\,dt \left[P_R \dot{R} + P_\Lambda \dot{\Lambda} + P_\varphi \dot{\varphi} - N \mathcal{H} - N^r C_r \right] + \int_{\partial M} (\ldots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\pi}^{ab} \tilde{\pi}_{ab} \ldots \tilde{\pi}^2 \right) - \sqrt{q}\, {}^{(3)}R + \frac{P_\varphi^2}{2\sqrt{q}} + \sqrt{q}\, q^{ab} \partial_a \varphi \partial_b \varphi$$

$$S = \int_M dr\,dt \left[P_R \dot{R} + P_\Lambda \dot{\Lambda} + P_\varphi \dot{\varphi} - N \mathcal{H} - N^r C_r \right] + \int_{\partial M} (\ldots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\pi}^{ab} \tilde{\pi}_{ab} - \frac{1}{2} \tilde{\pi}^2 \right) - \sqrt{q}\,{}^{(3)}R + \frac{P_\varphi^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \partial_a \varphi \partial_b \varphi$$

$$S = \int_M dr\,dt \left[P_R \dot{R} + P_\Lambda \dot{\Lambda} + P_\varphi \dot{\varphi} - N\mathcal{H} - N^r C_r \right] + \int_{\partial M} (\ldots)$$

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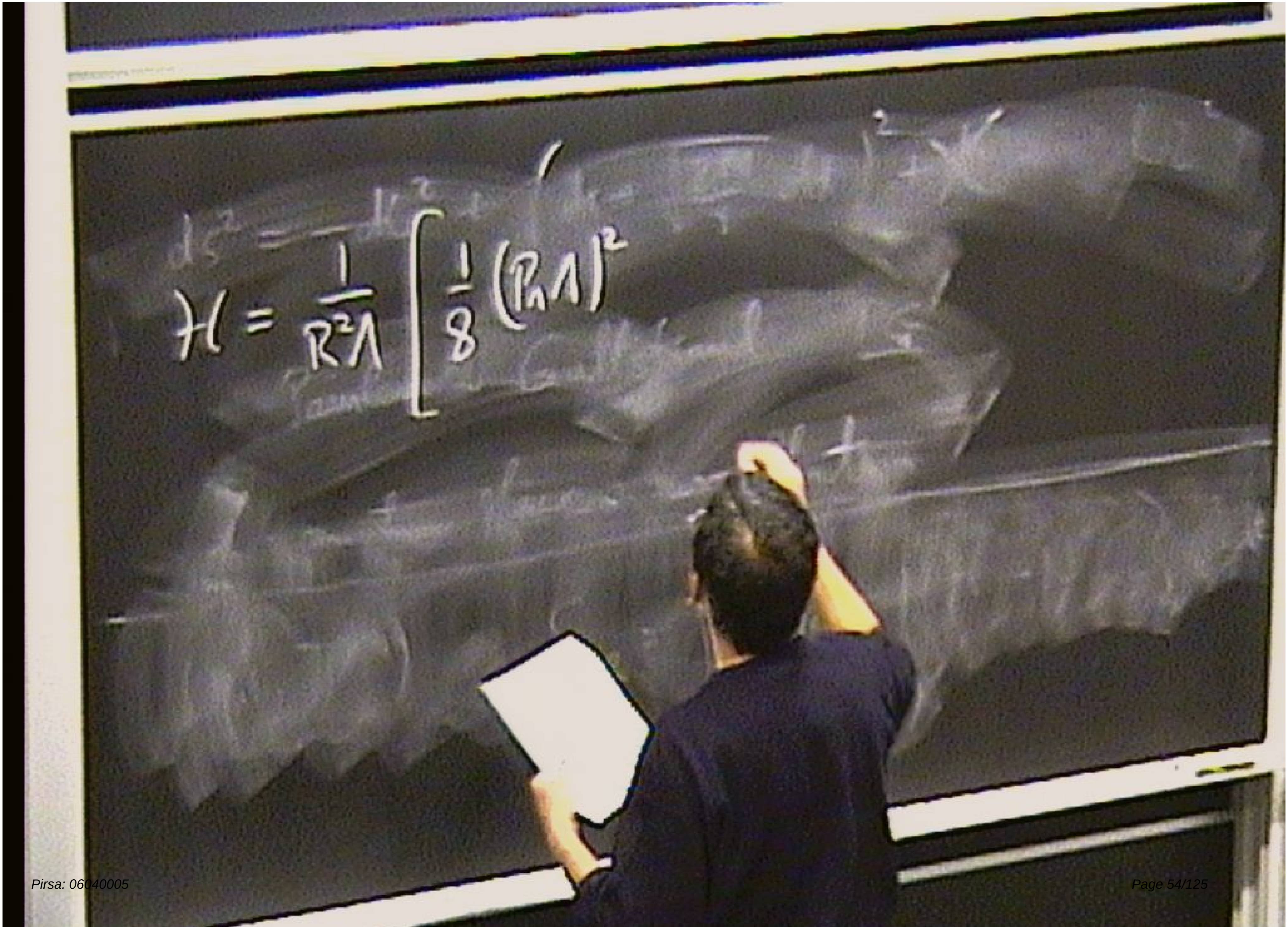
$$C_a = D_b \pi_a^b$$

$$S = \int_M dr\,dt \left[P_R \dot{R} + P_\Lambda \dot{\Lambda} + P_\varphi \dot{\varphi} - N\mathcal{H} - N^r C_r \right] + \int_{\partial M} (\dots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\pi}^{ab} \tilde{\pi}_{ab} - \frac{1}{2} \tilde{\pi}^2 \right) - \sqrt{q}\,{}^{(3)}R + \frac{P_\varphi^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \partial_a \varphi \partial_b \varphi$$

$$\tilde{\pi} = \pi^{ab} q_{ab}$$

$$C_a = D_b \pi_a{}^b$$


$\mathcal{H} = \frac{1}{R^2\Lambda}\left[\frac{1}{8}(P_n\Lambda)^2\right.$

$$\mathcal{H} = \frac{1}{R^2\Lambda}\left[\frac{1}{8}(P_\Lambda \Lambda)^2 - \frac{1}{4}(P_\Lambda\Lambda)(P_R R)\right] + \frac{2}{\Lambda^2}\left[2RR''\Lambda - 2RR'\Lambda' - \Lambda^3 + \Lambda R'^2\right] + \frac{P_\varphi^2}{2\Lambda R^2} + \frac{R^2}{2\Lambda}\varphi'^2 \simeq 0$$

$$\mathcal{H} = \frac{1}{R^2\Lambda}\left[\frac{1}{8}(P_\Lambda\Lambda)^2 - \frac{1}{4}(P_\Lambda\Lambda)(P_R R)\right]$$

$$+ \frac{2}{\Lambda^2}\left[2RR''\Lambda - 2RR'\Lambda' - \Lambda^3 + \Lambda R'^2\right]$$

$$+ \frac{P_\varphi^2}{2\Lambda R^2} + \frac{R^2}{2\Lambda}\varphi'^2 \simeq 0 \qquad ' \equiv \frac{d}{dr}$$

$$C_r = P_R R' - \Lambda P_\Lambda' + P_\varphi \varphi'_{\,\mathrm{OM}}$$

$$\mathcal{H} = \frac{1}{R^2\Lambda}\left[\frac{1}{8}(P_\Lambda \Lambda)^2 - \frac{1}{4}(P_\Lambda\Lambda)(P_R R)\right] + \frac{2}{\Lambda^2}\left[2RR''\Lambda - 2RR'\Lambda' - \Lambda^3 + \Lambda R'^2\right] + \frac{P_\varphi^2}{2\Lambda R^2} + \frac{R^2}{2\Lambda}\varphi'^2 \simeq 0 \qquad ' \equiv \frac{d}{dr}$$

(gr-qc 0503031)

$$C_r = P_R R' - \Lambda P_\Lambda' + P_\varphi \varphi'$$

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

e_{ab} – flat 3 metric

$$S = \int_0^\infty (\,.\;.\,) < \infty$$

$$\Pi^{ab} = \frac{g^{ab}(\theta,\varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

PG ($\varphi=0$)

$$q_{ab} = e_{ab}$$

$$\Pi^{ab} = \frac{C}{r^{3/2}}$$

$$g_{ab} = e_{ab} + \frac{f_{ab}(\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} – flat 3 metric

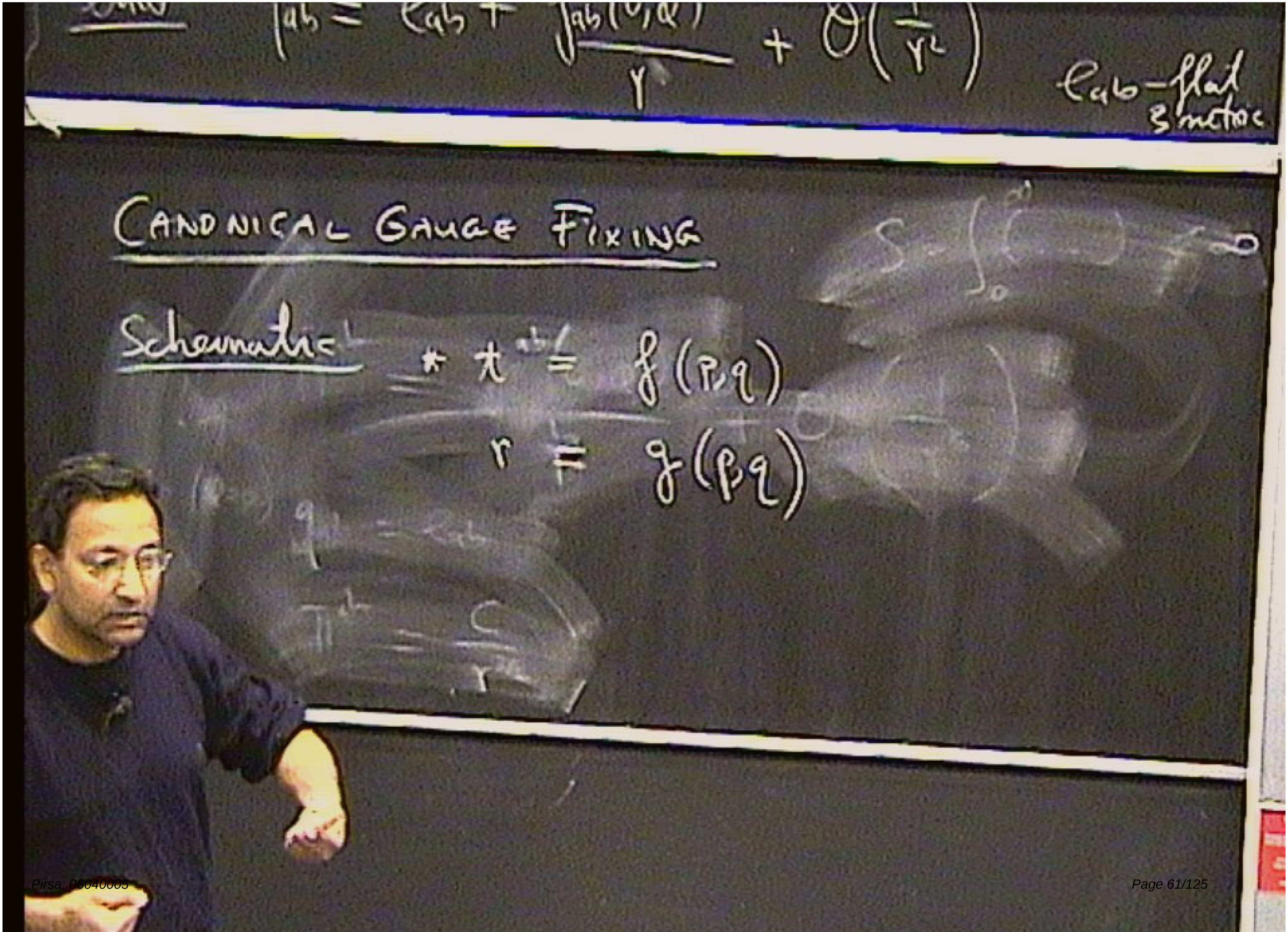
CANONICAL GAUGE FIXING

$$g_{ab} = e_{ab} + \frac{j_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

CANONICAL GAUGE FIXING

Schematic: $\pi^{ab} = f(p,q)$


q_ab = e_ab + f_ab(θ,φ)/r + O(1/r²)
e_ab – flat 3 metric
CANONICAL GAUGE FIXING
Schematic
π^ab = f(p,q)
r = g(p,q)

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

e_{ab} - flat 3 metric

CANONICAL GAUGE FIXING

Schematic

$$* \; t \triangleq f(p,q)$$

$$r = g(p,q)$$

$$\dot{t} = 1 = \left\{ f(p,q), \int N\mathcal{H} + N^a G_a \right\}$$

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\phi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

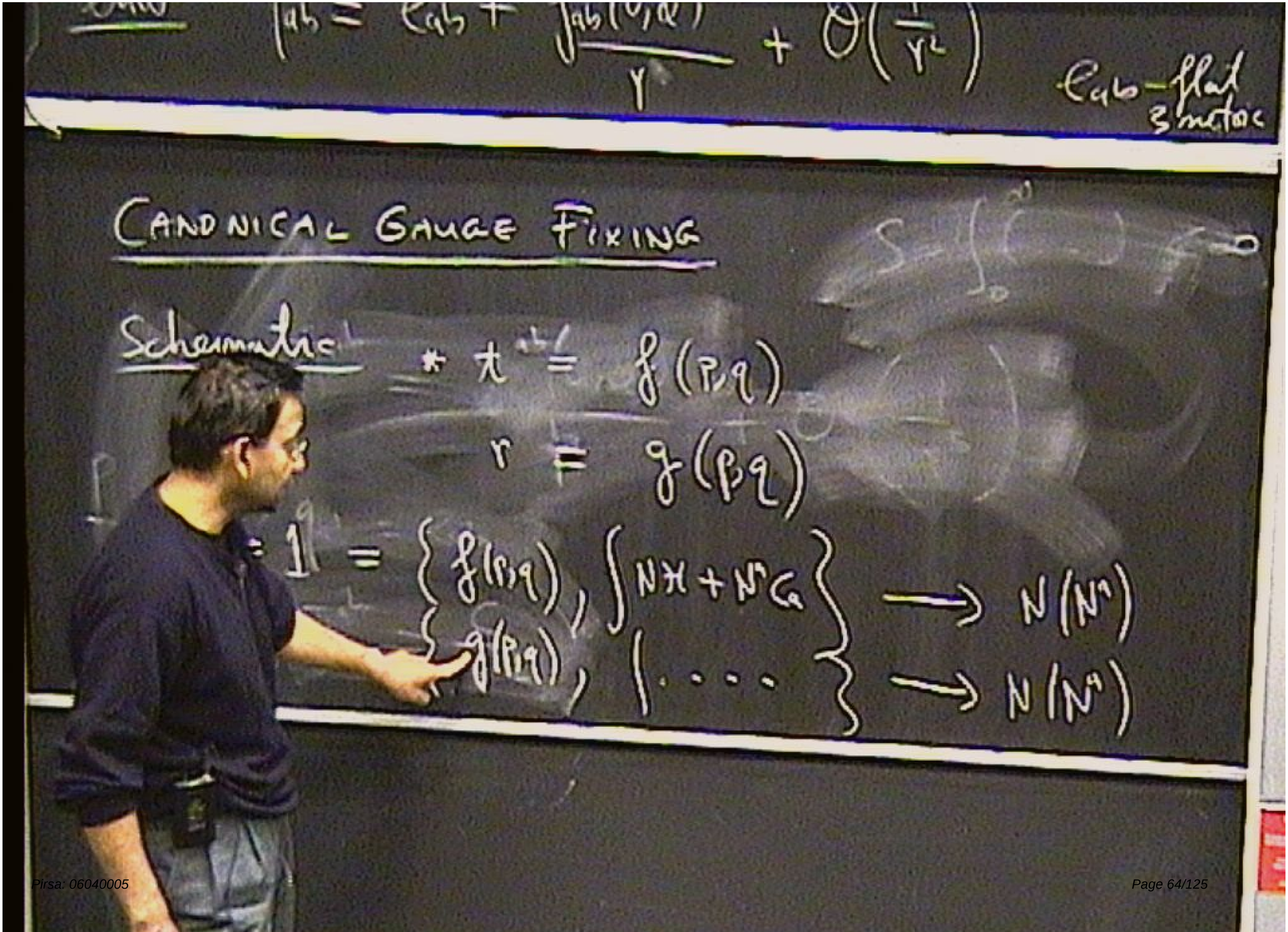
CANONICAL GAUGE FIXING

Schematic

$$* \; t = f(p,q)$$

$$r = g(p,q)$$

$$\dot{t} = 1 = \left\{ f(p,q), \int N\mathcal{H} + N^a G_a \right\} \longrightarrow N(N^a)$$


$g_{ab} = e_{ab} + \frac{f_{ab}(\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right)$
e_{ab} – flat 3 metric
CANONICAL GAUGE FIXING
Schematic
$\pi^{ab} = f(p,q)$
$r = g(p,q)$
$1 = \{f(p,q), \int N\mathcal{H} + N^a C_a\} \longrightarrow N(N^a)$
$\{g(p,q), \int \ldots\} \longrightarrow N(N^a)$

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

e_{ab} – flat 3 metric

CANONICAL GAUGE FIXING

Schematic

$$t = f(p,q)$$

$$r = g(p,q)$$

Soln → $N = ..$
$N^a = ..$

$$\left.\begin{array}{l} \dot{t} = 1 = \left\{ f(p,q), \int N\mathcal{H} + N^a C_a \right\} \longrightarrow N(N^a) \\ \dot{r} = 1 = \left\{ g(p,q), \int \dots \right\} \longrightarrow N(N^a) \end{array}\right\}$$

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

Canonical Gauge Fixing

Schematic

$$* \; t = f(p,q)$$

$$r = g(p,q)$$

Sln → $N = \ldots$, $N^a = \ldots$

$$\dot{t} = 1 = \left\{ f(p,q), \int N\mathcal{H} + N^a G_a \right\} \longrightarrow N(N^a)$$

$$r' = 1 = \left\{ g(p,q), \left(\ldots \right. \right\} \longrightarrow N(N^a)$$

$$\ldots_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} – flat 3 metric

CANONICAL GAUGE FIXING

Schematic

$$* \ t = f(p,q)$$

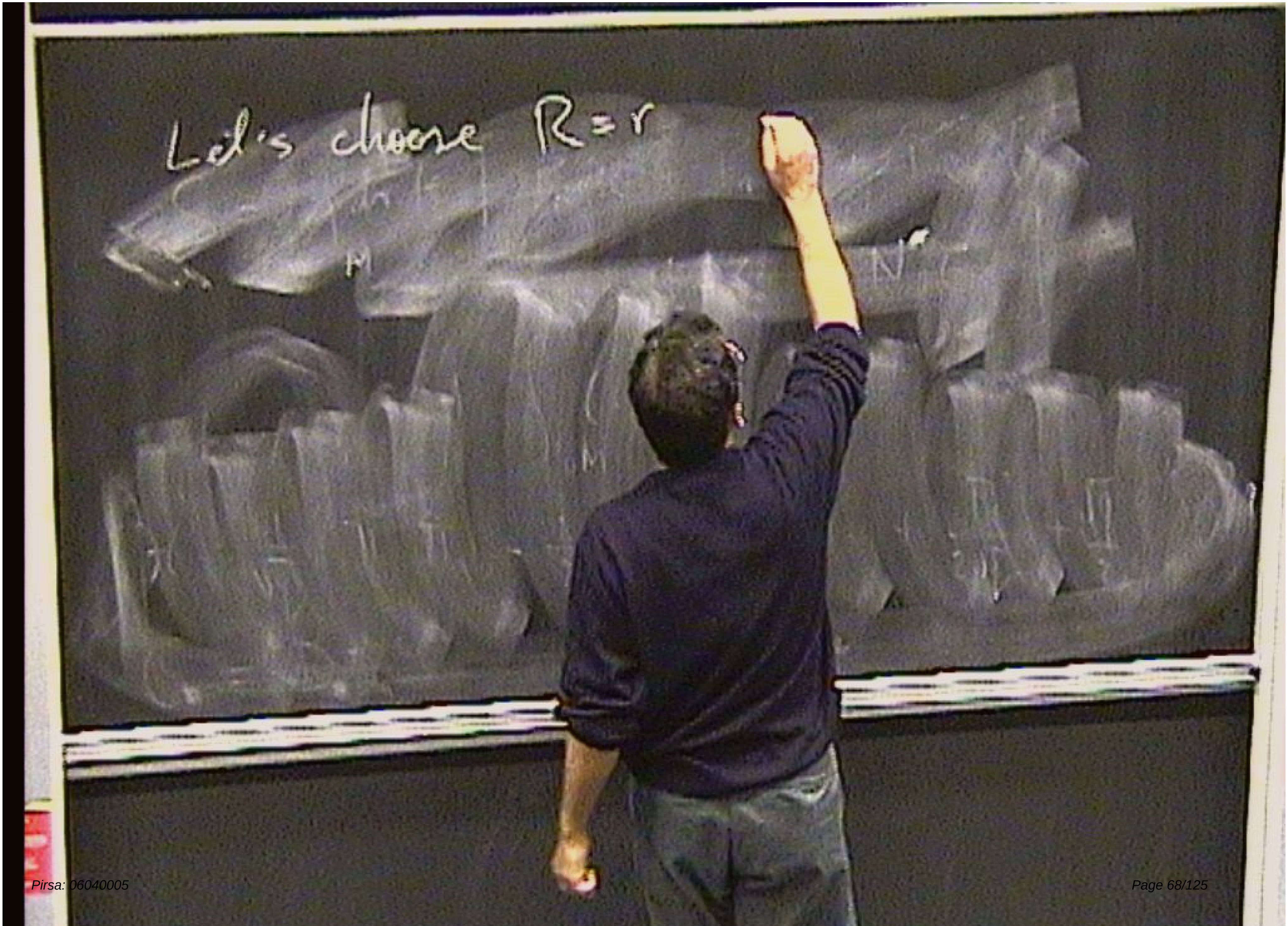
$$r = g(p,q)$$

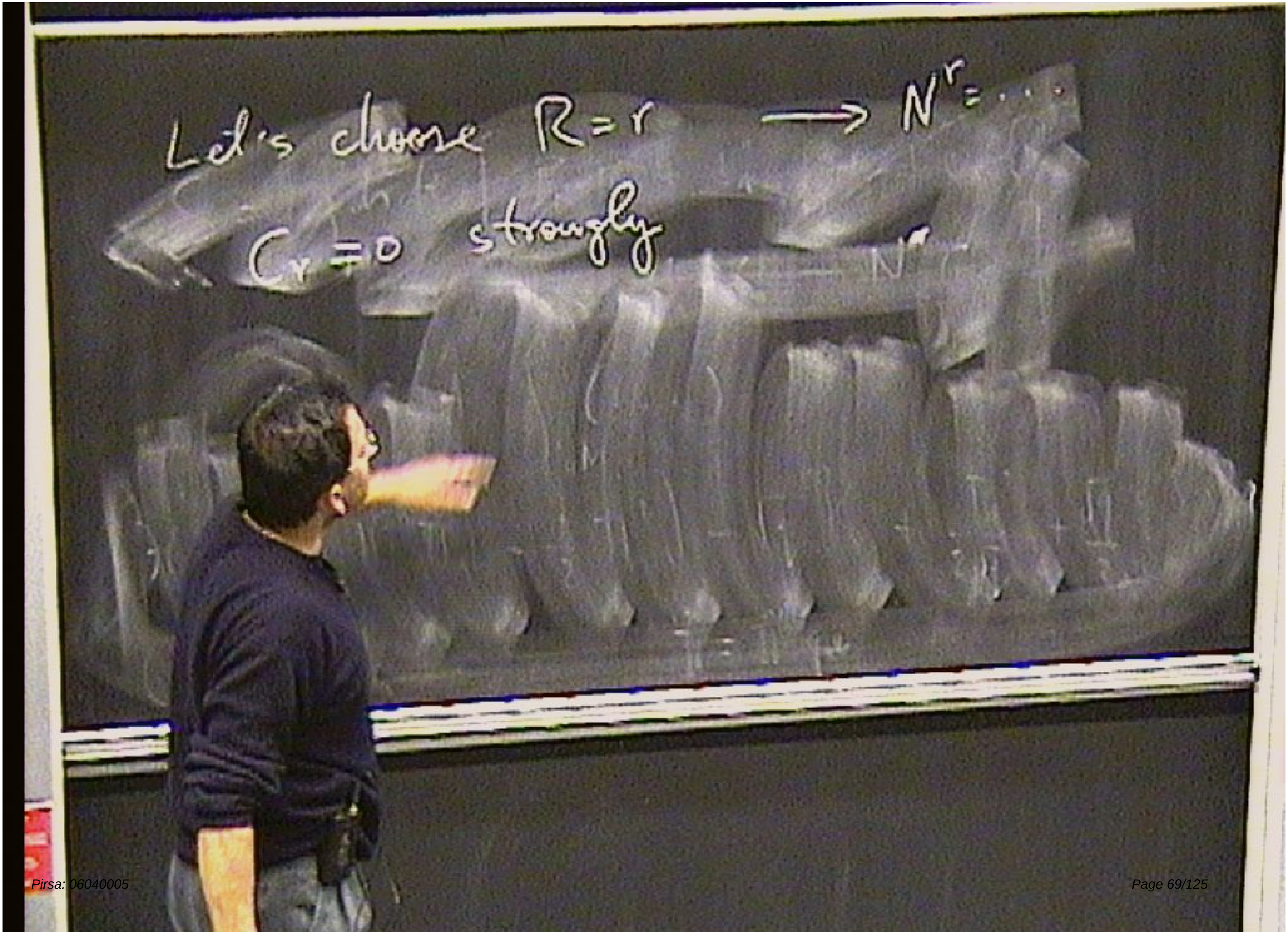
$$\dot{t} = 1 = \left\{ f(p,q), \int N\mathcal{H} + N^a C_a \right\} \longrightarrow N(N^a)$$

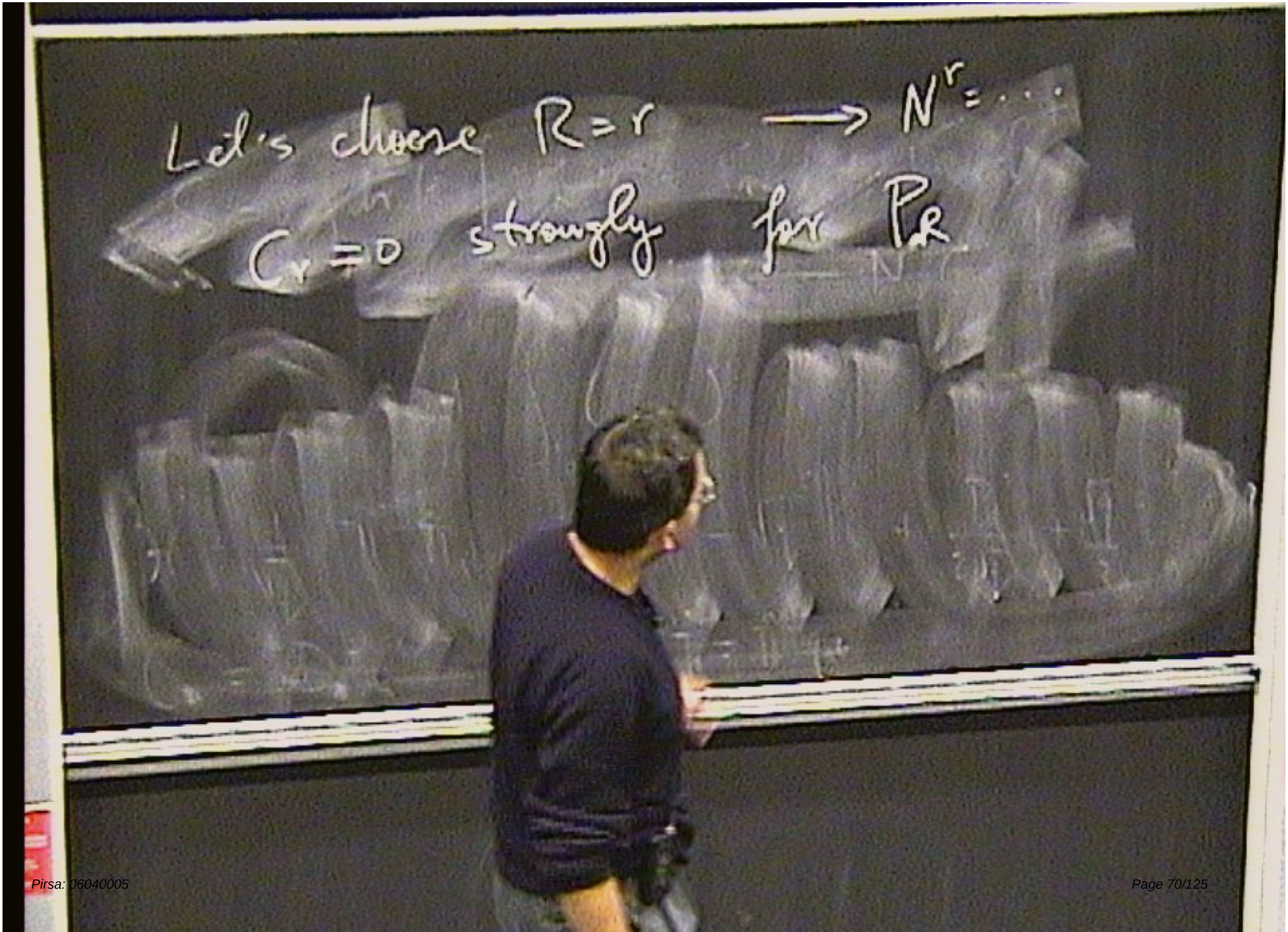
$$\dot{r} = 0 = \left\{ g(p,q), \int \ldots \right\} \longrightarrow N(N^a)$$

Sln $\to N = \ldots$

$N^a = \ldots$


Let's choose R=r

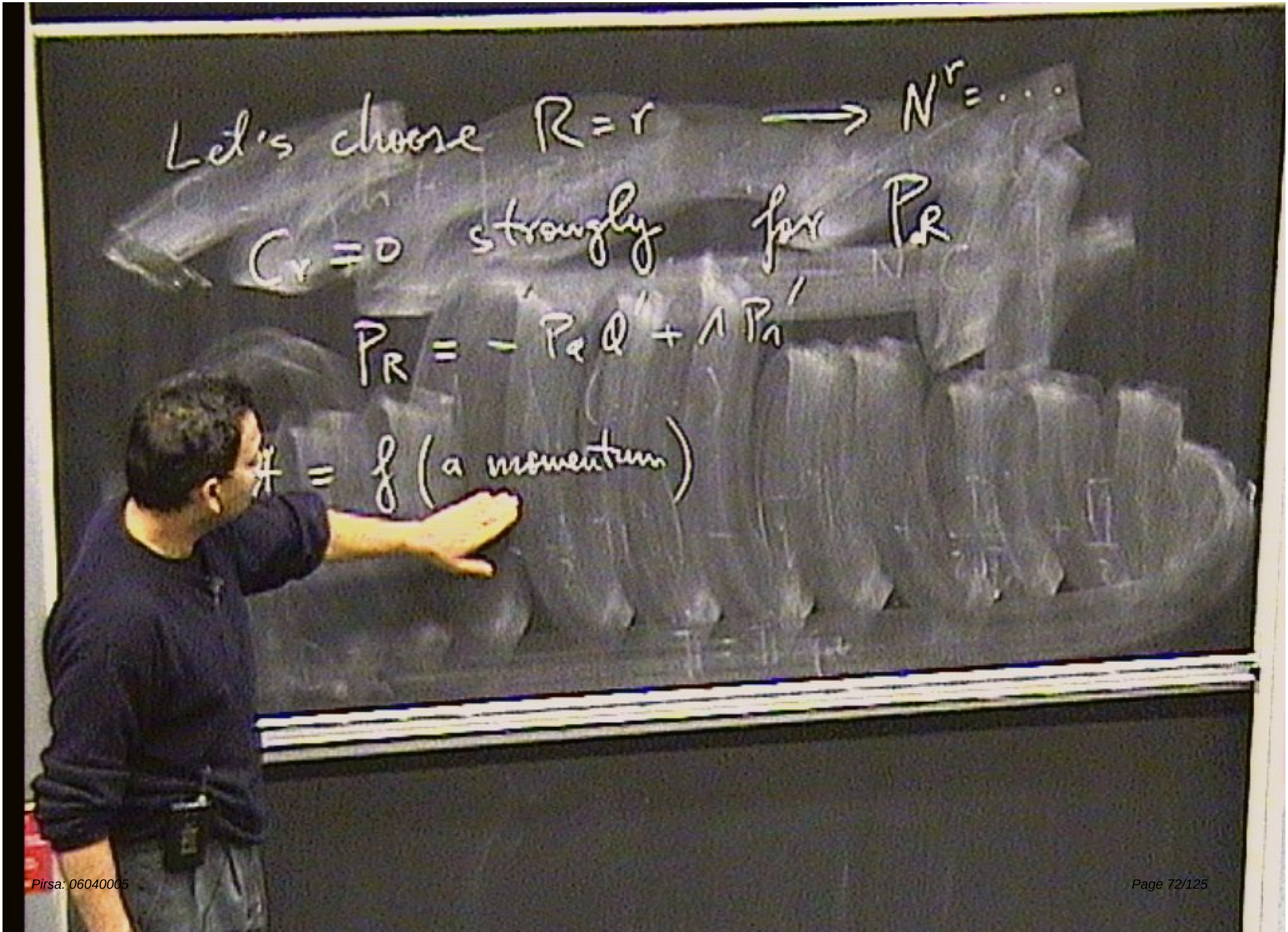

Let's choose R=r ⟶ N^r = . . .
C_r = 0 strongly


Let's choose R = r ⟶ N^r = . . .
C_r = 0 strongly for P_R

Let's choose $R = r \longrightarrow N^r = \ldots$

$C_r = 0$ strongly for P_R

$P_R = -P_e Q' + \Lambda P_1'$


Let's choose R = r ⟶ N^r = . . .
C_r = 0 strongly for P_R
P_R = − P_e Q' + Λ P_r'
# = f (a momentum)

Let's choose $R = r \longrightarrow N^r = \ldots$

$C_r = 0$ strongly for P_R

$$P_R = - P_e Q' + \Lambda P_1'$$

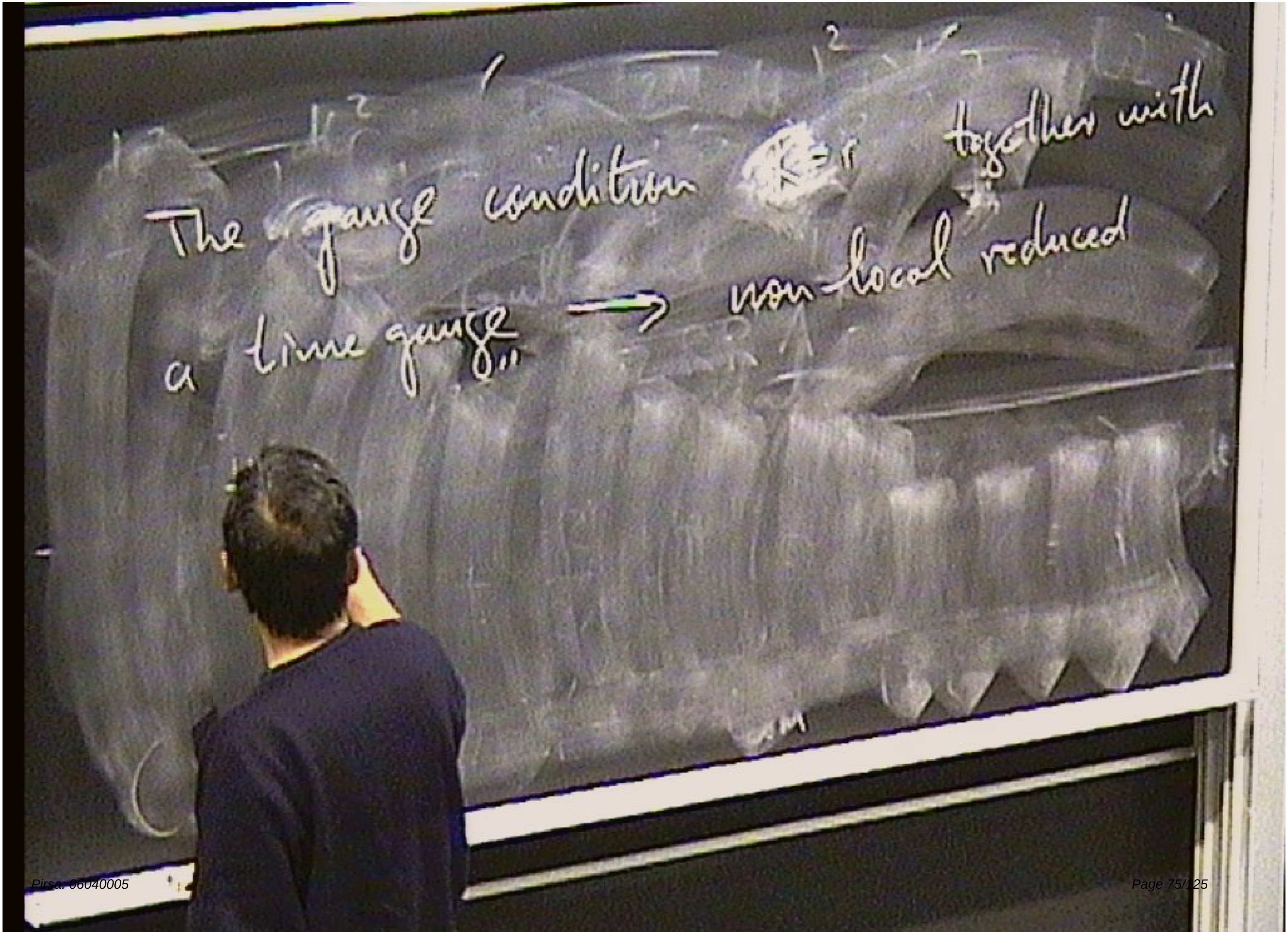
$$t = f(\text{a momentum})$$

Let's choose $\underline{R = r} \longrightarrow N^r = \ldots$

$C_r = 0$ strongly for P_R

$$P_R = -P_\varphi Q' + \Lambda P_\Lambda'$$

$$t = f(\text{a momentum})$$


The gauge condition K = r together with
a time gauge.. → non-local reduced

The gauge condition $R=r$ together with a time gauge $\longrightarrow$ non-local reduced Hamiltonian

* Do not fix $R=r$. Let R remain dynamical variable

The gauge condition $R = r$ together with a time gauge.. $\longrightarrow$ non-local reduced Hamiltonian

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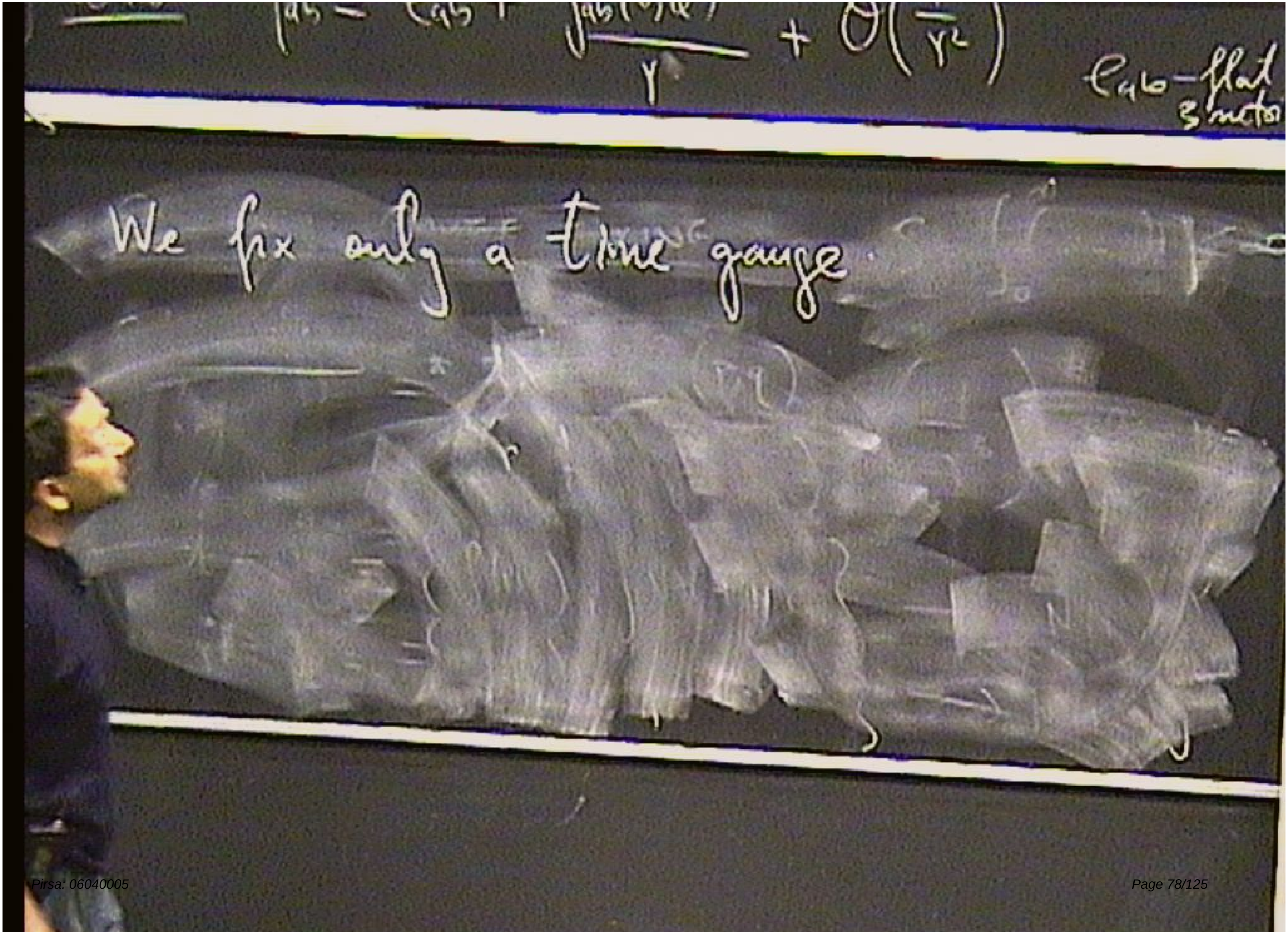

We fix only a time gauge

$$g_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} – flat 3 metric

We fix only a time gauge.

$N = 1$

Solve $\mathcal{H} = 0$ for p_r.

$$g_{ab} = e_{ab} + \frac{f_{ab}(\ldots)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

We fix only a time gauge

$N = 1$

Solve $H = 0$ for P_r —— a quadratic eqn for P_r

$$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$$

e_{ab} — flat 3 metric

We fix only a time gauge

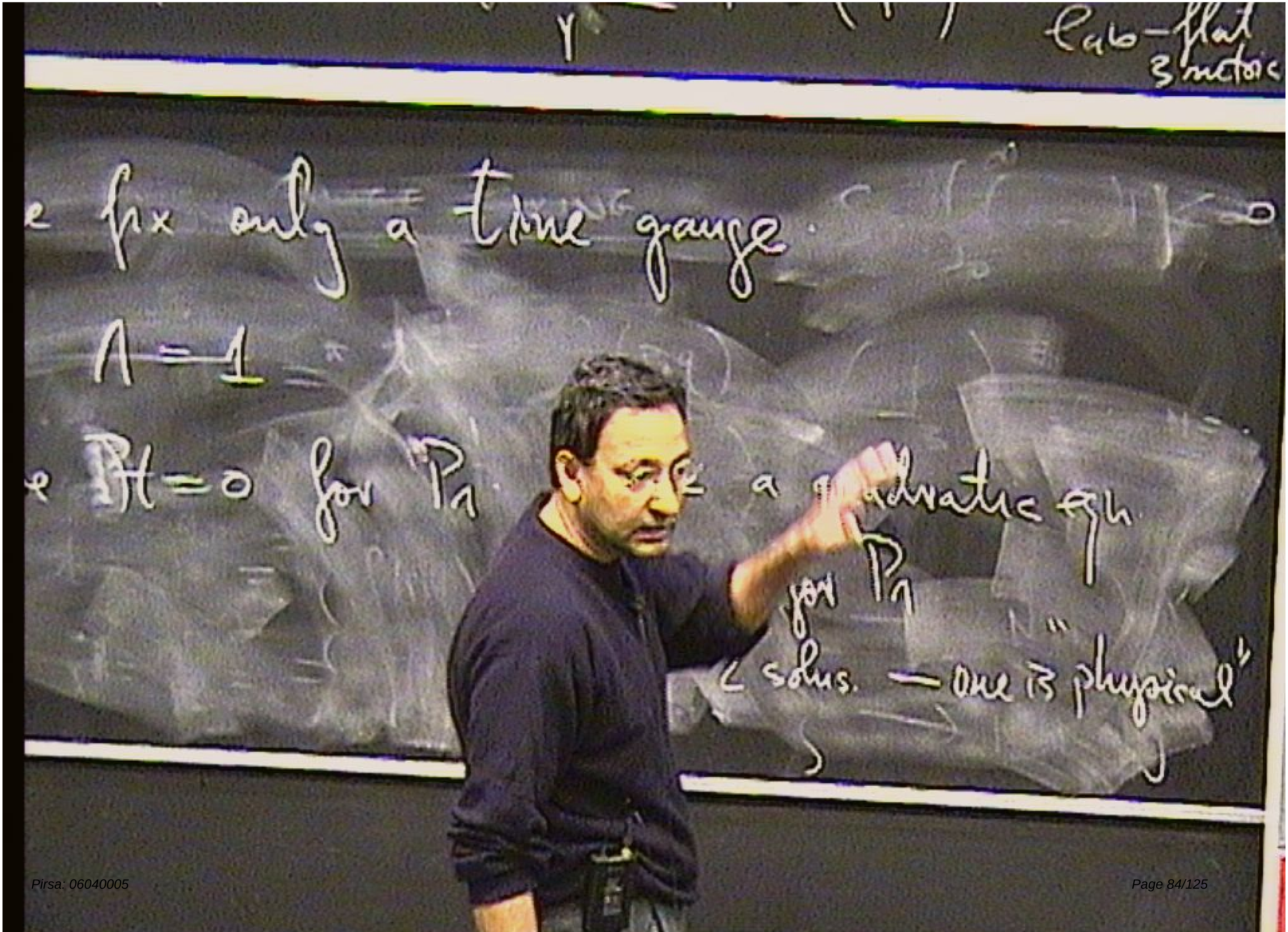
$N = 1$

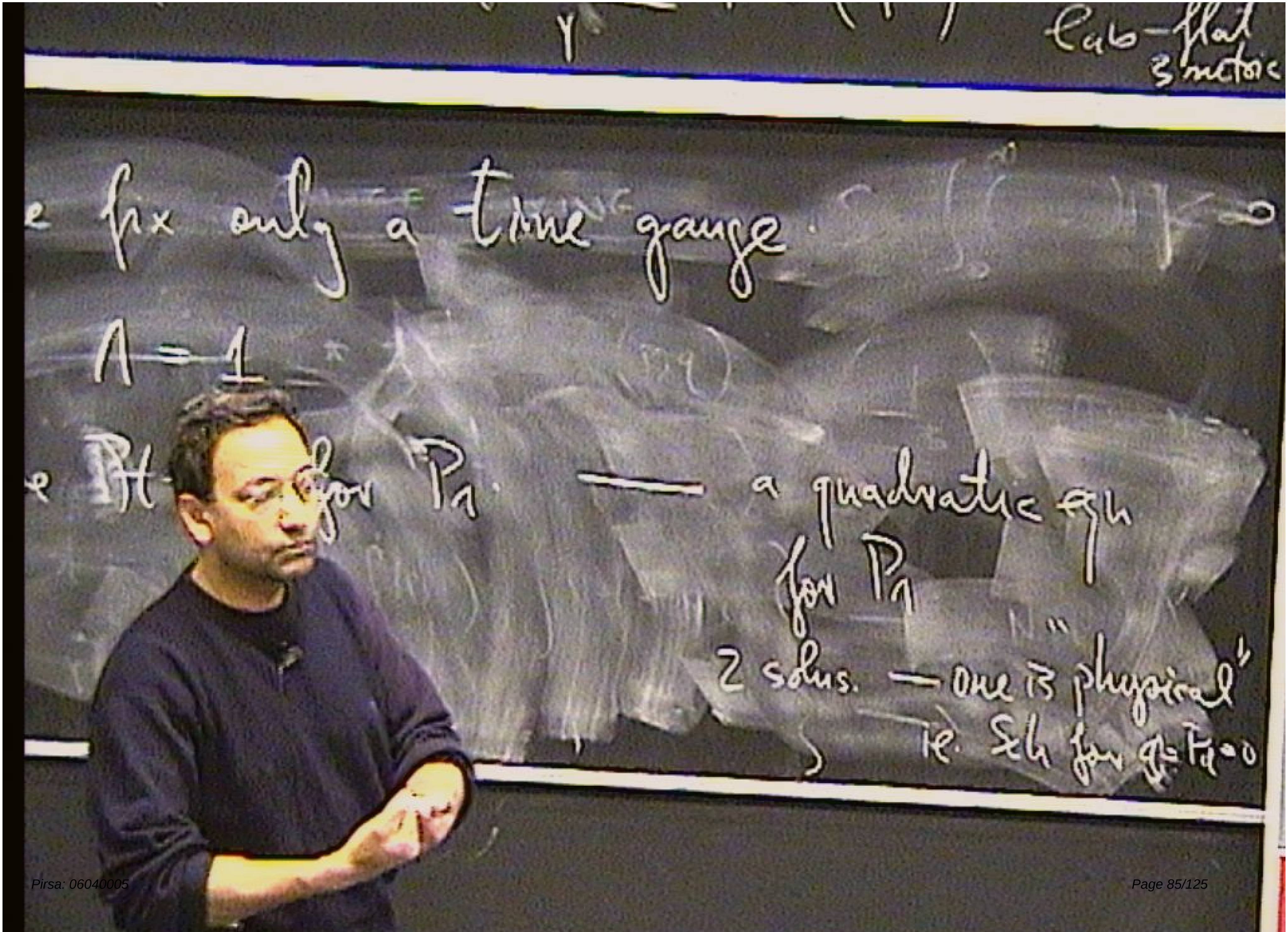
Solve $H = 0$ for P_r —— a quadratic eqn for P_r

$$S = \int dr\,dt \left[\dot{r} P_r + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt \,(\ldots)$$

$$S = \int dr\,dt \left[\dot{\Lambda} P_\Lambda + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt\,(\ldots)$$

$$S^{GF} = \int dr\,dt \left[\dot{\varphi} P_\varphi + \dot{R} P_R \right.$$


e fix only a time gauge
Λ = 1
e H = 0 for P1
a quadratic eqn
for P1
2 solns. — one is "physical"


flat
3 metric
fix only a time gauge
A = 1
for P1 — a quadratic eqn
for P1
2 solns. — one is "physical"

$$S = \int dr\,dt \left[\dot{\Lambda} P_\Lambda + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt \, (\cdots)$$

$$S^{GR} = \int dr\,dt \left[\dot{\varphi} P_\varphi + \dot{R} P_R - N^r C_r \right] + \int dt \, (\cdots$$

$$S = \int dr\,dt \left[\dot{\Lambda} P_\Lambda + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt\,(\ldots)$$

$$S^{GF} = \int dr\,dt \left[\dot{\varphi} P_\varphi + \dot{R} P_R - N^r C_r \right] + \int dt\,(\ldots)$$

$$S = \int dr\,dt \left[\dot{\Lambda} P_\Lambda + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt\,(\ldots)$$

$$S^{GF} = \int dr\,dt \left[\dot{\varphi} P_\varphi + \dot{R} P_R - N^r C_r \right] + \int dt\,(\ldots)$$

$$C_r \equiv P_R R' + P_\varphi \varphi' - \Lambda P_\Lambda' \approx 0$$

$$S = \int dr\,dt \left[\dot{\Lambda} P_\Lambda + \dot{\varphi} P_\varphi + \dot{R} P_R - N\mathcal{H} - N^r C_r \right] + \int dt\,(\ldots)$$

$$S^{GF} = \int dr\,dt \left[\dot{\varphi} P_\varphi + \dot{R} P_R - N^r C_r^{GF} \right] + \int dt\,(\ldots)$$

$$C_r \equiv P_R R' + P_\varphi \varphi' - \Lambda P_\Lambda' \approx 0 \qquad C_r^{GF} = P_R R' + P_\varphi \varphi' - P_\Lambda'(R, P_R, \varphi, P_\varphi)$$

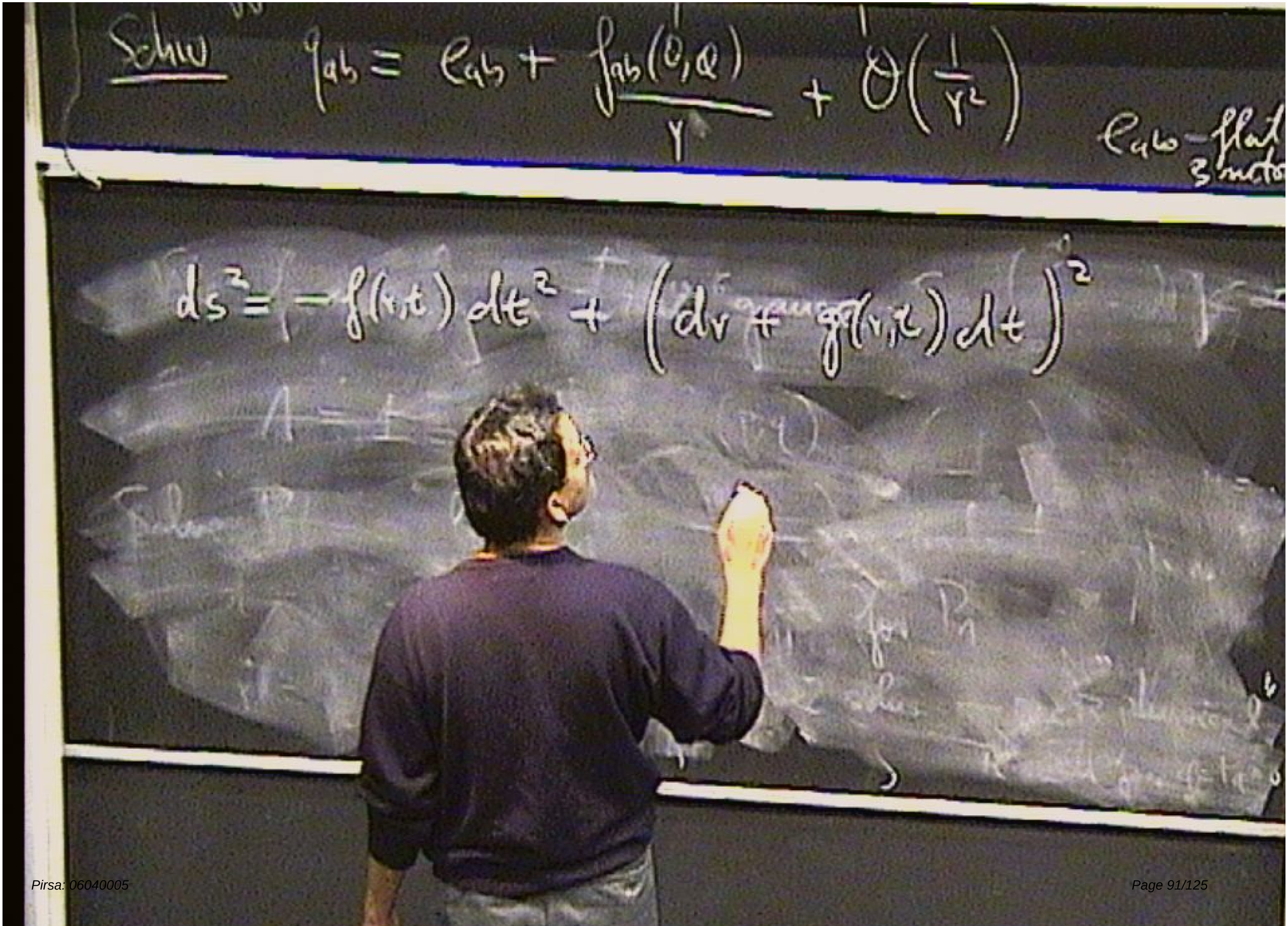
X Let's choose $R = r$ $\longrightarrow$ $N^r = \dots$

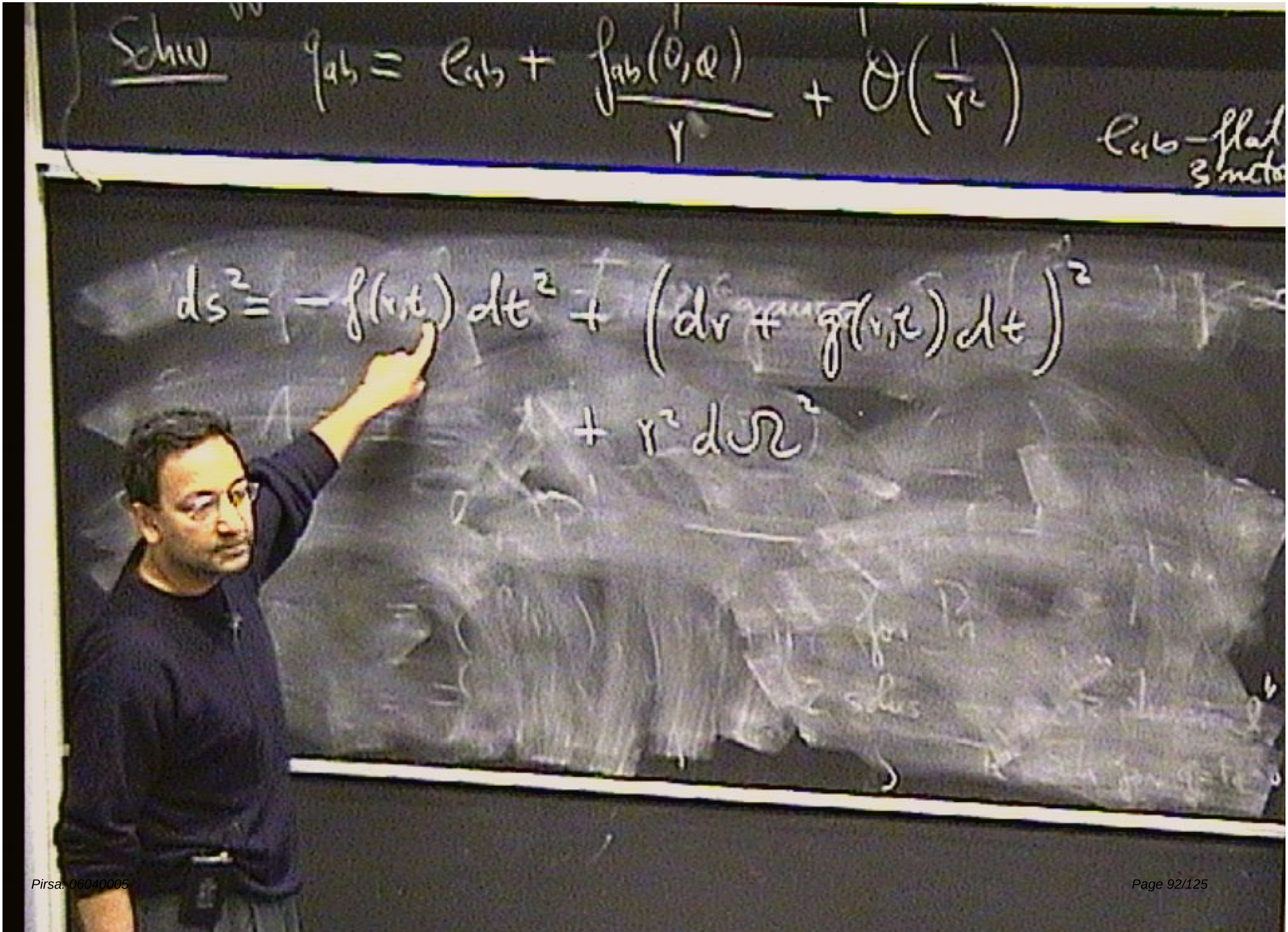
$C_r = 0$ strongly for P_R

$$P_R = -P_Q Q' + \Lambda P_\Lambda'$$

QUANTISE

$$t = f(\text{a momentum})$$


Schw
$g_{ab} = e_{ab} + \frac{f_{ab}(\theta,\phi)}{r} + O\left(\frac{1}{r^2}\right)$
e_{ab} – flat
$ds^2 = -f(r,t)\,dt^2 + \left(dr + g(r,t)\,dt\right)^2$


Schw
$q_{ab} = e_{ab} + \frac{f_{ab}(\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right)$
e_{ab} – flat 3 metric
$ds^2 = -f(r,t)\,dt^2 + \left(dr + g(r,t)\,dt\right)^2 + r^2 d\Omega^2$

X Let's choose $R = r$ $\longrightarrow N^r = \ldots$

$C_r = 0$ strongly for P_R

$$P_R = -P_a Q' + \Lambda P_\Lambda'$$

$$A = f\,(\text{a momentum})$$

QUANTISE

S^{GF}

$(a, P_a)\ (R, P_R)$

$C_r \simeq 0$

Schw $\quad g_{ab} = e_{ab} + \dfrac{f_{ab}(\theta,\varphi)}{r} + O\left(\dfrac{1}{r^2}\right)$

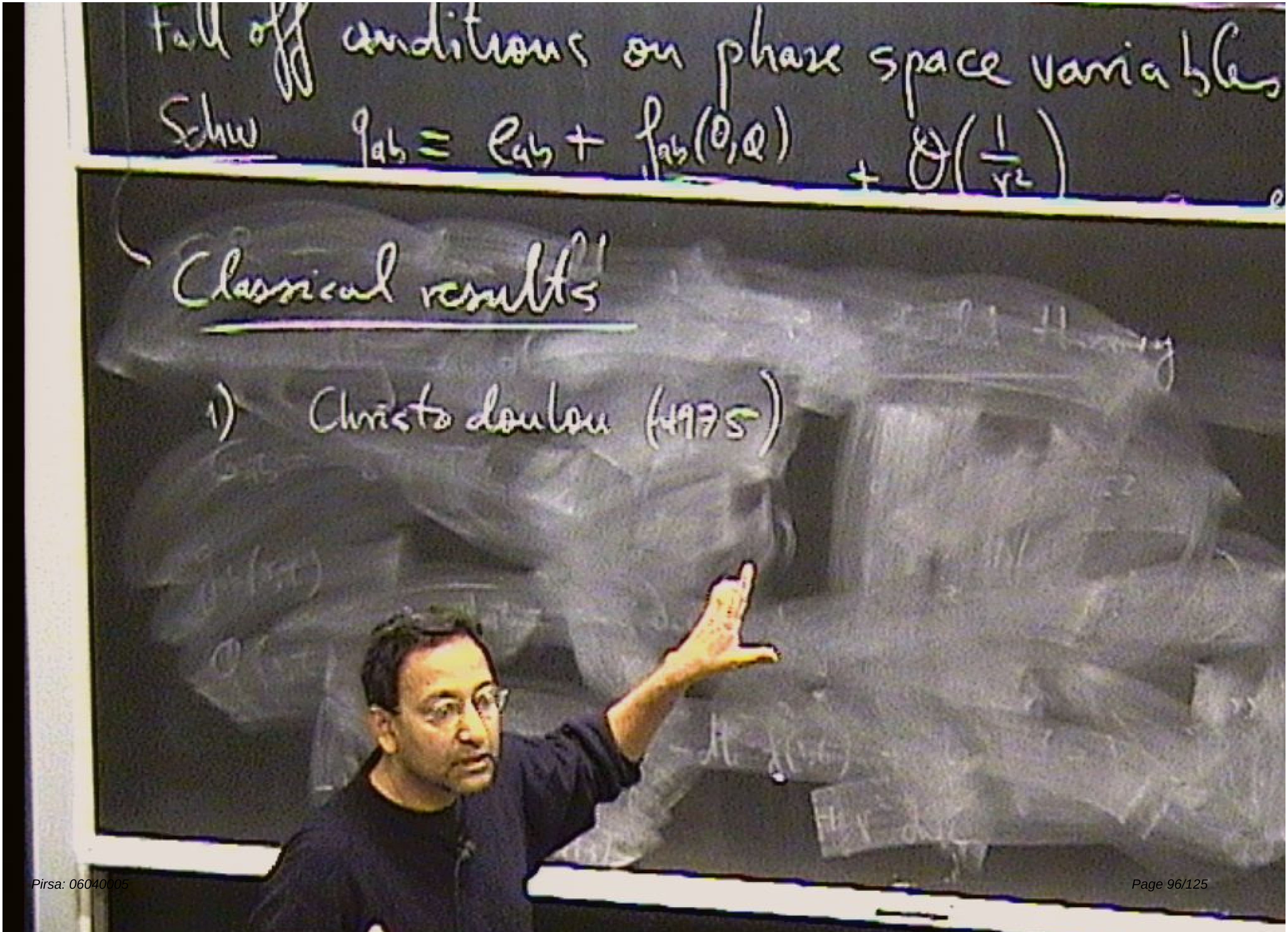
$$ds^2 = -f(r,t)\,dt^2 + \left(dr + g(r,t)\,dt\right)^2 + r^2\,d\Omega^2$$

$$ds^2 = -f^2(r,t)\,dt^2 + g^2(r,t)\,dr^2 + r^2\,d\Omega^2$$

Fall off conditions on phase space variables

Schw $\quad g_{ab} = e_{ab} + f_{ab}(\theta,\varphi) + \mathcal{O}\left(\frac{1}{r^2}\right)$

Classical results!


fall off conditions on phase space variables
Schw
$q_{ab} = e_{ab} + f_{ab}(\theta,\varphi) + O\left(\frac{1}{r^2}\right)$
Classical results!
1) Christodoulou (1975)

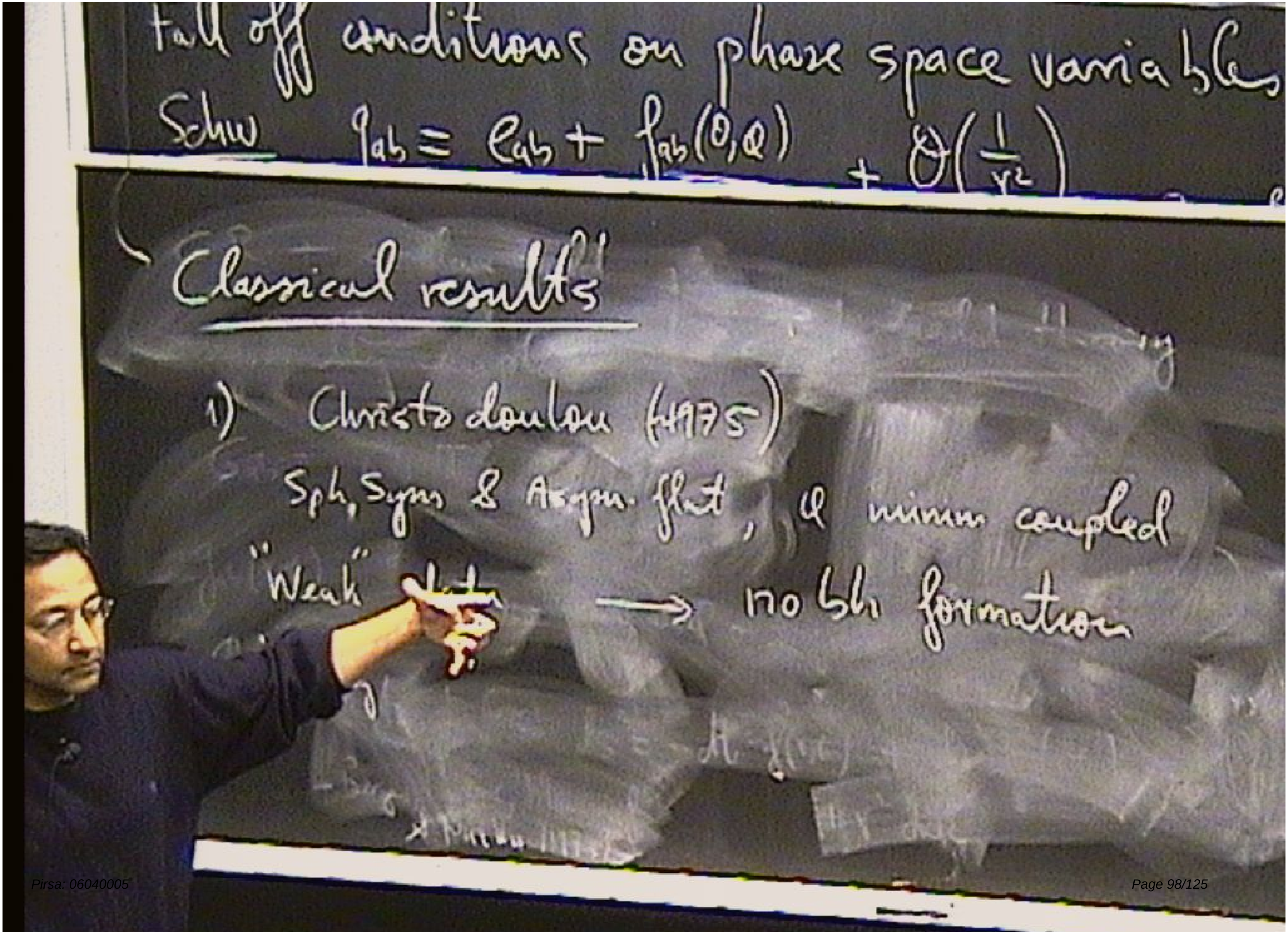
fall off conditions on phase space variables

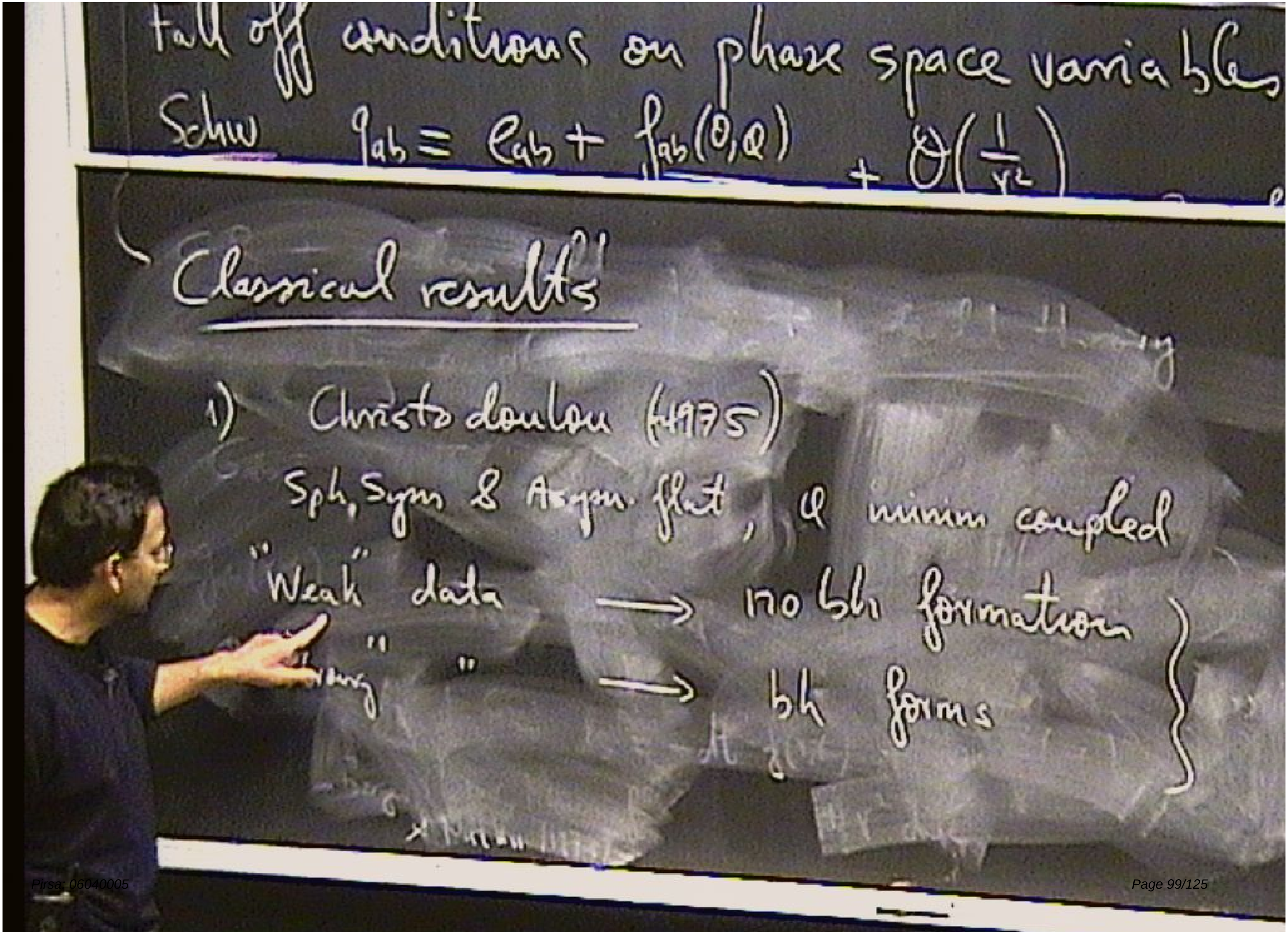
Schw $q_{ab} = e_{ab} + f_{ab}(\theta,\varphi) + \mathcal{O}\left(\frac{1}{r^2}\right)$

Classical results!

1) Christodoulou (+1975)

Sph. Sym & Asym. flat, φ minim coupled


fall off conditions on phase space variables
Schw $q_{ab} = e_{ab} + f_{ab}(\theta,\varphi) + O\left(\frac{1}{r^2}\right)$
Classical results
1) Christodoulou (1975)
Sph. Sym. & Asym. flat, φ minim coupled
"Weak" data → no bh formation


fall off conditions on phase space variables
Schw qab ≡ eab + fab(θ,φ) + O(1/r²)
Classical results
1) Christodoulou (~1975)
Sph. Sym & Asym. flat, φ minim coupled
"Weak" data → no bh formation
"Strong" " → bh forms

Fall off conditions on phase space variables

Schw $\quad q_{ab} = e_{ab} + f_{ab}(\theta, \varphi) + \mathcal{O}\left(\frac{1}{r^2}\right)$

"Classical results"

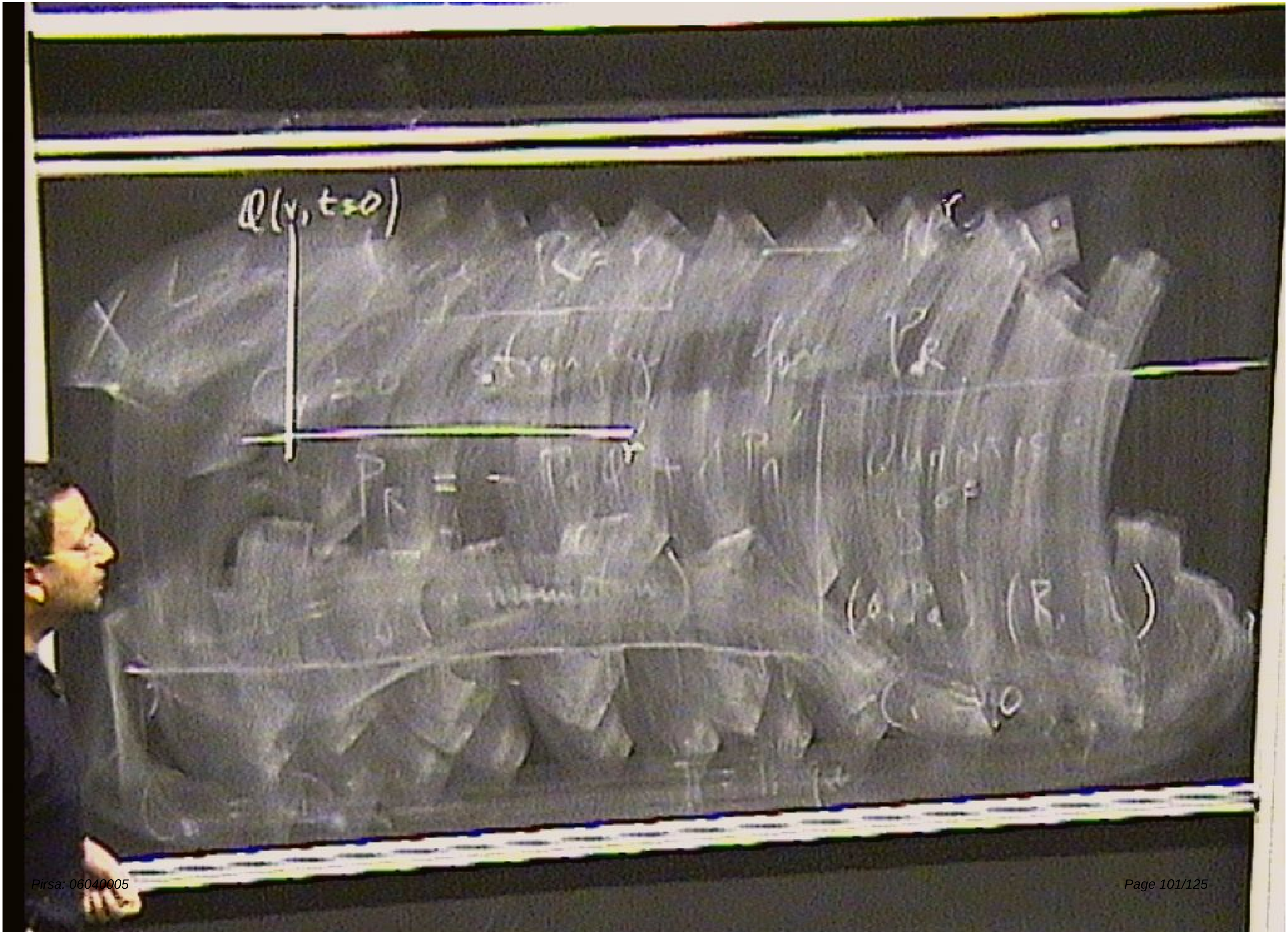
1) Christodoulou (1975)

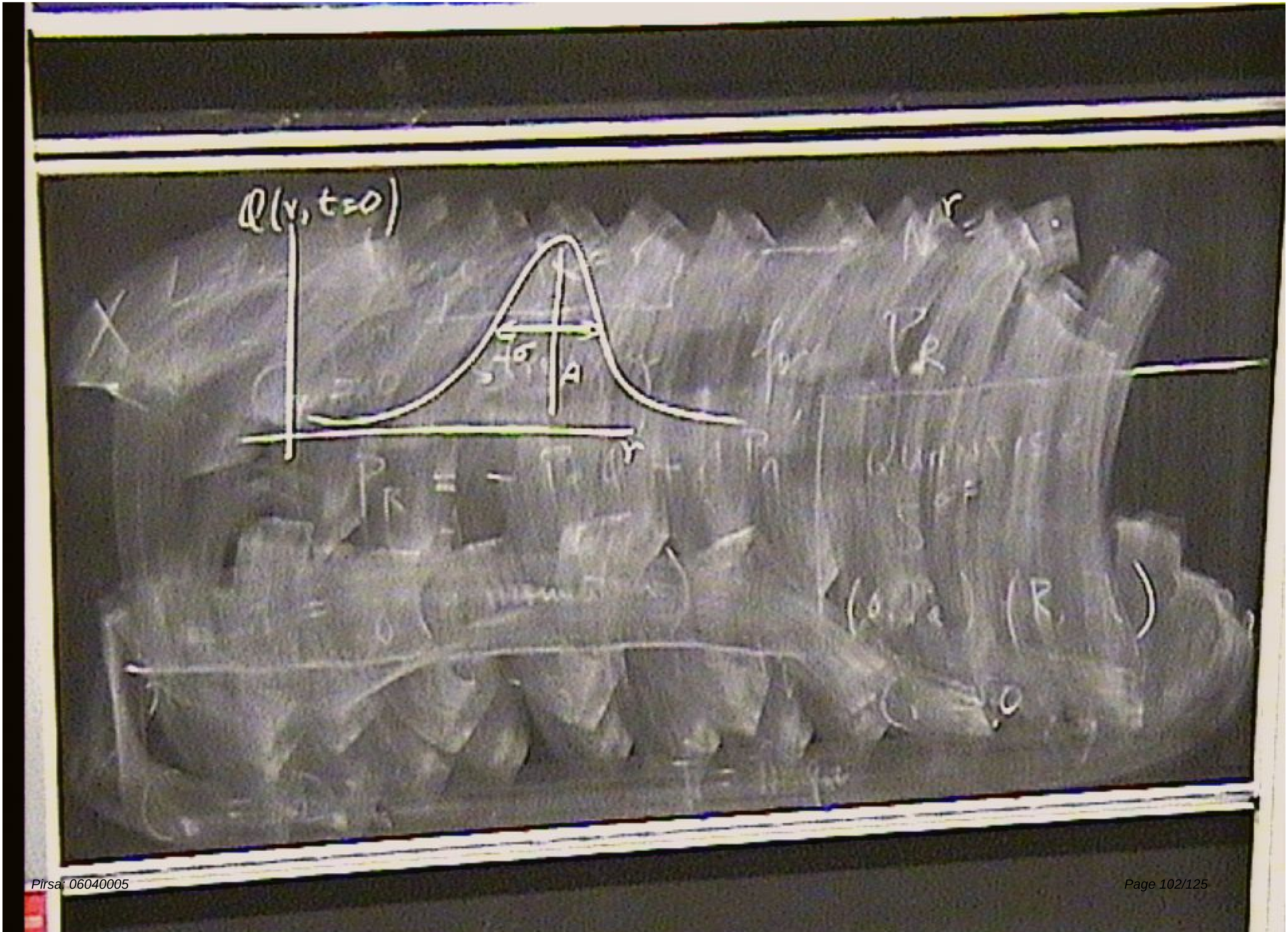
Sph. Sym & Asym. flat, φ minim coupled

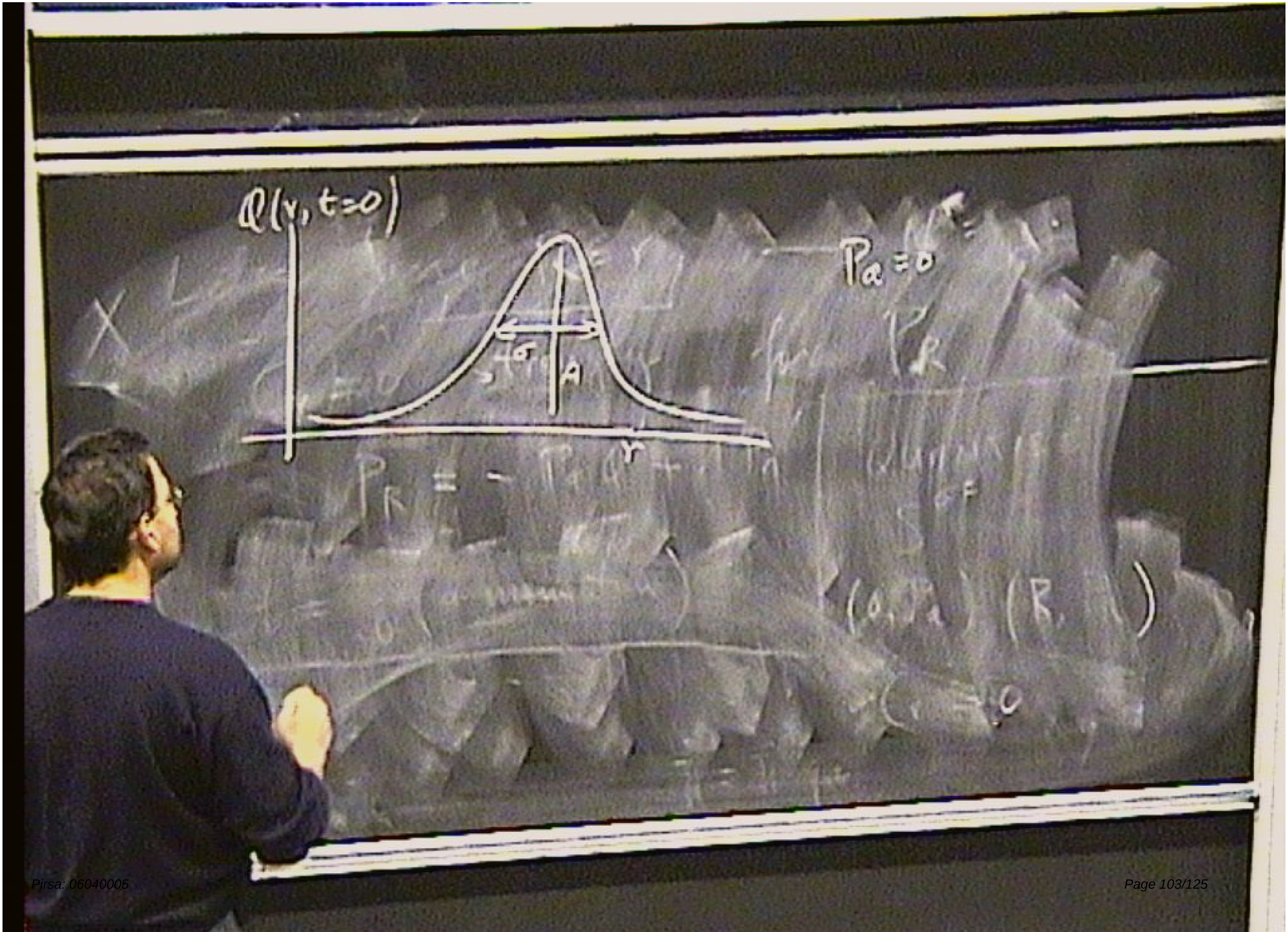
"Weak" data $\longrightarrow$ no bh formation

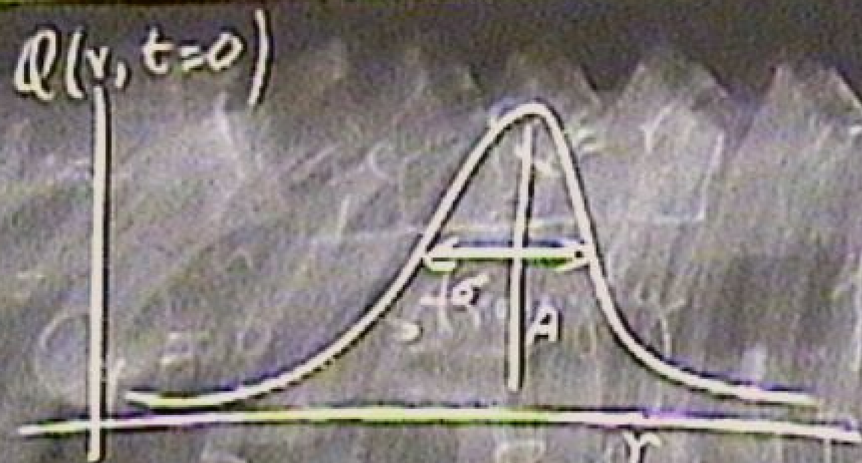
"Strong" " $\longrightarrow$ bh forms

What is the transition?


Q(r, t=0)


ρ(r, t=0)
r


Q(r, t=0)
Pa=0
4σ
A
r

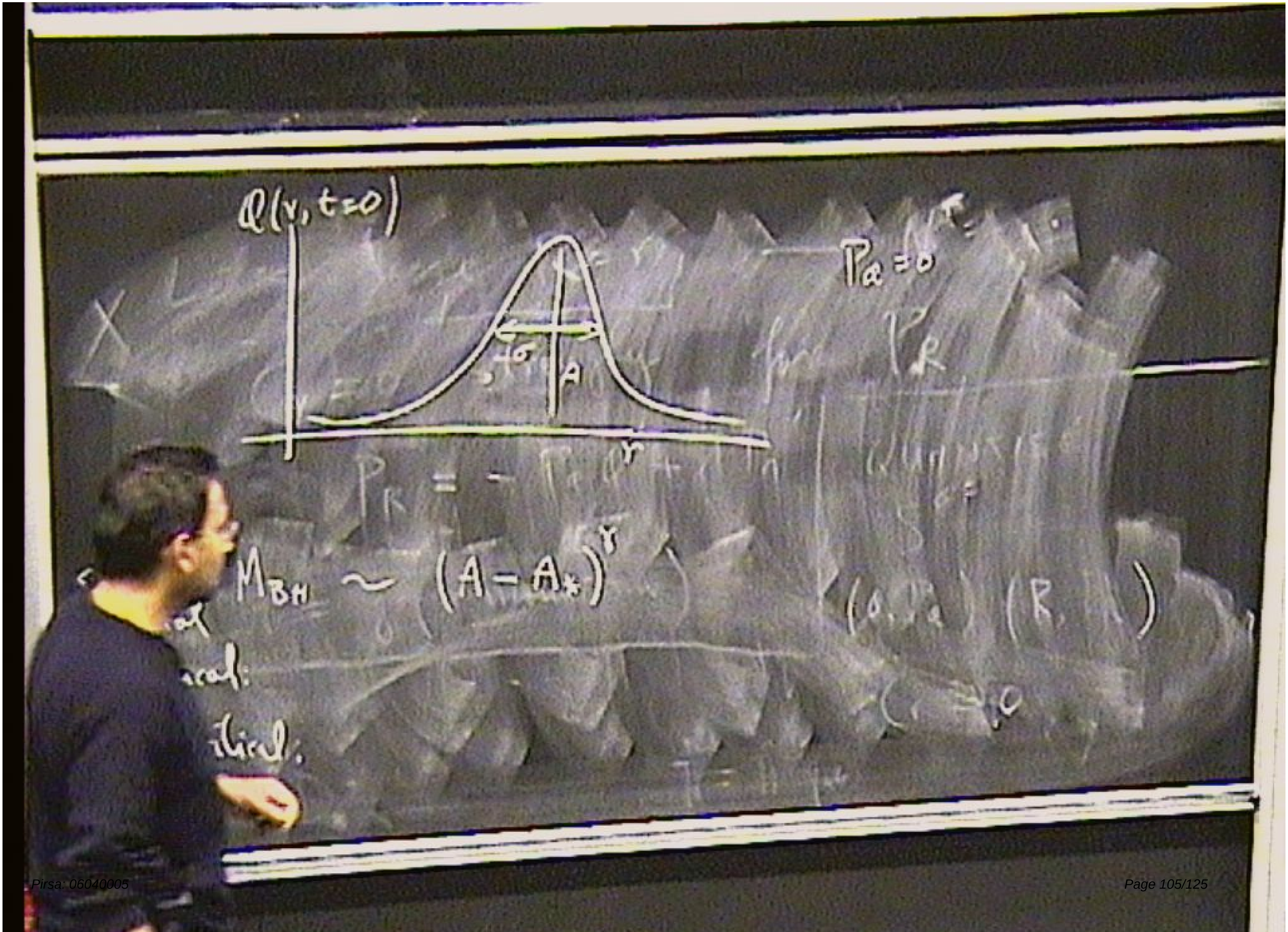


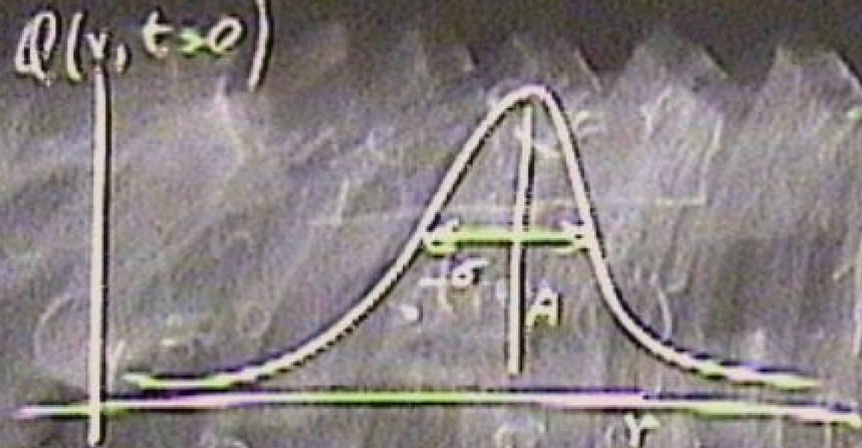
$P_\phi = 0$

P_R

$M_{BH} \sim (A - A_*)^\gamma$

$(0, P_R)$ $(R, \ )$


φ(r, t=0)
2σ
A
r
P_R = 0
M_BH ~ (A − A_*)^γ



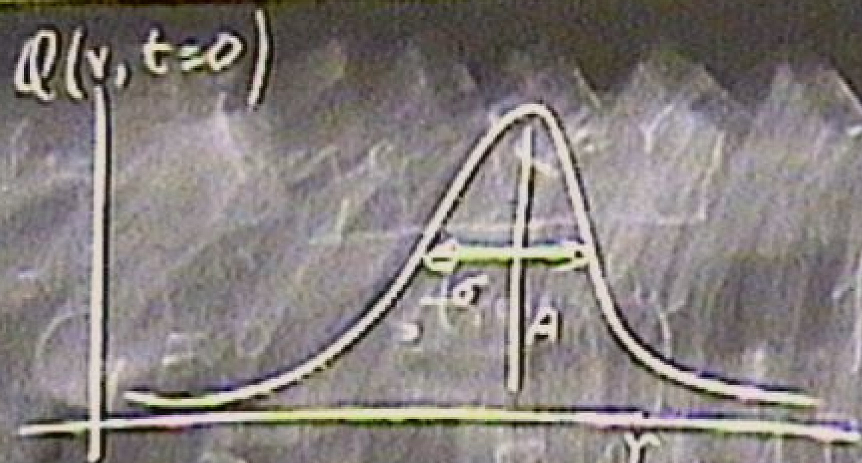
$\Pi_a = 0$

Super critical: $M_{BH} \sim (A - A_*)^\gamma$

Critical: Naked Singularity.

Sub critical: Scalar field scatters.

A_*, γ computed numerically.



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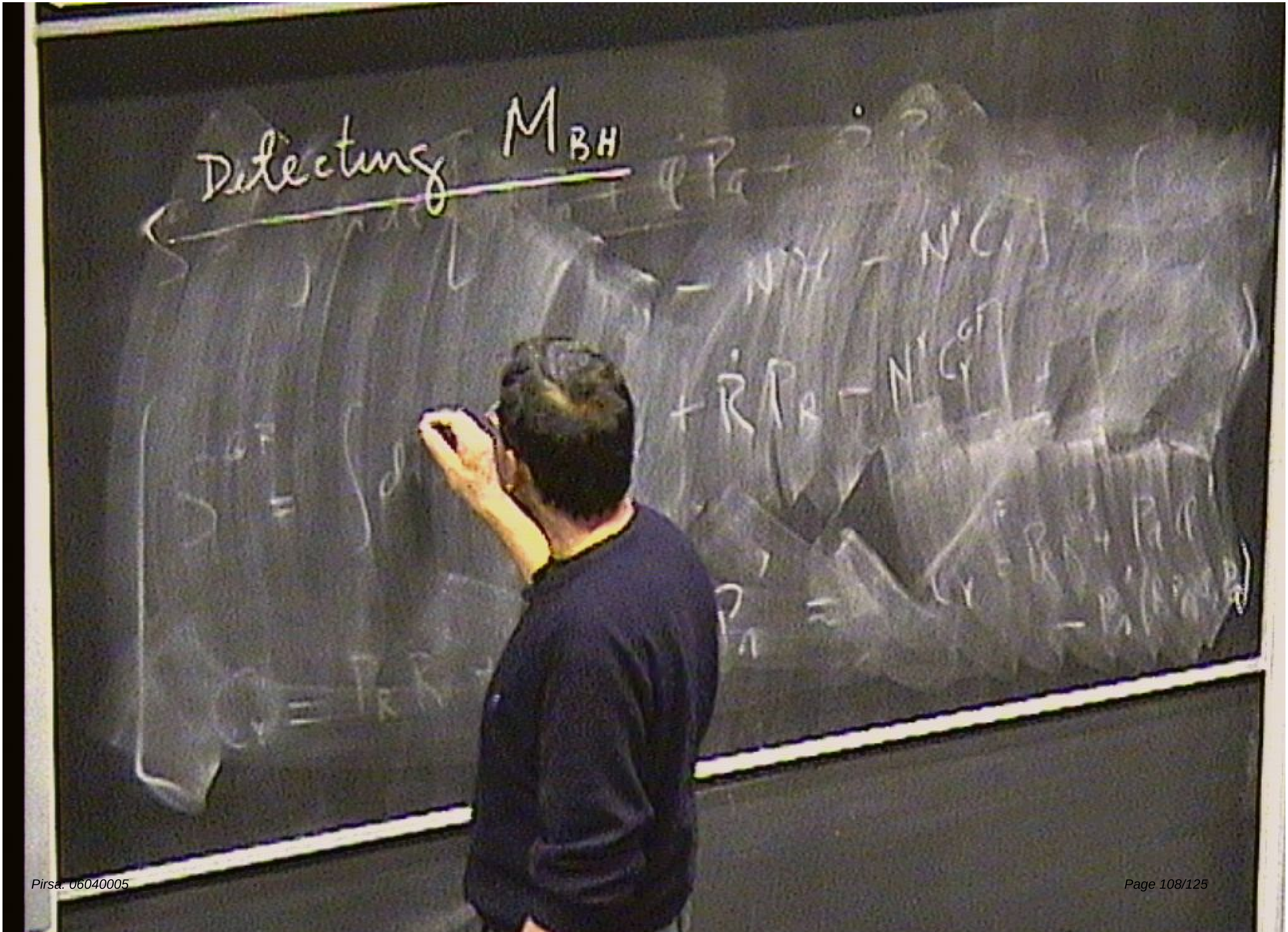
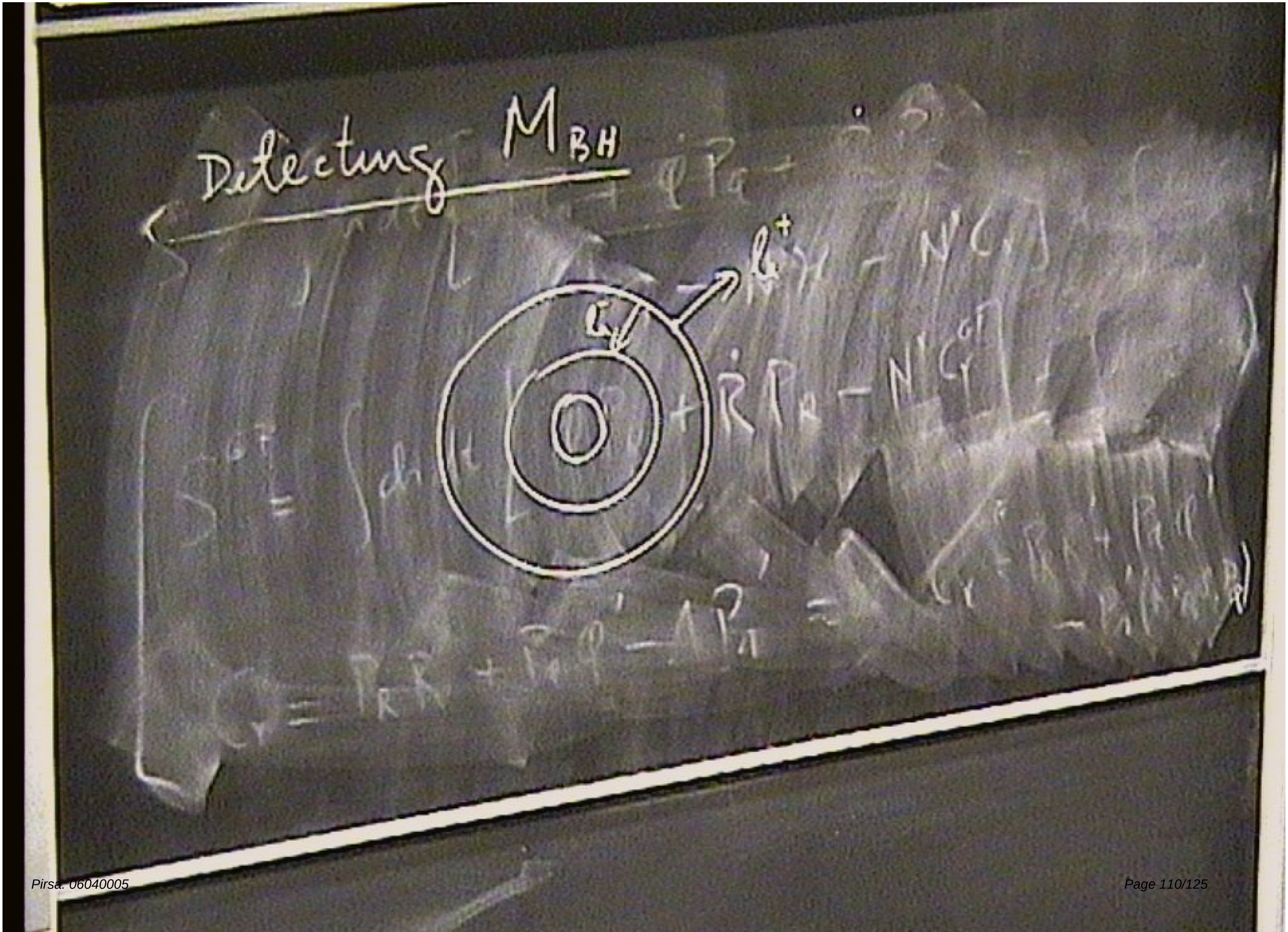
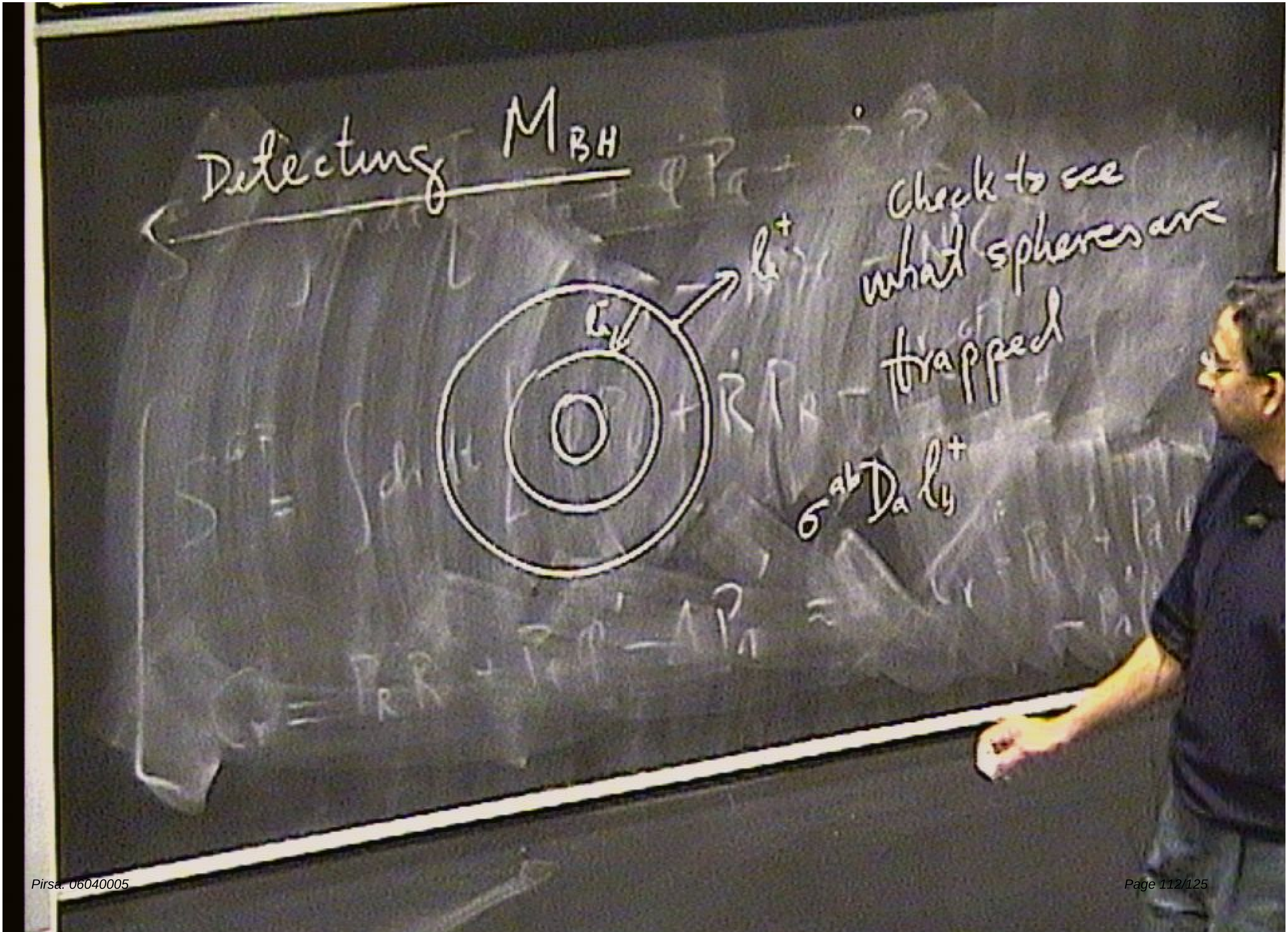

Detecting M_BH

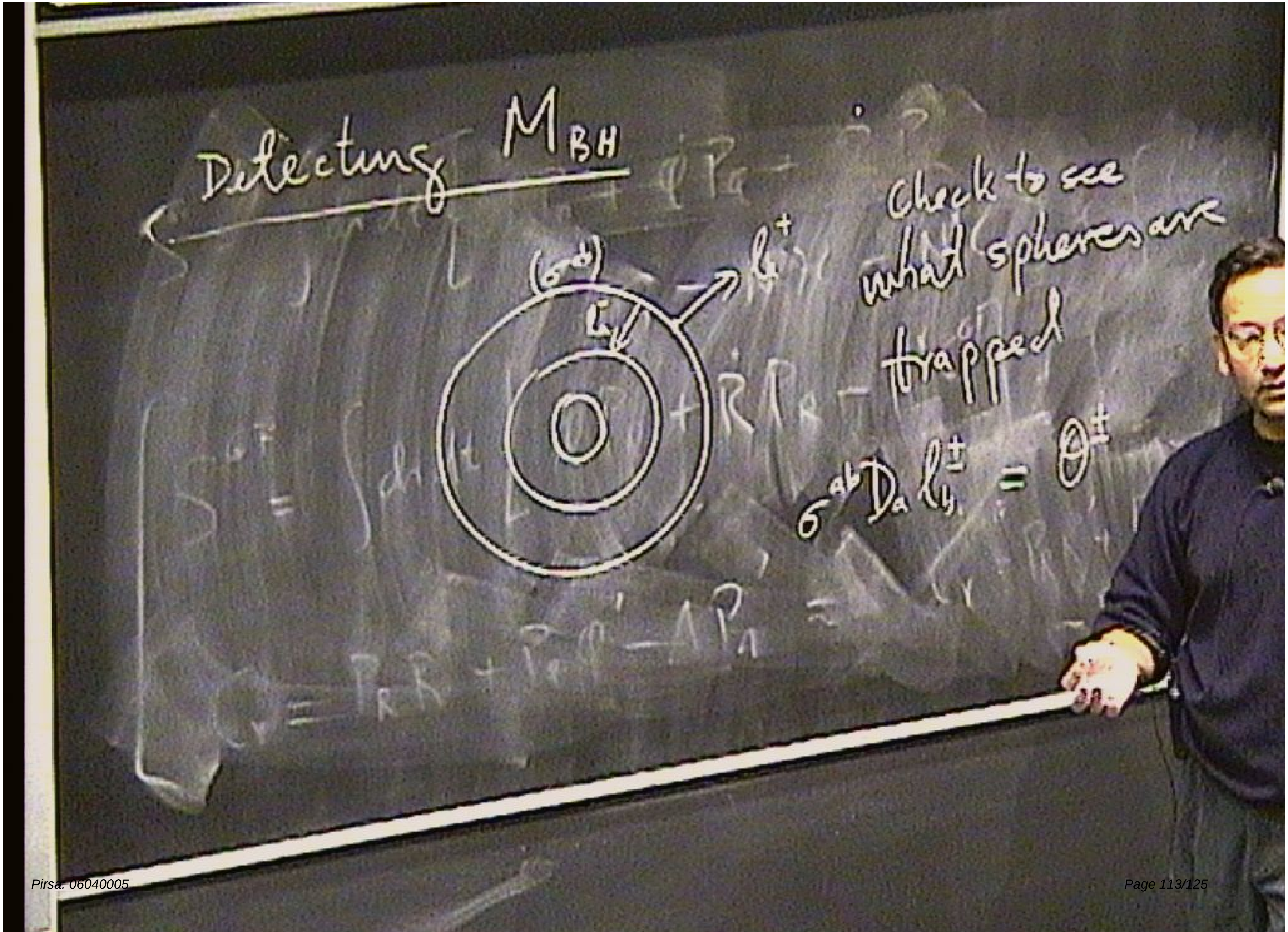

Detecting M_{BH}

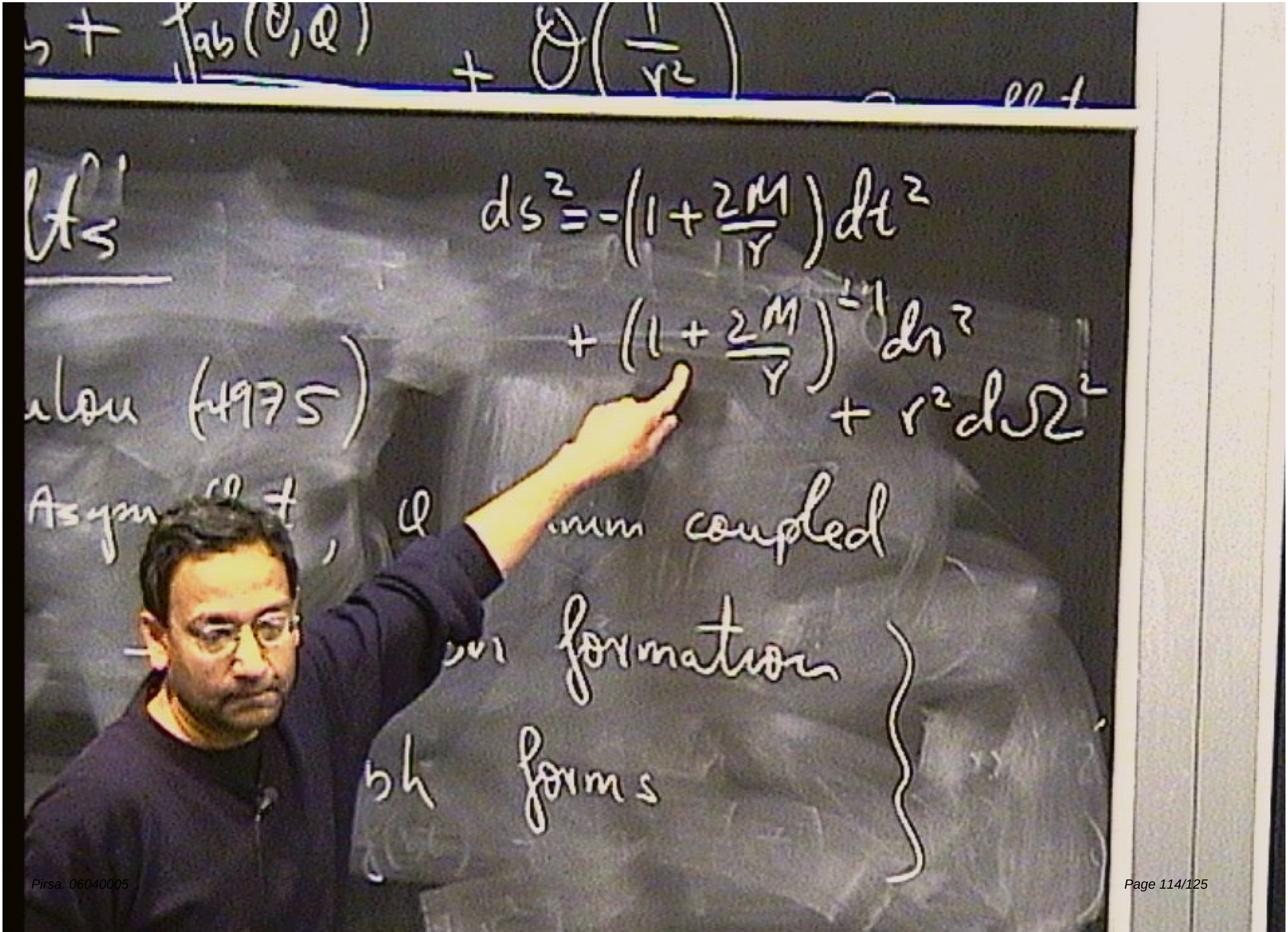

Detecting M_{BH}

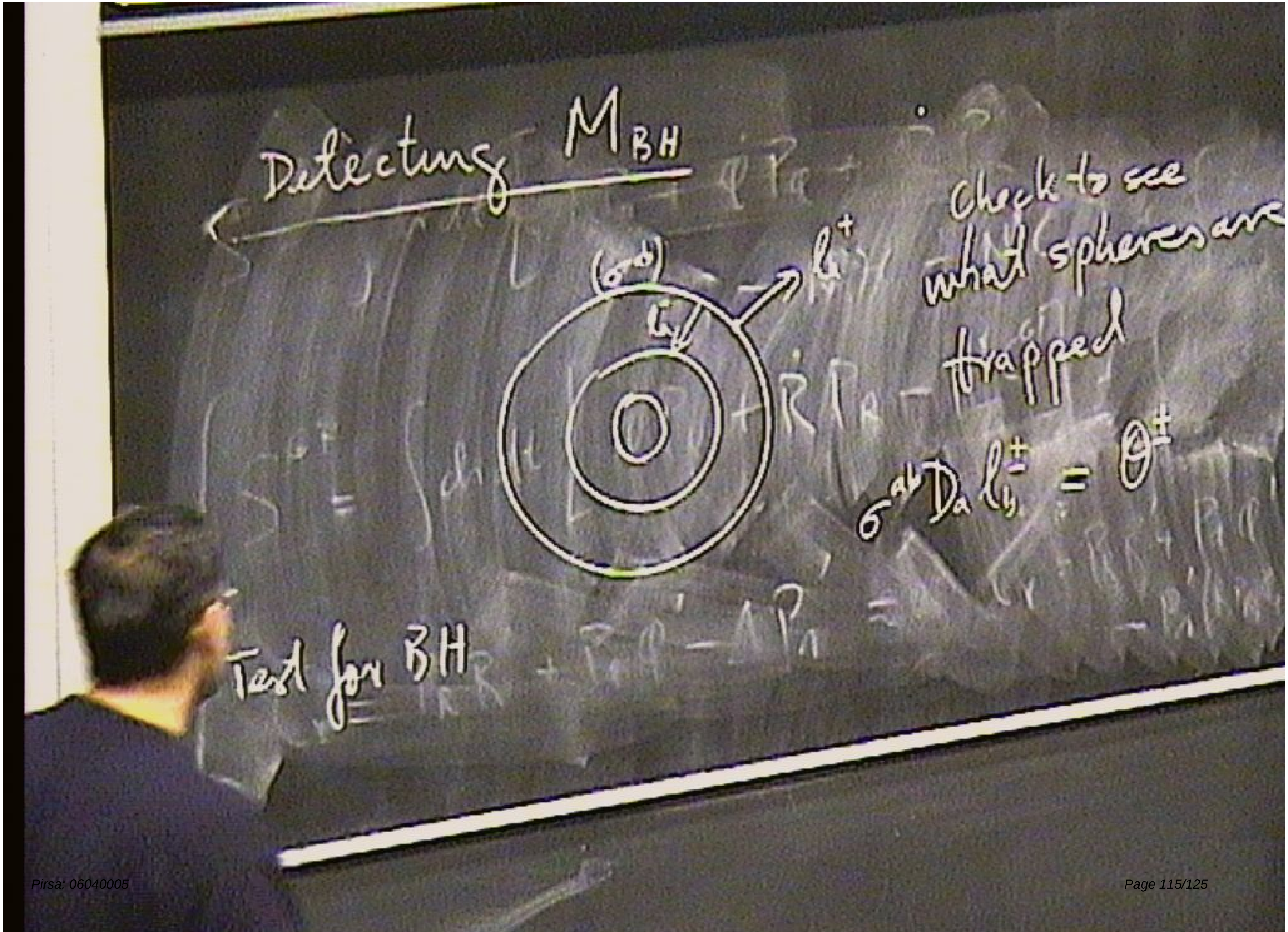
Detecting M_{BH}

Check to see what spheres are trapped


Detecting M_{BH}
Check to see what spheres are trapped
l_4^+
l_4
$\sigma^{ab} D_a l_4^+$


Detecting M_{BH}
l_b^+
Check to see what spheres are trapped
$\sigma^{ab} D_a l_b^{\pm} = \theta^{\pm}$


+ $f_{ab}(\theta, Q)$ + $O\left(\frac{1}{r^2}\right)$
$ds^2 = -\left(1+\frac{2M}{r}\right)dt^2$
$+ \left(1+\frac{2M}{r}\right)^{-1} dr^2$
$+ r^2 d\Omega^2$
ulou (1975)
coupled
formation
bh forms


Detecting M_{BH}
Check to see what spheres are trapped
$\sigma^{ab} D_a l_\nu^{\pm} = \theta^{\pm}$
Test for BH

Detecting M_{BH}

Check to see what spheres are trapped

$$\sigma^{ab} D_a l_b^{\pm} = \theta^{\pm}$$

Test for BH : Find the sphere(s) on which $\theta^+ = 0$, $\theta^- < 0$

Detecting M_{BH}

Check to see what spheres are trapped

Trapping:
$\theta_+ < 0$
$\theta_- < 0$

$$\sigma^{ab} D_a l_b^{\pm} = \theta^{\pm}$$

Test for BH : Find the sphere(s) on which $\theta^+ = 0$, $\theta^- < 0$) Marginal Trapping

Normal sphere: $\theta_+ > 0$, $\theta_- < 0$

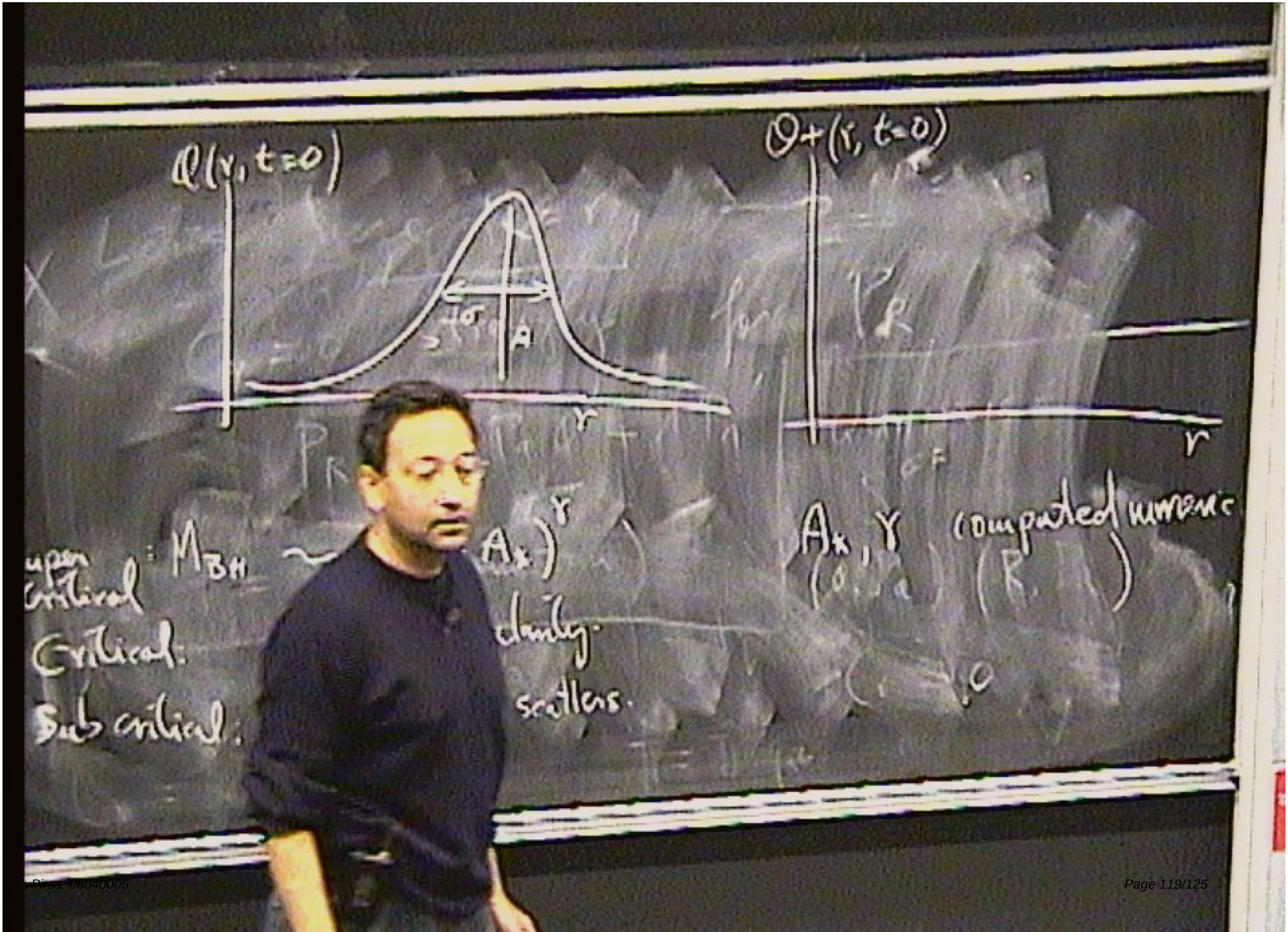
Detecting M_{BH}

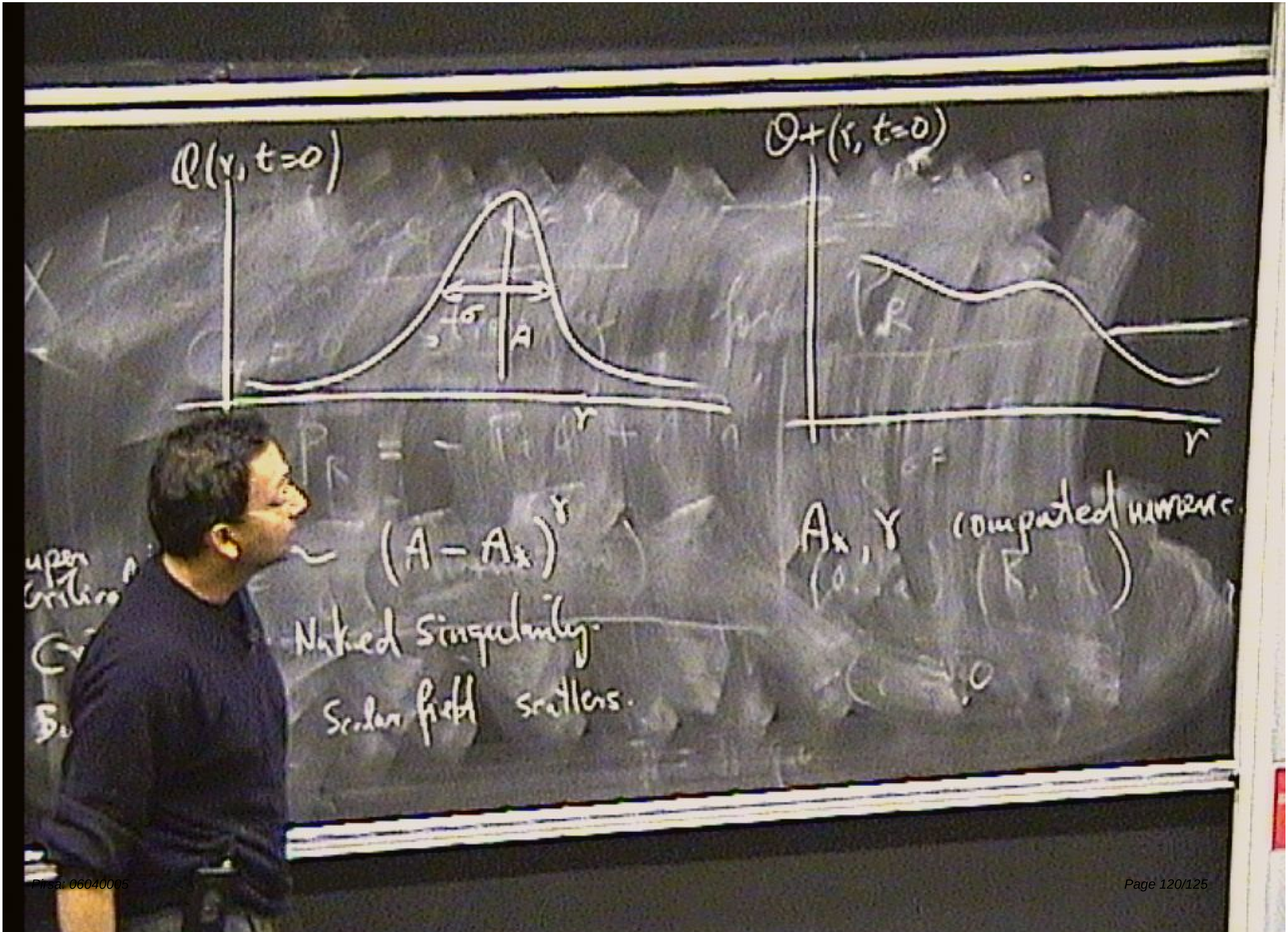
Check to see what spheres are trapped

Trapping:
$\theta_+ < 0$
$\theta_- < 0$

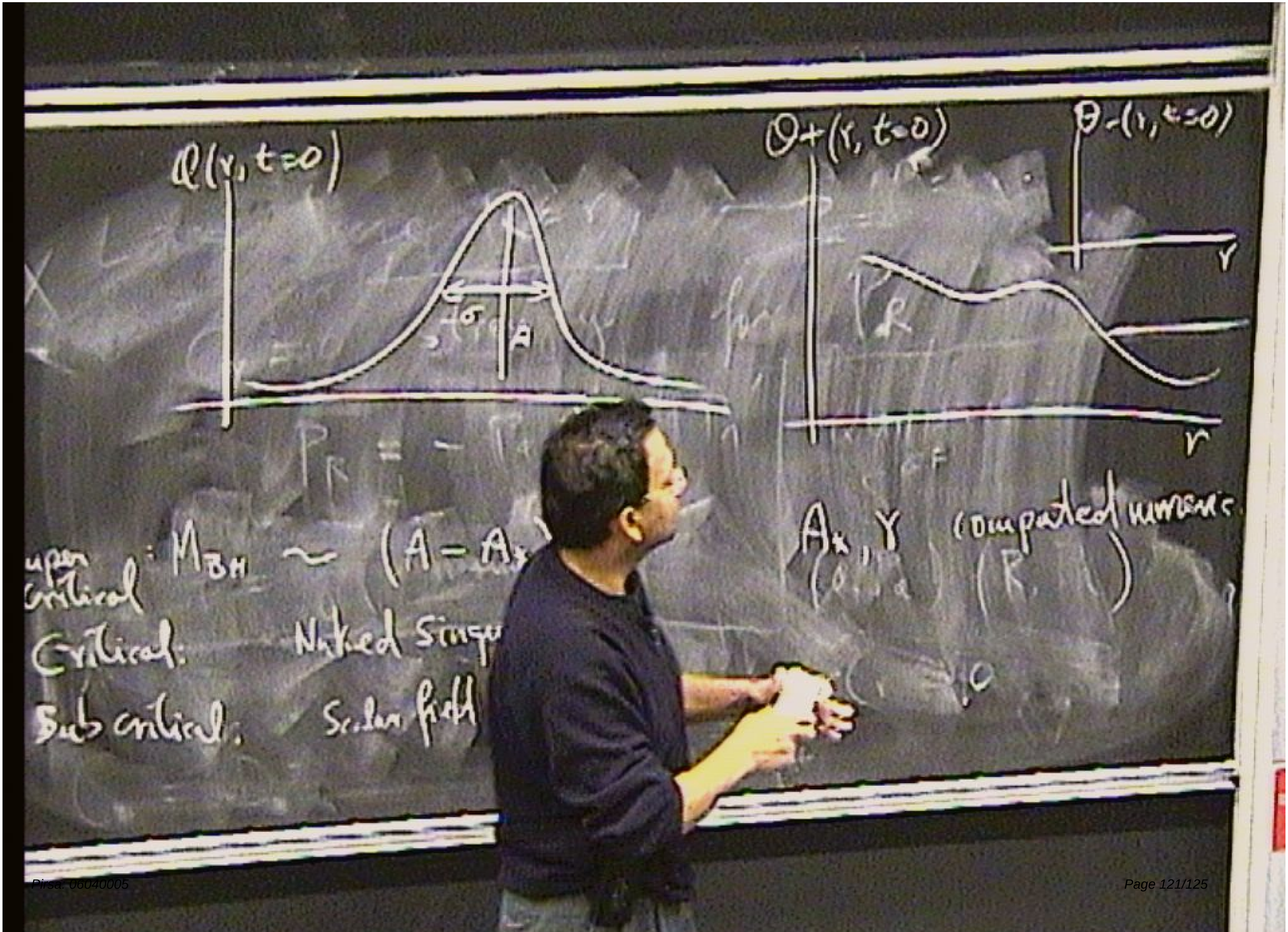
$$\sigma^{ab} D_a l_b^{\pm} = \theta^{\pm}$$

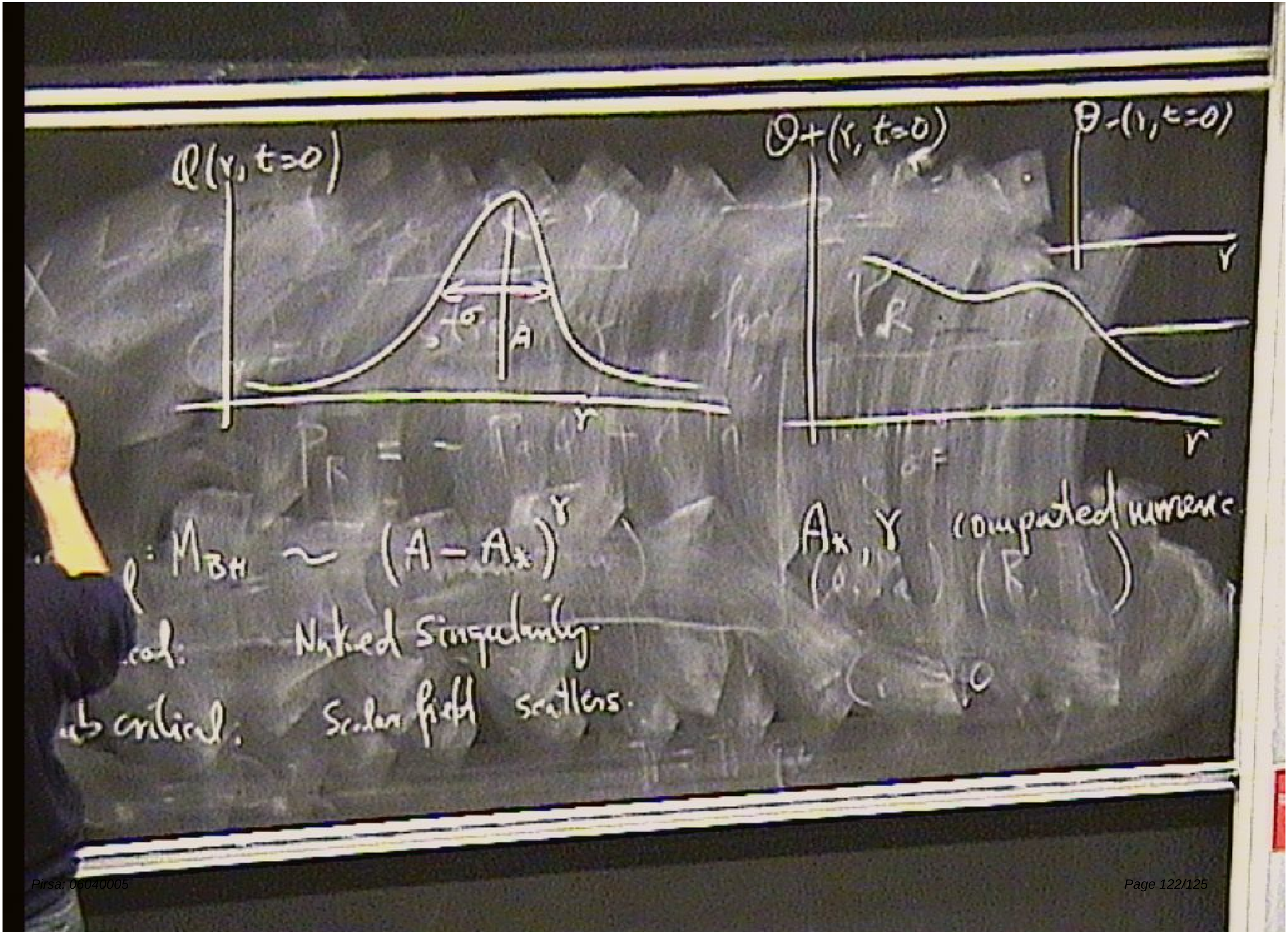
Test for BH: Find the sphere(s) on which $\theta^+ = 0$, $\theta^- < 0$ — Marginal Trapping


ρ(r, t=0)
2σ
A
r
θ+(r, t=0)
r
A*, γ
computed numeric
critical
M_BH ~
A*)^γ
Critical:
Sub critical:
scatters.



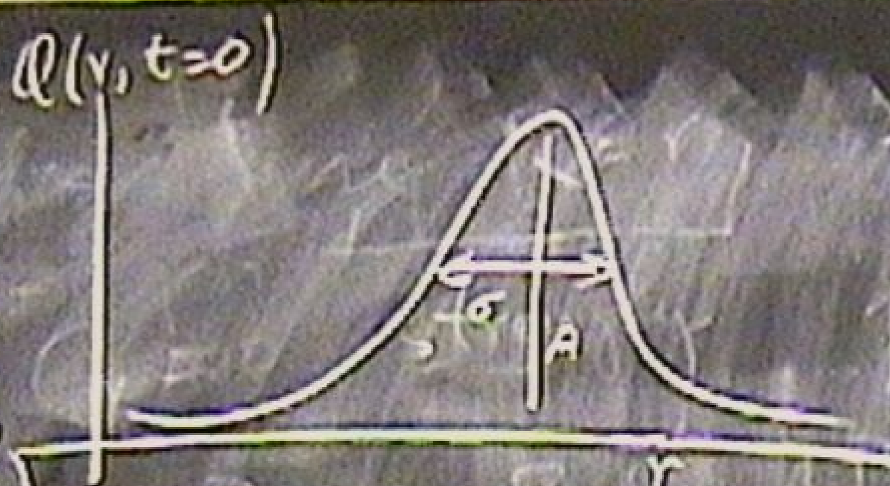
r
A
Ax, γ computed numeric.
Naked singularity.
Scalar field scatters.

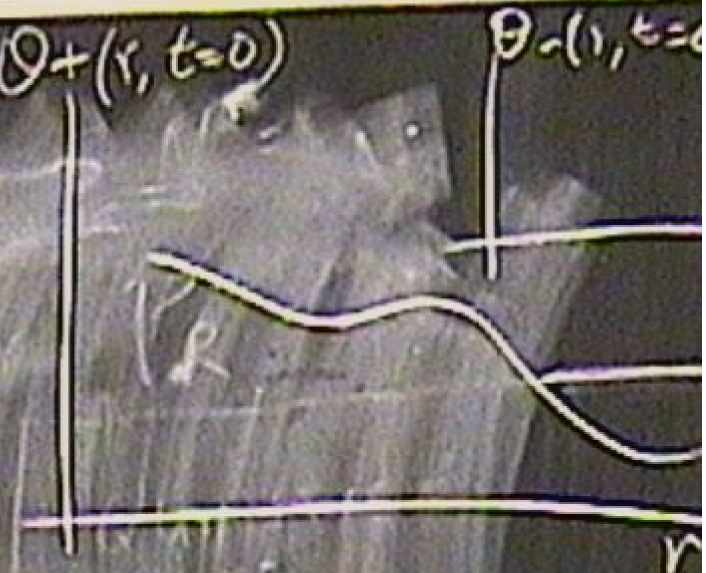

Φ(r, t=0)
Θ+(r, t=0)
Θ-(r, t=0)
r
r
A*, Y computed numeric
super critical: M_BH ~ (A − A*
Critical: Naked Singu
Sub critical: Scalar field



φ(r, t=0)
Θ+(r, t=0)
Θ-(r, t=0)
A
r
r
r
M_BH ~ (A − A*)^γ
A*, γ computed numeric
Naked singularity
sub critical: Scalar field scatters.

$\Theta_{\pm}(P_R, R, P_0, Q)$



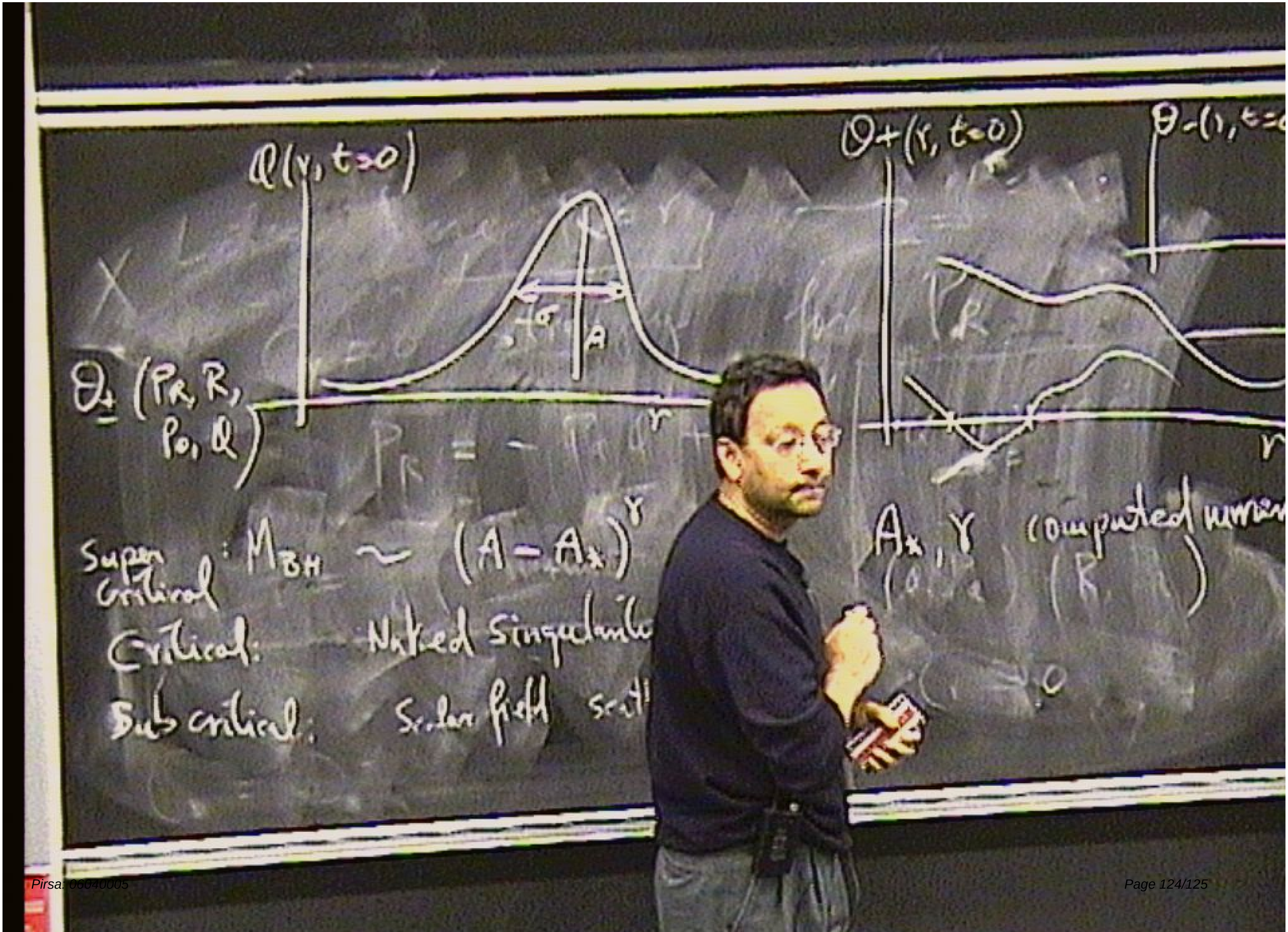


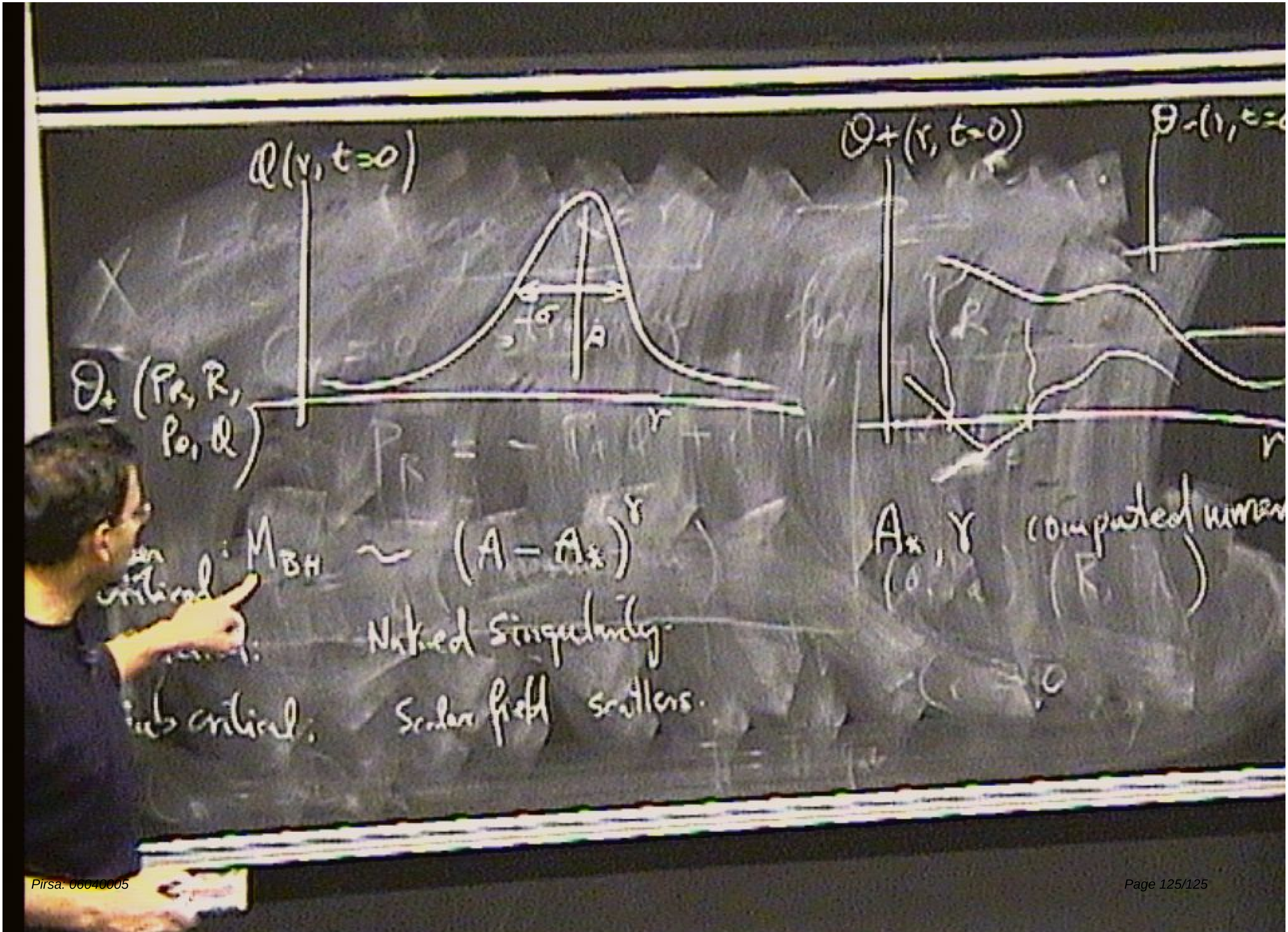
Super critical: $M_{BH} \sim (A - A_*)^{\gamma}$

Critical: Naked singularity.

Sub critical: Scalar field scatters.

A_*, γ (computed numer


φ(r, t=0)
Φ+(r, t=0)
Θ+ (P_R, R, P_0, φ)
Super critical: M_BH ~ (A − A*)^γ
A*, γ computed numerically
Critical: Naked Singularity
Sub critical: Scalar field scatt


Φ(r, t=0)
Θ+(r, t=0)
Θ+ (P_R, R, P_0, Φ)
r
critical: M_BH ~ (A − A*)^γ
Naked singularity.
sub critical: Scalar field scatters.
A*, γ computed