Title: D-branes in toroidal orbifolds and mirror symmetry

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Abstract:

D-BRANES IN TOROIDAL ORBIFOLDS AND MIRROR SYMMETRY

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[BASED ON hep-th/0512051]

+ WORK IN PROGRESS

MOTIVATION

- SIMPLE EXPLICIT EXAMPLE OF $\boxed{\text{MIRROR SYMMETRY}}$

  POTENTIALLY A SET-UP FOR FURTHER INVESTIGATION

- LARGE CLASS OF EXAMPLES:

  SUSY AND NON-SUSY TOROIDAL ORBIFOLDS
  IN VARIOUS DIMENSIONS
  \+ DEFORMATIONS OF THESE MODELS

- INTERPLAY BETWEEN DIFFERENT DESCRIPTIONS OF D-BRANES:

  GEOMETRY, BOUNDARY STATE, MATRIX FACTORIZATIONS

  (ALSO: TOPOLOGICAL vs. NON-TOPOLOGICAL
  i.e. "PHYSICAL" BRANES)

- POSSIBLE INTERESTING APPLICATIONS:
  * INVESTIGATION OF CFT / GEOMETRY RELATIONSHIP
    (MAIN EXAMPLE: K3)
  * INTERSECTING BRANES MODELS
  * PROBE OF CLOSED STRING TACHYON CONDENSATION
    IN COMPACT BACKGROUNDS
  * MORE MIRROR SYMMETRY TESTS (EX: INSTANTON COMPUTATIONS)
  * ...

OUTLINE:

- MIRROR SYMMETRY FOR TOROIDAL ORBIFOLDS
  (BULK THEORY)

[CHUN, LAUER, NILLES]

- AN EXAMPLE: $\boxed{T^2/\mathbb{Z}_4} \longleftrightarrow \boxed{W_{LG} = Y_1^4 + Y_2^4}$

  - DISCUSSION OF BRANES IN THESE MODELS
    - GEOMETRIC POINT OF VIEW
    - TOPOLOGICAL BRANES IN LG MODEL
    - BOUNDARY STATES IN GEPNER MODEL

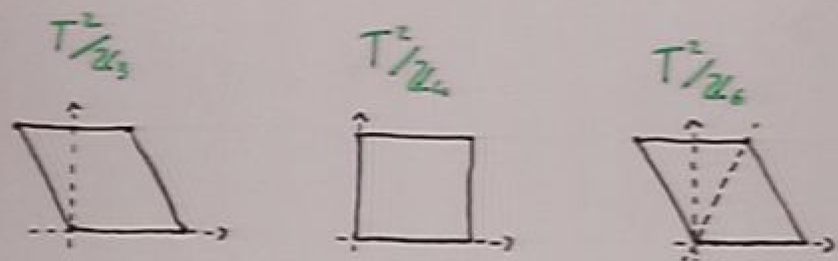
  $\longrightarrow$ RELATION (MIRROR SYMMETRY) BETWEEN THE VARIOUS DESCRIPTIONS

- MORE EXAMPLES, POSSIBLE FUTURE DIRECTIONS

MIRROR SYMMETRY FOR ORBIFOLDS OF T^2

THERE IS A FINITE LIST OF ORBIFOLDS, CLASSIFIED BY THE CRYSTALLOGRAPHIC GROUPS IN 2D

- BASIC EXAMPLES:



OTHER ORBIFOLDS ARE OBTAINED FROM THESE GAUGING VARIOUS (QUANTUM) SYMMETRIES

NOTE: THESE BACKGROUNDS ARE RIGID:
CAN'T CHANGE COMPLEX STRUCTURE PRESERVING THE SYMMETRY THAT IS GAUGED IN THE ORBIFOLDS

$\Rightarrow$ MIRRORS ARE NON-GEOMETRIC (NO KÄHLER PARAMETERS)

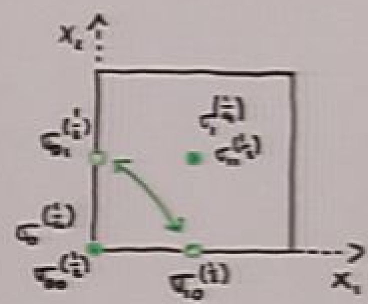
DESCRIPTIONS:

- AS ASYMMETRIC ORBIFOLDS (BY T-DUALITY)
- AS LANDAU-GINZBURG MODELS

$$\begin{cases} T^2/\mathbb{Z}_3 \longleftrightarrow W_{LG} = Y_1^3 + Y_2^3 + Y_3^3 + \alpha Y_1 Y_2 Y_3 \\ T^2/\mathbb{Z}_4 \longleftrightarrow W_{LG} = Y_1^4 + Y_2^4 + \alpha Y_1^2 Y_2^2 \\ T^2/\mathbb{Z}_6 \longleftrightarrow W_{LG} = Y_1^6 + Y_2^3 + \alpha Y_1^4 Y_2 \end{cases}$$

EXAMPLE: THE $T^2/\mathbb{Z}_4$ ORBIFOLD

● $\mathbb{Z}_4$ FIXED POINTS
○ $\mathbb{Z}_2$ FIXED POINTS



TWISTED SECTORS:

1st - $\sigma_1^{(\frac{1}{4})}$, $\sigma_0^{(\frac{1}{4})}$

2nd - $\sigma_1^{(\frac{2}{4})} \equiv \sigma_1^{(\frac{1}{2})}$, $\sigma_0^{(\frac{2}{4})} \equiv \sigma_{00}^{(\frac{1}{2})}$, $\sigma_{01}^{(\frac{2}{4})} = \frac{1}{\sqrt{2}}\left(\sigma_{01}^{(\frac{1}{2})} + \sigma_{10}^{(\frac{1}{2})}\right)$

3rd - $\sigma_1^{(\frac{3}{4})}$, $\sigma_0^{(\frac{3}{4})}$

EXTENDED SCA (FOR T^2 AT SELF-DUAL RADIUS):

$$\begin{cases} T = -\frac{1}{2}\left(\partial x^i \partial x^i + \psi^i \partial \psi^i\right) \\ G^{\pm} = \psi^{\pm} \partial x^{\mp} \equiv e^{\pm i B_L} \partial x^{\mp} \\ Q = i \partial B \end{cases} \qquad \begin{cases} \bar{T} = -\frac{1}{2}\left(\bar\partial x^i \bar\partial x^i + \bar\psi^i \bar\partial \bar\psi^i\right) \\ \bar{G}^{\pm} = \bar\psi^{\pm} \bar\partial x^{\mp} \equiv e^{\pm i B_R} \bar\partial x^{\mp} \\ \bar{Q} = -i \bar\partial B \end{cases}$$

+ ENHANCED $(U(1))^2$ CURRENTS

$$J = \frac{i}{2}\left[e^{i\sqrt{2}x_L^1} + e^{-i\sqrt{2}x_L^1} + e^{i\sqrt{2}x_L^2} + e^{-i\sqrt{2}x_L^2}\right]$$

AND $\bar{J}$

$$\boxed{T^2/\mathbb{Z}_4 \ \left(\begin{matrix}\text{SELF-DUAL}\\ \text{RADIUS}\end{matrix}\right) \xleftrightarrow{\text{MIRROR SYMMETRY}} A_2 \oplus A_2}$$

$\downarrow$

$K=2$ MINIMAL MODEL

$c = 3/2$

TO SEE THIS:

- <u>COMPARE CURRENTS</u>:

FOR ONE $\boxed{A_2}$ MINIMAL MODEL

$$\begin{cases} T_1 = -\frac{1}{2}\left(\partial\phi_1 \partial\phi_1 + \psi_1 \partial\psi_1\right) \\ G_1^{\pm} = \frac{1}{\sqrt{2}}\psi_1 e^{\pm i\sqrt{2}\phi_{1L}} \\ J_1 = \frac{i}{\sqrt{2}}\partial\phi_1 \end{cases} \qquad \begin{cases} \bar{T}_1 = -\frac{1}{2}\left(\bar\partial\phi_1 \bar\partial\phi_1 + \bar\psi_1 \bar\partial\bar\psi_1\right) \\ \bar{G}_1^{\pm} = \frac{1}{\sqrt{2}}\bar\psi_1 e^{\mp i\sqrt{2}\phi_{1R}} \\ \bar{J}_1 = \frac{i}{\sqrt{2}}\bar\partial\phi_1 \end{cases}$$

FOR $\boxed{A_2 \oplus A_2}$: $T = T_1 + T_2$, $G^{\pm} = G_1^{\pm} + G_2^{\pm}$

$J_{\pm} = J_1 \pm J_2$ <u>2 U(1) CURRENTS</u>

MAP TO $T^2/\mathbb{Z}_4$:

$$i\partial X^{\pm} = \frac{1}{\sqrt{2}}\left(\psi_1 e^{\mp iH_L} + \psi_2 e^{\pm iH_L}\right)$$

$$H_L = \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) \qquad H_R = -\frac{1}{\sqrt{2}}(\phi_1 - \phi_2)$$

$$B_L = \frac{1}{\sqrt{2}}(\phi_1 + \phi_2) \qquad B_R = -\frac{1}{\sqrt{2}}(\phi_1 + \phi_2)$$

$\downarrow$

MIRROR SYMMETRY

- <u>COMPARE SPECTRUM</u>: (NEXT PAGE)

MAP BETWEEN $A_2 \otimes A_2$ AND $T^2/\mathbb{Z}_4$ PRIMARY FIELDS

$A_2 \otimes A_2$ labels	T^2/Z_4 primary fields
$(1,1,0)\otimes(1,-1,0)$	$\frac{1}{\sqrt{2}}e^{-i\pi/4}(V^{Z_4}_{1,0;0,0}+iV^{Z_4}_{0,1;0,0})$
$(1,-1,0)\otimes(1,1,0)$	$\frac{1}{\sqrt{2}}e^{i\pi/4}(V^{Z_4}_{1,0;0,0}-iV^{Z_4}_{0,1;0,0})$
$(2,0,0)\otimes(0,0,0)$	$\frac{1}{2}(V^{Z_4}_{1,0;1,0}+V^{Z_4}_{0,1;0,1}+V^{Z_4}_{1,0;0,1}-V^{Z_4}_{0,1;1,0})$
$(0,0,0)\otimes(2,0,0)$	$\frac{1}{2}(V^{Z_4}_{1,0;1,0}+V^{Z_4}_{0,1;0,1}-V^{Z_4}_{1,0;0,1}+V^{Z_4}_{0,1;1,0})$
$(2,2,0)\otimes(2,-2,0)$	$\frac{1}{2}(+iV^{Z_4}_{1,0;1,0}-iV^{Z_4}_{0,1;0,1}+V^{Z_4}_{1,0;0,1}+V^{Z_4}_{0,1;1,0})$
$(2,-2,0)\otimes(2,2,0)$	$\frac{1}{2}(-iV^{Z_4}_{1,0;1,0}+iV^{Z_4}_{0,1;0,1}+V^{Z_4}_{1,0;0,1}+V^{Z_4}_{0,1;1,0})$
$(2,0,0)\otimes(2,0,0)$	$\frac{1}{2}(V^{Z_4}_{2,0;0,0}+V^{Z_4}_{1,1;-1,1}+V^{Z_4}_{0,2;0,0}+V^{Z_4}_{1,1;1,-1})$
$(1,\pm1,0)\otimes(0,0,0)$	$e^{\pm 3i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(\sigma_0^{\pm(\frac{1}{4})}-i\sigma_1^{\pm(\frac{1}{4})})$
$(0,0,0)\otimes(1,\pm1,0)$	$e^{\pm i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(-i\sigma_0^{\pm(\frac{1}{4})}+\sigma_1^{\pm(\frac{1}{4})})$
$(2,\pm2,0)\otimes(0,0,0)$	$e^{\pm 3i\pi/4}\frac{e^{\pm i\pi/2}}{2}(+i\sigma_0^{\pm(\frac{1}{2})}-i\sigma_1^{\pm(\frac{1}{2})}+\sqrt{2}\sigma_{01}^{\pm(\frac{1}{2})})$
$(0,0,0)\otimes(2,\pm2,0)$	$e^{\pm i\pi/4}\frac{e^{\pm i\pi/2}}{2}(-i\sigma_0^{\pm(\frac{1}{2})}+i\sigma_1^{\pm(\frac{1}{2})}+\sqrt{2}\sigma_{01}^{\pm(\frac{1}{2})})$
$(1,\pm1,0)\otimes(1,\pm1,0)$	$e^{\pm i\pi/2}\frac{e^{\pm i\pi/2}}{\sqrt{2}}(\sigma_0^{\pm(\frac{1}{2})}+\sigma_1^{\pm(\frac{1}{2})})$
$(1,\pm1,0)\otimes(2,\pm2,0)$	$e^{\pm i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(\sigma_0^{\pm(\frac{3}{4})}-i\sigma_1^{\pm(\frac{3}{4})})$
$(2,\pm2,0)\otimes(1,\pm1,0)$	$e^{\pm i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(-i\sigma_0^{\pm(\frac{3}{4})}+\sigma_1^{\pm(\frac{3}{4})})$
$(2,\pm2,0)\otimes(2,\pm2,0)$	$j^{\pm}\bar{j}^{\pm}$
$(1,\mp1,0)\otimes(2,\pm2,0)$	$e^{\pm 3i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(\tau_0^{\pm(\frac{1}{4})}-i\tau_1^{\pm(\frac{1}{4})})$
$(2,\pm2,0)\otimes(1,\mp1,0)$	$e^{\pm i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(-i\tau_0^{\pm(\frac{1}{4})}+\tau_1^{\pm(\frac{1}{4})})$
$(2,\pm2,0)\otimes(2,0,0)$	$e^{\pm 3i\pi/4}\frac{e^{\pm i\pi/2}}{2}(+i\tau_0^{\pm(\frac{1}{2})}-i\tau_1^{\pm(\frac{1}{2})}+\sqrt{2}\tau_{01}^{\pm(\frac{1}{2})})$
$(2,0,0)\otimes(2,\pm2,0)$	$e^{\pm i\pi/4}\frac{e^{\pm i\pi/2}}{2}(-i\tau_0^{\pm(\frac{1}{2})}+i\tau_1^{\pm(\frac{1}{2})}+\sqrt{2}\tau_{01}^{\pm(\frac{1}{2})})$

CHIRAL / ANTICHIRAL PRIMARIES

$\longrightarrow$ MARGINAL: KÄHLER / COMPLEX STRUCTURE DEFORMATION

NOTATION : A_2 LABELS ARE (ℓ, m, s)

$$V_{m_1,w_1;m_2,w_2} \equiv e^{\frac{i}{\sqrt{2}}(m_1 x^1 + m_2 x^2 + w_1 \tilde{x}^1 + w_2 \tilde{x}^2)}$$

SUPERSCRIPT $\mathbb{Z}_4$ DENOTES $\mathbb{Z}_4$-ORBIT OF OPERATORS

MIRROR SYMMETRY

$$T^2/\mathbb{Z}_4 \ \left(\text{SELF-DUAL RADIUS}\right) \longleftrightarrow A_2 \otimes A_2$$

$$\text{III}$$

$$W_{LG} = Y_1^4 + Y_2^4 \quad \text{LG MODEL}$$

$$\Big\downarrow \left(\text{KÄHLER DEFORMATION}\right) \qquad\qquad \Big\downarrow \left(\text{COMPLEX STRUCTURE DEFORMATION}\right)$$

$$\boxed{T^2/\mathbb{Z}_4 \ (\text{RADIUS } \underline{\underline{R}}) \longleftrightarrow W_{LG} = Y_1^4 + Y_2^4 + \underline{\underline{\alpha}}\, Y_1^2 Y_2^2}$$

USING THE LG DESCRIPTION OF $A_2 \otimes A_2$ THE EQUIVALENCE IS EXTENDED TO THE FULL MODULI SPACE

- CHIRAL RING:

$$\begin{cases} Y_1 \equiv \dfrac{e^{i\theta/4}}{\sqrt{2}}\left(\sigma_0^{(\frac{1}{4})} - i\,\sigma_1^{(\frac{1}{4})}\right) \\[2ex] Y_2 \equiv \dfrac{e^{i\theta/4}}{\sqrt{2}}\left(-i\,\sigma_0^{(\frac{1}{4})} + \sigma_1^{(\frac{1}{4})}\right) \end{cases}$$

THEN CHECK THAT THE $T^2/\mathbb{Z}_4$ CHIRAL PRIMARIES FORM THE RING

$$\boxed{\mathcal{R} = \frac{\mathbb{C}[Y_1, Y_2]}{\partial_1 W\ \partial_2 W}}$$

MAP BETWEEN $A_2 \otimes A_2$ AND $T^2/\mathbb{Z}_4$ PRIMARY FIELDS

$A_2 \otimes A_2$ labels	$T^2/\mathbb{Z}_4$ primary fields
$(1,1,0)\otimes(1,-1,0)$	$\frac{1}{\sqrt{2}}e^{-i\pi/4}(V^{\mathbb{Z}_4}_{1,0,0,0}+iV^{\mathbb{Z}_4}_{0,1,0,0})$
$(1,-1,0)\otimes(1,1,0)$	$\frac{1}{\sqrt{2}}e^{i\pi/4}(V^{\mathbb{Z}_4}_{1,0,0,0}-iV^{\mathbb{Z}_4}_{0,1,0,0})$
$(2,0,0)\otimes(0,0,0)$	$\frac{1}{2}(V^{\mathbb{Z}_4}_{1,0,1,0}+V^{\mathbb{Z}_4}_{0,1,0,1}+V^{\mathbb{Z}_4}_{1,0,0,1}-V^{\mathbb{Z}_4}_{0,1,1,0})$
$(0,0,0)\otimes(2,0,0)$	$\frac{1}{2}(V^{\mathbb{Z}_4}_{1,0,1,0}+V^{\mathbb{Z}_4}_{0,1,0,1}-V^{\mathbb{Z}_4}_{1,0,0,1}+V^{\mathbb{Z}_4}_{0,1,1,0})$
$(2,2,0)\otimes(2,-2,0)$	$\frac{1}{2}(+iV^{\mathbb{Z}_4}_{1,0,1,0}-iV^{\mathbb{Z}_4}_{0,1,0,1}+V^{\mathbb{Z}_4}_{1,0,0,1}+V^{\mathbb{Z}_4}_{0,1,1,0})$
$(2,-2,0)\otimes(2,2,0)$	$\frac{1}{2}(-iV^{\mathbb{Z}_4}_{1,0,1,0}+iV^{\mathbb{Z}_4}_{0,1,0,1}+V^{\mathbb{Z}_4}_{1,0,0,1}+V^{\mathbb{Z}_4}_{0,1,1,0})$
$(2,0,0)\otimes(2,0,0)$	$\frac{1}{2}(V^{\mathbb{Z}_4}_{2,0,0,0}+V^{\mathbb{Z}_4}_{1,1,-1,1}+V^{\mathbb{Z}_4}_{0,2,0,0}+V^{\mathbb{Z}_4}_{1,1,1,-1})$
$(1,\pm1,0)\otimes(0,0,0)$	$e^{\pm 3i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(\sigma_0^{\pm(\frac{1}{4})}-i\sigma_1^{\pm(\frac{1}{4})})$
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$(2,\pm2,0)\otimes(0,0,0)$	$e^{\pm 3i\pi/4}\frac{e^{\pm i\pi/2}}{2}(+i\sigma_0^{\pm(\frac{2}{4})}-i\sigma_1^{\pm(\frac{2}{4})}+\sqrt{2}\sigma_{01}^{\pm(\frac{2}{4})})$
$(0,0,0)\otimes(2,\pm2,0)$	$e^{\pm i\pi/4}\frac{e^{\pm i\pi/2}}{2}(-i\sigma_0^{\pm(\frac{2}{4})}+i\sigma_1^{\pm(\frac{2}{4})}+\sqrt{2}\sigma_{01}^{\pm(\frac{2}{4})})$
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$(2,\pm2,0)\otimes(1,\pm1,0)$	$e^{\pm 3i\pi/8}\frac{e^{\pm i\pi/4}}{\sqrt{2}}(-i\sigma_0^{\pm(\frac{3}{4})}+\sigma_1^{\pm(\frac{3}{4})})$
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$(2,\pm2,0)\otimes(2,0,0)$	$e^{\pm 3i\pi/4}\frac{e^{\pm i\pi/2}}{2}(+i\tau_0^{\pm(\frac{2}{4})}-i\tau_1^{\pm(\frac{2}{4})}+\sqrt{2}\tau_{01}^{\pm(\frac{2}{4})})$
$(2,0,0)\otimes(2,\pm2,0)$	$e^{\pm i\pi/4}\frac{e^{\pm i\pi/2}}{2}(-i\tau_0^{\pm(\frac{2}{4})}+i\tau_1^{\pm(\frac{2}{4})}+\sqrt{2}\tau_{01}^{\pm(\frac{2}{4})})$

CHIRAL / ANTICHIRAL PRIMARIES

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SUPERSCRIPT $\mathbb{Z}_4$ DENOTES $\mathbb{Z}_4$-ORBIT OF OPERATORS

NOW INTRODUCE BOUNDARIES.

WE WILL CONSIDER A-BRANES IN $T^2/\mathbb{Z}_4$ (PRESERVE HALF WORLDSHEET SUSY)

BY MIRROR SYMMETRY, THESE ARE IDENTIFIED WITH

B-BRANES IN $A_2 \otimes A_2$ / LG MODEL.

$\boxed{T^2/\mathbb{Z}_4}$

BOUNDARY STATES

(WORLDSHEET POINT OF VIEW)

BUT ALSO

SPACETIME INTERPRETATION

→ A-BRANES WRAP 1-CYCLES

+ POSSIBLE WILSON LINE

$\boxed{A_2 \otimes A_2}$

BOUNDARY STATES [RECKNAGEL, SCHOMERUS; RECKNAGEL]

$$\boxed{W_{LG} = Y_1^4 + Y_2^4 + \alpha Y_1^2 Y_2^2}$$

"MATRIX FACTORIZATIONS"

(BUT ONLY FOR TOPOLOGICAL DESCRIPTION OF BRANES)

↓

HOWEVER:
- EASY COMPUTATIONS
- APPROACH VALID EVERYWHERE IN MODULI SPACE

⟷

MIRROR SYMMETRY

|| GOAL : MAKE THIS MAP PRECISE

(AND POSSIBLY IN THE PROCESS LEARN SOMETHING MORE ABOUT BRANES IN THES MODELS)

BRIEF REVIEW OF BRANES IN LG MODELS

[KONTSEVICH, KAPUSTIN, LI, BRUNNER ET AL.]

- MODEL IS SPECIFIED BY A POLYNOMILA FUNCTION $W_{LG}(\Phi_i)$ OF THE CHIRAL SUPERFIELDS
- TOPOLOGICAL B-BRANES ARE CLASSIFIED BY (MATRIX) FACTORIZATIONS OF W_{LG}: $\boxed{F(\Phi_i)\cdot G(\Phi_i) = W_{LG}(\Phi_i)\cdot \mathbb{1}}$

$\downarrow$

- ABSTRACT CHARACTERIZATION — BUT JUSTIFIED FROM BOUNDARY ACTION ANALYSIS, WITH F AND G AS BOUNDARY POTENTIALS
- HOWEVER REDUCES COMPUTATIONS TO ALGEBRAIC MANIPULATIONS (NOT TOO HARD)

WHAT CAN WE COMPUTE ?

- INTERSECTION MATRIX BETWEEN TWO B-BRANES (BOUNDARY WITTEN INDEX $\mathrm{Tr}_{\alpha\beta}(-1)^F$)
- CORRELATORS OF BULK AND BOUNDARY OBSERVABLES (CHIRAL PRIMARIES)

EX: DISC 1-PT FUNCTION

$$\langle \mathcal{O} \rangle_D \propto \oint \frac{\mathcal{O}\cdot \mathrm{STr}[\partial_1 D\wedge\ldots\wedge\partial_n D]}{\partial_1 W \cdots \partial_n W} \quad \text{WITH} \quad D=\begin{pmatrix}0 & F\\ G & 0\end{pmatrix}$$

REMARK: IN THIS FORMALISM IT IS NATURAL TO LOOK FOR A MINIMAL SET OF B-BRANES

i.e. THE MOST BASIC SET OF BRANES FROM WHICH IT IS POSSIBLE TO CONSTRUCT ALL OTHER B-BRANES AS BOUND STATES

(SO THAT IN FACT IT IS ENOUGH TO WORK WITH A SMALL SUBSET OF ALL FACTORIZATIONS OF W_{LG})

B-BRANES IN $W_{LG} = Y_1^4 + Y_2^4$

THERE IS EVIDENCE THAT A MINIMAL SET OF BRANES IS:

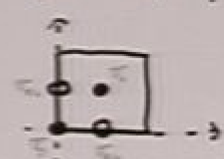
$$\boxed{W_{LG} = F_\eta \cdot G_\eta \quad \text{WITH} \quad F_\eta = Y_1 - \eta Y_2\,,\ G_\eta = \frac{W_{LG}}{F_\eta}\,,\ \eta^4 = -1}$$

QUESTION: WHAT ARE THESE BRANES GEOMETRICALLY IN THE $T^2/\mathbb{Z}_4$ MIRROR MODEL ?

SOME CLUES:
- WORKING OUT THE SPECTRUM OF BOUNDARY OBSERVABLES ONE FINDS THAT THESE BRANES HAVE NO MODULI.
- DISC 1PT-FUNCTIONS:

η	$q_0 = \frac{i}{2}\left(Y_1^2 - Y_2^2 - i\sqrt{2}\,Y_1 Y_2\right)$	$q_1 = \frac{i}{2}\langle -Y_1^2 + Y_2^2 - i\sqrt{2}\,Y_1 Y_2\rangle$	$q_{01} = \frac{1}{\sqrt{2}}\langle Y_1^2 + Y_2^2\rangle$
$e^{i\pi/4}$	0	$-i\sqrt{2}$	$-i$
$e^{-i\pi/4}$	$i\sqrt{2}$	0	i
$e^{i3\pi/4}$	0	$i\sqrt{2}$	$-i$
$e^{-i3\pi/4}$	$-i\sqrt{2}$	0	i

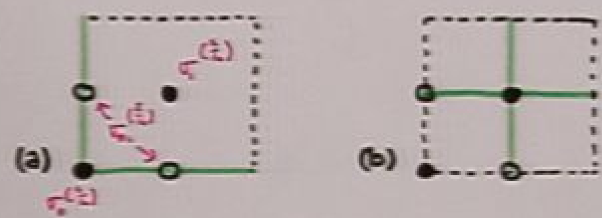
WITH $q_i = \langle e^{i\theta/2}\,\sigma_i^{\left(\frac{1}{4}\right)}\rangle$ FROM MIRROR MAP OF PRIMARY FIELDS



IN CONCLUSION: WE EXPECT THESE TO BE D1-BRANES (A-BRANES) OF $T^2/\mathbb{Z}_4$ THAT ARE CHARGED UNDER TWISTED SECTOR FIELDS AND CONSTRAINED TO THE FIXED POINTS.

TOPOLOGICAL A-BRANES IN $T^2/\mathbb{Z}_4$

THE MINIMAL SET OF LG B-BRANES MAPS TO :



WITH WILSON LINE

$\theta = 0, \pi$

FOR COMPARISON WITH LG COMPUTATIONS LOOK AT "TOPOLOGICAL TERMS" IN $T^2/\mathbb{Z}_4$ BOUNDARY STATES :

$$|a,\theta\rangle = \mathcal{N}\left[\sqrt{2}\, e^{i\theta} \|\sigma_1^{(\frac{1}{2})}\rangle\rangle + \|\sigma_{01}^{(\frac{1}{2})}\rangle\rangle\right]$$

$$|b,\theta\rangle = \mathcal{N}\left[-\sqrt{2}\, e^{i\theta} \|\sigma_0^{(\frac{1}{2})}\rangle\rangle + - \|\sigma_{01}^{(\frac{1}{2})}\rangle\rangle\right]$$

GENERALIZING ARGUMENT FROM [SEN, hep-th/9805019]

$\longrightarrow$ REPRODUCE CHARGES AND INTERSECTION MATRIX COMPUTED IN LG MODEL

- **NEXT** : NON-TOPOLOGICAL ANALYSIS

  $\hookrightarrow$ SO FAR WE DIDN'T NEED TO BE AT A SPECIFIC POINT IN MODULI SPACE, BUT NOW WE RESTRICT TO T^2 AT SELF-DUAL RADIUS

RECALL :

$$T^2/\mathbb{Z}_4 \left(\begin{smallmatrix}\text{SELF-DUAL}\\ \text{RADIUS}\end{smallmatrix}\right) \xleftrightarrow{\text{MIRROR}} W_{LG} = Y_1^4 + Y_2^4$$

OR $A_2 \otimes A_2$

B-BRANES IN $A_2 \otimes A_2$

- FOR A SINGLE A_2 MODEL:
  - PRIMARY FIELDS LABELED BY $(\ell, m, s) \sim (2-\ell, m+4, s+2)$

    $\ell = 0,1,2$; $m = -4, \ldots +3 \pmod 8$; $s = 0, \pm 1, 2 \pmod 4$

    $s = \pm 1 \Rightarrow$ R SECTOR

    $\quad = 0, 2 \Rightarrow$ NS SECTOR
  - ISHIBASHI STATES $\| \ell, m, s \rangle\rangle$
  - B-TYPE BOUNDARY STATES:

    BOUNDARY CONDITIONS $\begin{cases} (T_1 - \bar{T}_1) |B\rangle = 0 \\ (G_1^{\pm} + i\eta \bar{G}_1^{\pm}) |B\rangle = 0 \\ (J_1 + \bar{J}_1) |B\rangle = 0 \end{cases}$

    $$\longrightarrow \boxed{|L, S\rangle = \sum B^{L,S}_{\ell,s} \| \ell, 0, s \rangle\rangle}$$

- FOR $A_2 \otimes A_2$:

  $\longrightarrow$ TENSOR PRODUCT OF A_2 B-BRANES

  $\longrightarrow$ PERMUTATION BRANES [Recknagel]

  BOUNDARY CONDITIONS $\begin{cases} (T_1 - \bar{T}_2) |B\rangle = 0 \\ (G_1^{\pm} + i\eta G_2^{\pm}) |B\rangle = 0 \\ (J_1 + \bar{J}_2) |B\rangle = 0 \end{cases}$

  $$\boxed{|L, M, S_1, S_2\rangle = \sum C^{L,M,S_1,S_2}_{\ell,m,s_1,s_2} \| [\ell, m, s_1] \otimes [\ell, -m, -s_2] \rangle\rangle}$$

  $\downarrow$

  "PERMUTATION ISHIBASHI STATES"

- FOR A SINGLE $\boxed{A_2}$ MODEL :

  - PRIMARY FIELDS LABELED BY $(\ell, m, s) \sim (2-\ell, m+4, s+2)$

    $\ell = 0,1,2$; $m = -4, \ldots +3 \pmod 8$; $s = 0, \pm 1, 2 \pmod 4$

    $s = \pm 1 \Rightarrow$ R SECTOR

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  - ISHIBASHI STATES $\|\ell, m, s\rangle\rangle$

  - B-TYPE BOUNDARY STATES :

    BOUNDARY CONDITIONS $\begin{cases} (T_1 - \bar{T}_1)|B\rangle = 0 \\ (G_1^{\pm} + i\eta \bar{G}_1^{\pm})|B\rangle = 0 \\ (J_1 + \bar{J}_1)|B\rangle = 0 \end{cases}$

    $\longrightarrow \boxed{|L, S\rangle = \sum B_{\ell, s}^{L, S} \|\ell, 0, s\rangle\rangle}$

- FOR $\boxed{A_2 \otimes A_2}$:

  $\longrightarrow$ TENSOR PRODUCT OF A_2 B-BRANES

  $\longrightarrow$ PERMUTATION BRANES [Recknagel]

  BOUNDARY CONDITIONS $\begin{cases} (T_1 - \bar{T}_2)|B\rangle = 0 \\ (G_1^{\pm} + i\eta \bar{G}_2^{\pm})|B\rangle = 0 \\ (J_1 + \bar{J}_2)|B\rangle = 0 \end{cases}$

  $$\boxed{|L, M, S_1, S_2\rangle = \sum C_{\ell, m, s_1, s_2}^{L, M, S_1, S_2} \|[\ell, m, s_1] \otimes [\ell, -m, -s_2]\rangle\rangle}$$

  $\downarrow$

  "PERMUTATION ISHIBASHI STATES"

$T^2/\mathbb{Z}_4 \xleftrightarrow{\text{MIRROR}} \boxed{\begin{array}{c} W_{LG} = Y_1^4 + Y_2^4 \\ \updownarrow \\ A_2 \otimes A_2 \end{array}}$

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MINIMAL SET OF LG BRANES

$F_\eta = Y_1 - \eta Y_2$, $G_\eta = \frac{W_{LG}}{F_\eta}$

$\updownarrow$

PERMUTATION BOUNDARY STATES

$|0, M, 0, 0\rangle$, $M = -4, -2, 0, +2$

[ASHOK, DELL'AQUILA, DIACONESCU
BRUNNER, GABERDIEL
ENGER, RECKNAGEL, ROGGENKAMP]

SUMMARIZING:

- EARLIER WE ESTABLISHED A CORRESPONDENCE BETWEEN TOPOLOGICAL BRANES IN $T^2/\mathbb{Z}_4$ AND $W_{LG} = Y_1^4 + Y_2^4$:

  $F_\eta = Y_1 - \eta Y_2 \xleftrightarrow[\text{MIRROR}]{}$

  $\theta = 0, \pi$

- THERE IS A DESCRIPTION OF THE LG BRANES IN TERMS OF FULLY CONSISTENT (NOT JUST TOPOLOGICAL) BOUNDARY STATES IN $A_2 \otimes A_2$

  $F_\eta = Y_1 - \eta Y_2 \longleftrightarrow$ PERMUTATION BOUNDARY STATES $|0, M, 0, 0\rangle$

- NEXT STEP: MAP PERMUTATION BOUNDARY STATES TO $T^2/\mathbb{Z}_4$ BOUNDARY STATES

EXAMPLE:

PERMUTATION BOUNDARY STATE IN $A_1 \otimes A_1$

$$\|L,M,S_1,S_2\rangle\rangle = \sum_{l,m,s_1,s_2} C^{L,M,S_1,S_2}_{l,m,s_1,s_2} \, |[l,m,s_1] \otimes [l,-m,-s_2]\rangle\rangle^{\sigma}.$$

with coefficients

$$C^{L,M,S_1,S_2}_{l,m,s_1,s_2} = \frac{1}{2\sqrt{2}} e^{i\pi Mm/4} e^{-i\pi(S_1 s_1 - S_2 s_2)/2} \frac{\sin[\frac{\pi}{4}(L+1)(l+1)]}{\sin[\frac{\pi}{4}(l+1)]}.$$

- SET $S_1 = S_2 = 0$, $L = 0$
- USE MAP BETWEEN PRIMARY FIELDS (→ MAP BETWEEN ISHIBASHI STATES)

BOUNDARY STATE ON $T^2/\mathbb{Z}_4$

$$\begin{aligned}
&\|0,\{M=0,2\},0,0\rangle\rangle = \\
&\frac{1}{2\sqrt{2}}\Big[|0,0;0,0\rangle\rangle_{NS} + |2,0;0,0\rangle\rangle^{\mathbb{Z}_4}_{NS} + |1,1;-1,1\rangle\rangle^{\mathbb{Z}_4}_{NS} + |0,2;0,0\rangle\rangle^{\mathbb{Z}_4}_{NS} + |1,1;1,-1\rangle\rangle^{\mathbb{Z}_4}_{NS} \\
&\quad + |1,0;0,0\rangle\rangle^{\mathbb{Z}_4}_{NS} + (-1)^{\frac{M}{2}}|0,1;0,0\rangle\rangle^{\mathbb{Z}_4}_{NS} + (-1)^{\frac{M}{2}}|1,0;0,1\rangle\rangle^{\mathbb{Z}_4}_{NS} + (-1)^{\frac{M}{2}}|0,1;1,0\rangle\rangle^{\mathbb{Z}_4}_{NS} \\
&\quad + i|\sigma_{01}^{(\frac{1}{2})}\rangle\rangle_R - i\frac{(-1)^{\frac{M}{2}}}{\sqrt{2}}|\sigma_0^{(\frac{1}{2})}\rangle\rangle_R + i\frac{(-1)^{\frac{M}{2}}}{\sqrt{2}}|\sigma_1^{(\frac{1}{2})}\rangle\rangle_R - i\frac{(-1)^{\frac{M}{2}}}{\sqrt{2}}|\sigma_0^{-(\frac{1}{2})}\rangle\rangle_R - i\frac{(-1)^{\frac{M}{2}}}{\sqrt{2}}|\sigma_1^{-(\frac{1}{2})}\rangle\rangle_R \\
&\quad + i|\sigma_{01}^{(\frac{1}{2})}\rangle\rangle_R + i(-1)^{\frac{M}{2}}\sqrt{2}|\sigma_0^{(\frac{1}{2})}\rangle\rangle_R\Big]
\end{aligned}$$

$M=0 \rightarrow \theta=0$
$M=2 \rightarrow \theta=\pi$

SUMMARY:

- MIRROR SYMMETRY FOR D-BRANES IN A T^2 ORBIFOLD (BUT NOT HARD TO EXTEND TO OTHER EXAMPLES)
- CONNECTION BETWEEN

  GEOMETRIC PICTURE OF D-BRANES IN TOROIDAL COMPACTIFICATIONS

  AND

  BOUNDARY STATES IN GEPNER MODELS (FOR WHICH MANY COMPUTATIONS CAN AND HAVE BEEN DONE VERY PRECISELY)
- GEOMETRIC INTERPRETATION OF B-BRANES IN CERTAIN LG MODELS
- USE LG DESCRIPTION TO GET INFORMATION ABOUT DEFORMATIONS OF THE THEORY (?)

MORE EXAMPLES:

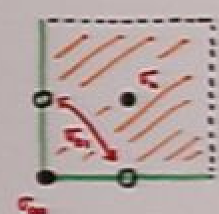
① DEFORMING $T^2/\mathbb{Z}_4$ AWAY FROM SELF-DUAL RADIUS

- TAKE $\boxed{W_{LG} = Y_1^6 + Y_2^6 + 2\alpha Y_1^2 Y_2^2}$ $\alpha \longleftrightarrow R$ (ROUGHLY)

- IT IS POSSIBLE TO CONSTRUCT A ONE PARAMETER FAMILY OF FACTORIZATIONS THAT, FOR $\alpha = 0$, REDUCE TO THE MINIMAL SET OF BRANES $F_\eta = Y_1 - \eta Y_2$.

- DISC ONE POINT-FUNCTIONS NOW WILL DEPEND ON α

AN EXAMPLE:



$$\left.\begin{array}{l} \langle \sigma_{00}^{(\frac{1}{2})} \rangle_\alpha \neq 0 \\ \langle \sigma_{01}^{(\frac{1}{2})} \rangle_\alpha \neq 0 \end{array}\right\} \text{EXPECTED}$$

$\langle \sigma_{11}^{(\frac{1}{2})} \rangle_\alpha \neq 0$! $\longrightarrow$ SIGN OF AN INSTANTON EFFECT IN A-MODEL ON $T^2/\mathbb{Z}_4$

(AT SELF-DUAL RADIUS INSTANTONS ARE SUPPRESSED BY U(1) SYMMETRY)

- POSSIBLE MIRROR SYMMETRY EXCERCISE:

  REPRODUCE LG RESULTS (VERY EASY COMPUTATION) ON A-MODEL SIDE (HARD)

② A 4D EXAMPLE : K3

$$T^4/\mathbb{Z}_4 = \left(T^2/\mathbb{Z}_4 \otimes T^2/\mathbb{Z}_4\right)/\mathbb{Z}_4 \xleftrightarrow{\text{"MIRROR"}} \left(W_4 = Y_1^4 + Y_2^4 + Y_3^4 + Y_4^4\right)/\mathbb{Z}_4$$

$$\begin{array}{ccc} & z_1 & z_2 \\ \mathbb{Z}_4 : & \omega & \omega^{-1} \end{array}$$

$\mathbb{Z}_4 \downarrow$ QUANTUM SYMMETRY

$\downarrow$ LARGE VOLUME (VIA GLSM)

$$Y_1^4 + Y_2^4 + Y_3^4 + Y_4^4 = 0 \quad \text{IN } \mathbb{CP}^3$$

- EXTENDED FRACTIONAL BRANES

QUESTIONS :
- CAN WE LEARN SOMETHING THAT WE DIDN'T ALREADY LEARN FROM POINT-LIKE FRACTIONAL BRANES IN NON-COMPACT ORBIFOLDS ?
- CAN WE FOLLOW THE BRANES ALONG SOME PATHS IN MODULI SPACE ?
- ...

RECENTLY [GRUNNER, GABERDIEL, KELLER]

③ MORE SUSY MODELS : $T^6/\mathbb{Z}_4\times\mathbb{Z}_4$, $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_6$, ... (INTERSECTING BRANES ?)

④ NON-SUSY MODELS : CAN WE LEARN SOMETHING ABOUT TACHYON CONDENSATION IN COMPACT SPACES ?