

Title: Introduction to quantum gravity - Part 21

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005
-Quantum gravityy with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048
-Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras

- 3d Ponzano-Regge model (geometry)
- idea of Feynman diagrams
- Group field theory
 - idea
 - general formalism
 - 3d example
 - outlook

- out look

$$\Sigma_{\bar{\Delta}} = \{$$



- out look

$$Z_{\bar{\Delta}} = \int D\bar{E}^T \int Dg e^{i \sum_F f_2(E_F g_F)}$$



- outlook

$$Z_{\bar{\Delta}} = \int dE^r \int dg^r e^{i \sum_f t_2(E_f g_f)} = \sum_{\bar{g}_r}$$



-outlook

$$Z_{\bar{\Delta}} = \int dE^r \prod g_i e^{i \sum_F t_2(E_F g_F)} = \sum_{\Sigma} \prod_{\epsilon} (2\pi\omega_{\epsilon}) \prod_F$$



-outlook

$$Z_{\Delta} = \int dE^T \prod g_i e^{i \sum_F T_2(E_F g_i)} = \sum_{\vec{J}_F} \prod_i (e^{i J_i - 1}) \prod_{k \neq l} \begin{Bmatrix} J_k & J_l & J_m \\ J_n & J_o & J_p \end{Bmatrix}$$

- out look

$$Z_{\bar{\Delta}} = \int D\bar{E}^T Dg e^{i \sum_F t_2(E_F g_F)} = \sum_{\bar{J}_F} \prod_F (2J_F + 1) \prod_F \begin{pmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{pmatrix}$$

F^* / $\sqrt{-1}$



- out look

$$\sum_{\Delta} = \int D\epsilon^T \int Dg \cdot e^{i \sum_F t_2(E_F g_F)} = \sum_{\Sigma_F} \prod_{e \in F} (2J_e + 1) \prod_{k \in F} \begin{pmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{pmatrix}$$


- out look

$$\mathcal{Z}_\Delta = \int dE^\tau \int dg^\tau e^{i\sum_F t_2(E_F g_F)} = \sum_{\vec{\tau}_F} \prod_{e \in F} (2\tau_e - 1) \prod_{k \in F} \begin{pmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{pmatrix}$$


- out look

$$Z_{\bar{\Delta}} = \left\{ D E^T \right\} P g e^{i \sum_F t_2(E_F g_F)} = \sum_{\Sigma} \prod_{F'} (e_{F'} - 1) \prod_{F''} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$$

• $\begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$



- outlook

$$Z_{\bar{\Delta}} = \left\{ \begin{matrix} DE^T \\ Pg \end{matrix} \right\} e^{\sum_F T_2(E_F g_F)} = \sum_{\Sigma} \prod_{F'} (e_{F'} \cdot 1) \prod_{F''} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

• $\left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$

$$L = C_{\mu} \Sigma_{\mu} (J_{\mu})$$


- out look

$$\sum_{\Delta} = \int dE^T \int dg \cdot e^{i \sum_F t_2(E_F g)} = \sum_{\Sigma} \prod_{e \in F} (2J_e \cdot 1) \prod_{k \in F} \left(\begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right)$$
$$L = L_{\mu \nu}(x, \dots)$$
$$\sim (\dots)$$


- out look

$$\sum_{\Delta} = \int D\mathbf{E}^T \int Dg e^{i \sum_F t_2(E_F g_F)} = \sum_{\Sigma} \prod_F (2J_F + 1) \prod_{k \in F} \left\{ \begin{array}{c} J_1 J_2 J_3 \\ J_4 J_5 J_6 \end{array} \right\}$$

. $\left\{ \begin{array}{c} J_1 J_2 J_3 \\ J_4 J_5 J_6 \end{array} \right\} \underset{J_i \rightarrow \infty}{\sim} () \cos S_{\text{proj}}$

$\zeta_{\Gamma(\Sigma+1)}$



- out look

$$\begin{aligned} Z_{\bar{\Delta}} &= \int D\bar{E}^T \int Dg^* e^{i \sum_F \bar{t}_2(E_F g_F)} = \sum_{\bar{J}_c} \prod_c \prod_{F'} (2J_c - 1) \prod_{F''} \left\{ \begin{array}{ccc} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{array} \right\} \\ &\cdot \left\{ \begin{array}{ccc} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{array} \right\} \underset{J_c \rightarrow \infty}{\sim} () \cos S_{\text{reg}, f_c}(J_c) \end{aligned}$$



- out look

$$\sum_{\vec{\Delta}} \left(\prod_{I \in \vec{\Delta}} P_I \right) e^{i \sum_{I \in \vec{\Delta}} t_I (\vec{E}_{I,0})} = \sum_{\vec{\Sigma}} \prod_{I \in \vec{\Sigma}} (e^{\vec{J}_I \cdot \vec{t}_I}) \prod_{I \in \vec{\Delta}} \frac{\prod_{j=1}^3 \left(\begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right)}{\prod_{I \in \vec{\Sigma}} \left(\begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right)}$$

$\sim \lim_{\vec{J}_I \rightarrow \infty} () \cos \sum_{I \in \vec{\Sigma}} \vec{J}_I \cdot \vec{t}_I$

$L_c = e_P(\langle \vec{J}_c \rangle)$



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- out look

$$\sum_{\vec{\Delta}} \left(\det \begin{pmatrix} E^T \\ P_B \end{pmatrix} e^{i \sum_F \tau_2(E_{FB})} \right) = \sum_{\vec{\Delta}_F} \prod_F (2\vec{\Delta}_F) \prod_{k \in F} \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix}$$

$\cdot \begin{Bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{Bmatrix} \xrightarrow[\vec{\Delta}_i \rightarrow \infty]{} (-)^{\epsilon_i} \cos S_{\vec{\Delta}_i}(J_i)$

$L_c = \langle \det \begin{pmatrix} E^T \\ P_B \end{pmatrix} \rangle$

$J(J_{1..6}) = \frac{1}{2}$

$L_c = \ell_P(\langle 2\vec{\Delta}_{c..1} \rangle)$

$$E_F \in \mathbb{R}$$

$$\neq E_F$$



$$E_F \in \mathbb{R}^3$$

$$|E_F| \quad L = \sqrt{E_F \cdot E_F}$$



$$E_r \in \mathbb{R}^3$$

J

$$\int E_r^2 \quad L = \sqrt{E_r \cdot E_r} = \sqrt{J \cdot J}$$

$$E_F \in \mathbb{R}^3$$

$$\vec{J}$$

$$\cancel{\sqrt{E_F^2}} \quad L = \sqrt{E_F \cdot E_F} = \sqrt{\vec{J} \cdot \vec{J}}$$

H

$$E_F \in \mathbb{R}^3$$

$$\vec{J}$$

$$\sqrt{E_F^2} = L = \sqrt{E_F \cdot E_F} = \sqrt{\vec{J} \cdot \vec{J}}$$

$$\mathcal{H} \quad E_F \rightarrow J_F(p)$$

$$\mathcal{H}^P$$



$$E_t \in \mathbb{R}^3$$

$$\vec{J}$$

$$\int E_t^2 \quad L = \sqrt{E_t \cdot E_t} = \sqrt{\vec{J}_t \cdot \vec{J}_t} = C(e)$$

$$\mathcal{H} \quad E_t \rightarrow J_t(p)$$

$$\mathcal{H}^P$$

$$E_r \in \mathbb{R}^3$$

$$\vec{J}$$

$$\cancel{E_r} \quad L = \sqrt{E_r \cdot E_r} = \sqrt{\vec{J}_r \cdot \vec{J}_r} = \sqrt{C(r)}$$

$$\mathcal{H} \quad E_r \rightarrow J_r(r)$$

$$\mathcal{H}^P$$

$$E_r \in \mathbb{R}^3$$

$$\vec{J}$$

$$\sqrt{E_r^2} = \sqrt{E_r \cdot E_r} = \sqrt{J_r \cdot J_r} = \sqrt{C(j)} = \sqrt{j(j)}$$

$$H \quad E_r \rightarrow J_r(P)$$

$$H^P$$

$$E_r \in \mathbb{R}^3$$

$$\vec{J}$$

$$\sqrt{E_r^2} = \sqrt{E_r \cdot E_r} = \sqrt{\vec{J}_r \cdot \vec{J}_r} = \sqrt{C(r)} = \sqrt{j(j_r)}$$

$$\mathcal{H} \quad E_r \rightarrow J_r(p)$$

$$\mathcal{H}^P$$



- out look

$$Z_{\bar{\Delta}} = \left\{ \begin{matrix} DE^T \\ Pg \end{matrix} \right\} e^{i \sum_{\epsilon} t_2 (\boxed{E_{\text{dd}}})} = \sum_{\Sigma} \prod_{\epsilon} (2J_{\epsilon+1}) \prod_{\epsilon} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

. $\left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \underset{J_1 \rightarrow \infty}{\sim} () \cos \sum_{k=1}^6 (J_k)$

$L_e = \ell_P(\boxed{\Sigma(J_{\epsilon+1})})$

$J(J_{\epsilon+1}) = \xi$

$L_e = \ell_P(2J_{\epsilon+1})$



- out look

$$\sum_{\vec{\Delta}} = \left\{ \begin{matrix} DE^T \\ Pg \end{matrix} \right\} e^{i \sum_{\vec{\epsilon}} \tau_2 (\vec{E}_{gg})} = \sum_{\vec{\Sigma}_e} \prod_{\vec{c}} (2J_e + 1) \prod_{\vec{f}} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$J_e \rightarrow \infty$ $\cos S_{\vec{\tau}_{gg}(\vec{\Sigma}_e)}$

$$L_e = \langle \vec{\Delta}(\vec{\Sigma}_e) \rangle$$
$$(J_{e+}) = \frac{1}{2}$$
$$L_e = \ell_p(2J_e + 1)$$


$$E_e \in \mathbb{R}^3$$

$$\mathcal{J}$$

$$E_r \quad L = \sqrt{E_r \cdot E_r} = \sqrt{\mathcal{J}_e \cdot \mathcal{J}_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$H \quad E_r \rightarrow \mathcal{J}_r(\rho)$$

$$H^\rho$$



$$E_e \in \mathbb{R}^3$$

J'

$$\cancel{E_r} \quad L = \sqrt{E_r \cdot E_r} = \sqrt{J_r \cdot J_r} = \sqrt{C(j)} = \sqrt{j(j..)}$$

$$H \quad E_r \rightarrow J_r(p)$$

J'



H^P

$$E_e \in \mathbb{R}^3$$

$$\vec{J}$$

$$/ E_e^j$$

$$L = \sqrt{E_e \cdot E_e} = \sqrt{\vec{J}_e \cdot \vec{J}_e} = \sqrt{C(j)} = \sqrt{j(j..)}$$

H

$$E_e \rightarrow J_e(p)$$



$$H^P$$

$$H^j.$$

$$J'$$

$$E_e \in \mathbb{R}^3$$

J'

$$\cancel{E_e} \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(p) \quad \mathcal{H}^p \quad \psi \in \mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3}$$



$$E_e \in \mathbb{R}^3 \quad J^1$$

$$\sqrt{E_e^2} \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j..)}$$

$$H \quad E_e \rightarrow J_e(\rho)$$



$$H^\rho \quad \psi \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$$



$$E_e \in \mathbb{R}^3$$

$$\vec{J}$$

$$|E_e| L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{C(s)} = \sqrt{j(j+1)}$$

$$H \quad E_e \rightarrow J_e(p)$$

$$H^P \quad \Psi \in H^{\frac{j}{2}} \otimes H^{\frac{j}{2}} \otimes H^{\frac{j}{2}}$$



$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow Inv_{SU(2)}$$

- outlook

$$Z_{\bar{\Delta}} = \left\{ \begin{matrix} DE^T \\ PG \end{matrix} \right\} e^{i \sum_{\epsilon} t_2 (\bar{E}_{\epsilon})} = \sum_{\Sigma} \prod_{\epsilon} (2J_{\epsilon} - 1) \prod_{\epsilon} \left\{ \begin{matrix} J_+ & J_- & J_0 \\ J_+ & J_- & J_0 \end{matrix} \right\}$$

. $\left\{ \begin{matrix} J_+ & J_- & J_0 \\ J_+ & J_- & J_0 \end{matrix} \right\} \underset{J_0 \rightarrow \infty}{\sim} () \cos \sum_{k=1}^n (J_k)$

$$L_e = e^{i \sum_{\epsilon} (J_{\epsilon})}$$
$$J(J_{\epsilon}) + \frac{1}{2}$$
$$L_e = e_P (2J_{e+1})$$


$$E_e \in \mathbb{R}^3$$

$$\vec{J}$$

$$/ E_e^j$$

$$L = \sqrt{E_e \cdot E_e} = \sqrt{\vec{J}_e \cdot \vec{J}_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$H$$


$$\rightarrow J_e(p)$$

$$H^p \quad \psi \in \left(H^{\frac{j}{2}} \otimes H^{\frac{j}{2}} \otimes H^{\frac{j}{2}} \right)^{J'}$$

$$\Rightarrow \psi = \rightarrow Inv_{SU(2)}$$

$$E_e \in \mathbb{R}^3$$

$$\sqrt{|E_e^2|} = L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(\rho) \quad \mathcal{H}^\rho \quad \psi \in \mathcal{T}_m \left(\mathcal{H}^{j_1} \otimes \mathcal{H}^{j_2} \otimes \mathcal{H}^{j_3} \right) \quad C_{n_1, n_2, n_3}^{j_1, j_2, j_3}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow Inv_{SU(2)}$$



- outlook

$$Z_{\bar{\Delta}} = \int D\vec{E}^T Dg e^{i \sum_{\vec{I}_c} t_2 (\vec{E}_{\vec{I}_c})} = \sum_{\vec{J}_c} \prod_{e} (2J_e + 1) \prod_{\vec{I}_c} \left\{ \begin{array}{ccc} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{array} \right\}$$

$\cdot \left\{ \begin{array}{ccc} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{array} \right\} \underset{J_c \rightarrow \infty}{\sim} () \cos S_{\mu_{J_c} \nu_{J_c}} (J_c)$

$$L_c = \ell_P(\Delta_c(\vec{J}_c))$$
$$J(J_{\perp\perp}) = \frac{1}{2}$$
$$L_c = \ell_P(2J_c + 1)$$


$$E_e \in \mathbb{R}^3 \quad J^I$$

$$\cancel{E_e} \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(\rho) \quad \mathcal{H}^P \quad \psi \in \text{Inv}\left(\mathcal{H}^{\frac{1}{2}} \otimes \mathcal{H}^{\frac{1}{2}} \otimes \mathcal{H}^{\frac{1}{2}}\right) \quad C_{n_1 n_2 n_3}^{j_1 j_2 j_3}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

$$E_e \in \mathbb{R}^3 \quad J^1$$

$$\cancel{E_e} \quad L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(g)} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(p) \quad \mathcal{H}^P \quad \Psi \in \mathcal{F}_{\text{inv}}\left(\mathcal{H}^{\frac{j_1}{2}} \otimes \mathcal{H}^{\frac{j_2}{2}} \otimes \mathcal{H}^{\frac{j_3}{2}}\right) \quad C_{n_1, n_2, n_3}^{j_1, j_2, j_3}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv}_{SU(2)}$$

$$E_e \in \mathbb{R}^3$$

$$J^L$$

$$\sqrt{E_e^2} = L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

H



$$E_e \rightarrow J_e(\rho)$$

$$H^\rho \quad \psi \in \Gamma_{\mathbb{M}}(H^{\frac{j_1}{2}} \otimes H^{\frac{j_2}{2}} \otimes H^{\frac{j_3}{2}}) \quad C_{n,m,n}$$

$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0 \rightarrow \text{Inv } SU(2)$$

- out look

$$Z_{\bar{\Delta}} = \int D\bar{E}^T Dg e^{i \sum_{\epsilon} t_{\epsilon} (\bar{E}_{\epsilon})} = \sum_{\Sigma} \prod_{\epsilon} \prod_{(2J_{\epsilon}, \dots)} \prod_{k \in \epsilon} \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\}$$

$\cdot \left\{ \begin{matrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{matrix} \right\} \underset{J_1 \rightarrow \infty}{\sim} () \cos S_{R_{J_1} J_2} (J_1)$

$L_c = \langle \bar{E}_c (J_{c,1}) \rangle$

$J(J_{c,1}) = \frac{1}{2}$

$L_c = \ell_p (J_{c,1})$



$$E_e \in \mathbb{R}^3$$

$$\sqrt{|E_e^2|} = L = \sqrt{E_e \cdot E_e} = \sqrt{J_e \cdot J_e} = \sqrt{\mathcal{C}(j)} = \sqrt{j(j+1)}$$

$$\mathcal{H} \quad E_e \rightarrow J_e(\rho) \quad \mathcal{H}^\rho \quad \psi \in \mathcal{T}_m \left(H^{j_1} \otimes H^{j_2} \otimes H^{j_3} \right) \quad C_{n_1, n_2, n_3}^{j_1, j_2, j_3}$$

$$+ \vec{E}_2 \cdot \vec{E}_3 = 0 \longrightarrow \text{Inv}_{SU(2)}$$

- out look

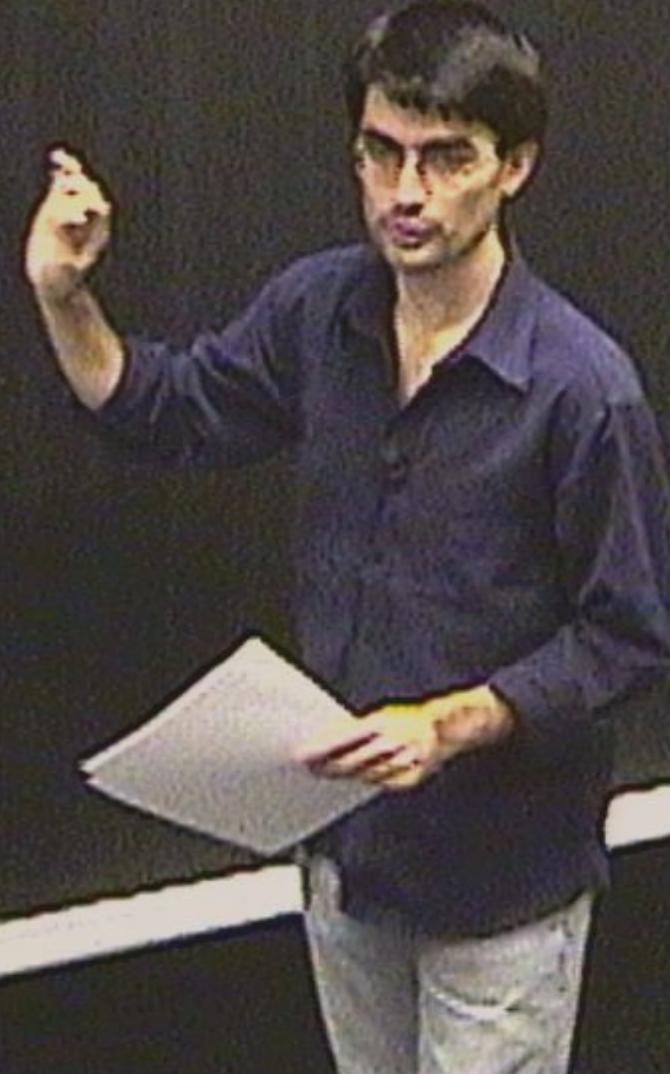
$$\sum_{\Delta} \left(\det \begin{pmatrix} P & E^T \\ E & g \end{pmatrix} e^{i \sum_{\epsilon} \tau_{\epsilon} (\text{Edd})} \right) = \sum_{\Sigma} \prod_{\epsilon} (2J_{\epsilon} + 1) \prod_{\epsilon} \begin{pmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{pmatrix}$$

$\cdot \begin{pmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \end{pmatrix} \underset{J_i \rightarrow \infty}{\sim} () \cos S_{\mu_1 \mu_2 \dots \mu_k} (J_i)$

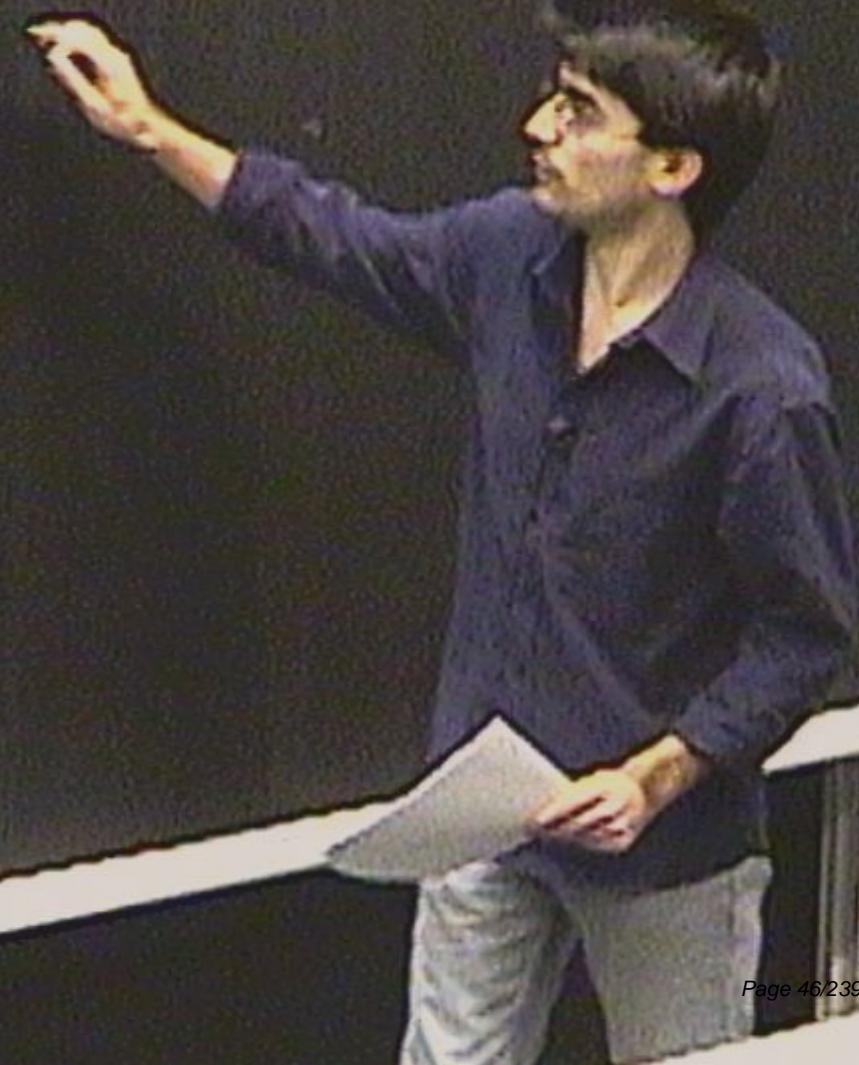
$L_c = e^{i \sum_{\epsilon} (\tau_{\epsilon} - \frac{1}{2})} \quad J(J_{\epsilon \epsilon}) = \frac{1}{2} \quad L_c = e_P (2J_{\epsilon \epsilon} + 1)$



$$Z = \int d\varphi e^{-S_{\text{d}\varphi}}$$



$$Z = \int D\varphi e^{-S[\varphi]} \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]$$



$$Z = \int D\varphi e^{-S_{\text{eff}} [\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi]}$$

$$Z = \int d\varphi e^{-S_0 \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$Z =$$

$$Z = \int D\varphi e^{-S[\varphi]} \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]$$

$$Z =$$

$$Z = \int D\varphi e^{-S[\varphi]} \quad \text{where} \quad S[\varphi] = \int d^4x \left[\frac{1}{2} \varphi (\partial^\mu + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]$$

$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 - \dots}$$

$$Z = \int d\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$= \int_{-\infty}^{\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q}$$

$$Z = \int_{-\infty}^{\infty} dq$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q}$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} \left(1 - \frac{\lambda}{4!} q^4 + \left(\frac{\lambda}{4!}\right)^2 q^8 \right)$$

$$\begin{aligned}
 Z &= \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]} \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^{1/2} q^8 \right) \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)
 \end{aligned}$$

$$Z = \int D\varphi e^{-\int d\lambda \left[\frac{1}{2} \varphi (\delta^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^4$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int D\varphi e^{-\int d\lambda \left[\frac{1}{2} \varphi (\delta^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$\int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^4 = \left(\frac{d}{dJ} \right)^4$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \left(\frac{d}{ds} \right)^4 \int_{-\infty}^{+\infty} d\tilde{q} e^{-\frac{1}{2} m^2 \tilde{q}^2}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^{1/2} q^2 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^n = \left(\frac{d}{ds} \right)^n$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{\lambda}{4!} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^4 = \left(\frac{d}{dq} \right)^4 \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int D\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\delta^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{4}{4} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^n = \left(\frac{d}{dq} \right)^n e^{\frac{J^2}{m^2}}$$

$$Z = \int D\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$= \int dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{\lambda}{4!} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} q^n = \left(\frac{d}{dq} \right)^n e^{\frac{J^2}{m^2}}$$

$$Z = \int D\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + \frac{J}{\lambda}} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{4}{4} q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) = e^{-\frac{\lambda}{4!} \left(\frac{J}{m} \right)^2} e^{\frac{1}{2} \frac{J^2}{m^2}}$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

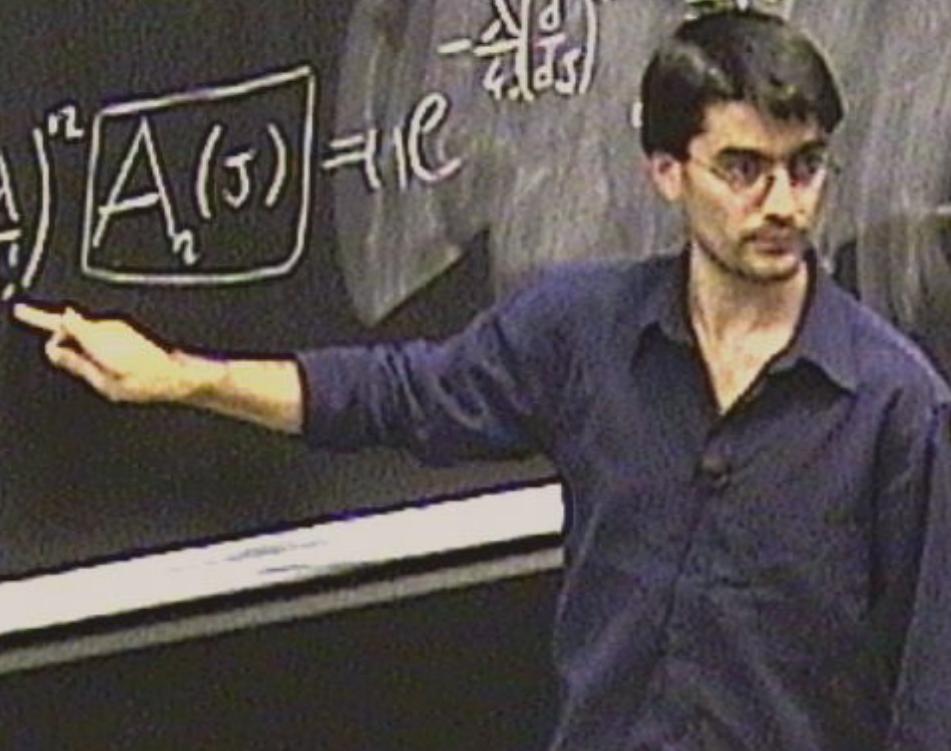
$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J) = \prod_n e^{-\frac{\lambda^2}{4!} \left(\frac{J}{m^2} \right)^2} e^{\frac{J^2}{2m^2}}$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{4}{4} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{(q-J)^2}{2m^2}} = \sqrt{\frac{\pi}{m^2}}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = \prod_n e^{-\frac{\lambda}{4!} J^n}$$



$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{4}{4} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{J^2}{m^2} \left(\frac{d}{ds} \right)^2} e^{\frac{J^2}{2m^2}} \times J^2$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = \prod_n e^{-\frac{\lambda}{4!} \left(\frac{J^2}{m^2} \right)^n} e^{\frac{J^2}{2m^2}}$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\delta^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \binom{4}{4} q^8 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \int_{-\infty}^{+\infty} ds e^{-\frac{J^2}{4m^2 s^2}} e^{\frac{J^2}{2m^2} s^2} = \left(\frac{1}{m^2} \right)^{\lambda/2} \lambda^{\lambda/2}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\partial^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + \dots \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)} = \prod_{i=1}^{\infty} e^{-\frac{\lambda}{m^2 i^2}} e^{\frac{\lambda J}{m^2}} \left(1 + \left(\frac{1}{m^2} \right) \lambda J \right)$$

$$= \sum_n (-1)^n \left[A_n(j) \right] = (-1)^j \left(1 - \left(\frac{1}{k} \right)^n \right) \lambda^j$$

T P

$$= \sum_n \left(-\frac{\lambda}{n!}\right) [A_n(j)] = ((C - \lambda) + \left(\lambda \left(\frac{1}{n^2}\right)\right)) \lambda^j j!$$

| p X



$$= \sum_n \left(-\frac{1}{4!} \right) [A_n(j)] = \left(-\frac{1}{4!} \right) \left(-\frac{1}{h^2} \right)^j \lambda j!$$

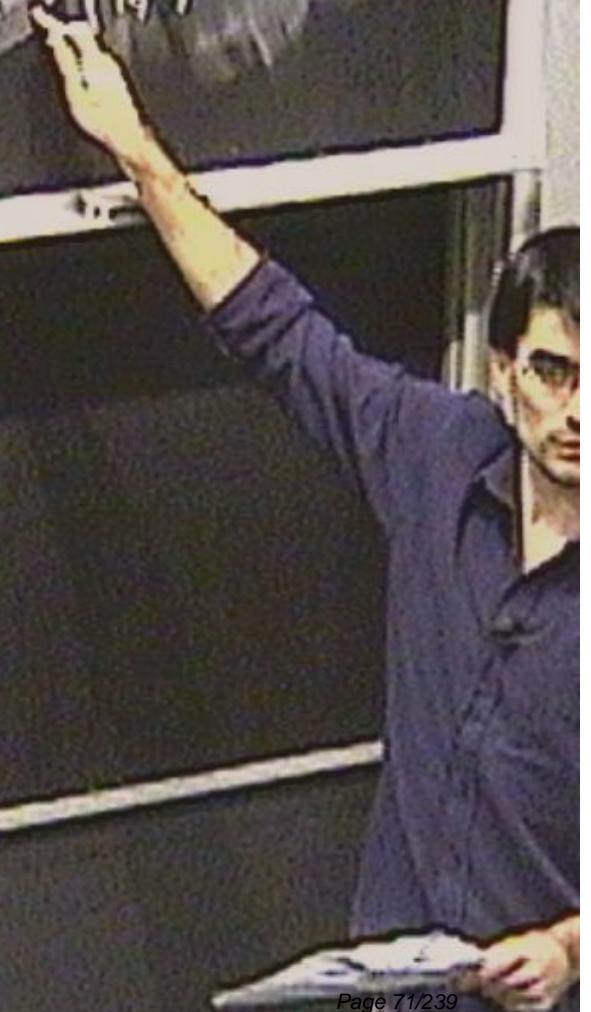
1 p X || | 8

$$L = \int_{-\infty}^{dq} e$$

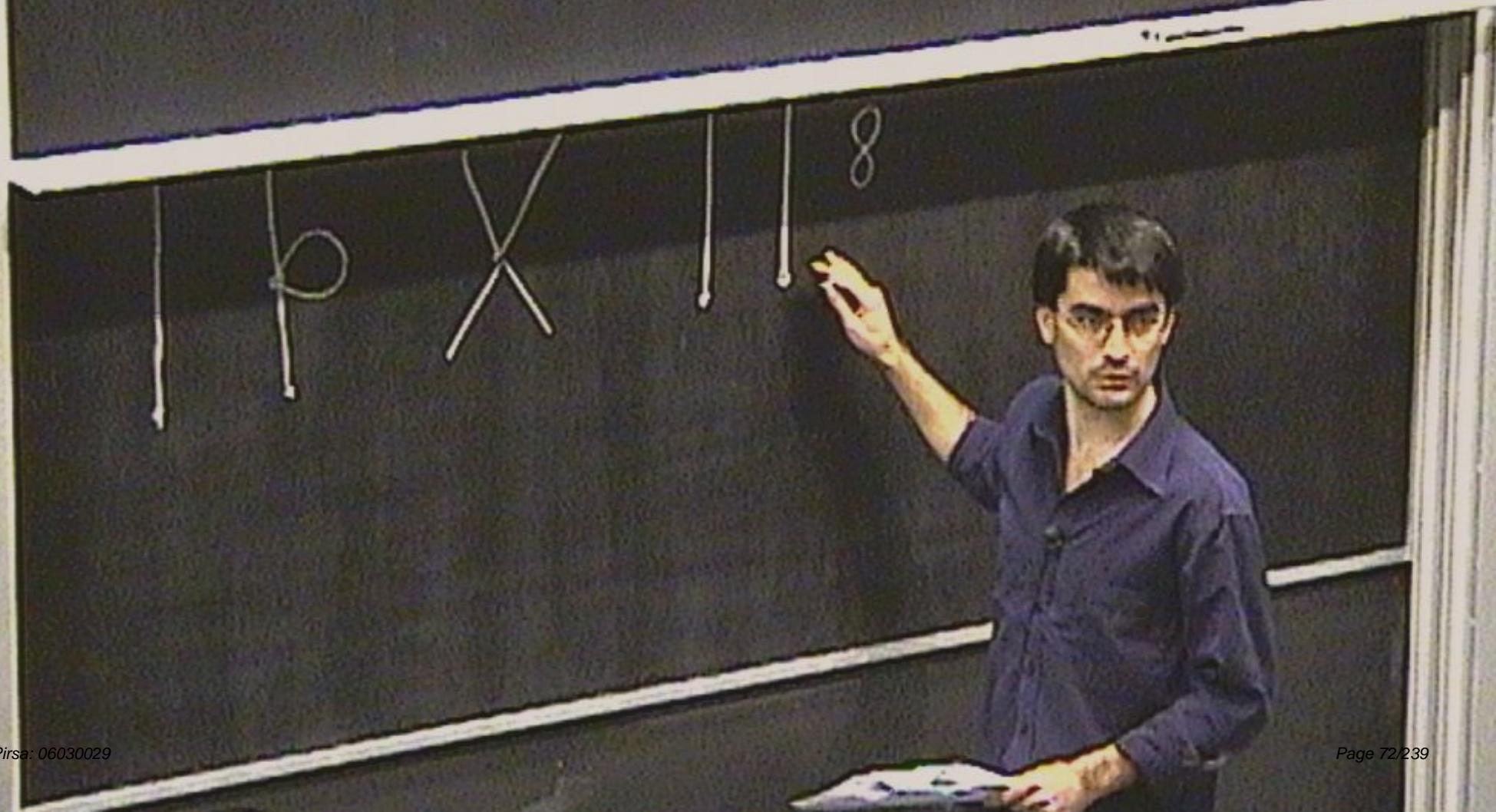
$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)}$$

$$= (-\lambda)^d \int_{-\infty}^d e^{-\lambda x} x^d \frac{x^d}{d!} dx$$

$$= \left(\lambda \left(\frac{1}{k!}\right)\right)^d \lambda^d$$

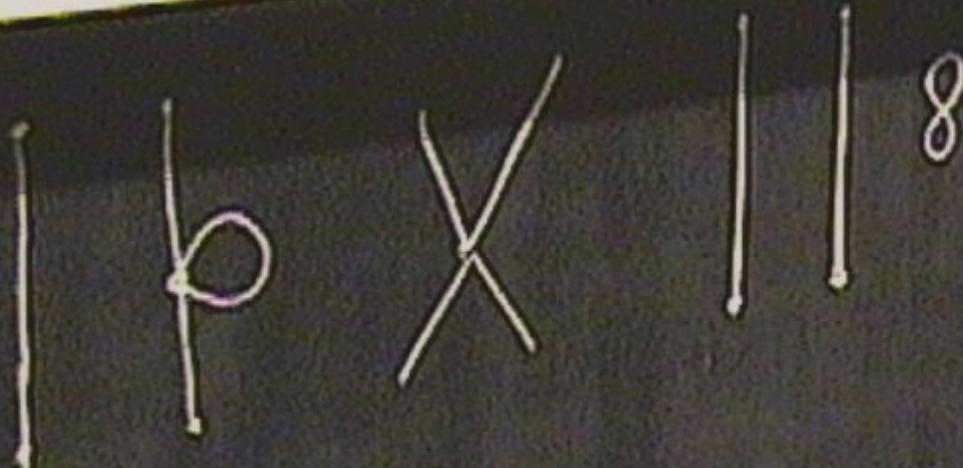


$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_4(j)} = (e^{-\frac{\lambda^2}{4!}}) e^{2\lambda j} (-1)^{\frac{j(j-1)}{2}} \lambda^j$$

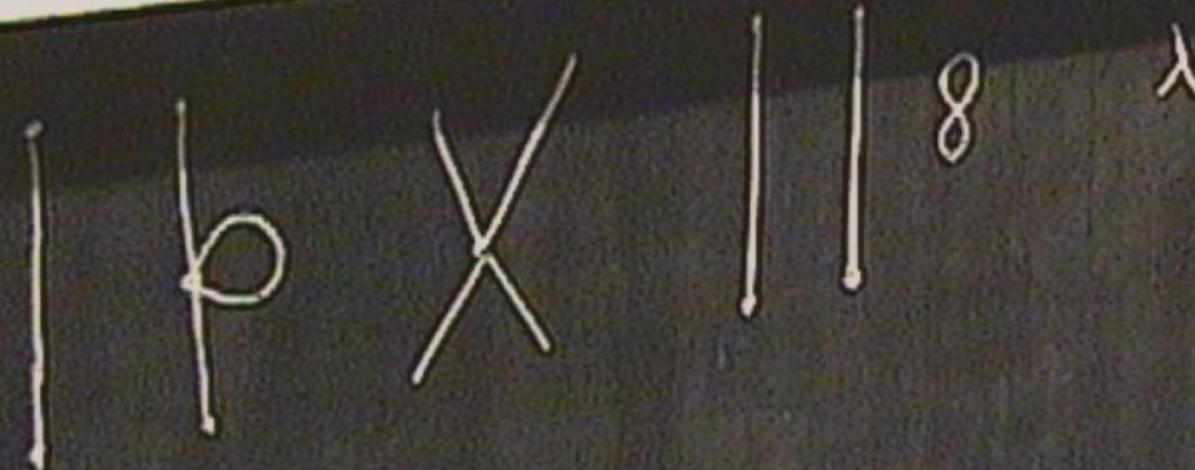


$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4 + \beta q^2} = \int dq e^{-\frac{1}{2}m^2q^2} e^{(\beta - \frac{\lambda}{4})q^2 + \frac{\lambda}{2}(q^4 - 1)}$$
$$= \sum_n (-\lambda)^n \boxed{A_n(\beta)} = \langle e^{-\frac{\lambda}{4}q^2} \rangle e^{\beta \langle q^2 \rangle} \left(\sqrt{\left(\frac{1}{m^2} \right)^n \lambda} \right)$$

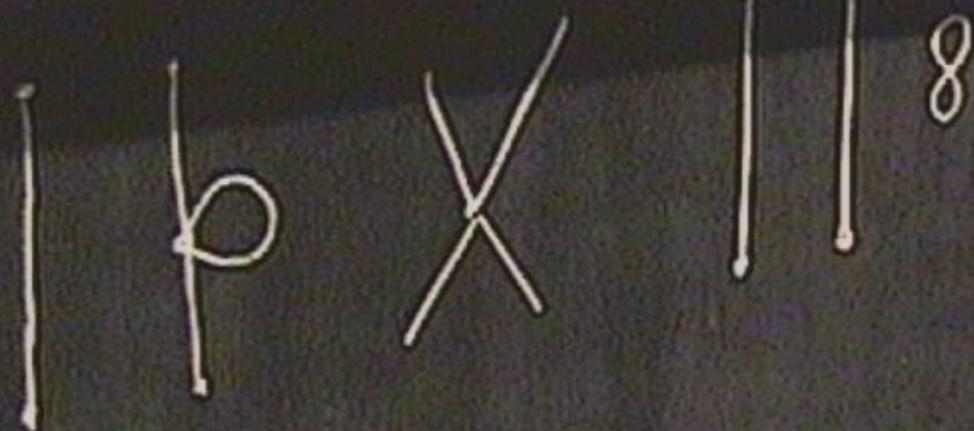
$$= \frac{2}{n} \left(-\frac{1}{4!} \right) A_4(3) + \dots + \left(-1 \right)^{\frac{n}{2}} \left(\frac{1}{n!} \right)^2 \lambda^{\frac{n}{2}}$$



$$= \frac{1}{n} \left(-\frac{1}{4} \right) A_n(3) - \left(\frac{1}{2} \right)^n \lambda^3$$



$$= \frac{2}{n} \left(-\frac{1}{4!} \right) A_n(3) \tilde{\psi}_n^3 \psi_n^3 = \left(-\frac{1}{4!} \left(\frac{1}{m^2} \right)^3 \lambda \right)$$



$$= \frac{2}{n} \left(-\frac{1}{4} \right) A_2(3) \tilde{\psi}_n^2 \left(\left(\frac{1}{m^2} \right)^{\frac{1}{2}} \lambda \right)$$

$$\begin{array}{cccccc} | & | & X & | & | & 8 & \lambda J\left(\frac{1}{m^2}\right) \\ | & | & | & | & | & | & \end{array}$$

$$Z = \int d\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi (\dot{\varphi}^2 + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4} \left(1 - \frac{\lambda}{4!} q^4 + \frac{J}{2} \int_{-\infty}^{+\infty} q^4 \right)$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2} e^{-\frac{\lambda}{4!} q^4} e^{J q}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(J)$$

$$Z = \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m\dot{q}^2 - \lambda q} = \int dq e^{-\frac{1}{2}m\dot{q}^2} e^{-\lambda q}$$
$$= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(j) = e^{-\frac{\lambda^2}{m^2}} e^{-\lambda j} = \left(\left(\frac{\lambda}{m^2}\right)^n \lambda j^n\right)$$



$$\begin{aligned}
 Z &= \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi(\delta+m) \varphi + \frac{\lambda}{4!} \varphi^4 \right]} \\
 &= \int dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda}{2(4!)} q^8 \right) \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} = \sqrt{\frac{2\pi}{m}} e^{-\frac{J^2}{m^2}} \\
 &= \sum_n \left(-\frac{J}{m} \right)^n A_n = \prod_n \left(1 - \left(\frac{J}{m} \right)^2 \right)^{\frac{1}{2}} \lambda^{\frac{n}{2}}
 \end{aligned}$$

$$\begin{aligned}
 Z &= \int D\varphi e^{-S[\varphi]} \quad \boxed{\left[\frac{1}{2} \varphi(\partial^2 + m^2) \varphi - \lambda \varphi^4 + J\varphi \right]} \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4} \left(1 - \frac{J^2}{4!} q^2 + \frac{J^4}{2!(4!)^2} q^4 \right) \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2}m^2q^2} \boxed{A_n(j)} = \boxed{e^{-\frac{m^2}{2}q^2}} \int_{-\infty}^{+\infty} dq e^{-\frac{m^2}{2}q^2} \boxed{A_n(j)} \\
 Z &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(j)
 \end{aligned}$$

$$\begin{aligned}
 Z &= \int d\varphi e^{-\int dx \left[\frac{1}{2} \varphi (\delta^2 + m^2) \varphi + \frac{\lambda}{4!} \varphi^4 + J \varphi \right]} \\
 &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + S_1 / 1! - \frac{\lambda}{4!} q^4 / \left(\frac{1}{4!} \int q^4 \right)} \\
 Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{\lambda}{4!} q^4 / \left(\frac{1}{4!} \int q^4 \right)} e^{S_1 / 1!} \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(S_1) = e^{-\frac{\lambda}{4!} \int q^4} e^{S_1 / 1!} \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(S_1)
 \end{aligned}$$

$$= \sum_{\lambda} (-1)^{\ell(\lambda)} A_{\lambda}^{(j)}$$

$$\begin{array}{c} | \\ | \\ | \\ | \\ | \\ | \\ | \\ | \end{array} b \quad \times \quad \begin{array}{c} | \\ | \\ | \\ | \end{array} 8 \quad (-1)^{\lambda_j} \left(\frac{1}{m^2} \right)$$



$$Z = \int d\varphi e^{-\frac{1}{2} \int dx \left[\dot{\varphi}^2 + m^2 |\varphi|^2 + \lambda \varphi^4 \right]}$$

$$Z = \int D\varphi e^{-\frac{1}{2} \int d^4x \left[\frac{1}{2} \varphi(x) (\partial_\mu m^2) \varphi(x) + J\varphi \right]} =$$
$$= e^{-\frac{1}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)}$$

$$\mathcal{L} = \int d\varphi \mathcal{E}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4 + \mathcal{J}q} = \int_{-\infty}^{+\infty} dq e^{\frac{-\frac{1}{2}m^2q^2 - \mathcal{J}q}{\lambda}} \left(1 - \frac{\lambda q^2}{4!}\right)^{\frac{1}{2}}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{\lambda}{4!}q^4 - \frac{\mathcal{J}}{\lambda}q^2}$$

$$= \sum_n (-\lambda)^{\frac{n(n+1)}{2}} \boxed{A_n(j)}$$

$$= \sum_n (-\lambda)^{\frac{n(n+1)}{2}} A_n(j)$$

$$Z = \int D\varphi e^{-I\left[\frac{1}{2}\int d^4x \left(\partial^\mu \varphi \partial_\mu \varphi + m^2 |\varphi|^2 + J\varphi\right)\right]} =$$
$$= e^{-\frac{1}{2} \int dx \int dy J(x) D(x-y) J(y)}$$

$$Z = \int d\varphi e^{-\frac{1}{2} \int d^4x \left[\bar{\varphi}(\lambda) (\partial^\mu m^2) \varphi(x) + \bar{\varphi} \right]} =$$

$$-\frac{1}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)$$

$$D(x-y) = \int d^4p \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}$$

$$Z = \int D\varphi e^{-\int d\lambda \left[\frac{1}{2} \varphi \lambda (\lambda^2 + m^2) \varphi_{,\lambda} + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

$$= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + \frac{J}{\lambda} q^4 / (1 - \frac{\lambda}{4!} q^2 + \frac{J}{2(4!)^2} q^8)}$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + \frac{J}{\lambda} q^4 / (1 - \frac{\lambda}{4!} q^2 + \frac{J}{2(4!)^2} q^8)}$$

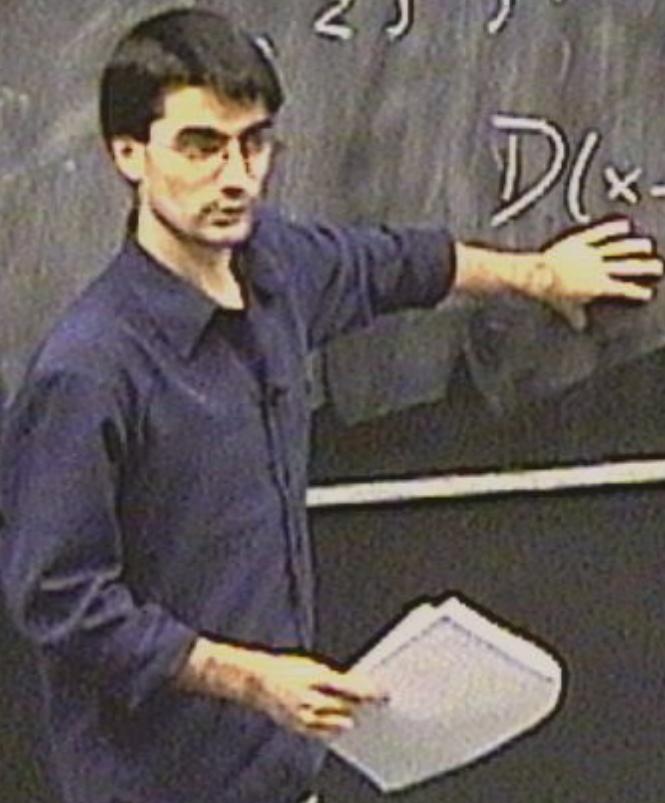
$$= \sum_n (-\frac{\lambda}{4!})^n A_n(J)$$

$$= \sum_n (-\frac{\lambda}{4!})^n A_n$$

$$Z = \int d\varphi e^{-\frac{1}{2} \int d^4x \left[\frac{1}{2} \varphi(\lambda) (\partial^\mu m^2) \varphi(x) + J\varphi \right]} =$$

$$-\frac{1}{2} \int d^4x \int d^4y J(x) D(x-y) J(y)$$

$$D(x-y) = \int d^4p \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}$$



$$= \sum_{n=0}^{\infty} (-\lambda)^n A_n(j)$$

$$\left| \begin{array}{c} | \\ | \\ p \\ | \end{array} \right| \times \left| \begin{array}{c} | \\ | \\ 8 \\ | \end{array} \right| + \lambda j \left(\frac{1}{m} \right)$$

$$= \sum_{j=0}^n (-1)^j A_{n,j}$$

$$\begin{array}{ccccccc} | & | & \times & | & | & 8 & (-1)^{\lambda j \left\lfloor \frac{1}{m} \right\rfloor} \\ | & | & \times & | & | & & \end{array}$$

$$\varphi(x) = \varphi(\textcolor{red}{x}) \varphi(\textcolor{blue}{y}) \varphi(\textcolor{violet}{z}) \varphi(\textcolor{brown}{w}) |\zeta(x-y) \dots$$

$$= \sum_{j=0}^n (-1)^j A_{r_n}(j)$$

$$\left| \begin{array}{c} p \\ | \\ \times \end{array} \right| \quad || \quad 8 \quad () \Delta^j \left(\frac{1}{m^2} \right)$$

$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y)} \dots$$

$$Z = \int D\varphi e^{-S[\varphi]} \left[\frac{1}{2} \varphi (\delta^2 + m^2) \varphi_{xx} - \frac{\lambda}{4!} \varphi^4 + J\varphi \right]$$

$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + J q} \left(1 - \frac{\lambda}{4!} q^4 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(j)}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(j)$$

$$\begin{aligned}
 Z &= \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4 + \beta q} = \int dq e^{-\frac{1}{2}m^2q^2 + \beta q} \left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}(\frac{\lambda}{4!}) \int q^4\right) \\
 &= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(\beta)} = e^{-\frac{1}{2}m^2q^2} e^{\beta q} \left(1 + \left(\frac{\lambda}{4!}\right) q^4 + \left(\frac{\lambda}{4!} \left(\frac{1}{m^2}\right)\right) \beta q^2\right)
 \end{aligned}$$

$$8 \quad (\Delta^2 \left(\frac{1}{m^2}\right))$$

$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y) \dots}$$

$$\begin{aligned}
 Z &= \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4 + \beta q} = \int dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4} \left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2!}\int_0^q s^4 ds\right) \\
 &= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(q) = e^{-\frac{1}{2}m^2q^2} e^{-\frac{\lambda}{3!}q^6} \left(1 - \left(\frac{\lambda}{4!}\right)^2 q^4\right) \\
 &= \sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(q)
 \end{aligned}$$

$$e^{-\frac{1}{2}m^2q^2} \left(1 - \frac{\lambda}{3!}q^6\right)$$

$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \dots$$

$$\begin{aligned}
 Z &= \int_{-\infty}^{\infty} dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4 + \beta q} = \int dq e^{-\frac{1}{2}m^2q^2 - \lambda q^4} \left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2!}\binom{\lambda}{2}q^8\right) \\
 &= \sum_n \left(\frac{-\lambda}{4!}\right)^n A_n(q) = e^{-\frac{1}{2}m^2q^2} e^{-\frac{\lambda}{24}q^4} \left(1 + \left(\frac{\lambda}{12}\right)^2 q^8\right)
 \end{aligned}$$

$$8 \quad (1 \triangle^j (\frac{1}{m^2}))$$

$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \boxed{\delta(x-y) \dots}$$

- 3d Ponzano-Regge Model (geometry)
- idea of Feynman diagrams
- Group field theory
 - idea
 - general formalism
 - 3d example
- D. ORITI , gr-qc/0512103
- L. FREIDEL, hep-th/0505046 outlook

$$\omega(\gamma_{\mu}) \cdot \bar{\epsilon}$$

$$e^{-\ell_p(\gamma_{\mu}, \dots)}$$

$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi R(\delta^2 + m^2) \varphi_{,0} + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

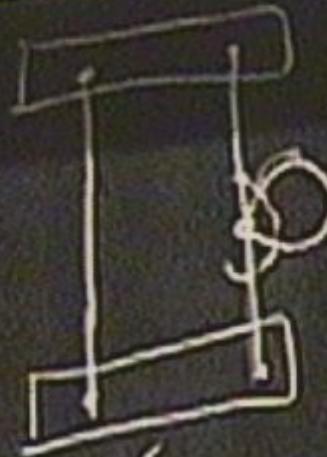
$$Z = \int_0^{+\infty} e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int_0^{+\infty} e^{-\frac{1}{2} m^2 q^2 + S_I} \left(1 - \frac{\lambda}{4!} q^4 + \frac{J}{2!} q^2 \right)$$

$$\sum_n \left(-\frac{\lambda}{4!}\right)^n A_n(j) = \lambda e^{-\frac{\lambda}{4!} j^4}$$

$$\sum_R \left(\frac{\lambda}{4!}\right)^n \overline{A}_R(j)$$

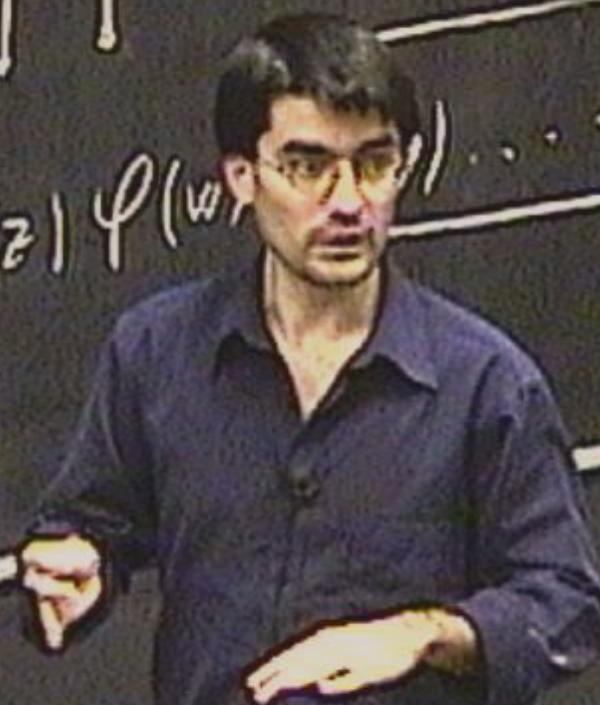
$$\begin{aligned}
 Z &= \int d\varphi e^{-\frac{i}{2} \int d\lambda \left[\bar{\varphi}(\lambda) \left(\partial_\lambda^2 m^2 / \varphi(x) + J\varphi \right) \right]} = \\
 &\quad - \frac{1}{2} \left(d^3x \int d^3y J(x) D(x-y) J(y) \right) \\
 &= e^{-\frac{1}{2} \int d^3p D(p) \frac{e^{ip \cdot (x-y)}}{p^2 - m^2 + i\epsilon}}
 \end{aligned}$$

$$= \sum_{n_k} (-\lambda)^{n_k} [A_{m_k}(j)]$$



$$8 \left(\Delta j \left(\frac{1}{m^2} \right) \right)$$

$$\varphi(x) = \varphi(x) \varphi(y) \varphi(z) \varphi(w) \dots$$



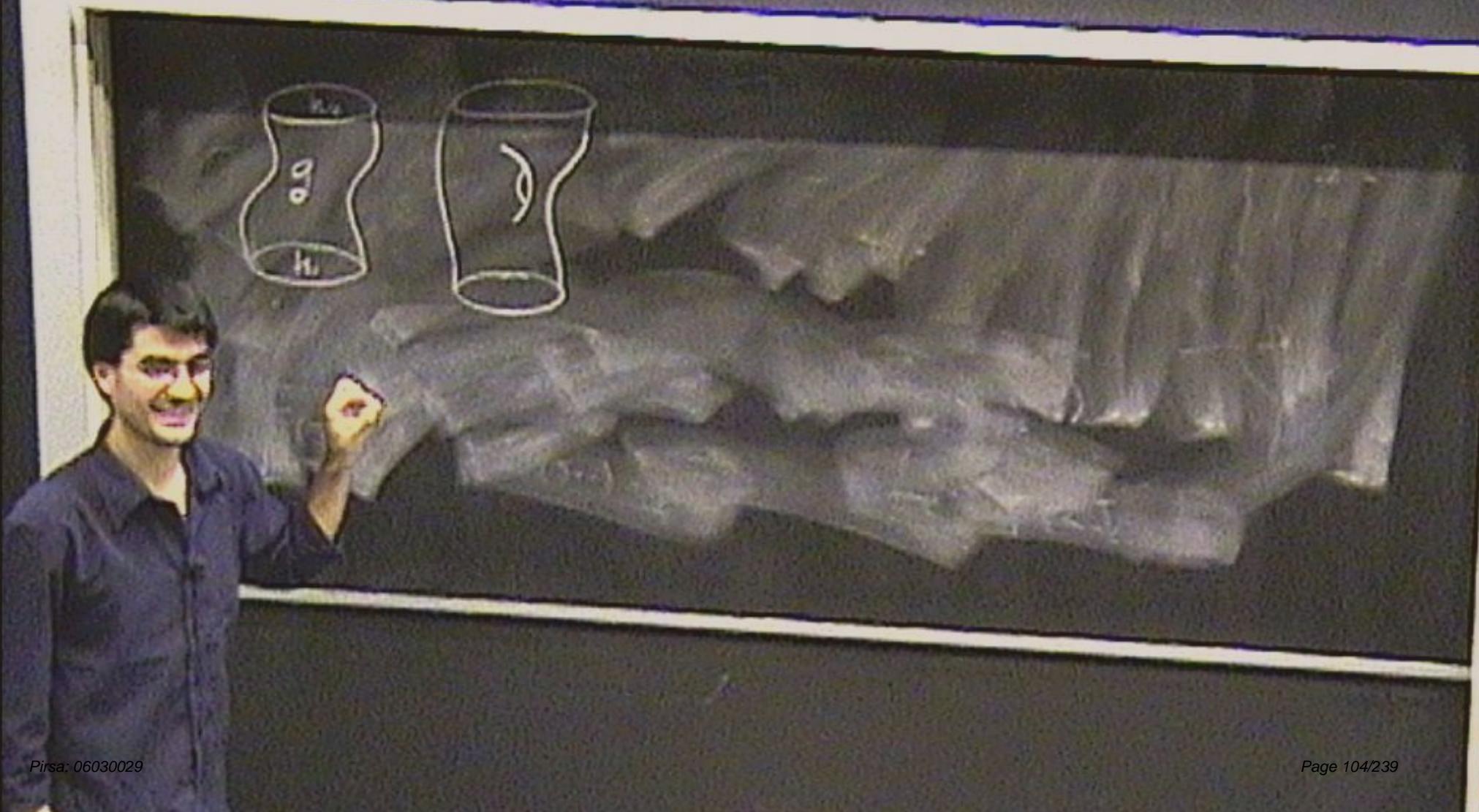
$$= \sum_{n=0}^{\infty} (-\frac{1}{4})^n A_n(j)$$

$$\psi(x) = \varphi(x) \psi_0 + \varphi(z) \psi_0 (x-z) \dots$$

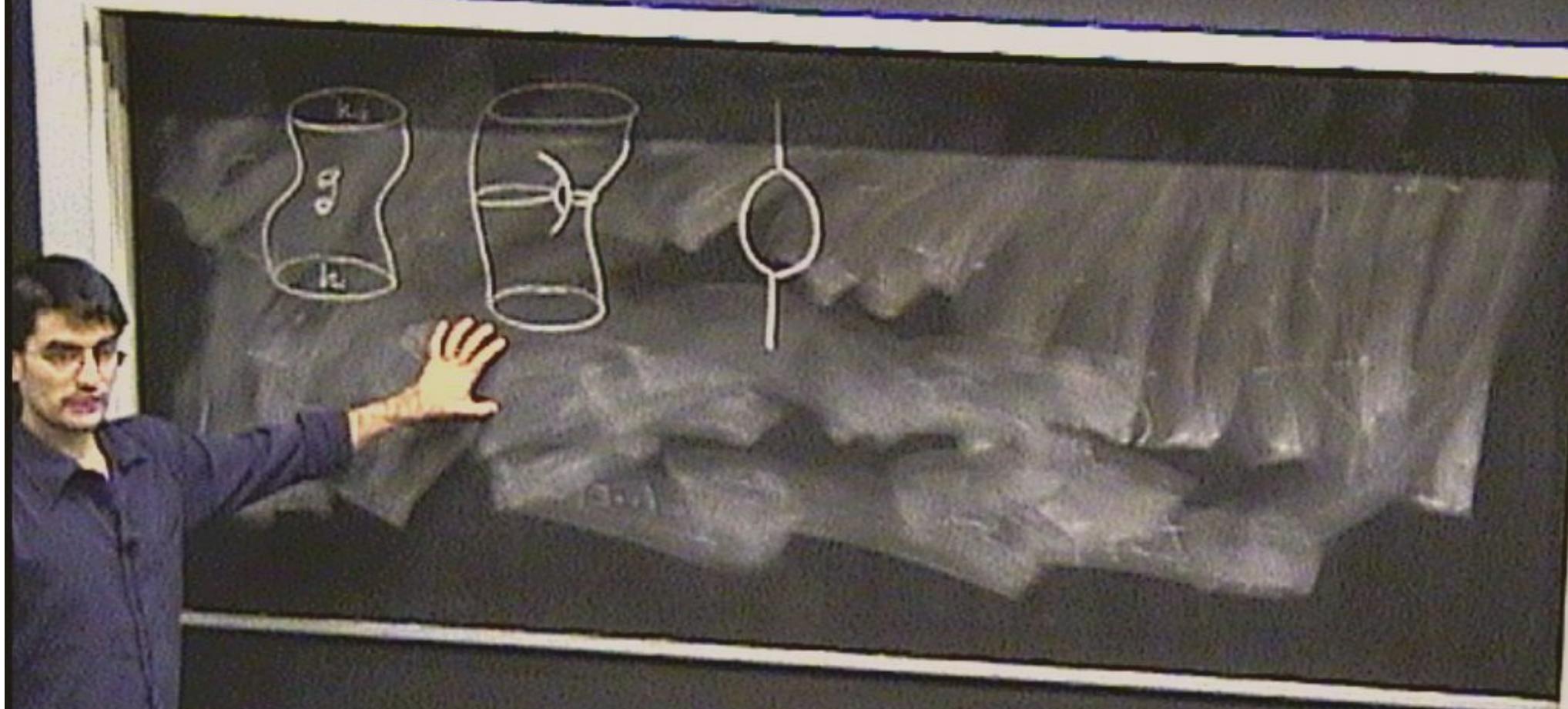
- L. FREIDEL, hep-th/0505016 out look — 5d example



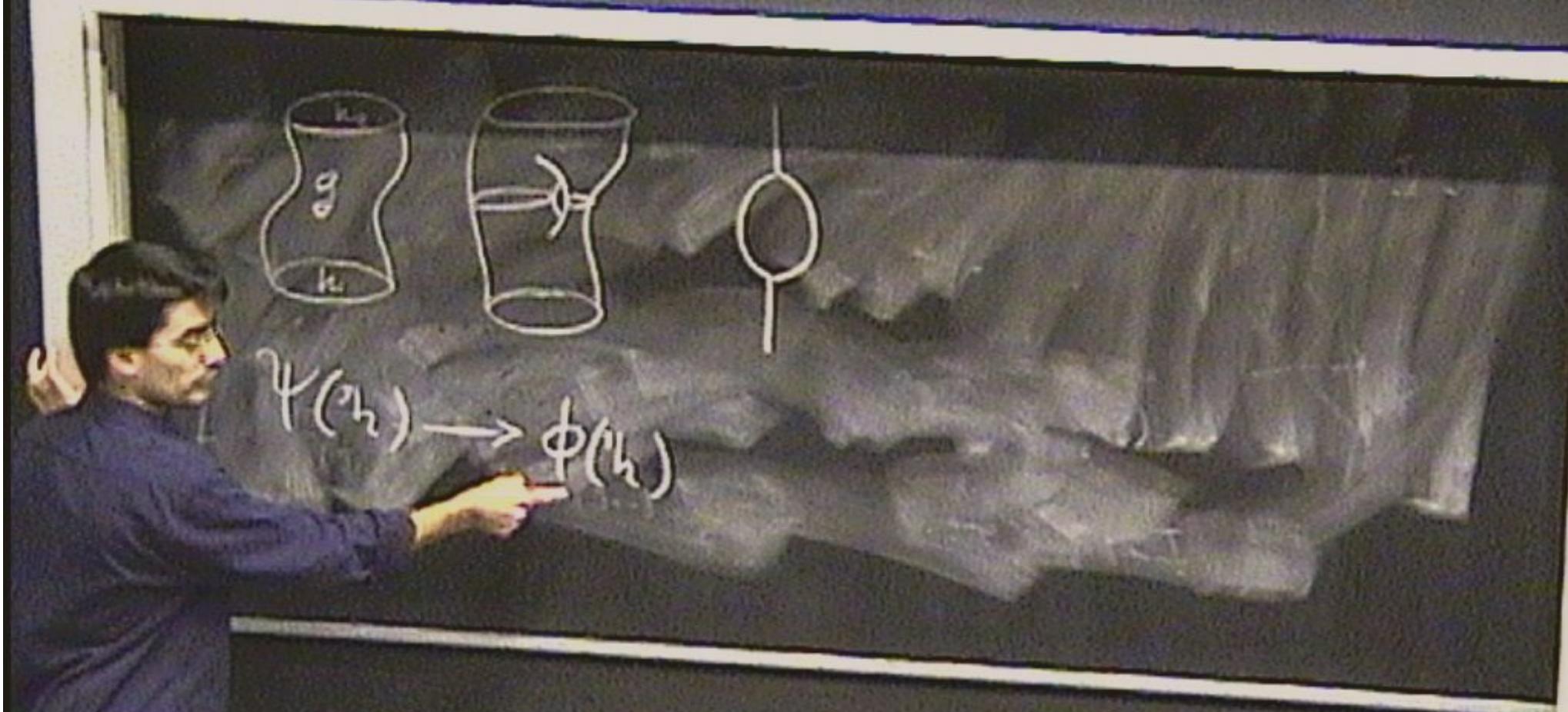
- L. FREI DEL, hep-th/0505016 outlook — 5d example



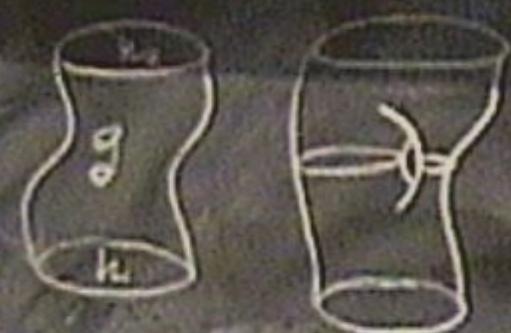
- L. FREIDEL, hep-th/0505016 outlook — 5d example



- L. FREIDEL, hep-th/0505016 outlook — 5d example



- L. FREIDEL, hep-th/0505016 outlook — 5d example

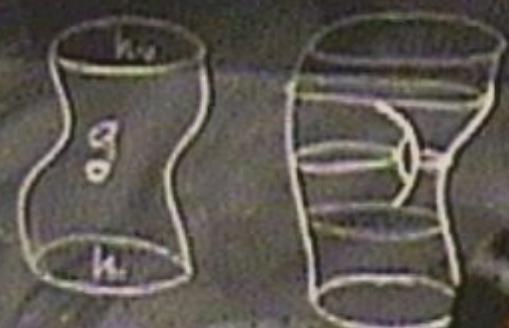


$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

Marco

- L. FREIDEL, hep-th/0505016 out look — So example

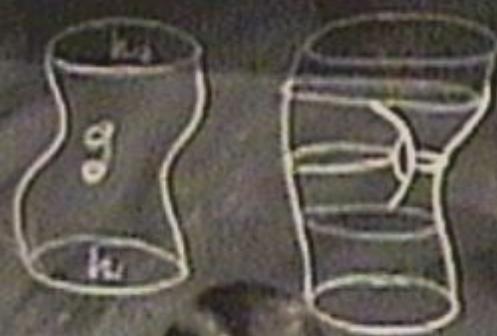


$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi$$



- L. FREIDEL, hep-th/0505016 outlook — So example



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H} Z_{(h_1, h_2)} = \delta S(h_1 - h_2)$$

$$= \frac{1}{R} \left(\frac{1}{f_1} \right) \left[A_{f_1}^{(3)} \right]$$

$$\psi(j_1, j_2, j_3)$$



$$= \frac{1}{R_h} \left(\frac{1}{f_1} \right) \left[\frac{1}{M_h} \right]^{(3)}$$

$$\psi(j_1, j_2, j_3)$$

$$\chi^j(g) = \sum_j f_{m\lambda}^j D_{m\lambda}^j(g)$$



$$= \lim_{n \rightarrow \infty} \left(\frac{1}{n!} \right) \left[A^{\otimes n} \right]$$

$$\psi(j_1, j_2, j_3)$$

$$\chi_j(g)$$

$$g(g) = \sum_j \int_{\text{mx}}^j D_{mk}^{jk} g$$

$$g(x) = \int dP \, g(p) e^{ip \cdot x}$$

$$= \frac{1}{R} \left(\frac{1}{\Delta} \right) \left[\Delta \frac{\partial}{\partial \Delta} \right]$$

$$\psi(j_1, j_2, j_3)$$



$$= \frac{A(A)}{R_k} \left[A \right]^{(s)}$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(j_1, g_1, g_2)$$



$$= \frac{1}{R_n} \left(\frac{1}{f_1} \right) \left[A \lambda \tau_n^{(3)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g, g, g, g, g)$$



$$= \frac{1}{R} \left(\frac{1}{f_1} \right) \left[A_{\text{M}} \right]^{f_2}$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g_1, g_2, g_3, g_4, g_5)$$

$$G \times G \times G \rightarrow \mathbb{R}$$



$$= \frac{1}{R_h} \left(\frac{1}{\Delta_1} \right) \left[\Delta_1 \Delta_{h_1}^{-1} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g_1, g_2, g_3, g_4, g_5)$$

$$\overline{\mathcal{C}} \times \mathcal{C} \times \mathcal{C} \rightarrow \mathbb{R}$$

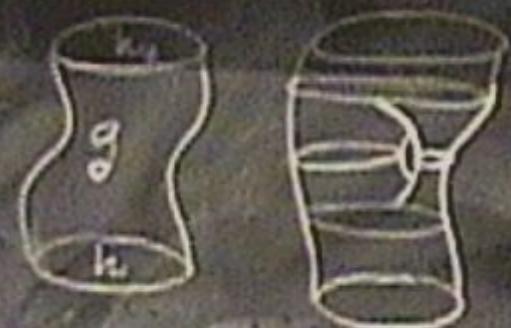
$$= \frac{1}{R} \left(\frac{1}{f_1} \right) \left[F \cdot \frac{1}{f_1} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g, g, g, g, g)$$

$$\Delta \rightarrow \varphi(j_1, j_2, j_3)$$

$$\overbrace{G \times G \times G}^G \rightarrow \mathbb{R}$$

- L. FREIDEL, hep-th/0505016 out look - So example



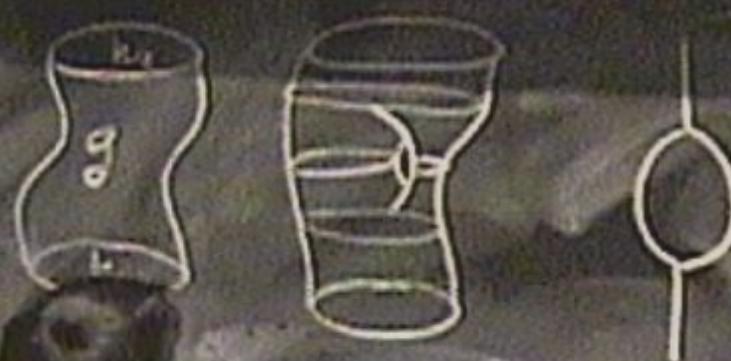
$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$H \Xi_{(h_1, h_2)} = \delta(h_1 - h_2)$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g, g, g)$$
$$G \times G \times G \xrightarrow[G]{} \mathbb{R}$$

- L. FREIDEL, hep-th/0505016 outlook — 5d example

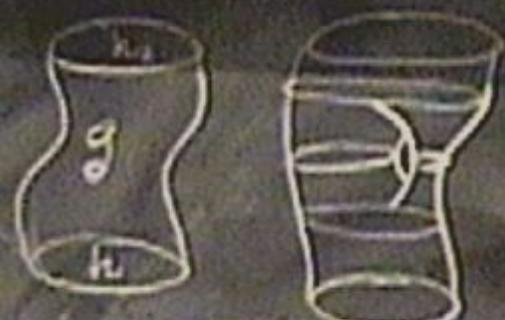


$$\phi(h)$$

$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$H \tilde{Z}(h_+, h_-) = i S(h_+ - h_-)$$

- L. FREIDEL, hep-th/0505016 outlook



$$Z = \int \prod g e^{iS(g)}$$

$$\psi(h) \rightarrow \phi(h)$$

$$H(\sum_{(h_+, h_-)} \psi S(h_+, h_-)) = \psi S(h_-, h_+)$$

$$= \frac{1}{R} \left(\frac{1}{4!} \right) \left[A_{\text{in}}^{(3)} \right]$$

$$\varphi(j_1, j_2, j_3) \rightarrow \varphi(g_1, g_2, g_3) = \varphi(g_1, g_2, g_3, g_4, g_5)$$

$$\Delta \rightarrow \varphi(j_1, j_2, j_3)$$

$$G \times G \times G \xrightarrow[G]{\varphi} \mathbb{R}$$



$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \psi(\varphi) (\dot{\varphi} + m^2) + \frac{\lambda}{4!} \varphi^4 + J\varphi \right]}$$

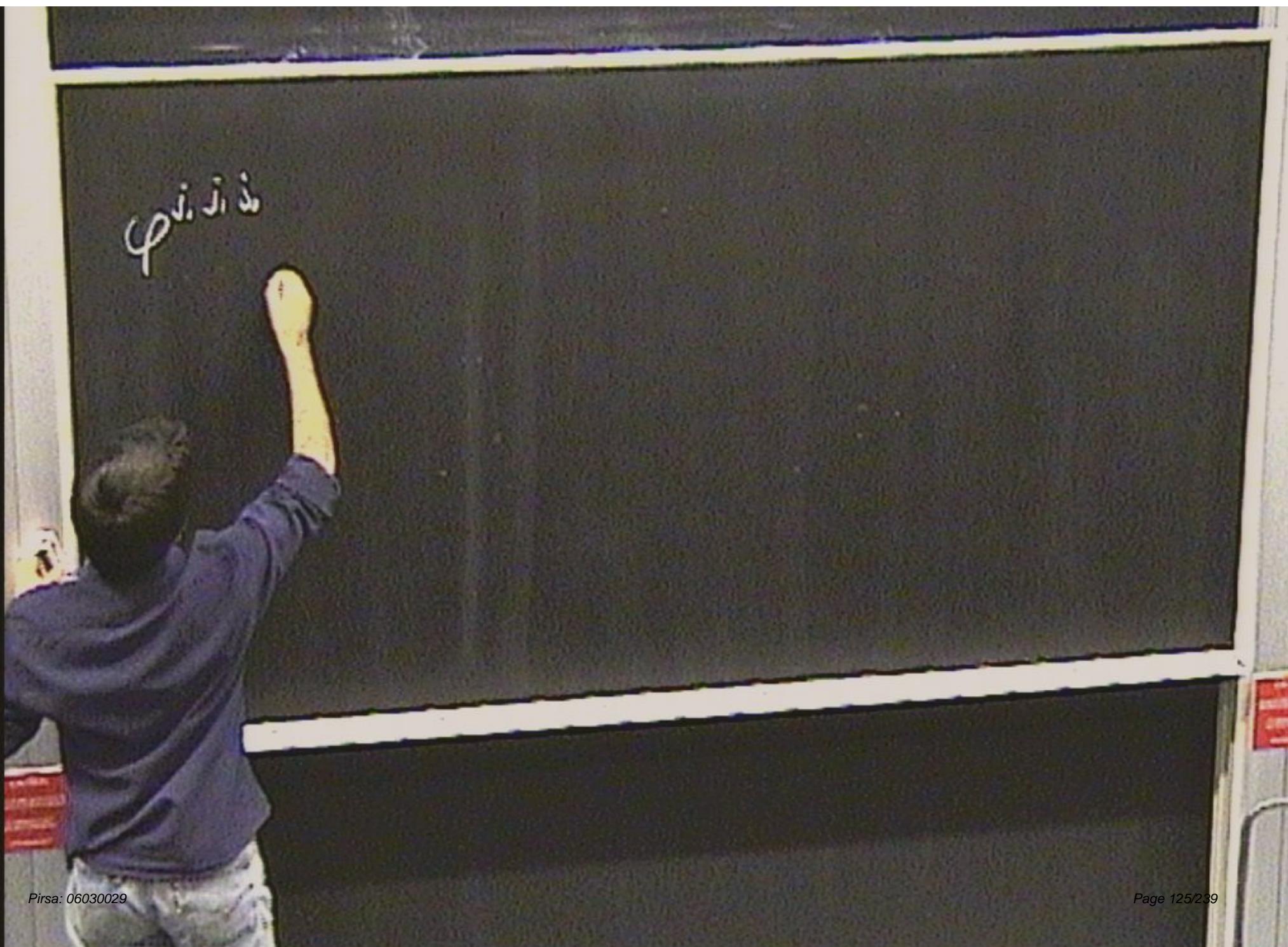
$$Z = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + \frac{\lambda}{4!} q^4} \left(1 - \frac{\lambda}{4!} q^4 + \frac{\lambda}{2(4!)} q^8 \right)$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(j) = (e^{-\frac{\lambda}{4!} q^4})^{\frac{1}{q^2}} e^{\frac{\lambda}{2(4!)^2} q^8}$$

$$= \sum_n \left(-\frac{\lambda}{4!} \right)^n A_n(j)$$

$$(e^{-\frac{\lambda}{4!} q^4})^{\frac{1}{q^2}} e^{\frac{\lambda}{2(4!)^2} q^8} = \left(\prod_{j=1}^{\infty} \left(1 - \frac{\lambda}{4!} j^4 \right)^{\frac{1}{j^2}} \right) e^{\frac{\lambda}{2(4!)^2} \sum_{j=1}^{\infty} j^8}$$

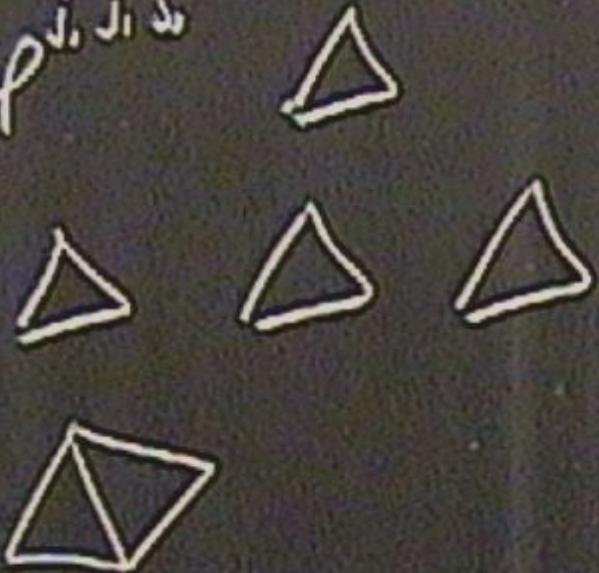
مذکور



مذکور



$\varphi_{j,j}$



$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$  $\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$

φ_{j_1, j_2, j_3}

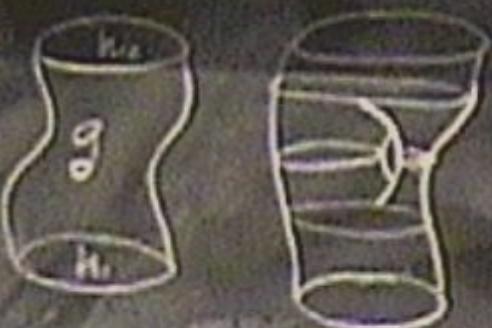


$\varphi_{j_1, j_2, j_3, j_4}$

K_1, K_2, K_3



- L. FREIDEL, hep-th/0505016 Outlook — So example



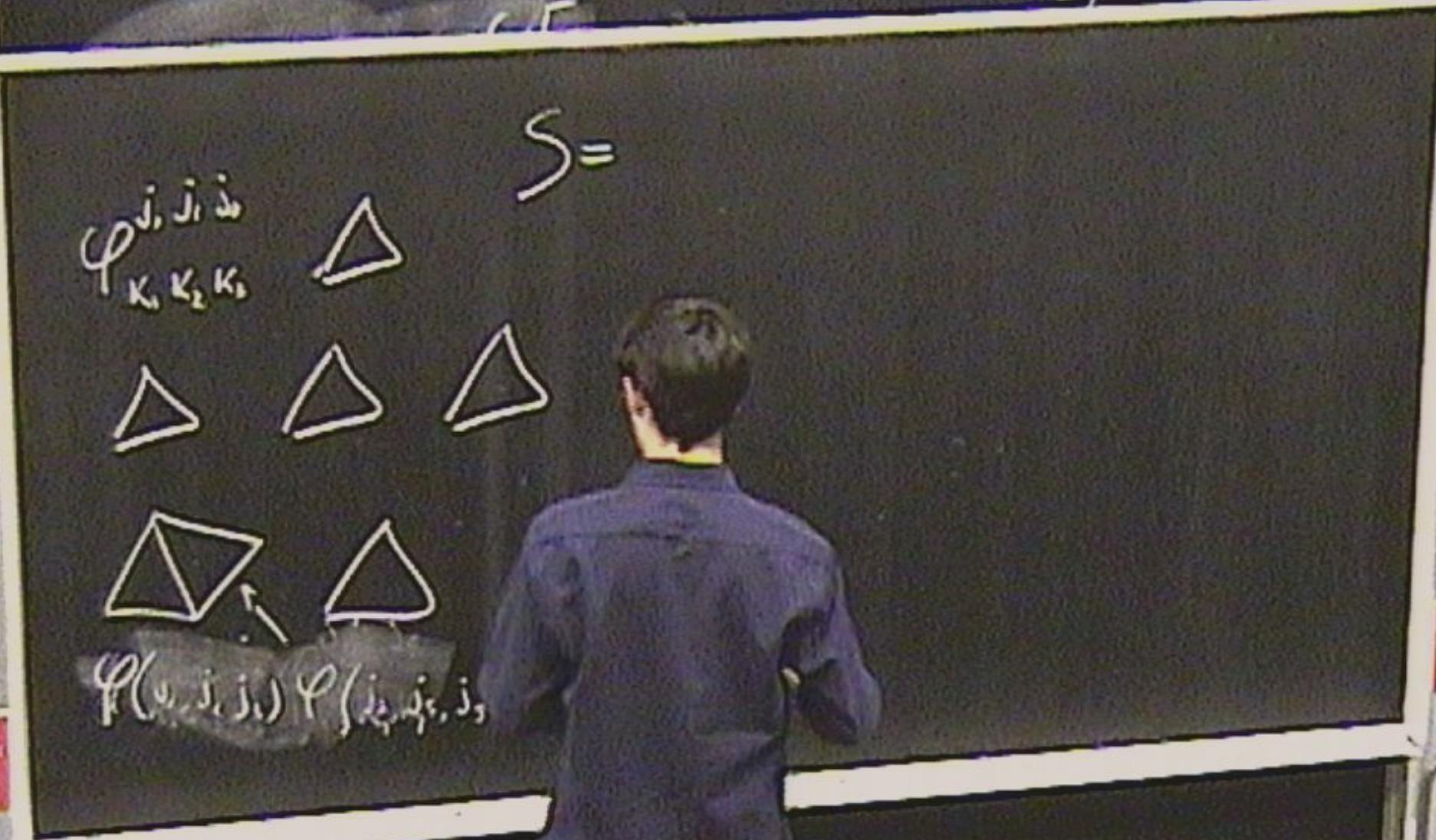
$$Z = \int Dg e^{iS(g)}$$

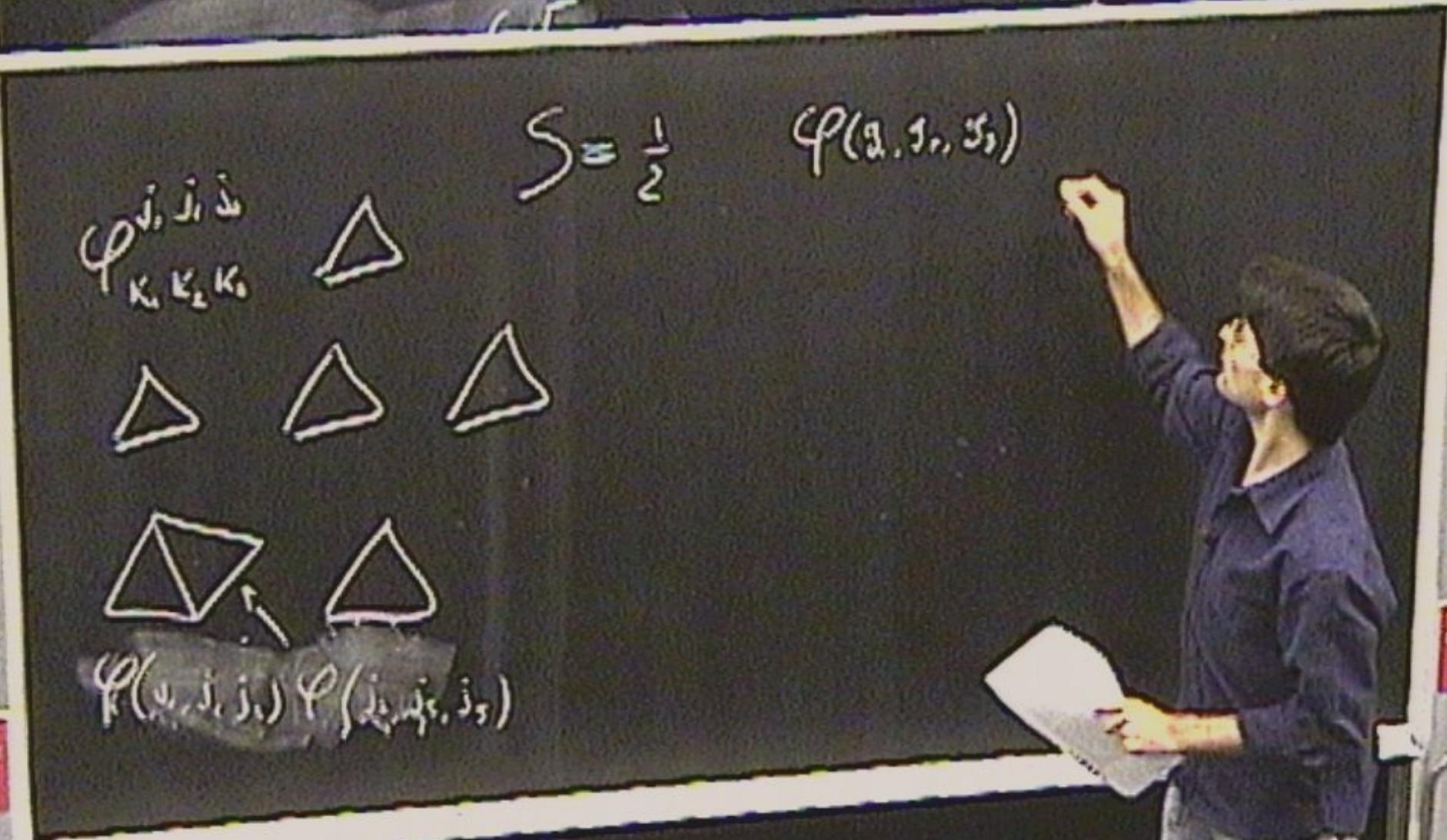
$$\psi(h) \rightarrow \phi(h)$$

$$\mathcal{H} Z_{(h_1, h_2)} = \delta S(h_1 - h_2)$$

φ_{j_1, j_2, j_3}  φ_{j_1, j_2, j_3} 

$\varphi_{K_1, K_2, K_3}^{j_1, j_2, j_3}$  $\varphi(j_1, j_2, j_3) \varphi(j_4, j_5, j_6)$





$$S = \frac{1}{2} \varphi(x_1, x_2, x_3) K(\quad) \varphi(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$$
$$\varphi_{K, K, K}$$
$$\varphi(y) \delta(\pm m^2)$$

A person in a blue shirt is writing on a chalkboard. The board contains several hand-drawn geometric shapes, specifically triangles, some filled with black ink and some outlined in yellow. There are also mathematical expressions written in white chalk, including the formula for a tensor S, terms involving the function phi and its arguments x and y, and a symbol resembling a K with a diagonal line through it. The person's arm is extended towards the right side of the board where the formula is being written.

$$S = \frac{1}{2} \varphi(g, j_1, s_1) K() \varphi(\tilde{g}, \tilde{j}_1, \tilde{s}_1)$$
$$\varphi(x) \varphi(y) (\delta^{j_1}_{x+m}) \delta(x-y) \varphi(y)$$

$\varphi_{K_1 K_2 K_3}$



φ_{j_1, j_2, j_3}



φ_{s_1}

j_1, j_2

$$S = \frac{1}{2} \varphi(z_1, z_2, z_3) \bar{\varphi}(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3)$$
$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$
$$\times \varphi(x) (\delta^2 + m^2) \delta(x-y) \varphi(y)$$


A person wearing a blue shirt is standing in front of a chalkboard, writing with a chalk stick. The chalkboard contains mathematical expressions and diagrams. At the top right, there is a formula involving three variables z_1, z_2, z_3 and their complex conjugates $\tilde{z}_1, \tilde{z}_2, \tilde{z}_3$. To the left of this, there is a formula involving indices j_1, j_2, j_3 and labels K_1, K_2, K_3 . Below these, there is a product of two functions, $\varphi(x) \varphi(y)$, multiplied by a term involving a delta function and a square root of a squared distance plus a mass squared. On the left side of the board, there are several diagrams: a single triangle, a double triangle, a triple triangle, and a pentagon-like shape. At the bottom left, there are two more formulas involving indices i_1, i_2, i_3 and i_4, i_5, i_6 .

$$S = \frac{1}{2} \int d^3x \varphi(x) K(x-y) \varphi(y)$$
$$\int dx \left[\varphi(x) \times \left[\delta^2 + m^2 \right] \delta(x-y) \right] \varphi(y)$$

$\varphi_{j_1 j_2 j_3}$

$\varphi_{k_1 k_2 k_3}$

$\varphi_{l_1 l_2 l_3}$

$\varphi_{m_1 m_2 m_3}$

$\varphi_{n_1 n_2 n_3}$

$\varphi_{o_1 o_2 o_3}$

$\varphi_{p_1 p_2 p_3}$

$\varphi_{q_1 q_2 q_3}$

$\varphi_{r_1 r_2 r_3}$

$\varphi_{s_1 s_2 s_3}$

$\varphi_{t_1 t_2 t_3}$

$\varphi_{u_1 u_2 u_3}$

$\varphi_{v_1 v_2 v_3}$

$\varphi_{w_1 w_2 w_3}$

$\varphi_{x_1 x_2 x_3}$

$\varphi_{y_1 y_2 y_3}$

$\varphi_{z_1 z_2 z_3}$

$\varphi_{\tilde{j}_1 \tilde{j}_2 \tilde{j}_3}$

$\varphi_{\tilde{k}_1 \tilde{k}_2 \tilde{k}_3}$

$\varphi_{\tilde{l}_1 \tilde{l}_2 \tilde{l}_3}$

$\varphi_{\tilde{m}_1 \tilde{m}_2 \tilde{m}_3}$

$\varphi_{\tilde{n}_1 \tilde{n}_2 \tilde{n}_3}$

$\varphi_{\tilde{o}_1 \tilde{o}_2 \tilde{o}_3}$

$\varphi_{\tilde{p}_1 \tilde{p}_2 \tilde{p}_3}$

$\varphi_{\tilde{q}_1 \tilde{q}_2 \tilde{q}_3}$

$\varphi_{\tilde{r}_1 \tilde{r}_2 \tilde{r}_3}$

$\varphi_{\tilde{s}_1 \tilde{s}_2 \tilde{s}_3}$

$\varphi_{\tilde{t}_1 \tilde{t}_2 \tilde{t}_3}$

$\varphi_{\tilde{u}_1 \tilde{u}_2 \tilde{u}_3}$

$\varphi_{\tilde{v}_1 \tilde{v}_2 \tilde{v}_3}$

$\varphi_{\tilde{w}_1 \tilde{w}_2 \tilde{w}_3}$

$\varphi_{\tilde{x}_1 \tilde{x}_2 \tilde{x}_3}$

$\varphi_{\tilde{y}_1 \tilde{y}_2 \tilde{y}_3}$

$\varphi_{\tilde{z}_1 \tilde{z}_2 \tilde{z}_3}$

$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$

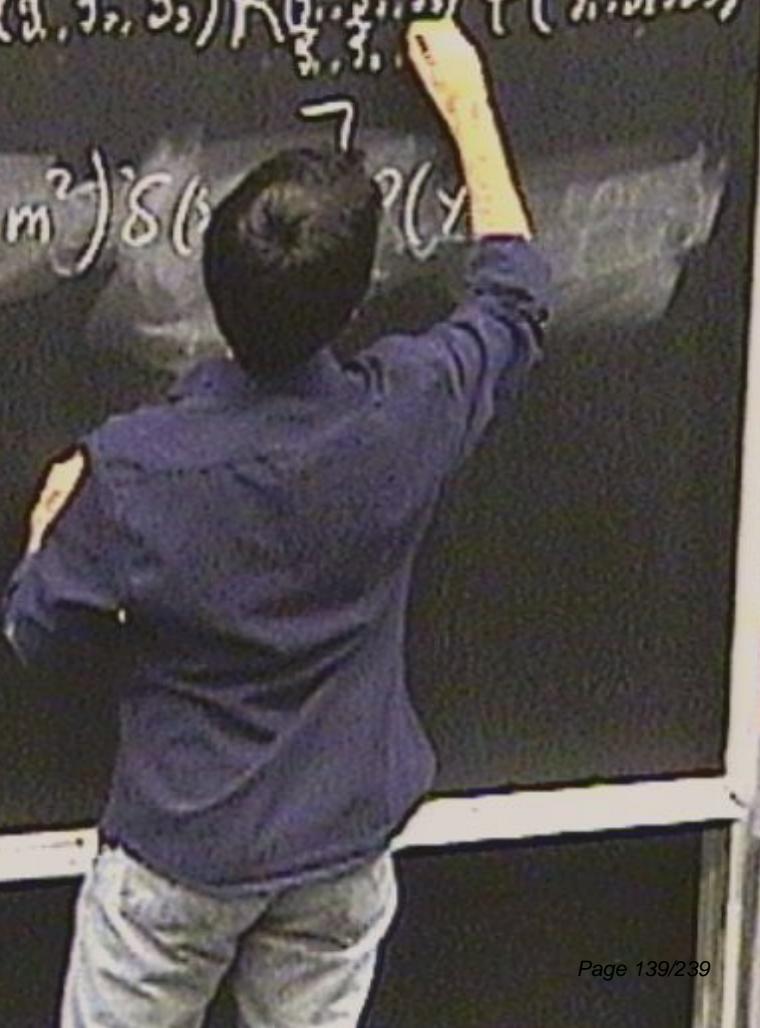


$$\varphi_{j_1 j_2 j_3} \varphi_{j_4 j_5 j_6}$$

$$S = \frac{1}{2}$$

$$\varphi_{(g, j_1, s_1)} K(g, j_1, s_1) \varphi(\tilde{s}_1, \tilde{j}_1, \tilde{s}_1)$$

$$\int dx \varphi(x) \left[\delta^2 + m^2 \right] \delta(x - y) \varphi(y)$$



$$S = \frac{1}{2} \varphi(g_1, g_2, g_3) K(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$
$$\left(\frac{dx}{dy} \varphi \times \left[(\delta^2 + m^2) \delta(x-y) \right] \varphi(y) \right)$$

$\varphi_{j_1 j_2 j_3}$

$\varphi_{\tilde{j}_1 \tilde{j}_2 \tilde{j}_3}$

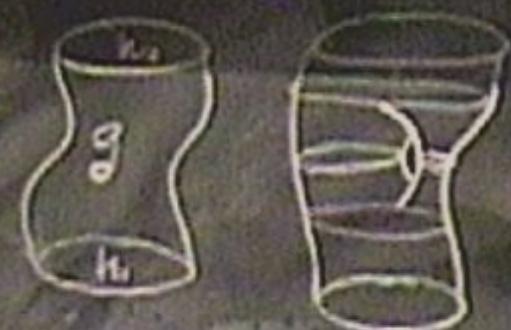
$\varphi_{j_1 j_2 j_3} \varphi_{\tilde{j}_1 \tilde{j}_2 \tilde{j}_3}$

The chalkboard contains several mathematical expressions and diagrams. At the top right is a string diagram with three external legs labeled j_1, j_2, j_3 and three internal legs labeled $\tilde{j}_1, \tilde{j}_2, \tilde{j}_3$. To its left is the equation $S = \frac{1}{2} \varphi(g_1, g_2, g_3) K(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$. Below this is another equation involving the derivative $\frac{dx}{dy}$ and the function φ . On the left side of the board, there are several diagrams of triangles and trapezoids, some with internal lines, and below them are two more mathematical expressions: $\varphi_{j_1 j_2 j_3}$ and $\varphi_{\tilde{j}_1 \tilde{j}_2 \tilde{j}_3}$. At the bottom left is the expression $\varphi_{j_1 j_2 j_3} \varphi_{\tilde{j}_1 \tilde{j}_2 \tilde{j}_3}$.

$$S = \frac{1}{2} \prod_{\text{SU}(2)} f^{j_1 j_2 j_3} \varphi(g_1, s_1, s_2) K(\tilde{g}_1, \tilde{s}_1, \tilde{s}_2) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$

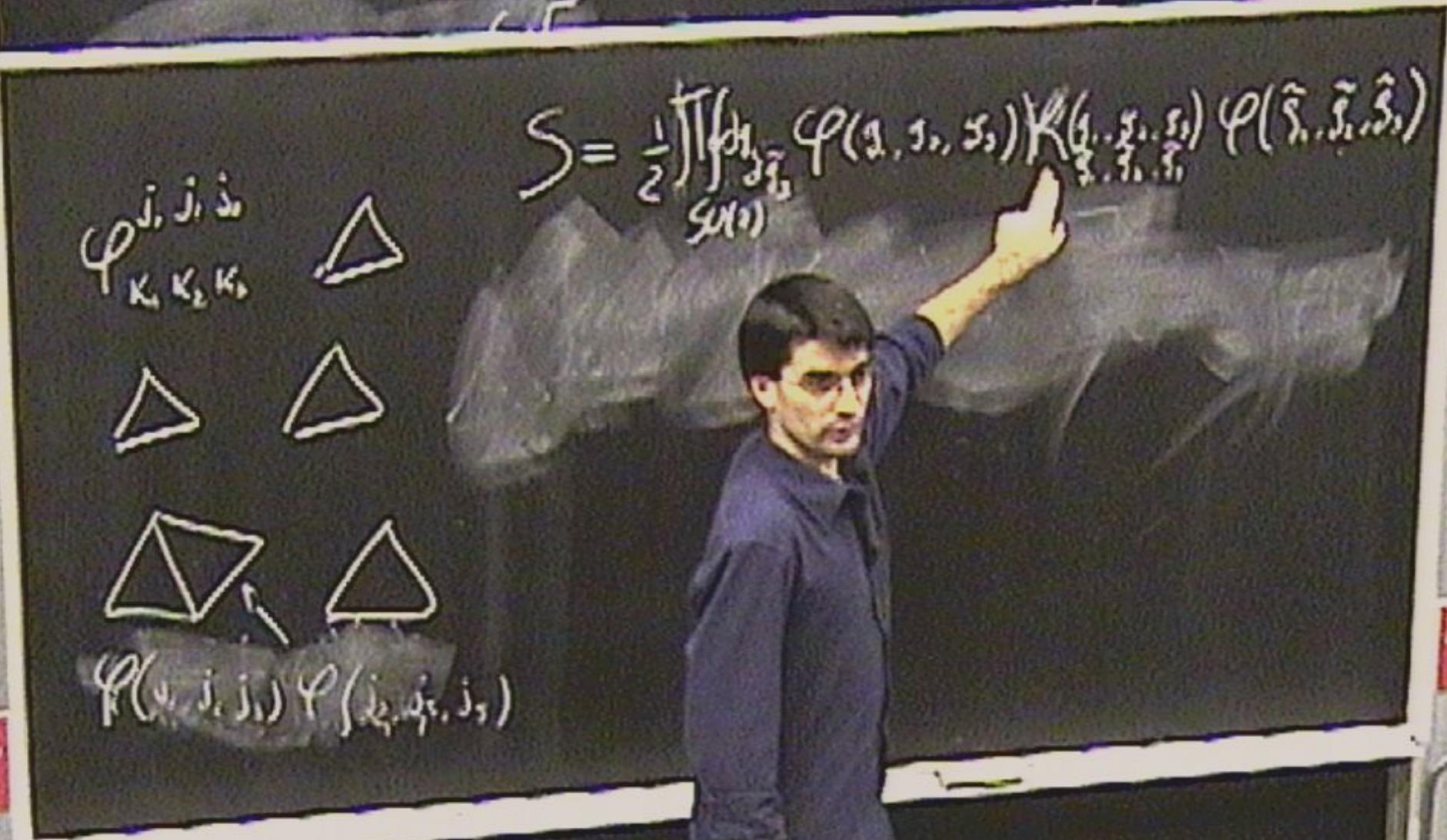
- L. FREIDL, hep-th/0505016 outlook - 5d example



$$Z = \int \mathcal{D}g e^{iS(g)}$$

$$\mathcal{H}(h) \rightarrow \phi(h)$$

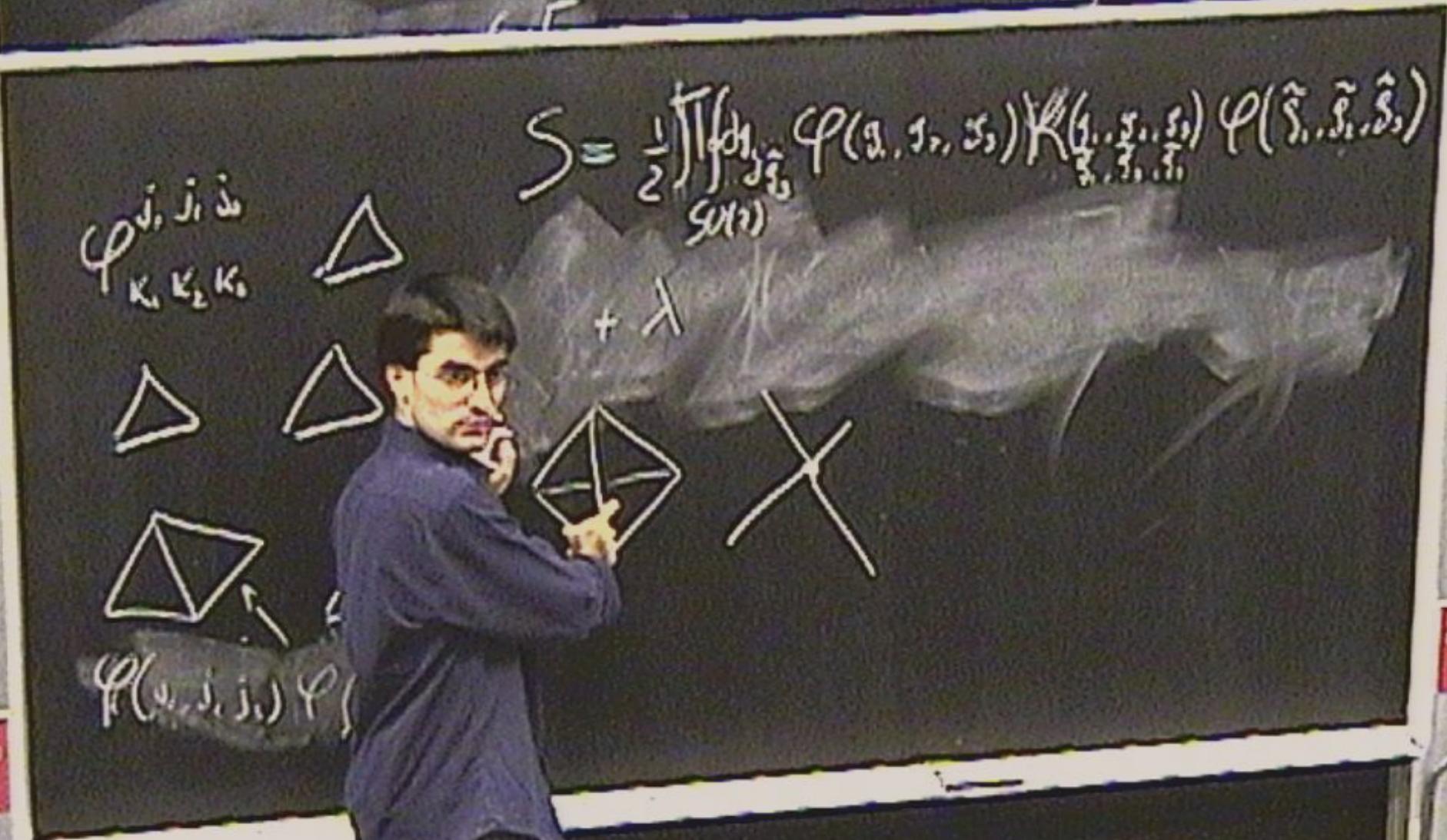
$$\mathcal{H}\tilde{Z}_{(h_1, h_2)} = iS(h_1 - h_2)$$



$$S = \frac{1}{2} \int \prod_{i=1}^3 f_{j_i, \tilde{j}_i} \varphi(g_i, j_i, \tilde{j}_i) K(g_i, j_i, \tilde{j}_i) \varphi(\tilde{g}_i, \tilde{j}_i, \tilde{\tilde{j}}_i)$$

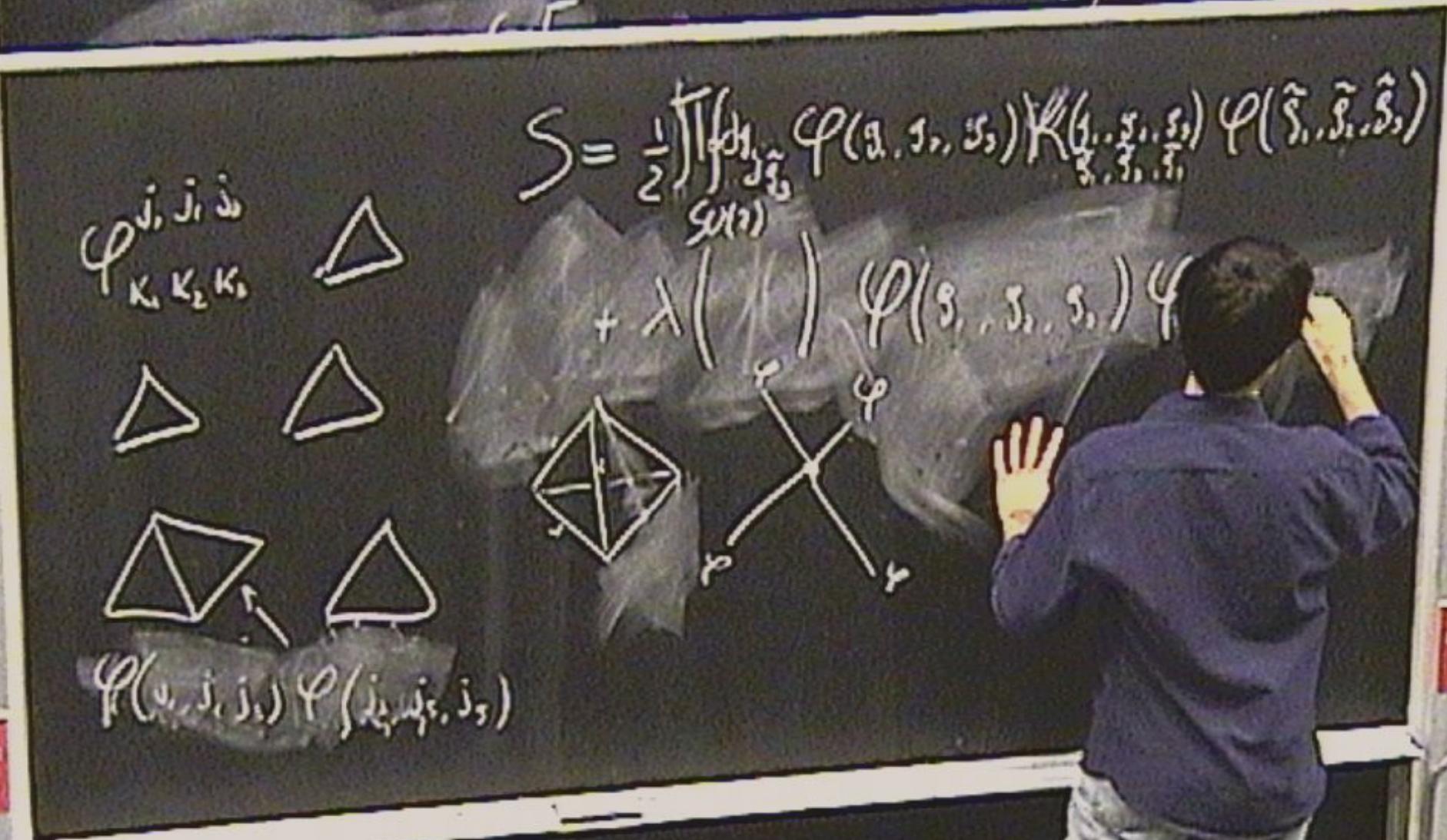
$\varphi_{K_1, K_2, K_3}^{j_1, j_2, \tilde{j}_3}$

$+ \lambda$



$$S = \frac{1}{2} \int_{SU(2)} f_{\alpha_1 \alpha_2 \alpha_3} \varphi(\alpha_1, \alpha_2, \alpha_3) K(\beta_1, \beta_2, \beta_3) \varphi(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$$

$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$



$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



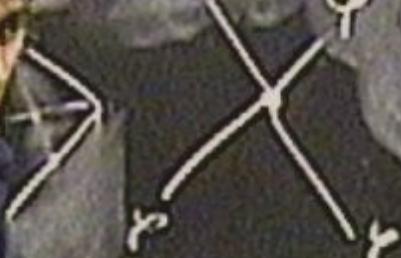
$$\varphi(s_1 s_2 s_3) \varphi(j_1 j_2 j_3)$$



$$S = \frac{1}{2} \int \int f_{\alpha_1 \alpha_2 \alpha_3} \varphi(j_1 j_2 j_3) K(s_1 s_2 s_3) \varphi(\tilde{s}_1 \tilde{s}_2 \tilde{s}_3)$$

$$+ \lambda \left(\dots \right) \varphi(s_1 s_2 s_3) \varphi(\tilde{s}_1 \tilde{s}_2 \tilde{s}_3)$$

$$\varphi \delta(s_1 s_3)$$



$$S = \frac{1}{2} \prod_{\substack{\text{factors} \\ \text{SU}(2)}} \varphi(s_1, \tilde{s}_1, s_2) \mathcal{K}(s_3, \tilde{s}_3, \tilde{s}_4) \varphi(\tilde{s}_5, \tilde{s}_1, \tilde{s}_6)$$

$$+ \lambda \left(\begin{array}{c} | \\ | \end{array} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi \delta(s_3, \tilde{s}_1) \varphi(\tilde{s}_5, \tilde{s}_2, \tilde{s}_6)$$

$$S = \frac{1}{2} \int \prod_{i=1}^4 \varphi(j_i, \tilde{j}_i) \varphi(g_i, s_i, \tilde{s}_i) K(s_i, \tilde{s}_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_i, \tilde{s}_i)$$

$SU(2)$

$$+ \lambda \left(\begin{array}{c} | \\ | \end{array} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$
$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$
$$\delta(s_2, \tilde{s}_2)$$

$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi(j_1, j_2, j_3)$$

$$S = \frac{1}{2} \int d^3 j_1 d^3 j_2 \varphi(j_1, j_2, j_3) K(j_1, j_2, j_3) \varphi(\tilde{j}_1, \tilde{j}_2, \tilde{j}_3)$$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$



$$\varphi \delta(s_1, s_2) \varphi(\tilde{s}_1, \tilde{s}_2, s_3)$$

$$\delta(s_2, \tilde{s}_3) \delta(s_1, \tilde{s}_3)$$

$$\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, s_2, s_3)$$

...

$$S = \frac{1}{2} \int \prod_{i=1}^3 \varphi(s_i, j_i, s_i) K(s_i, \tilde{s}_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{j}_i, \tilde{s}_i)$$

$$\varphi_{K_1 K_2 K_3}^{j_1, j_2, j_3}$$



$$+ \lambda \left(\quad \right) \varphi(s_1, j_1, s_1) \varphi(\tilde{s}_1, \tilde{j}_1, \tilde{s}_1)$$

$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{j}_1, \tilde{s}_1)$$

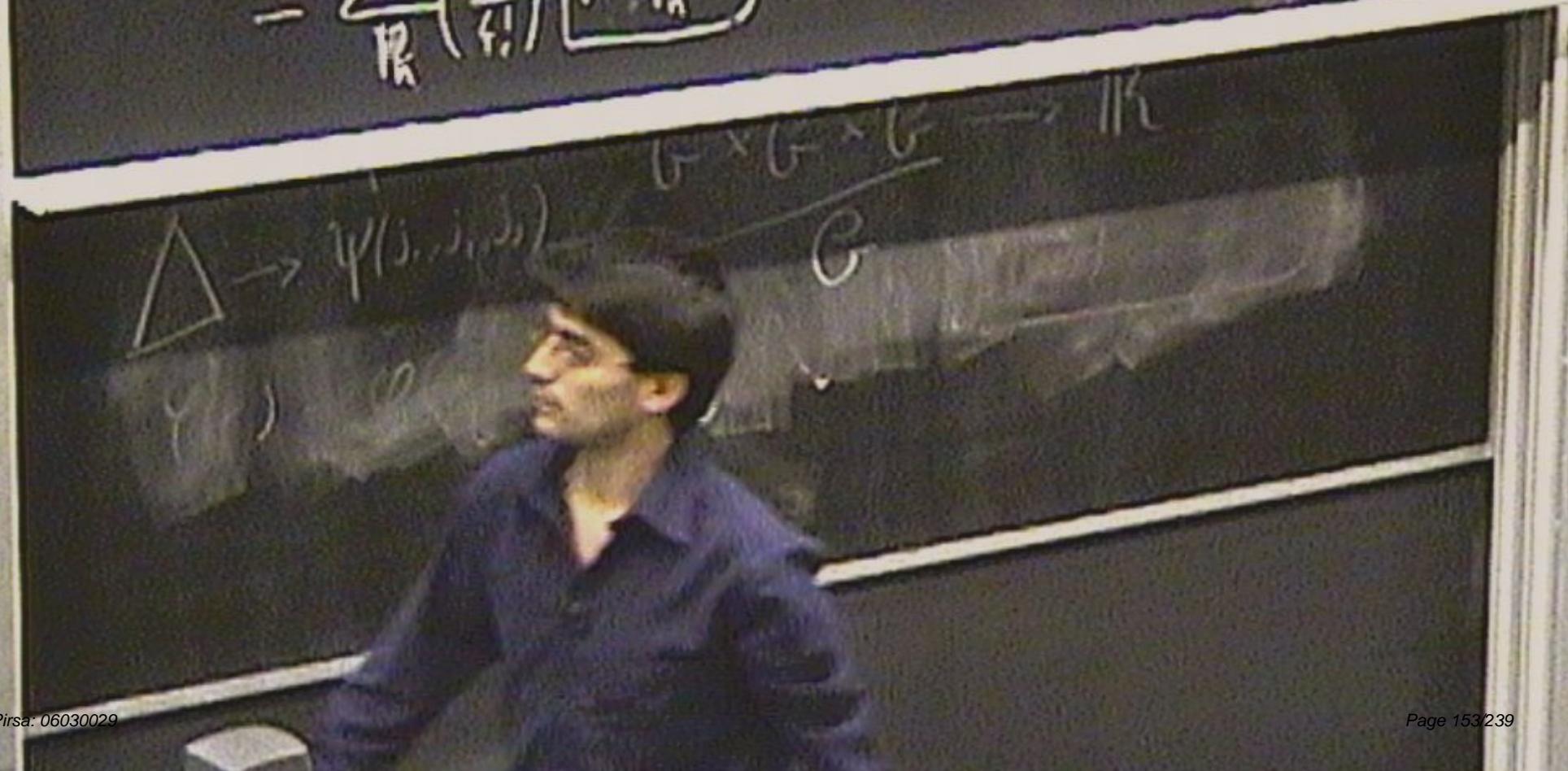
$$\delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{j}_1)$$

$$\varphi(\tilde{s}_1, \tilde{j}_1, \tilde{s}_1) \delta(\tilde{s}_1, \tilde{j}_1, \tilde{s}_1) \dots$$



$$\varphi(s_1, j_1, s_1) \varphi(s_2, j_2, s_2)$$

$$\begin{aligned} &= \sum_n \left(\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = e^{-\lambda/4!} e^{\lambda/4!} \cdot \left(\lambda \left(\frac{1}{4!}\right)\right)^j \lambda^j \\ &= \sum_n \left(\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} \end{aligned}$$



$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi_{s_1 s_2 s_3}$$

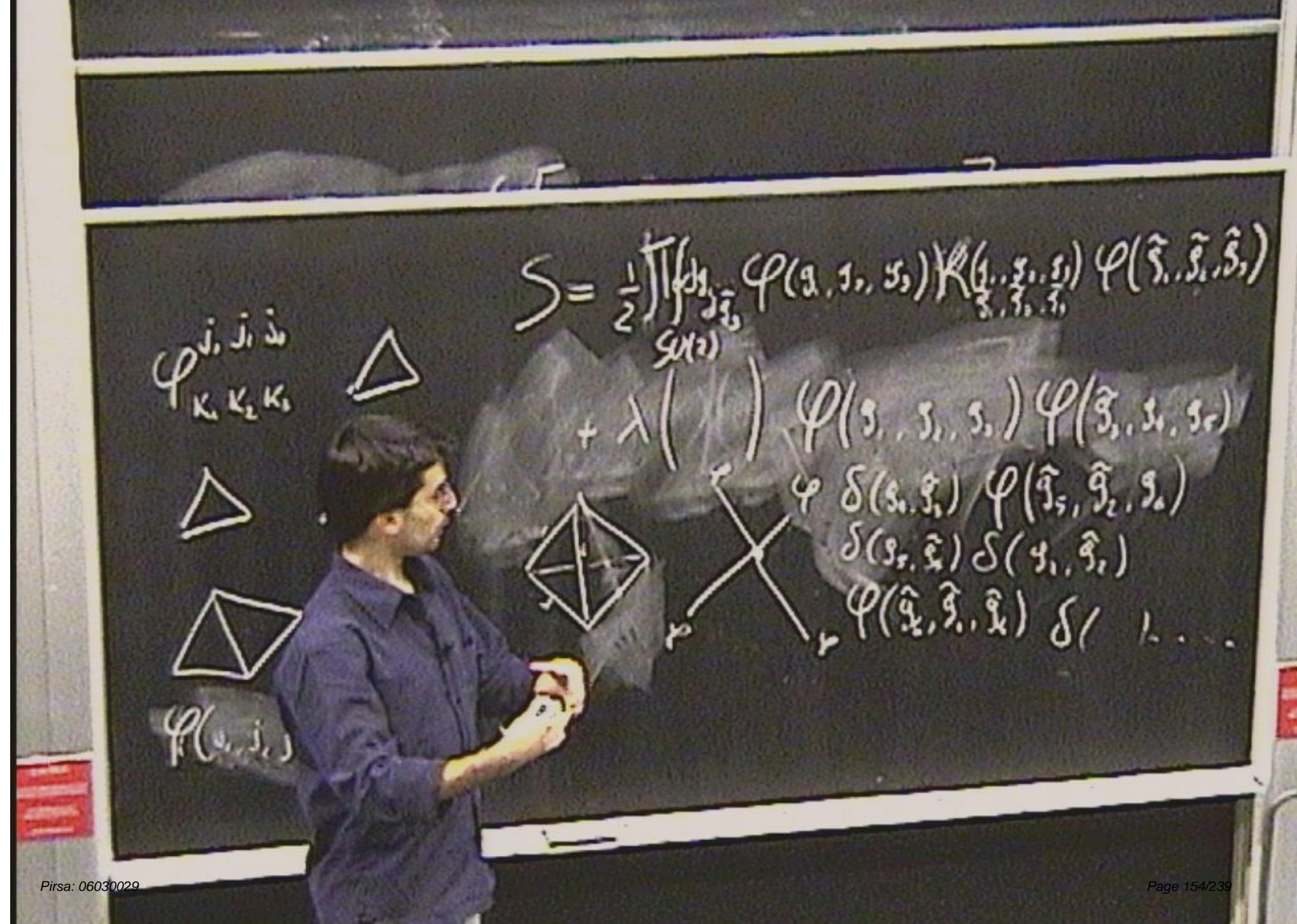
$$S = \frac{1}{2} \int \prod_{j=1}^3 \varphi(s_j, j_1, j_2) R(j_1, j_2, j_3) \varphi(\hat{s}_1, \hat{j}_1, \hat{j}_2)$$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3)$$

$$\varphi \delta(s_1, s_2) \varphi(\hat{s}_1, \hat{s}_2, s_3)$$

$$\delta(s_1, \hat{s}_2) \delta(s_1, \hat{s}_3)$$

$$\varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \delta(s_1, \dots)$$



$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$\varphi(s_1 s_2 s_3)$$



$$S = \frac{1}{2} \prod_{\substack{s_1, s_2, s_3 \\ S(1)}} \varphi(s_1 s_2 s_3) K(s_1 s_2 s_3) \varphi(\hat{s}_1 \hat{s}_2 \hat{s}_3)$$

$$+ \lambda \left(\dots \right) \varphi(s_1 s_2 s_3) \varphi(\hat{s}_1 \hat{s}_2 \hat{s}_3)$$

$$\varphi \delta(s_1 \hat{s}_1) \varphi(\hat{s}_2 \hat{s}_3 s_4)$$

$$\delta(s_1 \hat{s}_1) \delta(s_2 \hat{s}_2)$$

$$\varphi(\hat{s}_1 \hat{s}_2 \hat{s}_3) \delta(s_1 \hat{s}_1)$$

...

$$S = \frac{1}{2} \int \int f_{\alpha_1, \alpha_2, \alpha_3} \varphi(g_1, j_1, s_1) \delta(j_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$\underset{SU(2)}{\delta(g_1, j_1)}$

$$+ \lambda \left(\dots \right)$$


$$\varphi(j_1, j_2, s_1)$$

$$\varphi(j_2, j_3, s_2)$$

$$\varphi(j_3, j_1, s_3)$$

$$\varphi(j_1, j_2, j_3)$$

$$\varphi(j_2, j_3, j_1)$$

$$\varphi(j_3, j_1, j_2)$$

$$\delta(j_1, \tilde{s}_1)$$

$$\delta(j_2, \tilde{s}_2)$$

$$\delta(j_3, \tilde{s}_3)$$

$$\delta(\tilde{s}_1, \tilde{s}_2)$$

$$\delta(\tilde{s}_2, \tilde{s}_3)$$

$$\delta(\tilde{s}_3, \tilde{s}_1)$$

$$\varphi_{K_1 K_2 K_3}^{j_1 j_2 j_3}$$



$$\varphi(s_1, s_2, s_3)$$

$$S = \frac{1}{2} \int_{SU(2)} d\varphi_{j_1 j_2 j_3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_2)$$

$$\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_1) \dots$$



$$\varphi(s_1, s_2, s_3)$$

$$\varphi_{k_1 k_2 k_3}^{j_1 j_2 j_3}$$



$$S(j_1, j_2, j_3)$$

$$S = \frac{1}{2} \prod_{\substack{s_1, s_2, s_3 \\ \text{SU}(2)}} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$+ \lambda \left(\quad \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1)$$

$$\varphi(\tilde{s}_2, \tilde{s}_3, \tilde{s}_1) \delta(s_1, \tilde{s}_1)$$



$$S = \frac{1}{2} \int_{\text{SU}(2)} \varphi(s_1, \tilde{s}_1, s_2) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_1, \tilde{s}_2) \\ + \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\ \delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_1) \dots \\ \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \dots)$$



$$S = \frac{1}{2} \int \int_{\text{SU}(2)} \varphi(g_1, g_2, g_3) \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$

$$+ \lambda \left(\right) \varphi(g_1, g_2, g_3) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$

$$\varphi \delta(g_1, \tilde{g}_1) \varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$$

$$\delta(g_1, \tilde{g}_1) \delta(g_1, \tilde{g}_2)$$

$$\varphi(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3) \delta(g_1, \tilde{g}_1) \dots$$



$$S = \frac{1}{2} \int \prod_{j=1}^3 \varphi(s_j, \tilde{s}_j) \delta(s_j, \tilde{s}_j) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

s_1, s_2, s_3

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(s_1, s_4, s_5)$$

$\varphi \delta(s_1, s_2) \varphi(\tilde{s}_1, \tilde{s}_2, s_3)$
 $\delta(s_1, \tilde{s}_4) \delta(s_1, \tilde{s}_5)$
 $\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_4) \delta(s_1, \dots)$



$$S = \frac{1}{2} \int \int_{\text{SU}(2)} \varphi(s_1, s_2, s_3) \delta(s_1, \hat{s}_1) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3)$$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3)$$

$$\varphi \delta(s_1, \hat{s}_1) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3)$$

$$\delta(s_1, \hat{s}_1) \delta(s_1, \hat{s}_2)$$

$$\varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \delta(s_1, \dots)$$

$$S = \frac{1}{2} \int \prod_{j=1}^3 \varphi(s_j, \tilde{s}_j) \delta(s_j, \tilde{s}_j) \varphi(\tilde{s}_j, \tilde{s}_j, \hat{s}_j)$$

$\delta(s_1, \tilde{s}_1)$
 $\delta(s_2, \tilde{s}_2)$
 $\delta(s_3, \tilde{s}_3)$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(s_1, s_4, s_5)$$

$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_5, \tilde{s}_2, \tilde{s}_3)$
 $\delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1)$
 $\varphi(\tilde{s}_2, \tilde{s}_1, \tilde{s}_4) \delta(s_1, \dots)$



$$S = \frac{1}{2} \int \int f_{\alpha, \beta, \gamma} \varphi(\alpha, \beta, \gamma) \delta(\beta, \tilde{\beta}) \varphi(\tilde{\beta}, \tilde{\beta}, \tilde{\beta})$$

$\delta(\gamma, \tilde{\gamma})$
 $\delta(\beta, \tilde{\beta})$
 $\delta(\gamma, \tilde{\gamma})$

$$+ \lambda \left(\right) \varphi(\beta, \tilde{\beta}, \tilde{\beta}) \varphi(\tilde{\beta}, \tilde{\beta}, \tilde{\beta})$$

$\varphi \delta(\beta, \tilde{\beta}) \varphi(\tilde{\beta}, \tilde{\beta}, \tilde{\beta})$
 $\delta(\beta, \tilde{\beta}) \delta(\beta, \tilde{\beta})$
 $\varphi(\tilde{\beta}, \tilde{\beta}, \tilde{\beta}) \delta / \dots$

$\varphi(\tilde{x})$



$$S = \frac{1}{2} \int \int f_{\alpha_1 \alpha_2 \alpha_3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$\delta(s_1, \tilde{s}_1)$
 $\delta(s_2, \tilde{s}_2)$

$$+ \lambda \left(\right) \varphi(s_1, s_2, s_3) \varphi(s_1, s_2, s_3)$$

$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$
 $\delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1)$
 $\varphi(\tilde{s}_2, \tilde{s}_1, \tilde{s}_3) \delta(s_1, s_2, s_3)$



$$\varphi(\vec{x})$$



$$S = \frac{1}{2} \int \prod_{\substack{\text{curly lines} \\ \text{and vertices}}} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$+ \lambda \left(\begin{array}{|c|} \hline \text{curly lines} \\ \hline \end{array} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi(\vec{x})$$

$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\ \delta(s_2, \tilde{s}_2) \delta(s_3, \tilde{s}_3) \\ \varphi(\tilde{s}_2, \tilde{s}_3, \tilde{s}_1) \delta/ \dots$$

$$S = \frac{1}{2} \int \int f_{\delta_1, \delta_2, \delta_3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$+ \lambda \left(\quad \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

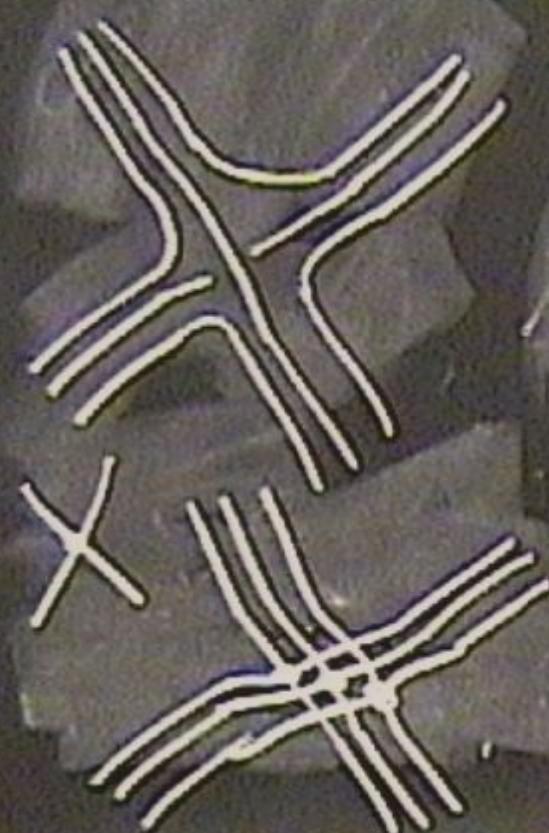
$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1)$$

$$\varphi(\tilde{s}_2, \tilde{s}_1, \tilde{s}_3) \delta(s_1, \tilde{s}_1)$$

...

$$\varphi(\tilde{x})$$



$$S = \frac{1}{2} \int \prod_{i,j,k} \varphi(s_i, s_j, s_k) \delta(s_i, \tilde{s}_j) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k)$$

$SU(2)$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(s_4, s_5, s_6)$$

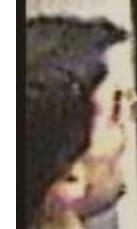
$$\varphi \delta(s_1, s_2) \varphi(\tilde{s}_5, \tilde{s}_2, s_6)$$

$$\delta(s_1, \tilde{s}_2) \delta(s_1, \tilde{s}_2)$$

$$\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_2) \delta(s_1, \dots)$$



$\varphi(x)$



$$S = \frac{1}{2} \int d^3x \left[\varphi(s_1, s_2, s_3) \delta(s_1, \hat{s}_1) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \right.$$

SU(2)

$$+ \lambda \left(\varphi(s_1, s_2, s_3) \varphi(s_1, \hat{s}_2, \hat{s}_3) \right.$$

$$\varphi \delta(s_1, s_2) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3)$$

$$\delta(s_1, \hat{s}_2) \delta(s_1, \hat{s}_2)$$

$$\left. \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \delta(s_1, s_2) \right]$$



$$\varphi(x)$$

$$S = \frac{1}{2} \int d^3 s_1 d^3 \tilde{s}_1 \varphi(s_1, \tilde{s}_1, s_2) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_1, \tilde{s}_2)$$

$S[11]$

$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$

$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, s_3)$$

$$\delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_2)$$

$$\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \dots)$$

$$S = \frac{1}{2} \int d^3x_1 d^3x_2 \varphi(g_{\mu\nu}) \delta(g_{\mu\nu}) \varphi(\tilde{g}_{\mu\nu}) \delta(\tilde{g}_{\mu\nu})$$

$$+ \lambda \left(\varphi(g_{\mu\nu}) \varphi(\tilde{g}_{\mu\nu}) \varphi(\tilde{\tilde{g}}_{\mu\nu}) \right) \varphi(g_{\mu\nu}) \delta(g_{\mu\nu}) \varphi(\tilde{g}_{\mu\nu}) \delta(\tilde{g}_{\mu\nu})$$

$$\varphi(g_{\mu\nu}) \delta(g_{\mu\nu}) \varphi(\tilde{g}_{\mu\nu}) \delta(\tilde{g}_{\mu\nu}) \varphi(\tilde{\tilde{g}}_{\mu\nu}) \delta(\tilde{\tilde{g}}_{\mu\nu})$$

$\varphi(\tilde{x})$

$$\mathcal{S} = \frac{1}{2} \int_{\text{SU}(2)} d\mathbf{s}_1 d\mathbf{s}_2 \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$
$$+ \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$
$$\varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$$
$$\delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_2)$$
$$\varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \dots)$$



$$\begin{aligned}
 S = & \frac{1}{2} \int_{SU(2)} d\tilde{s}_1 \varphi(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_1, \hat{s}_1) \\
 & \quad \delta(s_1, \tilde{s}_1) \\
 & + \lambda \left(\text{Diagram} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_2) \\
 & \quad \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_3) \dots
 \end{aligned}$$

$\varphi(\tilde{x})$

$$\begin{aligned}
 S = & \frac{1}{2} \int \prod_{i=1}^4 \varphi(s_i, \tilde{s}_i, s_i) \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_i, \tilde{s}_i) \\
 & \quad \delta(s_i, \tilde{s}_i) \\
 & + \lambda \left(\text{Diagram} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \delta(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1) \\
 & \quad \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_1) \dots
 \end{aligned}$$

$$\mathcal{S} = \frac{1}{2} \int_{\mathcal{M}} f_{j_1 j_2 j_3} \varphi(s, s_1, s_2) \delta(s, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ + \lambda \left(\dots \right) \varphi(s, s_1, s_2) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \varphi \delta(s, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \delta(s, \tilde{s}_1) \delta(s_1, \tilde{s}_2) \\ \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s, \dots)$$


$$\mathcal{D} = \frac{1}{2} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \varphi(s_1, s_2, s_3) \delta(s_1, \hat{s}_1) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \\ \delta(s_1, \hat{s}_1) \delta(s_2, \hat{s}_2) \delta(s_3, \hat{s}_3) + \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \\ \varphi \delta(s_1, \hat{s}_1) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \delta(s_2, \hat{s}_2) \delta(s_3, \hat{s}_3) \varphi(\hat{s}_1, \hat{s}_2, \hat{s}_3) \delta(s_1, \hat{s}_1) \dots$$

$$L = \int d\varphi \; e$$

$$Z = \int_{-\infty}^{+\infty} dq \; e^{-\frac{1}{2}m^2q^2 - \frac{\lambda}{4!}q^4 + \bar{J}q} = \int_{-\infty}^{+\infty} dq \; e^{-\frac{1}{2}m^2q^2 + \bar{J}q} \left(1 - \frac{\lambda}{4!}q^4 + \frac{1}{2}(\frac{\lambda}{4!})q^8\right)$$

$$\begin{aligned} &= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} = (e^{-\frac{\lambda}{4!}q^2})^{\frac{1}{2}} e^{\frac{\bar{J}q}{2}} \\ &= \sum_n \left(-\frac{\lambda}{4!}\right)^n \boxed{A_n(j)} \end{aligned}$$

$$\Delta \rightarrow \Psi(j_1, j_2, j_3)$$

$$\begin{aligned}
 S = & \frac{1}{2} \prod_{\substack{\text{S}, \tilde{\text{S}} \\ \text{SU}(2)}} \varphi(\text{s}_1, \text{s}_2, \text{s}_3) \delta(\text{s}_1, \tilde{\text{s}}_1) \varphi(\tilde{\text{s}}_1, \tilde{\text{s}}_2, \tilde{\text{s}}_3) \\
 & + \lambda \left(\right) \varphi(\text{s}_1, \text{s}_2, \text{s}_3) \varphi(\tilde{\text{s}}_1, \text{s}_4, \text{s}_5) \\
 & \quad \varphi \delta(\text{s}_1, \tilde{\text{s}}_1) \varphi(\tilde{\text{s}}_2, \tilde{\text{s}}_3, \text{s}_6) \\
 & \quad \delta(\text{s}_1, \tilde{\text{s}}_1) \delta(\text{s}_1, \tilde{\text{s}}_2) \\
 & \quad \varphi(\tilde{\text{s}}_1, \tilde{\text{s}}_2, \tilde{\text{s}}_3) \delta / \dots
 \end{aligned}$$

$$\text{Diagram} = \frac{1}{2} \int_{\mathcal{M}_{\tilde{s}_1, \tilde{s}_2}} \varphi(s_1, s_2, s_3) \delta(s, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ + \lambda \left(\text{Diagram} \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \varphi \delta(s, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_2) \\ \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s, \tilde{s}_1)$$

$\varphi(\tilde{s})$

$$\begin{aligned}
 S = & \frac{1}{2} \int \int \int \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \text{SUSY} \\
 & + \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \delta(s_1, \tilde{s}_1) \delta(s_2, \tilde{s}_2) \\
 & \quad \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_1) \\
 & \quad \dots
 \end{aligned}$$

$\varphi(\tilde{s})$



$$\text{Diagram} = \frac{1}{2} \int \prod_{i,j,k} \varphi(s_i, s_j, s_k) \delta(s_i, \tilde{s}_j) \varphi(\tilde{s}_i, \tilde{s}_k, \tilde{s}_j)$$
$$+ \lambda \left(\text{Diagram} \right) \varphi(s_i, s_j, s_k) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k)$$
$$\delta(s_i, \tilde{s}_j) \varphi(\tilde{s}_i, \tilde{s}_k, \tilde{s}_j)$$
$$\delta(s_i, \tilde{s}_k) \delta(s_i, \tilde{s}_j)$$
$$\varphi(\tilde{s}_i, \tilde{s}_k, \tilde{s}_j) \delta(s_i, s_j, s_k)$$

$$\mathcal{S} = \frac{1}{2} \int_{\mathbb{R}^3} f_{j_1, j_2} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \delta(s_1, \tilde{s}_1) \\ + \lambda \left(\dots \right) \varphi(s_1, s_2, s_3, s_4, s_5) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ \varphi(s_1, \tilde{s}_1) \varphi(\tilde{s}_5, \tilde{s}_1, s_4) \\ \delta(s_1, \tilde{s}_1) \delta(s_1, \tilde{s}_1) \\ \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \dots)$$

$$\mathcal{D} = \frac{1}{2} \int_{\Omega} \varphi_{\delta_1, \delta_2, \delta_3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ + \lambda \left(\int_{\Omega} \varphi(s_1, s_2, s_3) \varphi(s_3, \tilde{s}_2, \tilde{s}_3) \right. \\ \left. \varphi \delta(s_2, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \right) \\ \delta(s_2, \tilde{s}_1) \delta(s_1, \tilde{s}_1) \\ (\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, s_2, s_3)$$


$$\mathcal{D} = \frac{1}{2} \prod_{\text{sites } i,j} \varphi(s_i, s_j, s_3) \delta(s_i, \tilde{s}_j) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_3)$$
$$+ \lambda \left(\dots \varphi(s_i, s_j, s_3) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_3) \right.$$
$$\left. + \varphi(s_i, \tilde{s}_j, s_3) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_3) \right)$$

$$\begin{aligned}
 S = & \frac{1}{2} \int_{\text{S}^3} \varphi(s_1, \tilde{s}_1) \varphi(s_2, \tilde{s}_2) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \delta(s_2, \tilde{s}_2) \delta(s_3, \tilde{s}_3) \\
 & + \lambda \left(\right. \int_{\text{S}^3} \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_2, \tilde{s}_2, s_3) \\
 & \quad \delta(s_2, \tilde{s}_2) \delta(s_3, \tilde{s}_3) \\
 & \quad \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, s_2, s_3) \dots
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{D} = & \frac{1}{2} \int_{S^3} d^3x \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & + \lambda \left(\right. \int_{S^3} \varphi(s_1, s_2, s_3) \varphi(s_1, s_4, s_5) \right. \\
 & \quad \times \delta(s_2, \tilde{s}_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_4) \\
 & \quad \times \delta(s_4, \tilde{s}_5) \delta(s_1, \tilde{s}_3) \\
 & \quad \times \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_4) \delta(s_2, \tilde{s}_5) \dots
 \end{aligned}$$

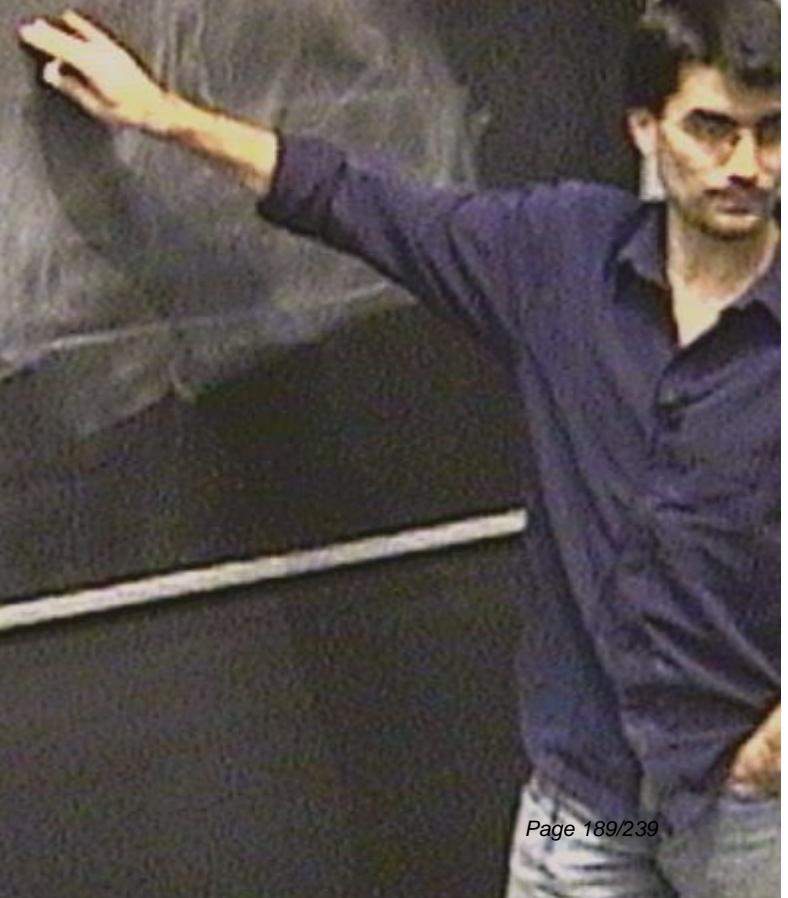
$$= \sum_{\mathbf{R}_k} (-1)^{\mathbf{r}_k^T} [\mathbf{A}_{\mathbf{R}_k}(\mathbf{j})]$$

$$\rangle = \delta(g, g, \tilde{g}, \tilde{g}') \cdot \delta(g, \tilde{g}, \tilde{g}, g')$$

$$\begin{aligned}
 S = & \frac{1}{2} \int d^4 s_1 d^4 \tilde{s}_1 \varphi(s_1, \tilde{s}_1, s_2) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & + \lambda \left(\right. \int d^4 s_1 \varphi(s_1, \tilde{s}_1, s_2) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\
 & \quad \times \delta(s_2, \tilde{s}_3) \varphi(\tilde{s}_3, \tilde{s}_4, s_4) \\
 & \quad - \delta(s_1, \tilde{s}_2) \delta(s_1, \tilde{s}_3) \\
 & \quad \left. - \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_4) \right. \dots
 \end{aligned}$$

$$= \sum_{\mu} (-1)^{\mu} [A_{\mu}(j)]$$

$$)) = \delta(g, g, \tilde{g}, \tilde{g}) \cdot \delta(\tilde{g}, \tilde{g}, g, g)$$



$$\begin{aligned}
 \mathcal{D} = & \frac{1}{2} \int_{S^3} d\mathbf{x}_1 d\mathbf{x}_2 \varphi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{s}_3) \delta(\mathbf{x}_1, \hat{\mathbf{x}}_1) \varphi(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{s}}_3) \\
 & + \lambda \left(\right) \varphi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{s}_3) \varphi(\mathbf{x}_3, \mathbf{x}_4, \mathbf{s}_5) \\
 & \quad \times \delta(\mathbf{x}_3, \hat{\mathbf{x}}_1) \varphi(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{s}}_3) \\
 & \quad \times \delta(\mathbf{x}_4, \hat{\mathbf{x}}_2) \varphi(\hat{\mathbf{x}}_3, \hat{\mathbf{x}}_4, \hat{\mathbf{s}}_5)
 \end{aligned}$$

$\varphi(x)$

$$= \sum_{\mathbf{r}} (-1)^{\mathbf{r}^T} [\mathbf{A}_{\mathbf{r}}(\mathbf{j})]$$

$$\mathcal{D} = \mathcal{H}\delta(y, \mathbf{j}, \hat{y}, \mathbf{j}') \dots \delta(s, \hat{s}, q, \hat{q}')$$

$$= \sum_{n_k} (-\lambda)^{n_k} [A_{n_k}(j)]$$

$$V = \delta(g, g, \hat{g}, \hat{g}) + \delta(g, \hat{g}, \hat{g}, g)$$

$$K = \delta(g, g, \hat{g}) + \delta(g, g, \hat{g}, \hat{g})$$

$$= \frac{1}{2} \int \int_{S(1)} f_{j_1 j_2 \bar{j}_3} \varphi(s_1, s_2, s_3) \delta(s_1, \bar{s}_3) \varphi(\bar{s}_1, \bar{s}_2, \bar{s}_3)$$

$$+ \lambda \left(\dots \right) \varphi(\varphi(s_1, s_2, s_3) \varphi(\bar{s}_1, \bar{s}_2, \bar{s}_3)$$

$$\varphi \delta(s_2, \bar{s}_3) \varphi(\bar{s}_1, \bar{s}_2, \bar{s}_3)$$

$$\delta(s_1, \bar{s}_3) \delta(s_1, \bar{s}_3)$$

$$\varphi(\bar{s}_1, \bar{s}_2, \bar{s}_3) \delta(s_1, \bar{s}_3) \dots$$

$$= \sum_{\mathbf{r}_k} (-1)^{\mathbf{r}_k^T} [A_{\mathbf{r}_k}(j)]$$

$$V = \prod_j \delta(g_j, \tilde{g}_j, g'_j) \cdot \delta(\tilde{g}_j, \tilde{g}'_j)$$

$$K = \int dg d\tilde{g} \delta(g_j, \tilde{g}_j, \tilde{g}') \cdot \delta(g_j, \tilde{g}'_j)$$

$$\begin{aligned}
 Z &= \int D\varphi e^{-S[\varphi]} \left[\frac{1}{2} \varphi_i (\partial^2 + m^2) \varphi_{i+} - \frac{\lambda}{4!} \varphi^4 + J\varphi \right] \\
 &= \int dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + Jq} = \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \left(1 - \frac{\lambda q^4}{4!} \frac{d^2}{dq^2} \right) \\
 Z &= \int dq e^{-\frac{1}{2} m^2 q^2 + Jq} \\
 &= \sum_n (-\lambda)^n \boxed{A_n(J)} = \lambda e^{-\frac{m^2}{2} J^2} \\
 &= \sum_n \left(-\frac{\lambda}{4!} \right)^n \boxed{A_n(J)}
 \end{aligned}$$

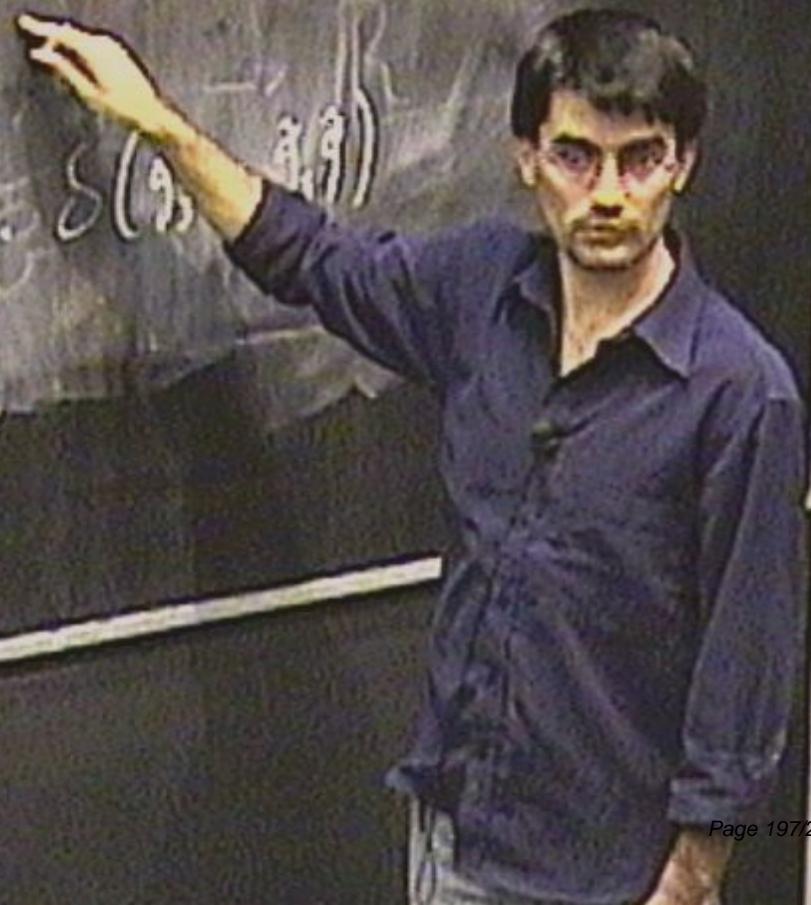
$$Z = \int D\varphi e^{-\int d\varphi \left[\frac{1}{2} \varphi R(\partial^2 + m^2) \varphi + \lambda \frac{\varphi^4}{4!} + J\varphi \right]}$$

$$\begin{aligned} Z &= \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 - \frac{\lambda}{4!} q^4 + J q} = \int_{-\infty}^{+\infty} dq e^{-\frac{1}{2} m^2 q^2 + S_1} \left(1 - \frac{\lambda}{4!} q^4 + \frac{1}{2} \left(\frac{\lambda}{4!} \right)^2 q^8 \right) \\ &= \sum_n (-\frac{\lambda}{4!})^n \boxed{A_n(j)} = \langle e^{-S_1} \rangle = \langle e^{-\frac{1}{2} m^2 p^2} \rangle = \langle \langle \left(1 - \left(\frac{1}{m^2} \right)^2 \lambda \right)^{-1} \rangle \rangle \end{aligned}$$

$$= \sum_{\mu} (\lambda) [A_\mu]^{(3)}$$

$$V = \int d^3x \delta(g, \dot{g}, \ddot{g}, \ddot{g}) \delta(\ddot{g}, \ddot{\dot{g}}, \ddot{\ddot{g}})$$

$$K = \int d^3x \delta(g, \dot{g}, \ddot{g}) \delta(\ddot{g}, \ddot{\dot{g}})$$



$$= \sum_{\mathbb{R}} (-1)^{\frac{1}{4}} [A_{\mathbb{R}}(3)]$$

$$V = \{ \delta(g, g, \tilde{g}, g') \cup \delta(g, \tilde{g}, g, g') \}$$

$$K = \{ \delta(g, g, g, \tilde{g}) \cup \delta(g, g, \tilde{g}, g) \} = P$$

$$= \sum_{\mathbf{r}_k} (-1)^{\frac{1}{4}} |\mathbf{A}_{\mathbf{r}_k}(3)\rangle$$

$$\mathcal{V} = \int d\mathbf{g} \delta(g, \hat{g}, \tilde{g}, g') \cdot \delta(\tilde{g}, \hat{g}, \tilde{g}, g'')$$

$$K = \int d\mathbf{g} d\hat{\mathbf{g}} \delta(g, g, \hat{g}, \tilde{g}) \cdot \delta(g, g, \hat{g}, \tilde{g}) = P$$

$$= \sum_{\mathbf{R}} (-1)^{\frac{1}{4}(\mathbf{R})} A_{\mathbf{R}}(3)$$

$$\rangle) = \text{if } S(g, g, \tilde{g}, \tilde{g}) \cdot S(\tilde{g}, \tilde{g}, \tilde{g}, \tilde{g})$$

$$K = \text{if } S(g, g, \tilde{g}, \tilde{g}) \cdot S(g, g, \tilde{g}, \tilde{g}) = P$$



$$\begin{aligned}
 Z &= \int_{-\infty}^{\infty} d\eta e^{-\frac{1}{2}\eta^2 - \lambda\eta + j\eta} \\
 Z &= \int_{-\infty}^{\infty} d\eta e^{-\frac{1}{2}\eta^2 - \lambda\eta + j\eta} = \int_{-\infty}^{\infty} d\eta e^{-\frac{1}{2}\eta^2 + j\eta} \left(1 - \frac{\lambda^2}{4}e^{-\frac{1}{2}\eta^2}\right) \\
 &= \sum_n \left(\frac{\lambda}{n!}\right)^n \boxed{A(n)} = e^{-\lambda} e^{\lambda} e^{-\lambda} = e^{-\lambda} \left(\frac{e^\lambda}{n!}\right) \lambda^n \\
 &= \sum_{R_k} \left(\frac{\lambda}{k!}\right)^k \boxed{A_k(k)}
 \end{aligned}$$

$$\begin{aligned}
 V &= \mu S(g_1, g_2) \cdot S(g_3, g_4) \\
 &= \mu S(g_1, g_2) \cdot S(g_3, g_4) = P
 \end{aligned}$$

$$= \sum_{\mathbf{R}} (-1)^{\mu} [A_{\mathbf{R}}(\mathbf{j})]$$

$$V = \int d^3\mathbf{r} \delta(g, g, \hat{g}, \hat{g}') \cdot \delta(g, \hat{g}, \hat{g}, \hat{g}'')$$

$$K = \int d^3\mathbf{r} \delta(g, g, g, g) \cdot \delta(g, g, g, g) = P$$

$$\sum_{\Delta \sim \Gamma}$$

$$= \sum_{\mathbf{R}} (-1)^n [A_{\mathbf{R}}(j)]$$

$$V = \int d^3g \delta(g, \hat{g}, \tilde{g}, \tilde{\hat{g}}) \cdot \delta(\hat{g}, \tilde{\hat{g}}, \tilde{g}, \hat{g})$$

$$K = \int d^3g \delta(g, \hat{g}, \tilde{g}, \tilde{\hat{g}})$$

$$Z_{\Delta \sim \Gamma} = \prod_i \int d^3g$$

$$\langle \dots \rangle_{SU(2)}$$

$$\langle \dots \rangle_{SU(2)} = P$$

$$= \sum_{\mathbf{R}} (-1)^{\ell} [A_{\mathbf{R}}(\mathbf{j})]$$

$$\lambda = \prod \delta(g, g, \hat{g}, \hat{g}) \cdot \delta(g, \hat{g}, \hat{g}, g)$$

$$K = \prod \delta(g, g, \hat{g}, \hat{g}) \cdot \delta(g, g, \hat{g}, \hat{g}) = P$$

$$Z_{\Delta \sim \Gamma} = \prod_f \frac{1}{|SU(f)|}$$



$$= \sum_{\mu} (-\lambda)^{\mu} [A_{\mu}]$$

$$J = \int d^4x \delta(g, \tilde{g}, \tilde{g}', g') \cdot \delta(g, \tilde{g}, \tilde{g}', g')$$

$$K = \int d^4x \delta(g, \tilde{g}, \tilde{g}, \tilde{g}) \cdot \delta(g, \tilde{g}, \tilde{g}, \tilde{g}) = P$$

$$Z_{\Delta \sim \Gamma} = \prod_I \int d^4g \prod_I \delta(g, \tilde{g})$$

$$\begin{aligned}
 S = & \frac{1}{2} \int f_{\tilde{s}_1, \tilde{s}_2, \tilde{s}_3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_2) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \text{S}(s_2, \tilde{s}_2) \\
 & + \lambda \left(\dots \right) \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_2, \tilde{s}_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \delta(s_1, \tilde{s}_2) \delta(s_1, \tilde{s}_3) \\
 & \quad \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \dots)
 \end{aligned}$$

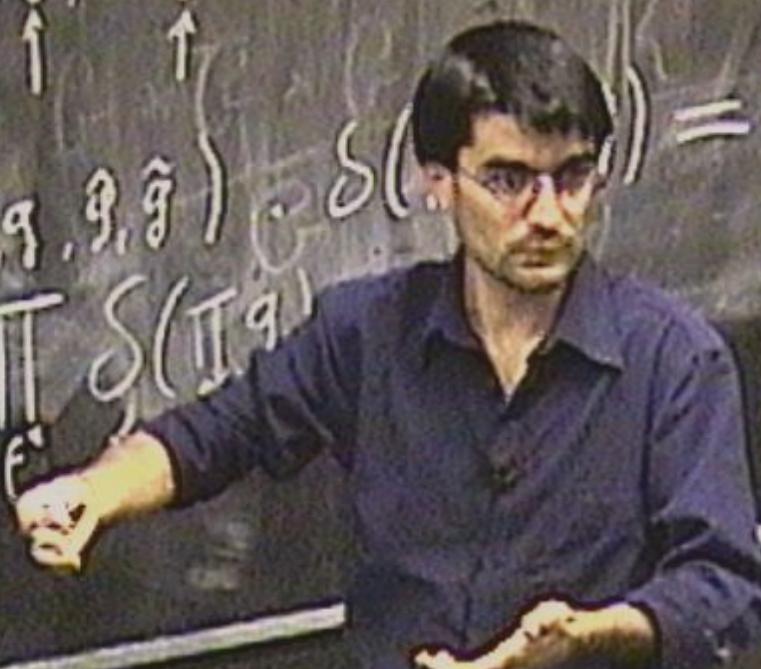
$\varphi(x)$

$$= \sum_{\mathbf{R}_n} (-\lambda)^{\ell^2} [\mathbf{A}_{\mathbf{R}_n}^{(\mathcal{I})}]$$

$$\mathcal{D} = \int d\mathbf{g} \delta(\mathbf{g}, \hat{\mathbf{g}}, \tilde{\mathbf{g}}, \hat{\tilde{\mathbf{g}}}) \delta(\mathbf{g}, \tilde{\mathbf{g}}, \mathbf{g}, \hat{\mathbf{g}})$$

$$K = \int d\mathbf{g} d\hat{\mathbf{g}} \delta(\mathbf{g}, \hat{\mathbf{g}}, \mathbf{g}, \hat{\mathbf{g}}) \delta(\mathbf{g}, \mathbf{g}, \mathbf{g}, \mathbf{g}) = P$$

$$Z_{\Delta \sim \Gamma} = \prod_r \int d\mathbf{g} \prod_{f \in \Gamma} S(f)$$



$$= \sum_{\mu} (-1)^{\mu} [A_{\mu}^{(3)}]$$

$$)=\int d^4g \delta(g,g,\hat{g},\hat{g}') S(\hat{g},\hat{g}',g,g')$$

$$K = \int d^4g \delta(g,g,\hat{g},\hat{g}) S(g,g)$$

$$Z_{\Delta-\Gamma} = \prod_I \int d^4g \prod_I S(\Gamma g)$$

$$= \sum_{\mu} (-1)^{\mu} [A_\mu^{(3)}]$$

$$V = \prod \delta(g, g, \tilde{g}, \tilde{g}) \quad \delta(g, \tilde{g}, g, \tilde{g})$$

$$K = \prod \delta(g, g, \tilde{g}, \tilde{g}) \quad \delta(g, g)$$

$$Z_{\Delta, \Gamma} = \prod_{f \in \Gamma} \prod_{g \in f} \prod_{\mu} S(\mu)$$

$$\begin{aligned}
 V &= \int d\vec{g} \delta(g, \tilde{g}, \tilde{g}, \tilde{g}) \cdot \delta(\tilde{g}, \tilde{g}, \tilde{g}, \tilde{g}) \\
 K &= \int d\vec{g} \delta(g, g, \tilde{g}, \tilde{g}) \cdot \delta(g, g, \tilde{g}, \tilde{g}) = P \\
 Z_{\Delta \sim \Gamma} &= \prod_I \int d\vec{g} \prod_I \delta(Ig)
 \end{aligned}$$

- L. FREIDEL, hep-th/0505016 outlook as example

$$Z = \int D\varphi_i e^{-S}$$



- L. FREIDEL, hep-th/0505016 outlook ^{as example}

$$Z = \int D\varphi e^{-S(\varphi)} =$$



- L. FREIDEL, hep-th/0505016 outlook as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta}$$



- L. FREIDEL, hep-th/0505016 outlook as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)}$$



- L. FREIDEL, hep-th/0505016 outlook ^{as example}

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{\text{w}(\Delta)}}{\text{Sym}(\Delta)} \xrightarrow{\text{SPIN Four}}$$

- L. FREIDLÉL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \quad \boxed{\text{SPIN FORT}}$$

$$\sum_{\Delta}^{\text{SF}} = \sum_{\{j\}}$$

- L. FREIDEL, hep-th/0505016 out look as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{\text{u}(\Delta)}}{\text{Sym}(\Delta)} \sum_{\Delta} \text{SF}$$

$$\sum_{\Delta}^{\text{SF}} = \sum_{\{ij\}} \prod_i A(i) \prod_j A(j)$$



- L. FREIDEL, hep-th/0505016 outlook as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \quad \boxed{\text{SPIN FERM}}$$

$$Z^{\text{SF}} = \sum_{\{\Delta\}} \prod_{f} A_f^{(1)} \prod_{e} A_e^{(2)} \prod_{v} A_v^{(3)}$$

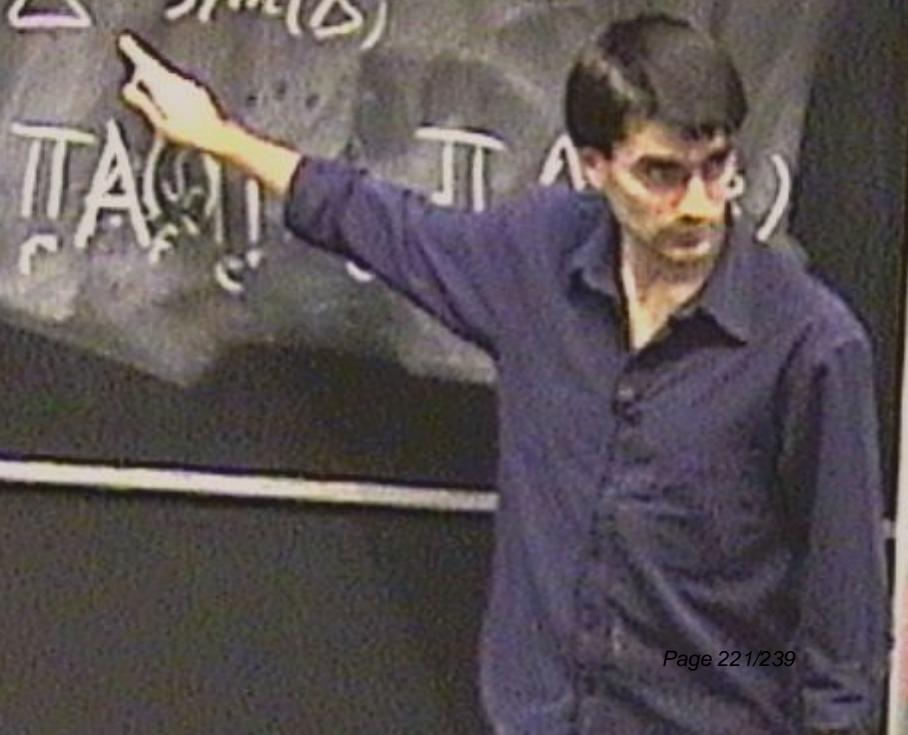


$$S = \frac{1}{2} \int_{\Omega} \int_{\partial\Omega} \varphi(s, s_1, s_2) \delta(s, \tilde{s}, \tilde{s}) \varphi(\tilde{s}, \tilde{s}_1, \tilde{s}_2) \\ S_{012} \\ + \lambda \left(\int_{\Omega} \int_{\Omega} \varphi(s, s_1, s_2) \varphi(s_1, s_3, s_4) \right. \\ \left. \varphi(s_3, s_5) \varphi(\tilde{s}_5, \tilde{s}_2, \tilde{s}_4) \right. \\ \left. \delta(s_1, \tilde{s}_1) \delta(s_2, \tilde{s}_2) \right. \\ \left. \delta(s_3, \tilde{s}_3) \delta(s_4, \tilde{s}_4) \right. \\ \left. \delta(s_5, \tilde{s}_5) \delta(\tilde{s}_1, \tilde{s}_2) \right. \\ \left. \delta(s_2, \tilde{s}_1) \delta(s_3, \tilde{s}_2) \right. \\ \left. \delta(s_4, \tilde{s}_3) \delta(s_5, \tilde{s}_4) \right)$$

$$\begin{aligned}
 D = & \frac{1}{2} \int_{S^2} d\Omega_{\hat{s}_1 \hat{s}_2} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & + \lambda \left(\int_{S^2} d\Omega \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \right. \\
 & \quad \times \delta(s_1, \tilde{s}_1) \delta(s_2, \tilde{s}_2) \varphi(\tilde{s}_3, \tilde{s}_1, \tilde{s}_2) \\
 & \quad \times \delta(s_1, \tilde{s}_2) \delta(s_3, \tilde{s}_1) \varphi(\tilde{s}_2, \tilde{s}_3, \tilde{s}_1) \\
 & \quad \times \left. \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_3) \dots \right)
 \end{aligned}$$

- L. FREIDEL, hep-th/0505016 as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \lambda^{\text{SF}} \xrightarrow[\text{Sym}(\Delta)]{\Delta} \text{FSPIN Four}$$

$$\sum_{\Delta}^{\text{SF}} = \sum_{\{ij\}} \prod_{e \in \Delta} A_e^{n_e}$$


$$\begin{aligned}
 S = & \frac{1}{2} \int_{S^2} d\Omega \delta_{ij} \varphi(g, s_i, s_j) \delta(s_i, \tilde{s}_i) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \\
 & + \lambda \left(\dots \right) \delta_{ij} \varphi(g, s_i, s_j) \varphi(\tilde{s}_i, \tilde{s}_j, \tilde{s}_k) \\
 & \quad \varphi \delta(s_k, \tilde{s}_k) \varphi(\tilde{s}_k, \tilde{s}_l, \tilde{s}_m) \\
 & \quad \delta(s_l, \tilde{s}_l) \delta(s_m, \tilde{s}_m) \\
 & \quad \varphi(\tilde{s}_k, \tilde{s}_l, \tilde{s}_m) \delta(s_k, s_l, s_m)
 \end{aligned}$$

$$V = \int d\vec{g} \delta(\vec{g}, \vec{g}, \hat{\vec{g}}, \vec{g}') \cdot \delta(\vec{g}, \vec{g}, \vec{g}, \vec{g}'')$$

$$K = \int d\vec{g} \delta(\vec{g}, \vec{g}, \vec{g}, \vec{g}) \cdot \delta(\vec{g}, \vec{g}, \vec{g}, \vec{g})$$

$$Z_{\Delta, \Gamma} = \prod_c \int d\vec{g} \prod_f \delta(\vec{g}, \vec{g})$$

$$\begin{aligned}
 \mathcal{D} = & \frac{1}{2} \int_{\text{SU}(2)} f_{j_1 j_2 j_3} \varphi(s, s_1, s_2) \delta(s, \tilde{s}_1) \varphi(\tilde{s}, \tilde{s}_1, \tilde{s}_2) \\
 & + \lambda \left(\right. \int_{\text{SU}(2)} \varphi(s, s_1, s_2) \varphi(\tilde{s}, \tilde{s}_1, \tilde{s}_2) \\
 & \quad \times \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\
 & \quad \times \delta(s_2, \tilde{s}_2) \delta(s_3, \tilde{s}_3) \\
 & \quad \times \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta(s_1, \tilde{s}_1) \dots
 \end{aligned}$$

- L. FREIDEL, hep-th/0505016 outlook as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{\text{w}(\Delta)}}{\text{Sym}(\Delta)} \xrightarrow{\text{SPIN FORT}}$$

$$\sum_{\Delta}^{\text{SF}} = \sum_{\{ij\}} \prod_{f} T_A(f) T_A(g) T_A(h)$$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{\text{u}(\Delta)}}{\text{Sym}(\Delta)} \quad \text{SPIN FERM}$$

$$\sum_{\Delta}^{\text{SF}} \sum_{\{ij\}} \prod_i T A_i \prod_j T A_j \prod_{ij} A_{ij}$$



- L. FREI DEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \lambda^{\text{u}(\Delta)} \frac{\text{Sym}(\Delta)}{\Delta} \xrightarrow{\text{[SPIN Four]}}$$


 $\geq^{\text{SF}} \sum_{\{\Delta\}} \prod_{f} A_f^{\text{u}(\Delta_f)} \prod_{g} A_g^{\text{u}(\Delta_g)} \prod_{V} A_V^{\text{u}(\Delta_V)}$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \lambda^{\text{u}(\Delta)} \frac{\text{sym}(\Delta)}{\Delta}$$

(SPIN FOUR)

$\Delta = \sum_{\{ij\}} \Delta^{SF}$

$\prod_{i,j} A_{ij} \sqrt{V} \{ \dots \}_{ij}$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \lambda^{\text{u}(\Delta)} \xrightarrow[\text{Sym}(\Delta)]{\text{FSIN Fari}} \text{Conf}(S_R(\omega)) \in \{ \dots \}$$

- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{|\Delta|}}{\text{Sym}(\Delta)} \int Dg e^{iS(g)}$$




- L. FREIDEL, hep-th/0505016 outlook

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{\text{Sym}(\Delta)}}{\text{Sym}(\Delta)} \int D\varphi e^{-S_{\Delta}(\varphi)}$$



- L. FREIDEL, hep-th/0505016 out look ^{as example}

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \cdot \sum_j A(j)$$



- L. FREIDEL, hep-th/0505016 out look

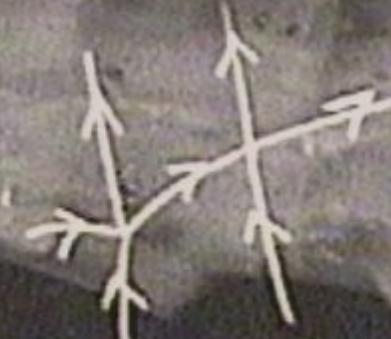
as example

$$Z = \int D\gamma e^{-S(\gamma)} = \sum_{\Delta} \frac{\lambda^{n(\Delta)}}{\text{Sym}(\Delta)} \cdot \sum_j A(j)$$



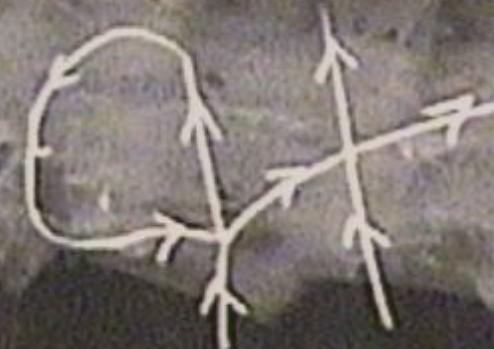
- L. FREIDEL, hep-th/0505016 out look ^{as example}

$$Z = \int D\psi e^{-S(\psi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \cdot \sum_j A(j)$$



- L. FREIDEL, hep-th/0505016 out look ^{as example}

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\lambda^{u(\Delta)}}{\text{Sym}(\Delta)} \cdot \sum_j A(j)$$

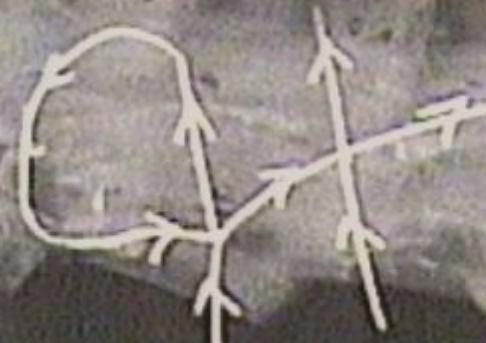


$$\begin{aligned}
 D = & \frac{1}{2} \int d\tilde{s}_1 d\tilde{s}_2 \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & + \lambda \left(\right. - \delta_{12} \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\
 & \quad \varphi \delta(s_2, \tilde{s}_3) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\
 & \quad \delta(s_1, \tilde{s}_2) \delta(s_1, \tilde{s}_3) \\
 & \quad \left. \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \delta_1 \dots \right)
 \end{aligned}$$



- L. FREIDEL, hep-th/0505016 *about look*

$$Z = \int D\gamma e^{-S(\gamma)} = \sum_{\Delta} \frac{\lambda^{|\Delta|}}{Sym(\Delta)} \cdot \left(\sum_j A(j) \right)$$



$$\begin{aligned} S = & \frac{1}{2} \prod_{\substack{\text{lines} \\ \text{in } S}} \int_{\mathbb{R}^3} \varphi(s_1, s_2, s_3) \delta(s_1, \tilde{s}_1) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \\ & + \lambda \left(\prod_{\substack{\text{lines} \\ \text{in } S}} \varphi(s_1, s_2, s_3) \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \right. \\ & \quad \times \delta(s_2, \tilde{s}_3) \varphi(\tilde{s}_1, \tilde{s}_2, s_3) \\ & \quad \times \delta(s_1, \tilde{s}_2) \delta(s_1, \tilde{s}_3) \\ & \quad \left. \varphi(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3) \right) \dots \end{aligned}$$

- L. FREIDEL, hep-th/0505016 as example

$$Z = \int D\varphi e^{-S(\varphi)} = \sum_{\Delta} \frac{\Delta^{n(\Delta)}}{\text{Sym}(\Delta)} \cdot \sum_j A(j)$$

