Title: Introduction to quantum gravity - Part 21

Date: Mar 31, 2006 01:30 PM

URL: http://pirsa.org/06030029

Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

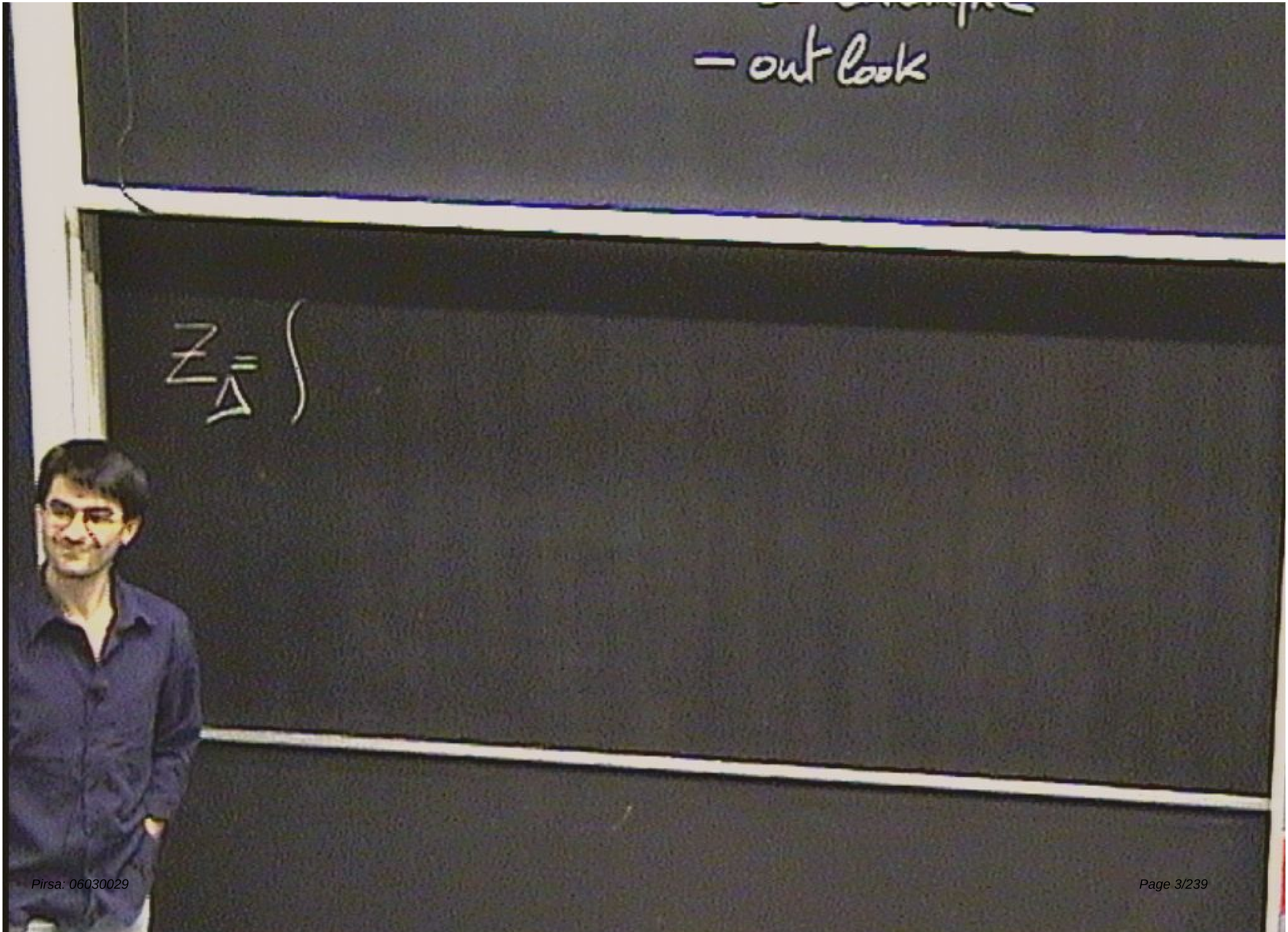
Basic texts:

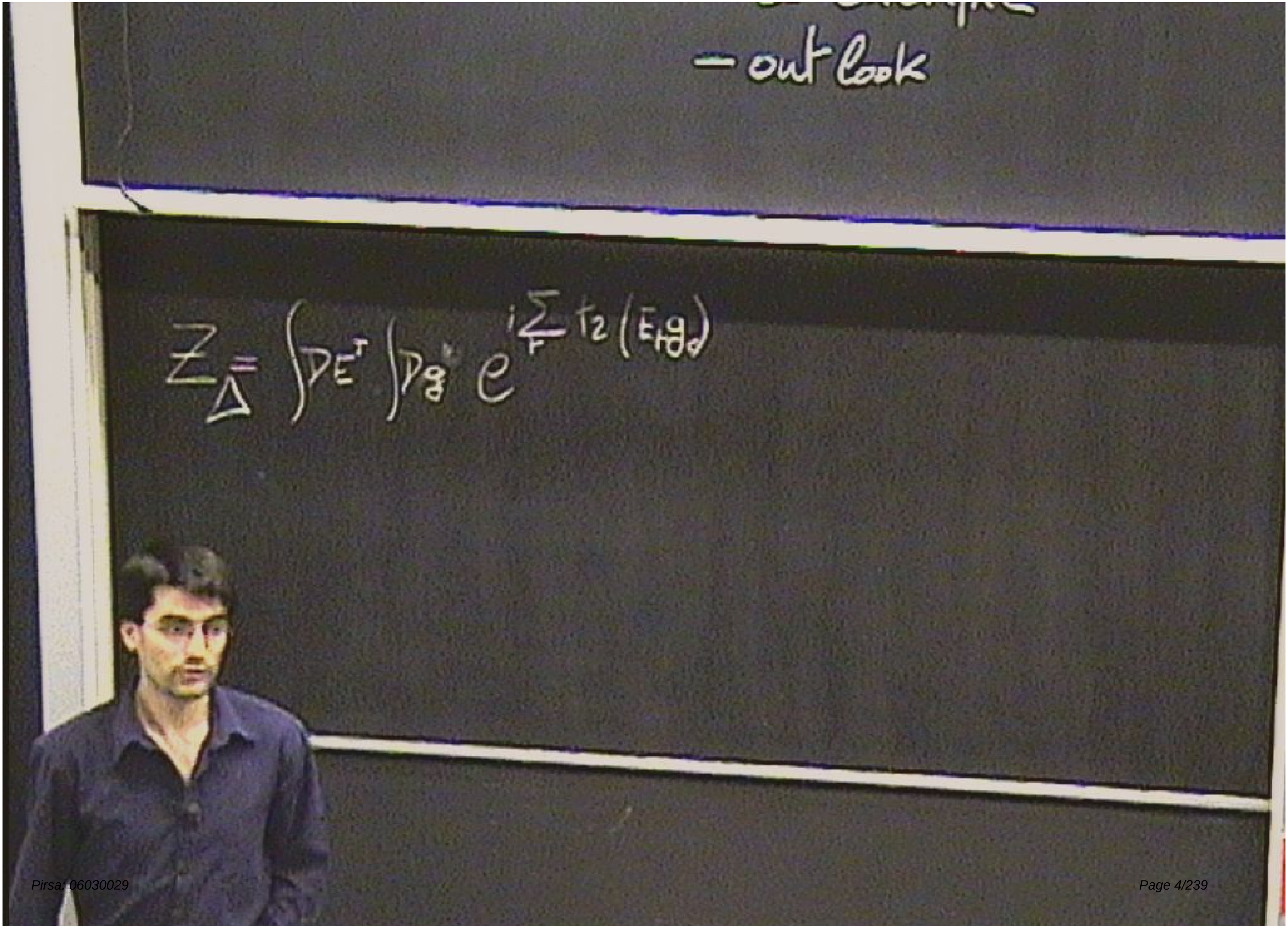
-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravityy with a positive cosmological constant, Lee Smolin, hep-th/0209079
-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

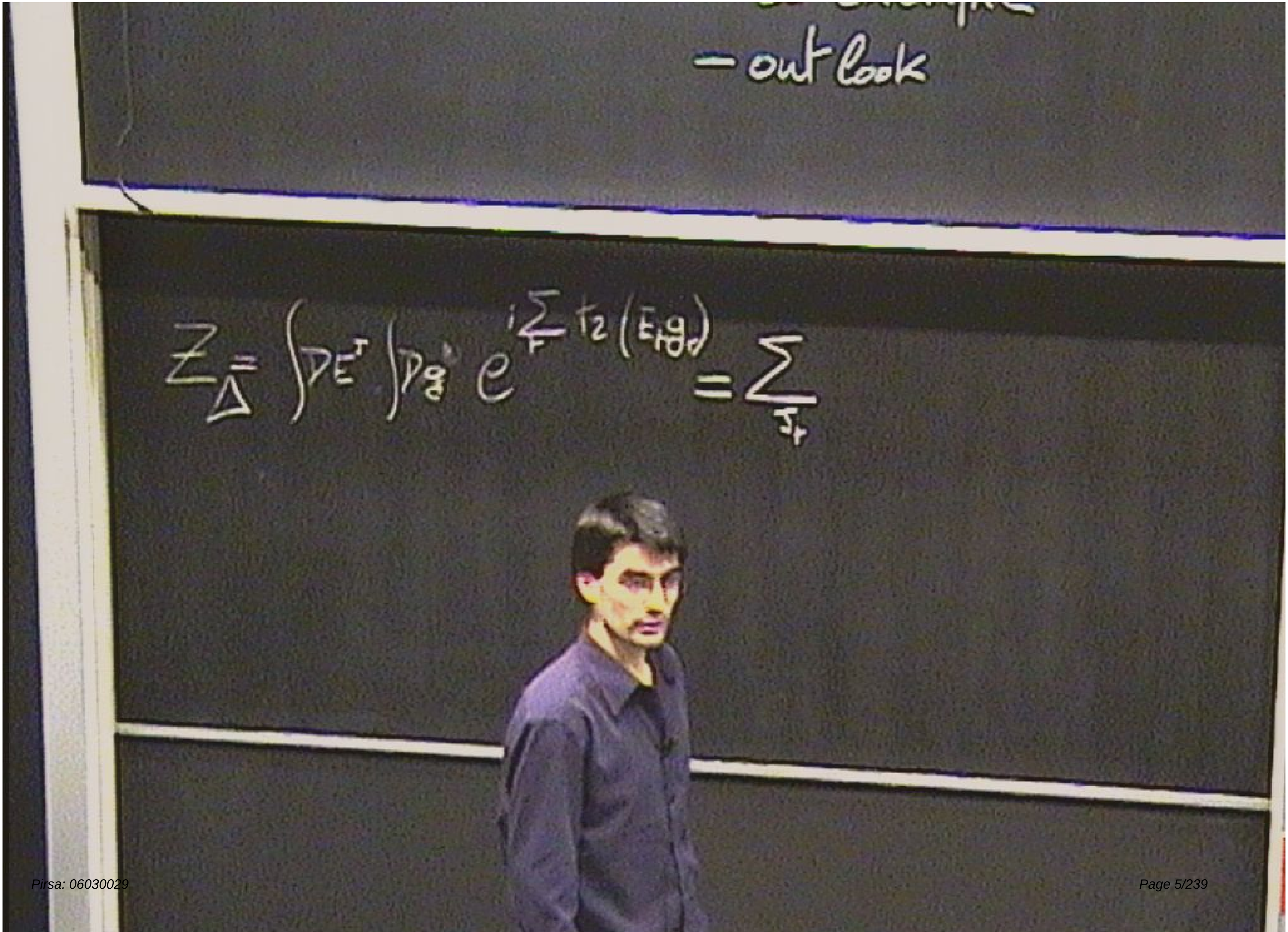
-undergraduate quantum mechanics
-basics of classical gauge field theories
-basic general relativity
-hamiltonian and lagrangian mechanics
-basics of lie algebras

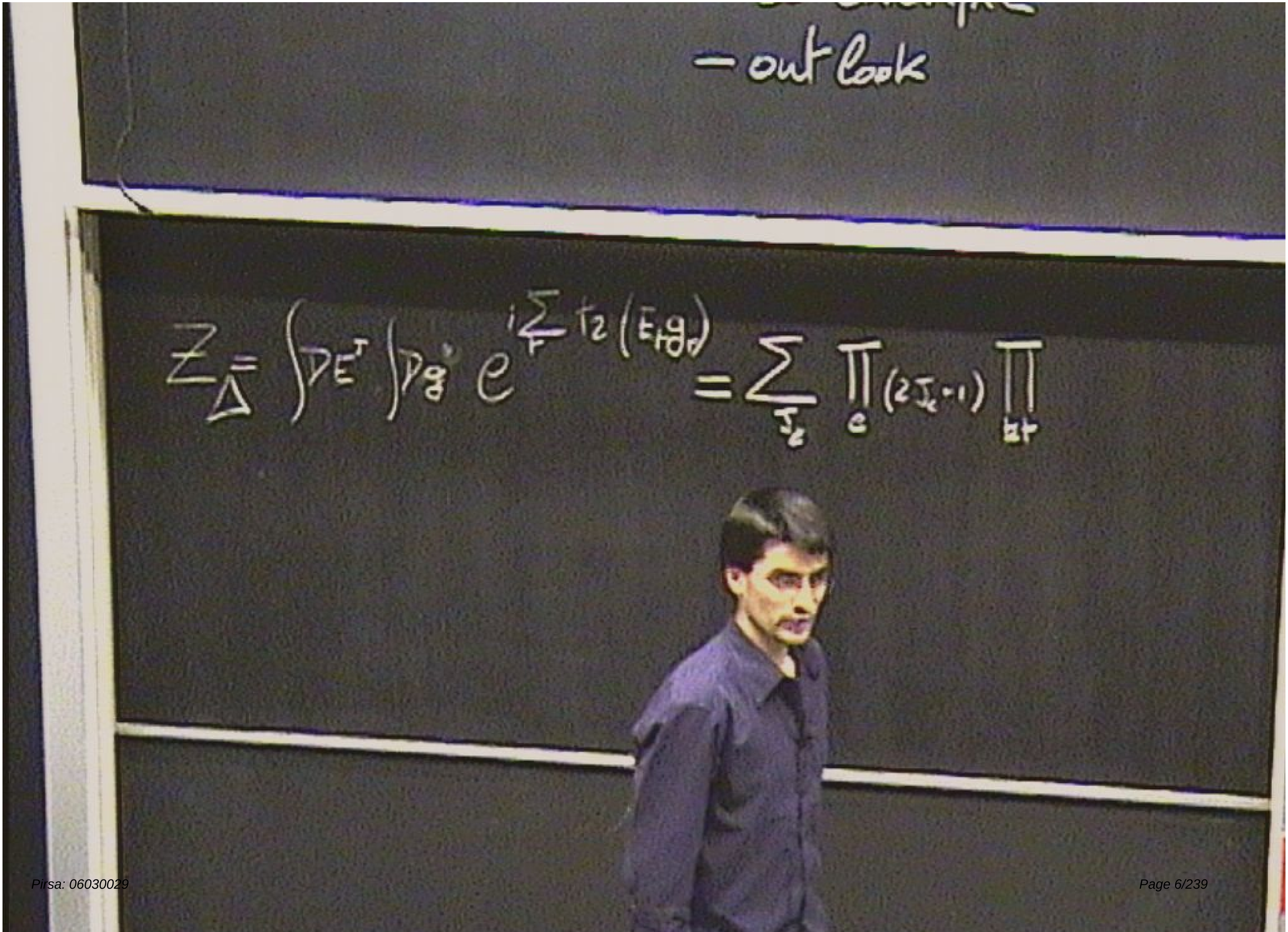
- 3d Ponzano-Regge model (geometry)
- idea of Feynman diagrams
- Group field theory
  - idea
  - general formalism
  - 3d example
  - outlook

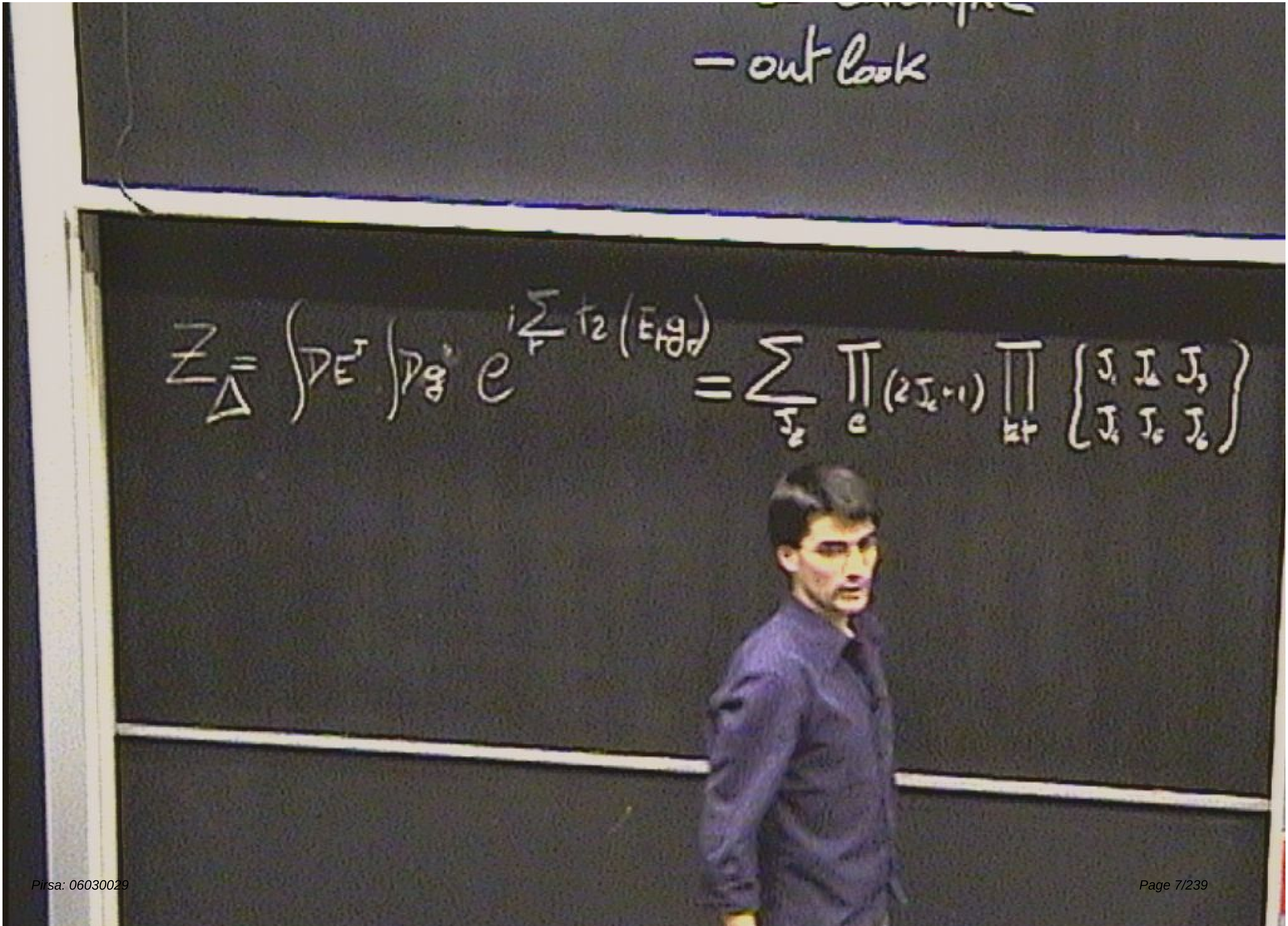

– out look
Z_Δ = ∫



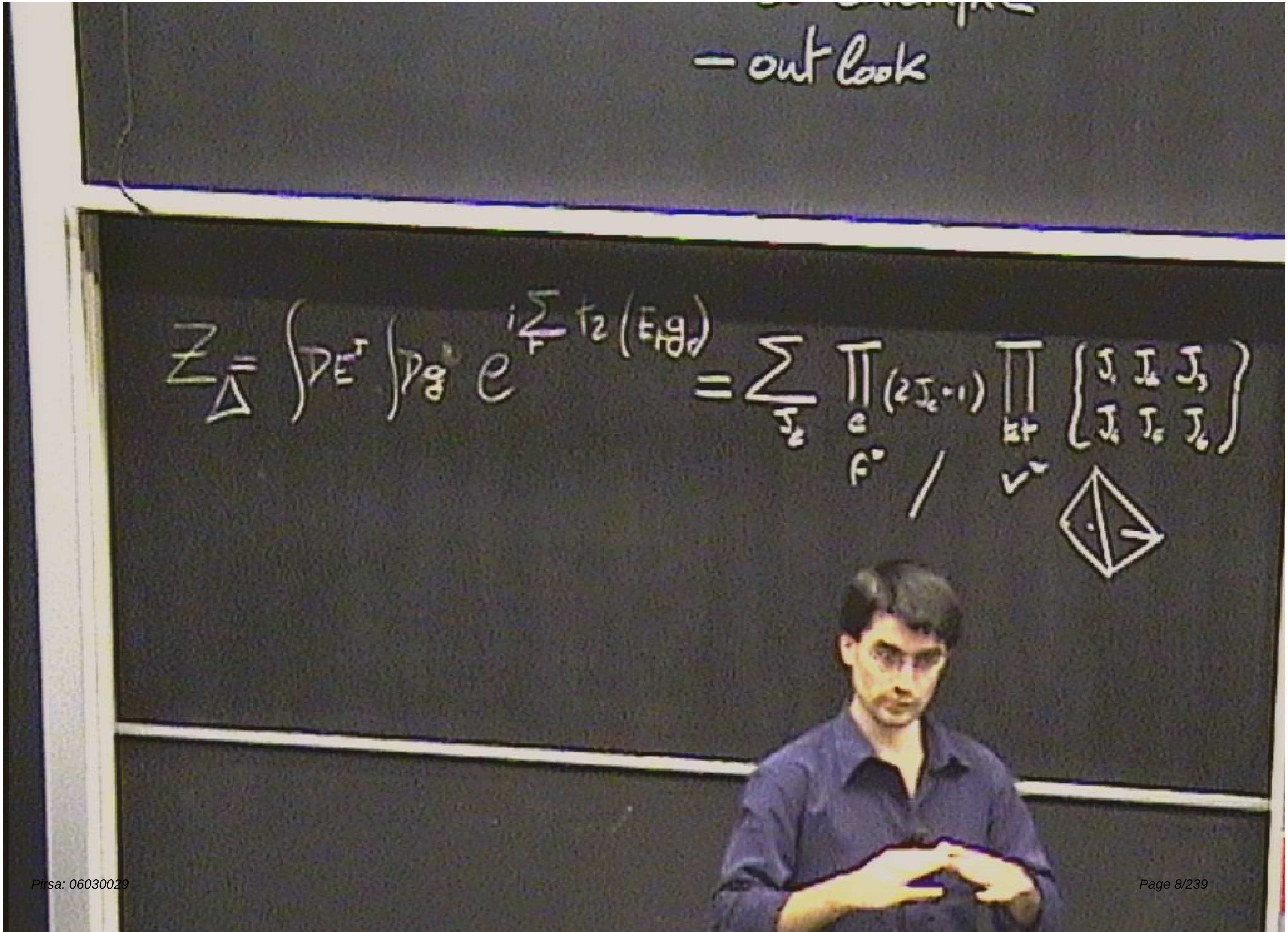
– out look
$Z_{\Delta} = \int DE^{T} \int Dg \; e^{i \sum_{F} tr(E_F g_F)}$

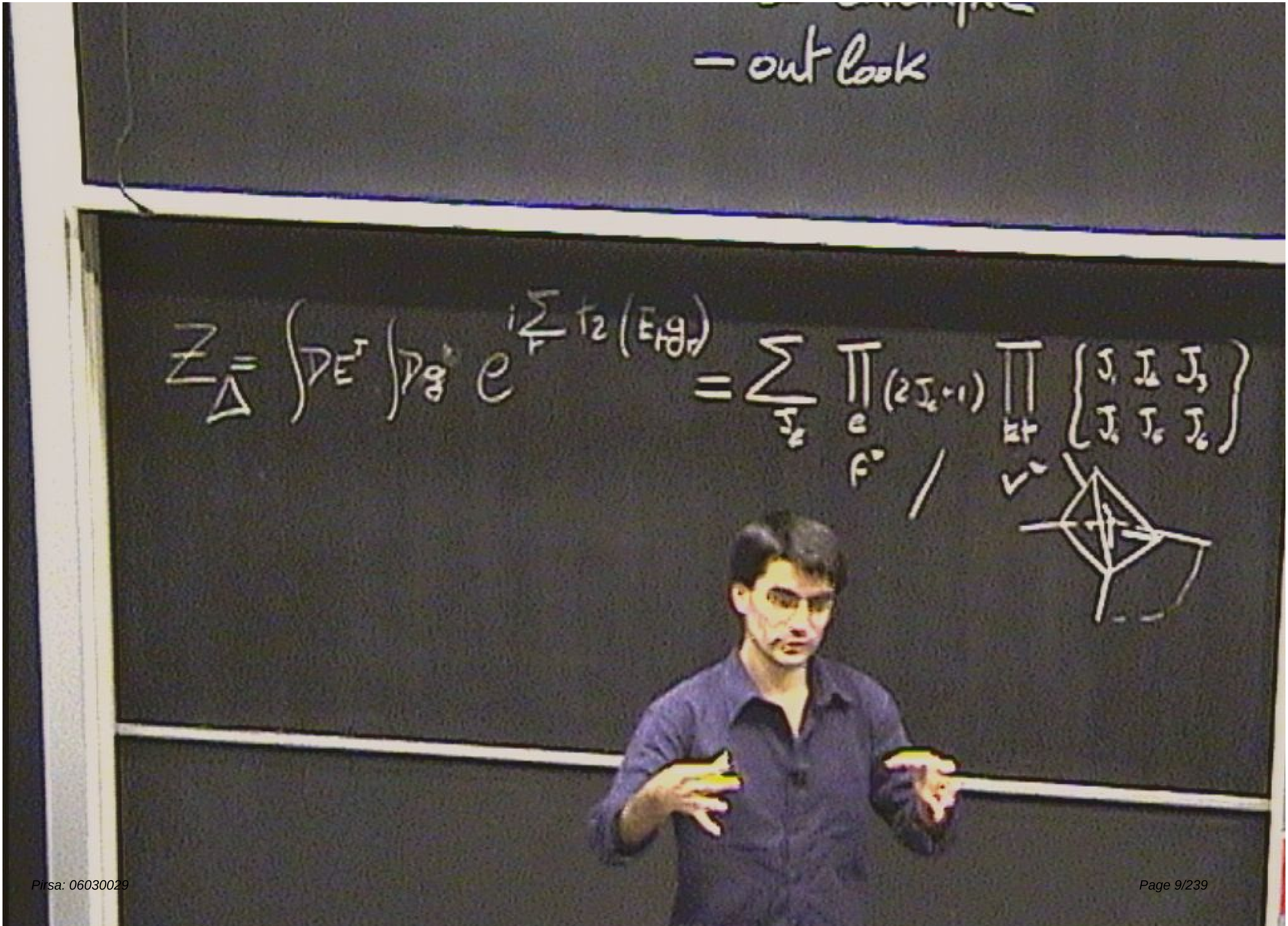

– out look
$Z_{\Delta} = \int DE^{\tau} \int Dg \, e^{i \sum_{F} \mathrm{tr}(E_F g_F)} = \sum_{j_F}$

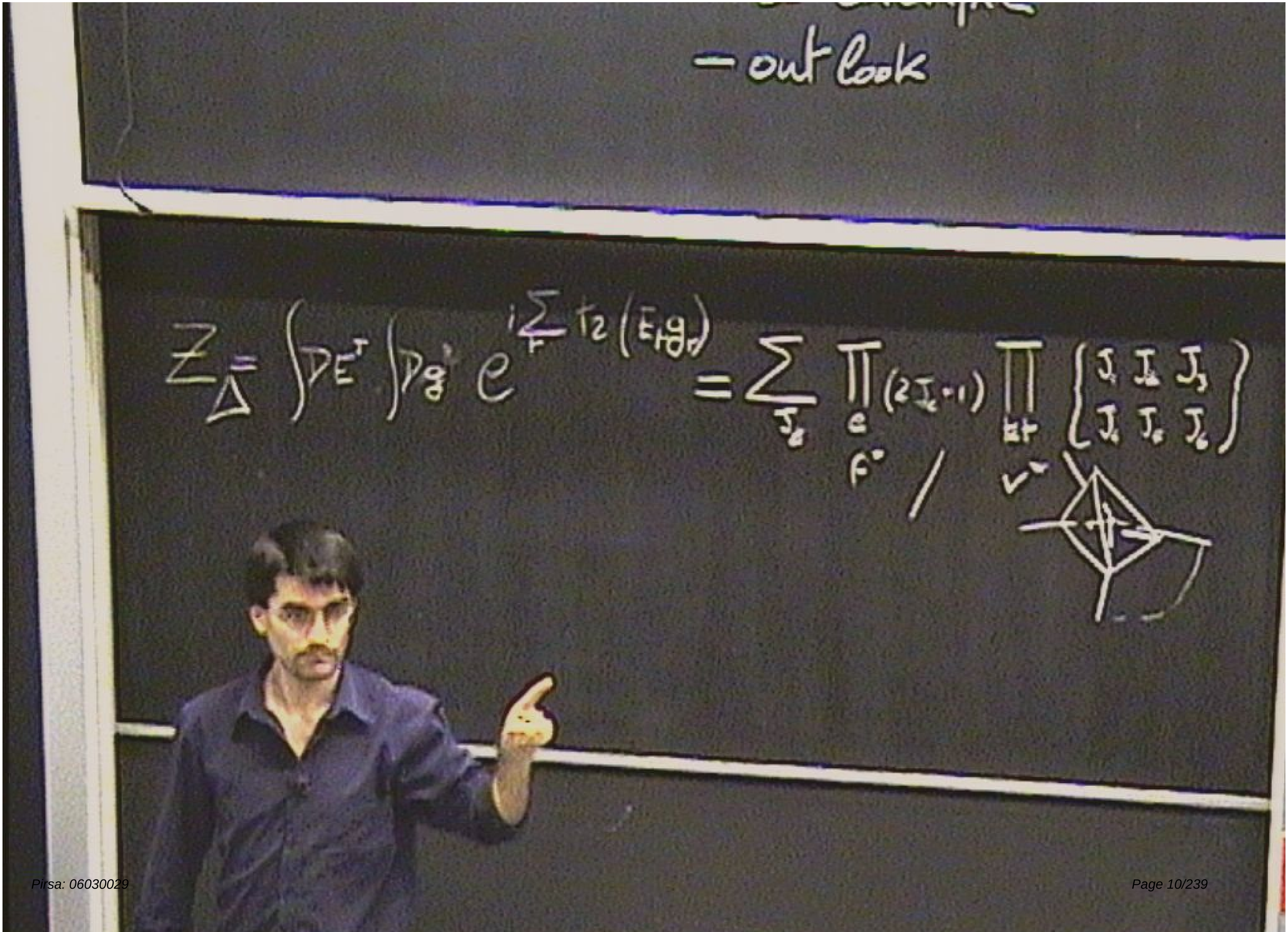

– out look
$Z_{\bar{\Delta}} = \int \mathcal{D}E^{T} \int \mathcal{D}g^{*}\, e^{i\sum_{f} \mathrm{tr}(E_f g_f)} = \sum_{J_e} \prod_{e} (2J_e+1) \prod_{tet}$

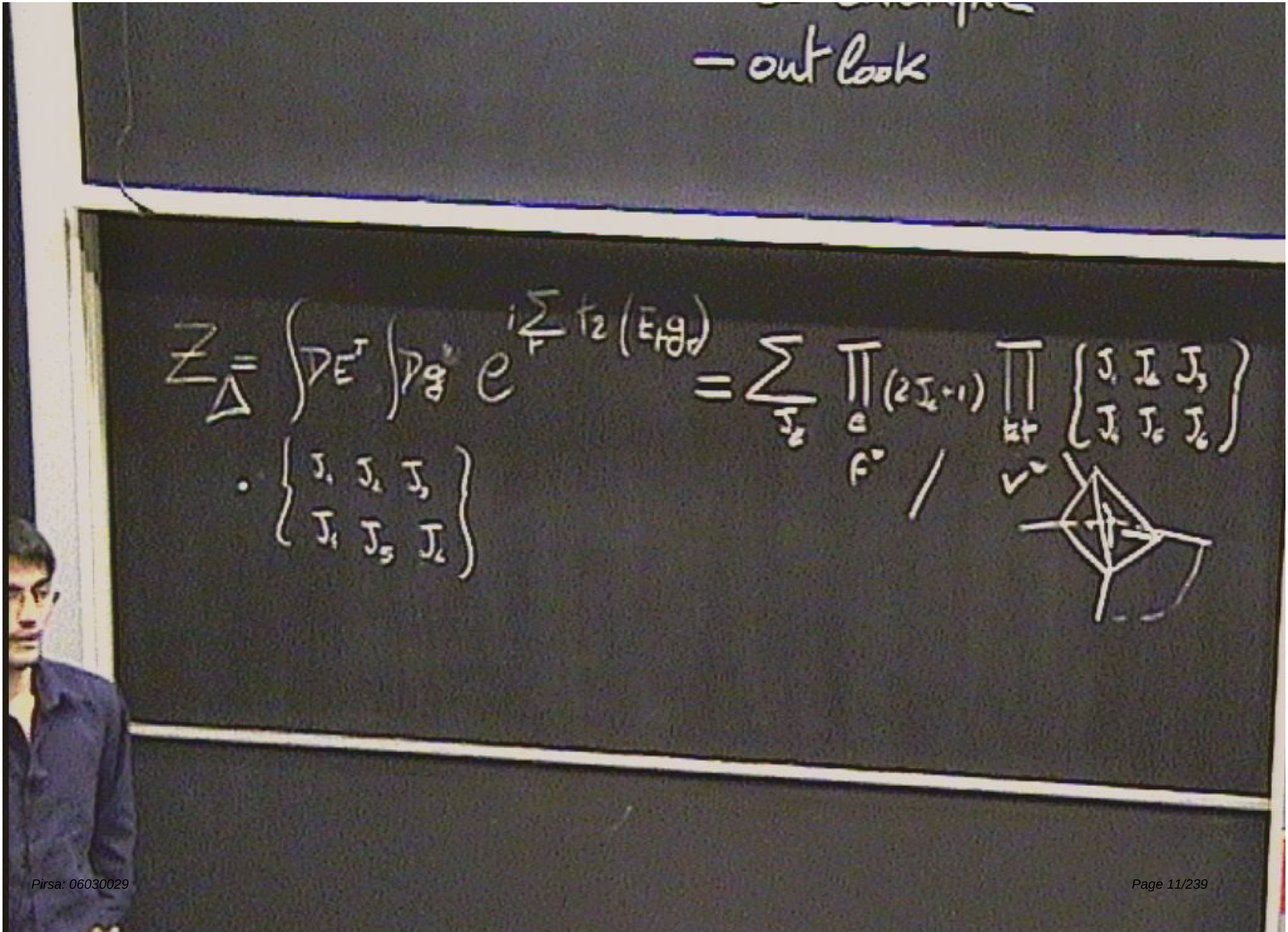


- out look
$Z_{\Delta} = \int DE^{\tau} \int Dg^{\tau} e^{i \sum_f tr(E_f g_f)} = \sum_{J_e} \prod_e (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$


– out look
Z_Δ = ∫DE^f ∫Dg^e e^{i Σ_f tr(E_f g_f)} = Σ_{J_e} Π_e (2J_e+1) Π_tet {J1 J2 J3 / J4 J5 J6}


– out look
$Z_\Delta = \int DE^f \int Dg^f \, e^{i\sum_f \mathrm{tr}(E_f g_f)} = \sum_{j_e} \prod_e (2j_e+1) \prod_{tet} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$


– out look
$Z_{\Delta} = \int DE^f \int Dg_f \, e^{i \sum_f tr_2 (E_f g_f)} = \sum_{J_e} \prod_e (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$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$Z_{\Delta} = \int DE^{f} \int Dg^{e}\, e^{i \sum_f tr(E_f g_f)} = \sum_{J_e} \prod_e (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$
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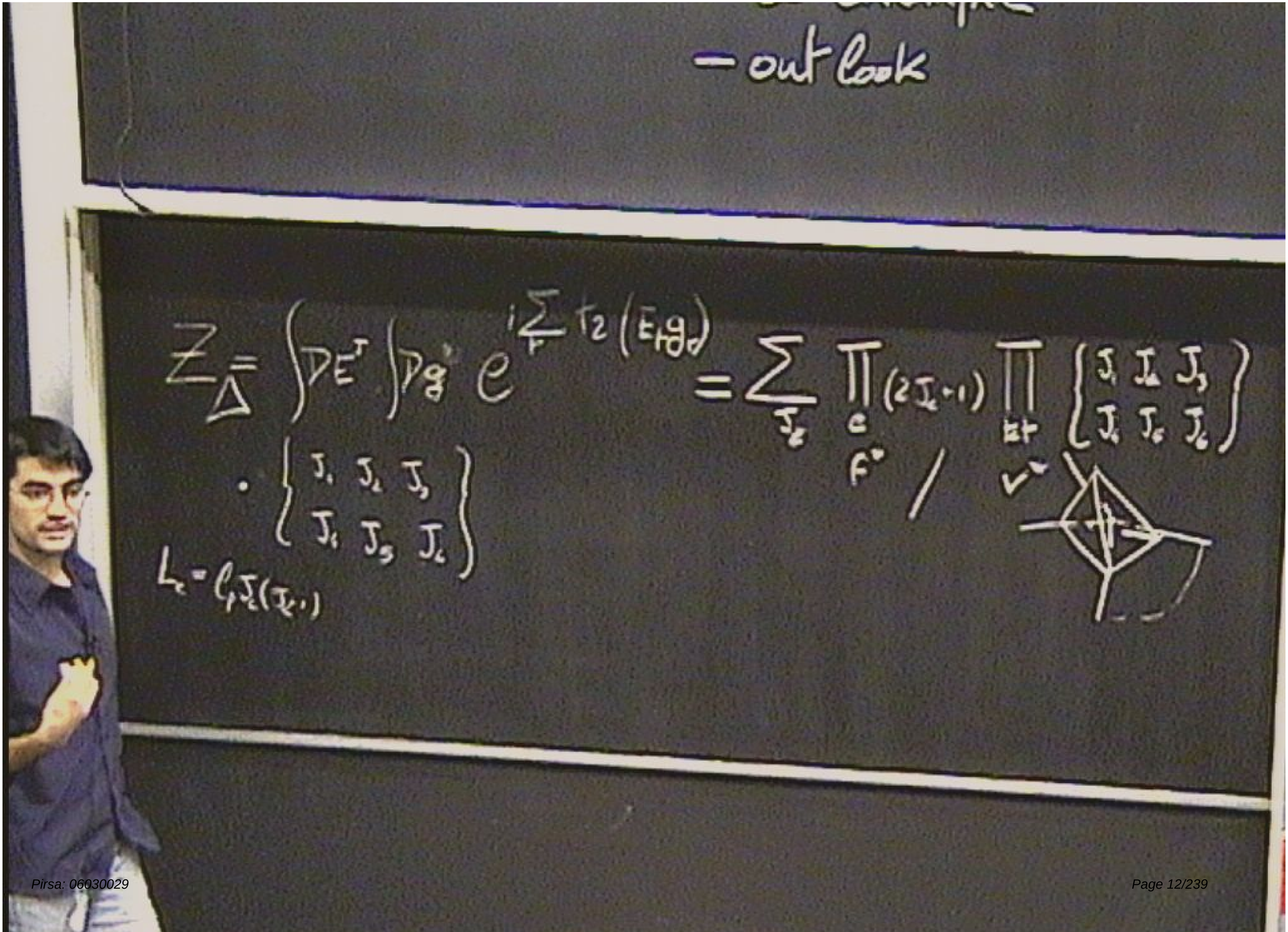
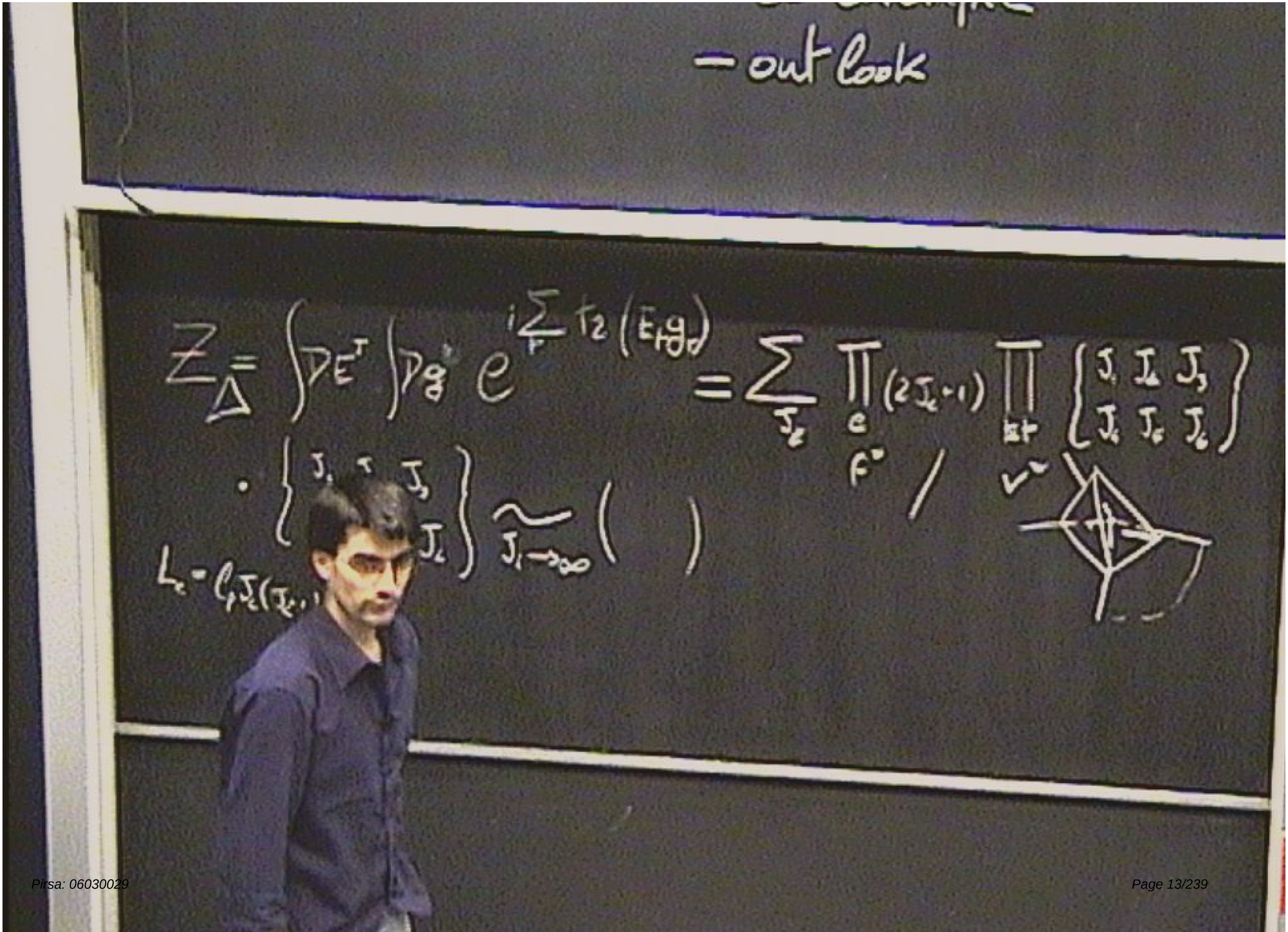

– out look
Z_Δ = ∫DE^f ∫Dg^e e^{i Σ_f tr(E_f g_f)} = Σ_{J_e} Π_e (2J_e+1) Π_{tet} {J_1 J_2 J_3 / J_4 J_5 J_6}
· {J_1 J_2 J_3 / J_4 J_5 J_6}
L_e = l_p J_e(J_e+1)
f*
v*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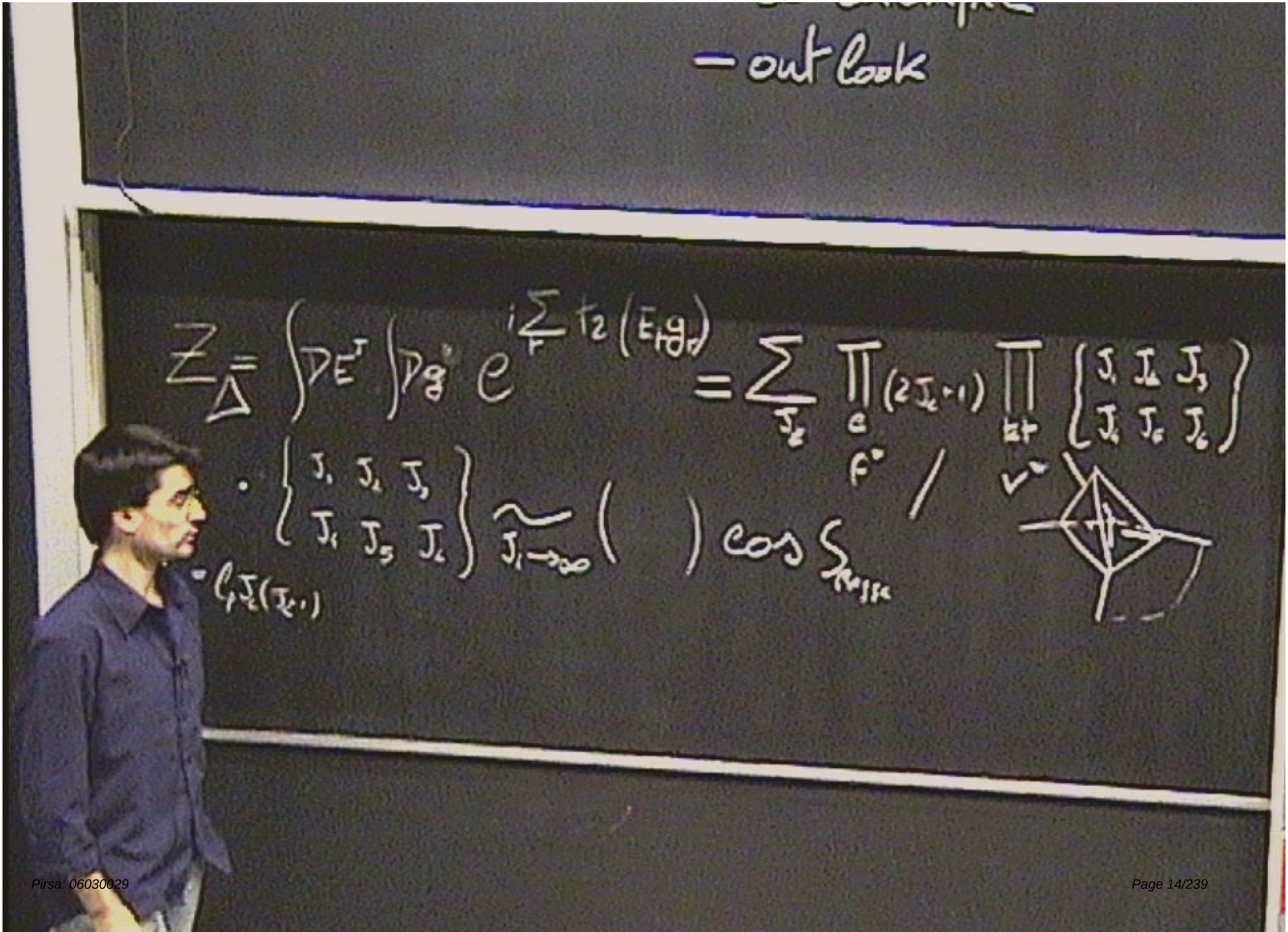
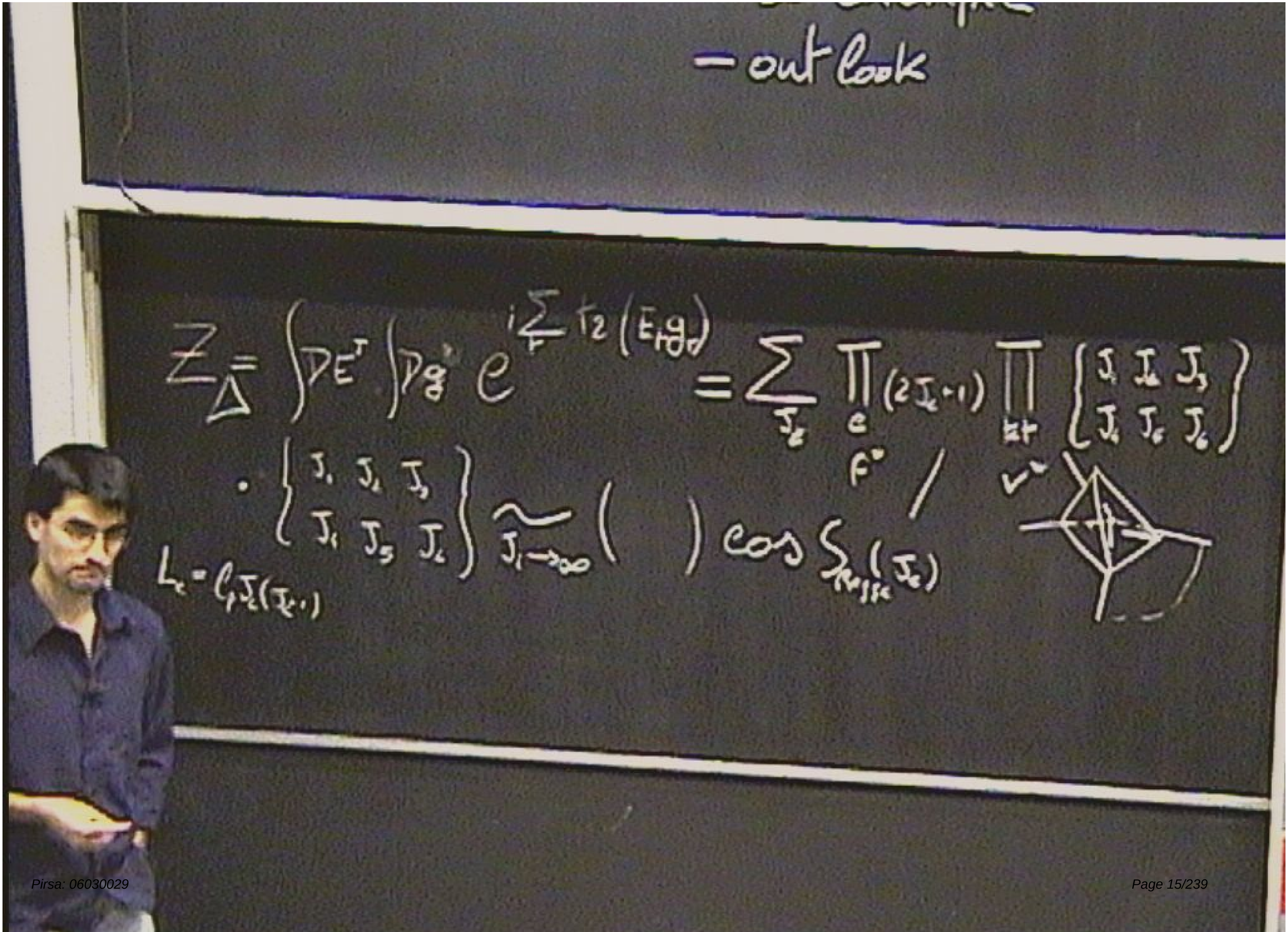

– out look
$Z_{\Delta} = \int DE^{\tau} \int Dg \, e^{i \sum_f \mathrm{tr}(E_f g_f)} = \sum_{J_e} \prod_e (2J_e+1) \prod \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$
$\begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \sim_{J_i \to \infty} (\quad) \cos S$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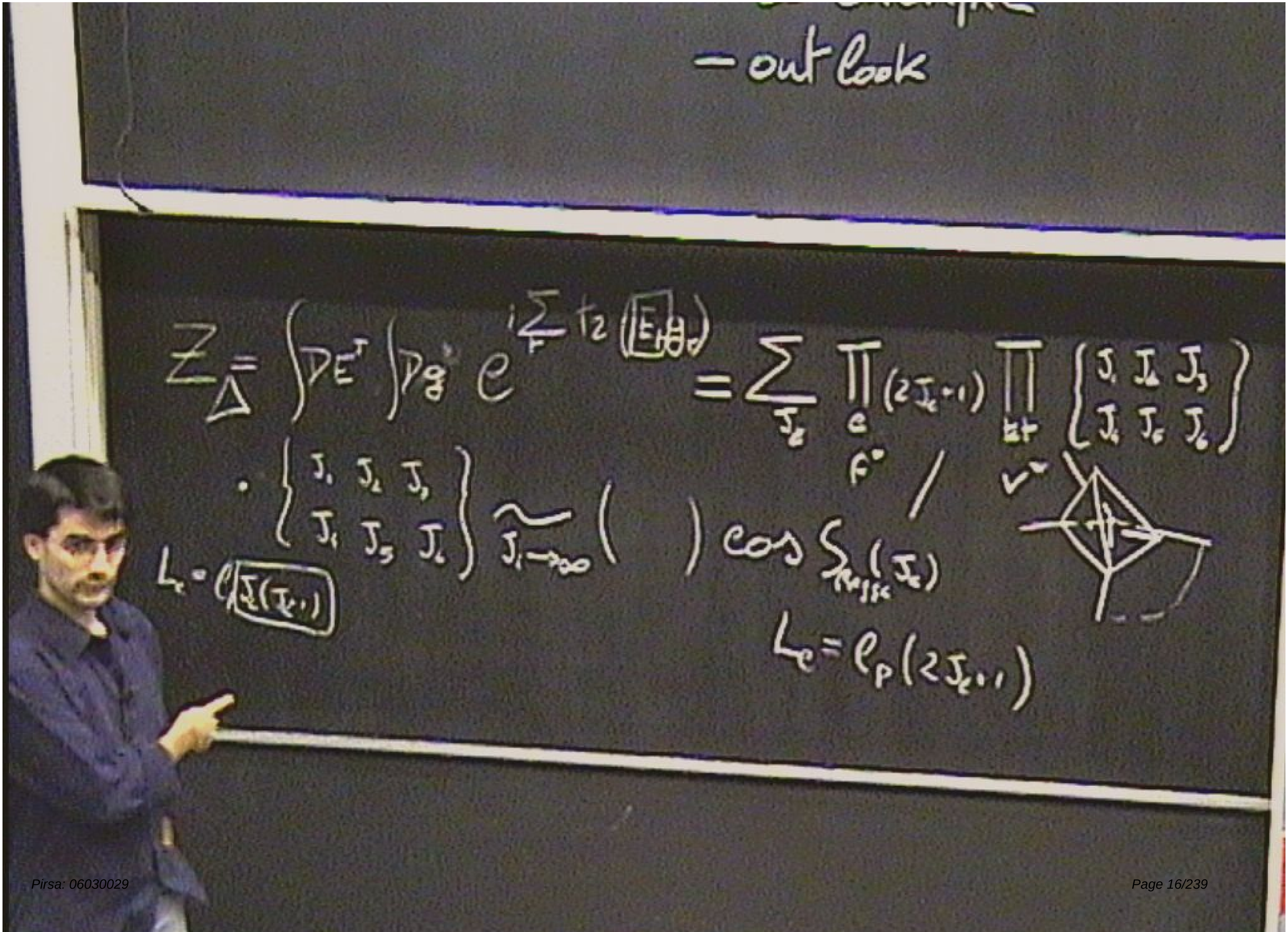
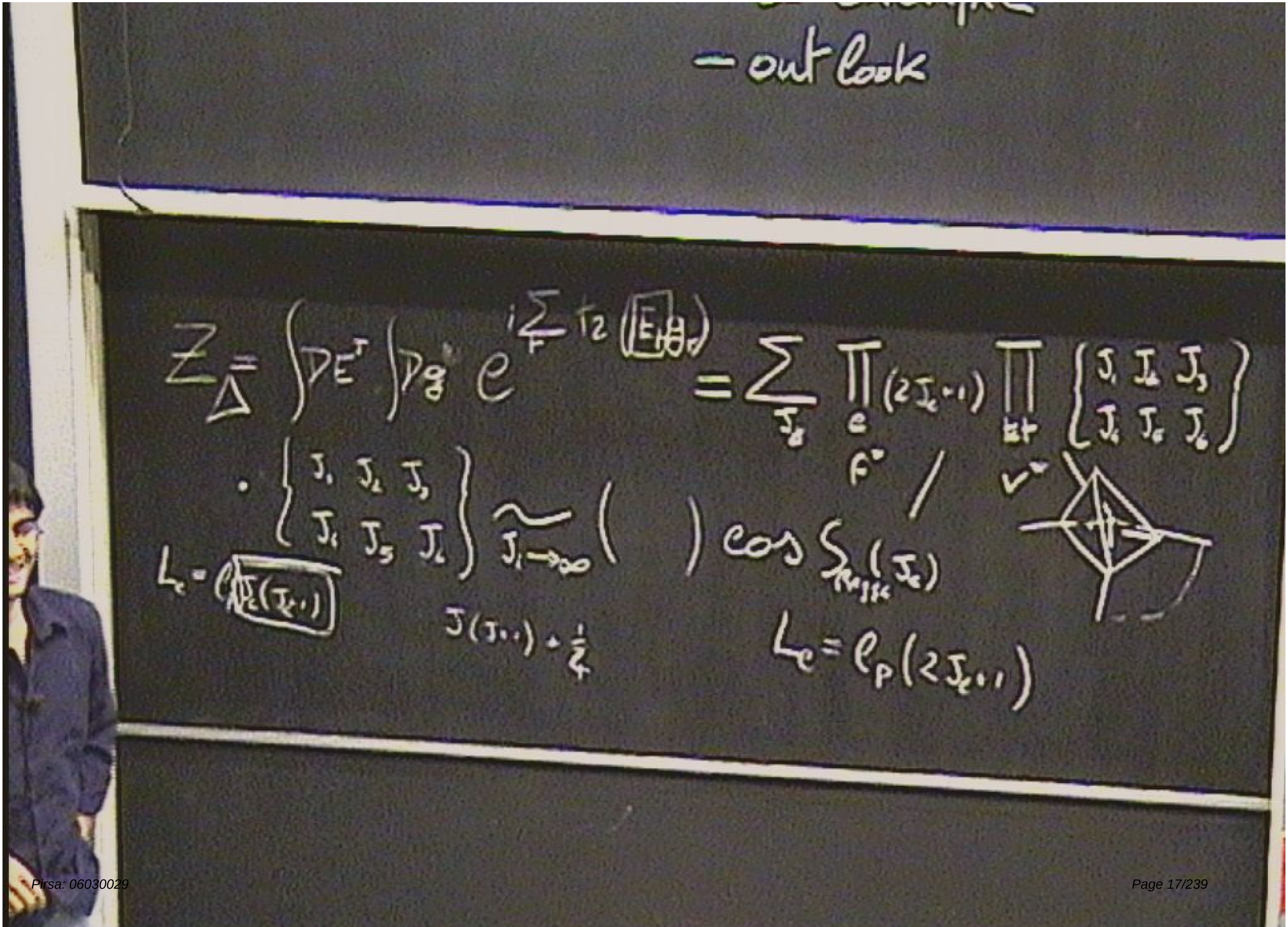
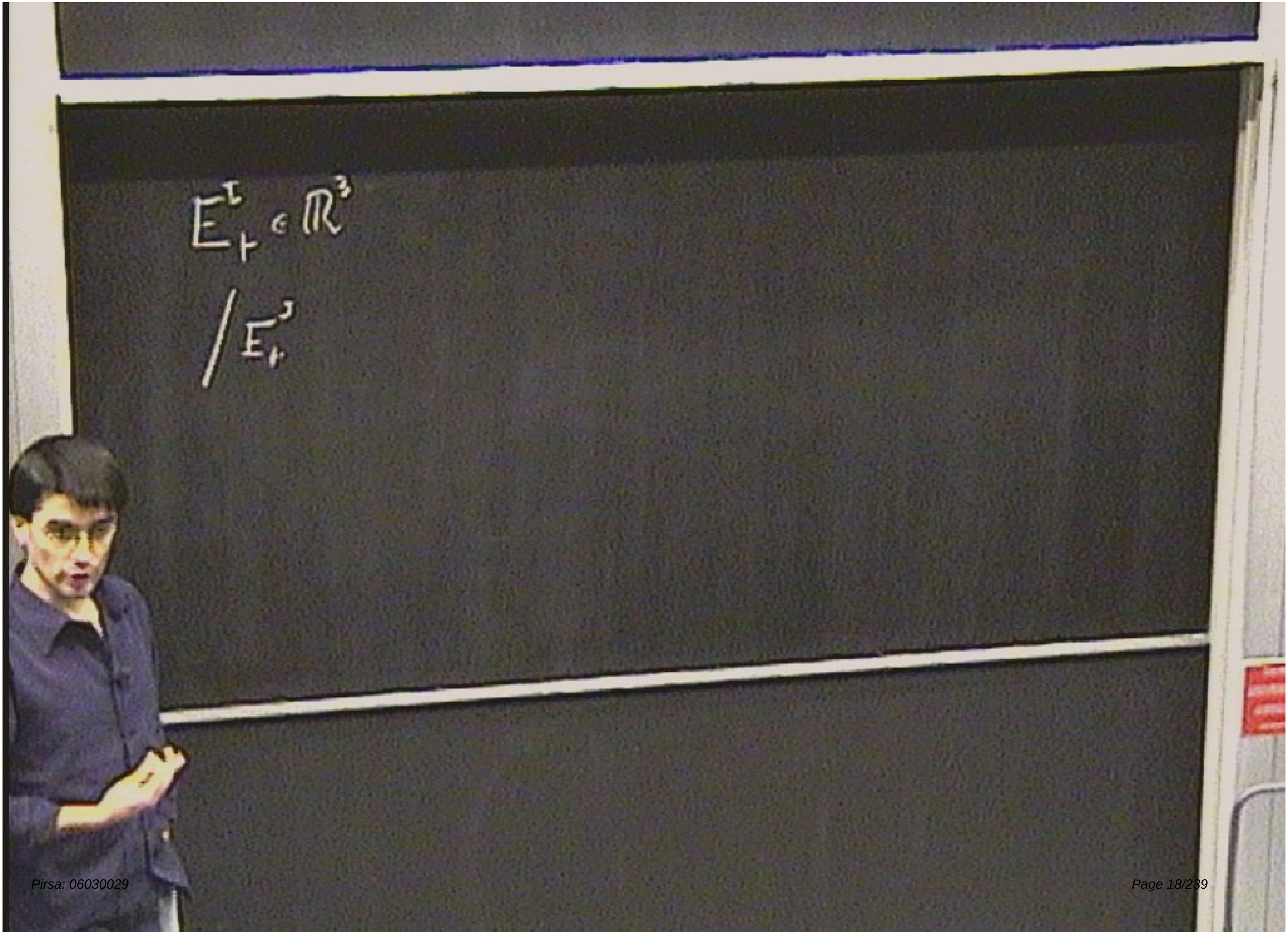
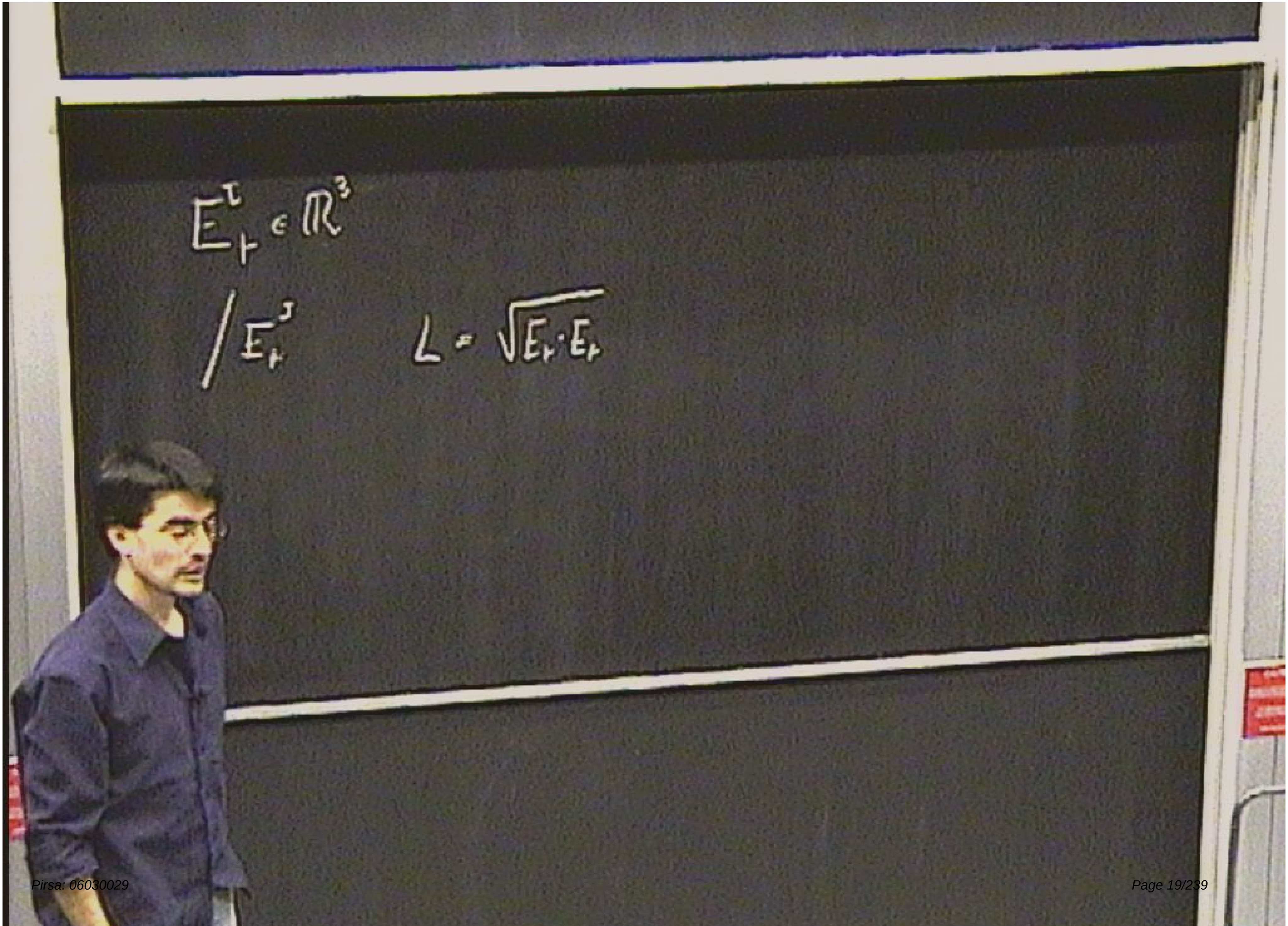
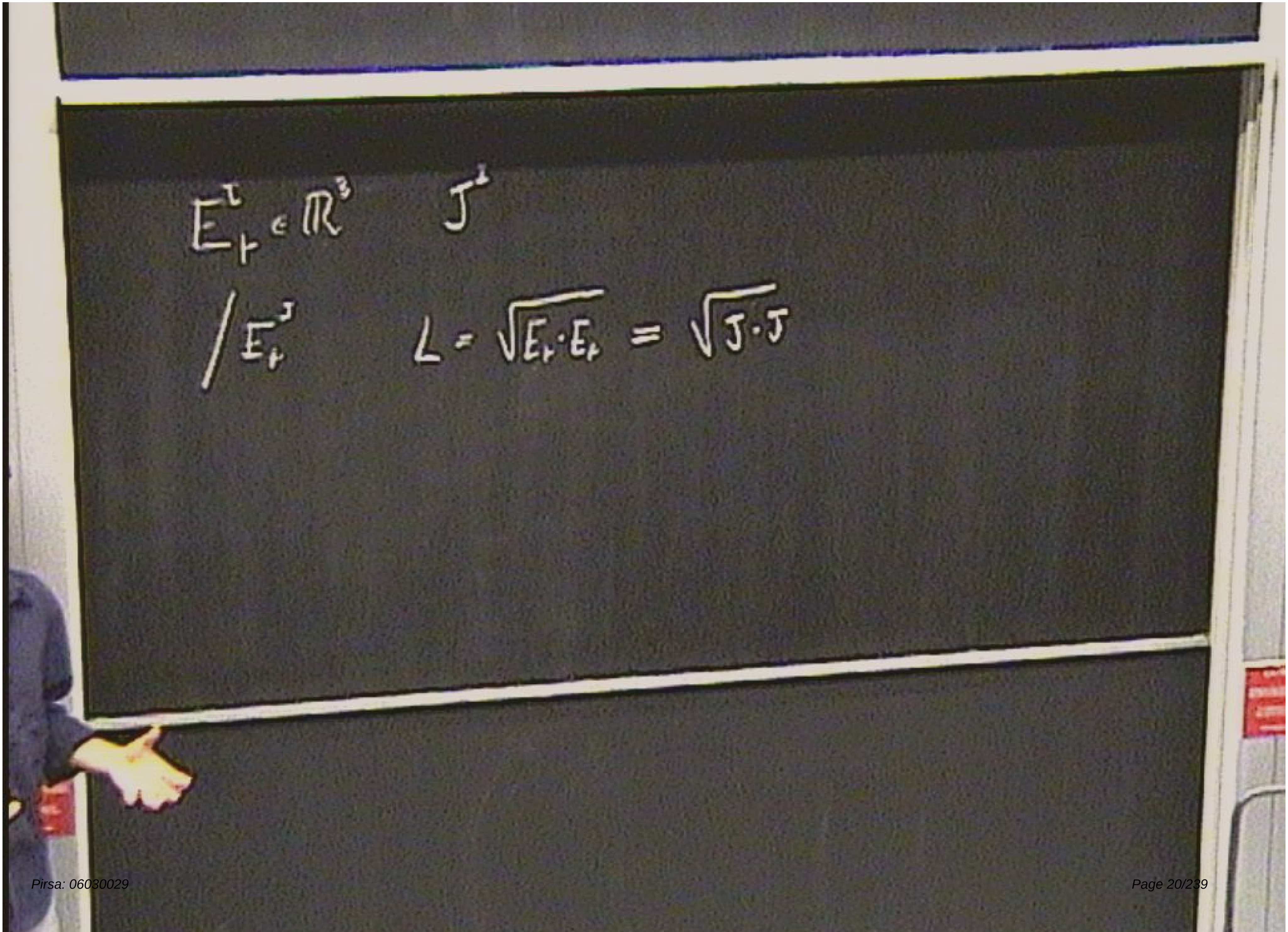
$Z_{\Delta} = \int DE^{f} \int Dg^{e}\, e^{i\sum_f \mathrm{tr}(E_f g_f)} = \sum_{J_e} \prod_e (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$
$\begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \underset{J_i \to \infty}{\sim} (\quad) \cos S_{Regge}(J_e)$
$L_e = l_p J_e (J_e+1)$

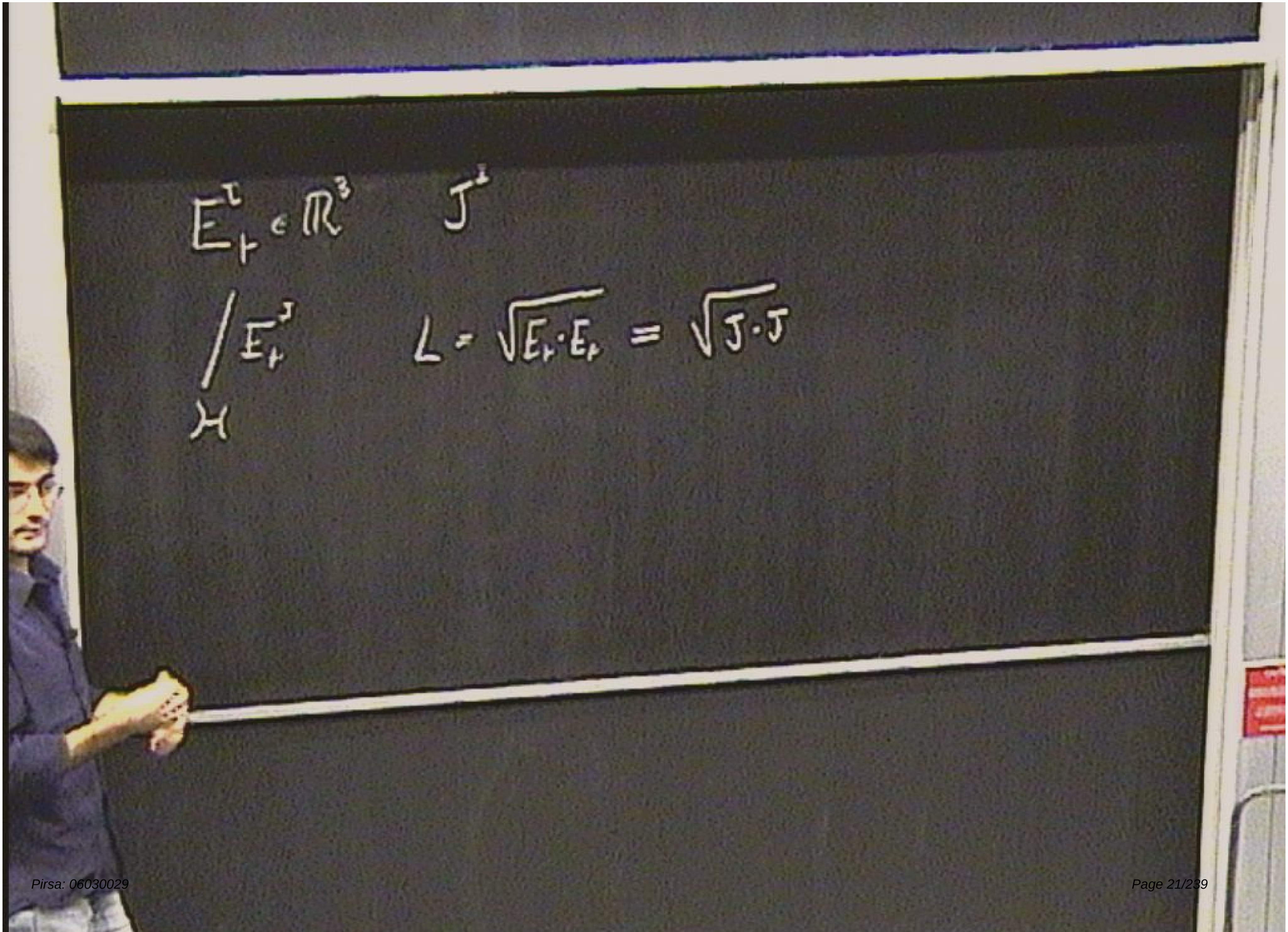

— out look
Z_Δ = ∫DE ∫Dg e^{i Σ_f tr(E ∧ F)} = Σ_{J_e} Π_e (2J_e+1) Π_tet {J_1 J_2 J_3 / J_4 J_5 J_6}
· {J_1 J_2 J_3 / J_4 J_5 J_6} ~ ( ) cos S_Regge(J_e)
J_i → ∞
L_e = l_P √(J_e(J_e+1))
L_e = l_P (2J_e+1)

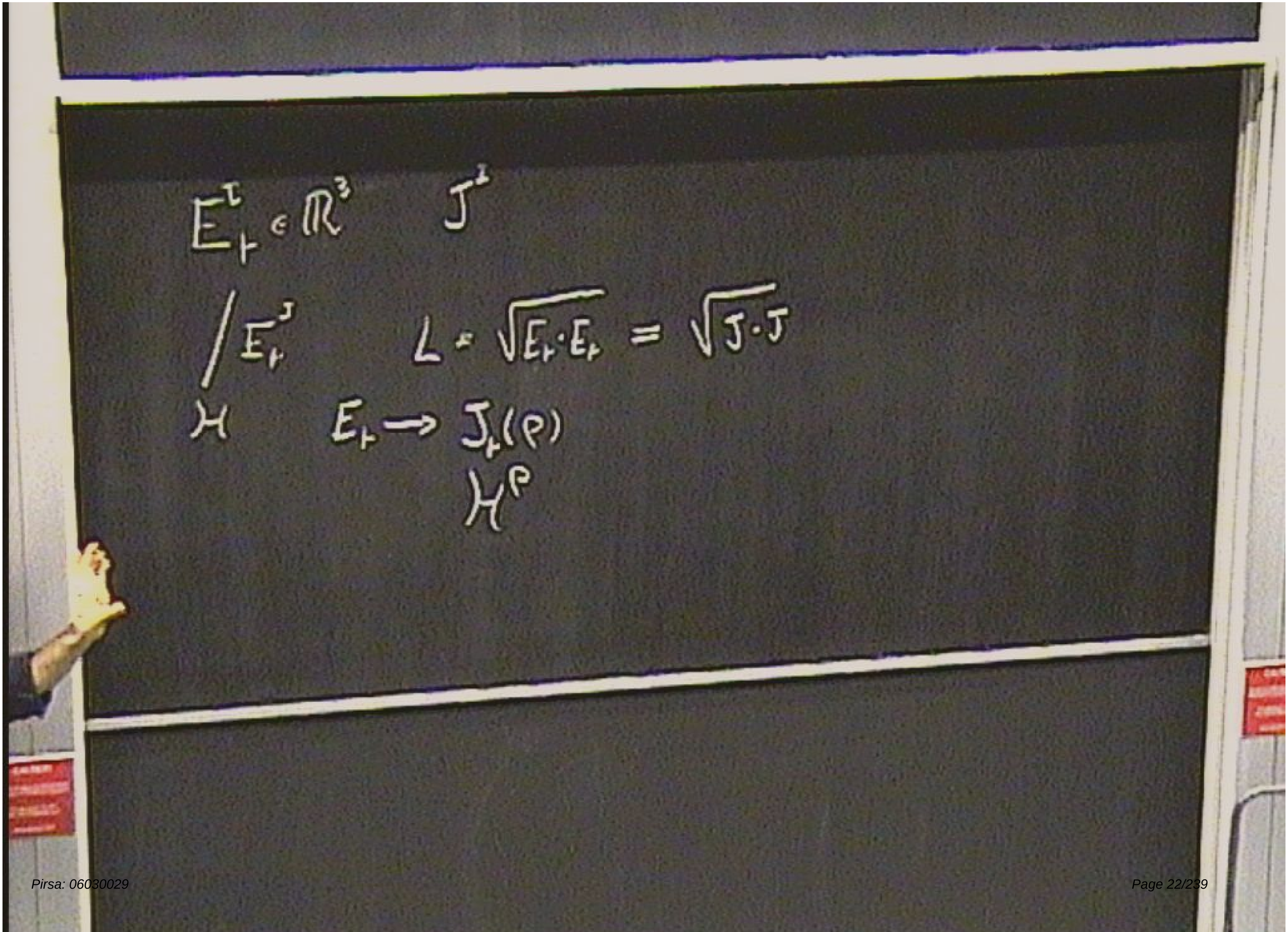

— out look
$L_e = \ell_P (2 J_e + 1)$

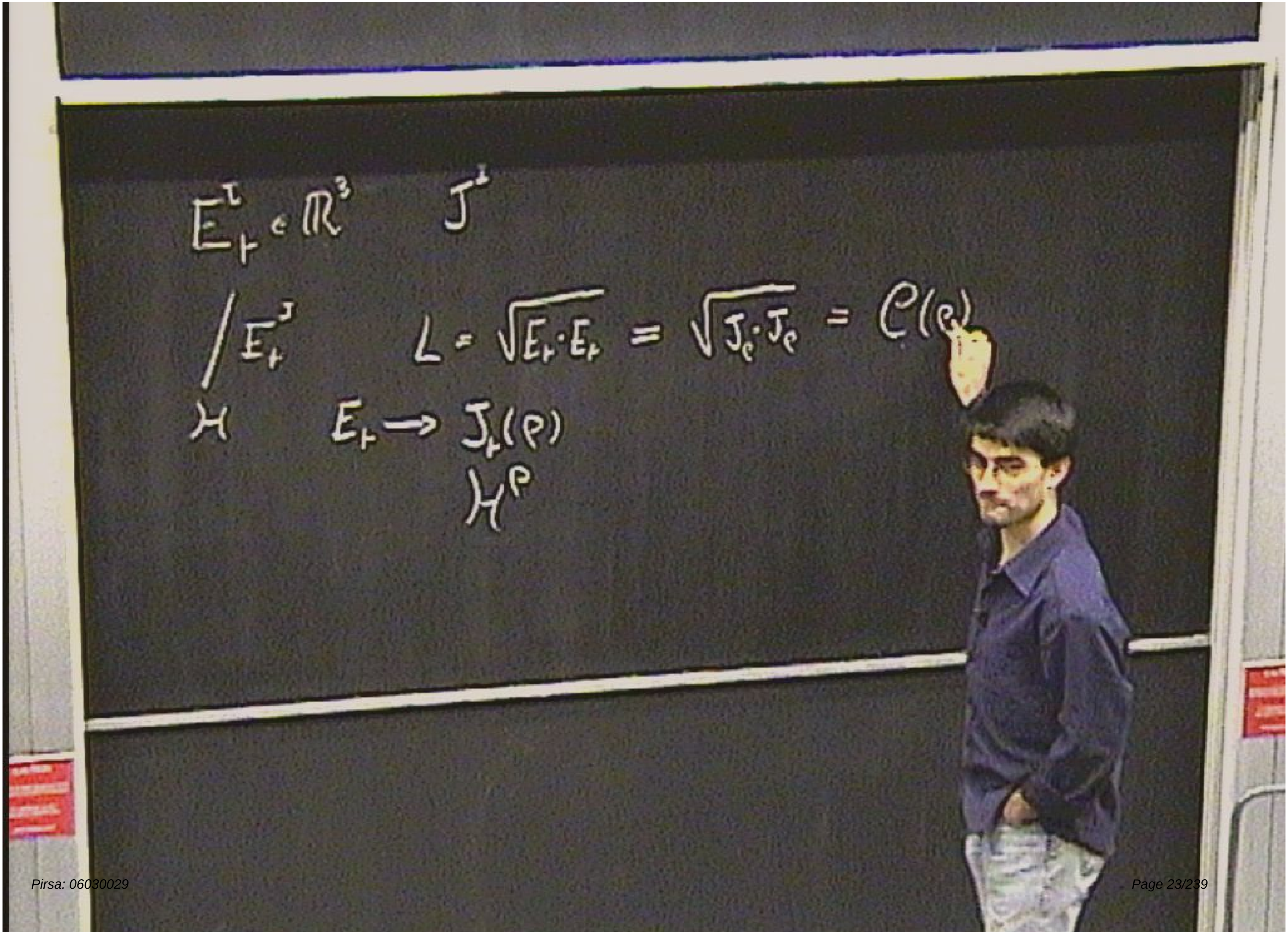

$E^I_t \in \mathbb{R}^3$
$/E^J_t$

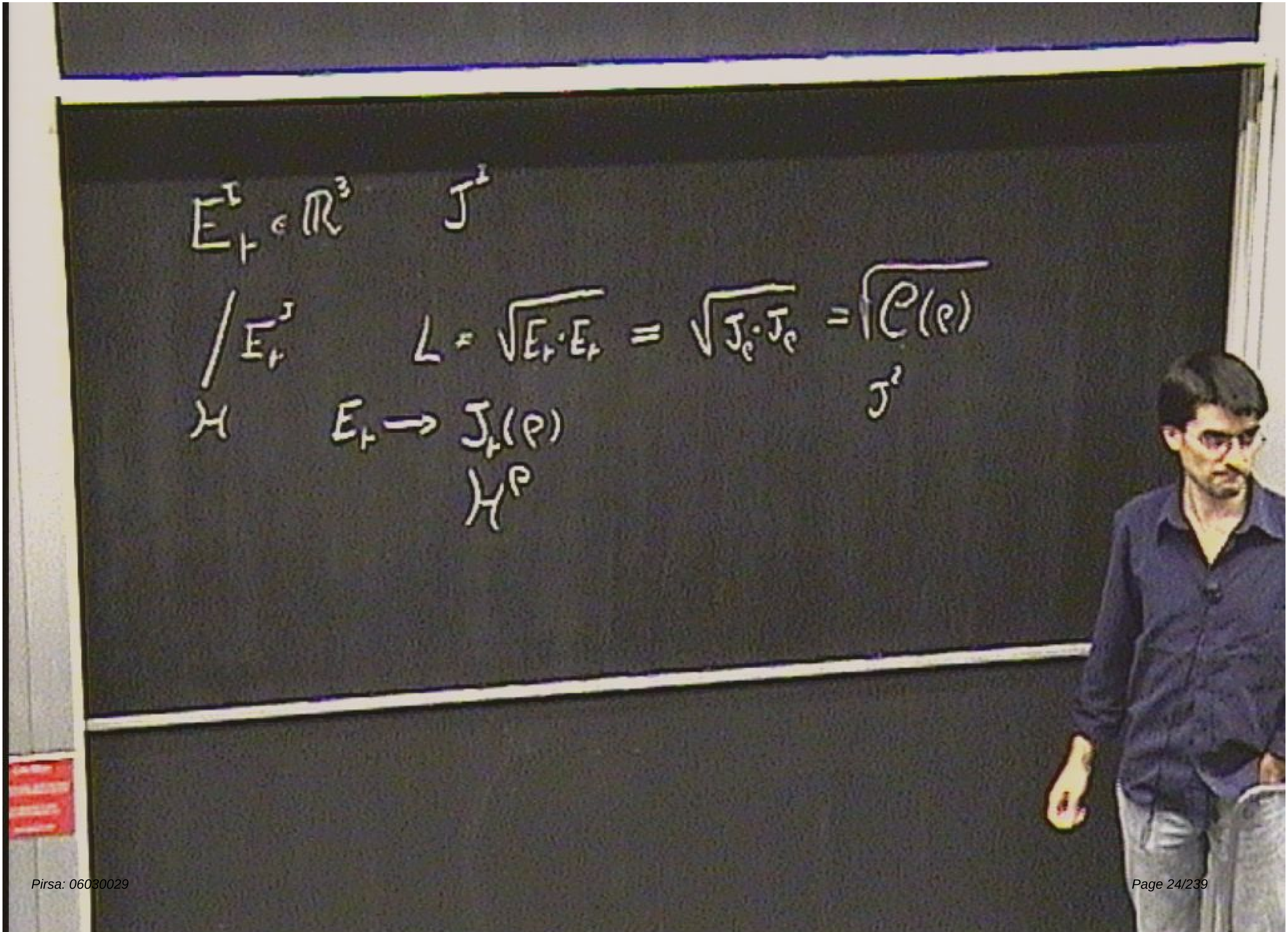

$E^i_r \in \mathbb{R}^3$
E^j_r
$L = \sqrt{E_r \cdot E_r}$

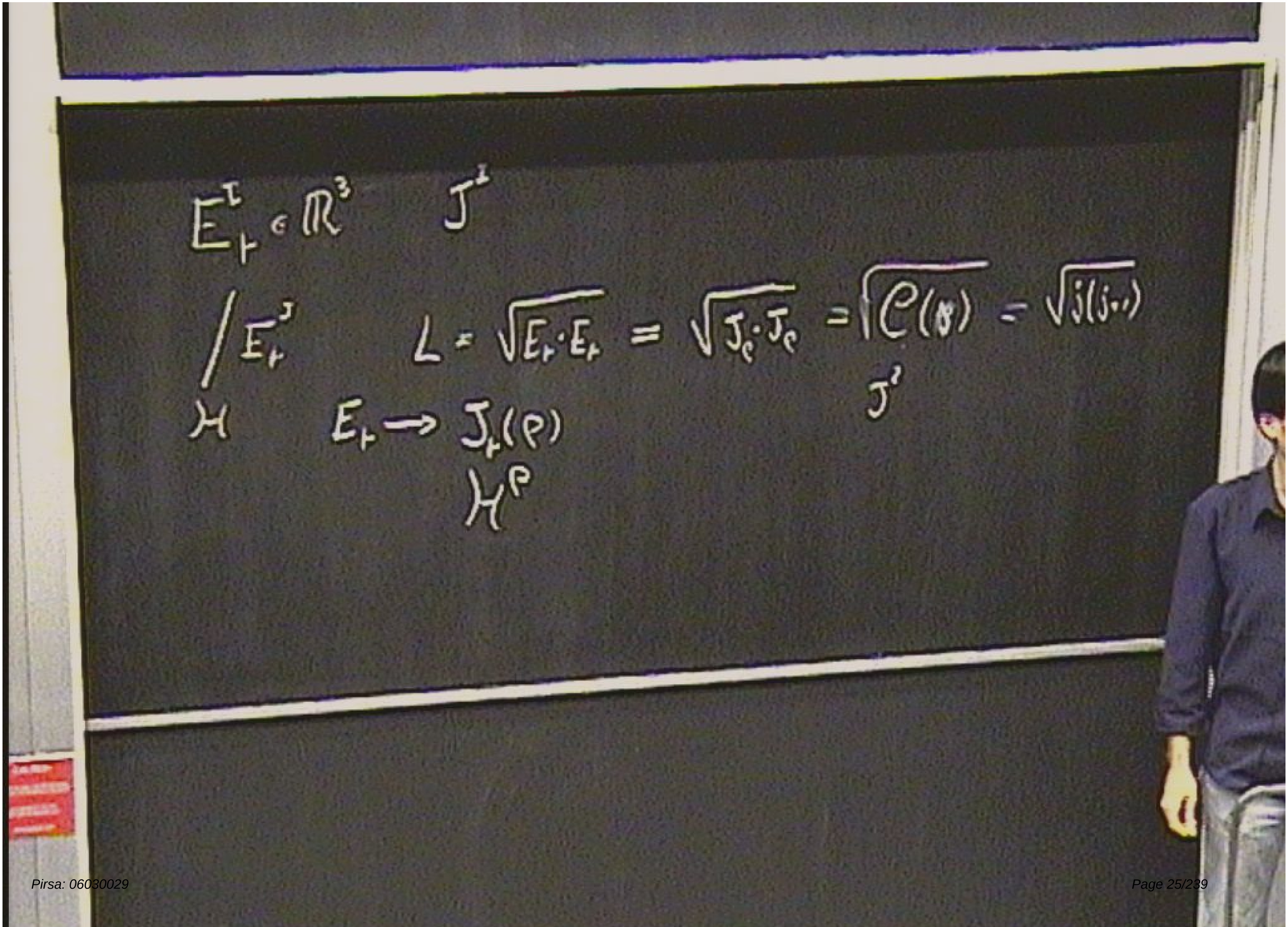

$E^i_r \in \mathbb{R}^3$ J^i
$/E^j_r$
$L = \sqrt{E_r \cdot E_r} = \sqrt{J \cdot J}$


$E^i_r \in \mathbb{R}^3$ J^i
E^j_r $L = \sqrt{E_r \cdot E_r} = \sqrt{J \cdot J}$
$\mathcal{H}$

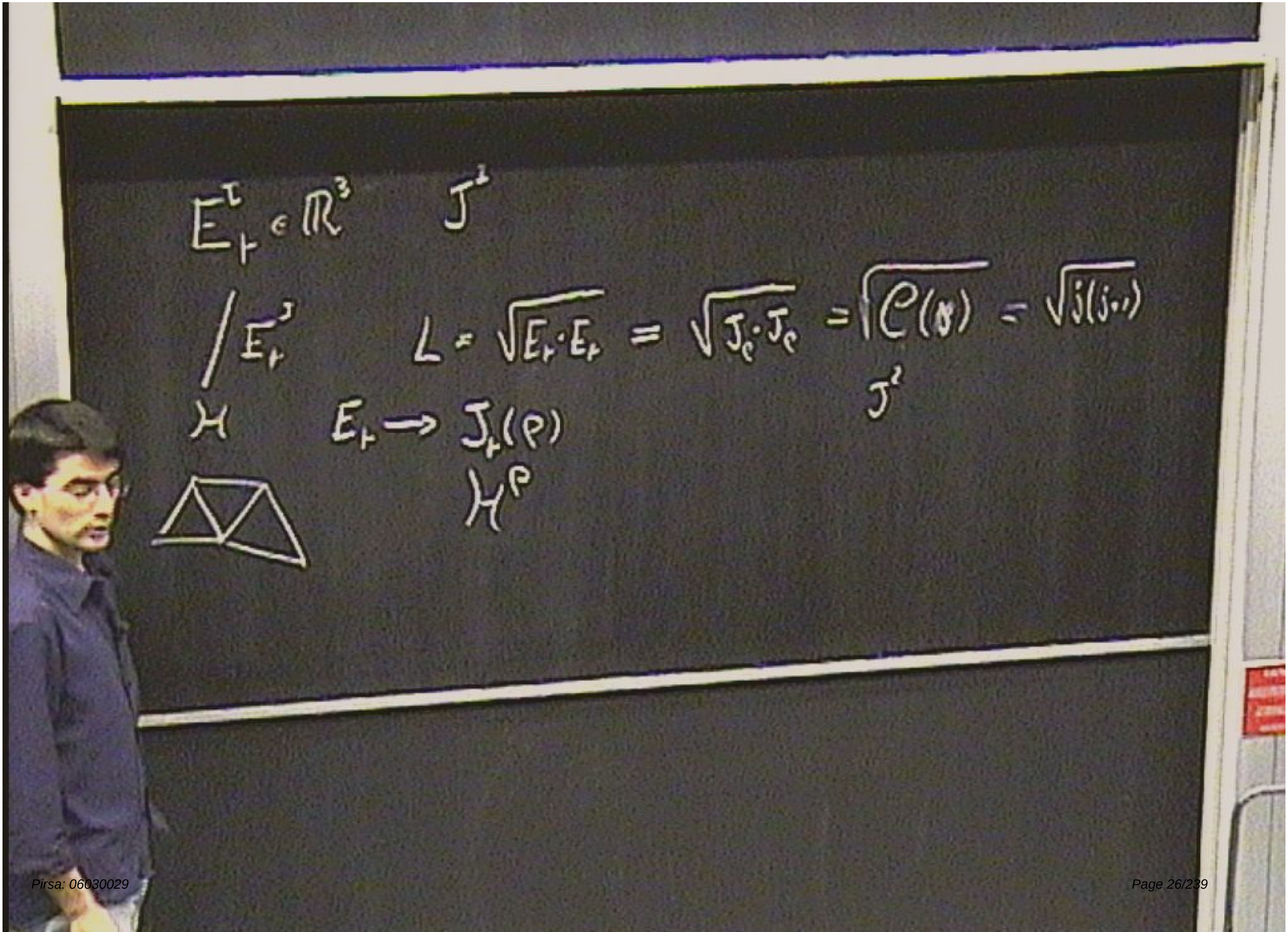

$E^I_t \in \mathbb{R}^3$
J^I
E^J_t
$L = \sqrt{E_t \cdot E_t} = \sqrt{J \cdot J}$
$\mathcal{H}$
$E_t \longrightarrow J_t(\rho)$
$\mathcal{H}^\rho$

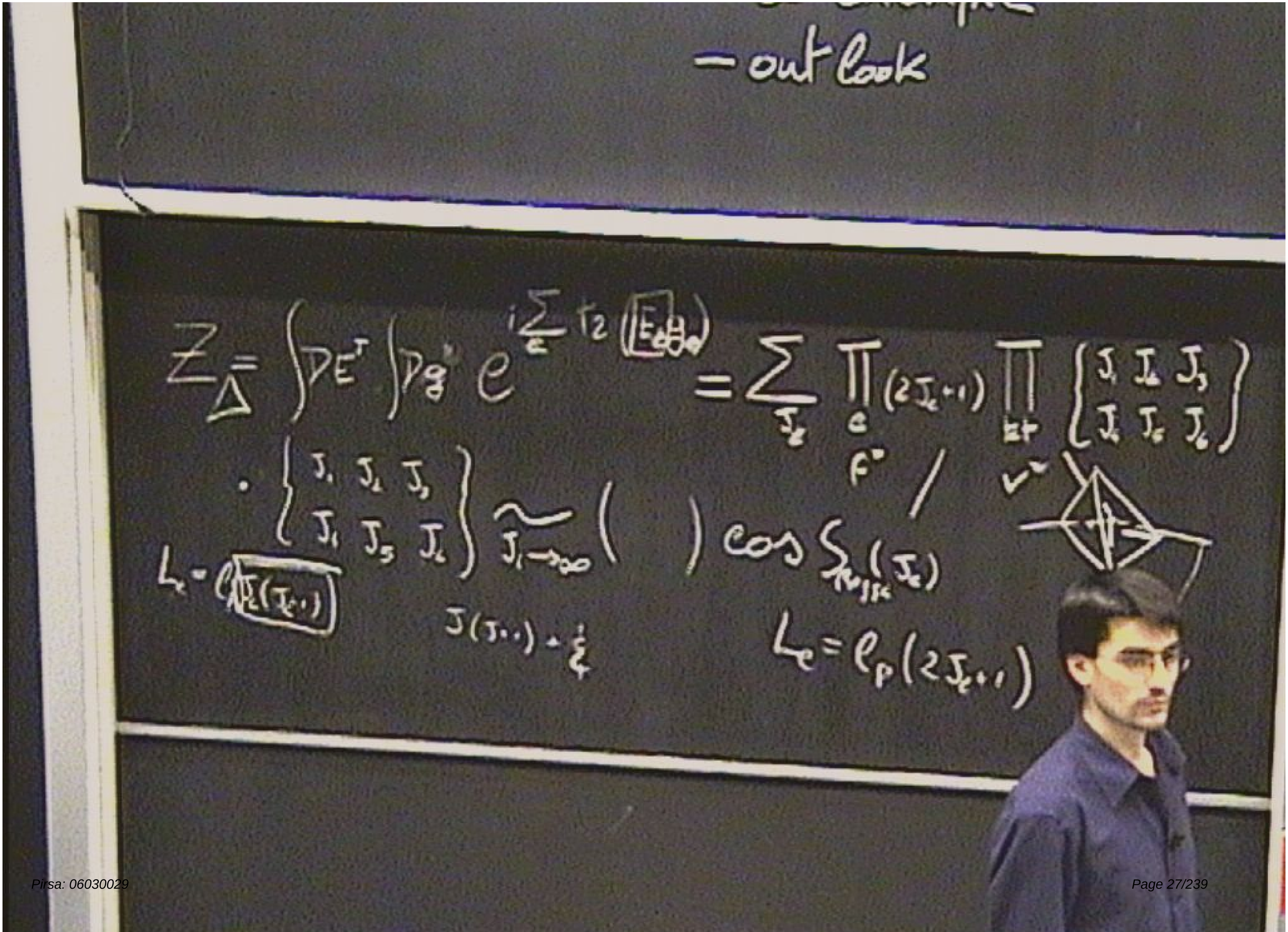

E^I_r ∈ ℝ^3
J^i
E^J_r
L = √(E_r · E_r) = √(J_ρ · J_ρ) = ℓ(ρ)
H
E_r → J_r(ρ)
H^ρ

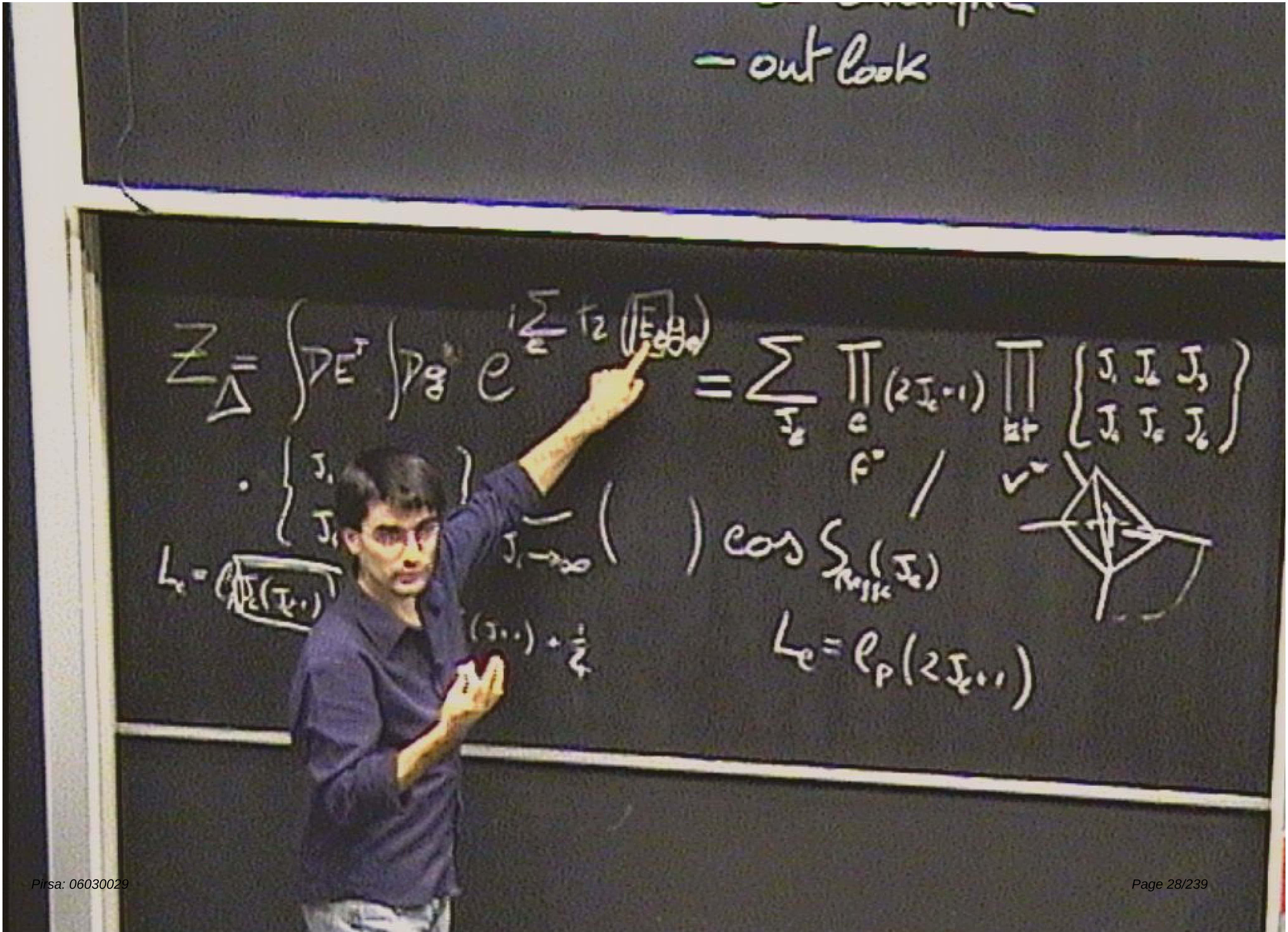

E^I_t ∈ ℝ^3   J^i
E^J_t
L = √(E_t · E_t) = √(J_e · J_e) = √(C(e))
J^2
H   E_t → J_t(e)
H^e

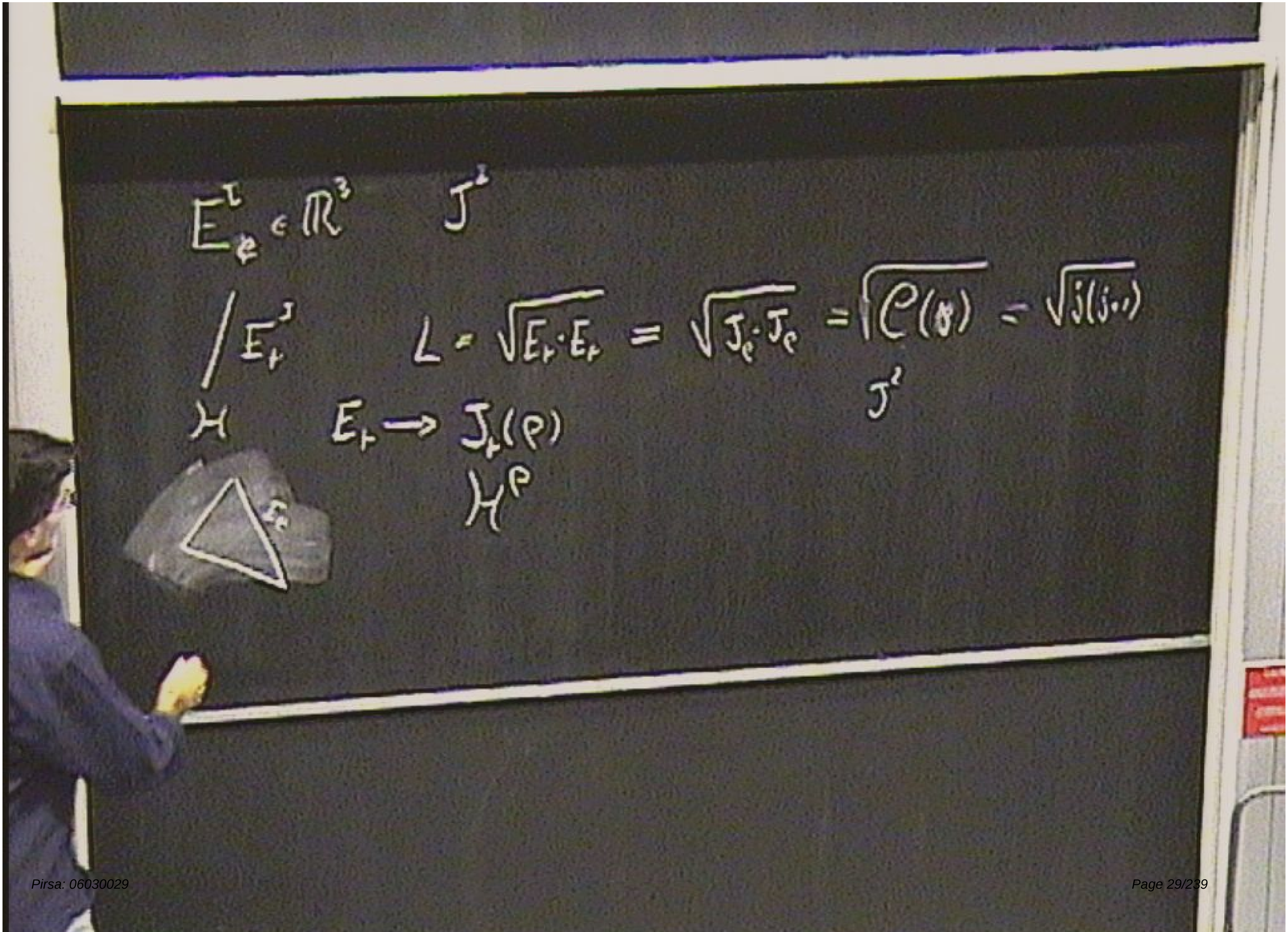


E^I_r ∈ ℝ^3   J^I
E^J_r   L = √(E_r · E_r) = √(J_e · J_e) = √C(…) = √(j(j+1))
J^2
H   E_r → J_r(ρ)
H^ρ

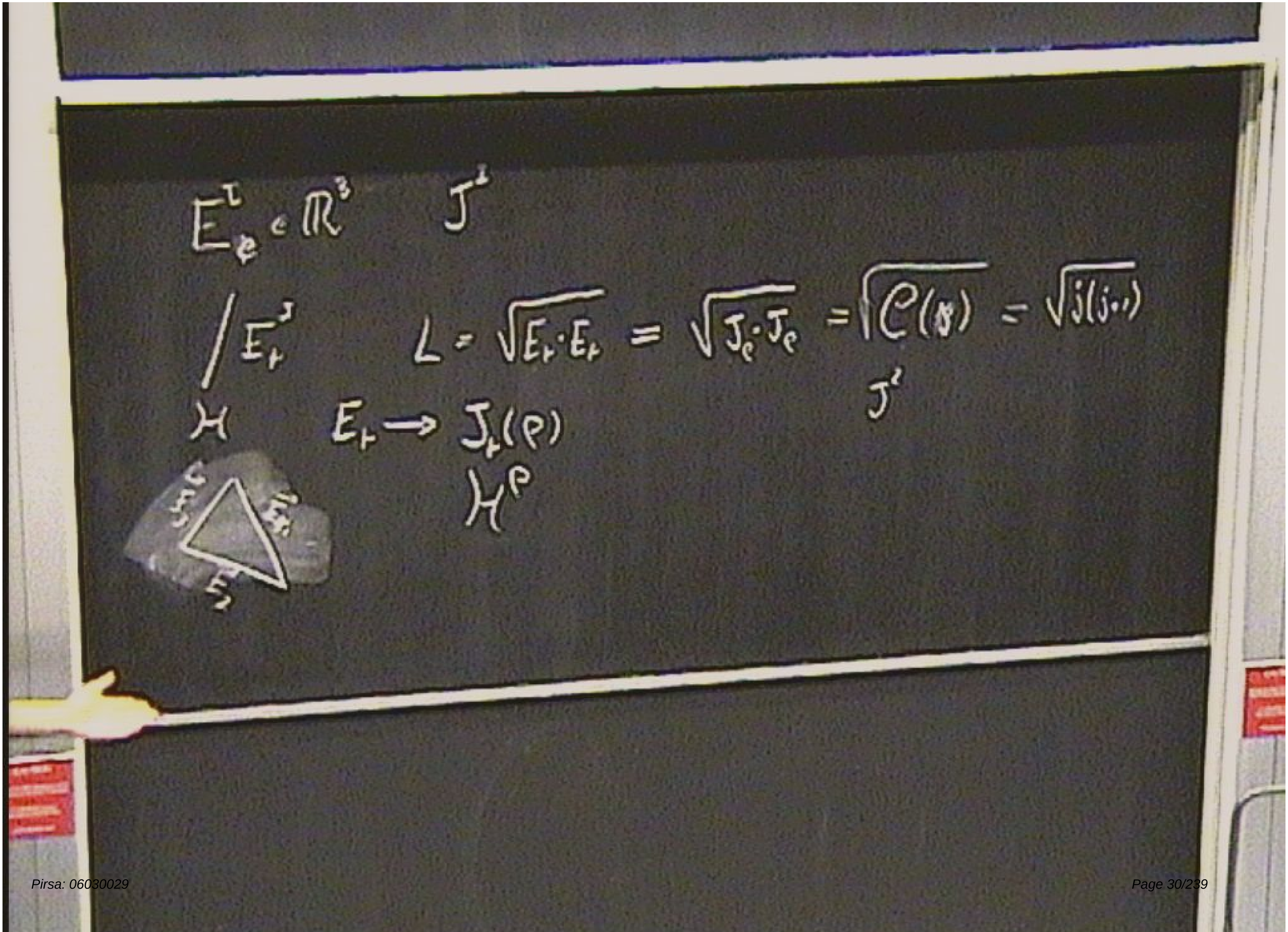

E_t^i ∈ ℝ^3
J^2
E_t^J
L = √(E_t·E_t) = √(J_e·J_e) = √(C(s)) = √(j(j+1))
J^2
H
E_t → J_t(ρ)
H^ρ

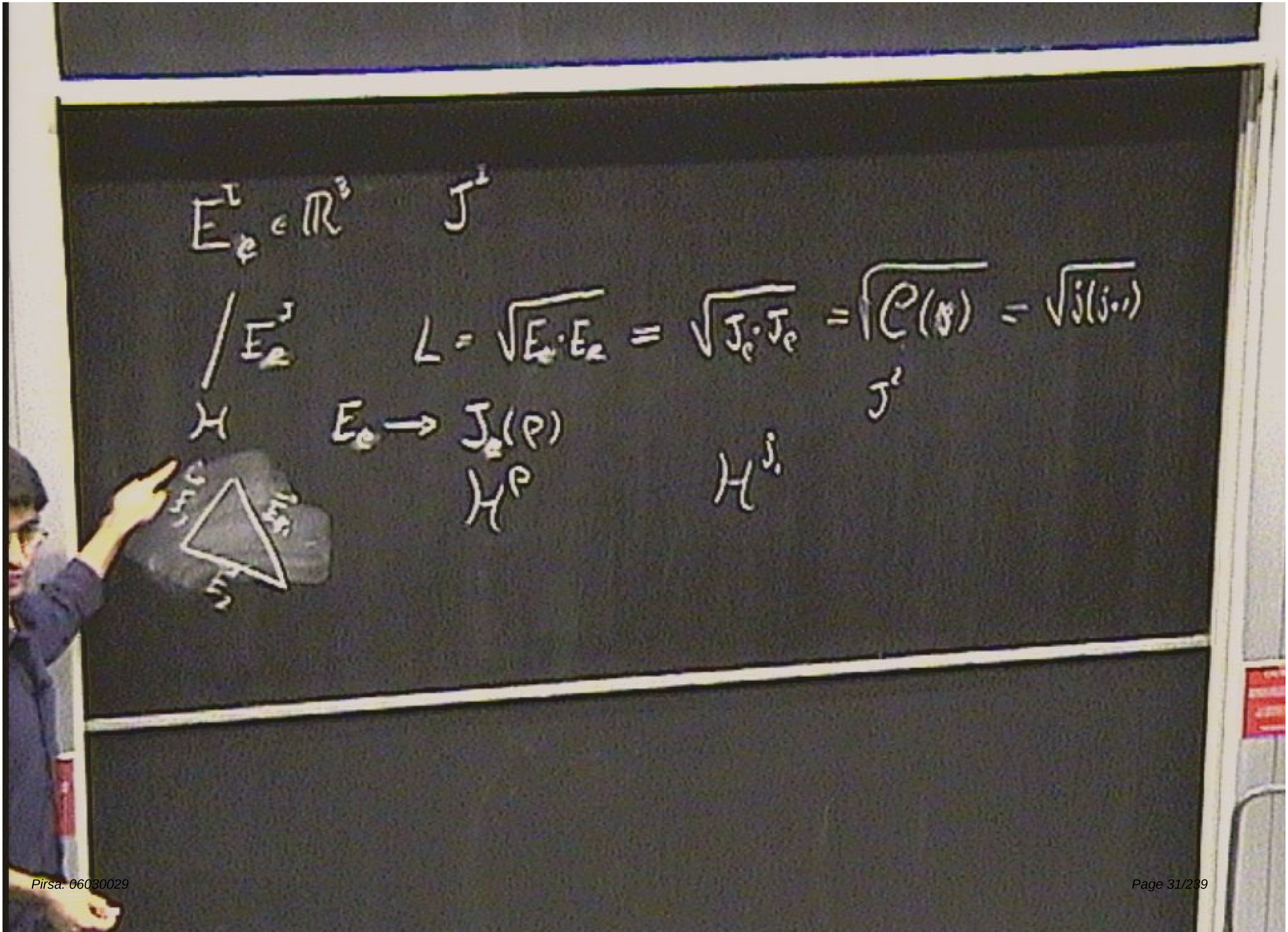

– out look
$Z_\Delta = \int DE^\tau \int Dg \; e^{i\sum_e tr(E_e g_e)} = \sum_{J_e} \prod_e (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$
$\cdot \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \sim_{J_i \to \infty} (\quad) \cos S_{Regge}(J_e)$
$L_e = \ell_p (2J_e+1)$

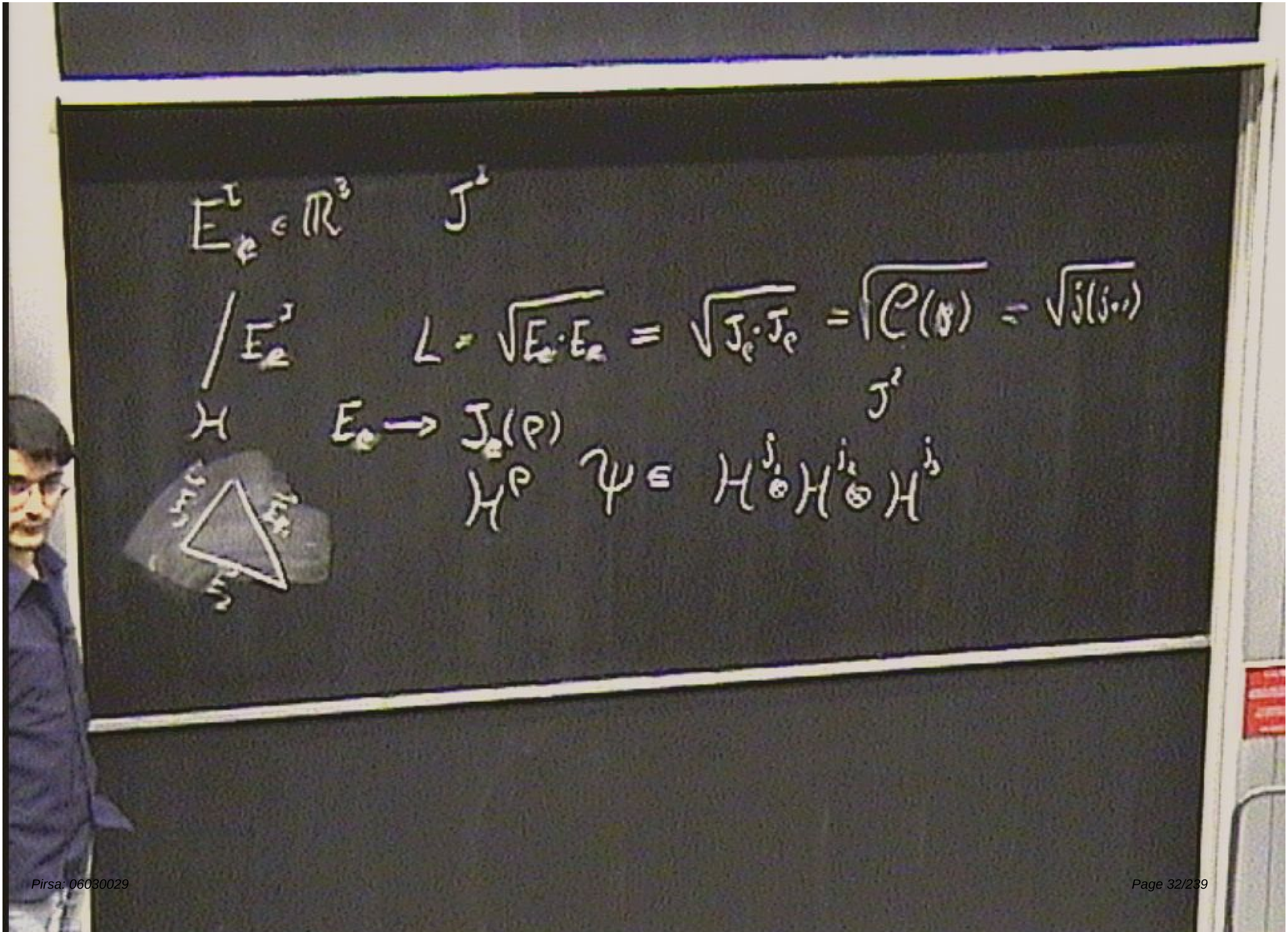

– out look
Z_Δ = ∫DE ∫Dg e^{i Σ_e tr(E g)} = Σ_{J_e} Π_e (2J_e+1) Π {J1 J2 J3 / J4 J5 J6}
cos S_Regge(J_e)
L_e = ℓ_P (2J_e+1)



$E^i_e \in \mathbb{R}^3$ J^i
E^J_t
$L = \sqrt{E_t \cdot E_t} = \sqrt{J_e \cdot J_e} = \sqrt{C(j)} = \sqrt{j(j+1)}$
J^2
$\mathcal{H}$ $E_t \longrightarrow J_t(\rho)$
$\mathcal{H}^\rho$
E_t


$E^I_e \in \mathbb{R}^3$
J^2
E^J_r
$L = \sqrt{E_r \cdot E_r} = \sqrt{J_e \cdot J_e} = \sqrt{C(j)} = \sqrt{j(j+1)}$
J^2
$\mathcal{H}$
$E_r \longrightarrow J_r(\rho)$
$\mathcal{H}^\rho$


E^i_e ∈ ℝ^3
J^2
E^J_e
L = √(E_e · E_e) = √(J_e · J_e) = √C(j) = √(j(j+1))
J^2
H
E_e → J_e(ρ)
H^ρ
H^j


E_e^i ∈ ℝ^3   J^i
E_e^j   L = √(E_e·E_e) = √(J_e·J_e) = √C(j) = √j(j+1)
J^2
H   E_e → J_e(ρ)
H^ρ   ψ ∈ H^{j_1} ⊗ H^{j_2} ⊗ H^{j_3}

$E^I_e \in \mathbb{R}^3 \qquad J^I$

$/\, E^J_e \qquad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\gamma)} = \sqrt{j(j+1)}$

J^2

$\mathcal{H} \qquad E_e \longrightarrow J_e(\rho)$

$\mathcal{H}^\rho \qquad \psi \in \mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3}$

$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$

$E^I_e \in \mathbb{R}^3$ $\quad J^I$

E^J_e

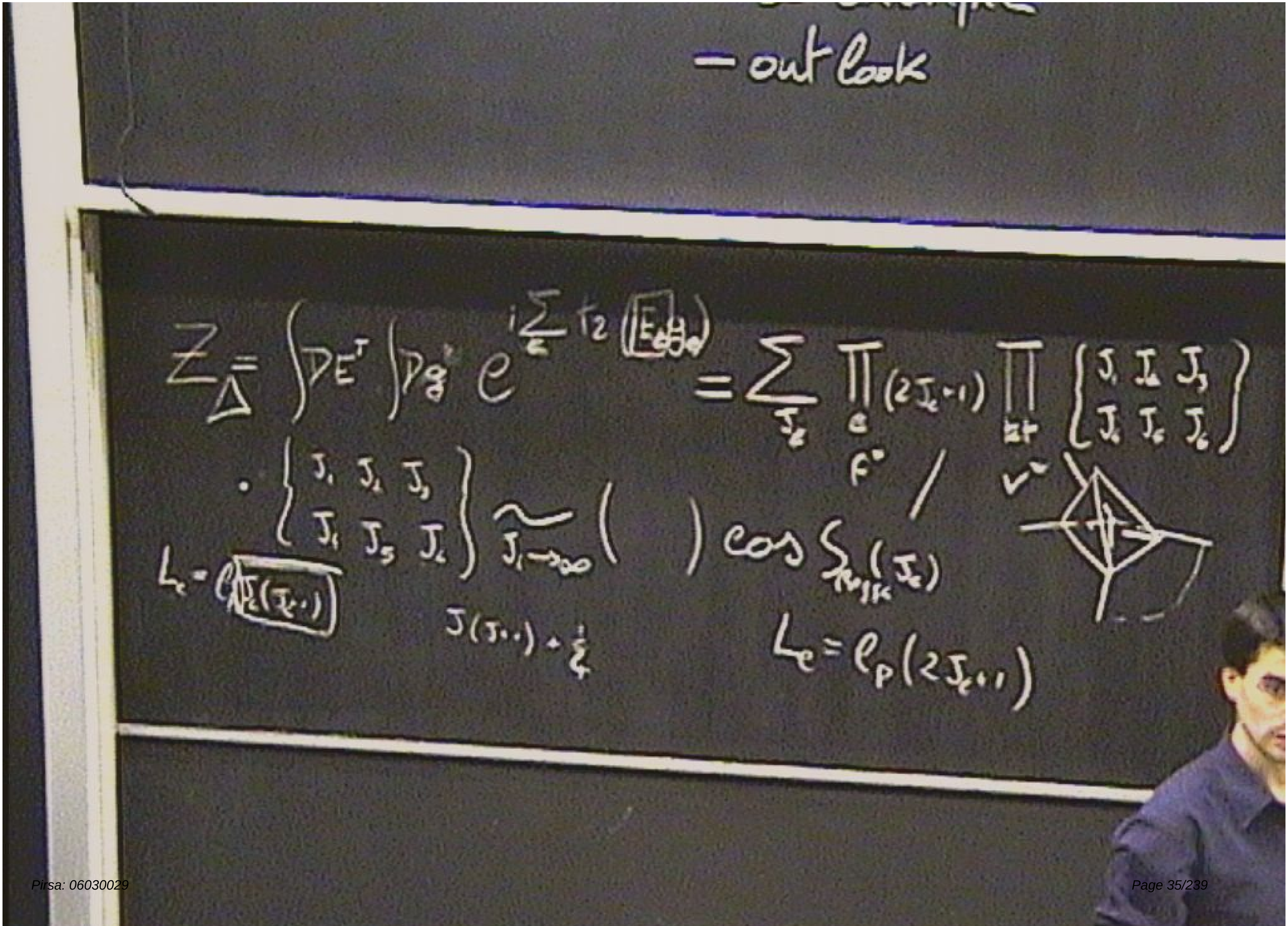
$$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\rho)} = \sqrt{j(j+1)}$$

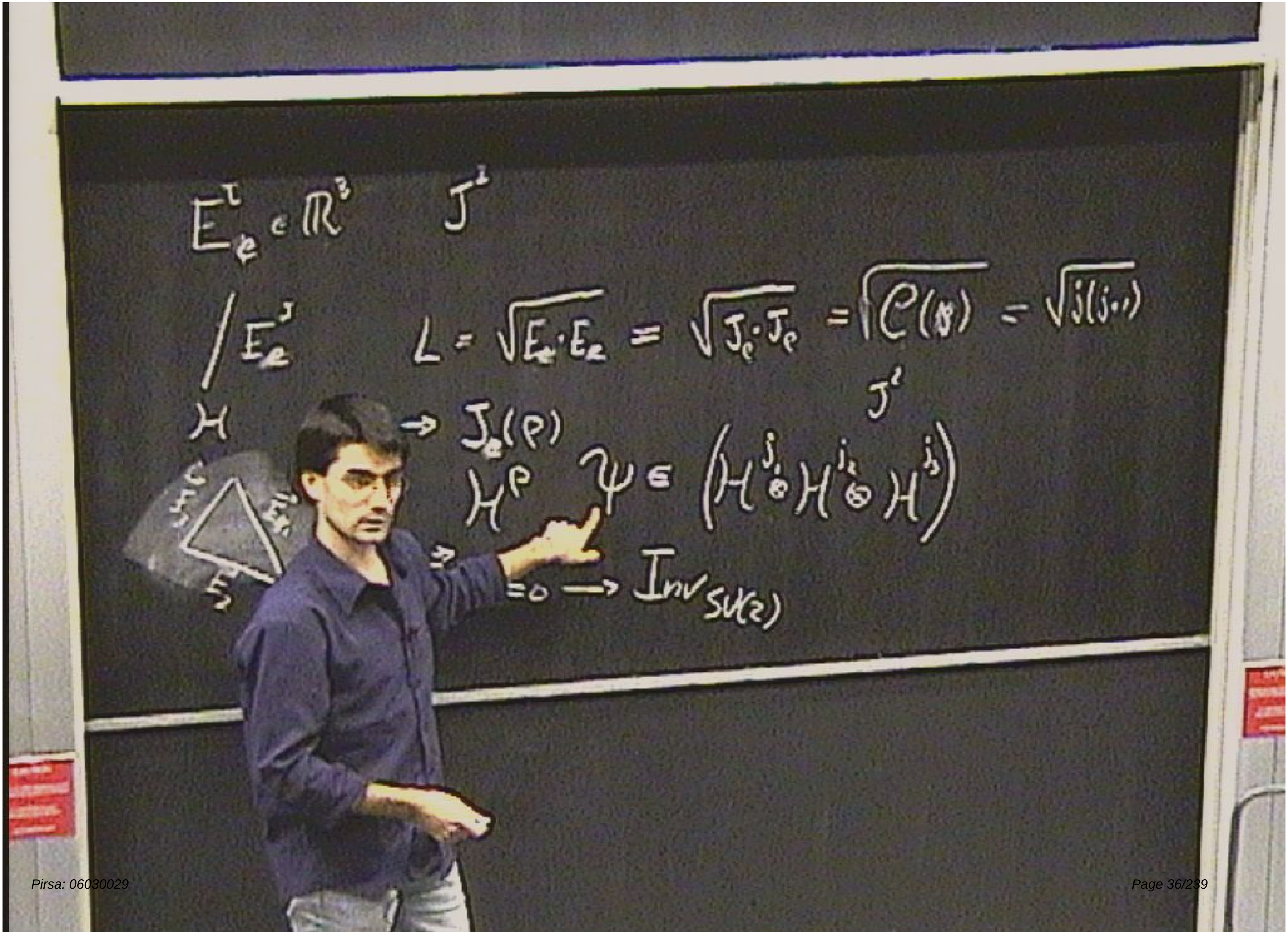
J^2

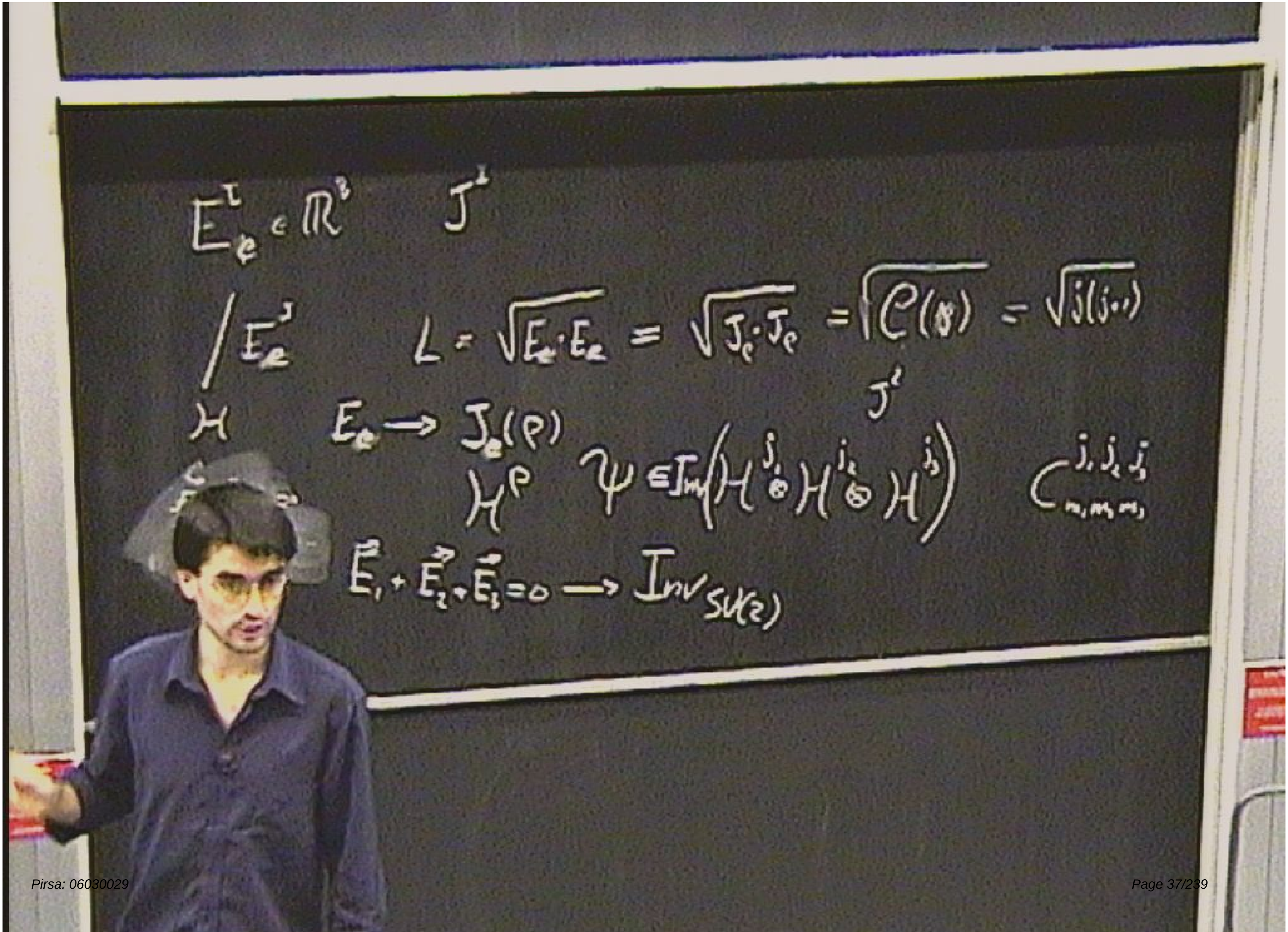
$\mathcal{H}$ $\quad E_e \longrightarrow J_e(\rho)$

$\mathcal{H}^\rho \quad \psi \in \mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3}$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \longrightarrow Inv_{SU(2)}$$


- out look
cos
L_e = l_p(2J_e+1)


$E_e^i \in \mathbb{R}^3$ J^i
$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(S)} = \sqrt{j(j+1)}$
$\mathcal{H} \to J_e(p)$
$\mathcal{H}^p$ $\psi \in (\mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3})$
$\to Inv_{SU(2)}$


$E^i_e \in \mathbb{R}^3$ J^i
E^i_e $L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(j)} = \sqrt{j(j+1)}$
J^i
$\mathcal{H}$ $E_e \rightarrow J_e(\rho)$
$\mathcal{H}^\rho$ $\Psi \in \mathrm{Inv}(\mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3})$ $C^{j_1 j_2 j_3}_{m_1 m_2 m_3}$
$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \longrightarrow \mathrm{Inv}_{SU(2)}$

— outlook

$$Z_\Delta = \int \mathcal{D}E^I \int \mathcal{D}g \; e^{i\sum_e \mathrm{tr}(E_e g_e)} = \sum_{J_e} \prod_{e} (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$$

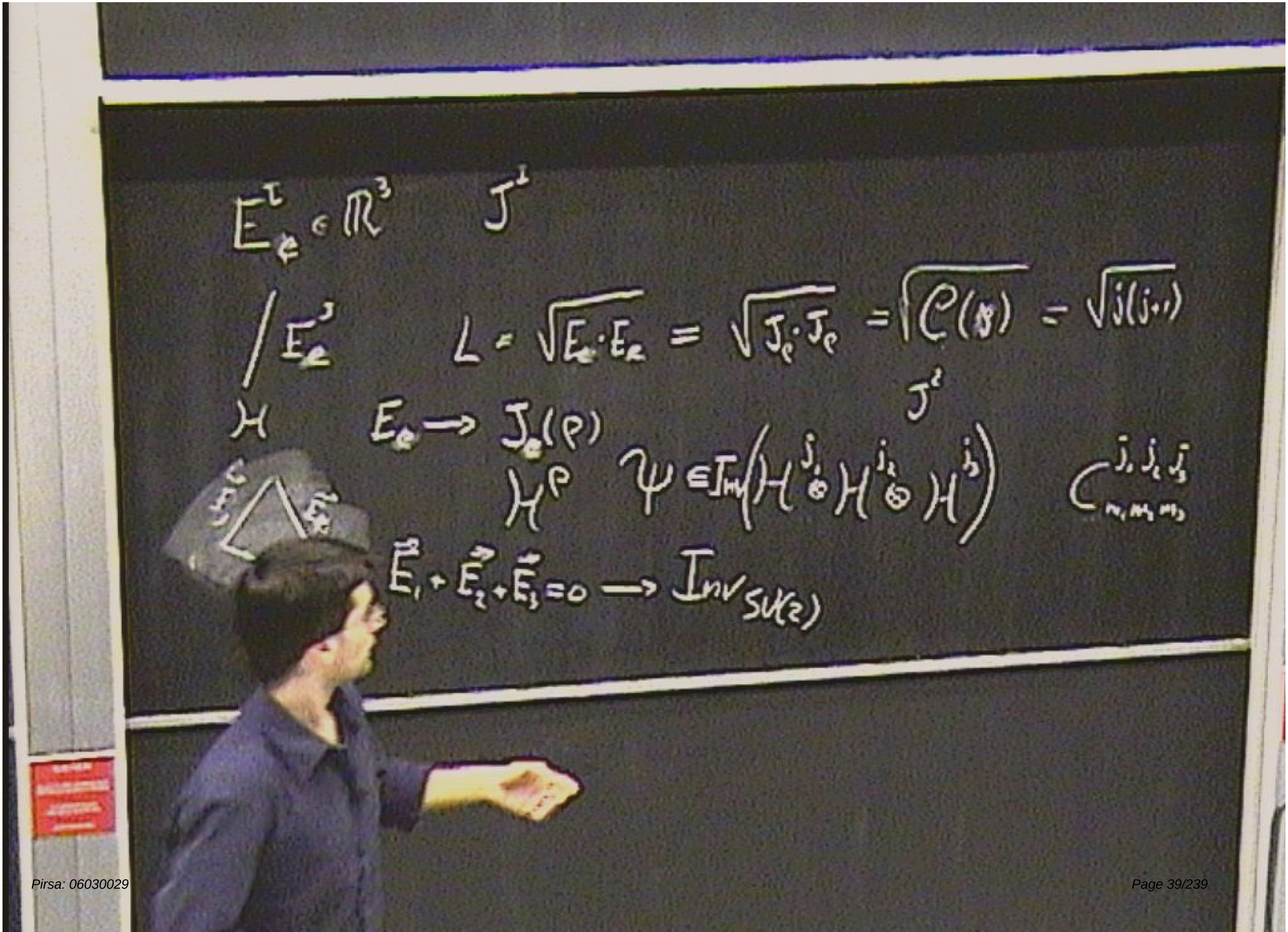
f^* / v^*

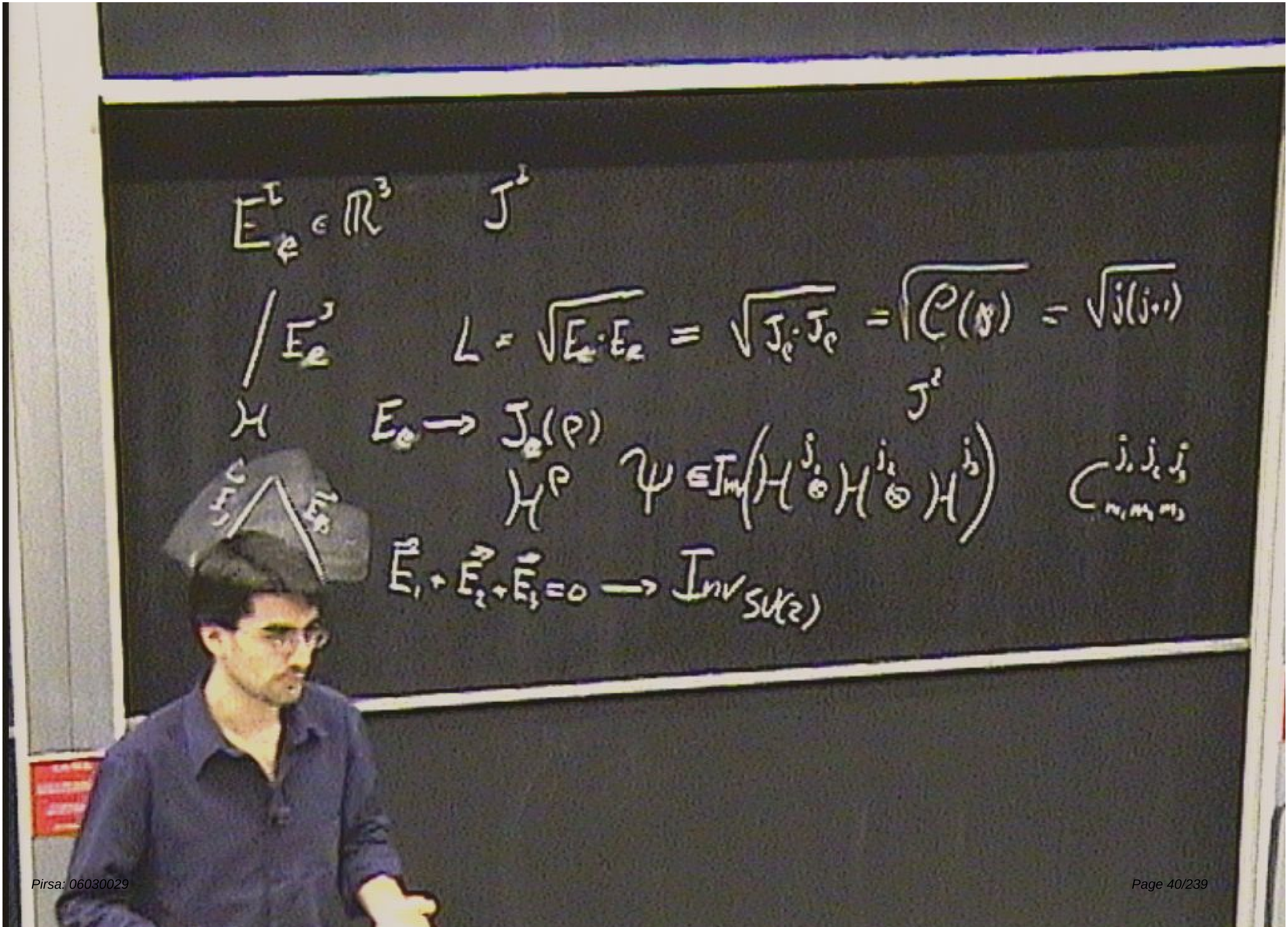
$$\cdot \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \underset{J_i \to \infty}{\sim} (\quad) \cos S_{Regge}(J_e)$$

$$L_e = \ell \sqrt{J_e(J_e+1)}$$

$$J(J+1) + \tfrac{1}{4}$$

$$L_e = \ell_P (2J_e+1)$$


E^I_e ∈ ℝ^3   J^I
L = √(E_e · E_e) = √(J_e · J_e) = √C(j) = √(j(j+1))
H   E_e → J_e(p)   J^2
H^P   Ψ ∈ Inv(H^{j1} ⊗ H^{j2} ⊗ H^{j3})   C^{j1 j2 j3}_{m1 m2 m3}
E1 + E2 + E3 = 0 → Inv_SU(2)


E^I_e ∈ ℝ^3   J^2
E_e   L = √(E_e · E_e) = √(J_e · J_e) = √(C(γ)) = √(j(j+1))
H   E_e → J_e(ρ)   J^2
H^ρ   ψ ∈ Inv(H^{j_1} ⊗ H^{j_2} ⊗ H^{j_3})   C^{j_1 j_2 j_3}_{m_1 m_2 m_3}
E_1 + E_2 + E_3 = 0 → Inv_{SU(2)}

$E^I_e \in \mathbb{R}^3 \qquad J^i$

E^J_e

$L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(\gamma)} = \sqrt{j(j+1)}$

J^2

$\mathcal{H} \qquad E_e \longrightarrow J_e(\rho)$

$\mathcal{H}^\rho \qquad \psi \in \mathrm{Inv}\left(\mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3}\right) \qquad C^{j_1 j_2 j_3}_{m_1 m_2 m_3}$

$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \longrightarrow \mathrm{Inv}_{SU(2)}$

– outlook

$$Z_{\Delta} = \int DE^{I} \int Dg^{I}\, e^{i\sum_{e} \mathrm{tr}_2 (E_e g_e)} = \sum_{J_e} \prod_{e} (2J_e+1) \prod_{tet} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$$

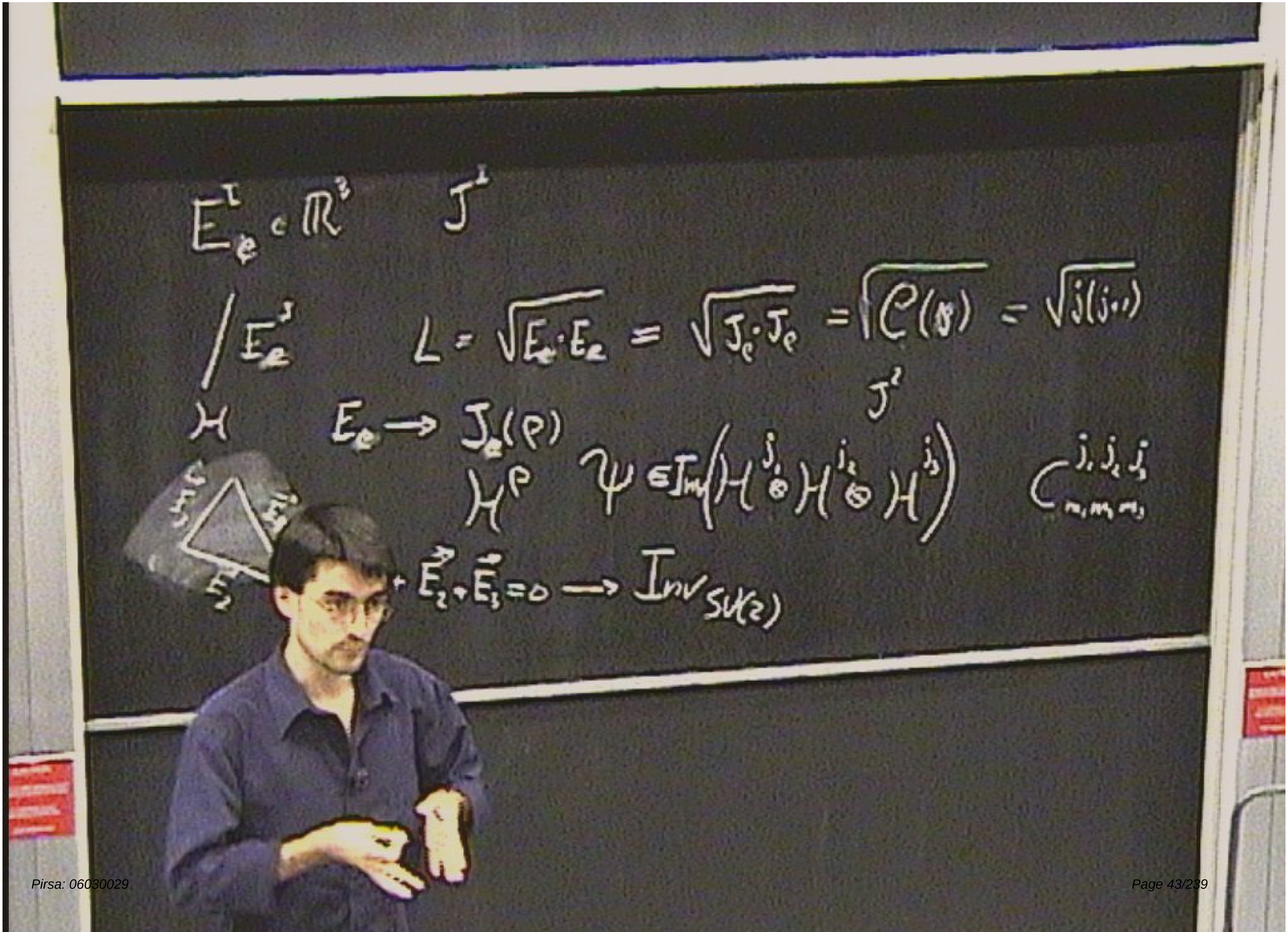
$$\cdot \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \underset{J_i \to \infty}{\sim} (\quad) \cos S_{Regge}(J_e)$$

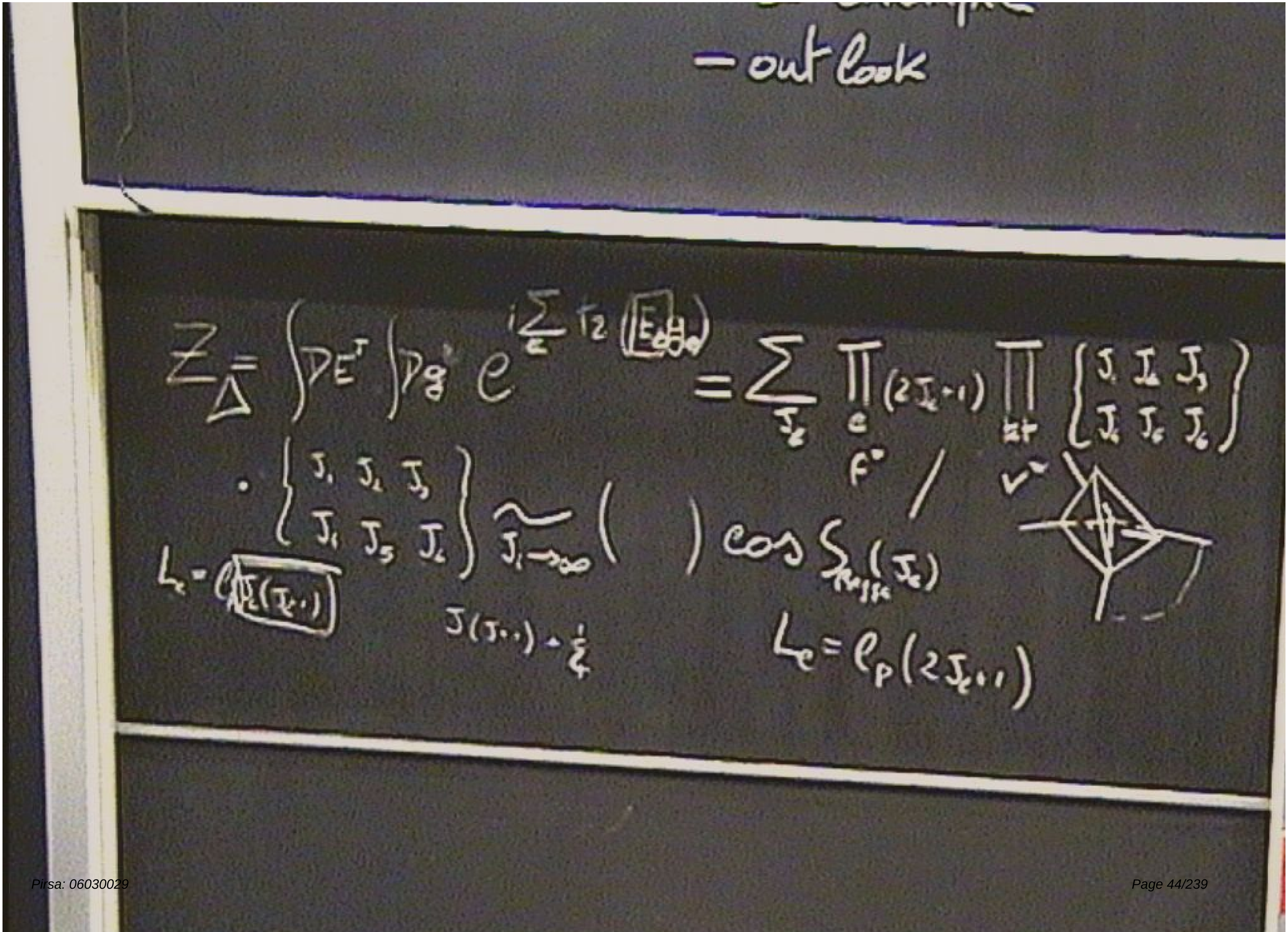
F°

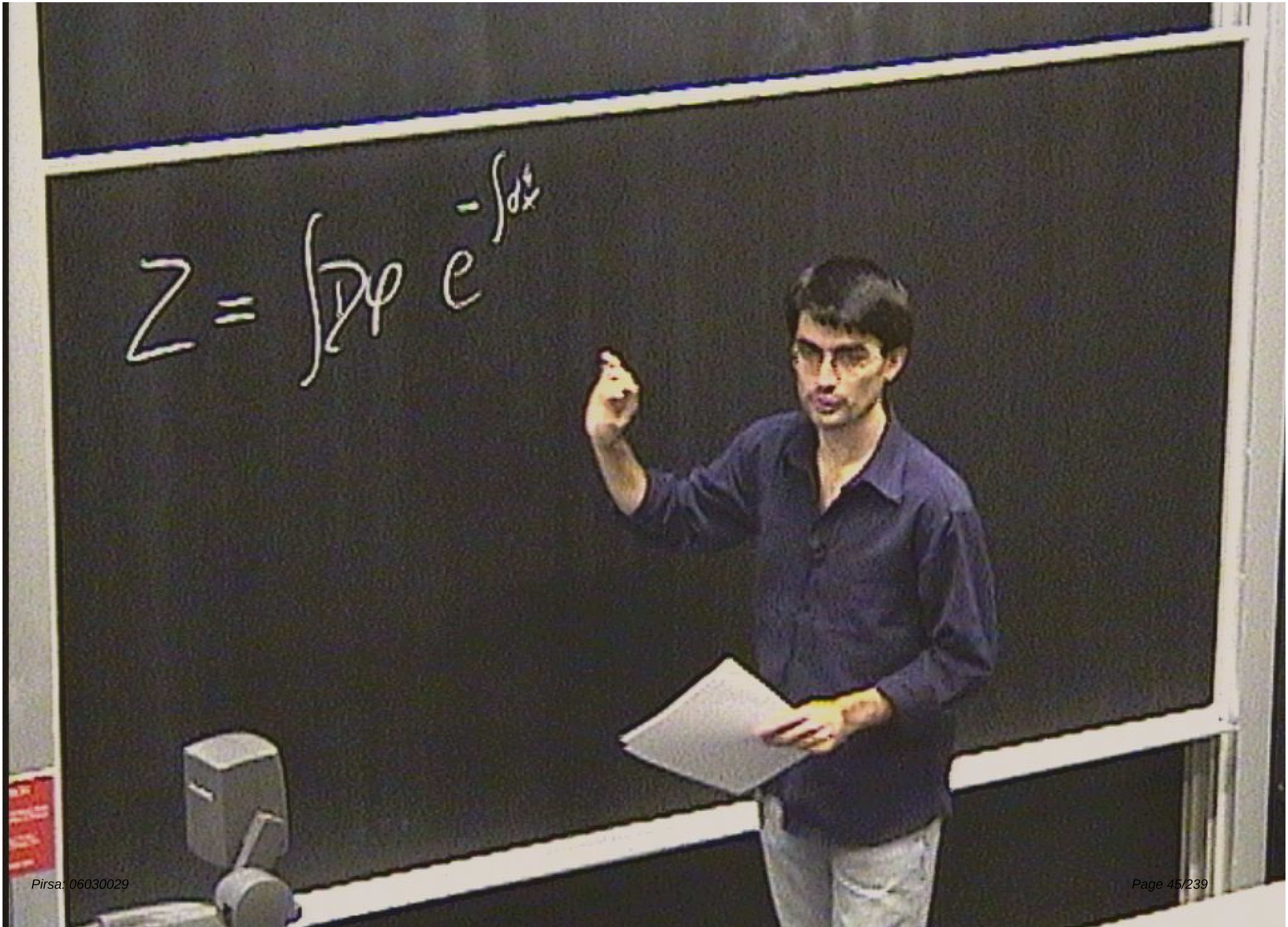
$L_e = \ell_P \sqrt{J_e(J_e+1)}$

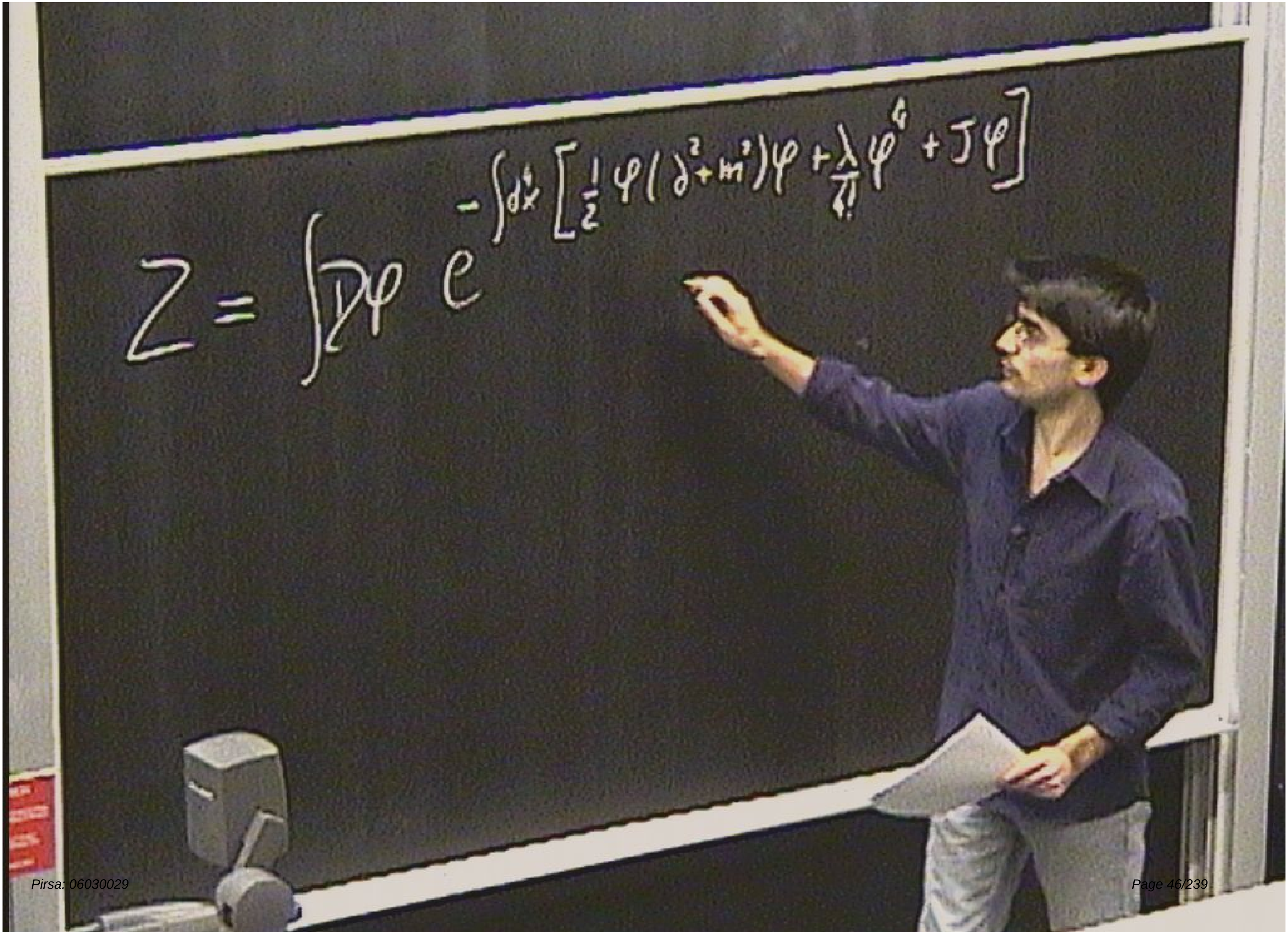
$J(J+1) + \frac{1}{4}$

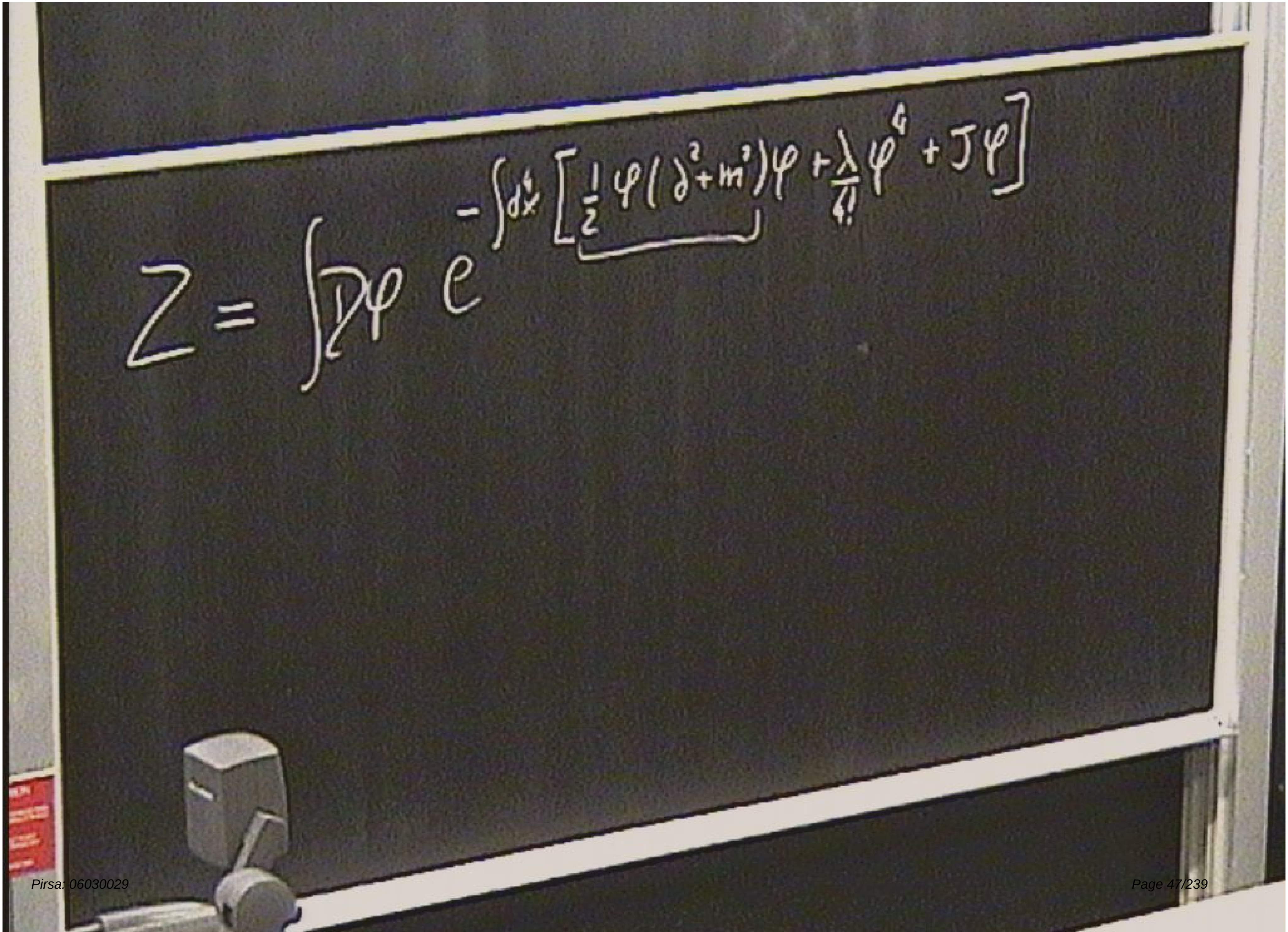
$L_e = \ell_P (2J_e+1)$

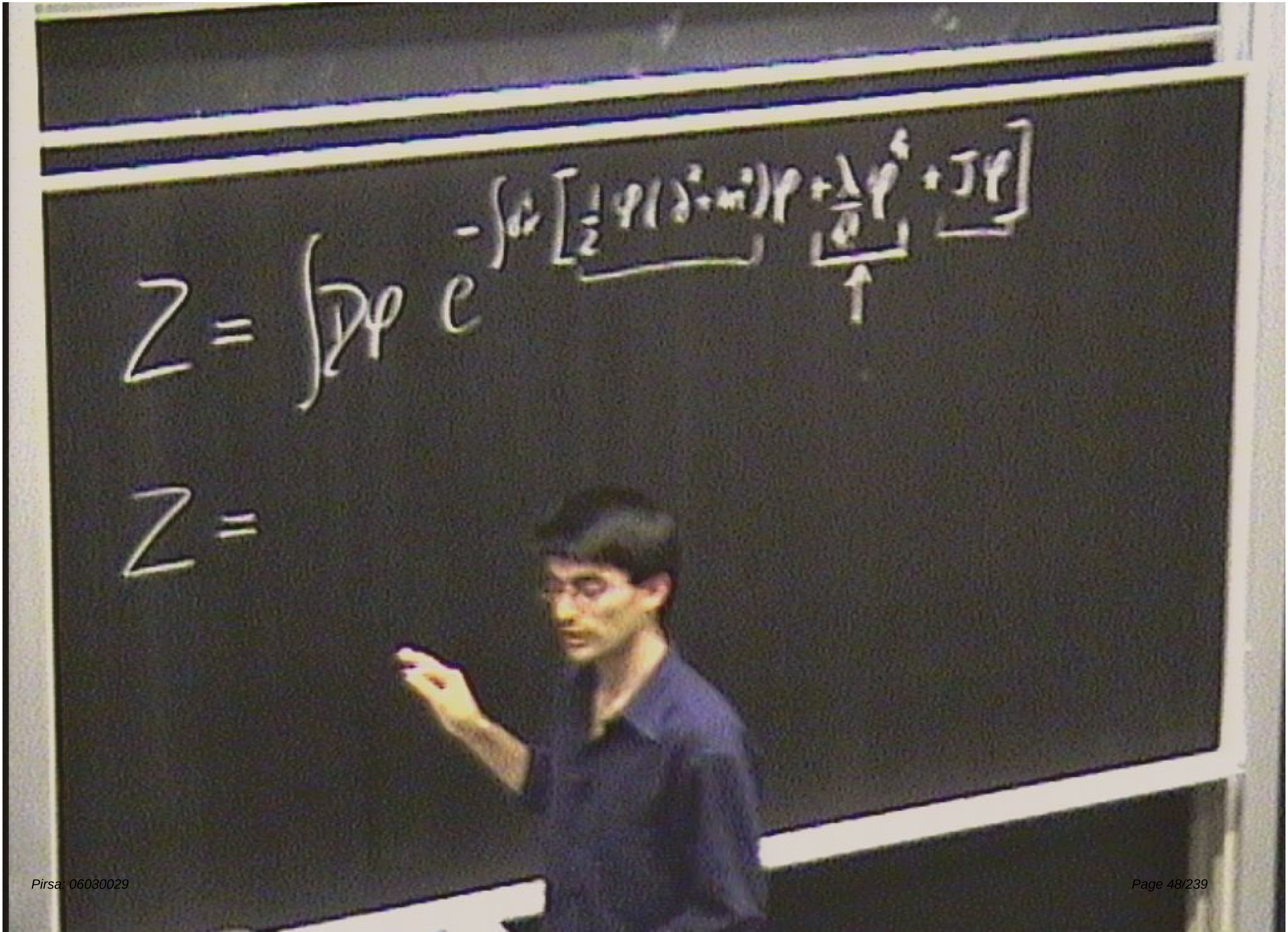

E^I_e ∈ ℝ^3   J^i
E^i_e   L = √(E_e · E_e) = √(J_e · J_e) = √(C(ρ)) = √(j(j+1))
J^2
H   E_e → J_e(ρ)
H^ρ   ψ ∈ Inv(H^{j1} ⊗ H^{j2} ⊗ H^{j3})   C^{j1 j2 j3}_{m1 m2 m3}
E_1 E_2 E_3
+ E_2 + E_3 = 0 → Inv_{SU(2)}

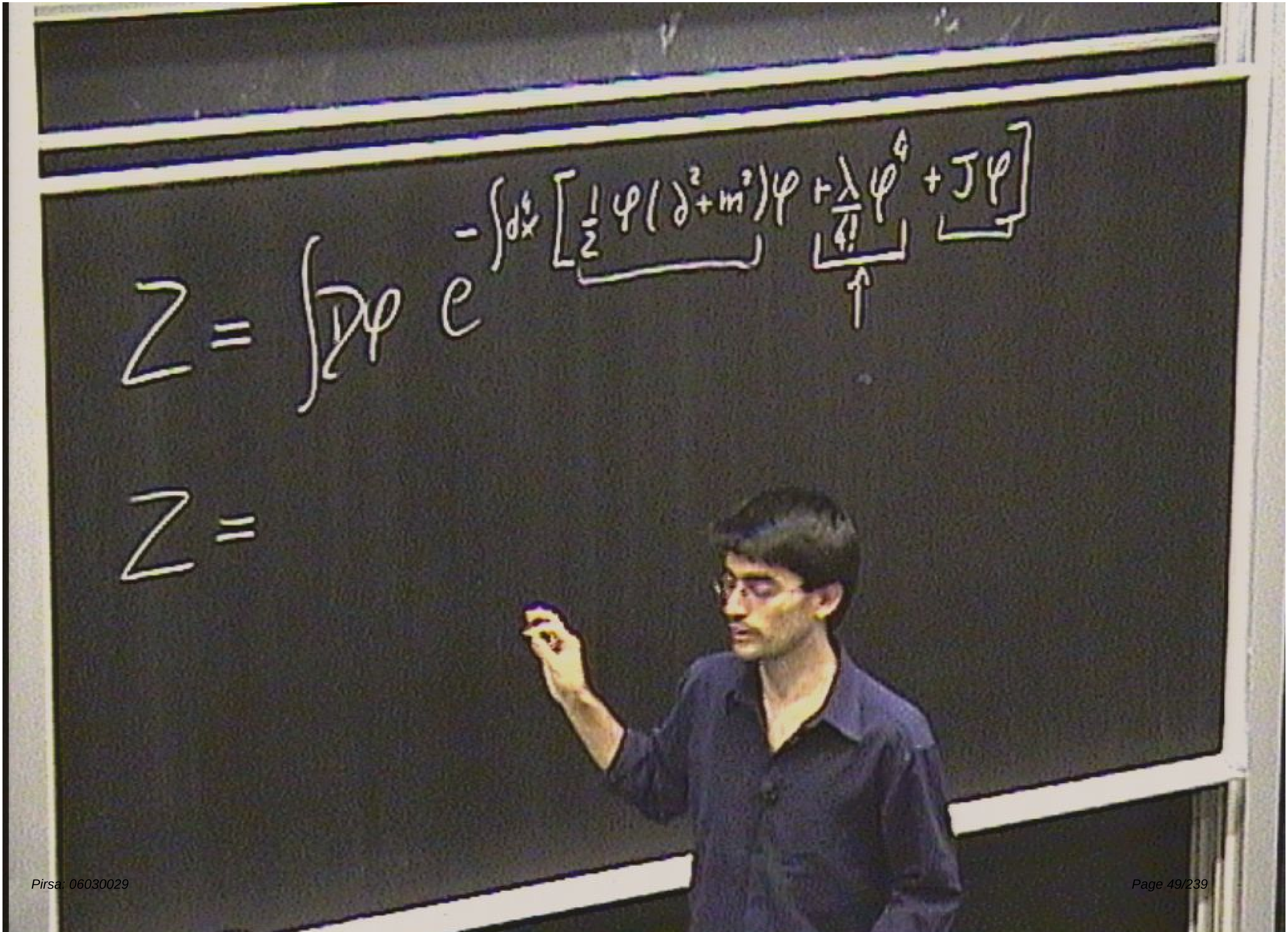

– out look
cos
L_e = ℓ_P (2 J_e + 1)


$Z = \int D\varphi \, e^{-\int d^4x}$


$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$


$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\underbrace{\frac{1}{2}\varphi(\partial^2+m^2)}\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi \right]}$


$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$
Z =


$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$
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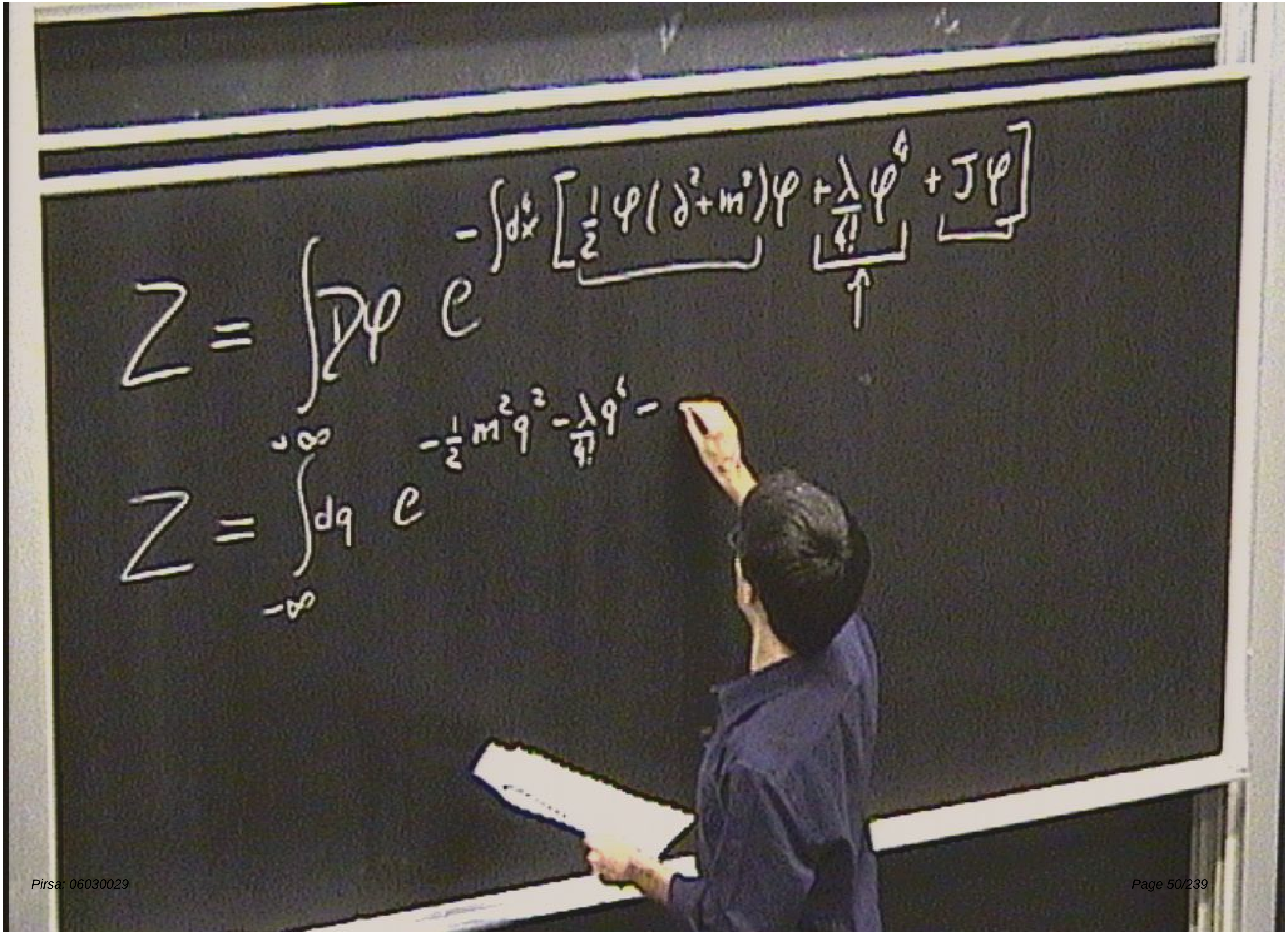
$Z = \int \mathcal{D}\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{\infty} dq \, e^{-\frac{1}{2}m^2 q^2 - \frac{\lambda}{4!} q^4 -}$

$$Z = \int D\varphi \; e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi \right]}$$

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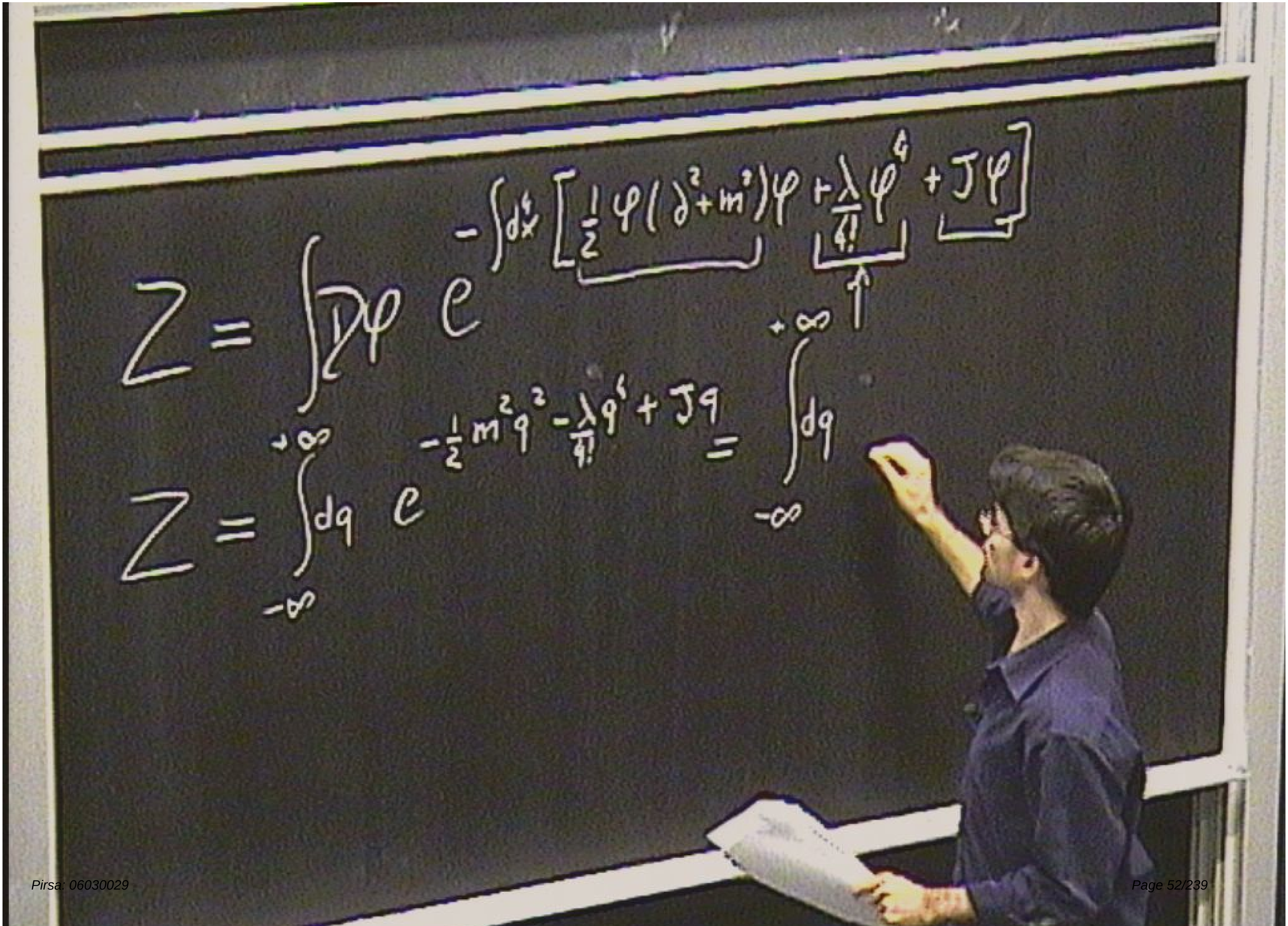
$Z = \int D\varphi \; e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{+\infty} dq \; e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq$

$$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2 q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2 q^2 + Jq} \left(1 - \frac{\lambda}{4!}q^4 + \left(\frac{\lambda}{4!}\right)^2 q^8 \right)$$

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$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$$

$$Z = \int D\varphi \; e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$$

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$$\int dq \; e^{-\frac{1}{2}m^2q^2 + Jq} \, q^4$$

$$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$$

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$$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$$

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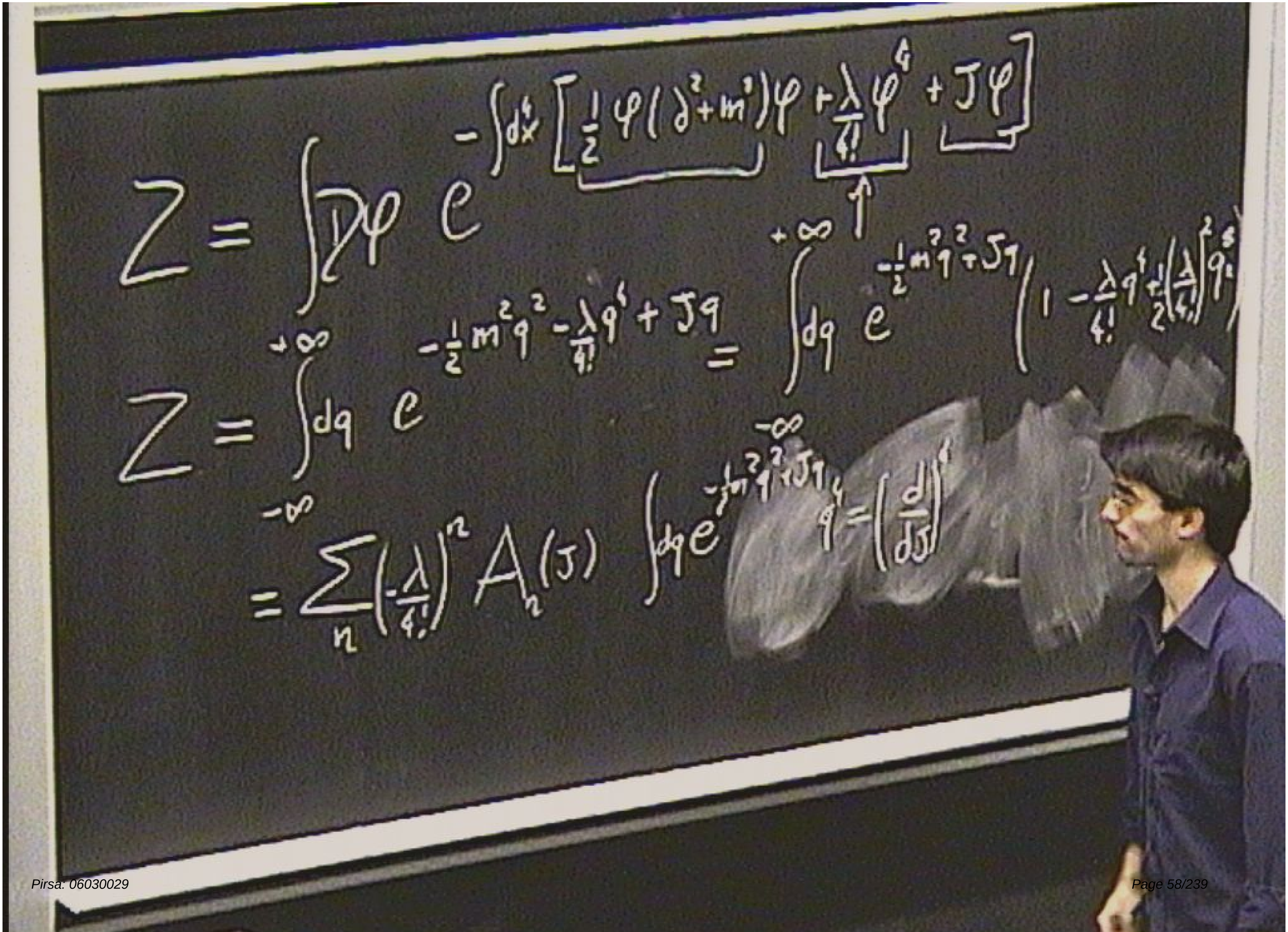
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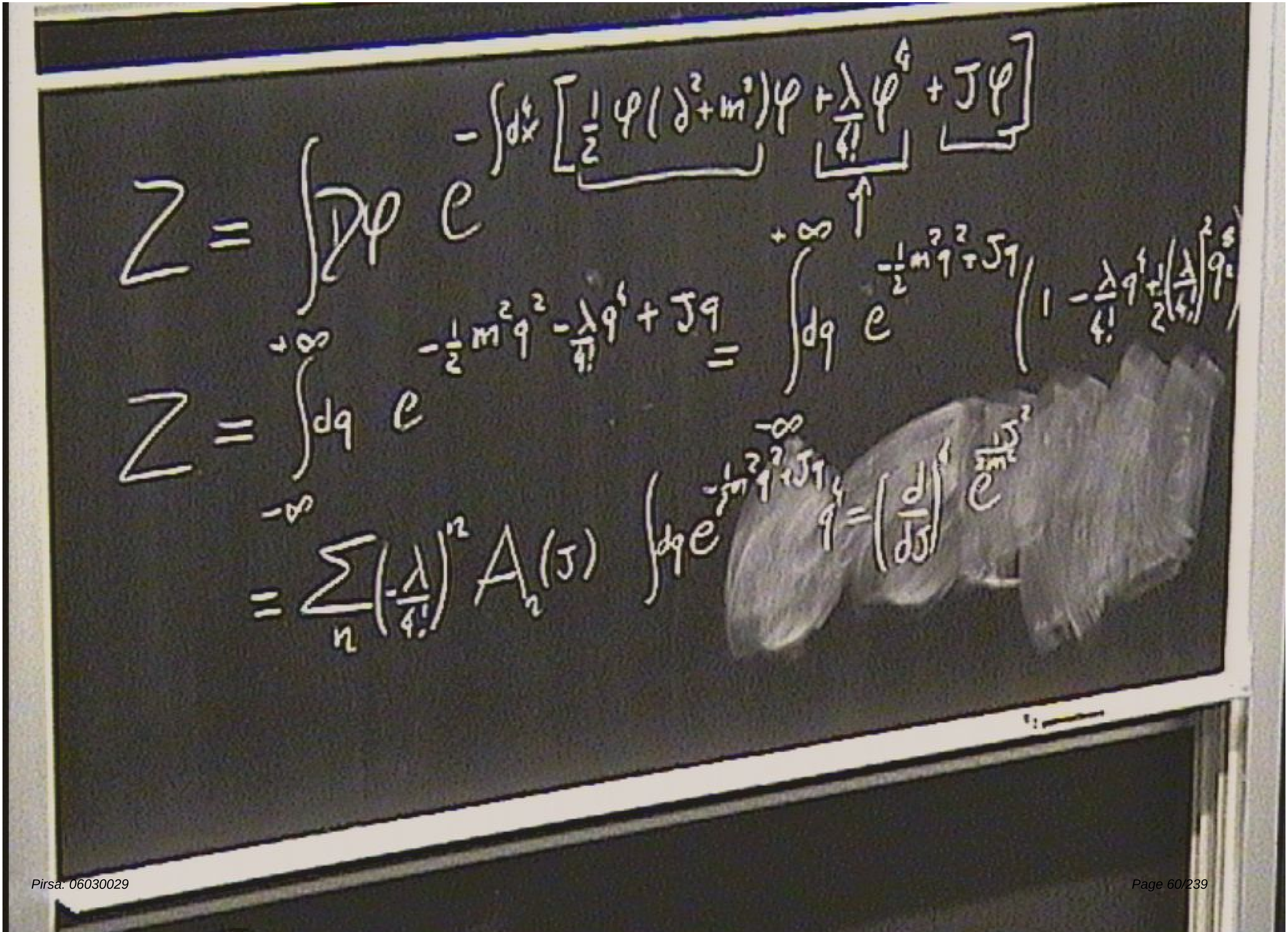
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$$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$$

$$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8 \cdots\right)$$

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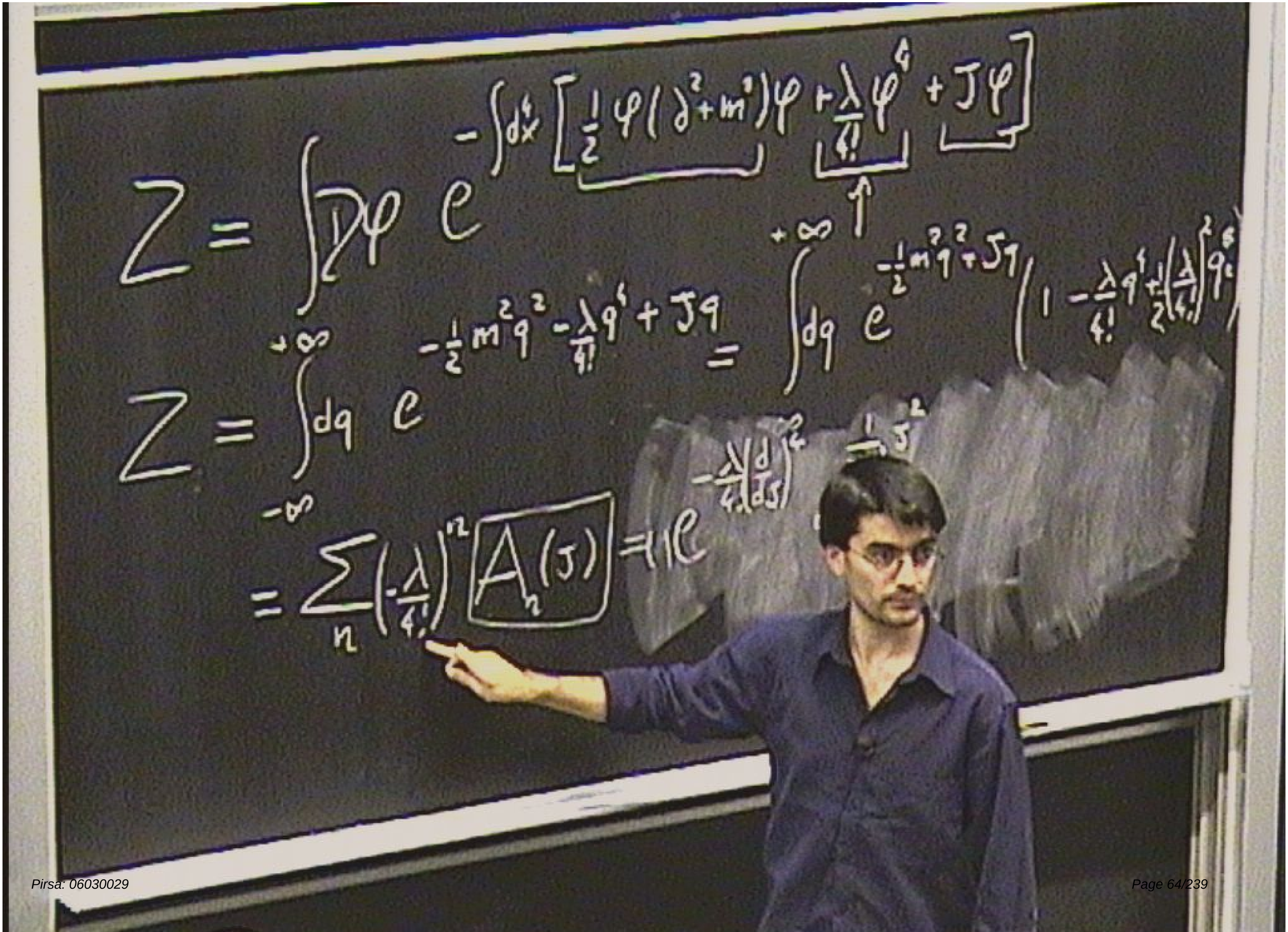
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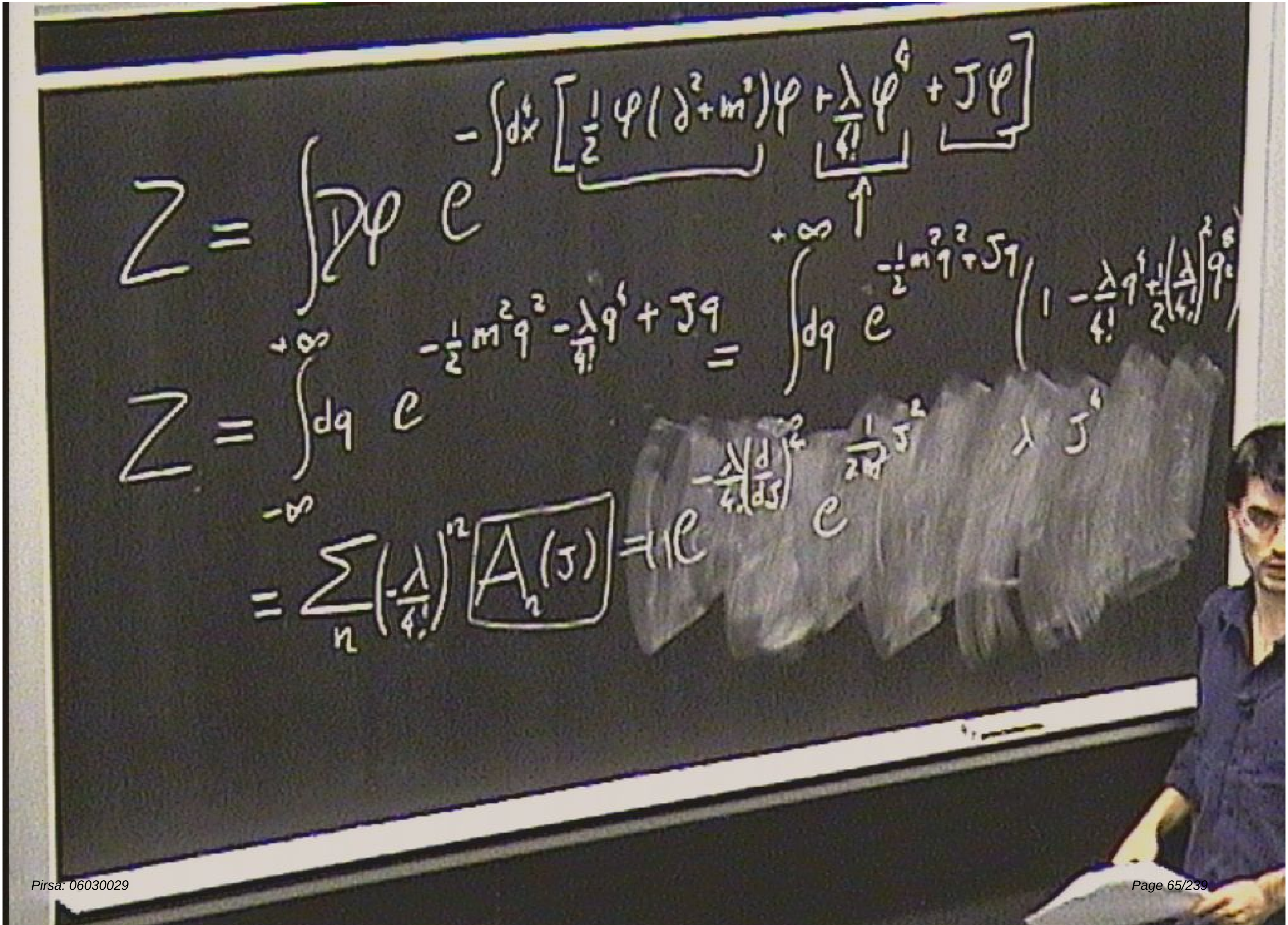
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$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$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J) = e^{-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2m^2}J^2}$
$\lambda \; J^4$

$$Z = \int D\varphi \; e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2 + m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq \; e^{-\frac{1}{2}m^2 q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \; e^{-\frac{1}{2}m^2 q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8 \ldots\right)$$

$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(J)} = e^{-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2m^2}J^2}$$

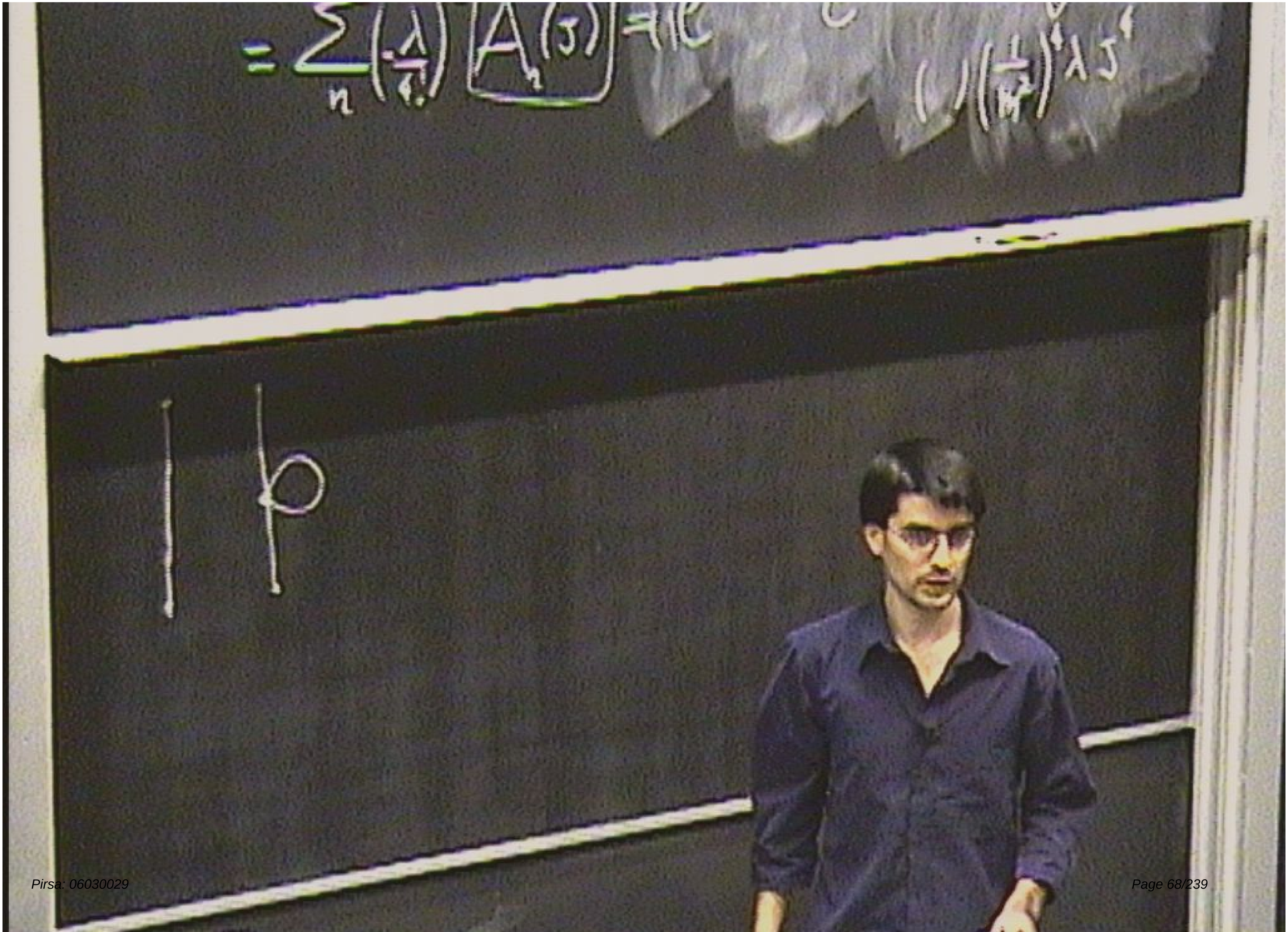
$$\lambda J^4 \;\downarrow\; \left(\frac{1}{m^2}\right)^4 \lambda J^4$$

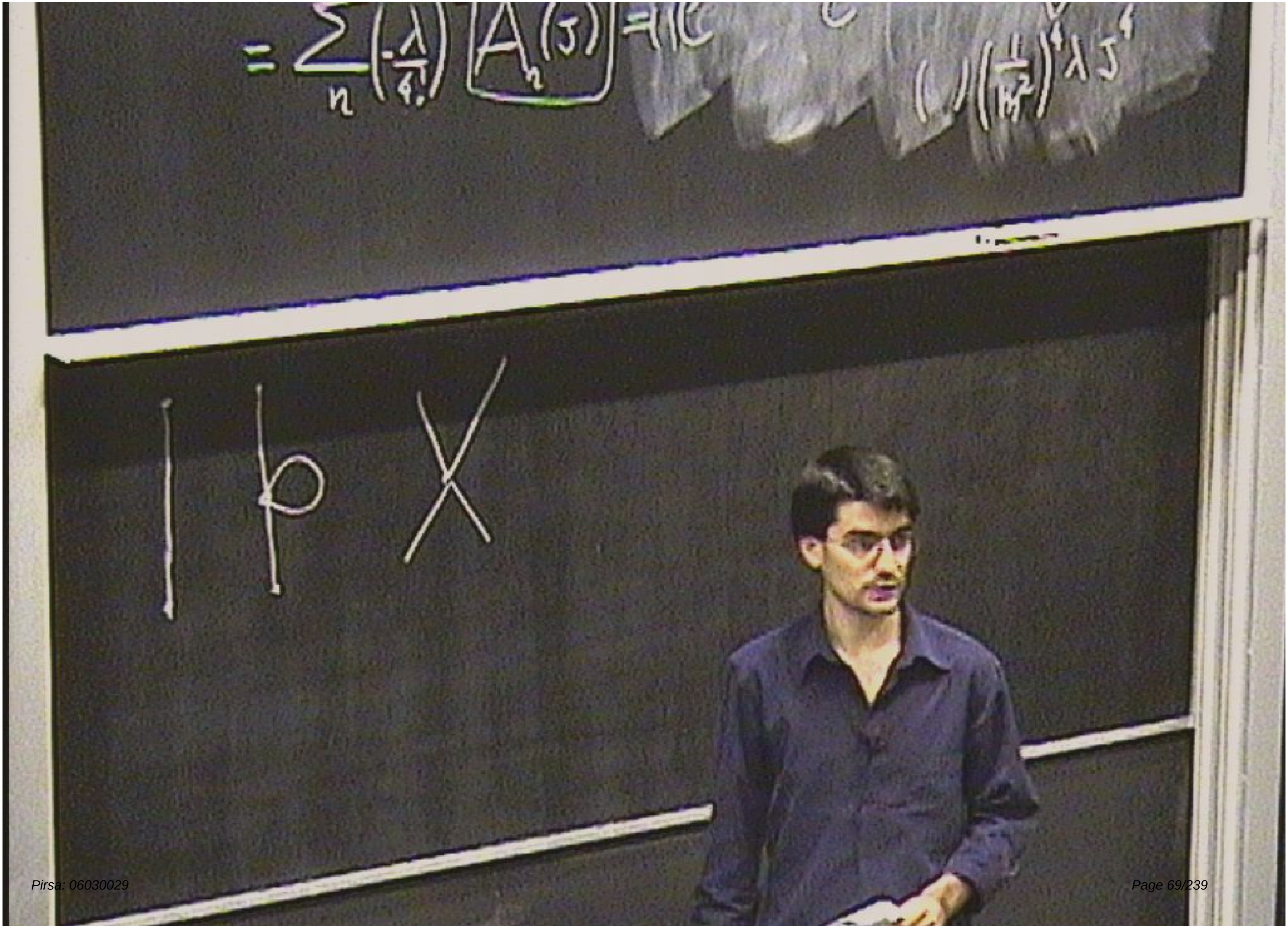
$$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$$

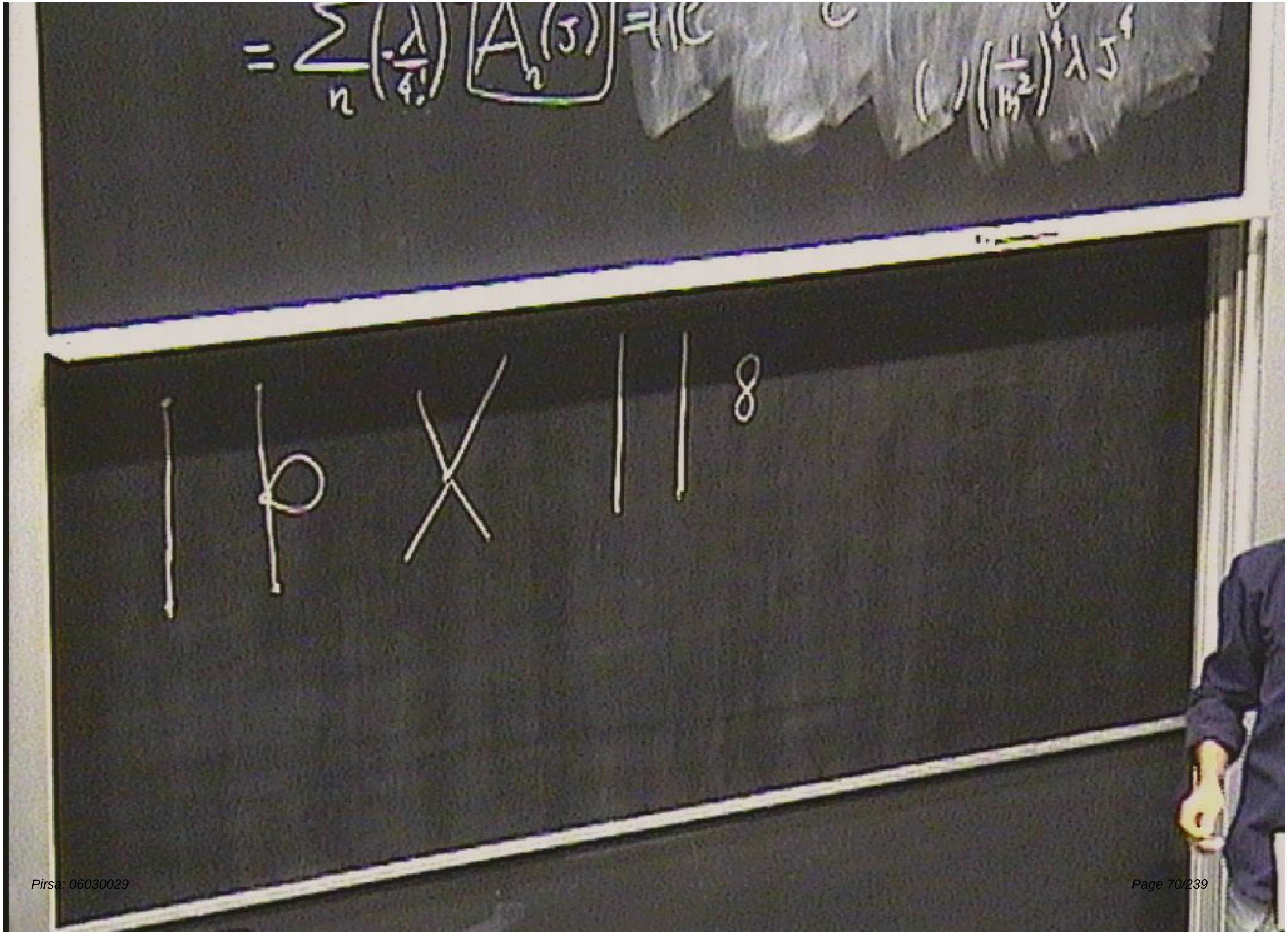
$$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8\right)$$

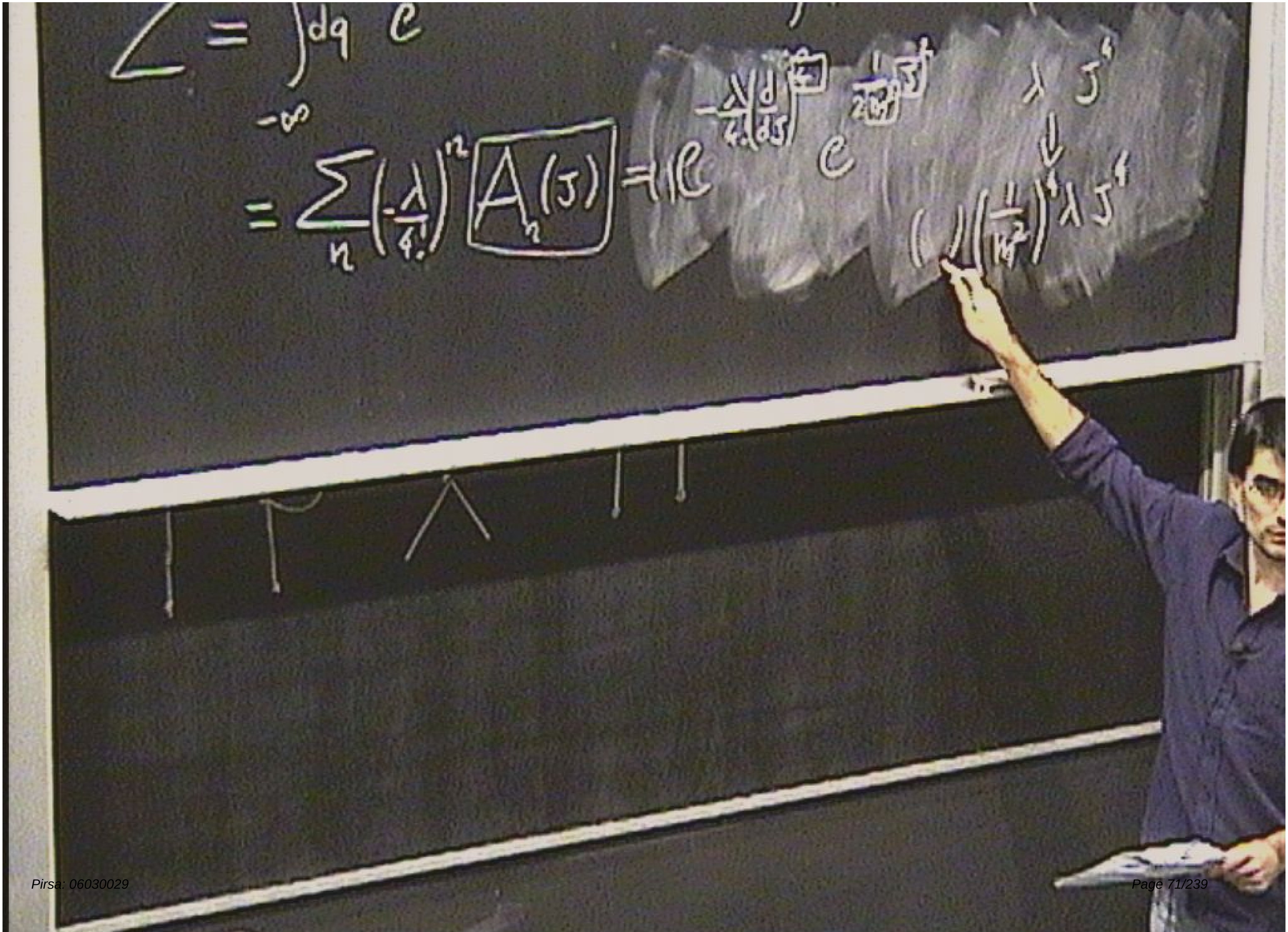
$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J) = \left(\right) e^{-\frac{\lambda}{4!}\left(\frac{\partial}{\partial J}\right)^4} e^{\frac{1}{2m^2}J^2}$$

$$\lambda J^4 \Downarrow \left(\;\right)\left(\frac{1}{m^2}\right)^4 \lambda J^4$$






= Σ_n (-λ/4!) A_n(J)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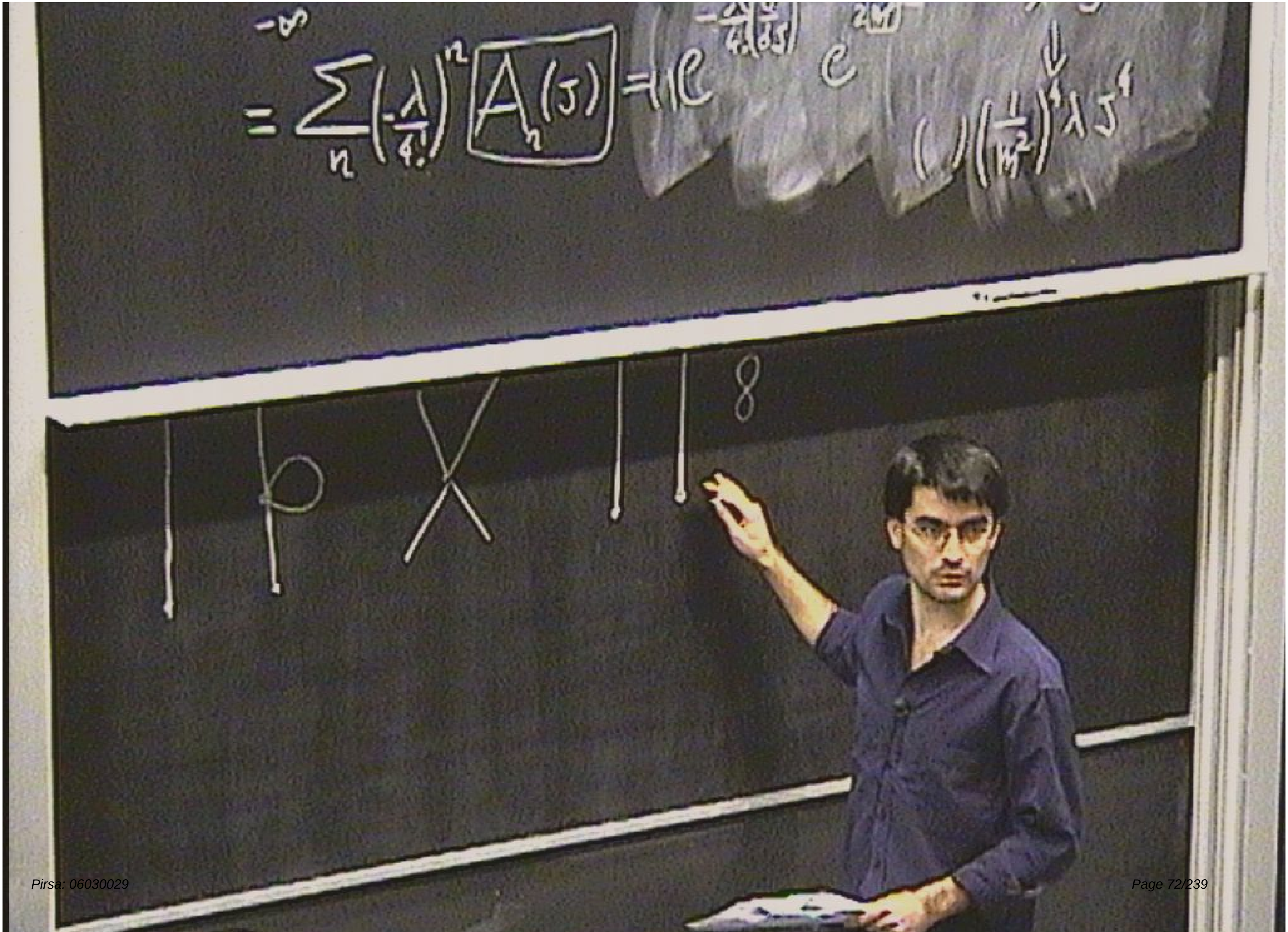
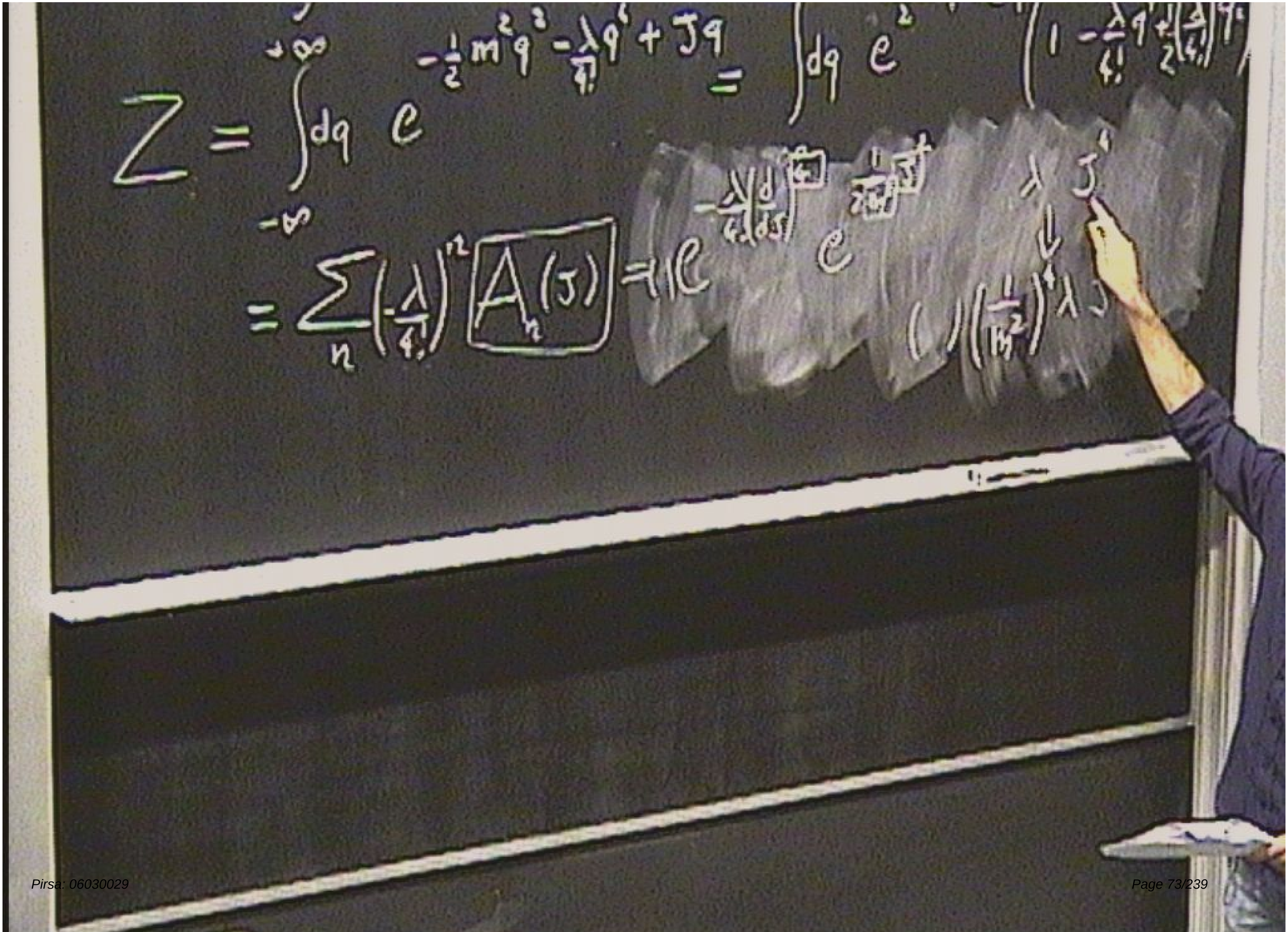
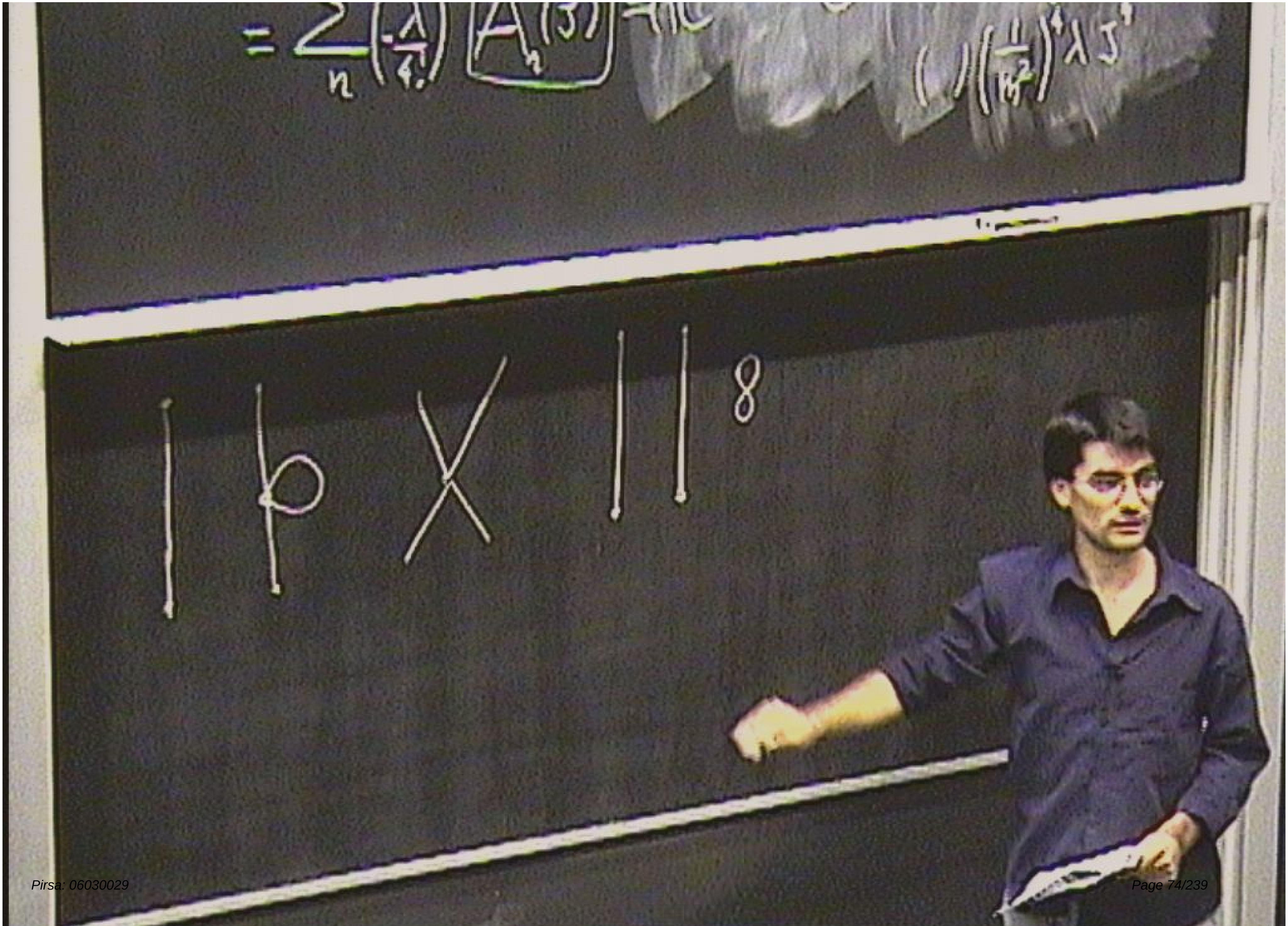
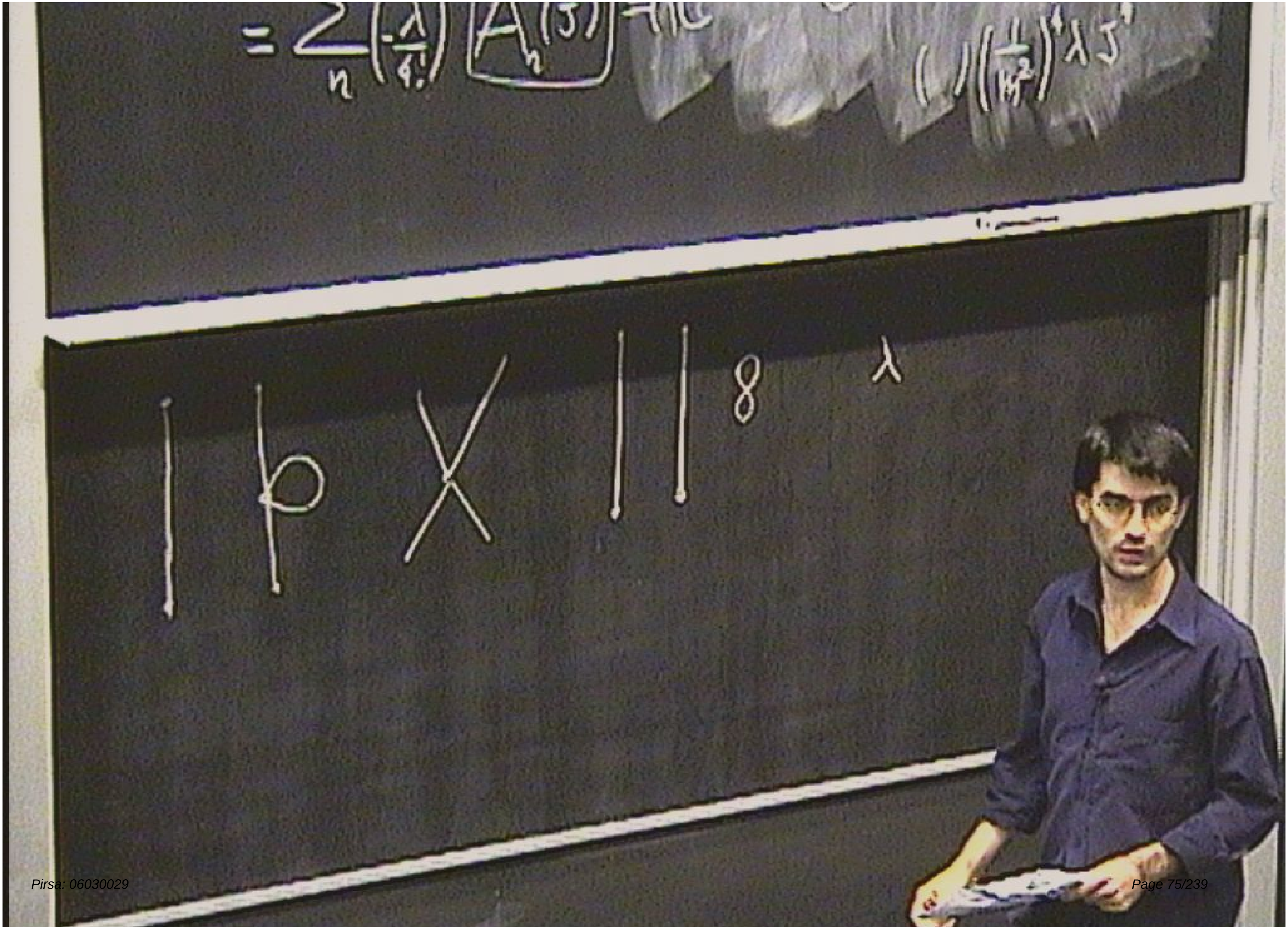
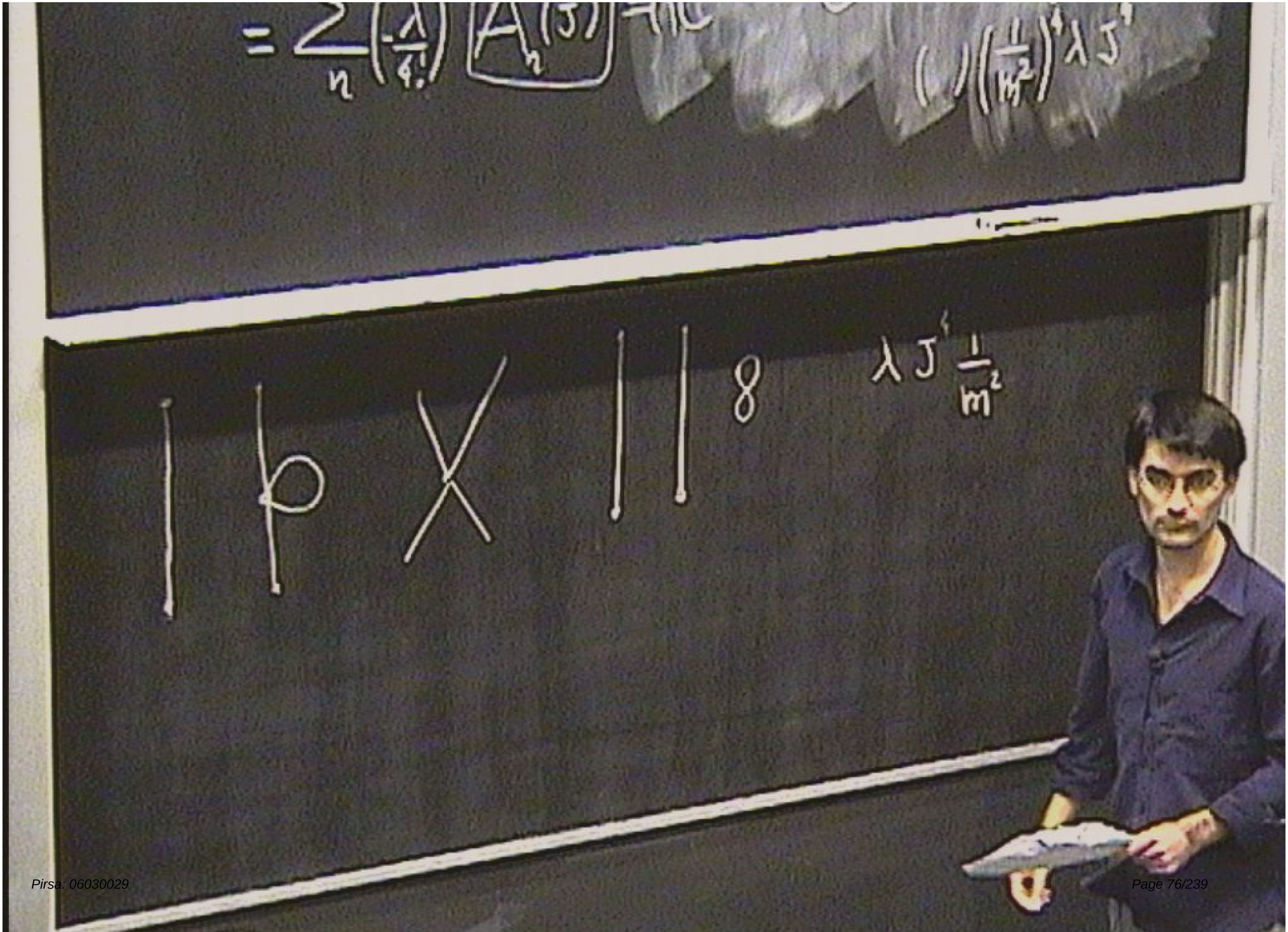
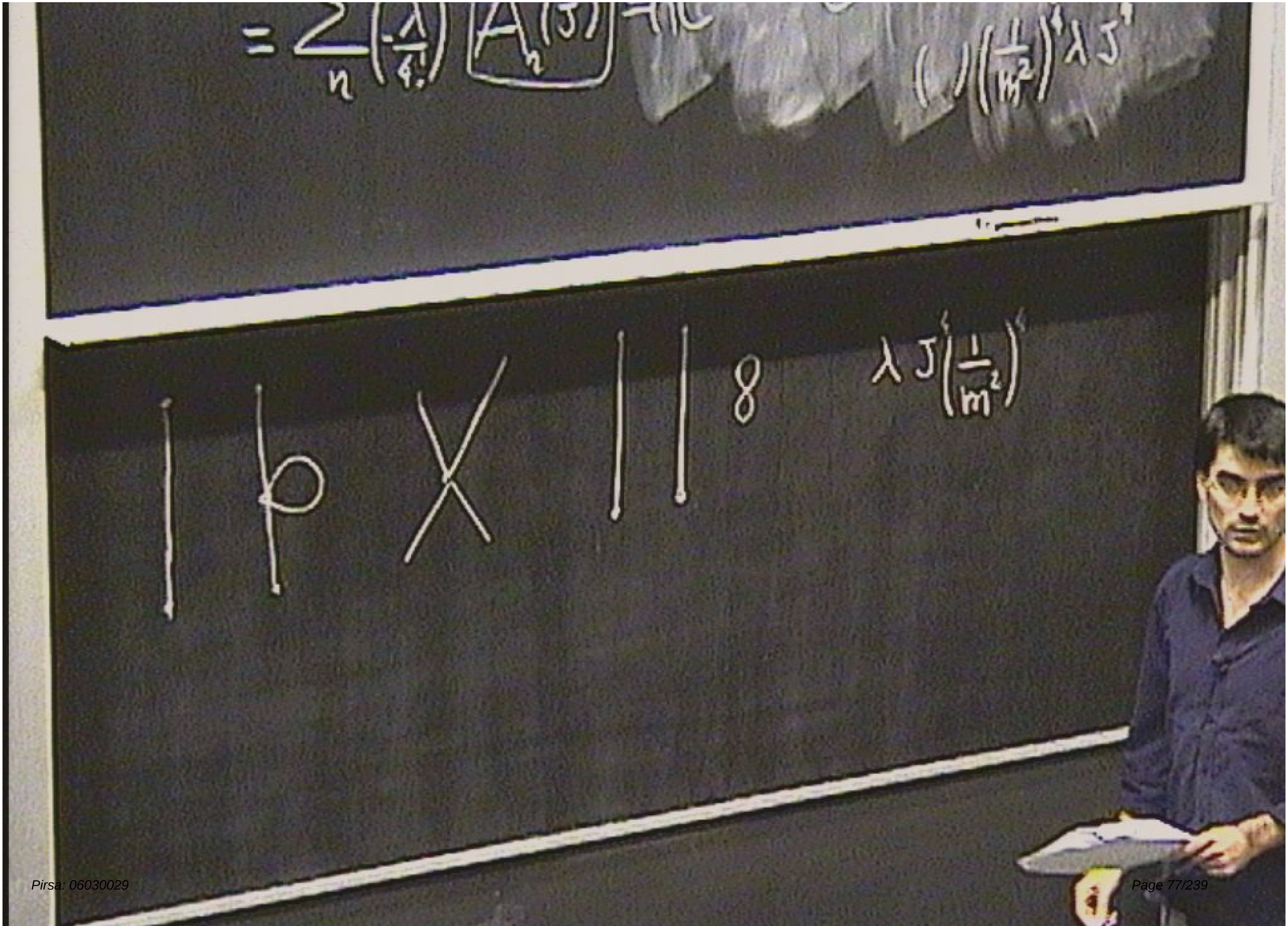


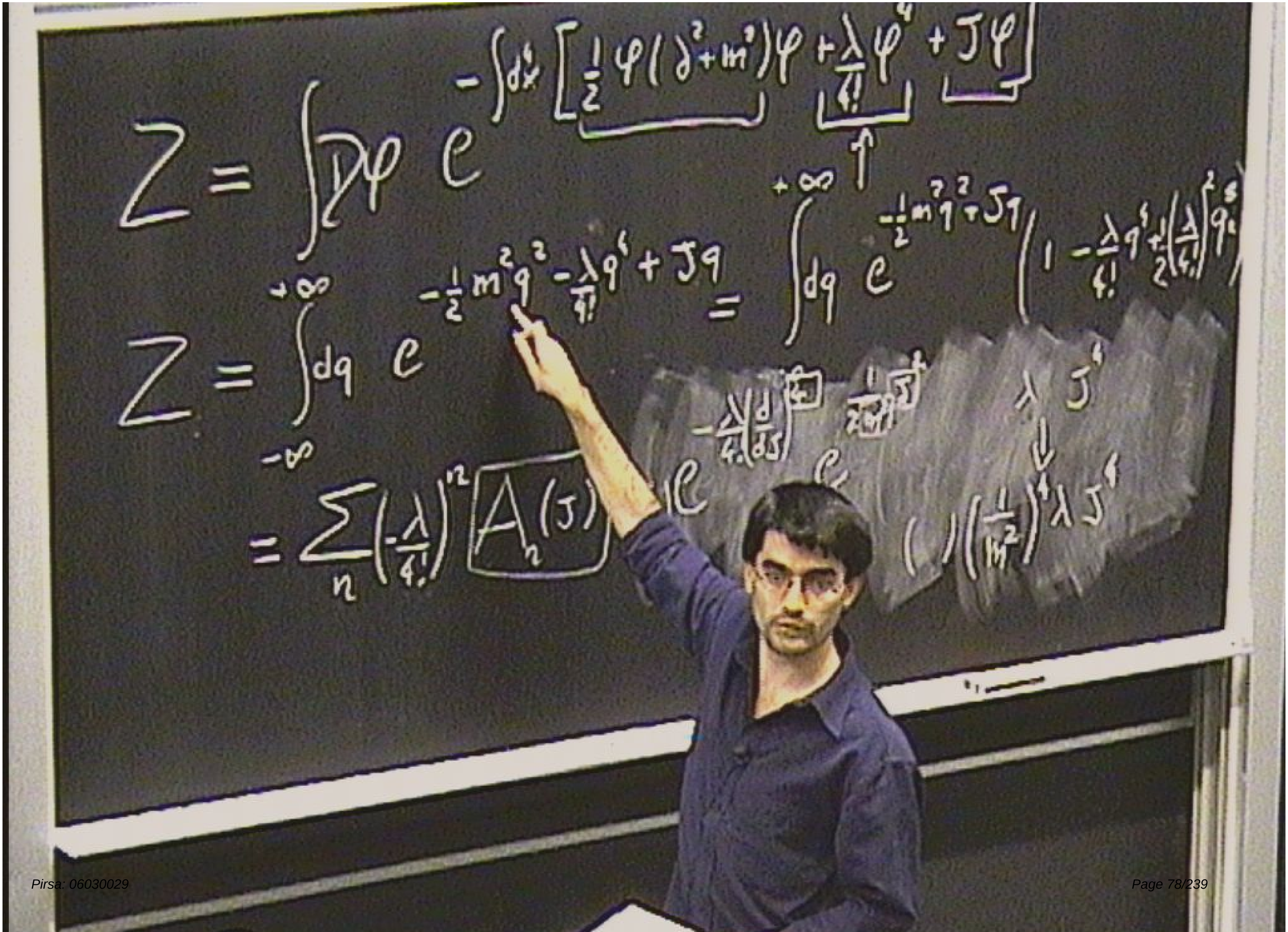

$Z = \int_{-\infty}^{\infty} dq\, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq}$
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$

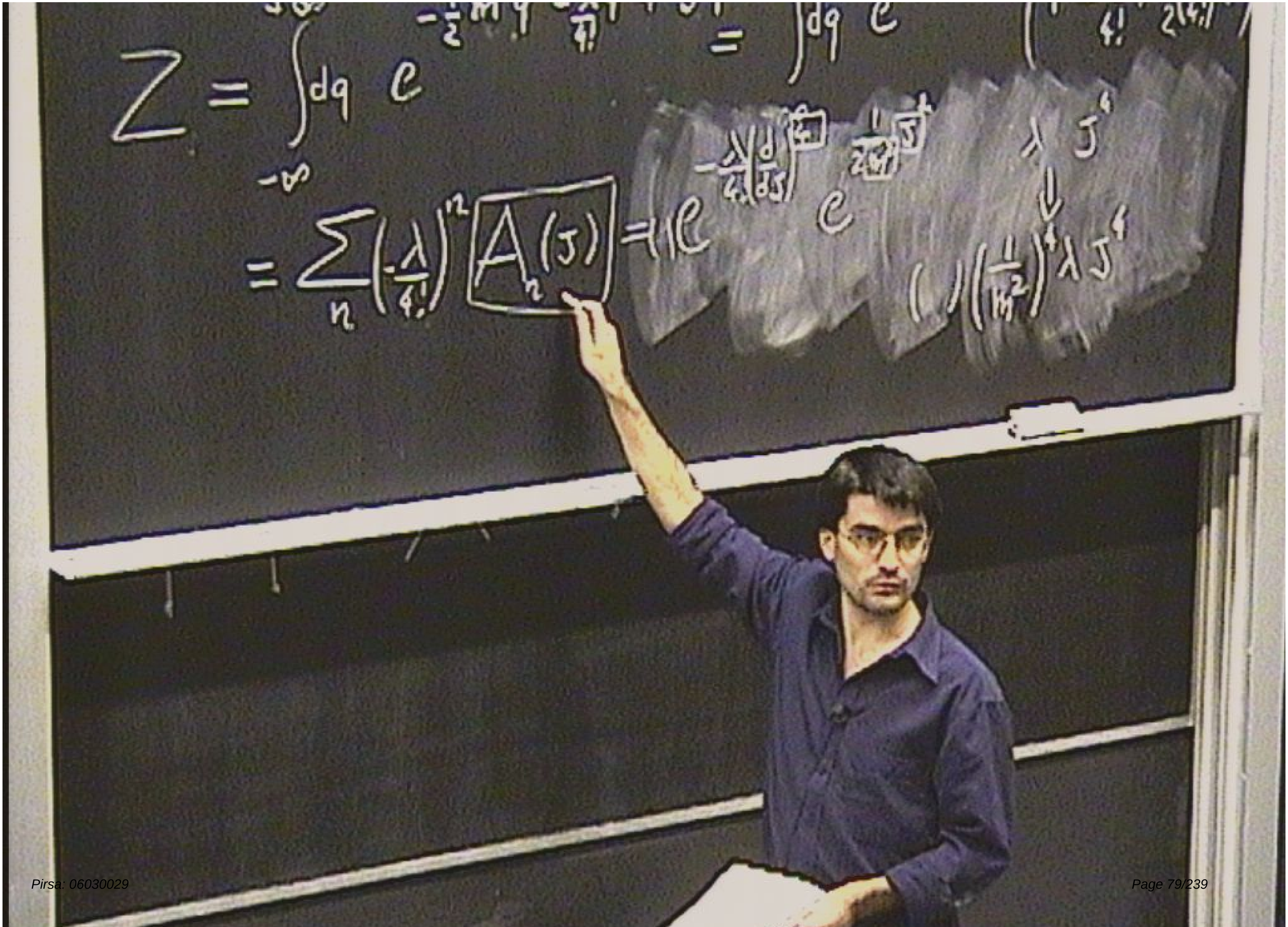


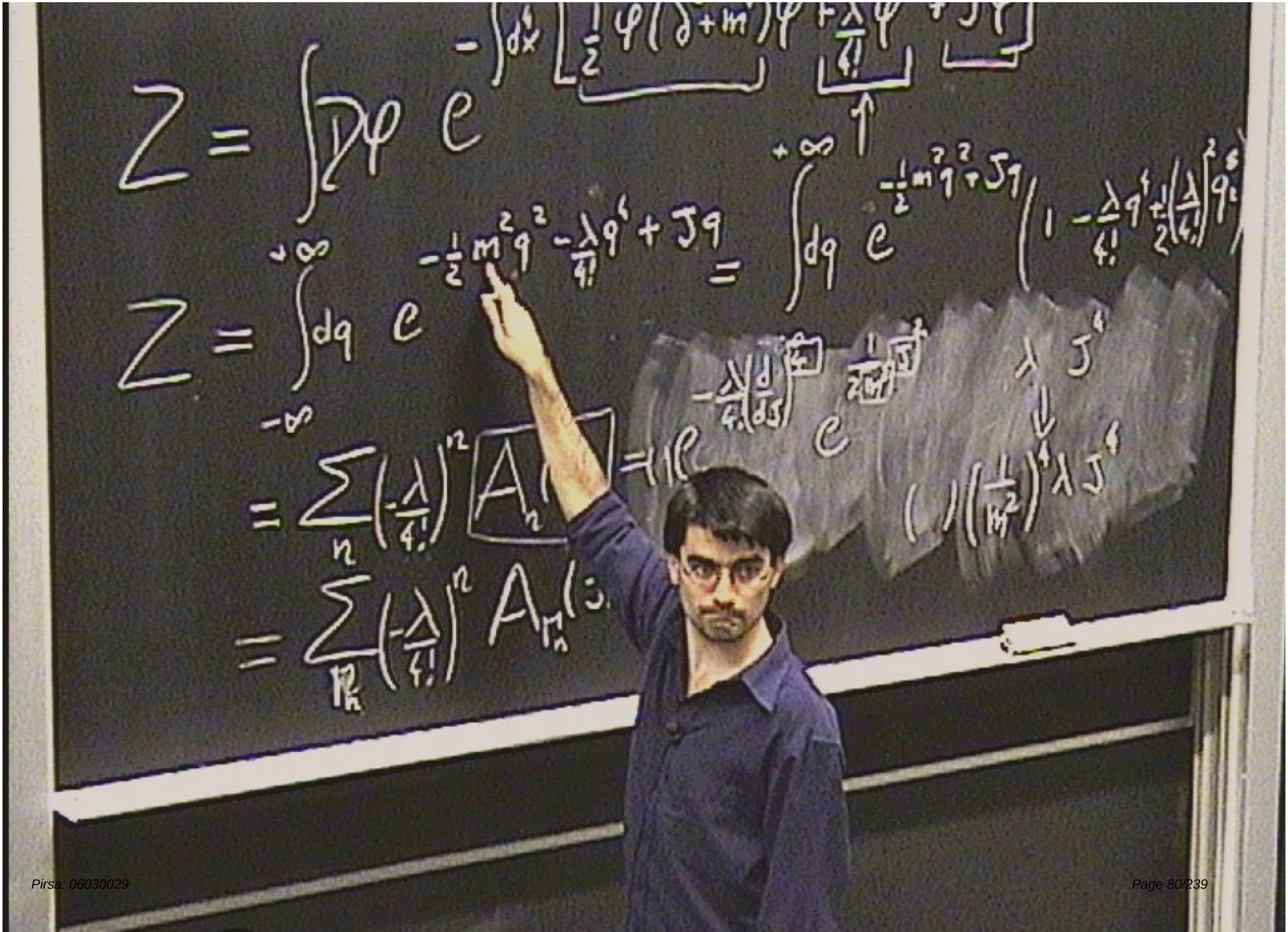



= $\sum_n \left(-\frac{\lambda}{4!}\right) A_n(J)$
$\left(\frac{1}{m^2}\right)^4 \lambda J^4$
8
$\lambda J^4 \frac{1}{m^2}$




Z = ∫Dφ e^{-∫d^4x [½ φ(∂² + m²)φ + λ/4! φ⁴ + Jφ]}
Z = ∫_{-∞}^{+∞} dq e^{-½ m² q² - λ/4! q⁴ + Jq} = ∫_{-∞}^{+∞} dq e^{-½ m² q² + Jq} (1 - λ/4! q⁴ + ½ (λ/4!)² q⁸
= Σ_n (-λ/4!)^n A_n(J)




Z = ∫Dφ e
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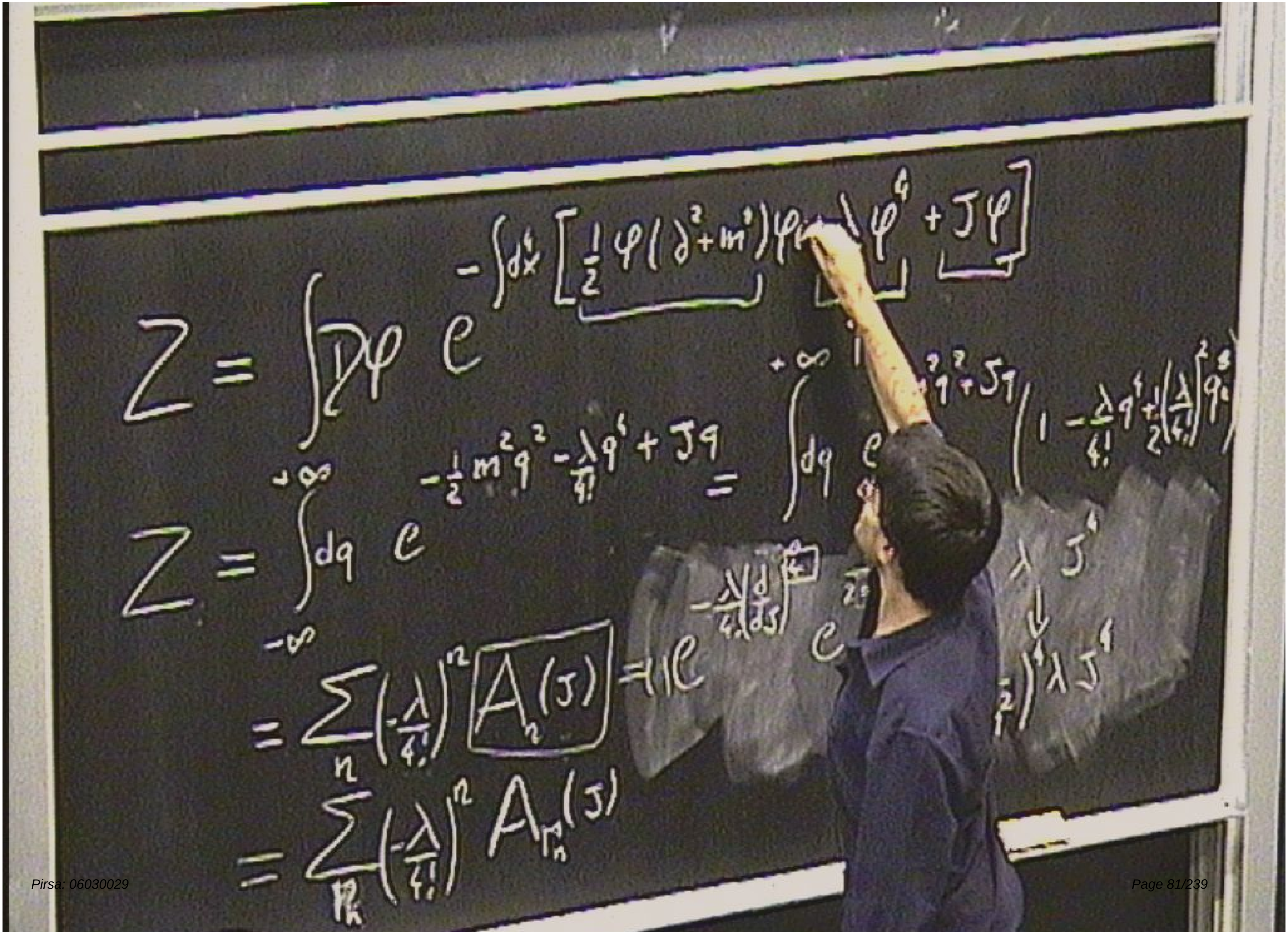
= Σ (-λ/4!)^n A_n
= Σ (-λ/4!)^n A_n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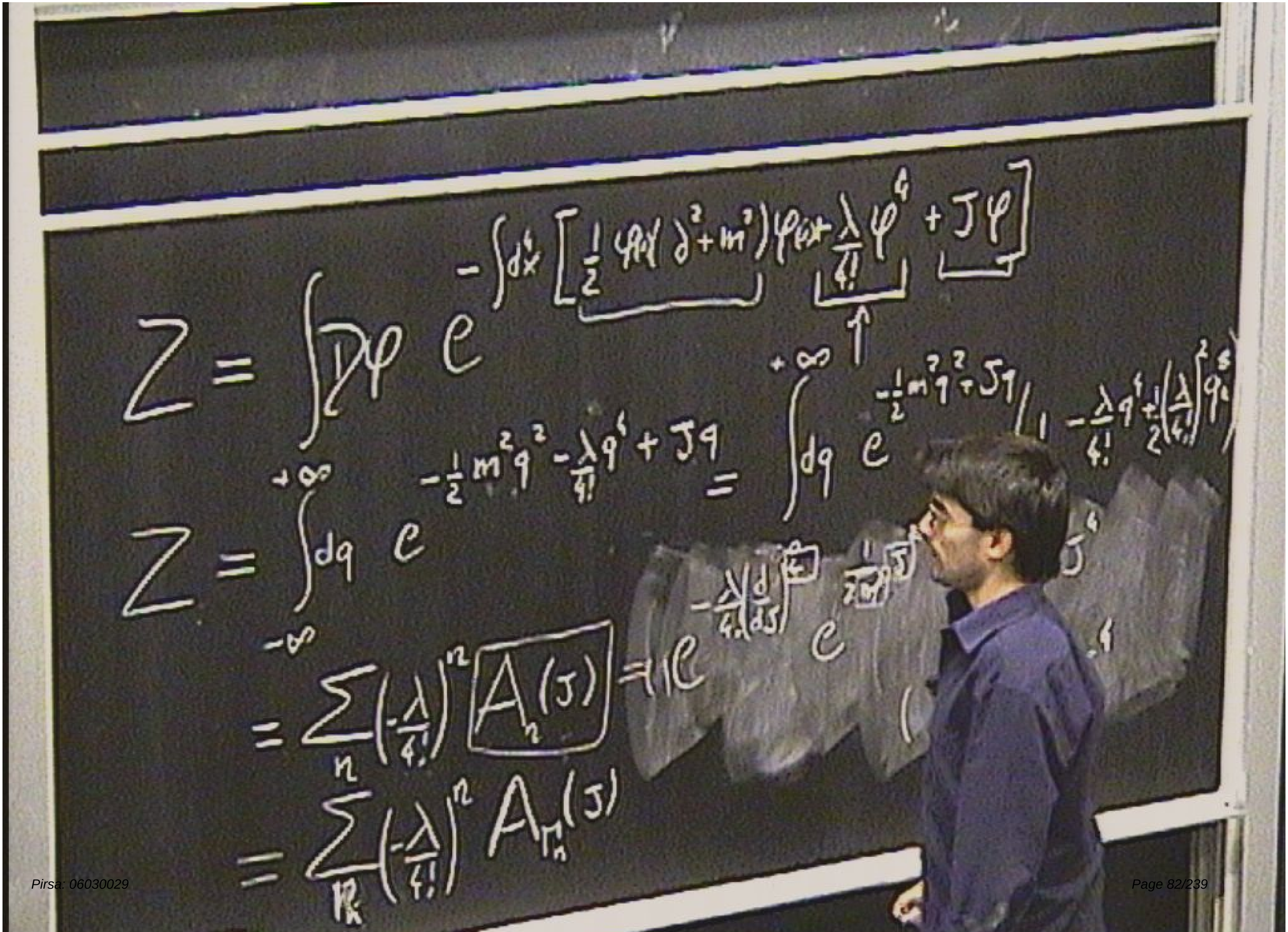
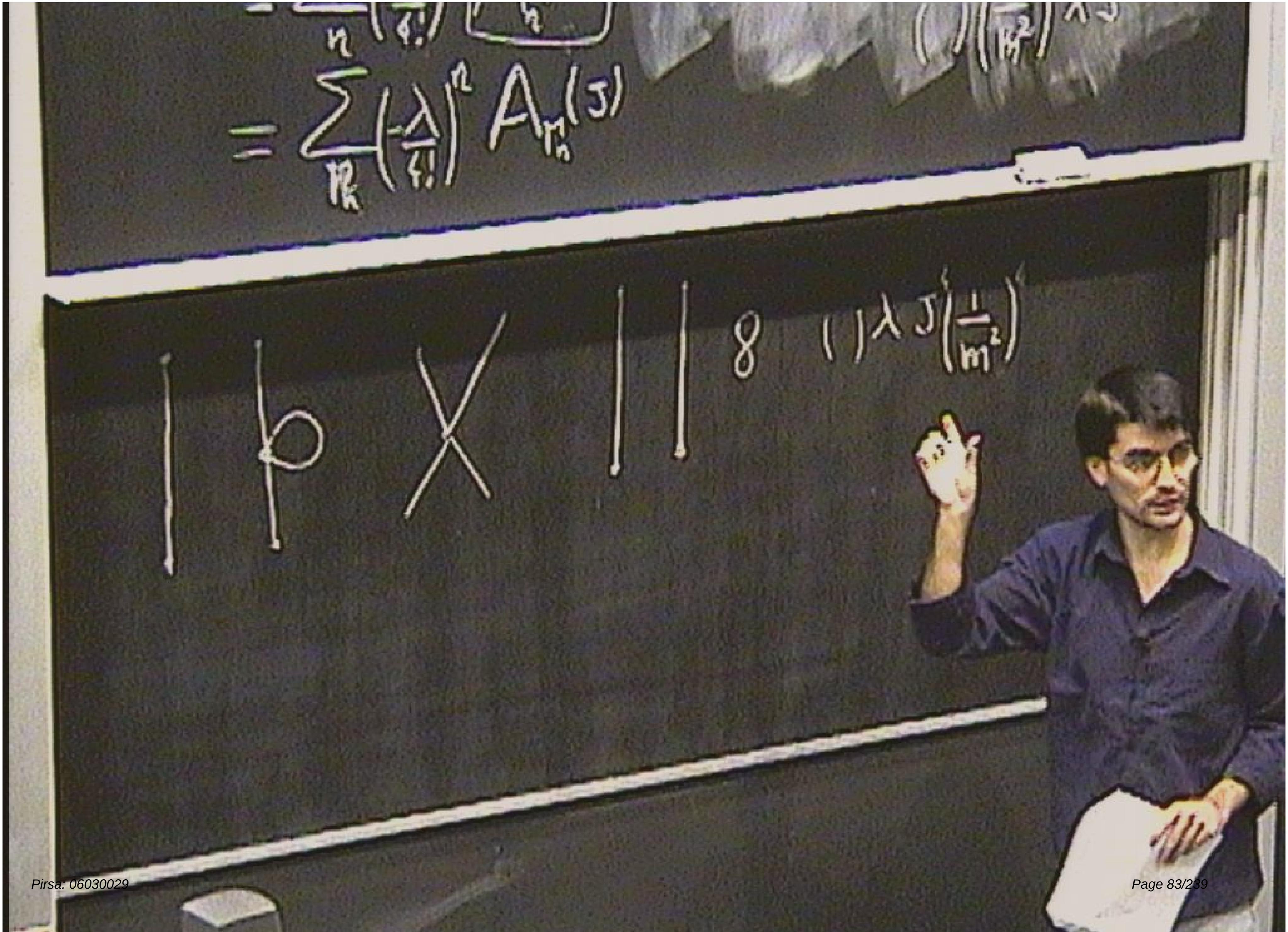
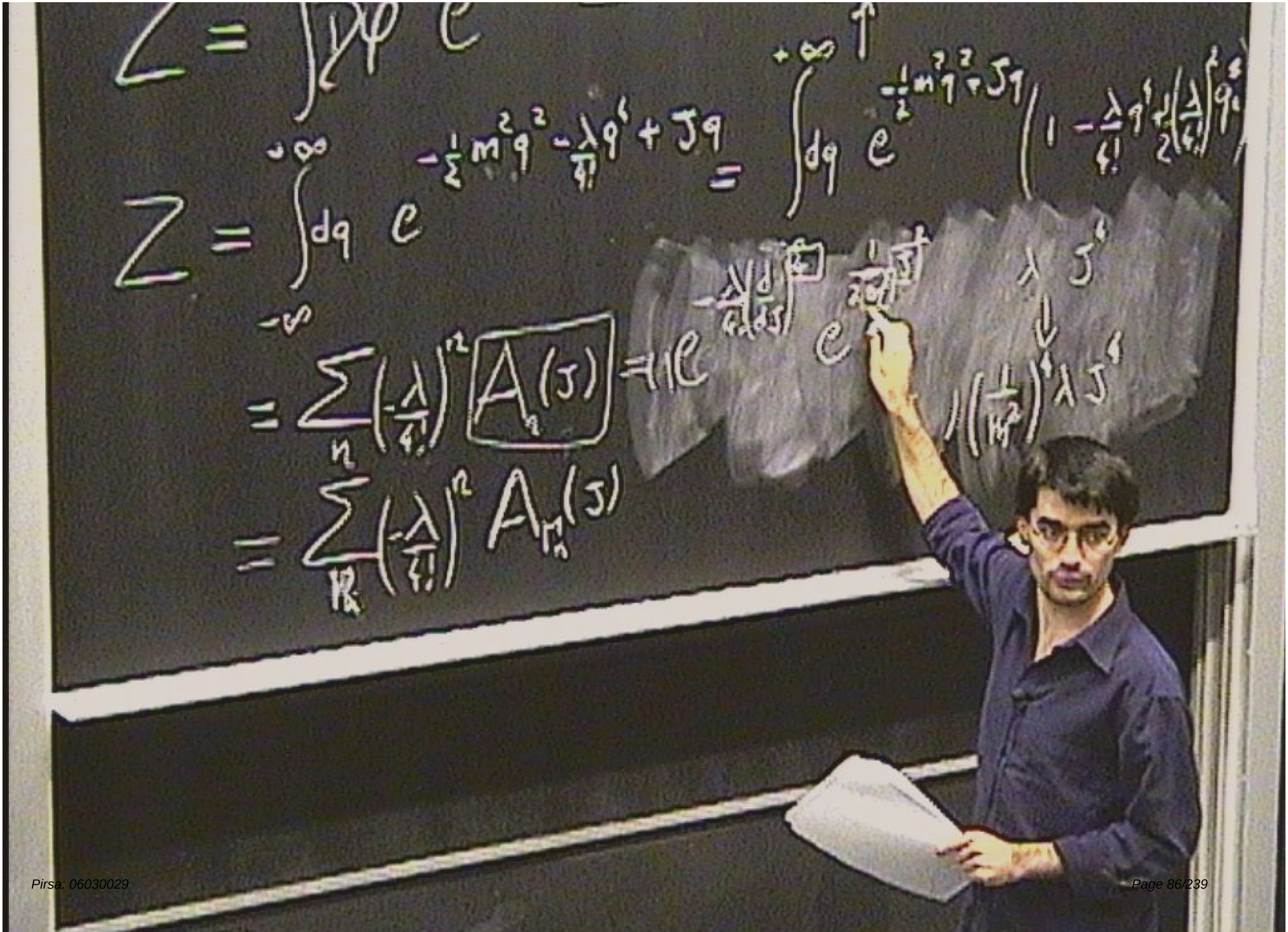

$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(x)(\partial^2+m^2)\varphi(x) + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8\right)$
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$
$= \sum_{n} \left(-\frac{\lambda}{4!}\right)^n A_n(J)$

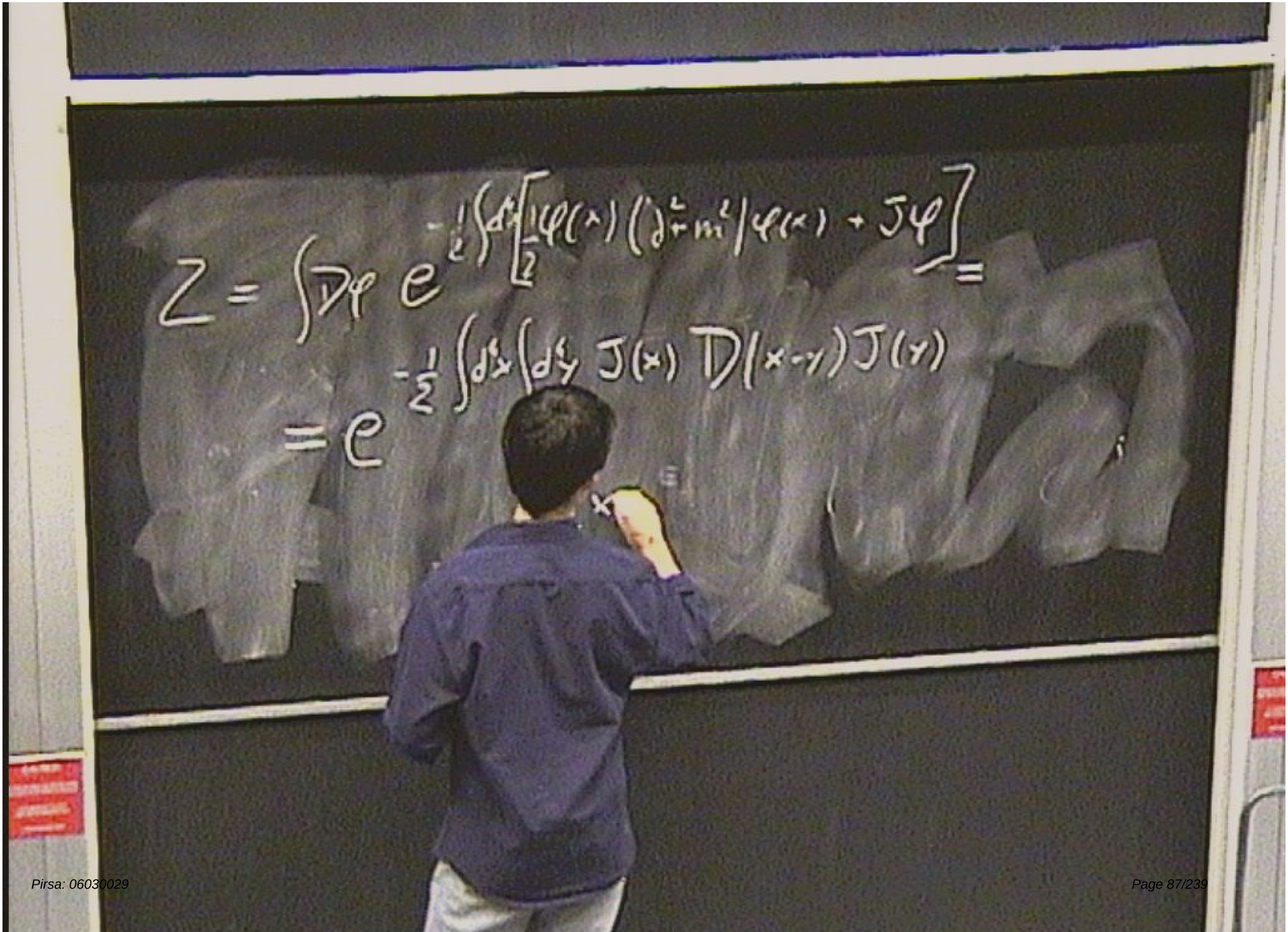


$$Z = \int \mathcal{D}\varphi \, e^{-i\int d^4x \left[\frac{1}{2}\varphi(x)\left(\partial^2 + m^2\right)\varphi(x) + J\varphi\right]} =$$

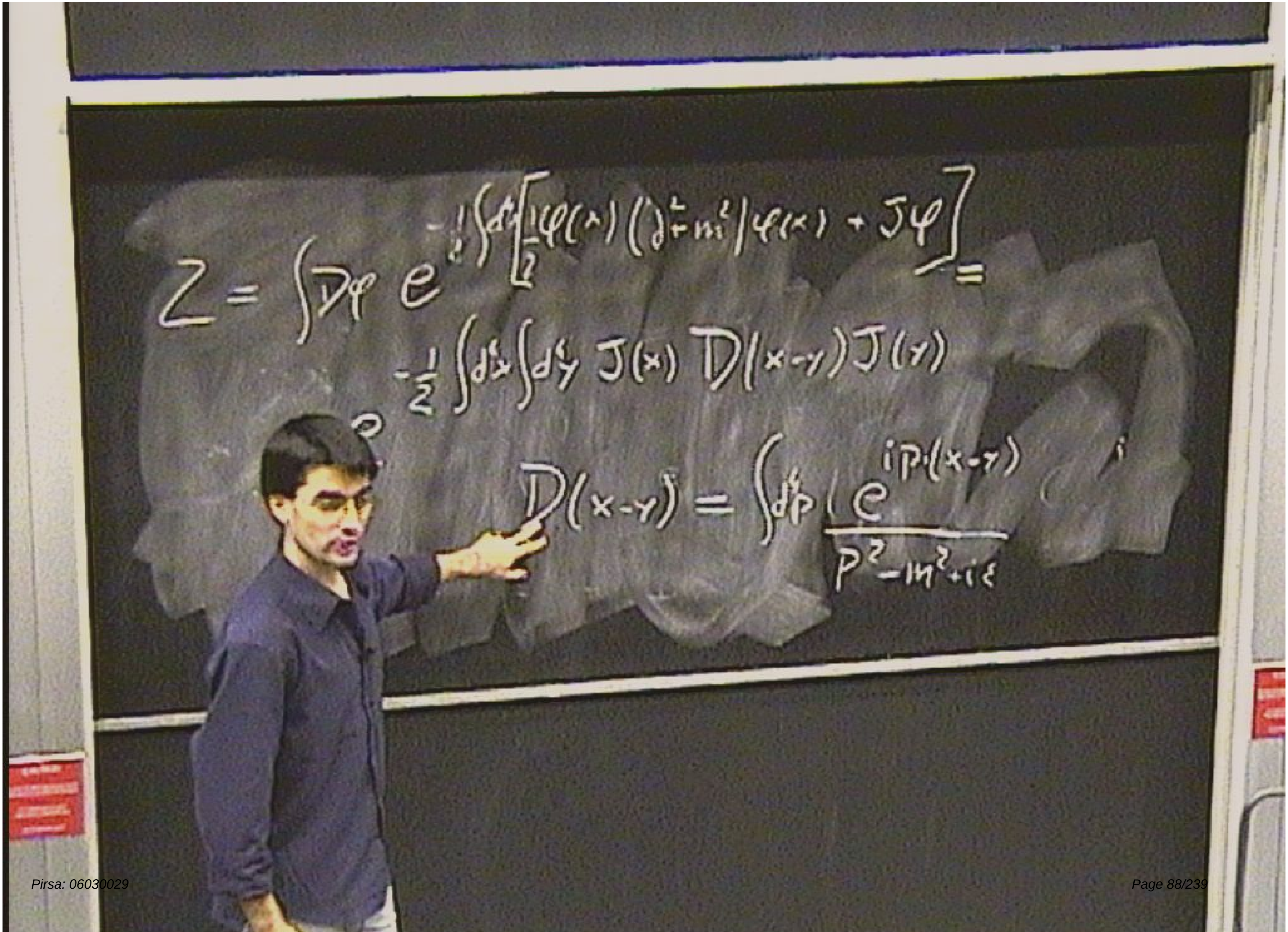
$$Z = \int D\varphi \, e^{-\frac{i}{\hbar}\int d^4x \left[\frac{1}{2}\varphi(x)\left(\partial^2 + m^2\right)\varphi(x) + J\varphi\right]} =$$

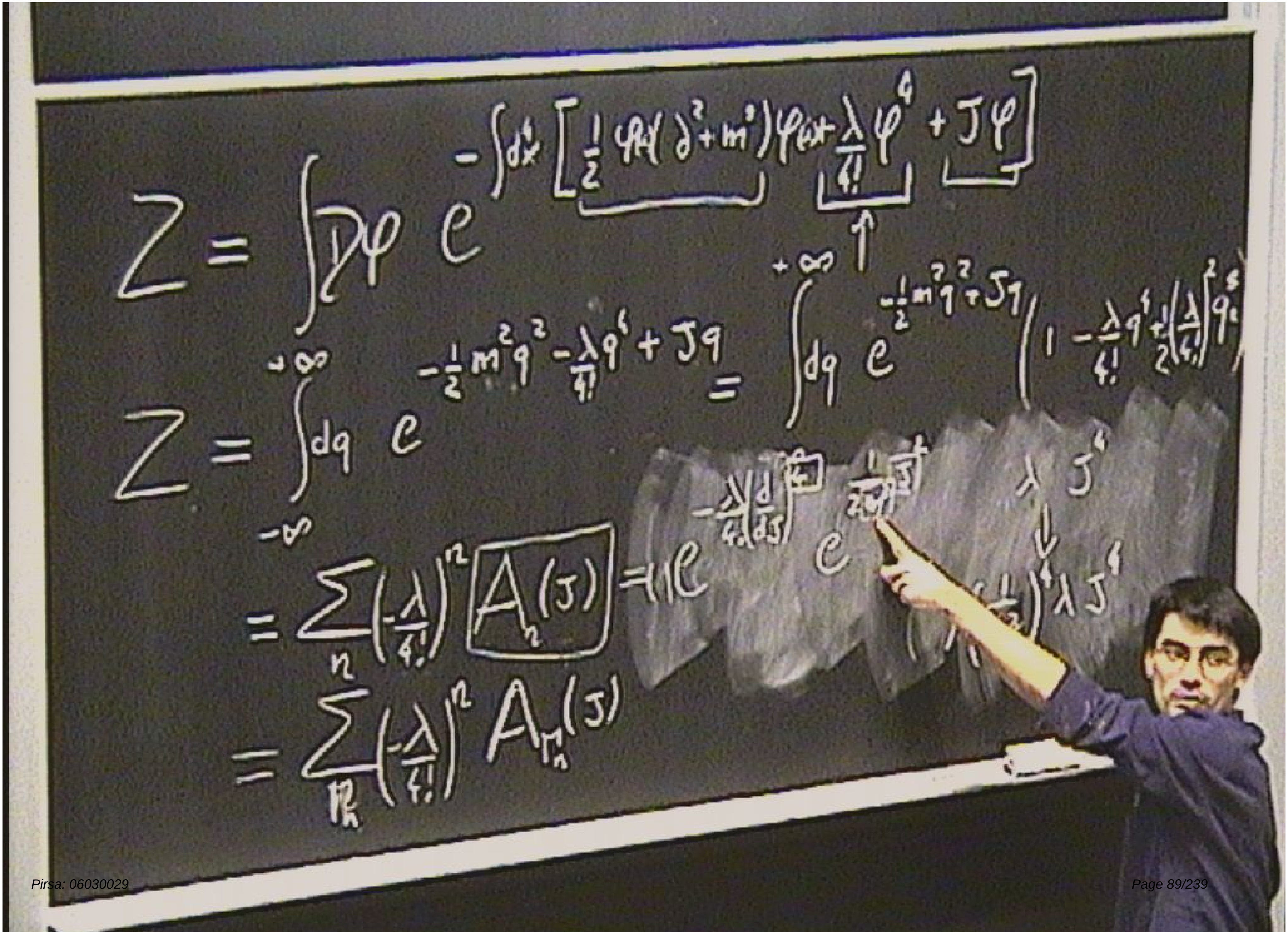
$$= e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x)\, D(x-y)\, J(y)}$$



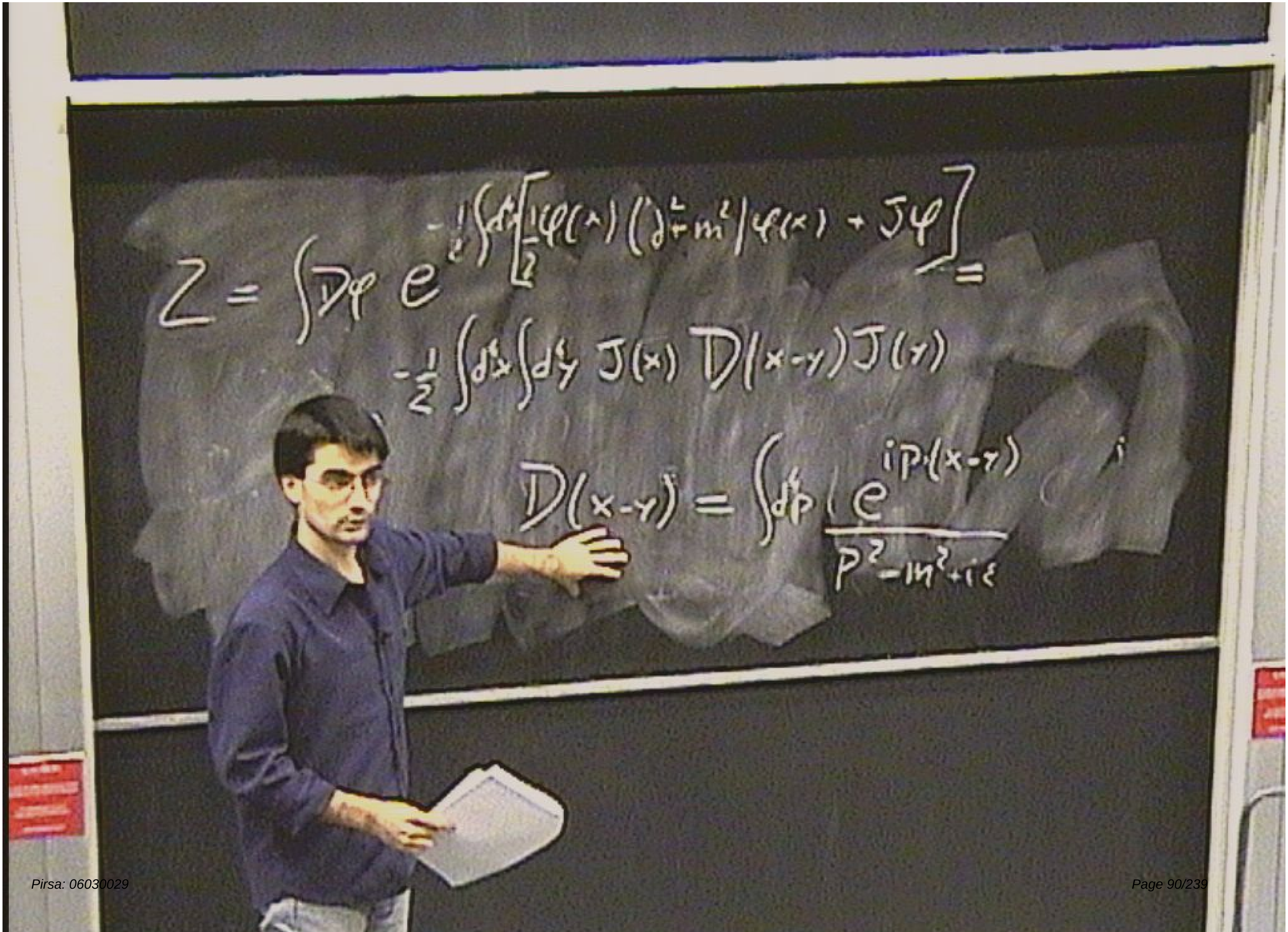


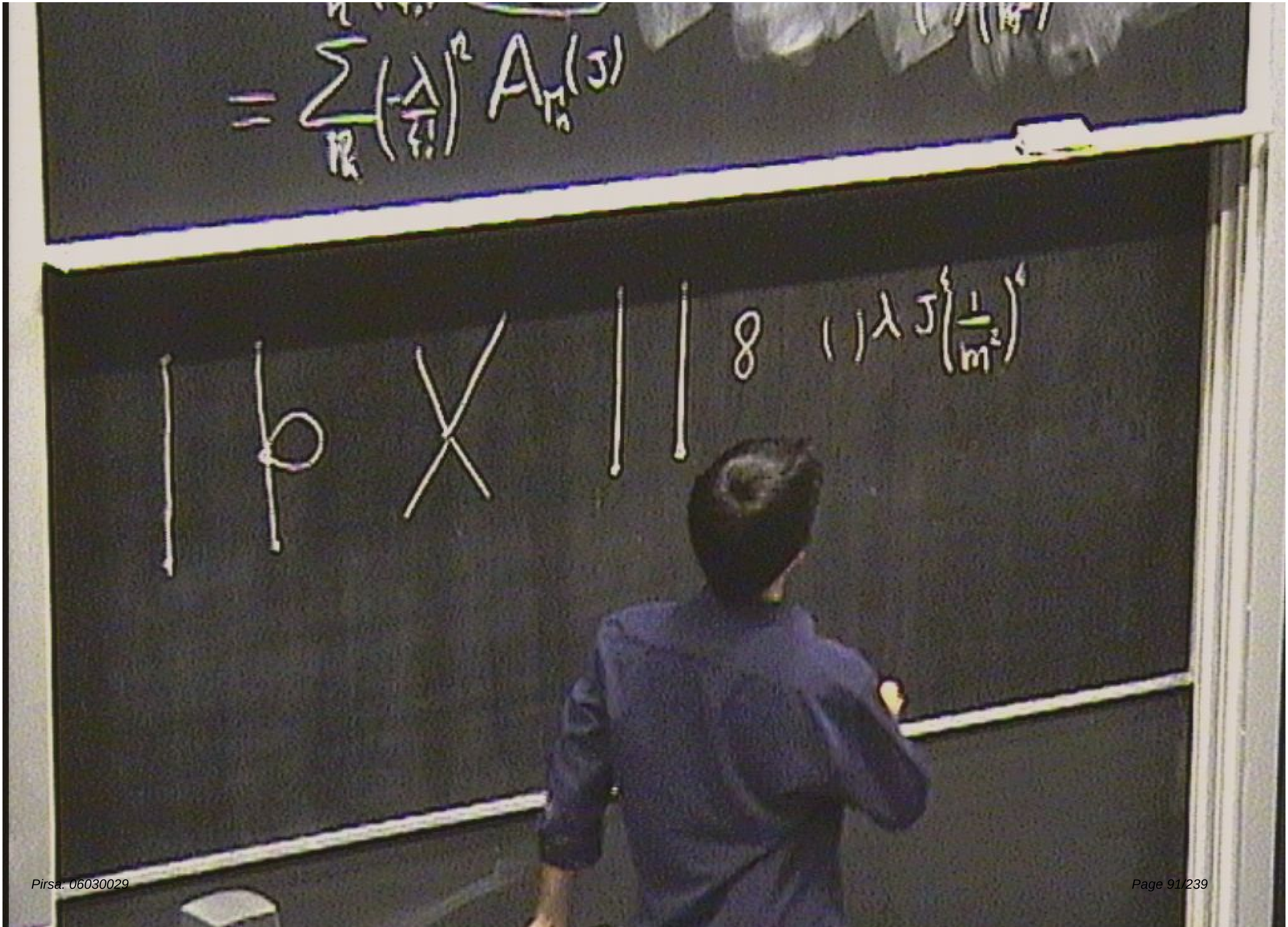
$Z = \int D\varphi \, e^{-\frac{1}{2}\int d^4x \left[\frac{1}{2}\varphi(x)(\partial^2+m^2)\varphi(x) + J\varphi\right]} =$
$= e^{-\frac{1}{2}\int d^4x \int d^4y \, J(x) D(x-y) J(y)}$

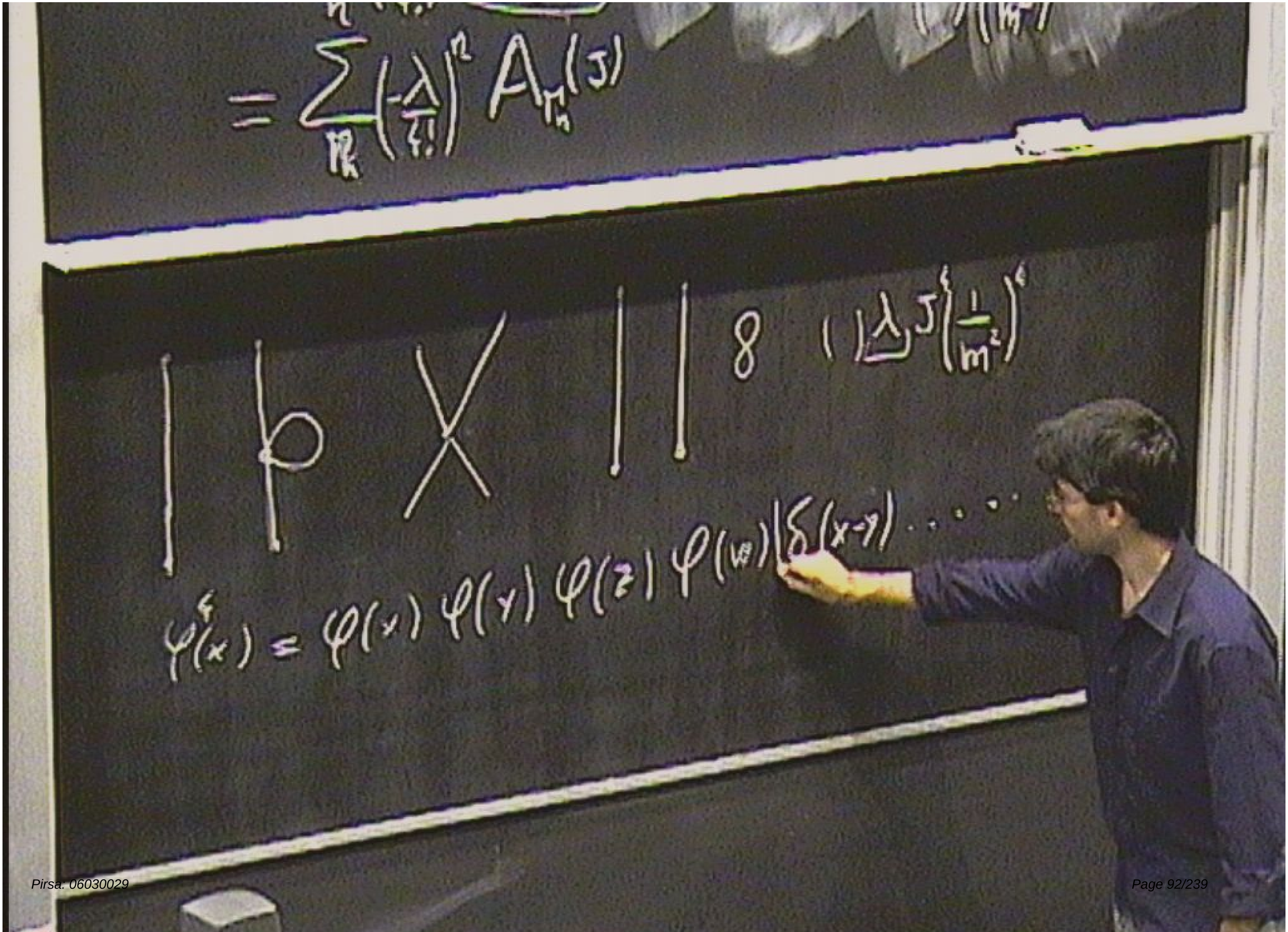

Z = ∫Dφ e^{i∫d⁴x[½φ(x)(∂²+m²)φ(x) + Jφ]} =
e^{-½∫d⁴x∫d⁴y J(x) D(x-y) J(y)}
D(x-y) = ∫d⁴p i e^{ip·(x-y)} / (p²-m²+iε)

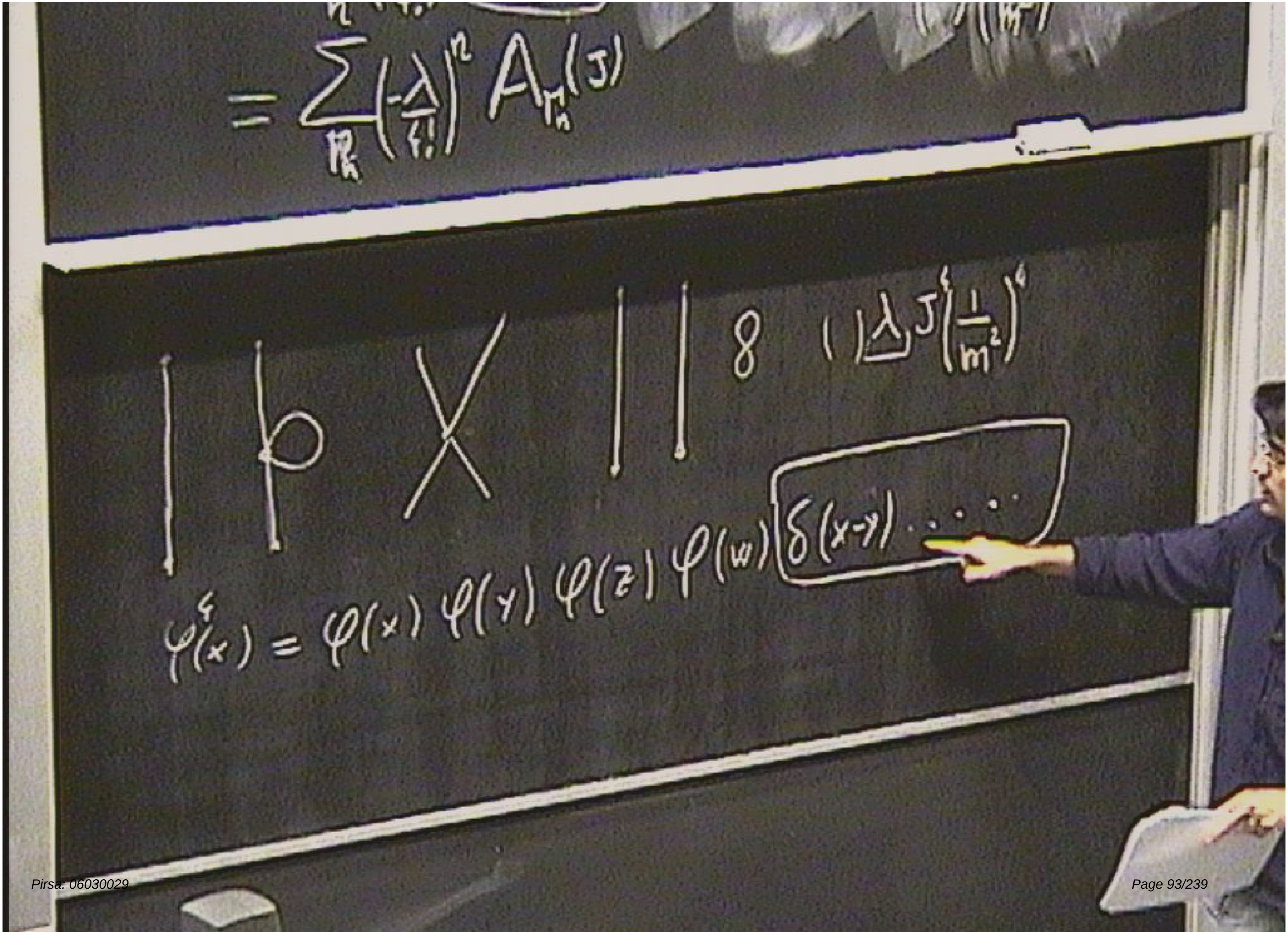


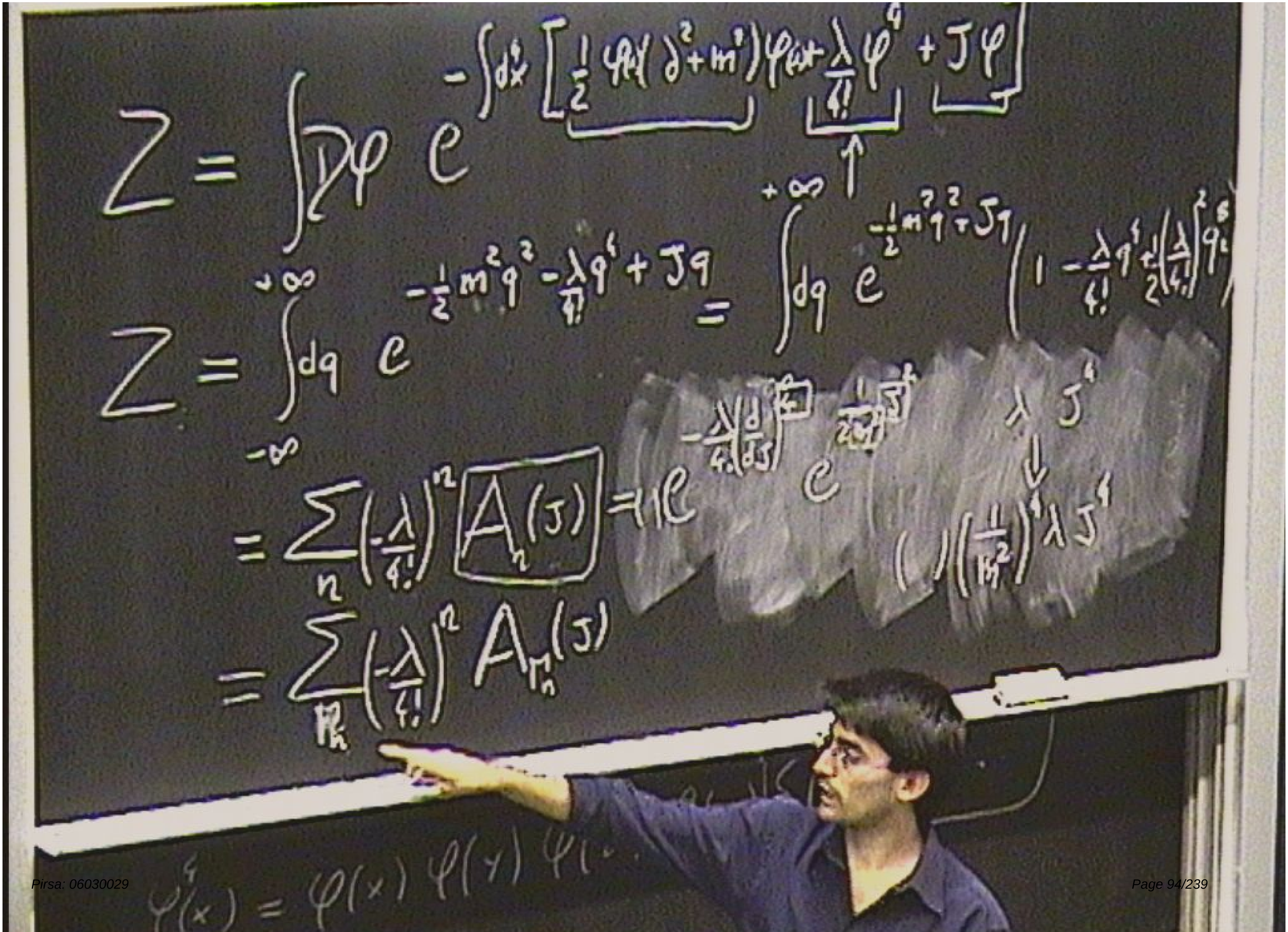
Z = ∫Dφ e^{-∫d⁴x [½ φ(x)(∂² + m²)φ(x) + λ/4! φ⁴ + Jφ]}
Z = ∫_{-∞}^{+∞} dq e^{-½m²q² - λ/4! q⁴ + Jq} = ∫_{-∞}^{+∞} dq e^{-½m²q² + Jq} (1 - λ/4! q⁴ + ½(λ/4!)² q⁸ …)
= Σ_n (-λ/4!)^n A_n(J)
= Σ_n (-λ/4!)^n A_n(J)


$Z = \int D\varphi \, e^{i\int d^4x \left[\frac{1}{2}\varphi(x)(\partial^2+m^2)\varphi(x) + J\varphi\right]} =$
$-\frac{1}{2}\int d^4x \int d^4y \, J(x)\, D(x-y)\, J(y)$
$D(x-y) = \int d^4p \, \frac{i\, e^{ip\cdot(x-y)}}{p^2-m^2+i\epsilon}$

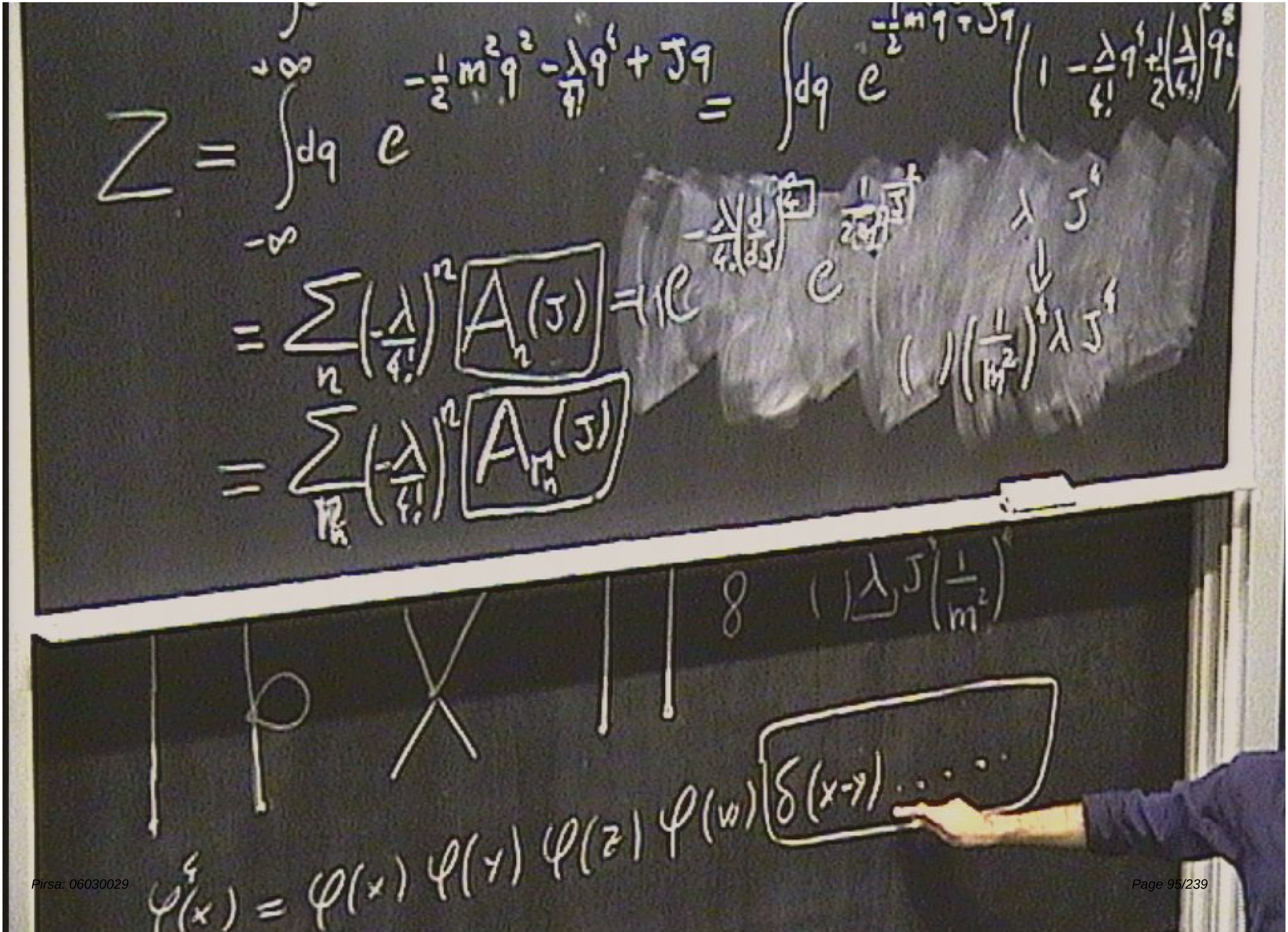


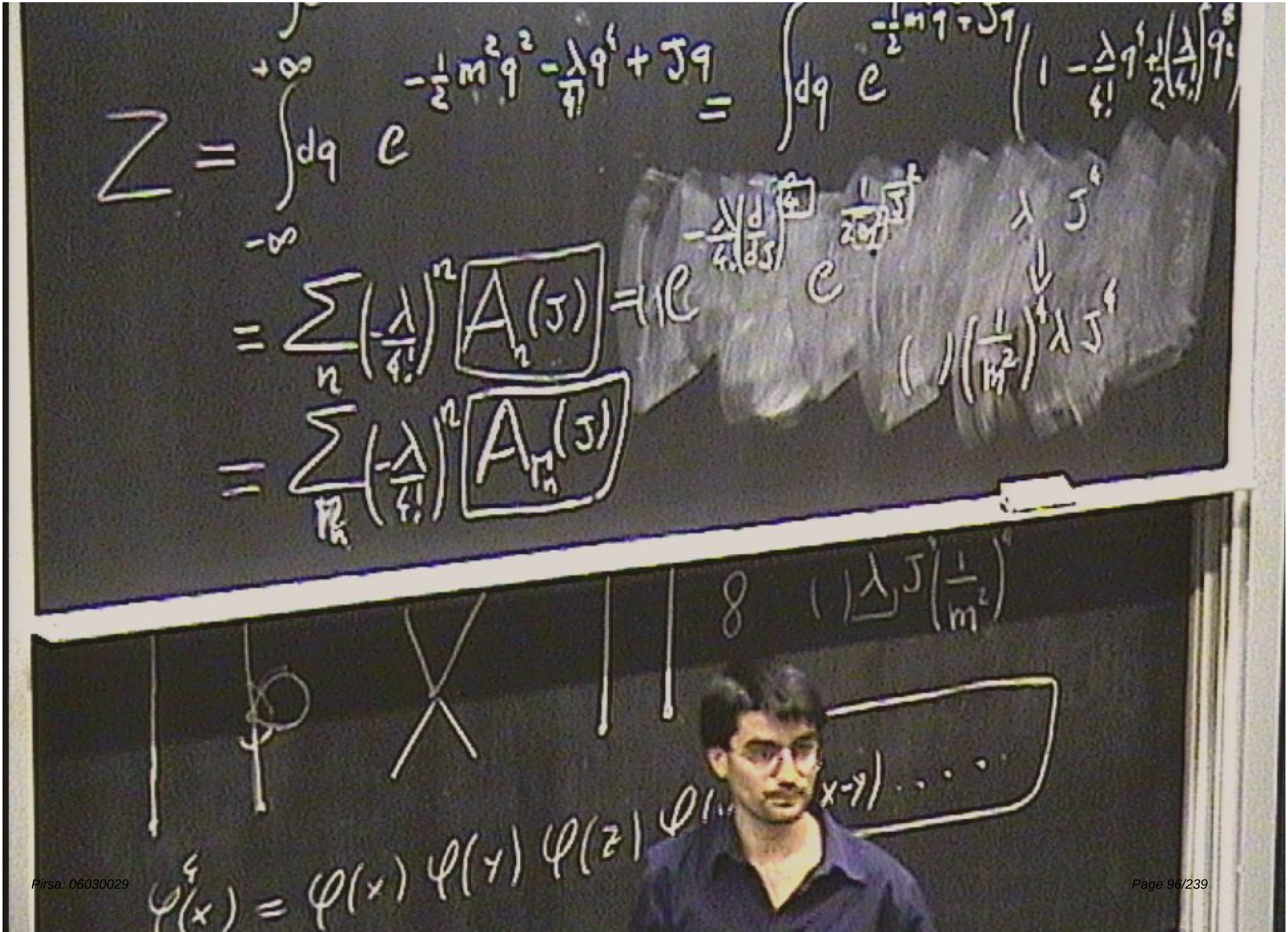

φ(x) = φ(x) φ(y) φ(z) φ(w) δ(x-y) . . . .

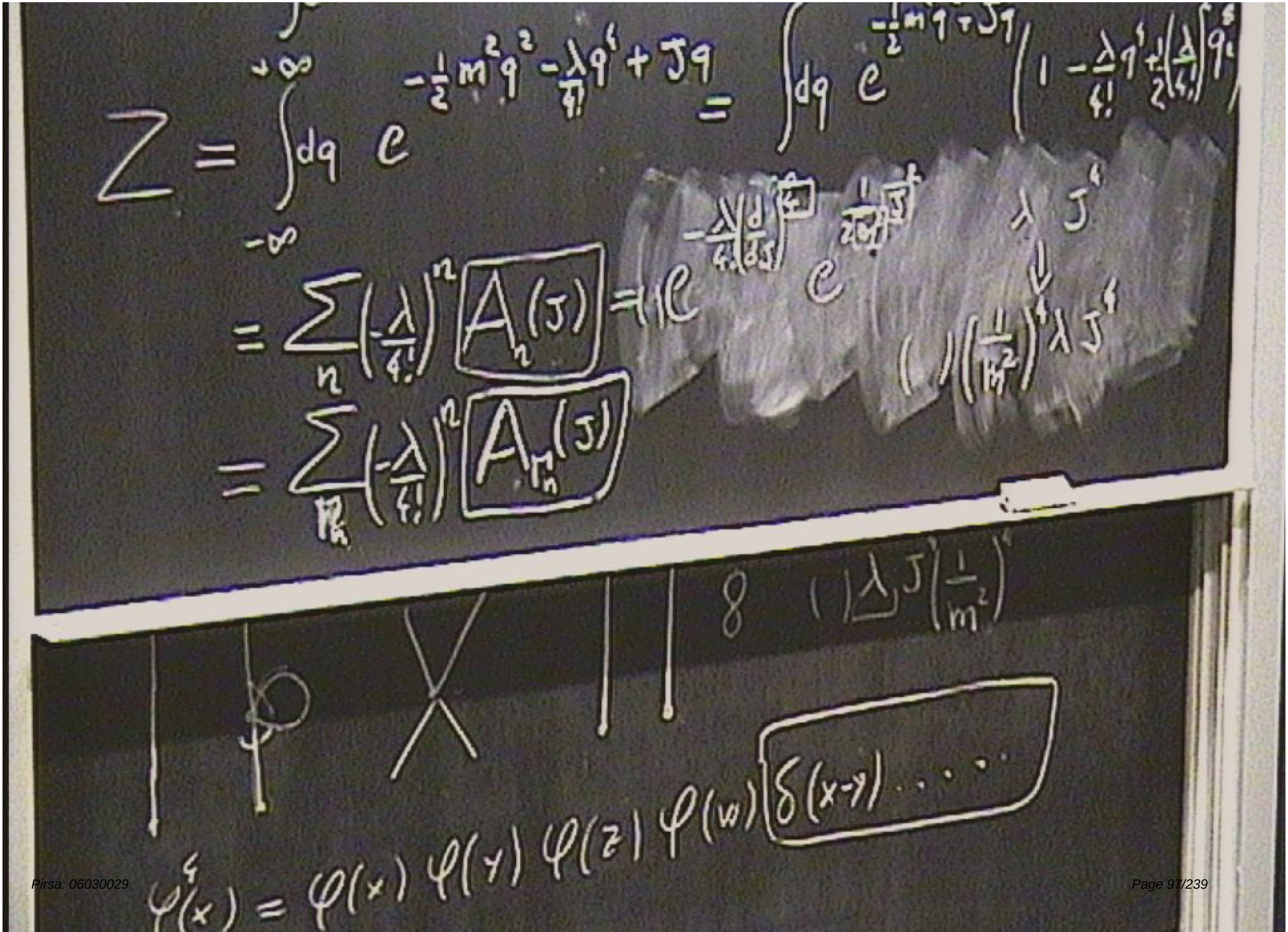

= Σ (-λ/4!)^n A_Γn (J)
8
φ^4(x) = φ(x) φ(y) φ(z) φ(w) δ(x-y) . . . .



Z = ∫Dφ e
Z = ∫dq e
= Σ (-λ/4!)^n A_n(J)
= Σ (-λ/4!)^n A_n(J)


Z = ∫ dq e^{-½m²q² - λ/4! q⁴ + Jq} = ∫ dq e^{-½m²q² + Jq} (1 - λ/4! q⁴ + ½(λ/4!)² q⁸ …)
= Σ_n (-λ/4!)^n A_n(J)
= Σ (-λ/4!)^n A_n(J)
λ J⁴
( )(1/m²)⁴ λ J⁴
8 ( ) λ J (1/m²)⁴
φ(x) φ(y) φ(z) φ(w) δ(x-y) . . . .
φ⁴(x) = φ(x) φ(y) φ(z)


Z = ∫dq e^{-½m²q² - λ/4! q⁴ + Jq}
= Σ_n (-λ/4!)^n A_n(J)

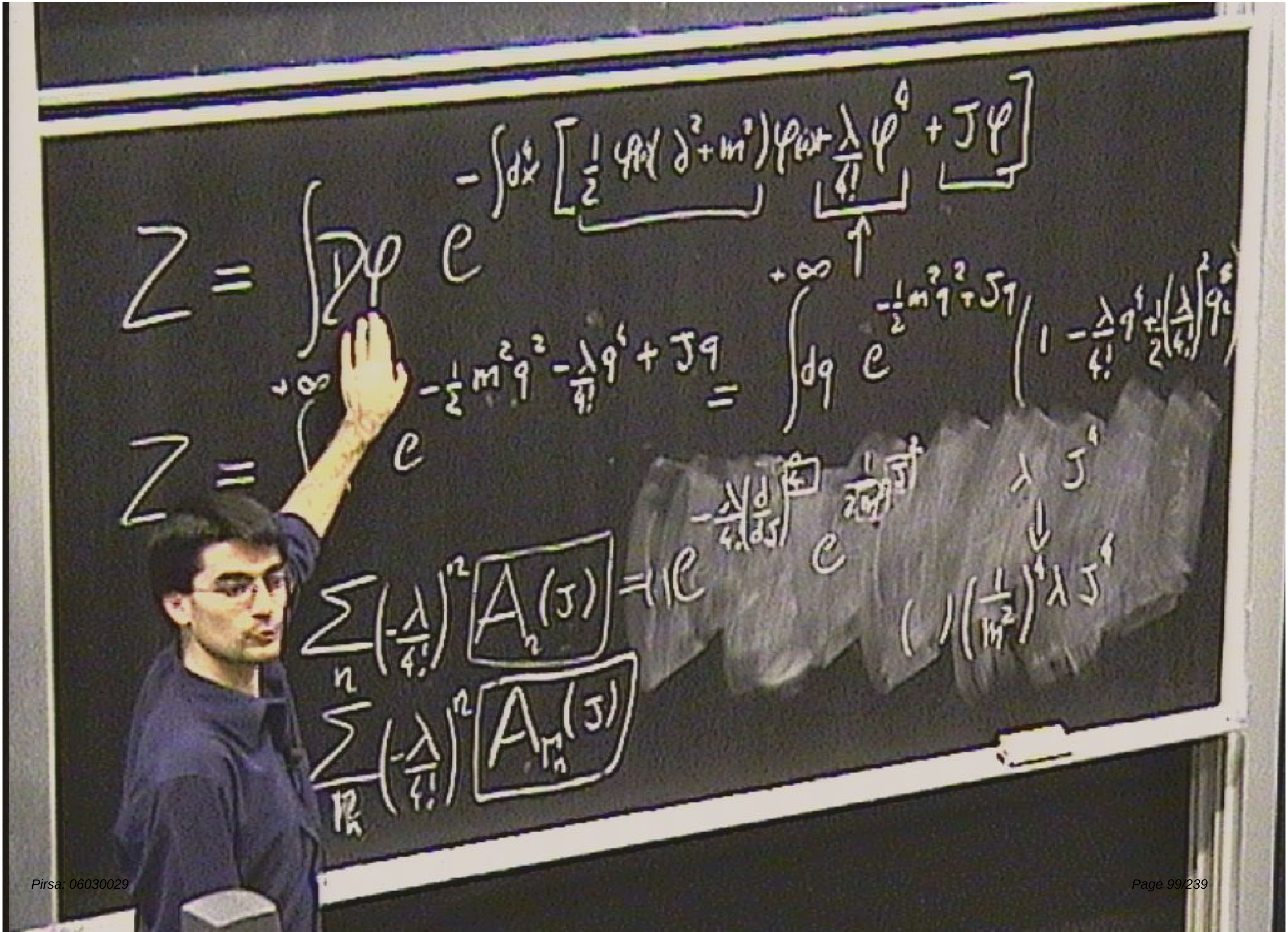


Z = ∫_{-∞}^{+∞} dq e^{-½m²q² - λ/4! q⁴ + Jq} = ∫dq e^{-½m²q² + Jq} (1 - λ/4! q⁴ + ½(λ/4!)² q⁸
= Σ_n (-λ/4!)^n A_n(J)
= Σ_n (-λ/4!)^n A_n(J)
φ⁴(x) = φ(x) φ(y) φ(z) φ(w) δ(x-y) . . . .

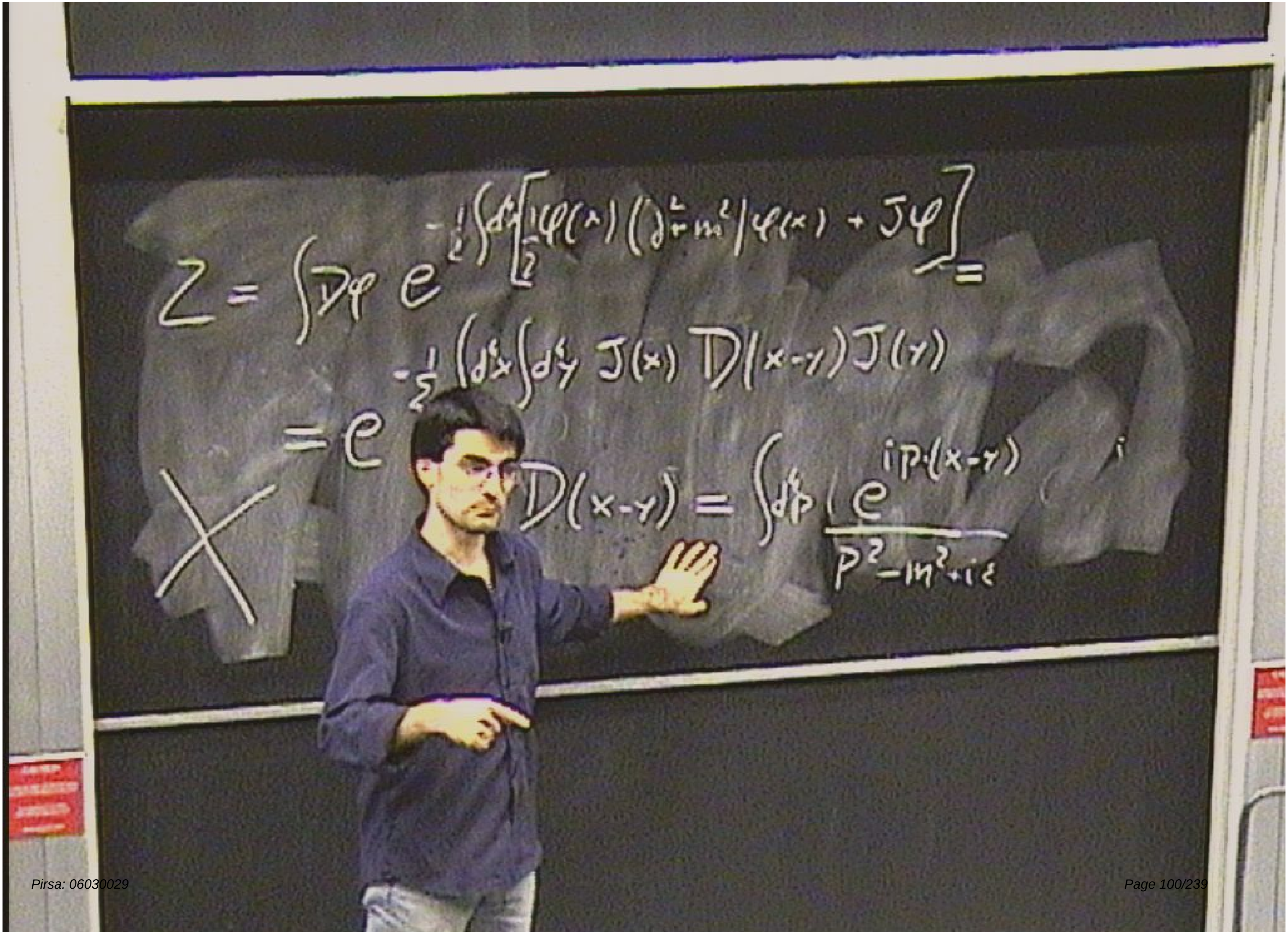
- 3d Ponzano-Regge model (geometry)
- idea of Feynman diagrams

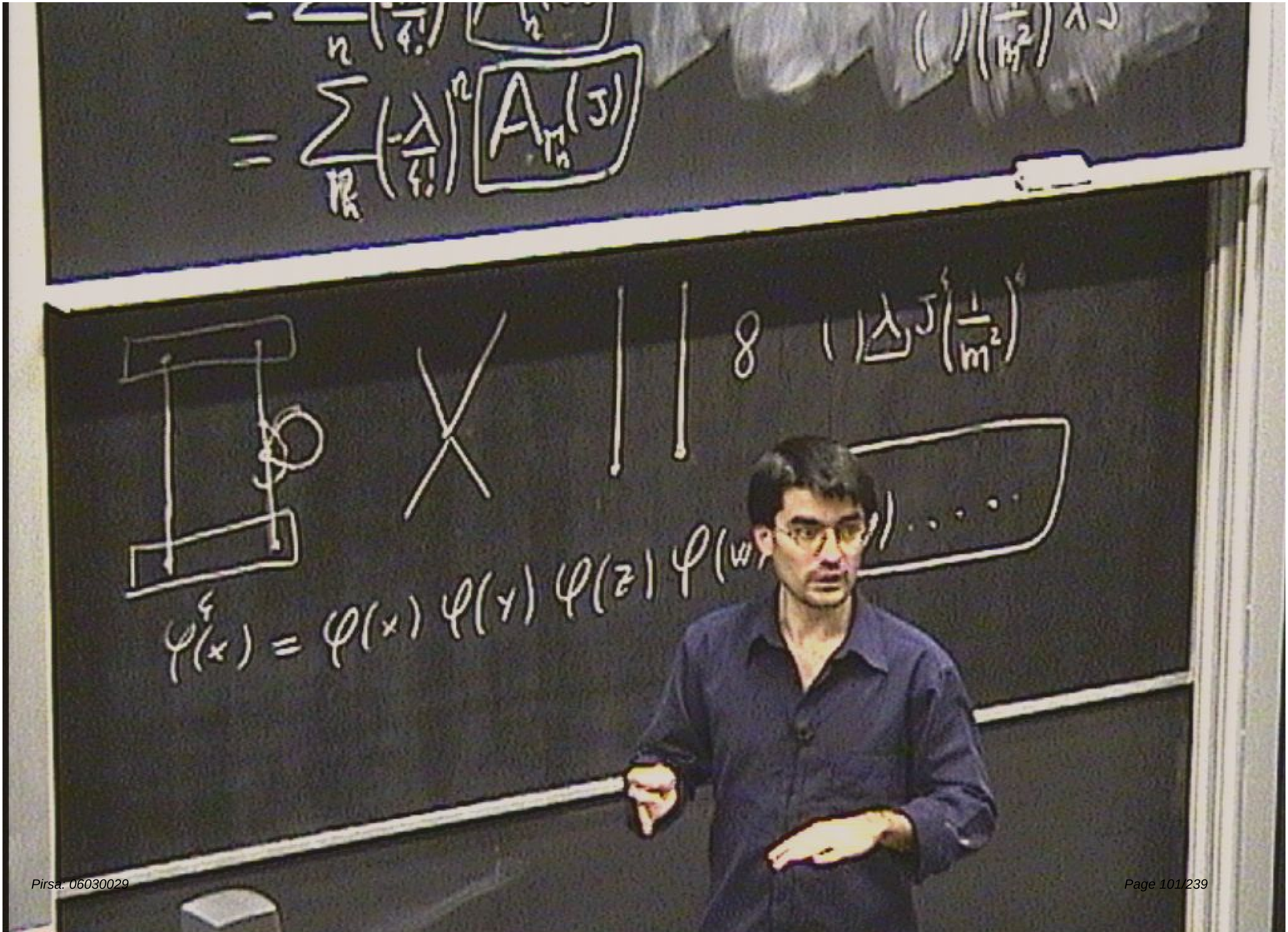
↳ Group field theory
- idea
- general formalism
- 3d example
- outlook

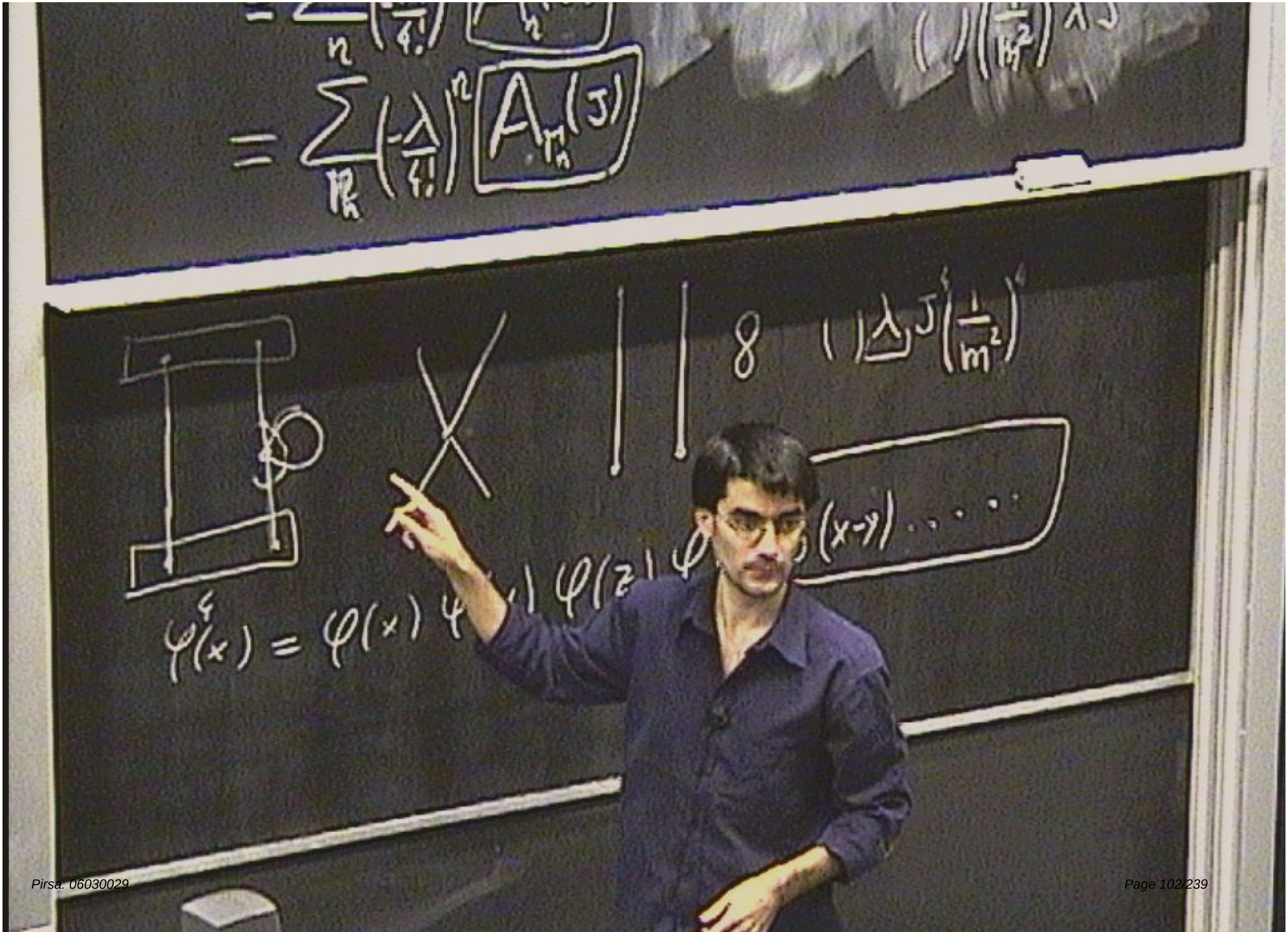
- D. ORITI, gr-qc/0512103
- L. FREIDEL, hep-th/0505016

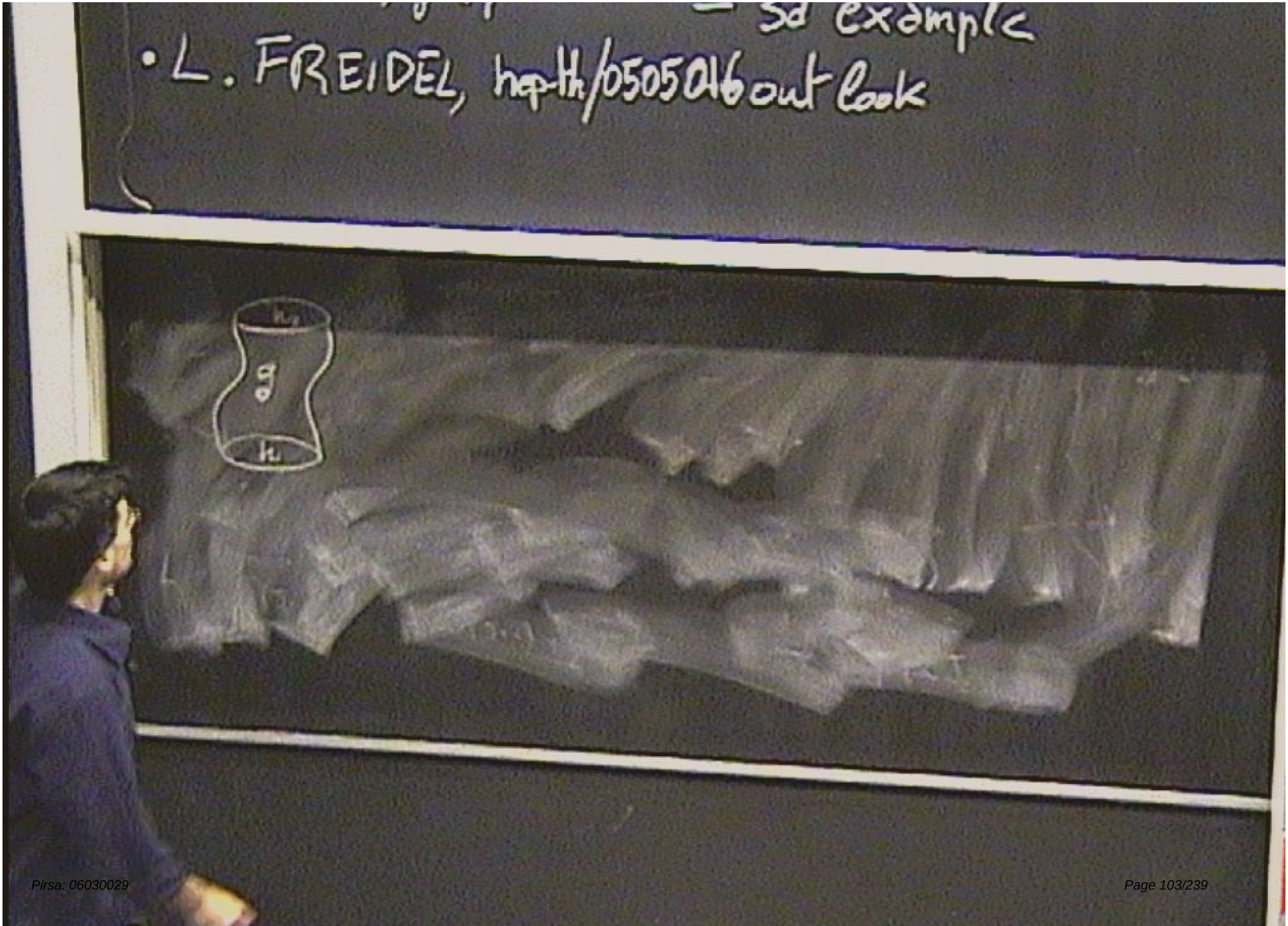


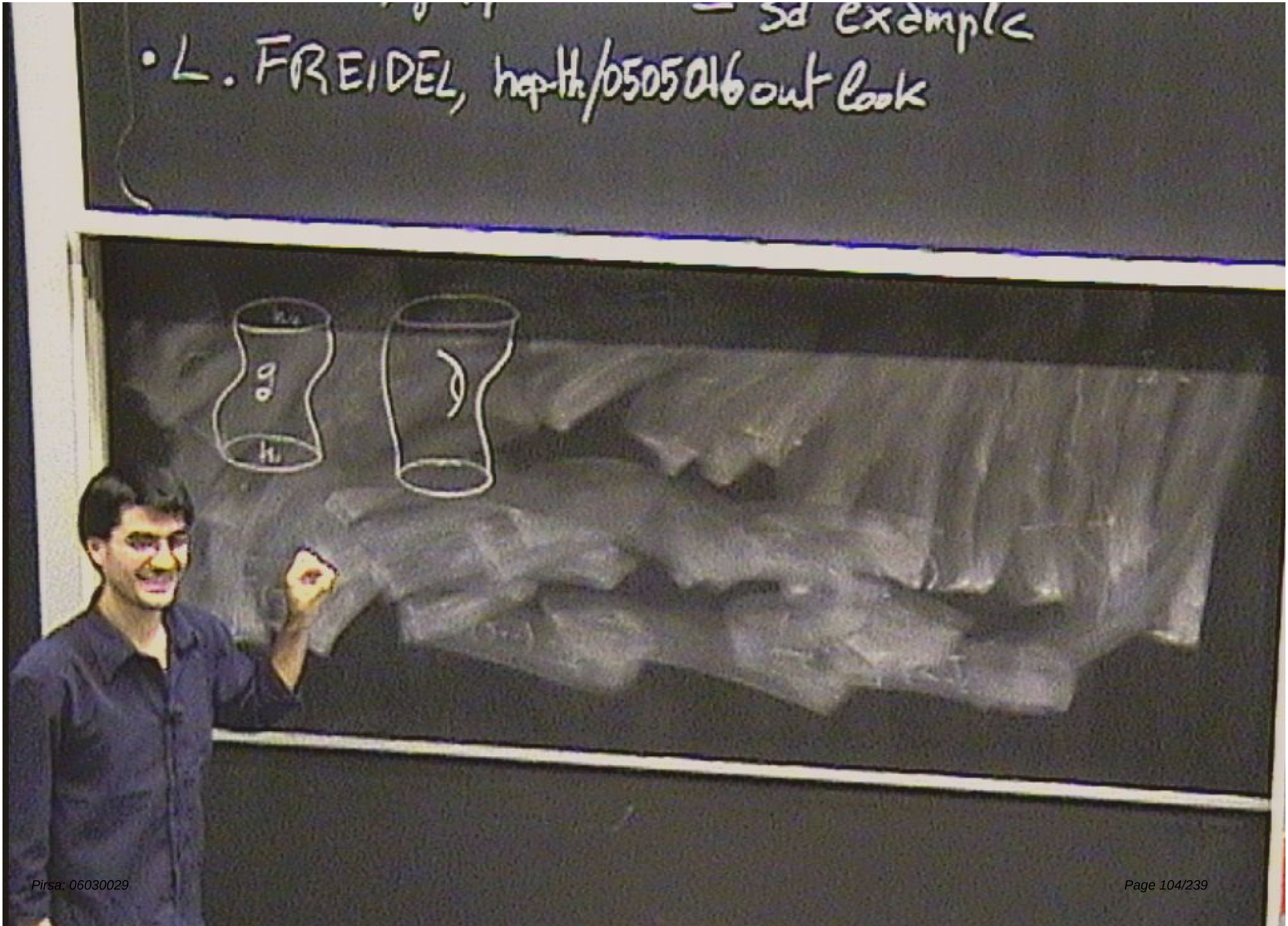
$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(x)(\partial^2 + m^2)\varphi(x) + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{+\infty} e^{-\frac{1}{2}m^2 q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int dq \, e^{-\frac{1}{2}m^2 q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8\right)$
$\sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$
λJ^4

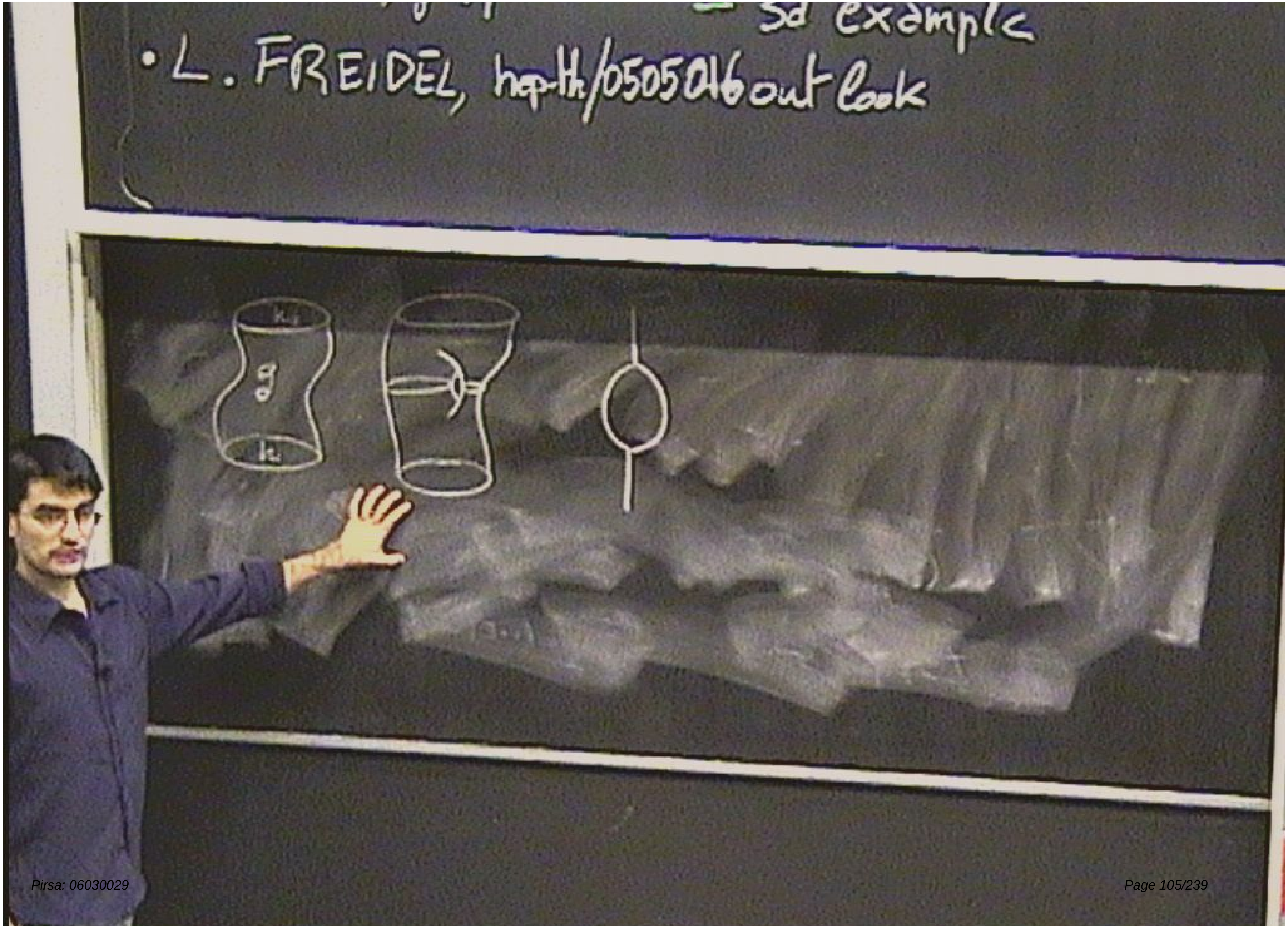

Z = ∫Dφ e^{-i/2 ∫d⁴x [½ φ(x)(∂² + m²)φ(x) + Jφ]} =
= e^{-i/2 ∫d⁴x ∫d⁴y J(x) D(x-y) J(y)}
D(x-y) = ∫d⁴p e^{ip·(x-y)} / (p² - m² + iε)

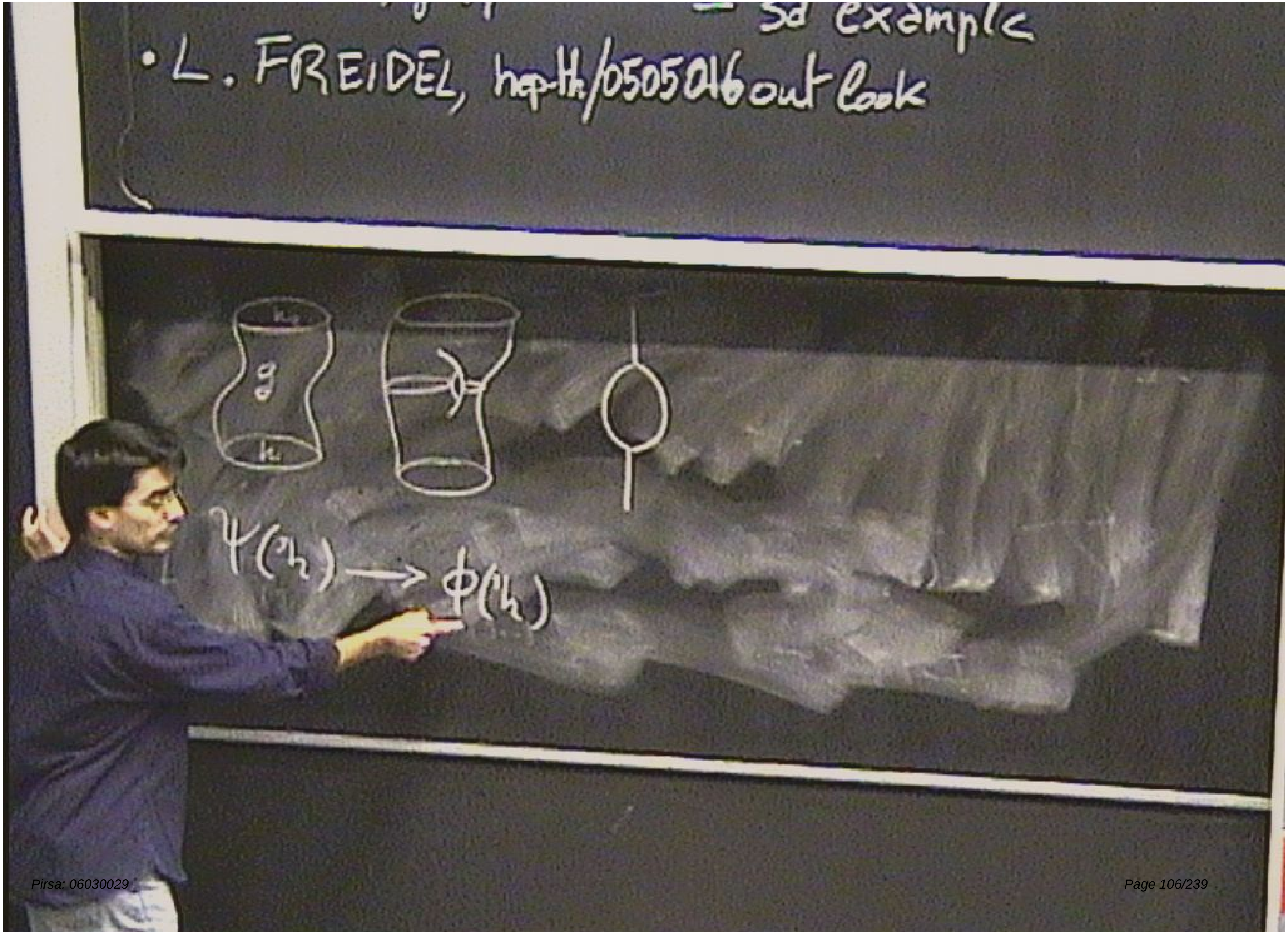


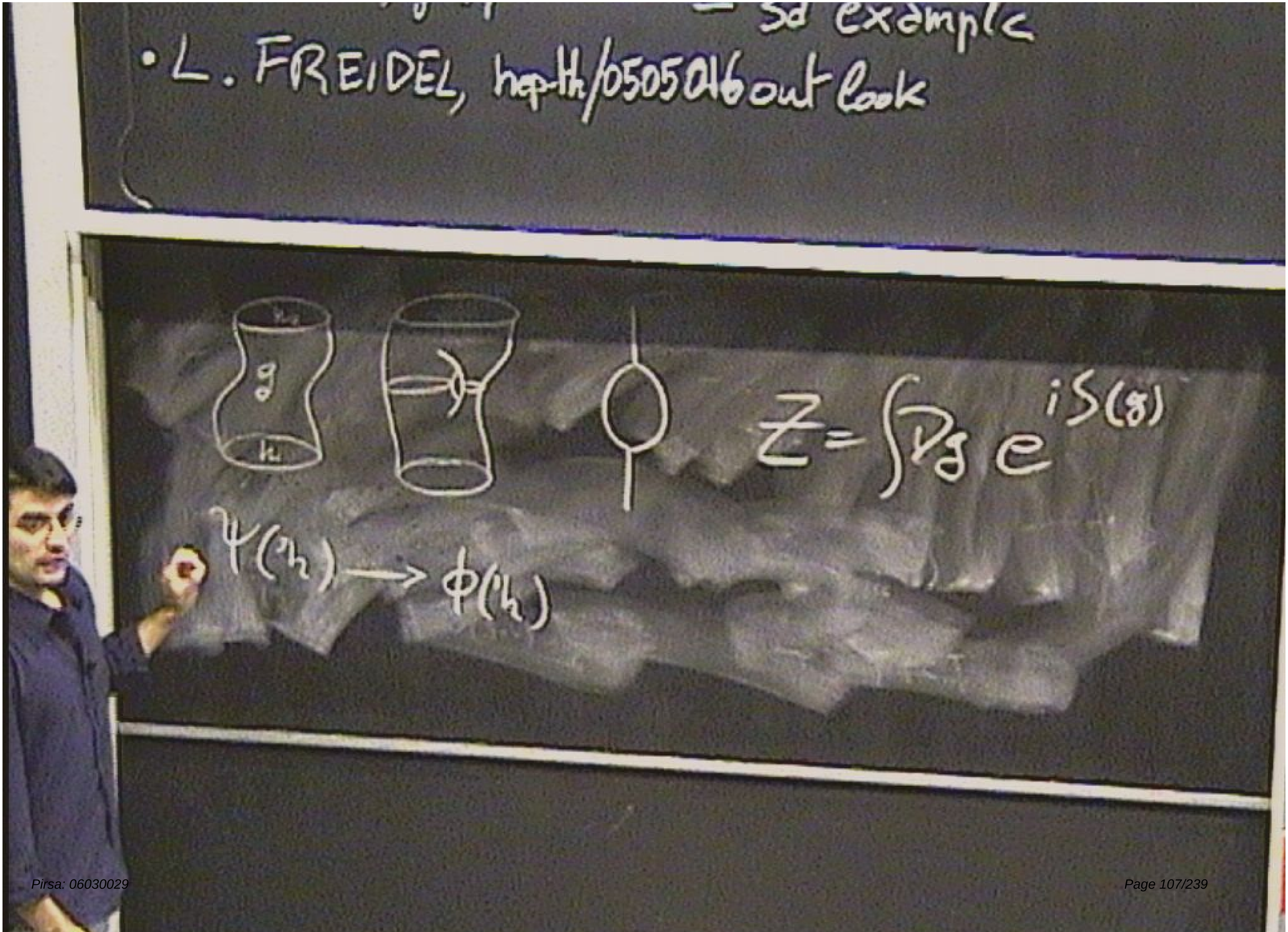


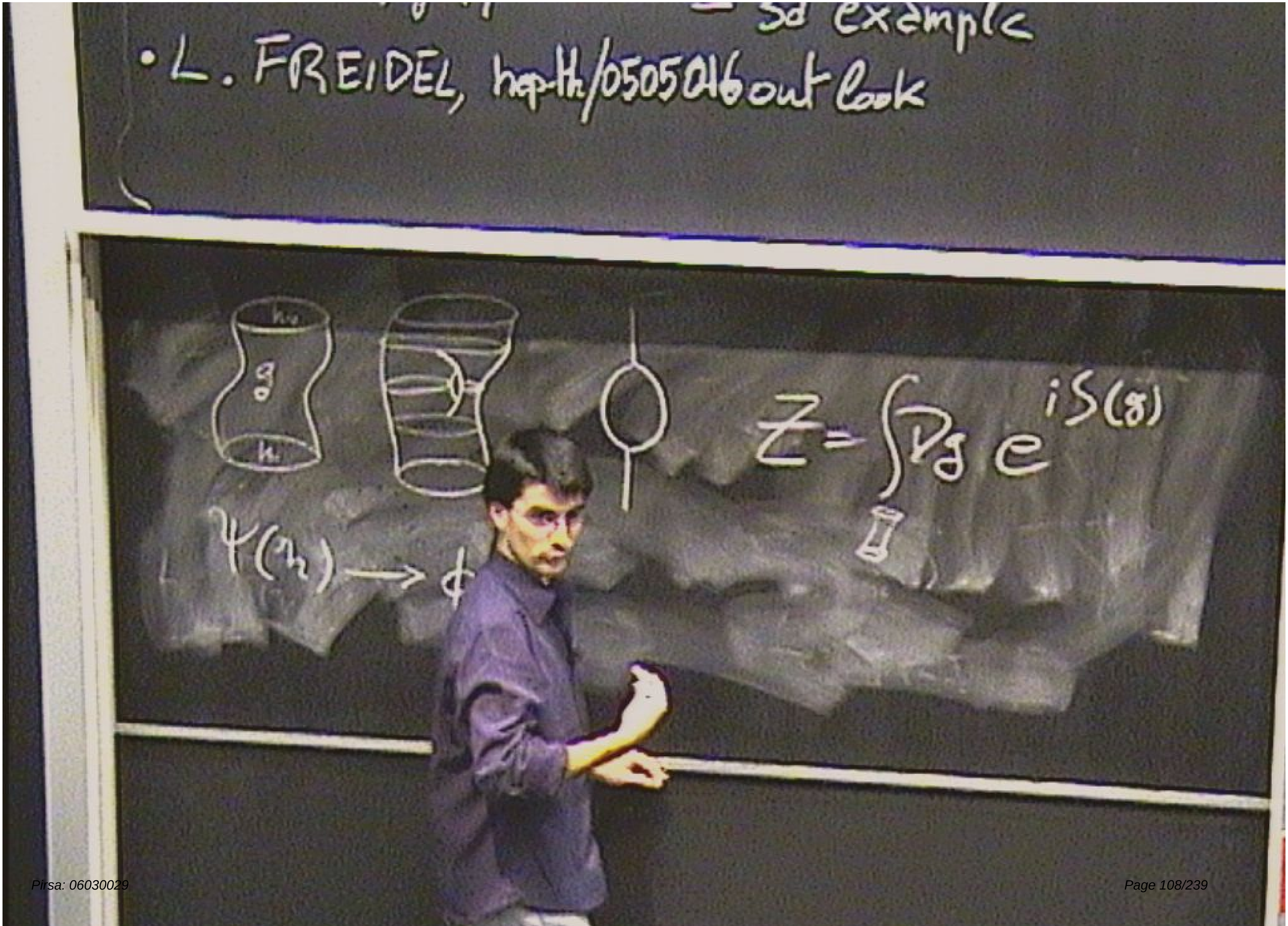

example
• L. FREIDEL, hep-th/0505016 out look
g
h


sd example
• L. FREIDEL, hep-th/0505016 out look

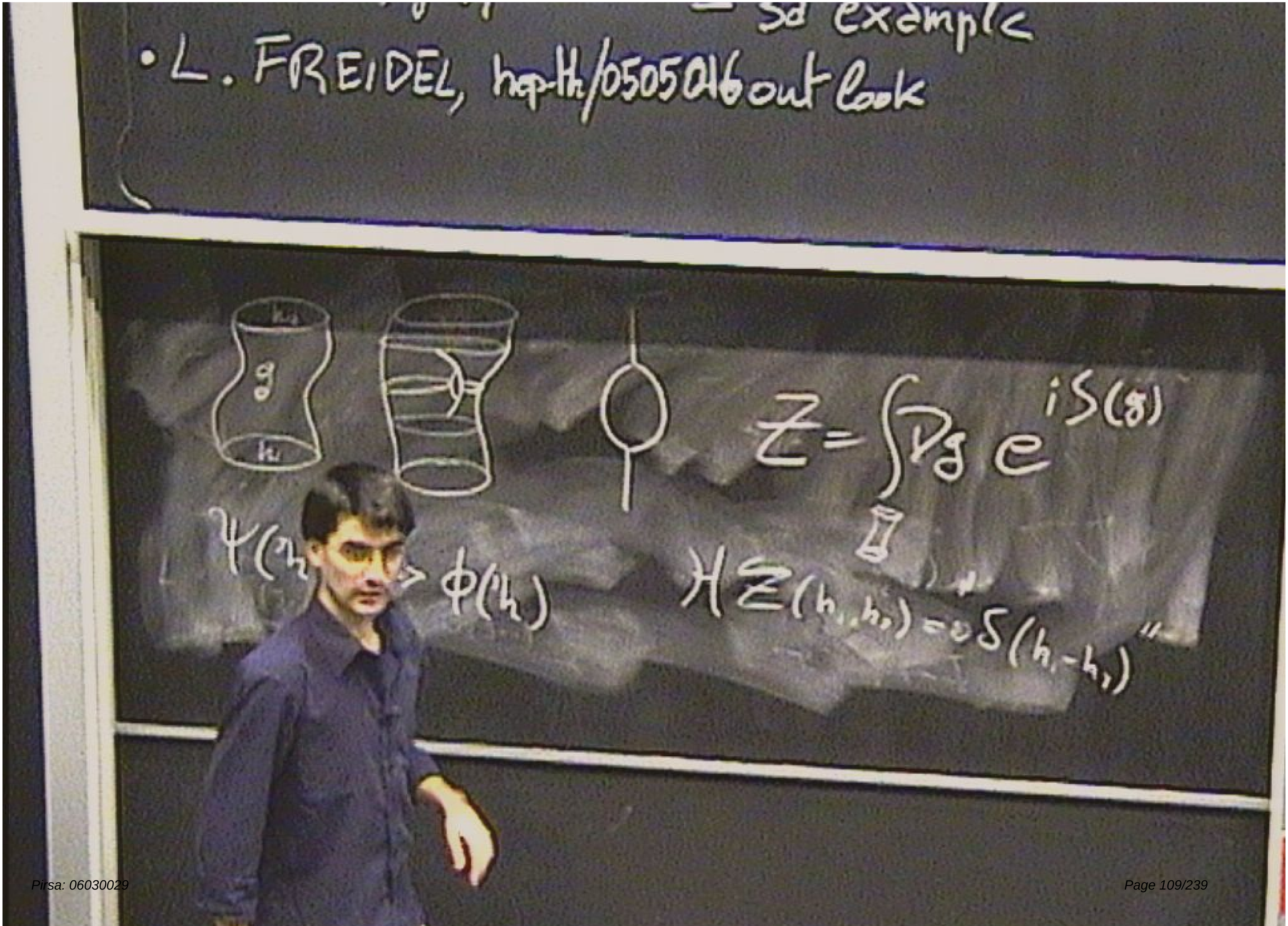

3d example
• L. FREIDEL, hep-th/0505016 outlook
g

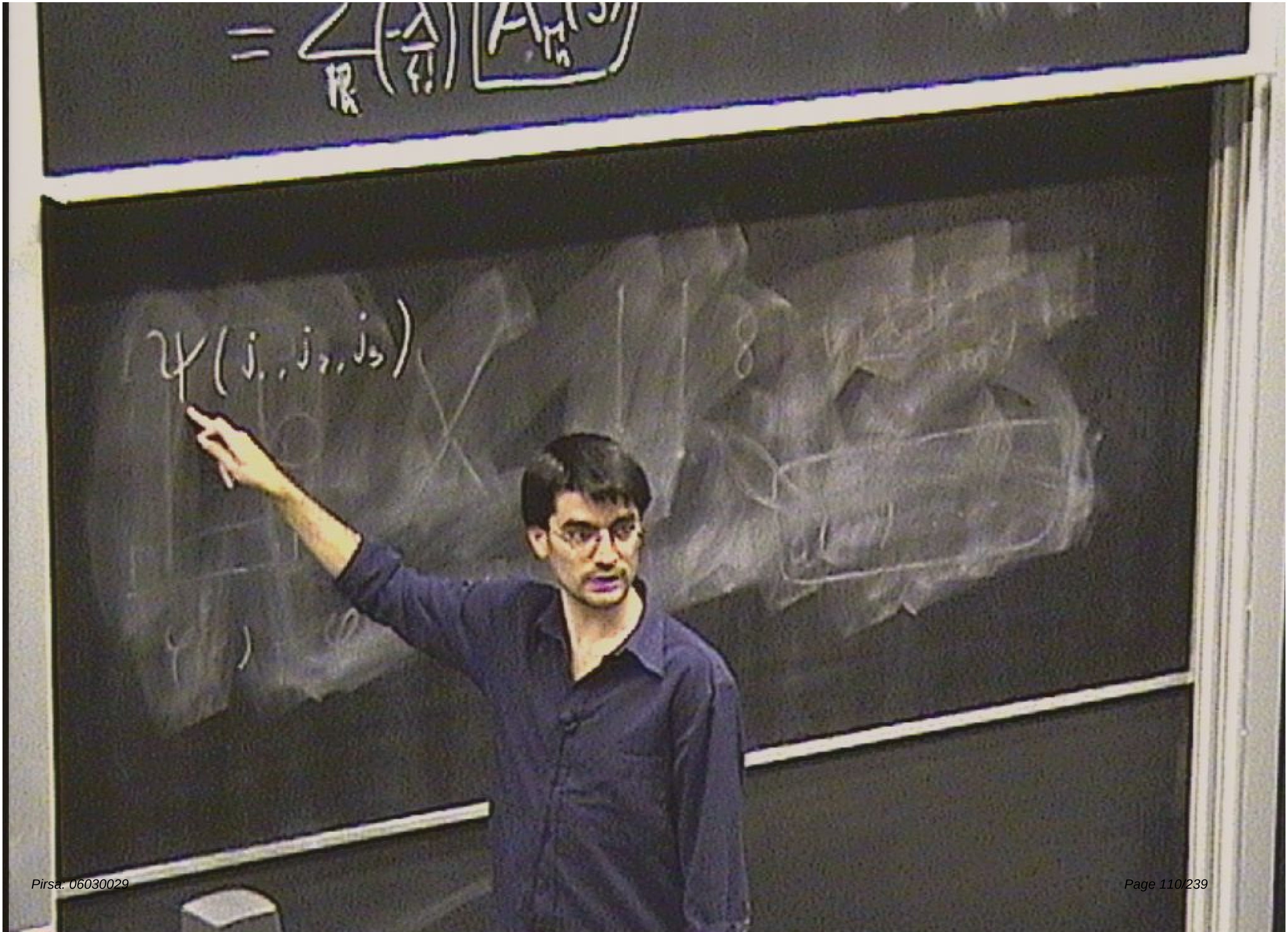

sd example
• L. FREIDEL, hep-th/0505016 out look
g
ψ(h) → φ(h)

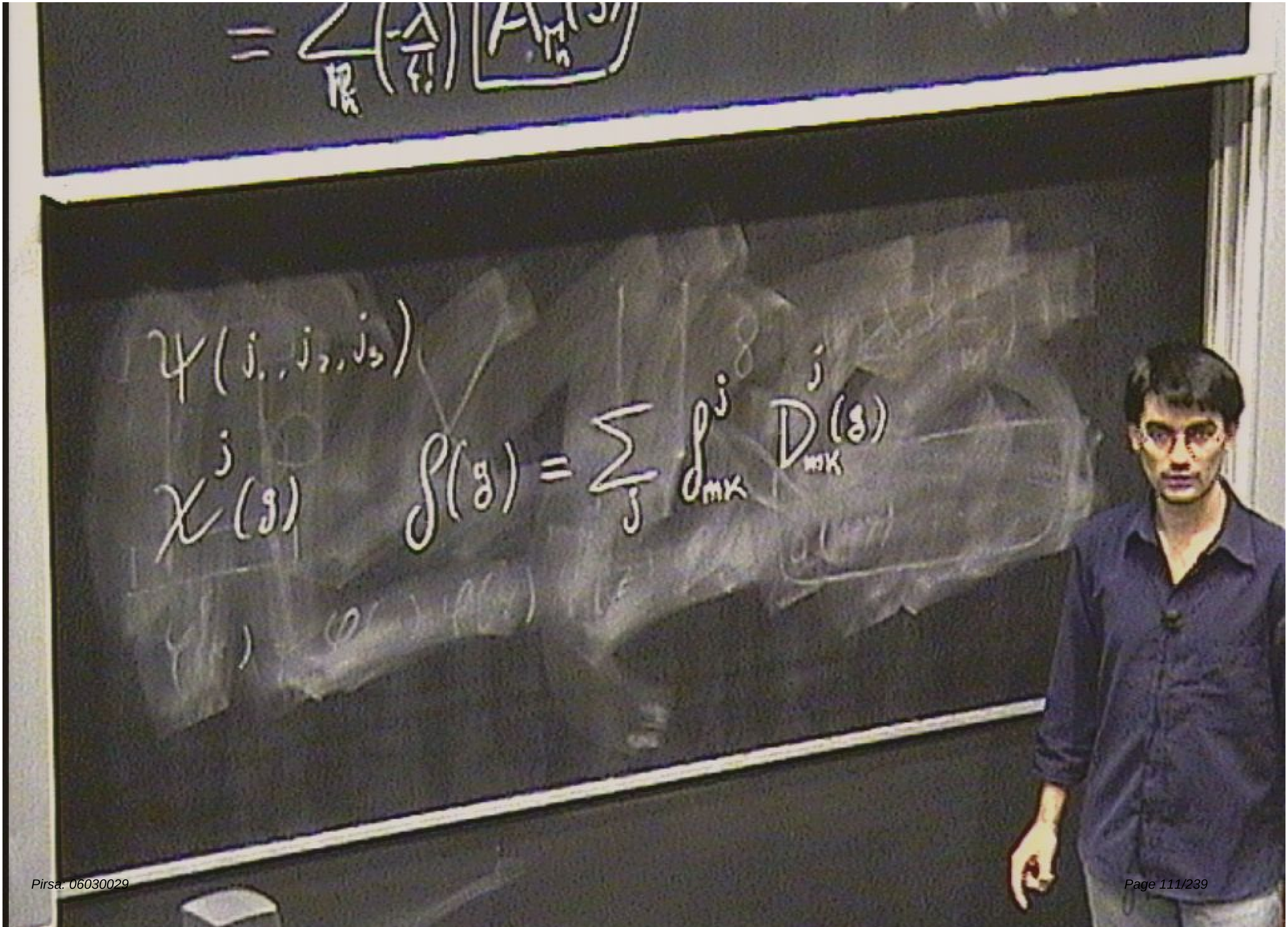

3d example
• L. FREIDEL, hep-th/0505016 outlook
g
Z = ∫Dg e^{iS(g)}

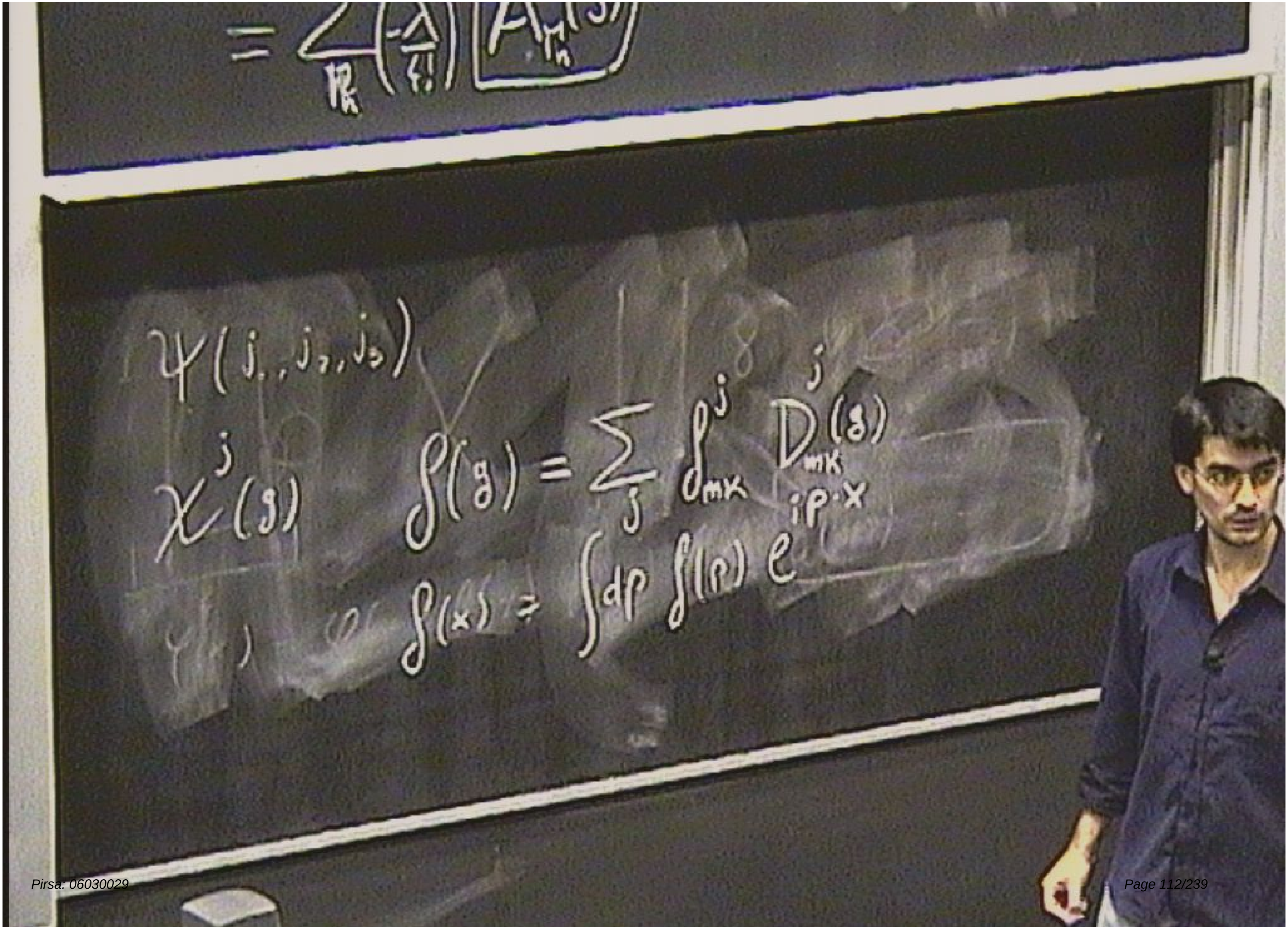


3d example
• L. FREIDEL, hep-th/0505016 out look
Z = ∫ Dg e^{iS(g)}
ψ(h) → φ

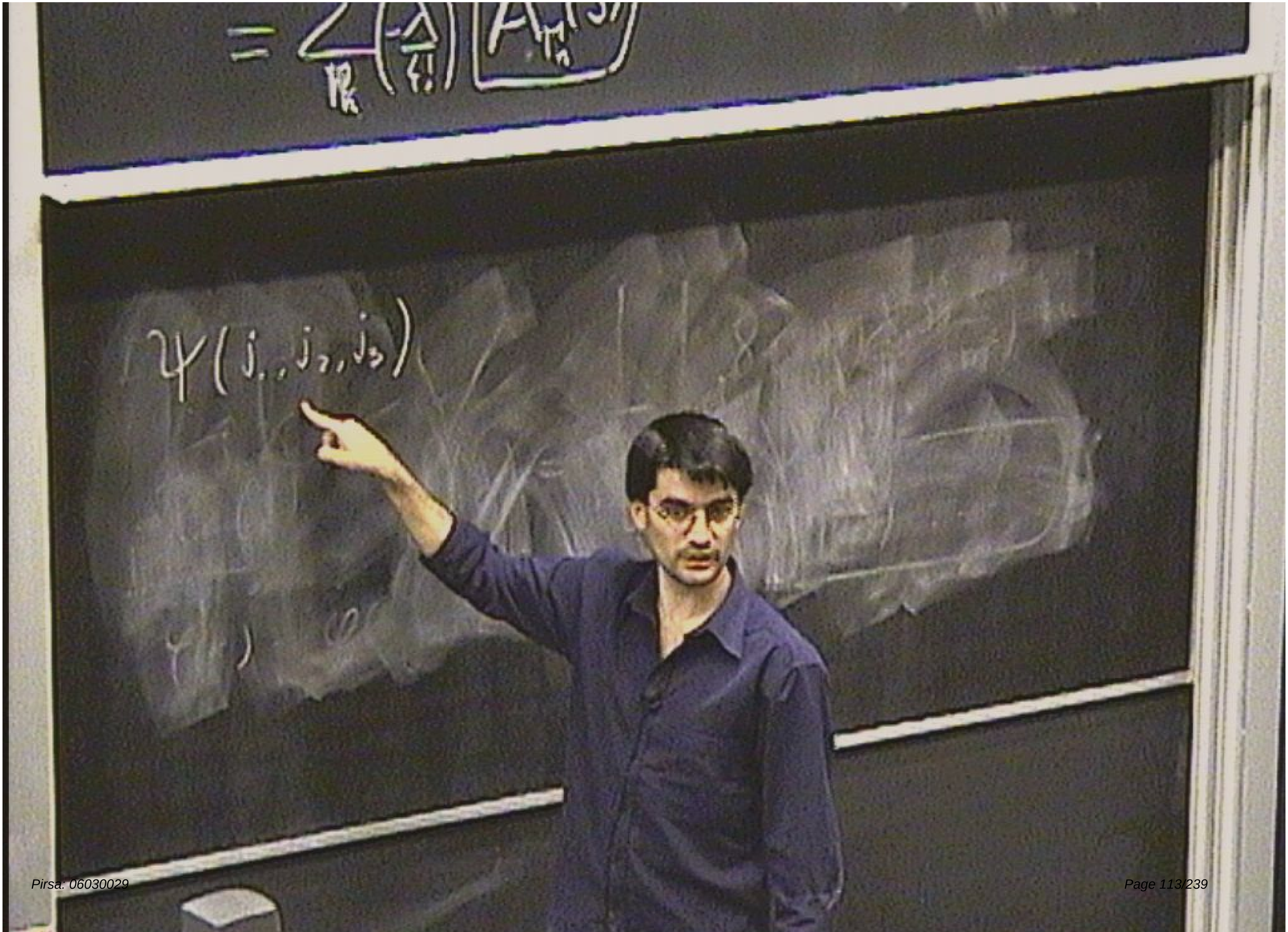

sd example
• L. FREIDEL, hep-th/0505016 outlook
Z = ∫ Dg e^{iS(g)}
H Z(h₁,h₂) = 0
δ(h₁-h₂)

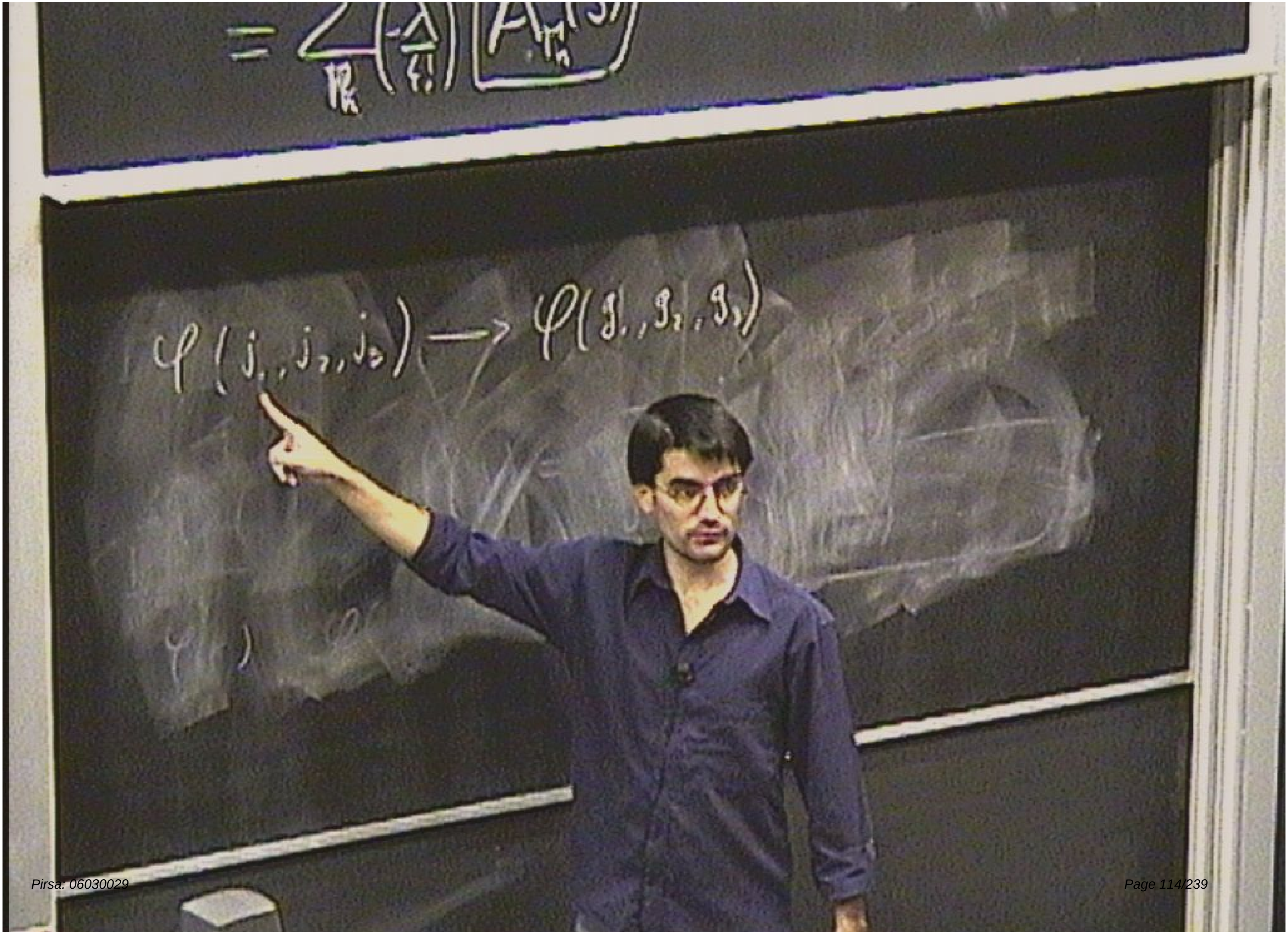

$\Psi(j_1, j_2, j_3)$

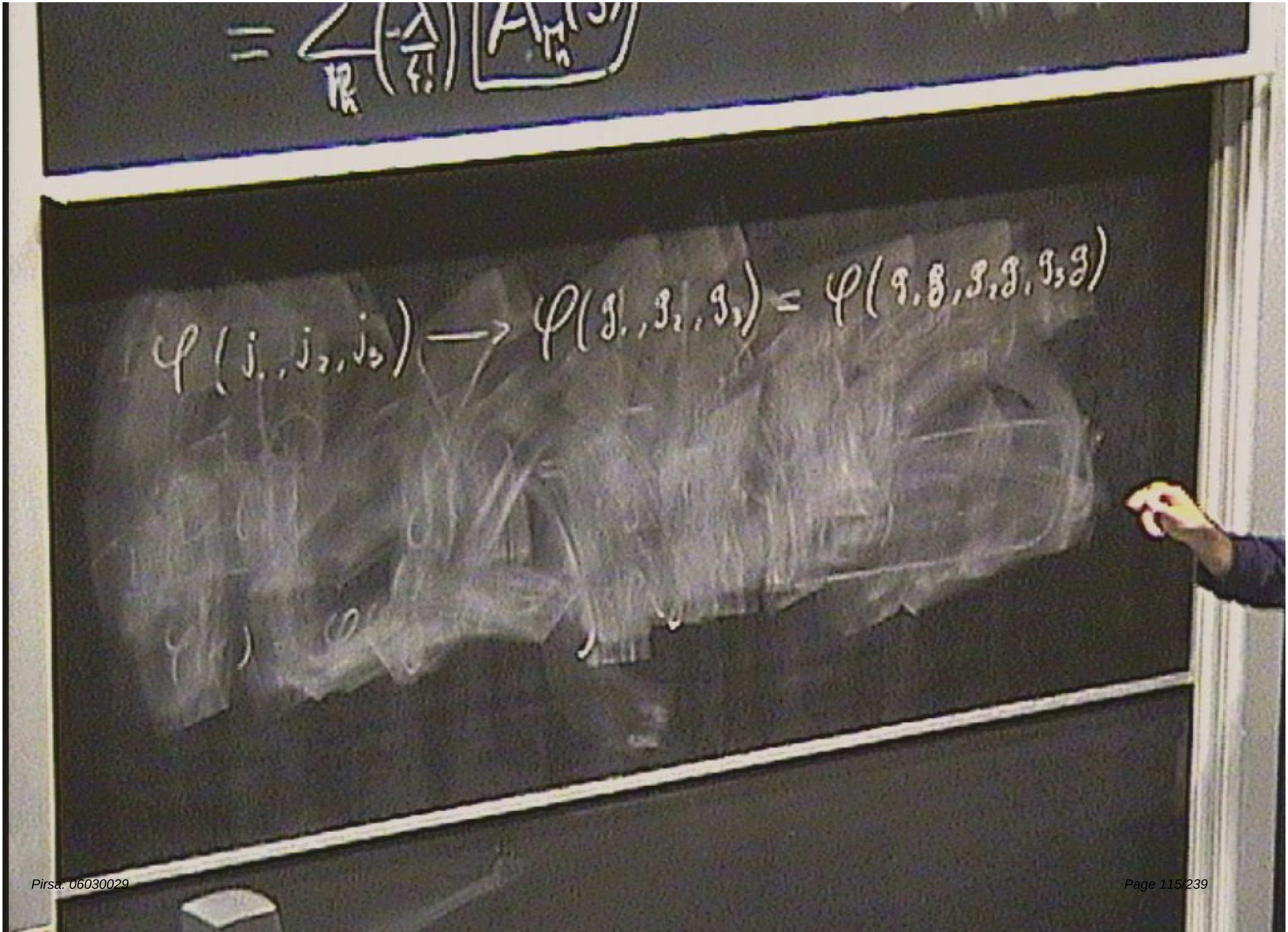

Ψ(j1, j2, j3)
χ^j(g)
δ(g) = Σ_j d^j_mk D^j_mk(g)

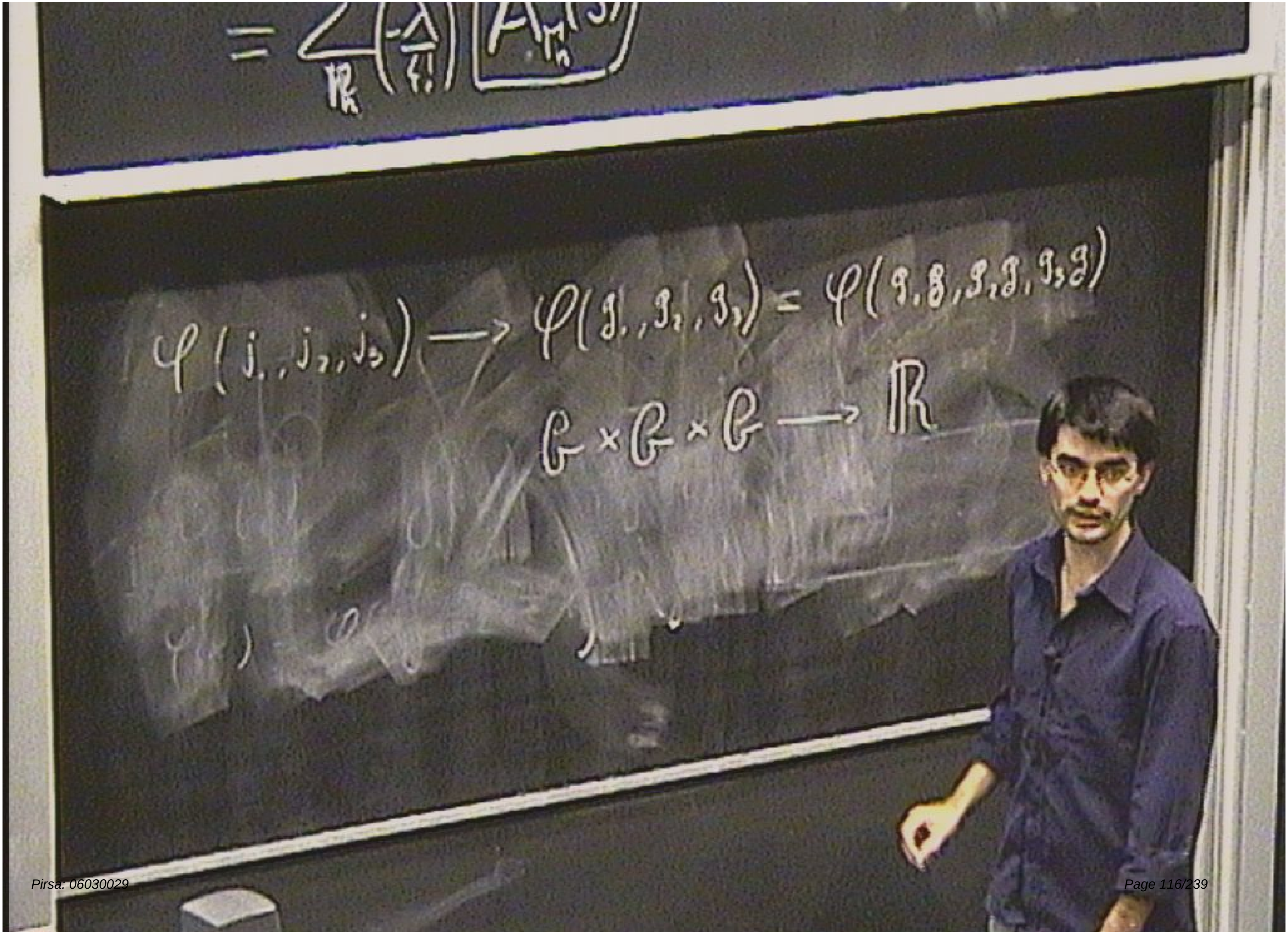


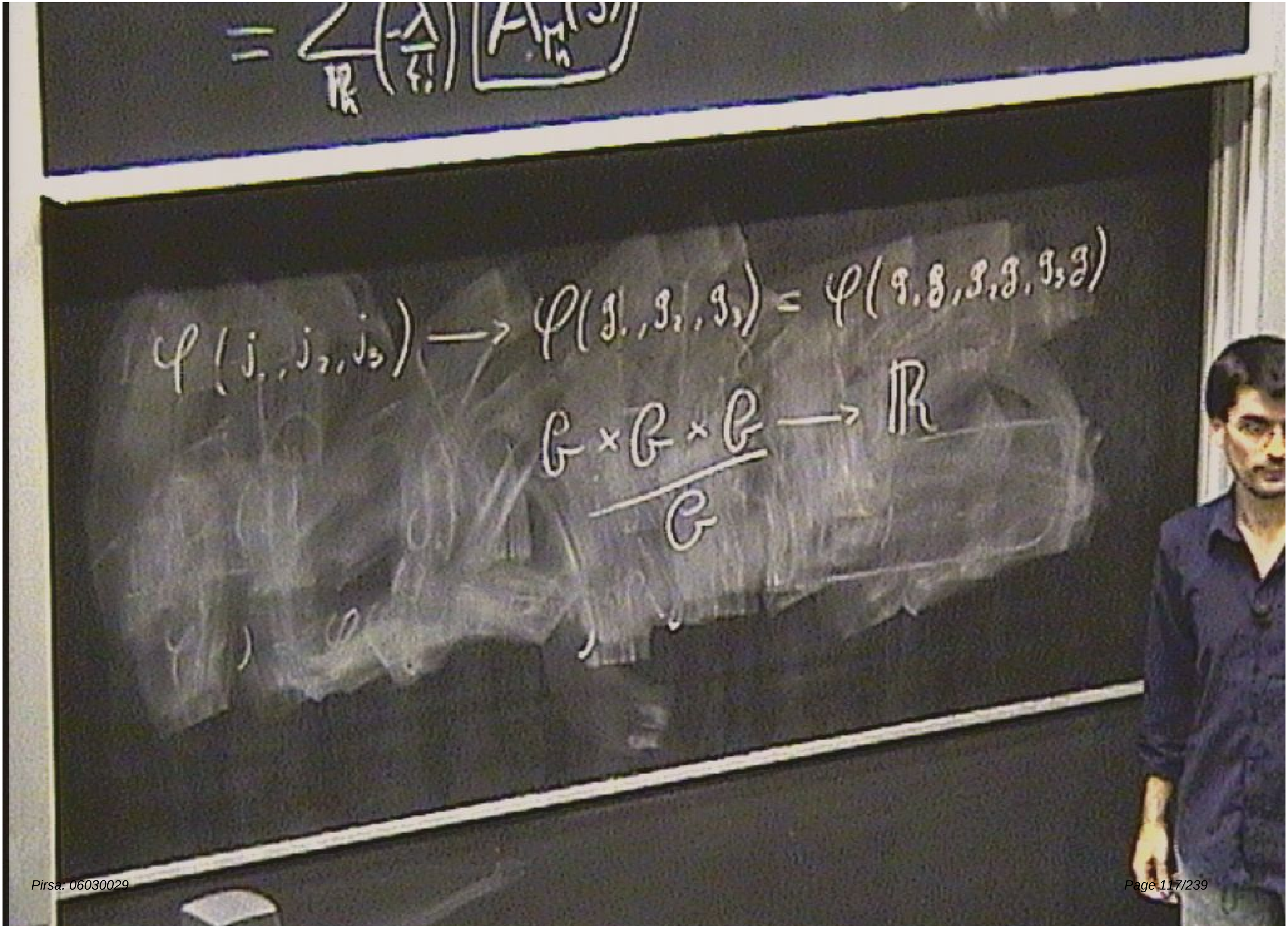
Ψ(j₁, j₂, j₃)
χʲ(g)
δ(g) = Σⱼ dʲ δ_mk Dʲ_mk(g)
δ(x) = ∫dp δ(p) e^{ip·x}


Ψ(j₁, j₂, j₃)


φ(j1, j2, j3) ⟶ φ(g1, g2, g3)


φ(j1, j2, j3) ⟶ φ(g1, g2, g3) = φ(g1g, g2g, g3g)


φ(j₁, j₂, j₃) ⟶ φ(g₁, g₂, g₃) = φ(g₁g, g₂g, g₃g)
G × G × G ⟶ ℝ

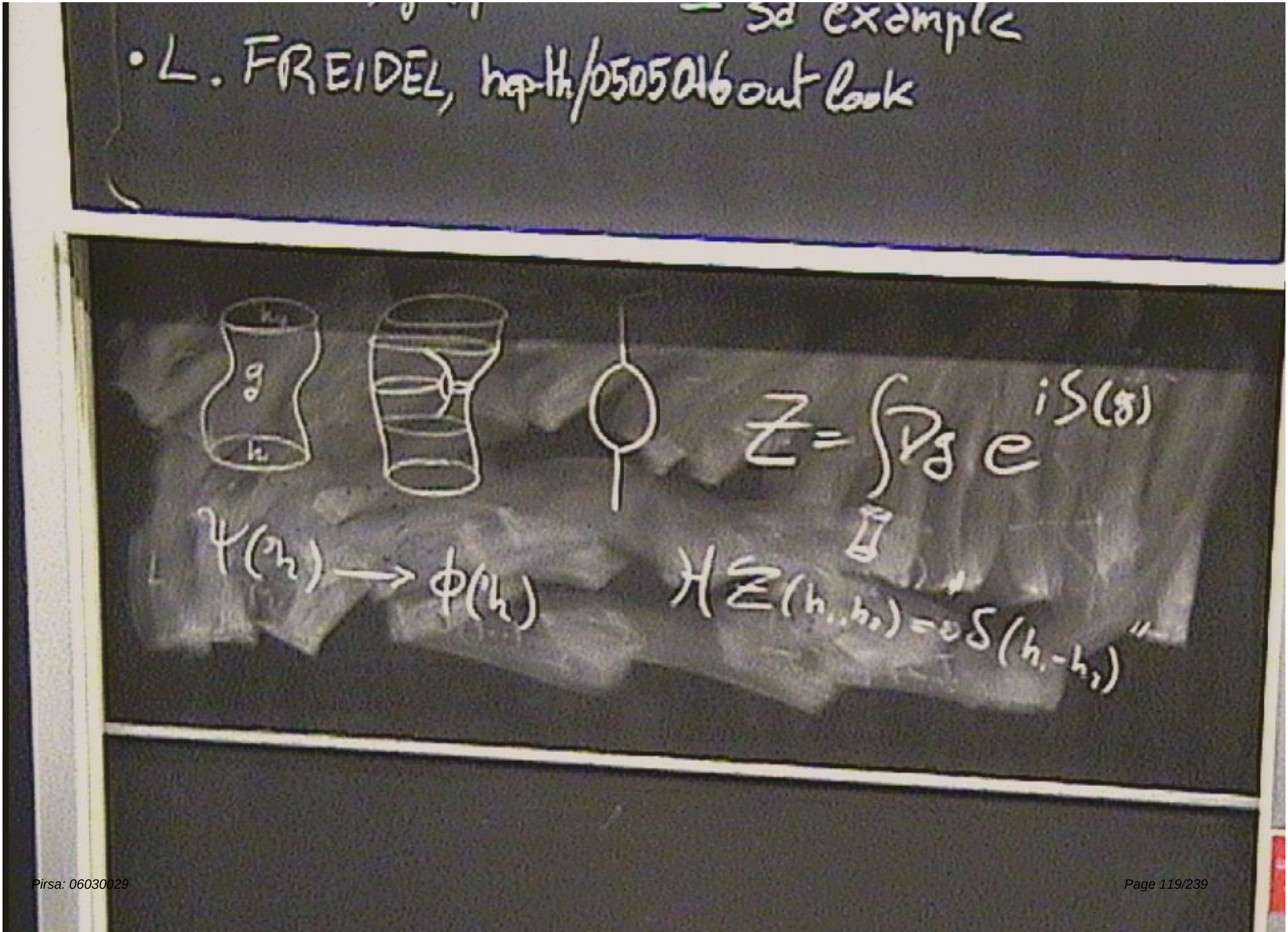

φ(j₁, j₂, j₃) ⟶ φ(g₁, g₂, g₃) = φ(g₁g, g₂g, g₃g)
G × G × G / G ⟶ ℝ

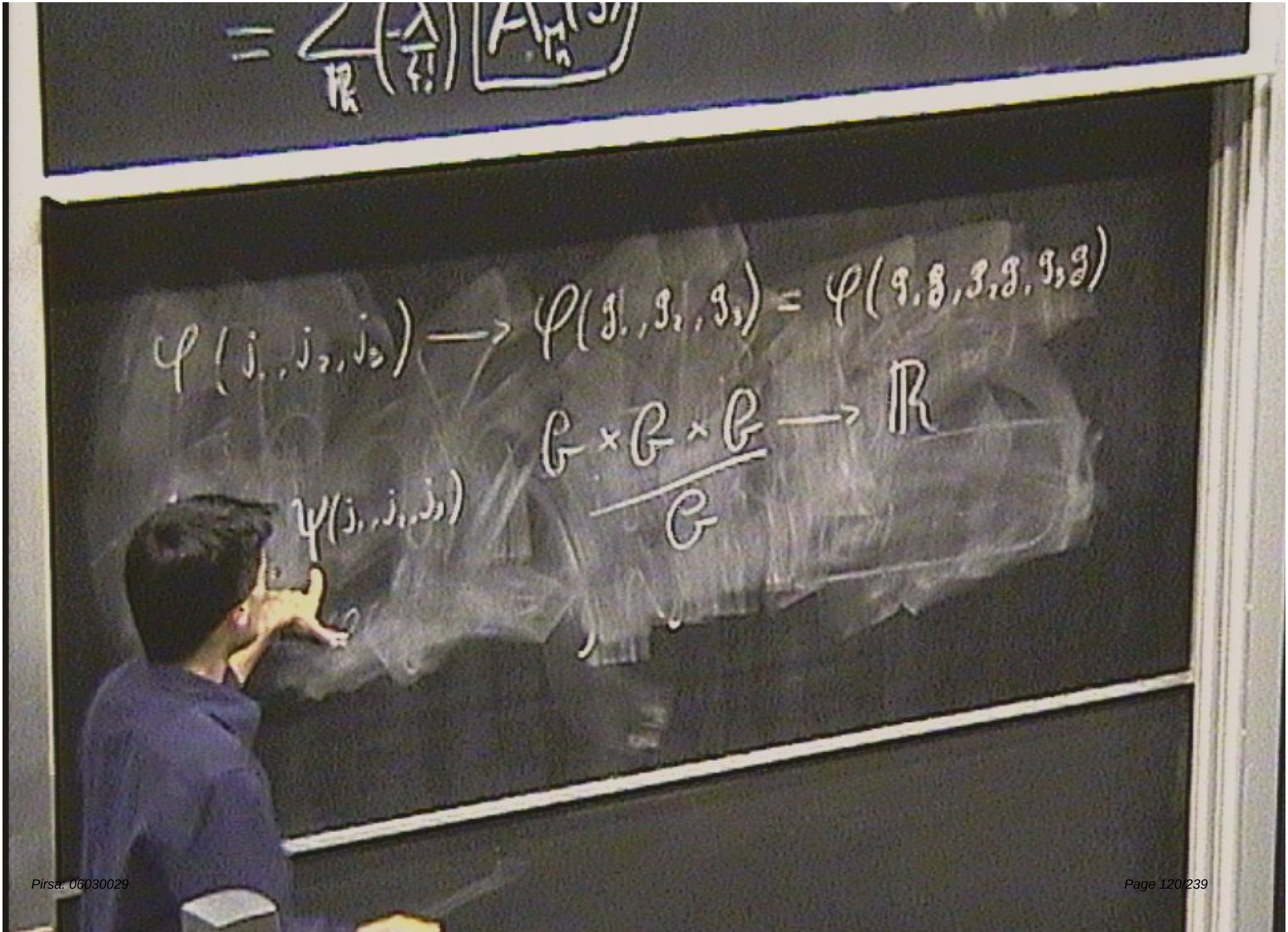
$$= \sum_{\mathbb{R}_n} \left(\frac{\Delta}{\Delta!}\right) \boxed{A_{\Gamma_n}(j)}$$

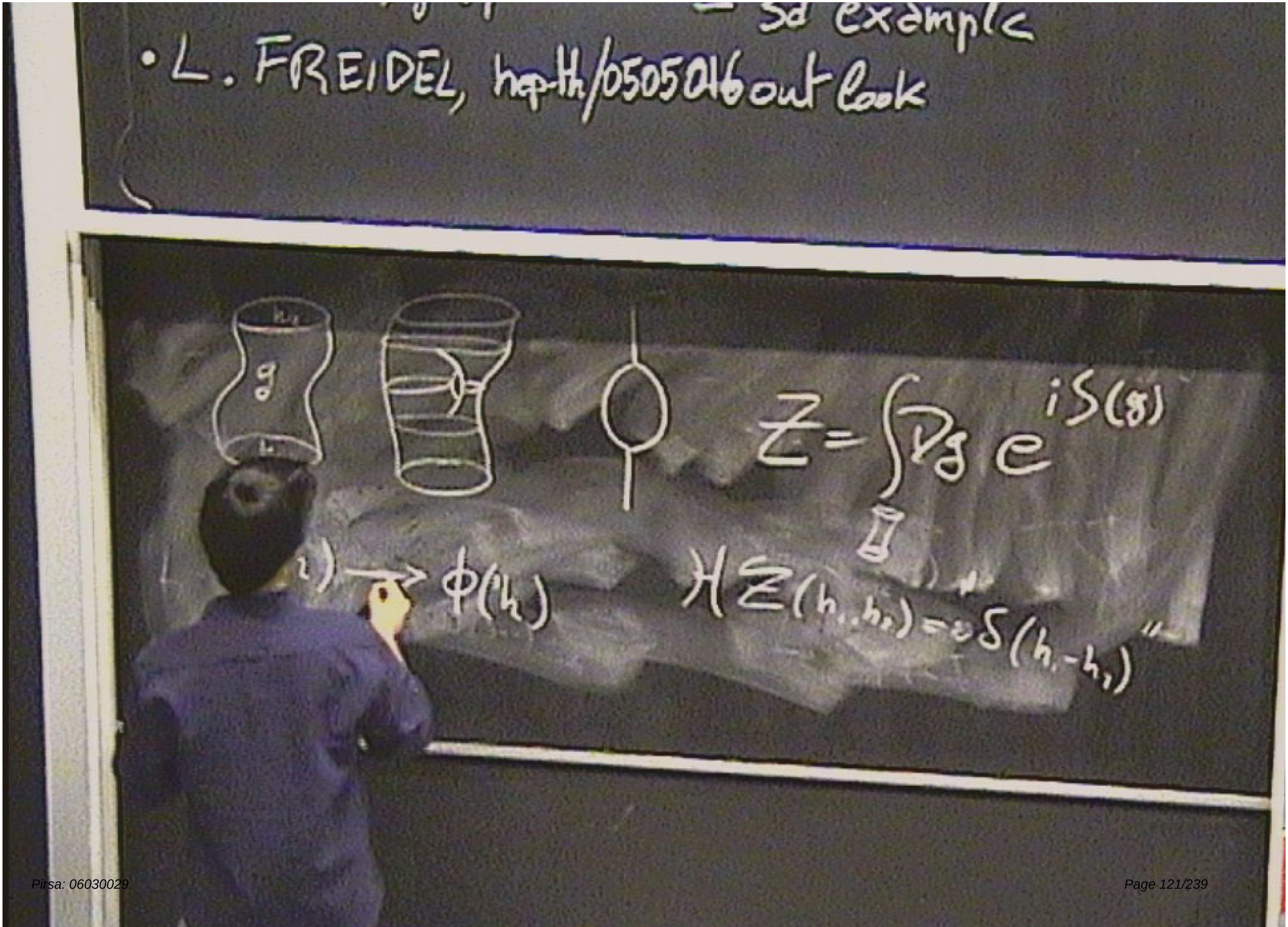
$$\varphi(j_1, j_2, j_3) \longrightarrow \varphi(g_1, g_2, g_3) = \varphi(g, g, g, g, g, g)$$

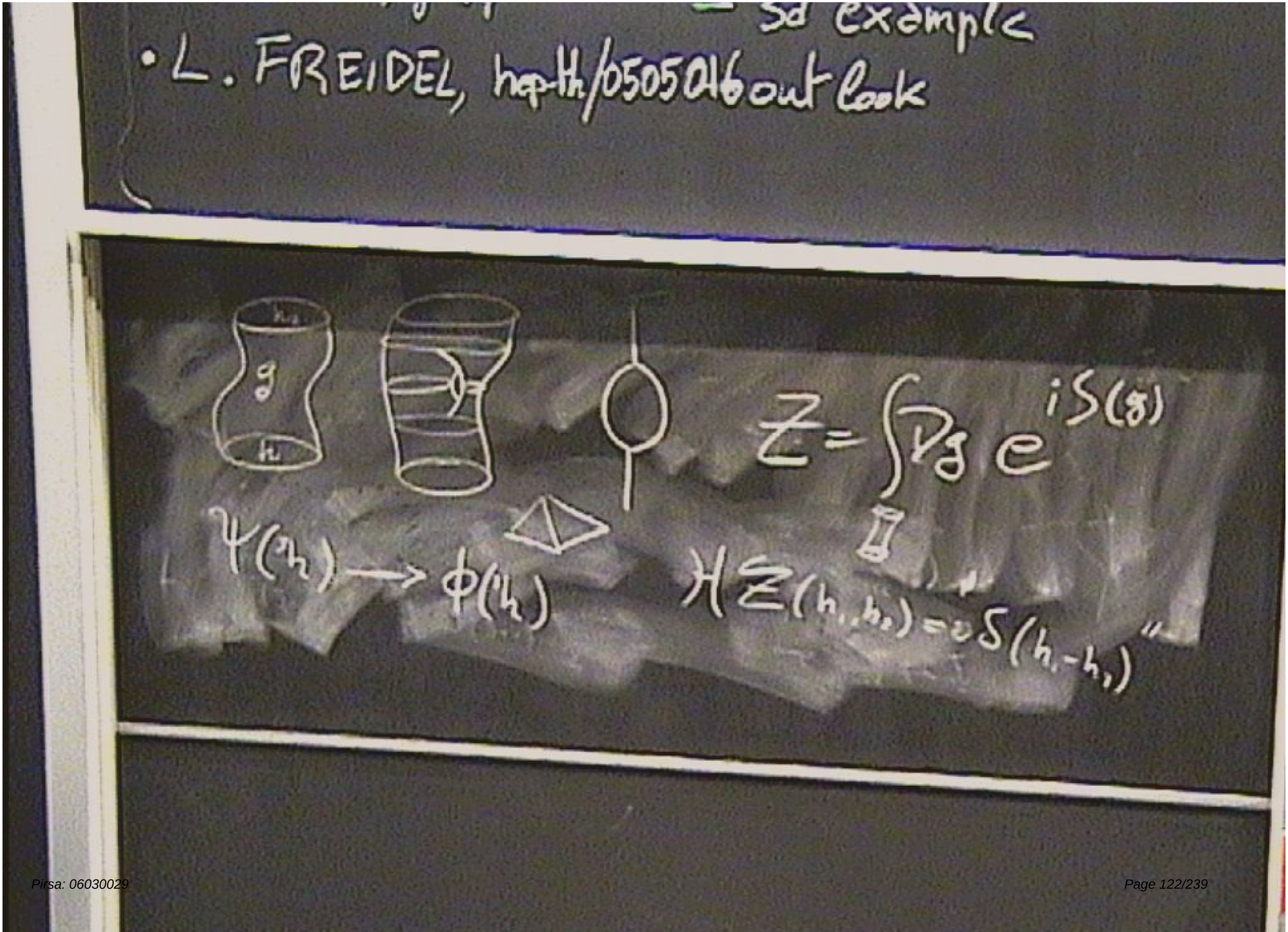
$$\frac{G \times G \times G}{G} \longrightarrow \mathbb{R}$$

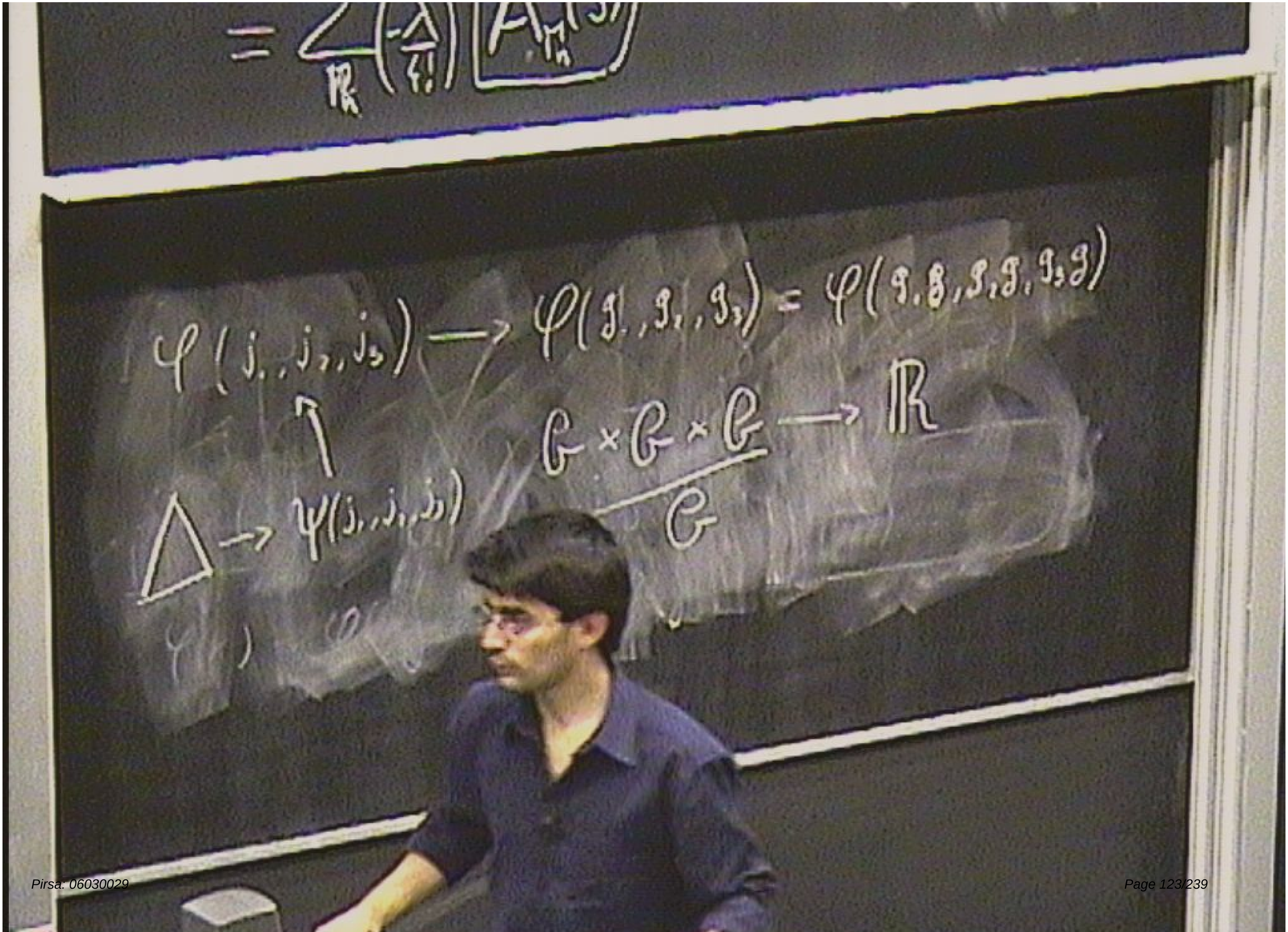
$$\Delta \to \psi(j_1, j_2, j_3)$$

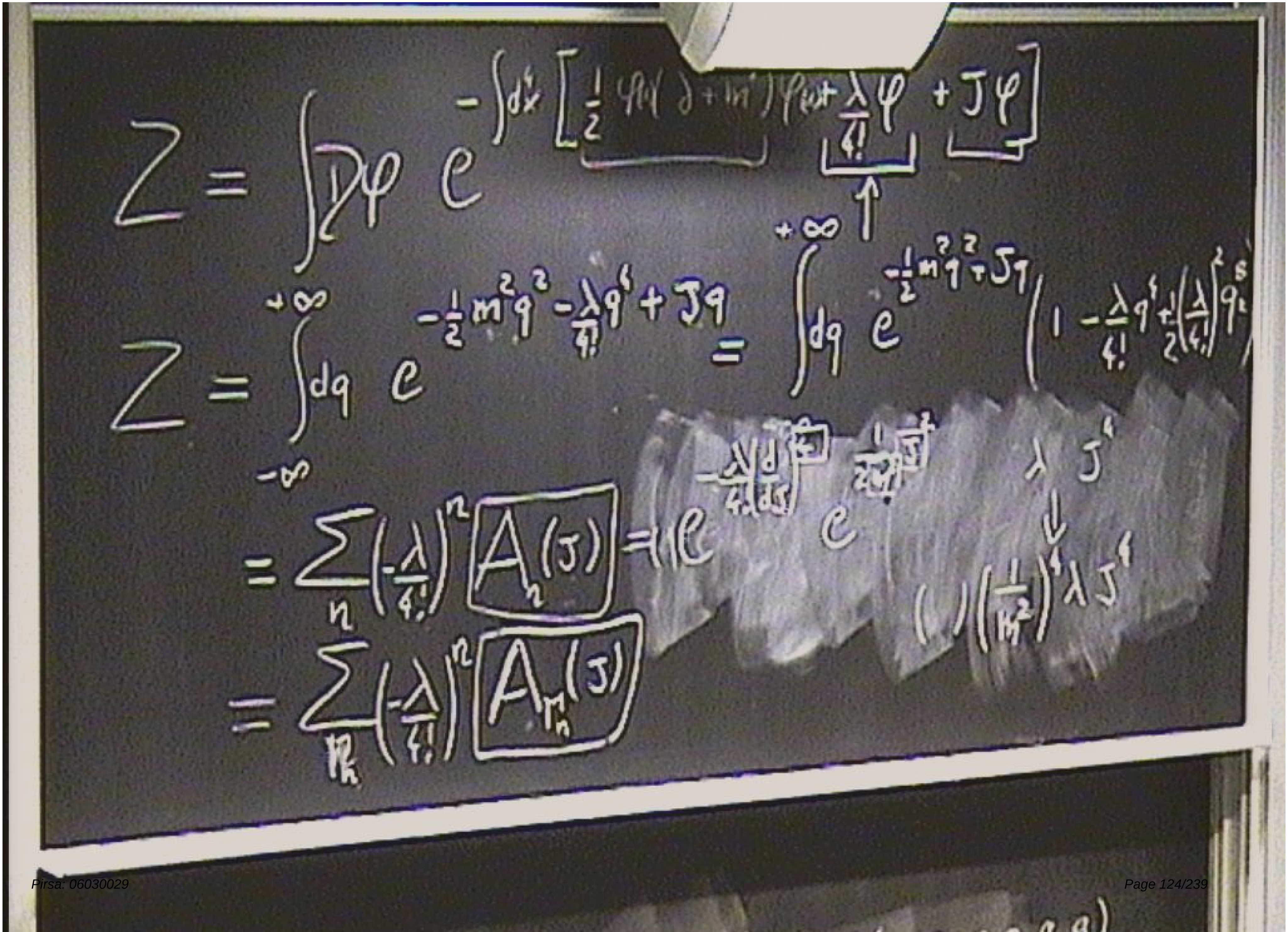

3d example
• L. FREIDEL, hep-th/0505016 out look
g
Z = ∫ Dg e^{iS(g)}
Ψ(h) → φ(h)
H Z(h1,h2) = "δ(h1-h2)"


$\varphi(j_1, j_2, j_3) \longrightarrow \varphi(g_1, g_2, g_3) = \varphi(g_1 g, g_2 g, g_3 g)$
$G \times G \times G \longrightarrow \mathbb{R}$
$\frac{G \times G \times G}{G}$
$\psi(j_1, j_2, j_3)$

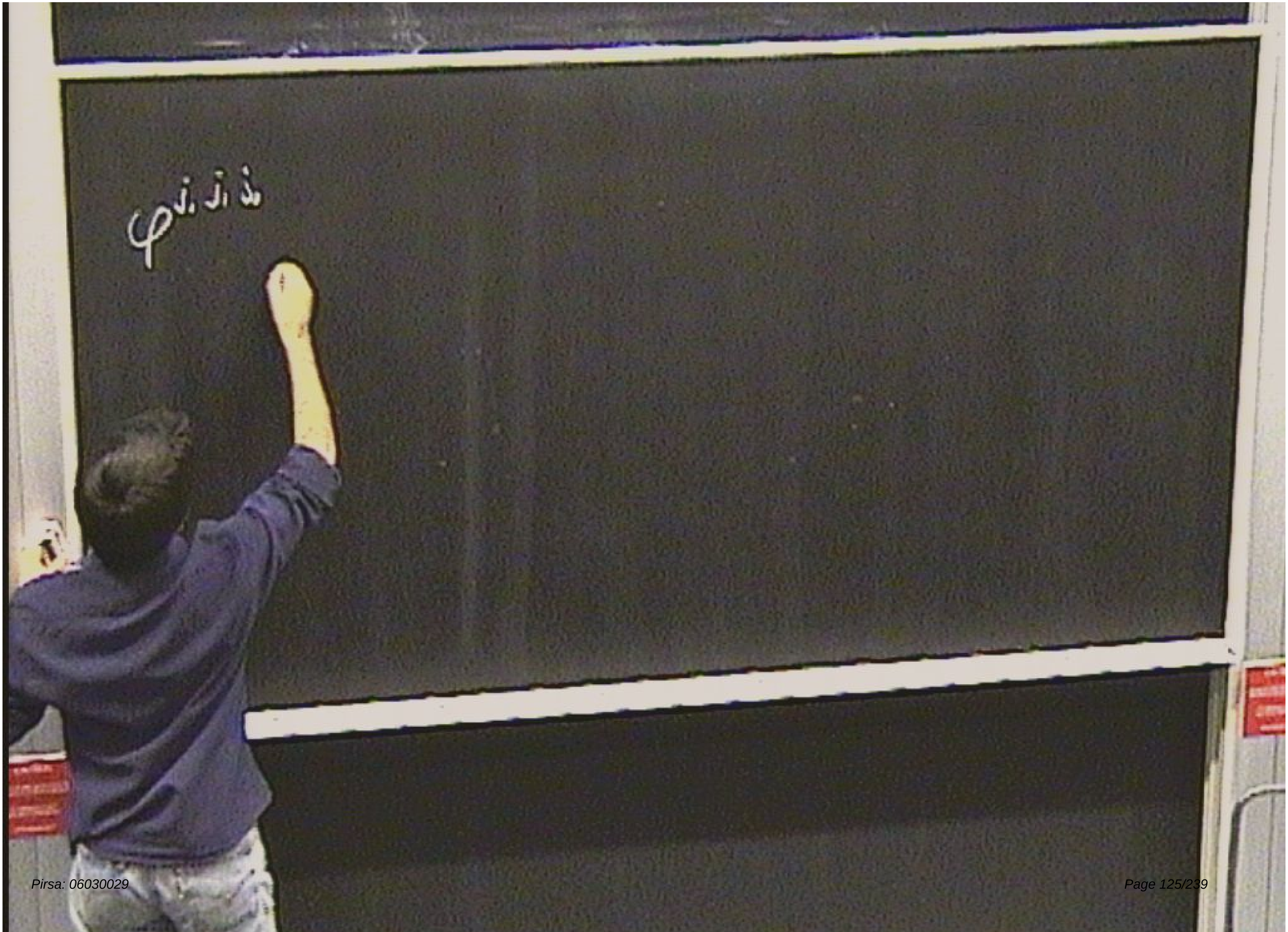

sd example
• L. FREIDEL, hep-th/0505016 out look
g
Z = ∫Dg e^{iS(g)}
φ(h₂)
Z(h₁,h₂) = δ(h₁-h₂)

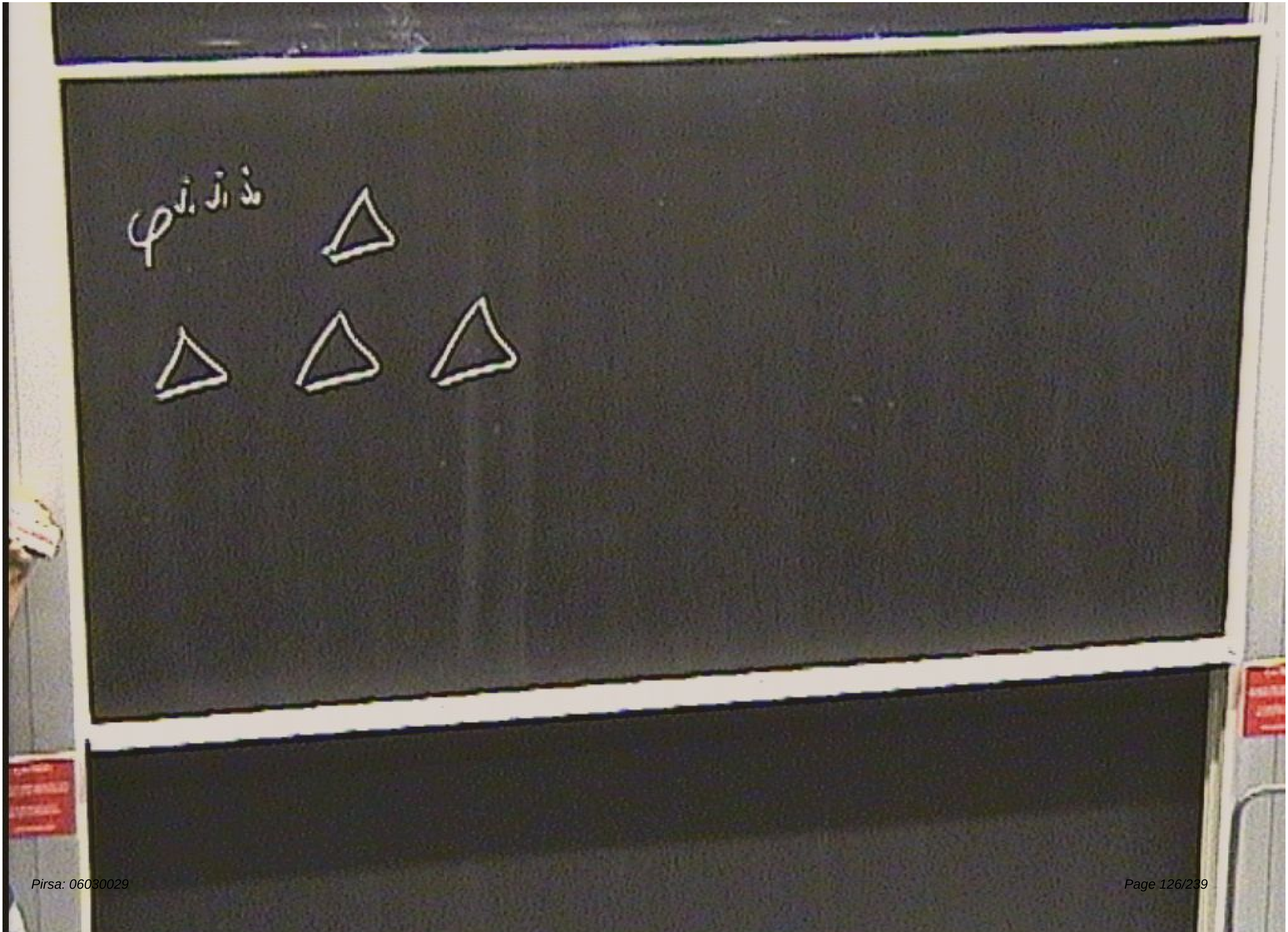

sd example
• L. FREIDEL, hep-th/0505016 outlook
g
Z = ∫Dg e^{iS(g)}
ψ(h) ⟶ φ(h)

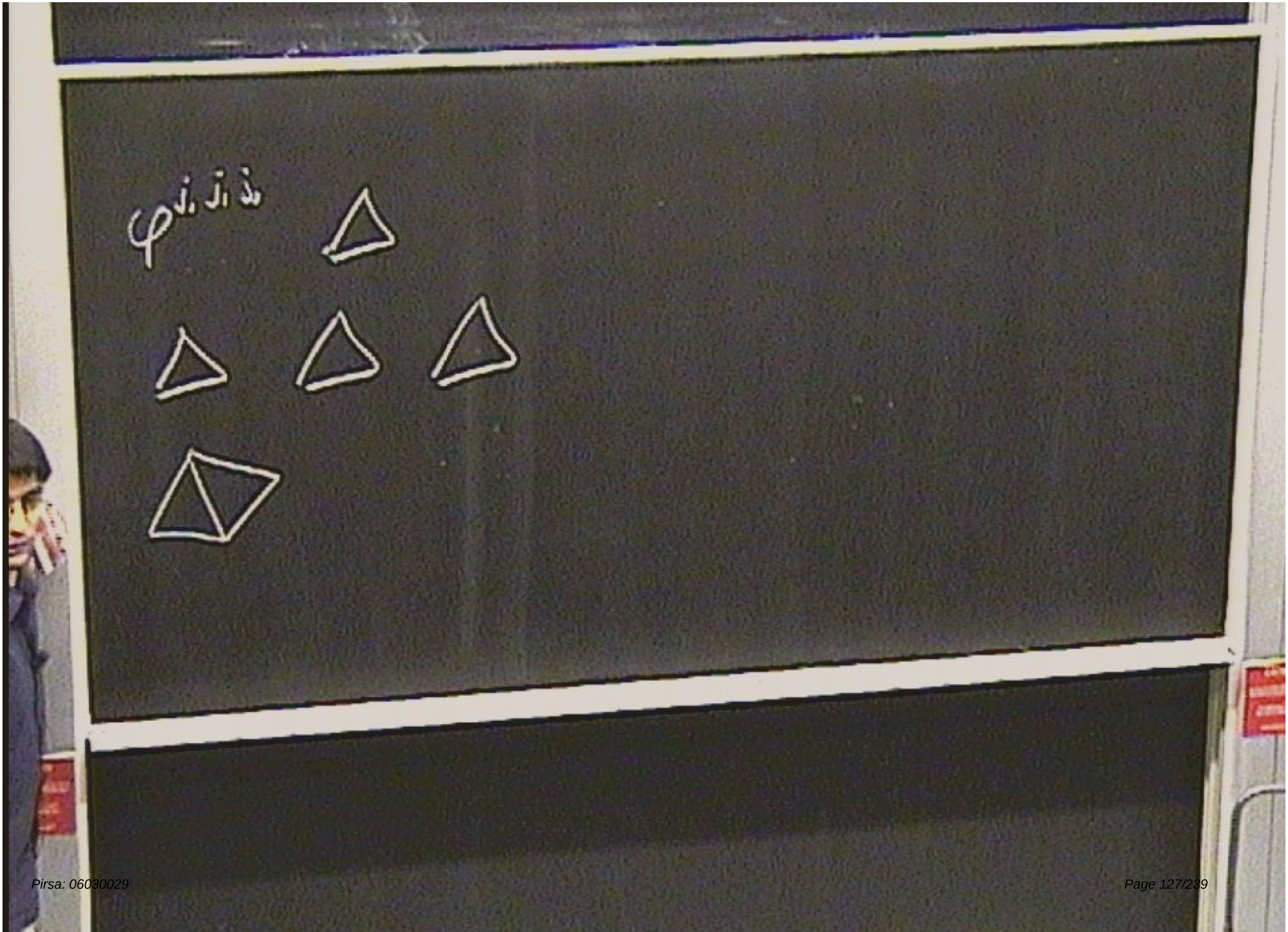

φ(j1, j2, j3) → φ(g1, g2, g3) = φ(g1g, g2g, g3g)
G × G × G → ℝ
G
Δ → ψ(j1, j2, j3)

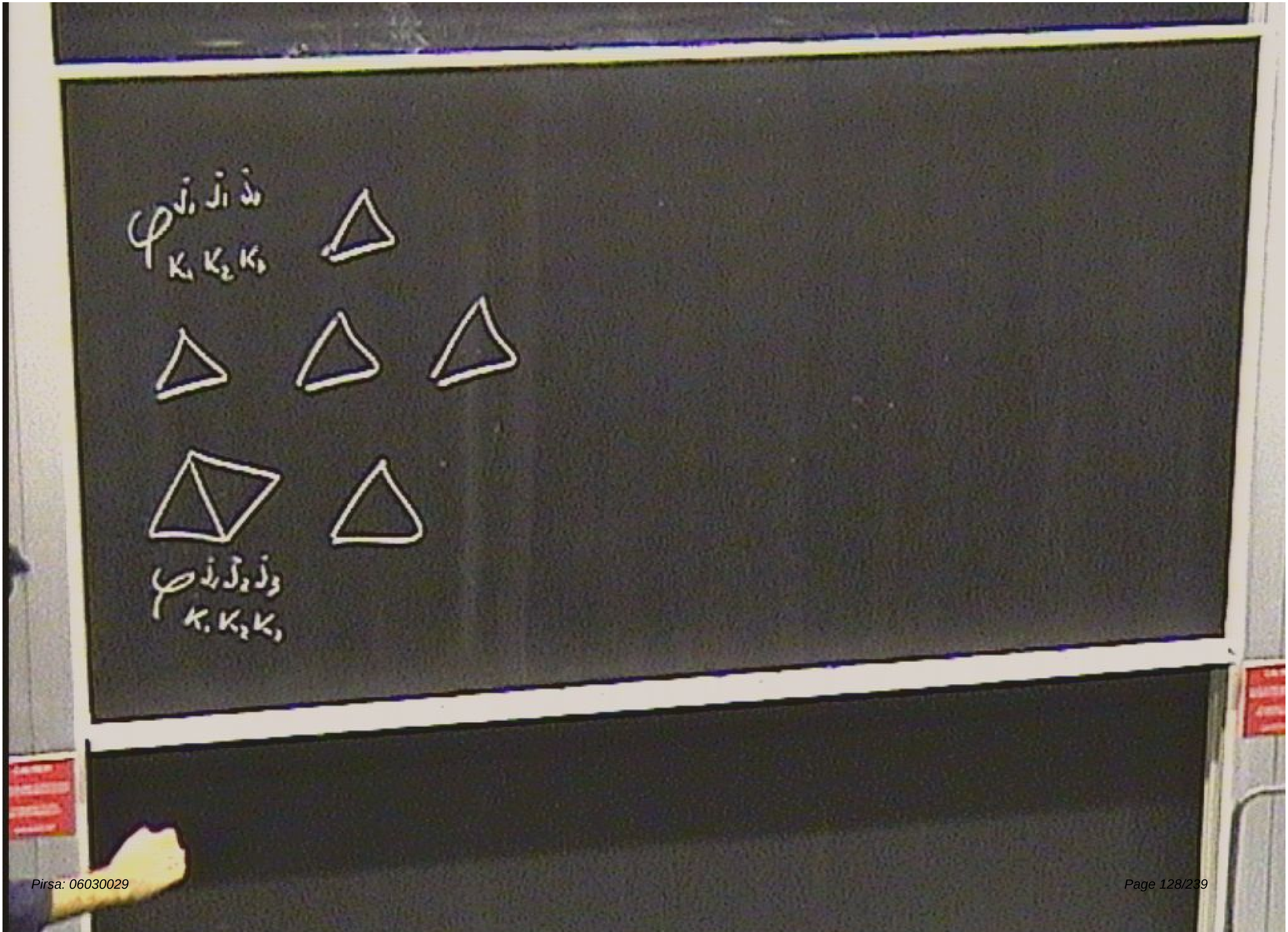


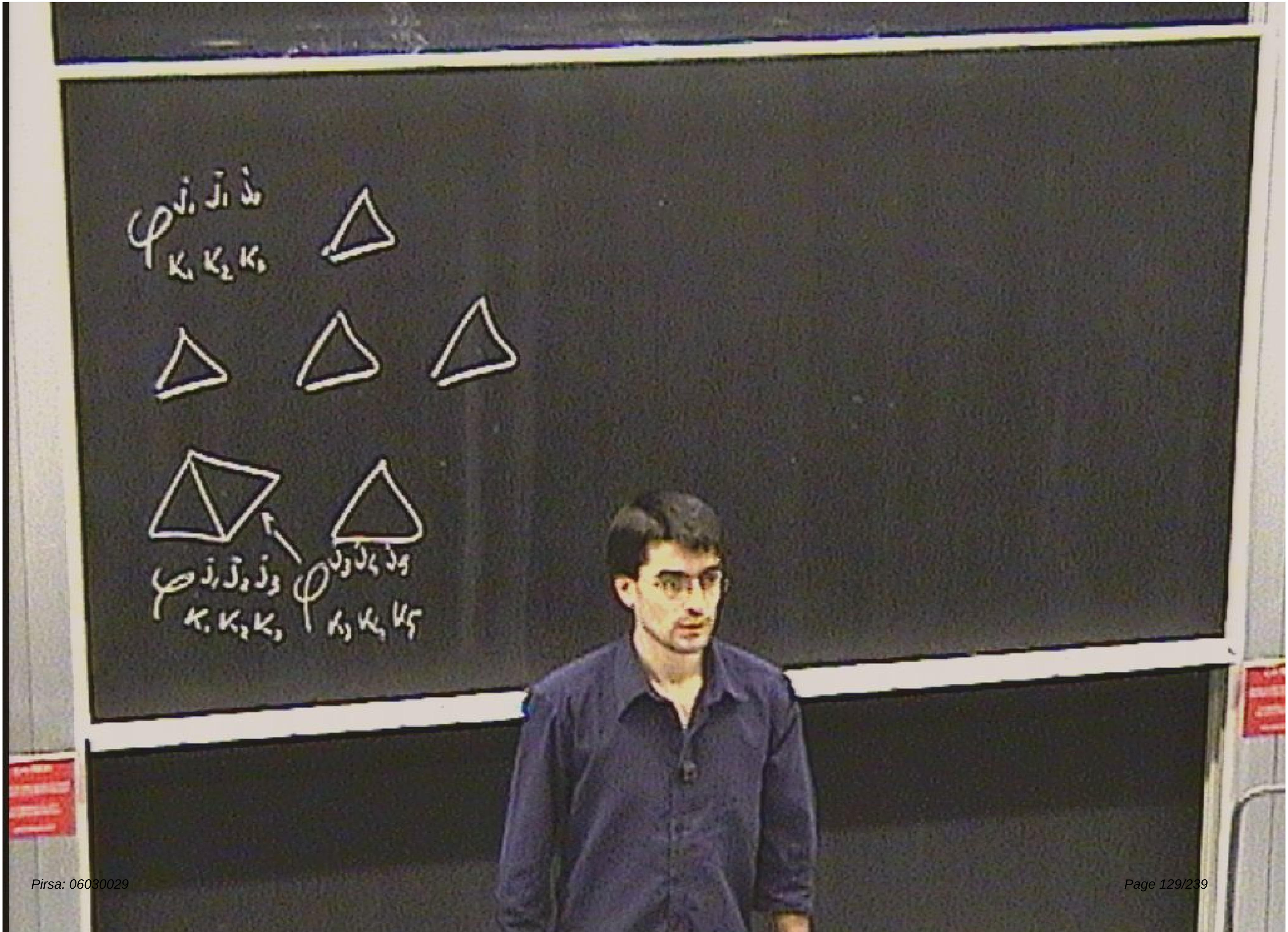
$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(x)(\partial^2 + m^2)\varphi(x) + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2 q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2 q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8 \ldots\right)$
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$
$= \sum_{n} \left(-\frac{\lambda}{4!}\right)^n A_{n}(J)$




φi,j,k


φi,j,k


φ j1 j2 j3
K1 K2 K3
φ j1 j2 j3
K1 K2 K3



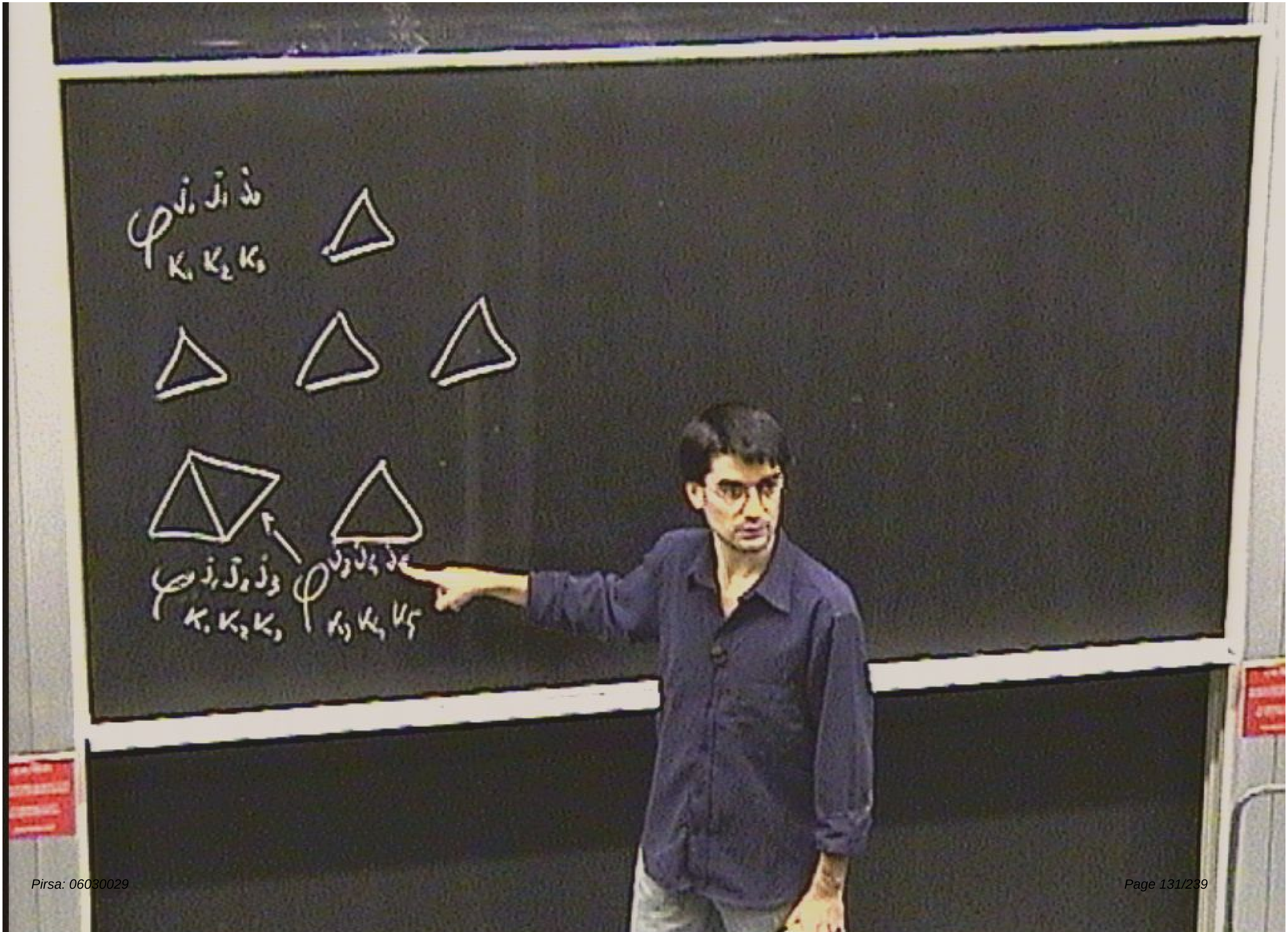
= 3d example

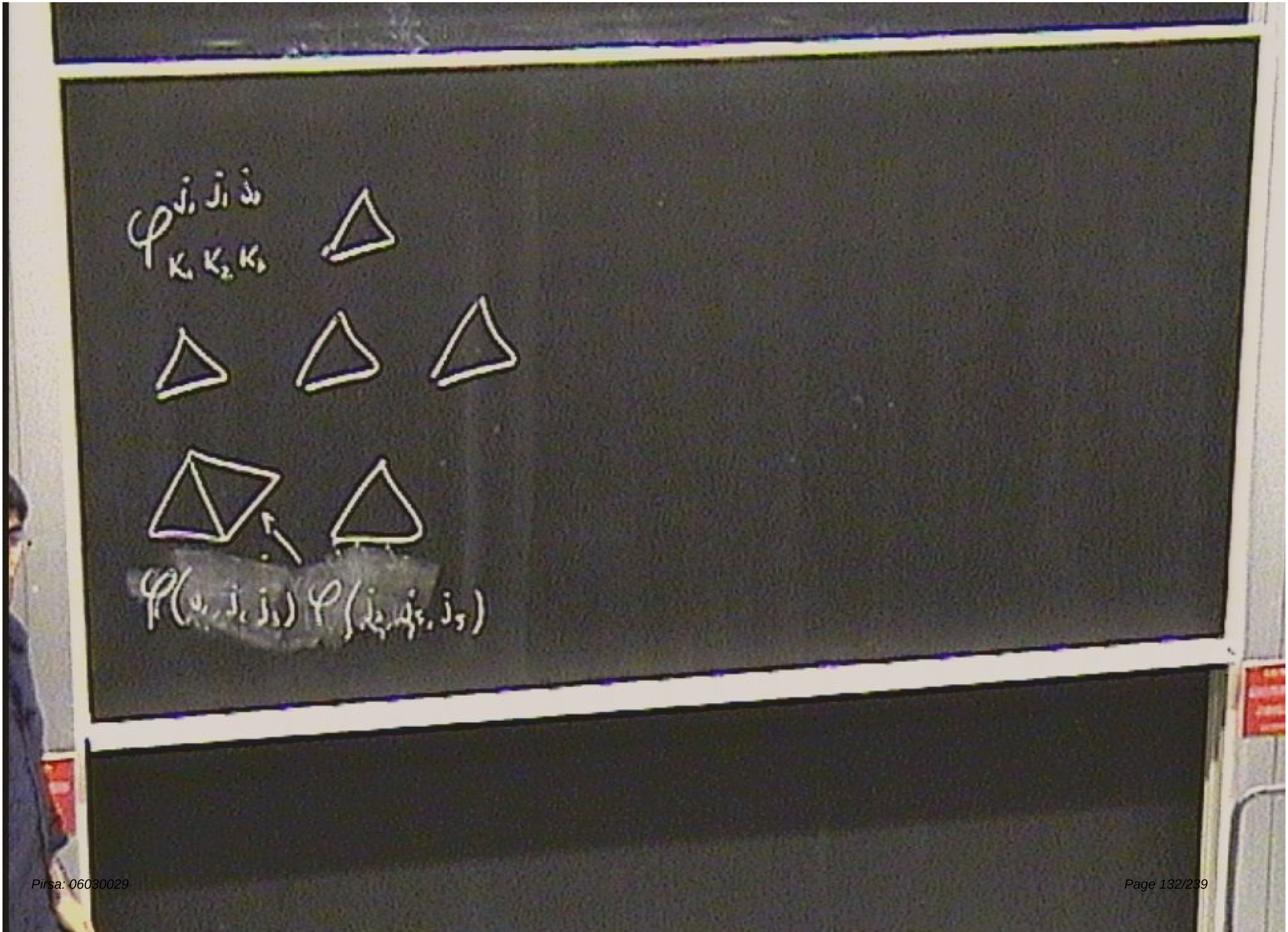
• L. FREIDEL, hep-th/0505016 out look

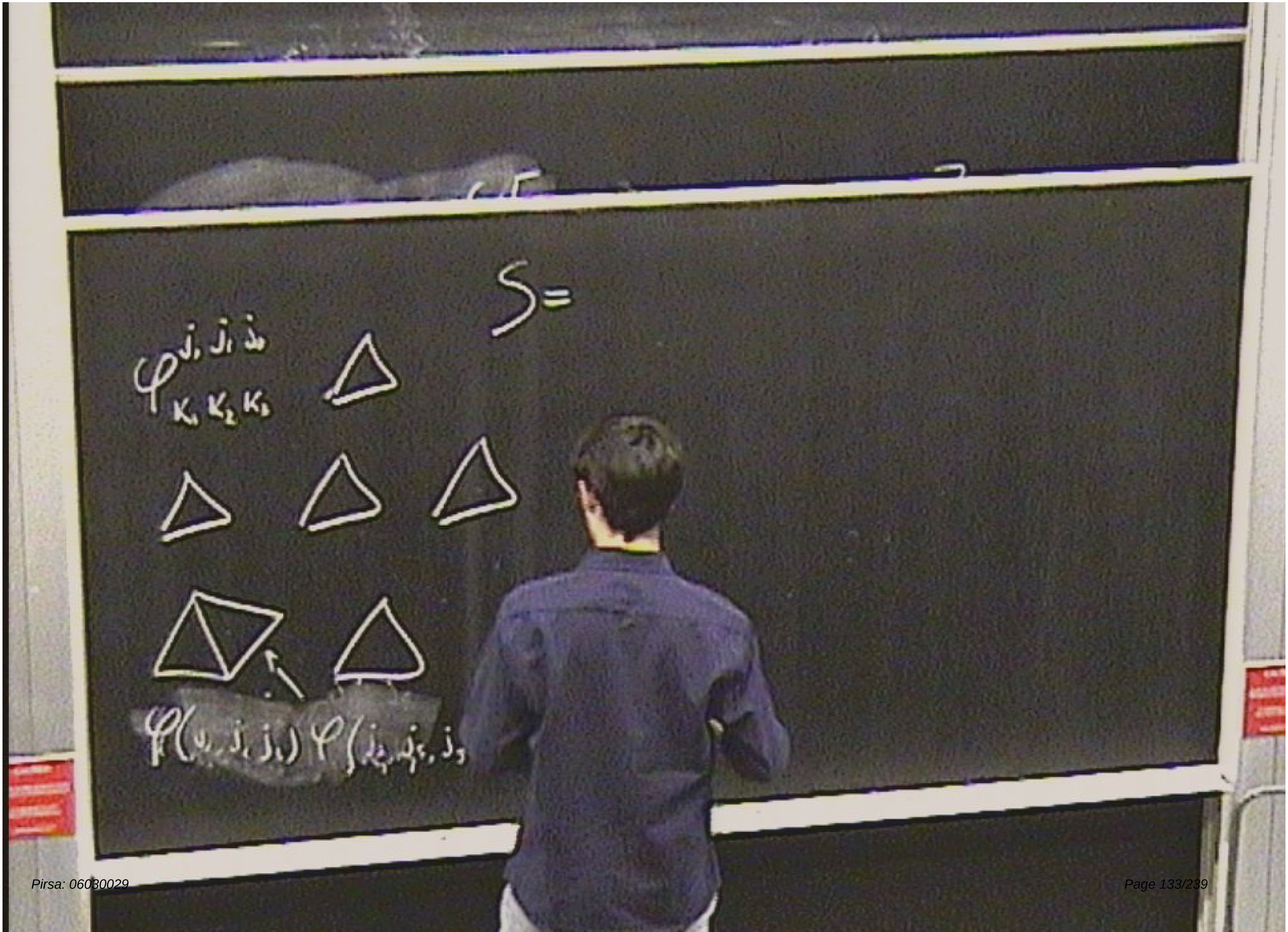
$$Z = \int \mathcal{D}g \, e^{iS(g)}$$

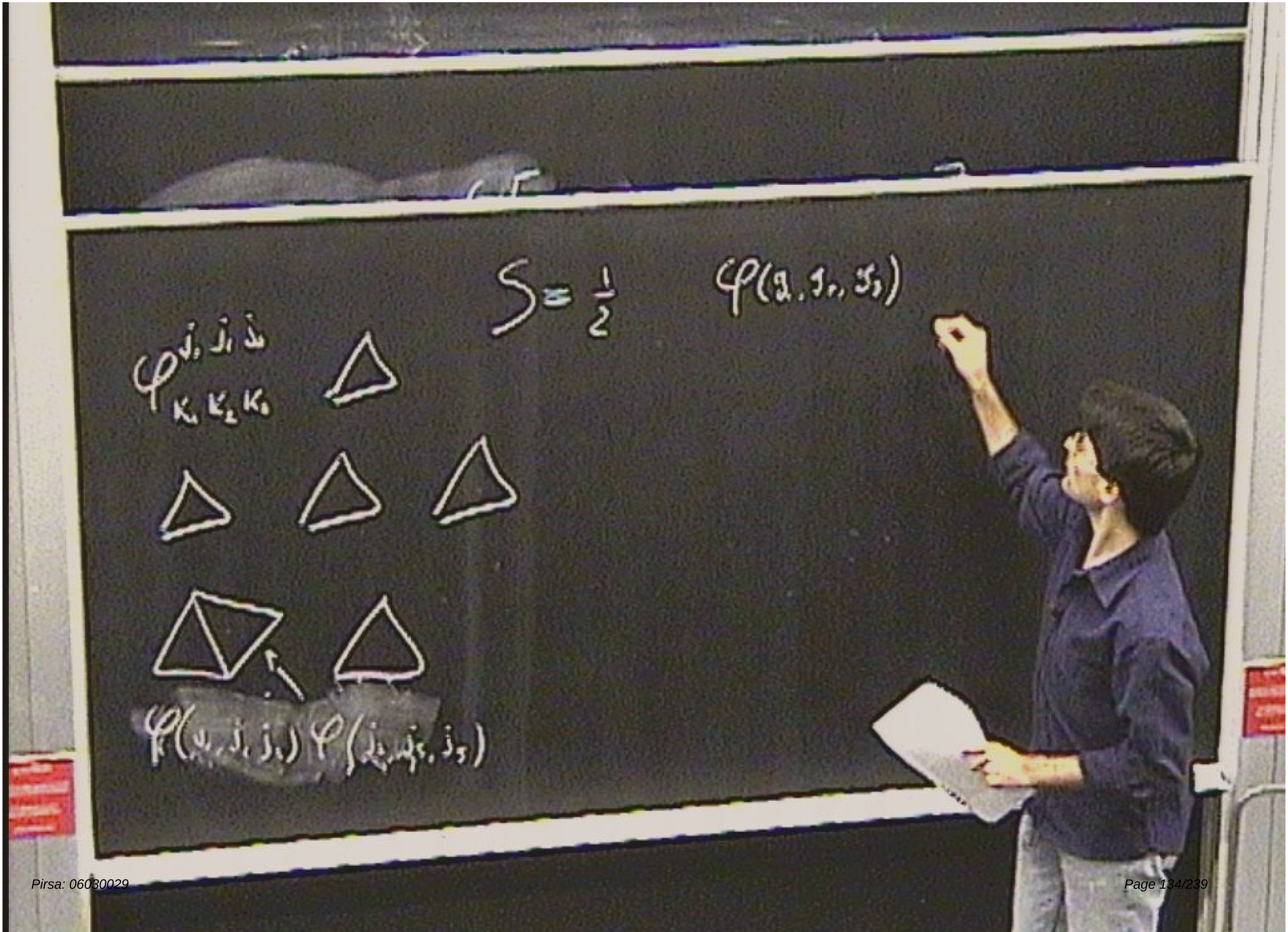
$\Psi(h) \longrightarrow \phi(h)$

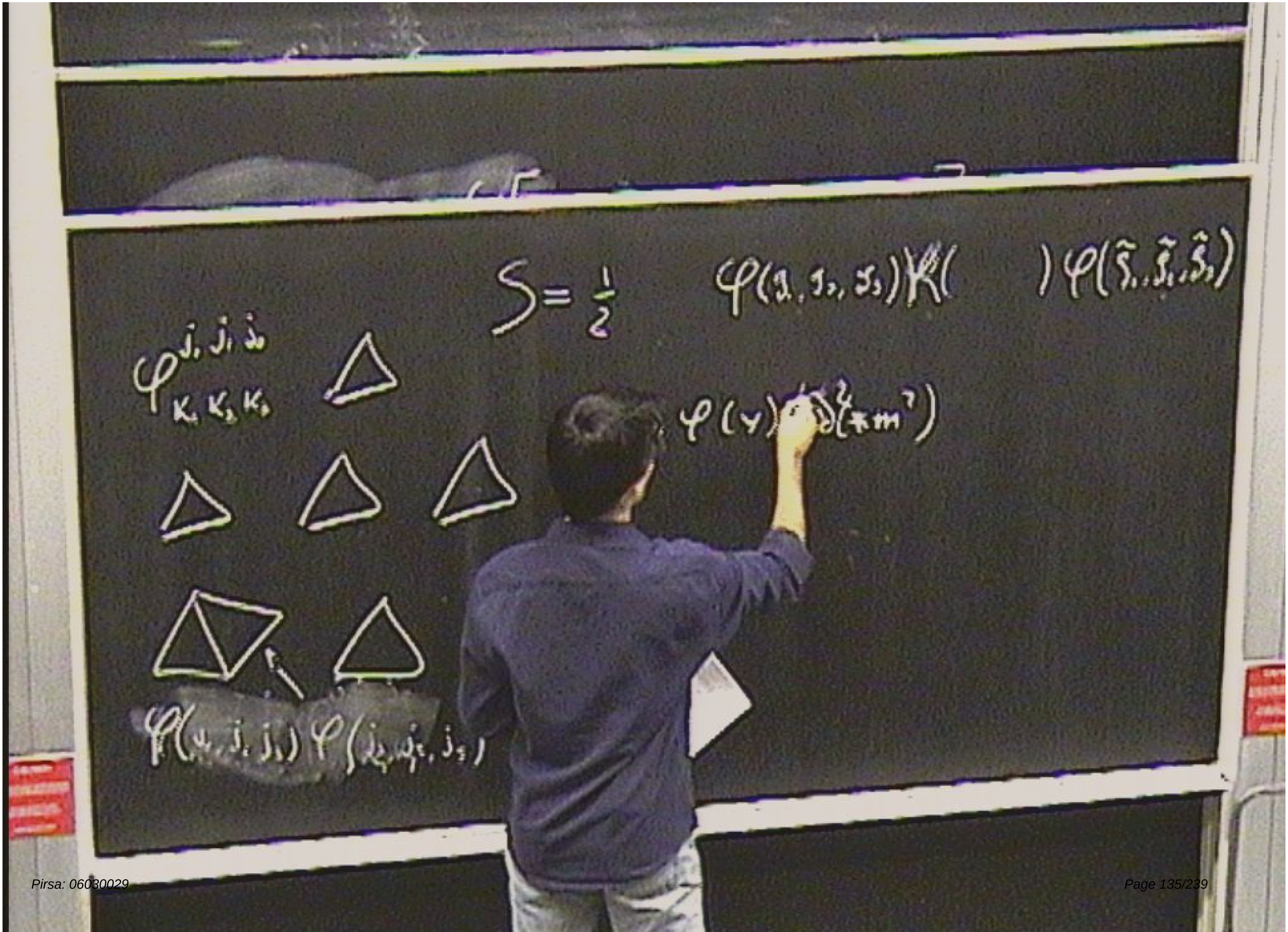
$\mathcal{H} Z(h_1, h_2) = \text{"}\delta(h_1 - h_2)\text{"}$

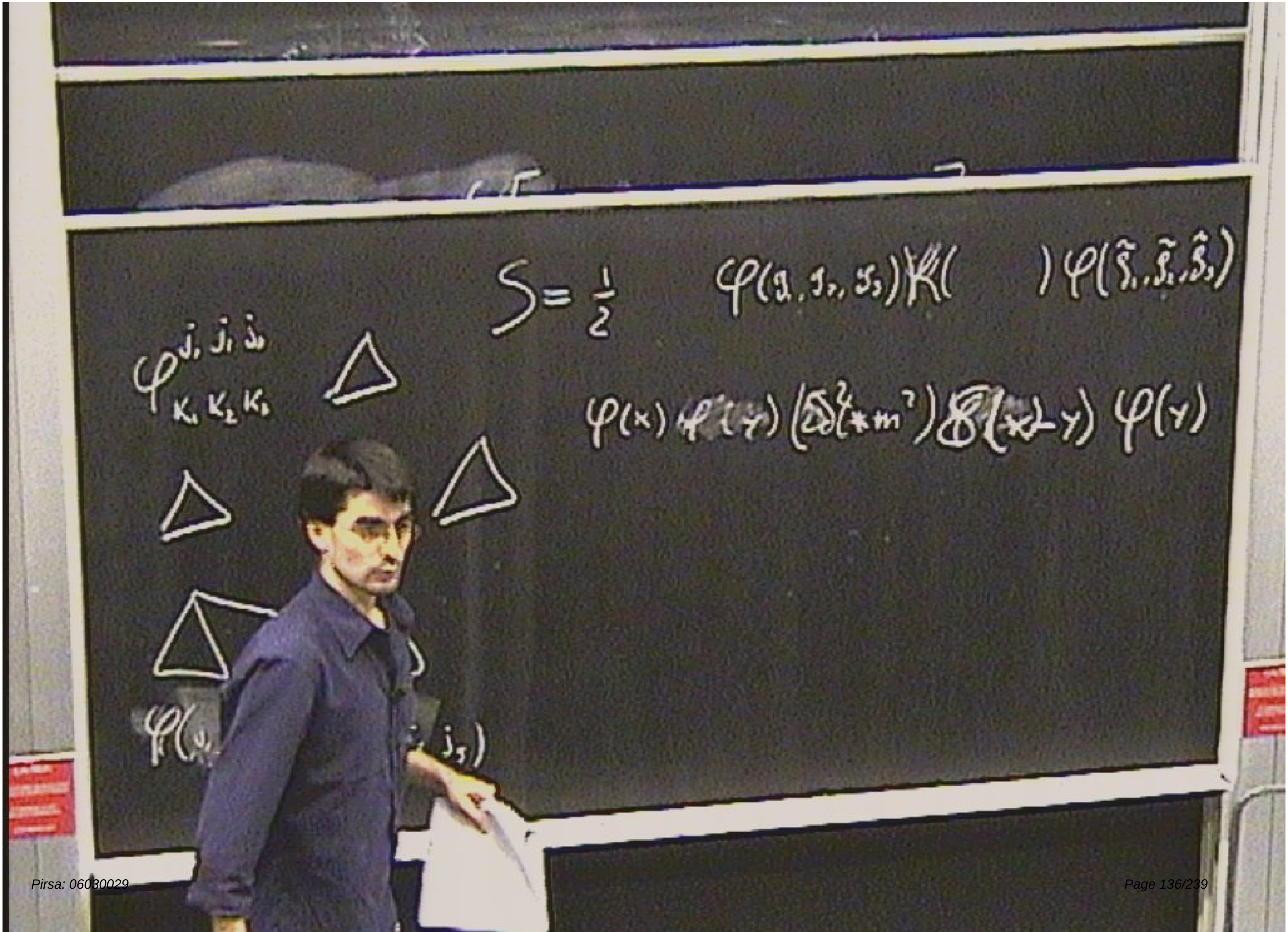


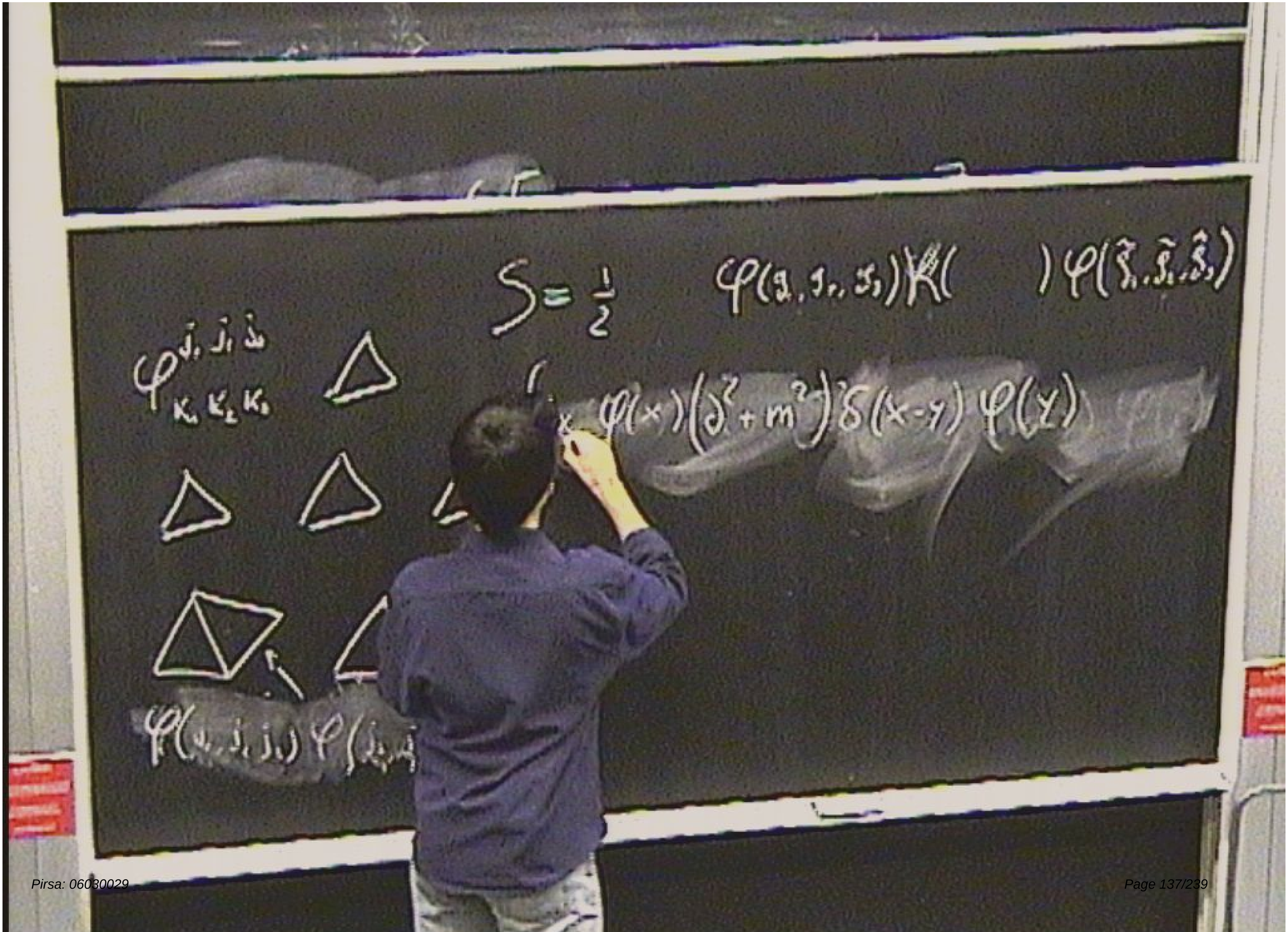


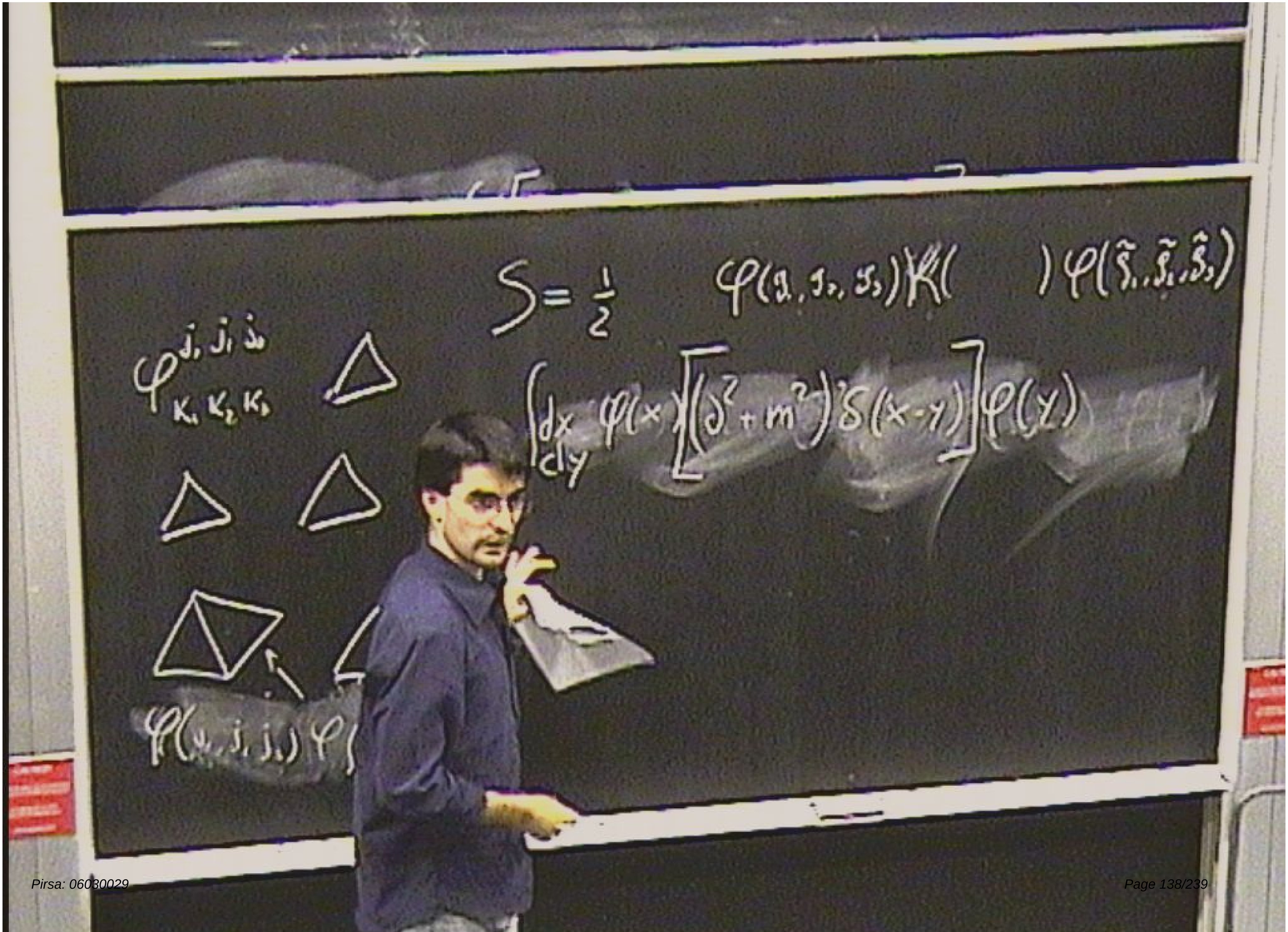

S=

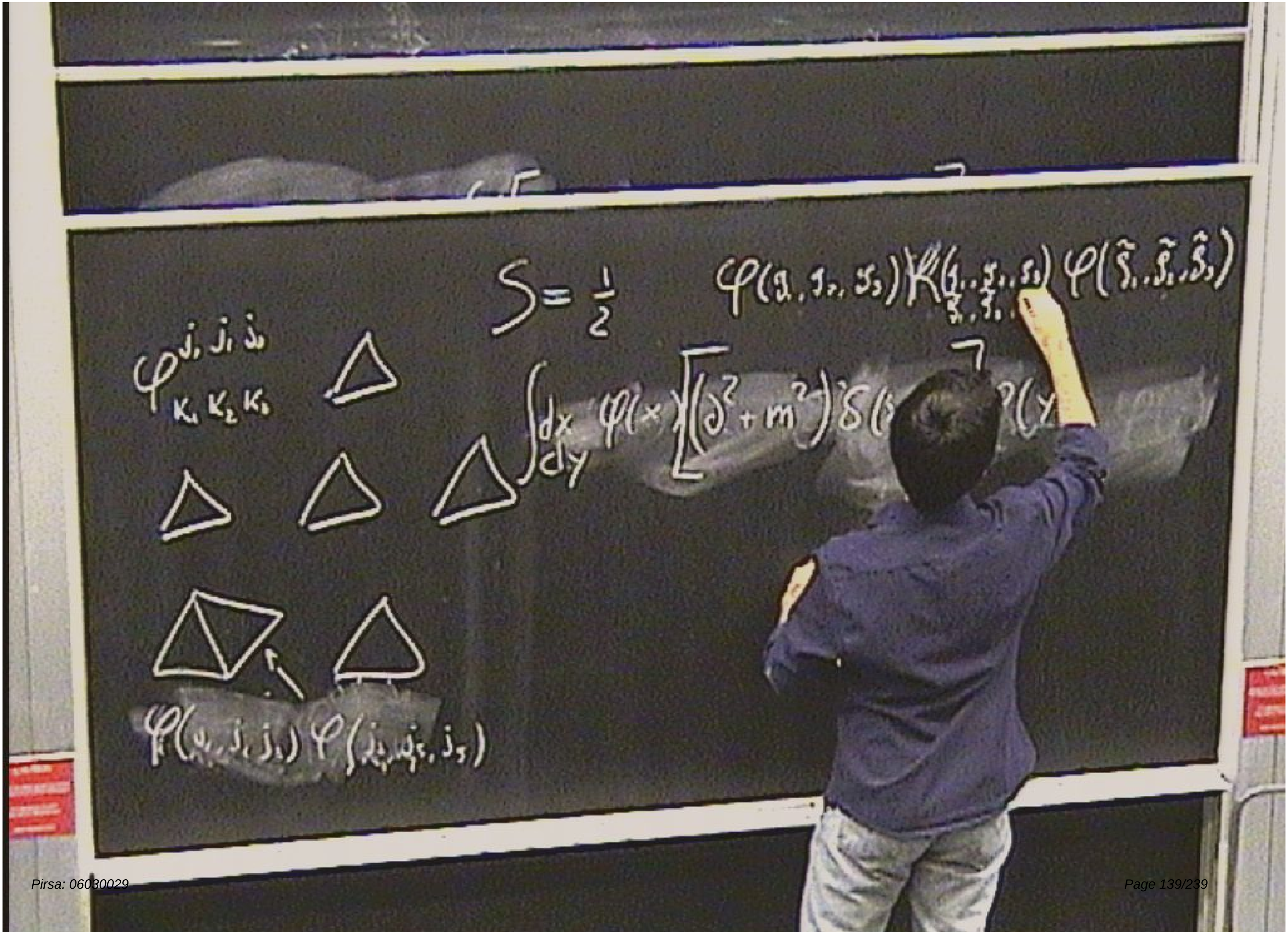

S = 1/2

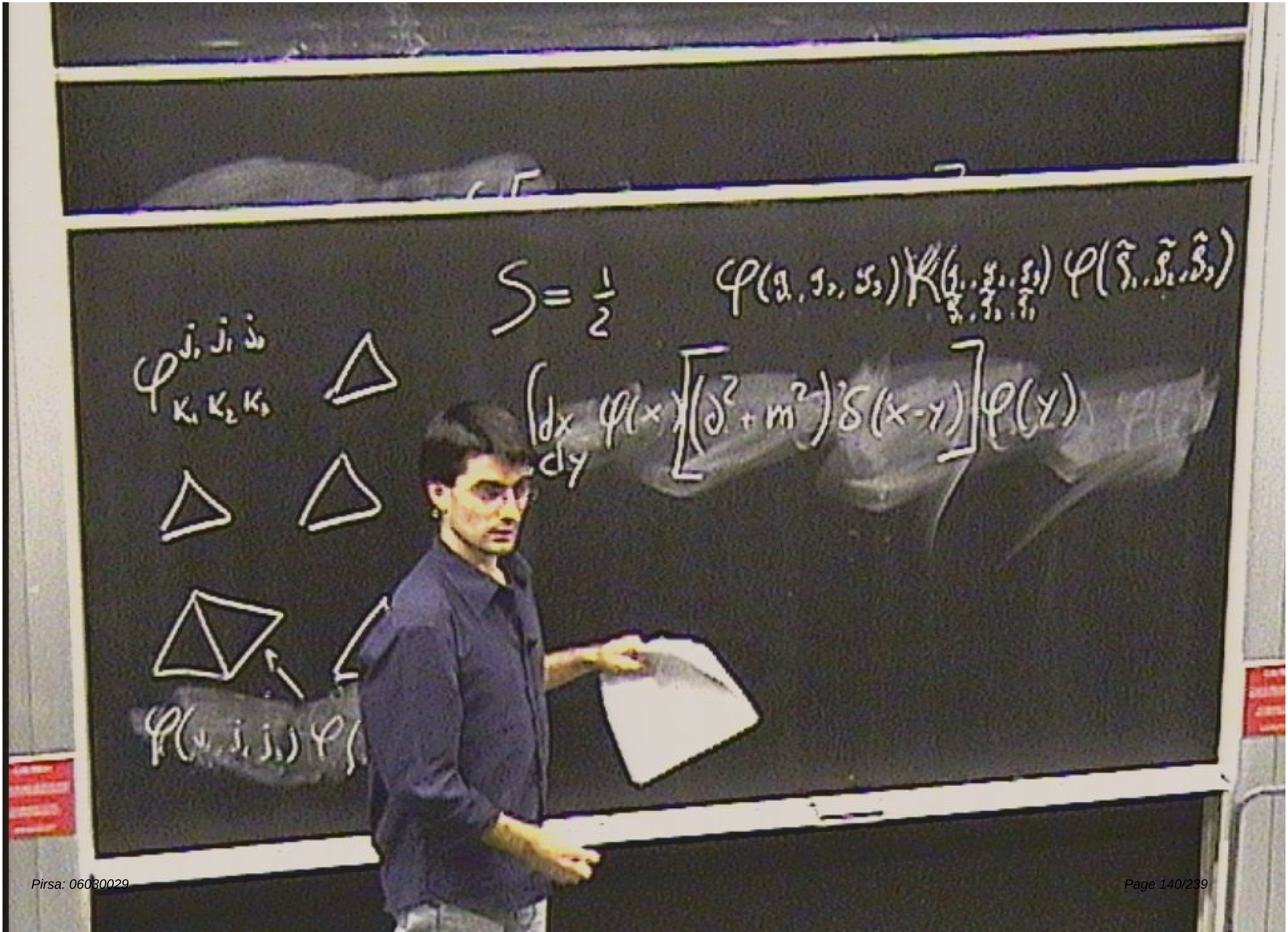

S = 1/2

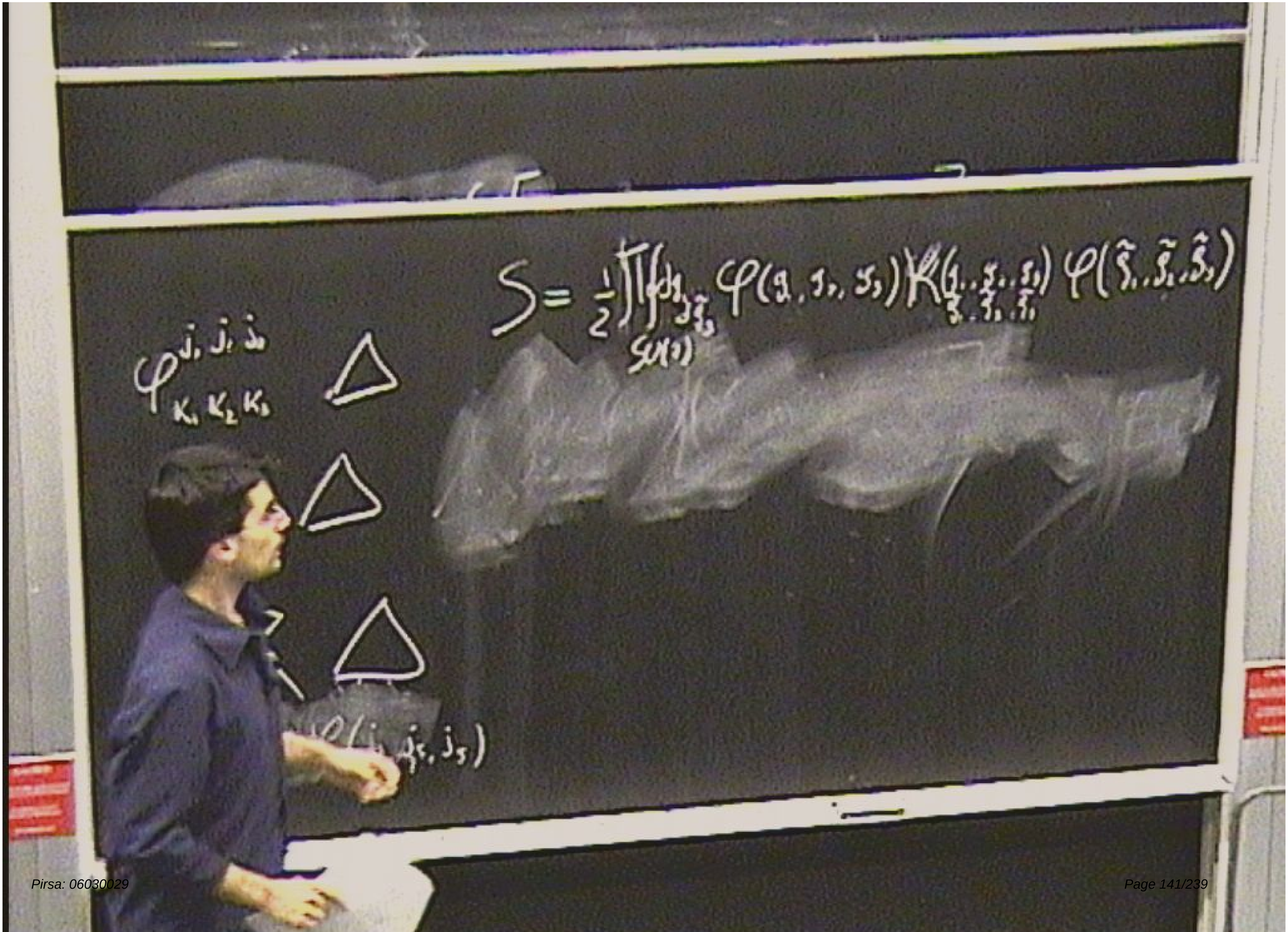


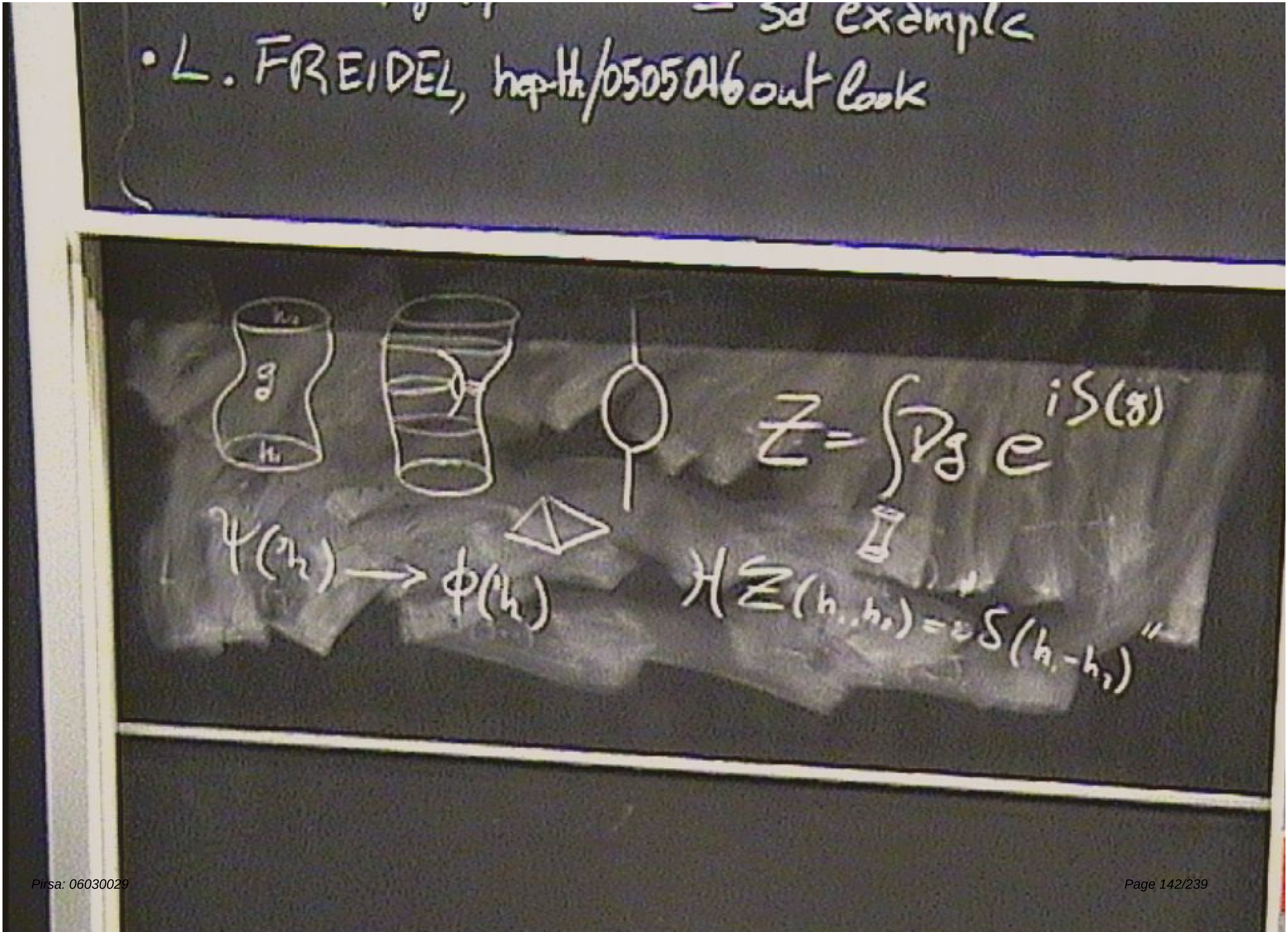

S = 1/2 φ(j1, j2, j3) K( ) φ(j̃1, j̃2, j̃3)
∫x φ(x)(∂² + m²)δ(x-y) φ(y)

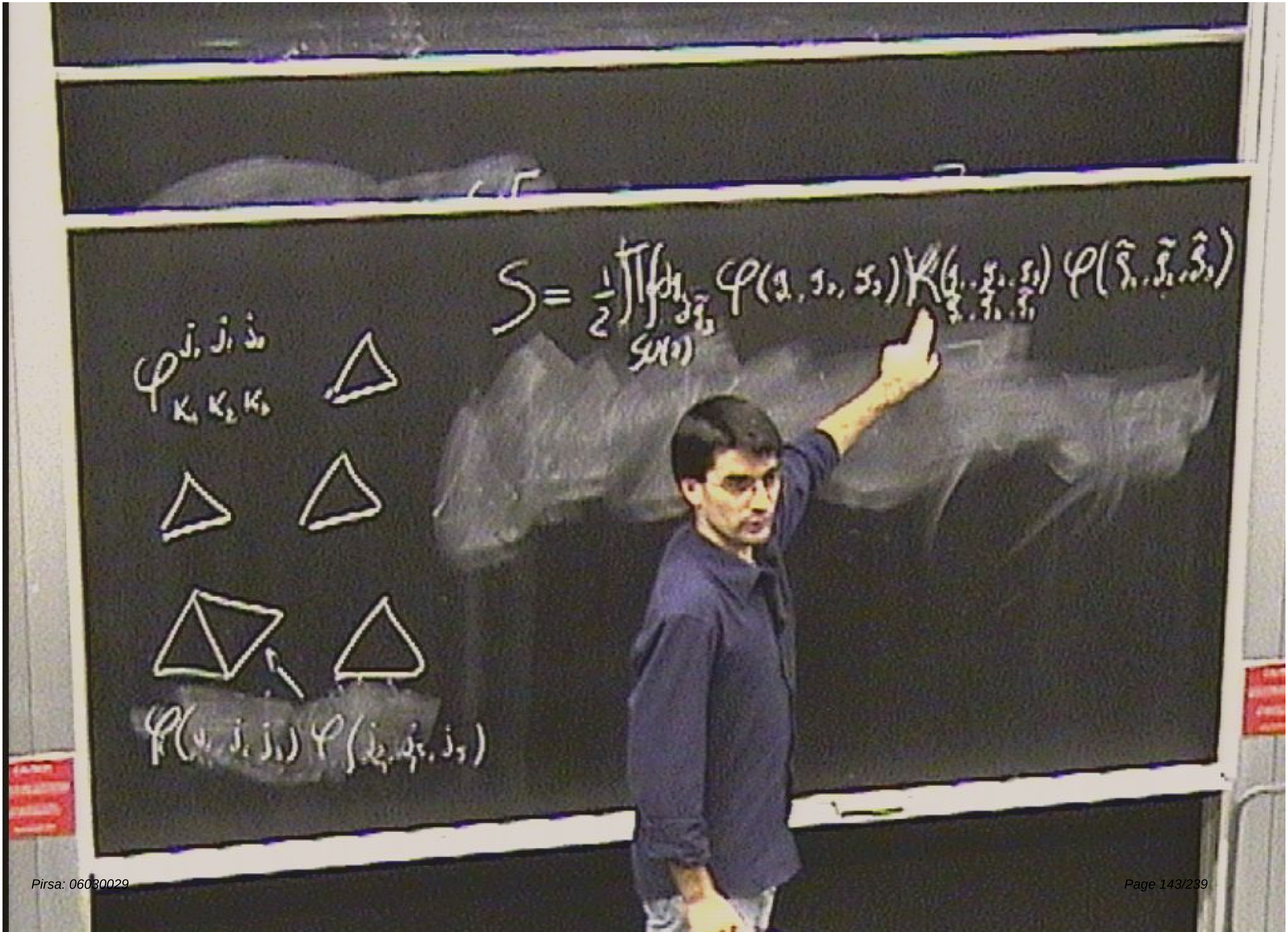

S = 1/2
∫dx dy φ(x)[(∂² + m²) δ(x − y)] φ(y)

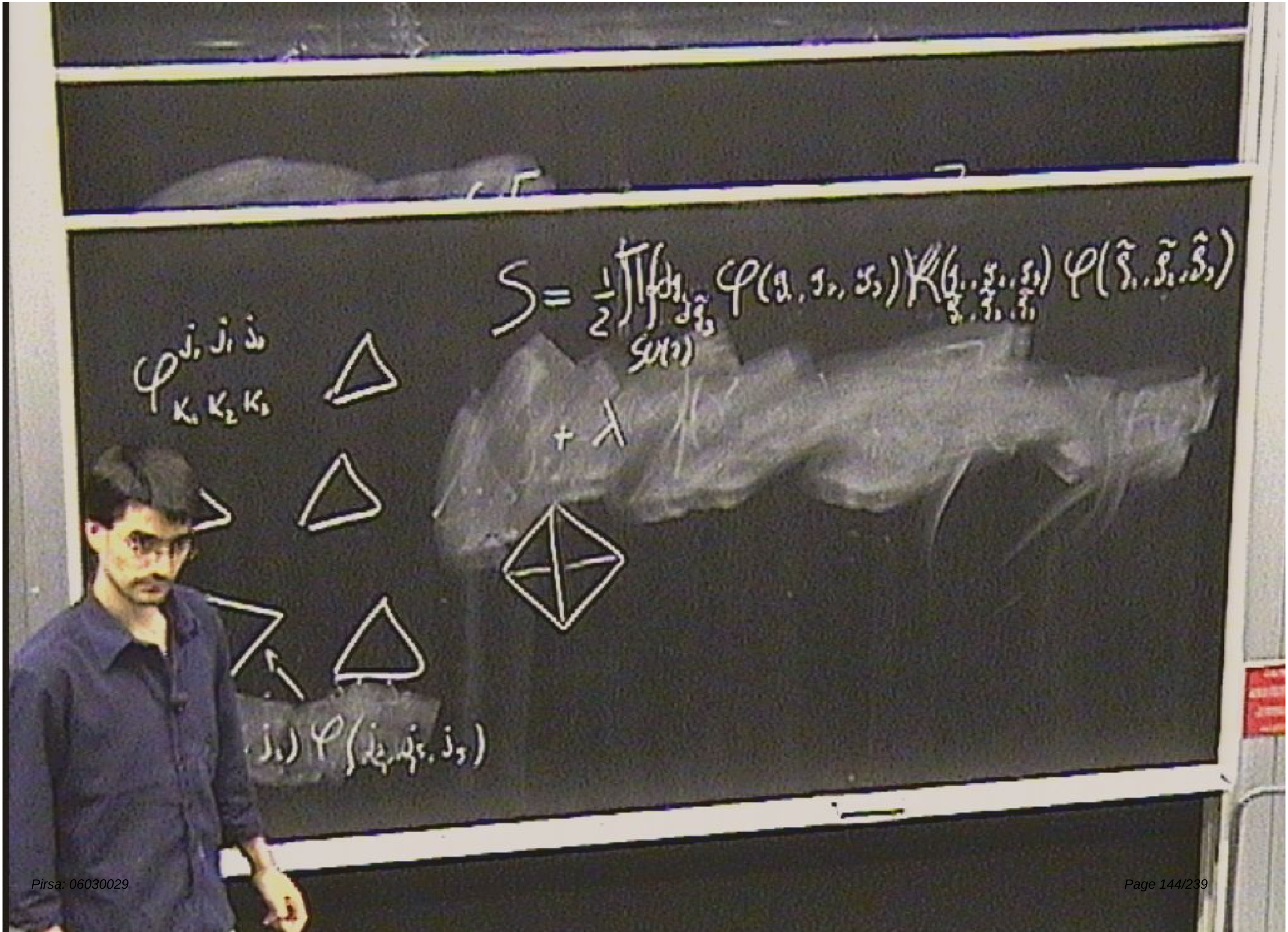


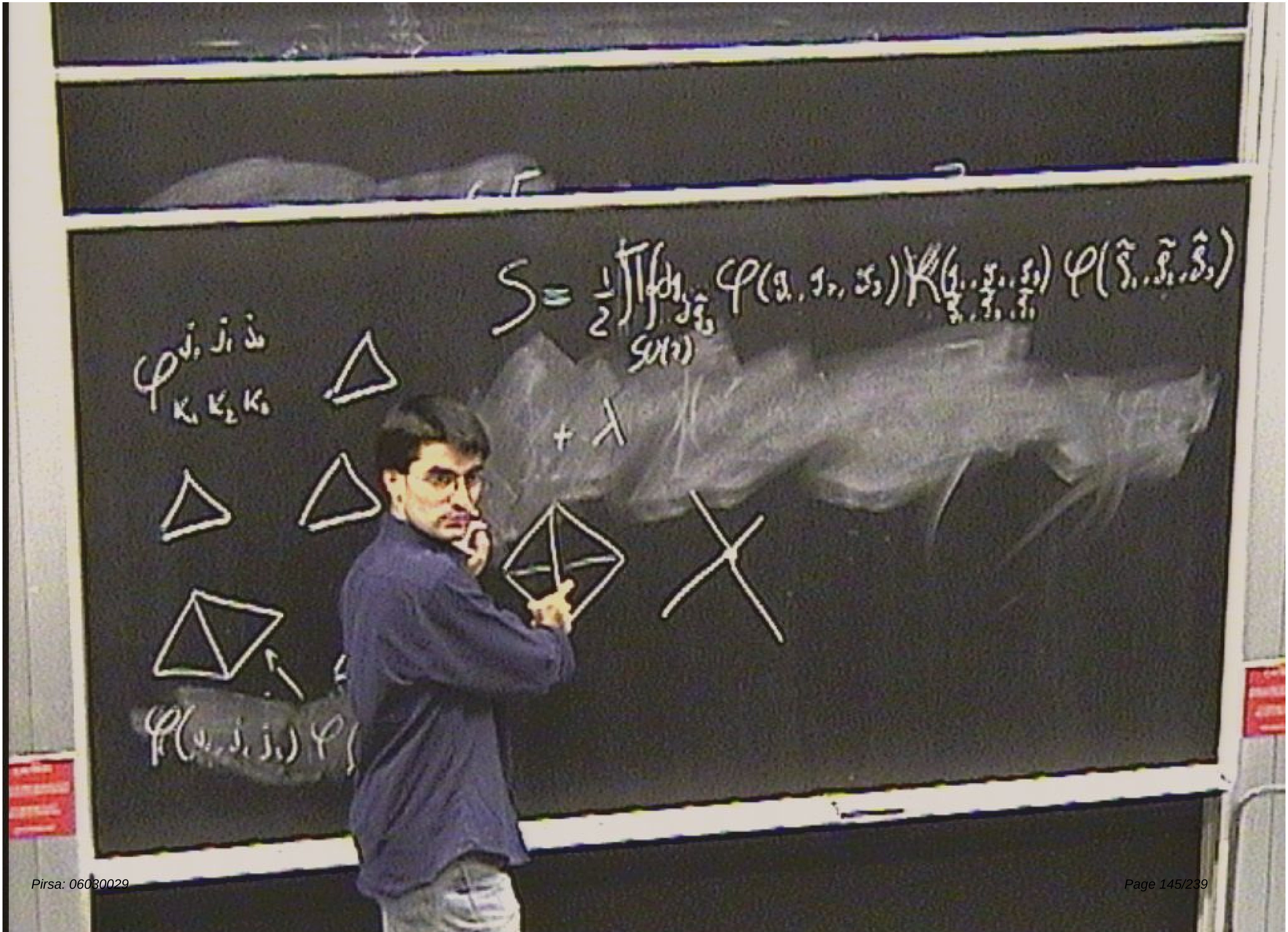


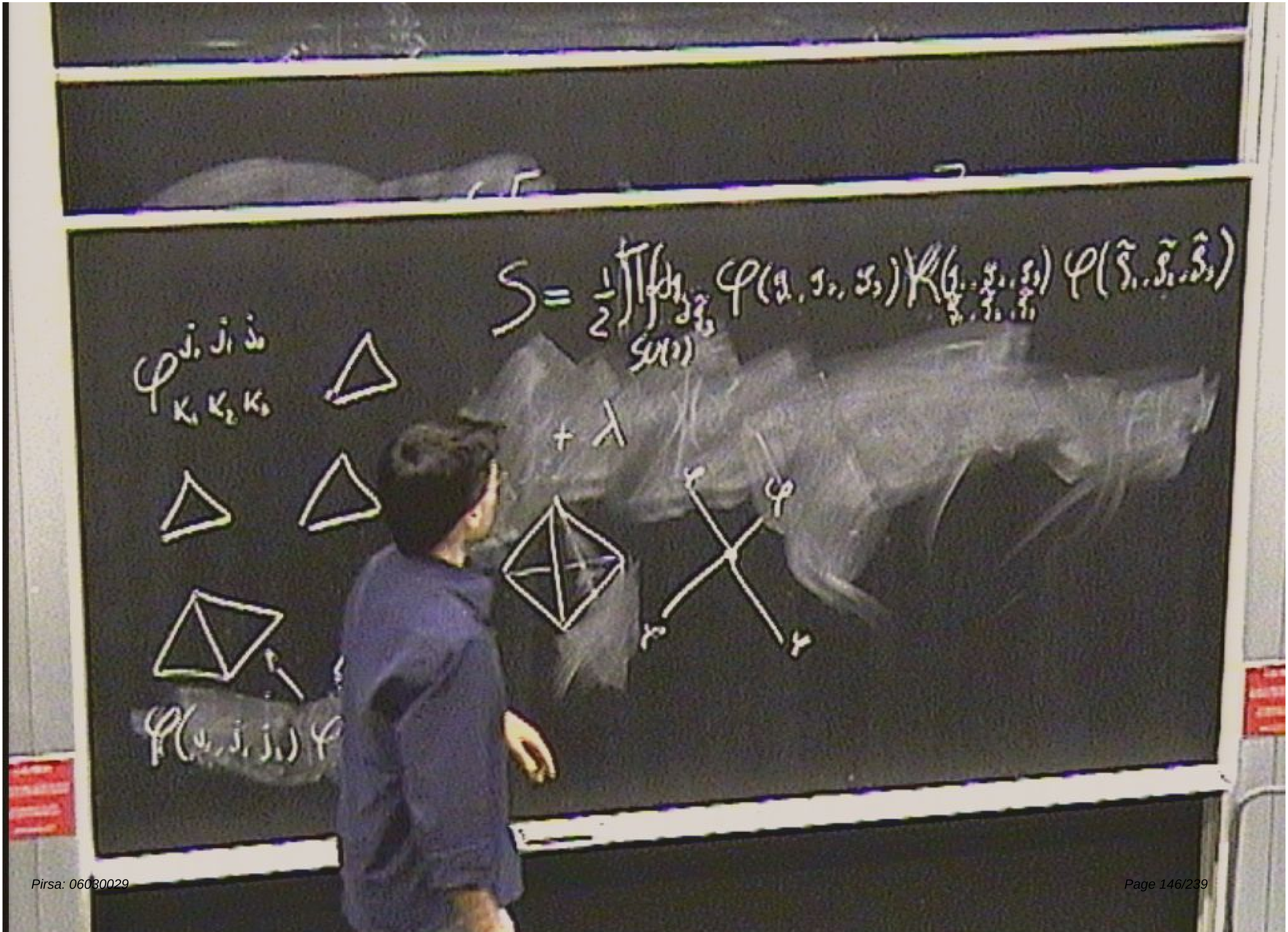


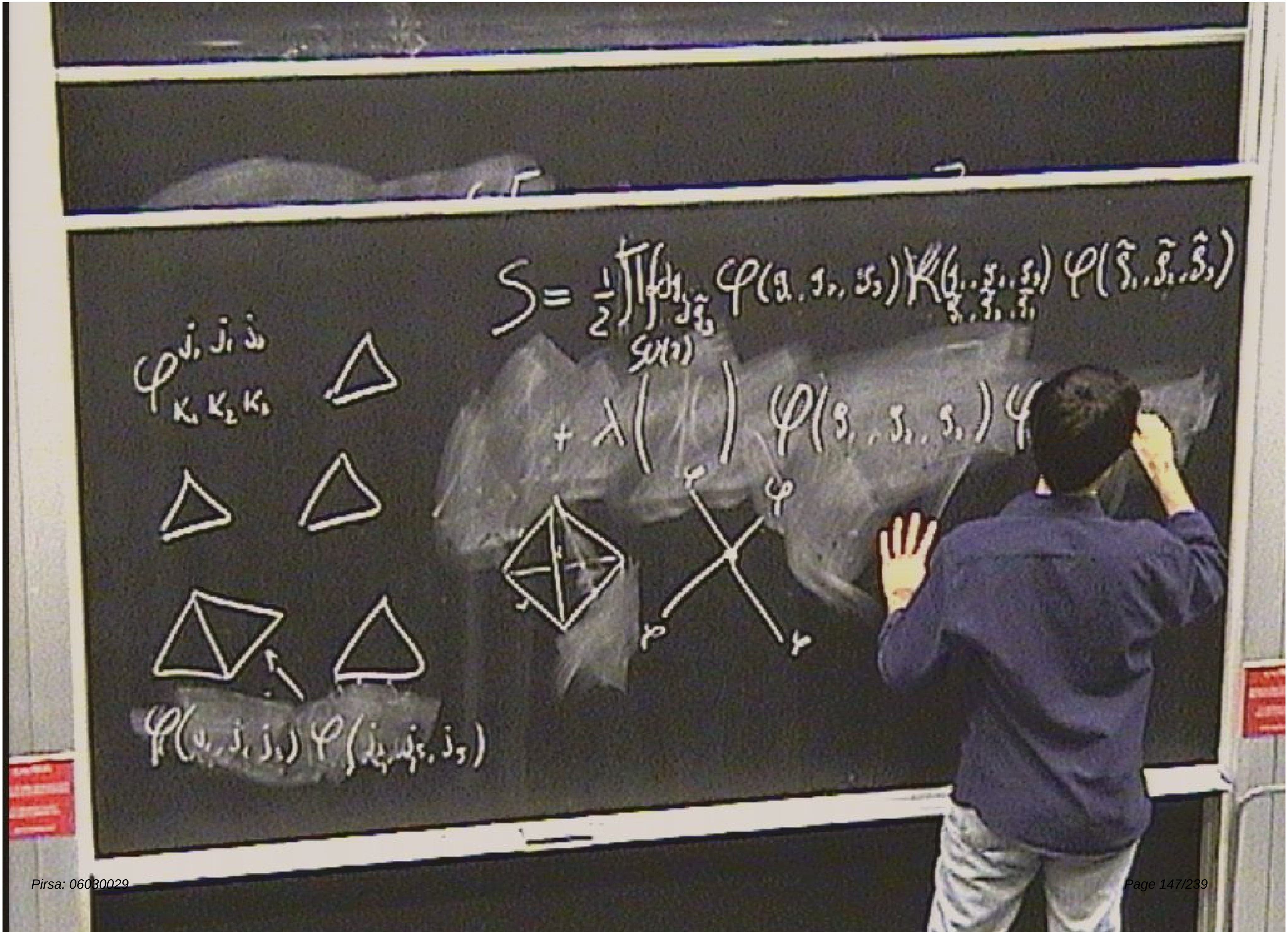

3d example
• L. FREIDEL, hep-th/0505016 out look
h_2
g
h_1
$Z = \int Dg\, e^{iS(g)}$
$\Psi(h_2) \longrightarrow \phi(h_2)$
"$H\, Z(h_1, h_2) = \delta(h_1 - h_2)$"

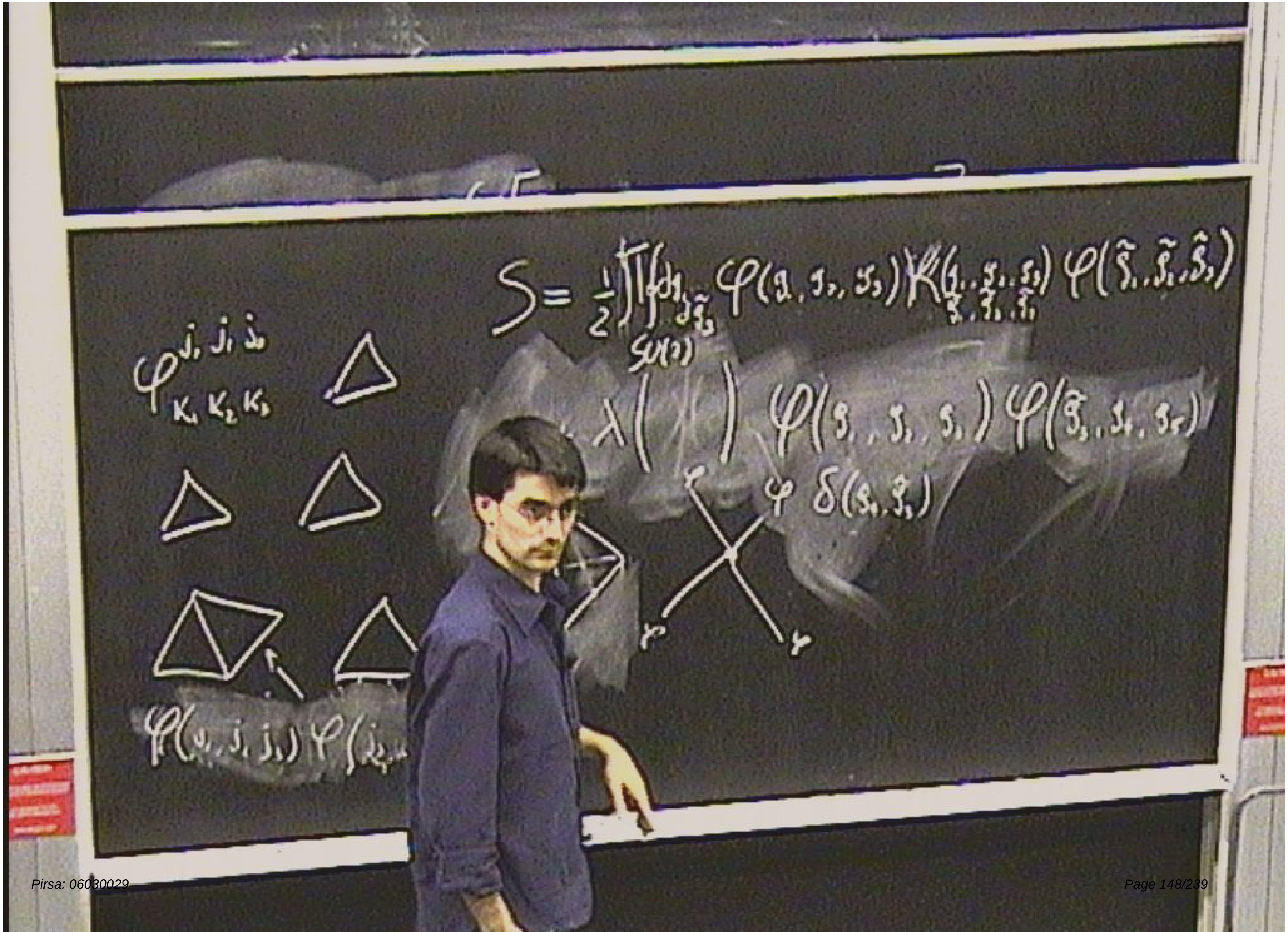


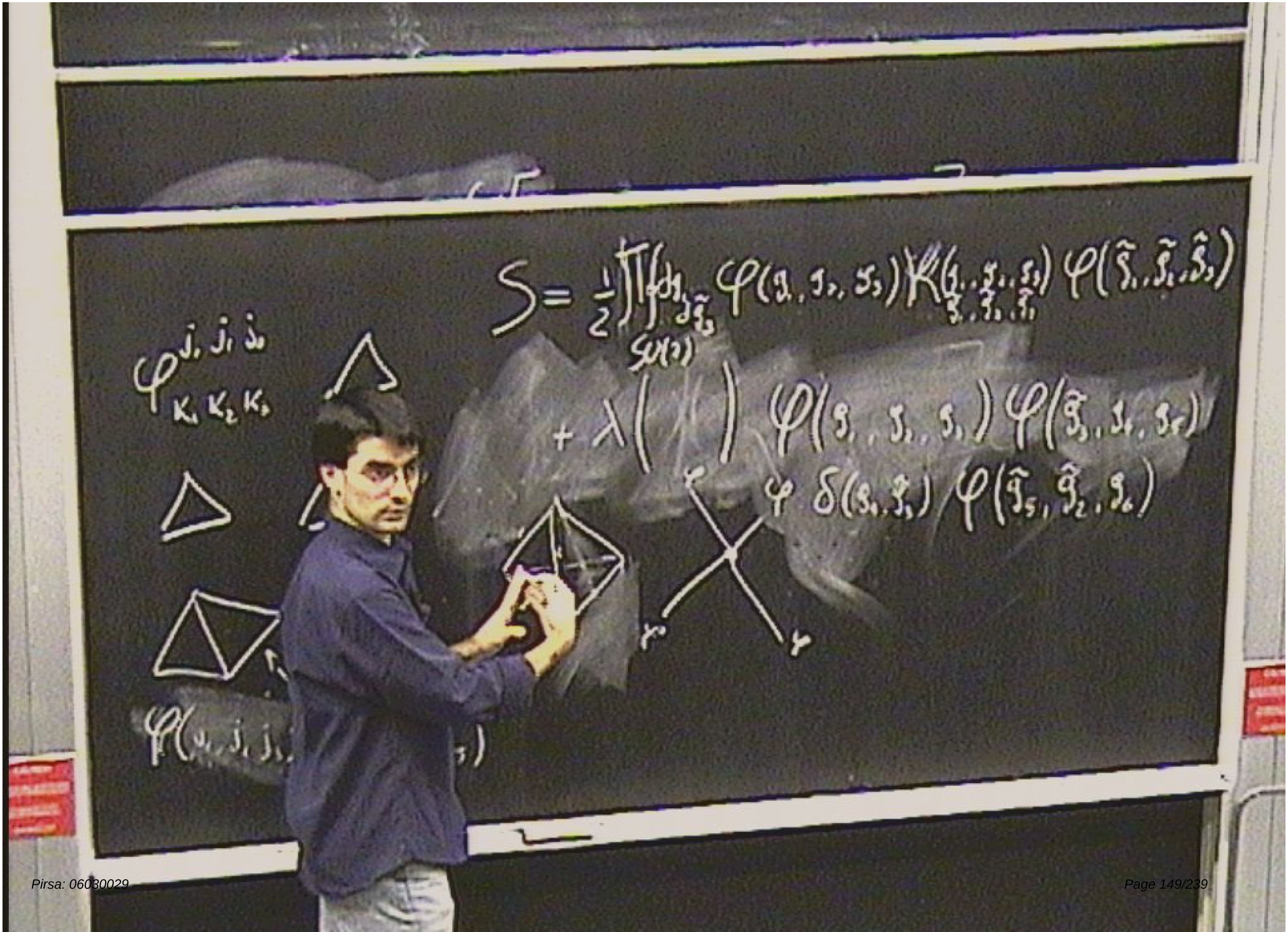


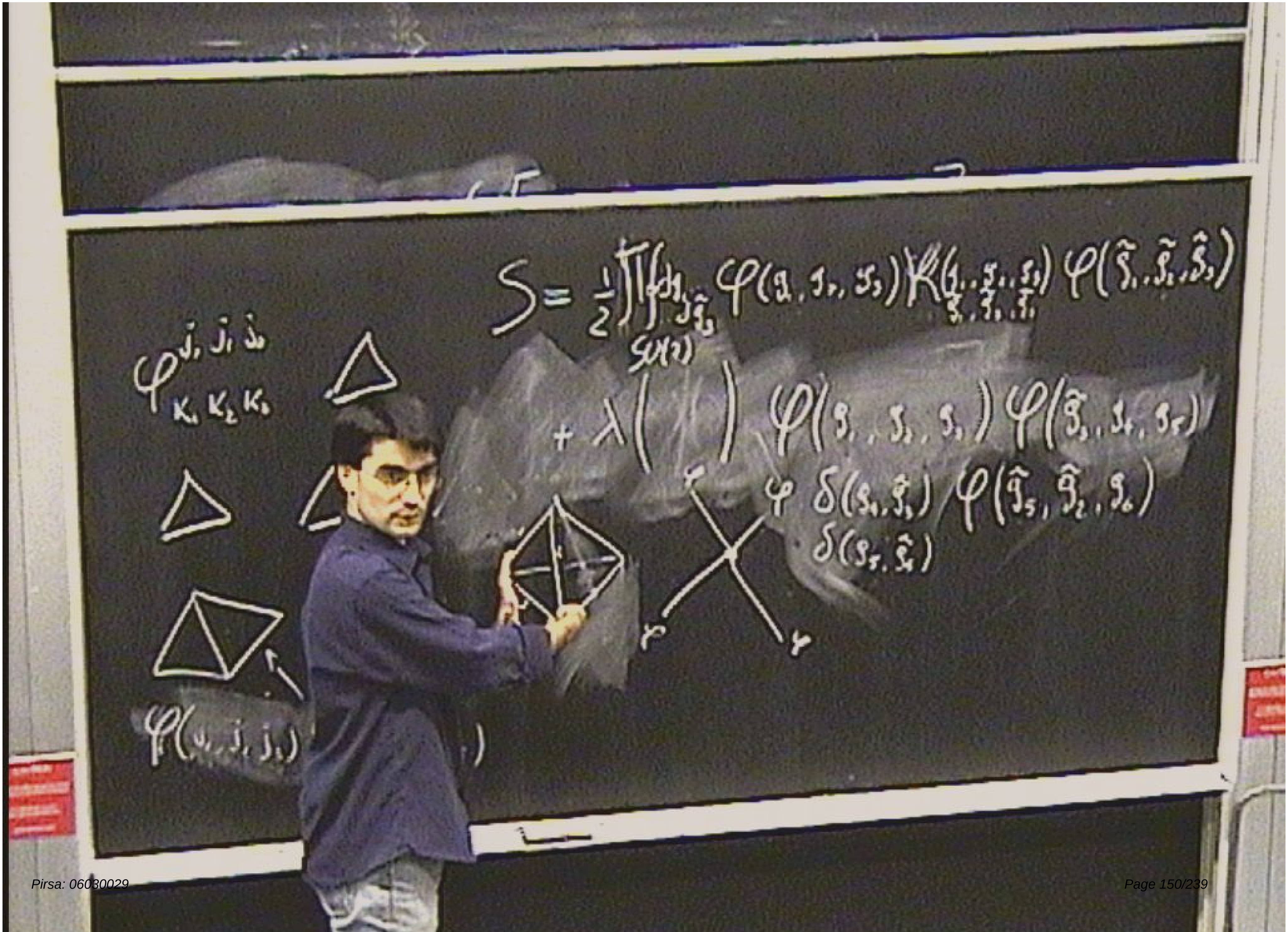


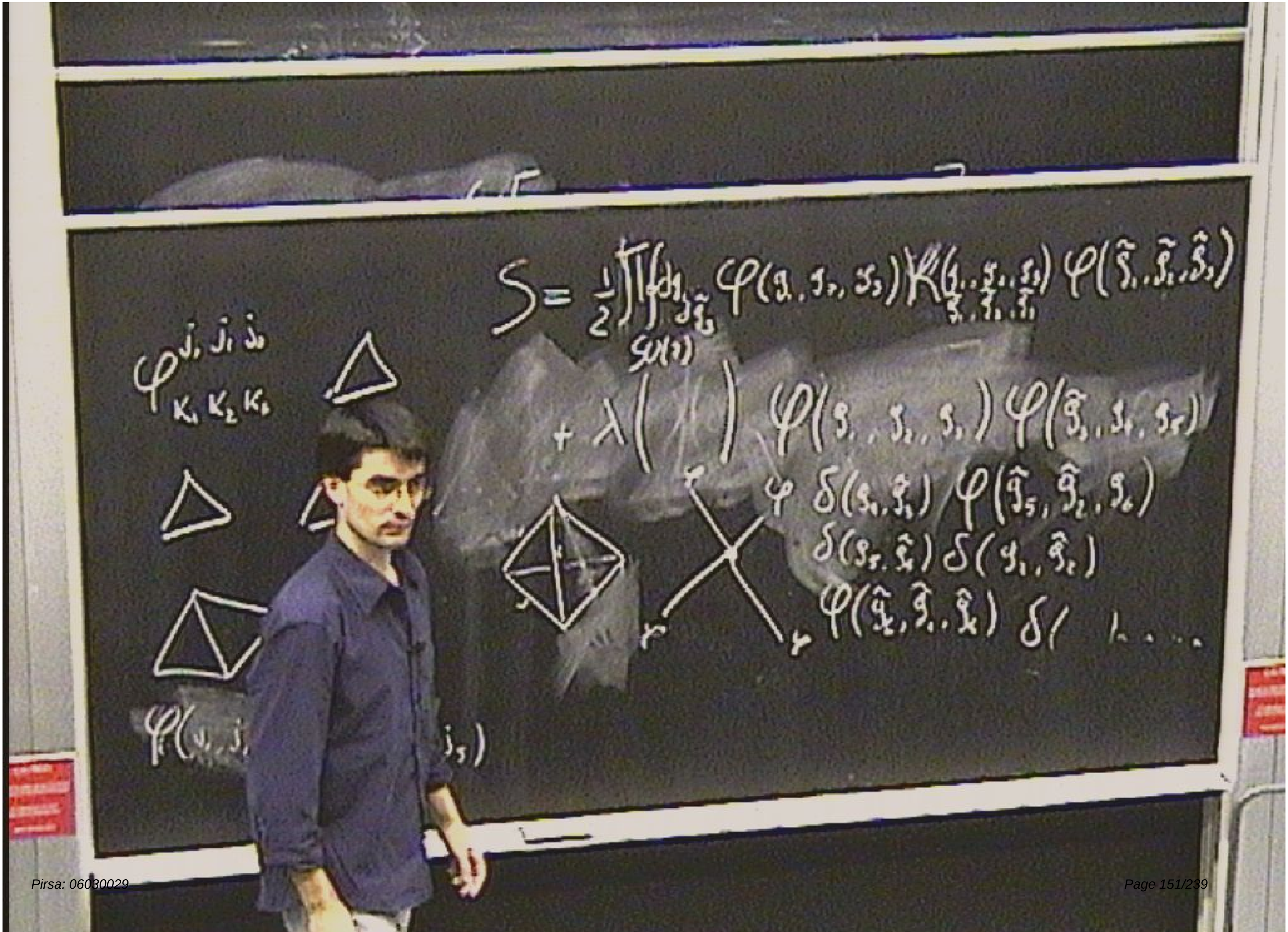

SU(2)
+ λ

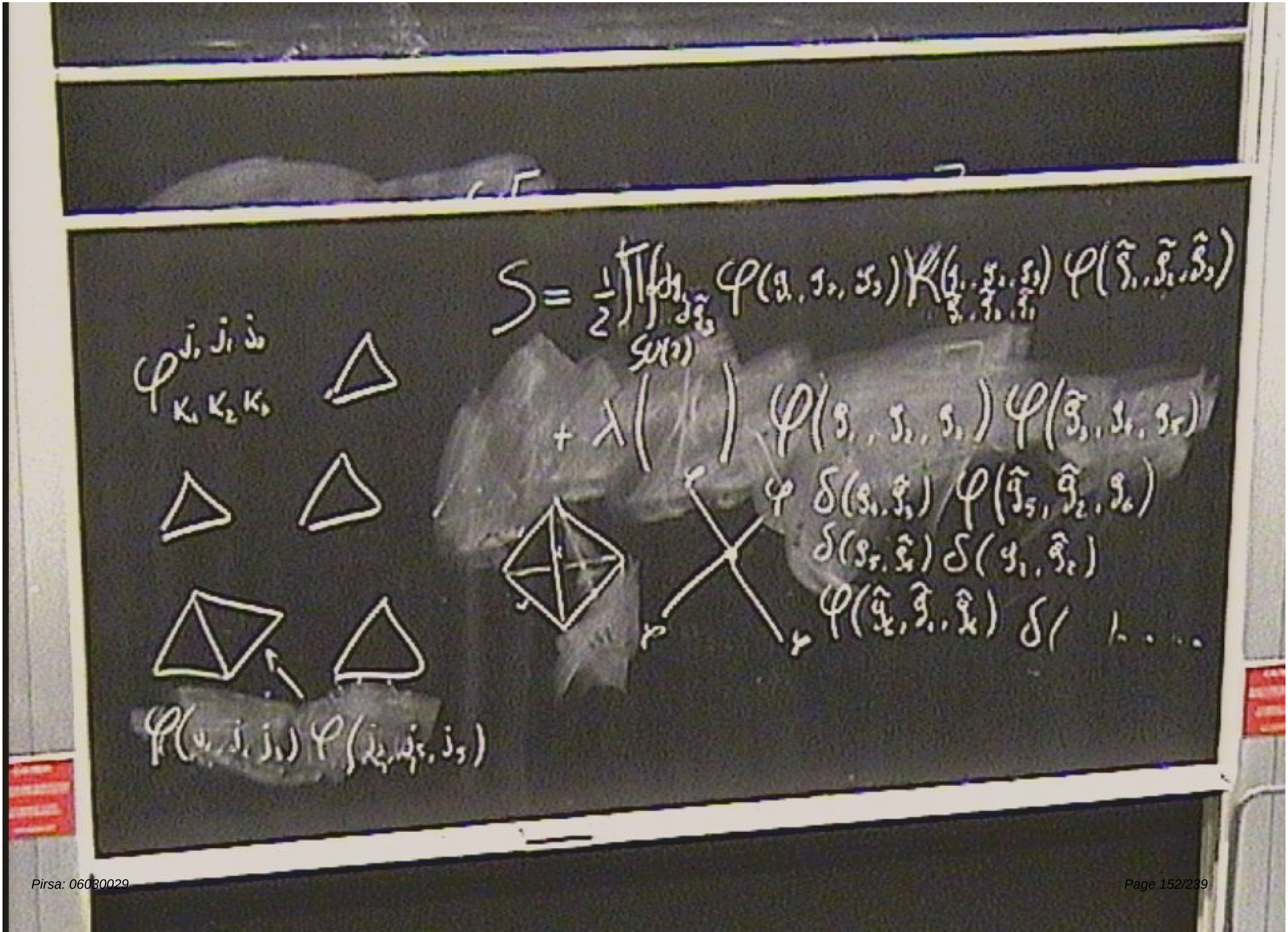


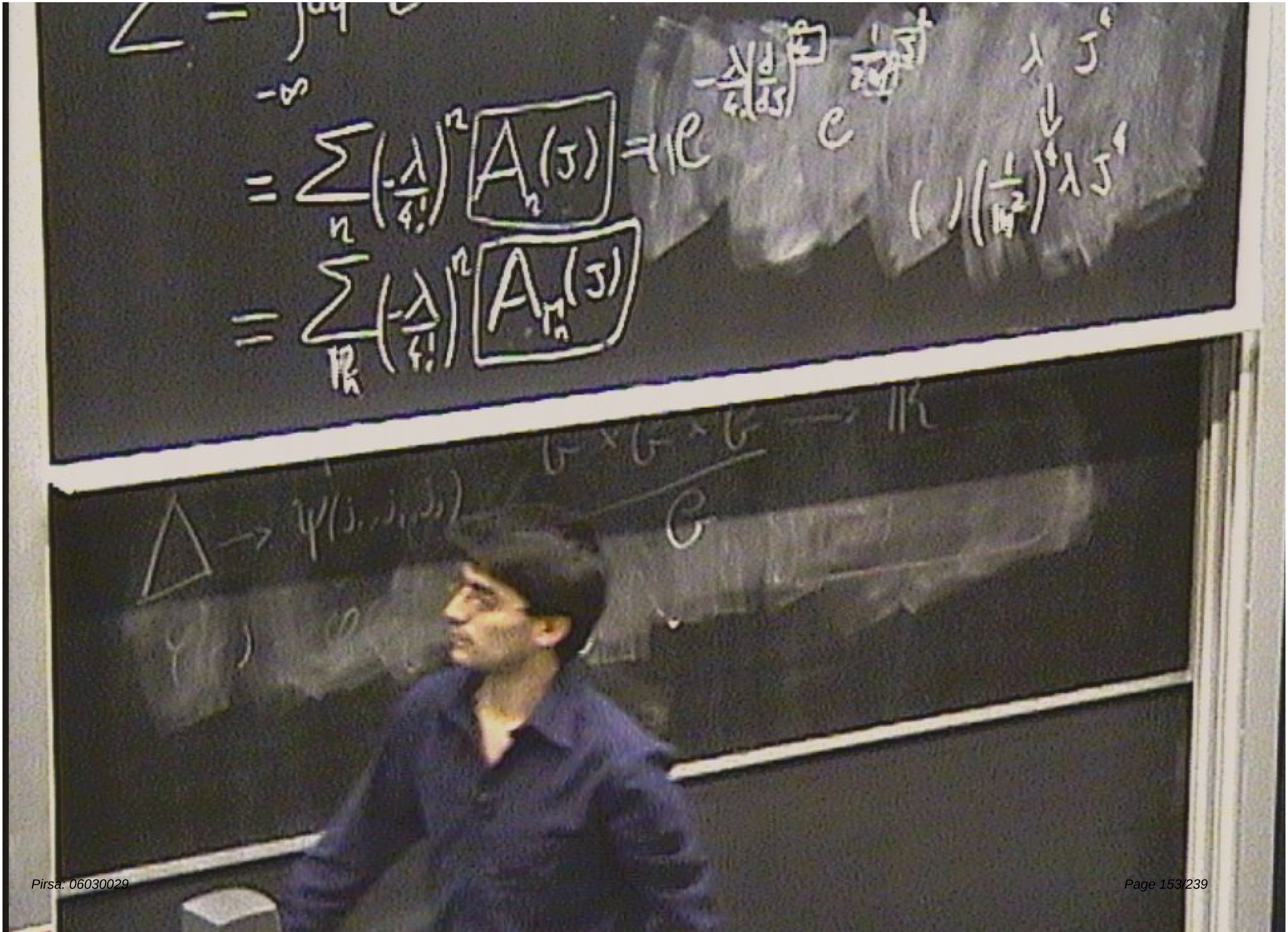


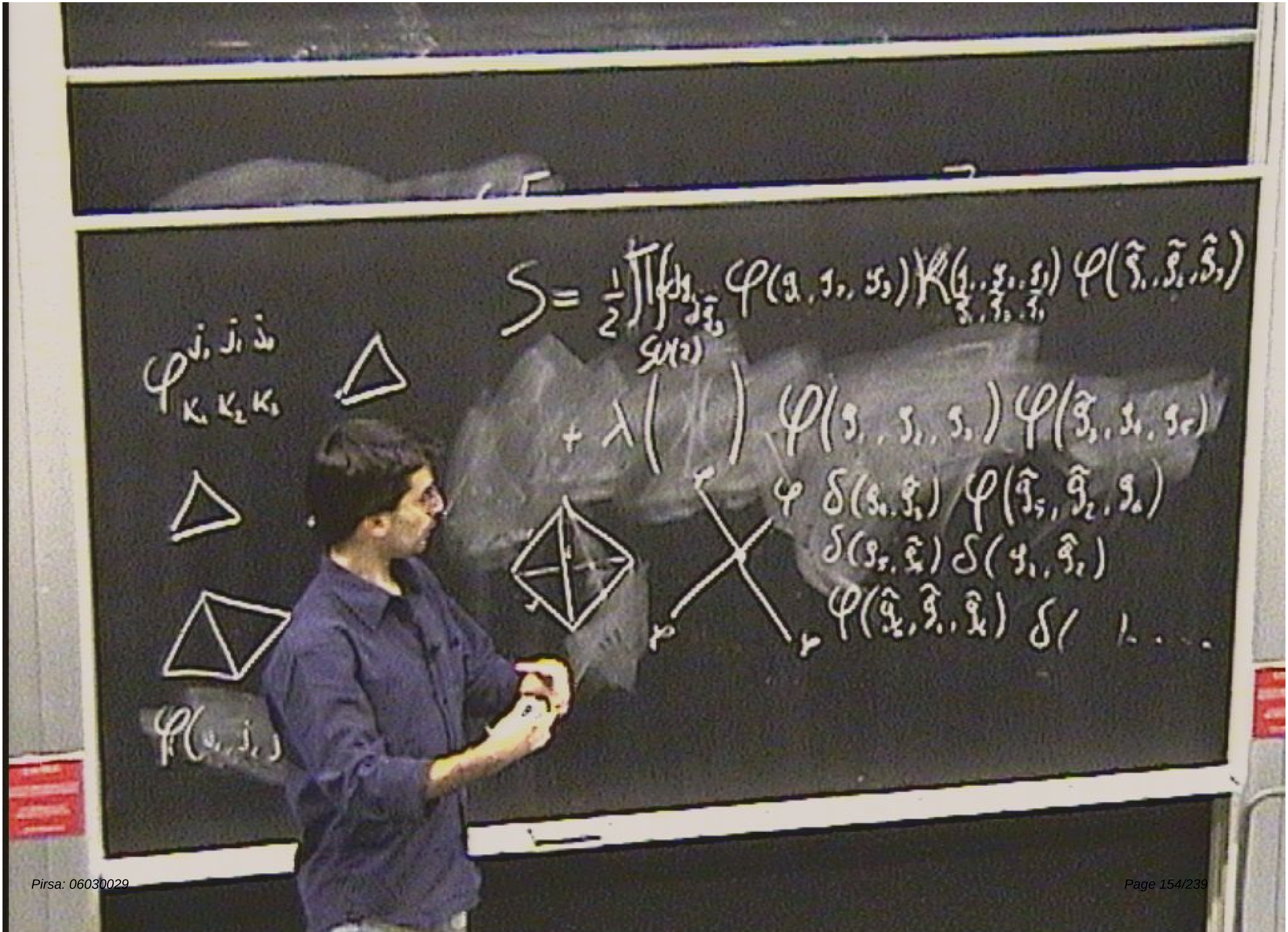


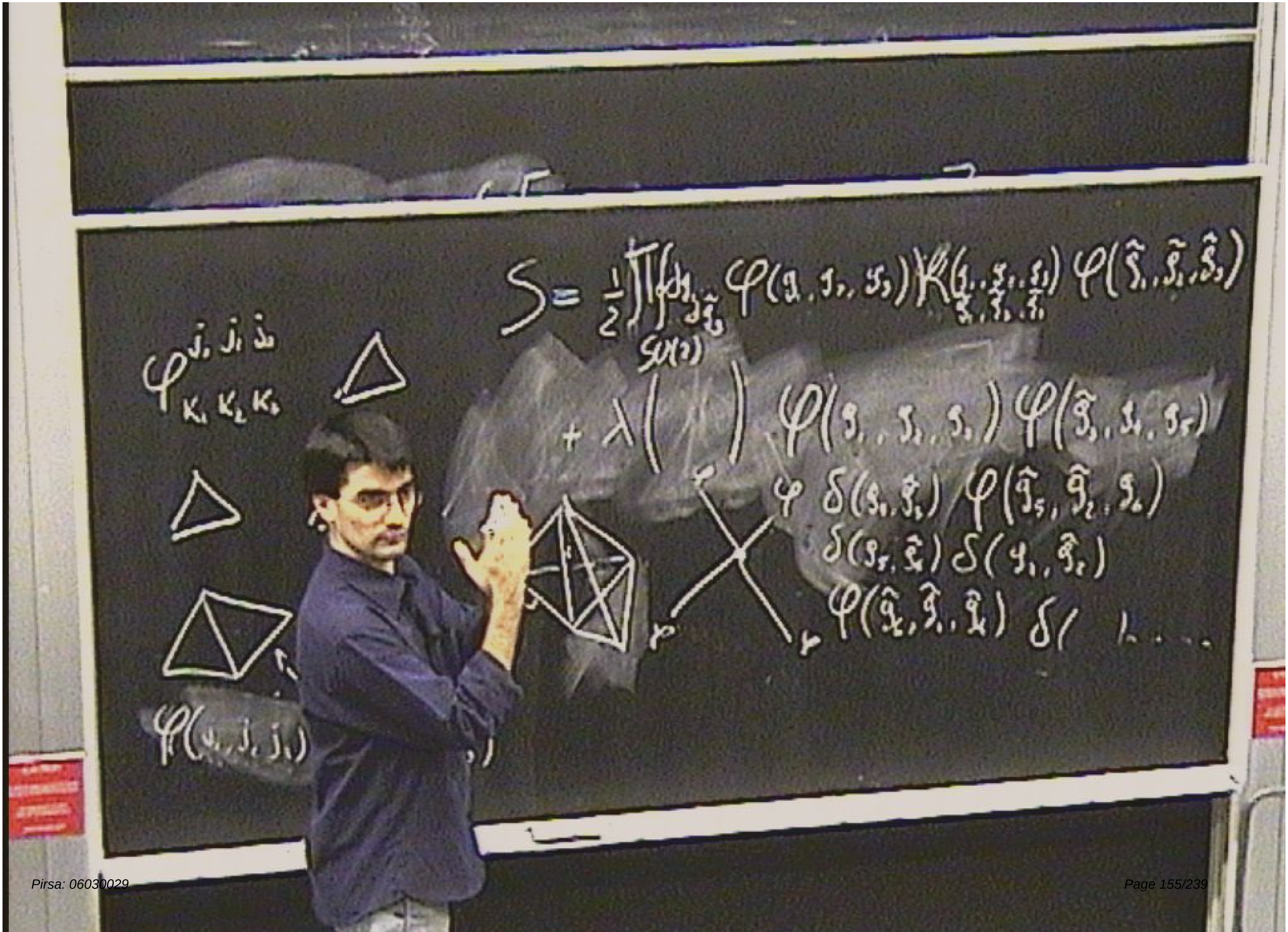


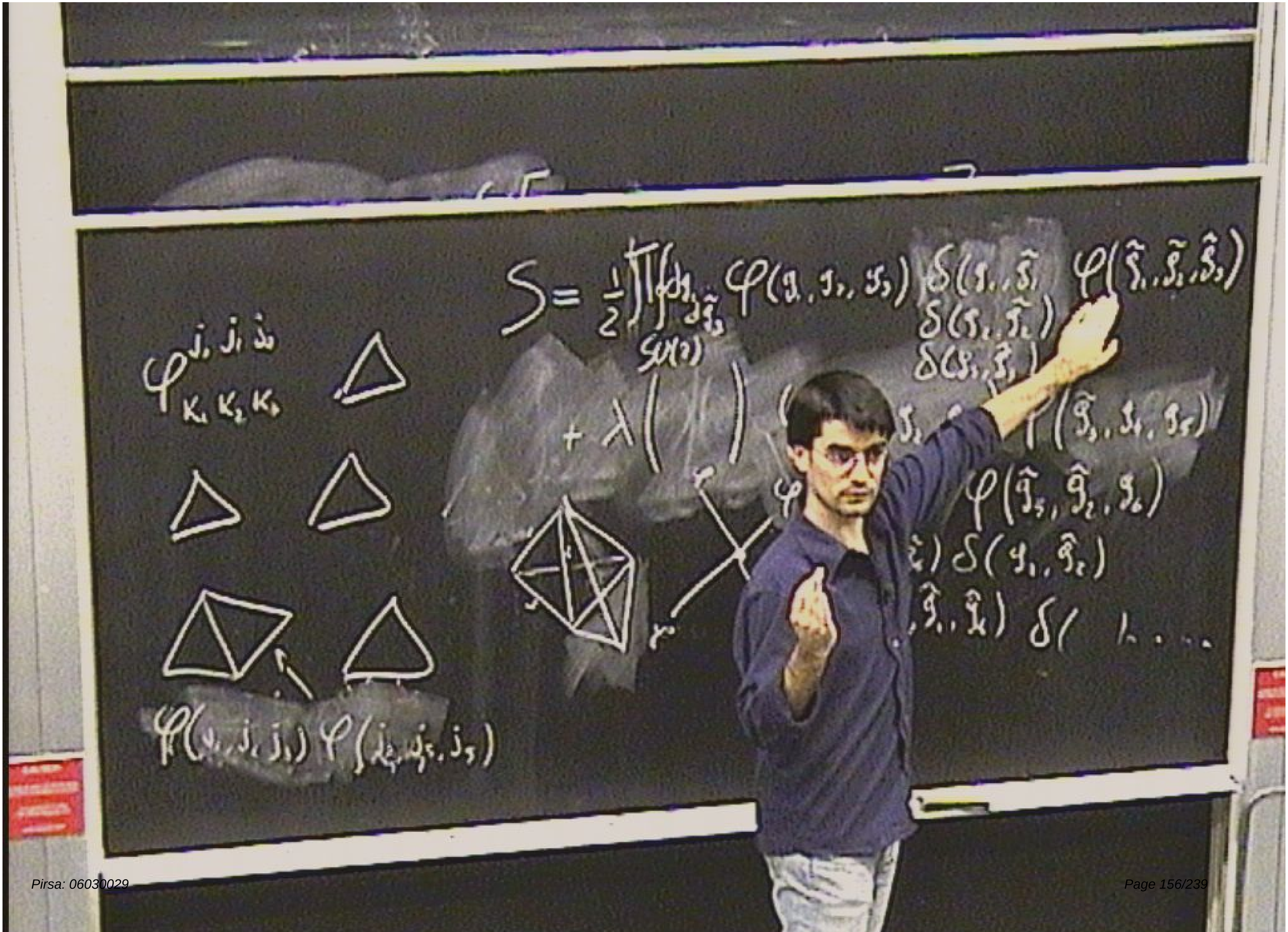


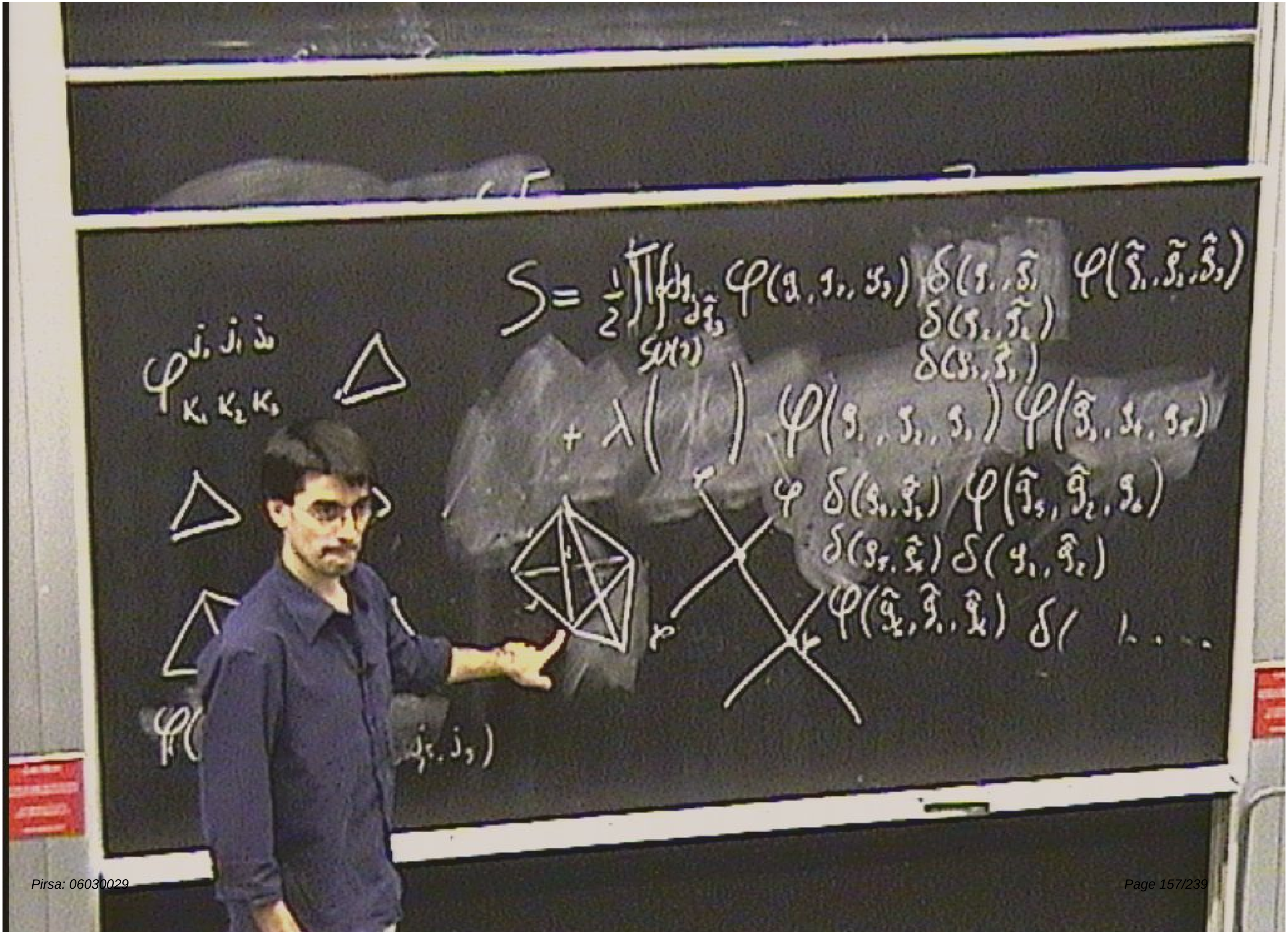


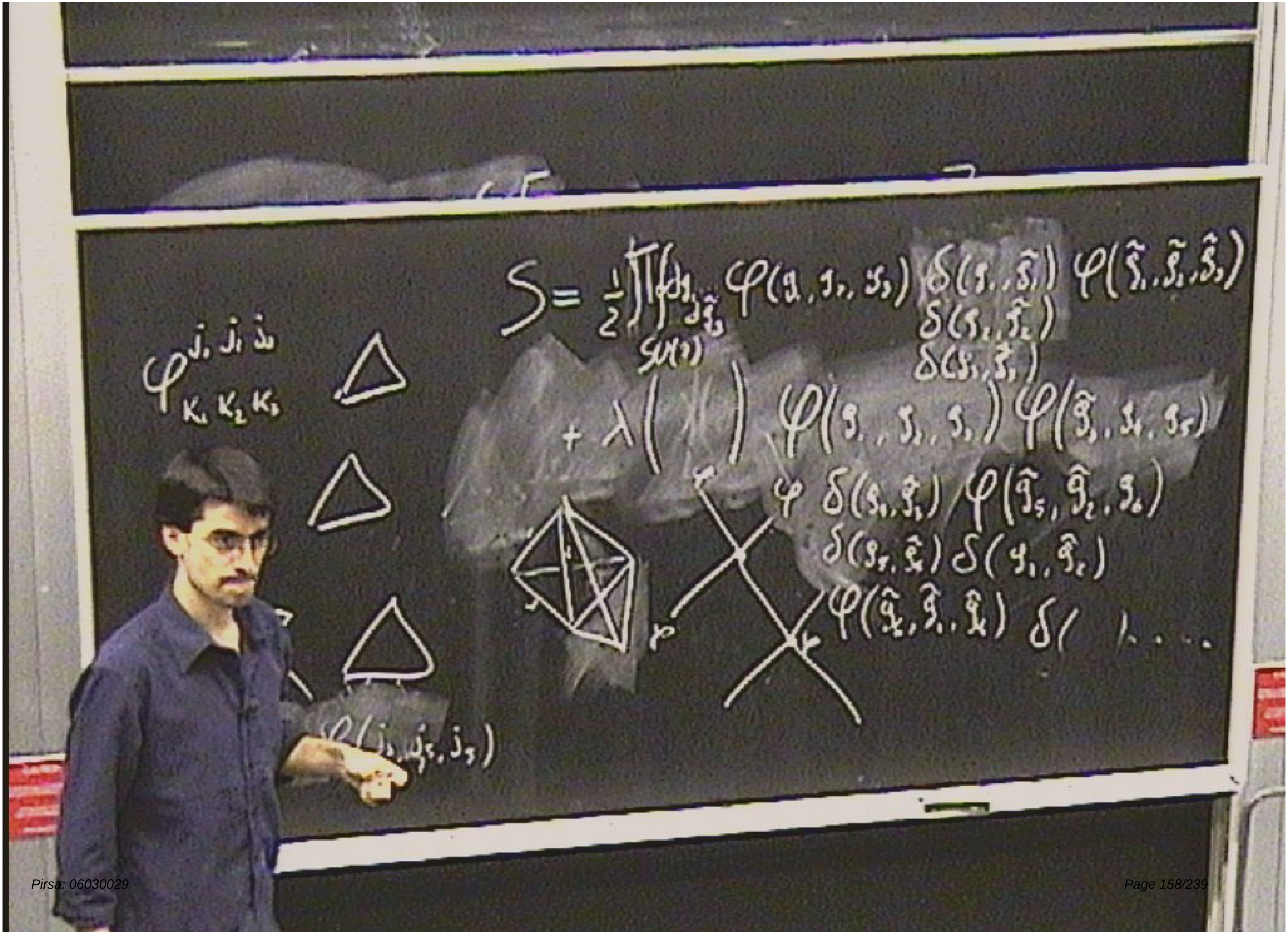


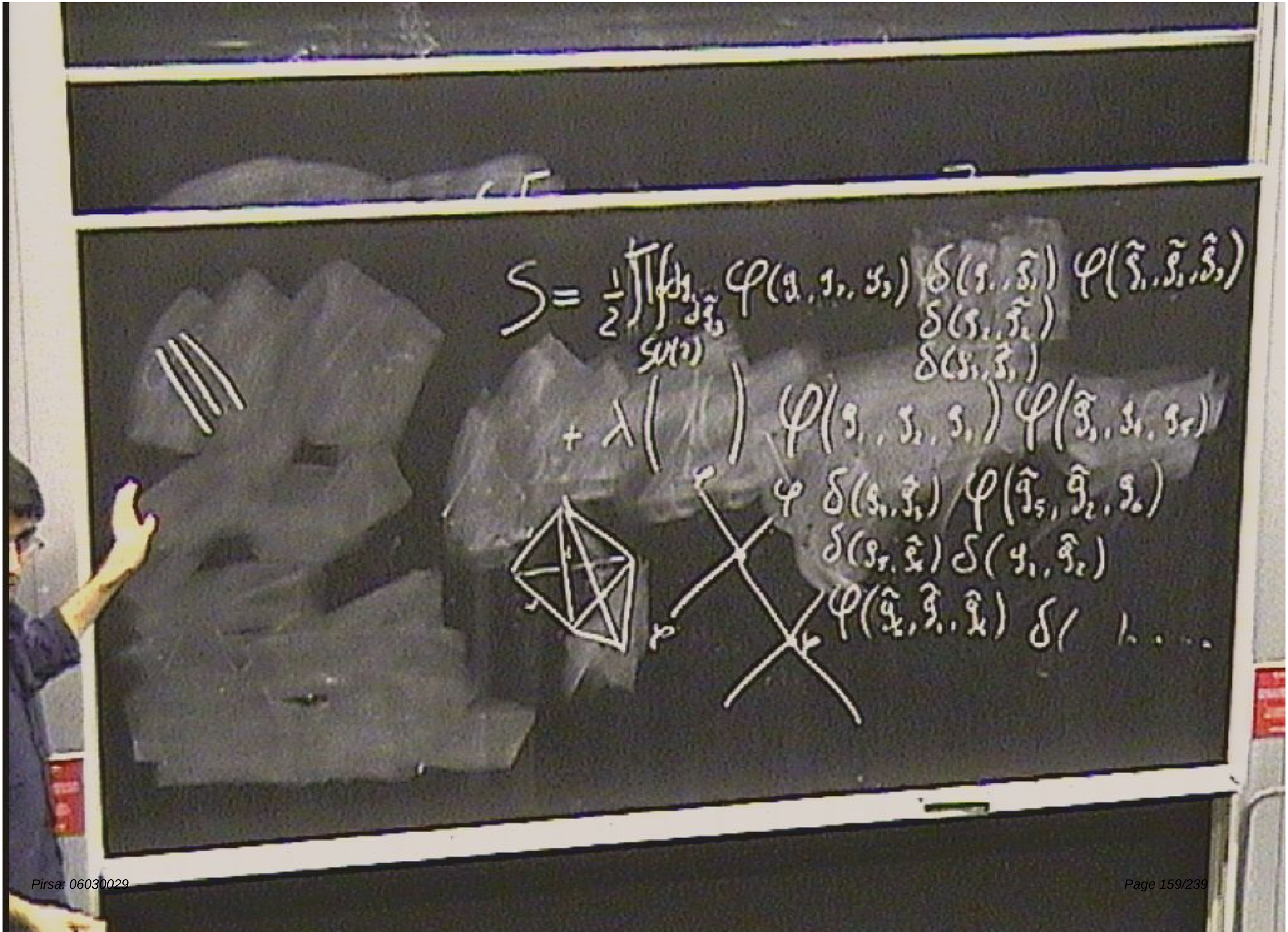


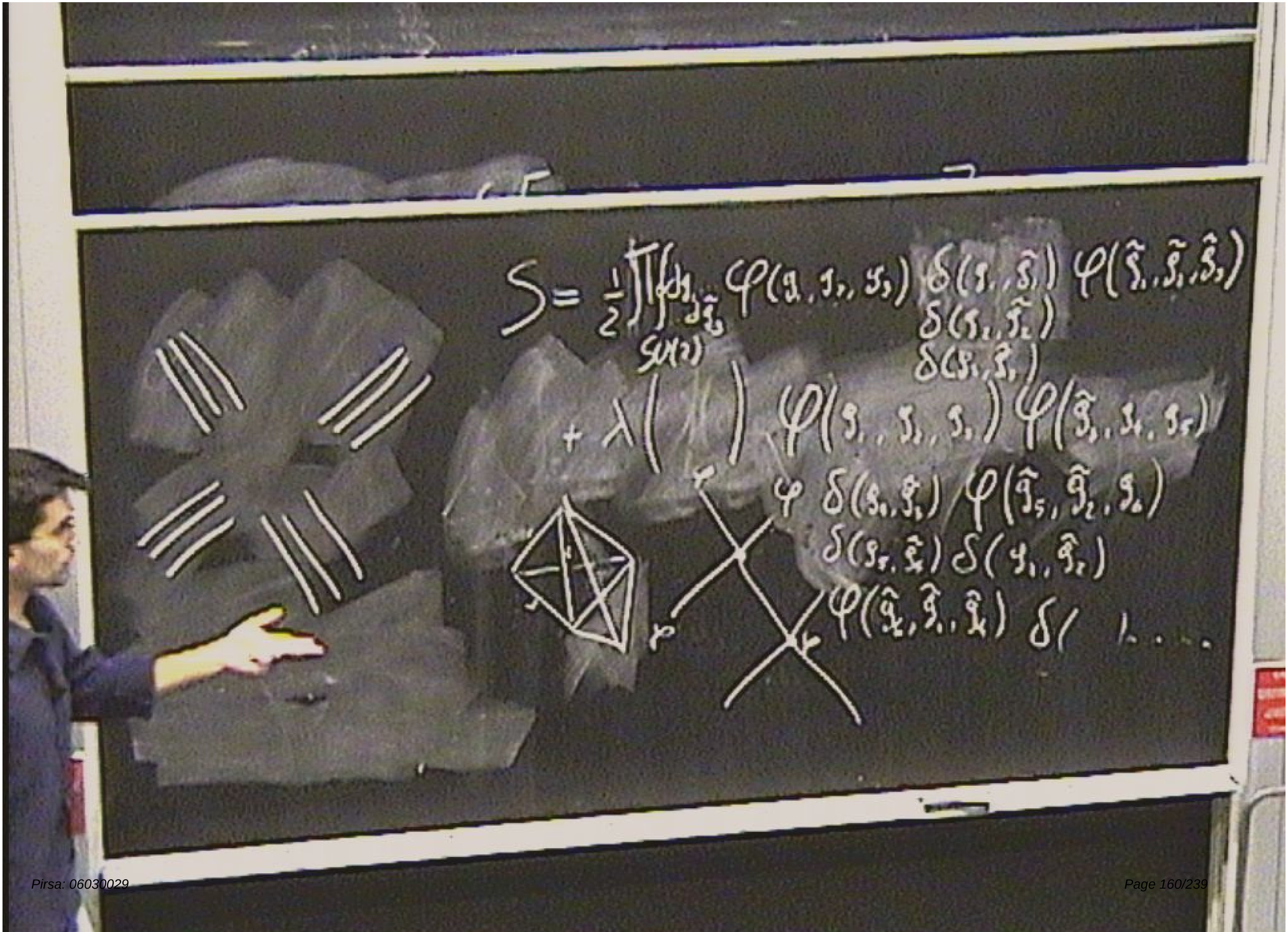


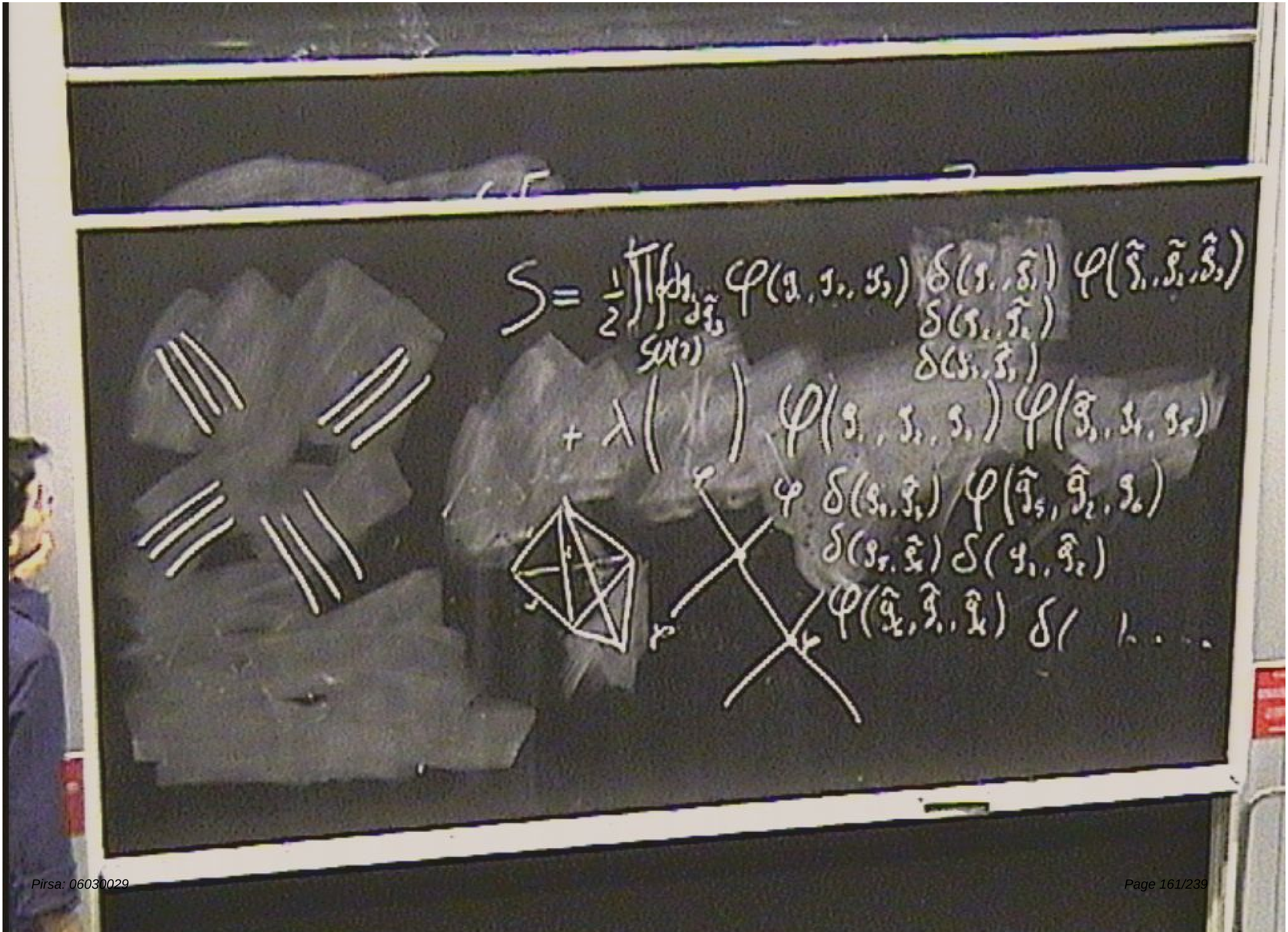


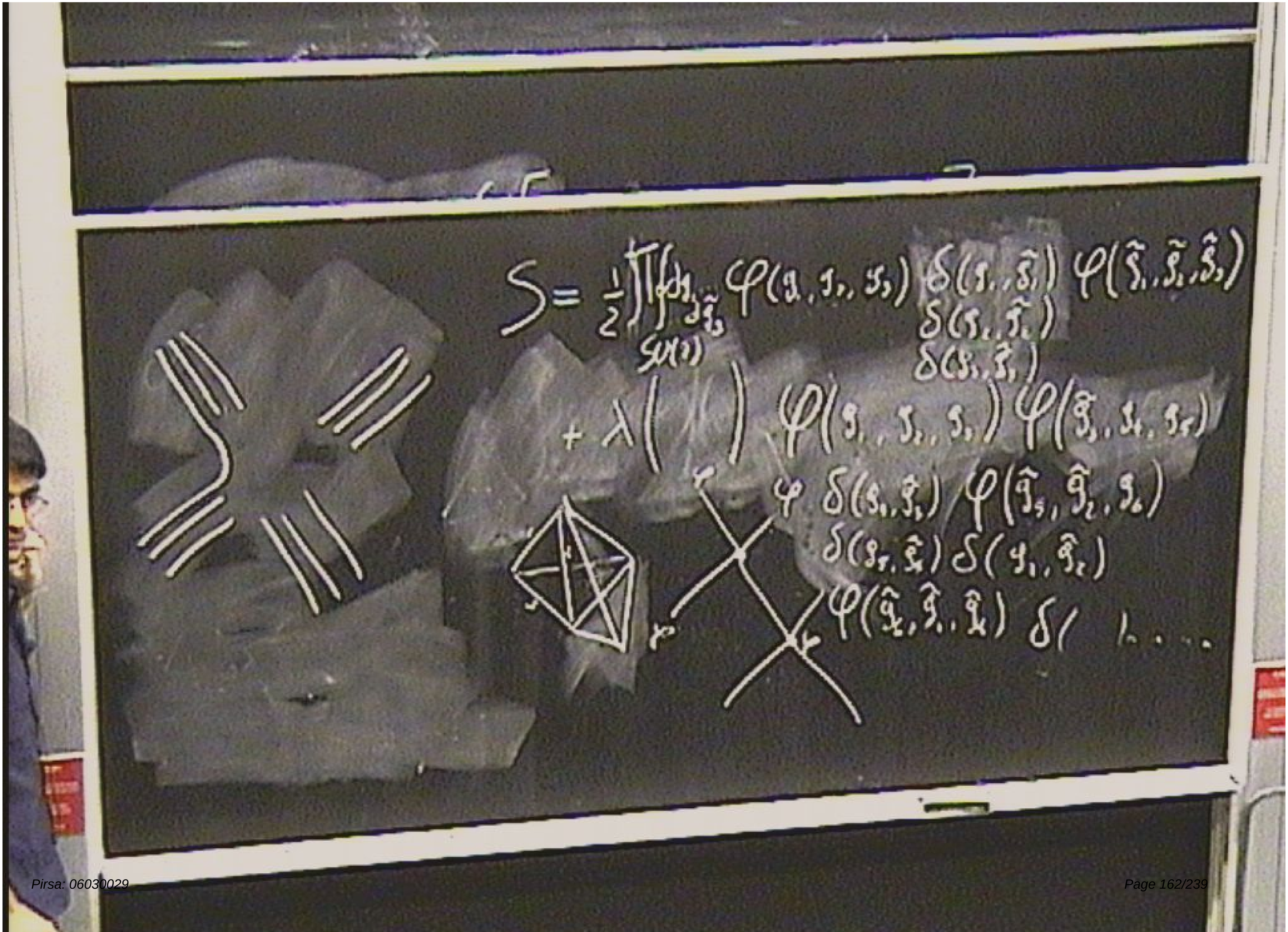


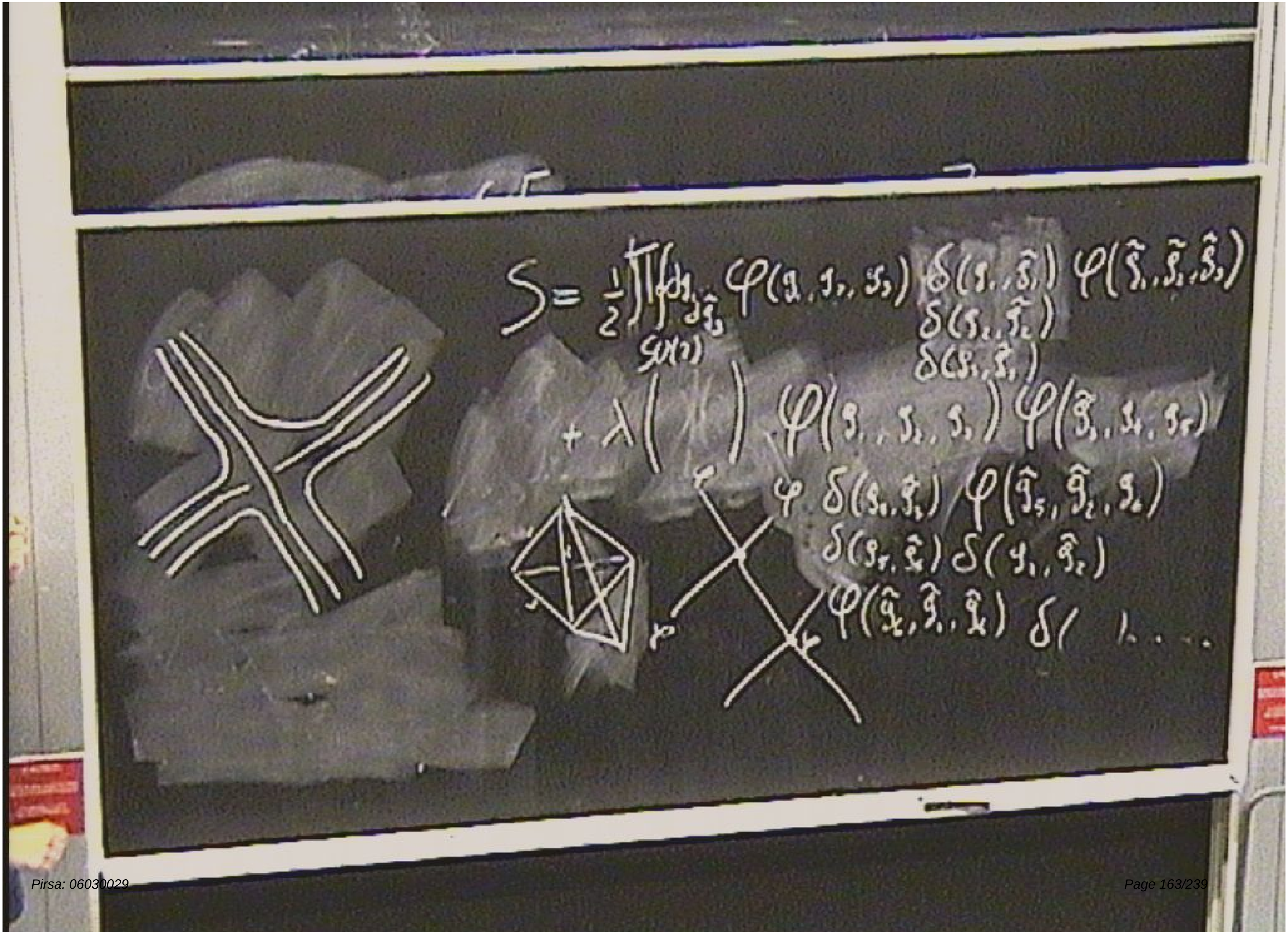


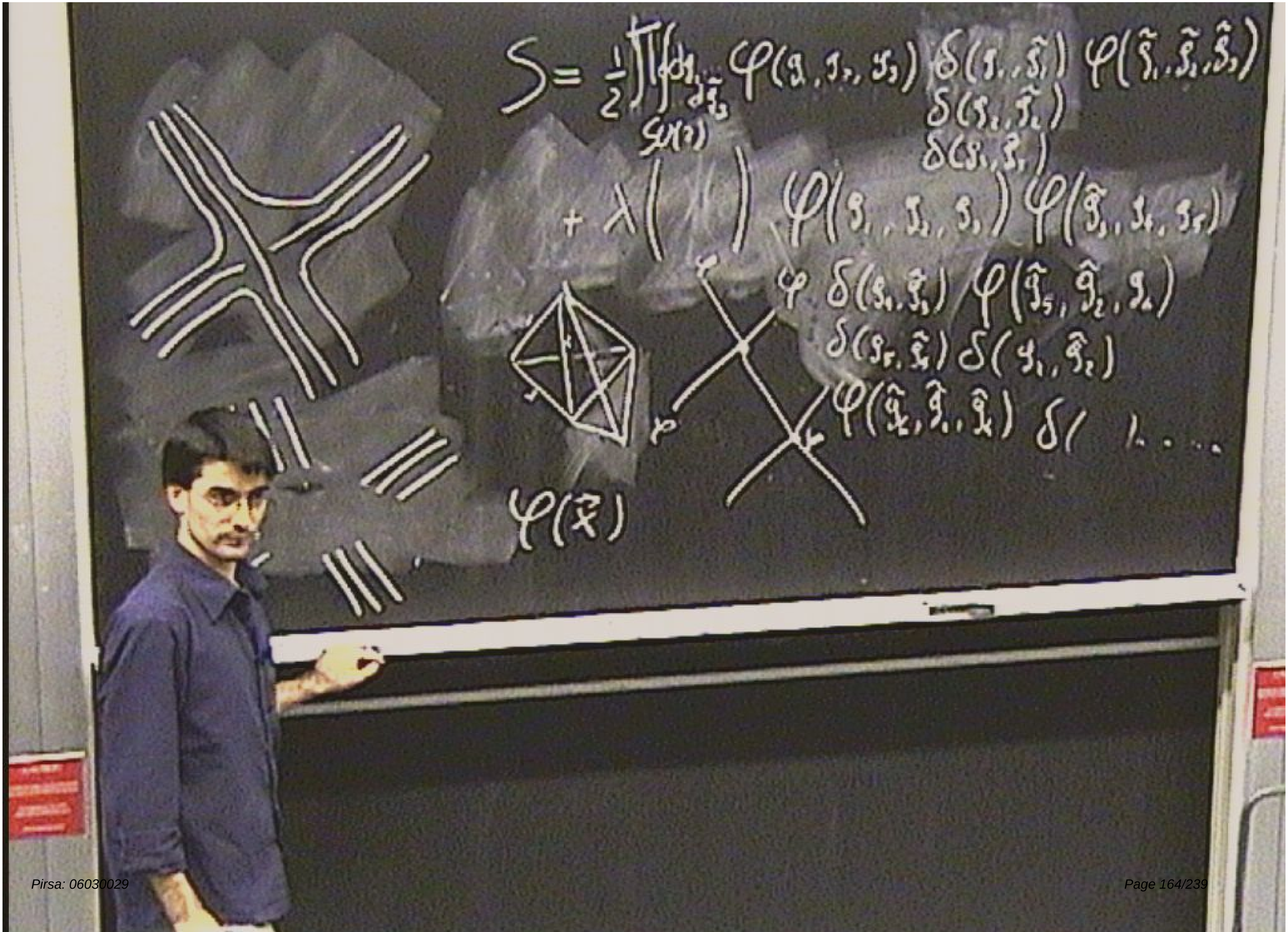


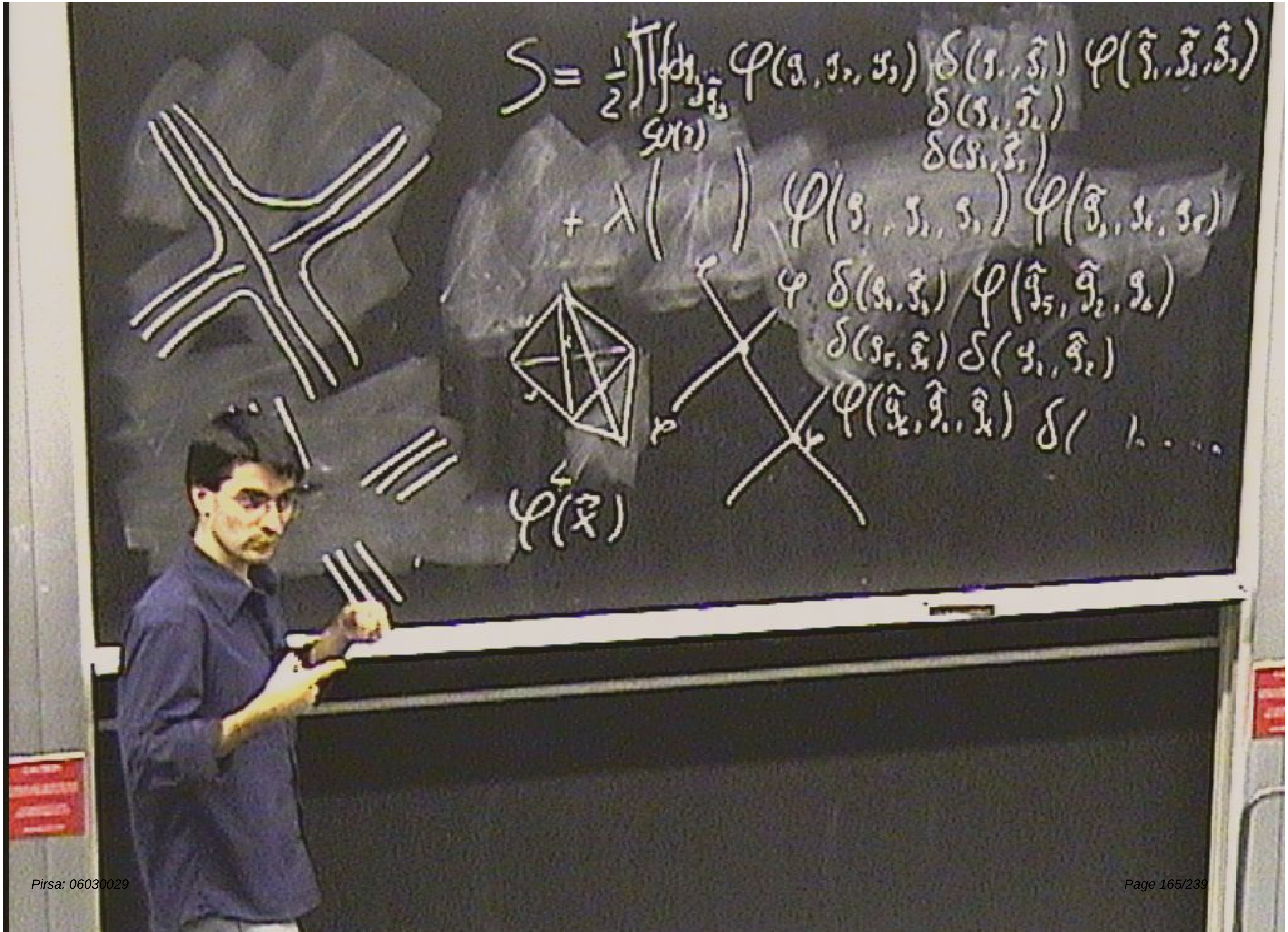


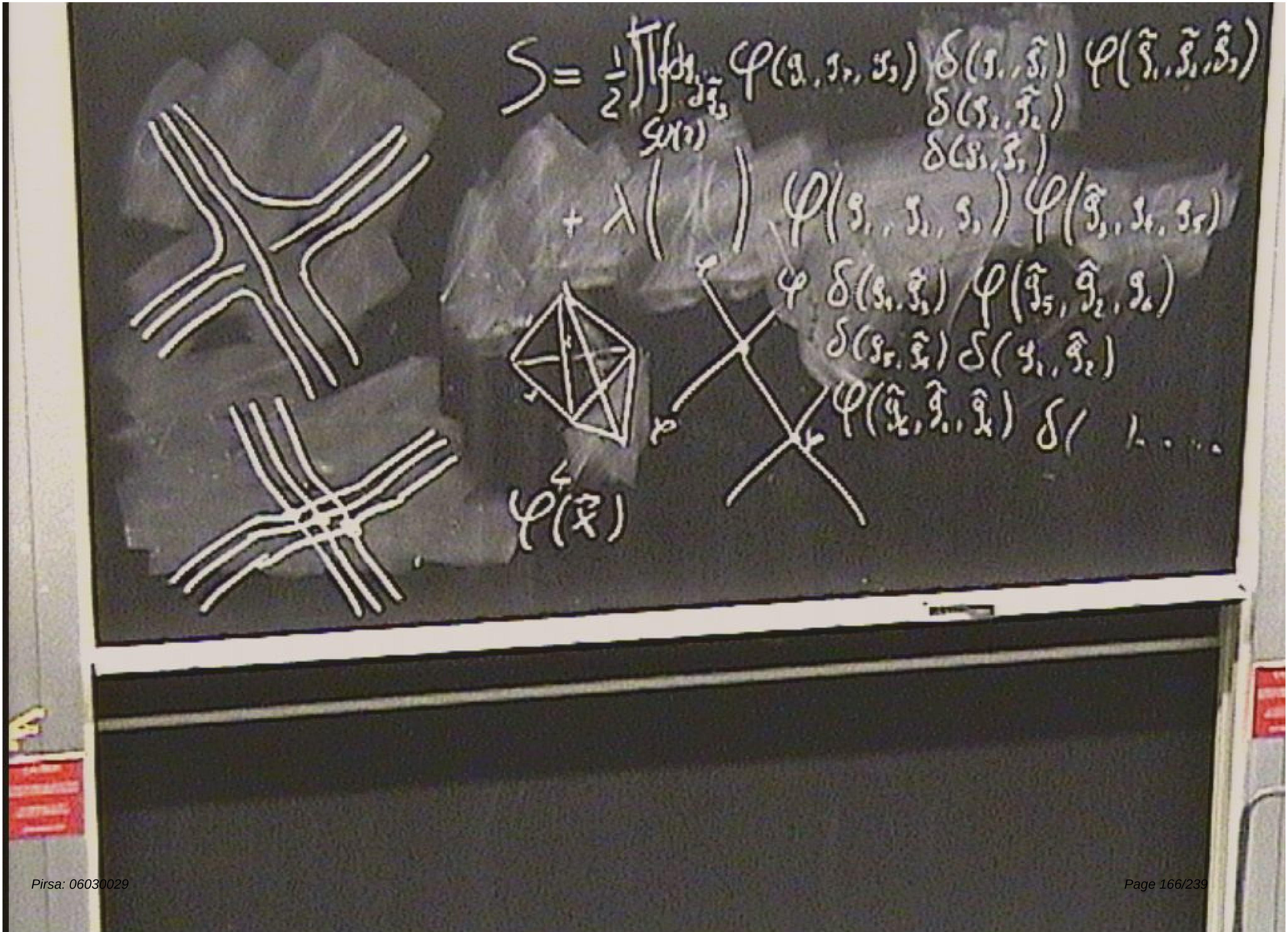

S = 1/2 ∫∏dg φ(g1, g2, g3) δ(g1, g̃1) φ(g̃1, g̃2, g̃3)
δ(g2, g̃2)
δ(g3, g̃3)
+ λ ( ) φ(g1, g2, g3) φ(g3, g4, g5)
φ δ(g1, ĝ1) φ(ĝ5, ĝ2, g6)
δ(g5, ĝ5) δ(g1, ĝ2)
φ(ĝ2, ĝ1, ĝ4) δ( ) ....

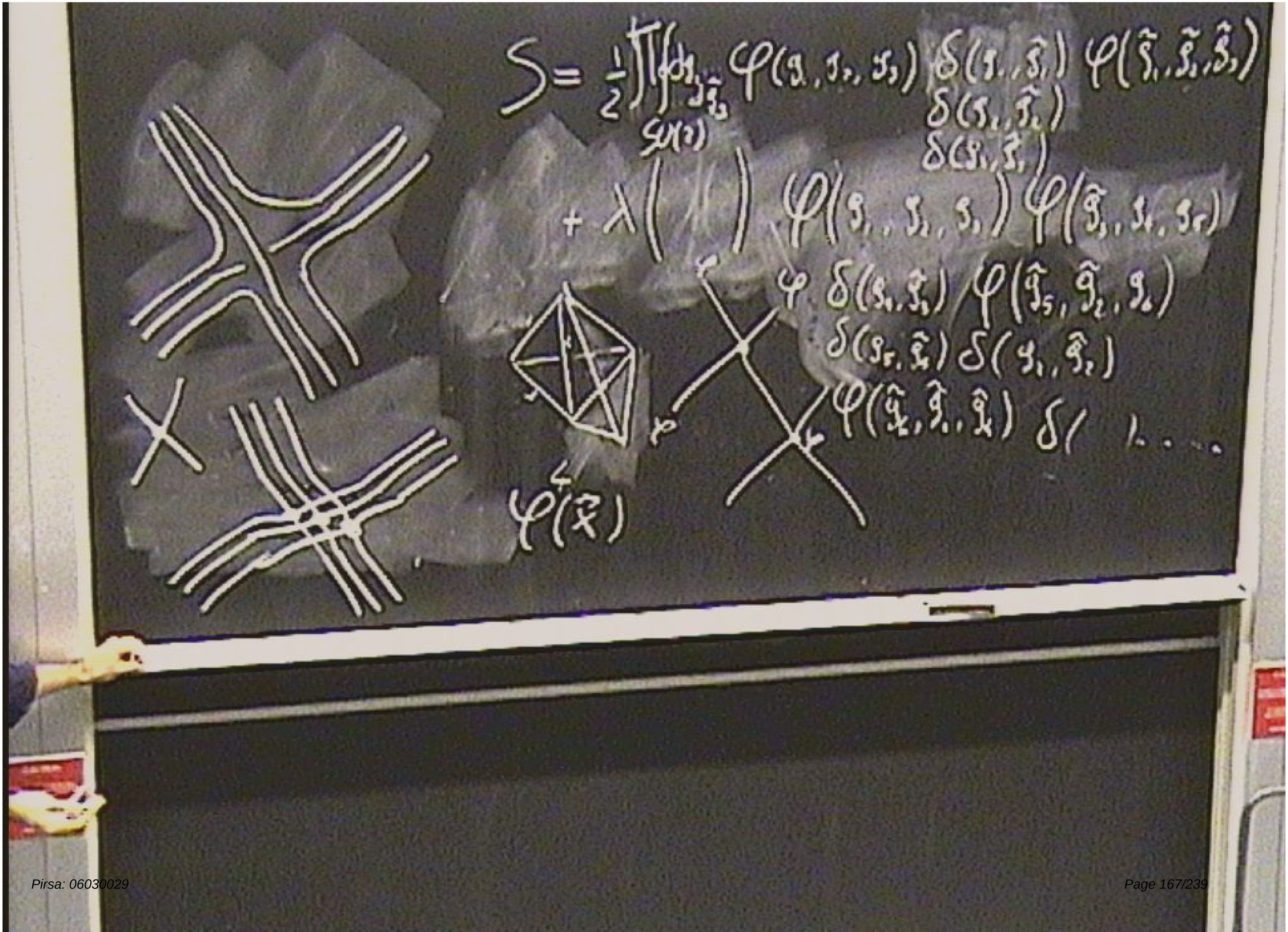

S = ½ ∫ φ(g1, g2, g3) δ(g1, g̃1) φ(g̃1, g̃2, g̃3)
δ(g2, g̃2)
δ(g3, g̃3)
SU(2)
+ λ ( ) φ(g1, g2, g3) φ(g̃3, g4, g5)
φ δ(g4, ĝ1) φ(ĝ5, ĝ2, g6)
δ(g5, ĝ5) δ(g1, ĝ2)
φ(ĝ2, ĝ3, ĝ4) δ( ) ....

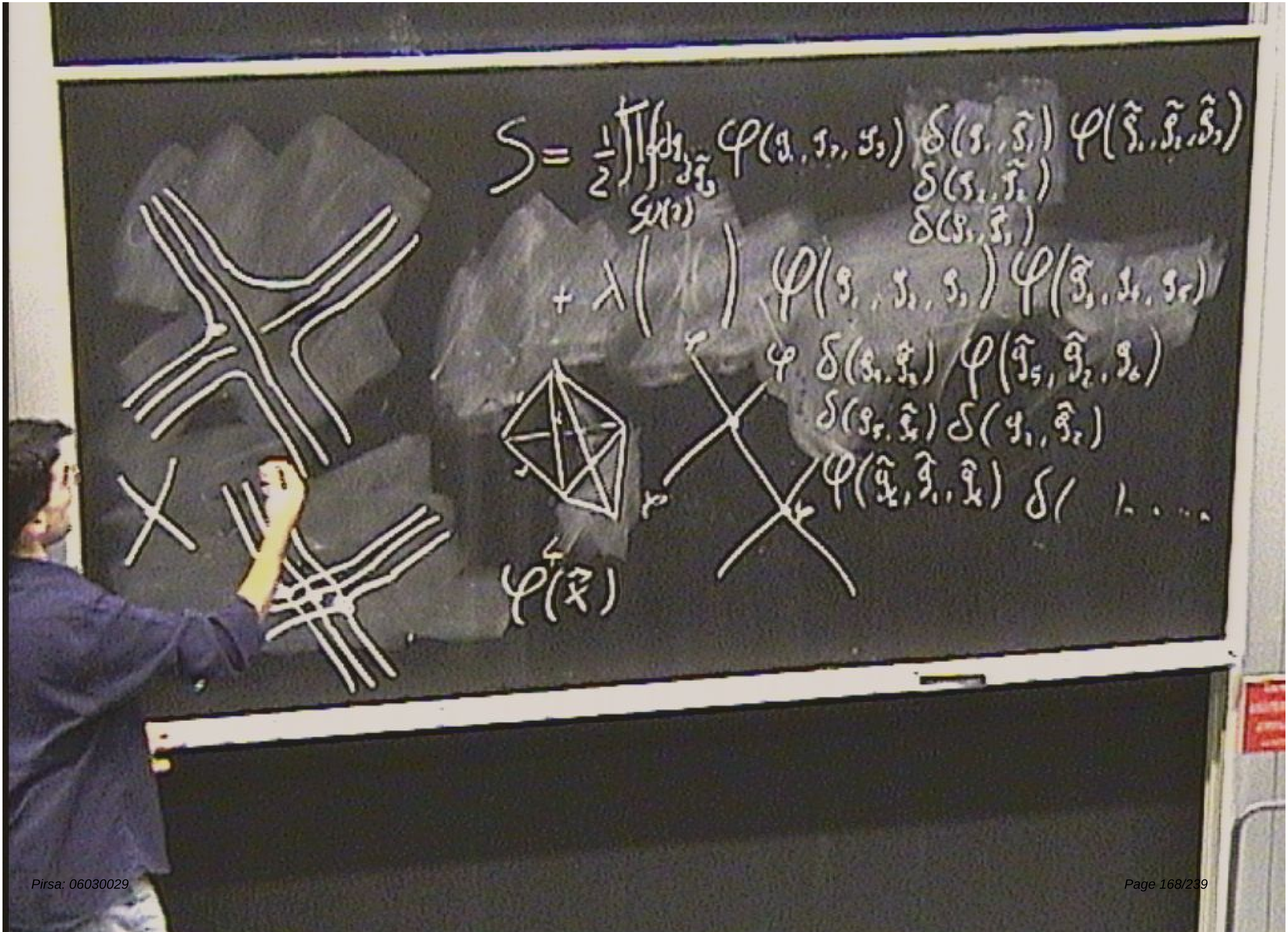


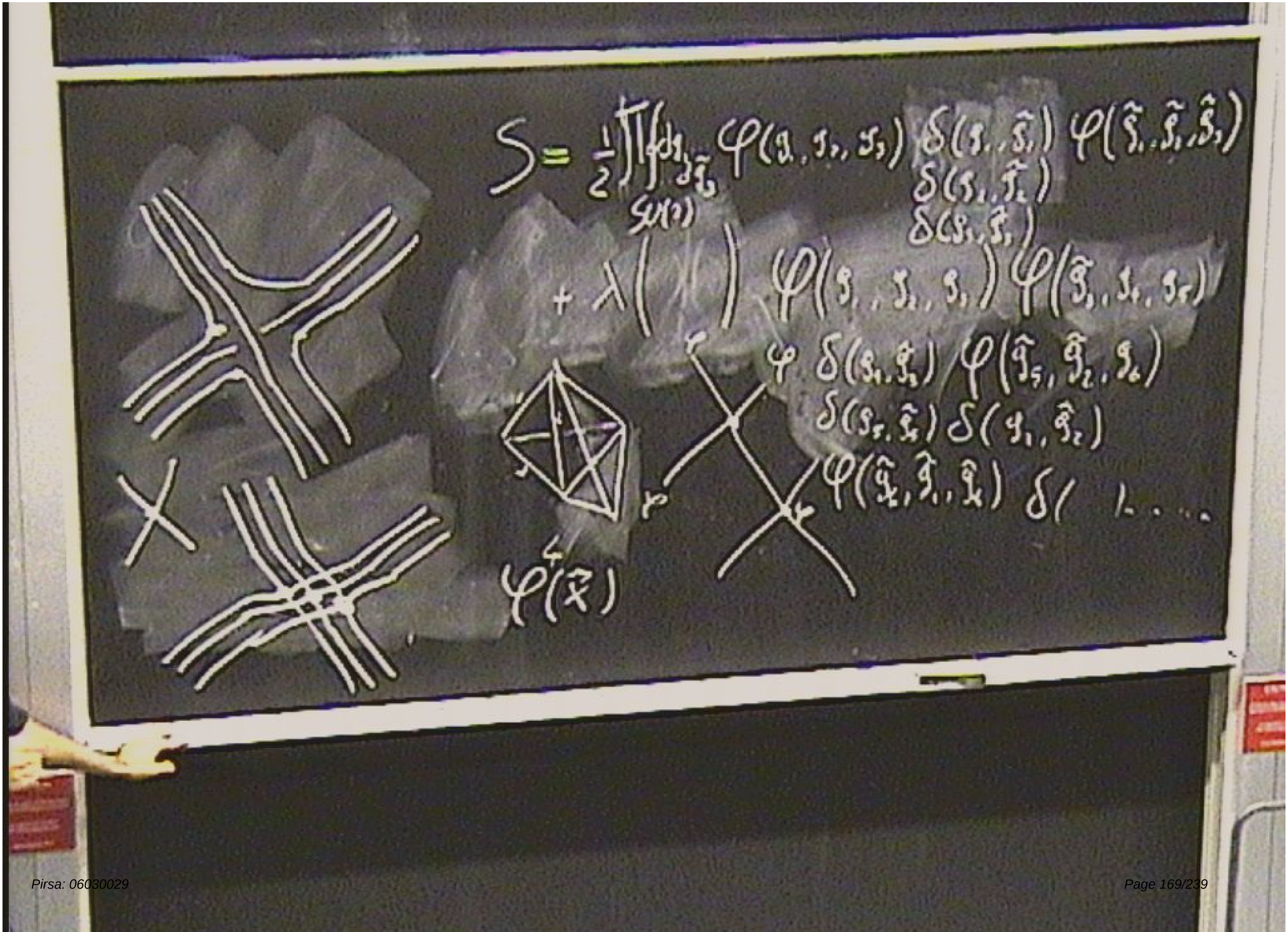


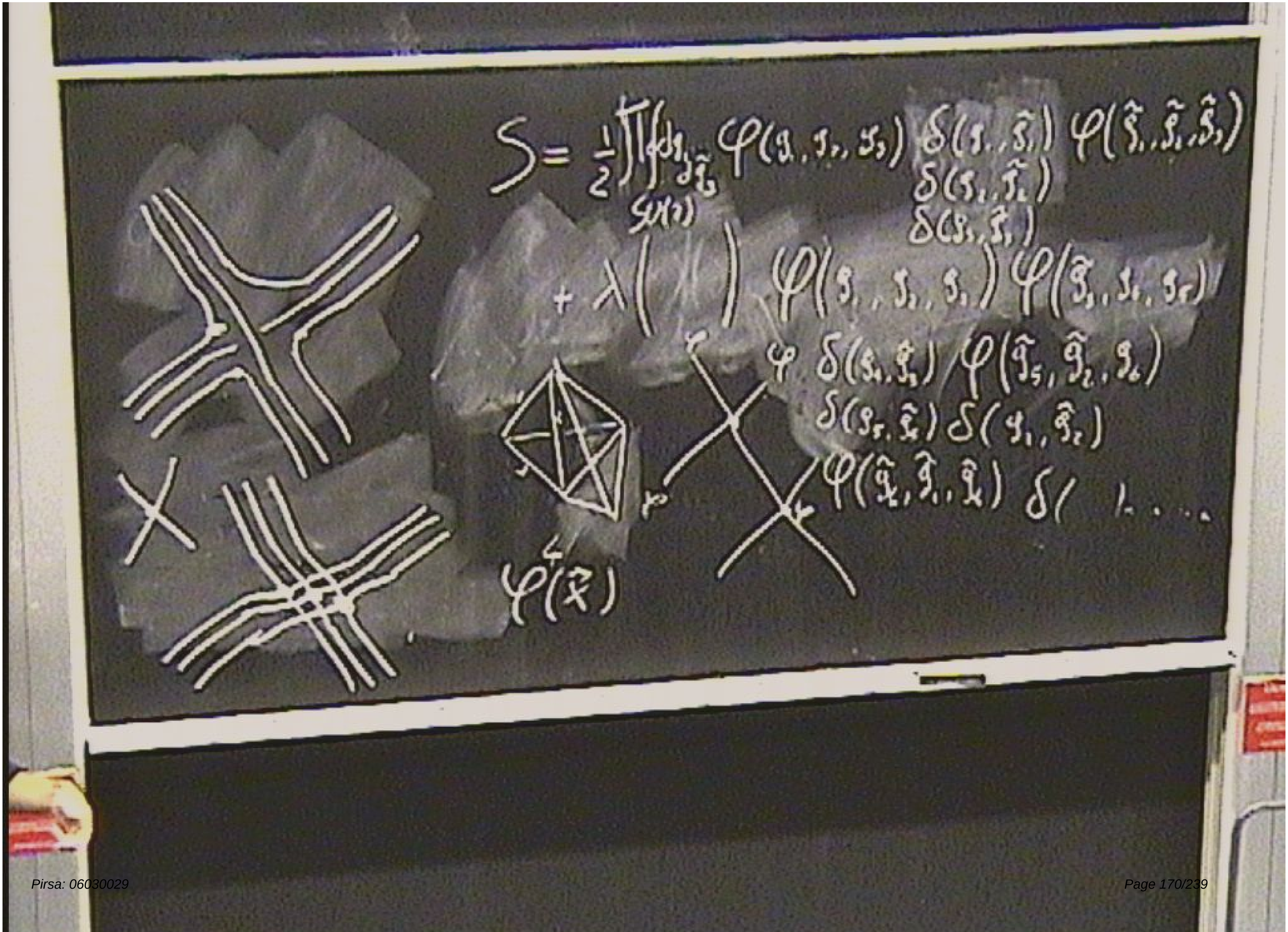


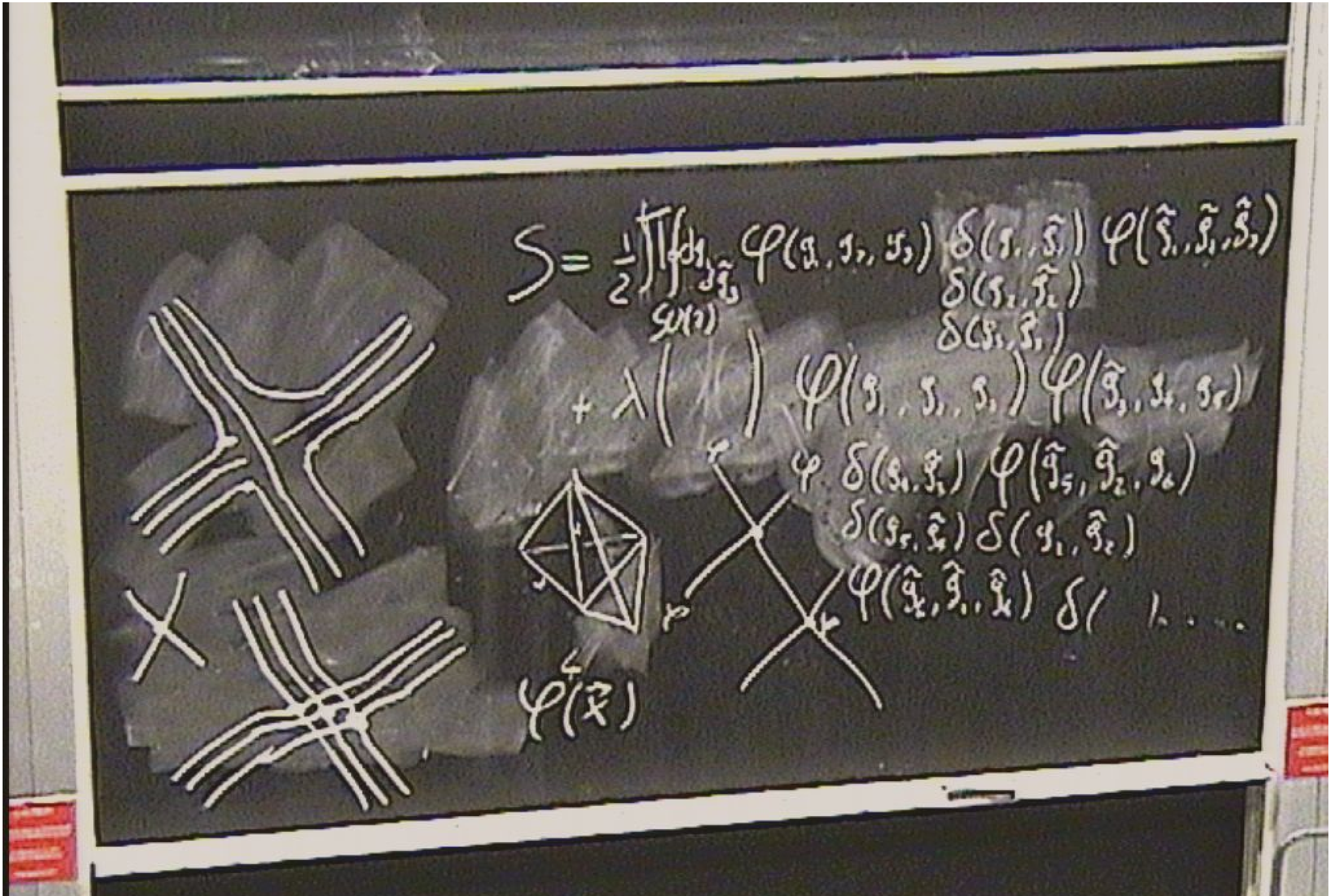


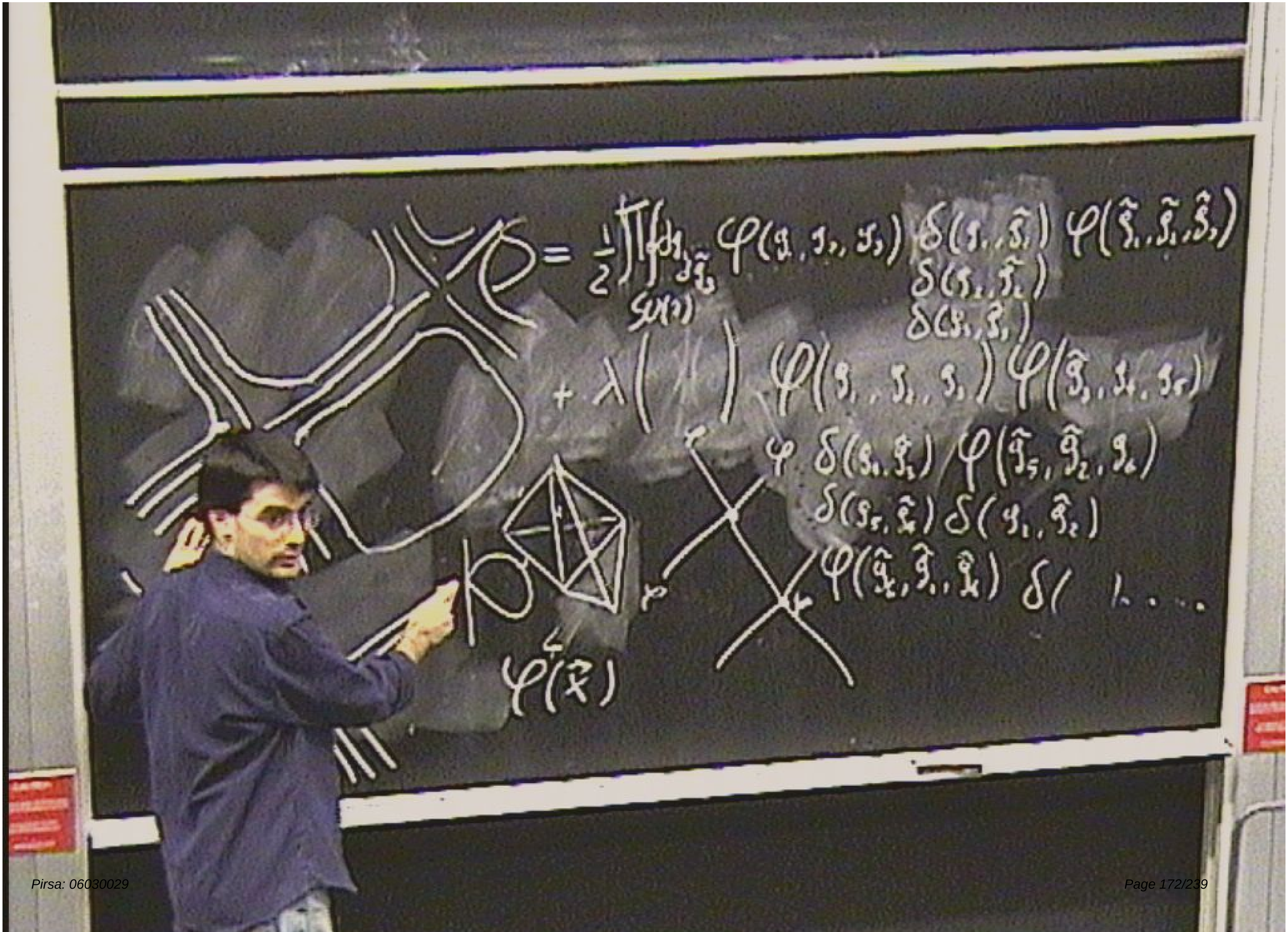


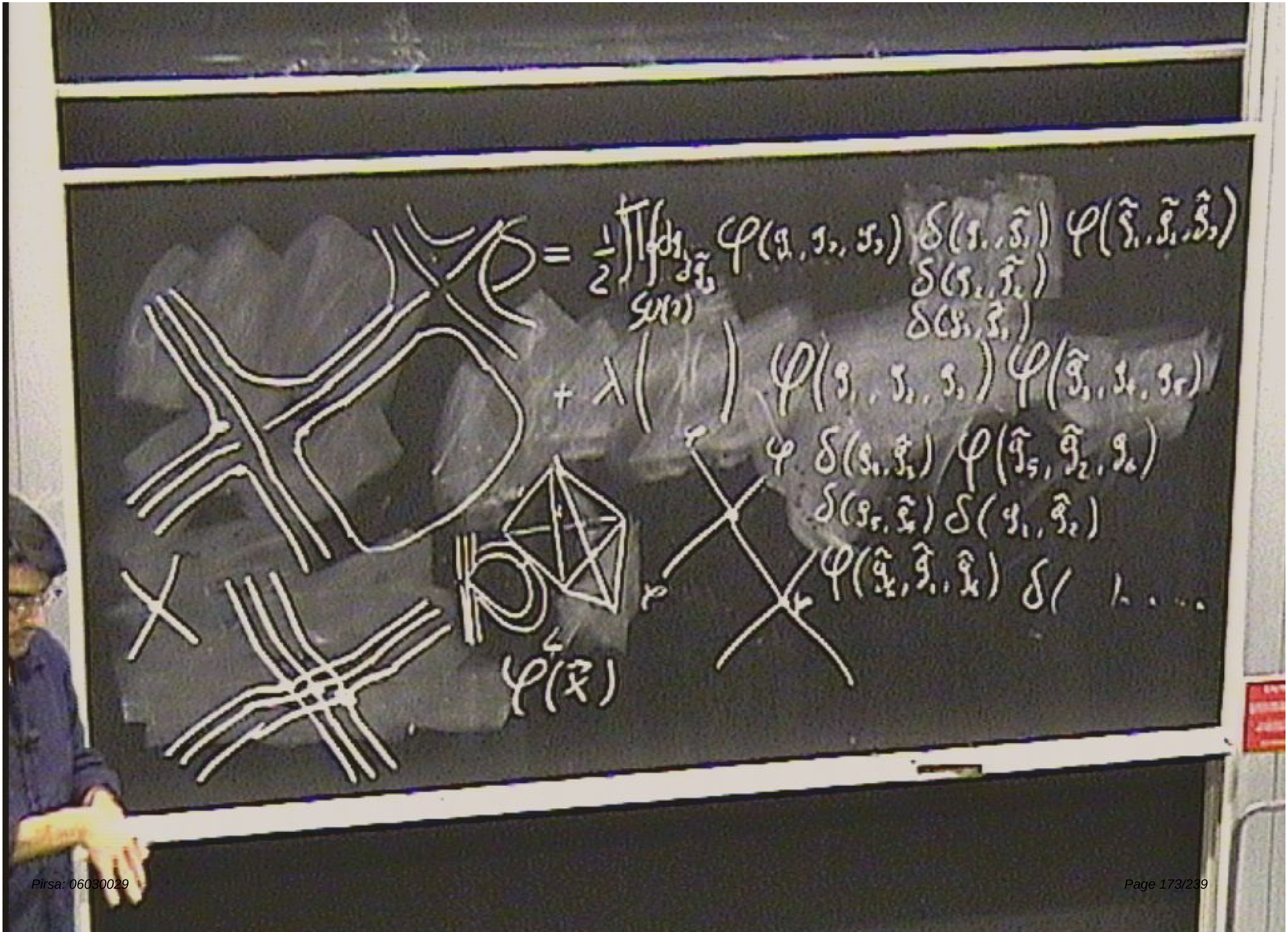



S = 1/2
SU(2)
+ λ
φ(ĝ)

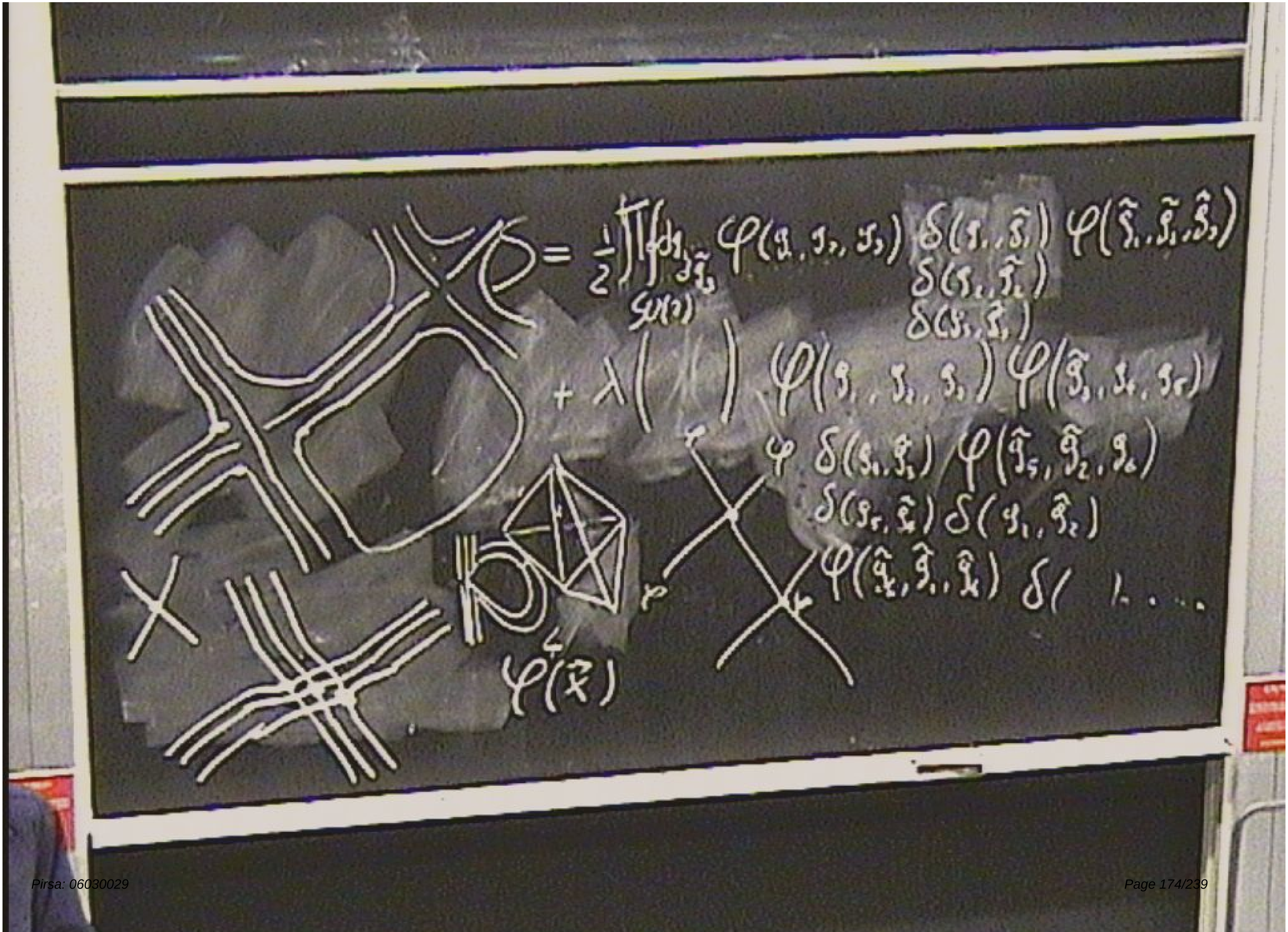


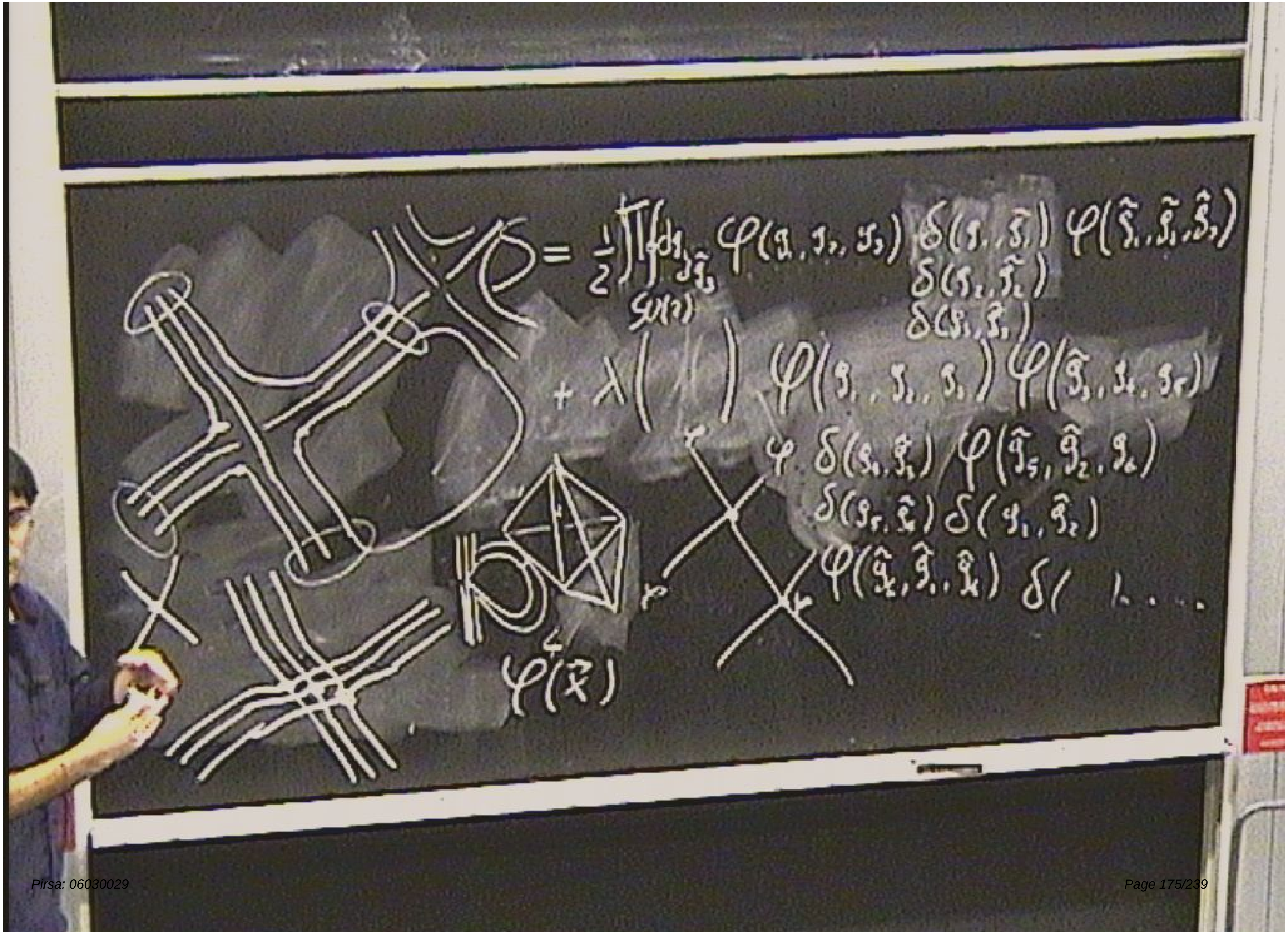


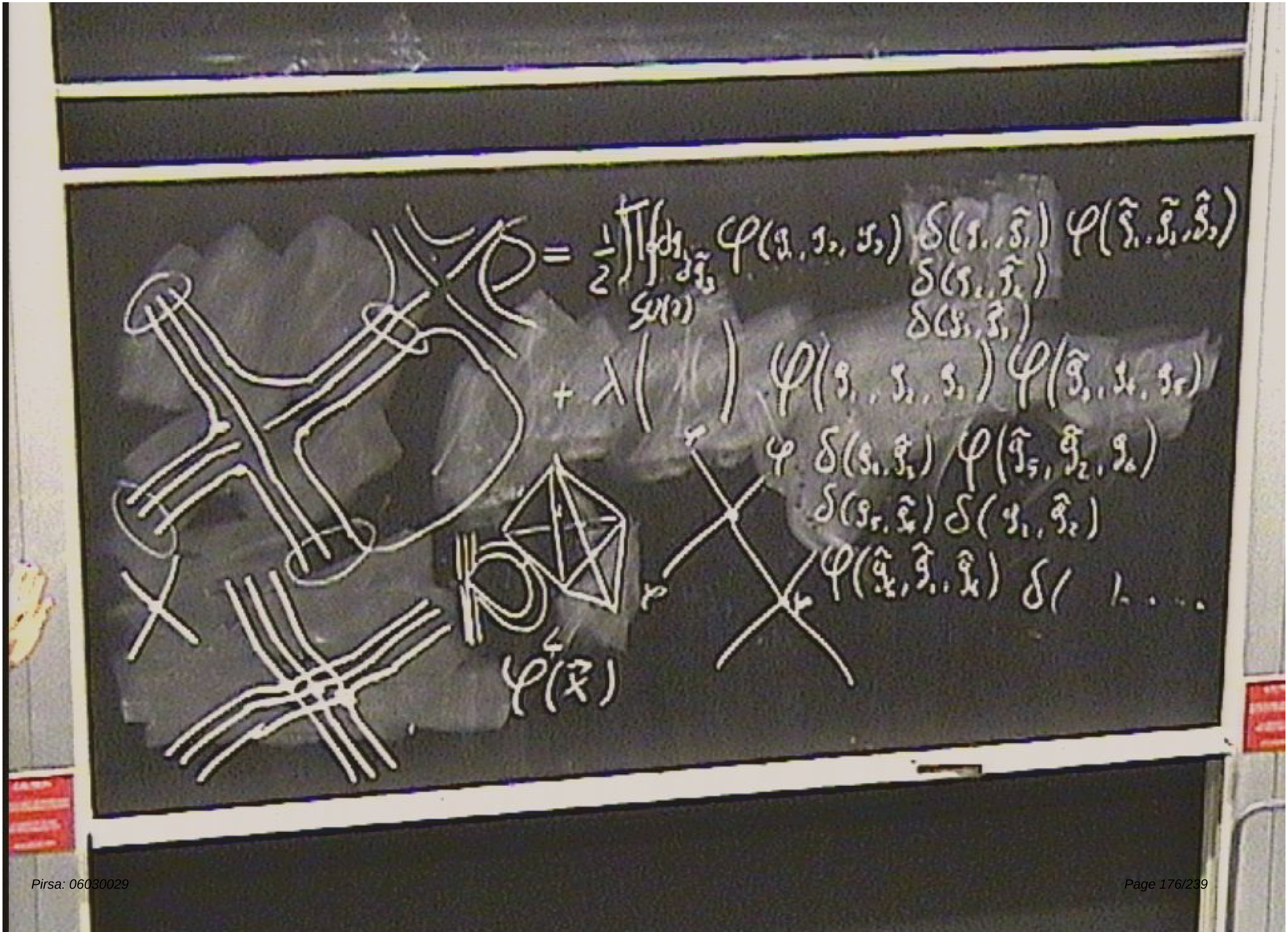


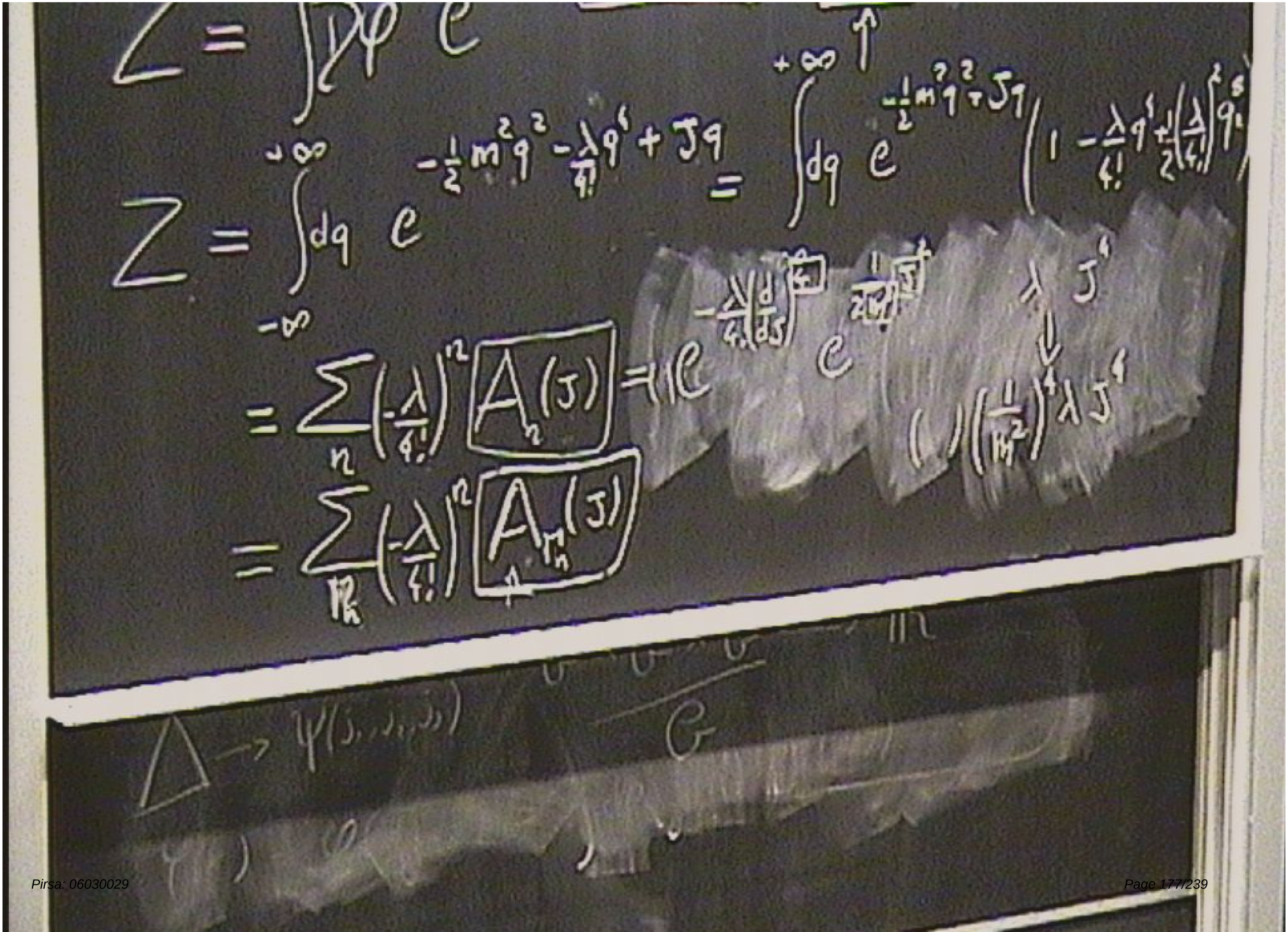


= ½ ∫ φ(g₁, g₂, g₃) δ(g₁, ĝ₁) φ(ĝ₁, ĝ₂, ĝ₃)
δ(g₂, ĝ₂)
δ(g₃, ĝ₃)
SU(2)
+ λ ( ) φ(g₁, g₂, g₃) φ(ĝ₁, g₄, g₅)
φ δ(g₄, ĝ₄) φ(ĝ₅, ĝ₂, g₆)
δ(g₅, ĝ₅) δ(g₁, ĝ₂)
φ(ĝ₄, ĝ₃, ĝ₆) δ( ) ....
φ(x̄)

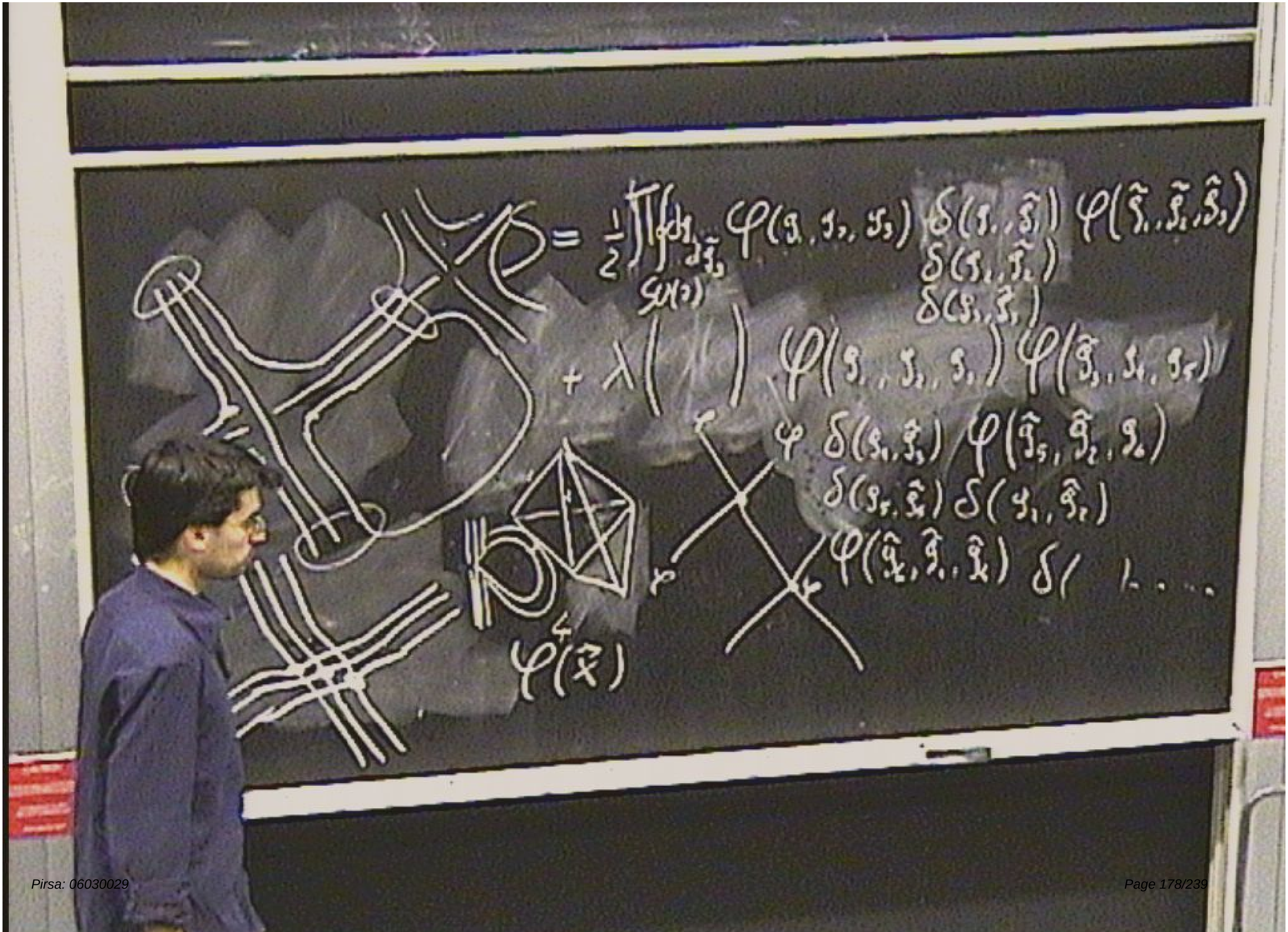


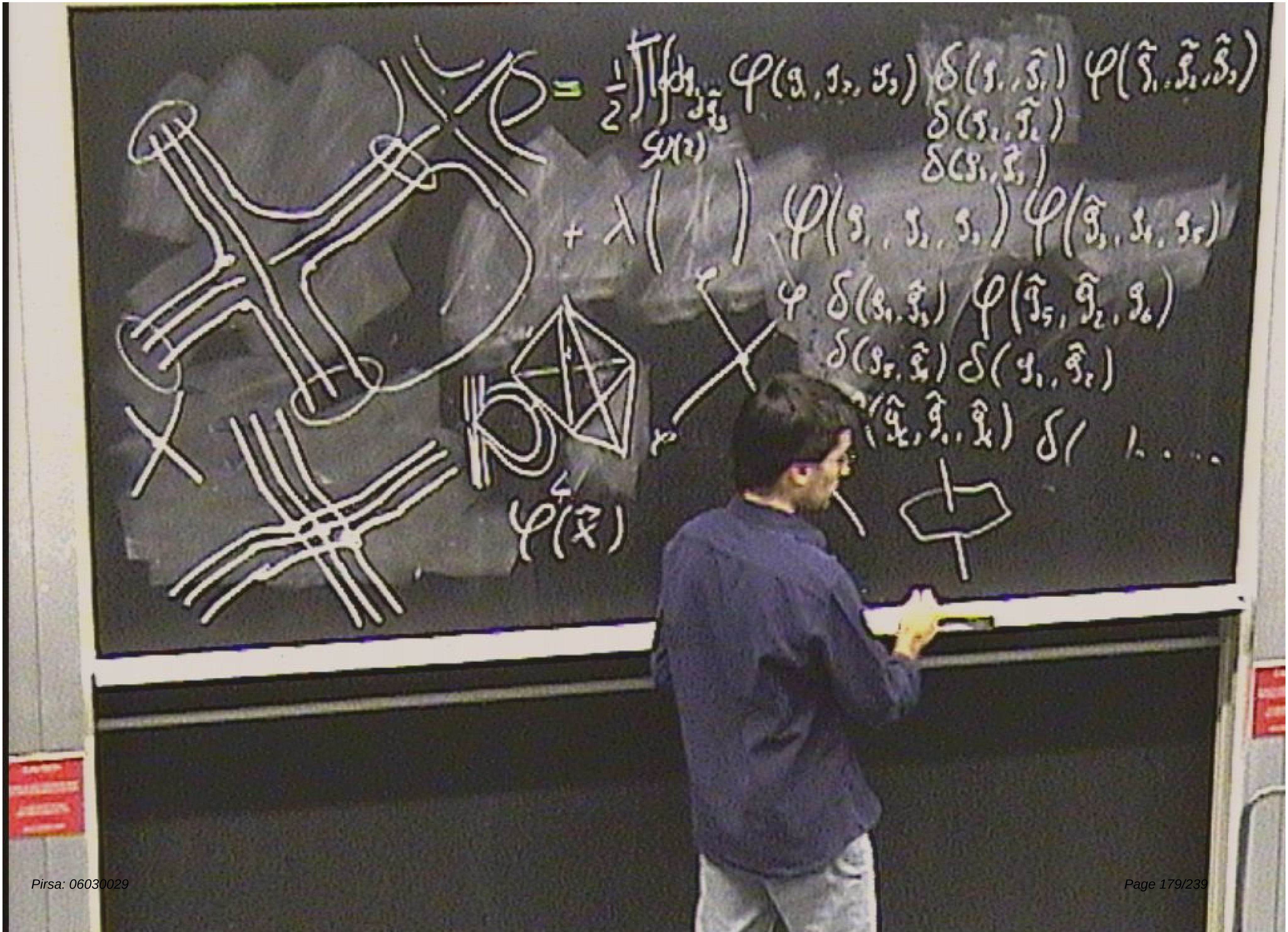


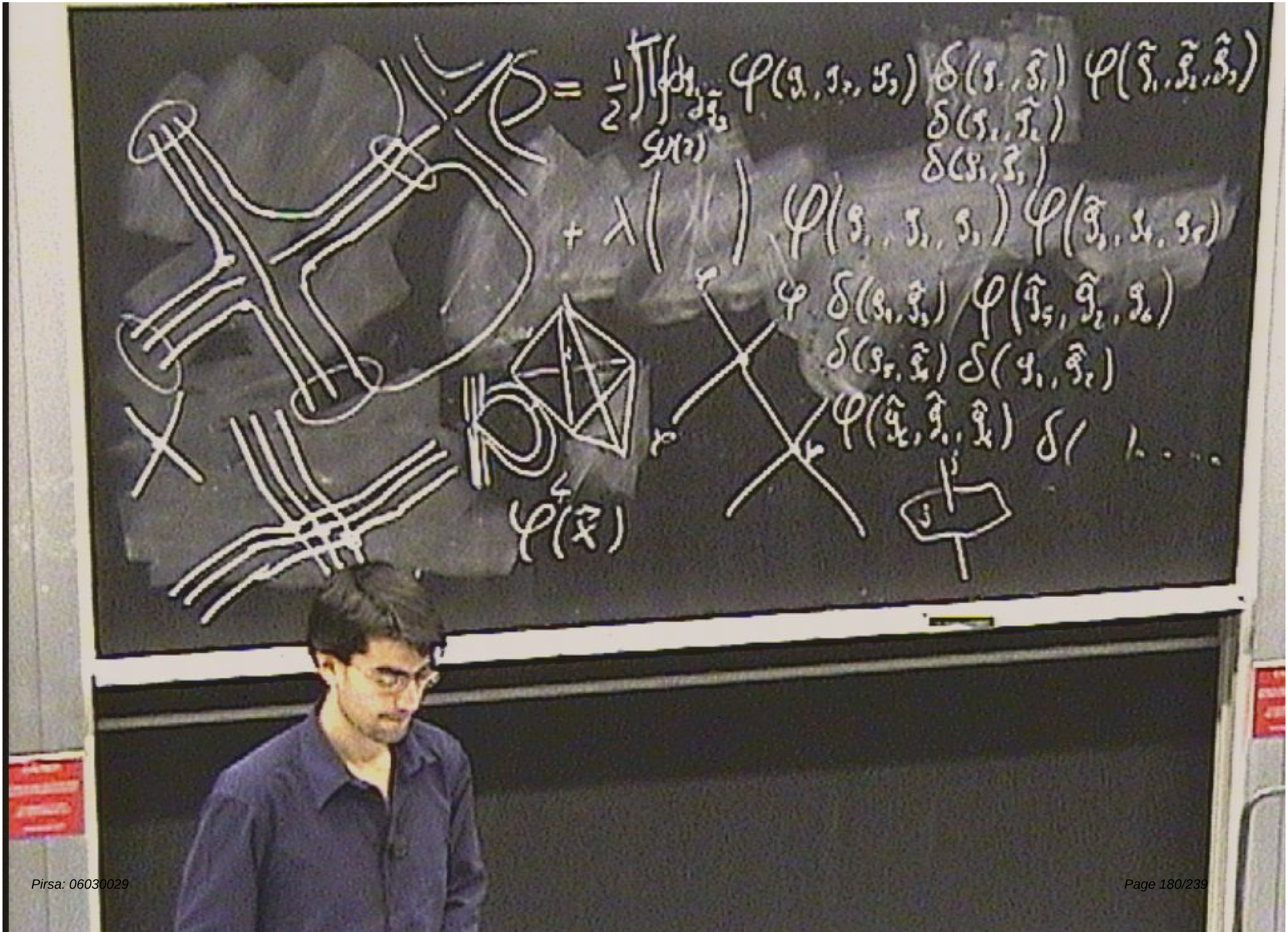


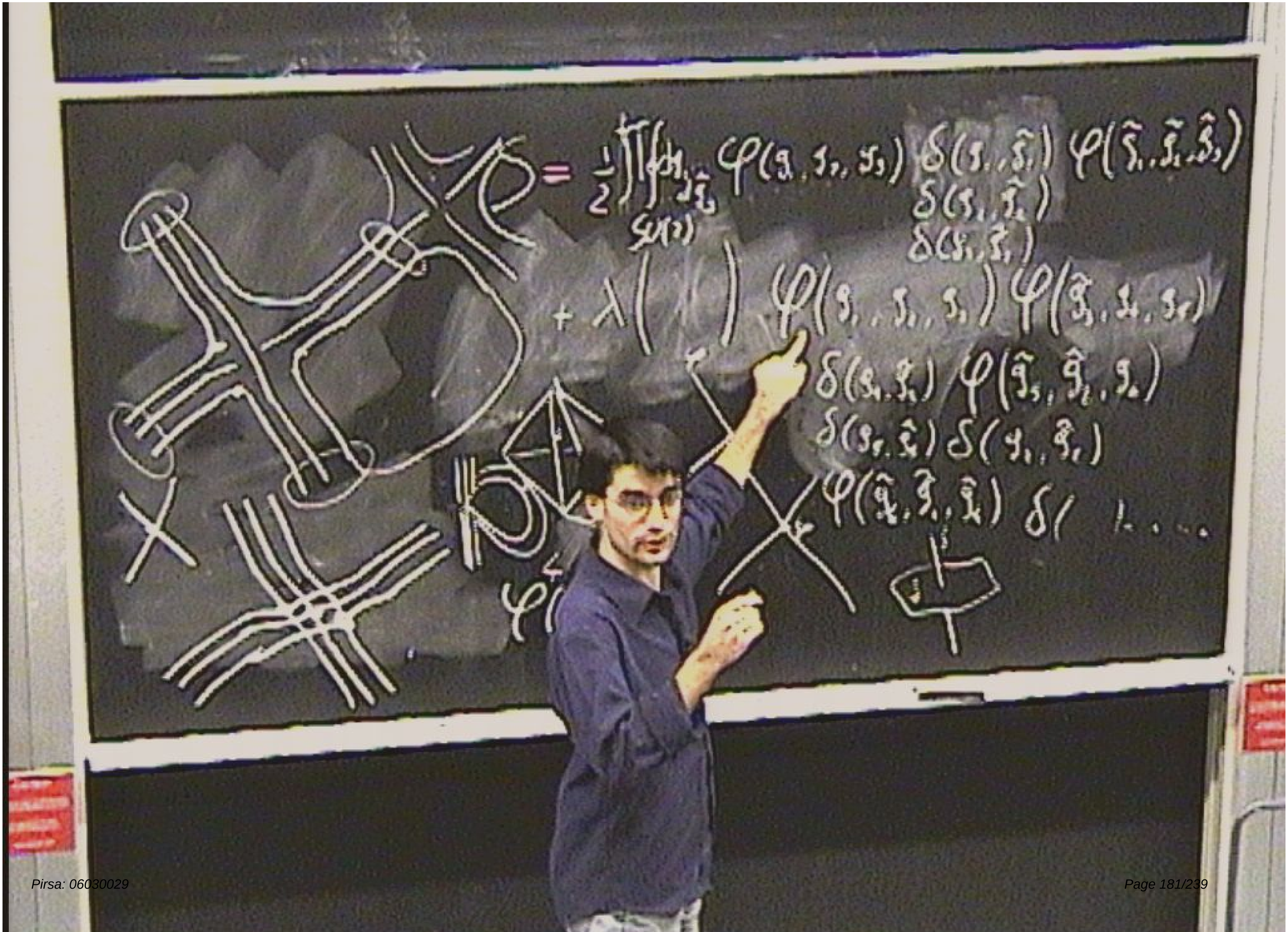


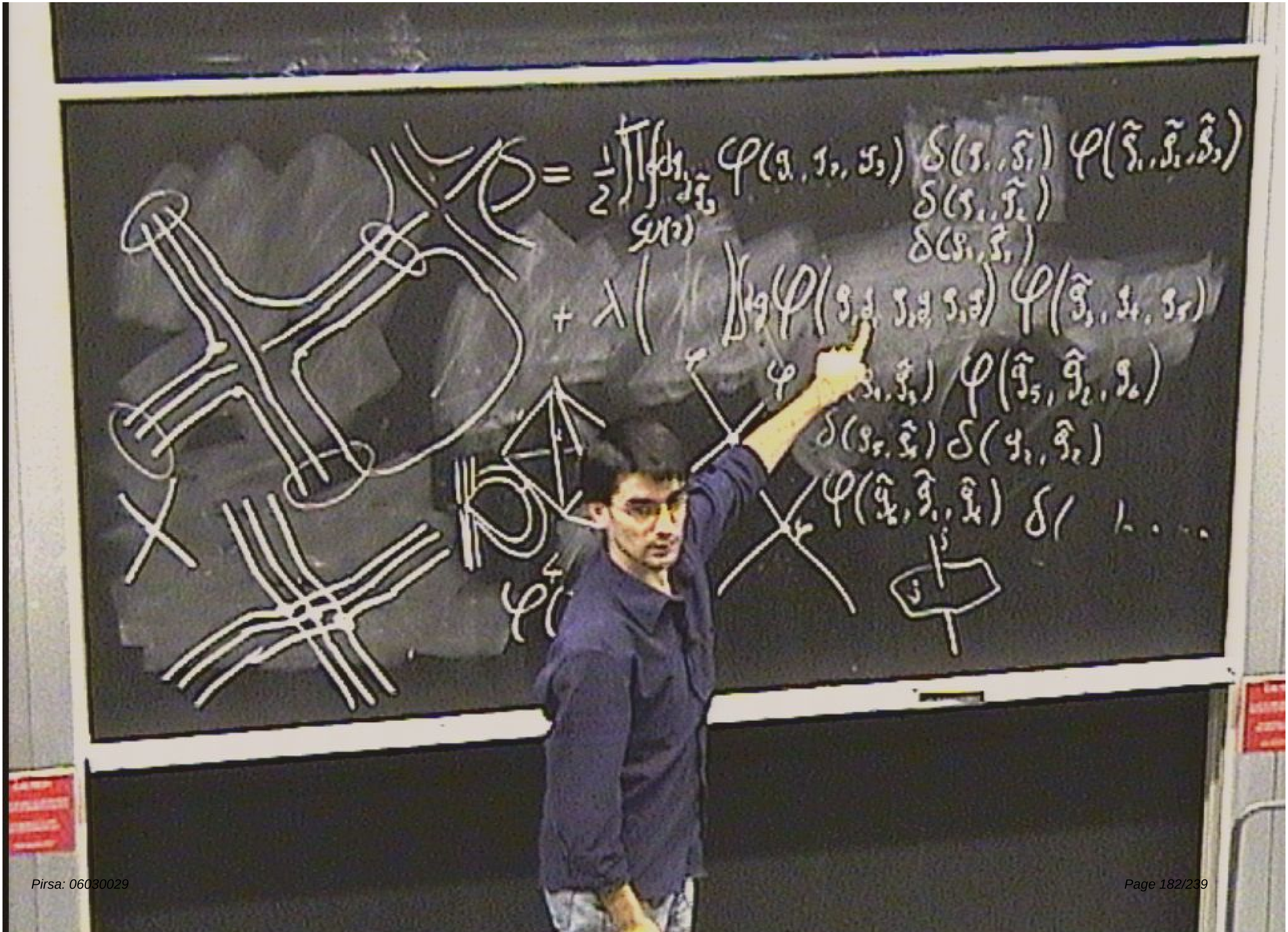
$Z = \int D\varphi \, e$
$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 + Jq}\left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}\left(\frac{\lambda}{4!}\right)^2 q^8\right)$
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J) = e^{-\frac{\lambda}{4!}\left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2m^2}J^2}$
$\lambda \; J^4$
$(\;)\left(\frac{1}{m^2}\right)^4 \lambda J^4$
$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(J)$
$\Delta \to \psi(j_1, j_2, j_3)$

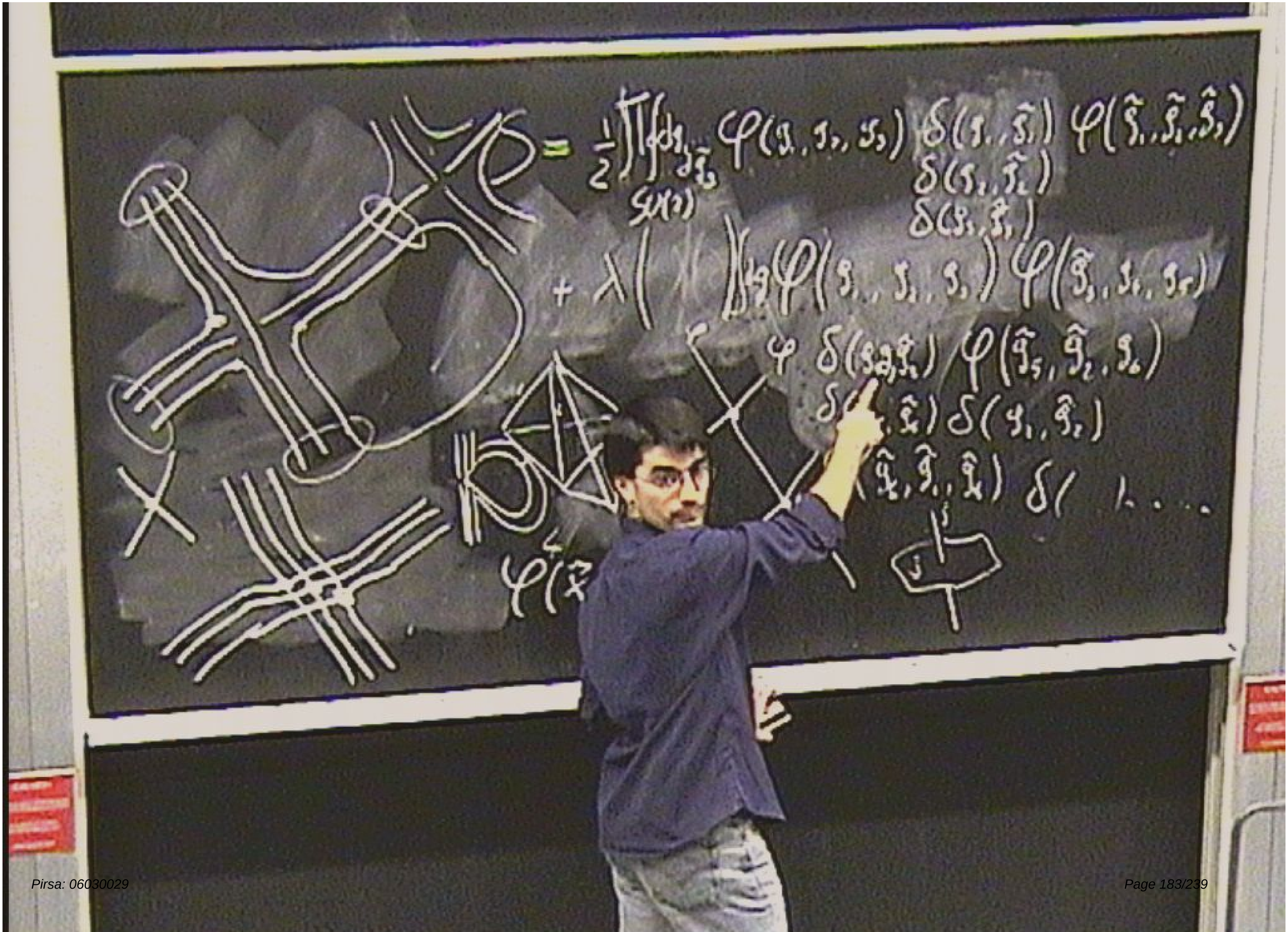


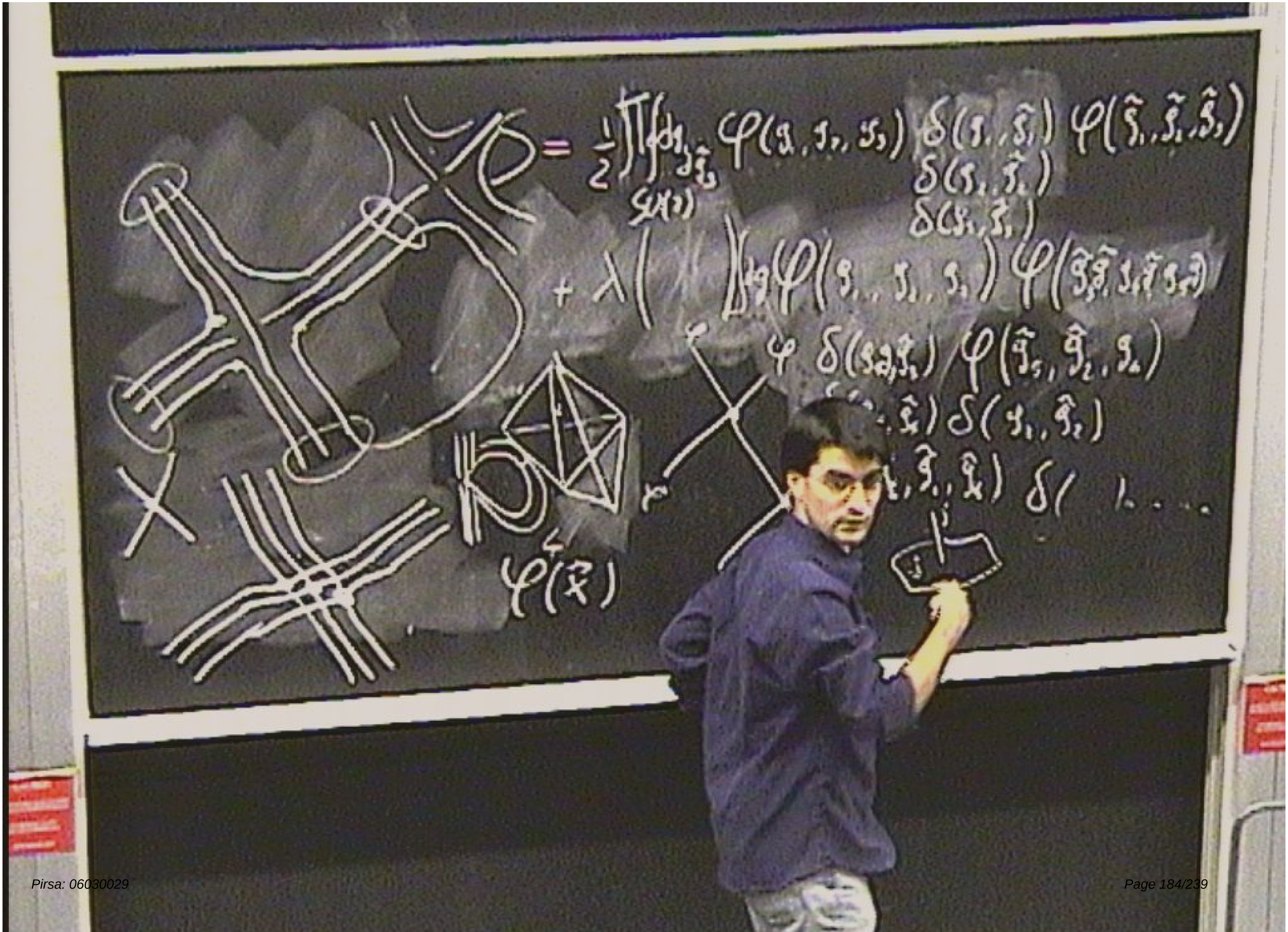


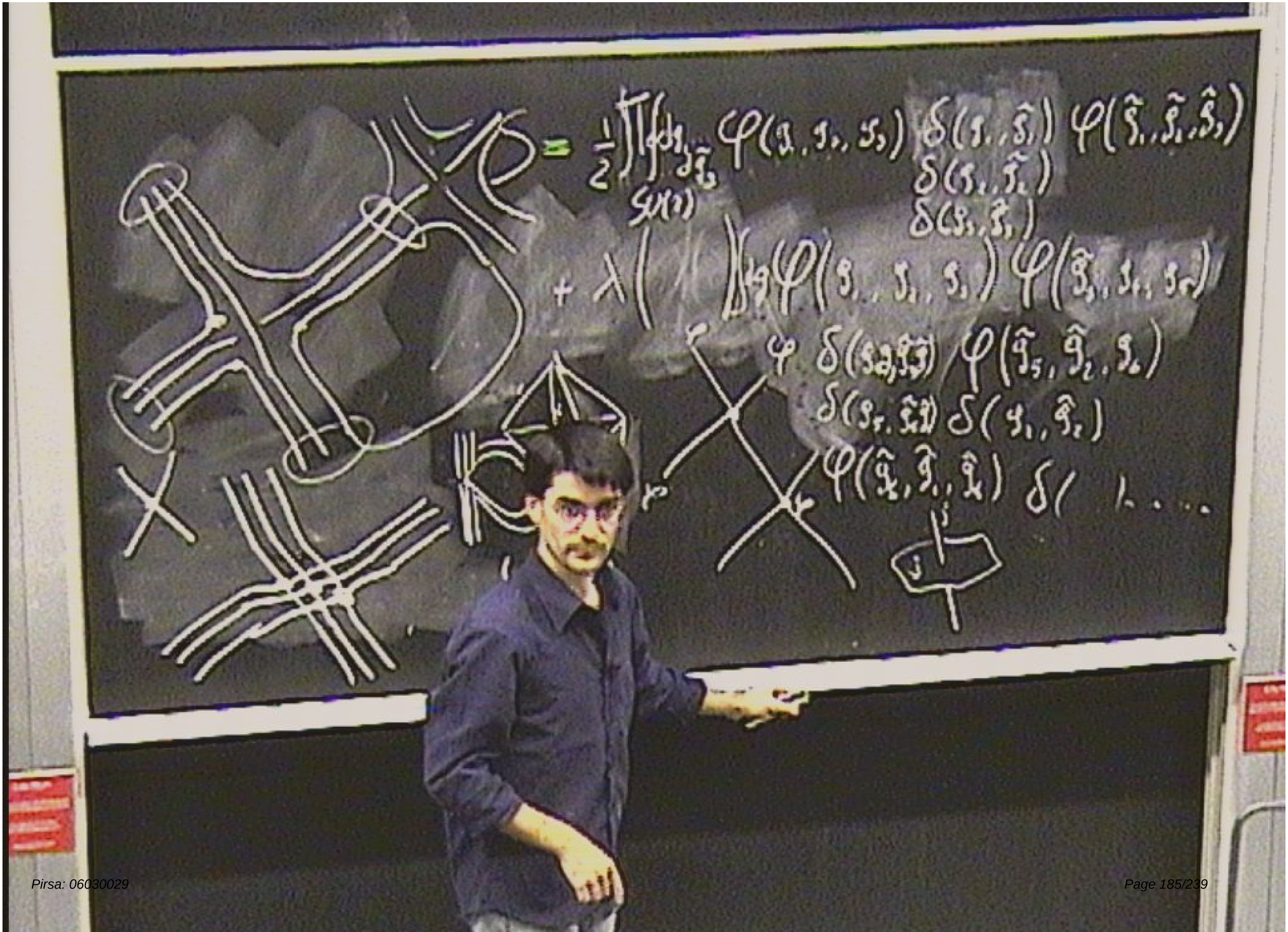


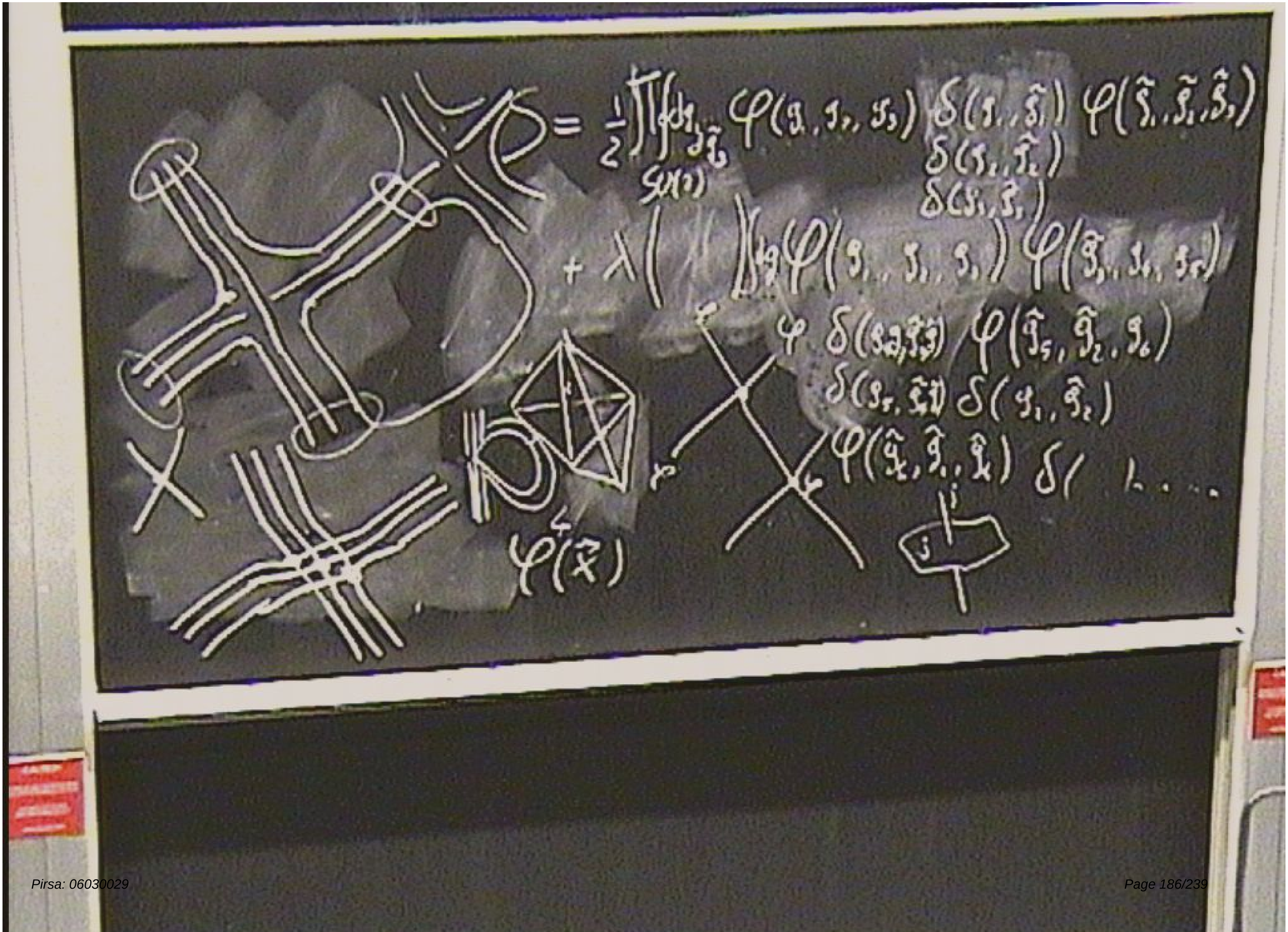


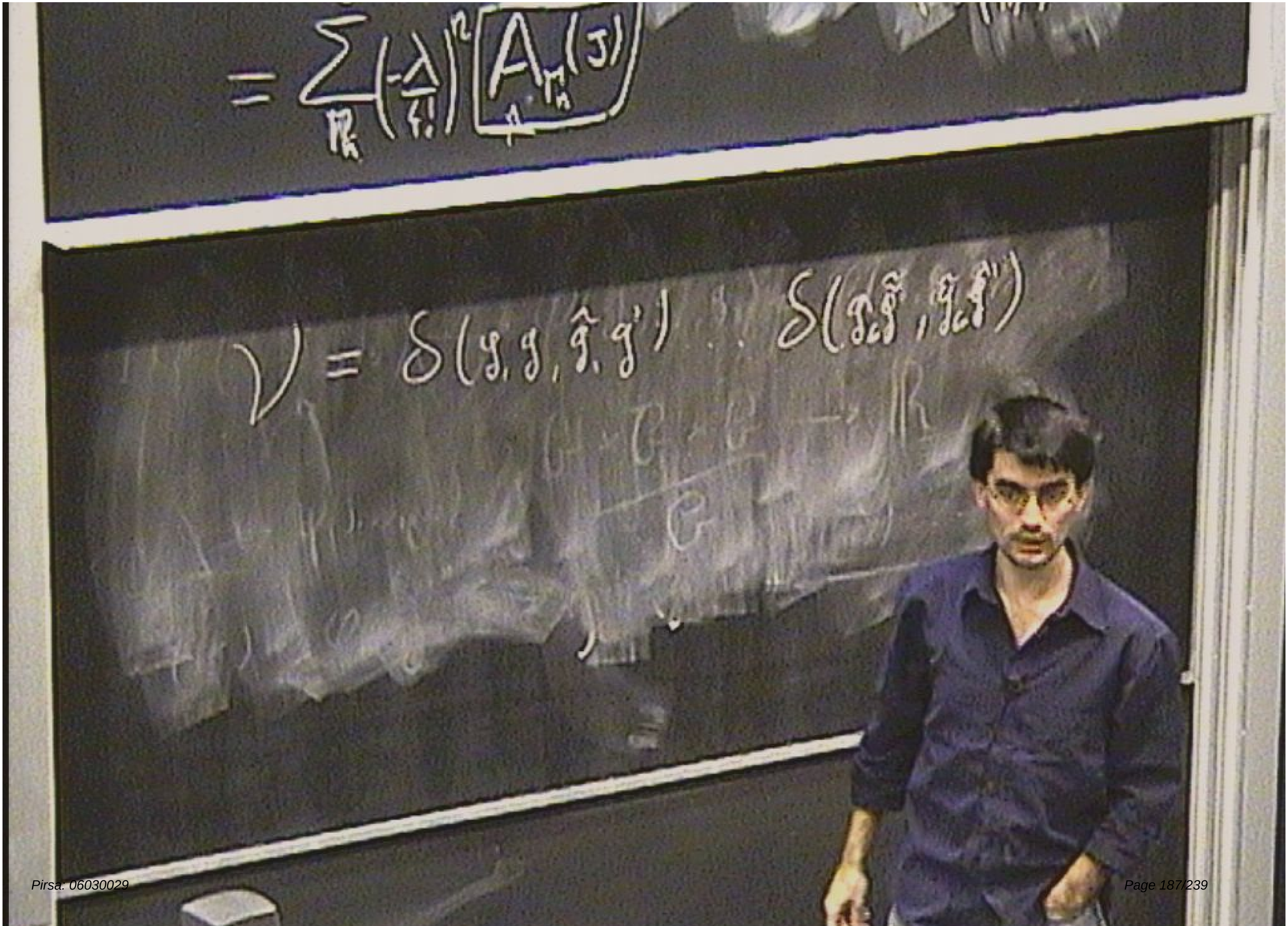


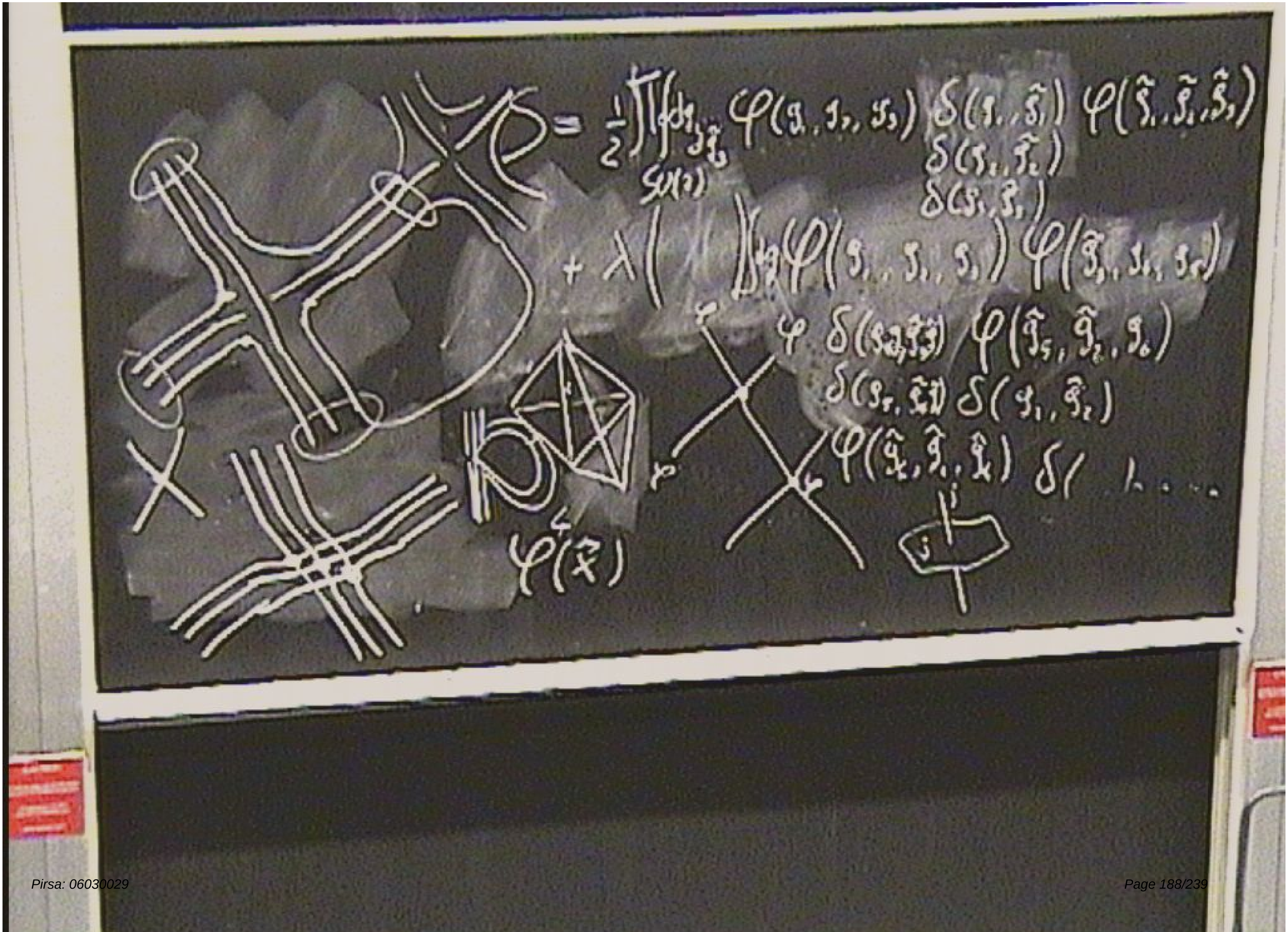


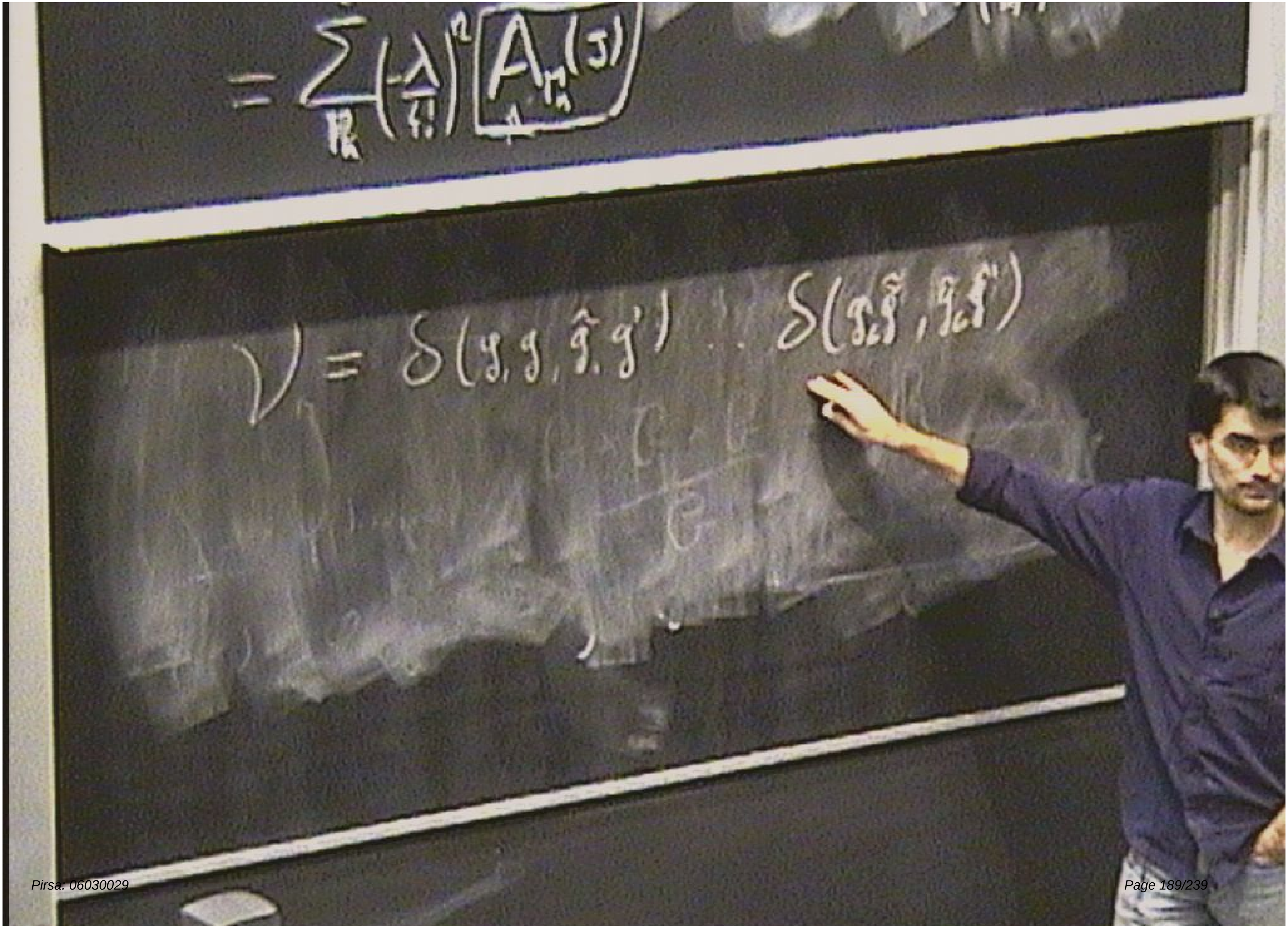


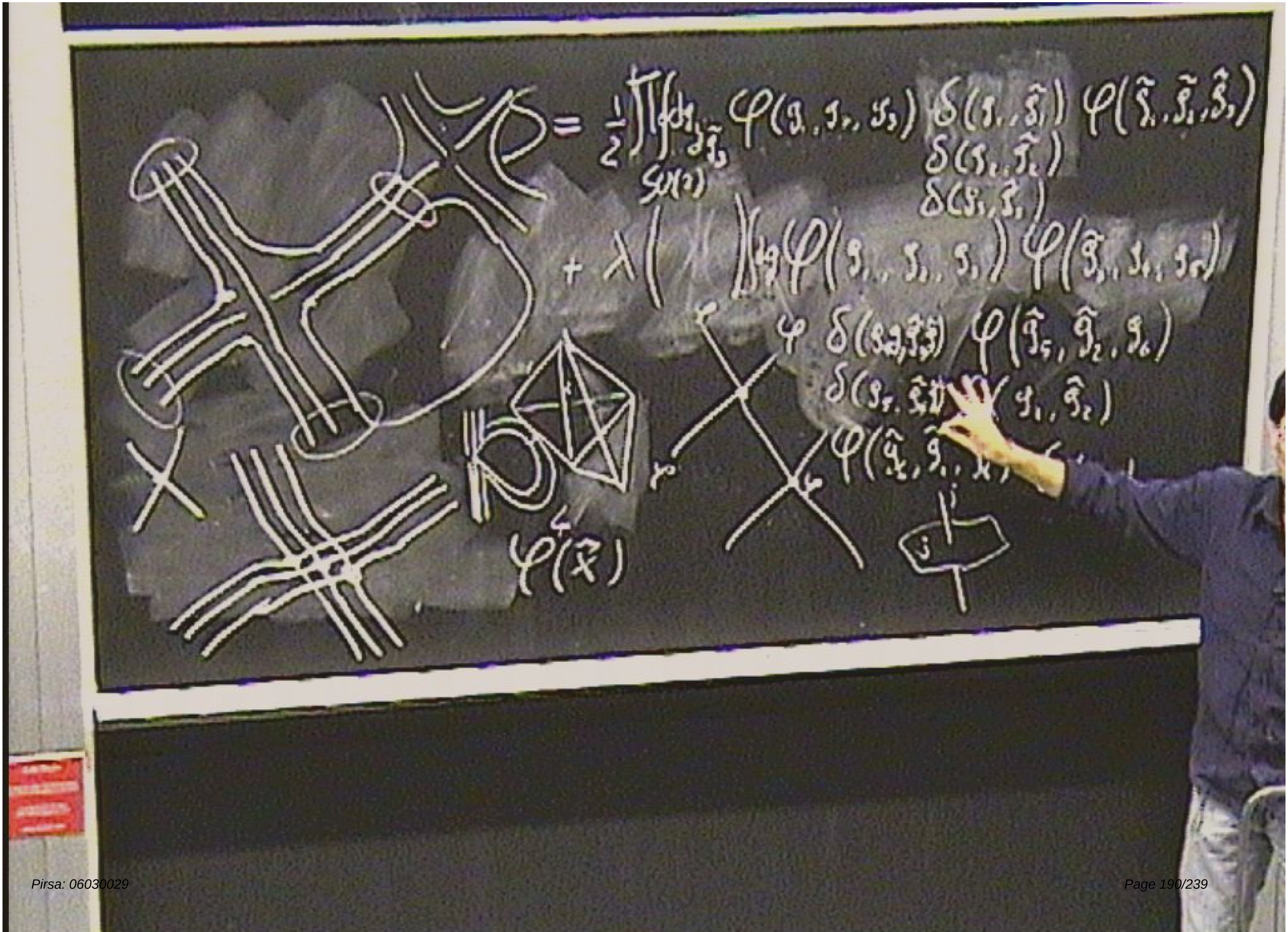


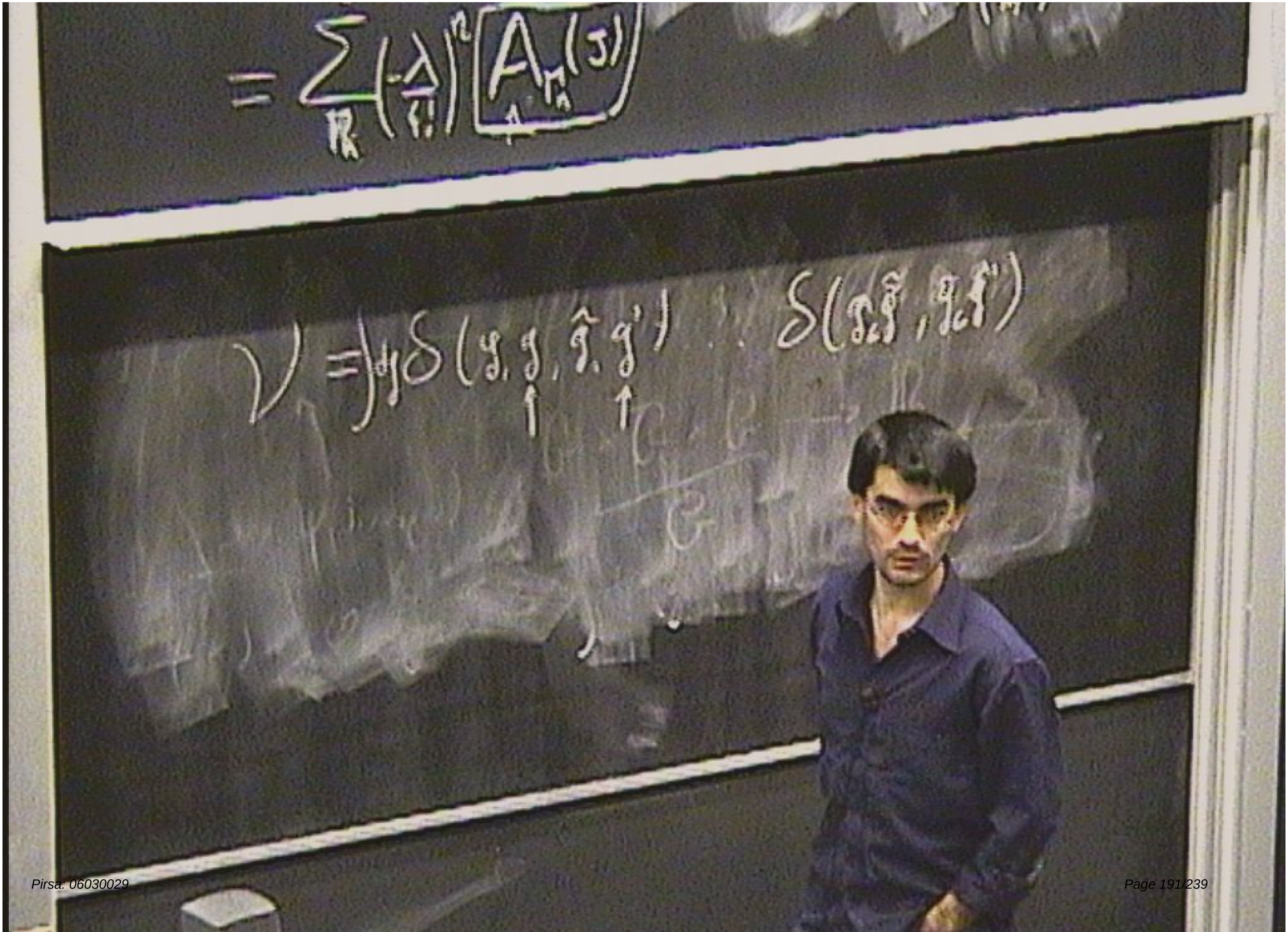


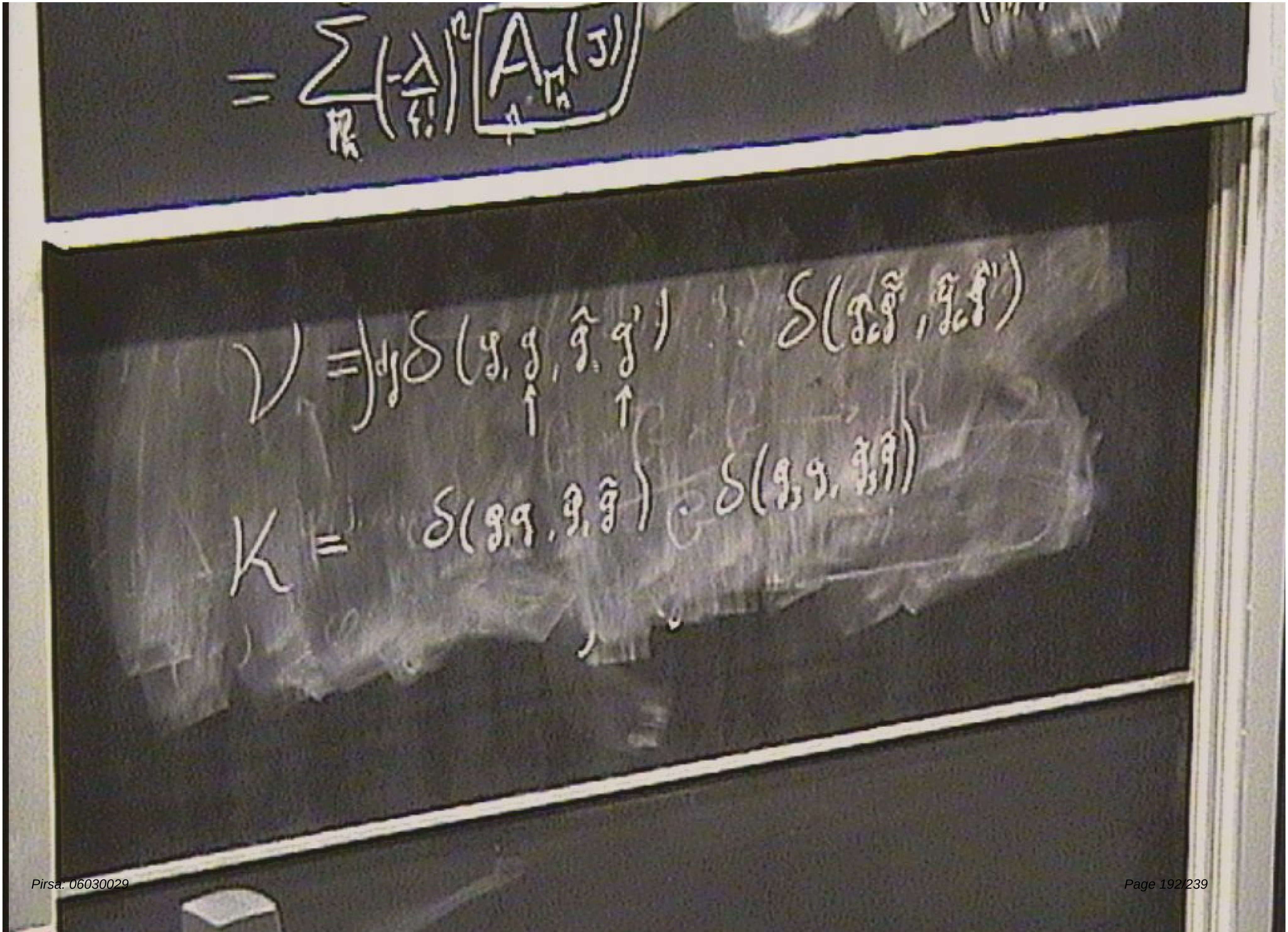


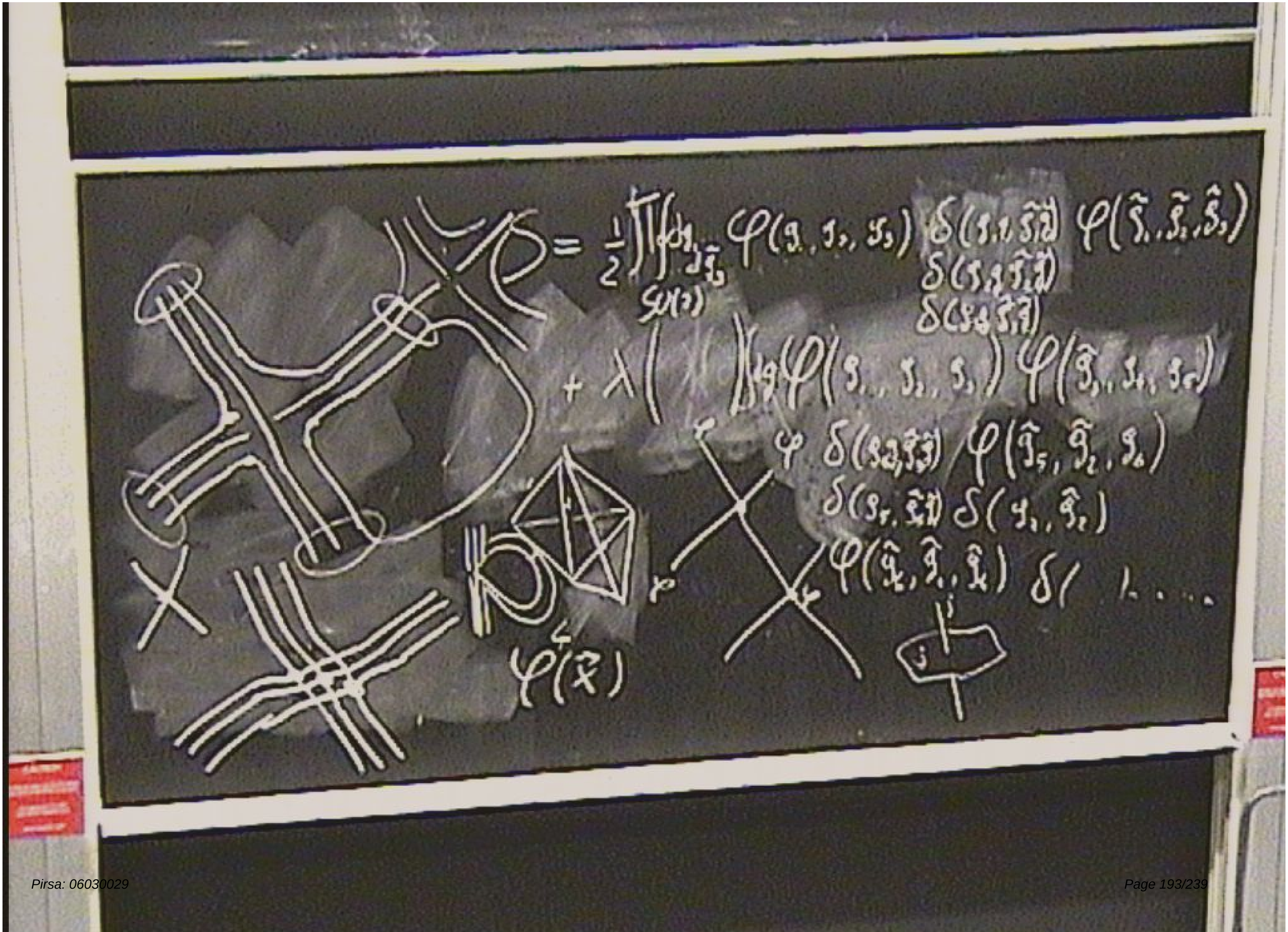


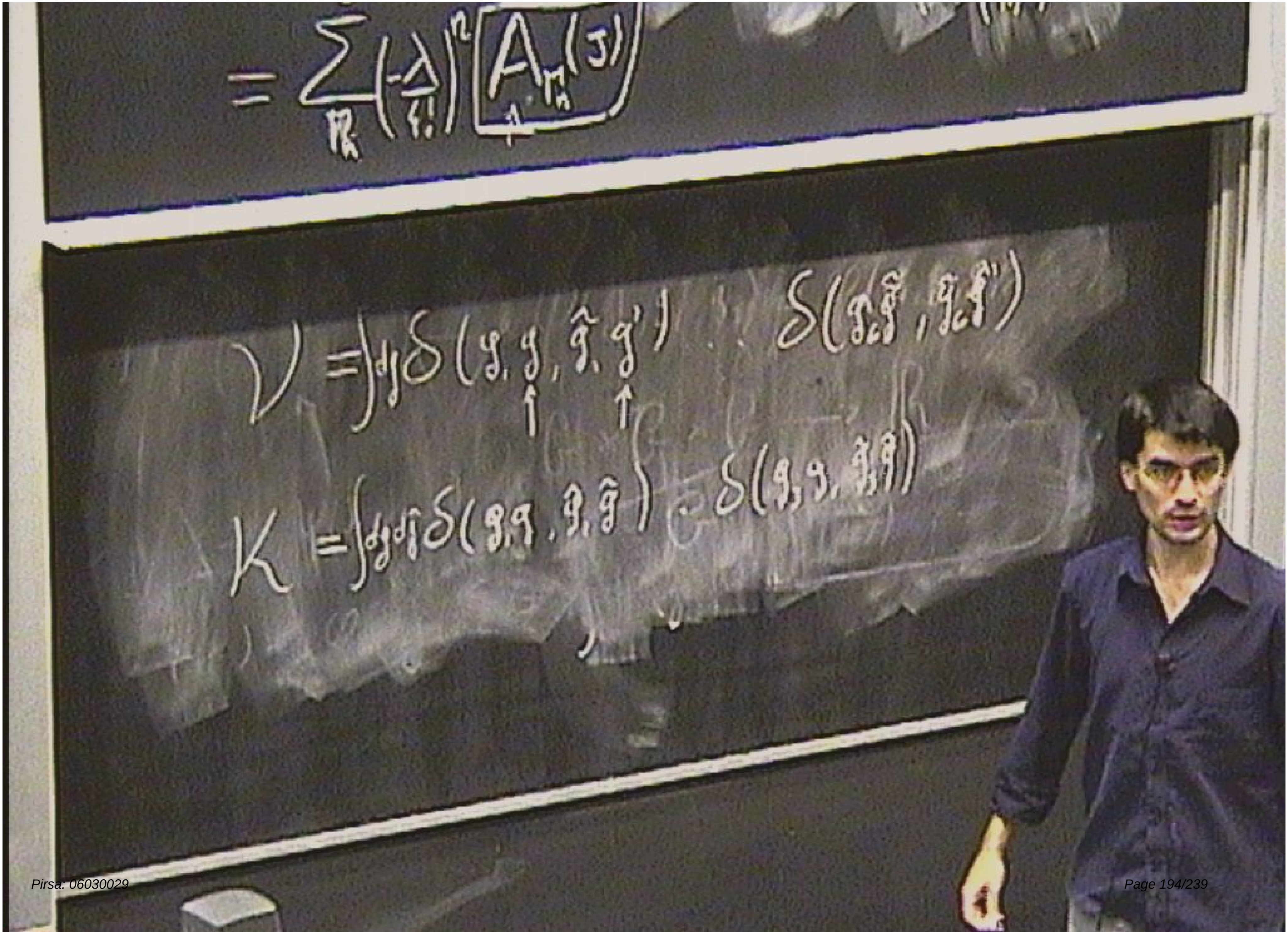


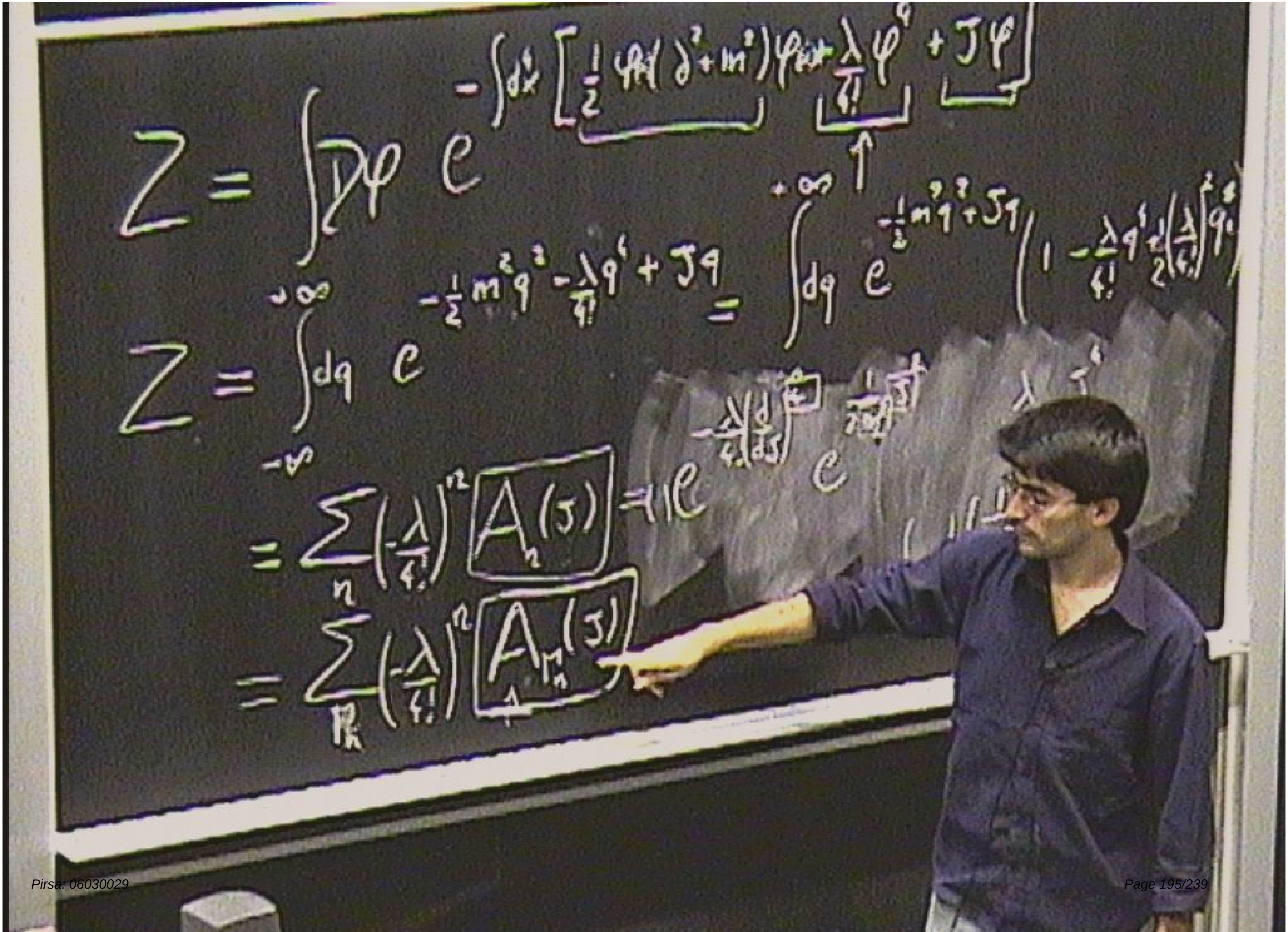




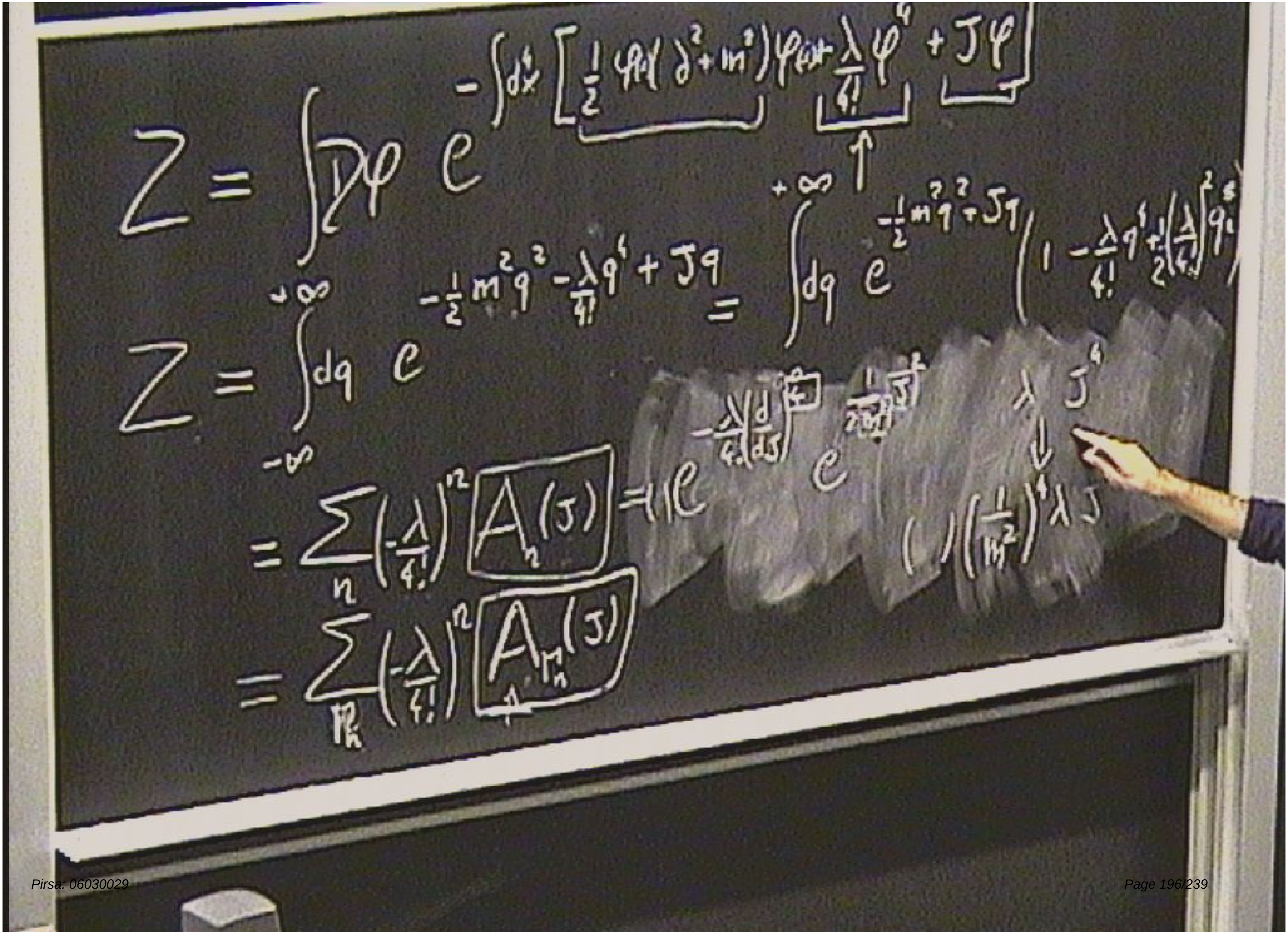


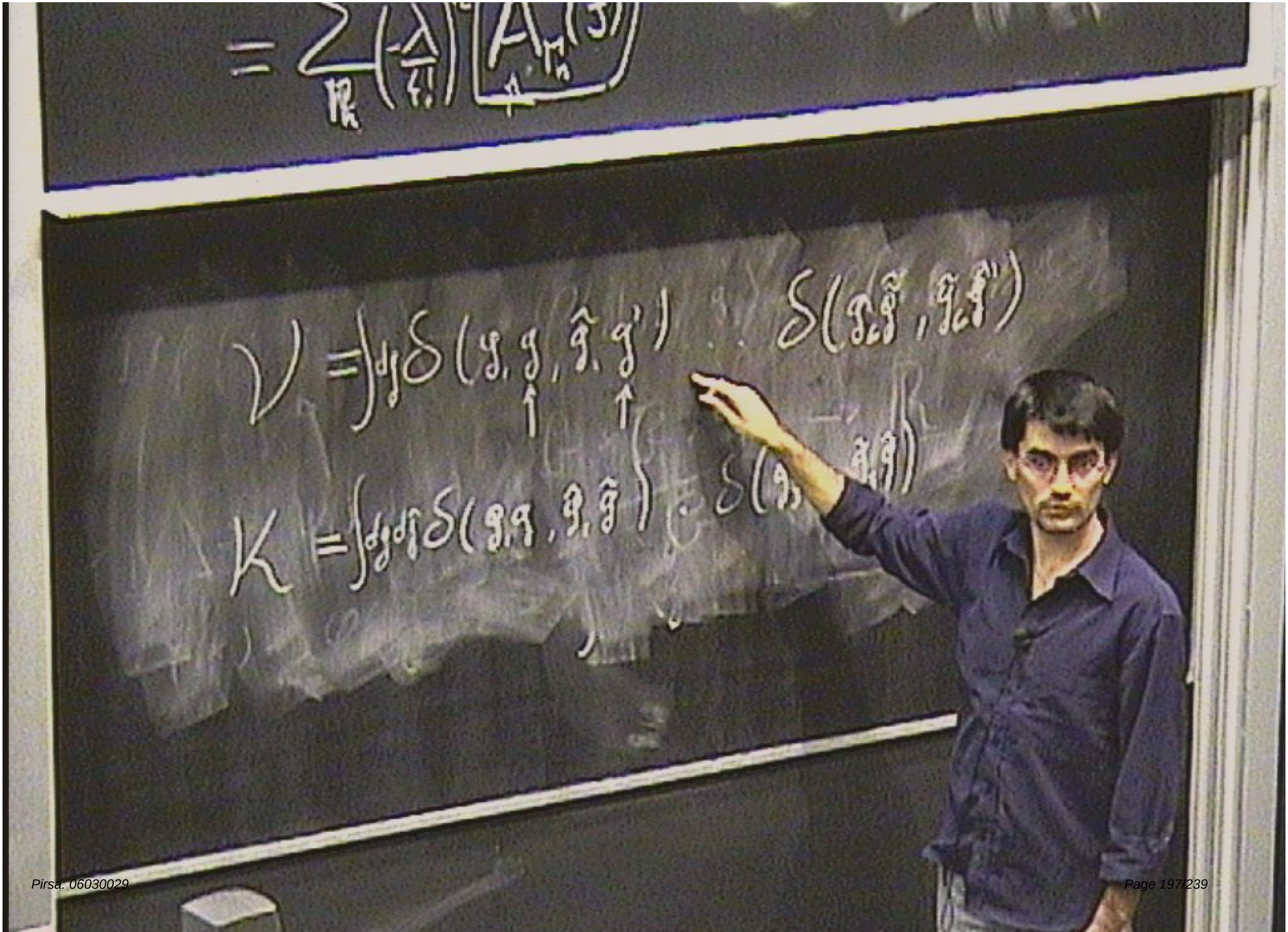


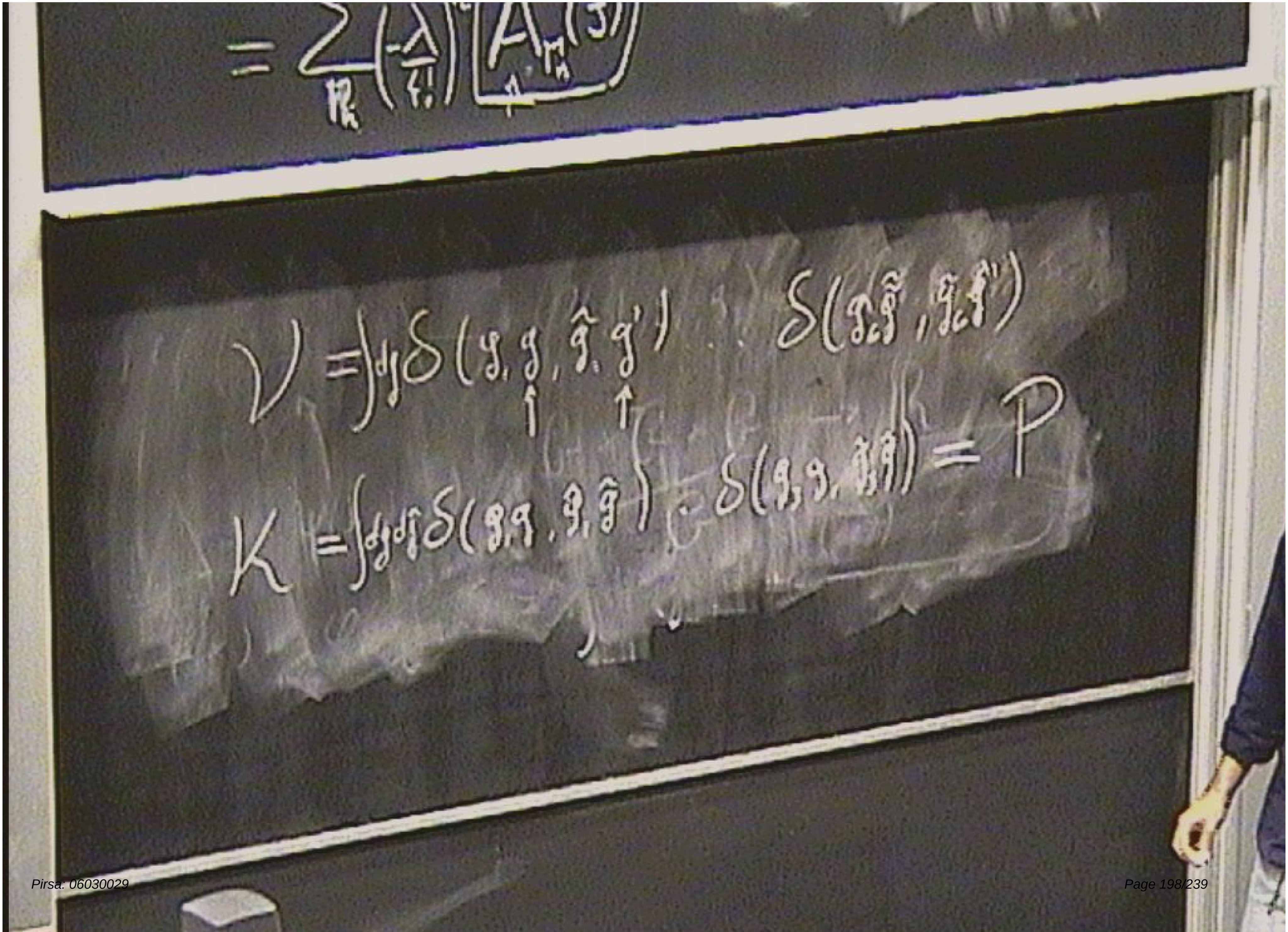


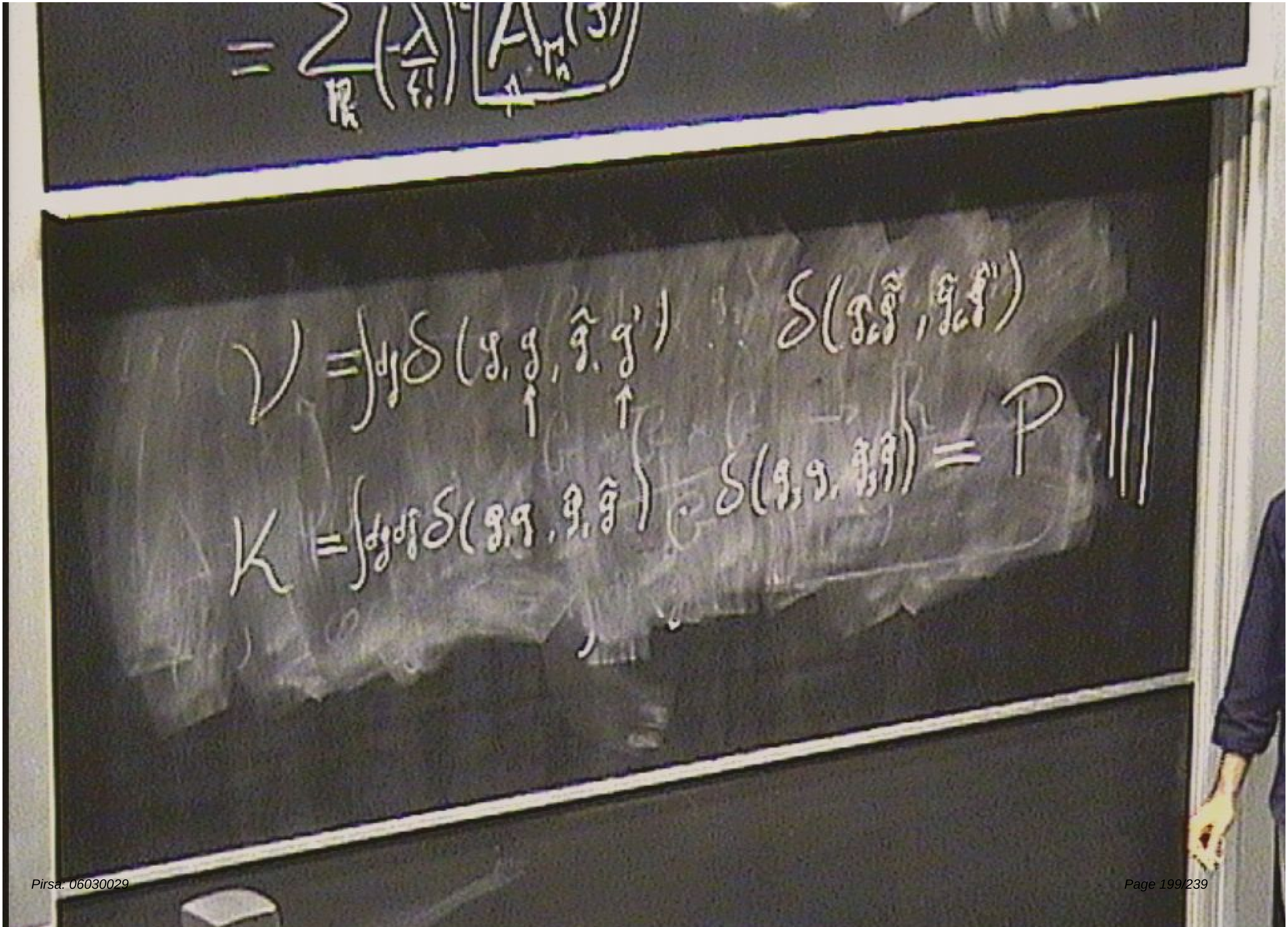


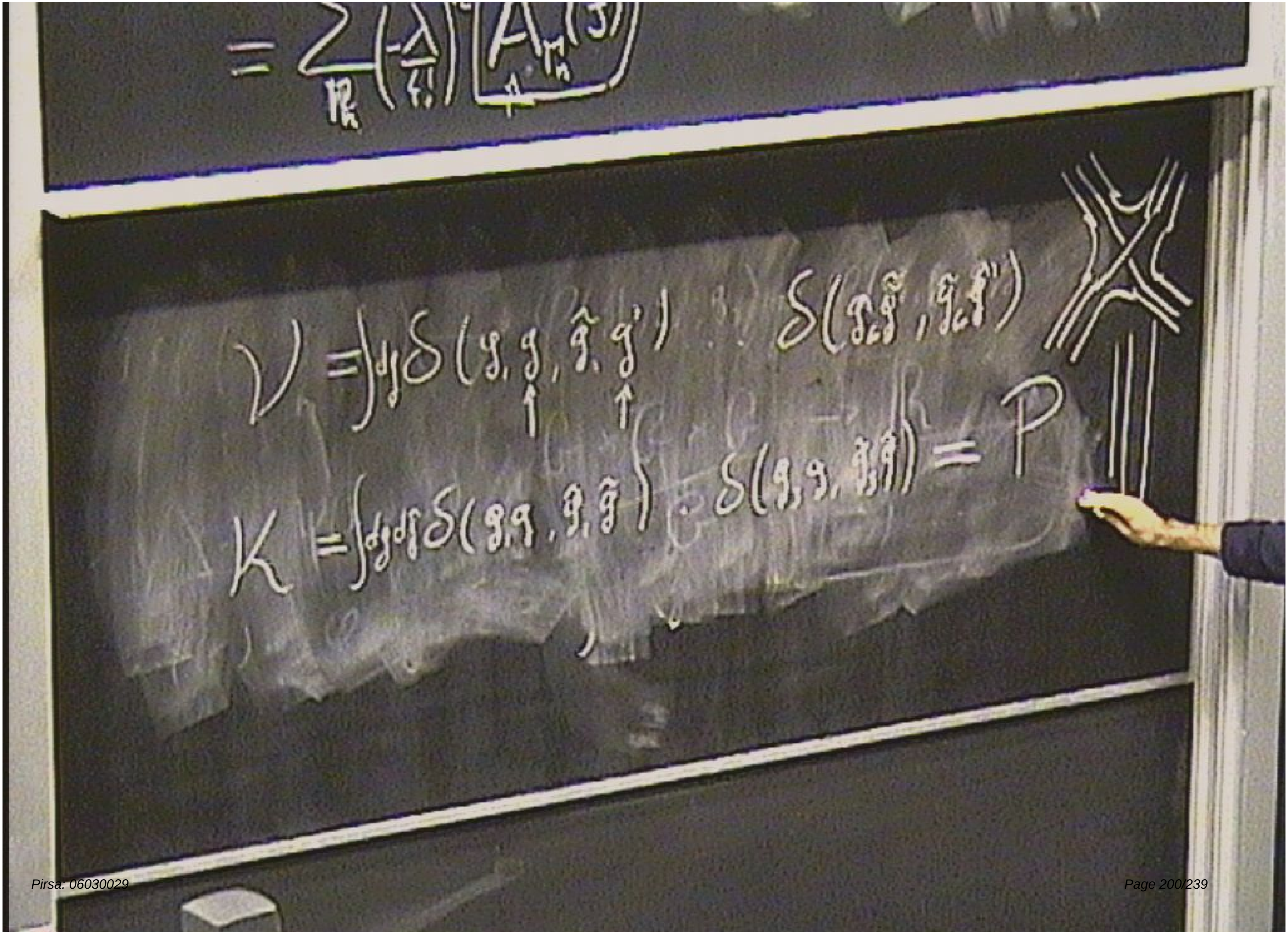
$Z = \int D\varphi \, e^{-\int d^4x \left[\frac{1}{2}\varphi(\partial^2+m^2)\varphi + \frac{\lambda}{4!}\varphi^4 + J\varphi\right]}$
$Z = \int_{-\infty}^{+\infty} dq \, e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq}$

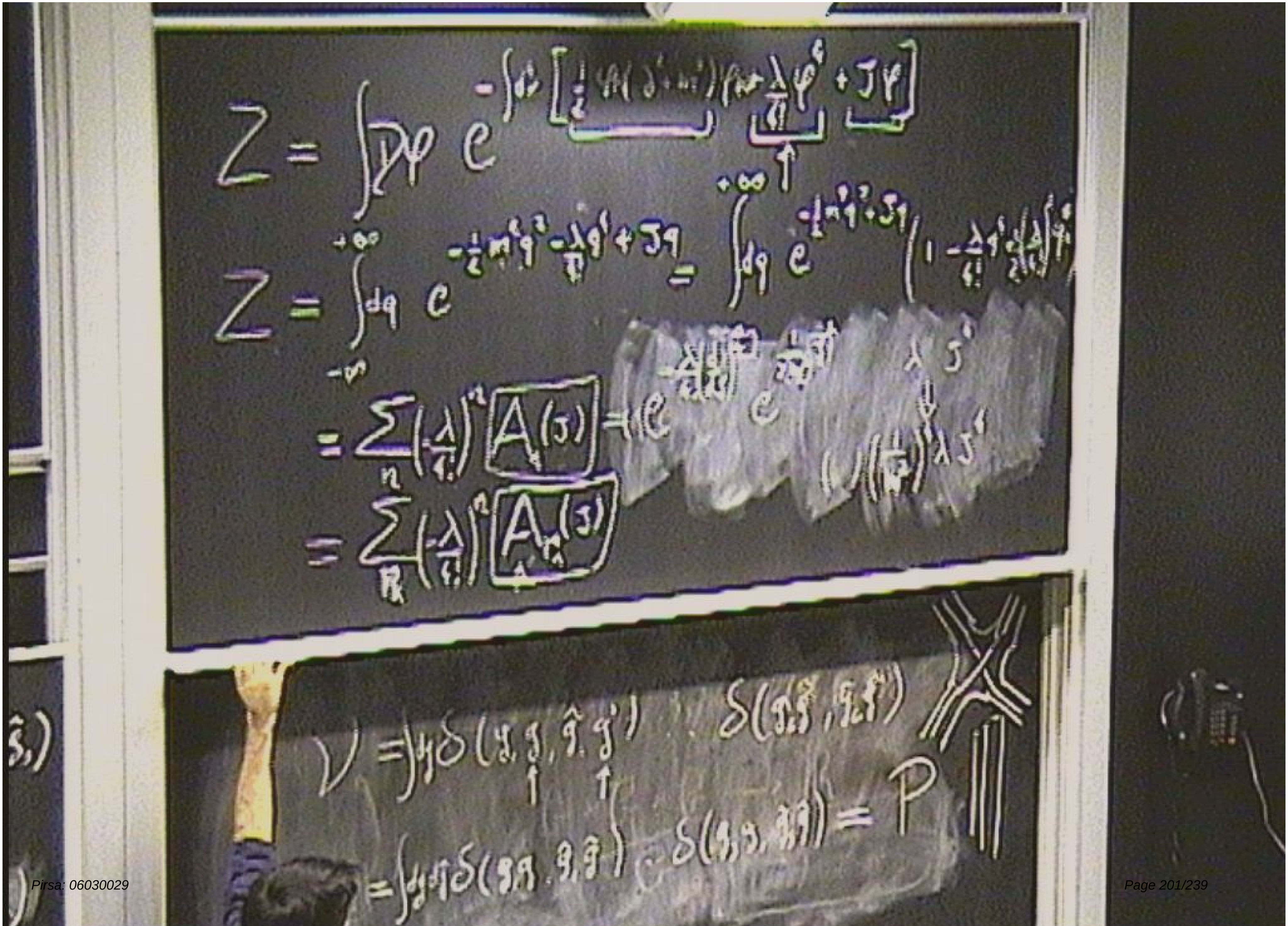

Z = ∫Dφ e^{-∫d^4x [½ φ(x)(∂² + m²)φ(x) + λ/4! φ⁴ + Jφ]}
Z = ∫_{-∞}^{+∞} dq e^{-½ m²q² - λ/4! q⁴ + Jq} = ∫_{-∞}^{+∞} dq e^{-½ m²q² + Jq} (1 - λ/4! q⁴ + ½ (λ/4!)² q⁸
= Σ_n (-λ/4!)^n A_n(J) = e^{-λ/4! (d/dJ)⁴} e^{(1/2m²) J²}
λ J⁴
( )(1/m²)⁴ λ J
= Σ_n (-λ/4!)^n A_n(J)

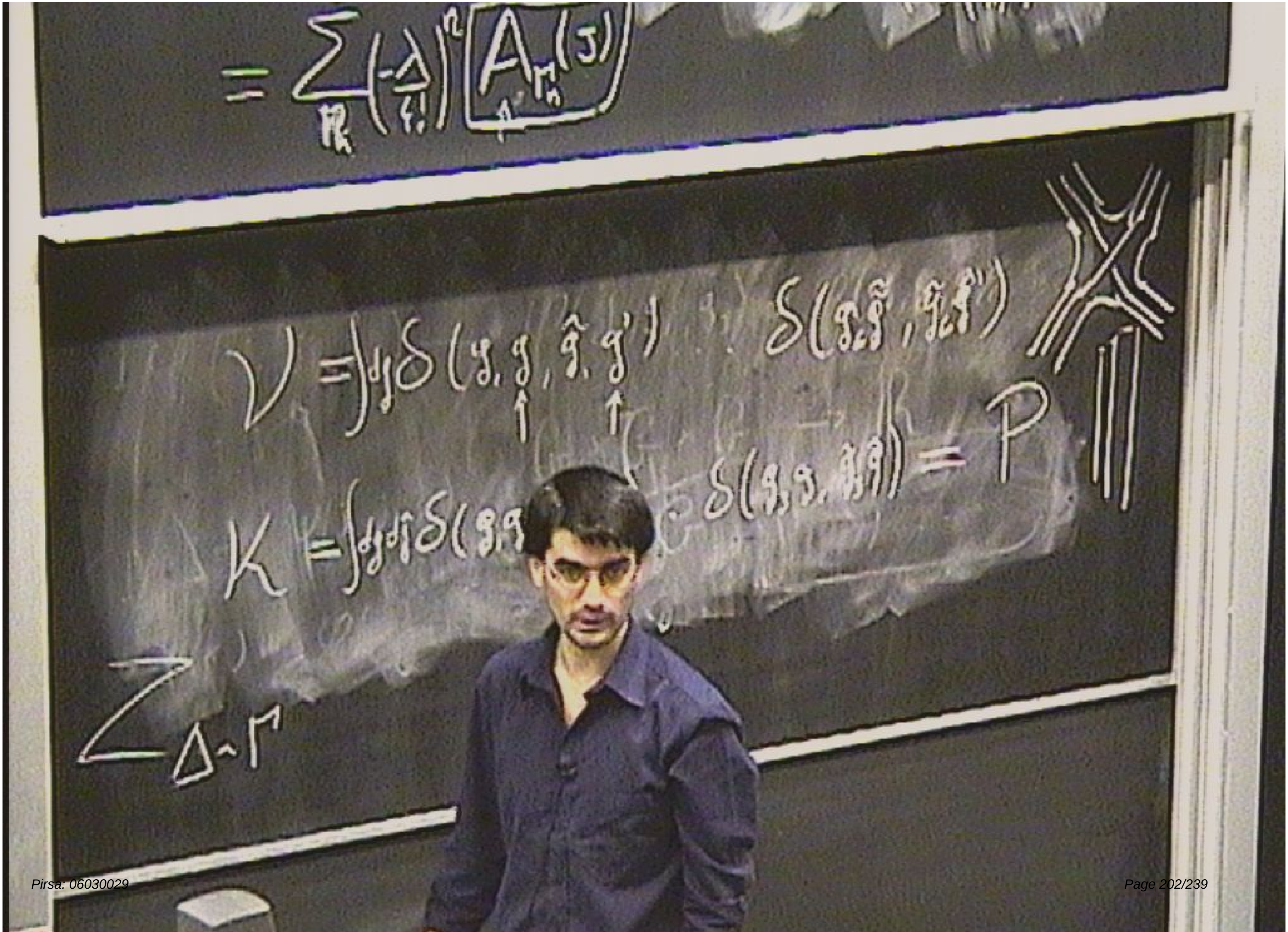


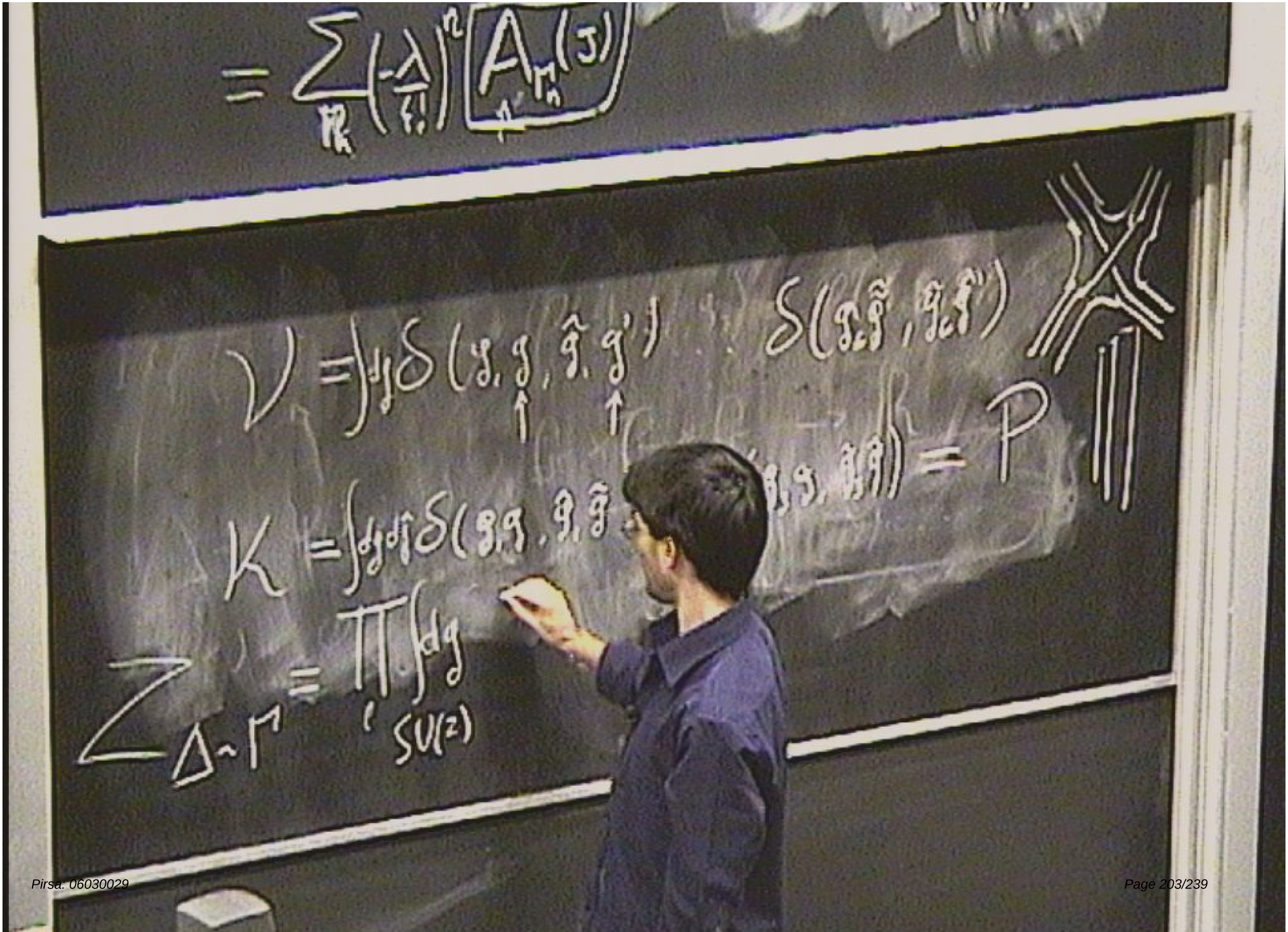










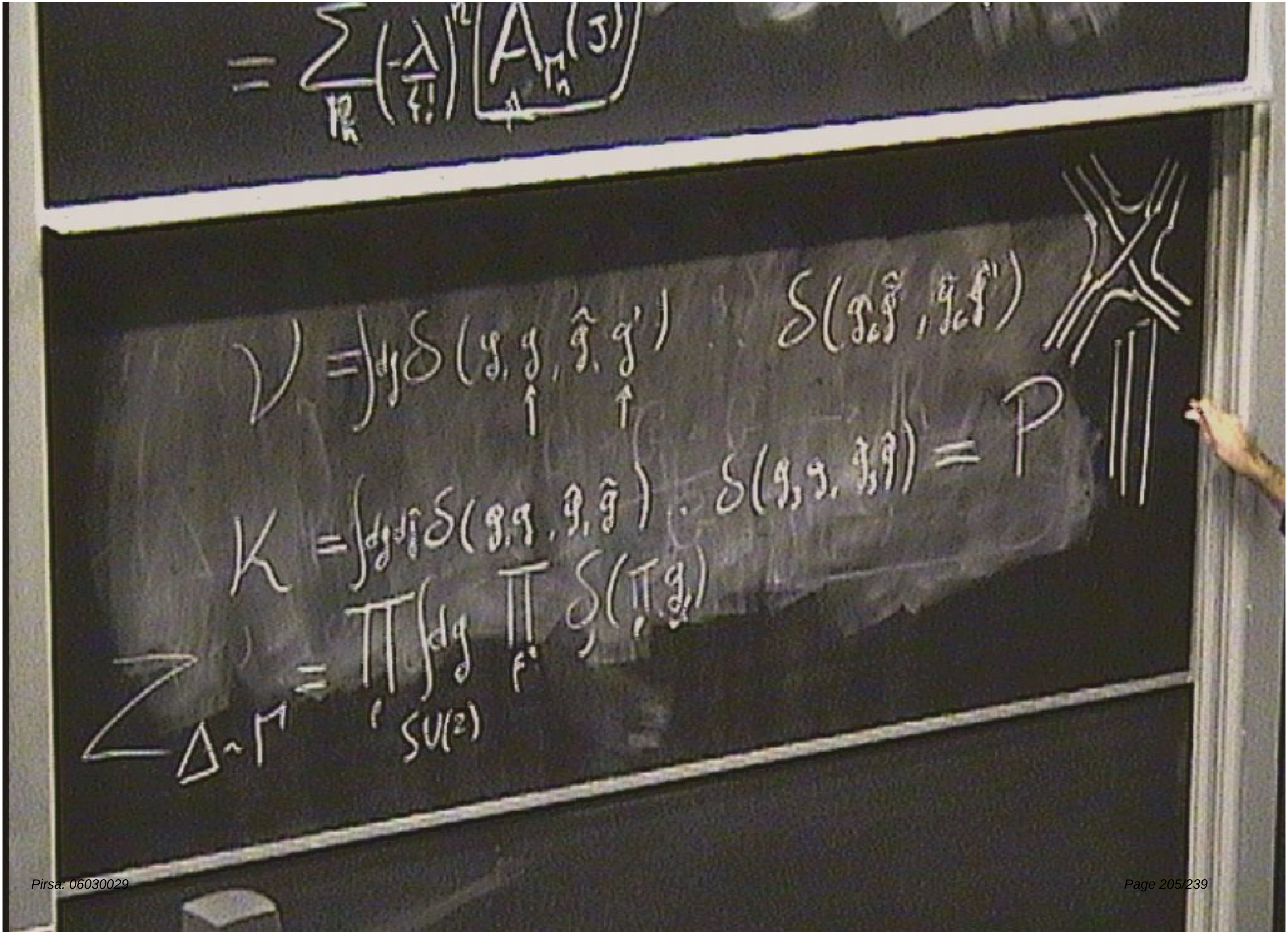


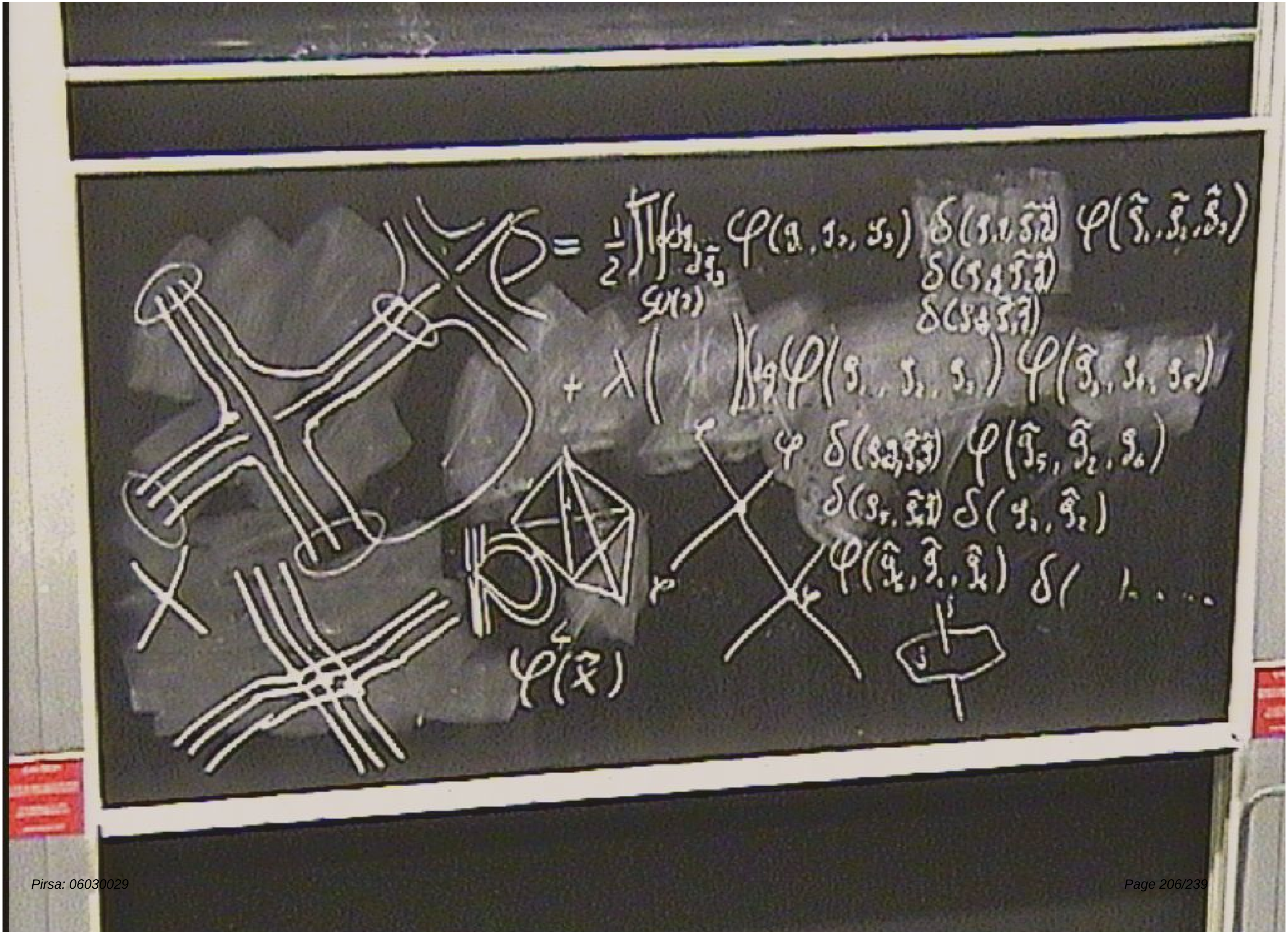
$$= \sum_{n} \left(-\frac{\lambda}{4!}\right)^n A_n(J)$$

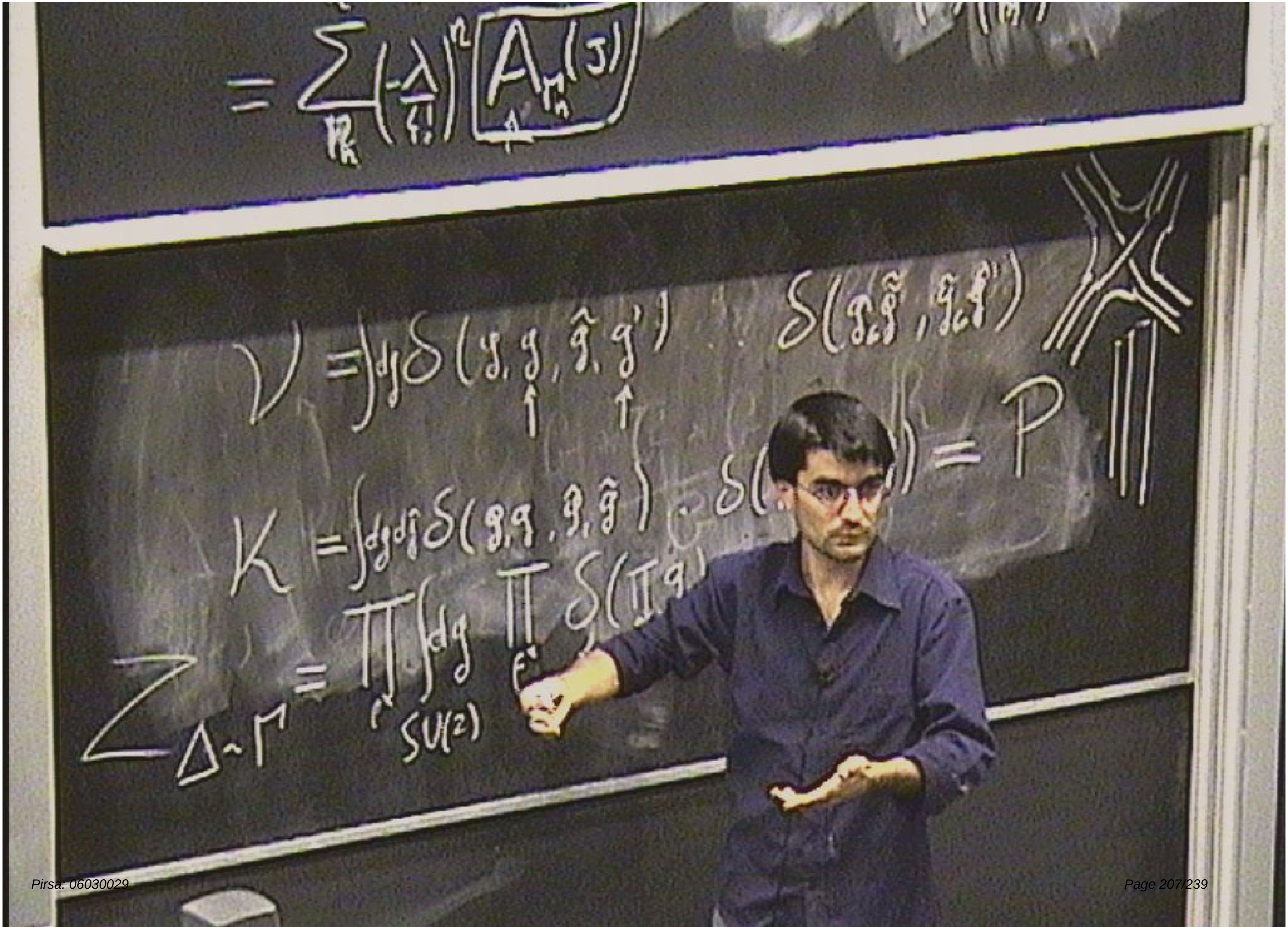
$$V = \int dg\, \delta(g_1 g, \hat{g}_1 g') \quad \delta(g_1 \hat{g}, g_2 \hat{g}')$$

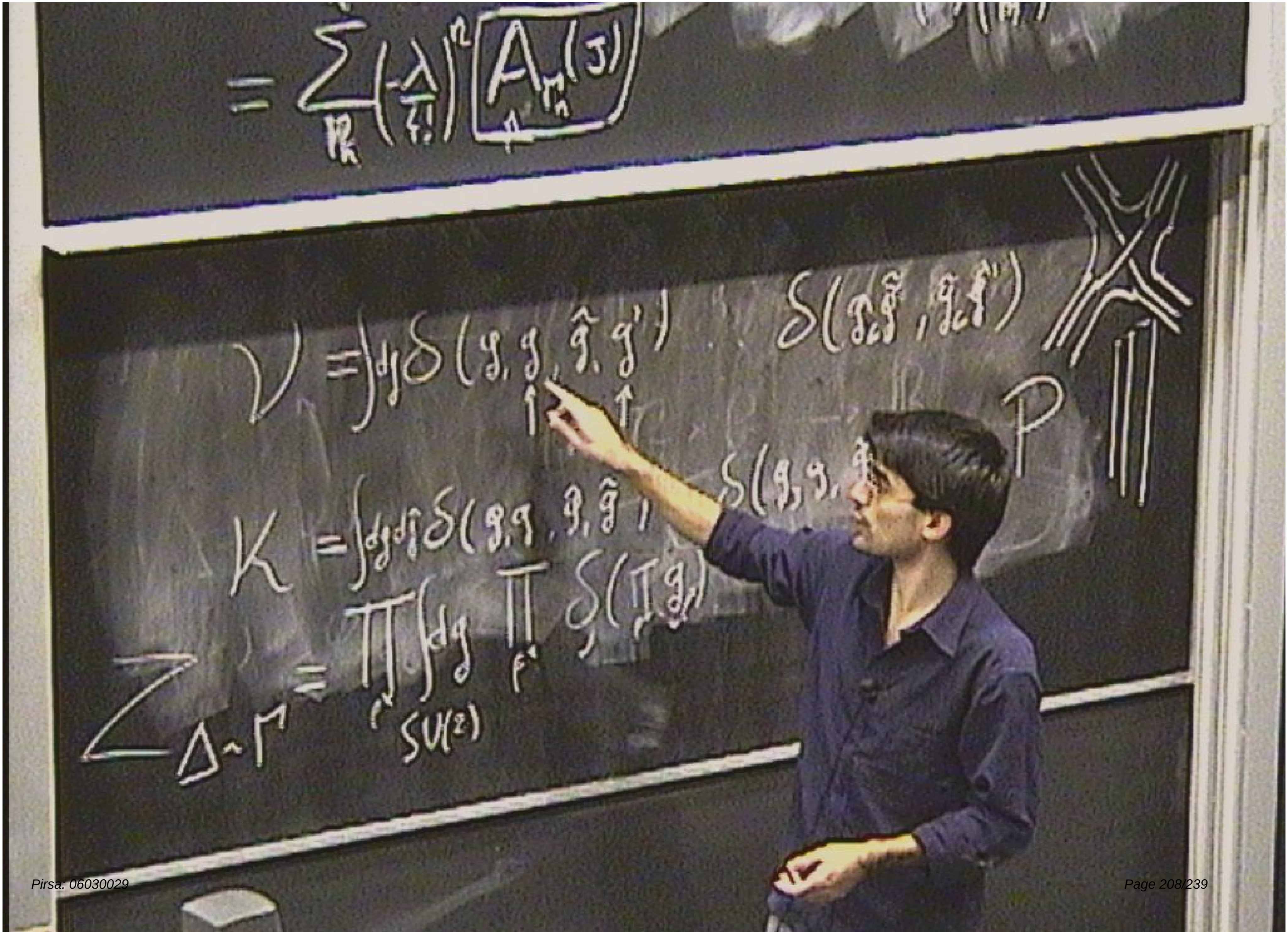
$$K = \int dg\, d\hat{g}\, \delta(g_1 g, \hat{g}_1 \hat{g}) \quad \delta(g_1 g, \hat{g}_1 \hat{g}) = P$$

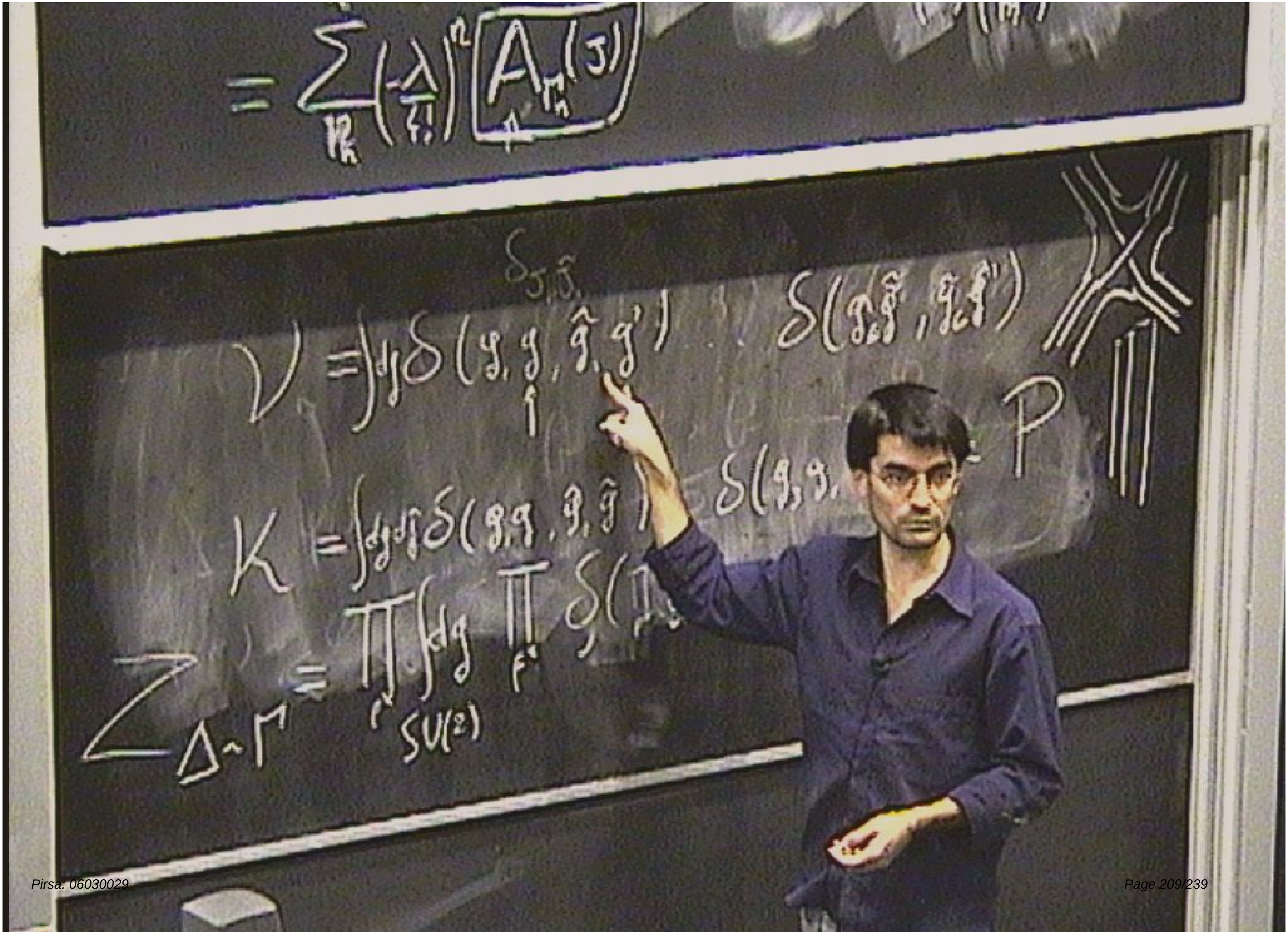
$$Z_{\Delta \sim \Gamma} = \prod_{\ell} \int_{SU(2)} dg \prod_{f}$$

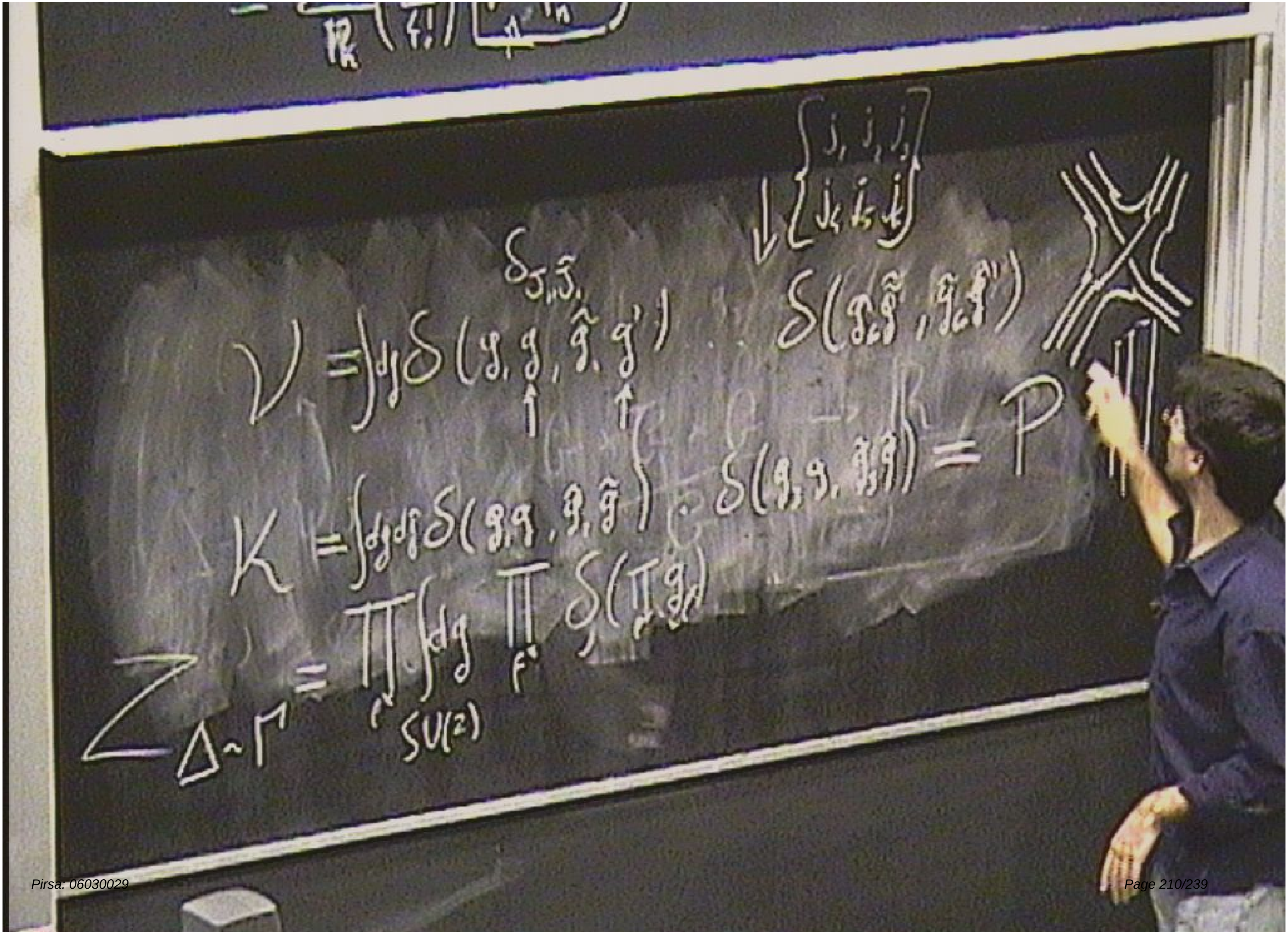

= Σ (-λ/4!)^n A_{Γ_n}(j)
V = ∫dg δ(g, g, ĝ, g')
δ(g ĝ, g ĝ')
= P
K = ∫dg dĝ δ(g, g, g, ĝ) · δ(g, g, g, g)
Z_{Δ~Γ} = ∏ ∫_{SU(2)} dg ∏_f δ(∏ g)

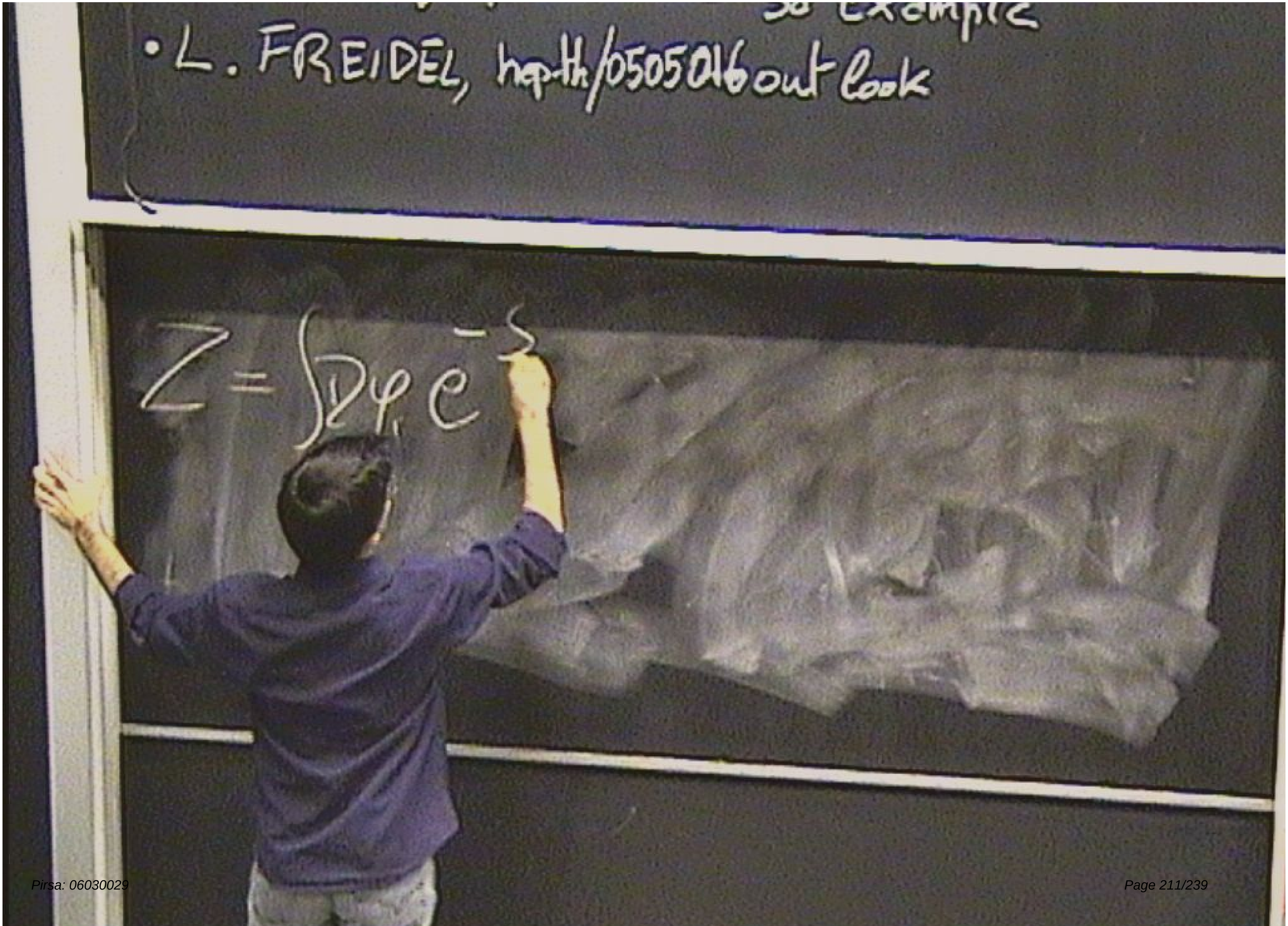


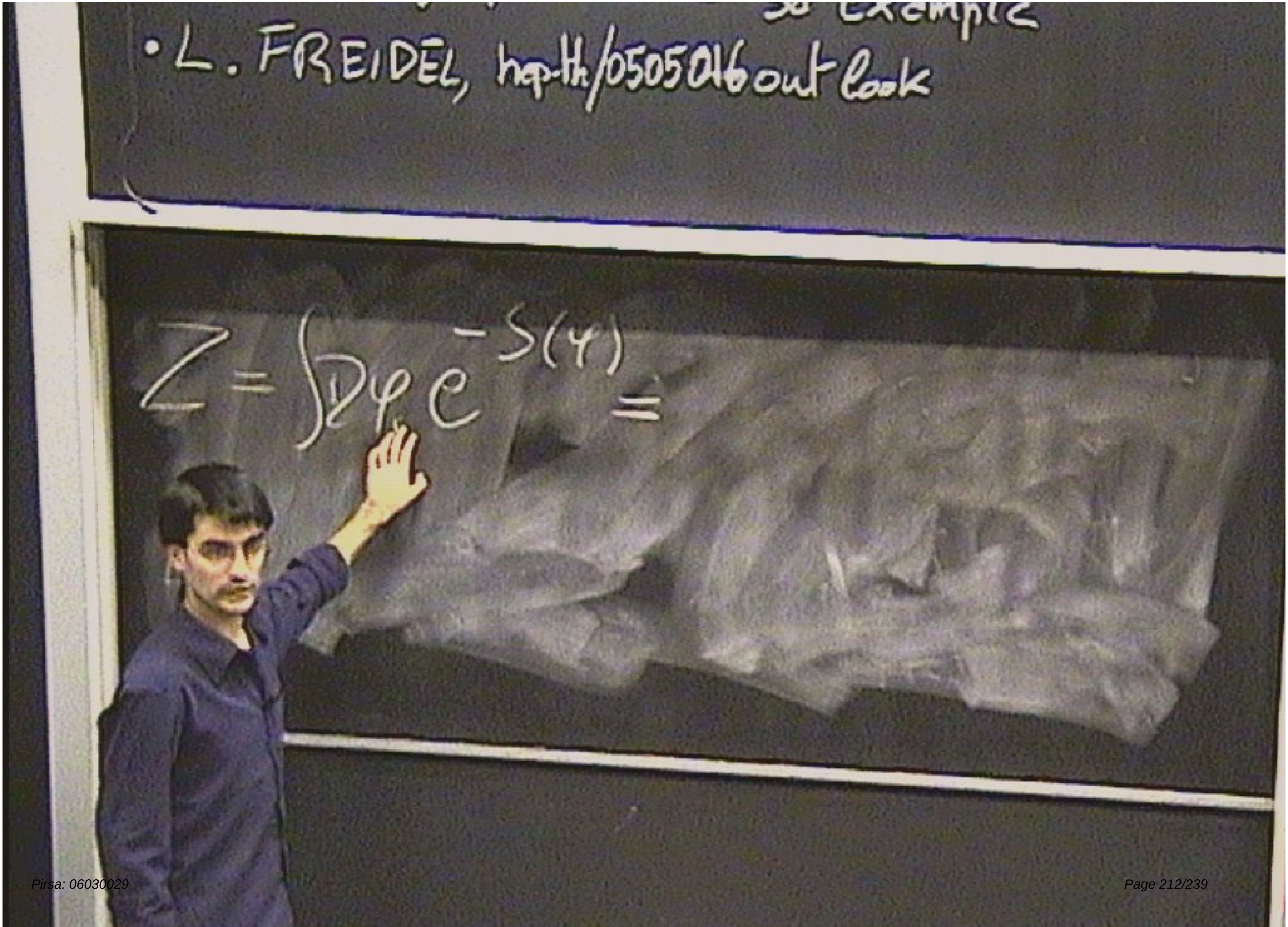


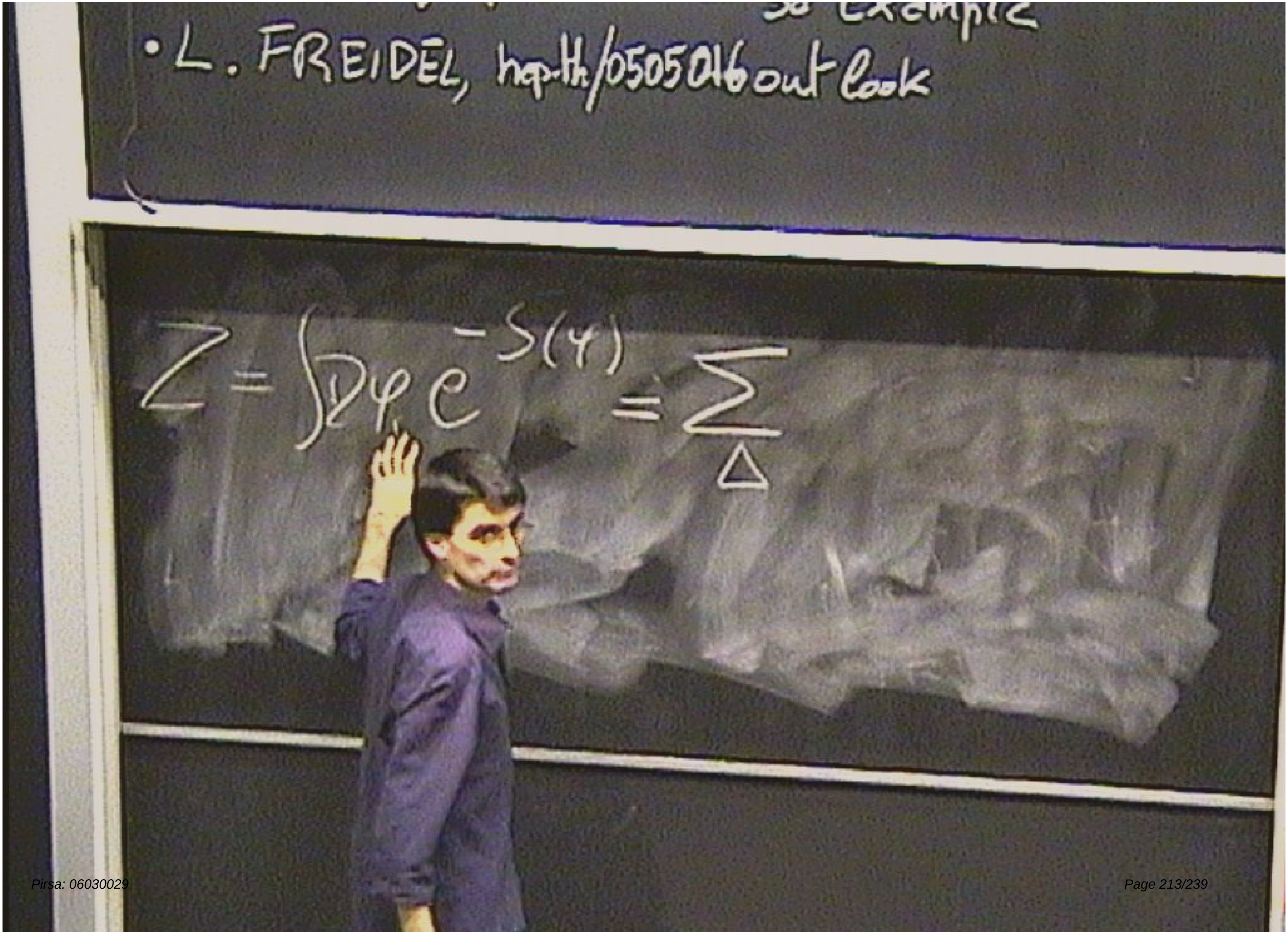

SU(2)
P

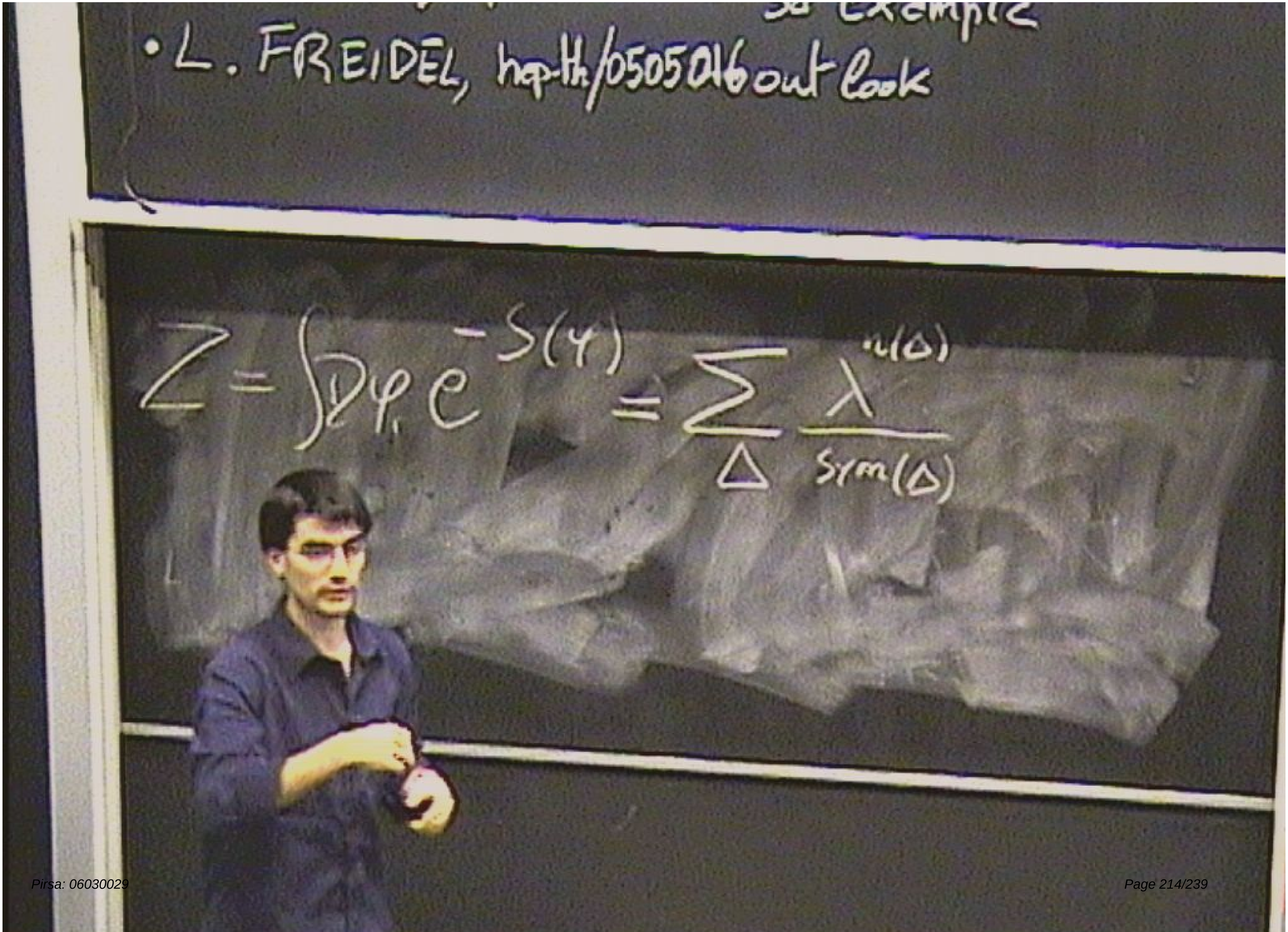


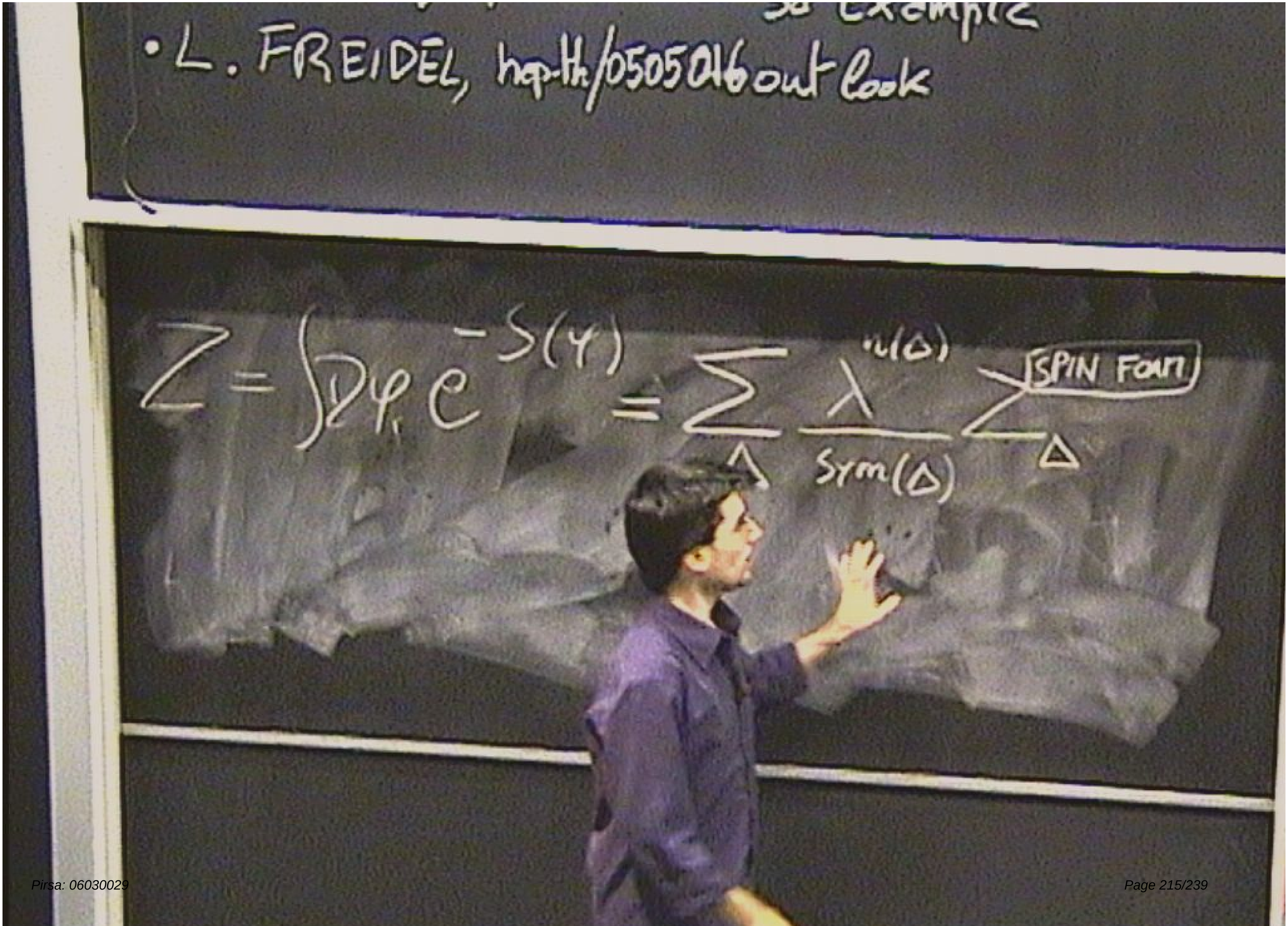

V =
K =
= P
SU(2)

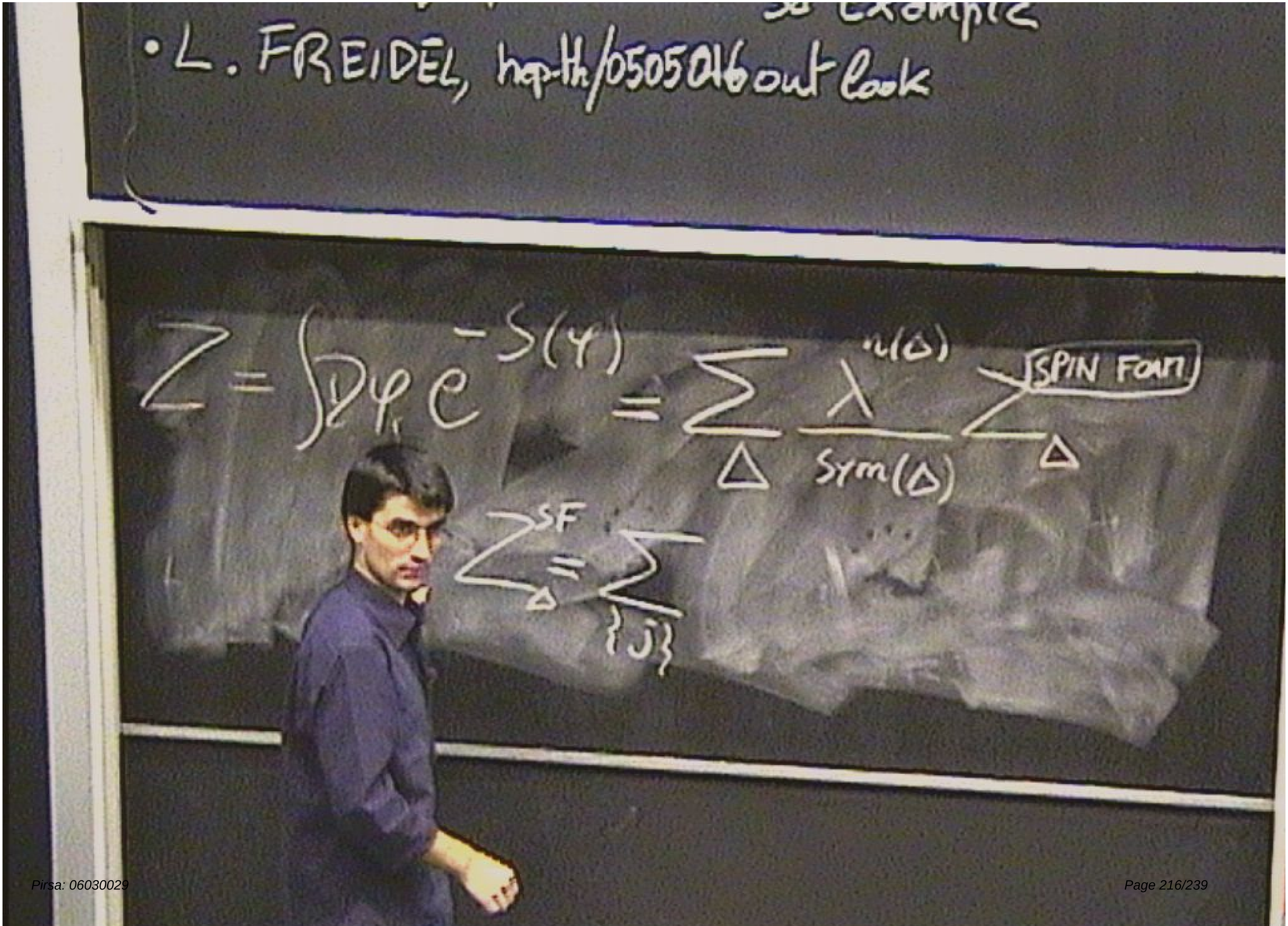

• L. FREIDEL, hep-th/0505016 outlook
Z = ∫Dφ e^{-S}

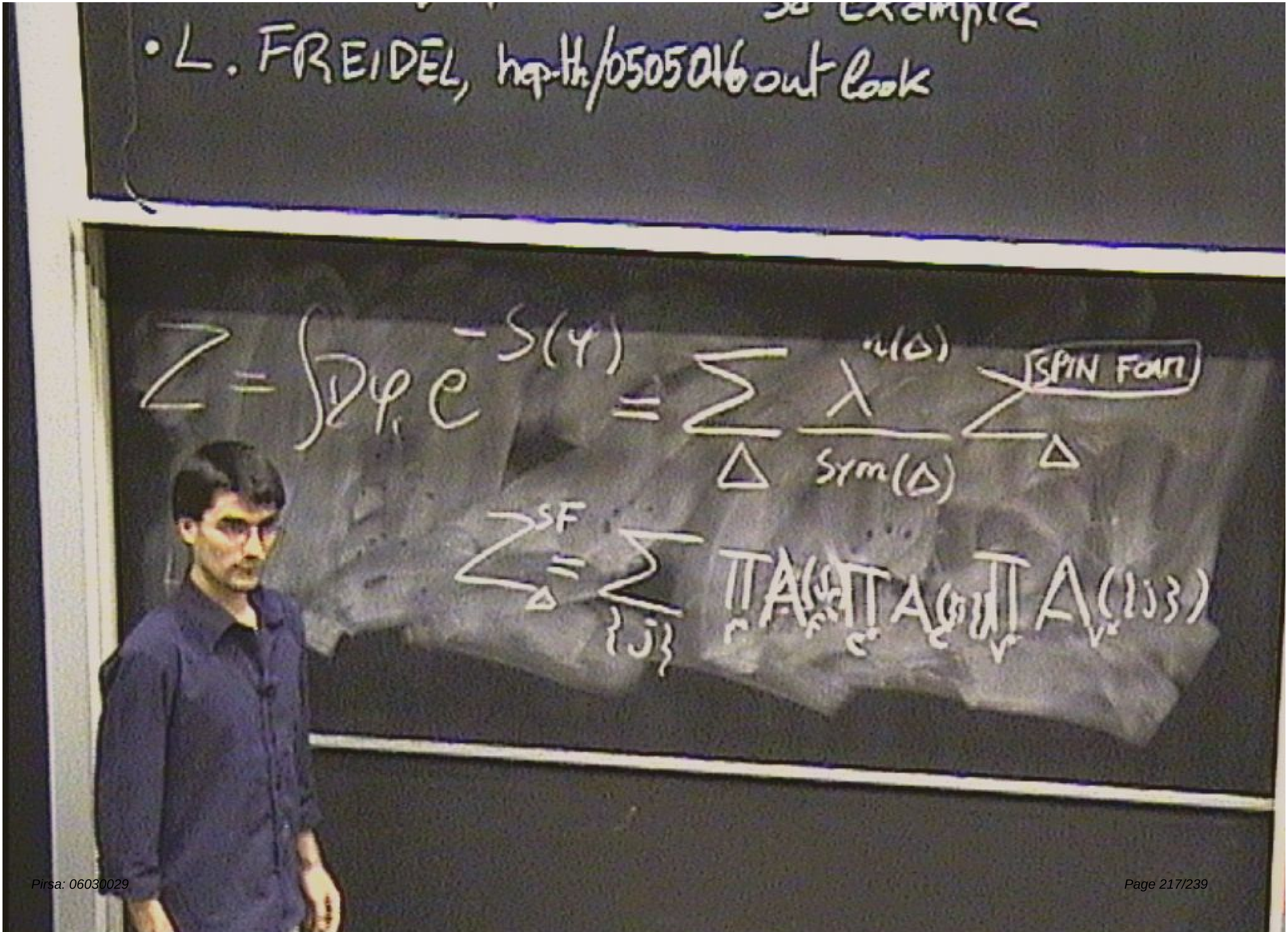

• L. FREIDEL, hep-th/0505016 outlook
Z = ∫Dφ e^{-S(φ)} =

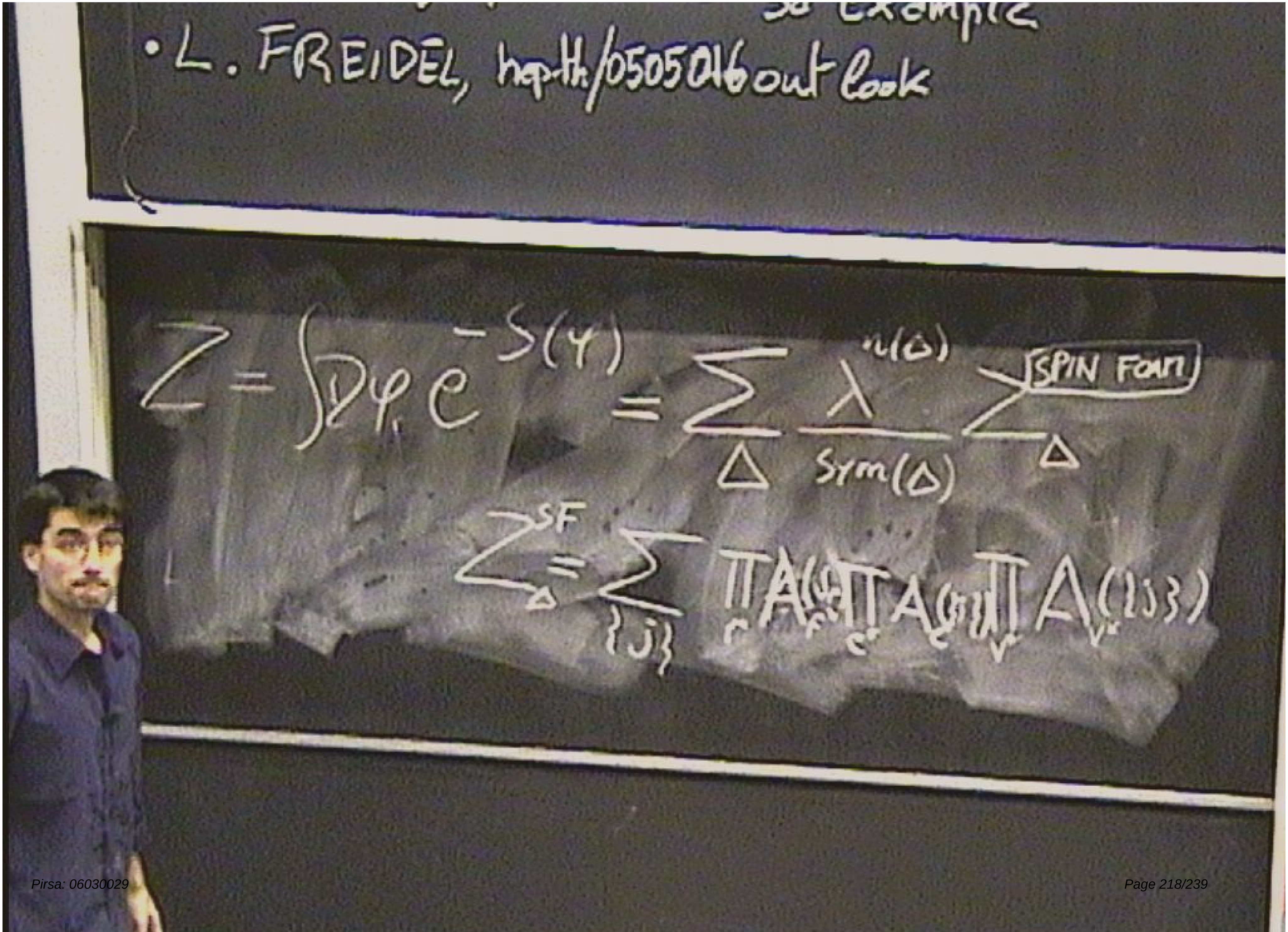

• L. FREIDEL, hep-th/0505016 outlook
$Z = \int D\varphi \, e^{-S(\varphi)} = \sum_{\Delta}$

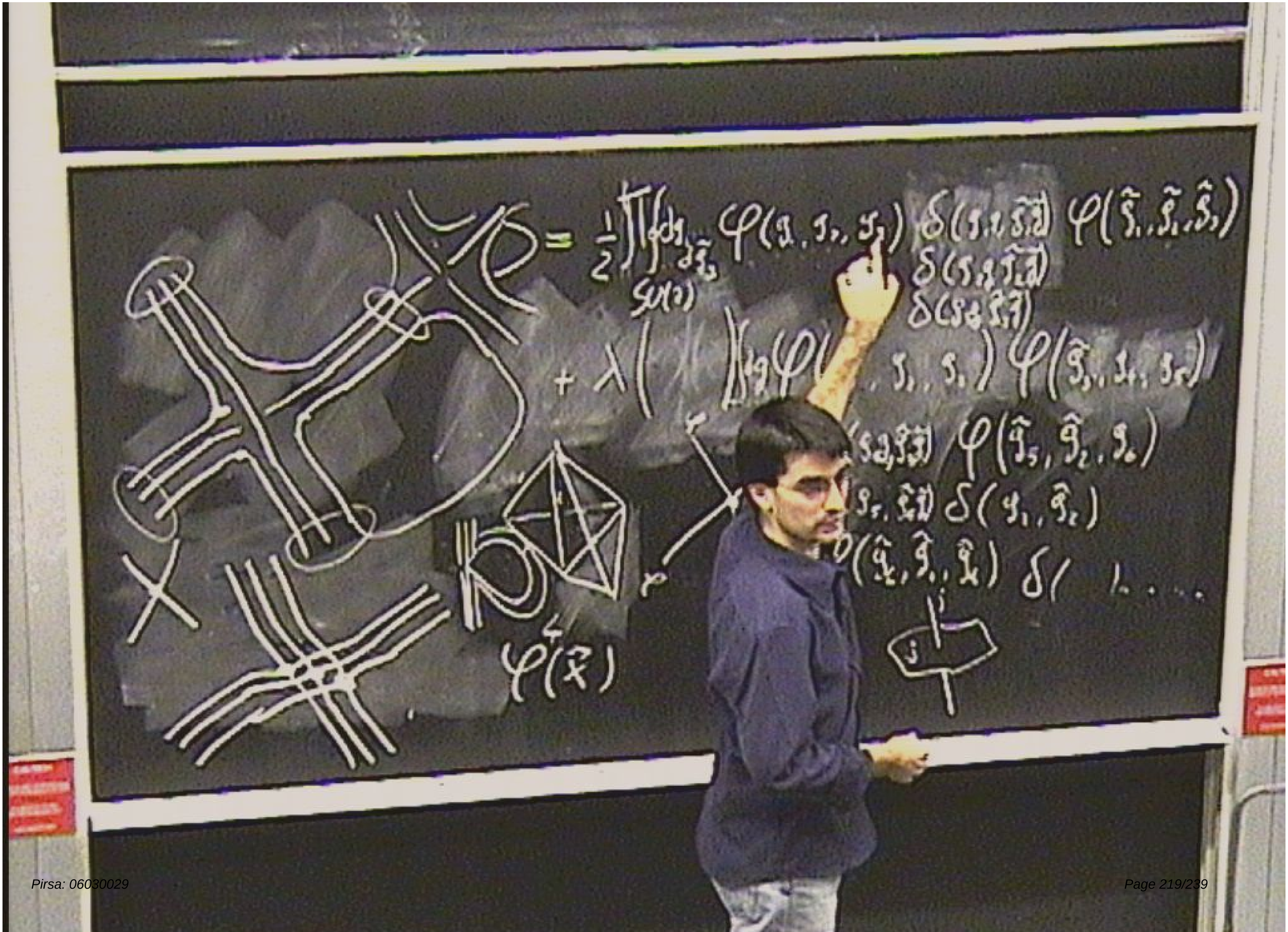

• L. FREIDEL, hep-th/0505016 outlook
$Z = \int D\varphi \, e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\mathrm{Sym}(\Delta)}$

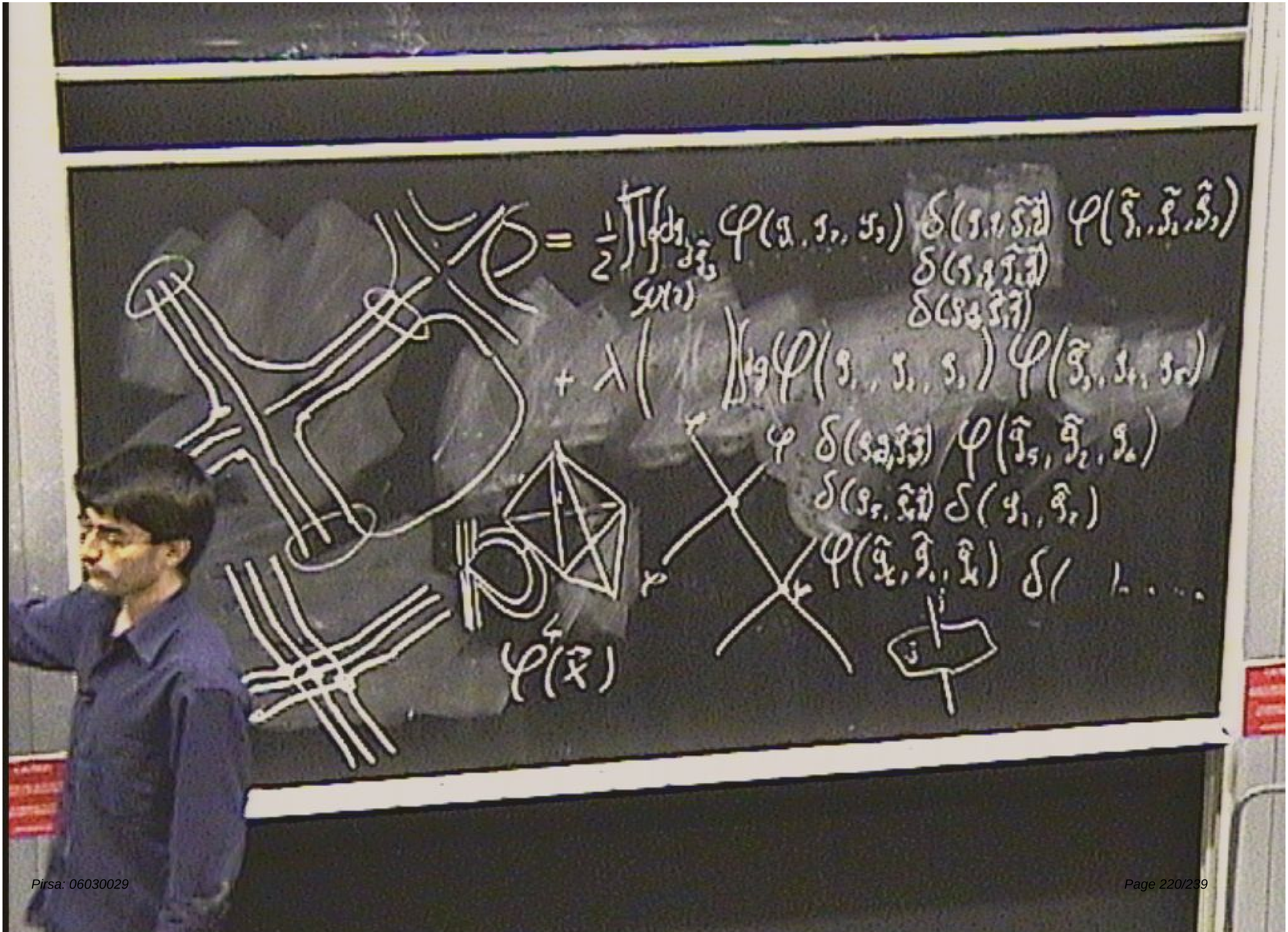

• L. FREIDEL, hep-th/0505016 outlook
$Z = \int D\varphi \, e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\mathrm{Sym}(\Delta)} \sum_{\Delta}$
SPIN Foam

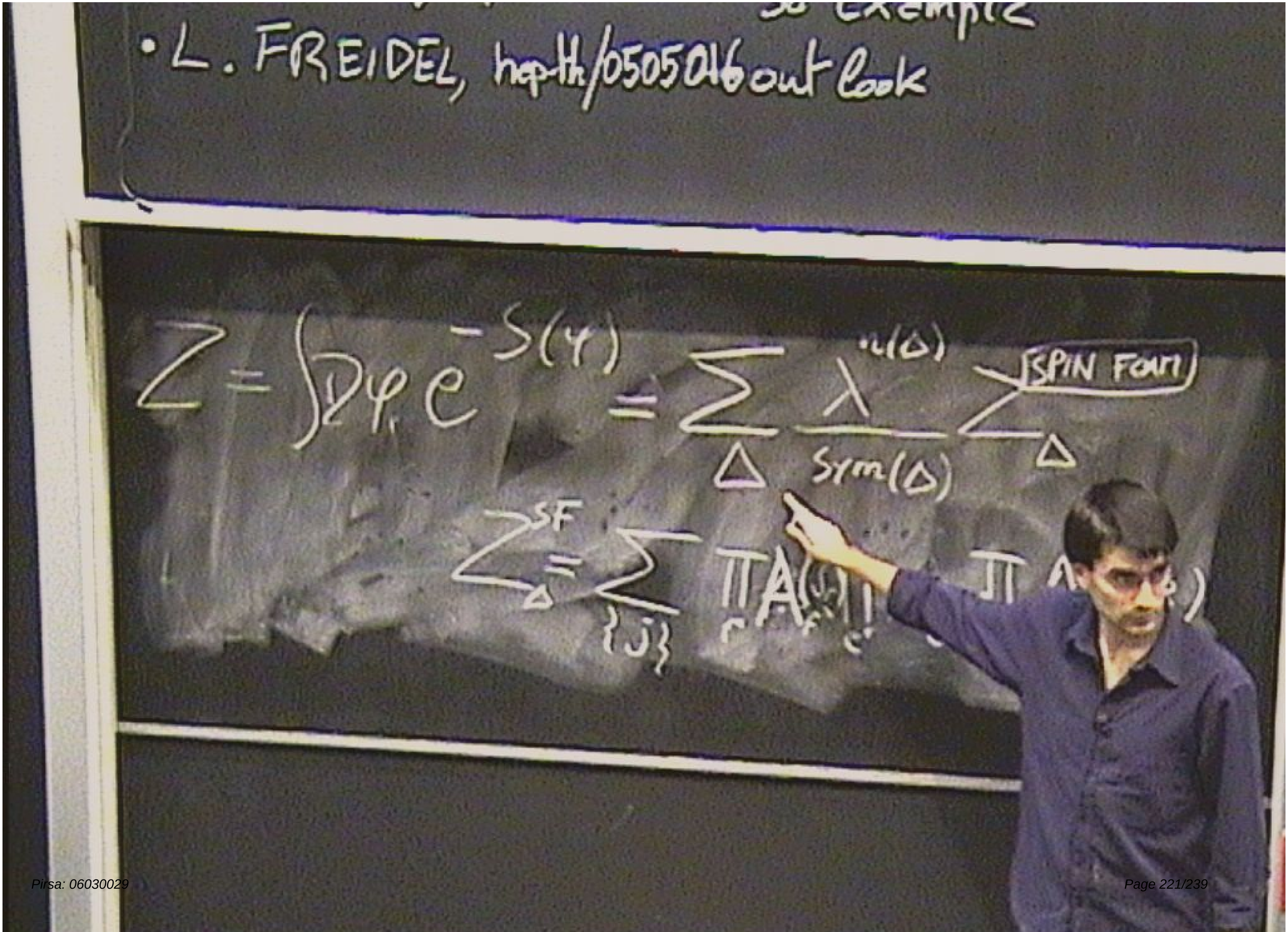

• L. FREIDEL, hep-th/0505016 out look
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / sym(Δ) Σ_Δ
SPIN FOAM
Σ_Δ^{SF} = Σ_{j}

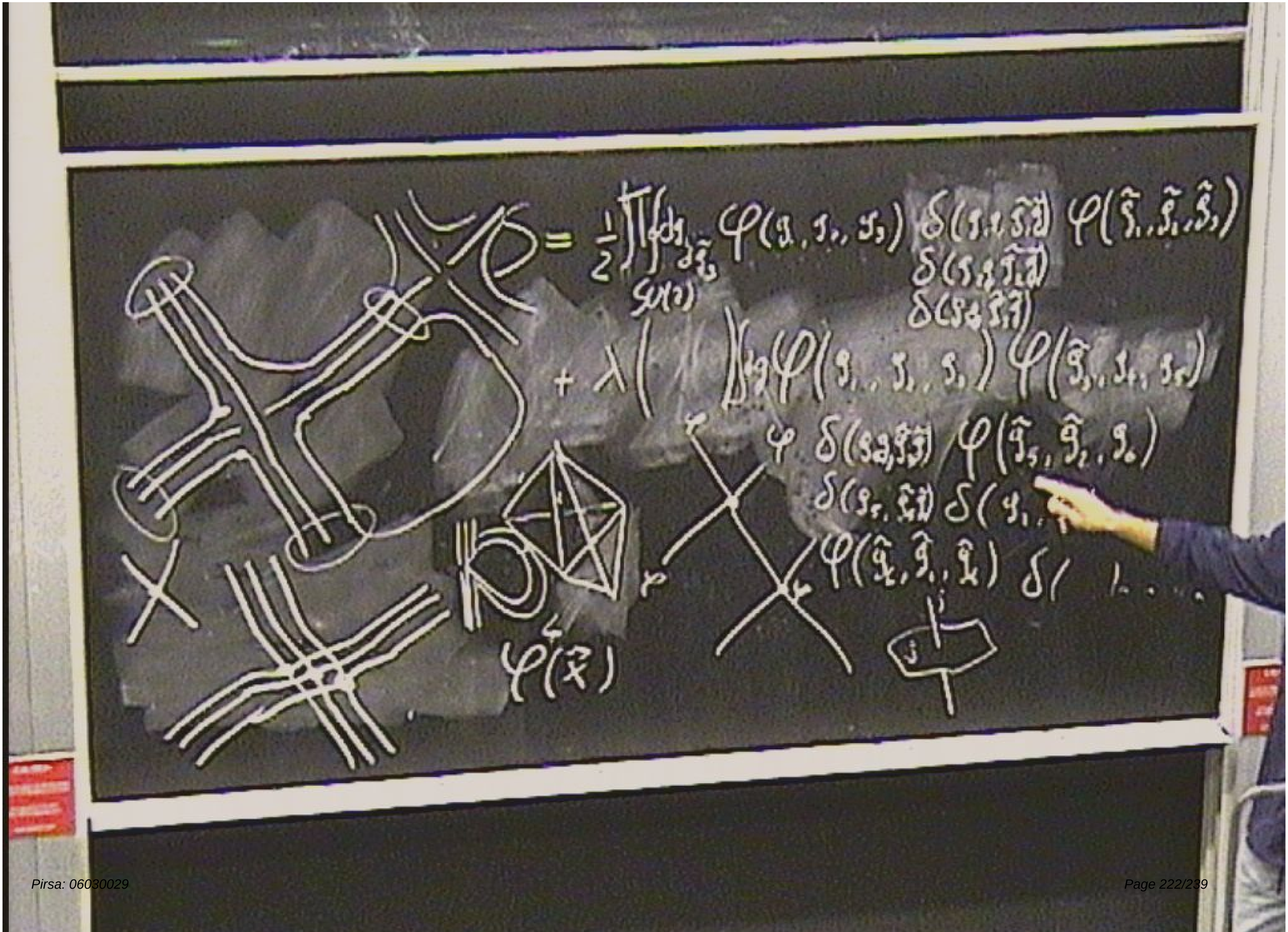

• L. FREIDEL, hep-th/0505016 outlook
SPIN Foam
Sym(Δ)

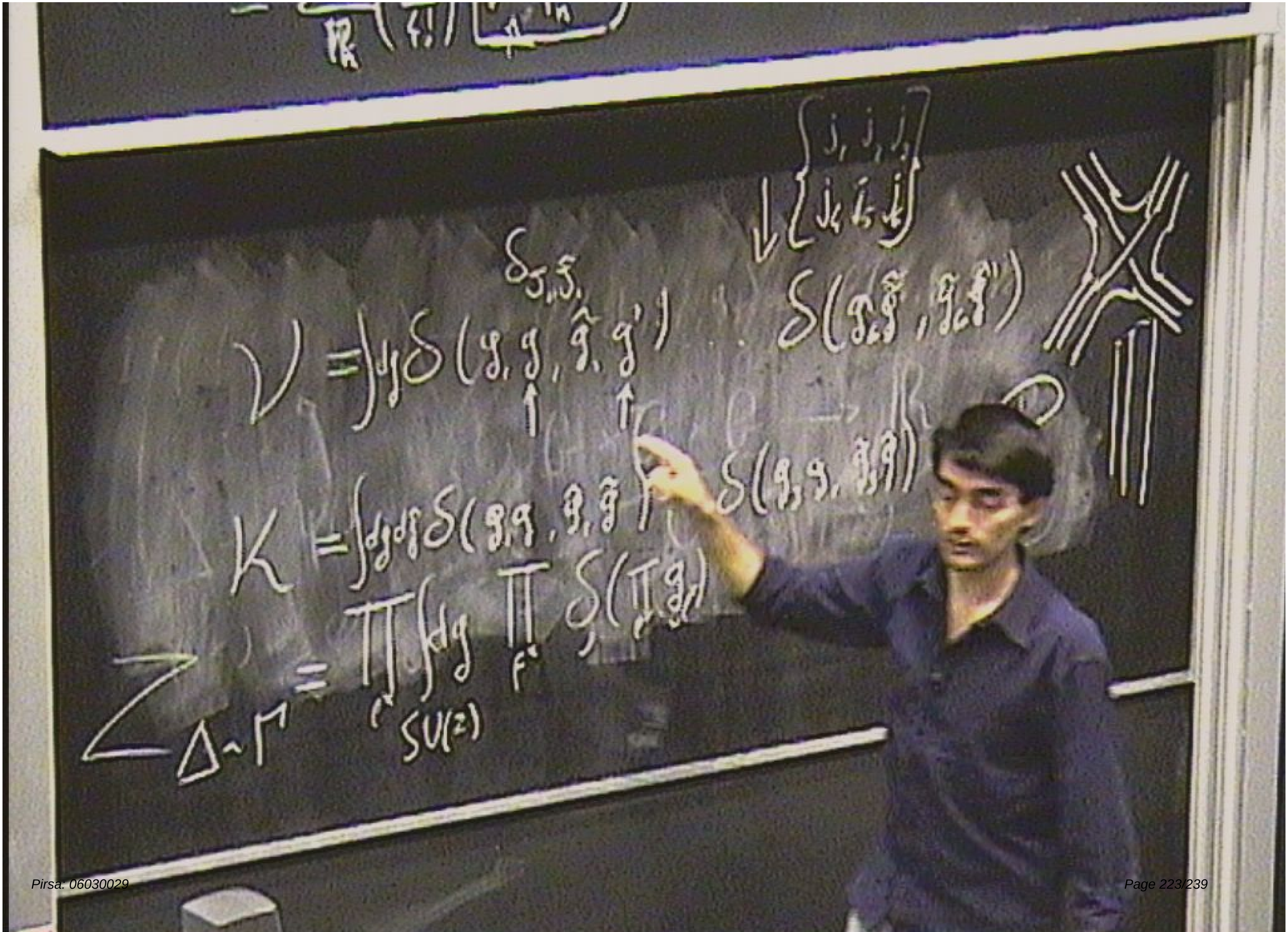

• L. FREIDEL, hep-th/0505016 outlook
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) Σ_Δ^{SPIN FOAM}
Σ_Δ^{SF} = Σ_{j} ∏_f A_f(j) ∏_e A_e(j) ∏_v A_v(j)

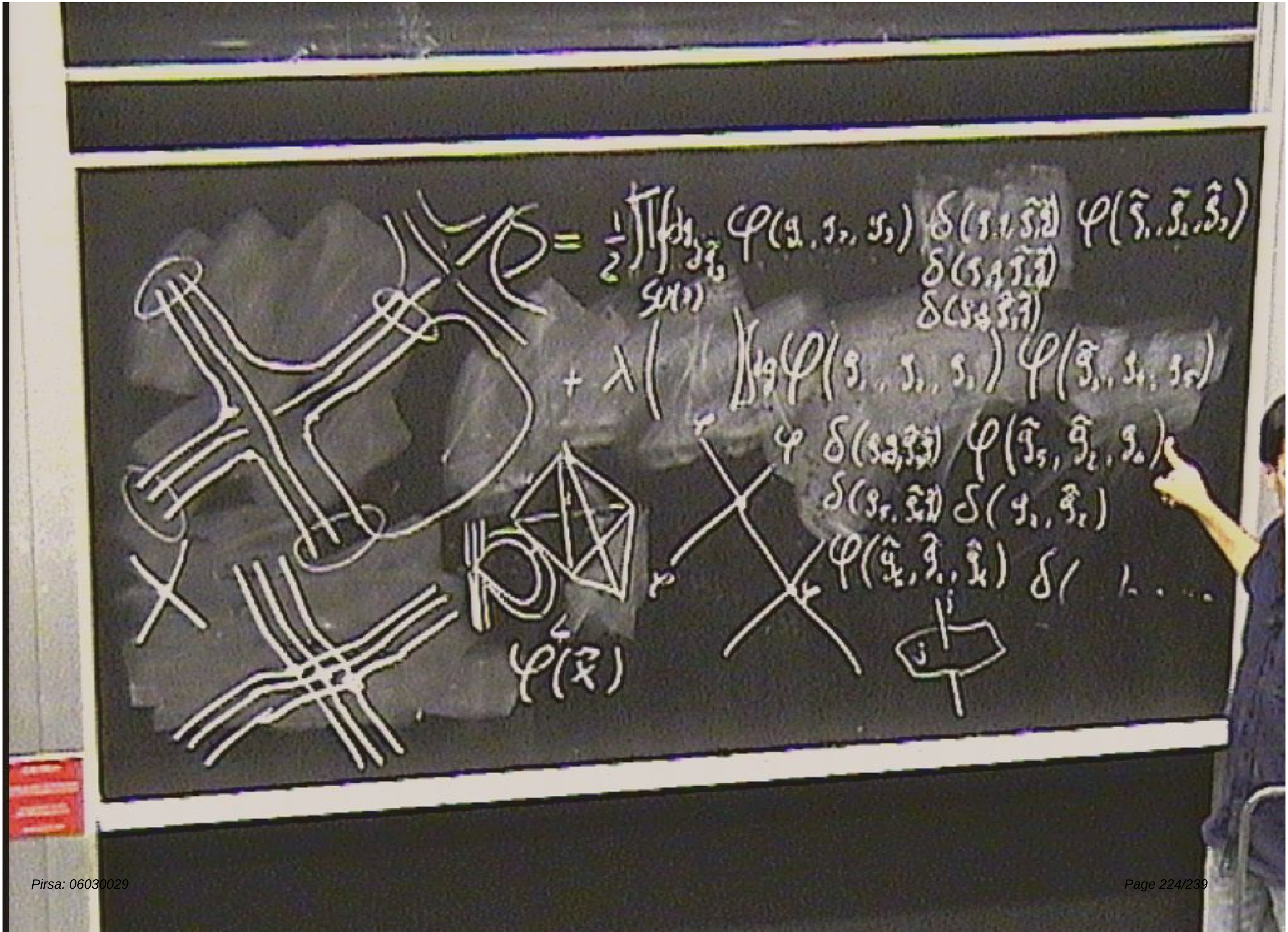





• L. FREIDEL, hep-th/0505016 outlook
SPIN Foam
Sym(Δ)
SF
{j}



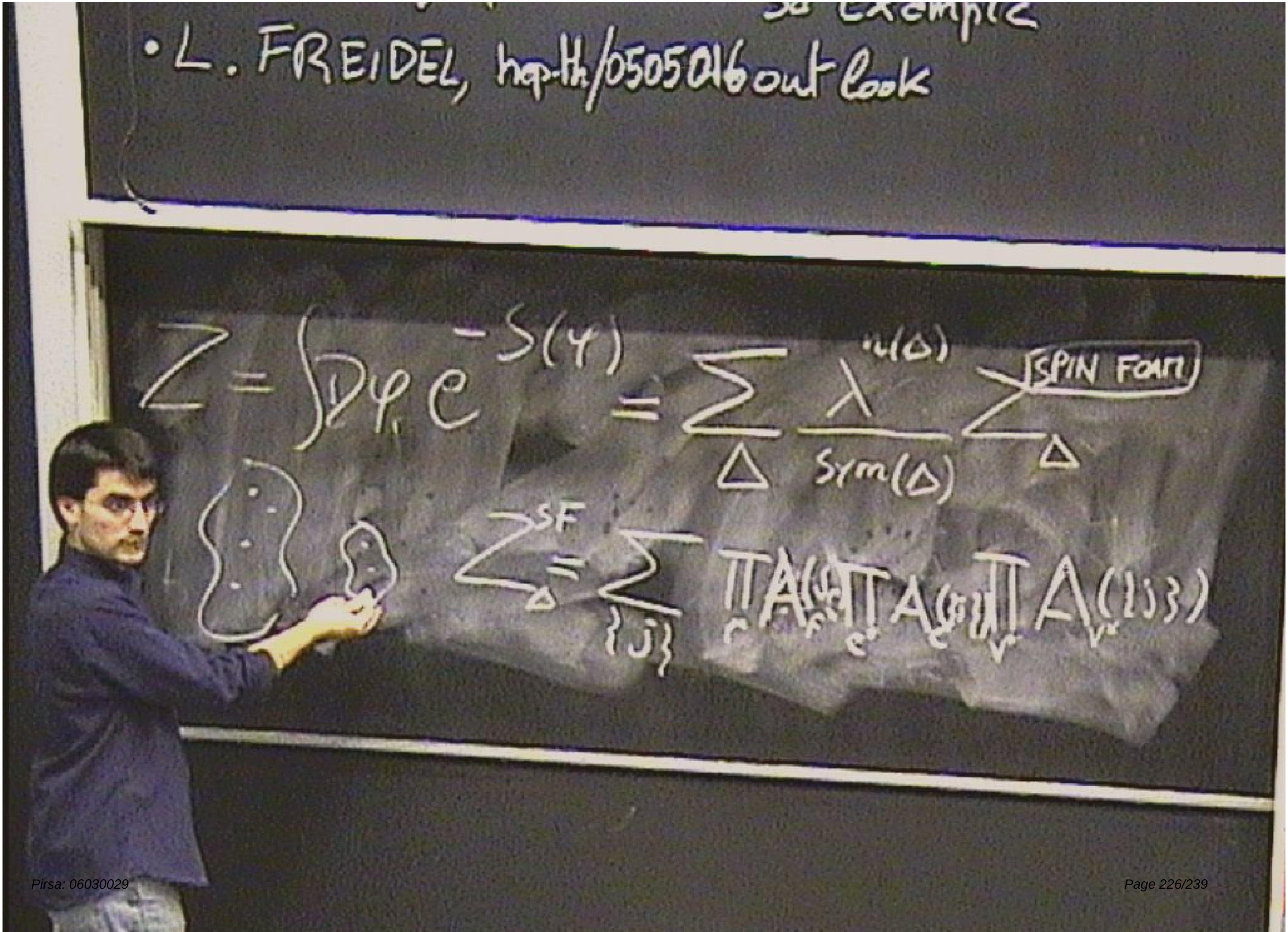


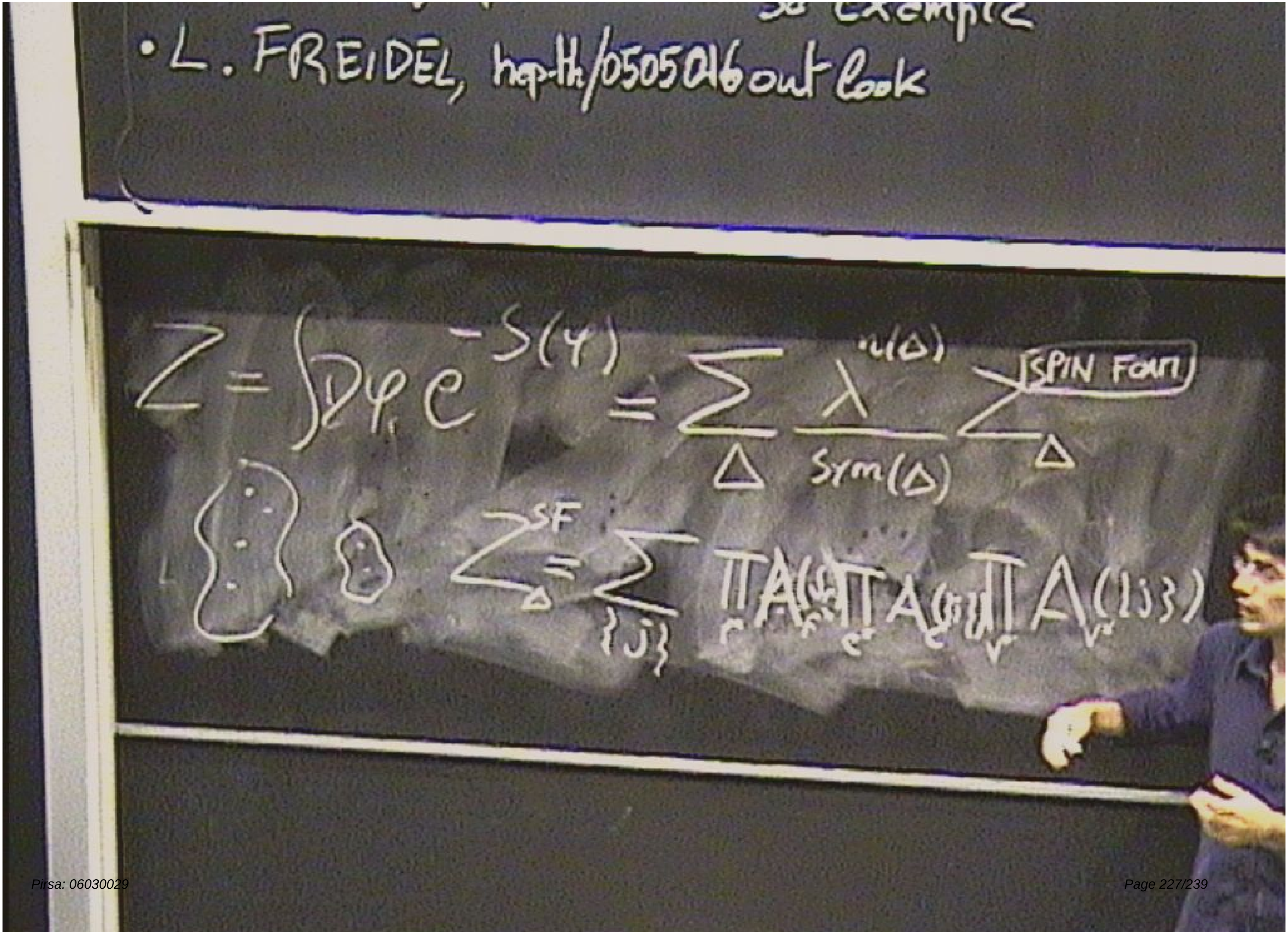


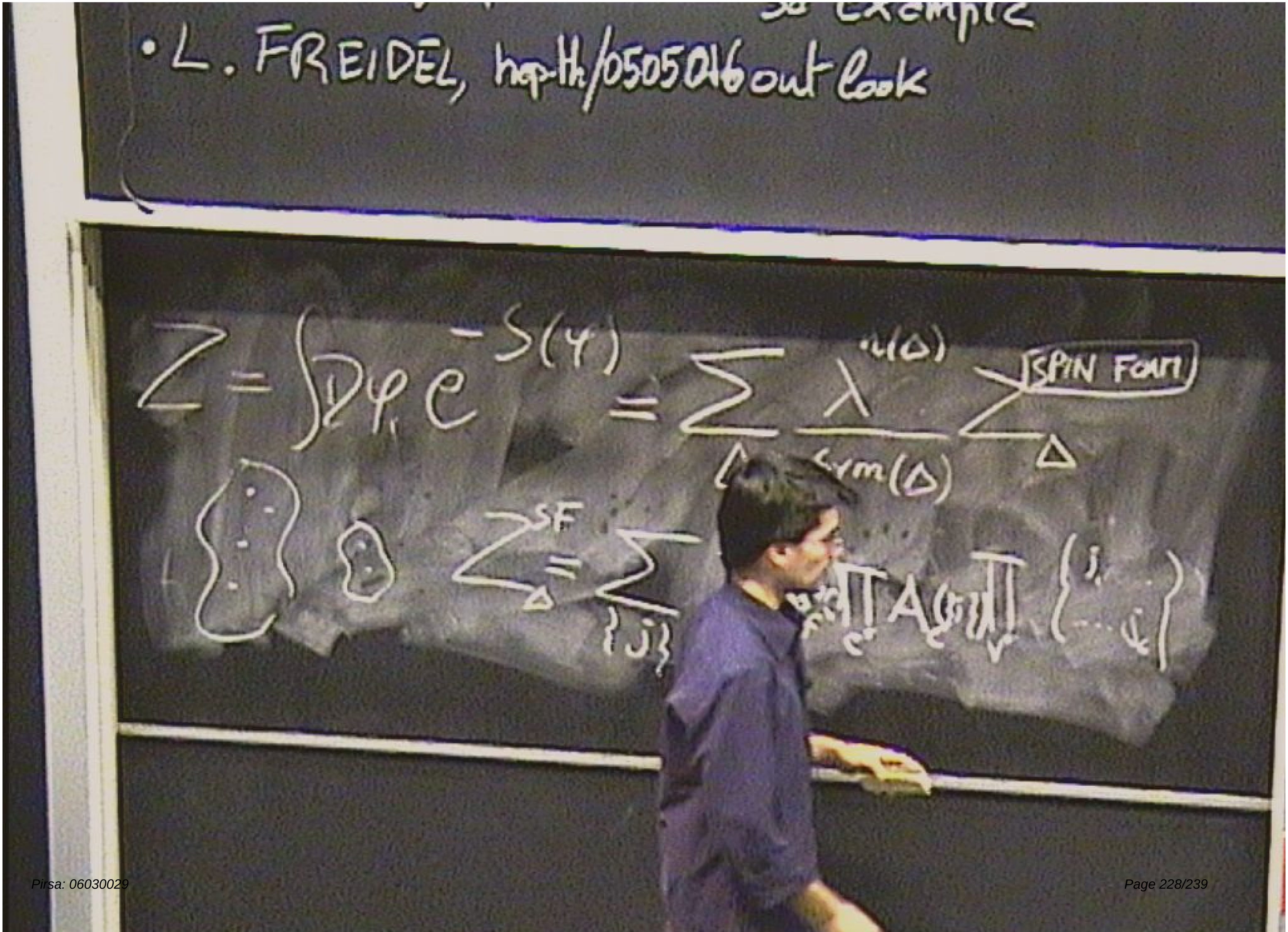
• L. FREIDEL, hep-th/0505016 outlook

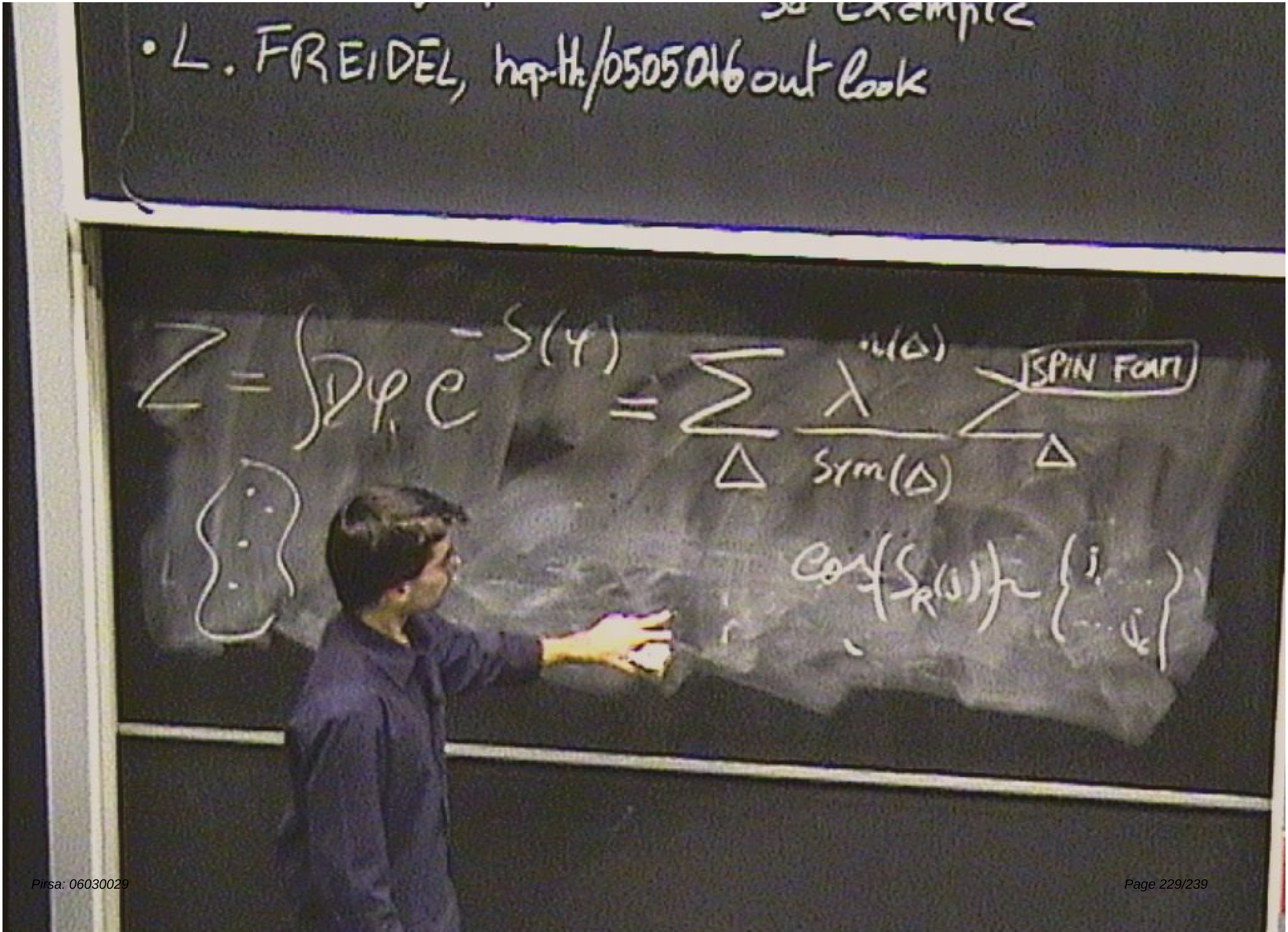
$$Z = \int \mathcal{D}\varphi \, e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{v(\Delta)}}{\mathrm{Sym}(\Delta)} \sum_{\Delta}^{\text{SPIN FOAM}}$$

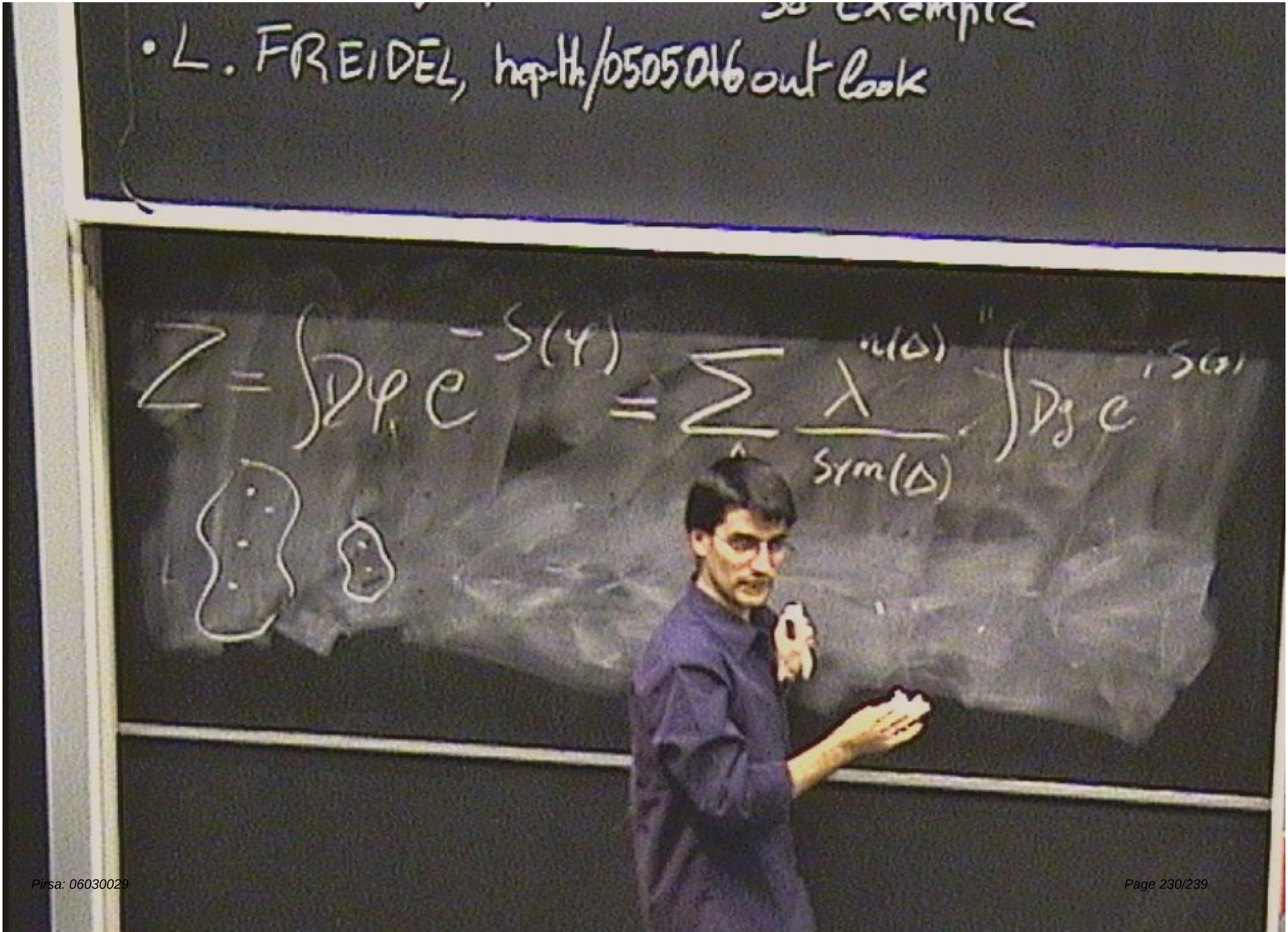
$$\sum_{\Delta}^{SF} = \sum_{\{j\}} \prod_{f} A(j_f) \prod_{e} A(j_e) \prod_{v} A(\{j\})$$

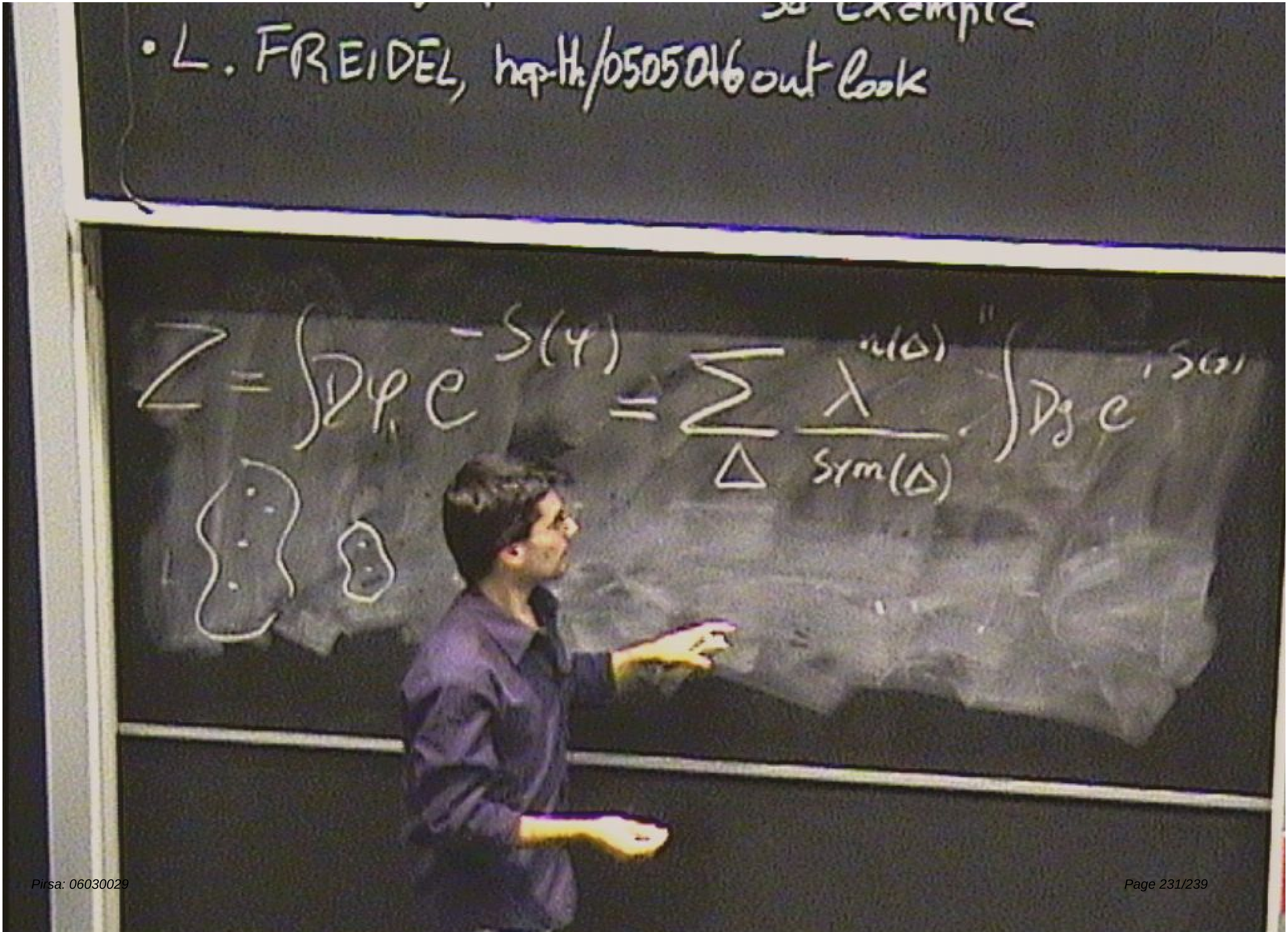

• L. FREIDEL, hep-th/0505016 outlook
SPIN Foam
Sym(Δ)

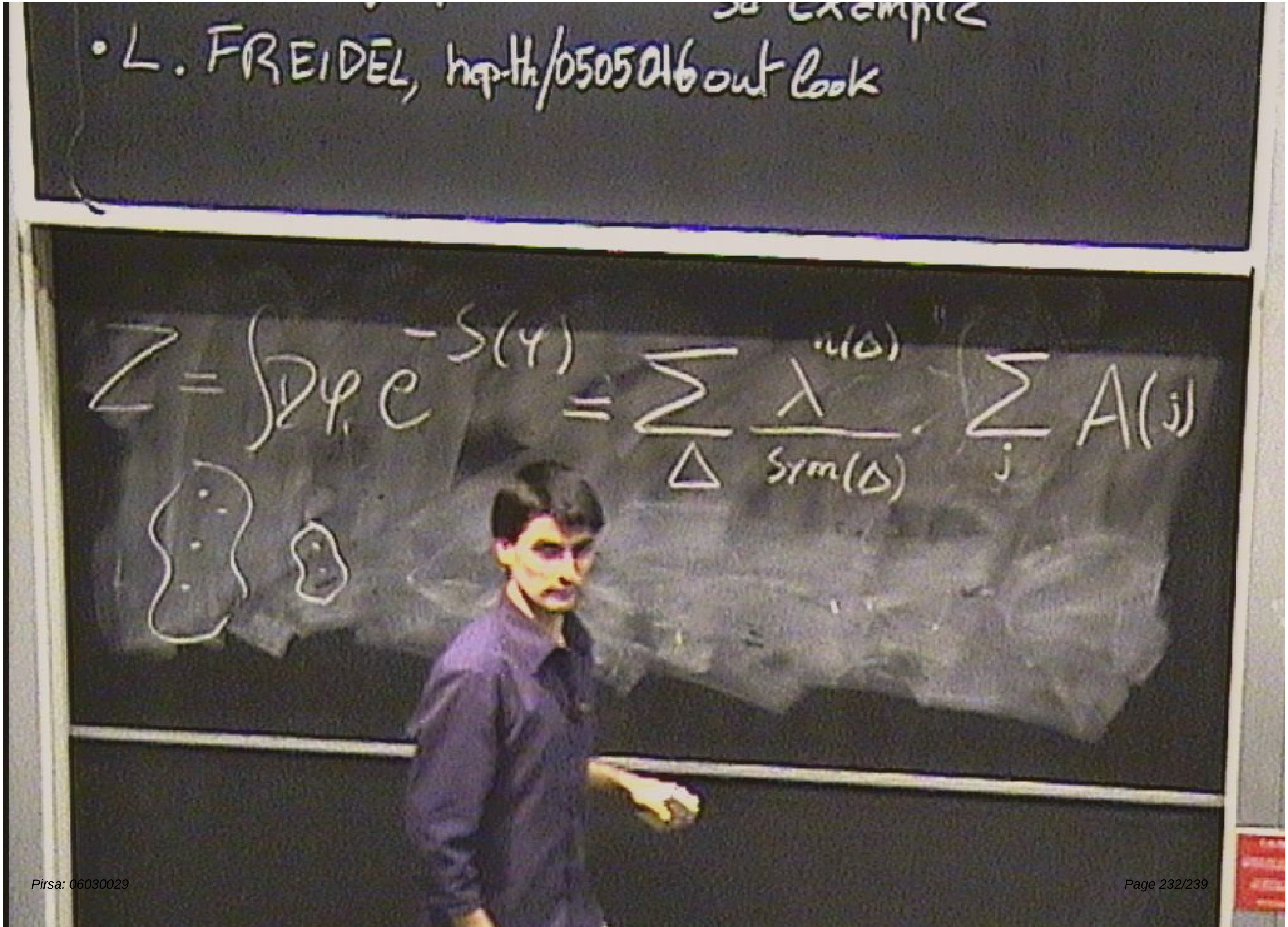

• L. FREIDEL, hep-th/0505016 out look
SPIN Foam
Sym(Δ)
SF
{j}

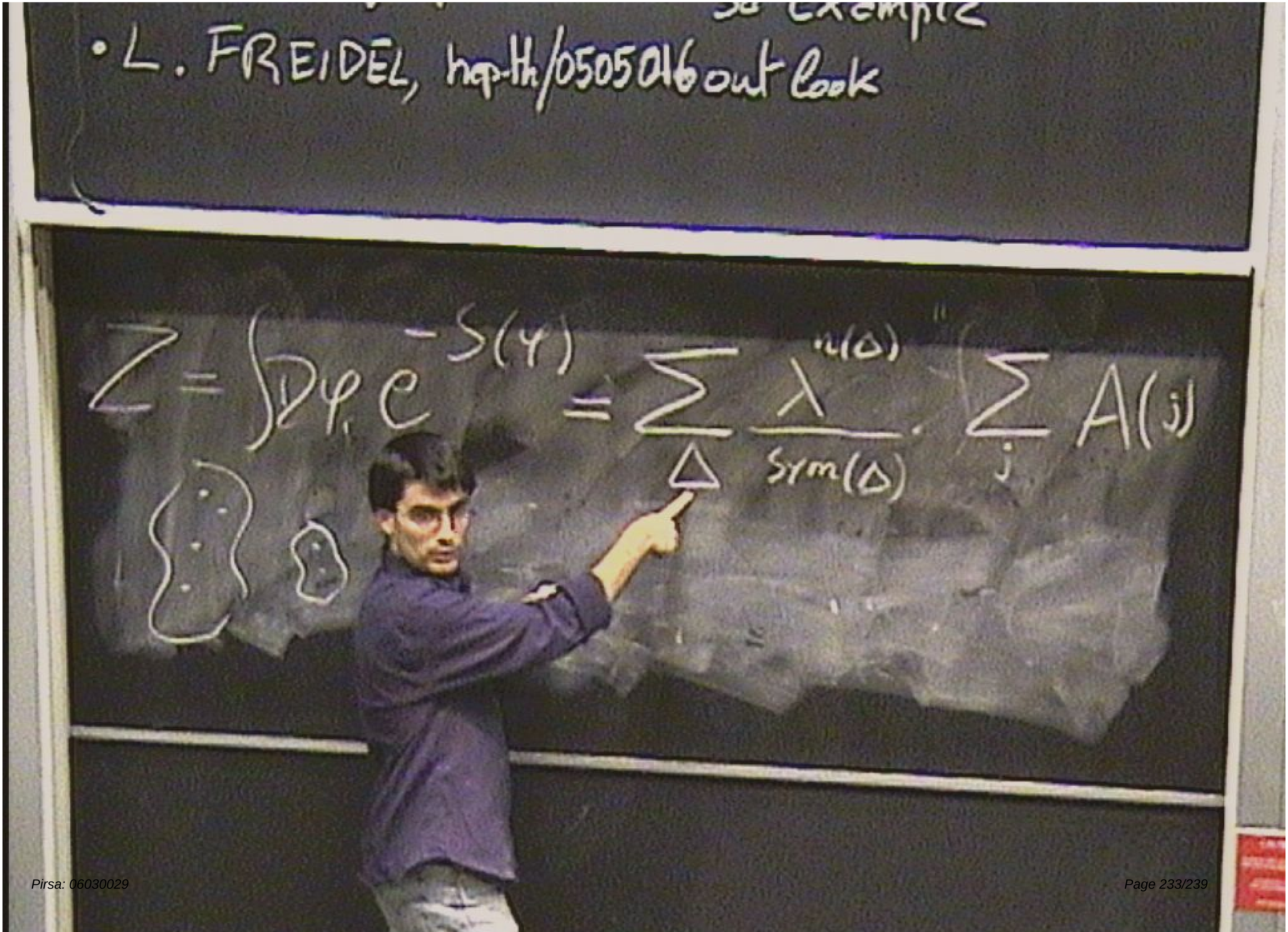

• L. FREIDEL, hep-th/0505016 out look
SPIN FOAM
SF

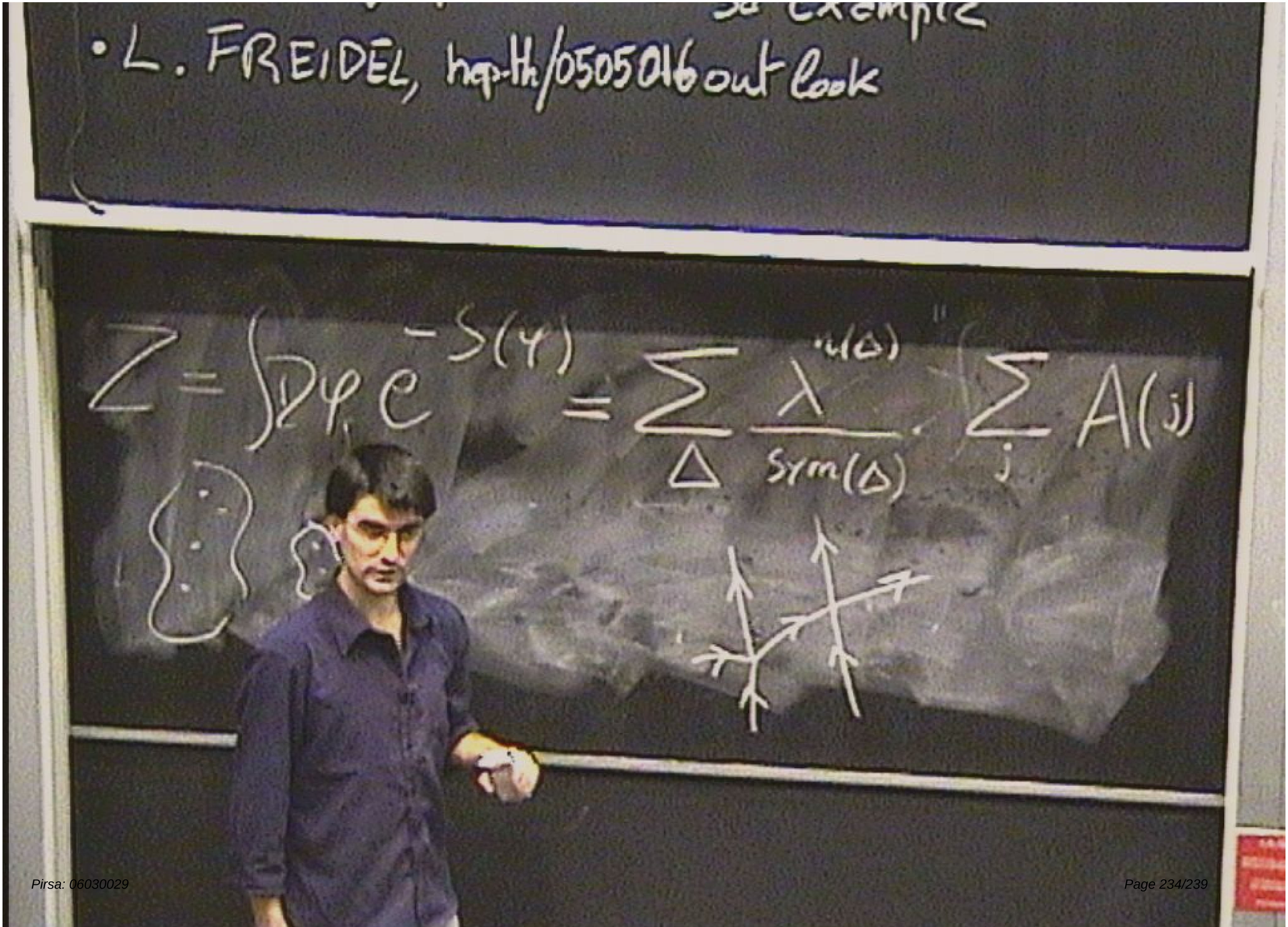

• L. FREIDEL, hep-th/0505016 outlook
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) Z_Δ
SPIN FOAM

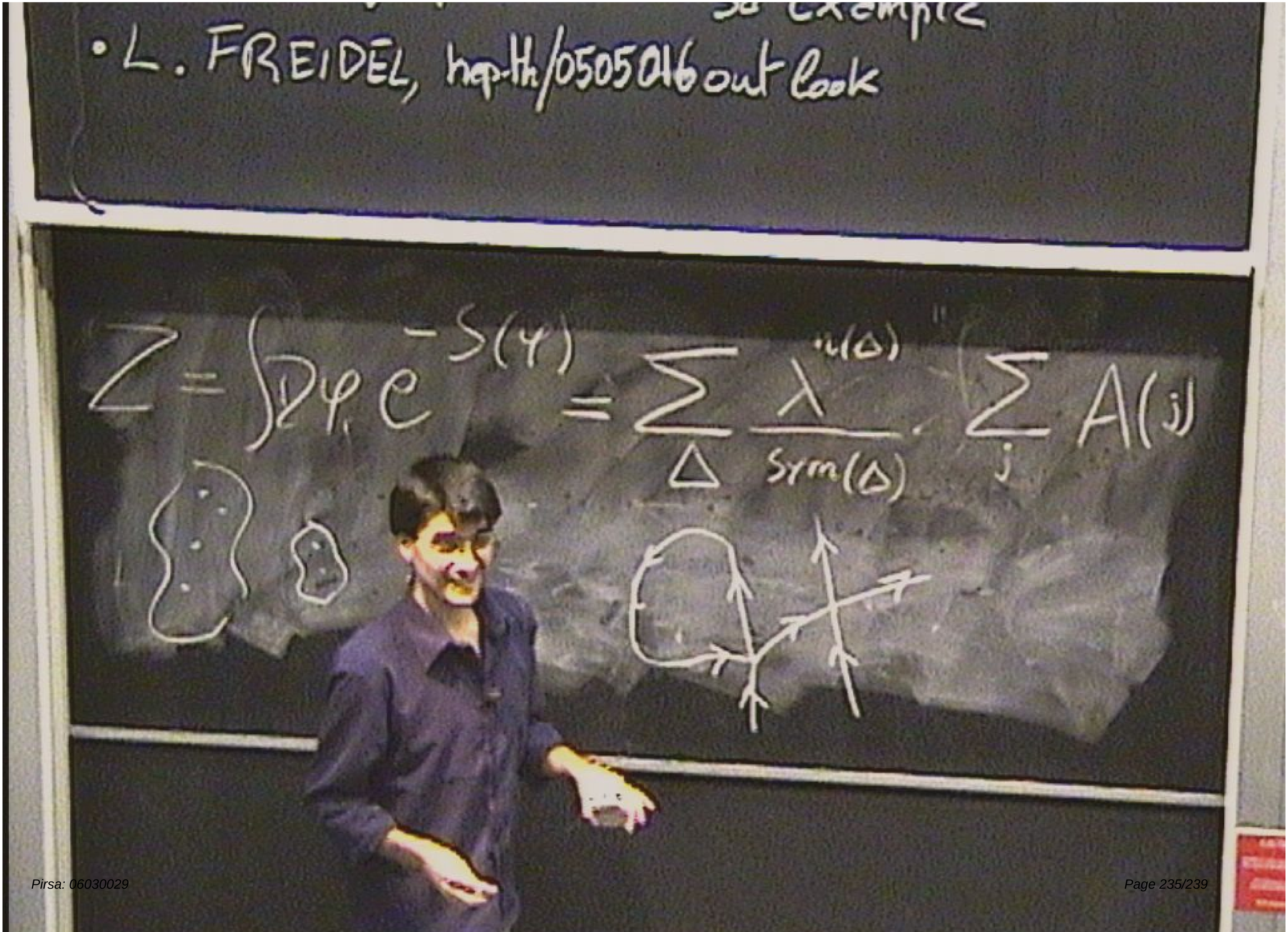

• L. FREIDEL, hep-th/0505016 out look

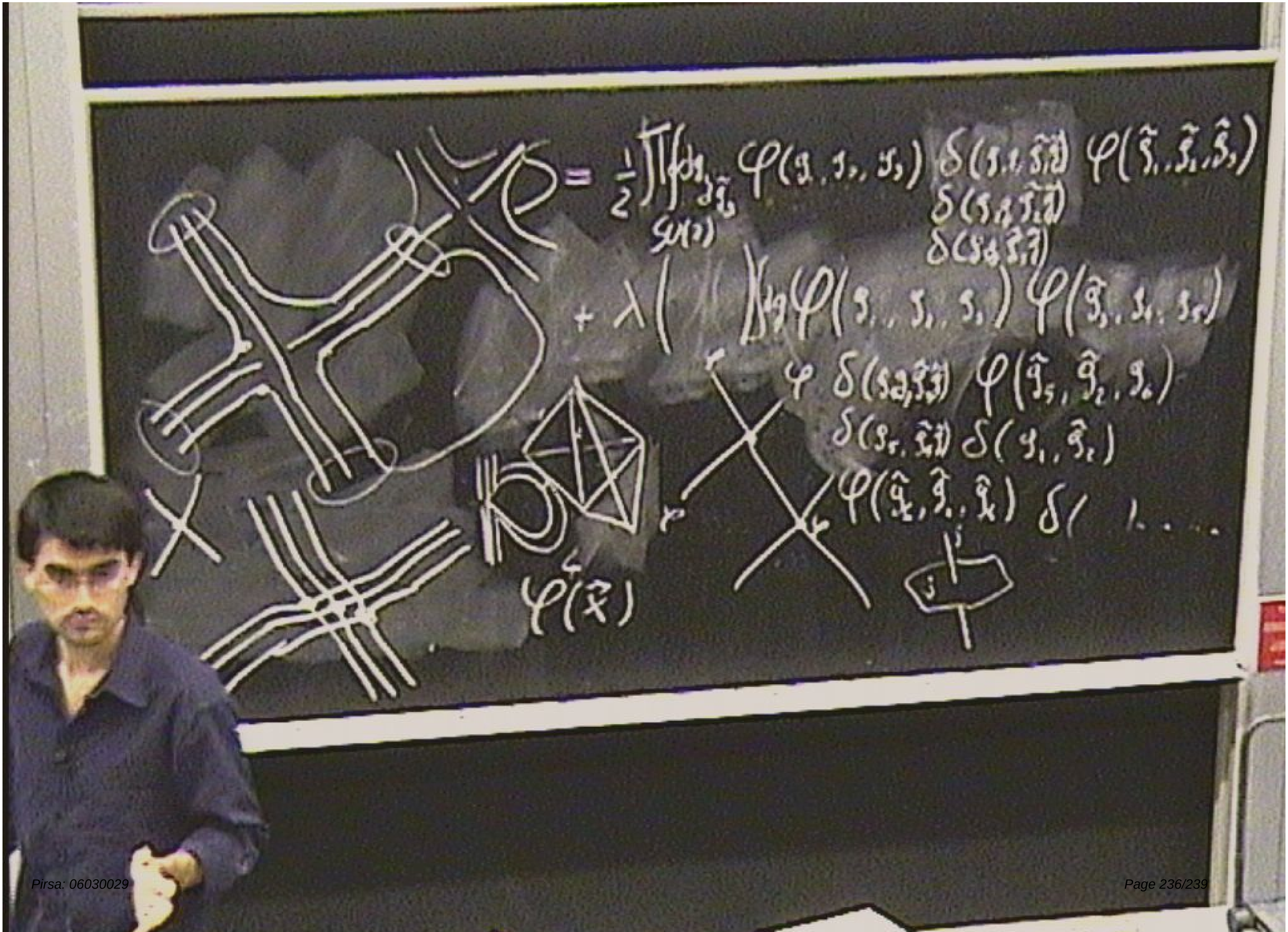

• L. FREIDEL, hep-th/0505016 outlook
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) ∫Dg e^{iS(g)}

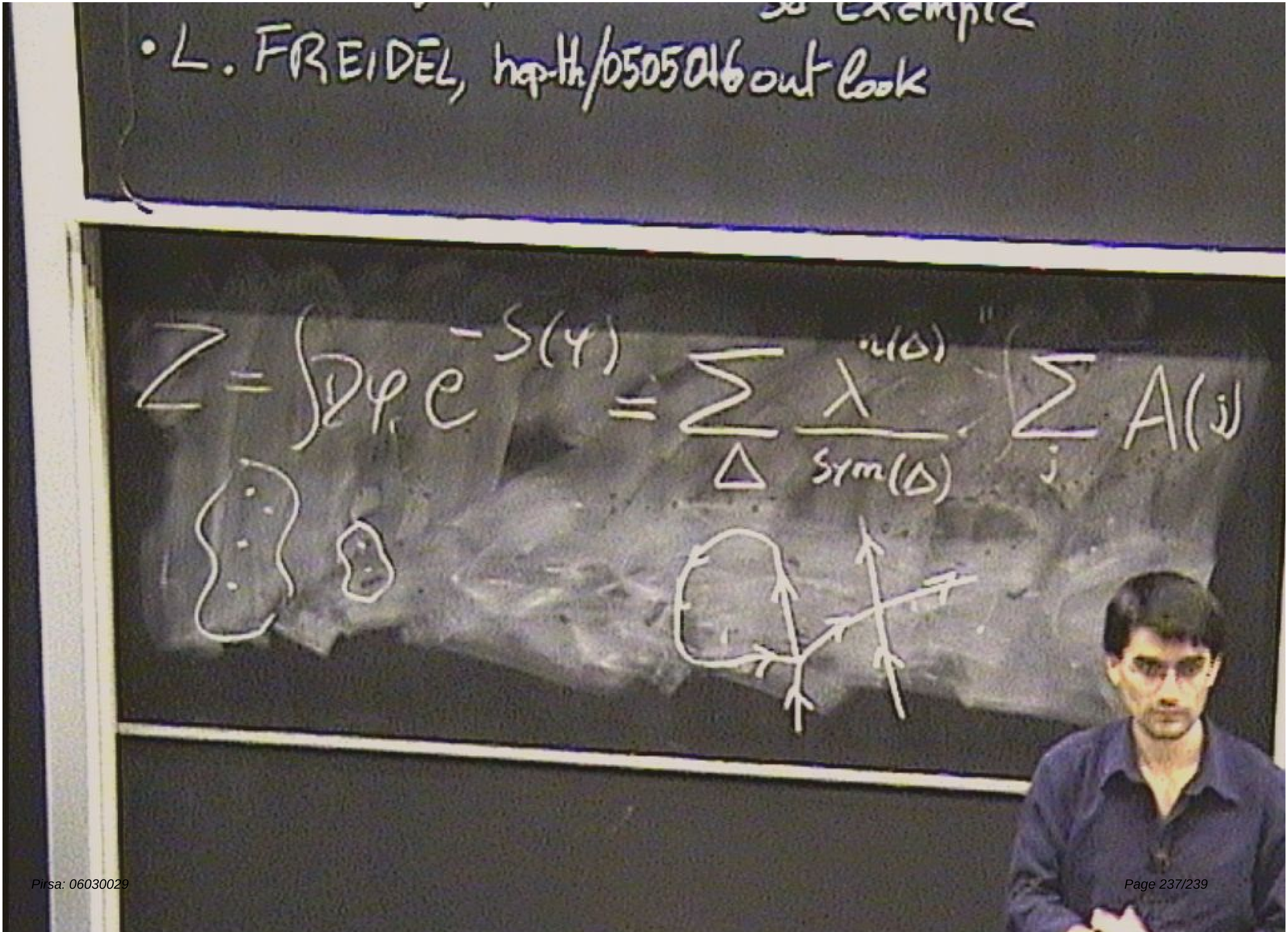

• L. FREIDEL, hep-th/0505016 out look
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)}/Sym(Δ) Σ_j A(j)

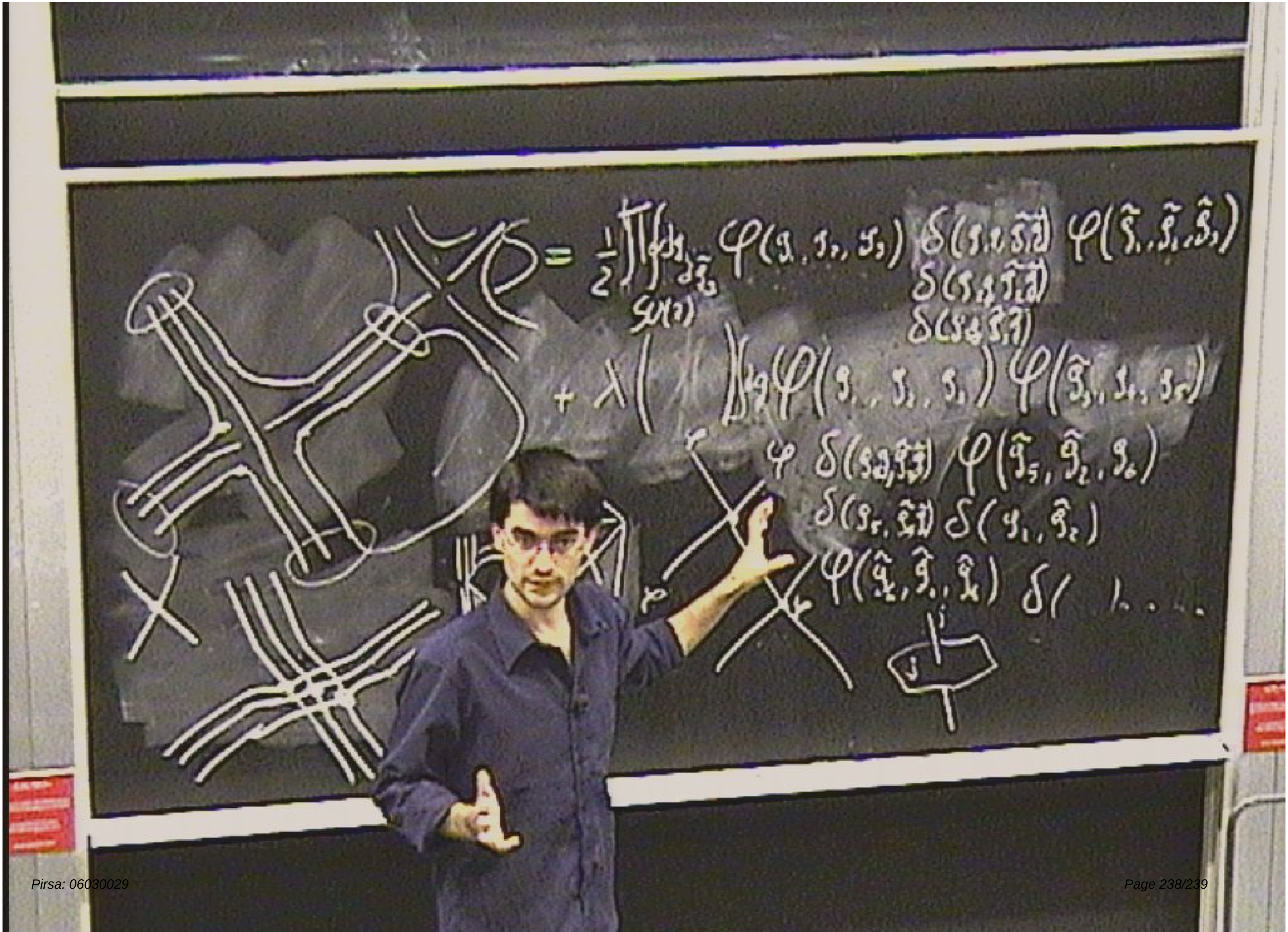

• L. FREIDEL, hep-th/0505016 out look
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) Σ_j A(j)

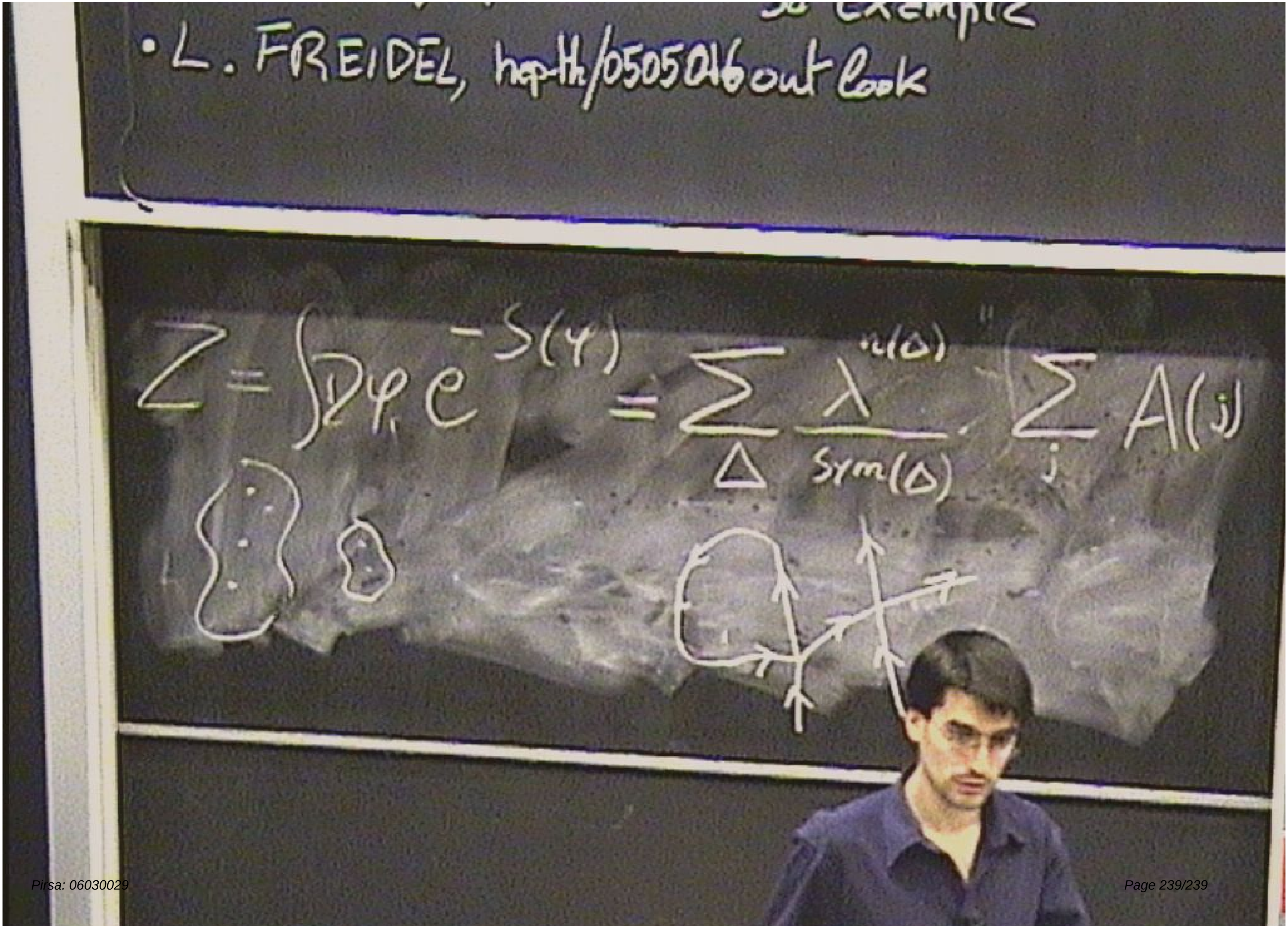

• L. FREIDEL, hep-th/0505016 out look
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) · Σ_j A(j)


• L. FREIDEL, hep-th/0505016 out look
$Z = \int D\varphi \, e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\mathrm{Sym}(\Delta)} \sum_{j} A(j)$




• L. FREIDEL, hep-th/0505016 out look
$Z = \int D\varphi\, e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\mathrm{sym}(\Delta)} \sum_{j} A(j)$




• L. FREIDEL, hep-th/0505016 out look
Z = ∫Dφ e^{-S(φ)} = Σ_Δ λ^{n(Δ)} / Sym(Δ) · Σ_j A(j)