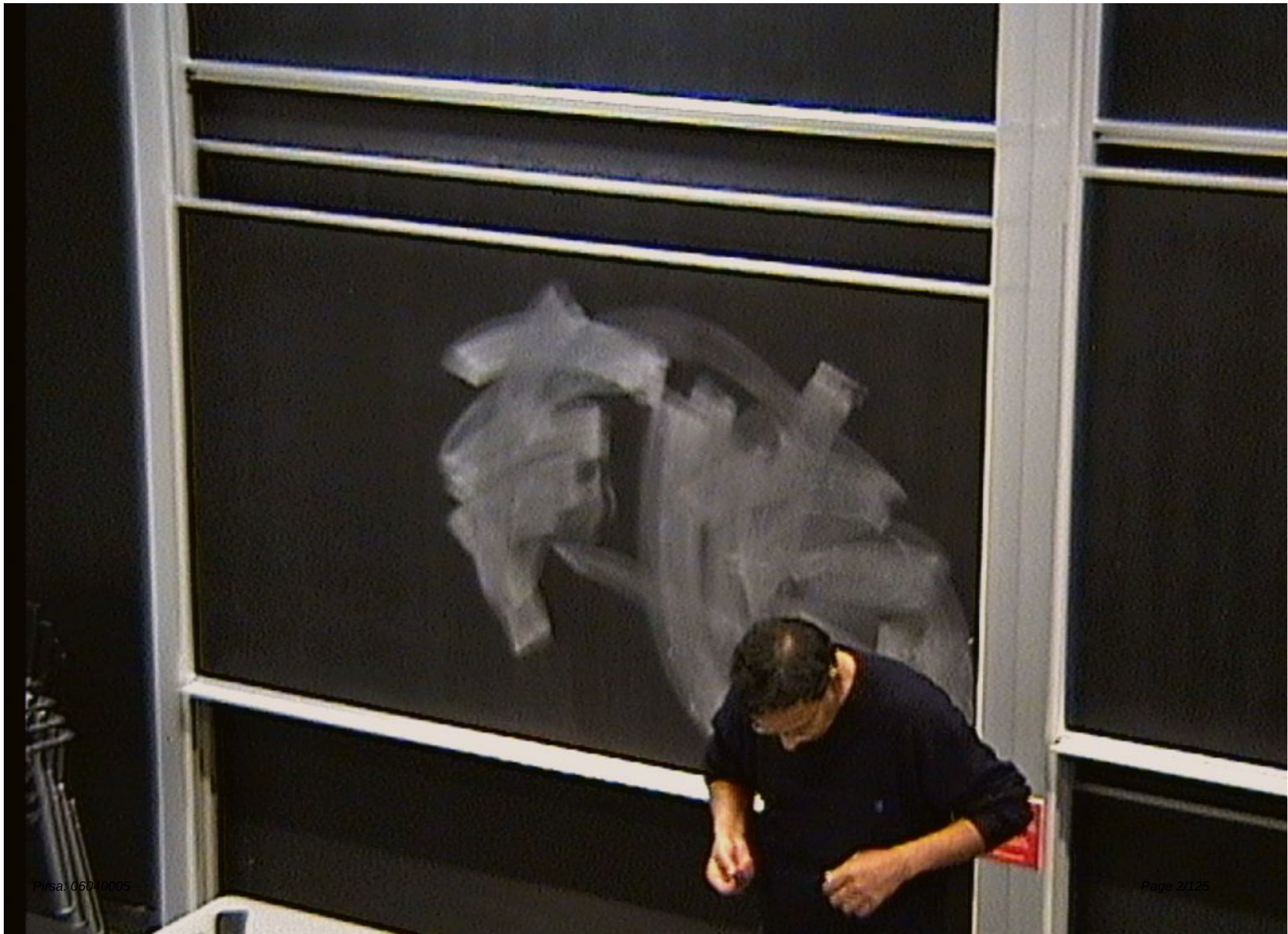


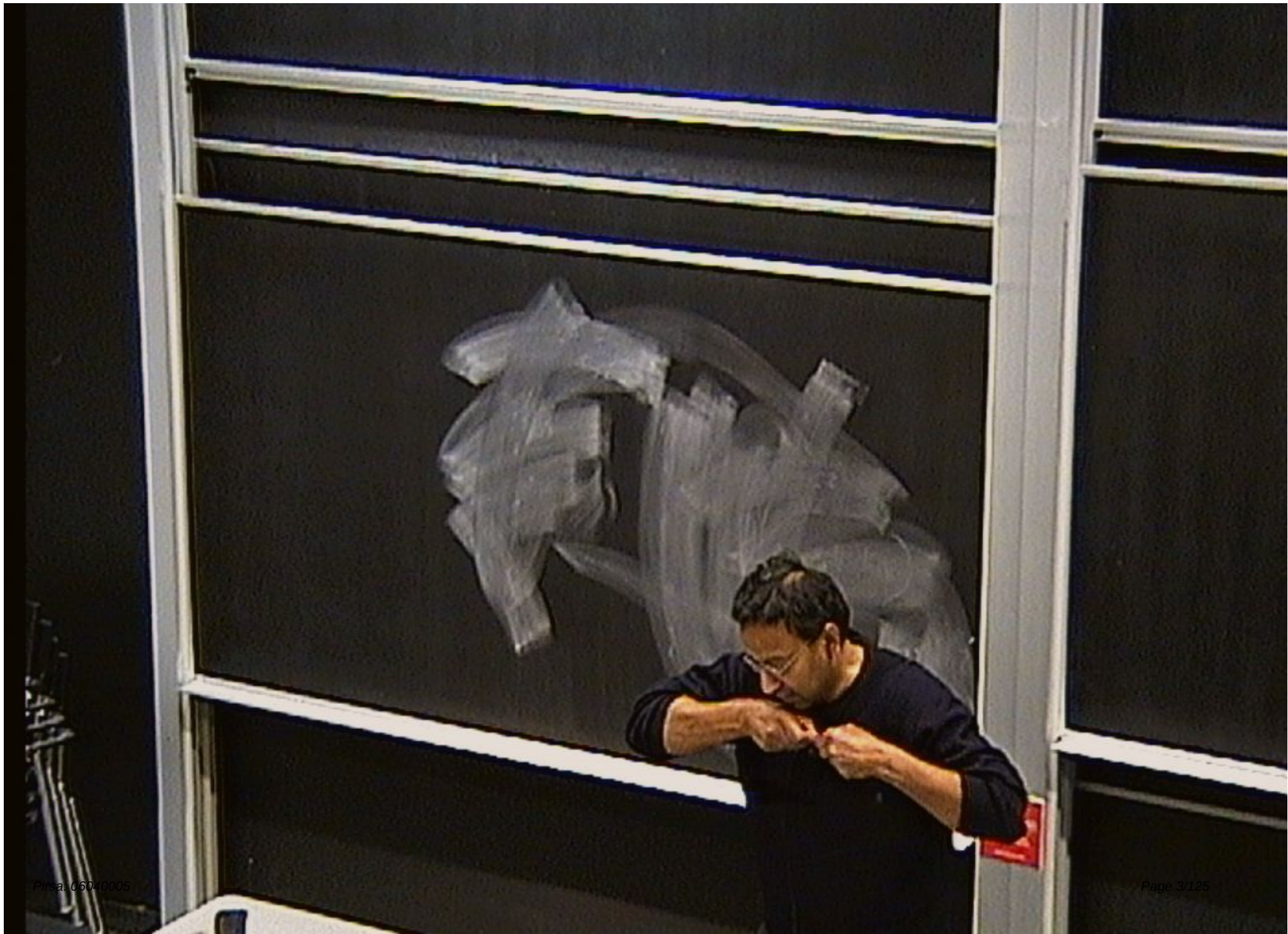
Title: Introduction to quantum gravity - Part 22

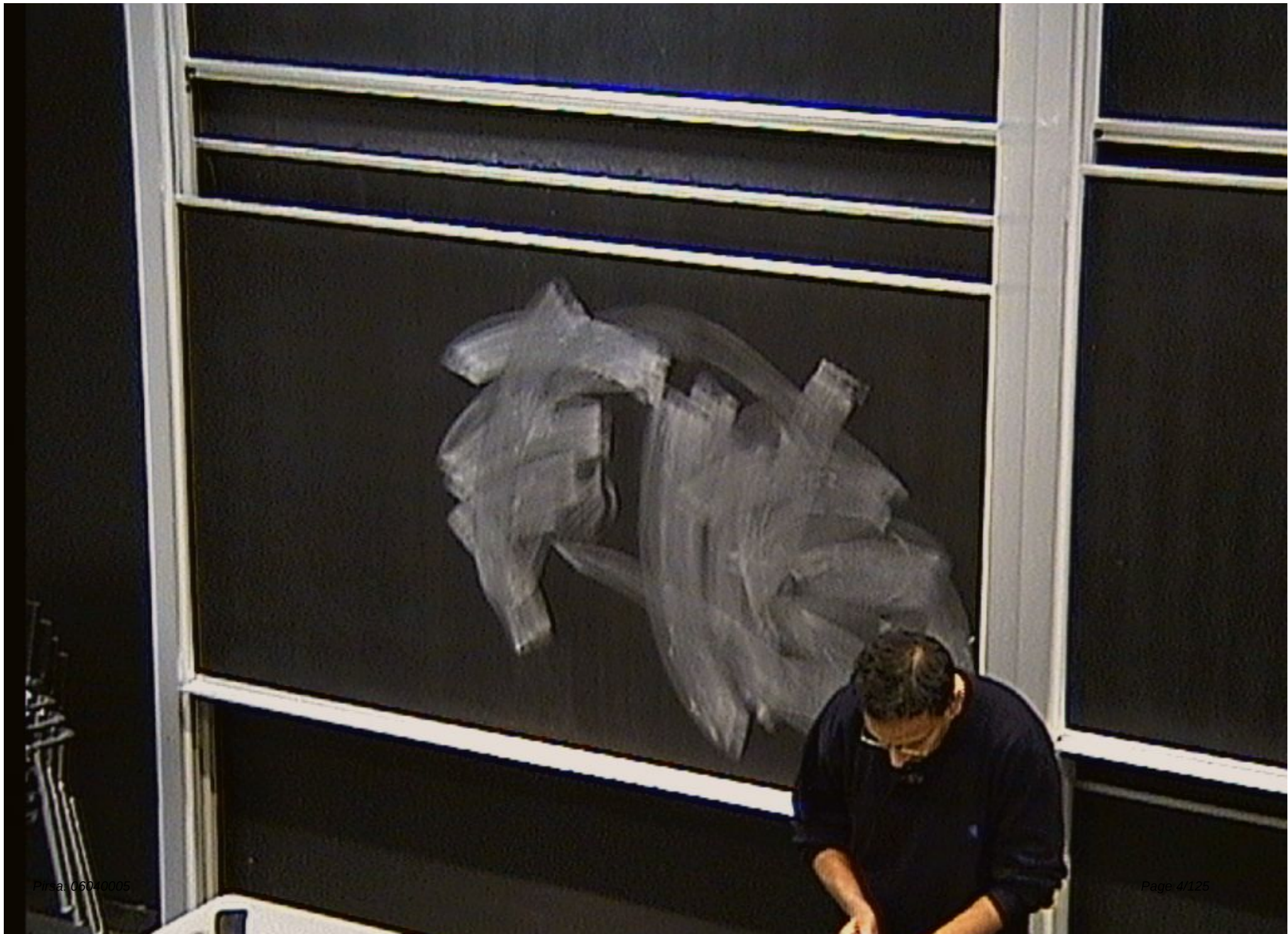
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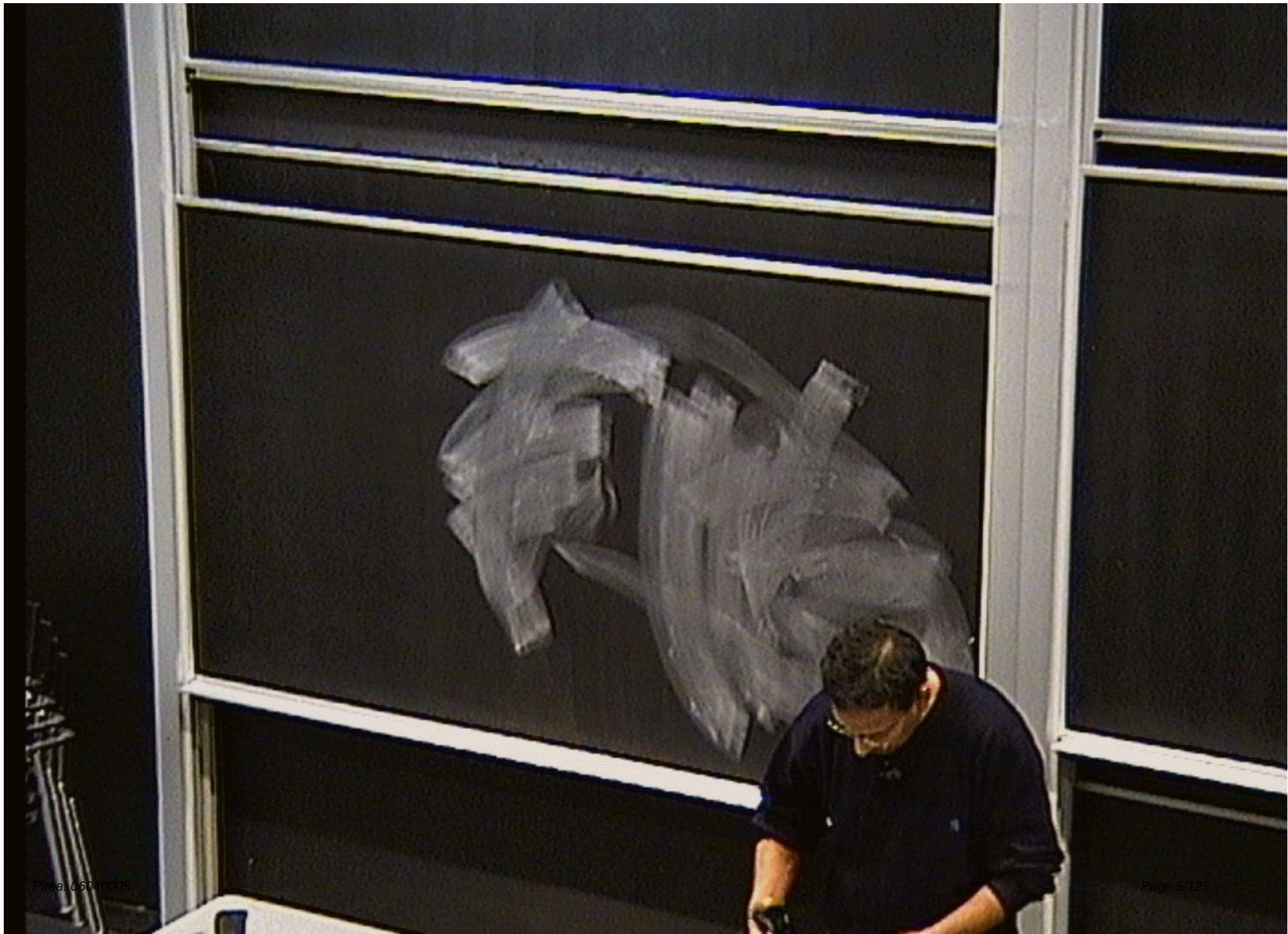
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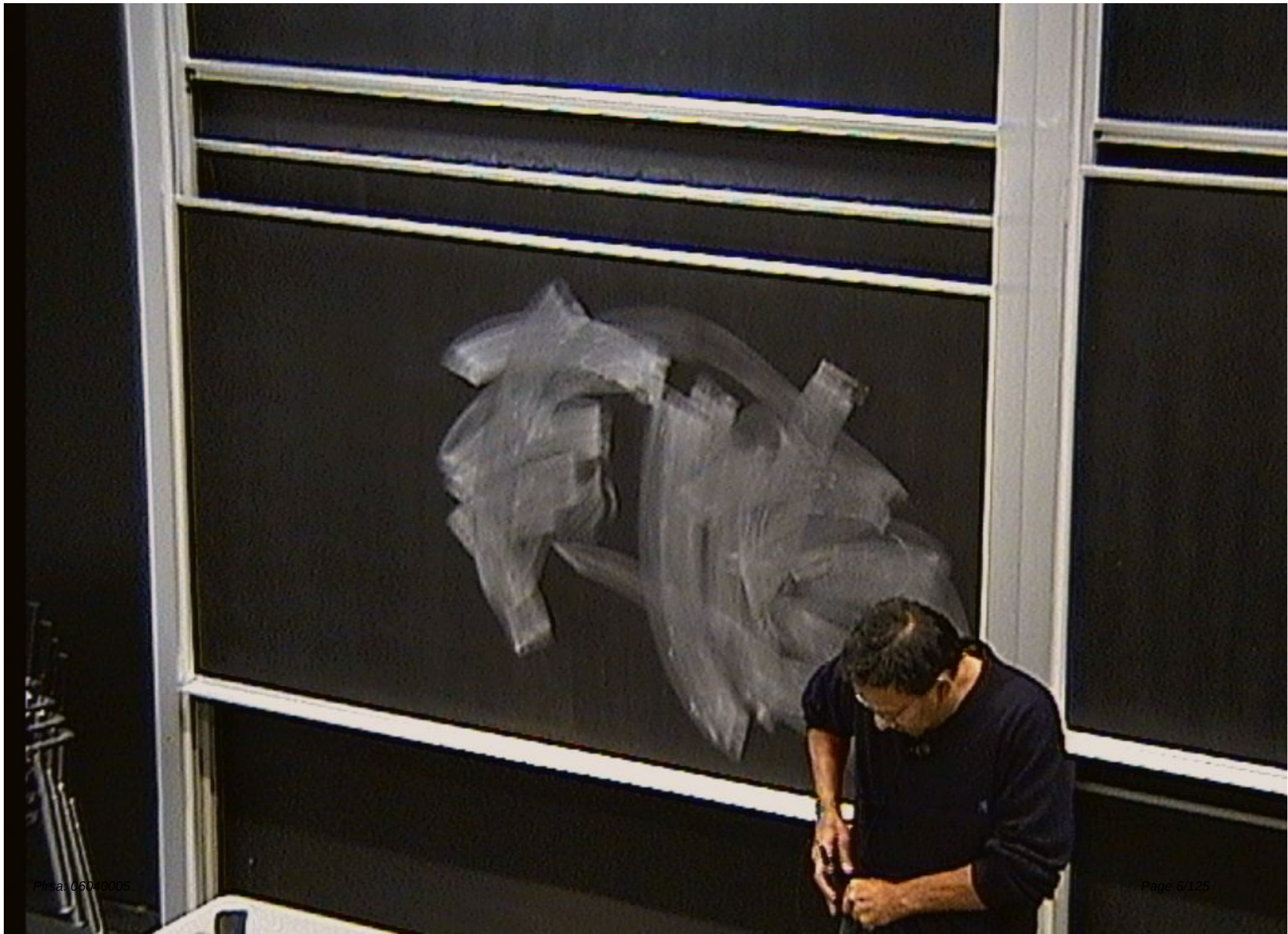
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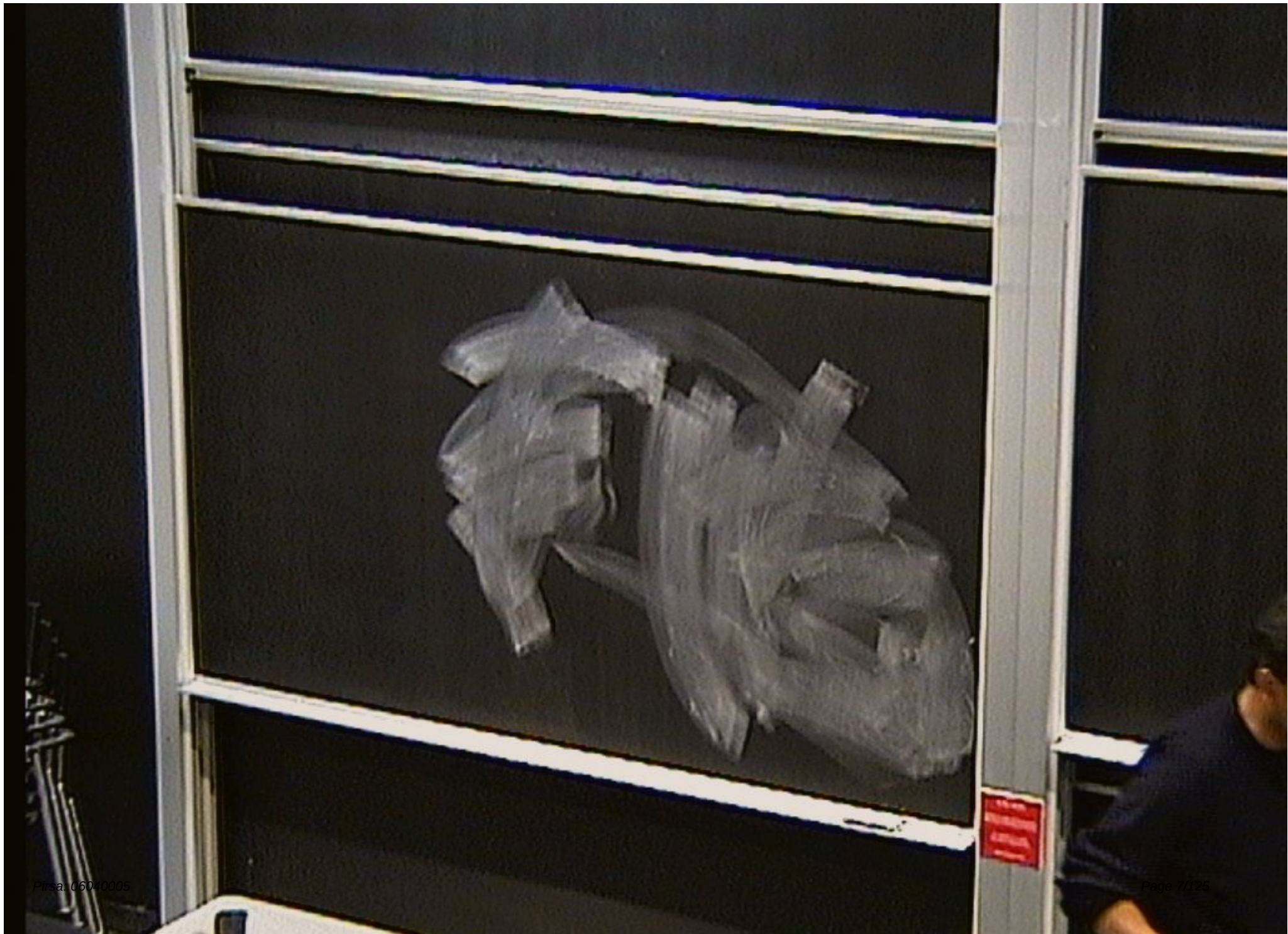


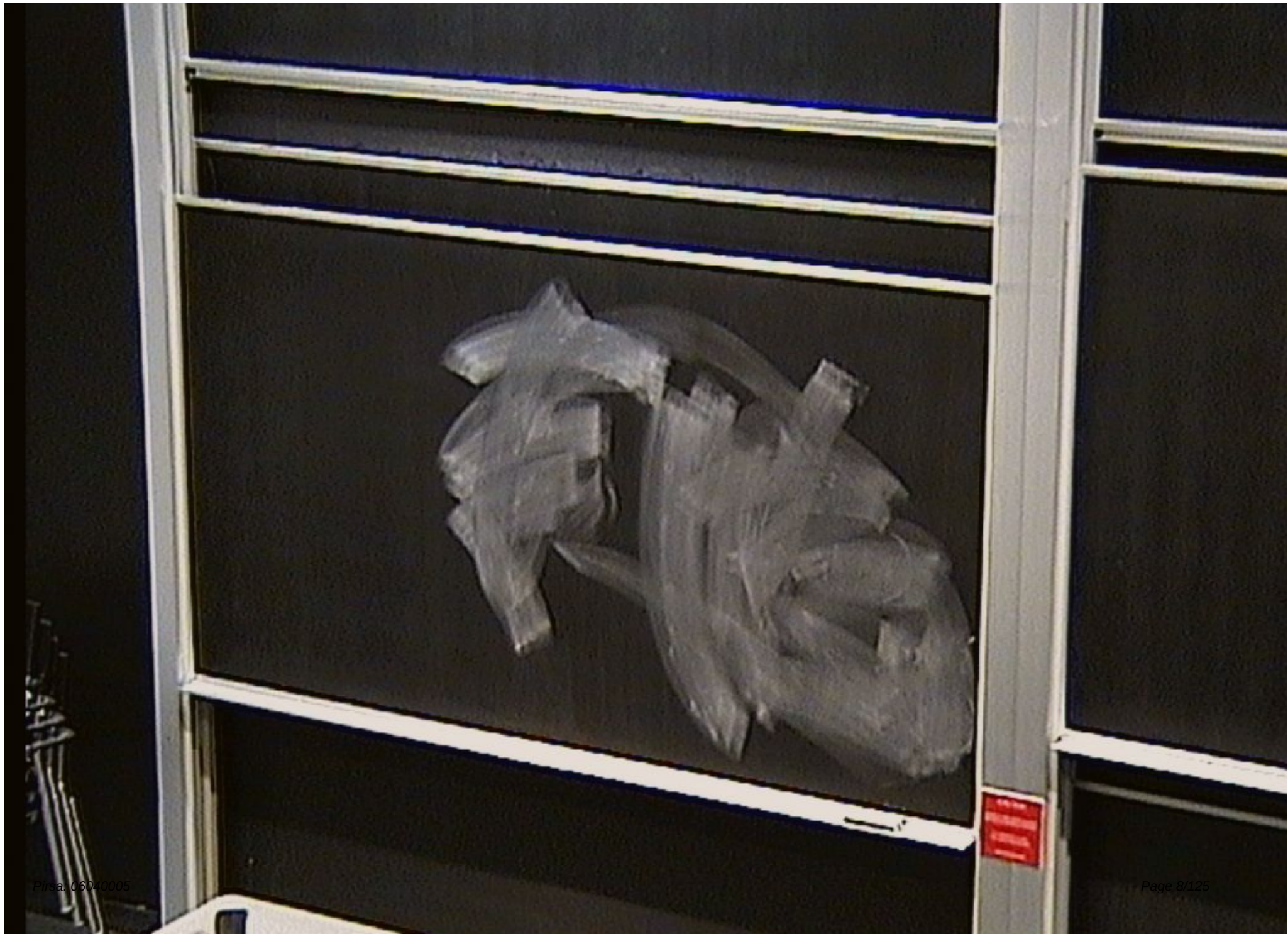


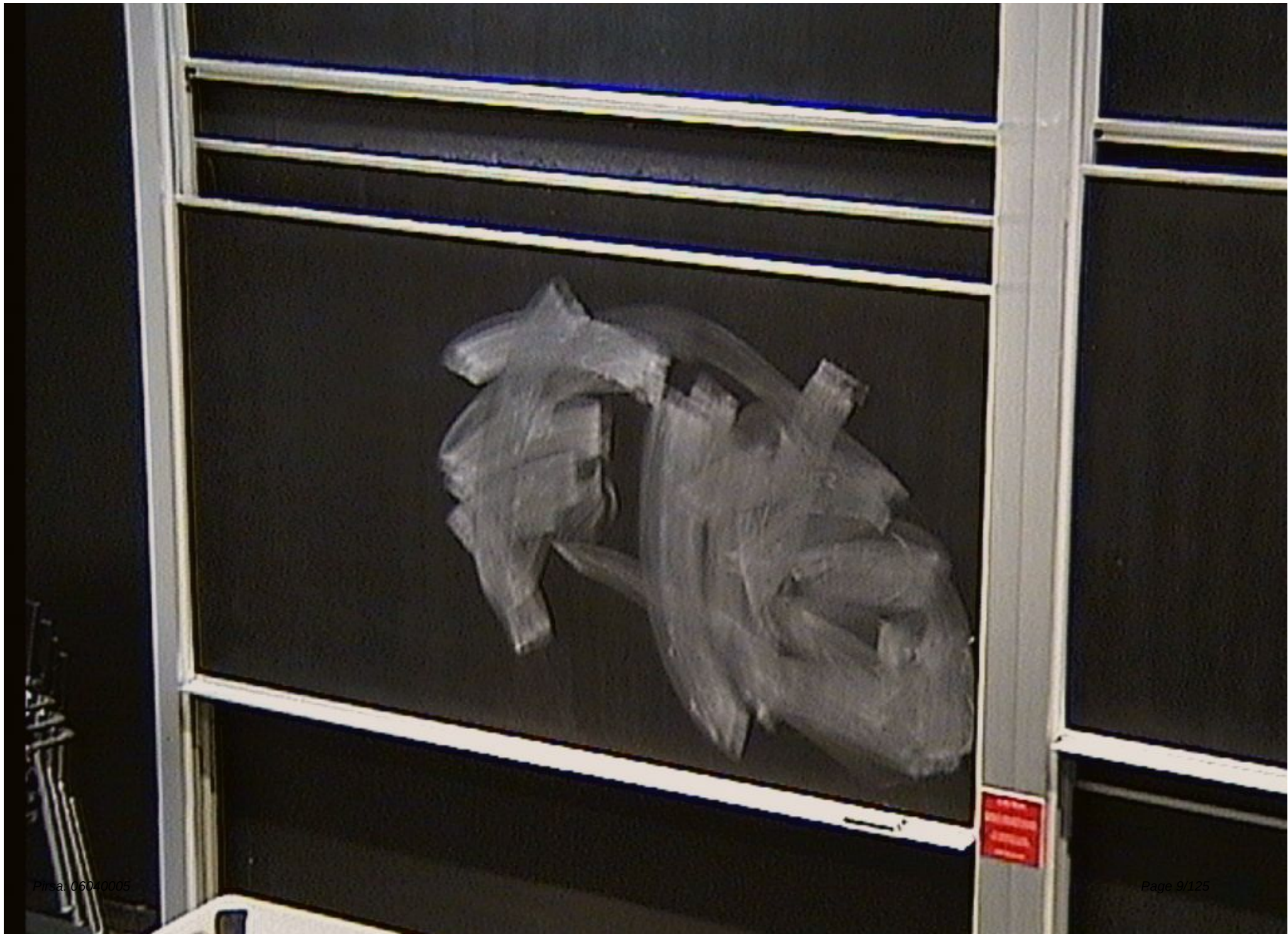


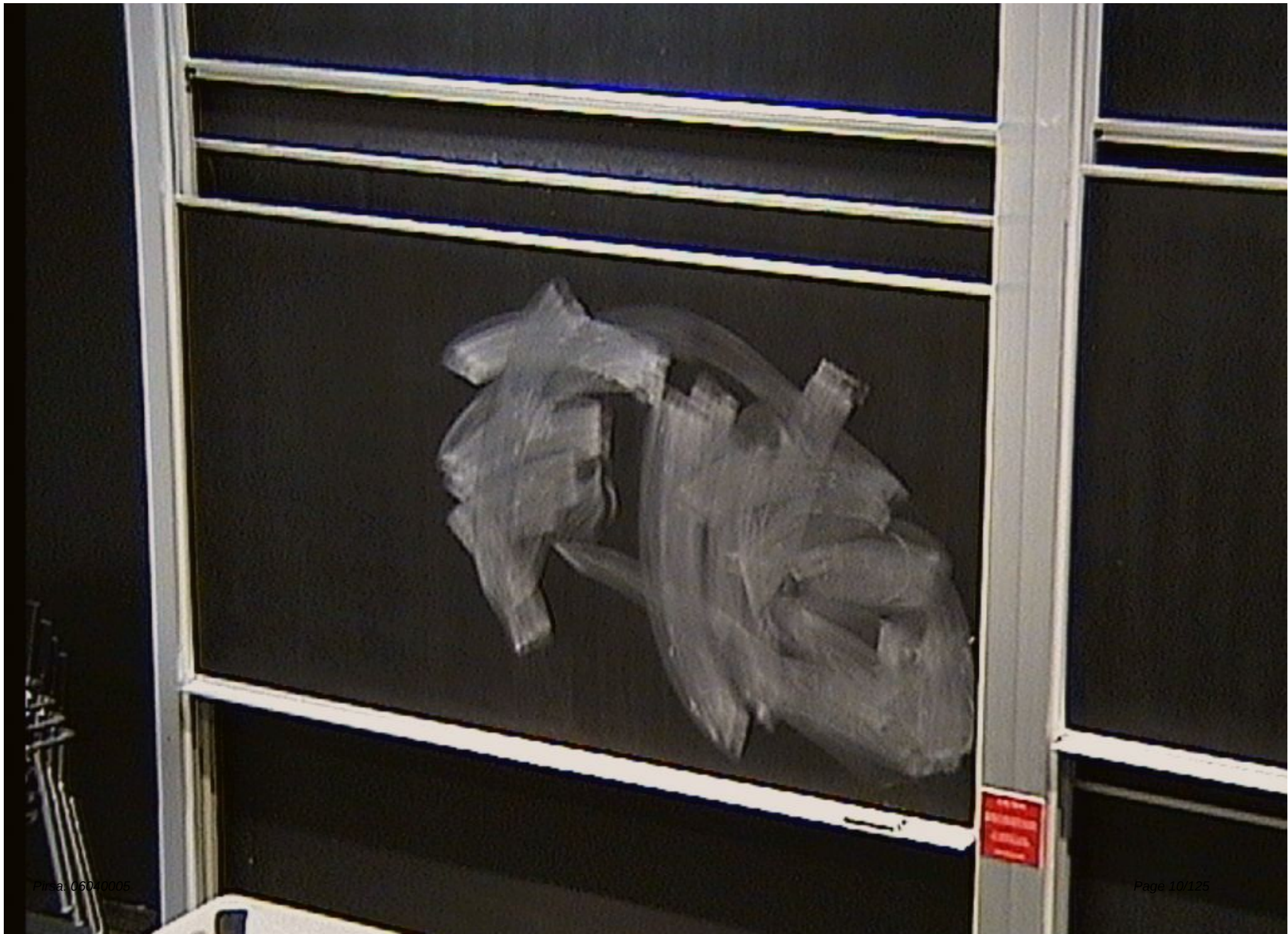


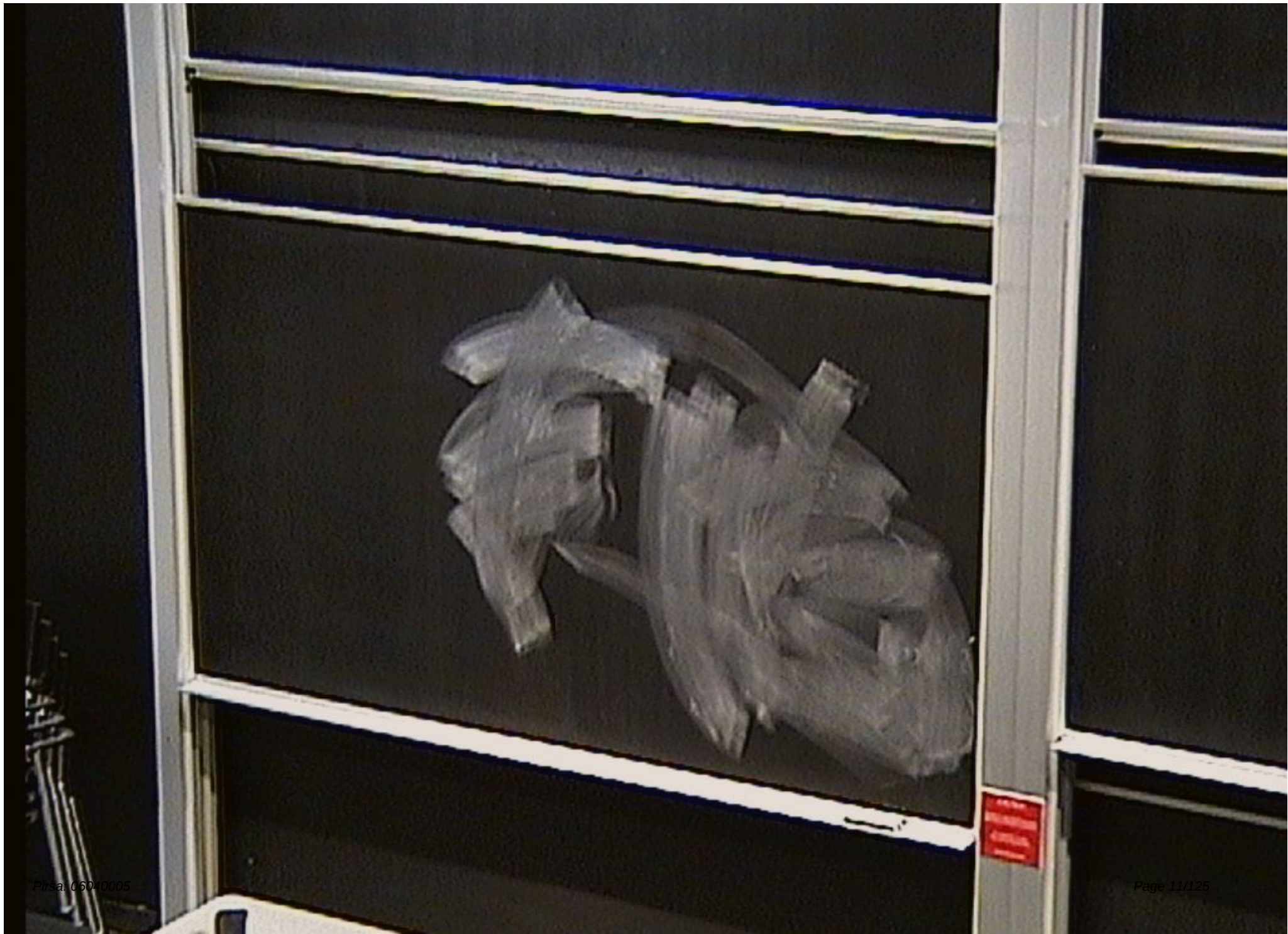


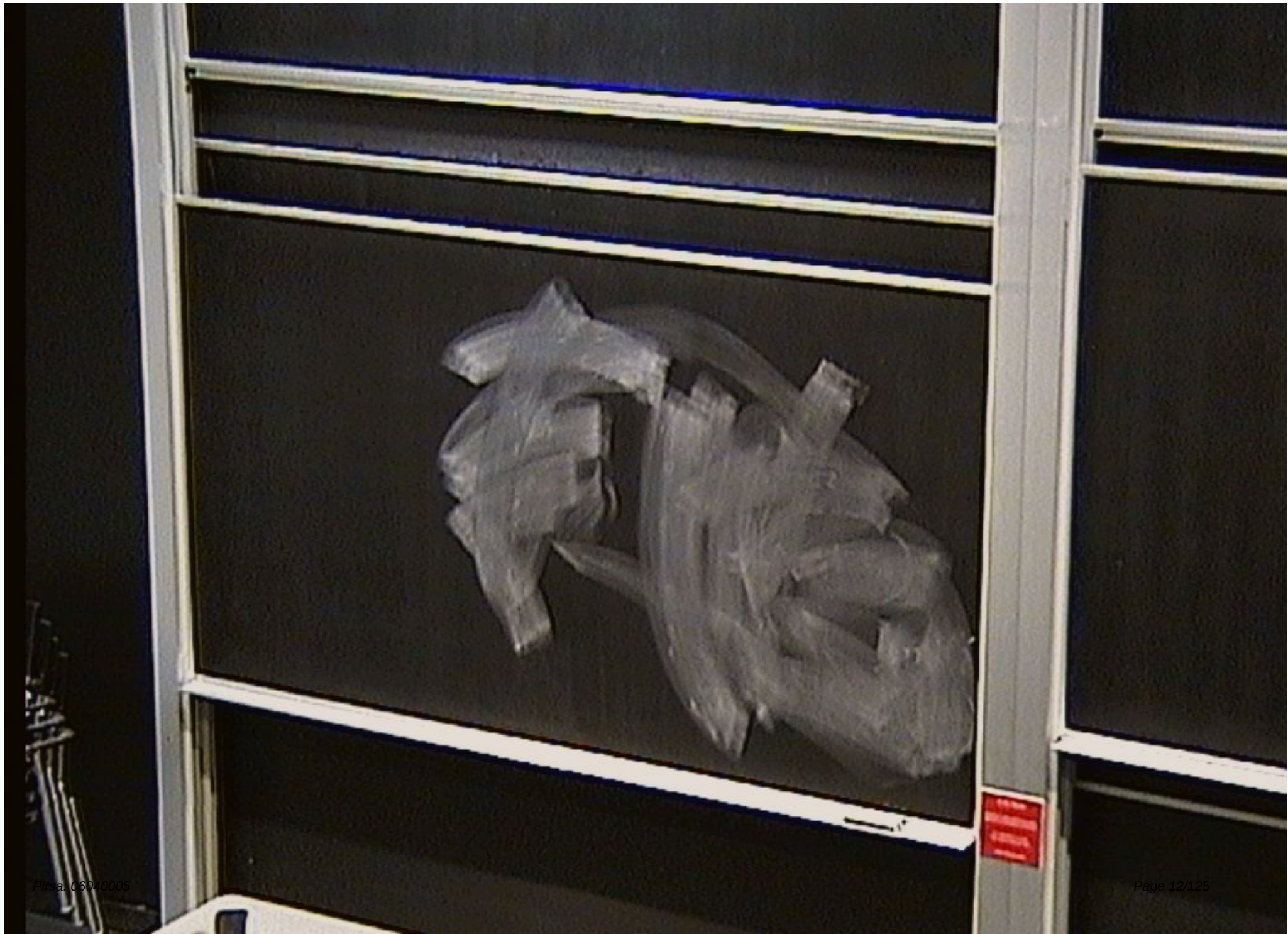












GR + scalar field
minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$$\rho(r, t)$$

$$Q(r, t)$$

$$T_{ab}$$

GR + scalar field
minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$$g_{ab}(r, t)$$

$$Q(r, t)$$

$$T_{ab} = \partial_a Q \partial_b Q - \frac{1}{2} g_{ab} (\partial Q)^2$$

GR + scalar field
minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$$g_{ab}(r,t)$$

$$Q(r,t)$$

Unruh

$$T_{ab} = \partial_a \phi \partial_b \phi - \frac{1}{2} g_{ab} (\partial \phi)^2$$

$$ds^2 = -dt^2 f(r) + dr^2 g(r) + r^2 d\Omega^2$$

GR + scalar field
minimally coupled

$$G_{ab} = 8\pi T_{ab}$$

$$g_{ab}(r,t)$$

$$Q(r,t)$$

$$T_{ab} = \partial_a Q \partial_b Q - \frac{1}{2} g_{ab} (\partial Q)^2$$

Unruh (1976) $ds^2 = -dt^2 f(r,t) + dr^2 g^2(r,t) + r^2 d\Omega^2$

Bergshoeff, Chiriac, Moncrief & Nulken (1973)

GR + scalar field
minimally coupled

field theory

$$G_{ab} = 8\pi T_{ab}$$

$$g_{ab}(r,t)$$

$$Q(r,t)$$

Un
Bergs,

$$\int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{ab} (\partial_a Q)^2 \right)$$

$$f = \int d^3x \left(\frac{1}{2} \dot{Q}^2 + \frac{1}{2} g^{ij} \partial_i Q \partial_j Q \right) + \int d^3x \sqrt{-g} \mathcal{L}_m$$

Fix time gauge.
Fix radial gauge } →

$$\left. \begin{array}{l} \text{Fix time gauge} \\ \text{Fix radial gauge} \end{array} \right\} \rightarrow H = \int_0^{\infty} dr f(a, p_a)$$

$$\left. \begin{array}{l} \text{Fix time gauge} \\ \text{Fix radial gauge} \end{array} \right\} \rightarrow H = \int_0^\infty dr f(r, P_r) e^{\int_0^r g(r, P_r) dr}$$

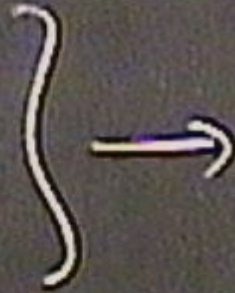
Fix time gauge.
Fix radial gauge } $\rightarrow$

$$H = \int_0^\infty dr \, \underline{f(r, P_r)} e^{\int_0^r \underline{g(r, P_r)} dr}$$

Non-local.

Fix time gauge.

Fix radial gauge



$$H = \int_0^\infty dr \, \underline{f(r, P_r)} e^{\int_0^r \underline{g(r, P_r)} dr}$$

Non-local.

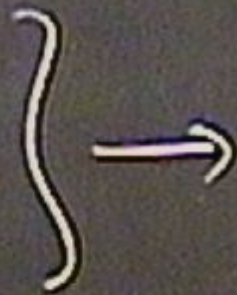
$$\left. \begin{array}{l} \text{Fix time gauge} \\ \text{Fix radial gauge} \end{array} \right\} \rightarrow H = \int_0^\infty dr \, \underline{f(r, p_r)} e^{\int_0^r \underline{g(r, p_r)} dr}$$

Non-local.

- Classical ADM formalism.

Fix time gauge.

Fix radial gauge



$$H = \int_0^\infty dr \, \underline{f(R, P_r)} e^{\int_0^r \underline{g(r, P_r)} dr}$$

Non-local.

classical ADM formalism — adapted to flat slice coordinates.

Fix time gauge.
 Fix radial gauge.

$$H = \int_0^\infty dr \, \underline{f(R, P_r)} e^{\int_0^r \underline{g(r, P_r)} dr}$$

Non-local.

- Classical ADM formalism — adapted to flat slice coordinates.
- Classical results on collapse.
- Quantization (LQG ideas)
- Quantum collapse

Fix time gauge.
 Fix radial gauge.

$$\left. \begin{array}{l} \text{Fix time gauge.} \\ \text{Fix radial gauge.} \end{array} \right\} \rightarrow H = \int_0^\infty dr \, \underline{f(R, P_r)} e^{\int_0^r \underline{g(R, P_r)} dr}$$

Non-local.

- Classical ADM formalism — adapted to flat slice coordinates.
- Classical results on collapse.
- Quantization (LQG ideas)
- Quantum collapse.

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand.

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand. PG

$t = \text{const}$

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand. PG

$t = \text{const}$ slice.

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand. PG

$t = \text{const}$ slices are flat

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand. PG

$t = \text{const}$ slices are flat

(g_{ab}, Π^{ab})

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + dr^2 + r^2 d\Omega^2$$

Painlevé & Gullstrand. PG

$t = \text{const}$ slices are flat

(q_{ab}, π^{ab})

$$S_{GR} = \int_M \left(\pi^{ab} \dot{q}_{ab} - N \mathcal{H} - N^a C_a \right) d^3x dt + \int_{\partial M} (\dots)$$

Plug in parametrization for

Plug in parametrization for
 q_{ab}, π^{ab} such that

$$\rightarrow \int dt dt \left(\dot{R} \dot{R} + \dot{\Phi} \dot{P}_\Phi + \dots \right)$$

Plug in parametrization for
 q_{ab}, π^{ab} such that

$$S \longrightarrow \int dt \left(\dot{R} P_R + \dot{Q} P_Q + \dots \right)$$

Plug in parametrization for
 q_{ab}, π^{ab} such that

$$S \rightarrow \int dt \left(\dot{R} P_R + \dot{Q} P_Q + \dots \right)$$

Fall off conditions on phase space variables.

Plug in parametrization for q_{ab}, π^{ab} such that

$$S \longrightarrow \int dt \left(\dot{R} P_R + \dot{Q} P_Q + \dots \right)$$

Fall off conditions on phase space variables.

Schw $q_{ab} = e_{ab} + \frac{f_{ab}(\theta, \varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$

$$c^2 + \left(dr + \sqrt{\frac{2M}{r}} dt \right)^2 + \cancel{r^2} + r^2 d\Omega^2$$

are & Gullstrand. PG

& slices are flat

$$S_{GR} = \int_M \left\{ \pi^{ab} \dot{q}_{ab} - N \mathcal{H} - N^a \mathcal{C}_a \right\} d^3x dt + \dots$$

Fall off conditions on phase space variables.
Schw $q_{ab} = e_{ab} + \frac{f_{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right)$


$$\pi^{ab} = \frac{g^{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

Fall off conditions on phase space variables.

Schw $q_{ab} = \ell_{ab} + \frac{f_{ab}(0, q)}{r^2} + \mathcal{O}\left(\frac{1}{r^2}\right)$

$$\pi^{ab} = \frac{g^{ab}(0, q)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

$$q_{ab} = \ell_{ab}$$

P
G

Fall off conditions on phase space variables.

Schw $q_{ab} = \ell_{ab} + \frac{f_{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^2}\right)$ ℓ_{ab} - flat metric

$$\pi^{ab} = \frac{g^{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

PG

$$q_{ab} = \ell_{ab}$$

Fall off conditions on phase space variables.

Schw $q_{ab} = \ell_{ab} + \frac{f_{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^2}\right)$ ℓ_{ab} -flat metric

$$\pi^{ab} = \frac{g^{ab}(\theta, \varphi)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

PG ($\varphi=0$) $q_{ab} = \ell_{ab}$

$$\pi^{ab} = \frac{c}{r^{3/2}}$$

Parametrization of phase space var

$$q_{ab} =$$

Non-Local

formalism - adapted to flat space coordinates

on collapse

(Higgs ideas)

collapse

Parametrization of phase space var.

For a single charge

$$q_{ab} = \Lambda^2(x) n_a n_b + \frac{R^2(x,t)}{v^2} (p_{ab} - n_a n_b)$$

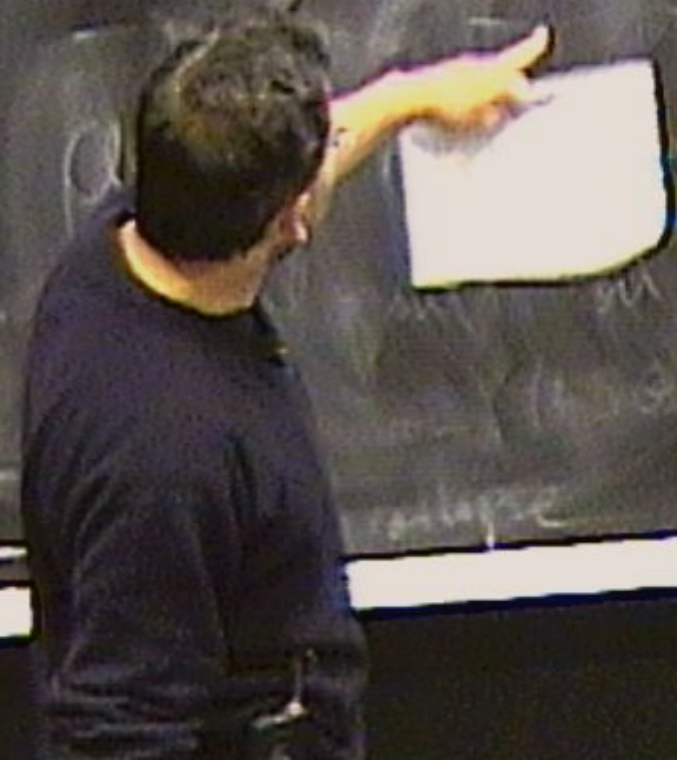
- Classical H.M. formulation
- Classical result on collapse
- Quantum ideas
- Quantum collapse

Parametrization of phase space var.

$$q_{ab} = \Lambda^2(x) n_a n_b + \frac{R^2(x,t)}{v^2} (e_{ab} - n_a n_b)$$

$$n_a = \frac{x_a}{r}$$

$$g = x^a x^b e_{ab}$$



Parametrization of phase space var.

$$g_{ab} = -\Lambda^2(r,t) n_a n_b + \frac{R^2(r,t)}{r^2} (e_{ab} - n_a n_b)$$

$$ds^2 = \Lambda^2(r,t) dr^2 + R^2 d\Omega^2$$

$$n^a = \frac{x^a}{r}$$

$$\gamma = x^a x^b e_a e_b$$

- classical field problem
- classical results not collapse
- quantum ideas
- quantum collapse

Parametrization of phase space var.

$$g_{ab} = \Lambda^2(r,t) n_a n_b + \frac{R^2(r,t)}{v^2} (e_{ab} - n_a n_b)$$

$$ds^2 = \Lambda^2(r,t) dr^2 + R^2 d\Omega^2$$

$$n^a = \frac{x^a}{r}$$

$$v = \sqrt{x^a x^a} e_{ab}$$

$$\tilde{\pi}^{ab} = \frac{P_1}{2\Lambda} n^a n^b + \frac{r^2 P_2}{4R} (e^{ab} - n^a n^b)$$

$$S = \int d^3x dt \left[P_R \dot{R} + P_A \dot{A} + P_\phi \dot{\phi} - N \mathcal{H} - N^r C_r \right]$$

$$S = \dots \int_M dt \left[P_R \dot{R} + P_A \dot{A} + P_\Phi \dot{\Phi} \right. \\ \left. - N \mathcal{H} - N^\alpha C_\alpha \right] \\ + \int_{\partial M} (\dots)$$

$$S = \int_M dt \left[P_R \dot{R} + P_A \dot{A} + P_\Phi \dot{\Phi} - N \mathcal{H} - N^r C_r \right]$$

$$+ \int_M (\dots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\pi}^a \tilde{\pi}_a \right) - \sqrt{q} R + \frac{P_a^2}{2\sqrt{q}} + \sqrt{q} \tilde{q}^4 \tilde{q}^4$$

$$S = \int_M d^4x dt \left[P_R \dot{R} + P_A \dot{A} + P_\Phi \dot{\Phi} \right. \\ \left. - N \mathcal{H} - N^a C_a \right]$$

$$+ \int_{\partial M} (\dots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\Pi}^{ab} \tilde{\Pi}_{ab} - \frac{1}{2} \tilde{\Pi}^2 \right) - \sqrt{q} {}^{(3)}R + \frac{P_A^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \partial_a \partial_b \Phi$$

$$S = \int_M d^4x dt \left[P_R \dot{R} + P_A \dot{A} + P_\Phi \dot{\Phi} \right. \\ \left. - N \mathcal{H} - N^a C_a \right]$$

$$+ \int_{\partial M} (\dots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\pi}^{ab} \tilde{\pi}_{ab} - \frac{1}{2} \tilde{\pi}^2 \right) - \sqrt{q} {}^{(3)}R + \frac{P_A^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \partial_a \partial_b \Phi$$

$$C_a = D_b \tilde{\pi}_a^b$$

$$S = \int_M d^4x dt \left[P_R \dot{R} + P_A \dot{A} + P_\Phi \dot{\Phi} \right. \\ \left. - N \mathcal{H} - N^a C_a \right]$$

$$+ \int_{\partial M} (\dots)$$

$$\mathcal{H} = \frac{1}{\sqrt{q}} \left(\tilde{\Pi}^{ab} \tilde{\Pi}_{ab} - \frac{1}{2} \tilde{\Pi}^2 \right) - \sqrt{q} {}^{(3)}R + \frac{P_A^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \delta_{,ab}$$

$$C_a = D_b \tilde{\Pi}_a^b$$

$$\tilde{\Pi} = \tilde{\Pi}^{ab} q_{ab}$$

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$
$$H = \frac{1}{R^2 \Lambda} \left[\frac{1}{8} (P_A)^2 \right]$$

$$\begin{aligned}
 ds^2 &= -dt^2 + \left[\frac{1}{R^2 \Lambda} \left(\frac{1}{8} (P_A \Lambda)^2 - \frac{1}{4} (P_A \Lambda) (P_R R) \right) \right. \\
 &\quad \left. + \frac{3}{\Lambda^2} \left[2 R R'' \Lambda - 2 R R' \Lambda' - \Lambda^3 + \Lambda R'^2 \right] \right. \\
 &\quad \left. + \frac{P_\phi^2}{2 \Lambda R^2} + \frac{R^2}{2 \Lambda} \phi'^2 \right] \simeq 0
 \end{aligned}$$

$$ds^2 = -dt^2 + \left[\frac{1}{R^2} \left(\frac{1}{8} (P_A)^2 - \frac{1}{4} (P_A)(P_R R) \right) \right]$$

$$+ \frac{3}{\Lambda^2} \left[2RR''\Lambda - 2RR'\Lambda' - \Lambda^3 + \Lambda R'^2 \right]$$

$$+ \frac{P_R^2}{2\Lambda R^2} + \frac{R^2}{2\Lambda} \phi'^2 \approx 0 \quad / \equiv \frac{d}{dr}$$

$$C_r = P_R R' - \Lambda P_A' + P_R \phi'$$

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

$$H = \frac{1}{R^2 \Lambda} \left[\frac{1}{8} (P_A \Lambda)^2 - \frac{1}{4} (P_A \Lambda) (P_R R) \right]$$

$$+ \frac{3}{\Lambda^2} \left[2 R R'' \Lambda - 2 R R' \Lambda' - \Lambda^3 + \Lambda R'^2 \right]$$

$$+ \frac{P_\phi^2}{2 \Lambda R^2} + \frac{R^2}{2 \Lambda} \phi'^2 \approx 0 \quad / \equiv \frac{d}{dr}$$

$$C_r = P_R R' - \Lambda P_\Lambda' + P_\phi \phi' \quad (\text{gr-qc 0503031})$$

$$g_{ab} = \ell_{ab} + \frac{g_{ab}(0, a)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

ℓ_{ab} - flat
3 metric

$$S = \int_0^\infty (\dots) < \infty$$

$$\pi^{ab} = \frac{g^{ab}(0, a)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)$$

PG ($Q=0$) $g_{ab} = \ell_{ab}$

$$\pi^{ab} = \frac{C}{r^{3/2}}$$

$$g_{ab} = \eta_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

η_{ab} - flat metric

CANONICAL GAUGE FIXING

$$g_{ab} = \eta_{ab} + \frac{g_{ab}(0, a)}{r} + O\left(\frac{1}{r^2}\right)$$

η_{ab} - flat metric

CANONICAL GAUGE FIXING

Schematic

$$x^{\mu} = f(p, q)$$

$$g_{ab} = \eta_{ab}$$

$$T_{ab} = \frac{C}{r^2}$$

$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

ϵ_{ab} -flat
metric

CANONICAL GAUGE FIXING

Schematic

$$*x^{\pm} = f(p, q)$$

$$r = g(p, q)$$



$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

ϵ_{ab} - flat metric

CANONICAL GAUGE FIXING

Schematic

$$* \pi = f(p, q)$$

$$r = g(p, q)$$

$$\dot{\pi} = 1 = \left\{ f(p, q), \int N \mathcal{H} + N^a \mathcal{G}_a \right\}$$

$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0,0)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

Curved flat
metric

CANONICAL GAUGE FIXING

Schematic

$$* \pi^{\pm} = f(p, q)$$

$$r = g(p, q)$$

$$\pi^{\pm} = \{ f(p, q), \int N_H + N_G \} \rightarrow N(N^3)$$

$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0,0)}{r} + O\left(\frac{1}{r^2}\right) \quad \epsilon_{ab} - \text{flat metric}$$

CANONICAL GAUGE FIXING

Schematic

$$*x = f(p, q)$$

$$r = g(p, q)$$

$$1 = \left\{ \begin{array}{l} f(p, q), \int N_H + N_G \\ g(p, q), \dots \end{array} \right\} \begin{array}{l} \longrightarrow N(N^a) \\ \longrightarrow N(N^a) \end{array}$$

$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

ϵ_{ab} - flat metric

CANONICAL GAUGE FIXING

Schematic

$$* \dot{x}^a = f(p, q)$$

$$\dot{r} = g(p, q)$$

Soln $\rightarrow N = \dots$
 $N^a = \dots$

$$\begin{aligned} \dot{x} = 1 &= \left\{ f(p, q), \int N \mathcal{H} + N^a \mathcal{C}_a \right\} \rightarrow N(N^a) \\ \dot{r} = 1 &= \left\{ g(p, q), \dots \right\} \rightarrow N(N^a) \end{aligned}$$

$$g_{ab} = \eta_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

Euler-flat
3 metric

CANONICAL GAUGE FIXING

Schematic

$$* \dot{x} = f(p, q)$$

$$r = g(p, q)$$

Soln $\rightarrow N = \dots$
 $N' = \dots$

$$\begin{aligned} \dot{x} = 1 &= \left\{ f(p, q), \int N \mathcal{H} + N' G \right\} \rightarrow N(N') \\ r = 1 &= \left\{ g(p, q), \dots \right\} \rightarrow N(N') \end{aligned}$$

$$g_{ab} = \epsilon_{ab} + \frac{g_{ab}(0, \alpha)}{r} + O\left(\frac{1}{r^2}\right)$$

ϵ_{ab} - flat metric

CANONICAL GAUGE FIXING

Schematic

$$* \dot{x}^a = f(p, q)$$

$$r = g(p, q)$$

Soln $\rightarrow N = \dots$
 $N^a = \dots$

$$\begin{aligned} \dot{x} = 1 &= \left\{ f(p, q), \int N \mathcal{H} + N^a \mathcal{G}_a \right\} \rightarrow N(N^a) \\ \dot{r} = 0 &= \left\{ g(p, q), \dots \right\} \rightarrow N(N^a) \end{aligned}$$

Let's choose $R=r$

Let's choose $R=r \rightarrow N^r = \dots$

$C_r = 0$ strongly

Let's choose $R=r \rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

Let's choose $R=r \rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_R Q' + A P_n'$$

Let's choose $R=r \rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_a Q' + \Lambda P_a'$$

$\# = f(\text{a momentum})$

Let's choose $R=r \rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_e Q' + \lambda P_n'$$

$\lambda = f(\text{a momentum})$

Let's choose $R=r$ $\rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_e Q' + \Lambda P_n'$$

$\Lambda = f$ (a momentum)

The gauge condition $\vec{E} = -\vec{\nabla}\phi$ together with
a time gauge $\rightarrow$ non-local reduced

The gauge condition $R=r$ together with
a time gauge $\rightarrow$ non-local reduced

Hamiltonian

* Do not fix $R=r$. Let R remain dynamical
variable!

The gauge condition $R=r$ together with
a "time gauge" $\rightarrow$ non-local reduced

Hamiltonian

* Do not fix $R=r$. Let R remain dynamical
variable!

$$g_{ab} = \eta_{ab} + \frac{\int g_{ab}(\vec{x})}{r} + O\left(\frac{1}{r^2}\right)$$

ϵ_{ab} -flat
3 metric

We fix only a time gauge.



$$g_{ab} = \eta_{ab} + \frac{g_{ab}(y/a)}{r} + O\left(\frac{1}{r^2}\right)$$

Cur-flat
3 metric

We fix only a time gauge.

$$\Lambda = 1$$

Solve $\mathcal{H} = 0$ for P_1 .

$$h_{ab} = \eta_{ab} + \frac{h_{ab}(r)}{r} + O\left(\frac{1}{r^2}\right)$$

Calc - flat
3 metric

We fix only a time gauge.

$$\Lambda = 1$$

the $H=0$ for P_1

— a quadratic eqn
for P_1

$$\frac{1}{r} - \frac{1}{r} + \frac{1}{r} + O\left(\frac{1}{r^2}\right)$$

Cur-flat
3 metric

We fix only a time gauge.

$$\Lambda = 1$$

Solve $\mathcal{H} = 0$ for P_1 — a quadratic eqn
for P_1

$$S = \int dr dt \left[\dot{n} P_n + \dot{\phi} P_\phi + \dot{R} P_R \right. \\ \left. - N \mathcal{H} - N' C_V \right] + \int dt (\dots)$$

$$S = \int dr dt \left[\dot{\eta} P_{\eta} + \dot{\phi} P_{\phi} + \dot{R} P_R \right. \\ \left. - N \mathcal{H} - N' C_V \right] + \int dt (\dots)$$

$$S^{GF} = \int dr dt \left[\dot{\phi} P_{\phi} + \dot{R} P_R \right]$$

Lab-flat
3 metric

fix only a time gauge.

$$\Lambda = 1$$

$H=0$ for P_1

a quadratic eqn.
for P_1
2 solns. — one is "physical"

Lab-flat
metric

fix only a time gauge

$$\Lambda = 1$$

for P_1

— a quadratic eqn
for P_1

2 solns. — one is "physical"
ie. Soln for $g_{tt} = 0$

$$S = \int dt \left[\dot{\alpha} P_{\alpha} + \dot{\phi} P_{\phi} + \dot{R} P_R - N \mathcal{H} - N' C_r \right] + \int dt (\dots)$$

$$S^{GF} = \int dt \left[\dot{\phi} P_{\phi} + \dot{R} P_R - N' C_r \right] + \int dt (\dots)$$

$$S = \int dr dt \left[\dot{\pi} P_{\pi} + \dot{\phi} P_{\phi} + \dot{R} P_R - N \mathcal{H} - N^i C_i \right] + \int dt (\dots)$$

$$S^{GF} = \int dr dt \left[\dot{\phi} P_{\phi} + \dot{R} P_R - N^i C_i \right] + \int dt (\dots)$$

$$S = \int dr dt \left[\dot{\eta} P_{\eta} + \dot{\phi} P_{\phi} + \dot{R} P_R - N \mathcal{H} - N^r C_r \right] + \int dt (\dots)$$

$$S^{GF} = \int dr dt \left[\dot{\phi} P_{\phi} + \dot{R} P_R - N^r C_r \right] + \int dt (\dots)$$

$$C_r = P_R R' + P_{\phi} \phi' - \Lambda P_{\eta}' \approx 0$$

$$S = \int dt \left[\dot{\Lambda} P_{\Lambda} + \dot{\Phi} P_{\Phi} + \dot{R} P_R - N \mathcal{H} - N^i C_i \right] + \int dt (\dots)$$

$$S^{GF} = \int dt \left[\dot{\Phi} P_{\Phi} + \dot{R} P_R - N^i C_i^{GF} \right] + \int dt (\dots)$$

$$C_i = P_R R' + P_{\Phi} \Phi' - \Lambda P_{\Lambda}' \approx 0 \quad C_i^{GF} = P_R R' + P_{\Phi} \Phi' - P_{\Lambda}'(R, P_R, \Phi, P_{\Phi})$$

X Let's choose $R=r$ $\rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_Q Q' + \lambda P_n' \quad \text{QUANTISE}$$

$$\lambda = f(\text{a momentum})$$

Schw $q_{ab} = e_{ab} + \frac{f_{ab}(0, a)}{r} + O\left(\frac{1}{r^2}\right)$ e_{ab} - flat 3-metric

$$ds^2 = -f(r, t) dt^2 + \left(dr + g(r, t) dt \right)^2$$

$$A = \pm \frac{1}{r}$$

Solution P_1 for P_1 is a solution to the problem $\square u = 0$ in $\mathbb{R}^n$ with $u = 0$ on ∂B_R for $R > 0$ and $u = 0$ on ∂B_1 for $R > 0$.

Schw $q_{ab} = e_{ab} + \frac{f_{ab}(0, \varrho)}{r} + O\left(\frac{1}{r^2}\right)$

e_{ab} - flat
3 metric

$$ds^2 = -f(r,t) dt^2 + \left(dr + g(r,t) dt \right)^2 + r^2 d\Omega^2$$

X Let's choose $R=r$ $\rightarrow N^r = \dots$

$C_r = 0$ strongly for P_R

$$P_R = -P_Q Q' + A P_n'$$

$A = f(\text{a momentum})$

QUANTISE
 S_{eff}

$(Q, P_Q) (R, P_R)$

$(r \approx 0)$

Schw

$$g_{ab} = e_{ab} + \frac{f_{ab}(0,0)}{r} + O\left(\frac{1}{r^2}\right)$$

$e_{ab} = \delta_{ab}$

$$ds^2 = -f(r,t) dt^2 + (dr + g(r,t) dt)^2$$

$$+ r^2 d\Omega^2$$

$$ds^2 = -f^2(r,t) dt^2 + g^2(r,t) dr^2 + r^2 d\Omega^2$$

fall off conditions on phase space variables
Schw $I_{ab} = E_{ab} + f_{ab}(0, Q) + O(\frac{1}{r^2})$

Classical results

fall off conditions on phase space variables
Schw $\rho_{ab} = \rho_{ab} + \rho_{ab}(0,0) + O(\frac{1}{r^2})$

Classical results

- 1) Christodoulou (1975)

fall off conditions on phase space variables
 Schw $\rho_{ab} = \rho_{ab} + \rho_{ab}(0, Q) + O(\frac{1}{\sqrt{\epsilon}})$

Classical results

1) Christodoulou (1975)

Sph, Sym & Asym. flat, Q minim coupled

fall off conditions on phase space variables
Schw $\rho_{ab} = \rho_{ab} + f_{ab}(0, Q) + O(\frac{1}{r^2})$

Classical results

1) Christodoulou (1975)

Sph, Sym & Asym. flat, & minim coupled

"Weak" flat

→ no bh formation

fall off conditions on phase space variables
Schw $\rho_{ab} = \rho_{ab} + \rho_{ab}(\theta, \varphi) + O(\frac{1}{r^2})$

Classical results

1) Christodoulou (1975)

Sph, Sym & Asym. flat, & minim coupled

"Weak" data



no bh formation

"Strong" data



bh forms



fall off conditions on phase space variables
Schw $q_{ab} = e_{ab} + f_{ab}(\theta, \varphi) + \mathcal{O}(\frac{1}{r^2})$

Classical results

1) Christodoulou (1975)

Sph, Sym & Asym. flat, & minim coupled

"Weak" data $\longrightarrow$ no bh formation

"Strong" " $\longrightarrow$ bh forms

What is the transition?

$$Q(y, t=0)$$

N^C

$$P_R = P_0$$

strongly for P_R

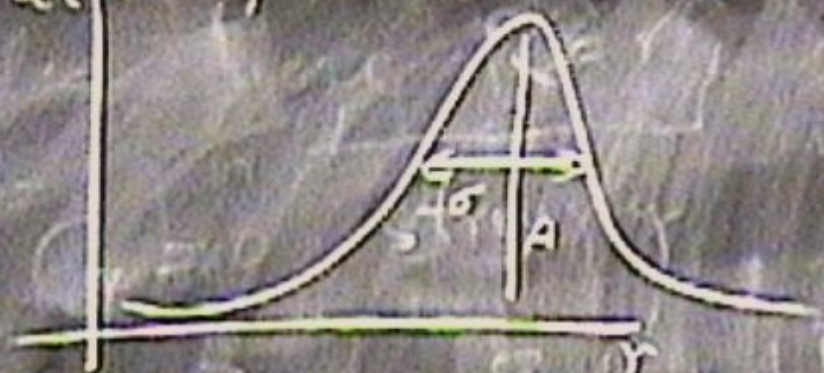
$$P_R = -P_0 \frac{d}{dt} + P_0$$

analysis of

$$(A, P_0) (R, 1)$$

$$C_1 = 40$$

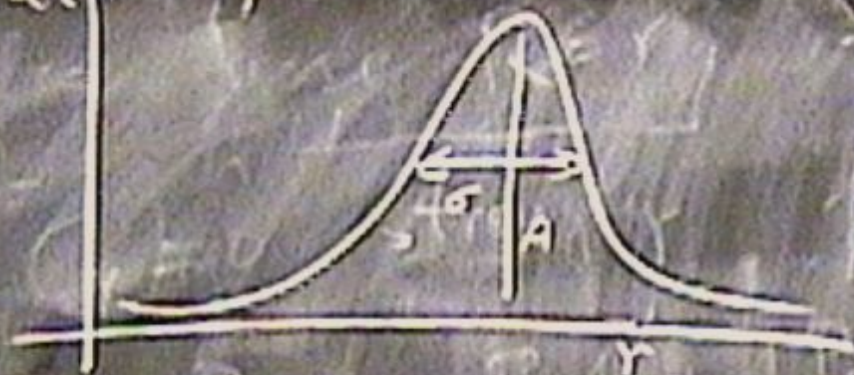
$Q(y, t=0)$



$$P_R = -T_R Q'' + (1/T_R)$$

$$(0, P_R) (R, 1)$$

$Q(y, t=0)$



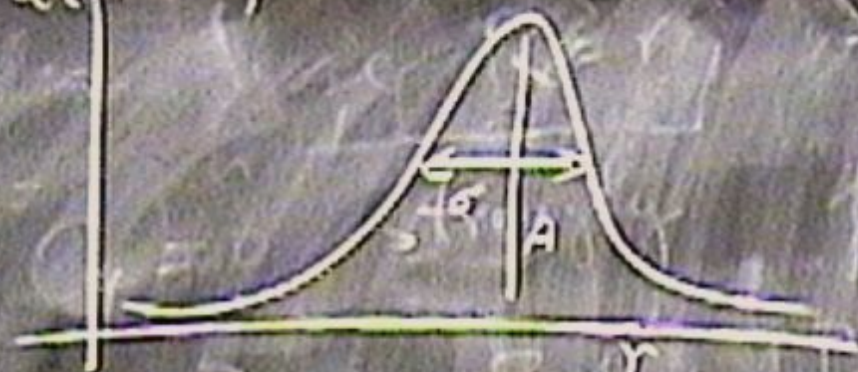
$$P_a = 0$$

$$P_R = -P_a \sigma^2 + \ln A$$

$$(0, P_a) \quad (R, 1)$$

$$y \rightarrow 0$$

$Q(y, t=0)$



$P_a = 0$

$$P_R = -P_1 \frac{dr}{dt} + \dots$$

$$M_{BH} \sim (A - A_x)^{\gamma}$$

$$(0, P_a) (R, \dots)$$

$Q(y, t=0)$



$P_a = 0$

P_R

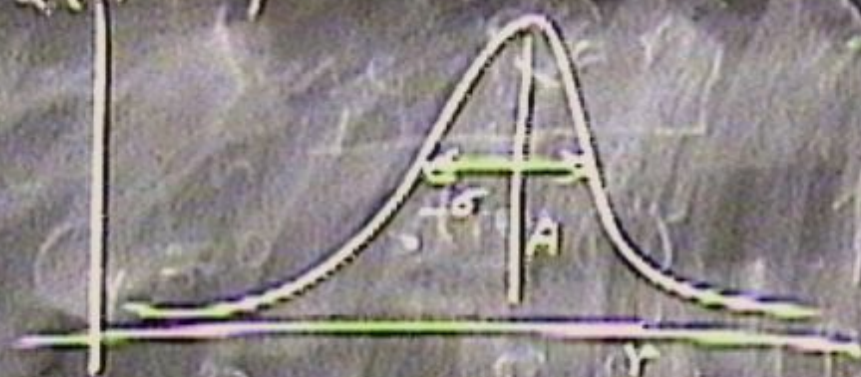
$$P_R = -P_a \ln r + \ln A$$

$$M_{BH} \sim (A - A_*)^{\gamma}$$

$(0, P_a) (R, P_R)$

$(r=0)$

$Q(v, t=0)$



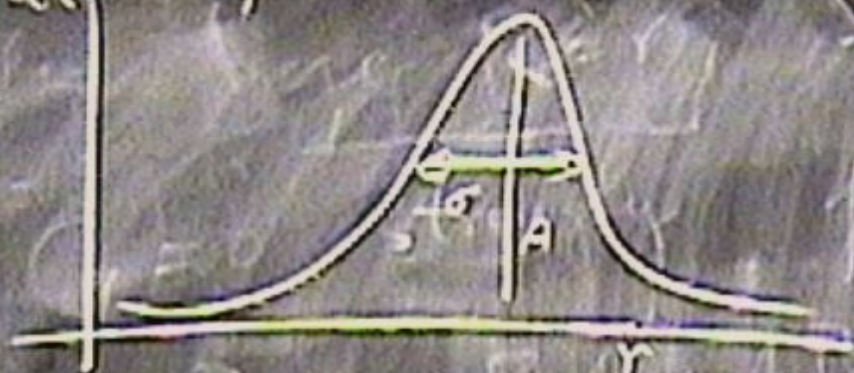
$P_R = 0$

$$P_R = -\frac{1}{2} \ln \left(\frac{A}{A_*} \right)$$

Supercritical: $M_{BH} \sim (A - A_*)^2$
 Critical: Naked Singularity
 Subcritical: Scalar field solitons

A_*, Y compared universe
 $(0, A_*) (R, Y)$

$Q(y, t=0)$



$P_R = 0$

$$P_R = -P_R \frac{dr}{dt} + \dots$$

Super Critical: $M_{BH} \sim (A - A_c)^{\gamma}$

Critical: Naked Singularity.

Sub critical: Scalar field scatterers.

A_c, γ computed numerically
 $(0, 1) (R, 1)$

Detecting M_{BH}

$\Phi P_a + \dots$

$$- N^i H - N^i C_i + \dot{R} P_a - N^i C_i$$

$$P_a = \dots$$

Detecting M_{BH}



Detecting M_{BH}



$L^+H - N^+C_1$

$+ R^+P_2 - N^+C_2$

$$G^+ = \int dr$$

$$C_2 = P_R P_2 + P_2 P_R - P_R P_2$$

$$C_2 = P_R P_2 + P_2 P_R - P_R P_2$$

Detecting M_{BH}



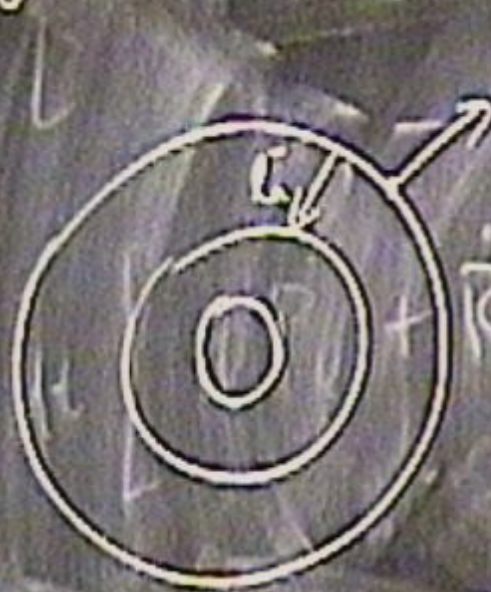
check to see
what spheres are
trapped

$$S^2 = \int d\Omega \sqrt{h}$$

$$E = P_R R + P_\theta \theta' - A P_\theta = 0$$

$$E = P_R R + P_\theta \theta' - A P_\theta = 0$$

Detecting M_{BH}



check to see
what spheres are
trapped

$$\sigma^{ab} D_a l_b^+$$

Detecting M_{BH}



check to see
what spheres are
trapped

$$\sigma^{ab} D_a \ell_b^\pm = \theta^\pm$$

$$+ \frac{1}{r^2} + O\left(\frac{1}{r^2}\right)$$

U.S.

$$ds^2 = -\left(1 + \frac{2M}{r}\right) dt^2$$

$$+ \left(1 + \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Alou (4975)

As you see, it is a minimally coupled

formation

bh forms

Detecting M_{BH}



check to see
what spheres are
trapped

$$\sigma^{ab} D_a \ell_b^\pm = \theta^\pm$$

Test for BH

Detecting M_{BH}



check to see
what spheres are
trapped

$$\sigma^{ab} D_a l_b^{\pm} = \theta^{\pm}$$

Test for BH: Find the sphere(s) on which $\theta^+ = 0$
 $\theta^- < 0$

Detecting M_{BH}

check to see
what spheres are
trapped

Trapping

$$\theta^+ < 0$$

$$\theta^- < 0$$



$$\sigma^{ab} D_a l_b^\pm = \theta^\pm$$

Marginal Trapping

Test for BH: Find the sphere(s) on which $\theta^+ = 0$
 $\theta^- < 0$

Detecting M_{BH}

Normal sphere: $\theta_+ > 0$
 $\theta_- < 0$

Trapping

$\theta_+ < 0$
 $\theta_- < 0$

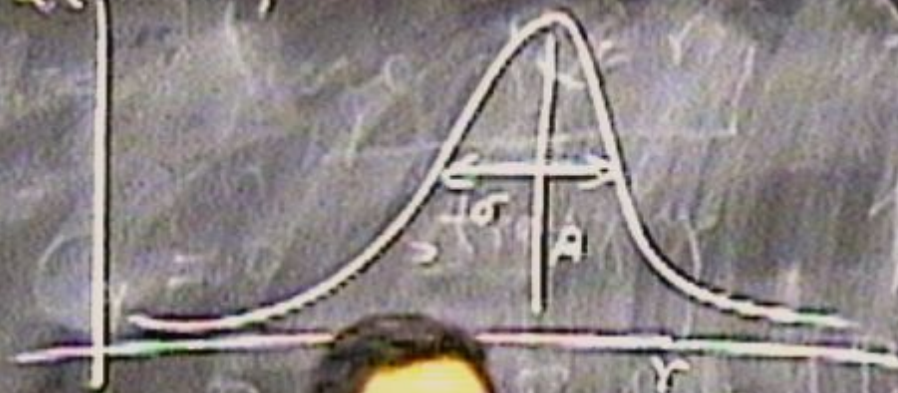


check to see
 what spheres are
 trapped

$\sigma^{ab} D_a l_b^\pm = \theta^\pm$ Marginal Trapping
 on which $\theta^+ = 0$
 $\theta^- < 0$

Test for BH: Find the sphere(s) on which

$$Q(r, t=0)$$



$$Q+(r, t=0)$$

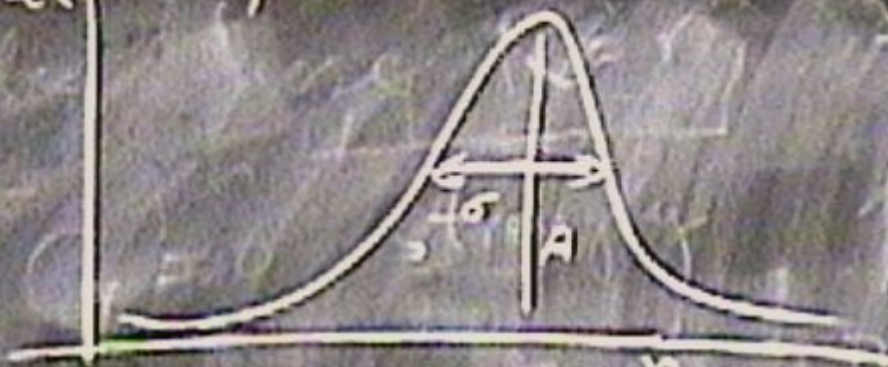


supercritical: M_{BH}
 Critical:
 Subcritical:

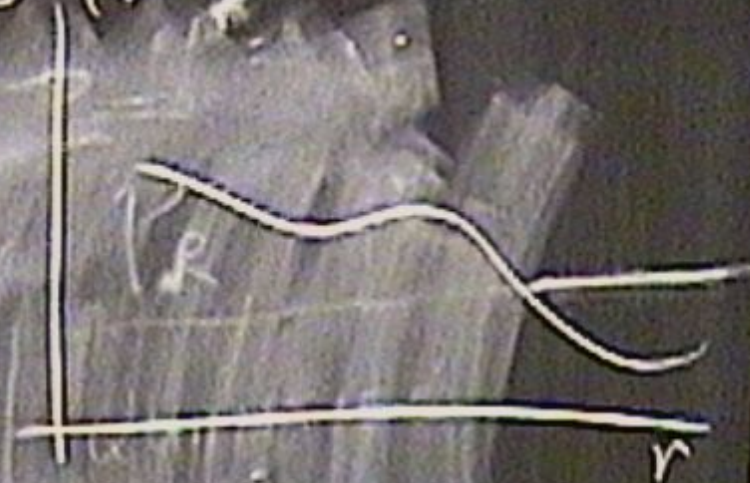
$A(r)$
 clearly
 scatterers.

$A(r, Y)$ (computed numerically)
 $(0, a)$ $(R, 1)$

$Q(r, t=0)$



$Q+(r, t=0)$



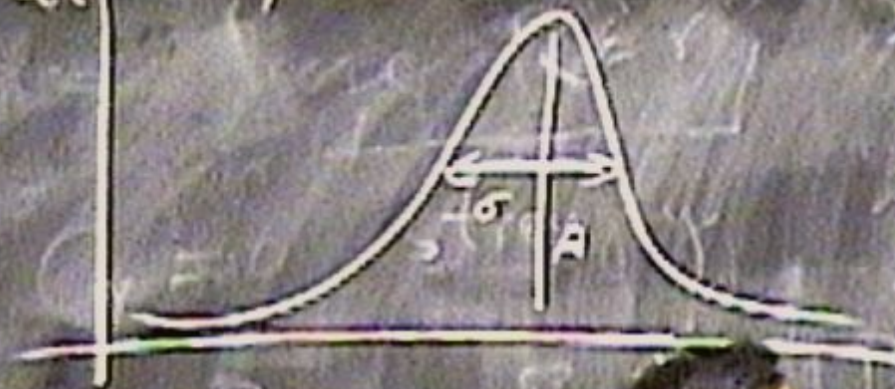
$$P_r = -P + Q^2 + A \ln$$

$$\sim (A - A_s)^r$$

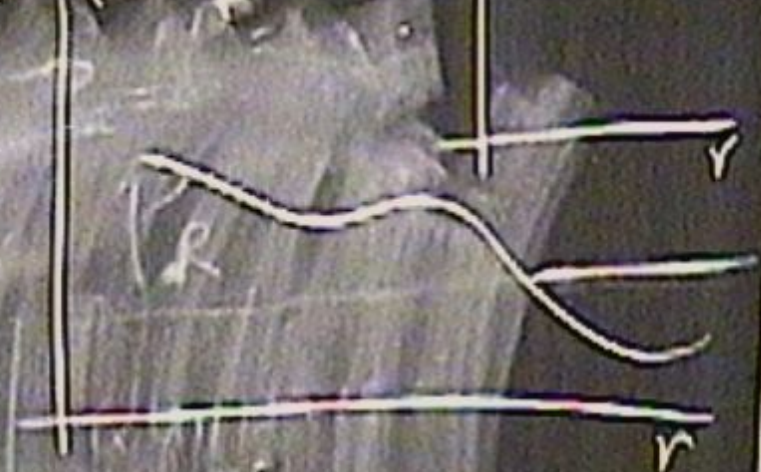
Naked Singularity.
Scalar field scatters.

A_s, Y computed numerically
(0.12) (R, 1)

$Q(r, t=0)$



$Q+(r, t=0)$



$Q-(r, t=0)$



$$P_R = -P_A$$

super critical: $M_{BH} \sim (A - A_*)$

Critical: Naked Singularity

Sub critical: Scalar field

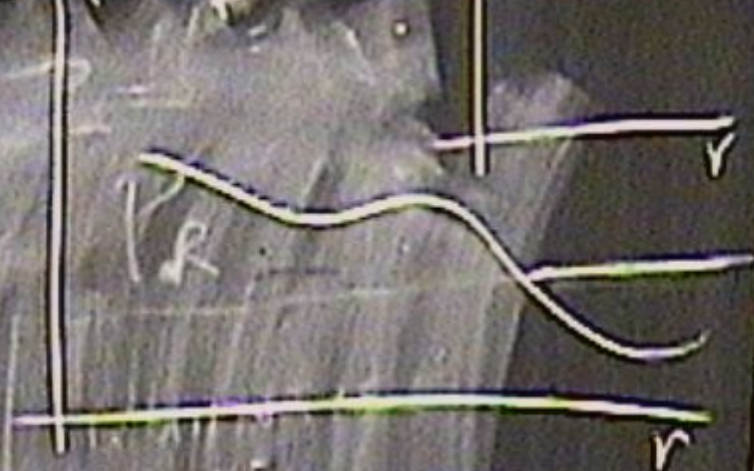
A_*, Y (computed numerically)
(0.1, 0.1) (R, 1)

$$C = 10$$

$Q(r, t=0)$



$Q+(r, t=0)$



$Q-(r, t=0)$



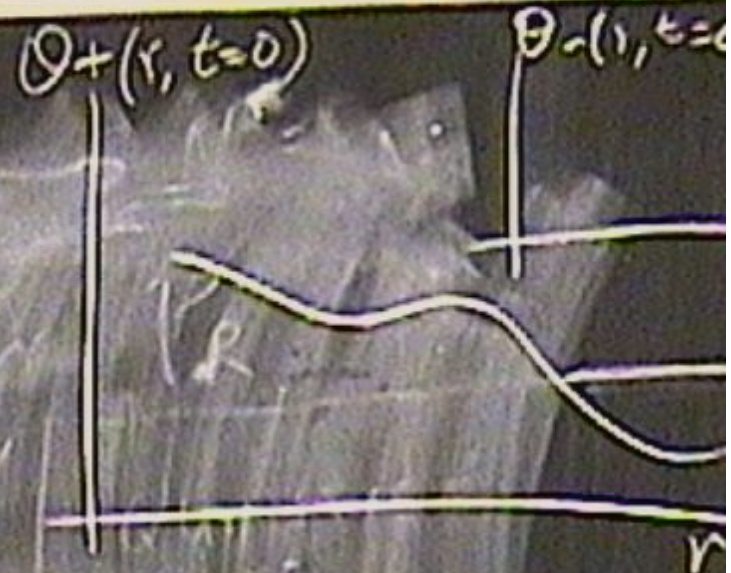
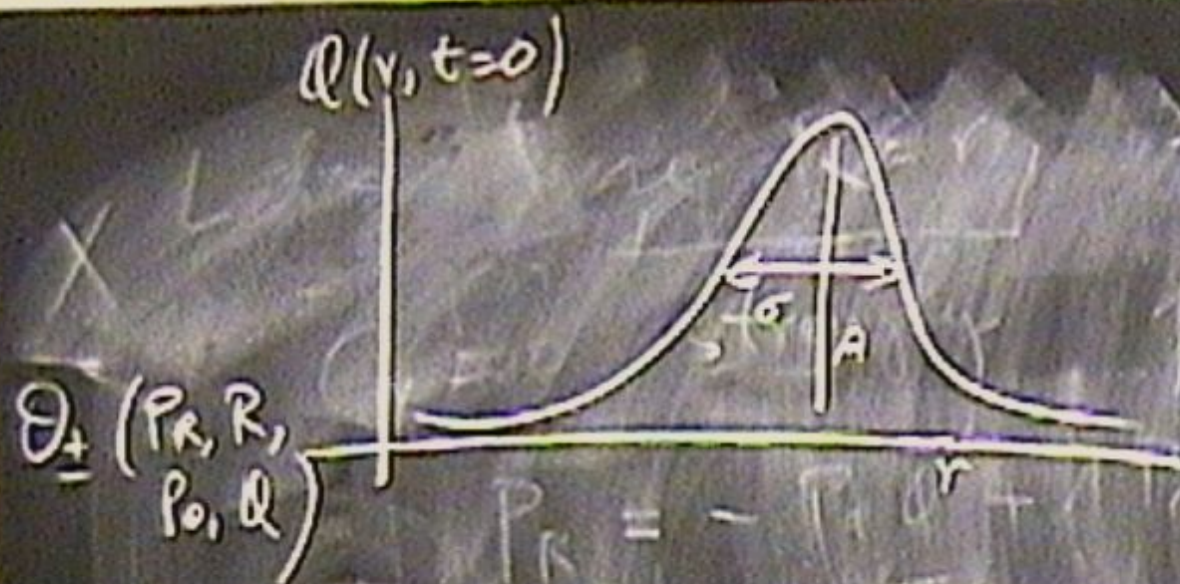
$$P_R = -\frac{1}{2} \ln \left(\frac{A}{A_*} \right)$$

$$q = M_{BH} \sim (A - A_*)^{\gamma}$$

A_*, γ computed numerically
(0.1, 0.2) (R, 1)

cal: Naked Singularity.

critical: Scalar field scatters.



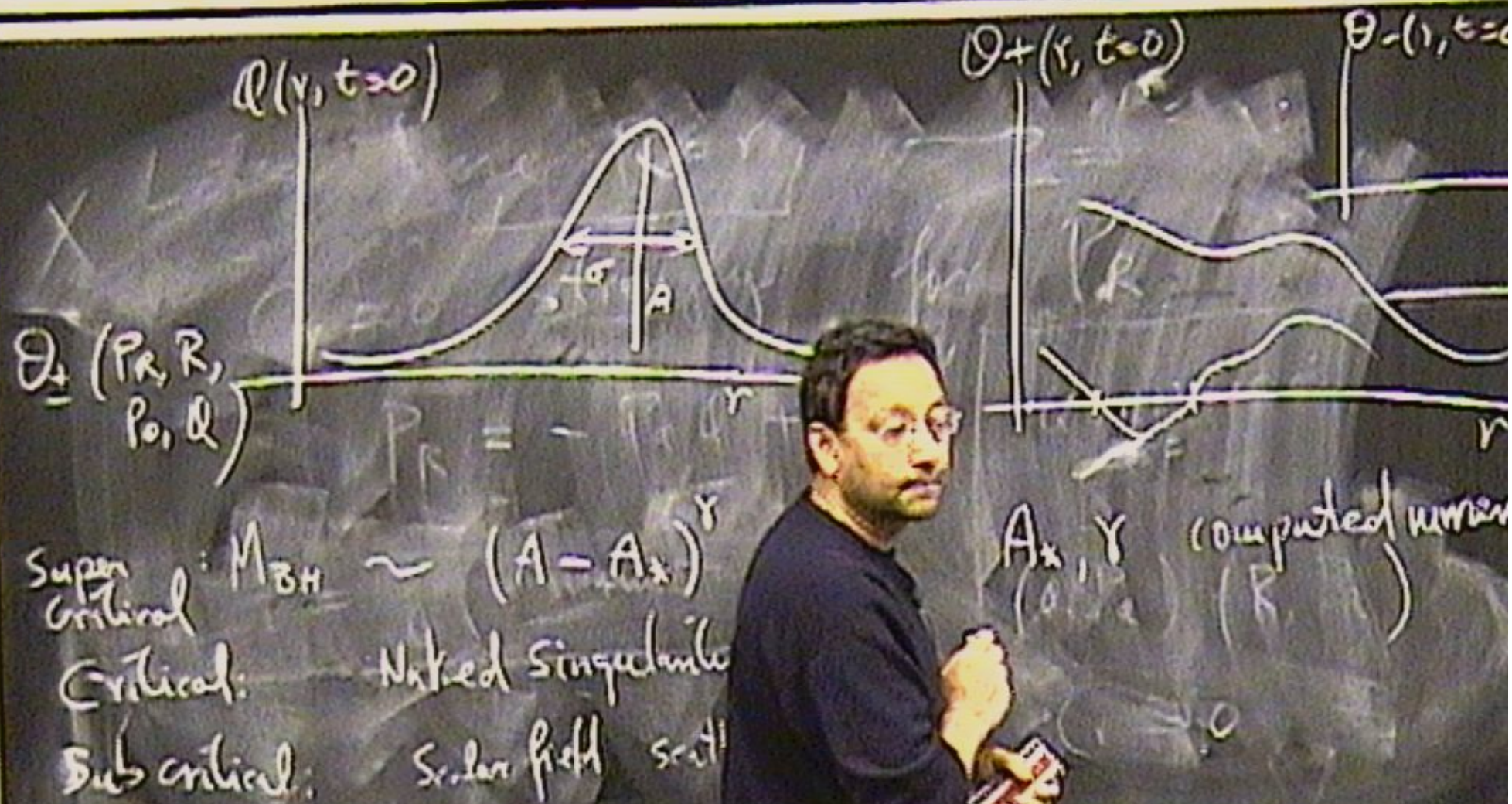
Super Critical: $M_{BH} \sim (A - A_*)^{\gamma}$

Critical: Naked Singularity.

Sub critical: Scalar field scatterers.

A_*, γ (computed numerically)

$(0, 1) (R, 1)$



$Q(r, t=0)$

$Q_+(r, t=0)$

$Q_-(r, t=0)$

$\partial_+ (P_R, R, P_0, Q)$

$$P_R = -\frac{1}{r} \frac{d}{dr} (r^2 \frac{dQ}{dr})$$

$$M_{BH} \sim (A - A_*)^{\frac{1}{2}}$$

Naked Singularity.

Subcritical: Scalar field scatters.

A_*, γ computed numerically
(0.14) (R, 1)