

Title: Introduction to quantum gravity - Part 21

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras

- 3d Ponzano-Regge model (geometry)
- idea of Feynman diagrams
- Group Field theory
 - idea
 - general formalism
 - 3d example
 - outlook

- outlook

$$Z_{\Delta} = \int$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_F t_2(E, g)}$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^{\mu\nu} e^{i \sum_T t_2 (E_T g_T)} = \sum_{\mathcal{F}_T}$$



- out look

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^i e^{i \sum_T t_2 (E_T g_T)} = \sum_{\mathbb{Z}_2} \prod_C (2\mathbb{Z}_2 - 1) \prod_{\text{bt}}$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^i e^{i \sum_T t_2(E, g)} = \sum_{\mathcal{I}_2} \prod_c (2\mathcal{I}_c + 1) \prod_{k \neq c} \left\{ \begin{matrix} \mathcal{I}_k & \mathcal{I}_c & \mathcal{I}_k \\ \mathcal{I}_k & \mathcal{I}_c & \mathcal{I}_k \end{matrix} \right\}$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^{\mu\nu} e^{i \sum_F t_2(E, g)} = \sum_{\mathcal{I}_E} \prod_{\substack{e \\ F^+}} (2\mathcal{I}_e + 1) \prod_{\substack{kt \\ V^-}} \left\{ \begin{matrix} \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\ \mathcal{I}_4 & \mathcal{I}_5 & \mathcal{I}_6 \end{matrix} \right\}$$


- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_T t_2(E, g)} = \sum_{\mathbb{Z}_2} \prod_{\substack{e \\ F^+}} (2\mathbb{Z}_2 + 1) \prod_{\substack{e \\ F^-}} \left\{ \begin{matrix} \mathbb{Z}_2 & \mathbb{Z}_2 & \mathbb{Z}_2 \\ \mathbb{Z}_2 & \mathbb{Z}_2 & \mathbb{Z}_2 \end{matrix} \right\}$$



- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^T e^{i \sum_T t_2 (E_T g_T)} = \sum_{\mathcal{J}_\Delta} \prod_{\substack{e \\ f}} (2\mathcal{I}_e - 1) \prod_{\substack{e \\ f}} \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{I}_1 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{I}_4 & \mathcal{J}_6 \end{matrix} \right\}$$



- out look

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_T t_2 (E_T g_T)} = \sum_{\mathbb{J}_2} \prod_{\substack{e \\ f}} (2\mathbb{J}_e + 1) \prod_{k+} \left\{ \begin{matrix} \mathbb{J}_1 & \mathbb{J}_2 & \mathbb{J}_3 \\ \mathbb{J}_4 & \mathbb{J}_5 & \mathbb{J}_6 \end{matrix} \right\}$$

• $\left\{ \begin{matrix} \mathbb{J}_1 & \mathbb{J}_2 & \mathbb{J}_3 \\ \mathbb{J}_4 & \mathbb{J}_5 & \mathbb{J}_6 \end{matrix} \right\}$



- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^T e^{i \sum_T t_2(E_T g_T)} = \sum_{J_2} \prod_{\substack{e \\ F^+}} (2J_e + 1) \prod_{\substack{e \\ F^-}} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$
 $L_e = \ell_{J_2}(J_e)$



- out look

$$Z_{\Delta} = \int \mathcal{D}E^T \mathcal{D}g^{\mu\nu} e^{i \sum_F t_2(E, g)} = \sum_{\{J_i\}} \prod_{\square} (2J_i + 1) \prod_{\square} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \approx \left(\right)$
 $L = \mathcal{L}_{J_1, J_2, \dots}$



- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_T t_2(E, g)} = \sum_{J_2} \prod_{\substack{e \\ F^*}} (2J_e + 1) \prod_{k \in T} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \sim_{J_1 \rightarrow \infty} () \cos S_{\text{max}}$$

$$= \mathcal{L}_{J_2}(J_2)$$



- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_T t_2(E, g)} = \sum_{J_2} \prod_{\substack{e \\ F^*}} (2J_e + 1) \prod_{k \in F^*} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \approx_{J_1 \rightarrow \infty} () \cos S_{\text{eff}}(J_2)$$

$L_2 = \mathcal{L}_{J_2}(J_2)$



- out look

$$Z_{\Delta} = \left(\mathcal{D} E^T \right) \mathcal{D} g^* e^{i \sum_F t_2 (\boxed{E_H \theta})} = \sum_{J_2} \prod_{\substack{e \\ F^*}} (2J_e + 1) \prod_{\substack{e \\ \checkmark}} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \widetilde{\sum_{J_1 \rightarrow \infty}} (\quad) \cos S_{\text{res}}(J_2)$$

$$L_e = \boxed{\ell_e(J_e, 1)}$$

$$L_e = \ell_p(2J_e + 1)$$



- out look

$$Z_{\Delta} = \left(\mathcal{D} E^T \right) \mathcal{D} g \cdot e^{i \sum_F t_2 (E_H)} = \sum_{J_0} \prod_{\substack{e \\ F^+}} (2J_e + 1) \prod_{\substack{e \\ F^-}} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \sim_{J_i \rightarrow \infty} () \cos S_{\text{bulk}}(J_e)$$

$$L_e = \left(\frac{1}{2J_e + 1} \right) \quad J(J+1) \rightarrow \frac{1}{4} \quad L_e = \ell_p(2J_e + 1)$$



$$E_t^i \in \mathbb{R}^3$$
$$/E_t^j$$

$$E_t^i \in \mathbb{R}^3$$

$$/E_t^j$$

$$L = \sqrt{E_t^i \cdot E_t^j}$$

$$E_t \in \mathbb{R}^3 \quad J$$

$$/E_t \quad L = \sqrt{E_t \cdot E_t} = \sqrt{J \cdot J}$$

$$E_t \in \mathbb{R}^3 \quad J^i$$

$$\frac{1}{\hbar} E_t$$

$$L = \sqrt{E_t \cdot E_t} = \sqrt{J \cdot J}$$

$$E_t \in \mathbb{R}^3 \quad J^1$$

$$/E_t \quad L = \sqrt{E_t \cdot E_t} = \sqrt{J \cdot J}$$

$$\mathcal{H} \quad E_t \rightarrow J_t(p) \\ \mathcal{H}^p$$

$$E_t \in \mathbb{R}^3 \quad J_t$$

$$/E_t \quad L = \sqrt{E_t \cdot E_t} = \sqrt{J_t \cdot J_t} = C(\rho)$$

$$\mathcal{H} \quad E_t \rightarrow J_t(\rho) \\ \mathcal{H}^P$$

$$E_t \in \mathbb{R}^3 \quad J^1$$

$$/E_t \quad L = \sqrt{E_t \cdot E_t} = \sqrt{J_t \cdot J_t} = \sqrt{C(p)}$$

$$\mathcal{H} \quad E_t \rightarrow J_t(p)$$

$$\mathcal{H}^p$$

$$J^2$$

$$E_t \in \mathbb{R}^3 \quad J^1$$

$$/E_t^j$$

$\mathcal{H}$

$$E_t \rightarrow J_t(p)$$

$\mathcal{H}^p$

$$L = \sqrt{E_t \cdot E_t} = \sqrt{J_t \cdot J_t} = \sqrt{C(\gamma)} = \sqrt{j(j+1)}$$

J^2

$$E_t \in \mathbb{R}^3 \quad J^1$$

$$/E_t^j$$

$\mathcal{H}$



$$E_t \rightarrow J_t(p)$$

$\mathcal{H}^p$

$$L = \sqrt{E_t \cdot E_t} = \sqrt{J_t \cdot J_t} = \sqrt{C(x)} = \sqrt{j(j+1)}$$

J^2

- out look

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^* e^{i \sum_{\ell} t_2 \langle E_{\ell} H_{\ell} \rangle} = \sum_{J_{\ell}} \prod_{\ell} (2J_{\ell} + 1) \prod_{\ell \neq \ell'} \left\{ \begin{matrix} J_{\ell} & J_{\ell'} & J_{\ell''} \\ J_{\ell} & J_{\ell'} & J_{\ell''} \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_1 & J_3 & J_4 \end{matrix} \right\} \sim \left(\right) \cos S_{\ell_{1/2}}(J_{\ell})$$

$$L_{\ell} = \langle E_{\ell}(J_{\ell+1}) \rangle$$

$$J(J+1) = \frac{1}{2}$$

$$L_{\ell} = \ell_p(2J_{\ell+1})$$



- outlook

$$Z_{\Delta} = \left(\mathcal{D} E^T \right) \mathcal{D} g^* e^{i \sum_{\ell} t_2 \left(\sqrt{E_{\ell} H_{\ell}} \right)} = \sum_{\mathcal{I}_{\ell}} \prod_{\ell} (2\mathcal{I}_{\ell} + 1) \prod_{k \neq \ell} \left\{ \begin{matrix} \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\ \mathcal{I}_4 & \mathcal{I}_5 & \mathcal{I}_6 \end{matrix} \right\}$$

$$L_e = \left(\frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right) \cos S_{\text{eff}}(\mathcal{I}_{\ell})$$

$$L_e = \ell_p(2\mathcal{I}_{\ell+1})$$



$$E_t \in \mathbb{R}^3 \quad J^1$$

$$/E_t$$

$\mathcal{H}$



$$E_t \rightarrow J_t(p)$$

$\mathcal{H}^p$

$$L = \sqrt{E_t \cdot E_t} = \sqrt{J_t \cdot J_t} = \sqrt{C(t)} = \sqrt{j(j+1)}$$

J^2

$$E_e \in \mathbb{R}^3 \quad J^1$$

$$/E_t$$

$\mathcal{H}$



$$E_t \rightarrow J_t(p)$$

$\mathcal{H}^p$

$$L = \sqrt{E_t \cdot E_t} = \sqrt{J_e \cdot J_e} = \sqrt{C(y)} = \sqrt{j(j+1)}$$

J^1

$$E_e \in \mathbb{R}^3 \quad J^i$$

$$/E_e^j$$

H

$$E_e \rightarrow J_e(p)$$

H^p

H^{j_i}

$$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(j)} = \sqrt{j(j+1)}$$

J^j



$$E_e \in \mathbb{R}^3 \quad J'$$

$$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\theta)} = \sqrt{j(j+1)}$$

H

$$E_e \rightarrow J_e(p)$$

H^p

$$\psi \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$$



$$E_e^i \in \mathbb{R}^3 \quad J^i$$

$$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\gamma)} = \sqrt{j(j+1)}$$

$\mathcal{H}$

$$E_e \rightarrow J_e(p)$$

$\mathcal{H}^p$

$$\psi \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$$



$$E_e^i \in \mathbb{R}^3 \quad J^i$$

$$/E_e^j \quad L = \sqrt{E_1 \cdot E_2} = \sqrt{J_1 \cdot J_2} = \sqrt{C(\vartheta)} = \sqrt{j(j+1)}$$

$\mathcal{H}$

$$E_e \rightarrow J_e(P)$$

$\mathcal{H}^P$

$$\psi \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$$



$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{\text{SU}(2)}$$

- out look

$$Z_{\Delta} = \left(\mathcal{D} E^T \right) \mathcal{D} g^* e^{i \sum_{\ell} t_2 \left(\boxed{E_{\ell} H_{\ell}} \right)} = \sum_{\mathcal{J}_{\ell}} \prod_{\ell} (2\mathcal{J}_{\ell} + 1) \prod_{\ell} \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\}$$

$\cdot \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\} \sim \left(\right) \cos S_{\mathcal{H}_{1K}}(\mathcal{J}_{\ell})$

$L_e = \boxed{\ell_e(\mathcal{J}_{\ell+1})}$

$\mathcal{J}(\mathcal{J}_{\ell+1}) = \frac{1}{2}$

$L_e = \ell_p(2\mathcal{J}_{\ell+1})$



$$E_e \in \mathbb{R}^3 \quad J^i$$

$$/E_e^j$$

$$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\mathcal{H})} = \sqrt{j(j+1)}$$

$\mathcal{H}$

$$\rightarrow J_e(p)$$

$\mathcal{H}^p$

$$\psi \in \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right)$$

$$\rightarrow 0 \rightarrow \text{Inv}_{\text{SU}(2)}$$



$$E_e \in \mathbb{R}^3 \quad J'$$

$$/E_e \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\mathcal{G})} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(p) \quad \mathcal{H}^p \quad \psi \in \text{Inv} \left(H \begin{pmatrix} j_1 & \\ & 0 \end{pmatrix} H \begin{pmatrix} j_2 & \\ & 0 \end{pmatrix} H \begin{pmatrix} j_3 & \\ & 0 \end{pmatrix} \right) \quad \begin{pmatrix} j_1, j_2, j_3 \\ m_1, m_2, m_3 \end{pmatrix}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{\text{SU}(2)}$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g^T e^{i \sum_{\ell} t_{\ell} (\overline{E_{\ell} H_{\ell}})} = \sum_{\mathcal{J}_{\ell}} \prod_{\ell} (2\mathcal{J}_{\ell+1}) \prod_{\ell} \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\} \sim_{\mathcal{J}_1 \rightarrow \infty} () \cos S_{\text{reg}}(\mathcal{J}_{\ell})$$

$$L_e = \boxed{\mathcal{L}_{\ell}(\mathcal{J}_{\ell+1})}$$

$$\mathcal{J}(\mathcal{J}_{\ell+1}) = \frac{1}{\mathcal{J}_{\ell+1}}$$

$$L_e = \ell_p(2\mathcal{J}_{\ell+1})$$



$$E_e \in \mathbb{R}^3 \quad J^1$$

$$/E_e \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\mathcal{P})} = \sqrt{j(j+1)}$$

$$H \quad E_e \rightarrow J_e(p) \quad H^p \quad \psi \in \text{Inv} \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right) \quad \begin{matrix} j_1, j_2, j_3 \\ m_1, m_2, m_3 \end{matrix}$$



$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

$$E_e^T \in \mathbb{R}^3 \quad J^T$$

$$/E_e^T \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e^T \cdot J_e} = \sqrt{C(y)} = \sqrt{j(j+1)}$$

$\mathcal{H}$

$$E_e \rightarrow J_e(p)$$

$\mathcal{H}^p$

$$\psi \in \text{Inv} \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right)$$

$$\subset \begin{matrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{matrix}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

$$E_e^T \in \mathbb{R}^3 \quad J^T$$

$$/E_e^J \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\delta)} = \sqrt{j(j+1)}$$

$$H \quad E_e \rightarrow J_e(p) \quad \psi \in \text{Inv} \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right) \quad C_{m_1 m_2 m_3}^{j_1 j_2 j_3}$$



$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

- outlook

$$Z_{\Delta} = \int \mathcal{D}E^T \int \mathcal{D}g \, e^{i \sum_{\square} t_2 (\square)} = \sum_{\mathcal{I}_e} \prod_{\square \in \mathcal{F}^+} (2\mathcal{I}_e + 1) \prod_{\square \in \mathcal{F}^-} \left\{ \begin{matrix} \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\ \mathcal{I}_4 & \mathcal{I}_5 & \mathcal{I}_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} \mathcal{I}_1 & \mathcal{I}_2 & \mathcal{I}_3 \\ \mathcal{I}_4 & \mathcal{I}_5 & \mathcal{I}_6 \end{matrix} \right\} \sim \left(\right) \cos S_{\text{bulk}}(\mathcal{I}_e)$$

$$L_e = \langle \mathcal{I}_e(\mathcal{I}_{e+1}) \rangle$$

$$\mathcal{I}(\mathcal{I}_{e+1}) = \frac{1}{2}$$

$$L_e = \ell_P(2\mathcal{I}_{e+1})$$



$$E_e \in \mathbb{R}^3 \quad J^1$$

$$|E_e| \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\theta)} = \sqrt{j(j+1)}$$

$$H \quad E_e \rightarrow J_e(p) \quad \psi \in \text{Inv} \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right) \quad \begin{matrix} j_1, j_2, j_3 \\ m_1, m_2, m_3 \end{matrix}$$



$$\vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

- out look

$$Z_{\Delta} = \left(\mathcal{D} E^T \right) \mathcal{D} g^* e^{i \sum_{\ell} t_{\ell} \left(\overline{E_{\ell} H_{\ell}} \right)} = \sum_{\mathcal{J}_{\ell}} \prod_{\ell} (2\mathcal{J}_{\ell} + 1) \prod_{\ell} \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\}$$

$$\cdot \left\{ \begin{matrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{J}_3 \\ \mathcal{J}_4 & \mathcal{J}_5 & \mathcal{J}_6 \end{matrix} \right\} \widetilde{\mathcal{J}_{\ell} \rightarrow \infty} () \cos S_{\mathcal{J}_{\ell}}(\mathcal{J}_{\ell})$$

$$L_c = \left(\overline{A_{\ell}(\mathcal{J}_{\ell})} \right)$$

$$\mathcal{J}(\mathcal{J}_{\ell}) = \frac{1}{\mathcal{J}_{\ell}}$$

$$L_c = \ell_p(2\mathcal{J}_{\ell} + 1)$$



$$Z = \int \mathcal{D}\varphi e^{-\int \mathcal{L} d^4x}$$



$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\text{kinetic + mass}} + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z =$$



$$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z =$$

$$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{\infty} dq \, e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 - \dots}$$

$$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\substack{+\infty \\ \uparrow}} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\substack{+\infty \\ \uparrow}} + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\substack{\uparrow \\ -\frac{1}{2} m^2 q^2}} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\substack{\uparrow \\ -\frac{\lambda}{4!} q^4}} + \underbrace{J\varphi}_{\substack{\uparrow \\ Jq}} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} q^{4n}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \quad \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} q^4 = \left(\frac{d}{dJ} \right)^4$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\uparrow} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\uparrow} + \underbrace{J\varphi}_{\uparrow} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$\int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} q^4 = \left(\frac{d}{dJ} \right)^4 \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} q^n = \left(\frac{d}{dJ} \right)^n$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\underbrace{\frac{1}{2} \varphi (\partial^2 + m^2) \varphi}_{\text{quadratic}} + \underbrace{\frac{\lambda}{4!} \varphi^4}_{\text{quartic}} + \underbrace{J\varphi}_{\text{linear}} \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} = \left(\frac{d}{dJ} \right)^4 \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} = \left(\frac{d}{dJ} \right)^4 e^{\frac{J^2}{2m^2}}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} = \left(\frac{d}{dJ} \right)^4 e^{\frac{J^2}{2m^2}}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} = \left(\frac{d}{dJ} \right)^n e^{\frac{J^2}{2m^2}}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2!} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq}$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n A_n(J) = e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^4} e^{\frac{1}{2m^2} J^2}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2!} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq}$$

$$= \sum_n \left(\frac{\lambda}{4!} \right)^n A_n(J) = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^2} e^{\frac{1}{2m^2} J^2}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^2} e^{\frac{1}{2} J^2}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^2} e^{\frac{1}{2m^2} J^2} \lambda J^4$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 \left(q - \frac{J}{m^2} \right)^2} e^{\frac{1}{2} \frac{J^2}{m^2}} = e^{\frac{1}{2} \frac{J^2}{m^2}} \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \frac{J^4}{m^4}} e^{\frac{1}{2m^2} J^2} \left(1 - \left(\frac{1}{m^2} \right)^4 \lambda J^4 \right)$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right) [A_1(j)] = 1/c$$

$$(1) \left(\frac{1}{4!} \right) \lambda j^4$$

1 p

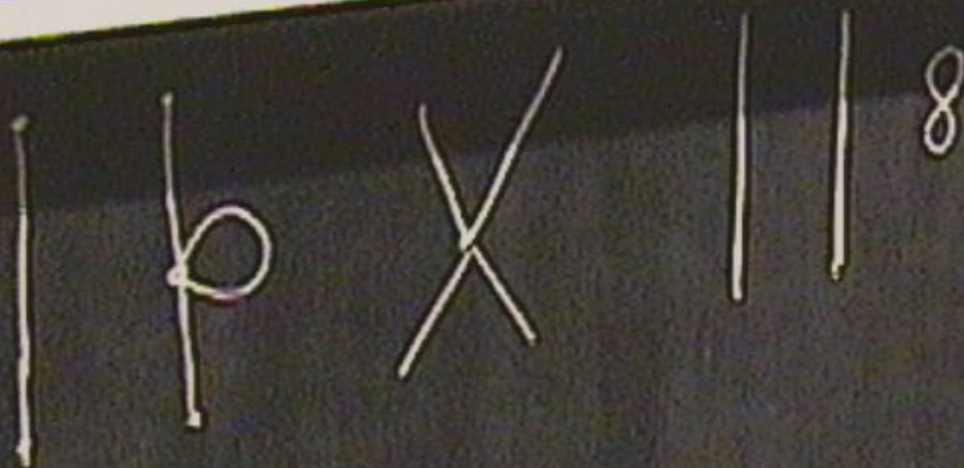


$$= \sum_n \left(-\frac{\lambda}{4}\right) [A_n(j)] = 110$$

$$(1) \left(\frac{1}{14^2}\right)^4 \lambda j^4$$

1 p X

$$= \sum_n \left(\frac{\lambda}{4!} \right) \boxed{A_n(j)} = 110 \quad \left(\frac{1}{16^2} \right)^4 \lambda j^4$$



$$Z = \int_{-\infty}^{\infty} dq e$$

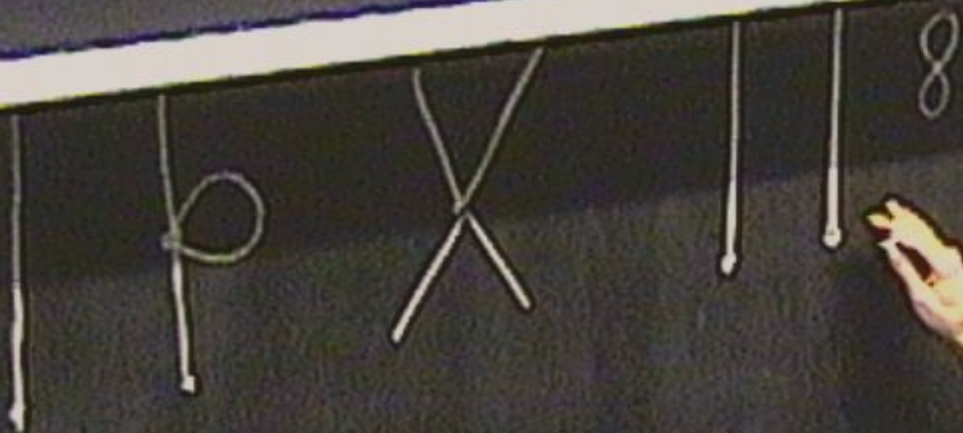
$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = 1 e$$

$$e^{-\frac{\lambda}{4!} j^4} e^{\frac{\lambda}{2!} j^2} e^{\lambda j}$$

$$\left(-\frac{\lambda}{4!}\right)^n \lambda j^4$$

$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = 1 e^{-\frac{\lambda}{4!} j^4} e^{2\lambda j^2}$$

$(1) \left(\frac{1}{4!}\right)^4 \lambda^4 j^4$

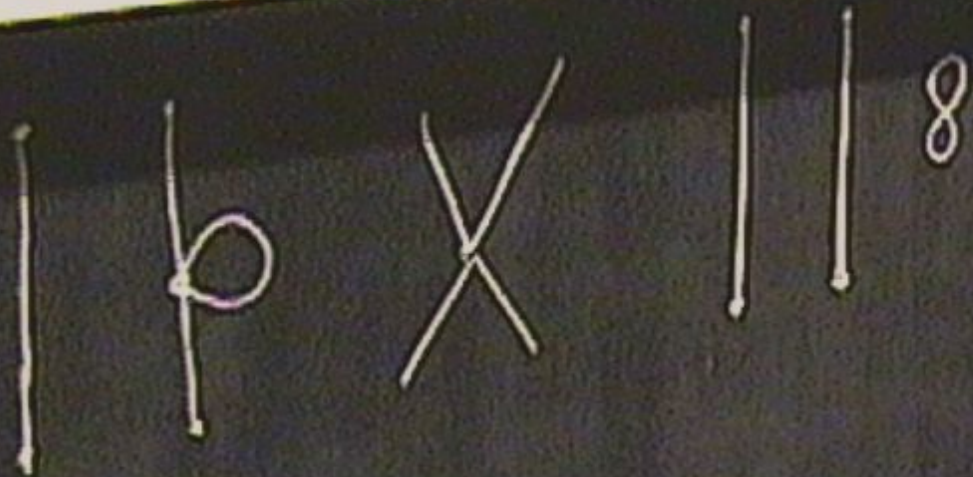


$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{\left(1 - \frac{\lambda}{4!} q^2 + \frac{1}{2} \left(\frac{\lambda}{4!}\right) q^2\right)}$$

$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(J)} = 1$$

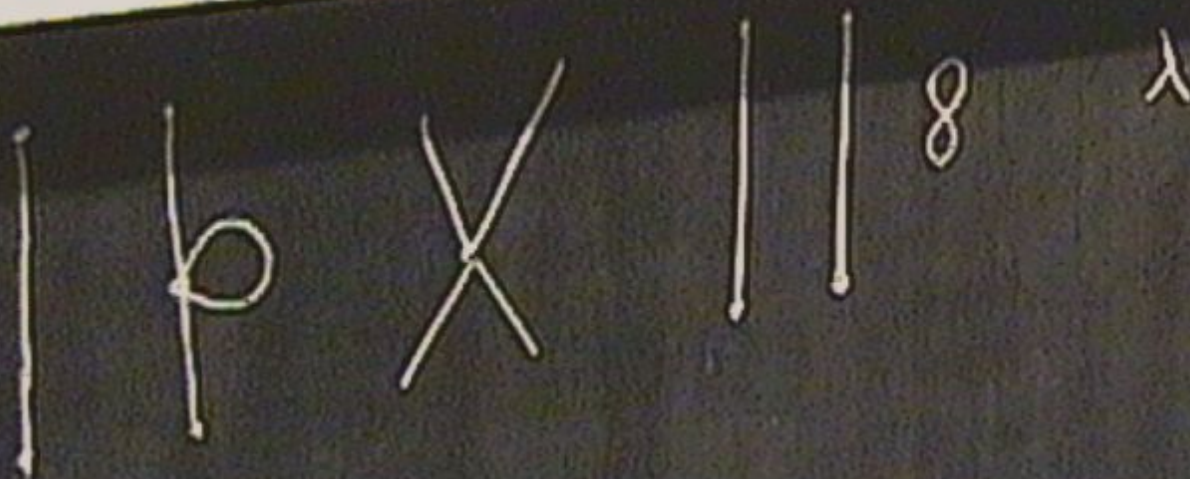
$e^{-\frac{\lambda}{4!} q^2} e^{\frac{1}{2} \left(\frac{\lambda}{4!}\right) q^2}$
 $(1) \left(\frac{1}{16^2}\right) \lambda J$

$$= \sum_n \left(\frac{-\lambda}{4!} \right) [A_1(3)] \dots \left(\frac{-\lambda}{4!} \right)^4 \lambda^5$$



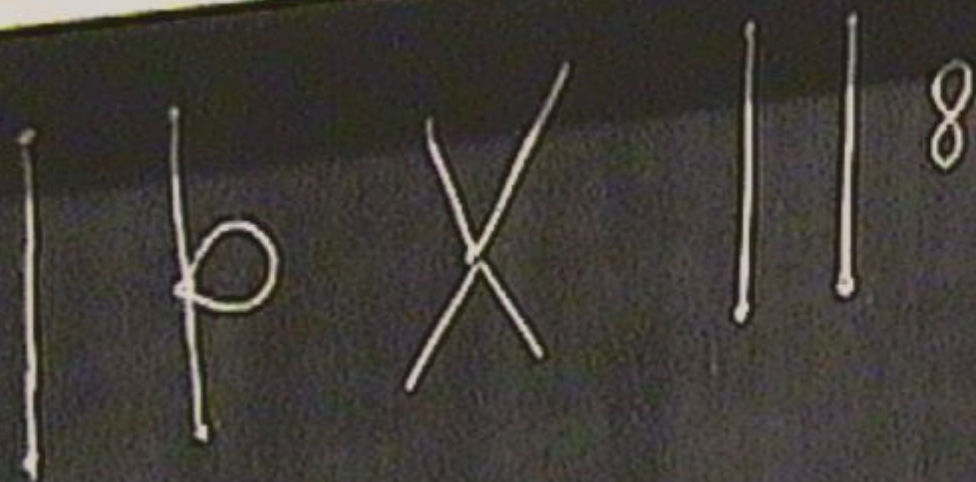
$$= \sum_n \left(\frac{-\lambda}{4!} \right) \left[A_n(3) \right] \frac{1}{n!}$$

$$\left(\frac{-\lambda}{4!} \right)^4 \lambda^5$$



$$= \sum_n \left(\frac{\lambda}{4!} \right) \left[A_n(\beta) \right] \frac{1}{n!}$$

$$\left(\frac{1}{m^2} \right)^4 \lambda J'$$



$$\lambda J \left(\frac{1}{m^2} \right)^4$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} e^{-\frac{1}{2} m^2 q^2 + Jq}$$

$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} \lambda q^2} = \int dq e^{-\frac{1}{2} \lambda q^2}$$

$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = 11e$$

$-\frac{\lambda}{4!} \int dq q^4 e^{-\frac{1}{2} \lambda q^2}$
 $\downarrow$
 $(-1) \left(\frac{1}{16}\right)^4 \lambda j^4$

$$\begin{aligned}
 Z &= \int \mathcal{D}\varphi e^{-\int dx^4 \left[\frac{1}{2} \varphi (\partial + m)^2 \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]} \\
 Z &= \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right) \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n} = 1 \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(0)
 \end{aligned}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi - \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 11e$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi(x) (\partial^2 + m^2) \varphi(x) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} e^{-\frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right) q^4}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} e^{-\frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right) q^4}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^2} e^{\frac{1}{2} \left(\frac{\lambda}{4!} \right) \left(\frac{d}{dJ} \right)^2}$$

$$= \sum_{n \in \mathbb{R}} \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n A_n(j)$$

$$1 \quad p \quad X \quad || \quad 8 \quad (1) \lambda J \left(\frac{1}{m^2} \right)$$



$$Z = \int \mathcal{D}\varphi e^{-i \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi)^2 - m^2 \varphi^2 + \mathcal{L}(\varphi) \right]}$$

$$Z = \int \mathcal{D}\varphi e^{-\frac{i}{2} \int d^4x \left[\varphi(x) (\partial^2 + m^2) \varphi(x) + J(x) \varphi(x) \right]}$$

$$= e^{-\frac{i}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)}$$

$$Z = \int \mathcal{D}\varphi e$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda^2}{2(4!)^2} q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1$$

$$= \sum_{n \in \mathbb{R}} \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int \mathcal{D}\varphi e^{-\frac{i}{2} \int d^4x \left[\frac{1}{2} \varphi(x) (\partial_\mu^2 + m^2) \varphi(x) + J\varphi \right]} =$$

$$= e^{-\frac{i}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)}$$



$$Z = \int \mathcal{D}\varphi e^{-\frac{i}{2} \int d^4x \left[\varphi(x) (\partial^2 + m^2) \varphi(x) + J(x) \varphi(x) \right]}$$

$$= e^{-\frac{i}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)}$$

$$D(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1e$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int \mathcal{D}\varphi e^{-\frac{i}{2} \int d^4x \left[\varphi(x) (\partial^2 + m^2) \varphi(x) + J\varphi \right]} =$$

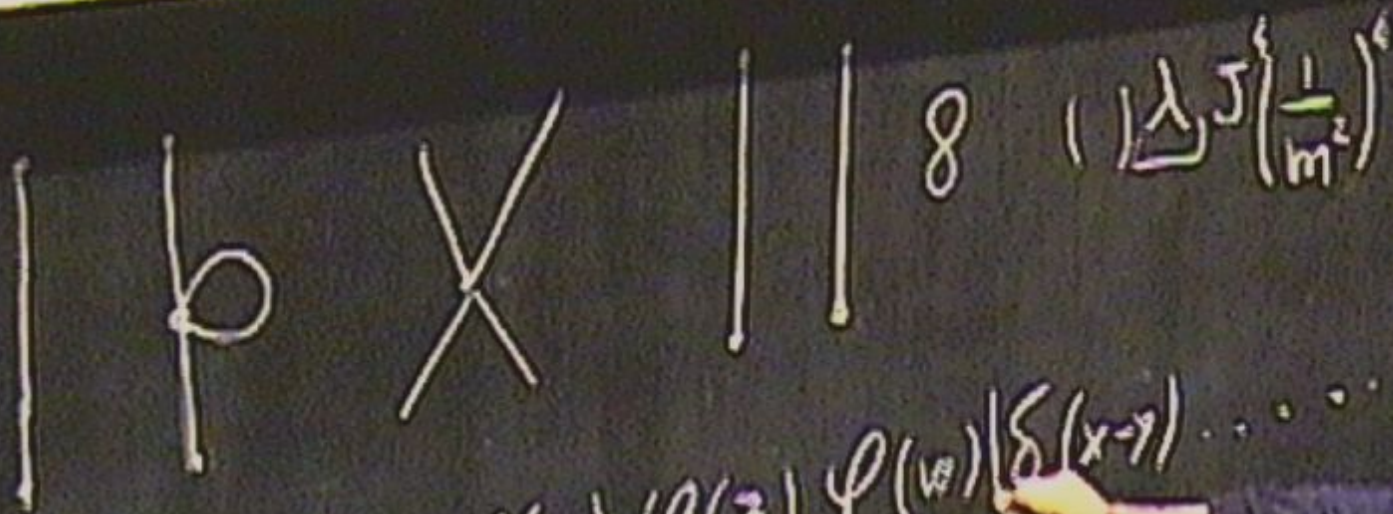
$$-\frac{i}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)$$

$$D(x-y) = \int d^4p \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n A_n(j)$$

$$1 \quad p \quad X \quad || \quad 8 \quad (1) \lambda j \left(\frac{1}{m^2} \right)^4$$

$$= \sum_{n \in \mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n A_n(j)$$



$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \delta(x-y) \dots$$

$$= \sum_n \left(-\frac{\Delta}{4!}\right)^n A_n(j)$$

$\downarrow$ $\downarrow$ $\times$ $\downarrow$ $\downarrow$ 8 $(1)\Delta^j \left(\frac{1}{m^2}\right)^4$

$$\varphi^4(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y) \dots}$$

$$\begin{aligned}
 Z &= \int \mathcal{D}\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]} \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2!} \left(\frac{\lambda}{4!} \right)^2 q^8 \right) \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^4} e^{\frac{1}{2} J^2} \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \quad \left(\left(\frac{1}{m^2} \right)^4 \lambda J^4 \right)
 \end{aligned}$$

$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda^2}{2(4!)^2} q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} J^4} e^{\frac{1}{2} J^2}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

8 $(\lambda J)^4 \left(\frac{1}{m^2} \right)^4$

$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y) \dots}$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda^2}{2(4!)^2} q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

8 $(\lambda J (\frac{1}{m^2}))^4$

$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \dots$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda^2}{2(4!)^2} q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} J^4}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

$$8 \left(-\frac{\lambda}{4!} J^4 \left(\frac{1}{m^2} \right)^4 \right)$$

$$\varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y) \dots}$$

- 3d Ponzano-Regge model (geometry)
- idea of Feynman diagrams
- Group Field theory — idea
 - general formalism
- D. ORITI, [gr-qc/0512103](#) — 3d example
- L. FREIDEL, [hep-th/0505016](#) outlook

$$j(j+1) = \frac{1}{4}$$

$$L_c = (p(2j_{c+1}))$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$= \int_{-\infty}^{+\infty} d^4q e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} d^4q e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e^{-\frac{\lambda}{4!} \left(\frac{d}{dJ} \right)^4} e^{\frac{1}{2m^2} J^2}$$

$$\sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

$$\left(1 - \left(\frac{\lambda}{m^2} \right)^4 J^4 \right)$$

$$Z = \int \mathcal{D}\varphi e^{-\frac{i}{2} \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi)^2 - m^2 \varphi^2 + \mathcal{J}\varphi \right]}$$

$$= e^{-\frac{i}{2} \int d^4x \int d^4y \mathcal{J}(x) D(x-y) \mathcal{J}(y)}$$

$$= e$$

$$D(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(j)}$$

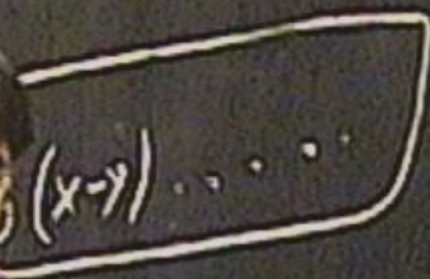
$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \dots$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(j)}$$



8

$$1) \Delta^j \left(\frac{1}{m^2} \right)$$

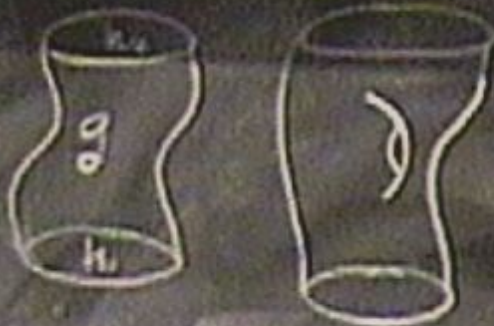


$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \varphi(x-y) \dots$$

- 3d example
- L. FREIDEL, hep-th/0505016 out look



sd example
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• L. FREIDEL, hep-th/0505016 out look



$$\psi(h) \rightarrow \phi(h)$$

- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi$$

- 3d example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H}(\Sigma(h_1, h_2)) = \delta(h_1 - h_2)$$

$$= \sum_{\mathbb{R}} \left(\frac{-1}{4!} \right) \left[A_{\mathbb{R}}^{(4)} \right]$$

$$\psi(j_1, j_2, j_3)$$

$$= \sum_k \left(\frac{-1}{k!} \right) \left[A_{\mu}^{(k)} \right]$$

$$\psi(j_1, j_2, j_3)$$

$$\chi^j(g)$$

$$\rho(g) = \sum_j \int_{mK}^j D_{mK}^j(g)$$

$$= \sum_{\mathbf{R}} \left(\frac{-\Delta}{4!} \right) \left[A_{\mathbf{R}}^{(4)} \right]$$

$$\psi(j_1, j_2, j_3)$$

$$\chi^j(\mathbf{g})$$

$$f(\mathbf{g}) = \sum_j \int_{mK}^j D_{mK}^j(\mathbf{g}) e^{i\mathbf{p} \cdot \mathbf{x}}$$

$$f(x) = \int dp f(p) e^{ipx}$$

$$= \sum_k \left(\frac{-\Delta}{4!} \right) \left[A_{\mu_1 \mu_2 \mu_3}^{(3)} \right]$$

$$\psi(j_1, j_2, j_3)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) [A_{\mathbb{R}}^{(3)}]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(s_1, s_2, s_3)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) \left[A_{\mathbb{R}}^{(3)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(3, 3, 3) = \varphi(9, 8, 3, 3, 3, 3)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) [A_{\mathbb{R}}^{(4)}]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(1, 1, 1) = \varphi(1, 1, 1, 1, 1, 1)$$

$$\mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{R}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) \left[A_{\mathbb{R}}^{(4)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(3, 3, 3) = \varphi(9, 8, 3, 3, 3, 2)$$

$$\frac{\mathbb{C} \times \mathbb{C} \times \mathbb{C}}{\mathbb{C}} \rightarrow \mathbb{R}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) \left[A_{\mathbb{R}}^{(4)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(3, 3, 3) = \varphi(3, 3, 3, 3, 3, 3)$$

$$\Delta \rightarrow \psi(j_1, j_2, j_3) \quad \frac{\mathbb{C} \times \mathbb{C} \times \mathbb{C}}{\mathbb{C}} \rightarrow \mathbb{R}$$

3d example
 • L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H}(\Sigma(h_1, h_2)) = \delta(h_1 - h_2)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{2!} \right) \left[A_{\mathbb{R}}^{(2)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(3, 3, 3) = \varphi(9, 8, 3, 3, 3, 3)$$

$$\begin{array}{ccc} \psi(j_1, j_2, j_3) & G \times G \times G & \rightarrow \mathbb{R} \\ & \underline{G} & \end{array}$$

- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

2) $\rightarrow \phi(h)$

$$\mathcal{H}(\Sigma(h_1, h_2)) = \delta(h_1 - h_2)$$

- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H}(\Sigma(h_1, h_2)) = \delta(h_1 - h_2)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\Delta}{4!} \right) \left[A_{\mathbb{R}}^{(4)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g, g, g, g)$$

$$\begin{array}{ccc} \Delta \rightarrow \psi(j_1, j_2, j_3) & \uparrow & \mathbb{C} \times \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{R} \\ & & \mathbb{C} \end{array}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = e^{-\frac{\lambda}{4!} \frac{J^4}{m^4}} e^{\frac{1}{2} \frac{J^2}{m^2}}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

ذی قیظ

فیزیکی



$\varphi_{i,j,i}$



$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$

$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H}(\Sigma(h_1, h_2)) = e^{iS(h_1 - h_2)}$$

$$\varphi_{\substack{j_1, j_2, j_3 \\ K_1, K_2, K_3}}$$



$$\varphi_{\substack{j_1, j_2, j_3 \\ K_1, K_2, K_3}}$$



$$\varphi_{\substack{j_1, j_2, j_3 \\ K_1, K_2, K_3}}$$



φ_{j_1, j_2, j_3}
 K_1, K_2, K_3



$\varphi(j_1, j_2, j_3)$ $\varphi(j_1, j_2, j_3)$

$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$



$S =$



$\varphi(j_1, j_2, j_3) \varphi(j_4, j_5, j_6)$

$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$



$$S = \frac{1}{2}$$

$\varphi(z_1, z_2, z_3)$



$\varphi(z_1, z_2, z_3) \varphi(z_1, z_2, z_3)$

$$\varphi_{K_1, K_2, K_3}^{j_1, j_2, j_3}$$



$$S = \frac{1}{2}$$

$$\varphi(a_1, a_2, a_3) K(1) \varphi(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$$



$$\varphi(\gamma) \cdot \gamma^2 (\pi m^2)$$



$$\varphi(a_1, a_2, a_3) \varphi(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$$

$$\varphi_{j_1, j_2, j_3}^{k_1, k_2, k_3}$$



$$\varphi(x)$$

$$\varphi(y)$$

$$S = \frac{1}{2}$$

$$\varphi(x, y, z) K(\quad) \varphi(\hat{x}, \hat{y}, \hat{z})$$

$$\varphi(x) \varphi(y) (\delta^4(x-y)) \delta(x-y) \varphi(y)$$

$$\varphi_{\vec{j}_1, \vec{j}_2, \vec{j}_3}^{j_1, j_2, j_3}$$



$$\varphi(\vec{j}_1, \vec{j}_2, \vec{j}_3) \varphi(\vec{j}_4, \vec{j}_5, \vec{j}_6)$$

$$S = \frac{1}{2}$$

$$\varphi(\vec{j}_1, \vec{j}_2, \vec{j}_3) K(1) \varphi(\vec{j}_4, \vec{j}_5, \vec{j}_6)$$

$$\varphi(x) (\partial^2 + m^2) \delta(x-y) \varphi(y)$$

$$\varphi_{j_1 j_2 j_3}^{k_1 k_2 k_3}$$



$$\varphi_{j_1 j_2 j_3} \varphi_{j_4 j_5 j_6}$$

$$S = \frac{1}{2} \int d^4x \varphi(x) \left[(\partial^2 + m^2) \delta(x-y) \right] \varphi(y)$$

$$\int d^4x \varphi(x) \left[(\partial^2 + m^2) \delta(x-y) \right] \varphi(y)$$

$$\varphi_{j_1 j_2 j_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1, j_2, j_3) \varphi(j_1, j_2, j_3)$$

$$S = \frac{1}{2}$$

$$\varphi(x_1, x_2, x_3) K(x_1, x_2, x_3) \varphi(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$$

$$\int dx \frac{\varphi(x)}{dy} \left[(\partial^2 + m^2) \delta(x-y) \right]$$

$$\varphi_{j_1 j_2 j_3}^{k_1 k_2 k_3}$$



$$\varphi_{j_1 j_2 j_3} \varphi_{j_4 j_5 j_6}$$

$$S = \frac{1}{2}$$

$$\varphi(x_1, x_2, x_3) K(x_1, x_2, x_3) \varphi(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$$

$$\int dx \frac{d^3 y}{dy} \varphi(x) \left[(\partial^2 + m^2) \delta(x-y) \right] \varphi(y)$$

$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1, j_2, j_3)$$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

- sd example
- L. FREIDEL, hep-th/0505016 out look



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$



$$\mathcal{H}(\Sigma(h_1, h_2)) = \mathcal{H}(\Sigma(h_1 - h_2))$$

$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1 j_2 j_3) \varphi(j_4 j_5 j_6)$$

$$S = \frac{1}{2} \prod_{j_1 j_2 j_3} \varphi(j_1 j_2 j_3) K(j_1 j_2 j_3) \varphi(j_1 j_2 j_3)$$



$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$



$\varphi(j_1, j_2, j_3)$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$+ \lambda$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3} \varphi_{k_4 k_5 k_6}^{j_4 j_5 j_6}$$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3} K_{j_1 j_2 j_3} \varphi_{\tilde{k}_1 \tilde{k}_2 \tilde{k}_3}^{j_1 j_2 j_3}$$

$$+ \lambda$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3} K_{\left(\begin{smallmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{smallmatrix} \right)} \varphi_{\tilde{k}_1 \tilde{k}_2 \tilde{k}_3}^{j_1 j_2 j_3}$$

$$+ \lambda$$



$\varphi_{j_1 j_2 j_3}$
 $k_1 k_2 k_3$



$\varphi(j_1, j_2, j_3)$ $\varphi(j_1, j_2, j_3)$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(j_1, j_2, j_3)$$

$$+ \lambda \left(\begin{matrix} \varphi \\ \varphi \end{matrix} \right) \varphi(j_1, j_2, j_3) \varphi$$



$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi(s_1, s_2, s_3) \varphi(s_4, s_5, s_6)$$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(s_1, s_2, s_3) K_{\left(\begin{smallmatrix} j_1 & j_2 & j_3 \\ s_1 & s_2 & s_3 \end{smallmatrix} \right)} \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\lambda \left(\begin{smallmatrix} j_1 & j_2 & j_3 \\ s_1 & s_2 & s_3 \end{smallmatrix} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi \delta(s_1, \tilde{s}_1)$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$

$$S = \frac{1}{2} \prod_{j_1 j_2 j_3} \varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3} K_{j_1 j_2 j_3} \varphi_{\tilde{k}_1 \tilde{k}_2 \tilde{k}_3}^{j_1 j_2 j_3}$$

$$+ \lambda \left(\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3} \varphi_{\tilde{k}_1 \tilde{k}_2 \tilde{k}_3}^{j_1 j_2 j_3} \right)$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1, j_2, j_3)$$



$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \text{diagram} \end{array} \right) \varphi(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$



$$\varphi \delta(j_1, \tilde{j}_1) \varphi(\tilde{j}_2, \tilde{j}_3, j_3) \delta(j_2, \tilde{j}_2)$$

$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1 j_2 j_3)$$

$$j_3)$$

$$S = \frac{1}{2} \prod_{j_1 j_2 j_3} \varphi(j_1 j_2 j_3) K(j_1 j_2 j_3) \varphi(\hat{j}_1 \hat{j}_2 \hat{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \right)$$

$$\varphi(j_1 j_2 j_3) \varphi(\hat{j}_1 \hat{j}_2 \hat{j}_3)$$

$$\varphi \delta(j_1 \hat{j}_1) \varphi(\hat{j}_2 \hat{j}_3 j_3)$$

$$\delta(j_2 \hat{j}_2) \delta(j_3 \hat{j}_3)$$

$$\varphi(\hat{j}_1 \hat{j}_2 \hat{j}_3) \delta(1 \dots)$$



φ_{j_1, j_2, j_3}
 k_1, k_2, k_3



$\varphi(j_1, j_2, j_3)$ $\varphi(j_4, j_5, j_6)$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\hat{j}_1, \hat{j}_2, \hat{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \varphi(j_1, j_2, j_3) \varphi(\hat{j}_1, \hat{j}_2, \hat{j}_3) \end{array} \right)$$



$\varphi(j_1, j_2, j_3) \varphi(\hat{j}_1, \hat{j}_2, \hat{j}_3)$
 $\delta(j_1, \hat{j}_1) \delta(j_2, \hat{j}_2)$
 $\varphi(\hat{j}_1, \hat{j}_2, \hat{j}_3) \delta(j_3, \hat{j}_3)$

$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = 1e$$

$$= \sum_{\mathbb{R}} \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)}$$

$$e^{-\frac{\lambda}{4!} \frac{1}{j!}} e^{\frac{1}{j!}} \lambda j^4$$

$$\left(-\frac{\lambda}{4!}\right)^n \lambda j^4$$

$$\Delta \rightarrow \psi(j, j, j)$$

$$G \times G \times G \rightarrow \mathbb{R}$$

$$G$$

$$\varphi_{j_1 j_2 j_3}^{k_1 k_2 k_3}$$



$$\varphi_{j_1 j_2 j_3}$$

$$S = \frac{1}{2} \prod_{j_1 j_2 j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \right)$$

$$\varphi(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$$\varphi \delta(j_1, \tilde{j}_1) \varphi(\tilde{j}_2, \tilde{j}_3, j_3)$$

$$\delta(j_2, \tilde{j}_2) \delta(j_3, \tilde{j}_3)$$

$$\varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \delta(1, \dots)$$



$\varphi_{j_1 j_2 j_3}$
 $K_1 K_2 K_3$



$\varphi(j_1, j_2, j_3)$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \varphi(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \end{array} \right)$$

$$\varphi \delta(j_1, \tilde{j}_1) \varphi(\tilde{j}_2, \tilde{j}_3, j_3)$$

$$\delta(j_2, \tilde{j}_2) \delta(j_3, \tilde{j}_3)$$

$$\varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \delta(1, \dots)$$



φ_{K_1, K_2, K_3}



$\varphi(g_1, g_2, g_3) \varphi(g_1, g_2, g_3)$

$$S = \frac{1}{2} \prod_{g_i} \varphi(g_1, g_2, g_3) \delta(g_1, g_2) \delta(g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$

$$+ \lambda \left(\right) \left(\right)$$



$\varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$

$\varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$

$\delta(g_1, \tilde{g}_1)$

$\delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_3, \tilde{g}_1)$

$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$



φ

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(g_1, g_2, g_3) \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(g_2, \tilde{g}_2) \delta(g_3, \tilde{g}_3)$$

$$+ \lambda \left(\begin{array}{c} \text{diagram} \end{array} \right) \varphi(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$



$$\varphi \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(g_2, \tilde{g}_2) \delta(g_3, \tilde{g}_3) \varphi(\tilde{g}_2, \tilde{g}_3, \tilde{g}_1) \delta(\dots)$$

$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1, j_2, j_3)$$

$$S = \frac{1}{2} \prod_{j_1, j_2, j_3} \varphi(j_1, j_2, j_3) \delta(j_1, \tilde{j}_1) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \delta(j_2, \tilde{j}_2) \delta(j_3, \tilde{j}_3)$$

$$+ \lambda \left(\begin{array}{c} \varphi(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \end{array} \right)$$

$$\varphi \delta(j_1, \tilde{j}_1) \varphi(\tilde{j}_2, \tilde{j}_3, j_3) \delta(j_2, \tilde{j}_2) \delta(j_3, \tilde{j}_3)$$

$$\varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3) \delta(\dots)$$



$$S = \frac{1}{2} \prod_{i,j,k} \varphi(s_i, s_j, s_k) \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k)$$

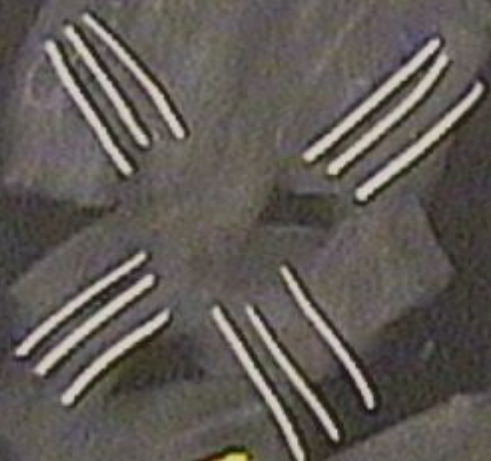
$$+ \lambda \left(\varphi(s_i, s_j, s_k) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \right)$$



$$\varphi \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k)$$

$$\delta(s_i, \tilde{s}_i) \delta(s_j, \tilde{s}_j)$$

$$\varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \delta(1, \dots)$$



$$S = \frac{1}{2} \prod_{i,j,k} \varphi(s_i, s_j, s_k) \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \delta(s_i, \tilde{s}_i) \delta(s_j, \tilde{s}_j) \delta(s_k, \tilde{s}_k)$$

$$+ \lambda \left(\varphi(s_i, s_j, s_k) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \right)$$



$$\varphi \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \delta(s_j, \tilde{s}_j) \delta(s_k, \tilde{s}_k)$$

$$\varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \delta(\dots)$$



$$S = \frac{1}{2} \prod_{i,j,k} \varphi(s_i, s_j, s_k) \delta(s_i, \hat{s}_i) \varphi(\hat{s}_i, \hat{s}_j, \hat{s}_k) \delta(s_i, \hat{s}_i) \delta(s_j, \hat{s}_j) \delta(s_k, \hat{s}_k) \\ + \lambda \left(\begin{array}{c} \varphi(s_i, s_j, s_k) \varphi(\hat{s}_i, \hat{s}_j, \hat{s}_k) \\ \varphi \delta(s_i, \hat{s}_i) \varphi(\hat{s}_i, \hat{s}_j, \hat{s}_k) \\ \delta(s_i, \hat{s}_i) \delta(s_j, \hat{s}_j) \\ \varphi(\hat{s}_i, \hat{s}_j, \hat{s}_k) \delta(\dots \end{array} \right)$$





$$S = \frac{1}{2} \prod_{\substack{g, \tilde{g}, s \\ S(g)}} \varphi(g, \tilde{g}, s) \delta(g, \tilde{g}) \varphi(\tilde{g}, \tilde{g}, \tilde{s})$$

$$+ \lambda \left(\varphi(g, \tilde{g}, s) \varphi(\tilde{g}, \tilde{g}, \tilde{s}) \right)$$



$$\varphi \delta(g, \tilde{g}) \varphi(\tilde{g}, \tilde{g}, \tilde{s})$$

$$\delta(g, \tilde{g}) \delta(g, \tilde{g})$$

$$\varphi(\tilde{g}, \tilde{g}, \tilde{s}) \delta(1, \dots)$$



$$S = \frac{1}{2} \prod_{\substack{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3 \\ S(\mathbf{g}_1)}} \varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \frac{\delta(\mathbf{g}_1, \hat{\mathbf{g}}_1)}{\delta(\mathbf{g}_2, \hat{\mathbf{g}}_2)} \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3)$$

$$+ \lambda \left(\varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \right)$$



$$\varphi \delta(\mathbf{g}_1, \hat{\mathbf{g}}_1) \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3)$$

$$\delta(\mathbf{g}_2, \hat{\mathbf{g}}_2) \delta(\mathbf{g}_3, \hat{\mathbf{g}}_3)$$

$$\varphi(\hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3, \hat{\mathbf{g}}_1) \delta(1, \dots)$$



$$S = \frac{1}{2} \prod_{\substack{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3 \\ S(\mathbf{g}_i)}} \varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \delta(\mathbf{g}_1, \hat{\mathbf{g}}_1) \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2) \delta(\hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\hat{\mathbf{g}}_3, \hat{\mathbf{g}}_1) \\ + \lambda \left(\varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\mathbf{g}_1, \hat{\mathbf{g}}_1) \delta(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2) \delta(\hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\hat{\mathbf{g}}_3, \hat{\mathbf{g}}_1) \delta(\mathbf{g}_2, \hat{\mathbf{g}}_2) \delta(\mathbf{g}_3, \hat{\mathbf{g}}_3) \right) \\ \varphi(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\mathbf{g}_1, \hat{\mathbf{g}}_1) \delta(\mathbf{g}_2, \hat{\mathbf{g}}_2) \delta(\mathbf{g}_3, \hat{\mathbf{g}}_3) \delta(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2) \delta(\hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3) \delta(\hat{\mathbf{g}}_3, \hat{\mathbf{g}}_1) \delta(\mathbf{g}_1, \mathbf{g}_2) \delta(\mathbf{g}_2, \mathbf{g}_3) \delta(\mathbf{g}_3, \mathbf{g}_1) \dots$$



$\varphi(\vec{x})$



$$S = \frac{1}{2} \prod_{\substack{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3 \\ S(\mathbf{g}_i)}} \varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \delta(\mathbf{g}_1, \tilde{\mathbf{g}}_1) \varphi(\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_3) \delta(\mathbf{g}_2, \tilde{\mathbf{g}}_2) \delta(\mathbf{g}_3, \tilde{\mathbf{g}}_3) \\ + \lambda \left(\varphi(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) \varphi(\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_3) \right. \\ \left. \varphi \delta(\mathbf{g}_1, \tilde{\mathbf{g}}_1) \varphi(\tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_3) \delta(\mathbf{g}_2, \tilde{\mathbf{g}}_2) \delta(\mathbf{g}_3, \tilde{\mathbf{g}}_3) \right. \\ \left. \varphi(\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \tilde{\mathbf{g}}_3) \delta(\dots) \right)$$

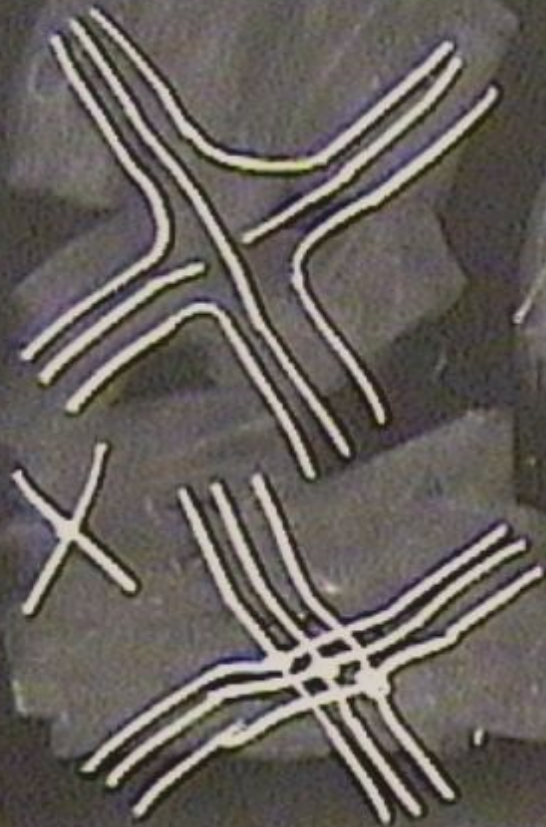


$\varphi(\tilde{\mathbf{x}})$



$$S = \frac{1}{2} \prod_{\substack{\vec{g}_1, \vec{g}_2, \vec{g}_3 \\ S(\vec{g}_1)}} \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \delta(\vec{g}_1, \vec{g}_2) \varphi(\vec{g}_2, \vec{g}_3, \vec{g}_1) \delta(\vec{g}_2, \vec{g}_3) \delta(\vec{g}_3, \vec{g}_1) \\ + \lambda \left(\begin{array}{c} \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \varphi(\vec{g}_2, \vec{g}_3, \vec{g}_4) \\ \varphi \delta(\vec{g}_1, \vec{g}_2) \varphi(\vec{g}_3, \vec{g}_4, \vec{g}_1) \\ \delta(\vec{g}_2, \vec{g}_3) \delta(\vec{g}_4, \vec{g}_1) \\ \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \delta(\dots \end{array} \right)$$



$$S = \frac{1}{2} \prod_{i,j,k} \varphi(g_i, g_j, g_k) \delta(g_i, \tilde{g}_i) \varphi(\tilde{g}_i, \tilde{g}_j, \tilde{g}_k) \delta(g_i, \tilde{g}_i) \delta(g_j, \tilde{g}_j) \delta(g_k, \tilde{g}_k)$$

$$+ \lambda \left(\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \right) \varphi(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$

$$\varphi \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(g_2, \tilde{g}_2) \delta(g_3, \tilde{g}_3)$$

$$\varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(1, \dots)$$

$\varphi(\tilde{x})$



$$S = \frac{1}{2} \int d\tau \int d\vec{x} \varphi(x, \tau, \vec{x}) \begin{pmatrix} \delta(x, \vec{x}) \\ \delta(x, \vec{y}) \\ \delta(x, \vec{z}) \end{pmatrix} \varphi(\vec{x}, \vec{y}, \vec{z})$$

$$+ \lambda \binom{21}{1} \varphi(s, s, s) \varphi(\bar{s}, \bar{s}, \bar{s})$$

$$\varphi(\delta_1, \delta_2) \varphi(\hat{g}_1, \hat{g}_2, g_2)$$

$$\varphi(\tilde{x}, \tilde{y}, \tilde{z}) \delta(1 - \dots)$$



$$\varphi(x)$$

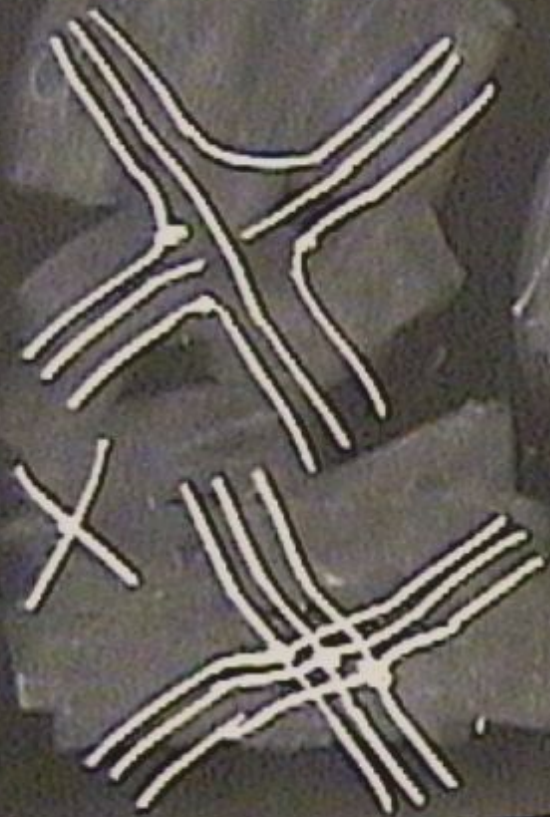




$$S = \frac{1}{2} \prod_{i,j,k} \varphi(z_i, z_j, z_k) \delta(z_i, \tilde{z}_i) \varphi(\tilde{z}_i, \tilde{z}_j, \tilde{z}_k) \delta(\tilde{z}_i, \tilde{z}_j) \delta(\tilde{z}_j, \tilde{z}_k) \delta(\tilde{z}_k, \tilde{z}_i)$$

$$+ \lambda \left(\begin{array}{c} \varphi(z_i, z_j, z_k) \varphi(\tilde{z}_i, \tilde{z}_j, \tilde{z}_k) \\ \varphi \delta(z_i, \tilde{z}_i) \varphi(\tilde{z}_i, \tilde{z}_j, \tilde{z}_k) \\ \delta(z_j, \tilde{z}_j) \delta(z_i, \tilde{z}_i) \\ \varphi(\tilde{z}_i, \tilde{z}_j, \tilde{z}_k) \delta / 1 \dots \end{array} \right)$$



$$S = \frac{1}{2} \prod_{i,j,k} \varphi(g_i, g_j, g_k) \delta(g_i, \tilde{g}_i) \varphi(\tilde{g}_i, \tilde{g}_j, \tilde{g}_k)$$

$$+ \lambda \left(\varphi(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \right)$$



$$\varphi(\tilde{x})$$

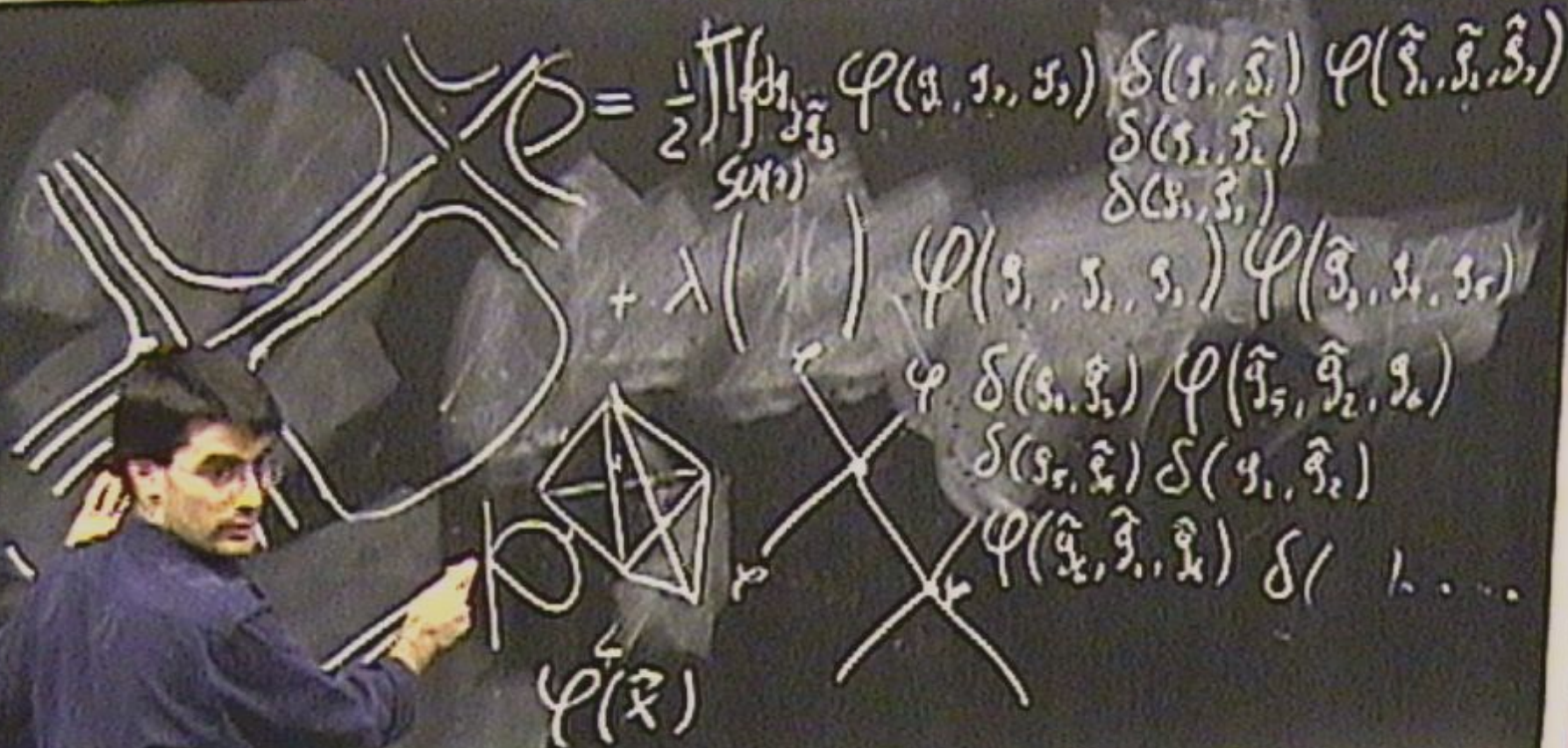


$$\varphi \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, g_2) \delta(g_2, \tilde{g}_2) \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_2, \tilde{g}_1, \tilde{g}_1) \delta / 1 \dots$$



$$\begin{aligned}
 S = & \frac{1}{2} \prod_{\substack{\vec{g}, \vec{g}_1, \vec{g}_2 \\ S(1)}} \varphi(\vec{g}, \vec{g}_1, \vec{g}_2) \delta(\vec{g}, \vec{g}_1) \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \\
 & + \lambda \left(\begin{array}{c} \varphi(\vec{g}, \vec{g}_1, \vec{g}_2) \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \\ \varphi \delta(\vec{g}_1, \vec{g}_2) \varphi(\vec{g}_2, \vec{g}_3, \vec{g}_4) \\ \delta(\vec{g}_2, \vec{g}_3) \delta(\vec{g}_3, \vec{g}_4) \\ \varphi(\vec{g}_4, \vec{g}_5, \vec{g}_6) \delta / \dots \end{array} \right)
 \end{aligned}$$





$$S = \frac{1}{2} \prod_{\tilde{g}_i} \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_1, \tilde{g}_2) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_3, \tilde{g}_1) + \lambda \left(\varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_3, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_3, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(1 \dots) \right)$$

$\varphi(\tilde{x})$

$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \prod_{\substack{\vec{g}_1, \vec{g}_2, \vec{g}_3 \\ \text{sum}}} \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \frac{\delta(\vec{g}_1, \vec{g}_1)}{\delta(\vec{g}_1, \vec{g}_2)} \frac{\delta(\vec{g}_2, \vec{g}_2)}{\delta(\vec{g}_2, \vec{g}_1)} \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \\
 & + \lambda \left(\begin{array}{c} \diagup \diagdown \\ \diagdown \diagup \end{array} \right) \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \\
 & \varphi \frac{\delta(\vec{g}_1, \vec{g}_1)}{\delta(\vec{g}_1, \vec{g}_2)} \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \\
 & \frac{\delta(\vec{g}_1, \vec{g}_1)}{\delta(\vec{g}_1, \vec{g}_2)} \delta(\vec{g}_1, \vec{g}_2) \\
 & \varphi(\vec{g}_1, \vec{g}_2, \vec{g}_3) \delta(\dots
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \prod_{\vec{x}, \vec{y}, \vec{z}} \varphi(\vec{x}, \vec{y}, \vec{z}) \delta(\vec{x}, \vec{y}) \varphi(\vec{y}, \vec{z}, \vec{x}) \\
 & + \lambda \left(\varphi(\vec{x}, \vec{y}, \vec{z}) \varphi(\vec{y}, \vec{z}, \vec{x}) \right. \\
 & \quad \varphi \delta(\vec{x}, \vec{y}) \varphi(\vec{y}, \vec{z}, \vec{x}) \\
 & \quad \delta(\vec{x}, \vec{y}) \delta(\vec{y}, \vec{z}) \\
 & \quad \left. \varphi(\vec{x}, \vec{y}, \vec{z}) \delta(\vec{x}, \vec{y}) \dots \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \int \prod_{i,j} \varphi(x_i, y_j, z_k) \delta(x_i, \tilde{x}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & + \lambda \left(\int \varphi(x_i, y_j, z_k) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \right. \\
 & \quad \varphi \delta(x_i, \tilde{x}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & \quad \delta(x_i, \tilde{x}_i) \delta(y_j, \tilde{y}_j) \\
 & \quad \left. \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \delta / \dots \right)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \prod_{\substack{\vec{x}_1, \vec{x}_2, \vec{x}_3 \\ S(\vec{x})}} \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(\vec{x}_1, \vec{x}_2) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \\
 &\quad + \lambda \left(\begin{array}{c} \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \\ \varphi \delta(\vec{x}_1, \vec{x}_2) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \\ \delta(\vec{x}_1, \vec{x}_2) \delta(\vec{x}_1, \vec{x}_2) \\ \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(1 \dots) \end{array} \right)
 \end{aligned}$$

$$Z = \int \mathcal{D}\varphi e$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2!} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1 e$$

$$= \sum_{\mathbb{R}} \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

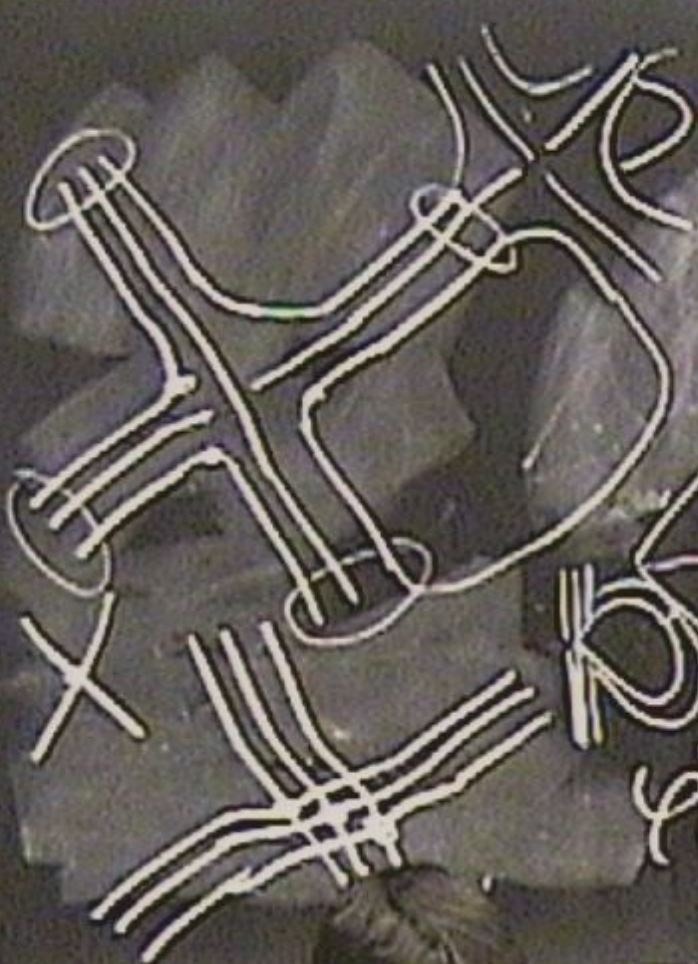
$$\Delta \rightarrow \psi(j, i, j)$$

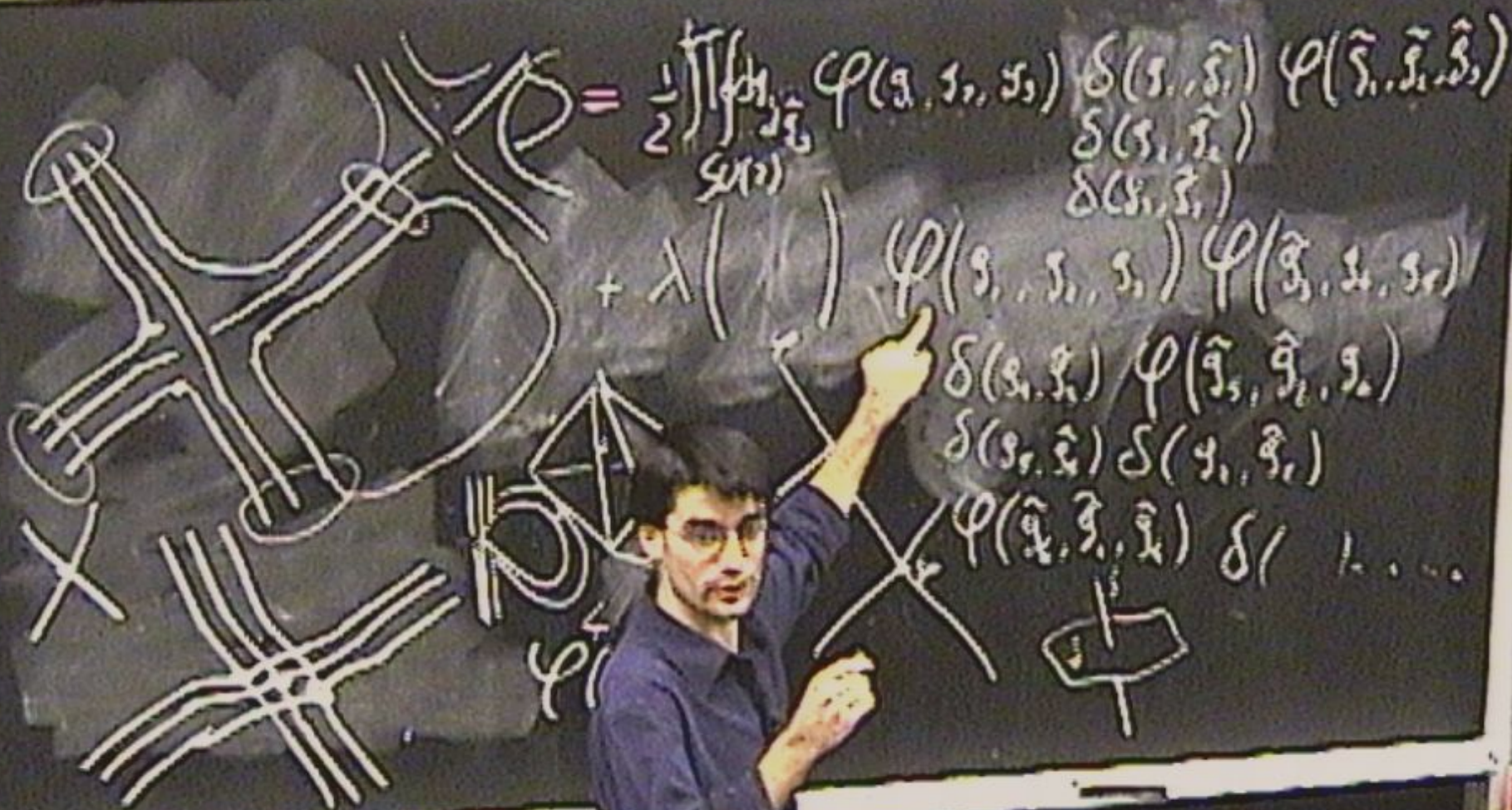
$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \prod_{i,j,k} \left(\frac{1}{S_{ij}} \right) \varphi(g_i, g_j, g_k) \delta(g_i, \hat{g}_i) \varphi(\hat{g}_i, \hat{g}_j, \hat{g}_k) \\
 & + \lambda \left(\frac{1}{S_{ij}} \right) \varphi(g_i, g_j, g_k) \varphi(\hat{g}_i, \hat{g}_j, \hat{g}_k) \\
 & \varphi \delta(g_i, \hat{g}_i) \varphi(\hat{g}_i, \hat{g}_j, g_k) \\
 & \delta(g_i, \hat{g}_i) \delta(g_j, \hat{g}_j) \\
 & \varphi(\hat{g}_i, \hat{g}_j, \hat{g}_k) \delta / 1 \dots
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \int \prod_{i=1}^3 \varphi(\mathbf{r}_i, \mathbf{r}_i, \mathbf{r}_i) \delta(\mathbf{r}_1, \tilde{\mathbf{r}}_1) \varphi(\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_1) \\
 & + \lambda \left(\varphi(\mathbf{r}_1, \mathbf{r}_1, \mathbf{r}_1) \varphi(\tilde{\mathbf{r}}_1, \mathbf{r}_1, \mathbf{r}_1) \right. \\
 & \quad \varphi \delta(\mathbf{r}_1, \tilde{\mathbf{r}}_1) \varphi(\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_1, \mathbf{r}_1) \\
 & \quad \delta(\mathbf{r}_1, \tilde{\mathbf{r}}_1) \delta(\mathbf{r}_1, \tilde{\mathbf{r}}_1) \\
 & \quad \left. (\tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_1, \tilde{\mathbf{r}}_1) \delta(1, \dots) \right)
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \prod_{i,j,k} \varphi(\mathfrak{g}_i, \mathfrak{g}_j, \mathfrak{g}_k) \delta(\mathfrak{g}_i, \hat{\mathfrak{g}}_i) \varphi(\hat{\mathfrak{g}}_i, \hat{\mathfrak{g}}_j, \hat{\mathfrak{g}}_k) \\
 & + \lambda \left(\begin{array}{c} \text{diagram} \end{array} \right) \varphi(\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3) \varphi(\hat{\mathfrak{g}}_1, \hat{\mathfrak{g}}_2, \hat{\mathfrak{g}}_3) \\
 & \varphi \delta(\mathfrak{g}_1, \hat{\mathfrak{g}}_1) \varphi(\hat{\mathfrak{g}}_1, \hat{\mathfrak{g}}_2, \hat{\mathfrak{g}}_3) \\
 & \delta(\mathfrak{g}_2, \hat{\mathfrak{g}}_2) \delta(\mathfrak{g}_3, \hat{\mathfrak{g}}_3) \\
 & \varphi(\hat{\mathfrak{g}}_1, \hat{\mathfrak{g}}_2, \hat{\mathfrak{g}}_3) \delta(\dots)
 \end{aligned}$$

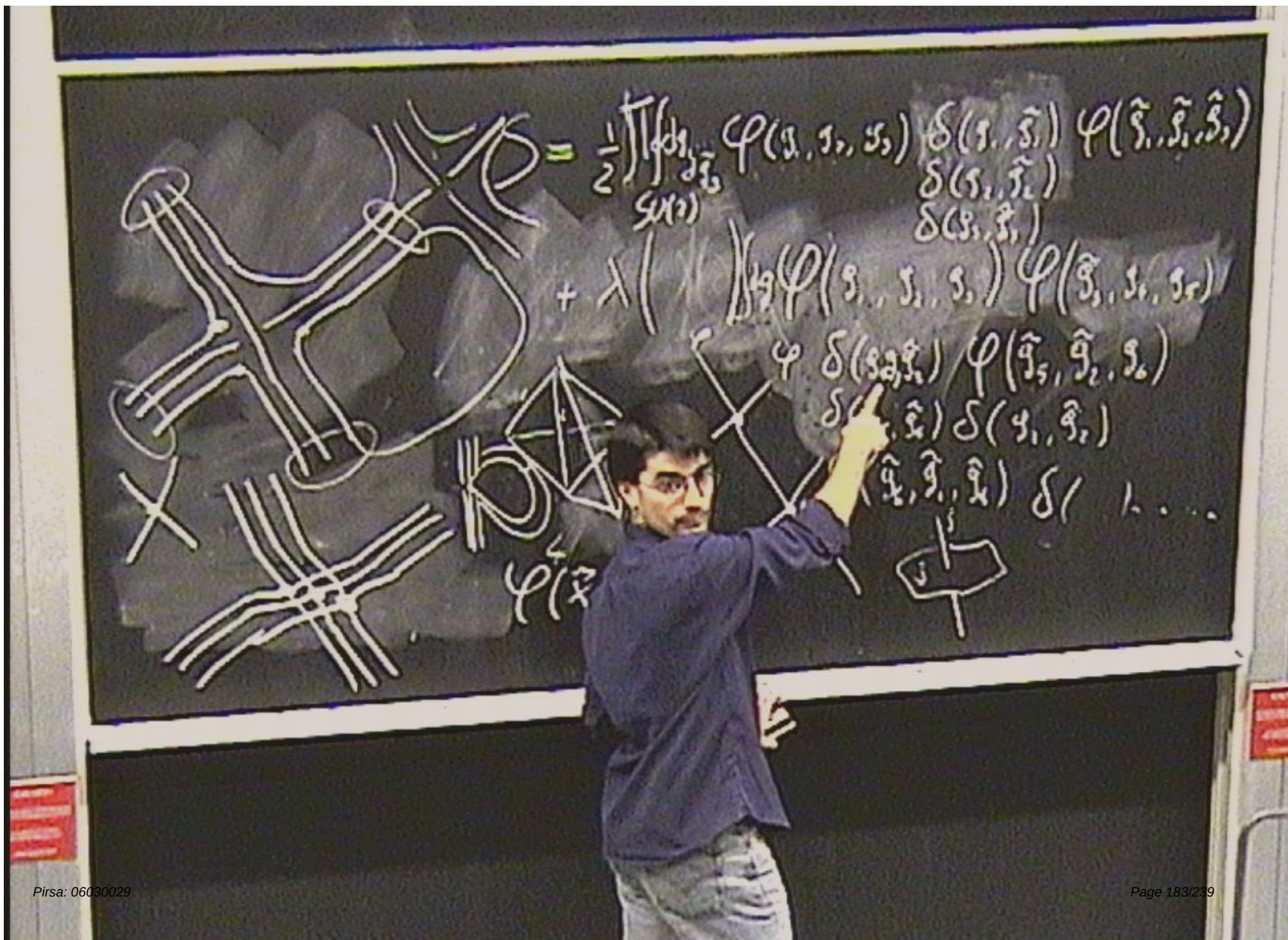




$$S = \frac{1}{2} \int_{\Sigma} \varphi(x, y, z) \delta(x, \hat{x}) \varphi(\hat{x}, \hat{y}, \hat{z}) \delta(x, \hat{x}) \delta(y, \hat{y}) \delta(z, \hat{z})$$

$$+ \lambda \left(\varphi(x, y, z) \varphi(\hat{x}, \hat{y}, \hat{z}) \delta(x, \hat{x}) \varphi(\hat{x}, \hat{y}, \hat{z}) \delta(y, \hat{y}) \delta(z, \hat{z}) \varphi(\hat{x}, \hat{y}, \hat{z}) \delta(x, \hat{x}) \delta(y, \hat{y}) \delta(z, \hat{z}) \right)$$

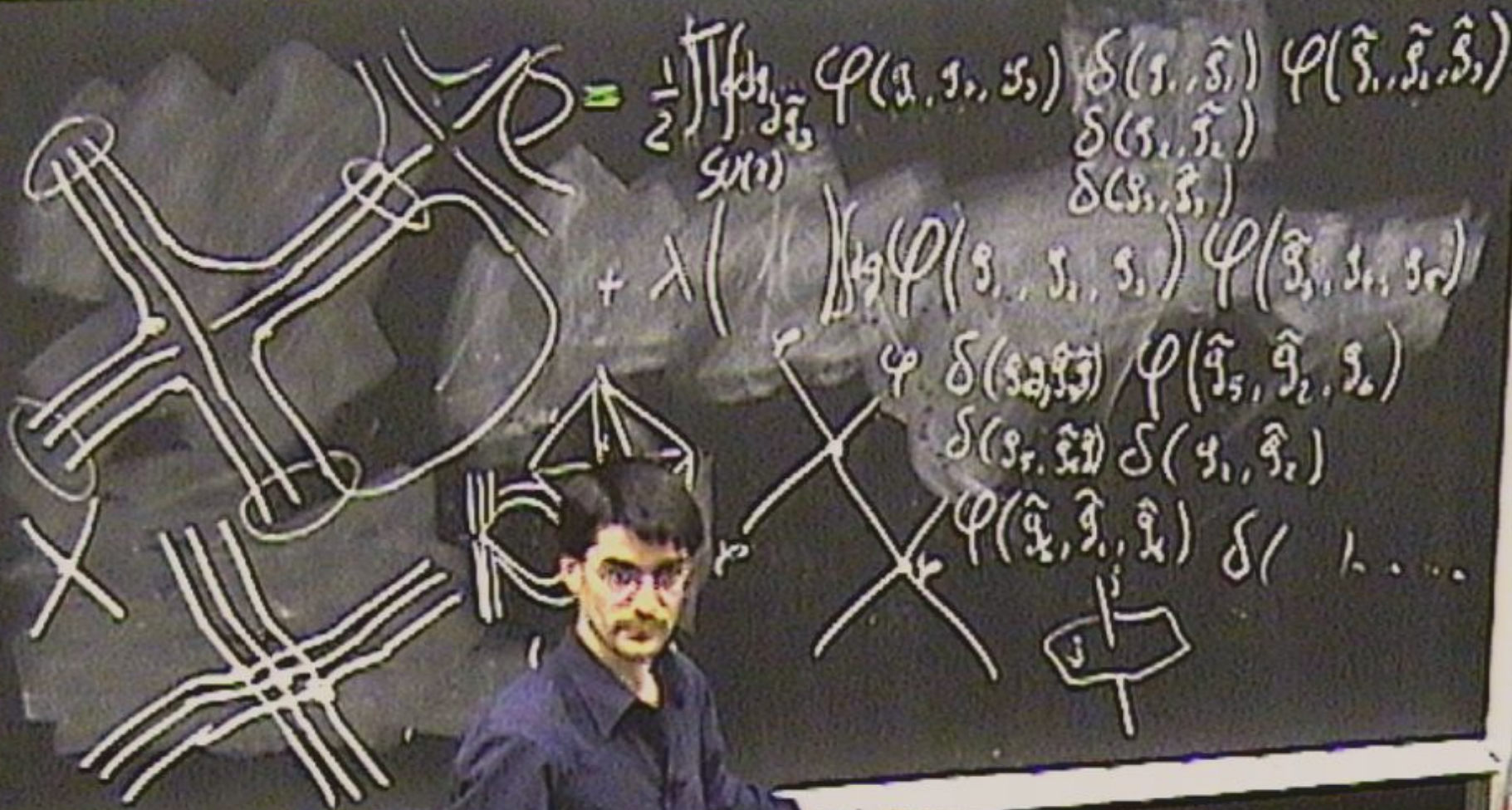
$$S = \frac{1}{2} \int \prod_{i,j} \varphi(x_i, y_j, z_k) \delta(x_i, \hat{x}_i) \varphi(\hat{x}_i, \hat{y}_i, \hat{z}_i) \delta(x_i, \hat{x}_i) \delta(y_i, \hat{y}_i) \delta(z_i, \hat{z}_i) + \lambda \left(\int \prod_{i,j} \varphi(x_i, y_j, z_k) \varphi(\hat{x}_i, \hat{y}_i, \hat{z}_i) \delta(x_i, \hat{x}_i) \delta(y_i, \hat{y}_i) \delta(z_i, \hat{z}_i) \varphi(\hat{x}_i, \hat{y}_i, \hat{z}_i) \delta(x_i, \hat{x}_i) \delta(y_i, \hat{y}_i) \delta(z_i, \hat{z}_i) \right)$$





$$\begin{aligned}
 \mathcal{Z} = & \frac{1}{2} \prod_{\vec{x}_1, \vec{x}_2} \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(\vec{x}_1, \vec{x}_2) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \\
 & + \lambda \left(\prod_{\vec{x}_1, \vec{x}_2, \vec{x}_3} \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \right. \\
 & \quad \left. \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(\vec{x}_1, \vec{x}_2) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \right. \\
 & \quad \left. \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(\vec{x}_1, \vec{x}_2) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \right) \dots
 \end{aligned}$$



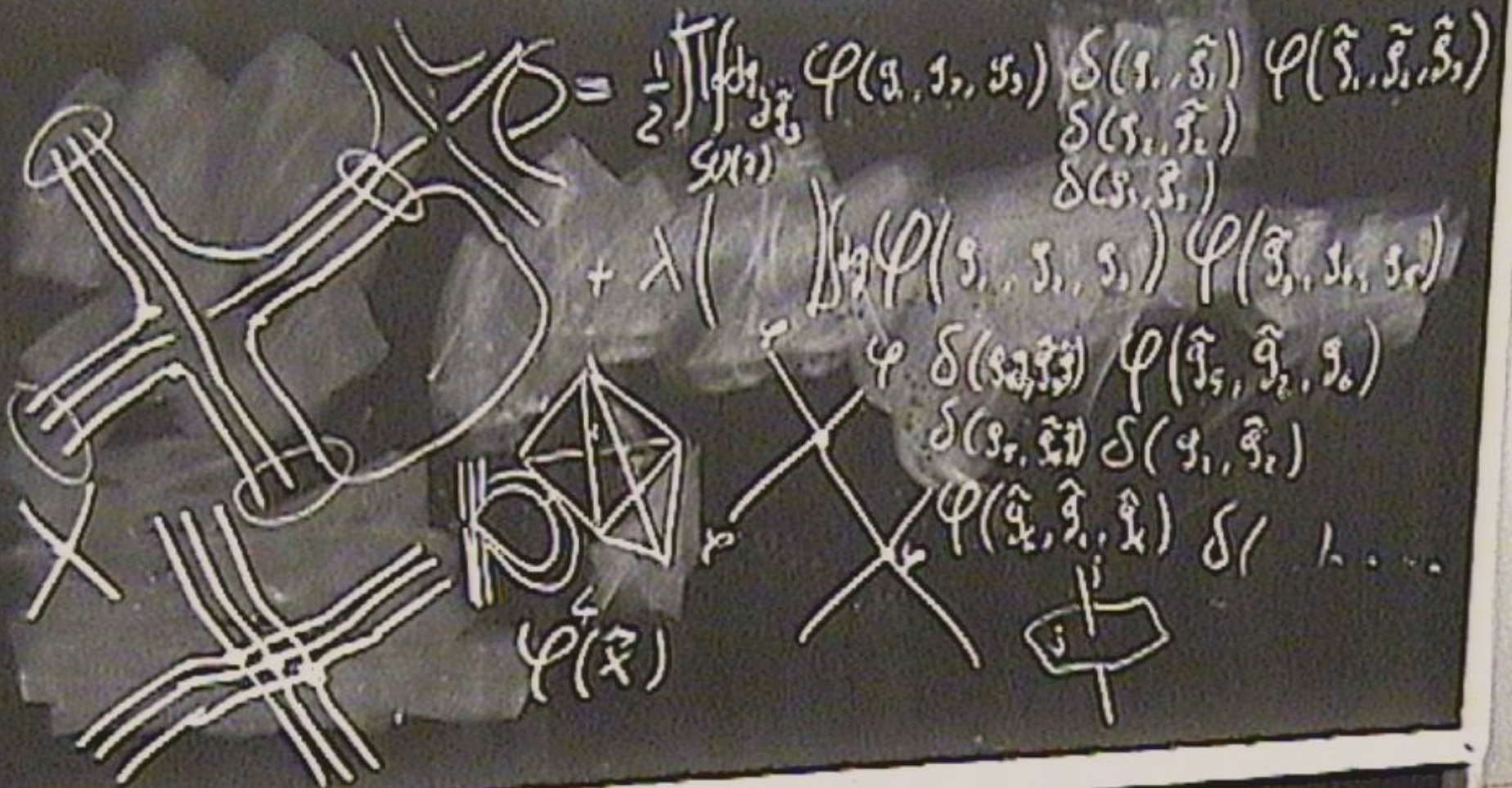


$$\begin{aligned}
 S = & \frac{1}{2} \int \prod_{i,j} \varphi(x_i, y_i, z_i) \delta(x_i, \tilde{x}_i) \delta(y_i, \tilde{y}_i) \delta(z_i, \tilde{z}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & + \lambda \left(\int \prod_{i,j} \varphi(x_i, y_i, z_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \right. \\
 & \quad \varphi \delta(x_i, \tilde{x}_i) \varphi(\tilde{y}_i, \tilde{y}_i, \tilde{z}_i) \delta(y_i, \tilde{y}_i) \delta(z_i, \tilde{z}_i) \\
 & \quad \left. \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \delta(1, \dots) \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \int \prod_{i,j,k} \varphi(x_i, y_j, z_k) \delta(x_i, \tilde{x}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & + \lambda \left(\int \varphi(x_1, y_1, z_1) \varphi(\tilde{x}_1, \tilde{y}_1, \tilde{z}_1) \right. \\
 & \quad \varphi \delta(x_2, \tilde{x}_2) \varphi(\tilde{x}_2, \tilde{y}_2, \tilde{z}_2) \\
 & \quad \delta(x_2, \tilde{x}_2) \delta(y_1, \tilde{y}_1) \\
 & \quad \left. \varphi(\tilde{x}_2, \tilde{y}_2, \tilde{z}_2) \delta(\dots) \right)
 \end{aligned}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \left[A_n(j) \right]$$

$$V = \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$



$$\begin{aligned}
 S = & \frac{1}{2} \int \prod_{i,j,k} \varphi(x_i, y_j, z_k) \delta(x_i, \tilde{x}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & + \lambda \left(\int \prod_{i,j,k} \varphi(x_i, y_j, z_k) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \right. \\
 & \quad \varphi \delta(x_i, \tilde{x}_i) \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \\
 & \quad \delta(x_i, \tilde{x}_i) \delta(y_j, \tilde{y}_j) \\
 & \quad \left. \varphi(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i) \delta / \dots \right)
 \end{aligned}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \left[A_n(j) \right]$$

$$V = \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$= \sum_R \left(\frac{-\lambda}{i!} \right)^n \left[A_n(j) \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, g, g, g')$$

$\uparrow \quad \quad \uparrow$
 $G \quad G \rightarrow R$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \left[A_n^{(j)} \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, g, g, g')$$

$$K = \delta(g, g, g, g) \cdot \delta(g, g, g, g')$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \left[A_n^{(j)} \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, g, g, g')$$

$$K = \int d^4x d^4y \delta(g, g, g, g) \cdot \delta(g, g, g, g')$$

$$\begin{aligned}
 Z &= \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]} \\
 &= \int_{-\infty}^{\infty} dq \, e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{\infty} dq \, e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda^2}{2(4!)^2} q^8 - \dots \right) \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1e \\
 &= \sum_{n \in \mathbb{R}} \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}
 \end{aligned}$$

$$Z = \int \mathcal{D}\varphi e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 - \dots \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = 1e$$

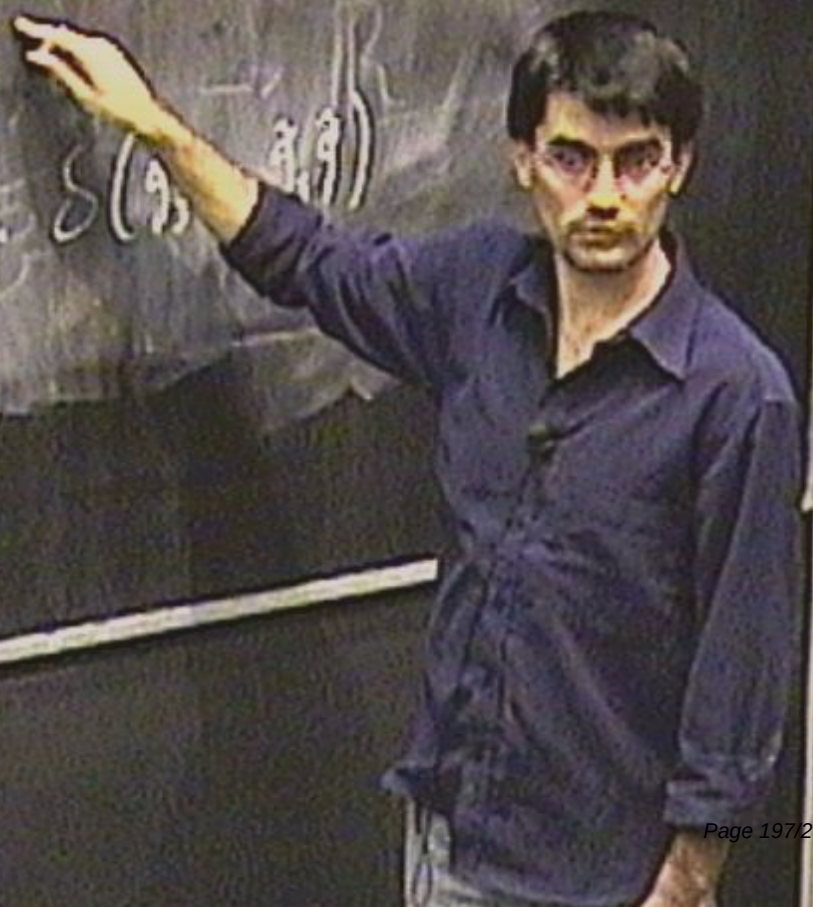
$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}$$

$\left(-\frac{\lambda}{4!} \right)^n \frac{1}{n!} \left(\frac{d}{dJ} \right)^n e^{-\frac{1}{2} m^2 q^2 + Jq}$
 $\left(-\frac{\lambda}{4!} \right)^n \frac{1}{n!} \left(\frac{d}{dJ} \right)^n e^{-\frac{1}{2} m^2 q^2 + Jq}$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \left[A_n^{(3)} \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g')$$

$$K = \int d^4x d^4y \delta(g, g, g, \hat{g}) \delta(g, \hat{g}, g')$$



$$= \sum_n \left(\frac{-\lambda}{4!} \right) \left[A_n^{(3)} \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int d^4x d^4y \delta(g, g, g, \hat{g}) \cdot \delta(g, g, \hat{g}, g') = P$$

$$= \sum_k \left(\frac{-\lambda}{4!} \right) \left[A_{\mu\nu}^{(3)} \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int d^4x d^4y \delta(g, g, g, \hat{g}) \cdot \delta(g, g, g, g') = P \quad |||$$

$$= \sum_k \left(\frac{-\lambda}{4!} \right) \left[A_{\mu\nu}(3) \right]$$

$$V = \int d^4x \delta(g, g, \hat{g}, g') \quad \delta(g, g, g, g')$$

$$K = \int d^4x d^4y \delta(g, g, g, g) \cdot \delta(g, g, g, g) = P$$



$$Z = \int \mathcal{D}\varphi e^{-\int d\tau \left[\frac{1}{2} m^2 (\dot{\varphi} + m\varphi)^2 + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} d\varphi e^{-\frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4 + J\varphi} = \int_{-\infty}^{+\infty} d\varphi e^{-\frac{1}{2} m^2 \varphi^2 + J\varphi} \left(1 - \frac{\lambda}{4!} \varphi^4 + \frac{\lambda^2}{2! (4!)^2} \varphi^8 - \dots \right)$$

$$= \sum_n \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(J)}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(J)}$$

$$\mathcal{V} = \int d\tau d\tau' \delta(\varphi(\tau), \varphi(\tau')) \delta(\varphi(\tau), \varphi(\tau'))$$

$$= \int d\tau d\tau' \delta(\varphi(\tau), \varphi(\tau')) \delta(\varphi(\tau), \varphi(\tau')) = P$$



$$= \sum_R \left(\frac{-\Delta}{4!} \right)^2 \left[A_R(j) \right]$$

$$V = \int d^4x \delta(g, \hat{g}, \hat{g}, g') \quad \delta(g, \hat{g}, \hat{g}, g')$$

$$K = \int d^4x d^4y \delta(g, g, \hat{g}, \hat{g}) = P$$



$$\Delta \sim \Gamma$$

$$= \sum_{\mathbb{R}} \left(-\frac{\lambda}{4!} \right)^2 \boxed{A_{\mathbb{R}}(j)}$$

$$V = \int dg \delta(g, \hat{g}, \hat{g}, g')$$

$$\delta(g, \hat{g}, g')$$



$$K = \int dg \delta(g, \hat{g}, \hat{g}, g')$$

$$= P$$

$$Z_{\Delta \sim \Gamma} = \prod_{SU(2)} \int dg$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{i} \right)^n \left[A_{\mathbb{R}}(j) \right]$$

$$V = \int dg \delta(g, \hat{g}, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int dg \delta(g, \hat{g}, g, \hat{g}) \quad \delta(g, \hat{g}, \hat{g}, g) = P$$

$$Z_{\Delta \sim \Gamma} = \prod_{SU(2)} \left(\int dg \prod_F \right)$$



$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \left[A_n(\mathcal{J}) \right]$$

$$\mathcal{V} = \int dg \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int dg \delta(g, g, g, \hat{g}) \cdot \delta(g, g, g, g) = P$$



$$Z_{\Delta \sim \Gamma} = \prod_{SU(2)} \int dg \prod_f \delta(\Pi, g)$$

$$\begin{aligned}
 \mathcal{S} = & \frac{1}{2} \int \prod_{i=1}^3 \varphi(\tilde{g}_i, \tilde{g}_i, \tilde{g}_i) \delta(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \\
 & + \lambda \left(\int \prod_{i=1}^3 \varphi(\tilde{g}_i, \tilde{g}_i, \tilde{g}_i) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \right. \\
 & \quad \varphi \delta(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \\
 & \quad \delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_1, \tilde{g}_3) \\
 & \quad \left. \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(1, \dots) \right)
 \end{aligned}$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(j)}$$

$$V = \int dg \delta(g, g, \hat{g}, g')$$

$$\delta(g, \hat{g}, g, g')$$



$$K = \int dg d\hat{g} \delta(g, g, g, \hat{g}) \cdot \delta(\dots)$$

$$\delta(\dots) = P$$

$$Z_{\Delta, \Gamma} = \prod_{\Gamma} \int_{SU(2)} dg \prod_{\Gamma} \delta(\Pi g)$$

$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(j)}$$

$$V = \int dg \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int dg \int \hat{g} \delta(g, g, g, \hat{g}) \quad \delta(g, g, g, \hat{g})$$

$$Z_{\Delta, \Gamma} = \prod_{\Gamma} \int dg \prod_{\Gamma} \delta(\prod_{\Gamma} g)$$



$$= \sum_{\mathbb{R}} \left(\frac{-\lambda}{4!} \right)^n \boxed{A_n(\mathbb{J})}$$

$$V = \int dg \delta(g, g, \hat{g}, g') \quad \delta(g, \hat{g}, g, g')$$

$$K = \int dg \delta(g, g, g, \hat{g}) \quad \delta(g, g, g, \hat{g})$$

$$Z_{\Delta, \Gamma} = \prod_{\Gamma} \int dg \prod_{\Gamma} \delta(g, \hat{g})$$



$$12 \left(\frac{4!}{1!} \right) \left(\frac{4!}{1!} \right)$$

$$V = \int dg \delta(g, g, \hat{g}, g')$$

$\delta_{\hat{g}, \hat{g}'}$
 $\uparrow \quad \uparrow$

$\downarrow \{j_1, j_2, j_3\}$
 $\downarrow \{i_1, i_2, i_3\}$
 $\delta(g, \hat{g}, g')$

$$K = \int dg dg' \delta(g, g, g, \hat{g}) \cdot \delta(g, g, g, g') = P$$

$$Z_{\Delta \sim \Gamma} = \prod_{\Gamma \in \text{SU}(2)} \int dg \prod_F \delta(\prod_F g)$$



• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S}$$

• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} =$$

• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta}$$

- Example
- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)}$$

- Example
- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \boxed{\text{SPIN FOUR}}$$

- Example
- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \quad \text{[SPIN FOUR]} \quad \Delta$$

$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}}$$

- Example
- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \boxed{\text{SPIN FOUR}}$$

$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_{\vec{r}} A_{\vec{r}}(\vec{r}) \prod_{\vec{r}} A_{\vec{r}}(\vec{r}) \prod_{\vec{r}} A_{\vec{r}}(\vec{r})$$

- Example
- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \text{SPIN FOUR}$$

$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_{\vec{r}} A(\vec{r}, j) \prod_{\vec{r}} A(\vec{r}, j) \prod_{\vec{r}} A(\vec{r}, j)$$

$$\begin{aligned}
 \mathcal{Z} = & \frac{1}{2} \int \prod_{i,j} \varphi(x_i, y_j, z_k) \delta(x_i, y_j, z_k) \varphi(\tilde{x}_i, \tilde{y}_j, \tilde{z}_k) \\
 & + \lambda \left(\int \prod_{i,j} \varphi(x_i, y_j, z_k) \varphi(\tilde{x}_i, \tilde{y}_j, \tilde{z}_k) \right. \\
 & \quad \varphi \delta(x_i, y_j, z_k) \varphi(\tilde{x}_i, \tilde{y}_j, \tilde{z}_k) \\
 & \quad \delta(x_i, \tilde{x}_i) \delta(y_j, \tilde{y}_j) \\
 & \quad \varphi(\tilde{x}_i, \tilde{y}_j, \tilde{z}_k) \delta / \dots \\
 & \quad \left. \varphi(\tilde{x}) \right)
 \end{aligned}$$

- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_{\text{SPIN FOUR}} \Delta$$

$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_{i \in j} \pi A_i(\varphi_i)$$

$$\begin{aligned}
 \text{Diagram} &= \frac{1}{2} \int \int_{\mathbb{R}^3} \varphi(x, y, z) \delta(x, y, z) \varphi(\tilde{x}, \tilde{y}, \tilde{z}) \\
 &+ \lambda \left(\int \int \varphi(x, y, z) \varphi(\tilde{x}, \tilde{y}, \tilde{z}) \right. \\
 &\quad \varphi \delta(\tilde{x}, \tilde{y}, \tilde{z}) \varphi(\tilde{x}, \tilde{y}, \tilde{z}) \\
 &\quad \delta(x, y, z) \delta(\tilde{x}, \tilde{y}, \tilde{z}) \\
 &\quad \varphi(\tilde{x}, \tilde{y}, \tilde{z}) \delta / \dots
 \end{aligned}$$

The diagram on the left consists of several interconnected loops and crossings, representing a complex topological structure. Below it, there are smaller diagrams including a cube-like structure labeled $\varphi(x)$ and a diagram with multiple lines crossing at a central point.

$$\mathbb{R} \left(\frac{1}{4!} \right) \left(\frac{1}{4!} \right)$$

$$V = \int dg \, \delta(g, g, \hat{g}, g')$$

$$\downarrow \left[\begin{matrix} j_1, j_2, j_3 \\ j_4, j_5, j_6 \end{matrix} \right]$$

$$\delta(g, g, g, g')$$



$$K = \int dg \, \delta(g, g, g, g')$$

$$\delta(g, g, g, g')$$

$$Z_{\Delta-1} = \prod_{g \in \text{SU}(2)} \int dg \prod_F \delta(\prod g)$$

$$\begin{aligned}
 \tau = & \frac{1}{2} \left(\prod_{i,j,k} \varphi(v_i, v_j, v_k) \delta(v_i, v_j) \varphi(\tilde{v}_i, \tilde{v}_j, \tilde{v}_k) \right. \\
 & + \lambda \left(\prod_{i,j,k} \varphi(v_i, v_j, v_k) \varphi(\tilde{v}_i, \tilde{v}_j, \tilde{v}_k) \right. \\
 & \quad \varphi \delta(v_i, \tilde{v}_j) \varphi(\tilde{v}_i, \tilde{v}_j, v_k) \\
 & \quad \delta(v_i, \tilde{v}_j) \delta(v_j, \tilde{v}_i) \\
 & \quad \varphi(\tilde{v}_i, \tilde{v}_j, \tilde{v}_k) \delta / \dots
 \end{aligned}$$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \boxed{\text{SPIN FOUR}}$$

$$\sum_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_r A_r \prod_{e^r} A_{e^r} \prod_{v^r} A_{v^r} \prod_{v^r} A_{v^r}$$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \boxed{\text{SPIN FOUR}}$$



$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_r A_r(\varphi_r) \prod_{e^+} A_{e^+}(\varphi_{e^+}) \prod_{v^-} A_{v^-}(\varphi_{v^-}) \prod_{v^+} A_{v^+}(\varphi_{v^+})$$

- 30 Example
- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \text{[SPIN FOUR]}$$



$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}} \prod_r A_r(\epsilon_r) \prod_e A_e(\epsilon_e) \prod_v A_v(133)$$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_{\text{SPIN FOUR}} \Delta$$



$$Z_{\Delta}^{\text{SF}} = \sum_{\{j\}}$$

$$\prod_{\vec{r}} A(\vec{r}) \prod_{\vec{r}} \{i, \dots, \vec{r}\}$$

- Example
- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \boxed{\text{SPIN FOUR}}$$



$$\cos(S_R(i)) \sim \left\{ \begin{matrix} i \\ \vdots \\ \psi \end{matrix} \right\}$$

• L. FREIDEL, hep-th/0505046 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \int \mathcal{D}g e^{iS(g)}$$



- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \int \mathcal{D}g e^{-S(g)}$$



• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_j A(j)$$



• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \left(\sum_j A(j) \right)$$



• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_j A(j)$$



- L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_j A(j)$$



$$\begin{aligned}
 \mathcal{Z} = & \frac{1}{2} \int \prod_{\vec{x}} \varphi(\vec{x}, \vec{x}, \vec{x}) \delta(\vec{x}, \vec{x}, \vec{x}) \varphi(\vec{x}, \vec{x}, \vec{x}) \\
 & + \lambda \left(\int \prod_{\vec{x}} \varphi(\vec{x}, \vec{x}, \vec{x}) \varphi(\vec{x}, \vec{x}, \vec{x}) \right. \\
 & \quad \varphi \delta(\vec{x}_2, \vec{x}_3) \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \\
 & \quad \delta(\vec{x}_1, \vec{x}_2) \delta(\vec{x}_1, \vec{x}_3) \\
 & \quad \varphi(\vec{x}_1, \vec{x}_2, \vec{x}_3) \delta(\vec{x}_1, \vec{x}_2) \delta(\vec{x}_1, \vec{x}_3) \dots \\
 & \quad \left. \varphi(\vec{x}) \right)
 \end{aligned}$$

- Example
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$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \sum_j A(j)$$



$$S = \frac{1}{2} \sum_{S(1,2)} \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\tilde{g}_1, \tilde{g}_2) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \\ + \lambda \sum_{S(1,2,3)} \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \\ \varphi \delta(\tilde{g}_1, \tilde{g}_2) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \\ \delta(\tilde{g}_1, \tilde{g}_2) \delta(\tilde{g}_1, \tilde{g}_2) \\ \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(\dots)$$

• L. FREIDEL, hep-th/0505016 out look

$$Z = \int \mathcal{D}\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \sum_j A(j)$$

