Title: Controlled dynamics in ultracold atomic systems

Date: Apr 05, 2006 04:00 PM

URL: http://pirsa.org/06040004

Abstract: I will investigate the creation and detection of multipartite entangled states in systems of ultracold neutral atoms trapped in an optical lattice. These setups are scalable, highly versatile and controllable at the quantum level. Thus they provide an ideal test bed for studying the properties of multipartite entangled states. I will first present methods exploiting incoherent dynamics for initializing an atomic quantum register. The immersion of an optical lattice in a Bose-Einstein condensate leads to spontaneous emission of phonons. This process can be used for irreversibly loading and cooling atoms within the lowest Bloch band of the lattice. I will describe loading and cooling schemes based on this mechanism and compare them to conventional loading schemes. I will then show how coherent dynamics in a very strongly interacting 1D optical lattice setup can be used for the efficient generation of arbitrary graph states in the atomic quantum register. This system can be mapped onto an XY spin chain which itself is equivalent to a system of non-interacting fermions. By exploiting the anticommutation relations between these fictitious fermions I will discuss how any graph state can be realized in an efficient and robust way. In the final part of my talk I will present a practical method for detecting and characterizing multipartite entangled states in atomic quantum registers. This scheme is based on measuring violations of entropic inequalities using simple quantum networks involving only two copies of the quantum state under consideration. I will investigate the performance of this method under realistic conditions taking into account the most common sources of experimental errors.

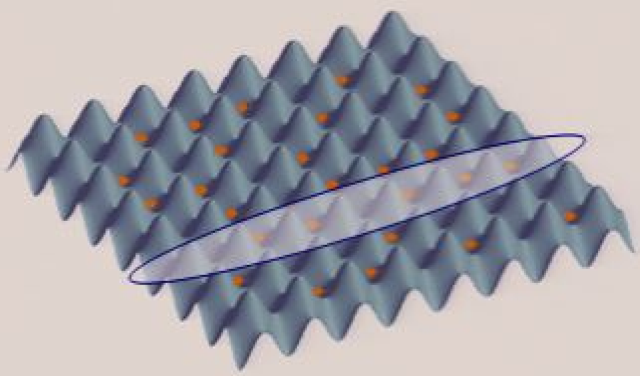
Controlled dynamics in ultracold atomic systems

Dieter Jaksch
University of Oxford

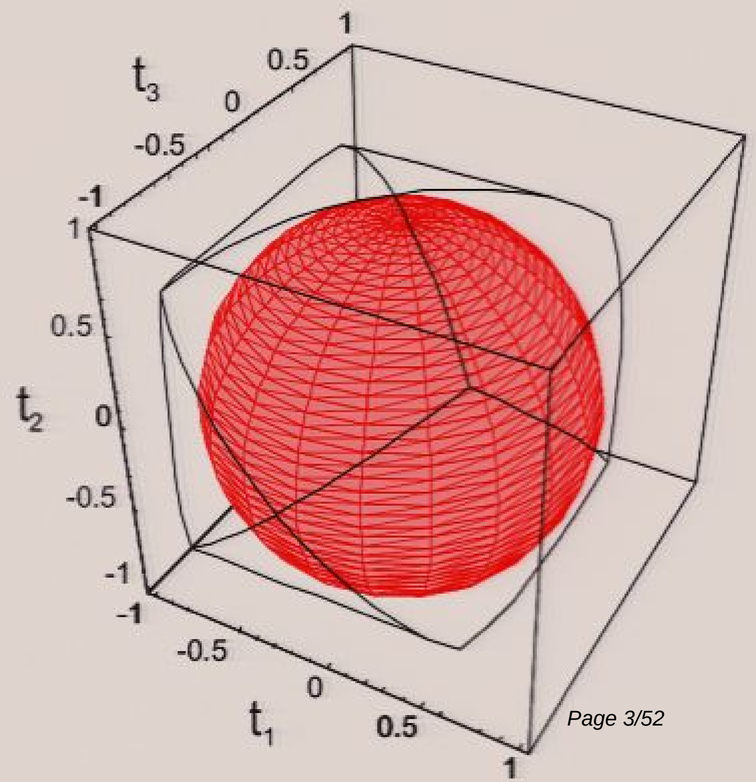
EU networks: OLAQUI, QIPEST

Overview

- Part I: Loading a lattice
  - ⇒ Cold atoms in optical lattices
  - ⇒ Loading an optical lattice

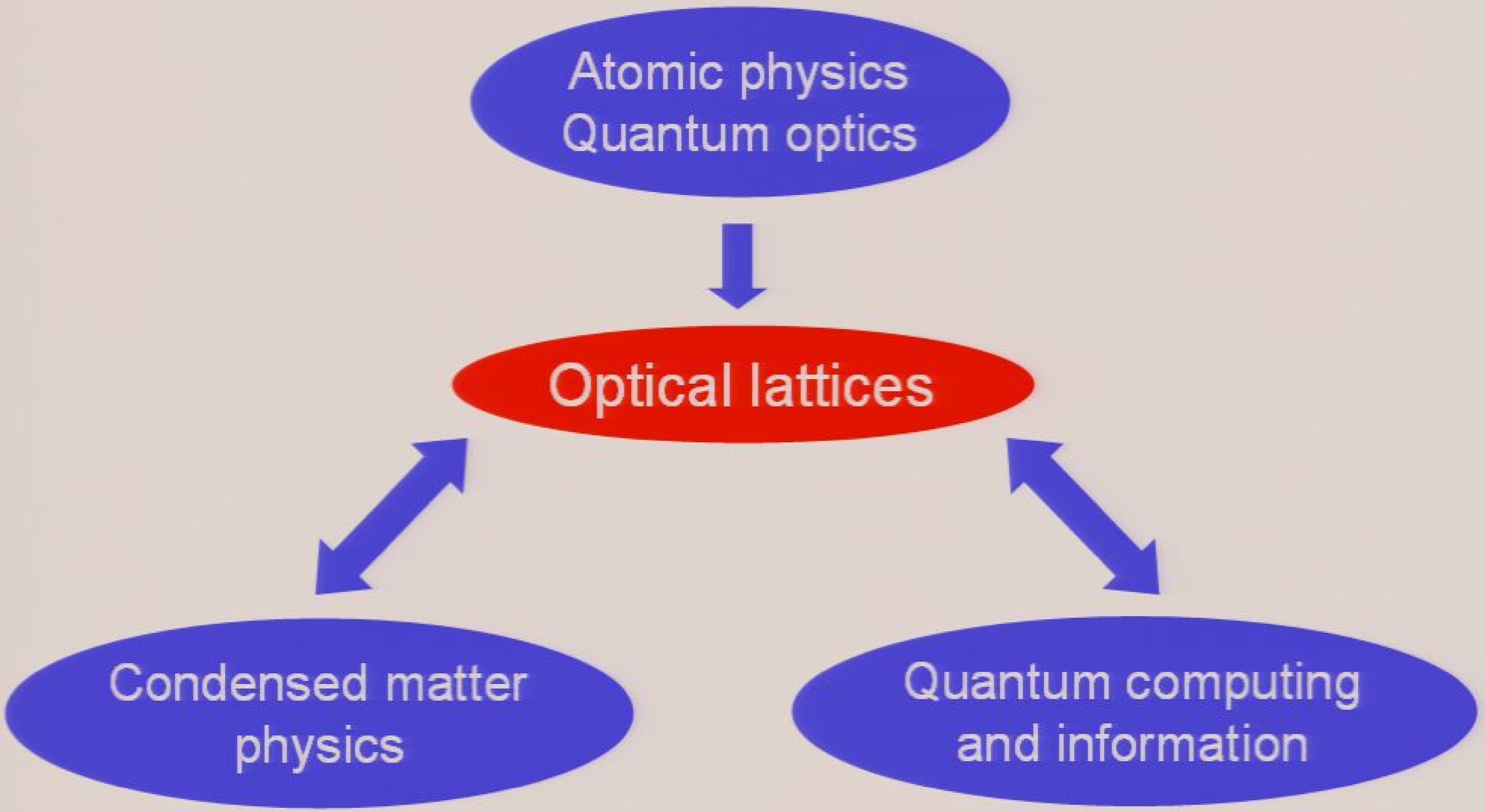


- Part II: Entanglement generation
  - ⇒ Creation of graph states in a strongly correlated chain



- Part III: Detection and Characterization of Multipartite Entanglement
  - ⇒ Simple entanglement detection networks
  - ⇒ Realistic in current experiments?
  - ⇒ Experimental imperfections
  - ⇒ Limited spatial resolution

Optical lattice physics



Optical lattice physics

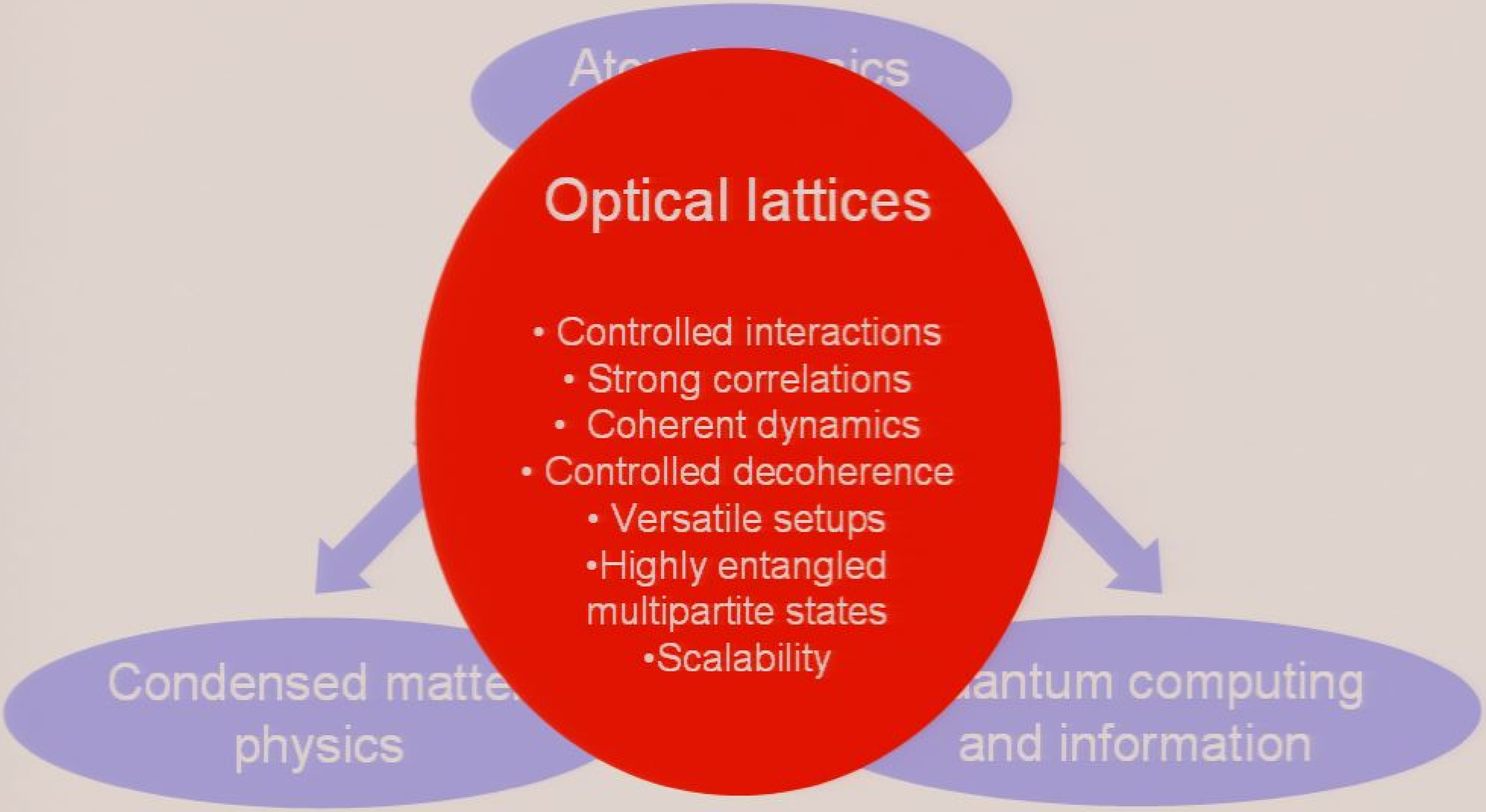
Atomic physics
Quantum optics

- Toolbox of building blocks
- Clean physical systems
- Well defined properties
- Control on the quantum level

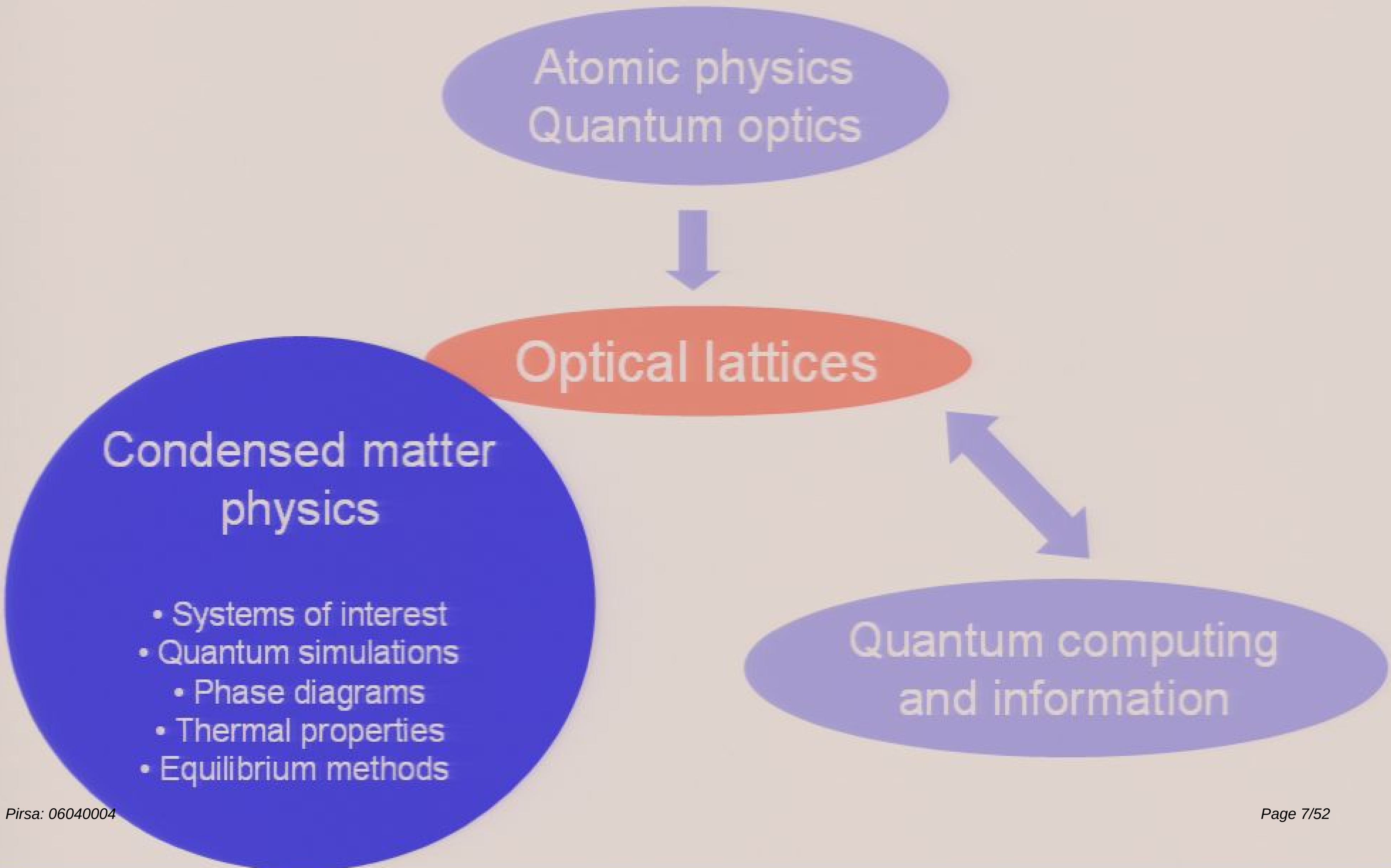
Condensed matter physics

Quantum computing and information

Optical lattice physics



Optical lattice physics



Optical lattice physics

Atomic physics
Quantum optics

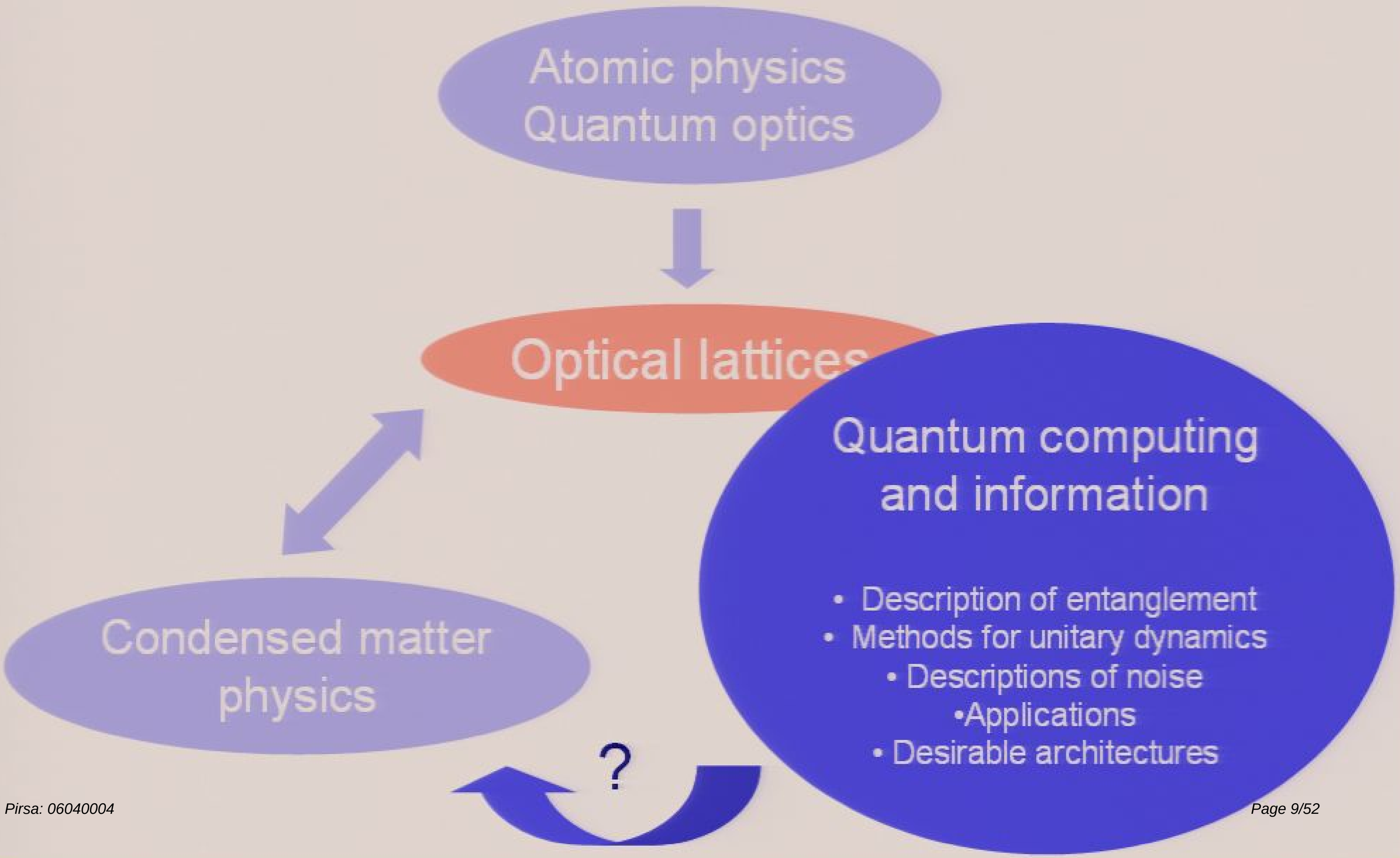
Optical lattices

Condensed matter physics

Quantum computing and information

- Description of entanglement
- Methods for unitary dynamics
- Descriptions of noise
- Applications
- Desirable architectures

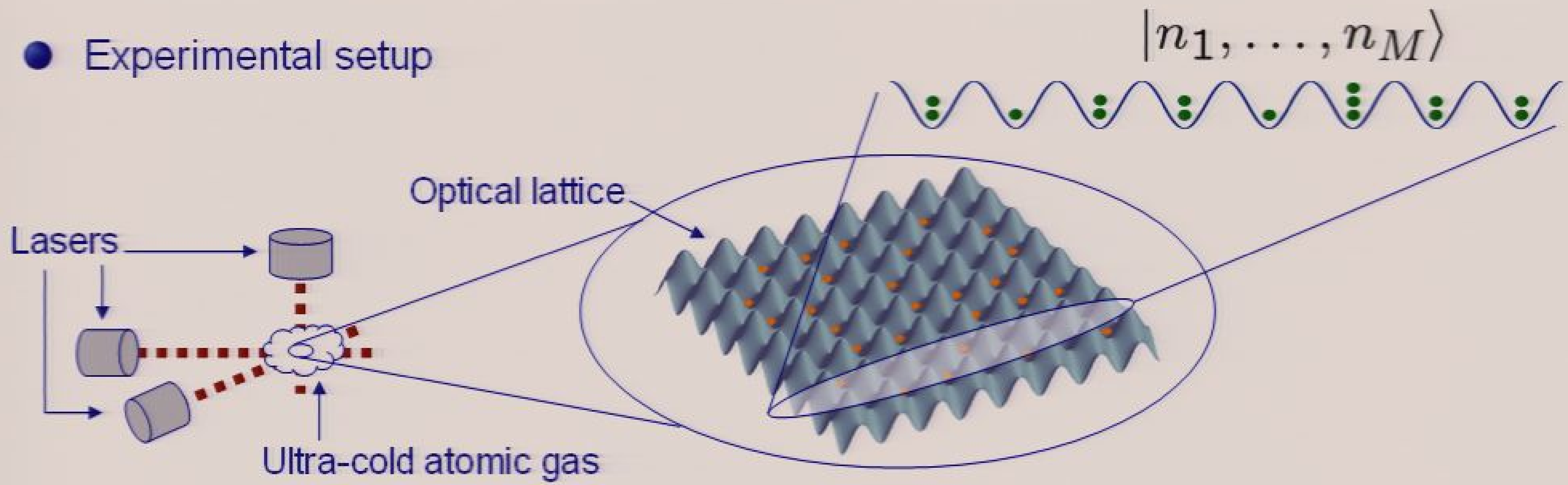
Optical lattice physics



Part I: Loading a lattice

Optical lattice

- Experimental setup



- Described by the Bose-Hubbard model

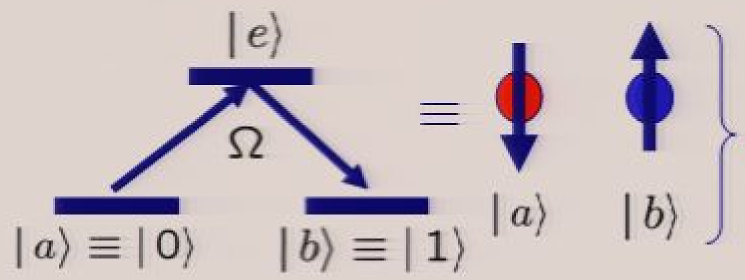
$$H = -J \sum_{\langle \alpha,\beta \rangle} (a_\alpha^\dagger a_\beta + \text{h.c.}) - \mu_\alpha a_\alpha^\dagger a_\alpha + \frac{U}{2} a_\alpha^\dagger a_\alpha^\dagger a_\alpha a_\alpha,$$

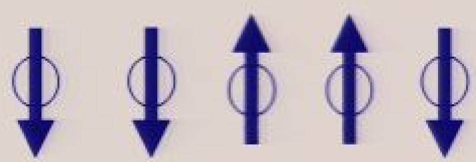
Hopping term J and interaction U are adjustable via the lattice depth

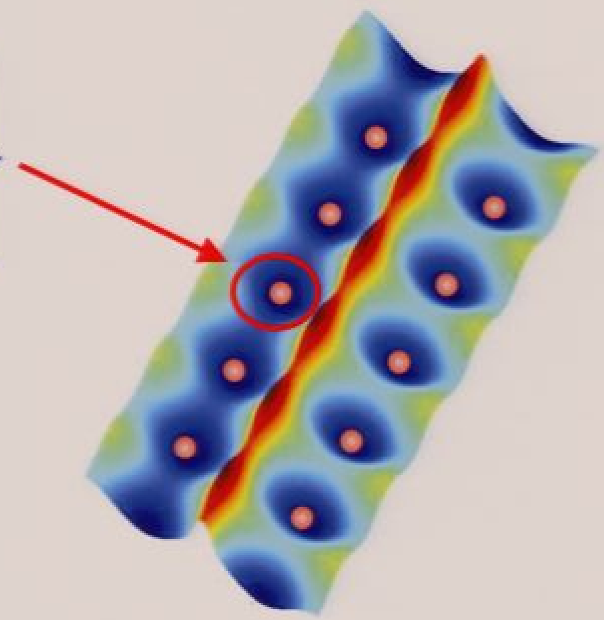
- a_α bosonic destruction operator for atoms in site α.
- Additional degrees of freedom provided by internal atomic levels → qubits

Realization of qubits in optical lattices

- Optical lattice setup







- A Mott insulating state of commensurate filling can be achieved for $U \gg J$
- In this limit a quantum register of neutral atoms is realized by each column of the atomic lattice

$$|00110\rangle = a_1^\dagger a_2^\dagger b_3^\dagger b_4^\dagger a_5^\dagger |\mathrm{vac}\rangle$$

Current loading methods

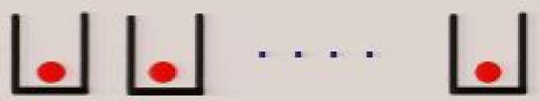
- Adiabatic loading in one sweep with almost no disorder

  ⇒ Arrange atoms by repulsion between bosons

Supefluid

D. J., C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

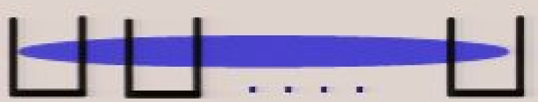
Mott insulator



M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).

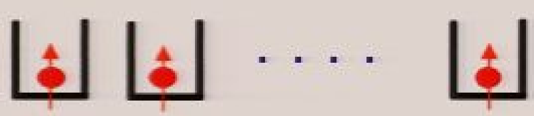
  ⇒ Arrange atoms by Fermi blocking

Fermi gas



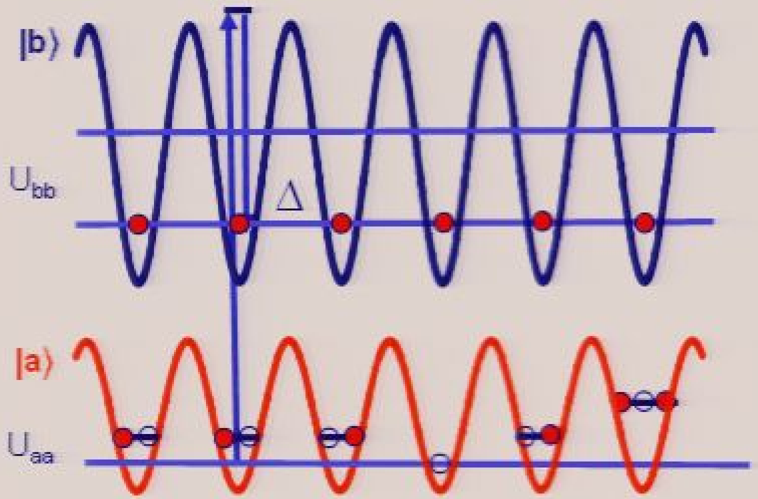
L. Viverit, C. Menotti, T. Calarco, A. Smerzi, Phys. Rev. Lett. **93**, 110401 (2004)

Single occupancy



Possible Improvements

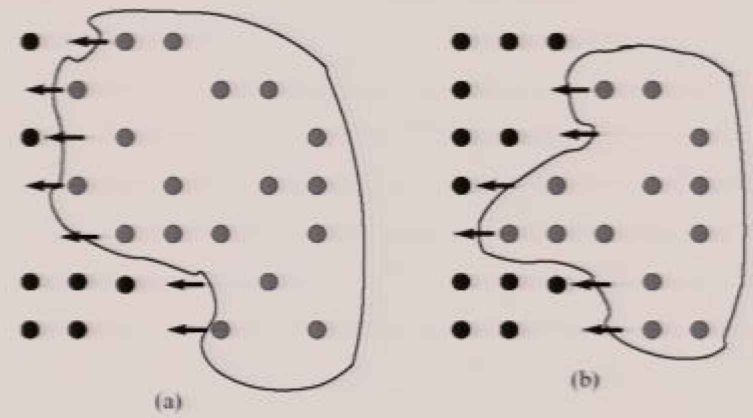
- Defect suppressed lattices



irregular → regular filling, i.e., mixed state → pure state → cooling

P. Rabl, A. J. Daley, P. O. Fedichev, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **91**, 110403 (2003).

- Measurement based schemes



Selective shift operations to close gaps

J. Vala, A.V. Thapliyal, S. Myrgren, U. Vazirani, D.S. Weiss, K.B. Whaley, Phys. Rev. A **71**, 032324 (2005).

Selective measurements of double occupancies

G.K. Brennen, G. Pupillo, A.M. Rey, C.W. Clark, C.J. Williams, Journal of Physics B **38**, 1687 (2005).

Repeatable irreversible loading schemes?

- Can we combine irreversible processes with repulsive and/or Fermi blocking for irreversible loading schemes?

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➡ Spontaneous emission of photons

- ☒ Large momentum kick → heating
- ☒ Large energies → no selectivity
- ☒ Reabsorption → heating

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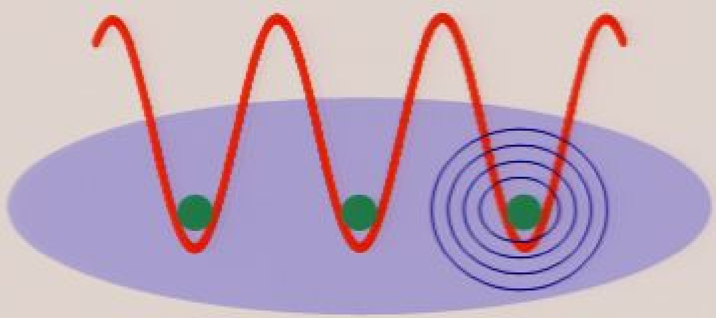
⇒ Spontaneous emission of phonons
- Loading of atoms into lattices
- Cooling within the lowest Bloch band
- Destroy spatial correlations
- Mediate interactions between sites



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N
PHO~~T~~ONS

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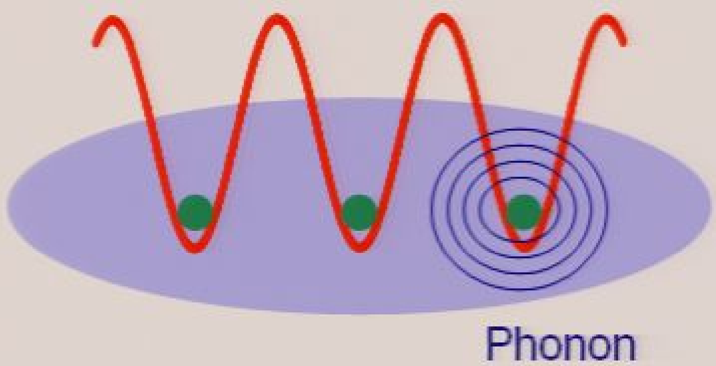

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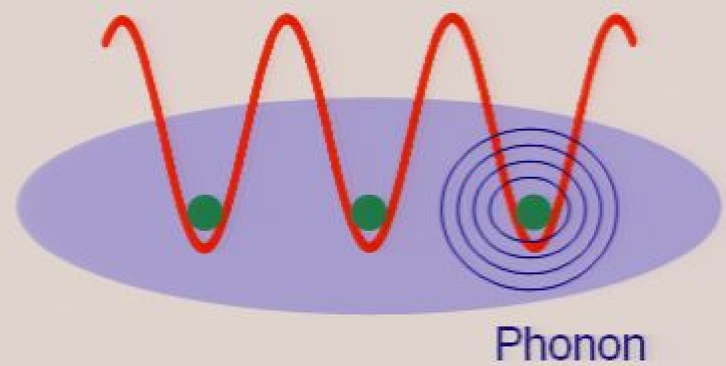
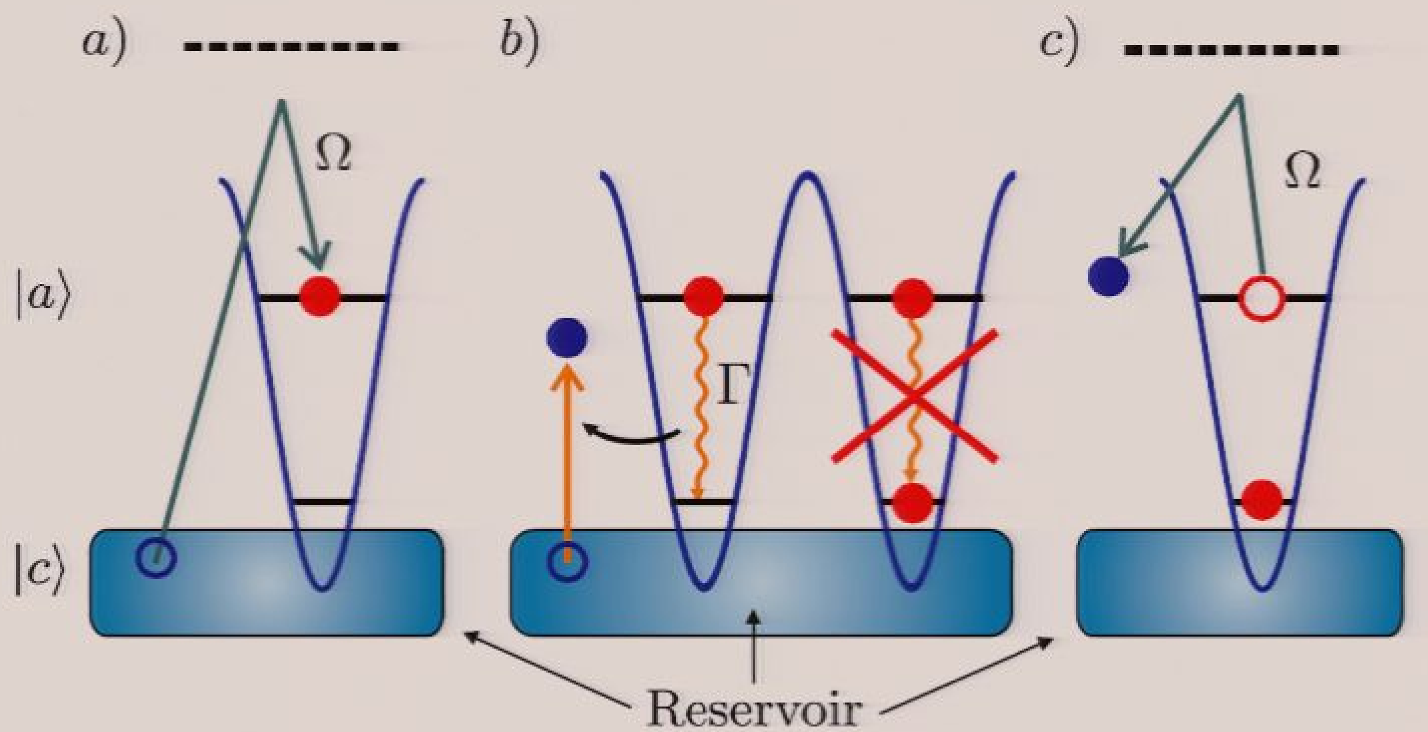


We combine irreversible processes with the blocking mechanisms

Initialization of a fermionic register

A. Griessner *et al.*, Phys. Rev. A **72**, 032332 (2005).

- We consider an optical lattice immersed in an ultracold Fermi gas



⇨ a) Load atoms into the first band

⇨ b) incoherently emit phonons into the reservoir

⇨ c) remove remaining first band atoms

Hamiltonians: Loading

Fermi sphere: $H_{\text{res}} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}}$

Optical Lattice: $H_{\text{sys}} = \sum_{\alpha,\mathbf{n}} (\omega_{\mathbf{n}} + \Delta) a_{\alpha,\mathbf{n}}^{\dagger} a_{\alpha,\mathbf{n}}$ No hopping

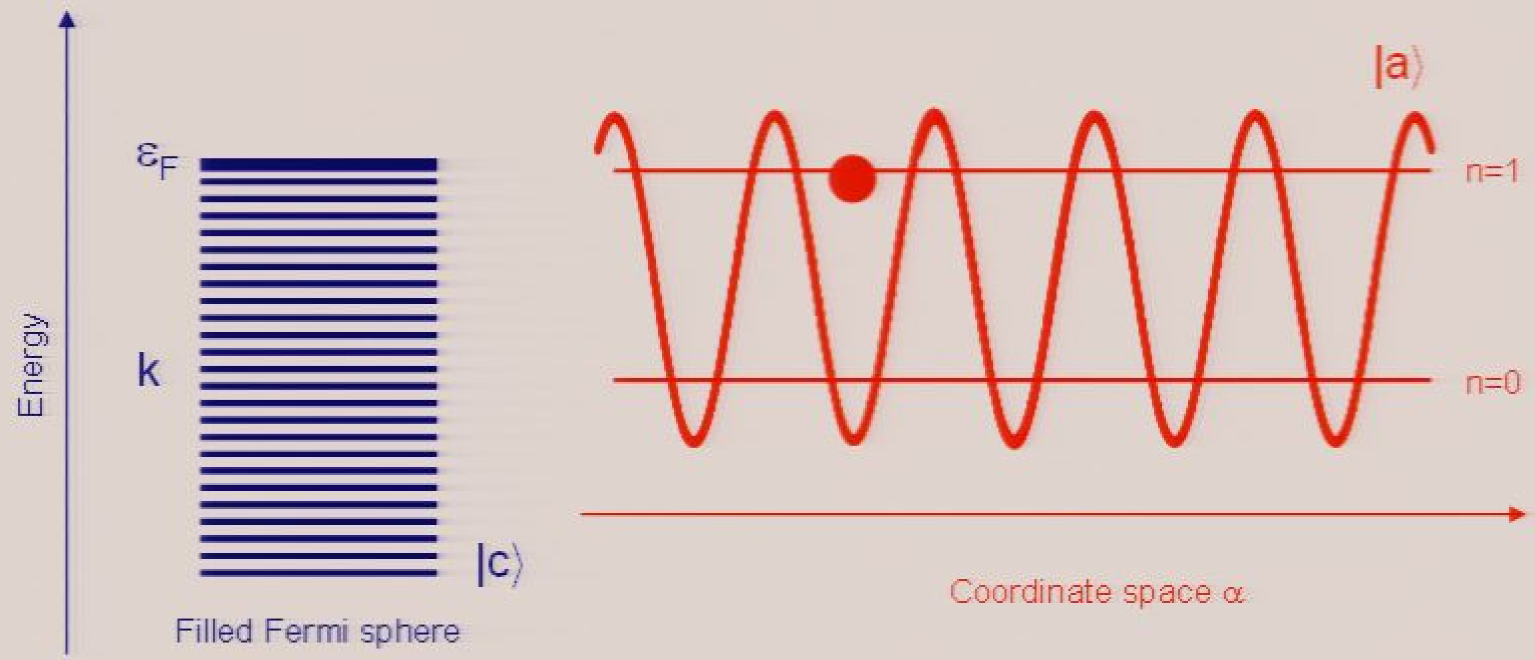
Raman Hamiltonian: $$H_{\text{RC}} = \frac{\Omega}{2} \sum_{\mathbf{k},\alpha,\mathbf{n}} (R_{\mathbf{k},\mathbf{n}} e^{-i\mathbf{k}\mathbf{x}_{\alpha}} c_{\mathbf{k}}^{\dagger} a_{\alpha,\mathbf{n}} + \text{h.c.})$$

$$R_{\mathbf{k},\mathbf{n}} = \frac{1}{\sqrt{V}} \int \mathrm{d}^3 x \mathrm{e}^{-i\mathbf{k}\mathbf{x}} w_{\mathbf{n}}(\mathbf{x})$$

- Slow loading: We consider the case where $|R_{k,n}|^{-1}$ is large compared to the time it takes a Fermi particle to move between the lattice sites $|R_{k,n}|^{-1} \gg \lambda/2\, v_F = T$, where v_F is the Fermi velocity and λ the optical lattice wave length
  - ⇒ This allows to "locally" increase the particle density in the loading process
  - ⇒ We can obtain energy selective loading of the lattice
- Fast loading: $|R_{k,n}|^{-1} \ll T$ laser is dominant and many bands will be occupied, no increase in density

Loading into the band n=1

- Consider the interactions of lattice particles with the Fermi sphere



This interaction can cause the creation of particle/hole excitations:

$$H_{\text{int}} = \sum_{\mathbf{k},\mathbf{k}'} \sum_{\alpha,\mathbf{n},\mathbf{n}'} g^{\mathbf{k},\mathbf{k}'}_{\alpha,\mathbf{n},\mathbf{n}'} c^\dagger_{\mathbf{k}} c_{\mathbf{k}'} a^\dagger_{\alpha,\mathbf{n}} a_{\alpha,\mathbf{n}'}$$

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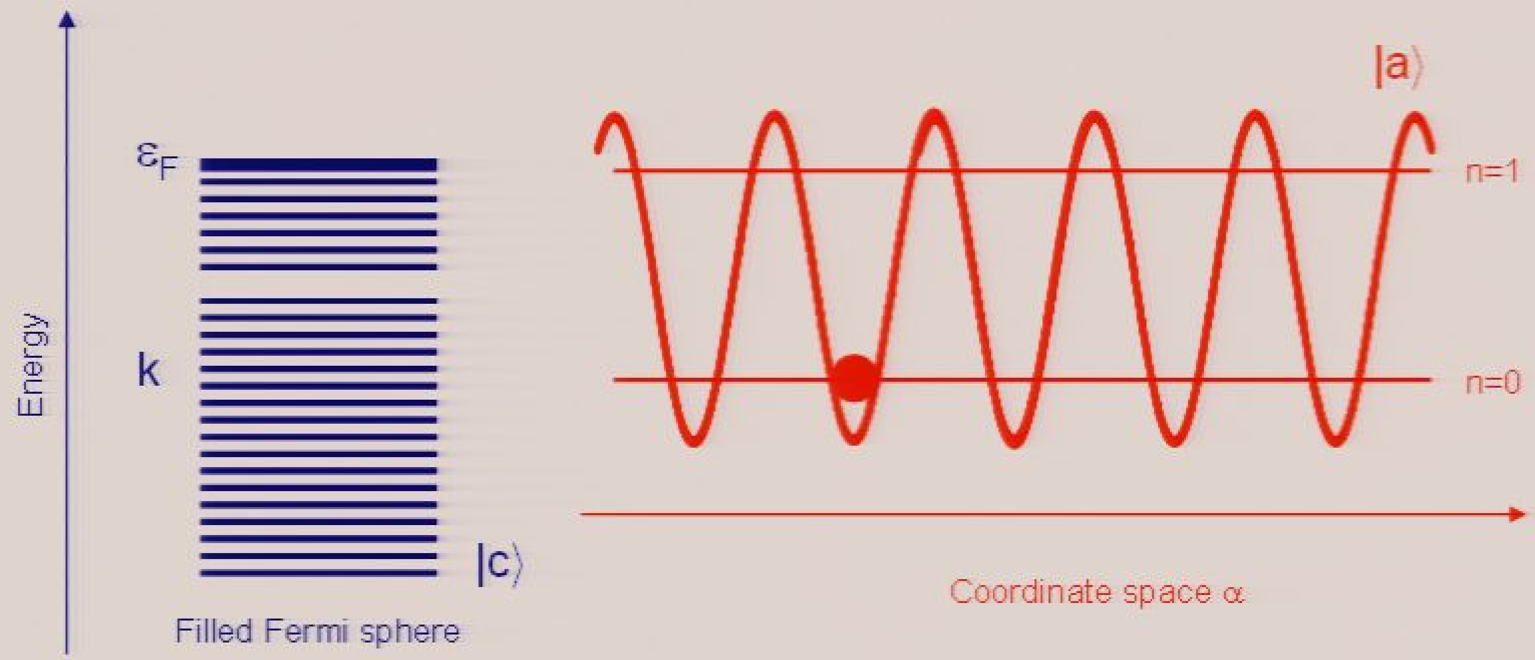

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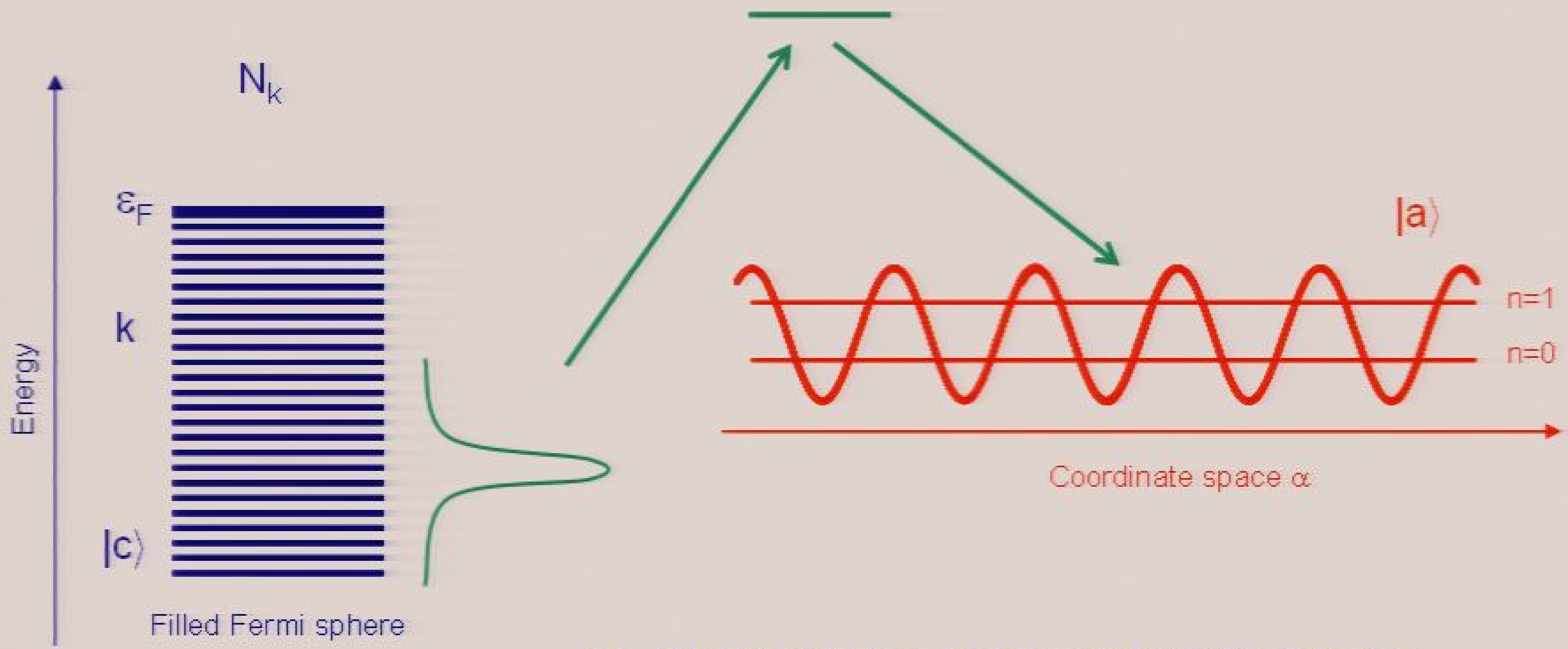
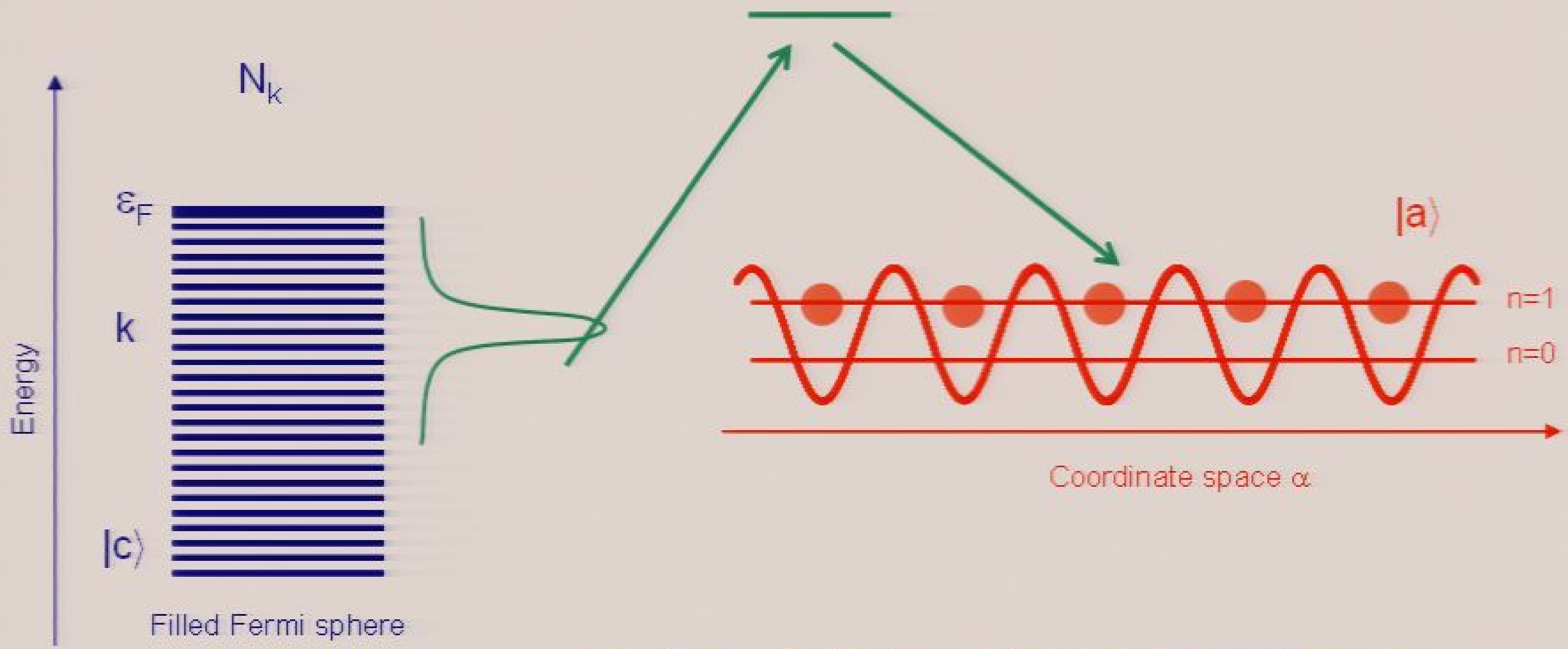
Linear sweep to load the whole lattice



- We change the laser detuning dynamically
  - a) Sweep through the Fermi sea to load the first band
  - b) Spontaneously emit phonons
  - c) Empty remaining atoms by tuning above the Fermi sea

Linear sweep to load the whole lattice



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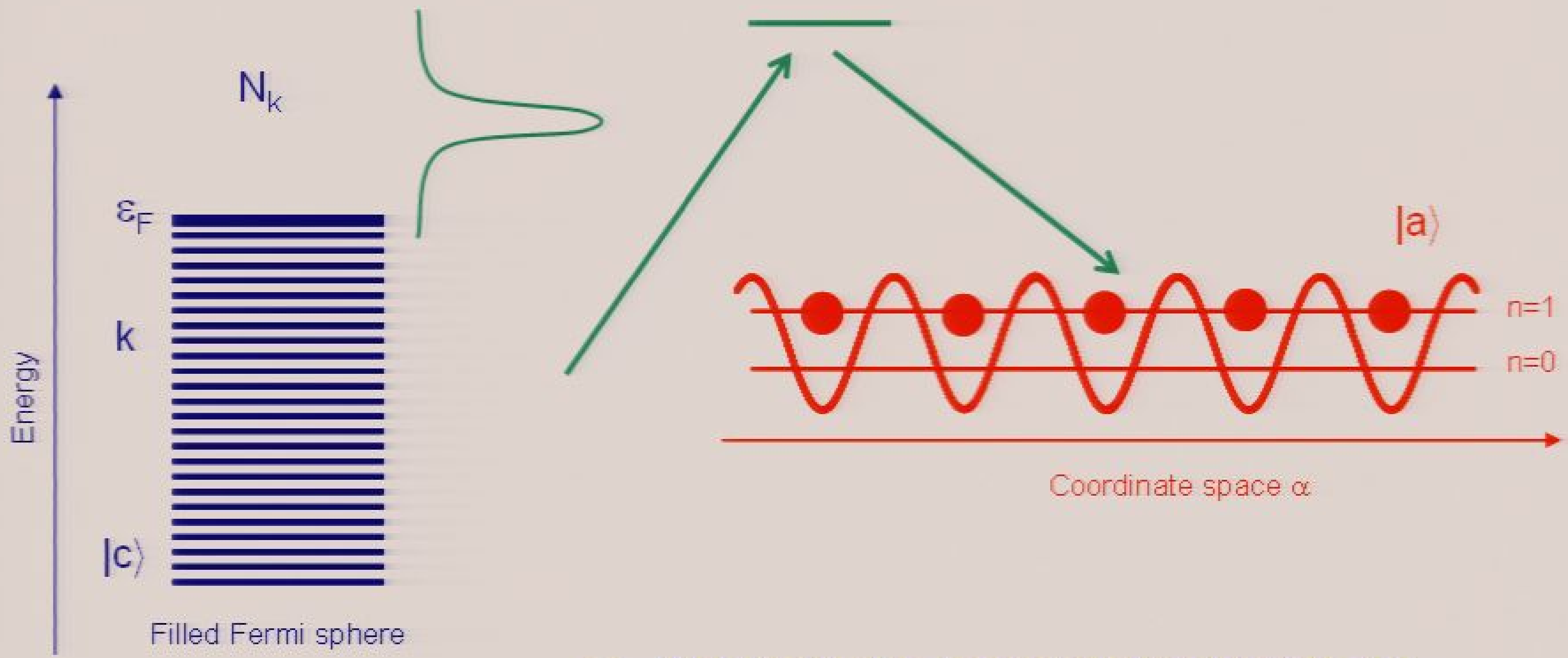

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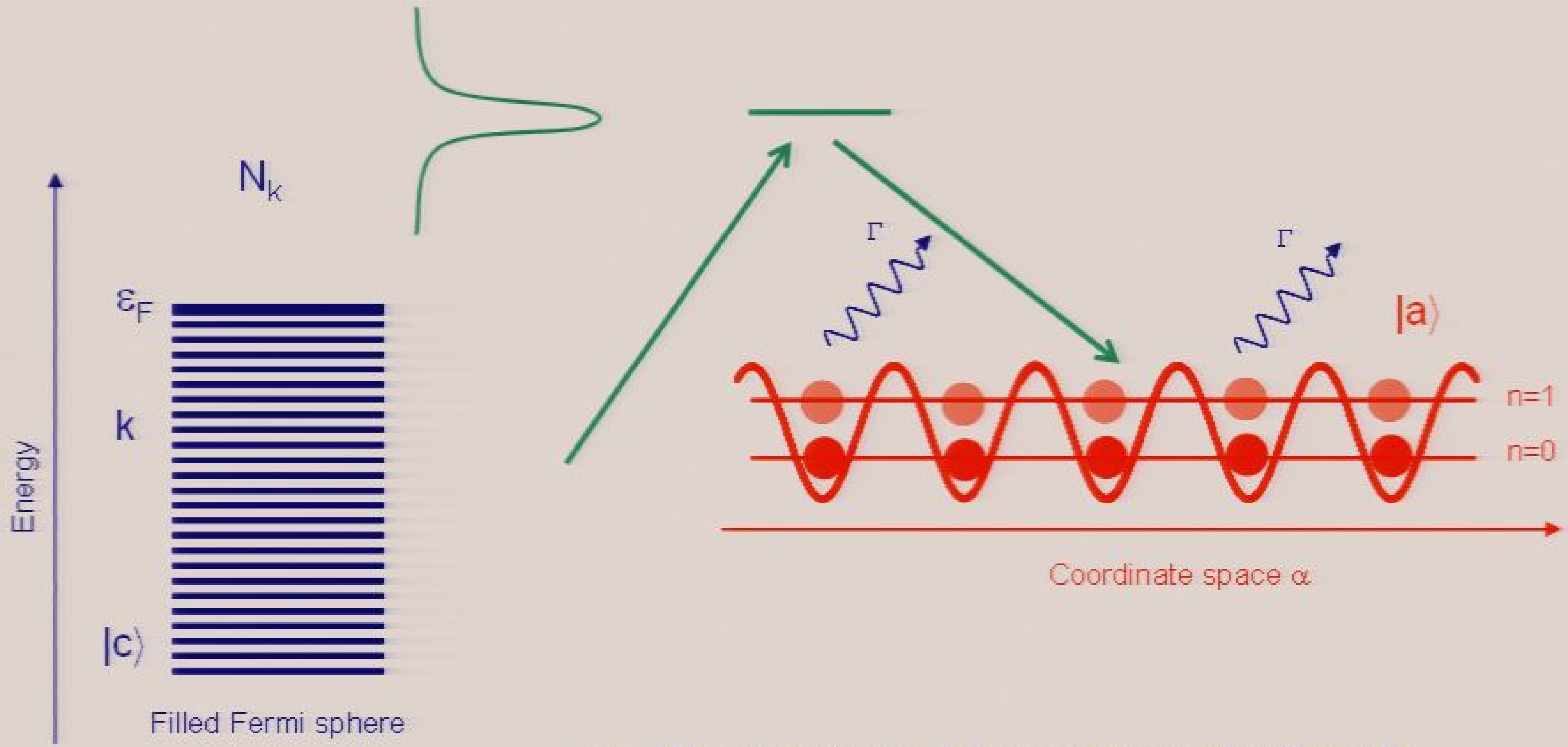

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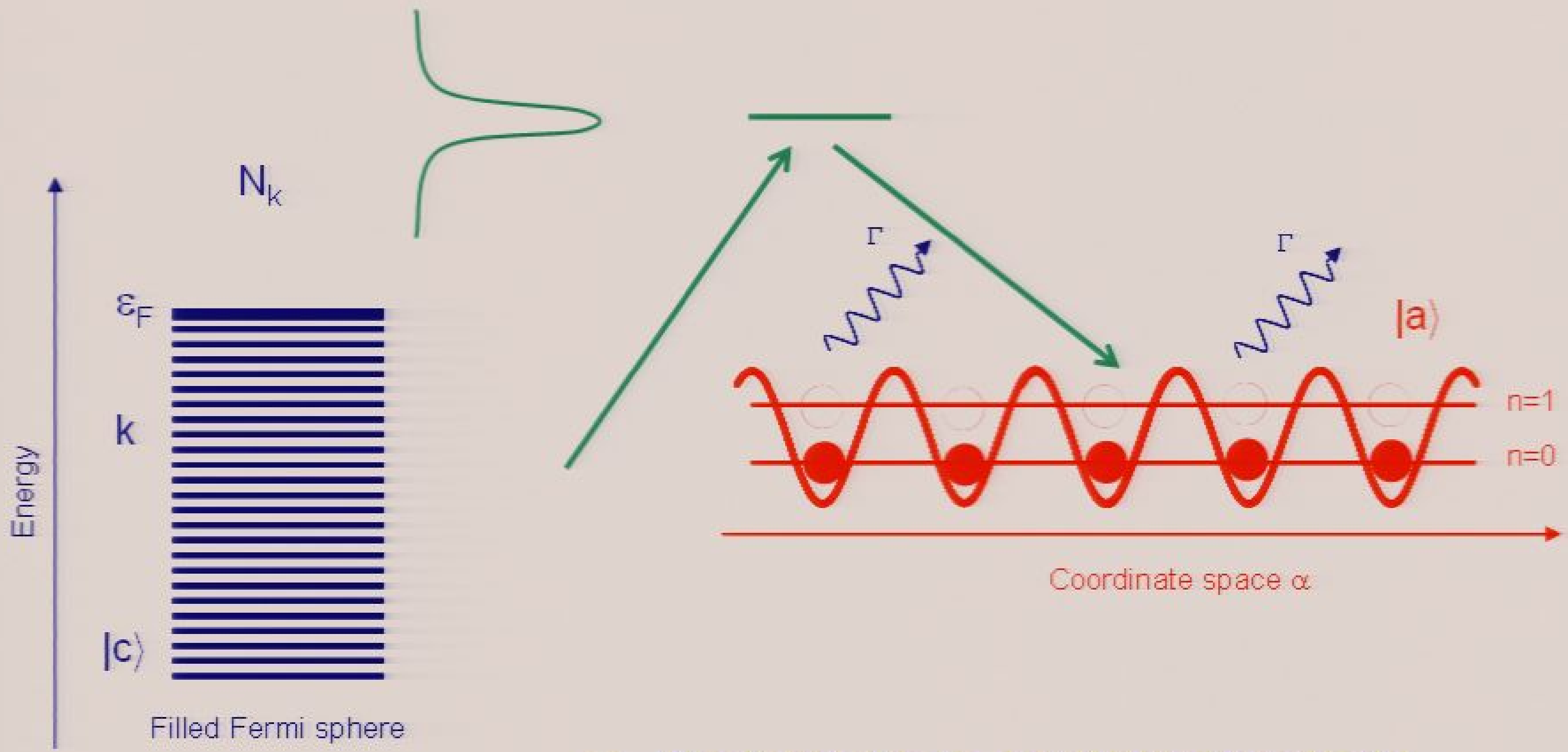

- We change the laser detuning dynamically
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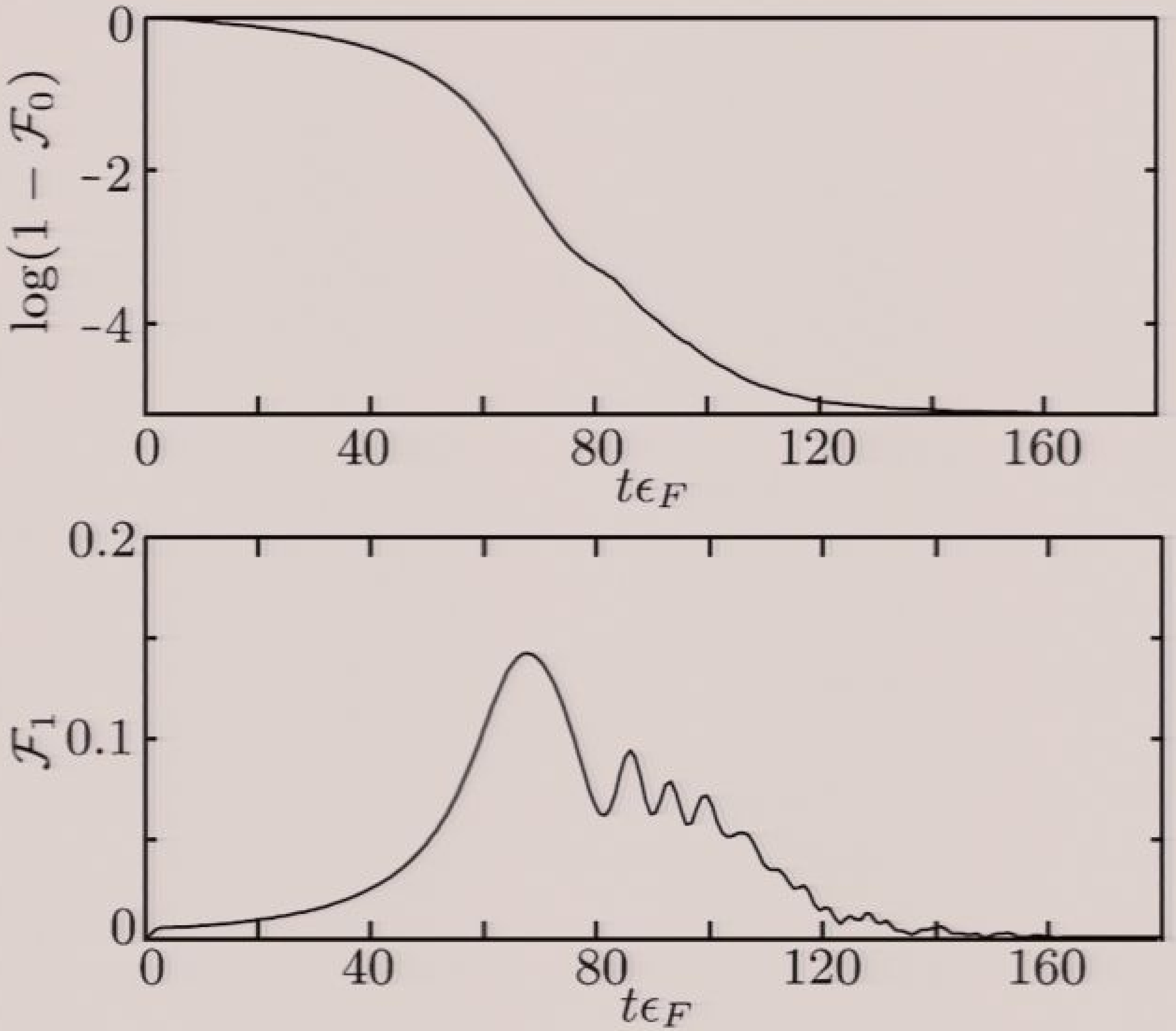
Result



- $\mathcal{F}_0$ is the filling in the lower Bloch band
- $\mathcal{F}_1$ is the filling in the first excited Bloch band

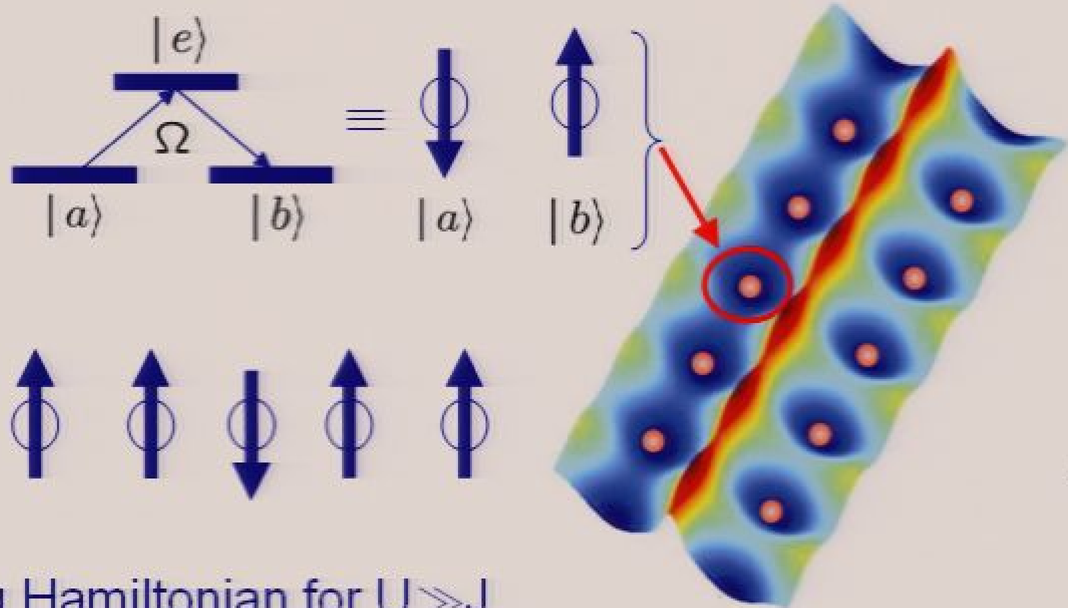
$N = 101,\ M = 5,\ \Omega/2 = 0.45\epsilon_F,\ n_{1\mathrm{D}}\lambda/2 = 1.4,\ \omega = 10\epsilon_F$ and $\Gamma = 0.1\epsilon_F$

Part II: Entanglement generation

Mapping a 1D optical lattice on a XY-model

S.R. Clark, C. Moura Alves, and D.J, New J. Phys. **7**, 124 (2005)

- Optical lattice setup



$$\mu_n = \frac{n+1}{N+1}$$

$$j_n = 2\sqrt{\mu_n(1-\mu_n)}$$

- Resulting Hamiltonian for U≫J

$$H = \frac{J(N+1)}{8}\sum_{n=0}^{N-1} j_n(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) - \frac{B}{2}\sum_{n=0}^{N}(\sigma_n^z - 1)$$

L.-M. Duan *et al.* Phys. Rev. Lett. **91**, 090402 (2003); M. Christandl *et al.* Phys. Rev. Lett. **92**, 187902 (2004); C. Albanese *et al.* quant-ph/0405029. M.-H. Yung *et al.* Quantum Inf. & Comp. **4**, 174 (2004); S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).

Finding the general solution

- Perform a Jordan Wigner transformation to obtain a non-interacting Fermi gas with known one particle spectrum and fermionic destruction operators c_k and basis states:

$$|\ell_1, \ldots, \ell_M\rangle = (c_1^\dagger)^{\ell_1} \ldots (c_M^\dagger)^{\ell_M} |\,\mathrm{vac}\rangle$$

- Calculate the one particle dynamics, i.e. the Heisenberg evolution for the creation operators to find (with $\bar{k}$ the mirror inverted position of k)

$$e^{-iH\tau} c_k^\dagger e^{iH\tau} = e^{i\xi} c_{\bar{k}}^\dagger, \quad \forall\ k$$

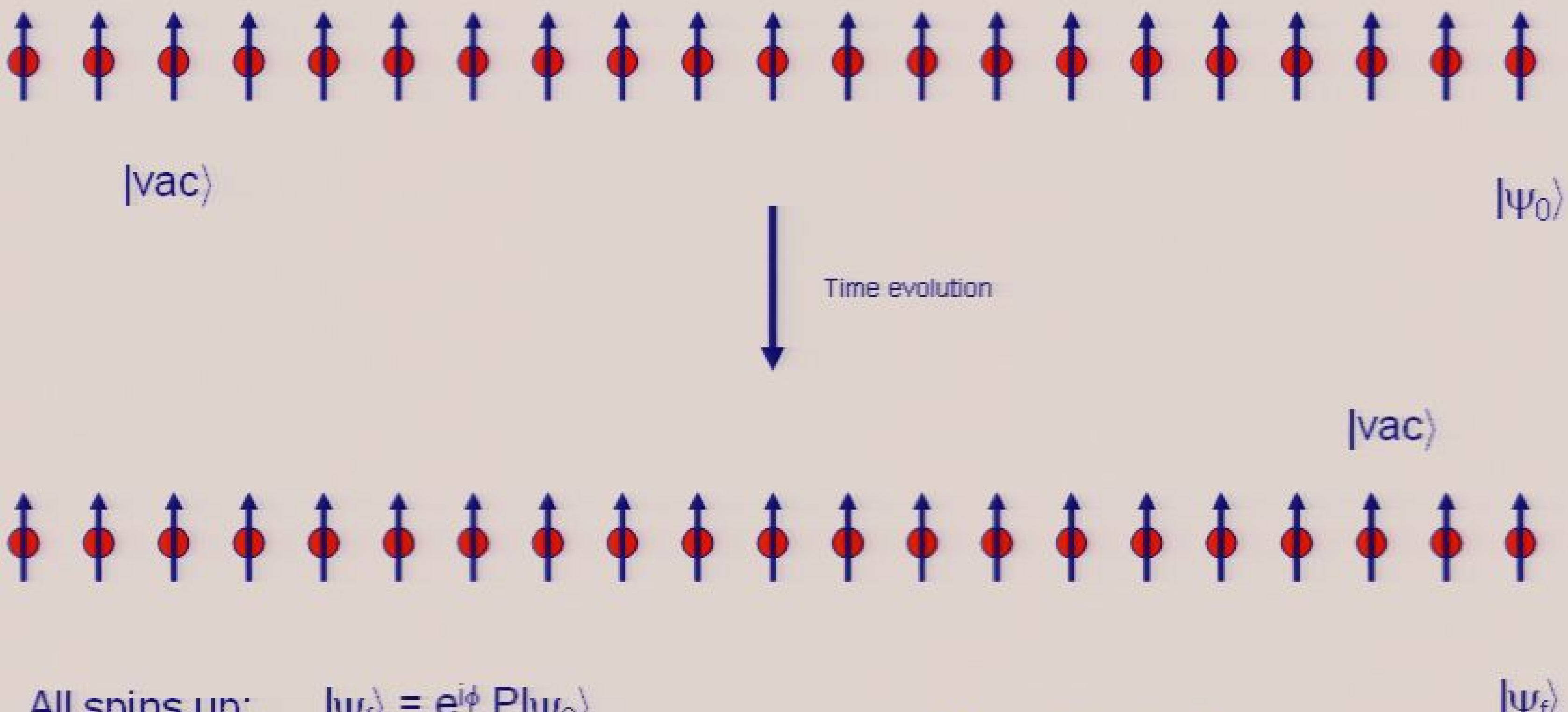
- This means that the initial state gets swapped and one obtains a nontrivial entanglement phase depending on the number of fermions

$$e^{-iH\tau} |\ell_1, \ldots, \ell_M\rangle = e^{-i\pi \Sigma_s} |\ell_M, \ldots, \ell_1\rangle \,. \quad (1)$$

$$\Sigma_s = s(s-1)/2 \qquad\qquad s = \langle \textstyle\sum_{k=1}^M c_k^\dagger c_k \rangle = \sum_{k=1}^M \ell_k$$

Dynamics for all spins up

- We solve the dynamics for the special time $\tau=\pi/J$ and find:



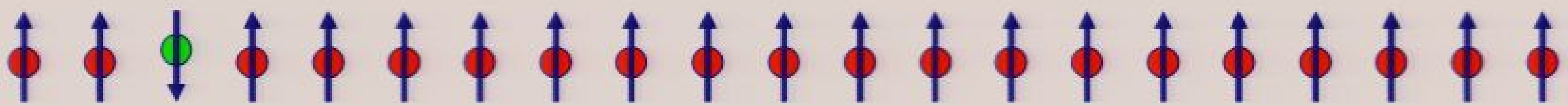
All spins up: $|\psi_f\rangle = e^{i\phi} P|\psi_0\rangle$

We get $\phi=\phi_0=0$

Where P inverts the spin positions

Dynamics for one spin down

- We solve the dynamics for the special time $\tau=\pi/J$ and find:



$c_3^\dagger\,|vac\rangle$ $\qquad\qquad$ $|\psi_0\rangle$

Time evolution like a spin S particle
with Hamiltonian $J\,S^x$

$c_{N-3}^\dagger\,|vac\rangle$

$|\psi_f\rangle$

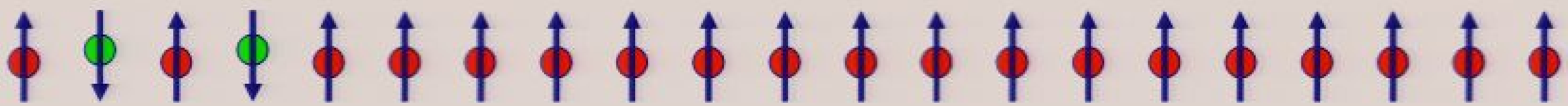
One spin down: $|\psi_f\rangle = e^{i\phi}\,P|\psi_0\rangle$

We get $\phi=\phi_1=\pi N/2$

Where P inverts the spin positions

Dynamics for two spins down

- We solve the dynamics for the special time $\tau=\pi/J$ and find:



$c_3^\dagger c_5^\dagger|vac\rangle$ $|\psi_0\rangle$

$\downarrow$ Time evolution

$c_{N-5}^\dagger c_{N-3}^\dagger\,|vac\rangle = -\,c_{N-3}^\dagger c_{N-5}^\dagger\,|vac\rangle$

$|\psi_f\rangle$

Two spins down: $|\psi_f\rangle = e^{i\phi}\,P|\psi_0\rangle$

We get $\phi=\phi_2=2\phi_1+\pi$

Where P inverts the spin positions

Dynamics for two spins down

- We solve the dynamics for the special time $\tau=\pi/J$ and find:

$c_3^\dagger c_5^\dagger|vac\rangle$ $|\psi_0\rangle$

Time evolution

Entanglement phase

$c_{N-5}^\dagger c_{N-3}^\dagger\ |vac\rangle = -\ c_{N-3}^\dagger c_{N-5}^\dagger\ |vac\rangle$

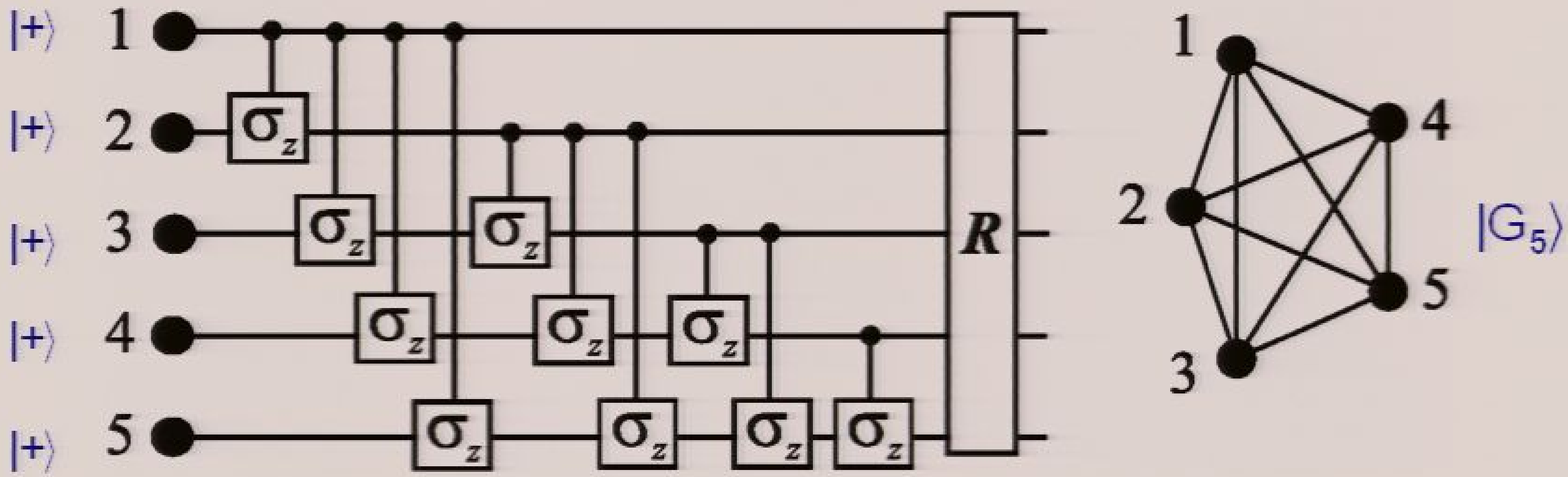
$|\psi_f\rangle$

Two spins down: $|\psi_f\rangle = e^{i\phi}\ P|\psi_0\rangle$

We get $\phi=\phi_2=2\phi_1+\pi$

Where P inverts the spin positions

Realizes a useful quantum network



- This dynamics corresponds to applying controlled σ_z between each pair of spins followed by an inversion of the spin positions
- When preparing all of the spins in the state $|+\rangle \propto |0\rangle + |1\rangle$ the resulting state is a fully connected graph state
- This model can be extended to realize arbitrary graph states

Part III: Detection of Multipartite Entanglement

Detecting entangled states

- Entropic inequalities: For separable states ρ

$$\rho_{123\ldots n} = \sum_{\ell} C_\ell \rho_1^\ell \otimes \rho_2^\ell \otimes \rho_3^\ell \otimes \ldots \otimes \rho_n^\ell$$

the reduced density operators satisfy the following inequalities

$$\begin{aligned} \mathrm{Tr}\,(\rho^2_{123\ldots n}) &\leq \mathrm{Tr}\,(\rho^2_{123\ldots n-1}) \leq \mathrm{Tr}\,(\rho^2_{123\ldots n-2}) \\ &\ldots \leq \mathrm{Tr}\,(\rho^2_{12}) \leq \mathrm{Tr}\,(\rho^2_1) \leq 1. \end{aligned}$$

- Each state which violates these inequalities is entangled. We can find the tr{ρ^2} of a reduced density operator if we can measure the symmetric and anti-symmetric components of ρ

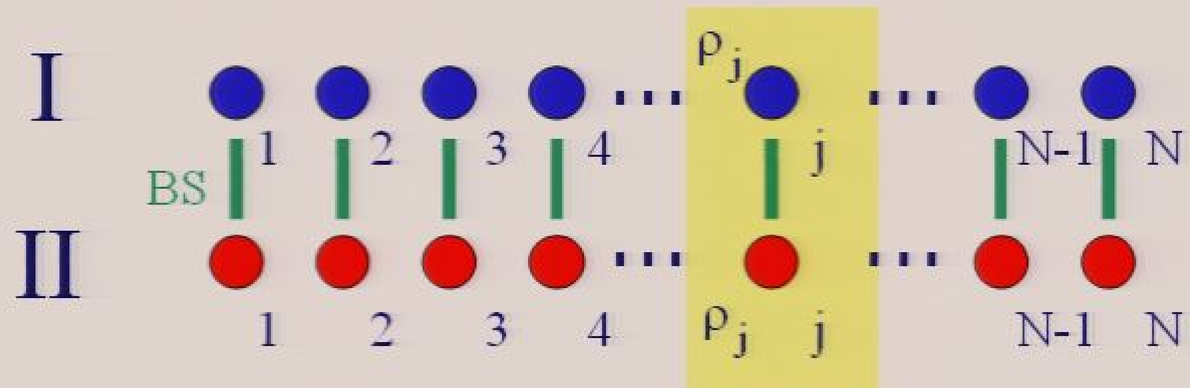
$$P_\pm = \frac{1}{2}\mathrm{Tr}\,\{(1 \pm \underbrace{\mathrm{V}}_{\text{Swap operator}})\rho \otimes \rho\} = \frac{1}{2}(1 \pm \mathrm{Tr}\{\rho^2\})$$

A.K. Ekert et al., Phys. Rev. Lett. **88**, 217901 (2002).

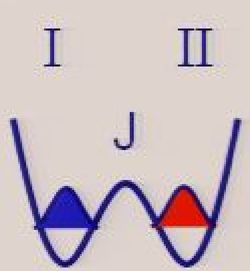
Quantum Networks for detecting entanglement

R.N. Palmer, C. Moura Alves and D.J., Phys. Rev. A **72**, 042335 (2005).
C. Moura Alves and D. J., Phys. Rev. Lett. **93**, 110501 (2004).

- We need two copies of a state ρ in a column of a lattice



- Beam splitters BS between the different atoms are realized by the hopping



$$H = -J(a_I^\dagger a_{II} + b_I^\dagger b_{II} + \text{h.c.})$$

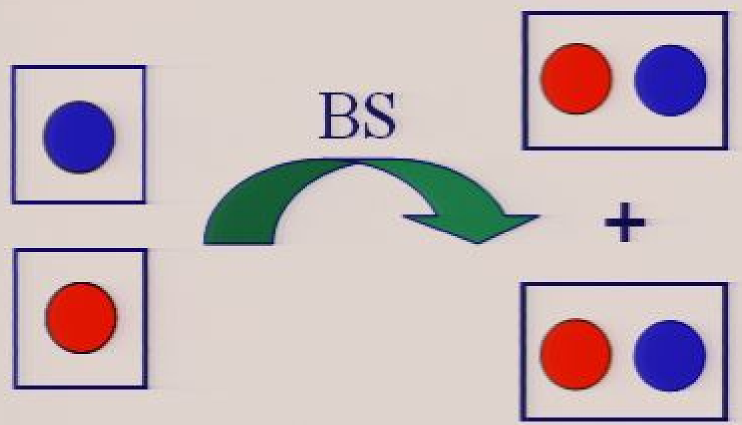
where a and b are bosonic destruction operators with indices I and II for the two rows.

$$a_{I,II} \rightarrow a_{I,II} - i a_{II,I}$$

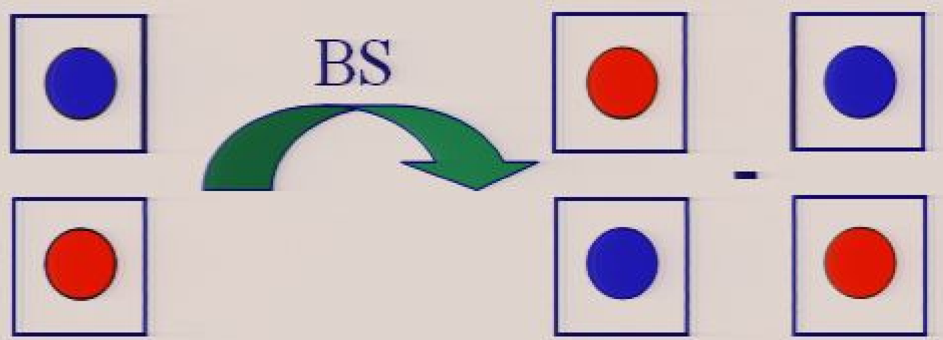
$$b_{I,II} \rightarrow b_{I,II} - i b_{II,I}$$

Action of the beam splitter

$\rho_j \otimes \rho_j$ is symmetric



$\rho_j \otimes \rho_j$ is antisymmetric



- Measuring the occupations after the beam splitters
  - ➡ Interference patterns
  - ➡ Recent experiment by A. Widera, *et al.* Phys. Rev. Lett. **92**, 160406 (2004)
  - ➡ Single atom transistor A. Micheli et al. Phys. Rev. Lett. **93**, 140408 (2004).
- These techniques can be used to detect violations of the entropic inequalities for entangled lattice states and characterize them

Measuring the purities of the reduced operators

- By measuring $P_{\pm}$ for each site j and correlating the respective results, we are able to determine the expectation value of the symmetric and antisymmetric projectors on $\rho_{1\ldots n} \otimes \rho_{1\ldots n}$, for all $n \leq N$.
- Example for n=3:

$$P_{\pm_1 \pm_2 \pm_3} = \frac{1}{2^3} Tr\left(\Pi_{i=1}^{3} \left(\mathrm{I} \pm_i V_i \right) \rho_{123} \otimes \rho_{123} \right)$$
$$= \frac{1}{8}\left(1 \pm_1 Tr\left(\rho_1^2\right) \pm_2 Tr\left(\rho_2^2\right) \pm_3 Tr\left(\rho_3^2\right) \pm_{1,2} Tr\left(\rho_{12}^2\right) \right.$$
$$\left. \pm_{1,3} Tr\left(\rho_{13}^2\right) \pm_{2,3} Tr\left(\rho_{23}^2\right) \pm_{123} Tr\left(\rho_{123}^2\right) \right)$$

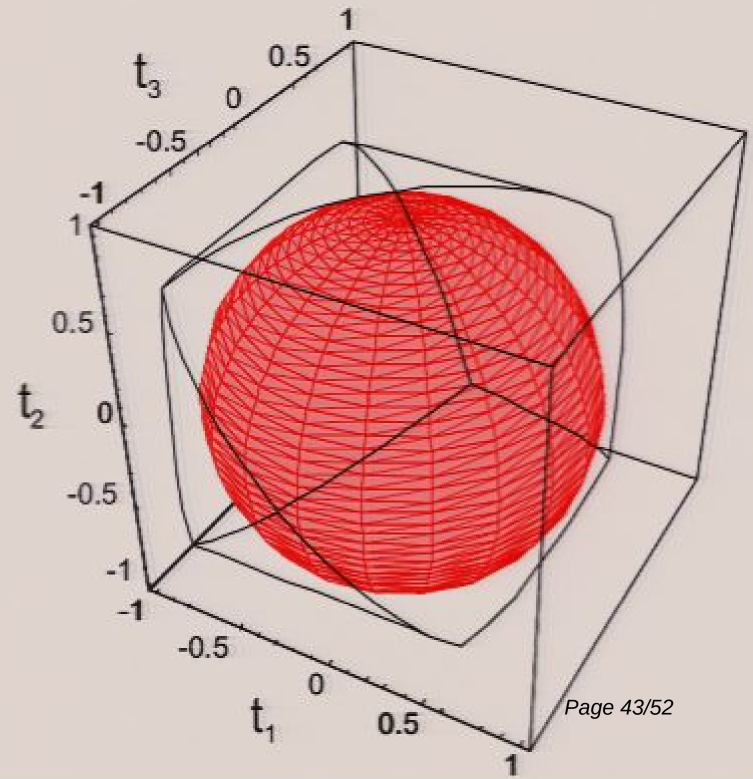
- This is a set of linear equations relating the purities of the reduced density operators to the measured lattice site occupations.
- Realistic experiment: No perfect spatial resolution
  Errors in atom detection
  Errors in the implementation of the beam splitters

Strength of entropic inequalities for two qubits

- For two qubits the state can be written as

$$\varrho = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \boldsymbol{a}\cdot\boldsymbol{\sigma}\otimes\mathbb{1} + \mathbb{1}\otimes\boldsymbol{b}\cdot\boldsymbol{\sigma} + \sum_{i,j} T_{ij}\,\sigma_i\otimes\sigma_j\right)$$

⇒ with $T_{ij} = \mathrm{diag}(t_1, t_2, t_3)$.



- *All* Bell (CHSH) inequalities detect entangled states only if they are outside intersection of three cylinders (black lines).
- The *entropic* inequalities detect all states outside the sphere (red). They do not constitute a sharp test.

No spatial resolution

- We find inequalities for the average traces

$$\mathrm{Tr}\left(\rho_n^2\right) \leq \overline{\mathrm{Tr}\left(\rho_{(n-1)}^2\right)} \leq \overline{\mathrm{Tr}\left(\rho_{(n-2)}^2\right)} \dots \leq \overline{\mathrm{Tr}\left(\rho_{(2)}^2\right)} \leq \overline{\mathrm{Tr}\left(\rho_{(1)}^2\right)} \leq 1$$

$$\overline{\mathrm{Tr}\rho_{(k)}^2} = \left[\binom{n}{k}\right]^{-1} \sum_{|B|=k} \mathrm{Tr}\rho_B^2$$

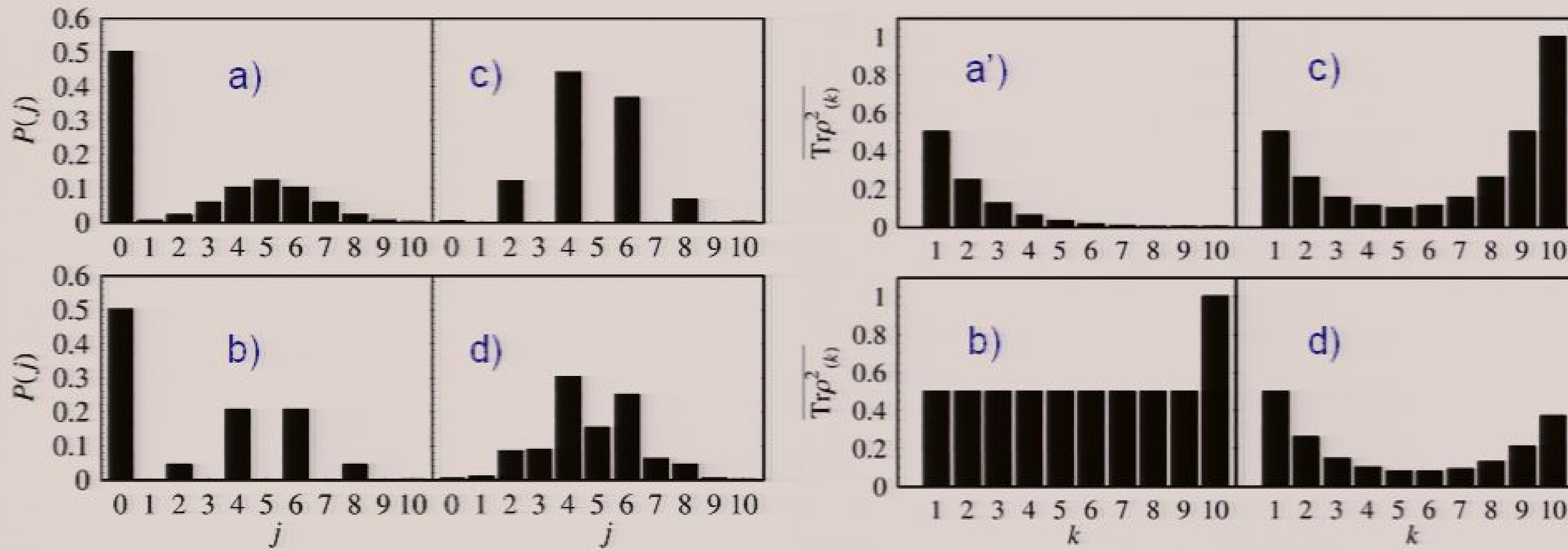
- We can only measure the probability of finding j singly occupied sites P(j), and they allow us to find the averaged traces in the form

$$\overline{\mathrm{Tr}\rho_{(k)}^2} = \left[\binom{n}{k}\right]^{-1} \sum_{j=0}^{n} P(j) \sum_i \binom{j}{i}\binom{n-j}{k-i}(-1)^i$$

- This is a linear relation and thus the average traces are uniquely defined by the P(j)'s.
- This calculation still assumes that there is no error in the measurement.

Detection: No spatial resolution

- We look at some states of interest with n=10 particles



a) classically correlated state
$\rho = (|a\rangle^n \langle a|^n + |b\rangle^n \langle b|^n)/2$

b) GHZ state

c) Cluster state

d) Cluster state with 10% phase noise

a') totally mixed state

b) GHZ state

c) Cluster state

d) Cluster state with 10% phase noise

Characterization: Cat states

- If the form of the entangled state is known, i.e. if it is determined by only a small number (< n) of parameters we can characterize the state without spatial resolution in the experiment.
- We consider a macroscopic superposition state

$$|\gamma_n\rangle = |0\rangle^n + \frac{(\gamma\,|0\rangle + \sqrt{1-|\gamma|^2}\,|1\rangle)^n}{\sqrt{2+\gamma^n+\bar{\gamma}^n}}$$

see W. Duer *et al.* Phys. Rev. Lett. **89**, 210402 (2002)

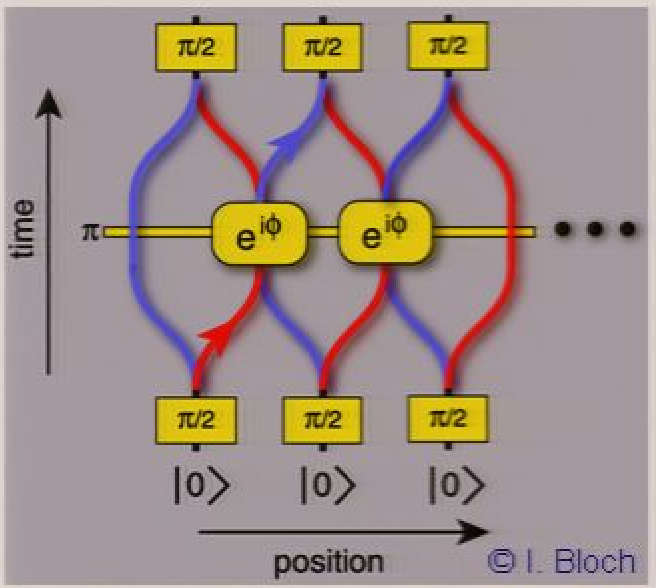
- This is product state for γ=1 and a maximally entangled state γ=0 and we find

$$\overline{\mathrm{Tr}\rho^2_{(k)}} = \frac{2+2\gamma^k\bar{\gamma}^k+2\gamma^{n-k}\bar{\gamma}^{n-k}}{(2+\gamma^n+\bar{\gamma}^n)^2} + \frac{4\gamma^n+4\bar{\gamma}^n+\gamma^{2n}+\bar{\gamma}^{2n}}{(2+\gamma^n+\bar{\gamma}^n)^2}.$$

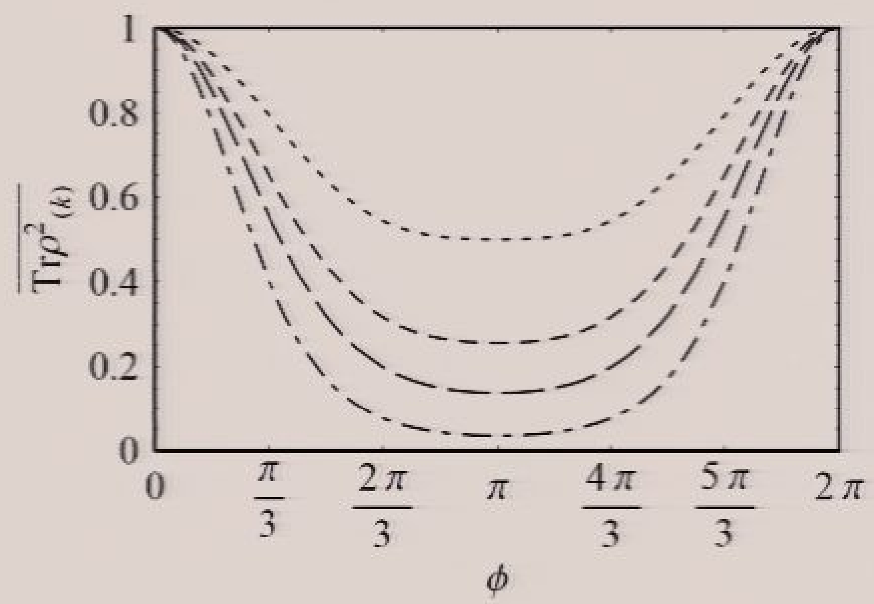
Characterization: Cluster states

- Similar calculation allows to measure the entanglement ϕ phase in a 1D cluster state.

- Experimental setup



- Resulting traces for n=15



dotted: k=1,14; dashed k=2,13; dash dotted 3,12.

Errors

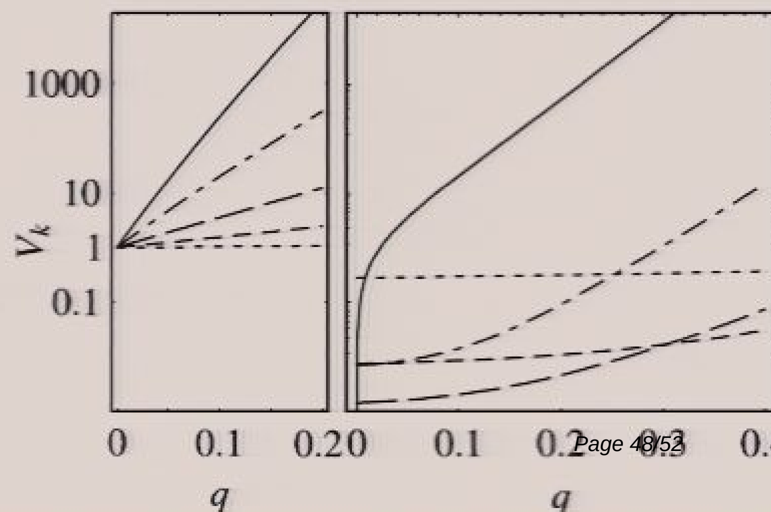
- We investigate beam splitter errors q and detection errors p as well as effects of limited spatial resolution. In general we find that the variance V_k in a single run has exponential bounds in the errors

$$V_k \leq e^{4kq} \qquad\qquad V_k \leq e^{8np}$$

- There is strong numerical evidence that the second bound can be made $\propto \exp(\alpha\, kp)$ instead of $\propto \exp(8np)$ and thus the practical limit for admissible errors which still allow a relatively small number of runs N is

$$p \approx q \approx 1/k$$

- Figure shows worst case variance (left) and variance for a cluster state with n=15 as a function of q
- The number of runs required to get accurate probabilities scales like $1/V_k$

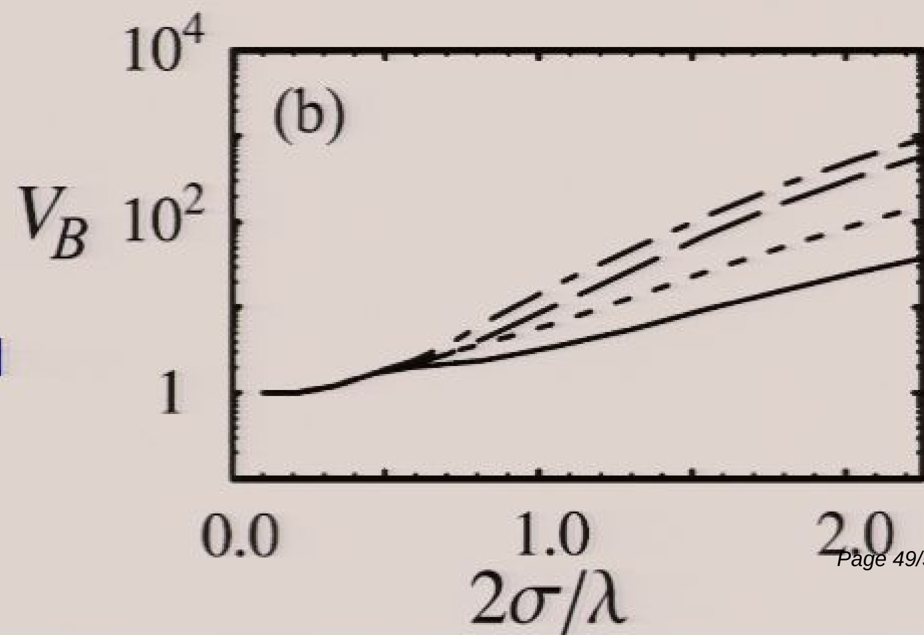


Limited spatial resolution

- A particle which should be detected at x will be found at y with probability f(x,y)
- The probability of detecting particles at sites described by the "set" A is then given by

$$P_{\exp}(A) = \sum_{i \in B} \sum_{\varsigma} \frac{1}{s(A)} \prod_{i=0}^{k} f(A_{\varsigma(i+k)}, B_i) f(A_{\varsigma(i)}, B_i) P(B)$$

  where B are the positions where the atoms should be found and ζ are all possible permutations
- We invert this set of equations to find P(B)
- The variance V_B of P(B) is shown for the worst case in as a function of the spatial resolution σ in f(x,y)
- Significant improvement compared to no spatial resolution is only achievable with $\sigma \approx \lambda$



Summary

- Initialization of a quantum register in optical lattices
  - ⇒ Irreversible scheme similar to optical pumping
  - ⇒ Very high fidelities achievable
- Optical lattices as a test bed for entanglement engineering
  - ⇒ Creation of multipartite quantum gates for the efficient creation of graph states
  - ⇒ Physical system ideally adapted to creating graph states
- Detection and characterization of multi partite entangled states
  - ⇒ Uses entropic inequalities (stronger than Bell) to detect entanglement
  - ⇒ Only limited resources are needed
  - ⇒ Works (to some extent) without spatial resolution in the presence of errors and imperfections

People

Oxford: S. Clark, R. Palmer, M. Bruderer, K. Surmacz, A. Klein, B. Vaucher, C. Moura Alves (→ finance)

U. Dorner, M. Pinilla

D. J.

Innsbruck: A. Griessner, A. Daley, P. Zoller

Open PhD positions: EU training network QIPEST starting this summer:

http://www.physics.ox.ac.uk/qubit/qipest/

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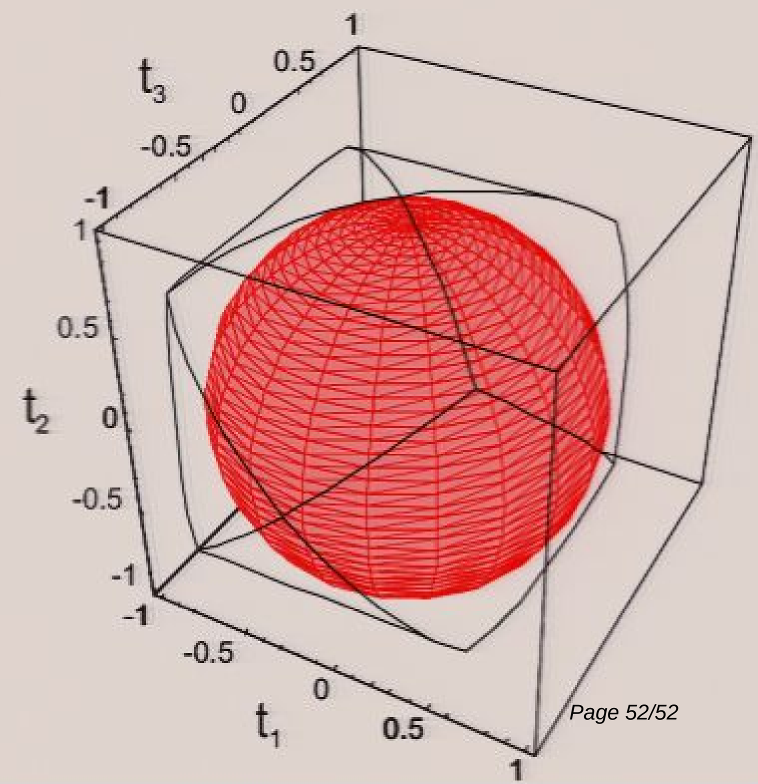

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