Title: Introduction to quantum gravity - Part 20

Date: Mar 29, 2006 08:00 PM

URL: http://pirsa.org/06030028

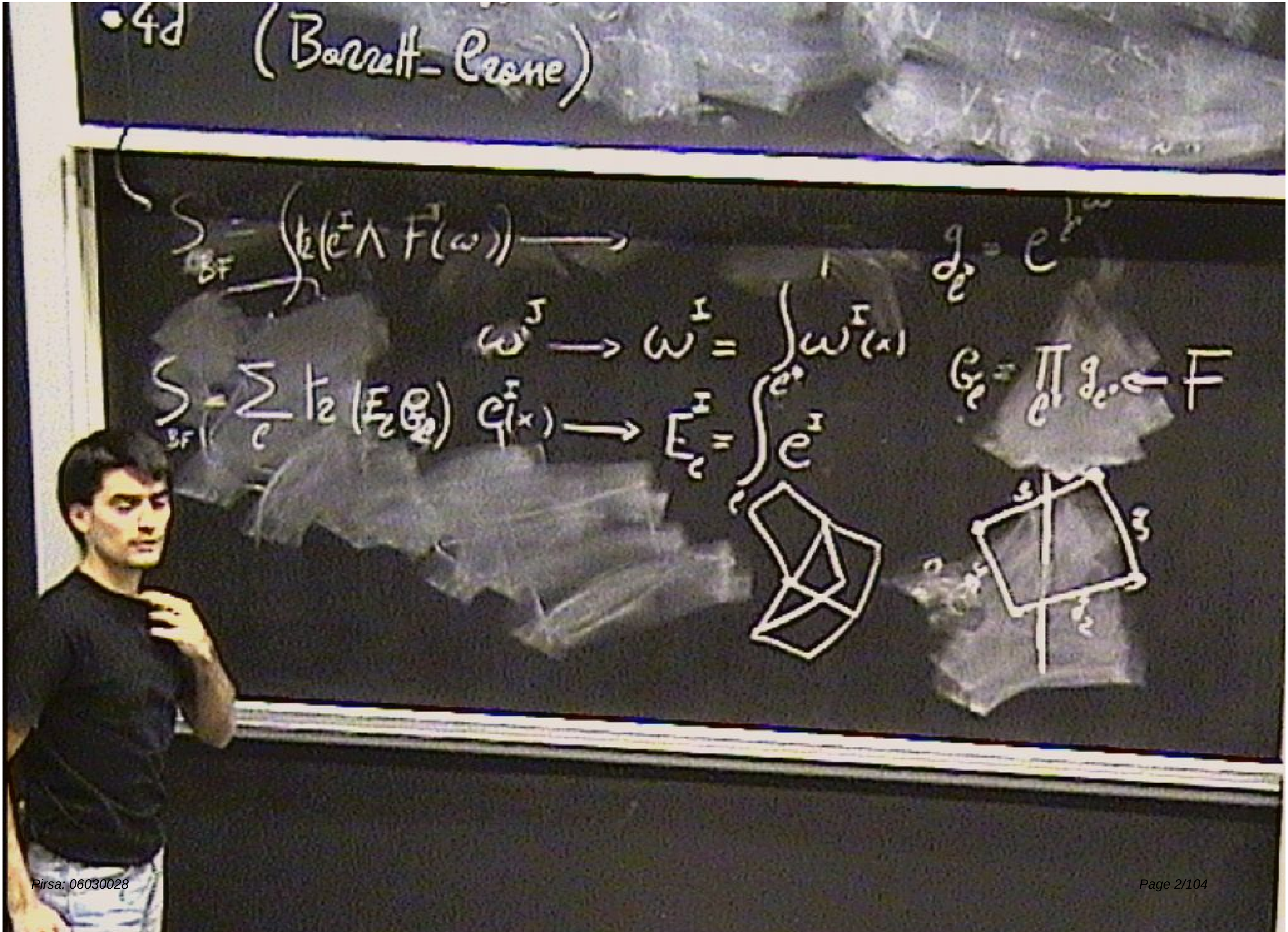
Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

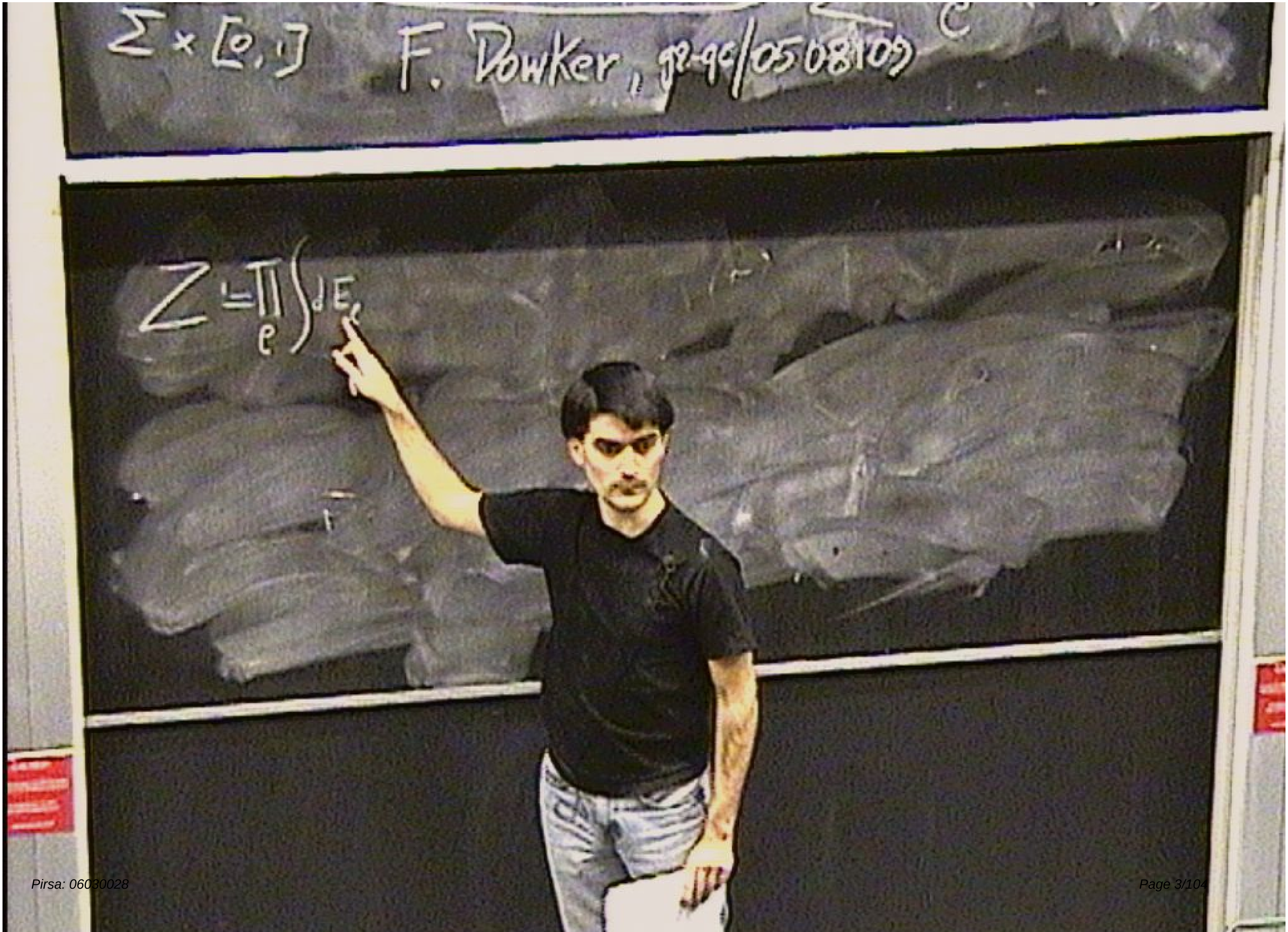
Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravityy with a positive cosmological constant, Lee Smolin, hep-th/0209079
-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

-undergraduate quantum mechanics
-basics of classical gauge field theories
-basic general relativity
-hamiltonian and lagrangian mechanics
-basics of lie algebras


• 4d (Barrett-Crane)
$S_{BF} = \int tr(e^I \wedge F^I(\omega))$
$\omega^I \longrightarrow \omega^I = \int_e \omega^I(x)$
$S_{BF} = \sum_e tr(E_e G_e)$
$e^I(x) \longrightarrow E_e^I = \int e^I$
$G_e = \prod g_e \leftarrow F$


Σ × [0,1]
F. Dowker, gr-qc/0508109
Z = ∏_e ∫ dE_e

$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109 C

$$Z \stackrel{?}{=} \prod_e \int dE_e^3$$

$$Z = \int De \int D\omega \, e^{iS_{HP}(e,\omega)}$$

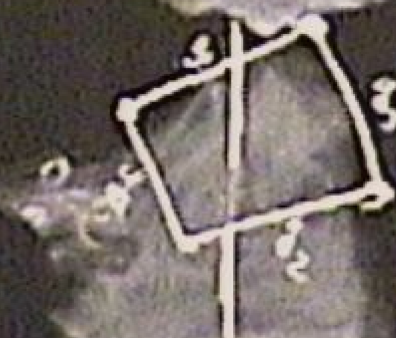
• 4d (Barrett–Crane)

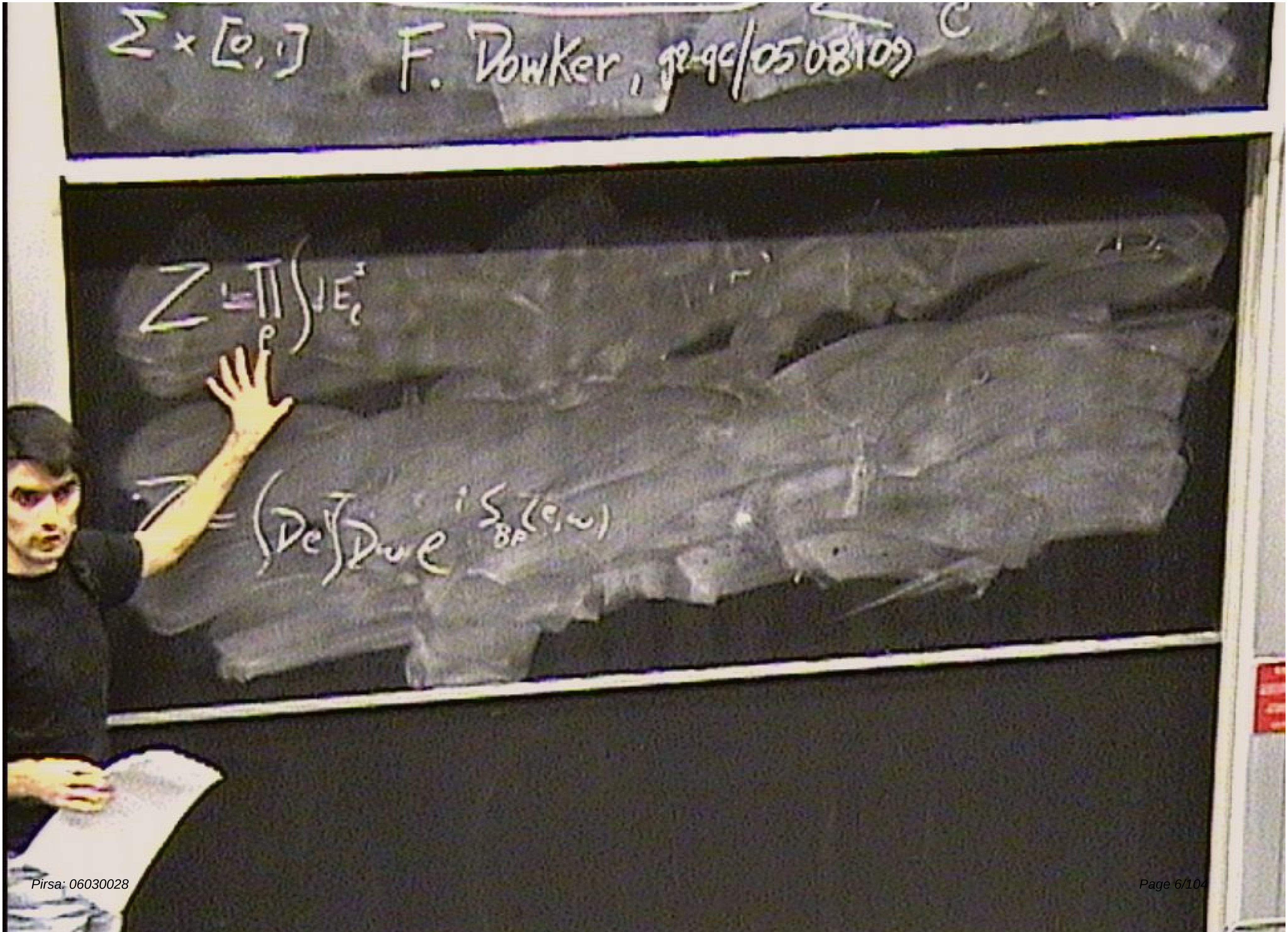
$$S_{BF} = \int tr(e^I \wedge F^I(\omega)) \longrightarrow \quad g_e = e^{\int \omega}$$

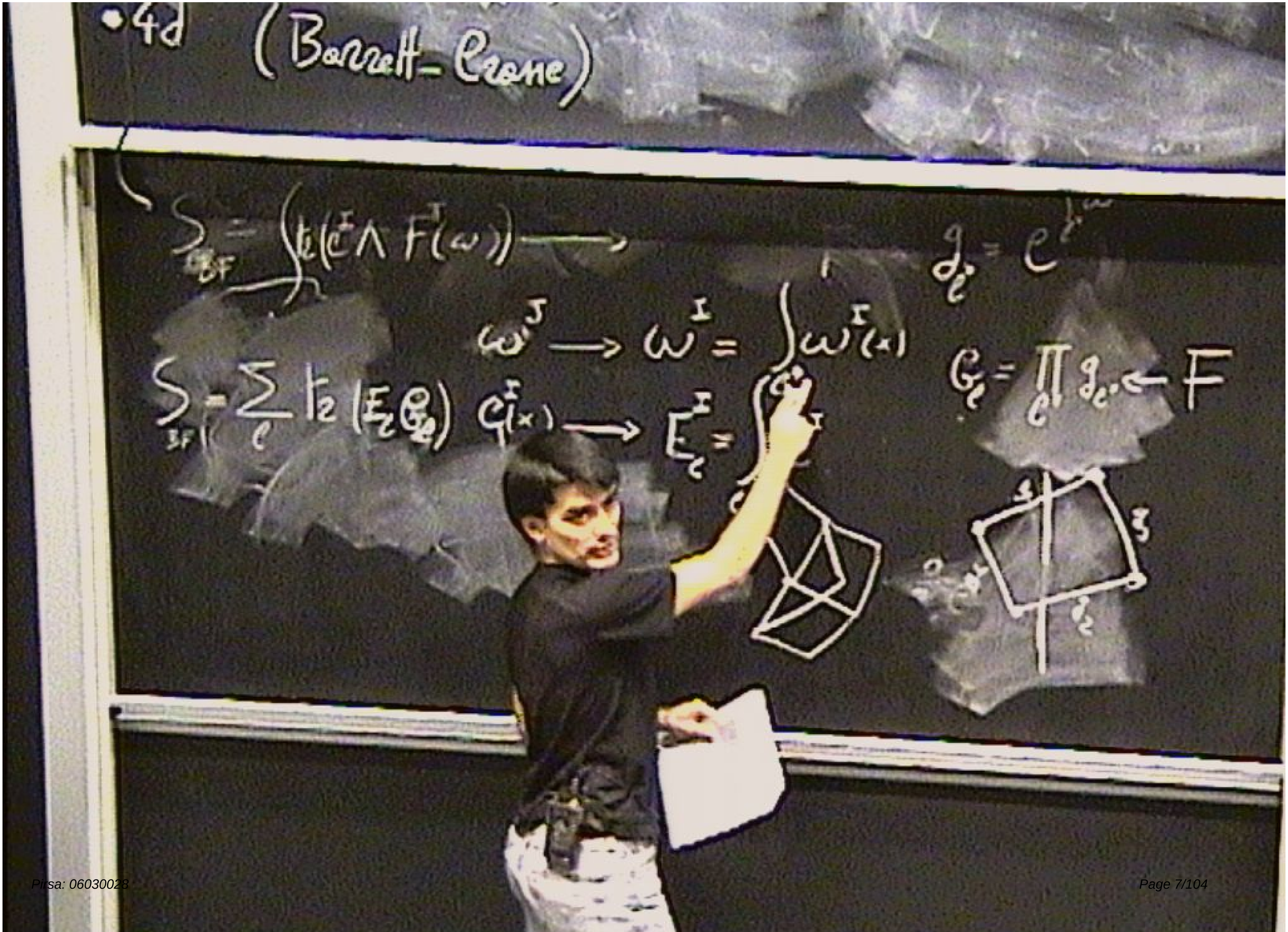
$$\omega^I \longrightarrow \omega^I_e = \int_e \omega^I(x)$$

$$S_{BF} = \sum_e tr(E_e G_e) \qquad e^I(x) \longrightarrow E_e^I = \int_e e^I$$

$$G_e = \prod_{e'} g_{e'} \leftarrow F$$




Σ × [0,1]
F. Dowker, gr-qc/0508109
Z = Π ∫ dE
Z = ∫De ∫Dω e^{iS_BF(e,ω)}

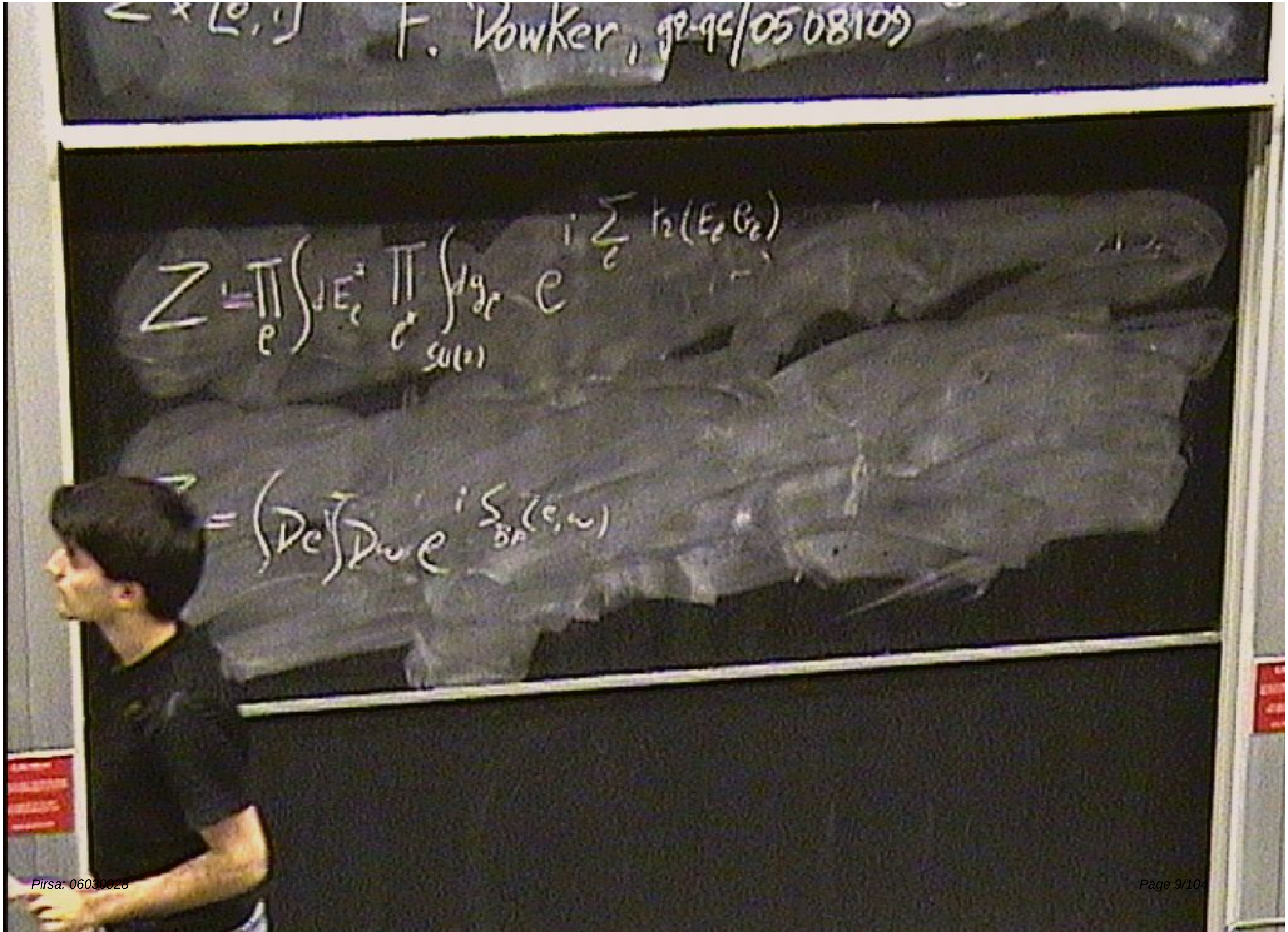

• 4d (Barrett-Crane)

• 4d (Barrett–Crane)

$$S_{BF} = \int \mathrm{tr}\left(e^I \wedge F^I(\omega)\right) \longrightarrow \qquad g_e = e^{\int \omega}$$

$$\omega^I \longrightarrow \omega^I_e = \int_e \omega^I(x) \qquad G_f = \prod_e g_e \longleftarrow F$$

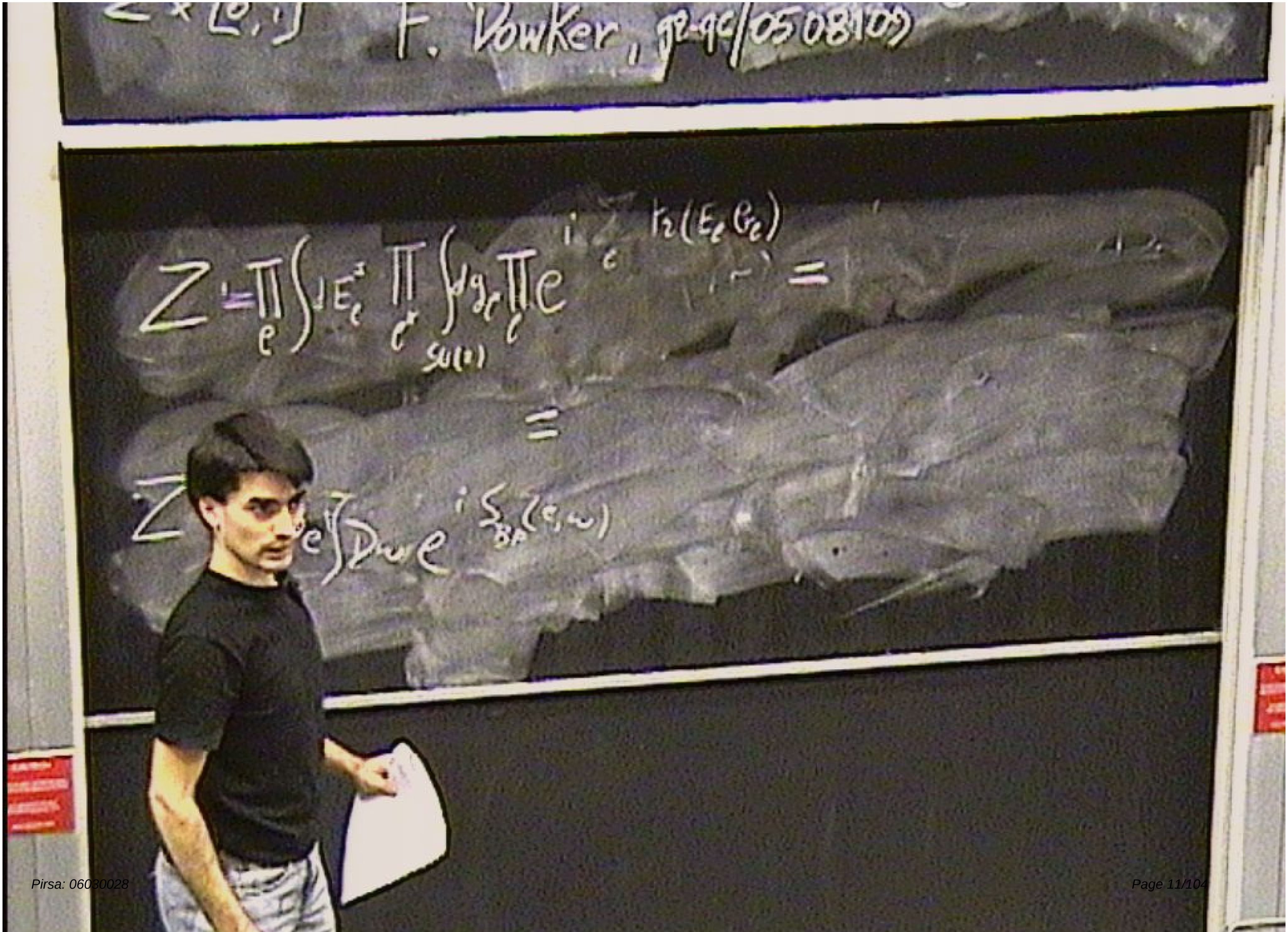
$$S_{BF} = \sum_f \mathrm{tr}\left(E_f G_f\right) \qquad e^I(x) \longrightarrow E^I_f = \int_f e^I$$


F. Dowker, gr-qc/0508109
$Z = \prod_{e} \int dE_e \prod_{e^*} \int_{SU(2)} dg_e \, e^{i \sum_{e} \mathrm{tr}(E_e \theta_e)}$
$Z = \int De \int D\omega \, e^{i S_{BF}(e,\omega)}$

$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

$$Z = \prod_{e} \int dE_e \prod_{e^*} \int_{SU(2)} dg_{e^*} \, e^{i \sum_{e} \mathrm{tr}(E_e G_e)}$$

$$Z = \int De \int D\omega \, e^{i S_{BF}(e,\omega)}$$

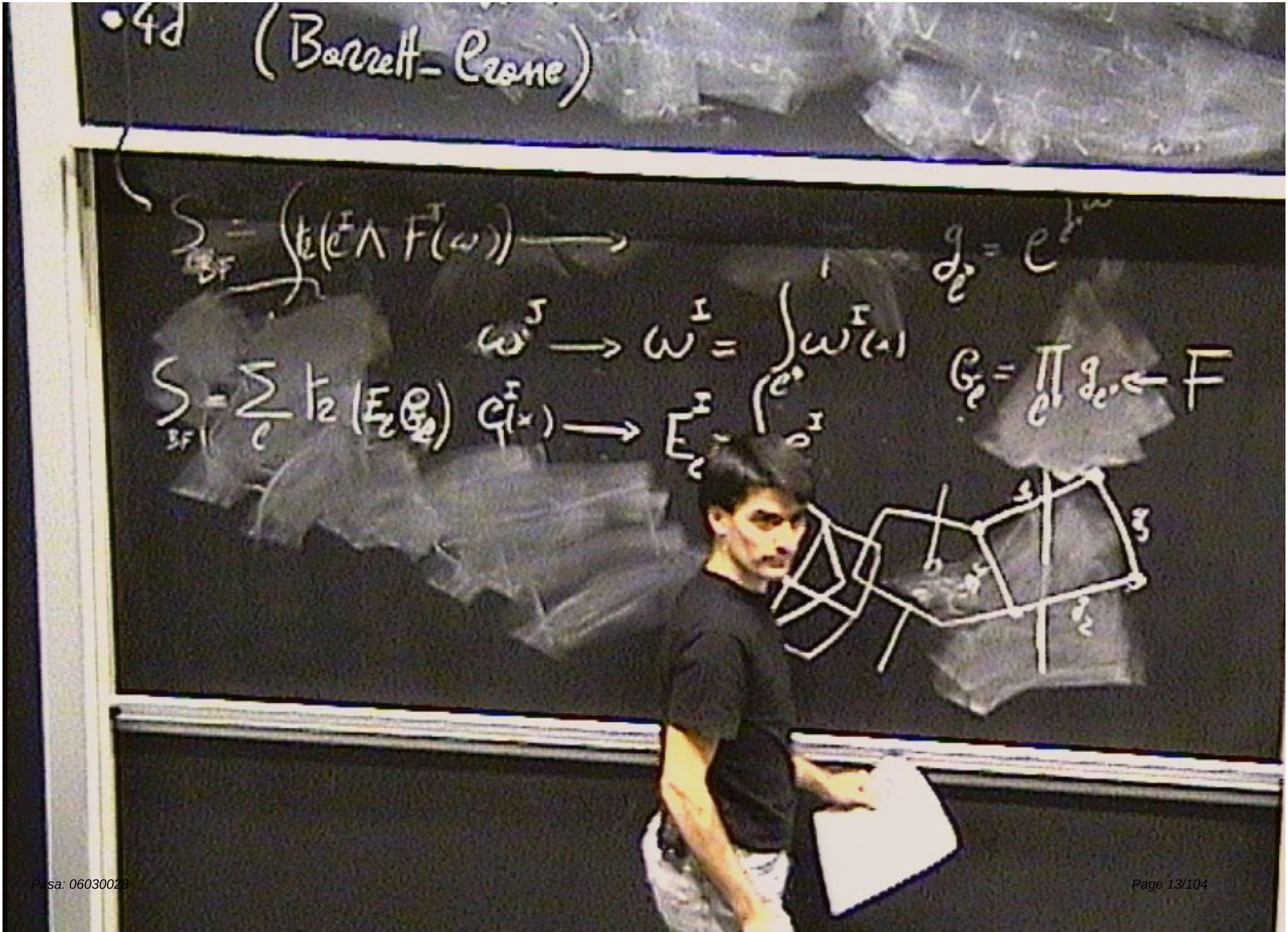

F. Dowker, gr-qc/0508109

$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

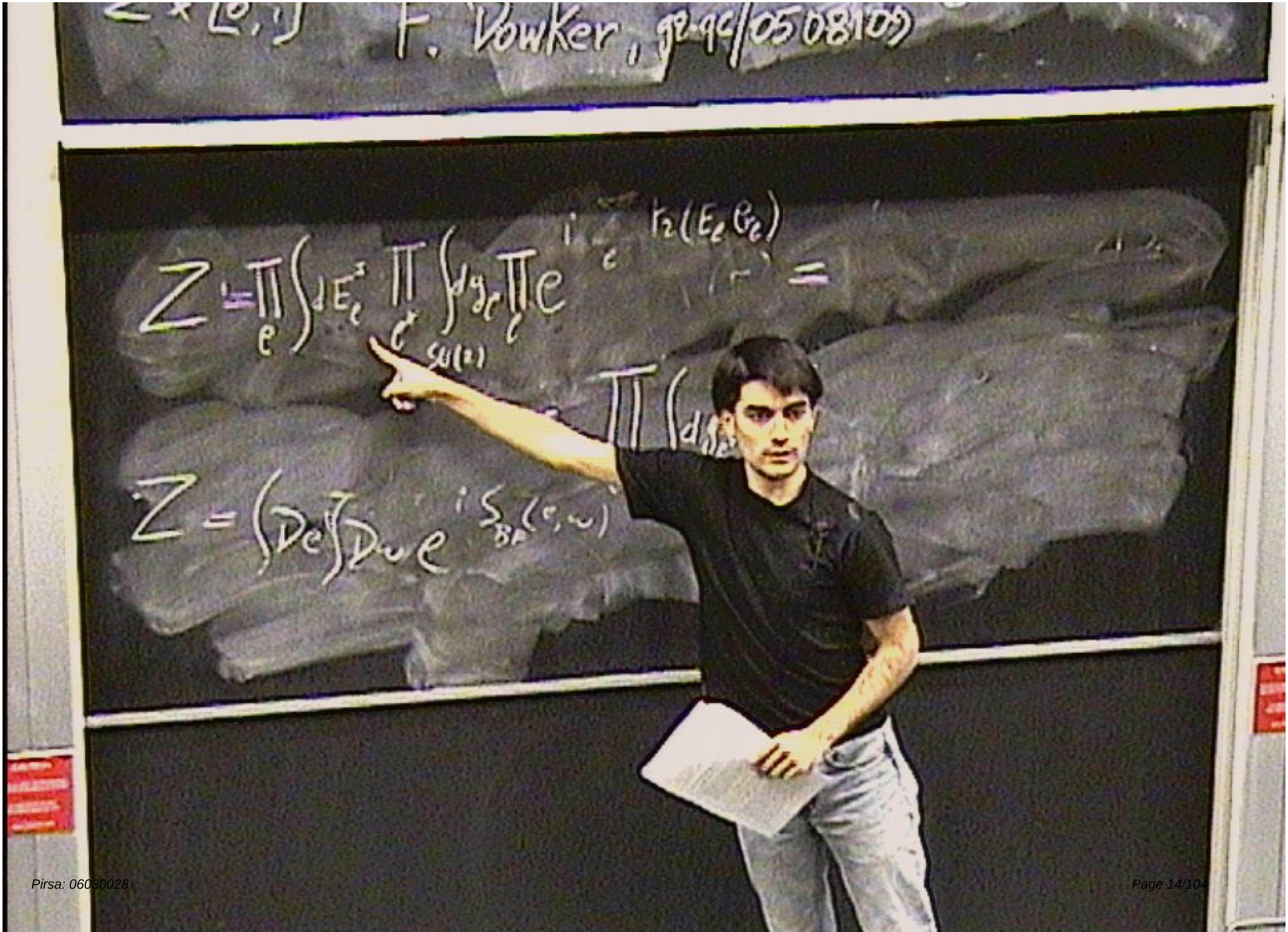
$$Z = \prod_{e} \int dE_e^i \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_{e} e^{i \sum_{e} \mathrm{tr}(E_e \theta_e)} =$$

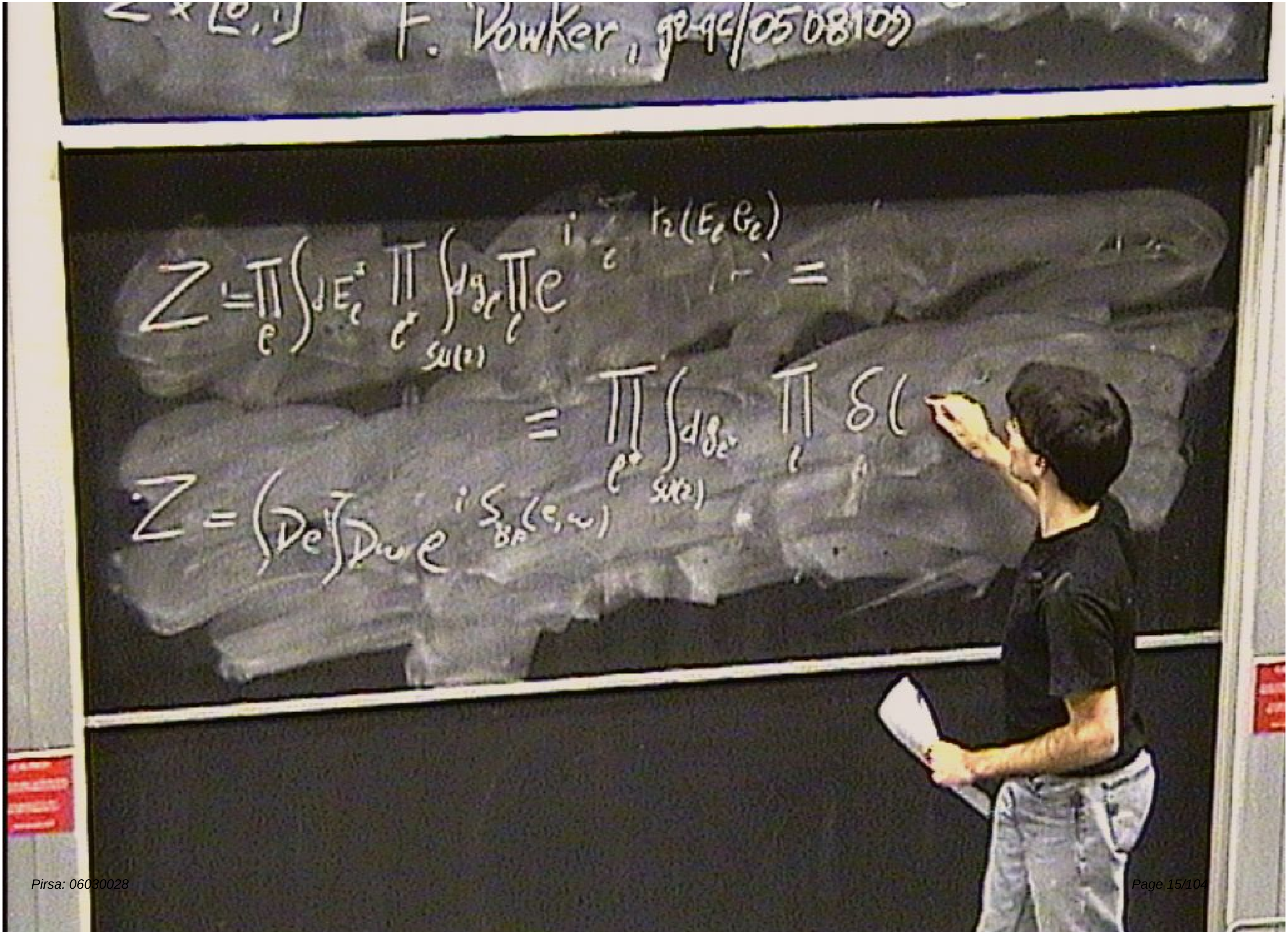
$$=$$

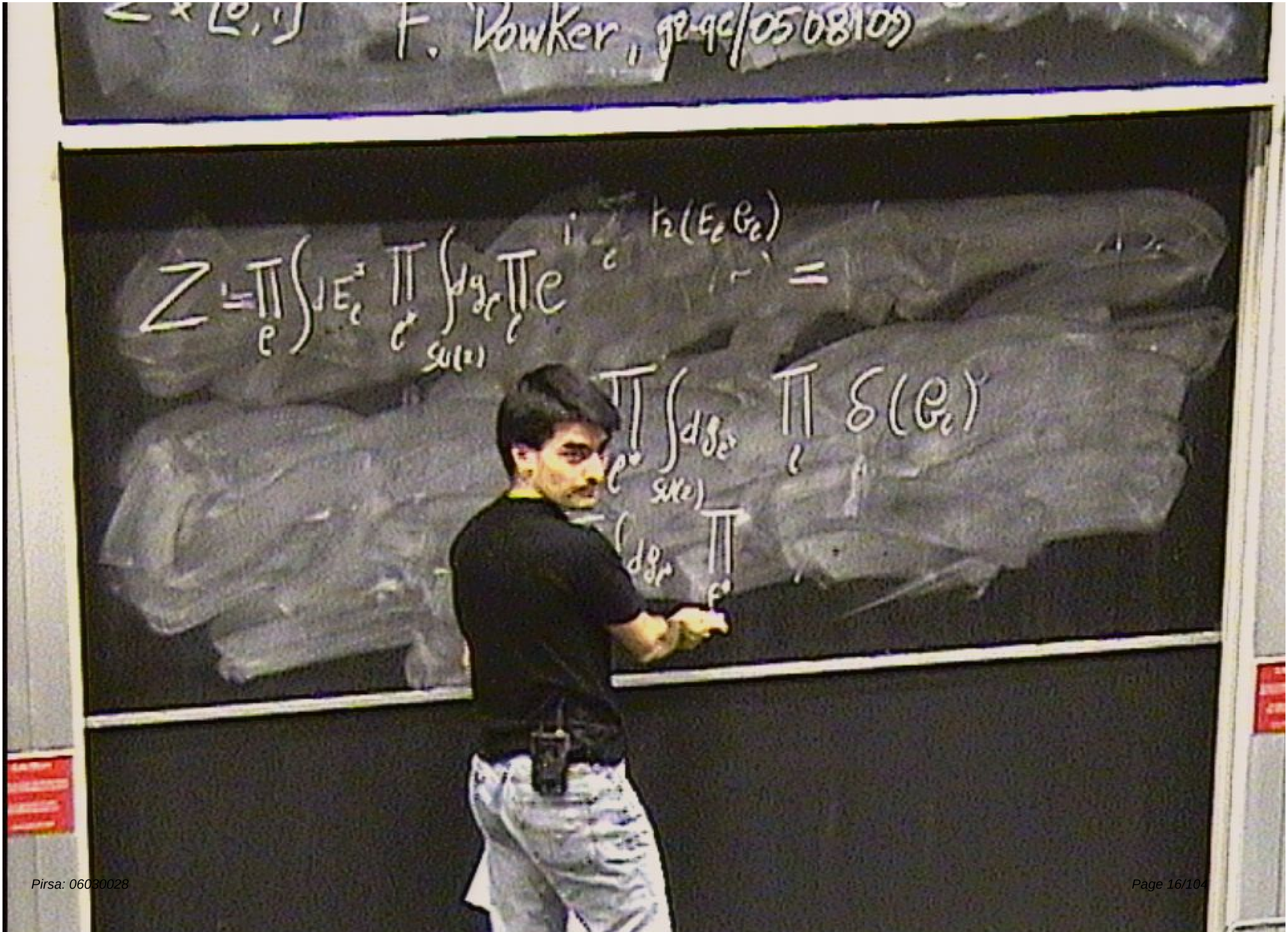
$$Z = \int De \int D\omega \, e^{i S_{BF}(e,\omega)}$$

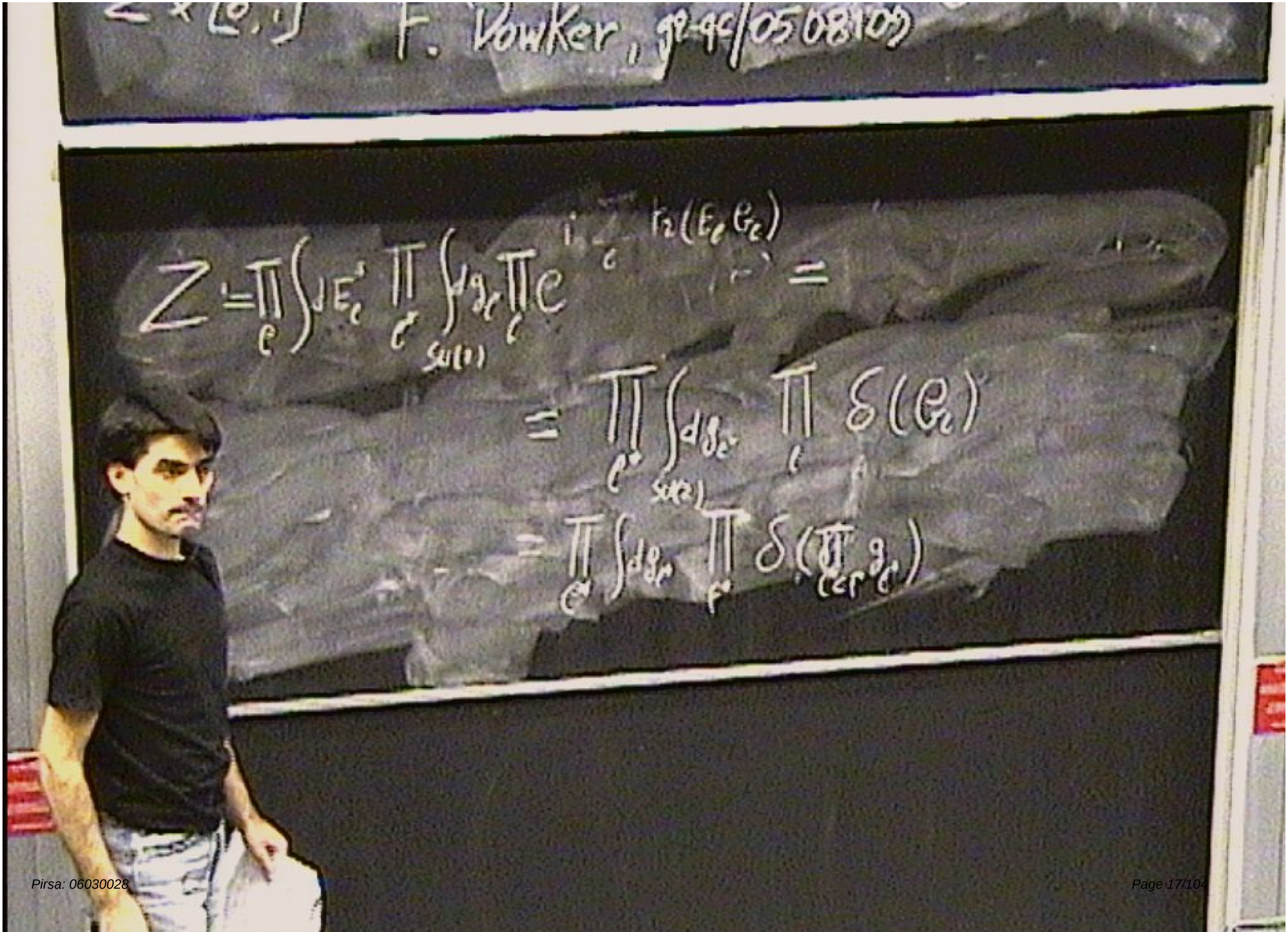


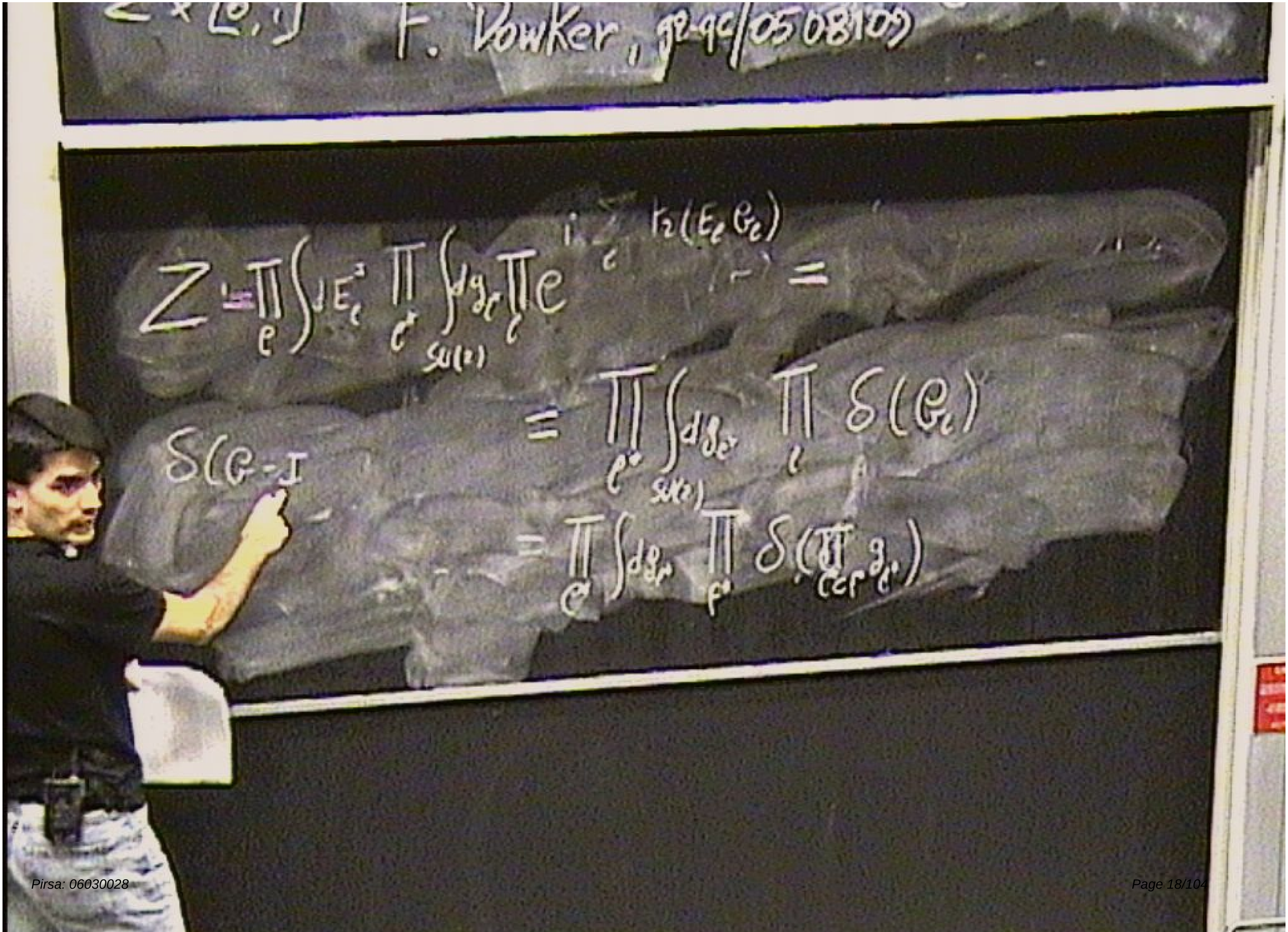
4d (Barrett-Crane)

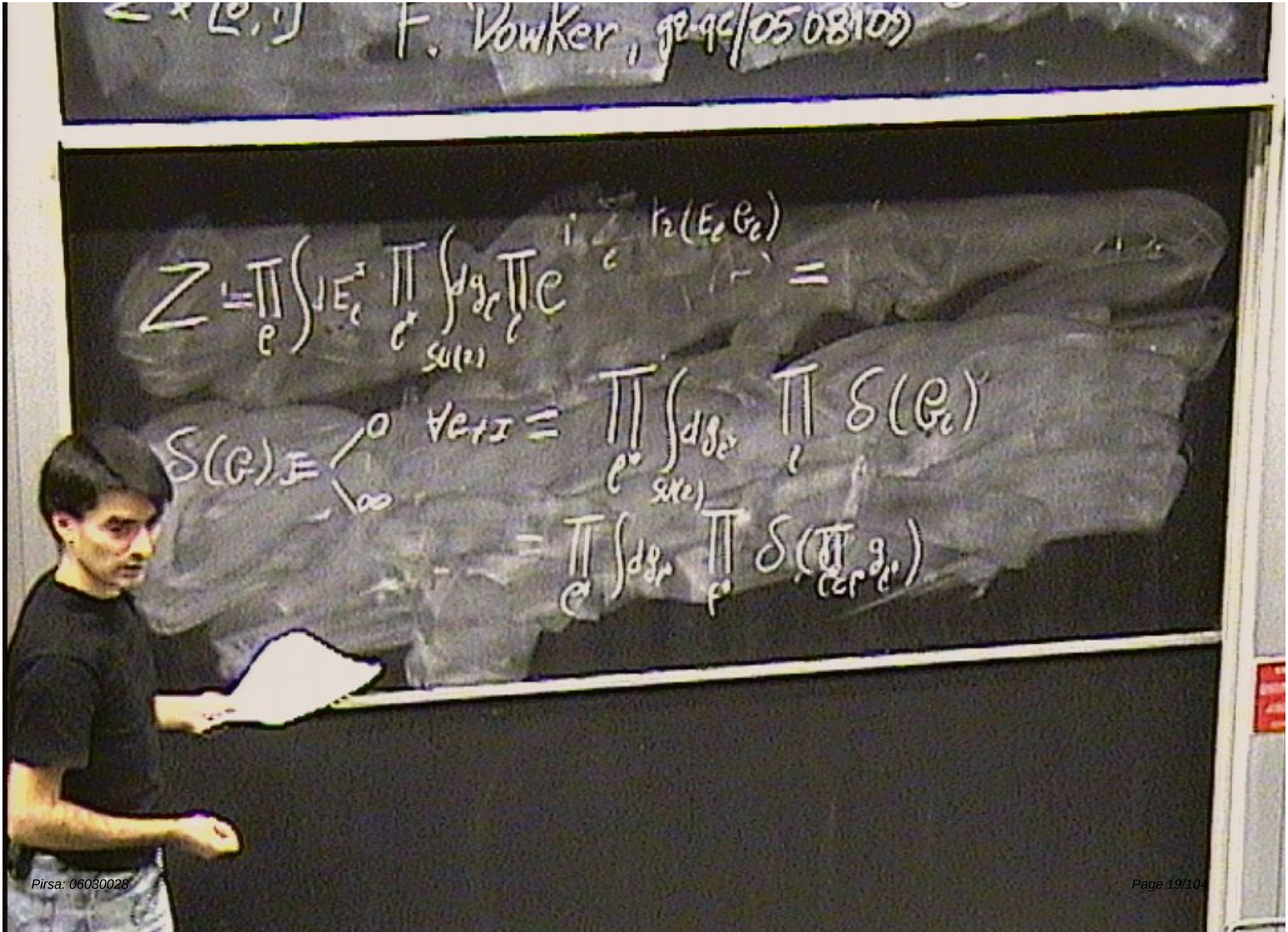

F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109
Z = ∏_ℓ ∫ dE_ℓ ∏_e* ∫_SU(2) dg_e* ∏_ℓ e^(i Σ_ℓ tr(E_ℓ G_ℓ)) =
= ∏_e* ∫_SU(2) dg_e* ∏_ℓ δ(
Z = ∫De ∫Dω e^(i S_BF(e,ω))

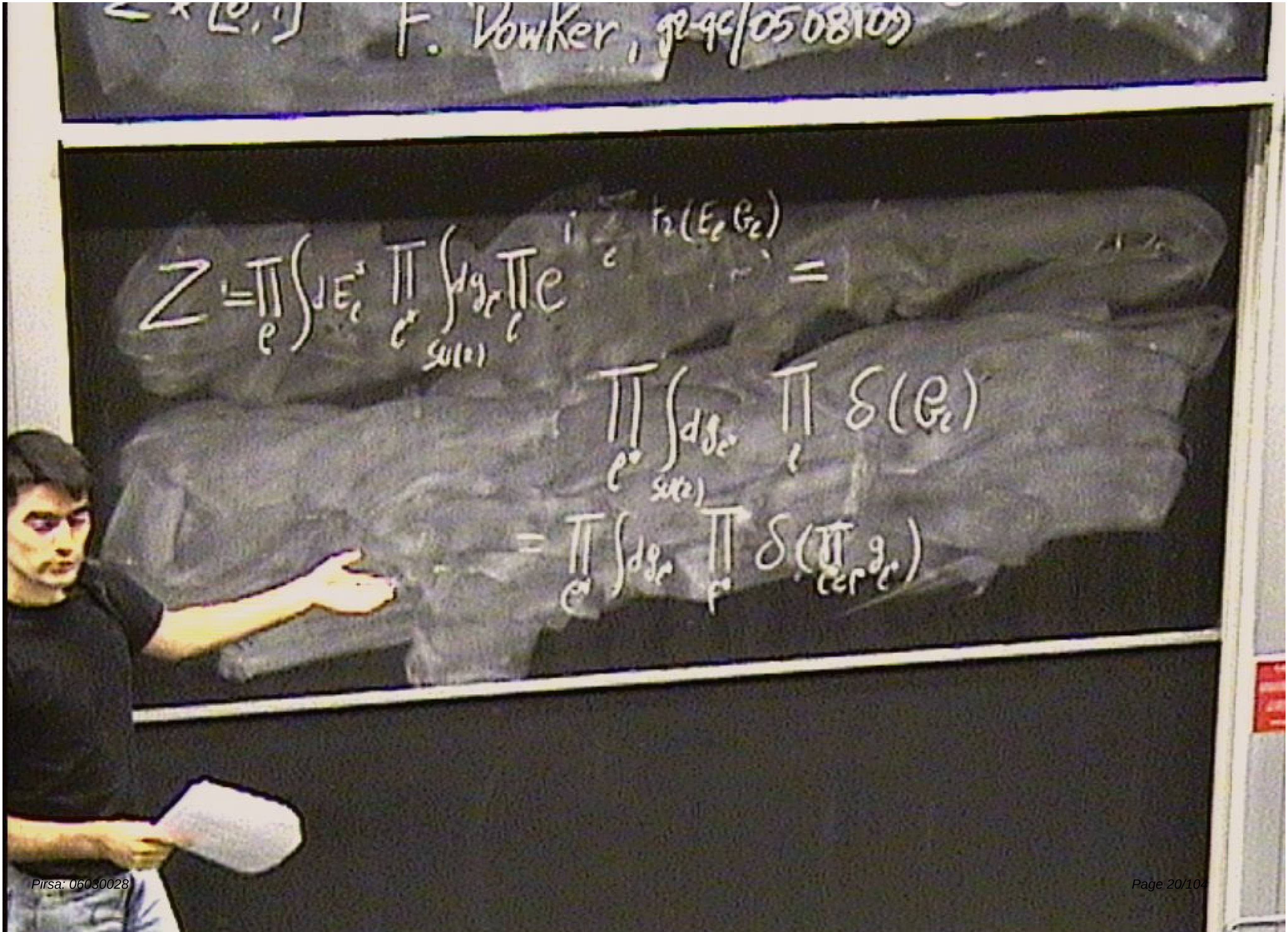

F. Dowker, gr-qc/0508109

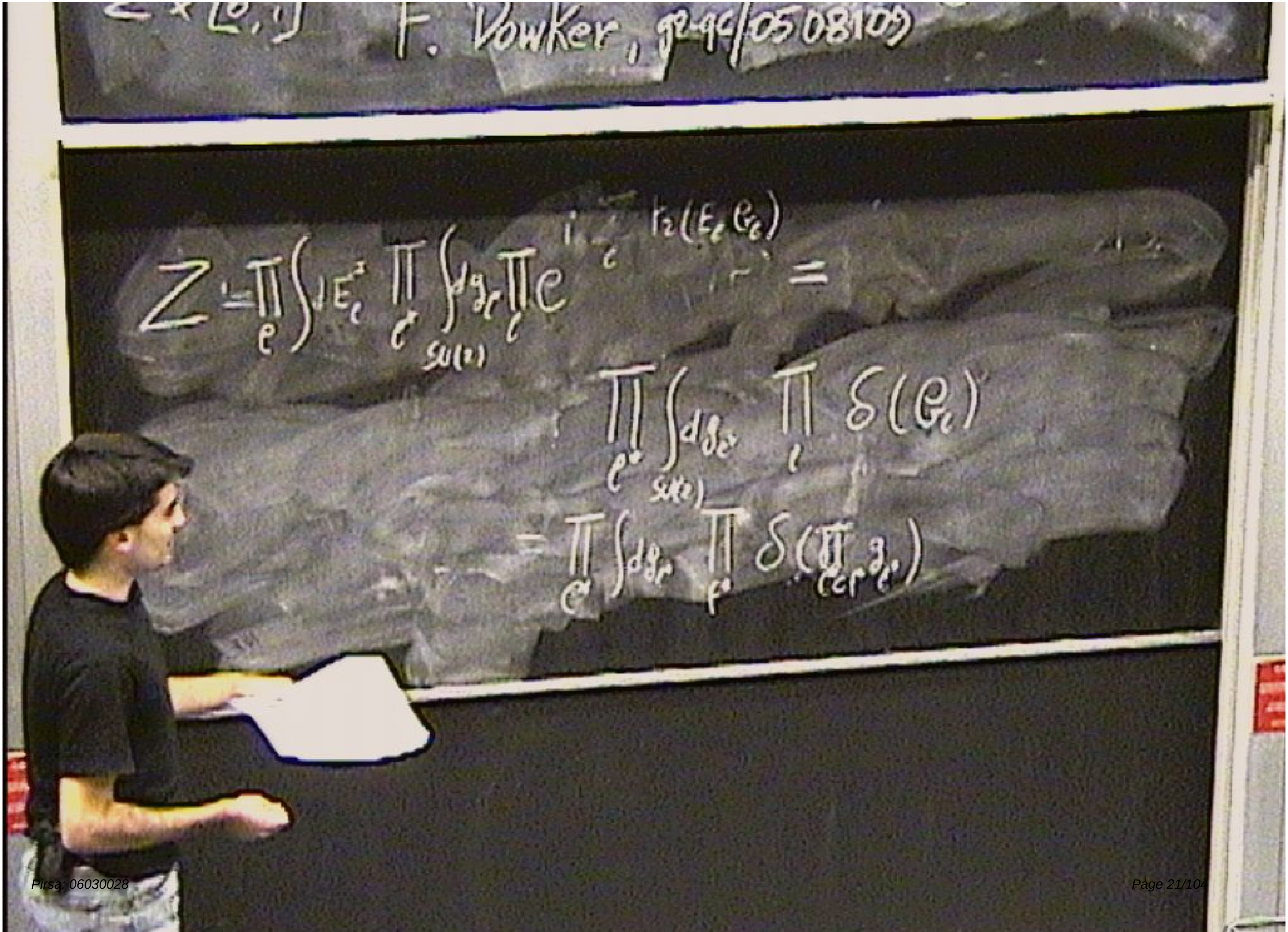

F. Dowker, gr-qc/0508109
$Z = \prod_e \int dE_e \prod_e \int_{SU(2)} dg_e \prod_f e^{i \sum_f \mathrm{tr}(E_f \theta_f)} =$
$= \prod_e \int_{SU(2)} dg_e \prod_f \delta(\theta_f)$
$= \prod_e \int dg_e \prod_f \delta(\prod_{e \in f} g_e)$


F. Dowker, gr-qc/0508109
$Z = \prod_e \int dE_e^i \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e \mathrm{tr}(E_e G_e)} =$
$= \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$
$\delta(G - I$
$= \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta(\prod_{e^* \subset f^*} g_{e^*})$



F. Dowker, gr-qc/0508109
$Z = \prod_e \int dE_e \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e tr(E_e G_e)} = \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$
$\delta(G) = 0 \ \forall G \neq I, \ \infty$
$= \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta(\prod_{e^* \in f^*} g_{e^*})$


F. Dowker, gr-qc/0508109
$Z = \prod_e \int dE_e \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e \mathrm{tr}(E_e G_e)} =$
$\prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$
$= \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta(\prod_{e \in f} g_e)$


F. Dowker, gr-qc/0508109
$Z = \prod_{l} \int dE_l \prod_{l^*} \int_{SU(2)} dg_{l^*} \prod_{l} e^{i \sum_{l} \mathrm{tr}(E_l \Theta_l)} =$
$\prod_{l^*} \int_{SU(2)} dg_{l^*} \prod_{l} \delta(\Theta_l)$
$= \prod_{l^*} \int dg_{l^*} \prod_{f^*} \delta(\prod_{l^* \in f^*} g_{l^*})$

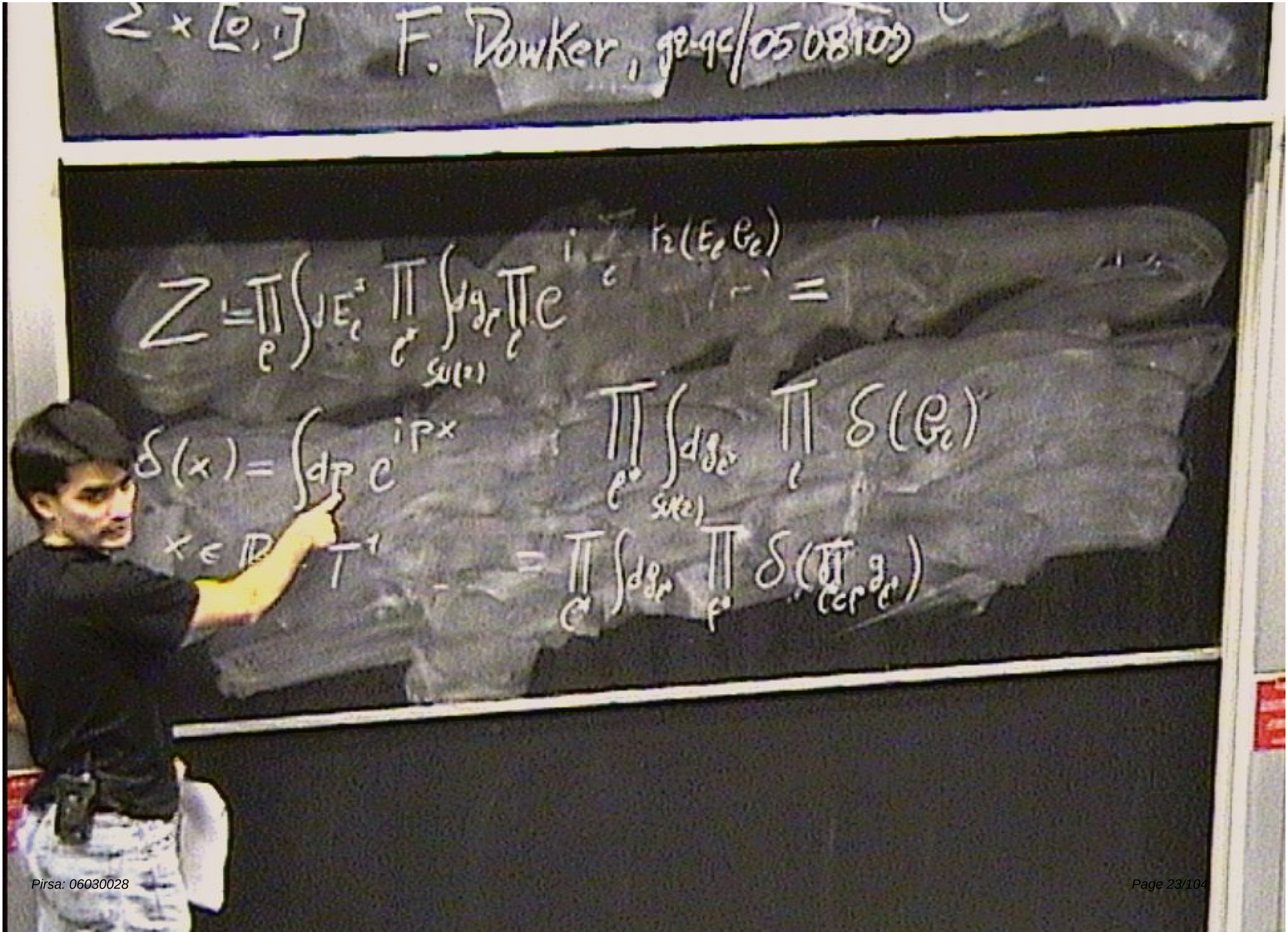
F. Dowker, gr-qc/0508109

$$Z \stackrel{\text{def}}{=} \prod_{e} \int dE_e \prod_{e} \int_{SU(2)} dg_e \prod_{e} e^{\,i \sum_{e} \mathrm{tr}_2 (E_e \Theta_e)} =$$

$$\delta(x) = \int dp\, e^{ipx}$$

$$\prod_{e} \int_{SU(2)} dg_e \prod_{e} \delta(\Theta_e)$$

$$= \prod_{e} \int dg_e \prod_{f} \delta\Big(\prod_{e \subset f} g_e\Big)$$

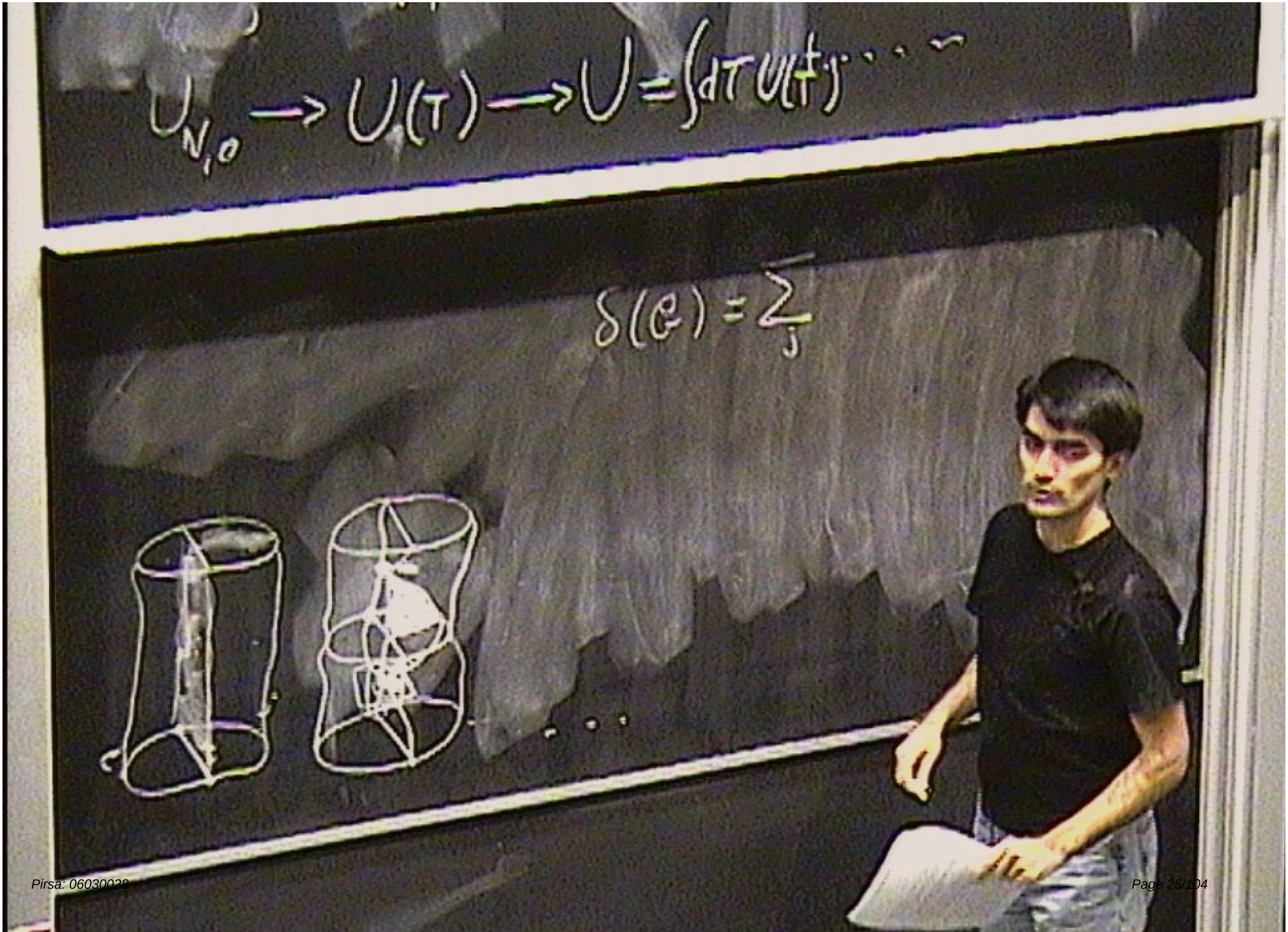

$\Sigma \times [0,1]$
F. Dowker, gr-qc/0508109
$Z = \prod_{\ell} \int dE_{\ell} \prod_{\ell^*} \int_{SU(2)} dg_{\ell^*} \prod_{\ell} e^{i \sum_{\ell} tr(E_{\ell} \theta_{\ell})} =$
$\delta(x) = \int dp \, e^{ipx}$
$\prod_{\ell^*} \int_{SU(2)} dg_{\ell^*} \prod_{\ell} \delta(\theta_{\ell})$
$x \in \mathbb{R}$ T^1
$= \prod_{\ell^*} \int dg_{\ell^*} \prod_{f^*} \delta(\prod_{\ell^* \in f^*} g_{\ell^*})$

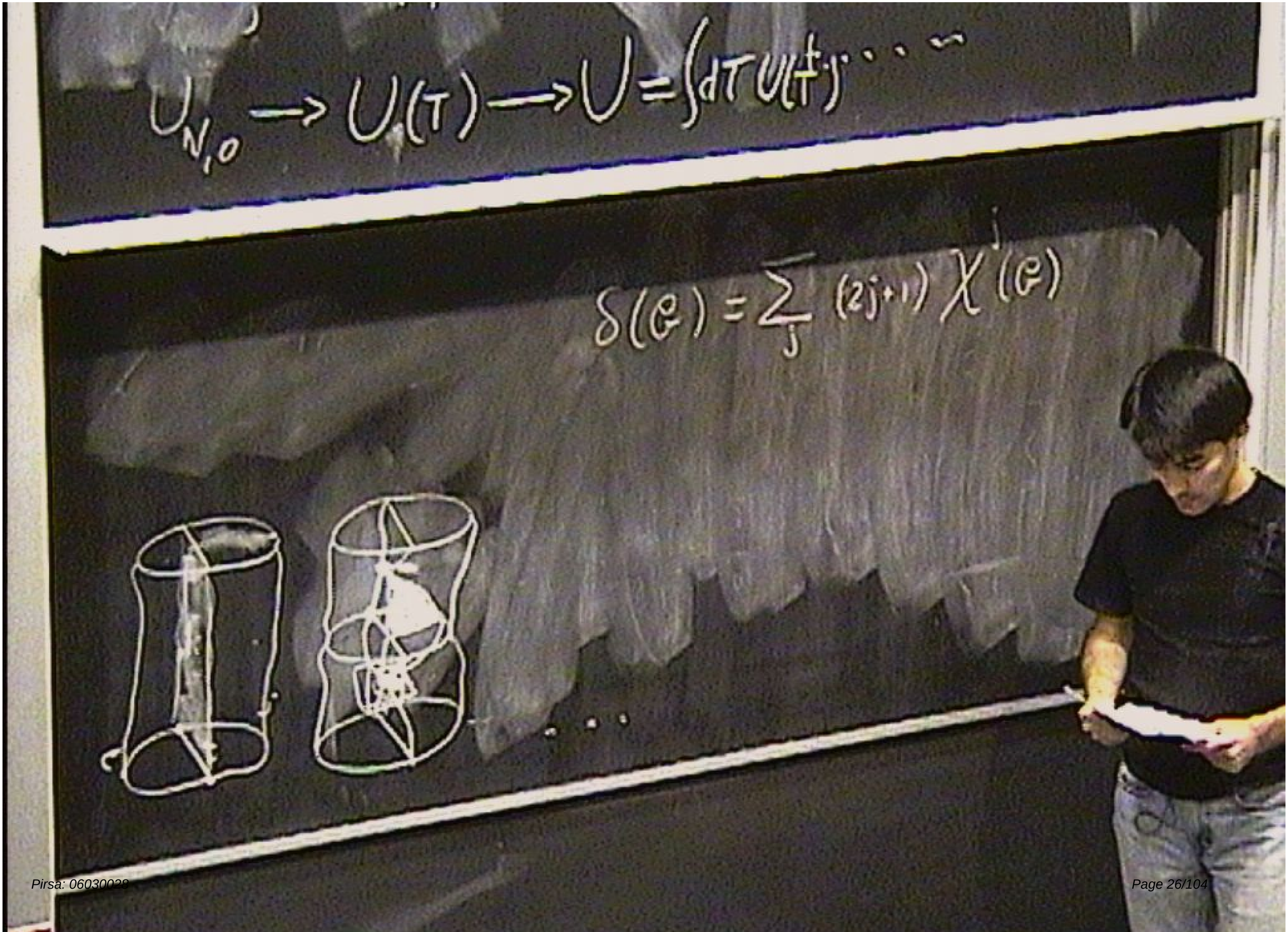
$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

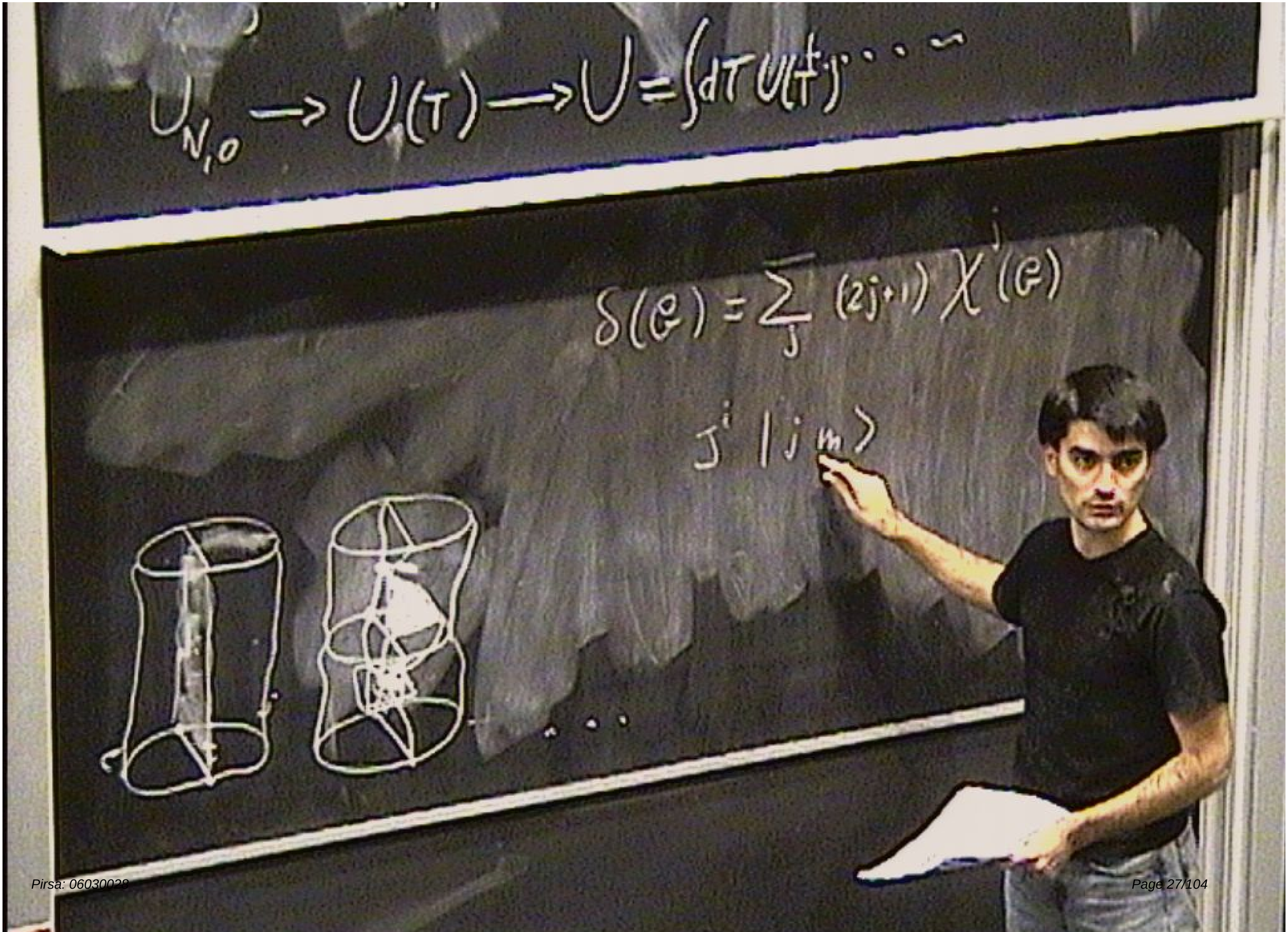
$$Z = \prod_{e} \int dE_e^i \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_{e} e^{i \sum_e \mathrm{tr}(E_e \theta_e)} = \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_{e} \delta(\theta_e)$$

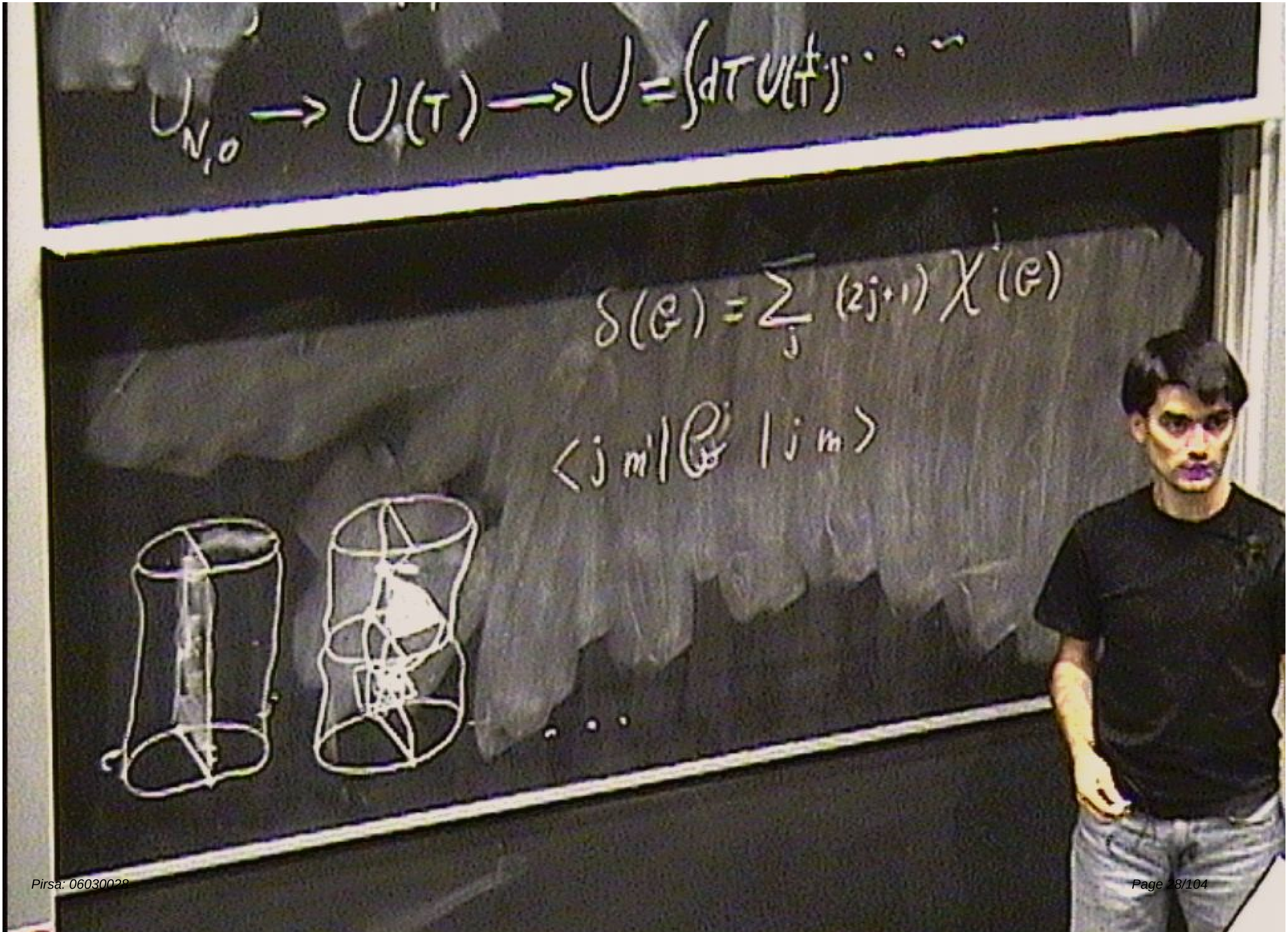
$$\delta(x) = \int dp\, e^{ipx}$$

$$x \in \mathbb{R} \sim T^1 \qquad = \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta\Big(\prod_{e^* \in f^*} g_{e^*}\Big)$$


$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$
$\delta(e) = \sum_j$


$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$
$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$


$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$
$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$
$J^i\, |j\, m\rangle$

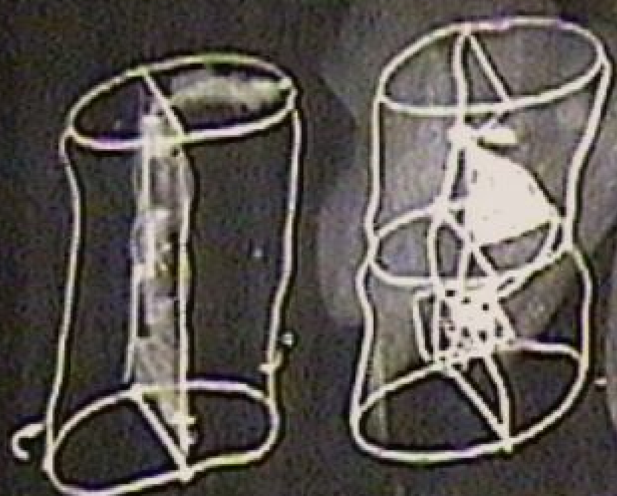


$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$
$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$
<j m'| g | j m>

$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$$

$$\langle j\, m' |\, g\, | j\, m \rangle = D^j_{mm'}(g)$$



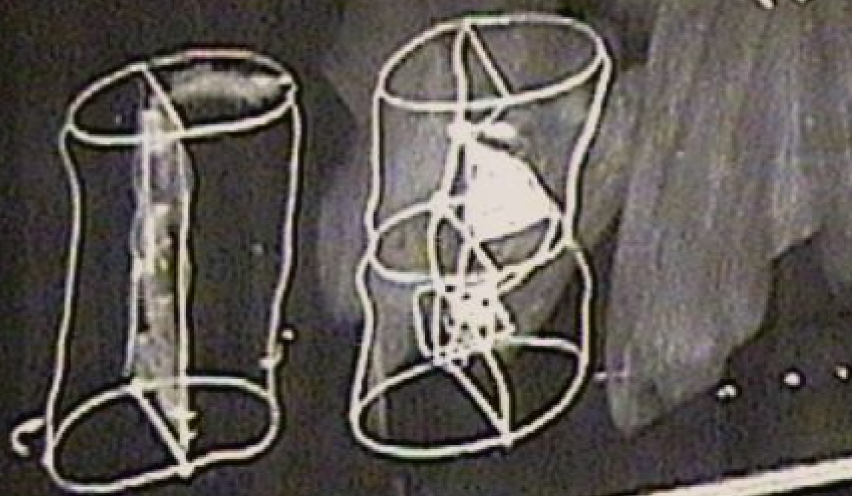
. . .

$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$$

$$\langle j\, m' |\, g\, | j\, m \rangle = D^j_{mm'}(g)$$

$$\chi^j(g) = \sum_m D^j_{mm}(g)$$



$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

$$Z = \prod_e \int dE_e^a \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e \mathrm{tr}(E_e G_e)} = \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$$

$$\delta(x) = \int dp\, e^{ipx}$$

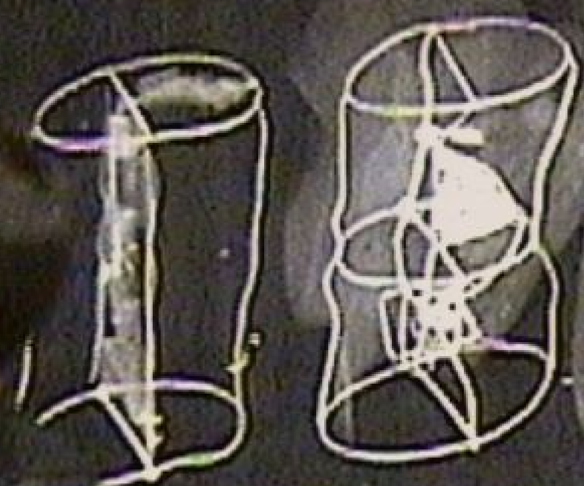
$$x \in \mathbb{R} \sim T^* \qquad = \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta\Big(\prod_{e^* \in f^*} g_{e^*}\Big)$$

$$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$$

$$\delta(g) = \sum_j (2j+1)\, \chi^j(g)$$

$$\langle j\, m' | g | j\, m \rangle = D^j_{m m'}(g)$$

$$\chi^j(g) = \sum_m D^j_{mm}(g)$$



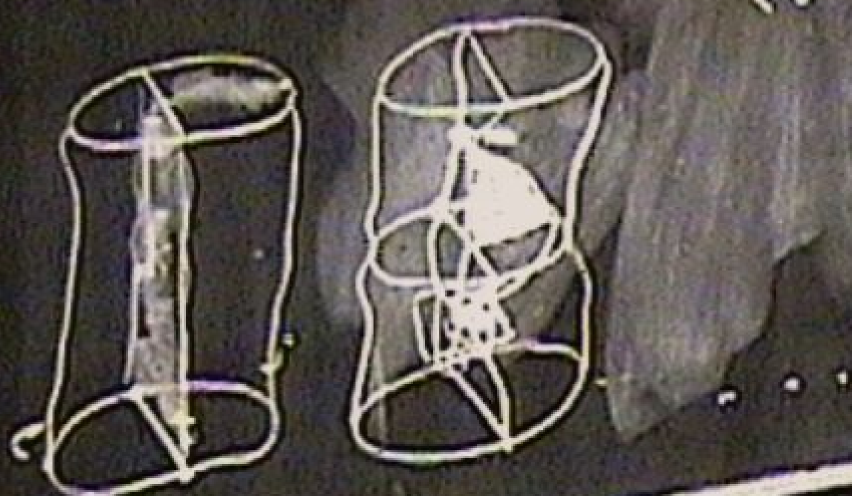
. . .

$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$\delta(g_{t^*}) = \sum_j (2j+1)\, \chi^j(g)$$

$$\langle j\, m' |\, g \,| j\, m \rangle = D^j_{mm'}(g)$$

$$\chi^j(g) = \sum_m D^j_{mm}(g)$$



$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

$$Z = \prod_e \int dE_e^i \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e \mathrm{tr}_2(E_e G_e)} =$$

$$\delta(x) = \int dp\, e^{ipx} \qquad \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$$

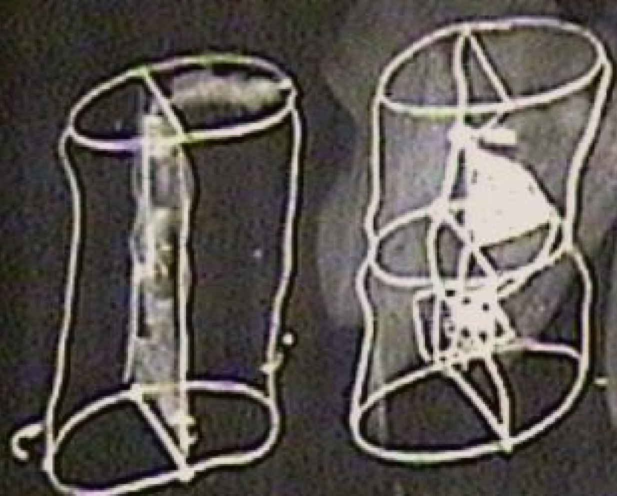
$$x \in \mathbb{R} \sim T^1 \qquad = \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta\Big(\prod_{e \subset f} g_{e^*}\Big)$$

$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$\delta(g_{\Gamma}) = \sum_j (2j+1)\, \chi^j(g)$$

$$\langle j\, m' |\, g \,| j\, m \rangle = D^j_{m'm}(g)$$

$$\chi^j(g) = \sum_m D^j_{mm}(g)$$



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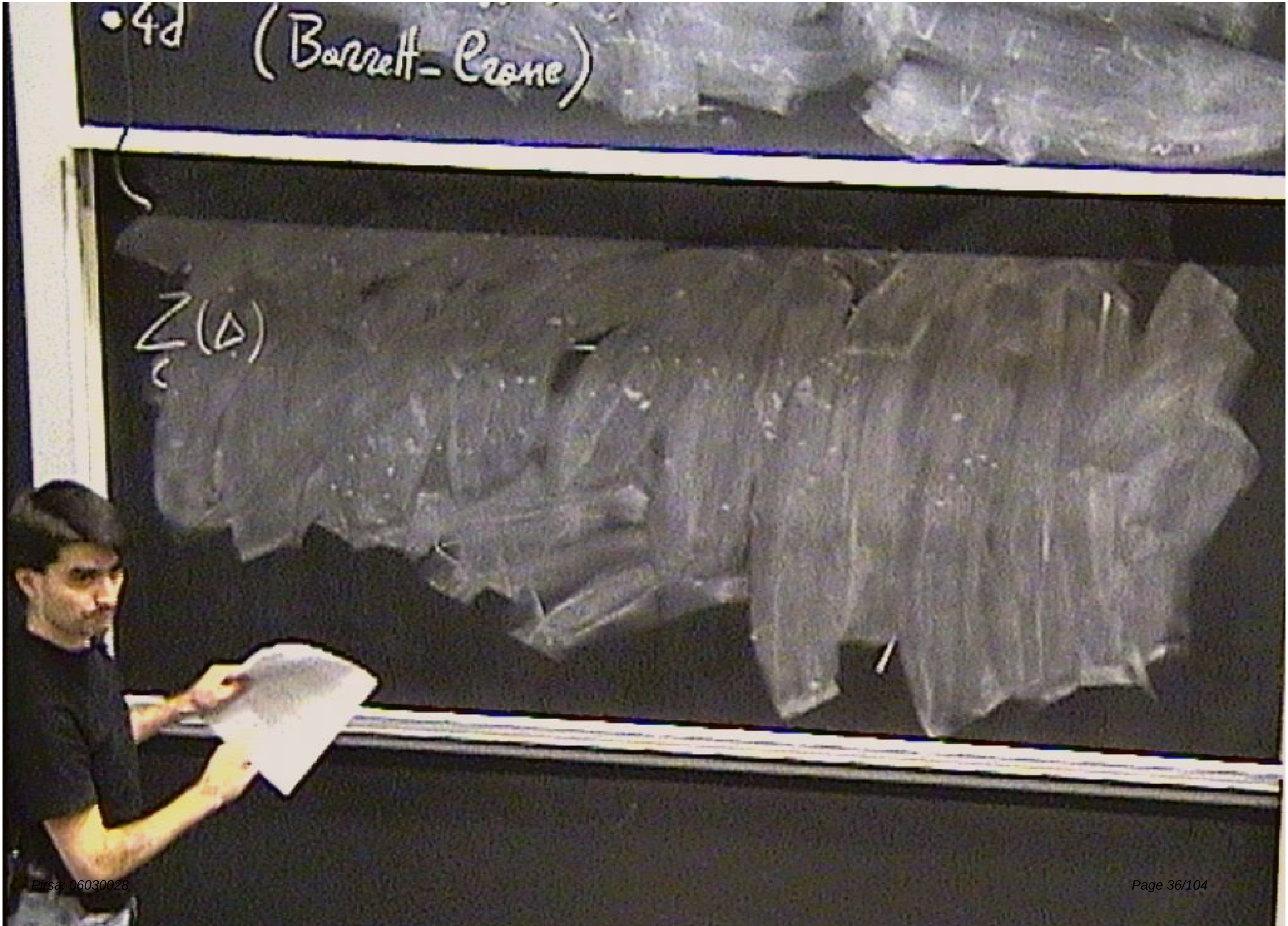
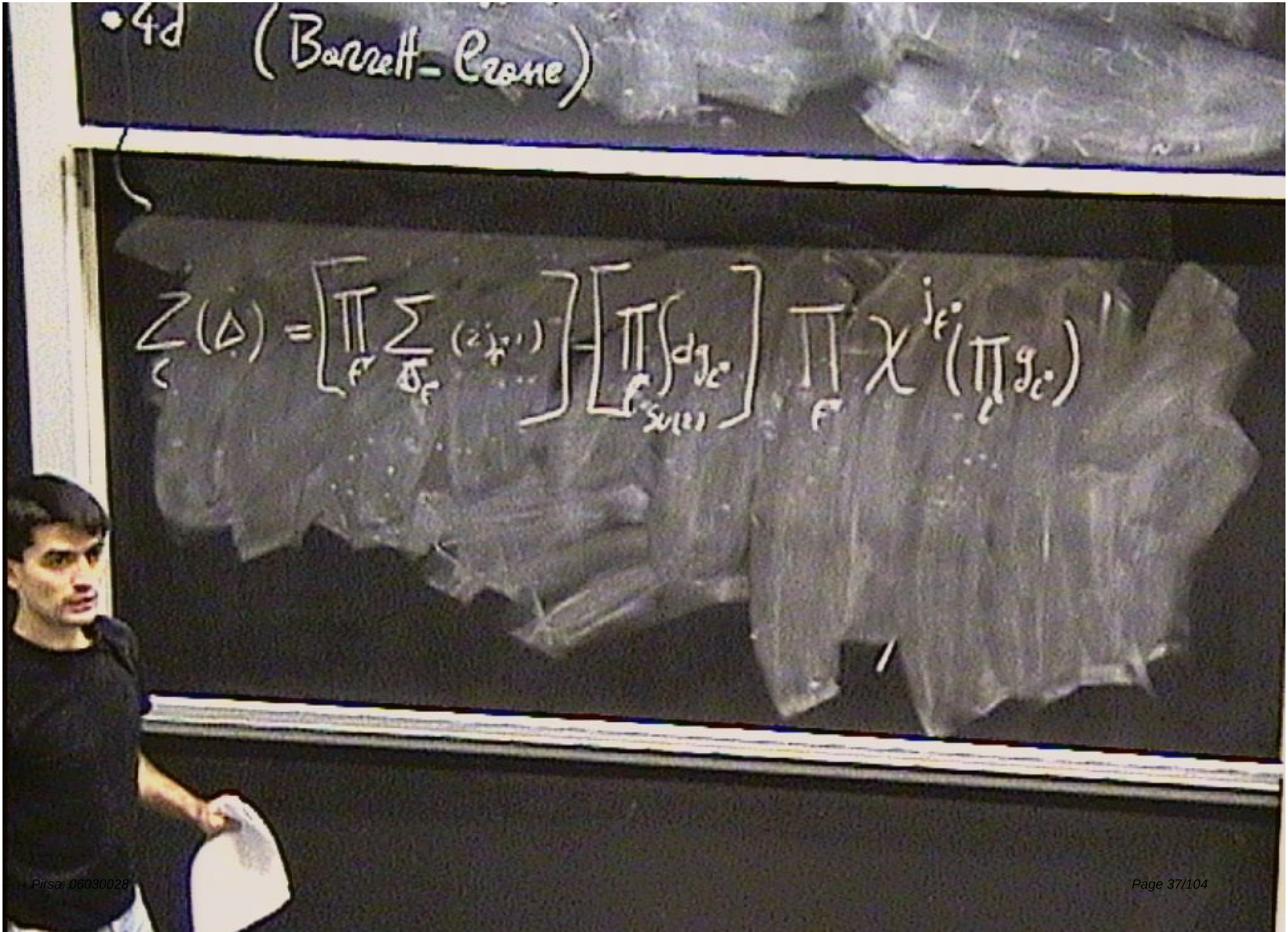
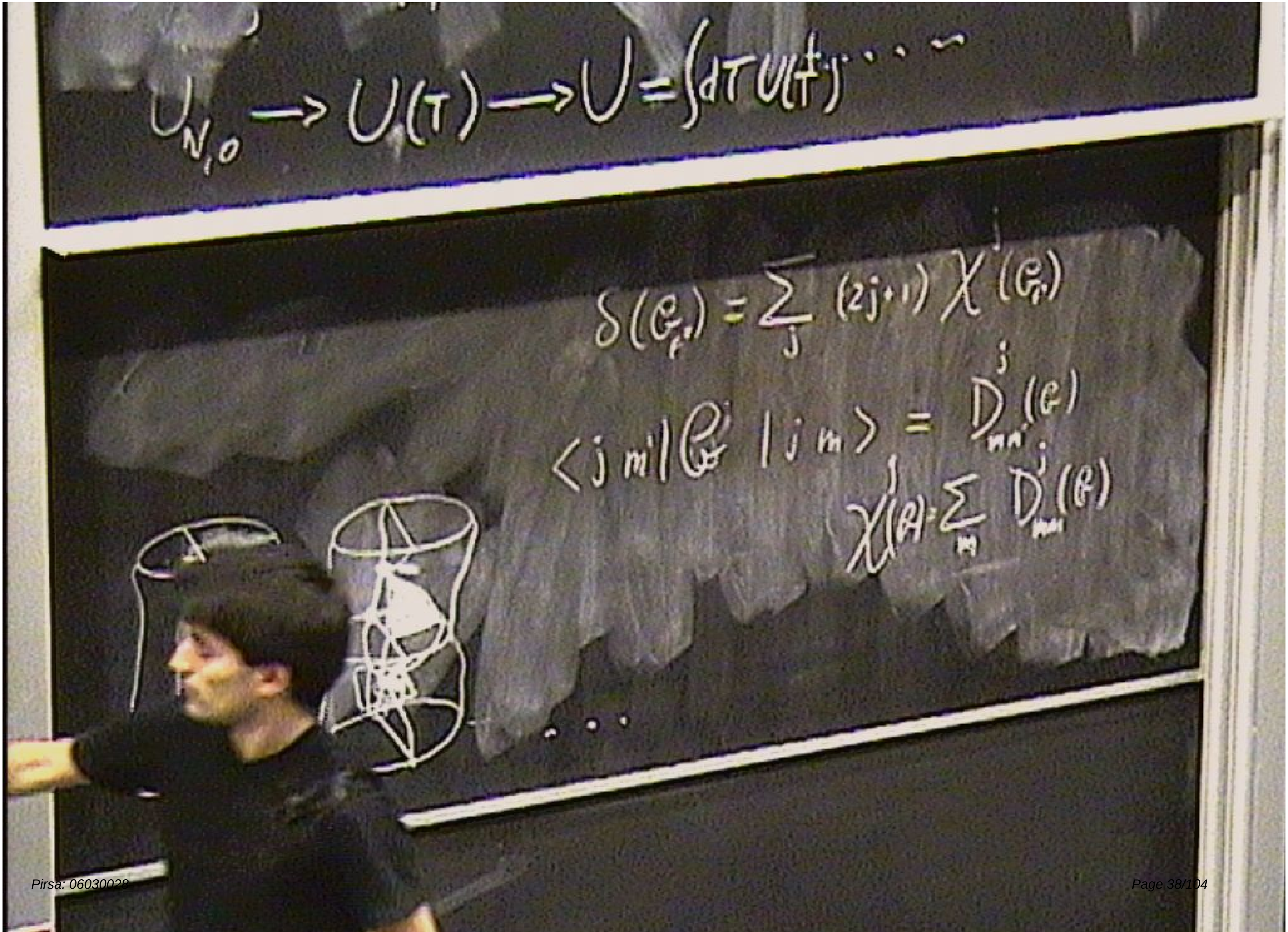
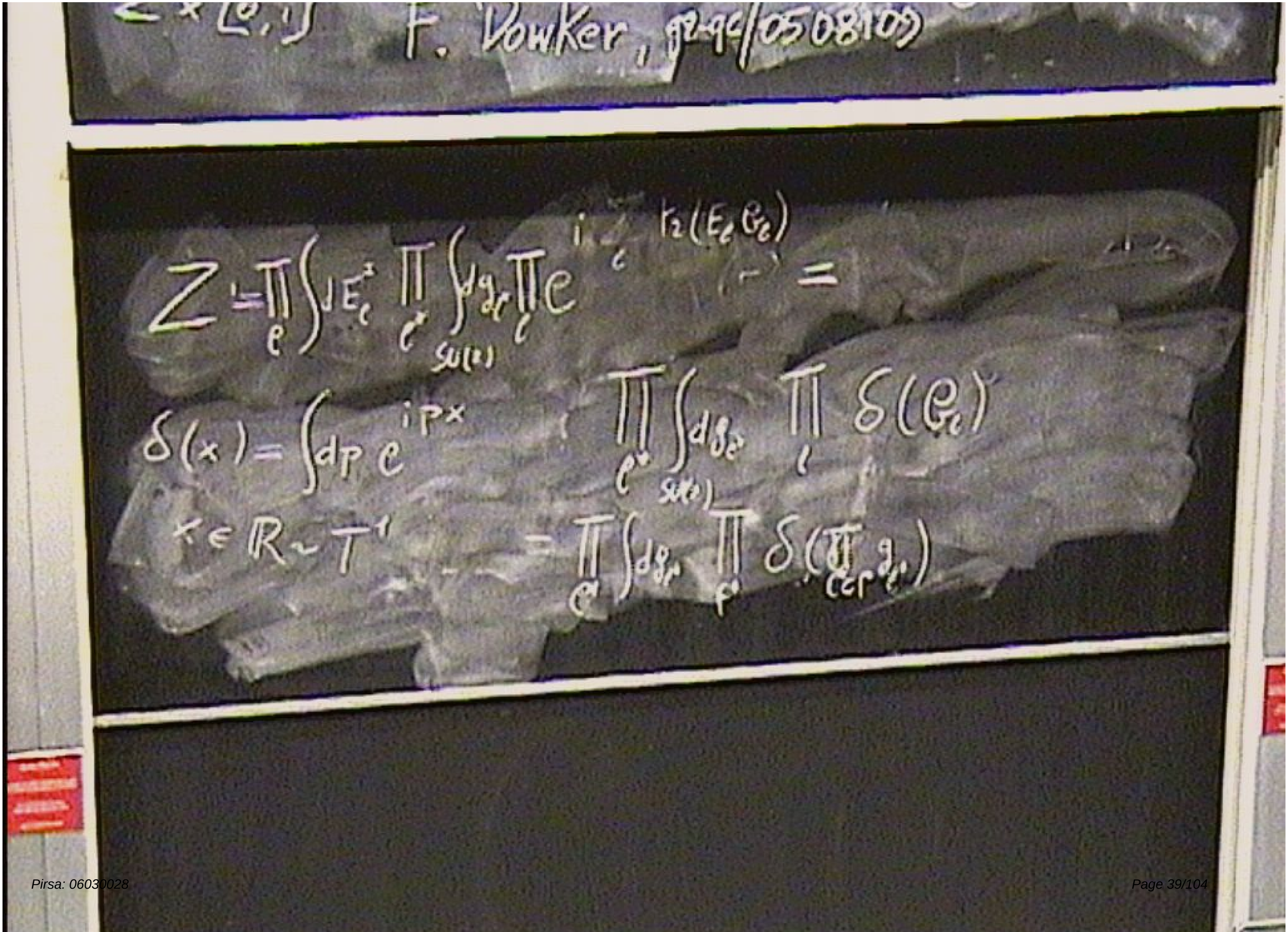
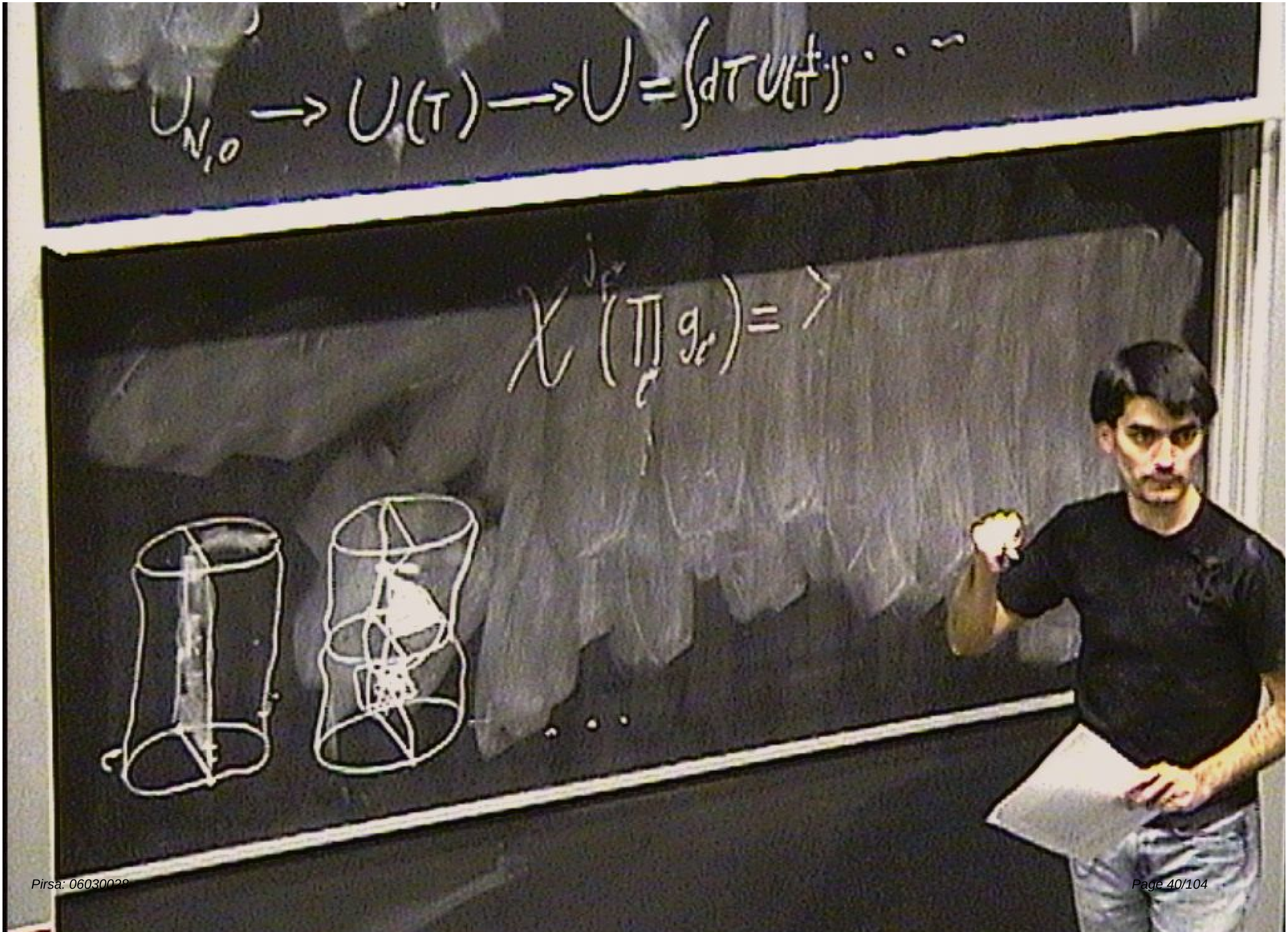
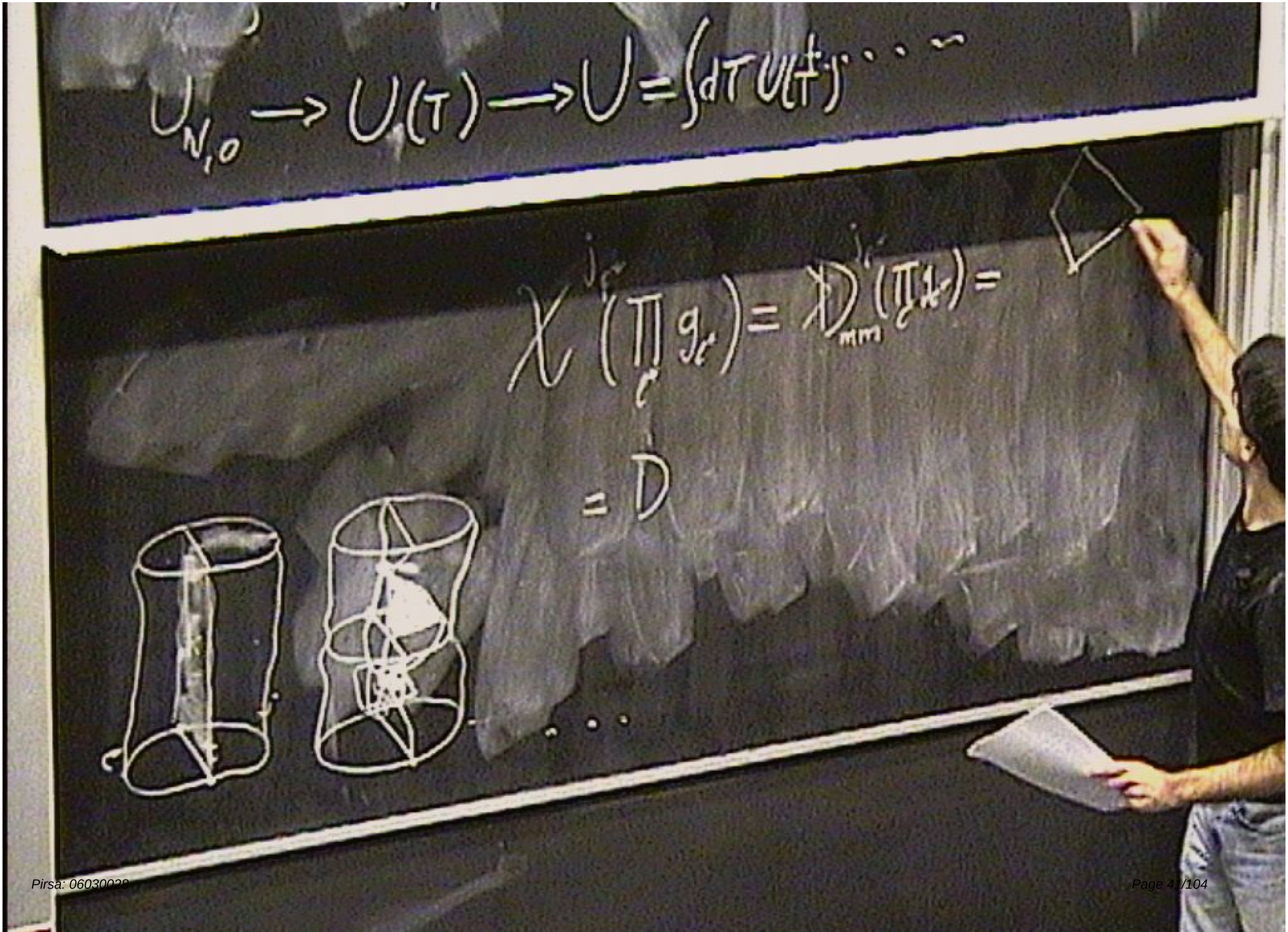

• 4d (Barrett_Crane)
Z(Δ)

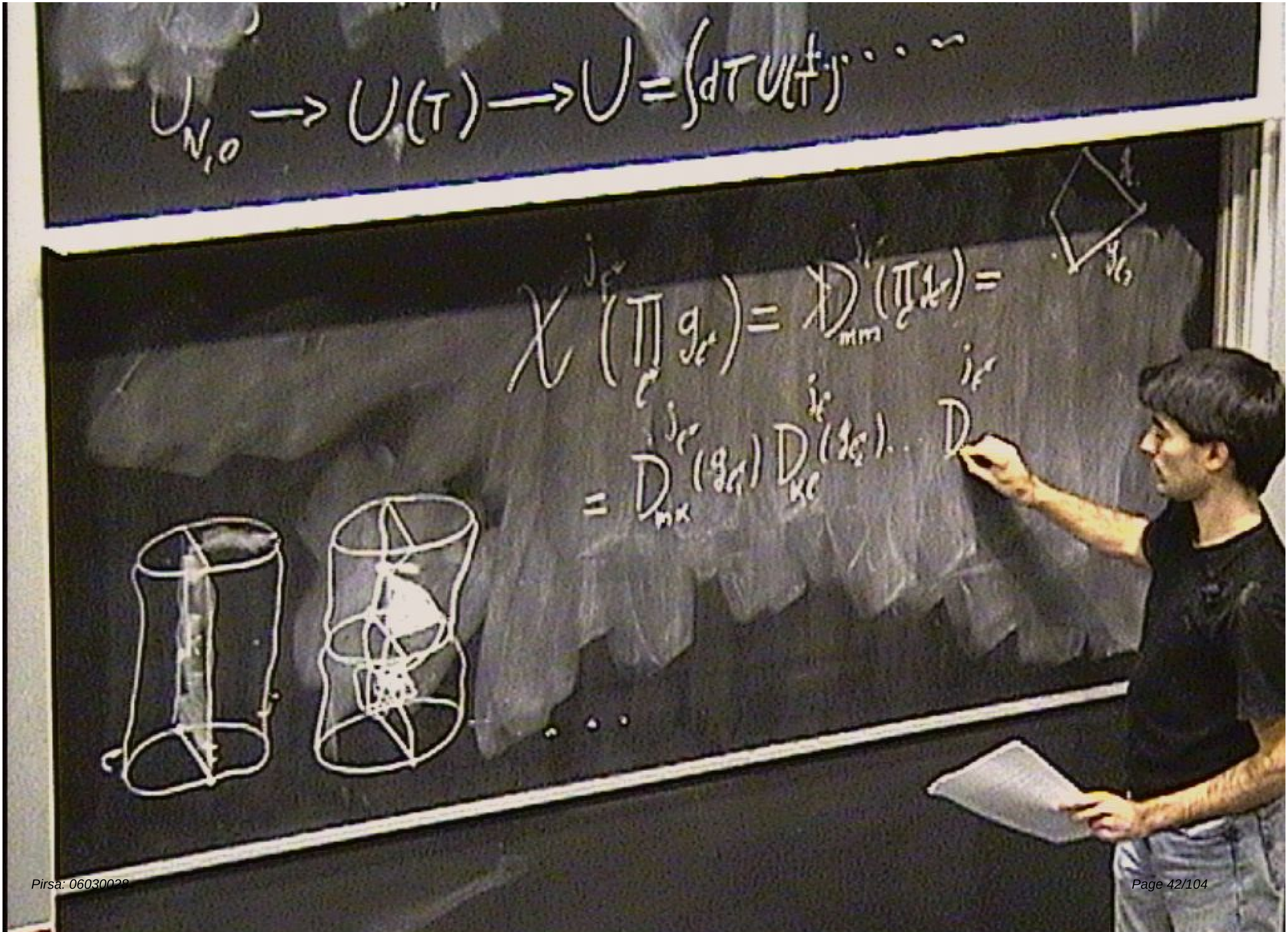

•4d (Barrett–Crane)
$Z(\Delta) = \left[\prod_f \sum_{\Delta_f} (2j_f+1)\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}(\prod_e g_e)$


$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$
$\delta(g_f) = \sum_j (2j+1)\, \chi^j(g_f)$
$\langle j\, m' | g | j\, m \rangle = D^j_{mm'}(g)$
$\chi^j(g) = \sum_m D^j_{mm}(g)$


F. Dowker, gr-qc/0508109
$Z = \prod_e \int dE_e \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e e^{i \sum_e \mathrm{tr}(E_e G_e)} =$
$\delta(x) = \int dp\, e^{ipx}$
$\prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_e \delta(G_e)$
$x \in \mathbb{R} \sim T^1$
$= \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta(\prod_{e \in f^*} g_{e^*})$


$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$
$\chi^{j_f}(\prod_\ell g_\ell) = \sum$
. . .

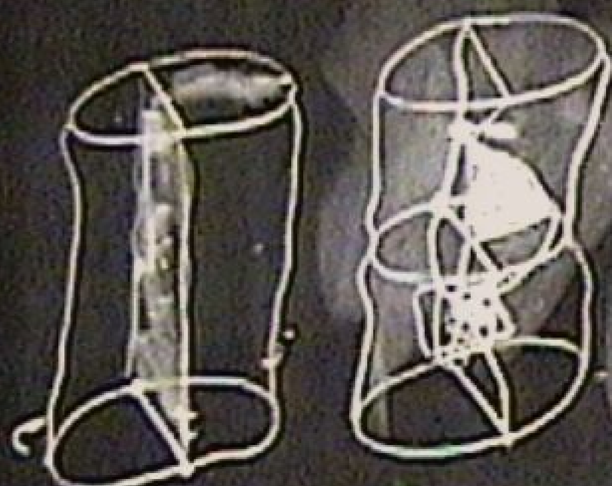

$U_{N,0} \to U(\tau) \to U = \int d\tau\, U(\tau) \dots$
$\chi^{j_\ell}(\prod_\ell g_\ell) = D^{j_\ell}_{mm}(\prod_\ell g_\ell) =$
$= D$


$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$
$\chi^{j_f}(\prod_e g_e) = D^{j_f}_{mm}(\prod_e g_e) =$
$= D^{j_f}_{mk}(g_{e_1}) D^{j_f}_{kc}(g_{e_2}) \ldots D^{j_f}$

$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$\chi^{j_\ell}\left(\prod_\ell g_\ell\right) = \chi D^{j_\ell}_{mm}\left(\prod_\ell g_\ell\right) =$$

$$= D^{j_\ell}_{mk}(g_{\ell_1})\, D^{j_\ell}_{kl}(g_{\ell_2}) \cdots D^{j_\ell}_{nm}(g_\ell)$$

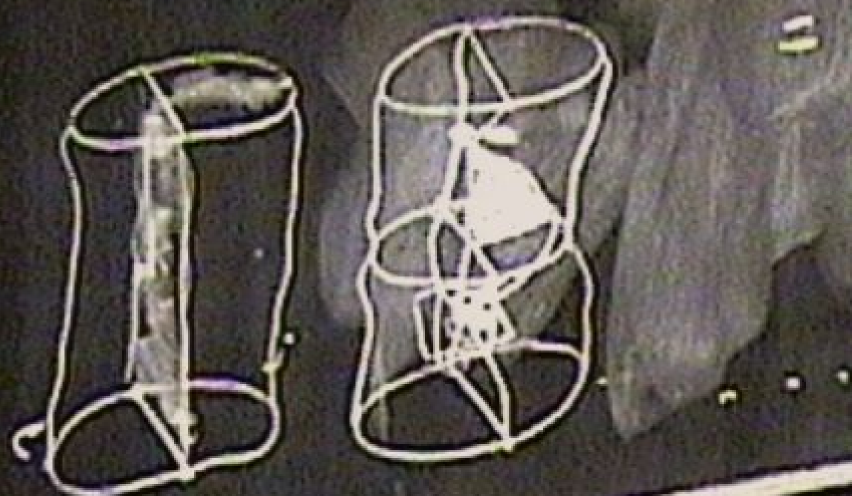


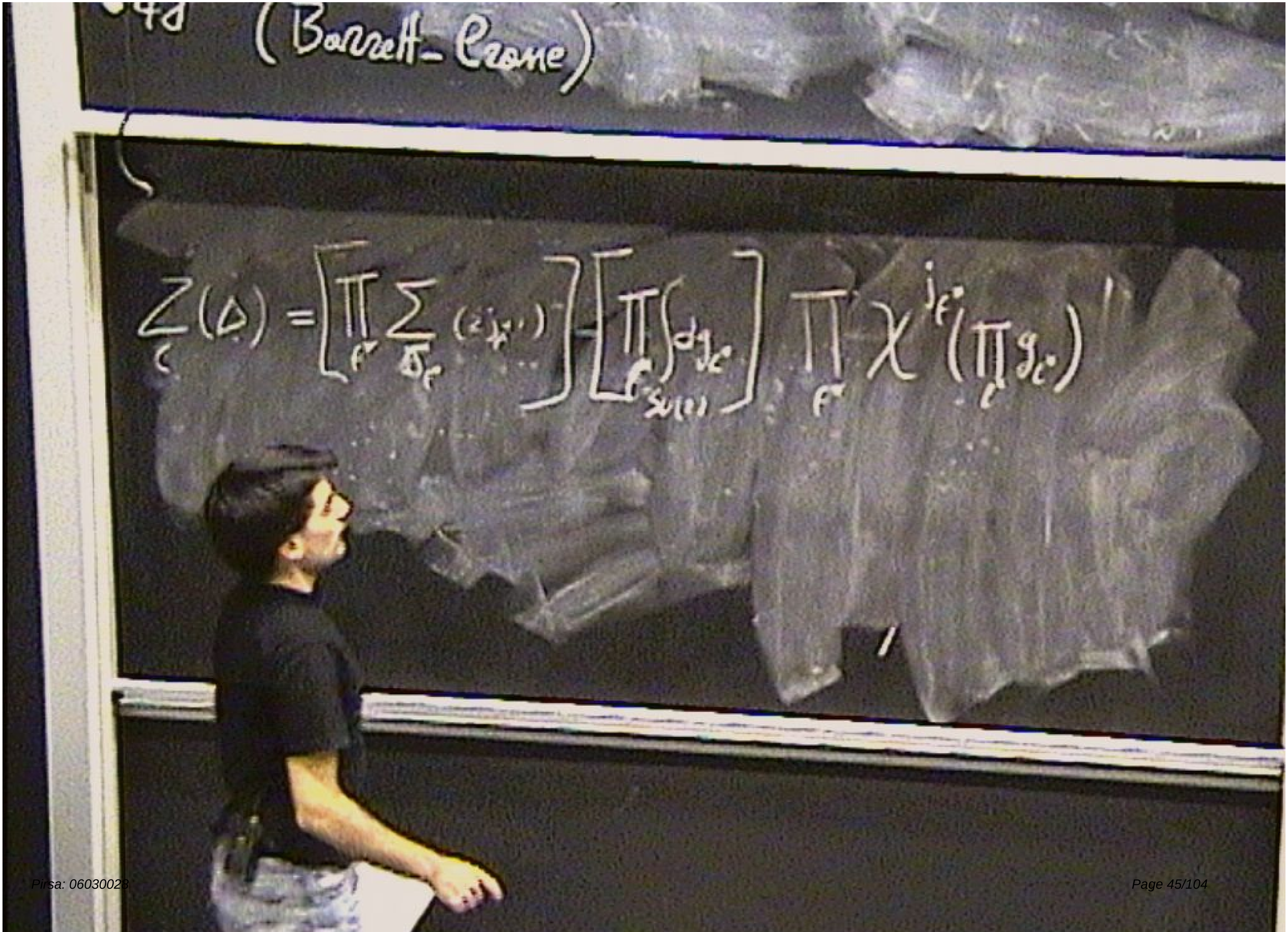
$\cdots$

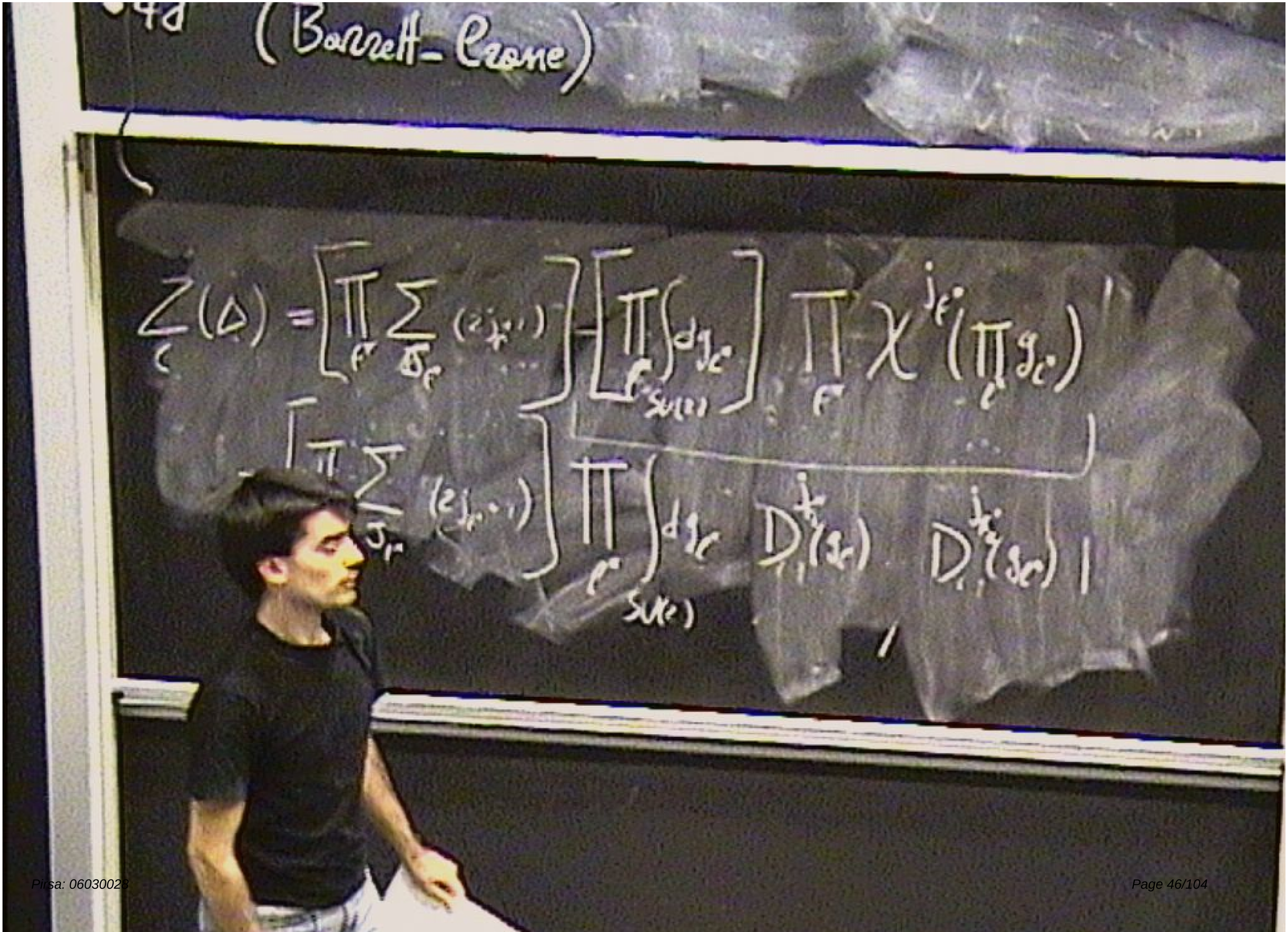
$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

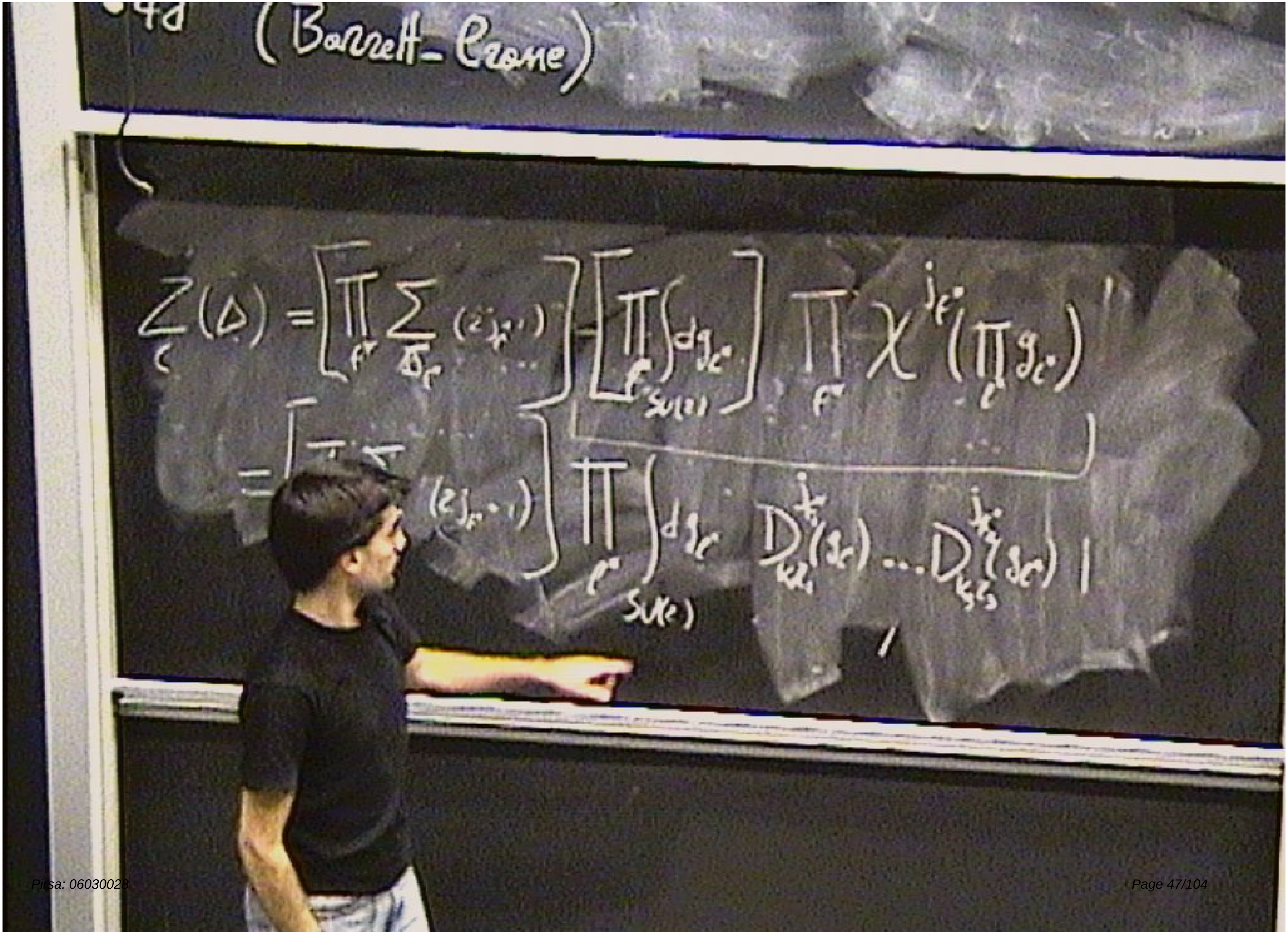
$$\chi^{j_{f^*}}\Big(\prod_{\ell} g_{\ell}\Big) = D^{j_{f^*}}_{mm}\Big(\prod_{\ell} g_{\ell}\Big) =$$

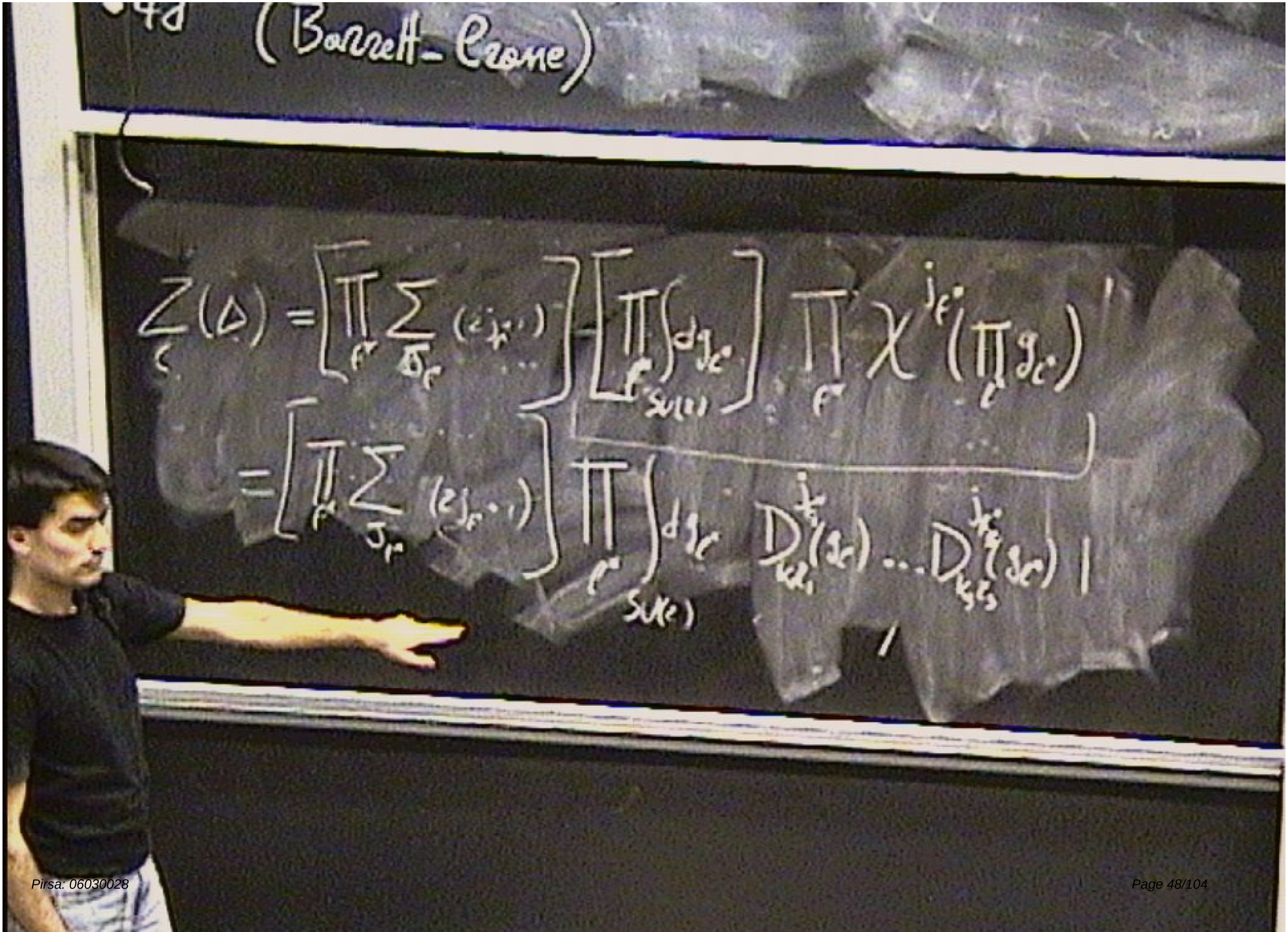
$$= D^{j_{f^*}}_{mk}(g_{\ell_1})\, D^{j_{f^*}}_{k\ell}(g_{\ell_2}) \cdots D^{j_{f^*}}_{nm}(g_{\ell_n})$$

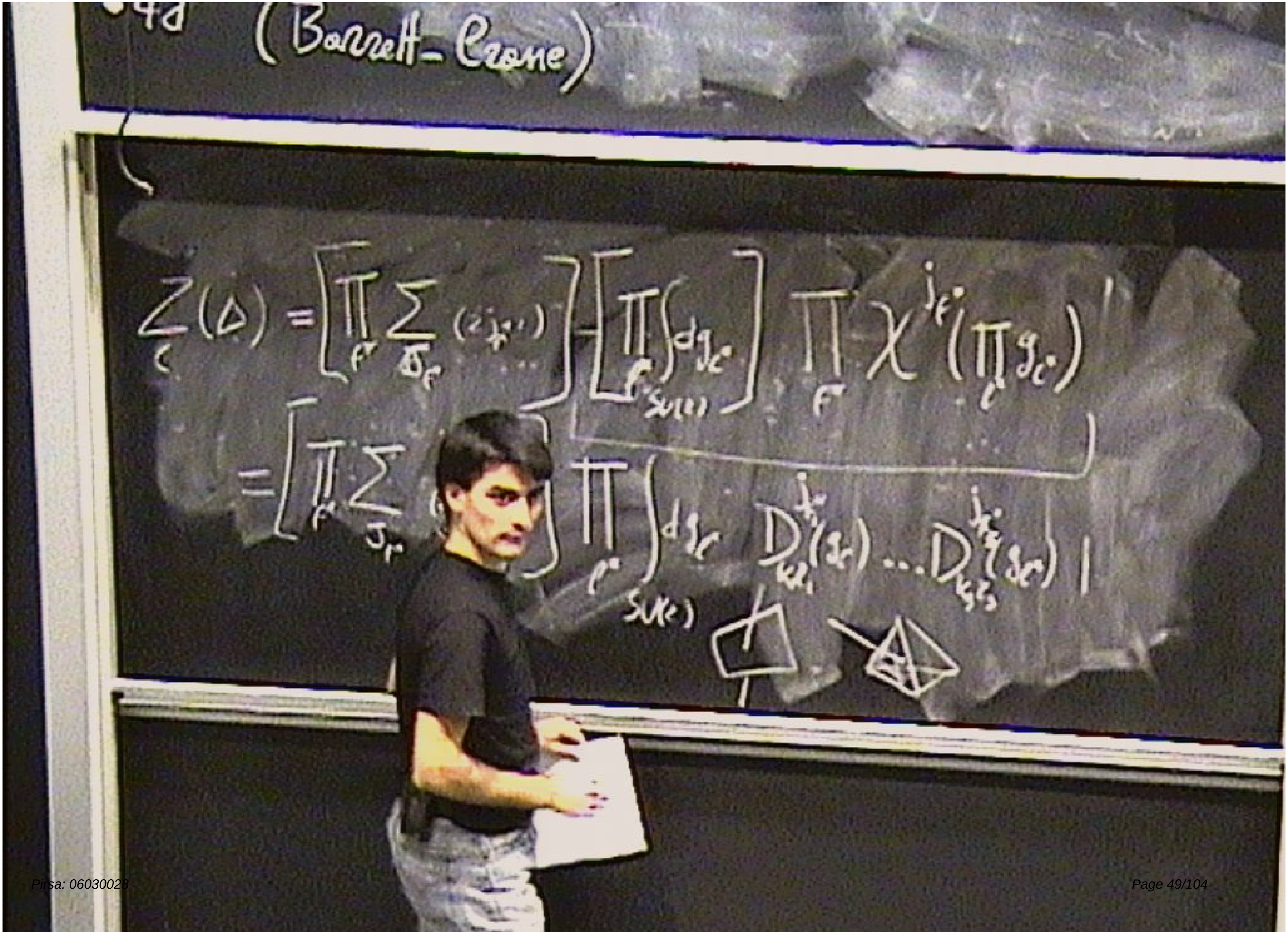



(Barrett_ Crane)
$Z_c(\Delta) = \left[\prod_f \sum_{\Delta_f} (2j_f+1)\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$


(Barrett_ Crane)
$Z_c(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right]\left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$
$\left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \; D^{j_f}_{11}(g_e) \; D^{j_f}_{11}(g_e) \; |$


(Barrett–Crane)
$Z_c(\Delta) = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \left[\prod_{e^*} \int_{SU(2)} dg_{e^*}\right] \prod_{f^*} \chi^{j_{f^*}}\left(\prod_{e^*} g_{e^*}\right)$
$= \left[\prod \sum (2j_{f^*}+1)\right] \prod_{e^*} \int_{SU(2)} dg_{e^*} \; D^{j_{f^*}}_{\alpha\alpha_1}(g_{e^*}) \dots D^{j_{f_3^*}}_{\xi\xi_3}(g_{e^*})$


(Barrett–Crane)
$Z_c(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$
$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \; D^{j_1}_{k_1 l_1}(g_e) \ldots D^{j_4}_{k_4 l_4}(g_e)$


(Barrett–Crane)

4d (Barrett–Crane)

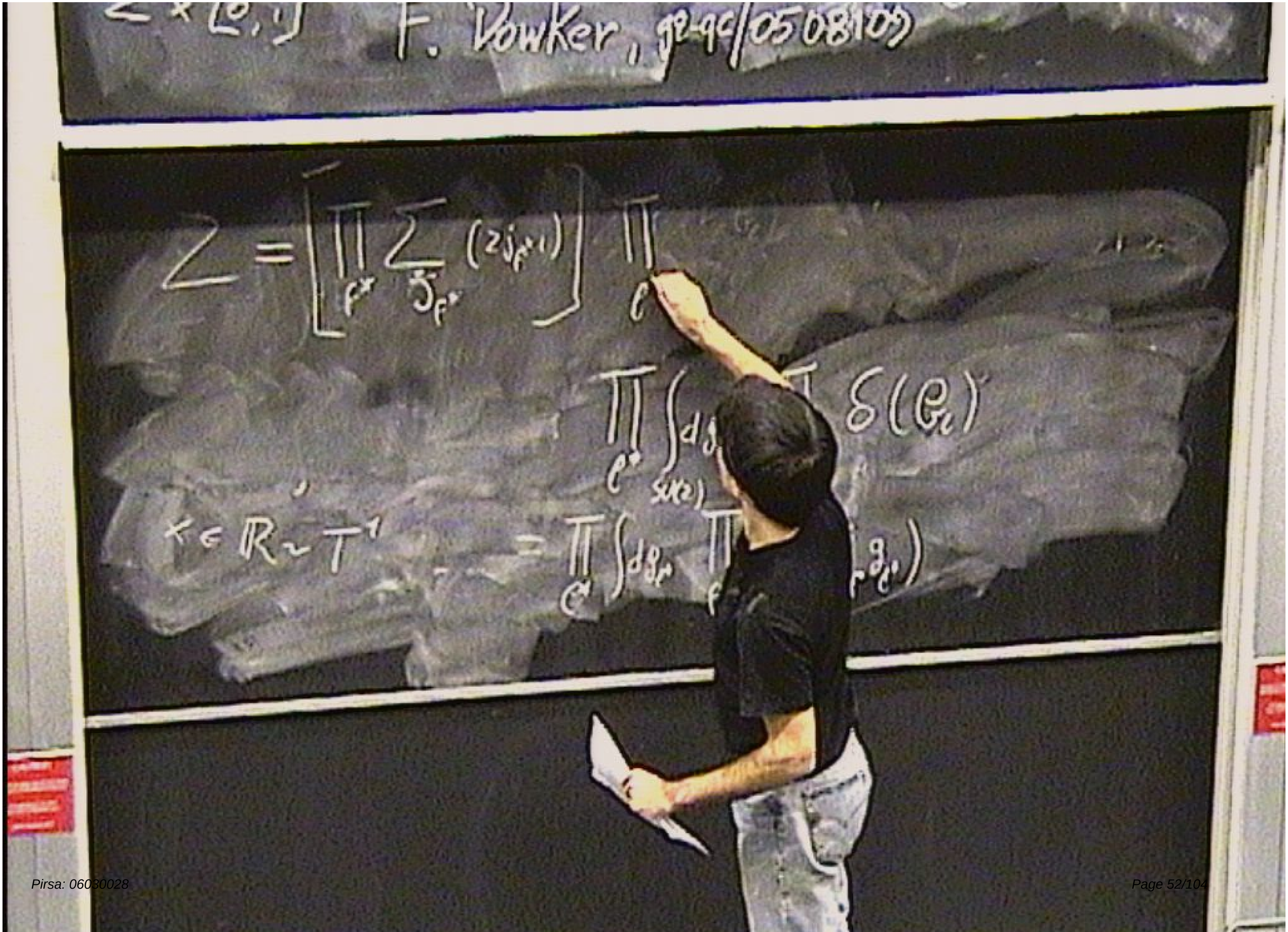
$$Z_c(\Delta) = \left[\prod_{f} \sum_{j_f} (2j_f+1) \cdots\right] \left[\prod_{e} \int_{SU(2)} dg_e\right] \prod_{f} \chi^{j_f}\Big(\prod_{e} g_e\Big)$$

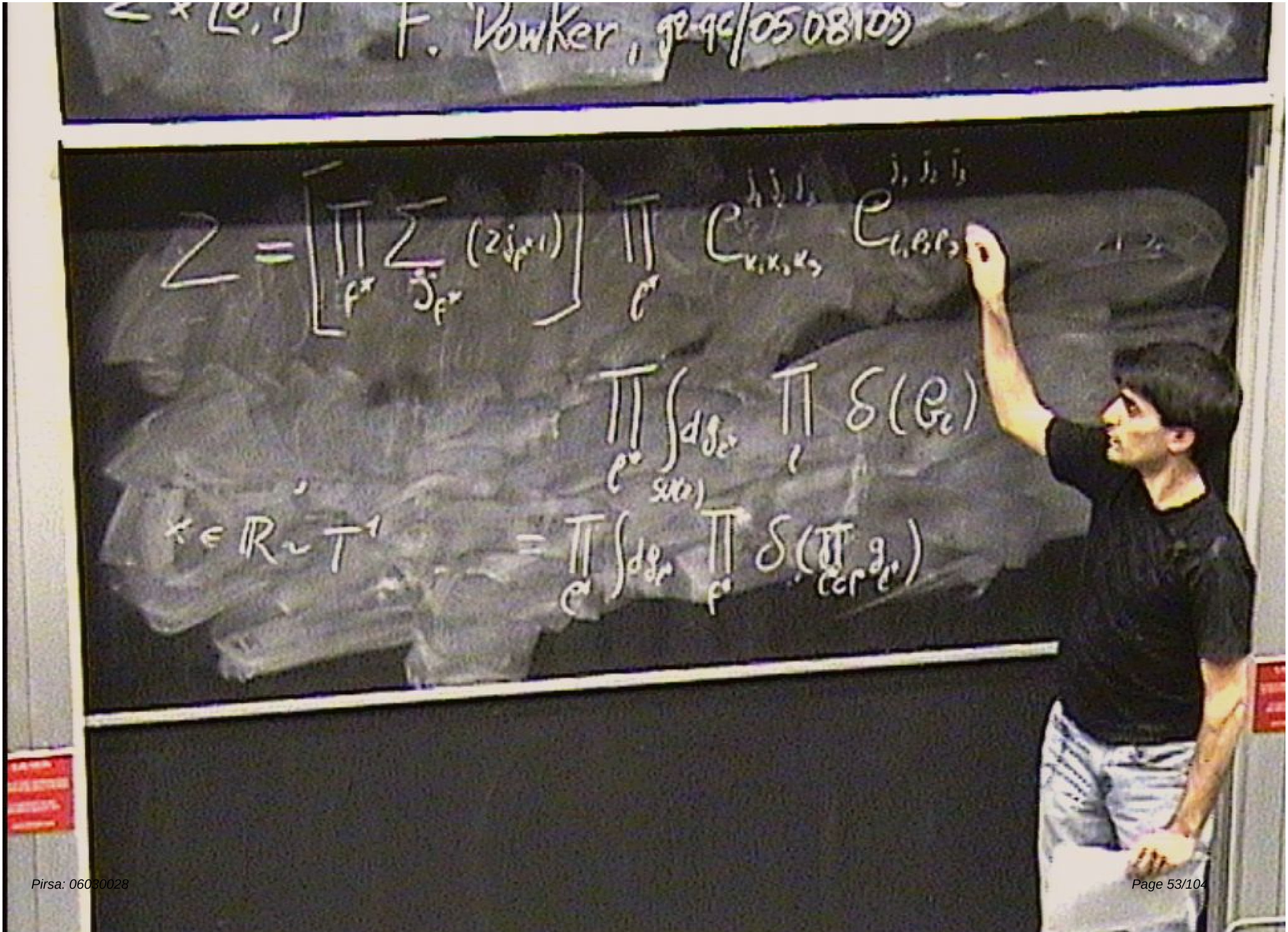
$$= \left[\prod_{f} \sum_{j_f} (2j_f+1)\right] \prod_{e} \int_{SU(2)} dg_e \; D^{j_1}_{k_1 l_1}(g_e) \cdots D^{j_5}_{k_5 l_5}(g_e) \;|$$

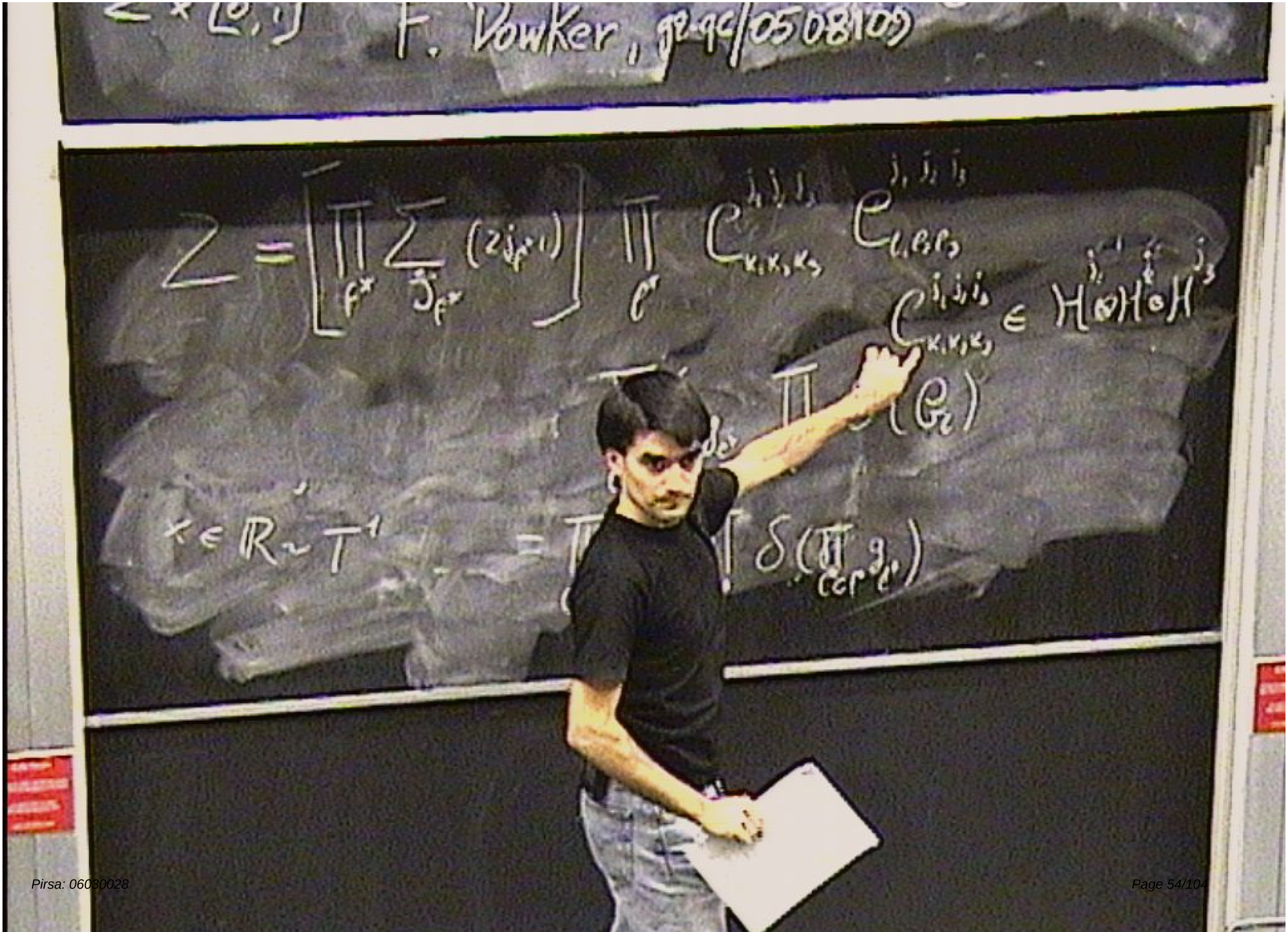
• 4d (Barrett–Crane)

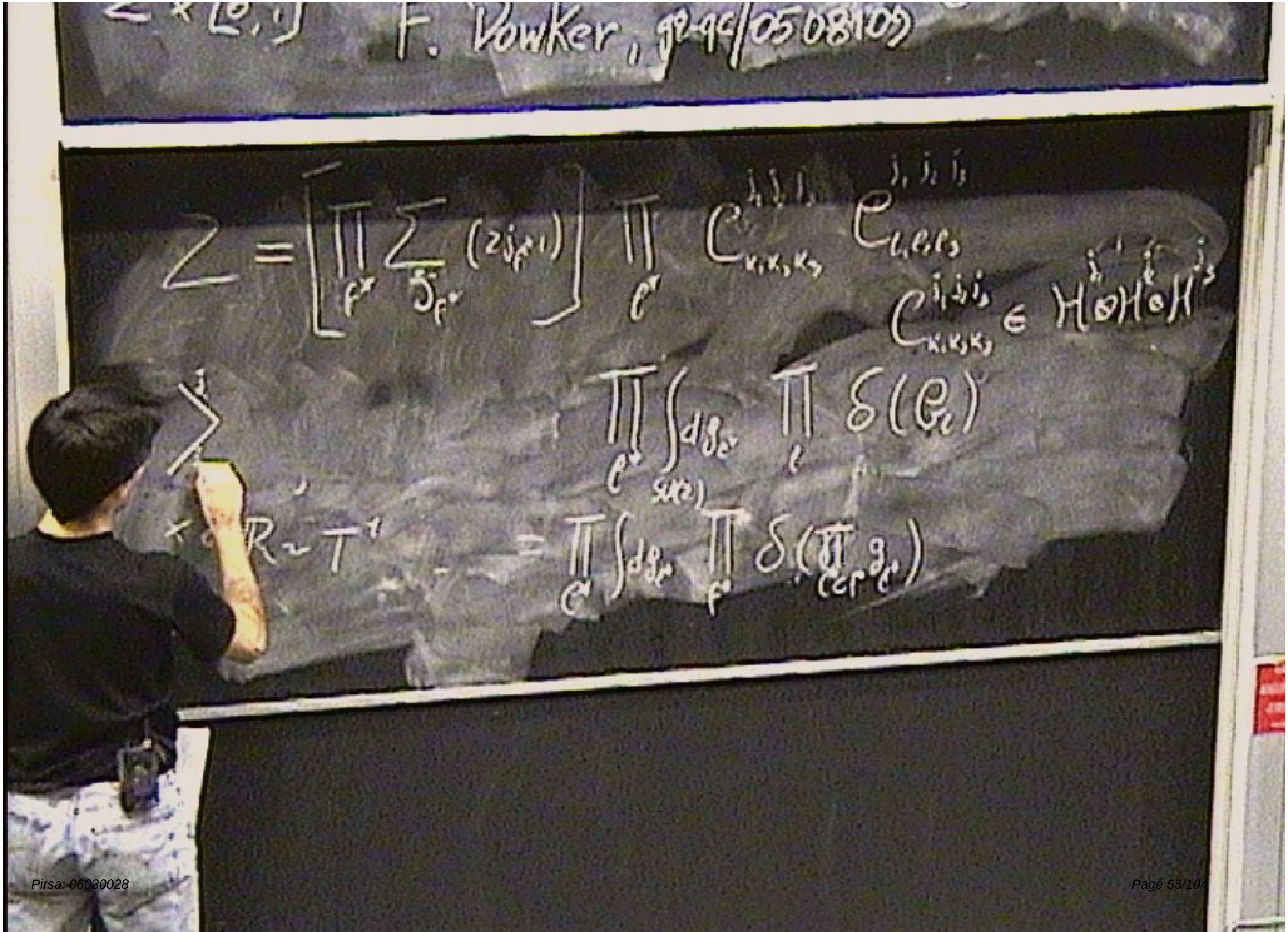
$$Z_c(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1) \ldots\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$$

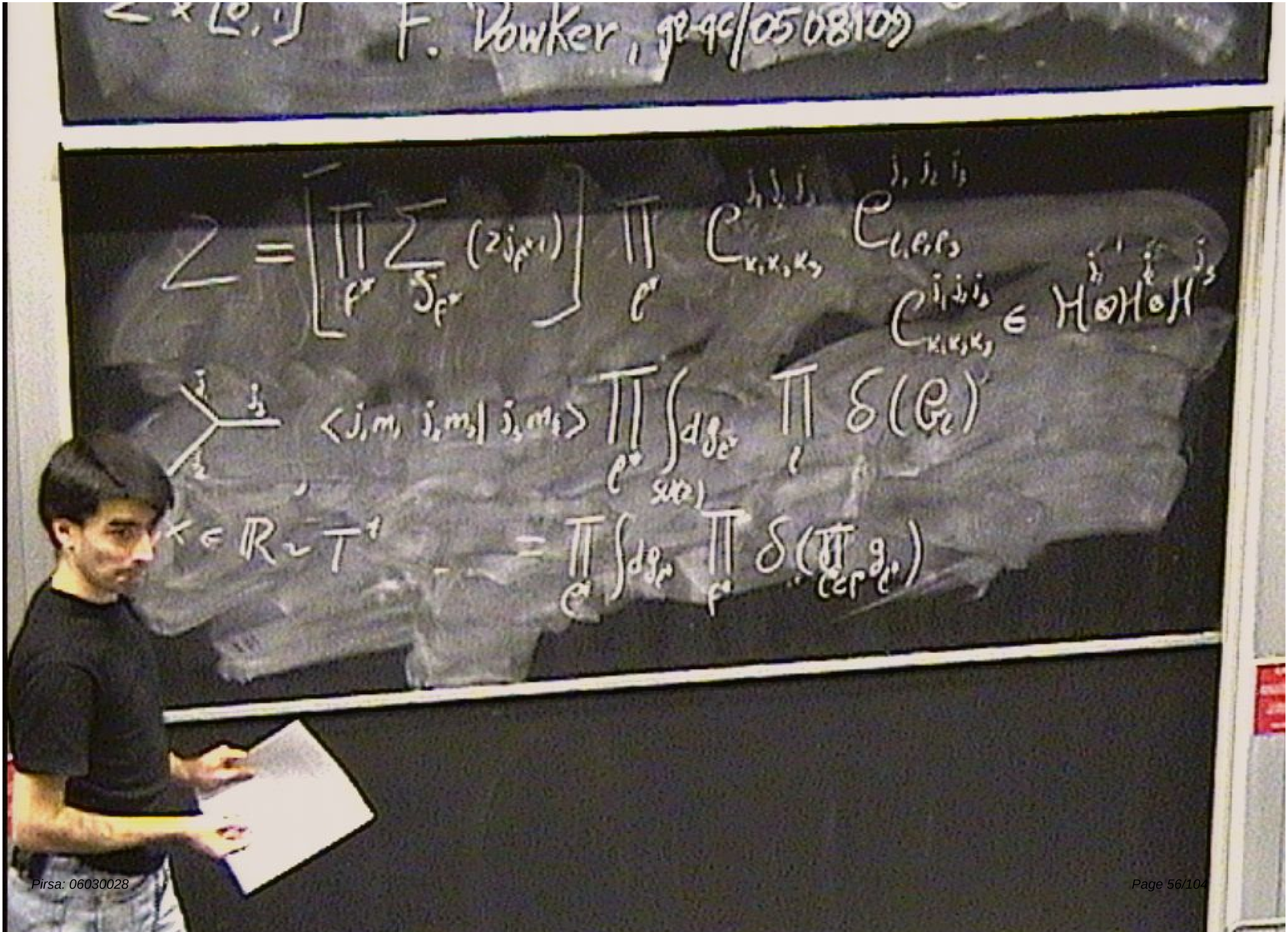
$$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \; D^{j_1}_{k_1 l_1}(g_e) \ldots D^{j_4}_{k_4 l_4}(g_e) \; |$$


F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} C^{j_1 j_2 j_3}_{\ell_1 \ell_2 \ell_3}$
$\prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_{\ell} \delta(G_\ell)$
$x \in \mathbb{R} \sim T^1$
$= \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta\left(\prod_{e^* \subset f^*} g_{e^*}\right)$


F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109



$\leq x\,[0,1]$
F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*}\sum_{j_{f^*}}(2j_{f^*}+1)\right]\prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3}\; \mathcal{C}^{j_1 j_2 j_3}_{\ell_1 \ell_2 \ell_3}$
$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1}\otimes H^{j_2}\otimes H^{j_3}$
j_1 j_2 j_3
$\langle j_1 m_1\; j_2 m_2 | j_3 m_3\rangle \prod_{e^*}\int_{SU(2)} dg_{e^*} \prod_{f} \delta(G_f)$
$\in \mathbb{R} \sim T^4$
$= \prod_{e^*}\int dg_{e^*} \prod_{f^*} \delta\left(\prod_{e\in f} g_{e^*}\right)$

$\ldots$ F. Dowker, gr-qc/0508109

$$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \, C^{j_1 j_2 j_3}_{e_1 e_2 e_3}$$

$$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$$

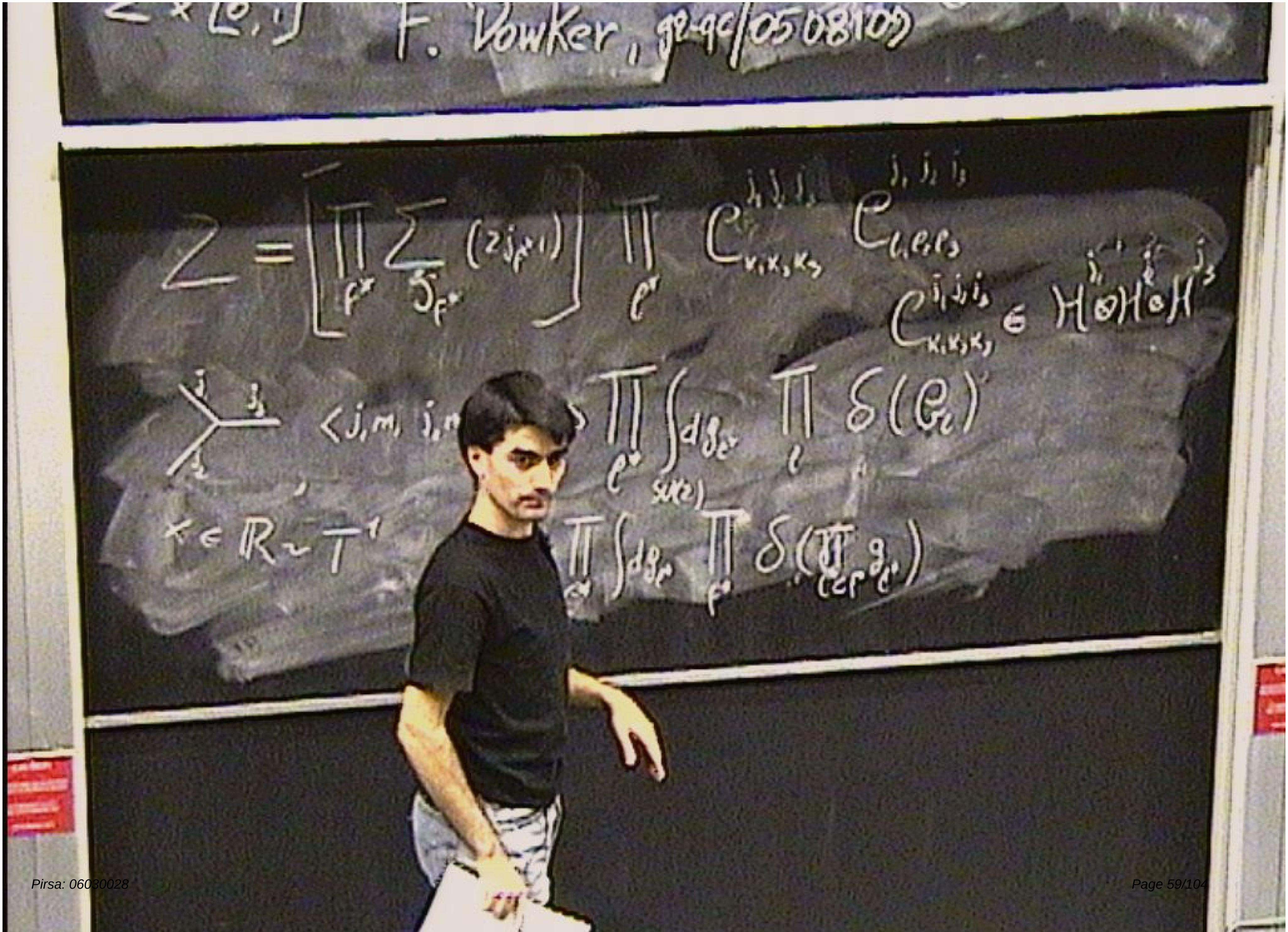
$$\langle j_1 m_1 \; j_2 m_2 | j_3 m_3 \rangle \prod_{e^*} \int_{SU(2)} dg_{e^*} \prod_{e} \delta(g_e)$$

$$x \in \mathbb{R} \sim T^4 \quad = \prod_{e^*} \int dg_{e^*} \prod_{f^*} \delta\Big(\prod_{e^* \in f^*} g_{e^*}\Big)$$

• 4d (Barrett–Crane)

$$Z_c(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1) \cdots\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$$

$$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \; D^{j_f}_{\alpha\alpha_1}(g_e) \cdots D^{j_f}_{\xi\xi}(g_e) \; |$$


F. Dowker, gr-qc/0508109

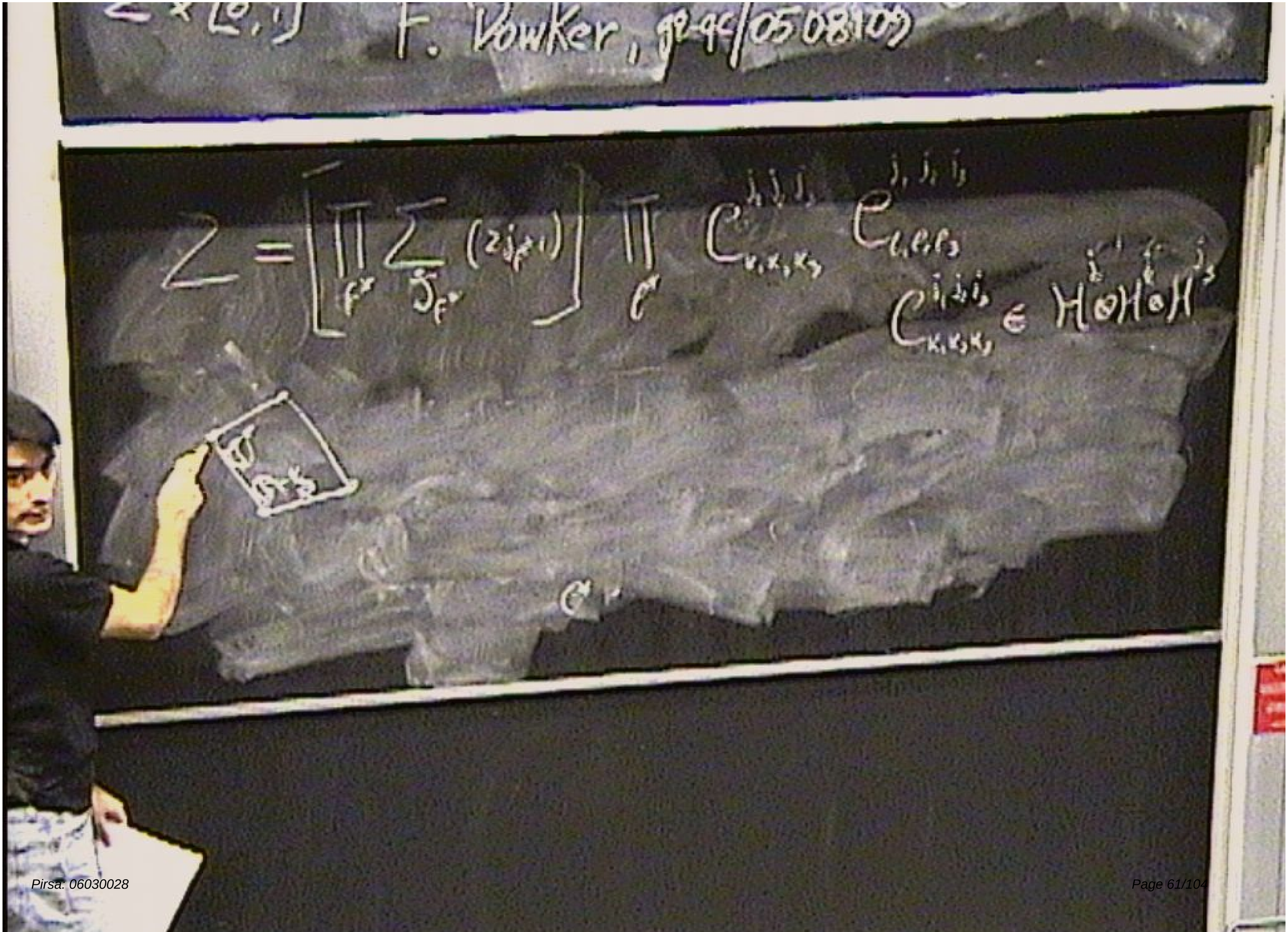
$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

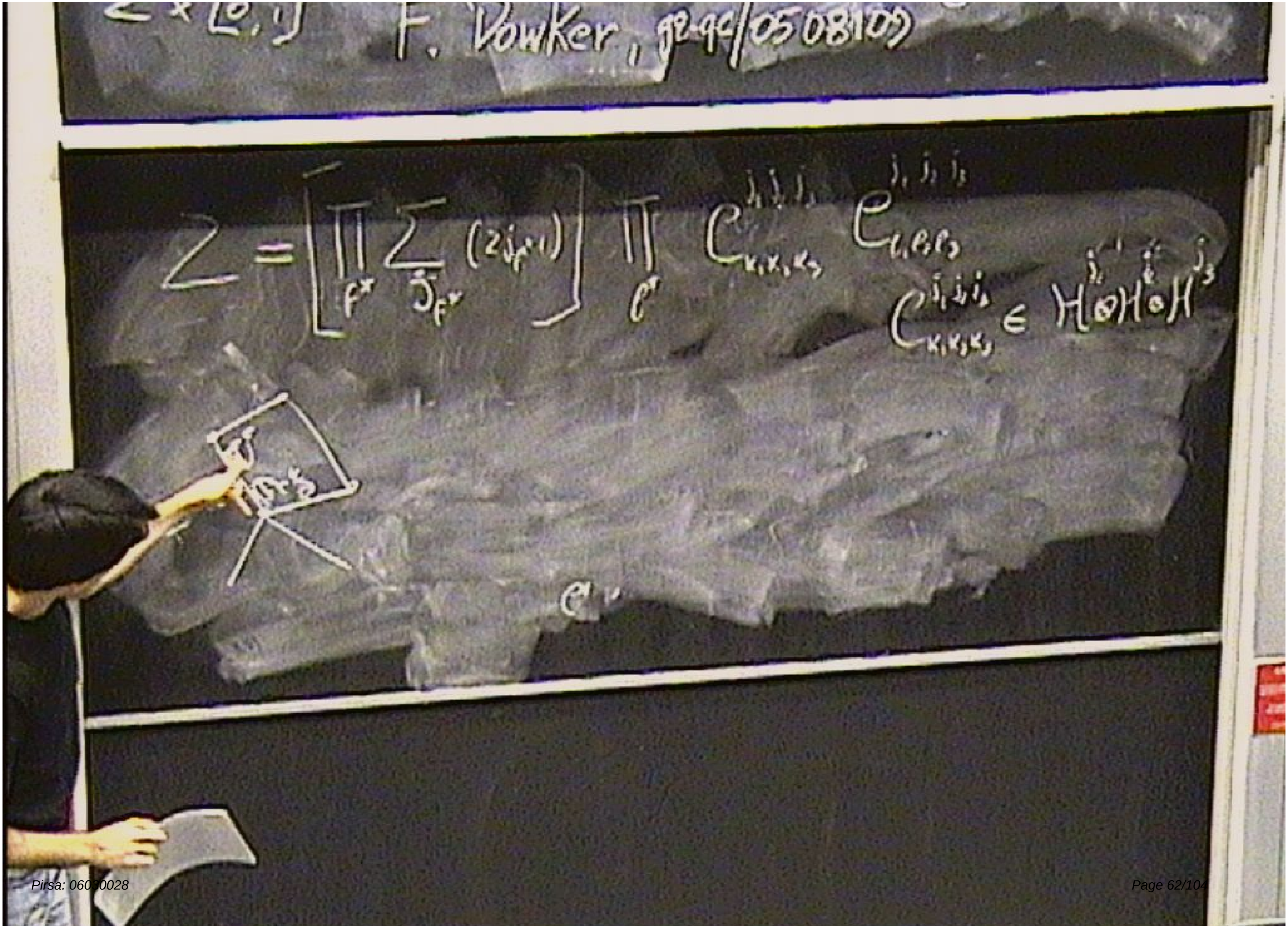
$$Z = \left[\prod_{f^*}\sum_{j_{f^*}}(2j_{f^*}+1)\right]\prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3}\; e^{j_1 j_2 j_3}_{\ell_1 \ell_2 \ell_3}$$

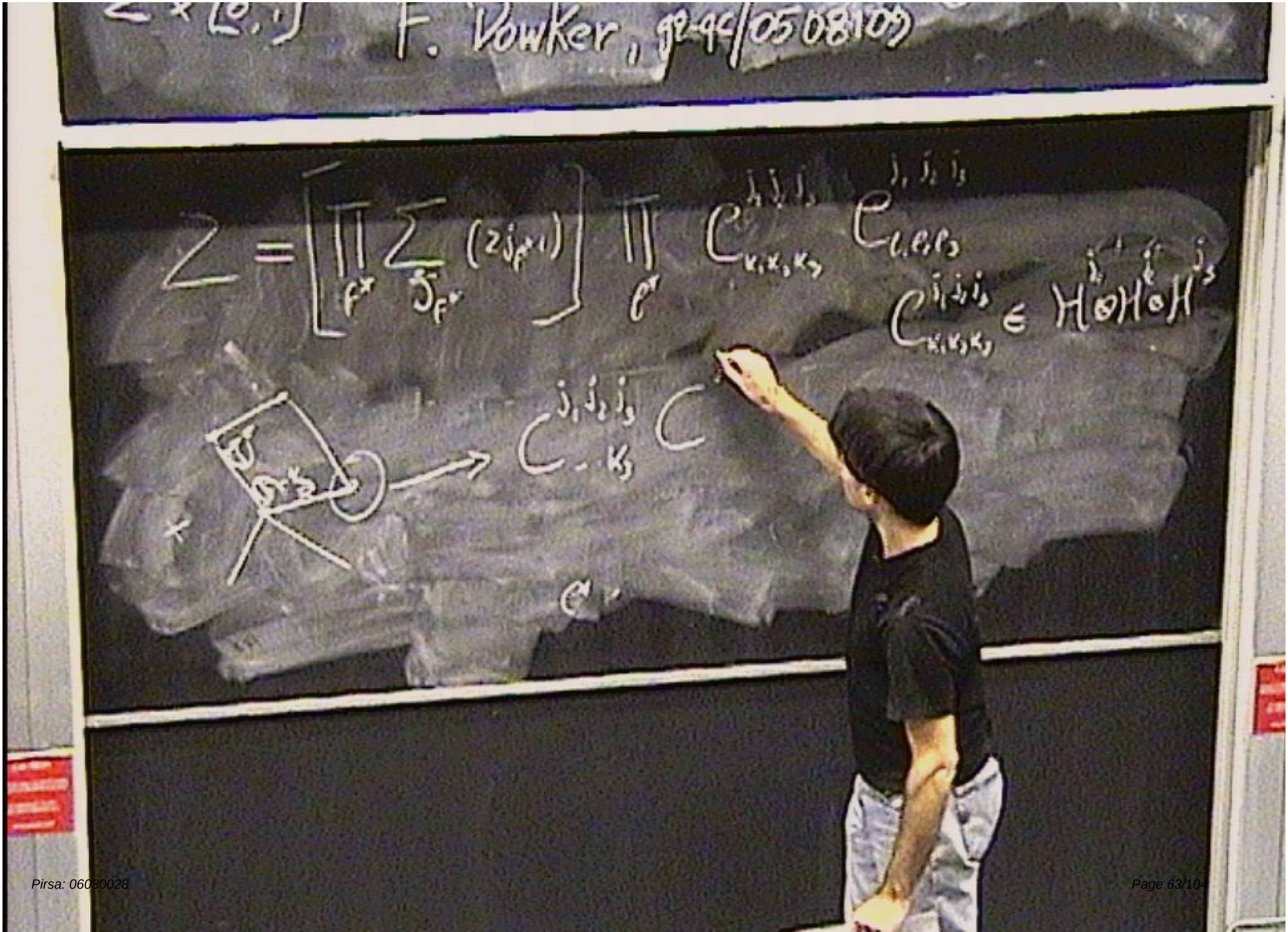
$$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1}\otimes H^{j_2}\otimes H^{j_3}$$

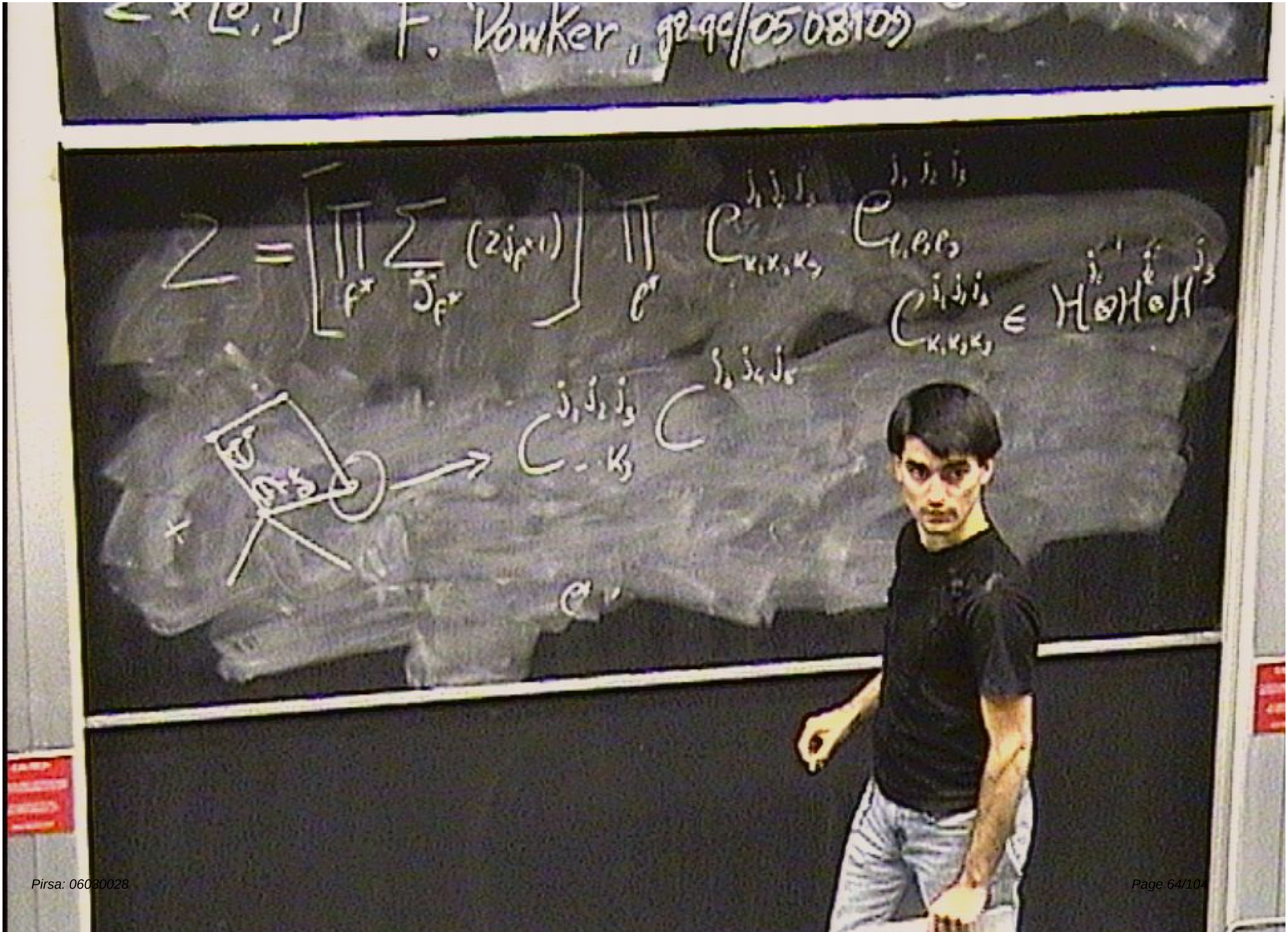
∂e

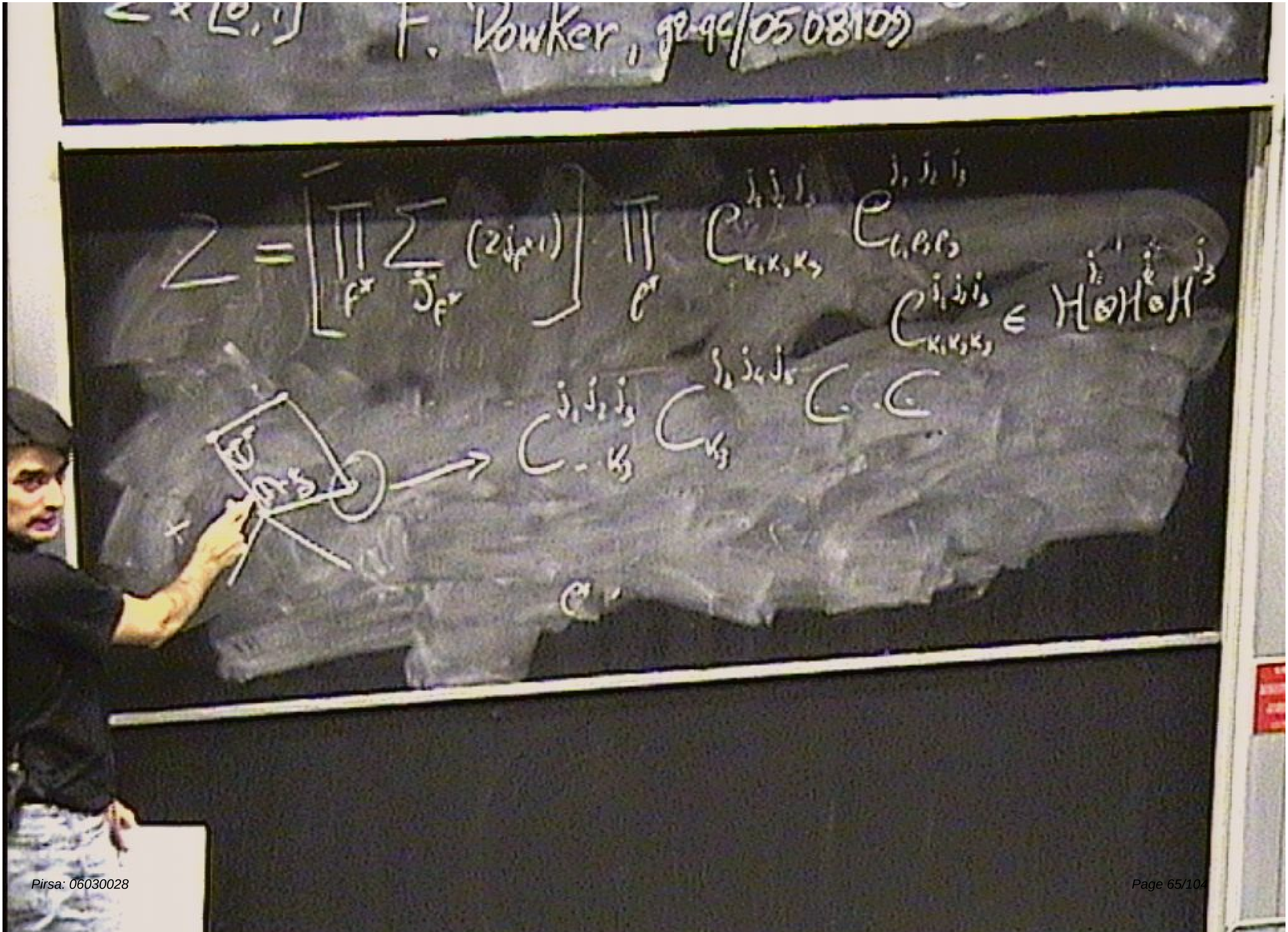
e


F. Dowker, gr-qc/0508109

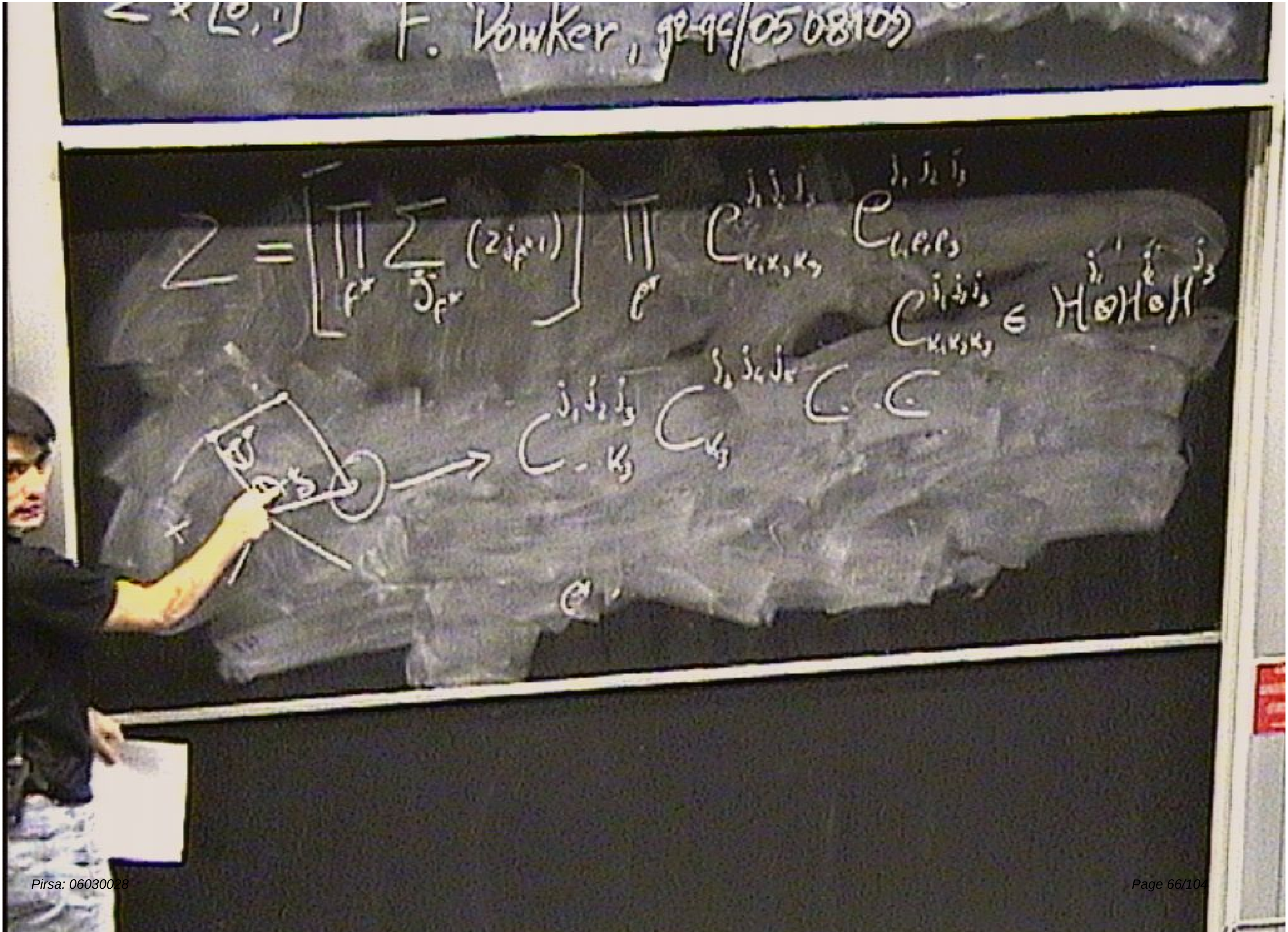

F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_f+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \, C^{j_1 j_2 j_3}_{l_1 l_2 l_3}$
$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$

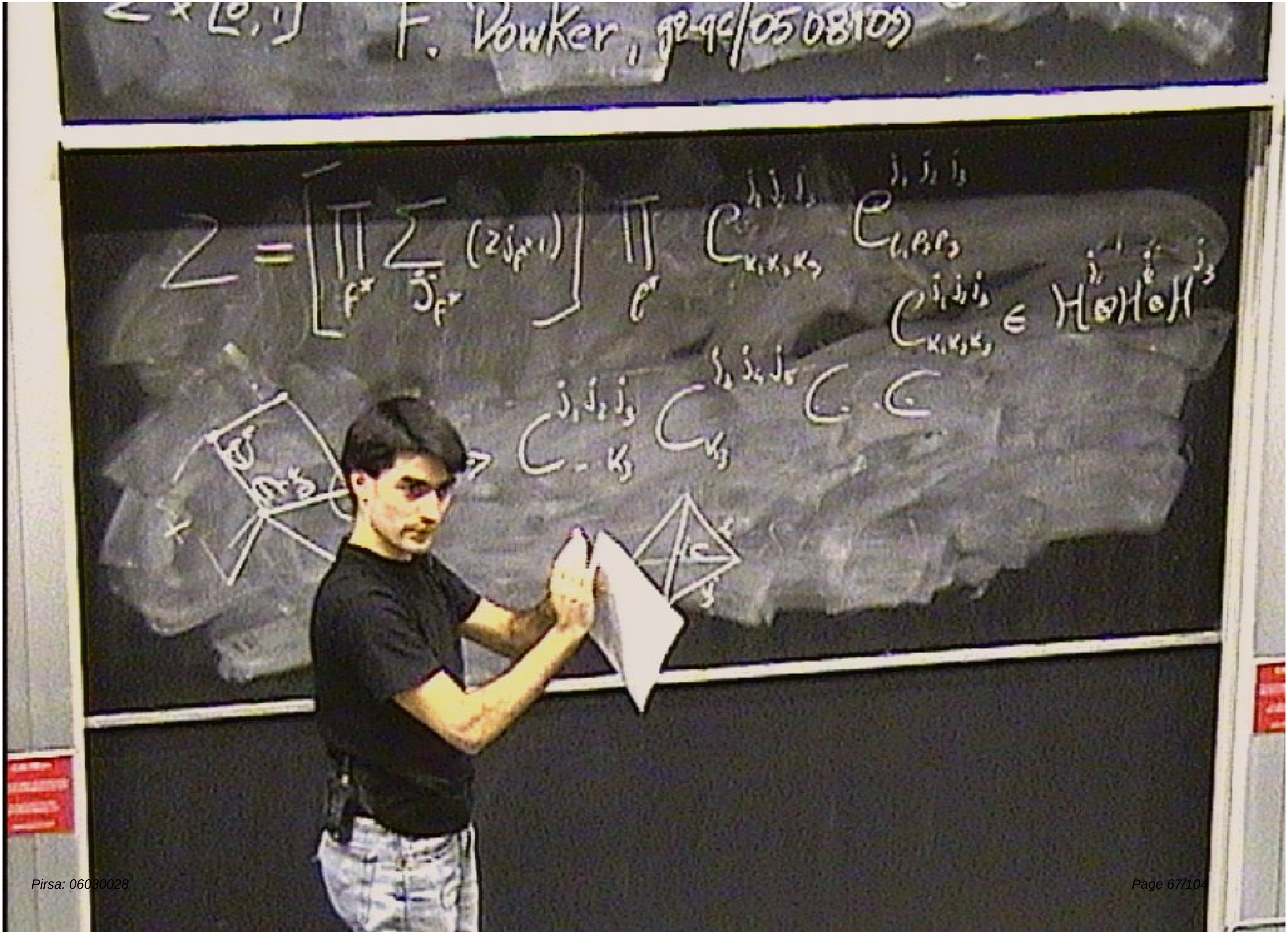

F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{K_1 K_2 K_3} C^{j_1 j_2 j_3}_{l_1 l_2 l_3}$
$C^{j_1 j_2 j_3}_{K_1 K_2 K_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$
$C^{j_1 j_2 j_3}_{\cdot\cdot K_3}$

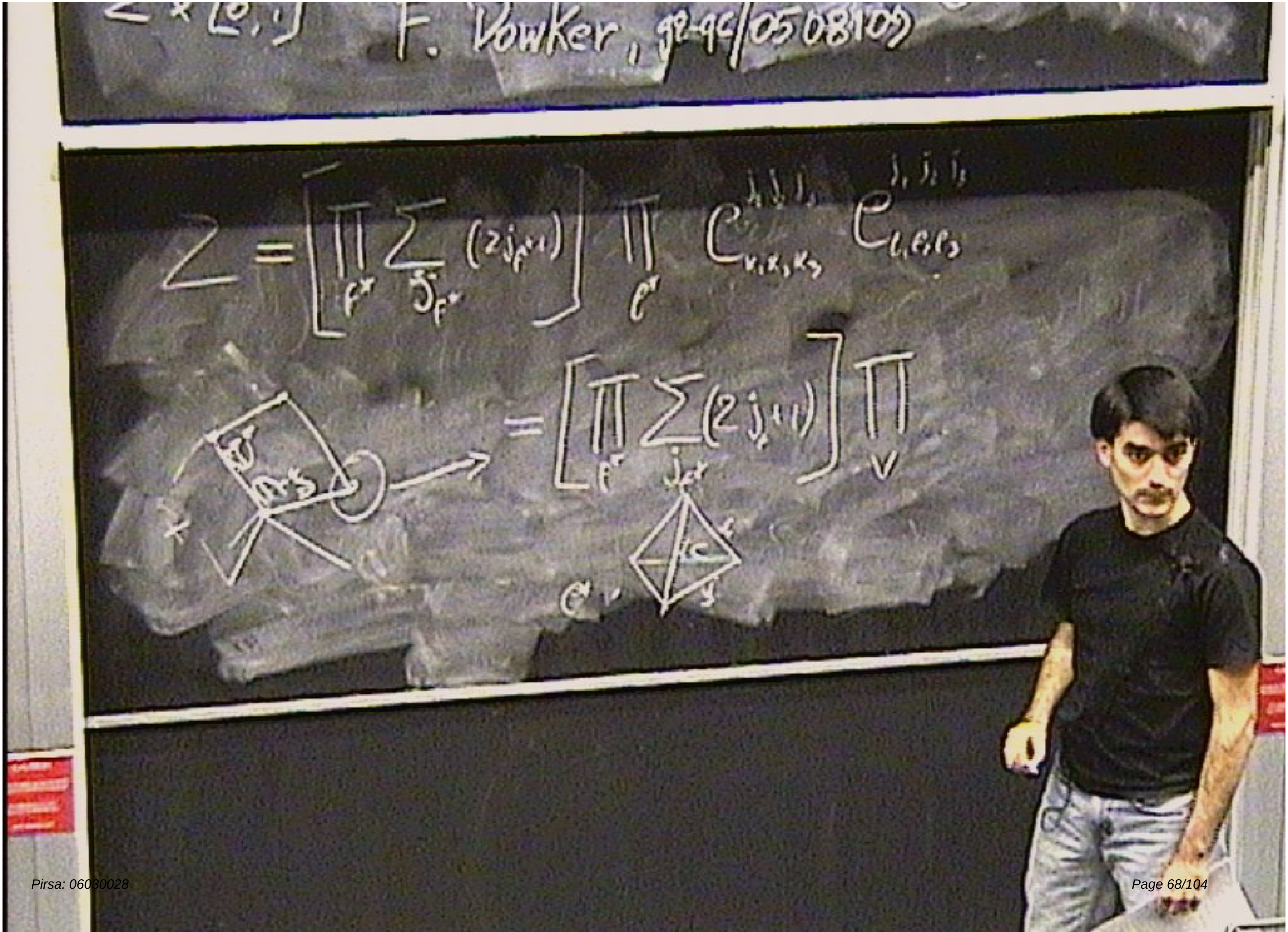

F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} C^{j_1 j_2 j_3}_{l_1 l_2 l_3}$
$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$
$C^{j_1 j_2 j_3}_{\cdot\cdot k_3} C^{j_3 j_4 j_5}$

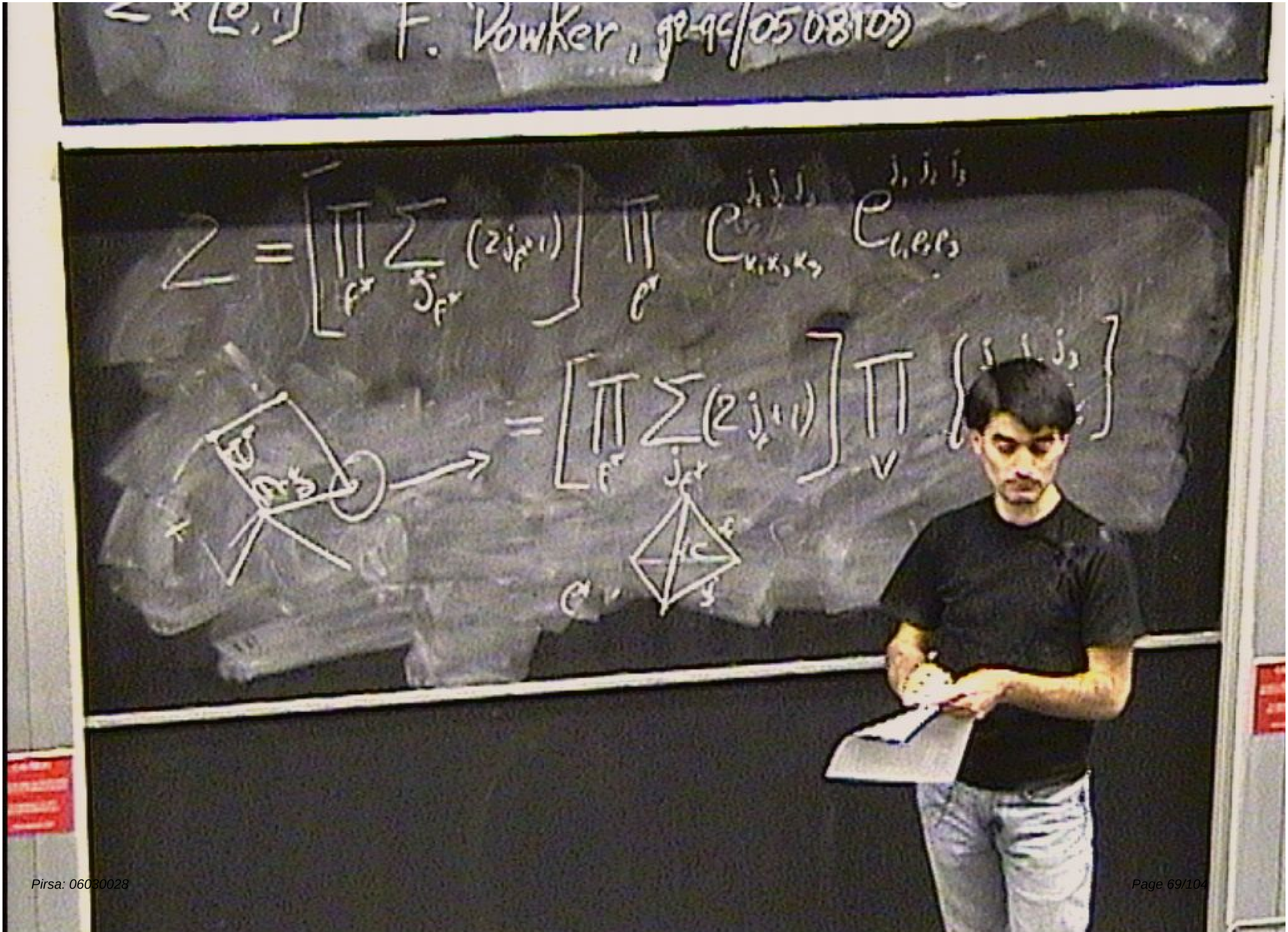


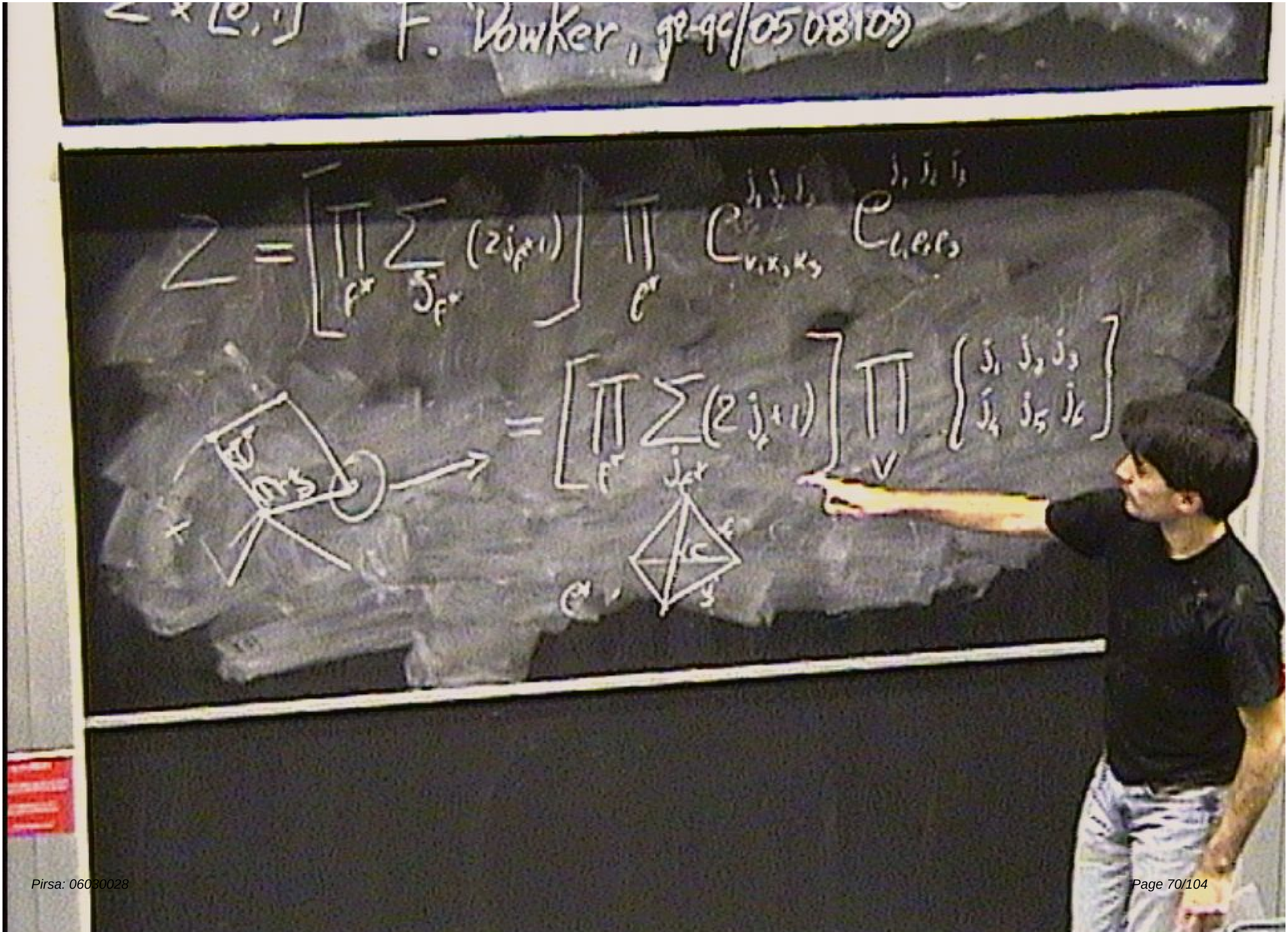
$\Sigma \times [0,1]$
F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_f+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \, e^{j_1 j_2 j_3}_{l_1 l_2 l_3}$
$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$
$C^{j_1 j_2 j_3}_{\cdot\cdot k_3} \, C^{j_3 j_4 j_5}_{k_3} \quad C \cdot C$

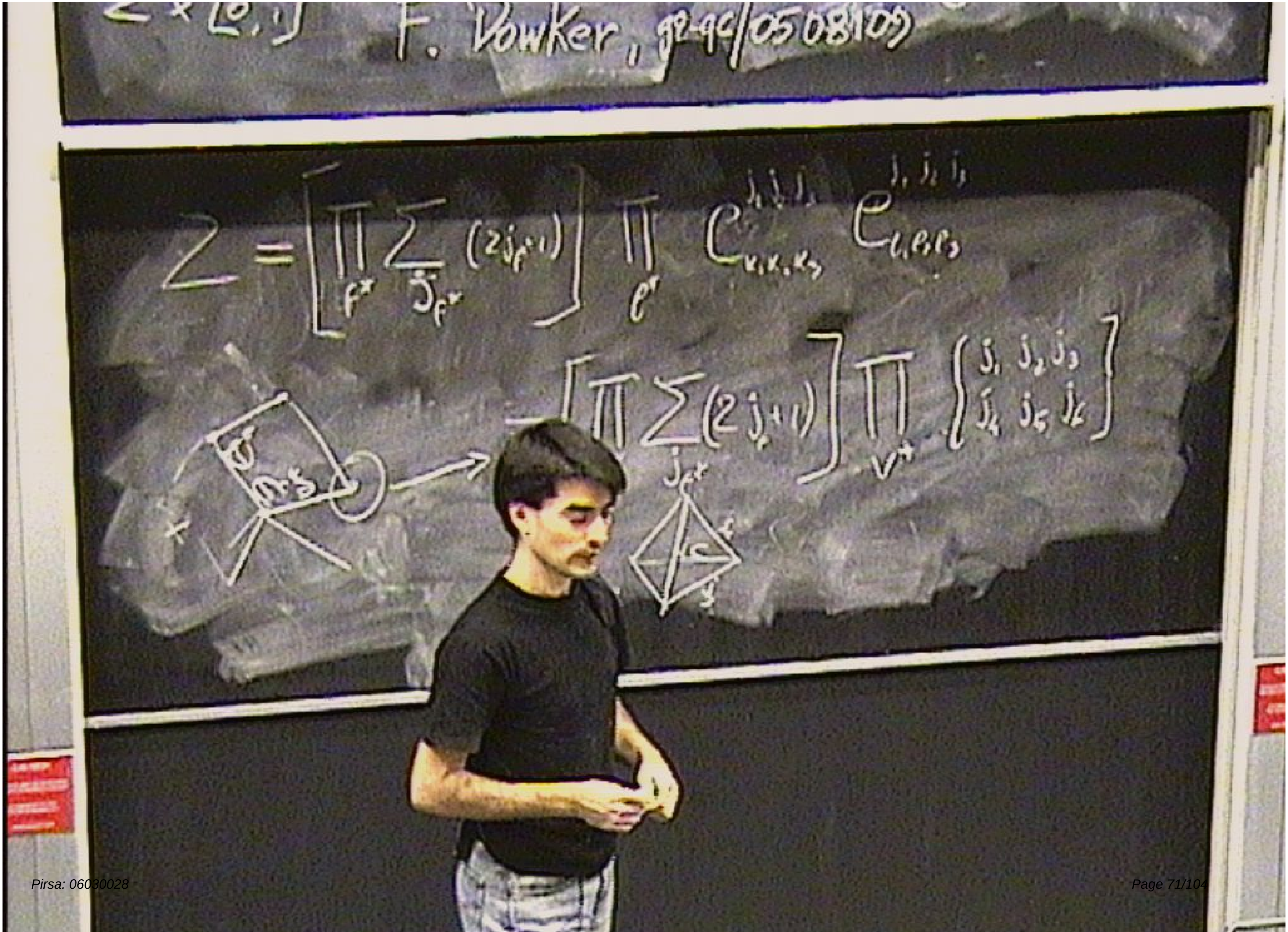

F. Dowker, gr-qc/0508109
$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_f+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3} C^{j_1 j_2 j_3}_{l_1 l_2 l_3}$
$C^{j_1 j_2 j_3}_{k_1 k_2 k_3} \in H^{j_1} \otimes H^{j_2} \otimes H^{j_3}$
$C^{j_1 j_2 j_3}_{\cdot\, k_3} C^{j_3 j_4 j_5}_{k_3} C \cdot C$

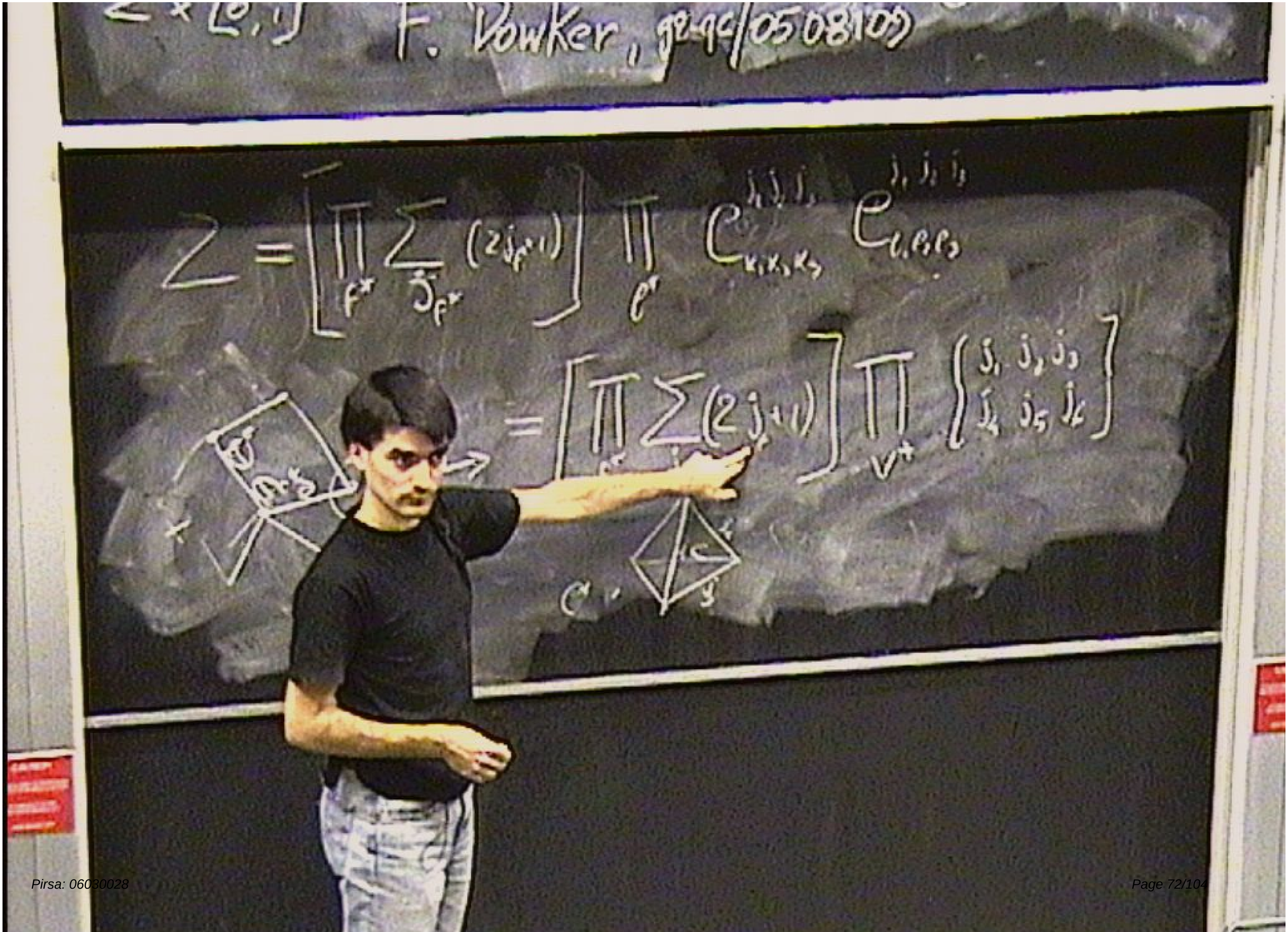

F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109


F. Dowker, gr-qc/0508109

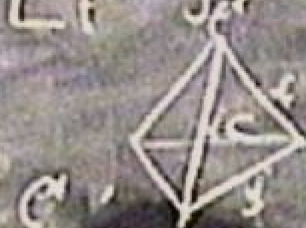

F. Dowker, gr-qc/0508109
Z = [∏ Σ (2j_f* + 1)] ∏ C C
= [∏ Σ (2j_f + 1)] ∏ {j1 j2 j3 / j4 j5 j6}

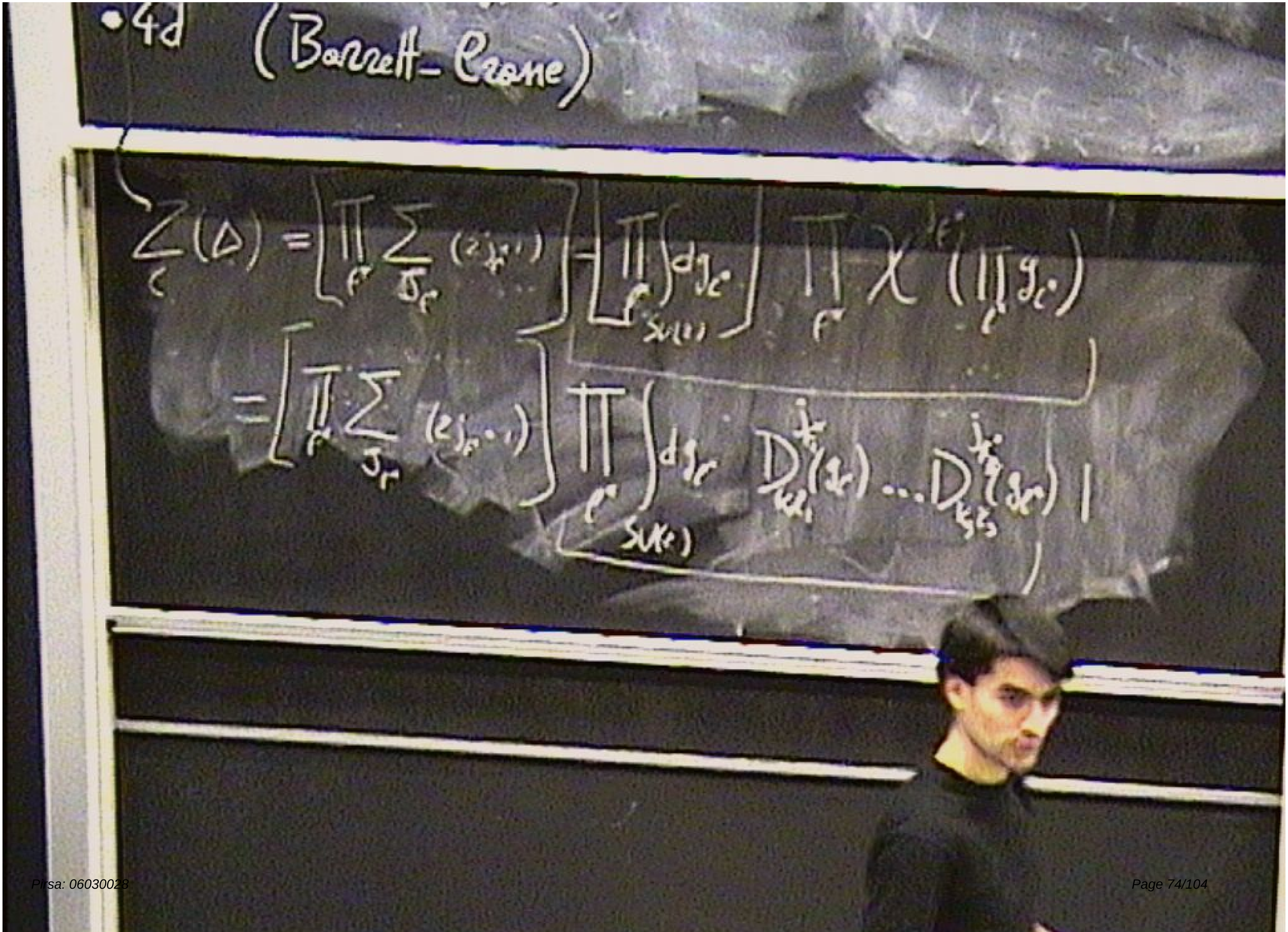

F. Dowker, gr-qc/0508109

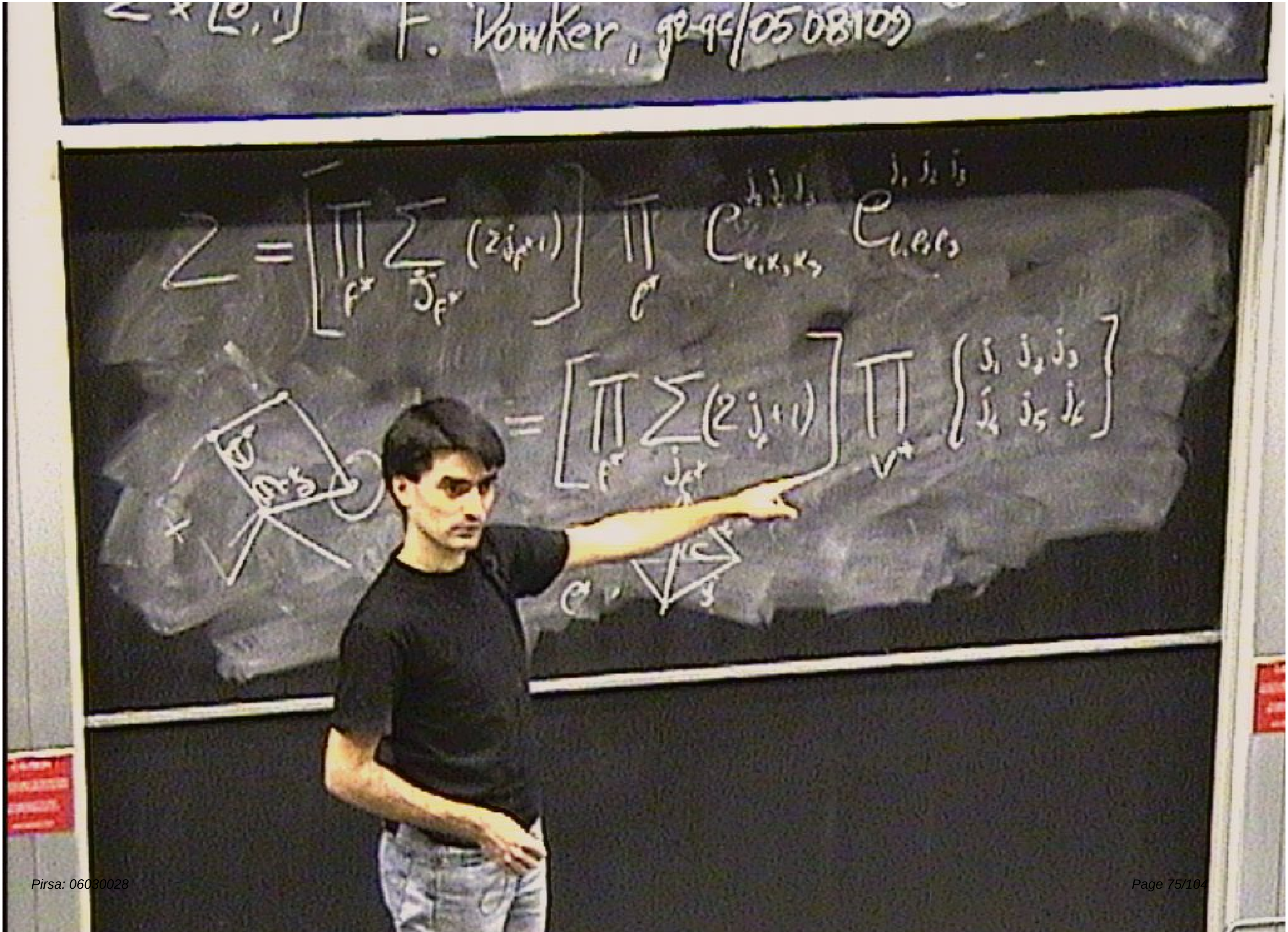
$\leq \times [0,1]$ F. Dowker, gr-qc/0508109

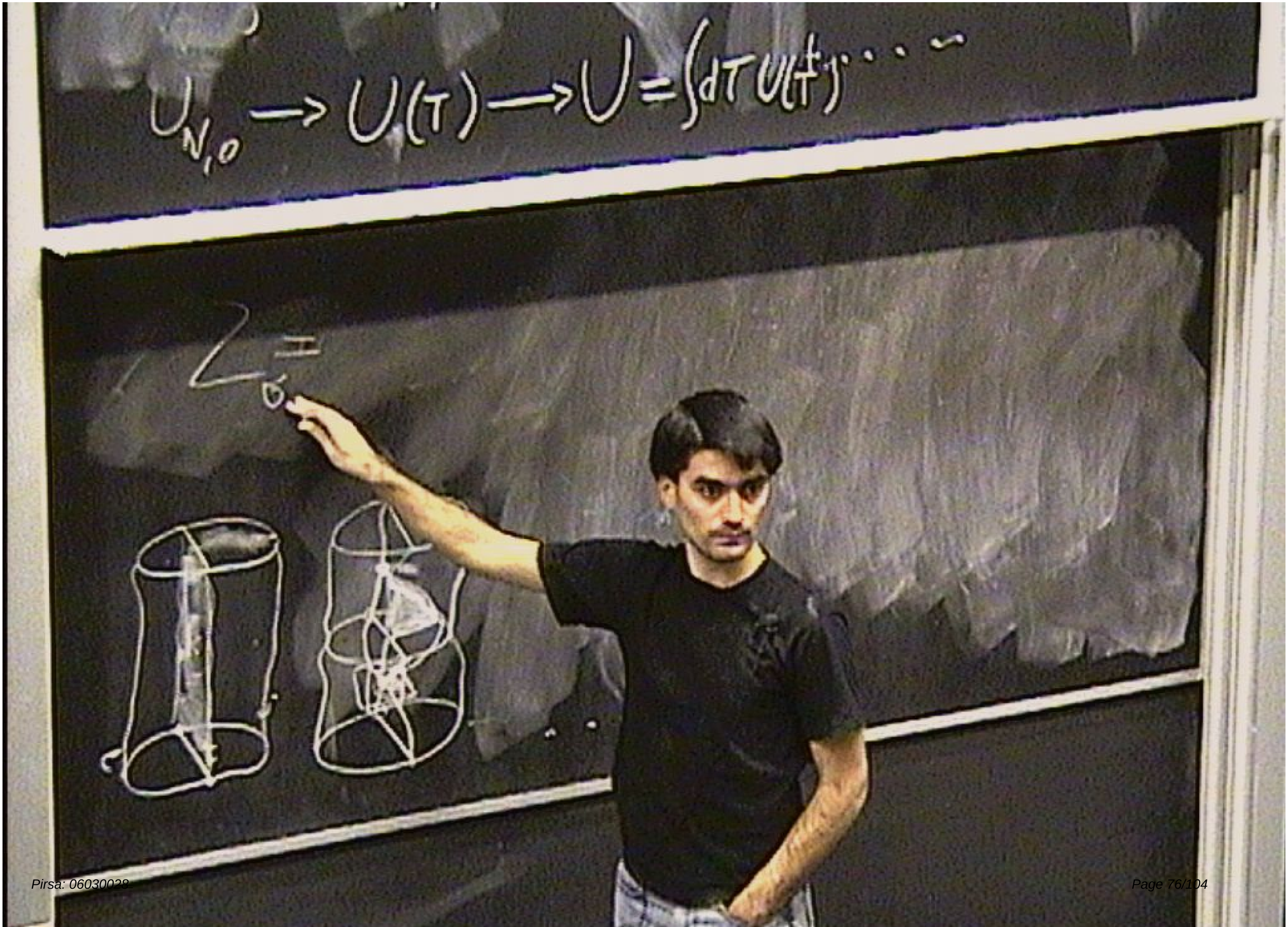
$$Z = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{x_1 x_2 x_3} \, C^{j_1 j_2 j_3}_{e_1 e_2 e_3}$$

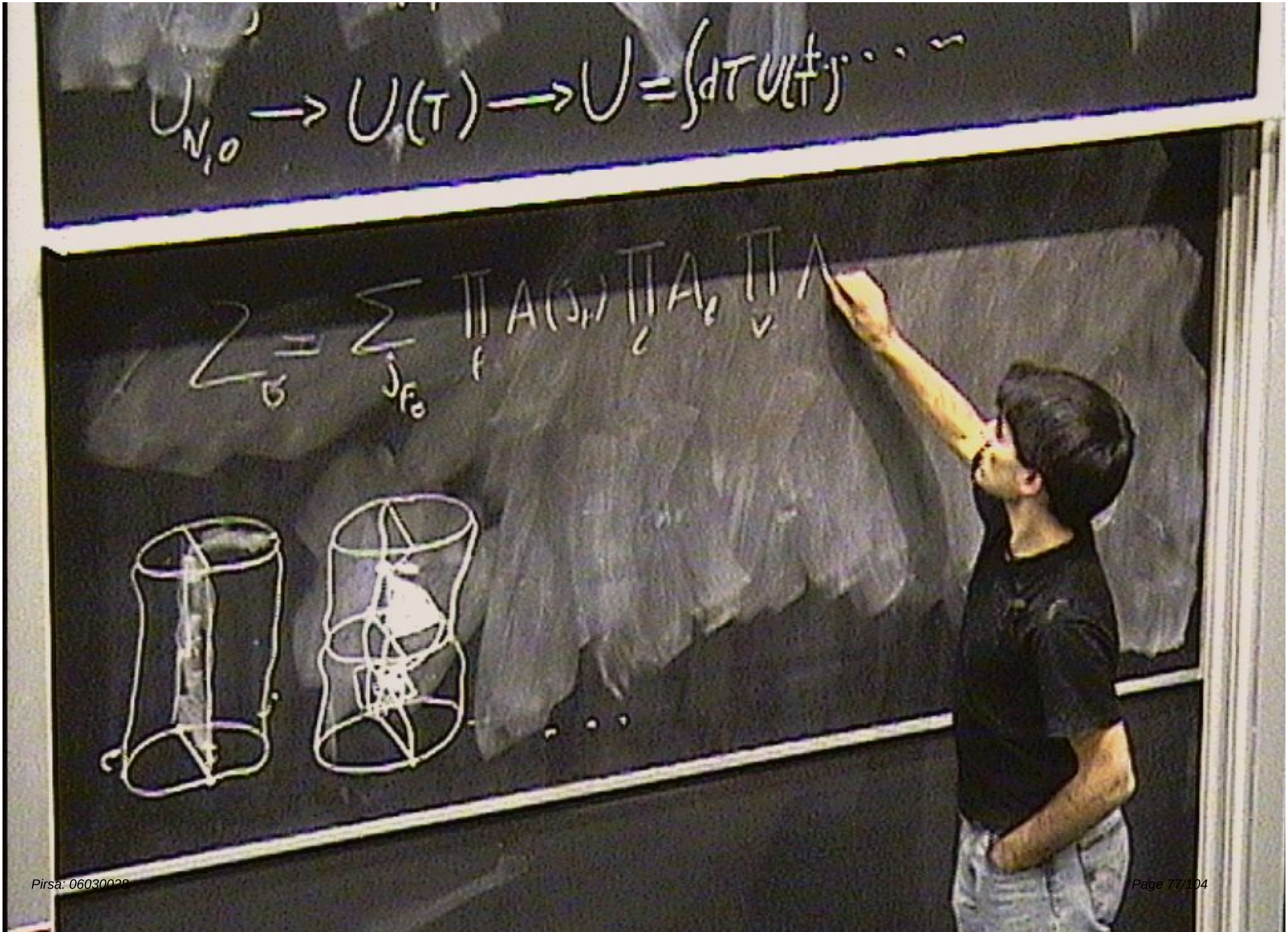
$$= \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{v^*} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

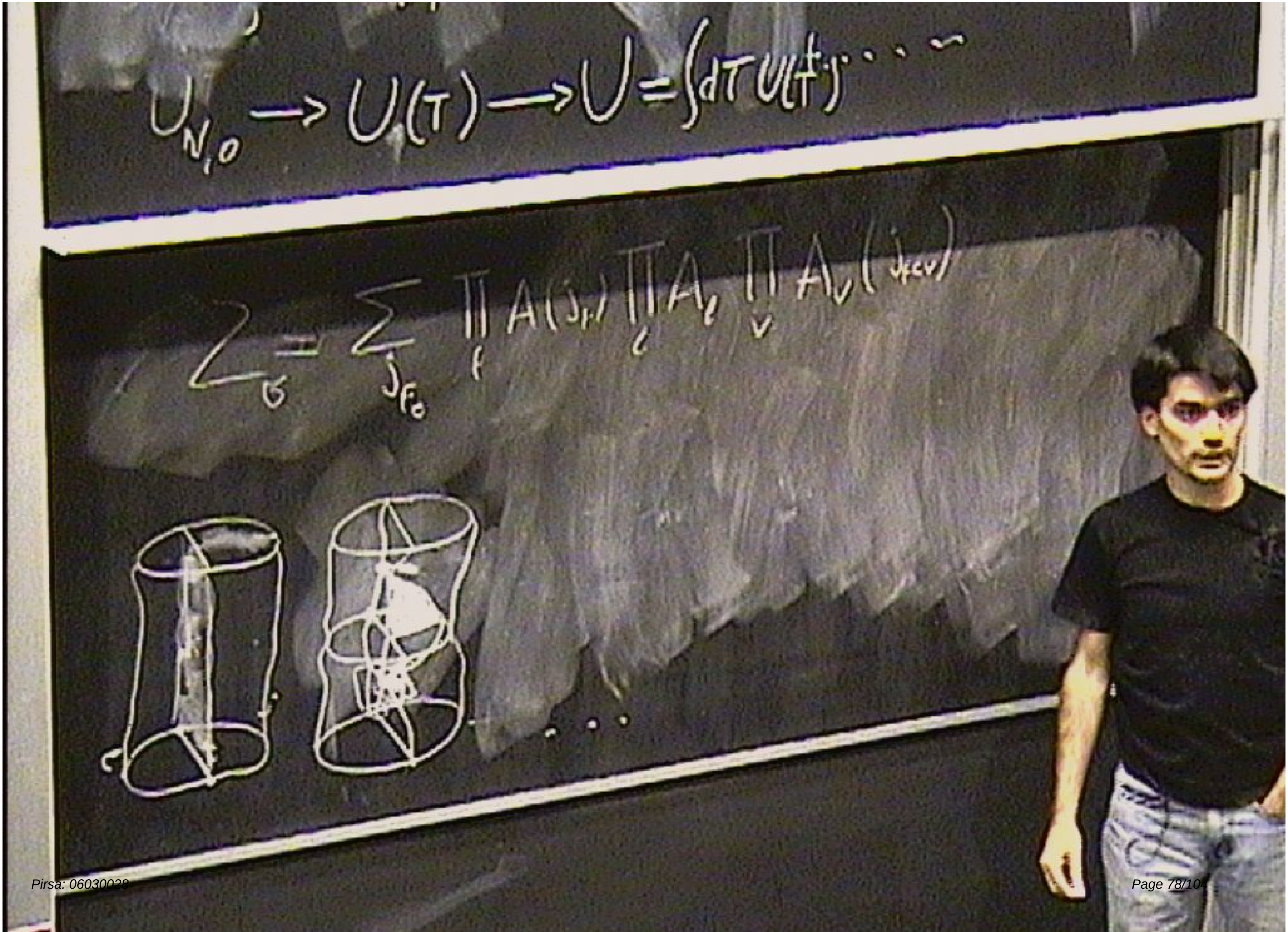



• 4d (Barrett-Crane)
$Z(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right]\left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$
$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \, D^{j_{f_1}}_{k_1 l_1}(g_e) \dots D^{j_{f_3}}_{k_3 l_3}(g_e) \mid$

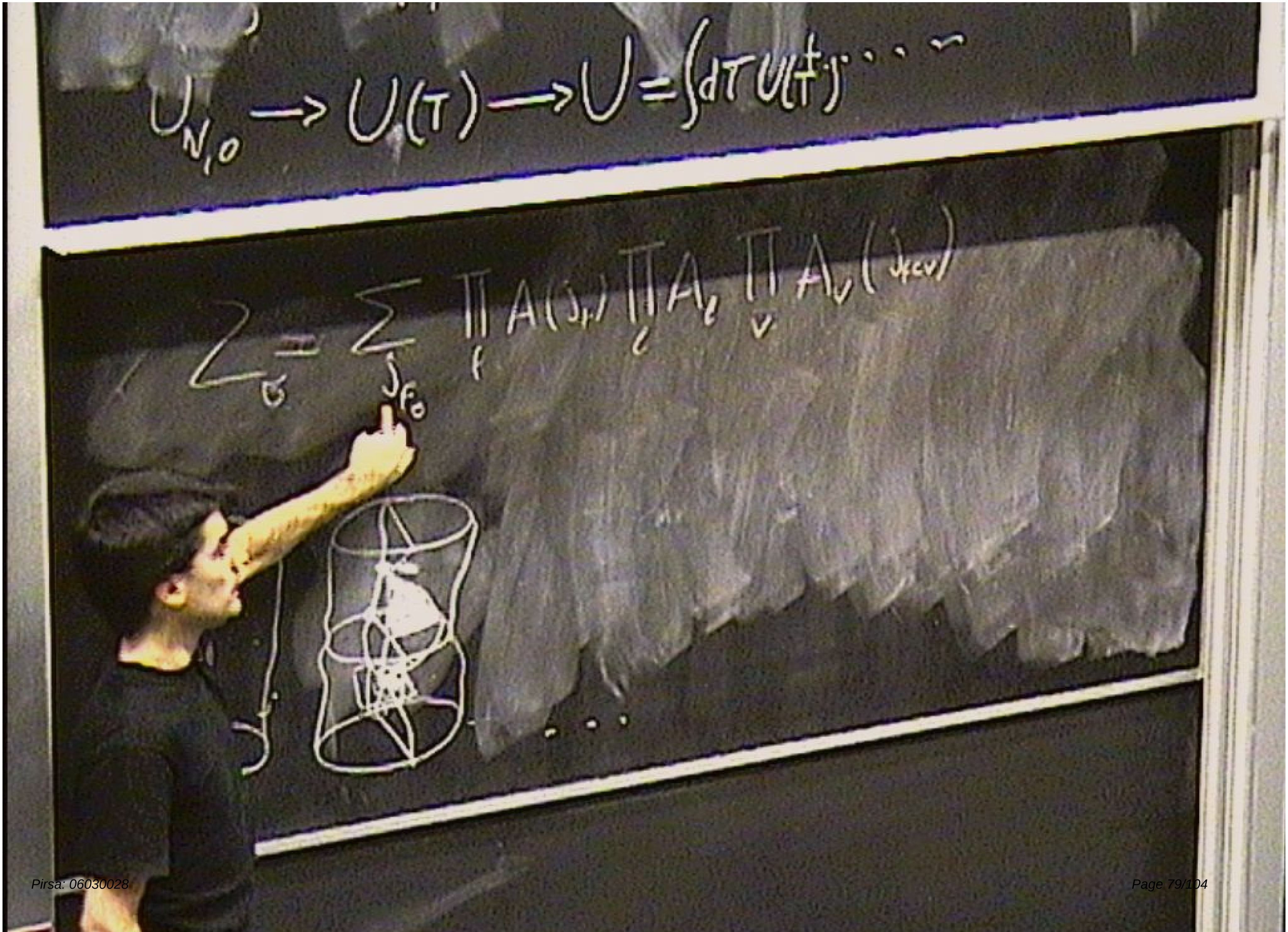

F. Dowker, gr-qc/0508109

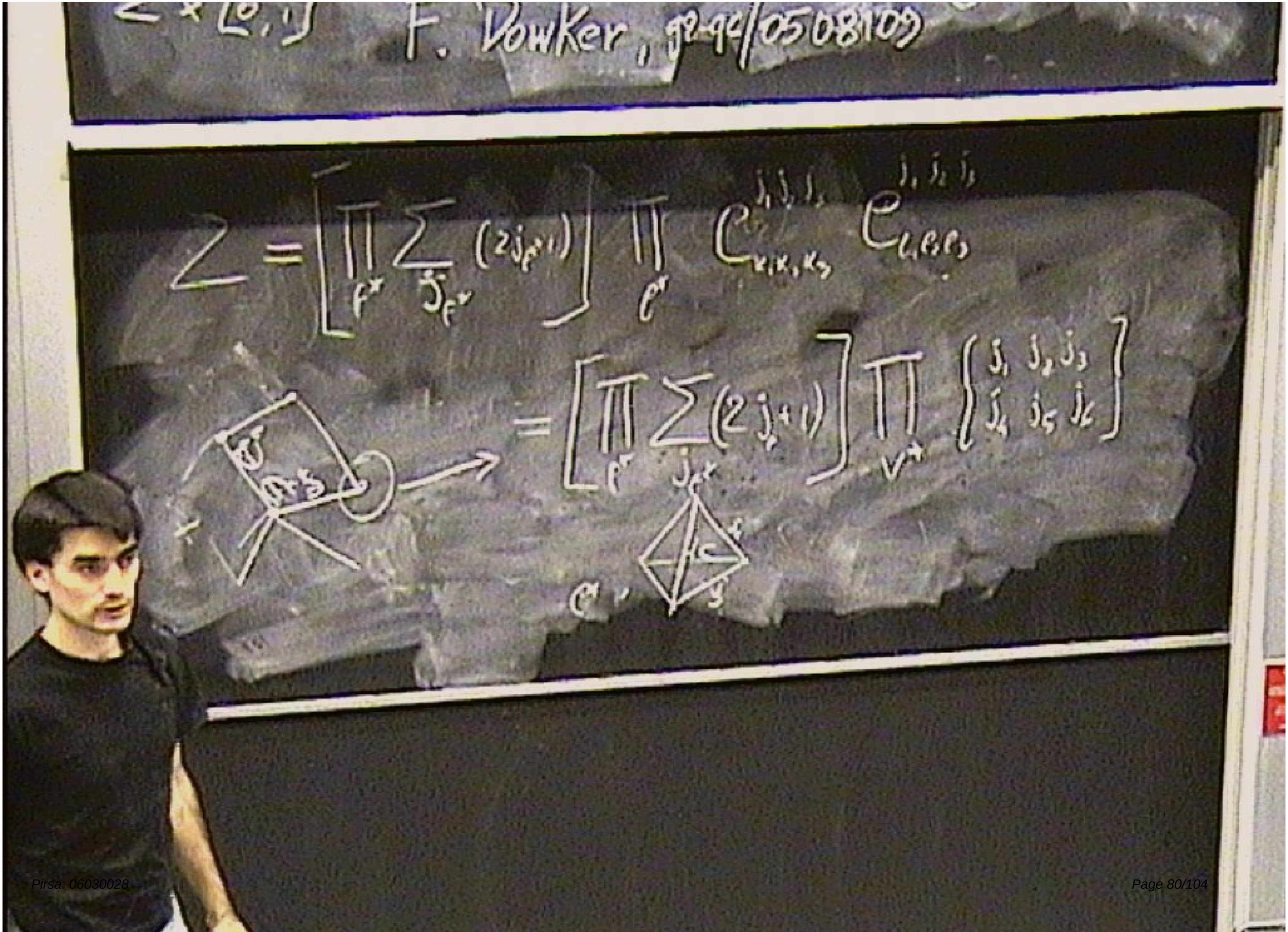

$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$


U_{N,0} → U(T) → U = ∫dT U(T) . . .
Σ_σ = Σ_{j_f} ∏_f A(j_f) ∏_l A_l ∏_v A
. . .



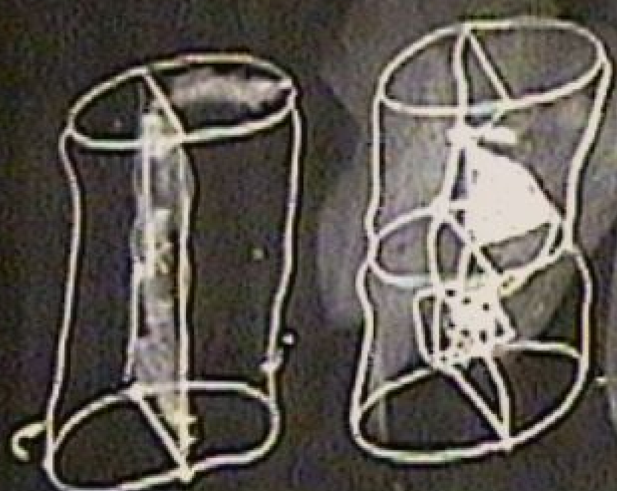
$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$
$Z_\sigma = \sum_{j_{f_\sigma}} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{f \in v})$


$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$
$Z_\sigma = \sum_{j_{f_0}} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{f(v)})$

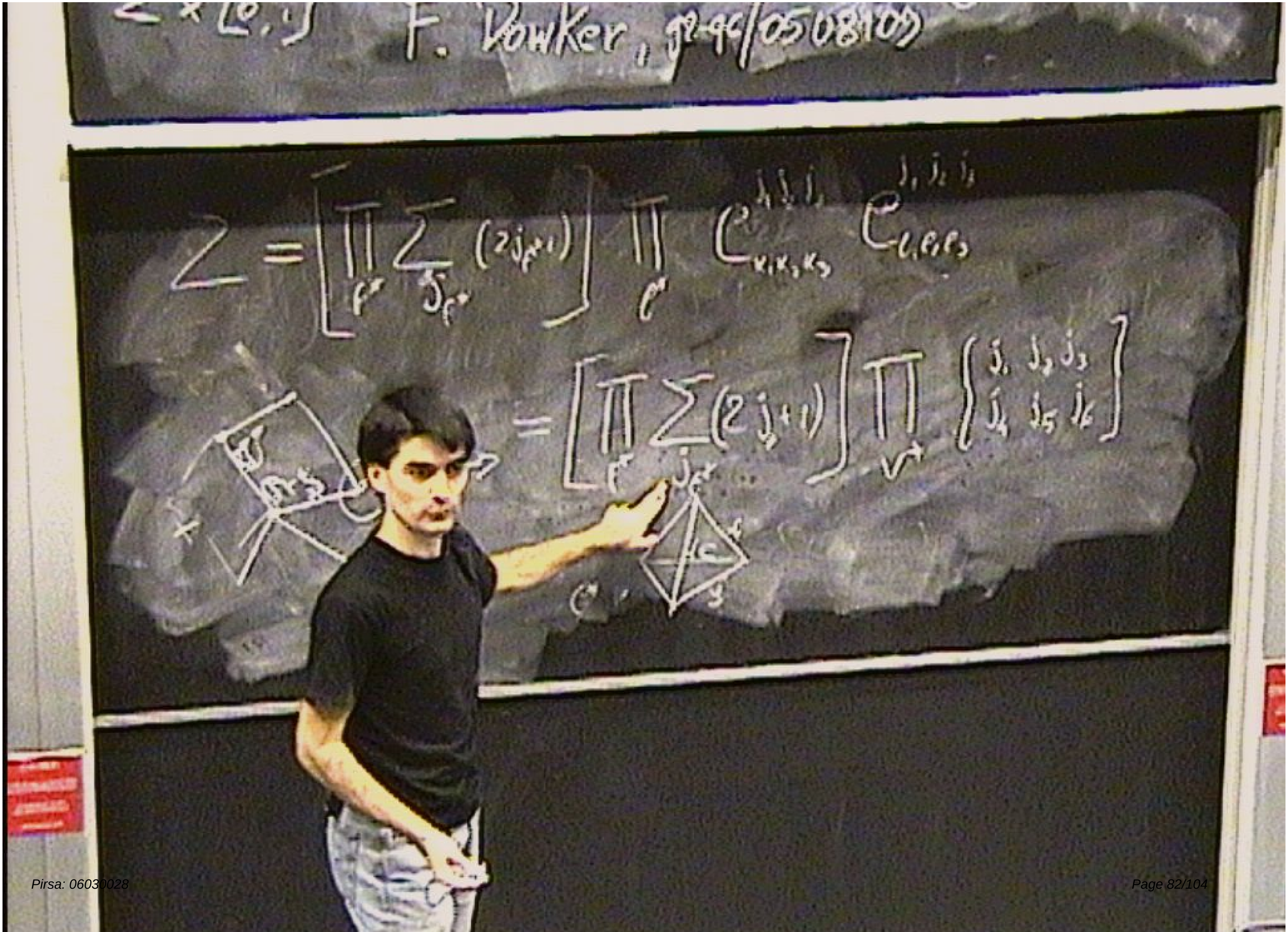

F. Dowker, gr-qc/0508109

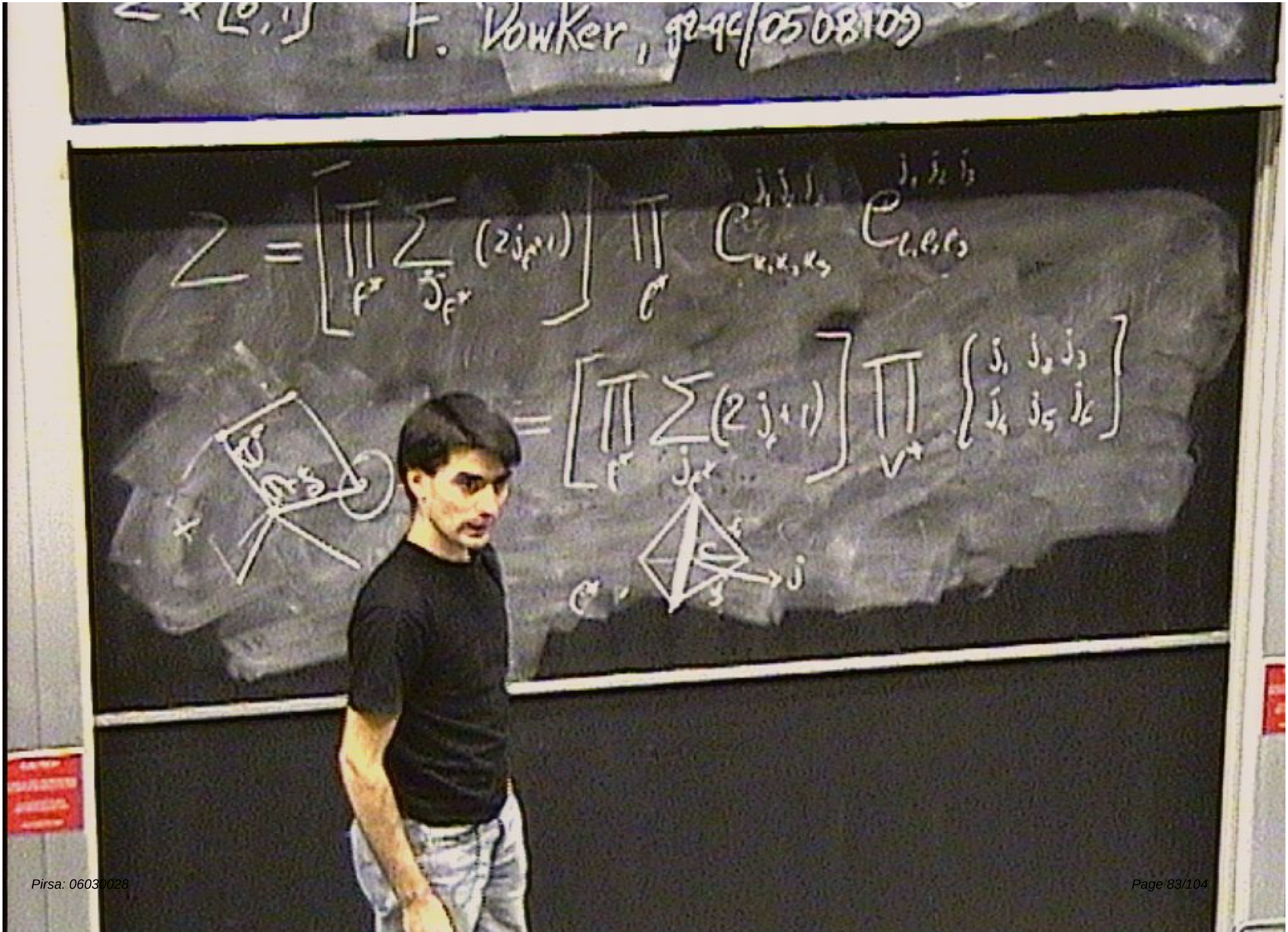
$$U_{N,0} \longrightarrow U(T) \longrightarrow U = \int dT\, U(T) \cdots$$

$$Z_\sigma = \sum_{j_{f}} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{fev})$$



. . .


F. Dowker, gr-qc/0508109

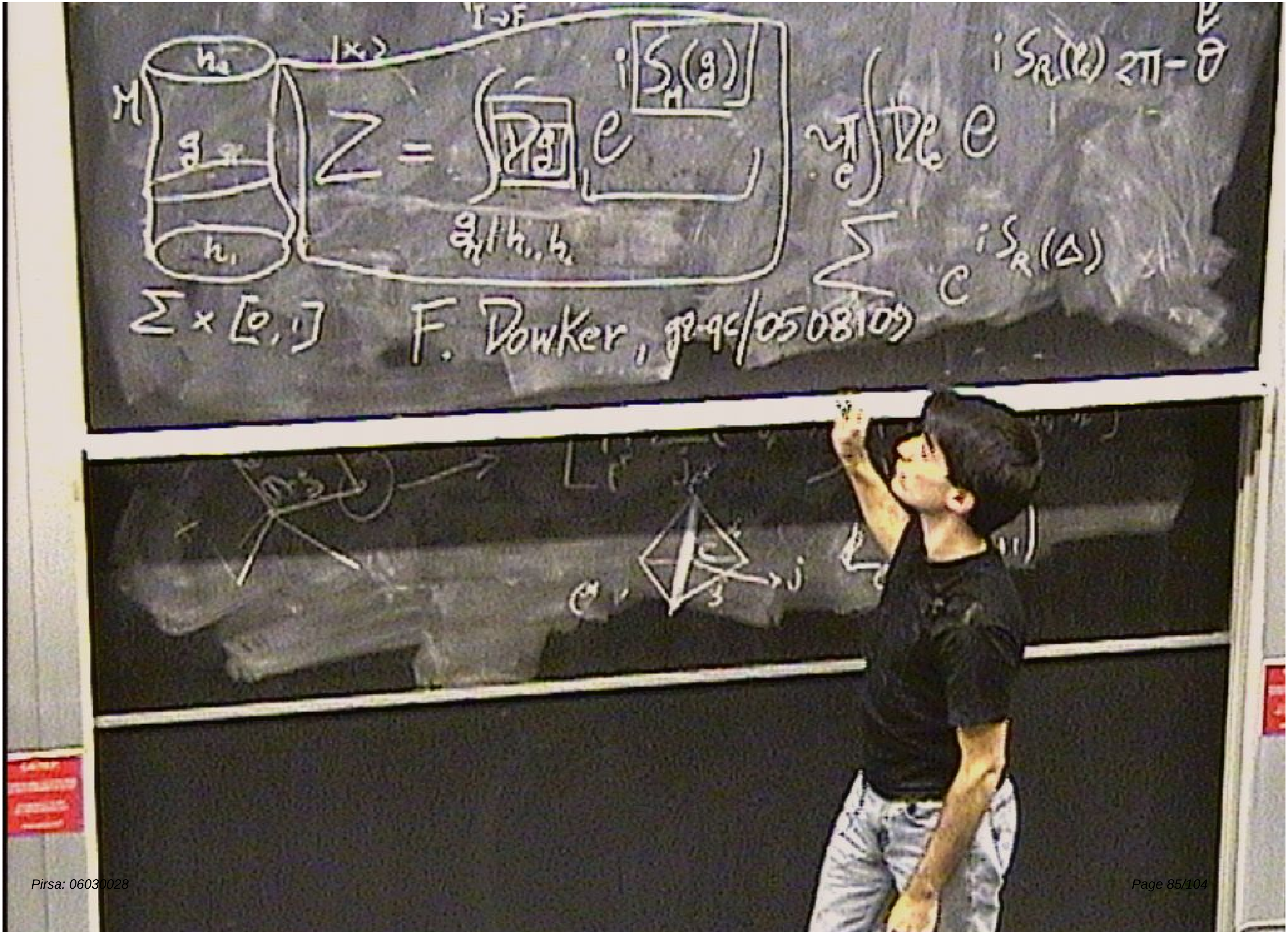

F. Dowker, gr-qc/0508109

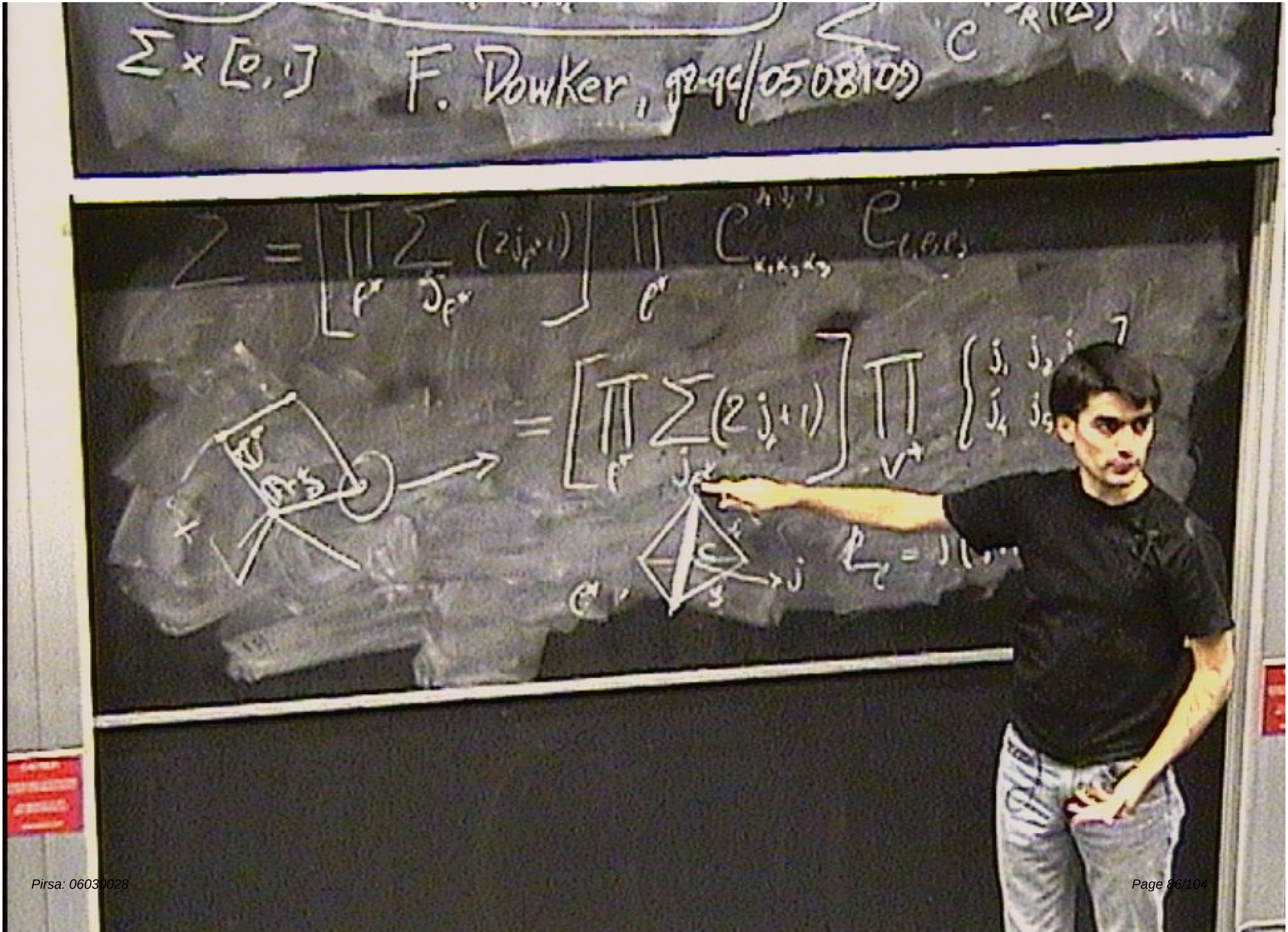
$\Sigma \times [0,1]$ F. Dowker, gr-qc/0508109

$$\Sigma = \left[\prod_{f^*} \sum_{j_{f^*}} (2j_{f^*}+1)\right] \prod_{e^*} C^{j_1 j_2 j_3}_{k_1 k_2 k_3}\, C^{j_1 j_2 j_3}_{\ell_1 \ell_2 \ell_3}$$

$$= \left[\prod_{f^*} \sum_{j_{f^*}} (2j_f+1)\right] \prod_{v^*} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$\ell_e = j(j+1)$

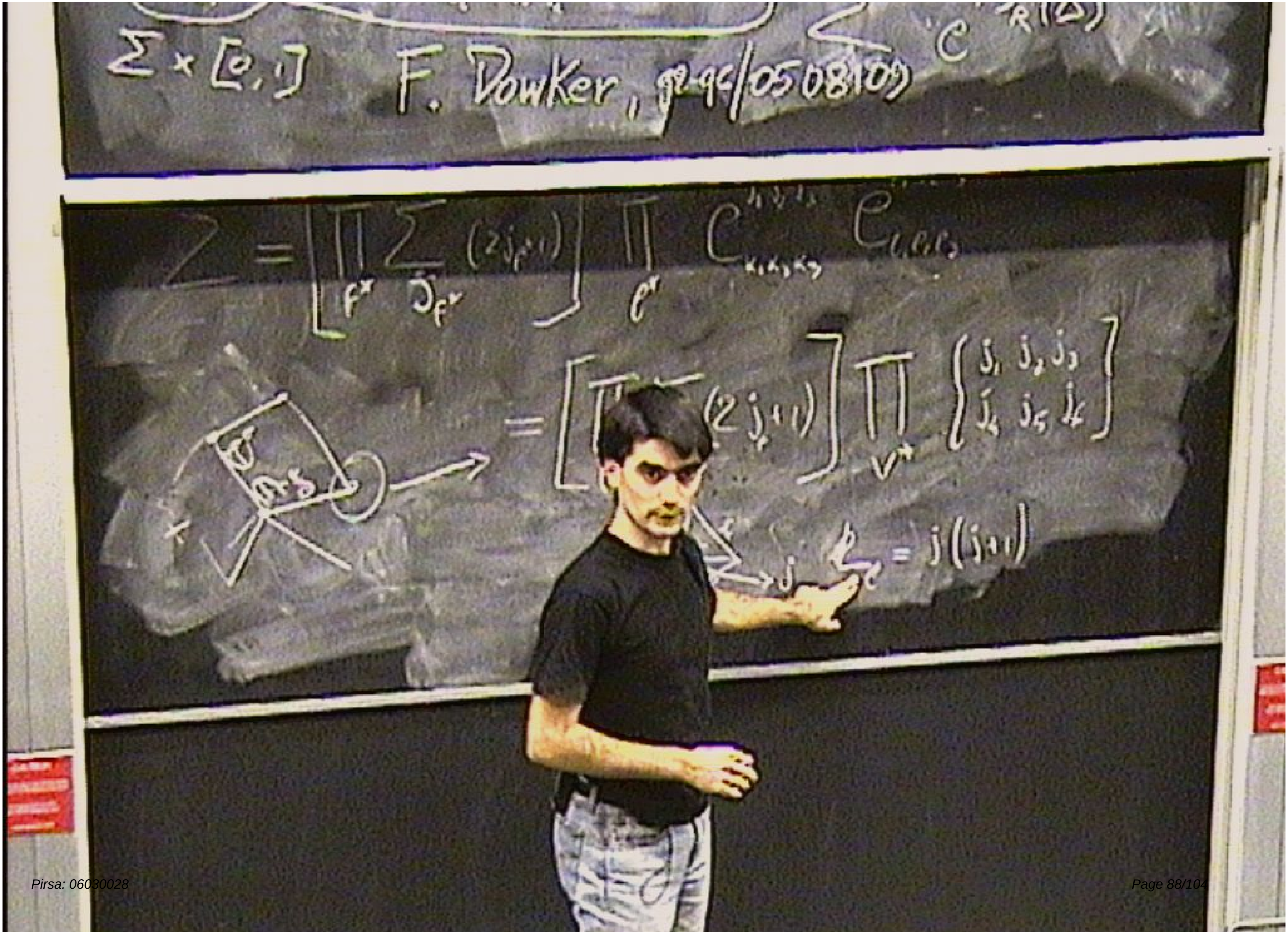

Z =
Σ × [0,1]
F. Dowker, gr-qc/0508109

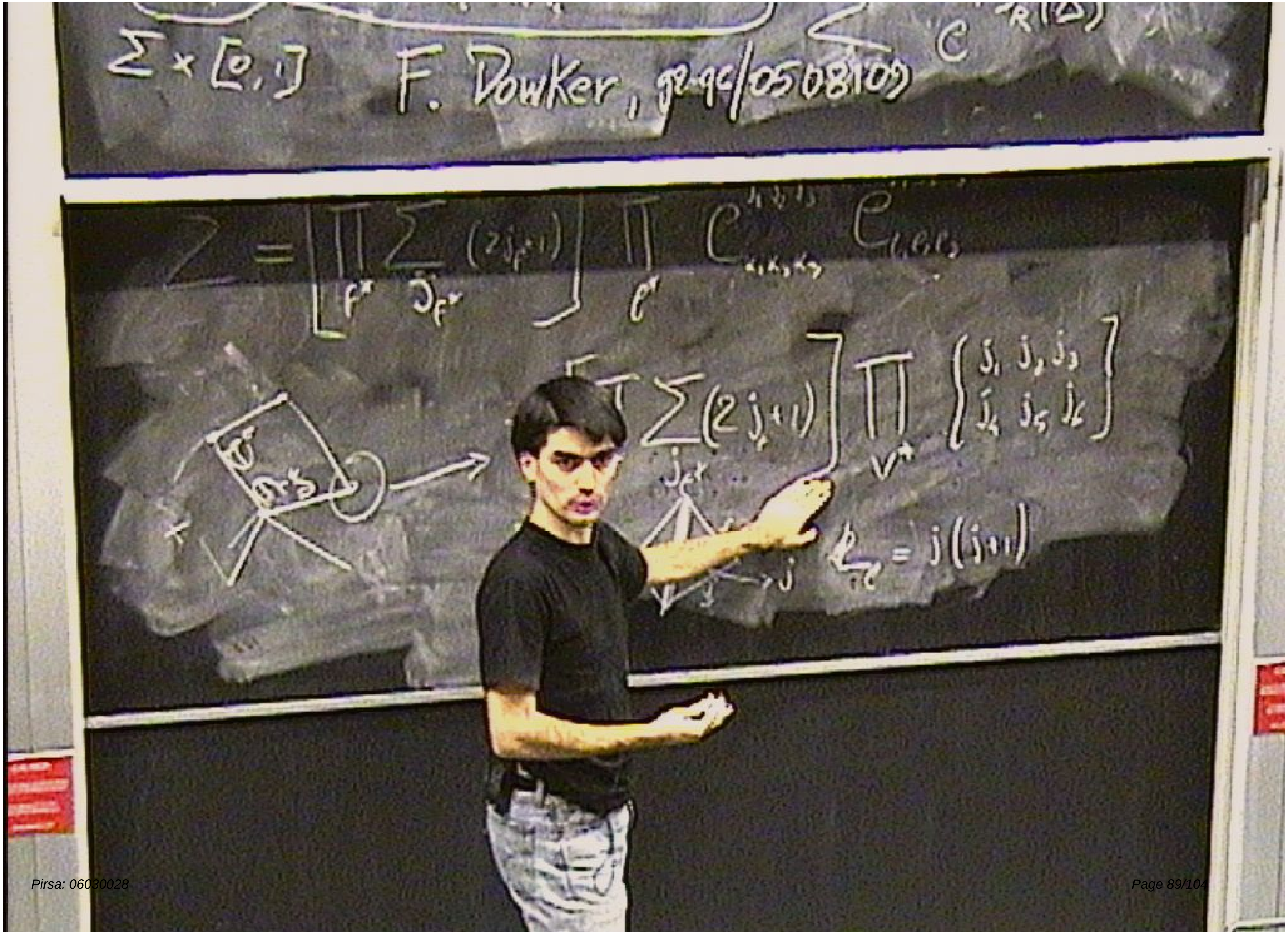

Σ × [0,1]
F. Dowker, gr-qc/0508109

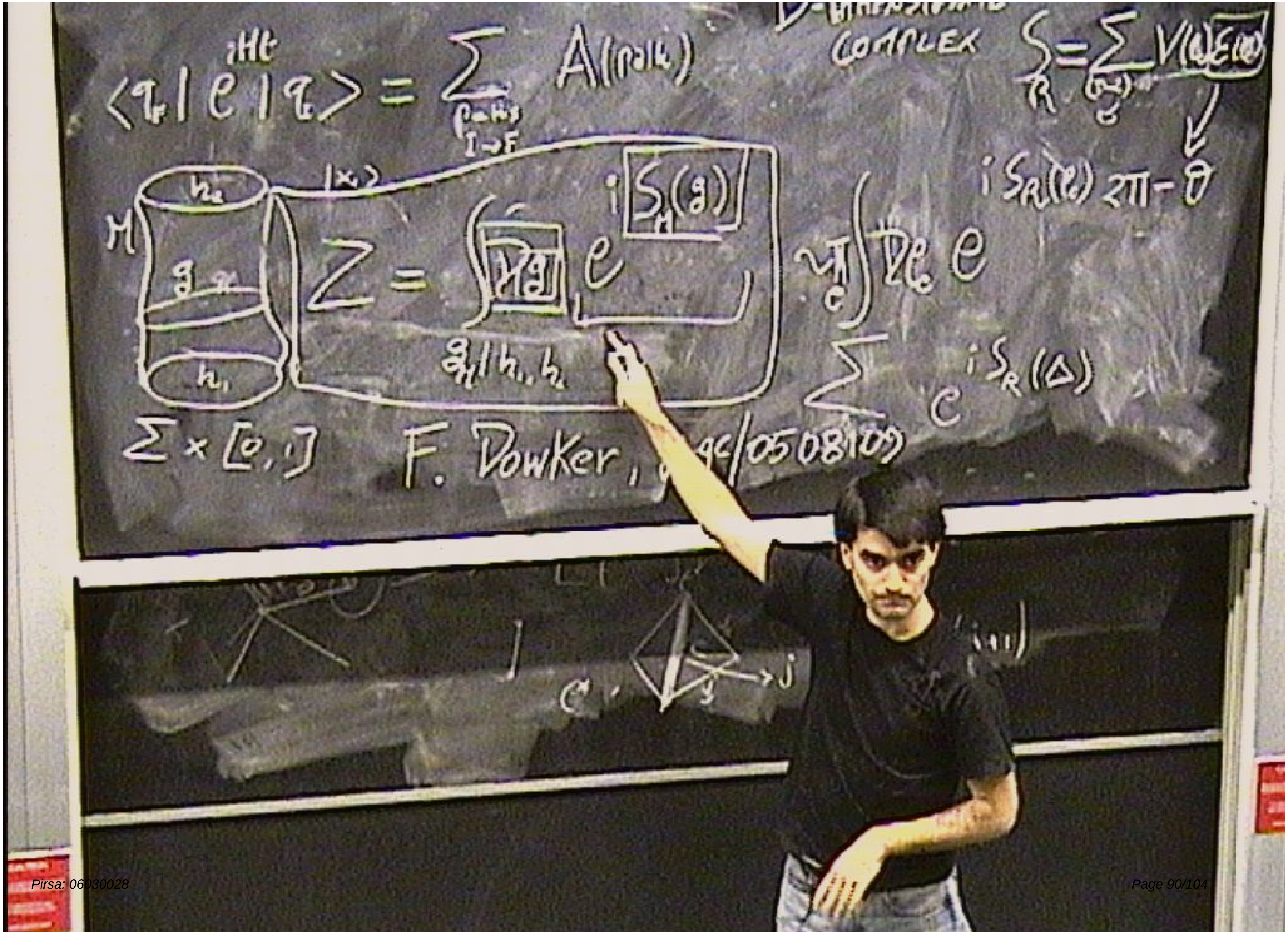
• 4d (Barrett–Crane)

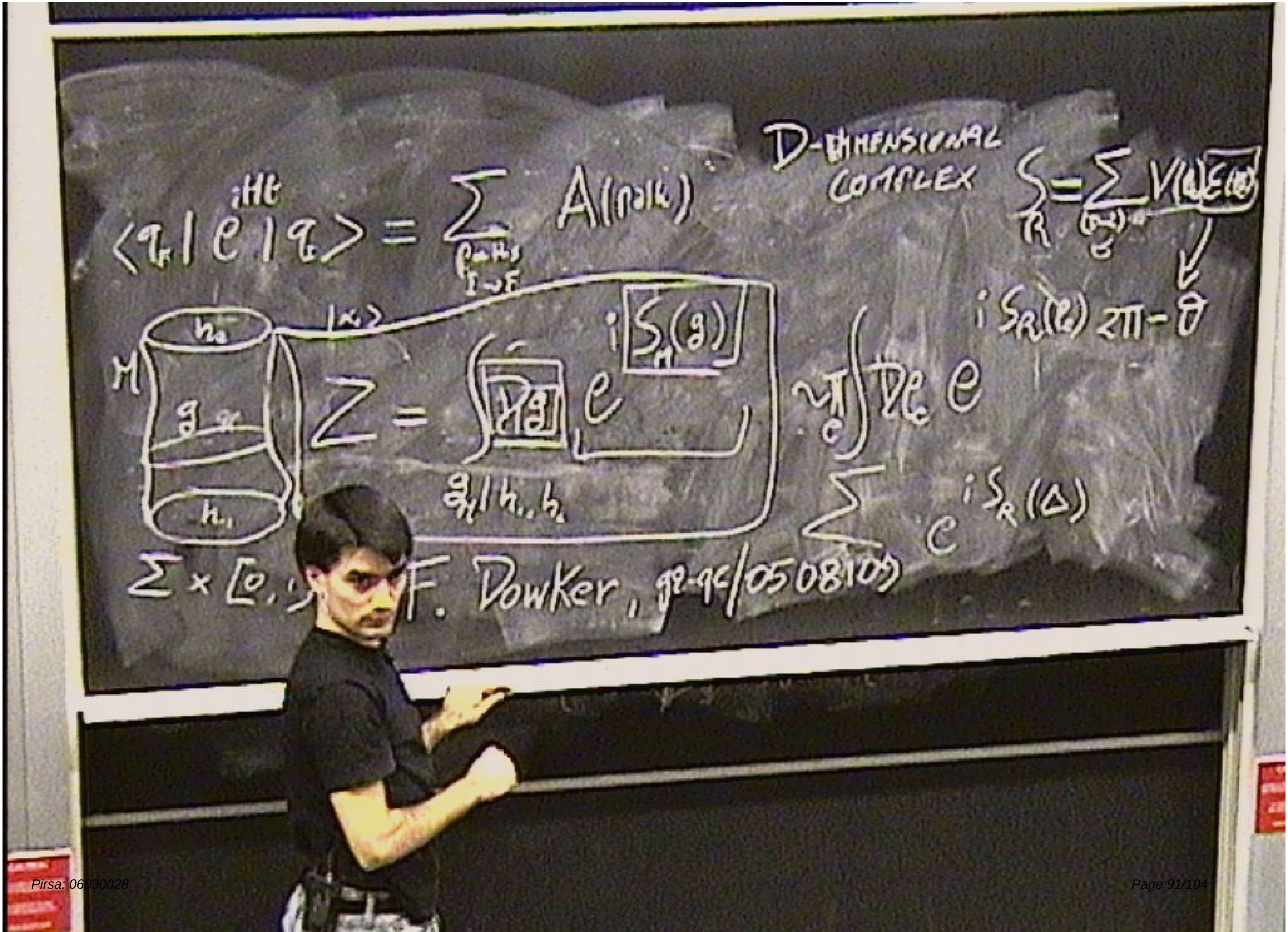
$$Z(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right]\left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$$

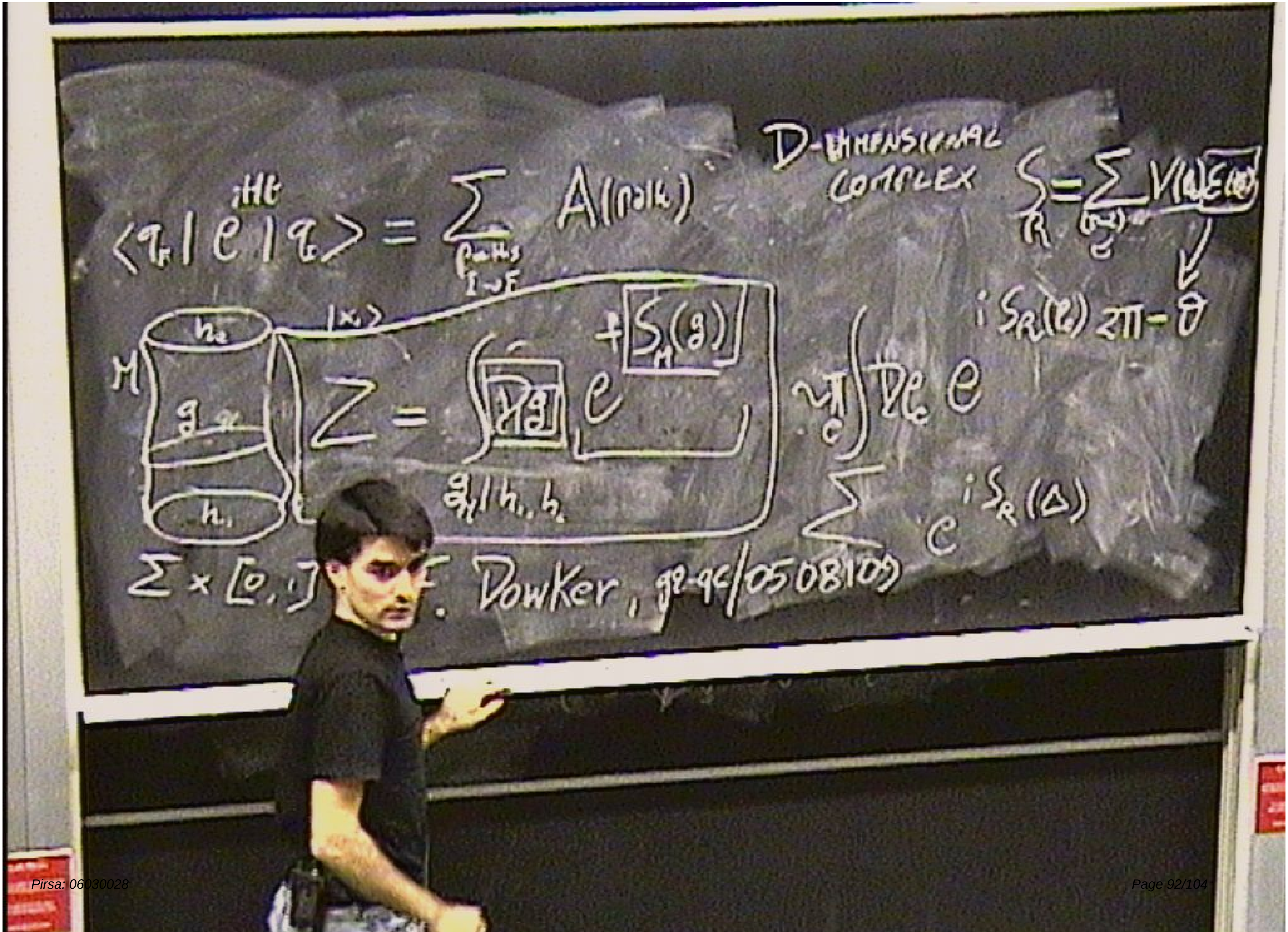
$$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e \; D^{j_{f_1}}_{k_1 l_1}(g_e) \cdots D^{j_{f_4}}_{k_4 l_4}(g_e)$$

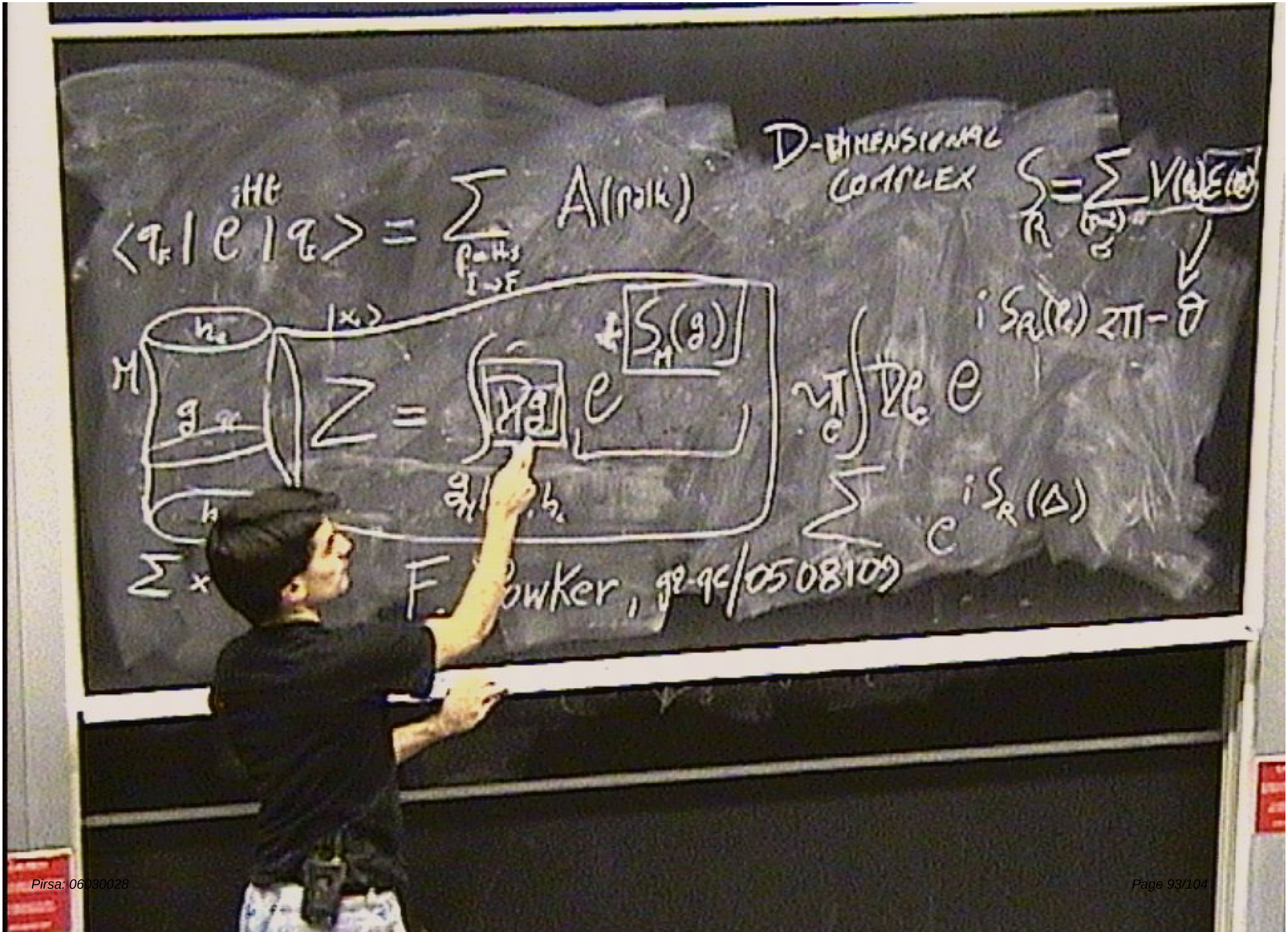

Σ × [0,1]
F. Dowker, gr-qc/0508109
= j(j+1)


Σ × [0,1]
F. Dowker, gr-qc/0508109
Z = [∏ Σ (2j_f+1)] ∏ C e
l = j(j+1)


COMPLEX
Z =
Σ × [0,1]
F. Dowker


D-DIMENSIONAL
COMPLEX
F. Dowker, gr-qc/0508109


D-DIMENSIONAL
COMPLEX
paths
I→F
Σ × [0,1]
F. Dowker, gr-qc/0508109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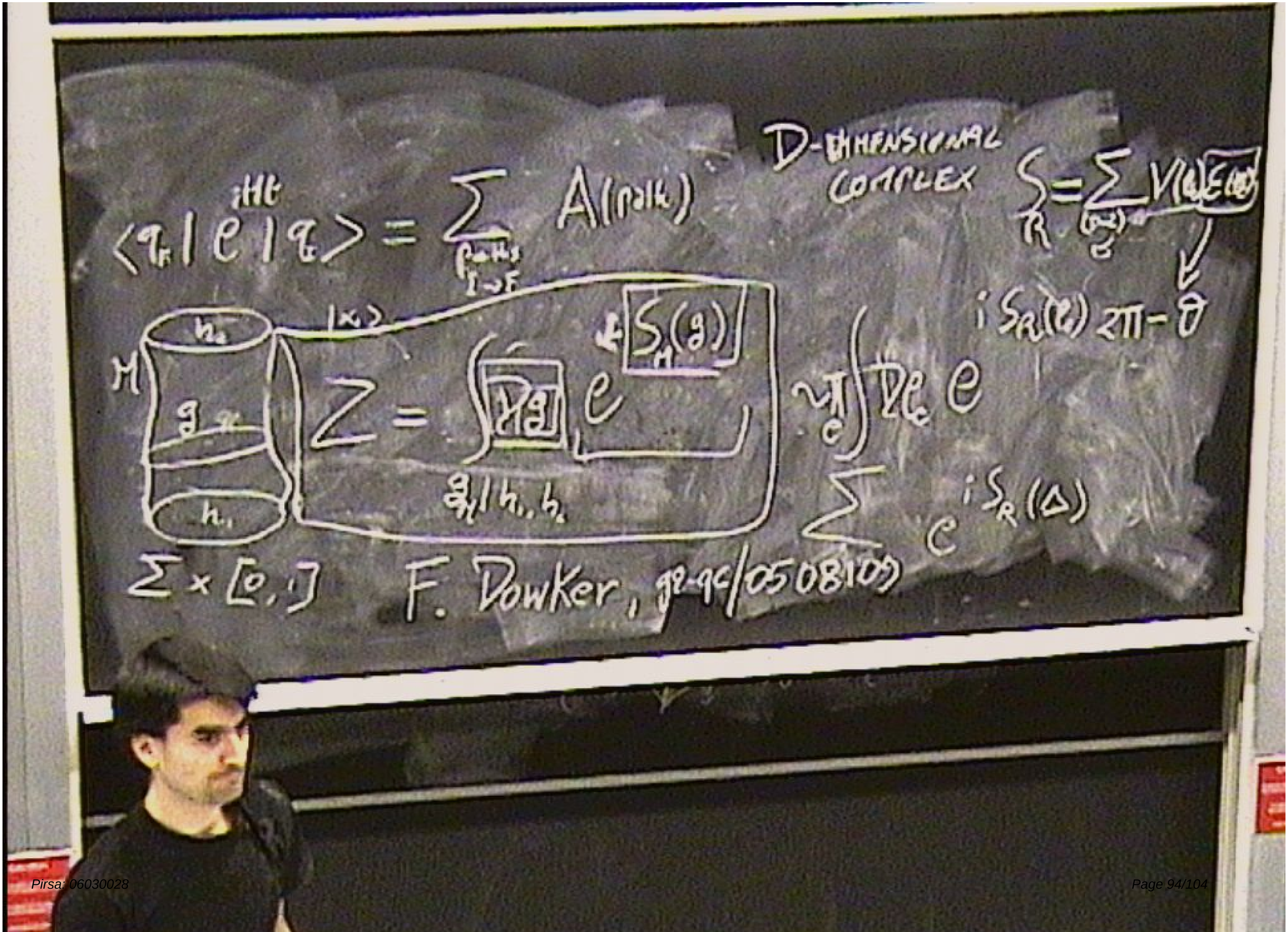
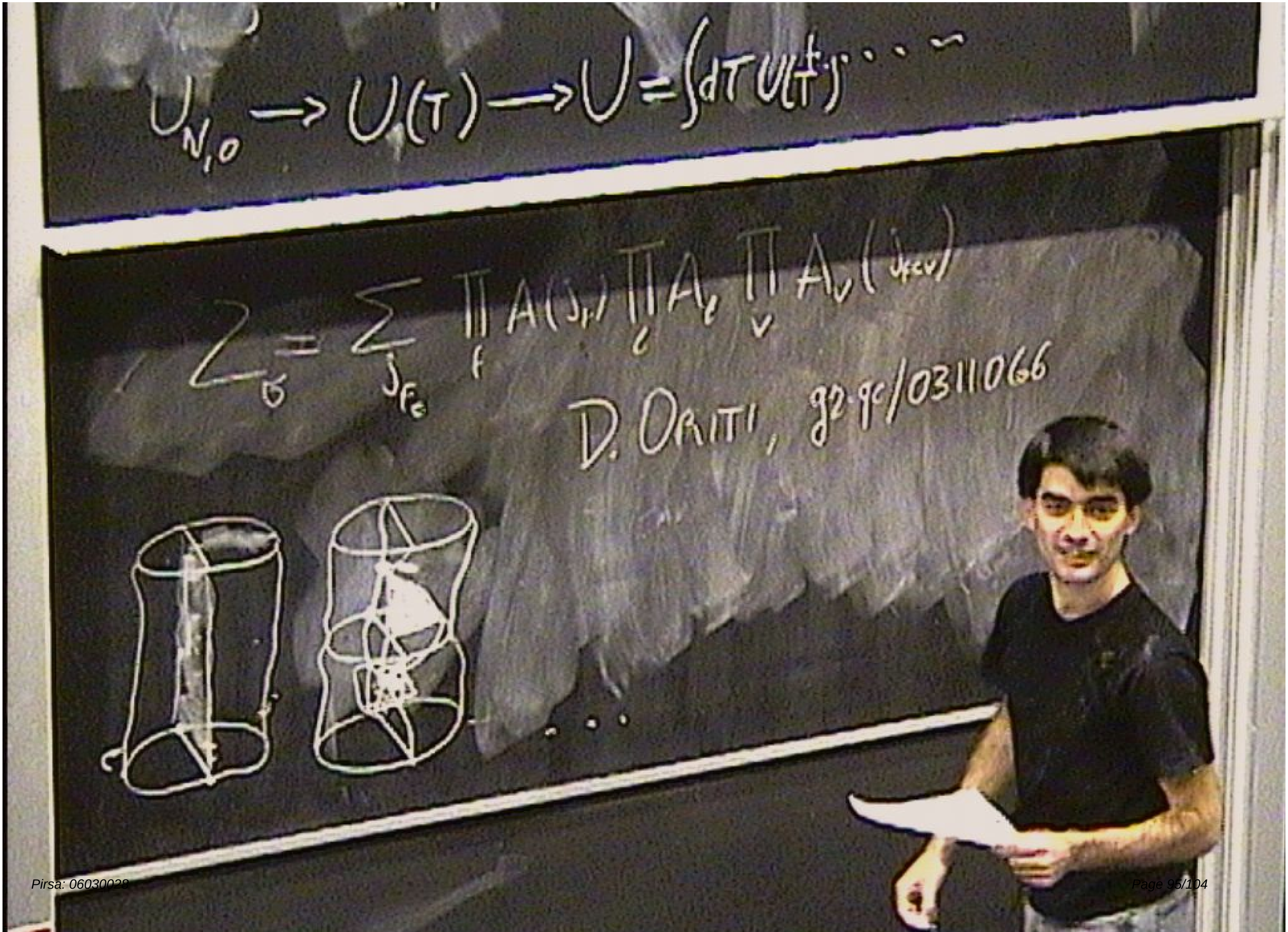
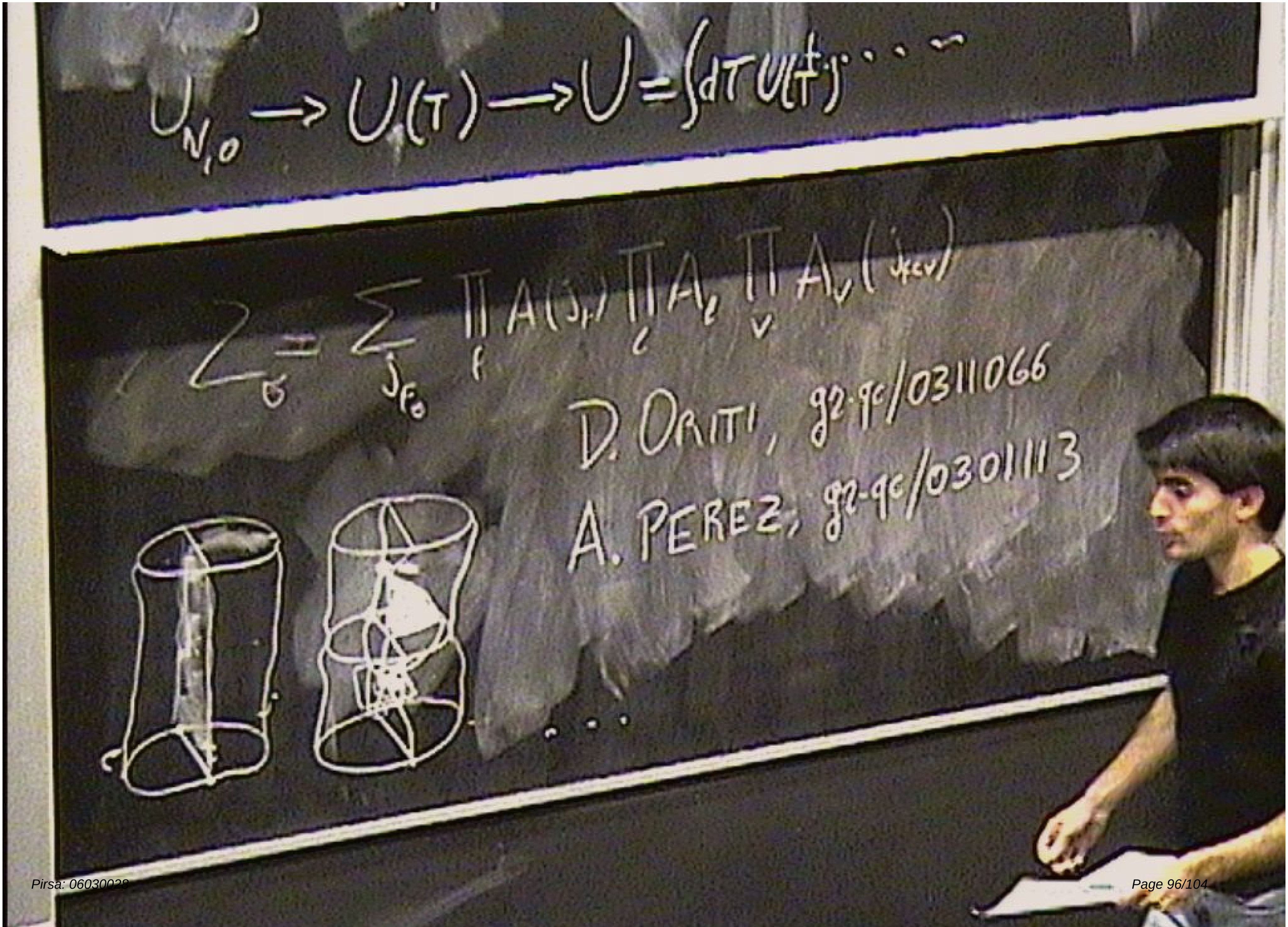
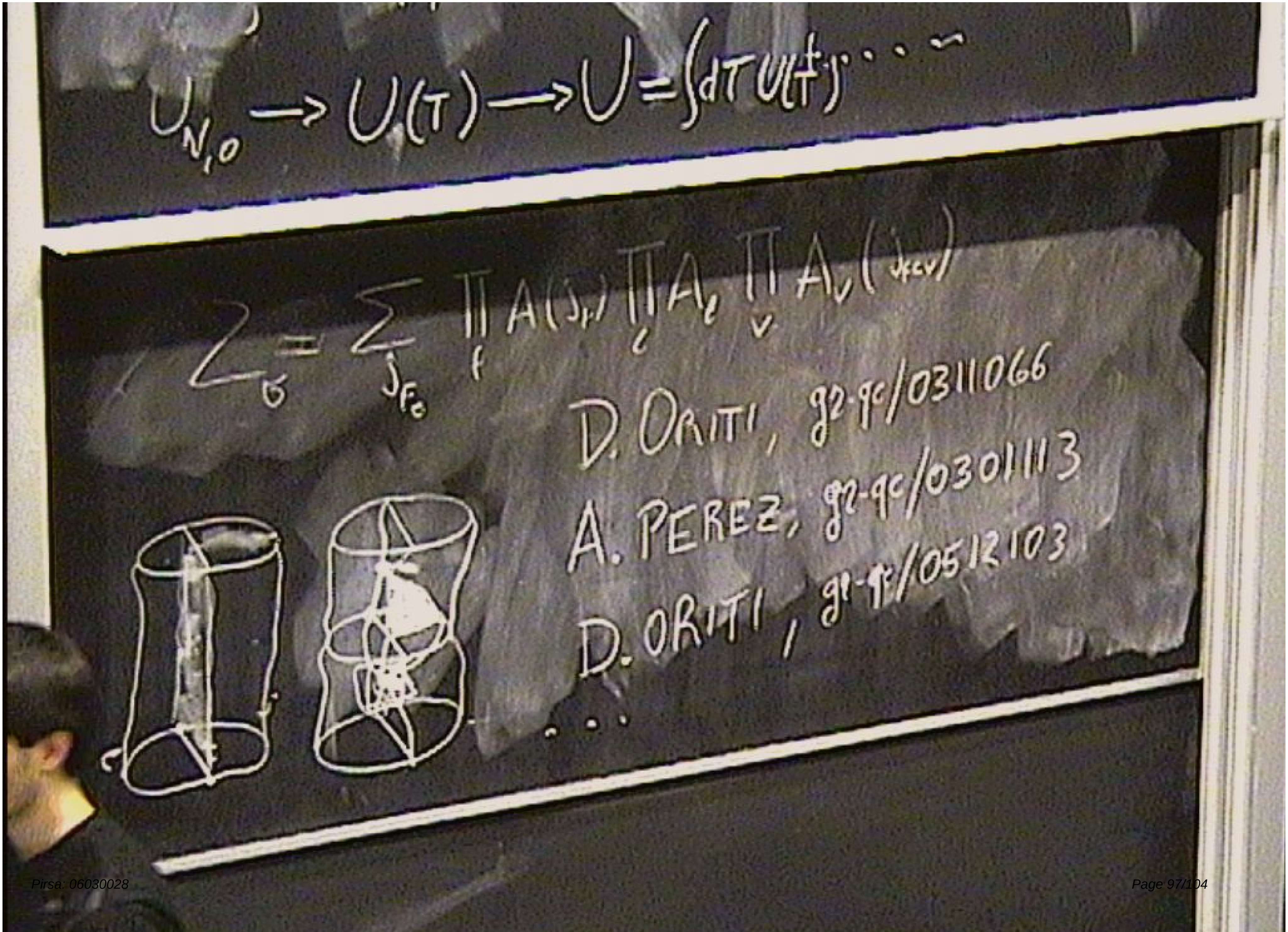
COMPLEX
F. owker, gr-qc/0508109

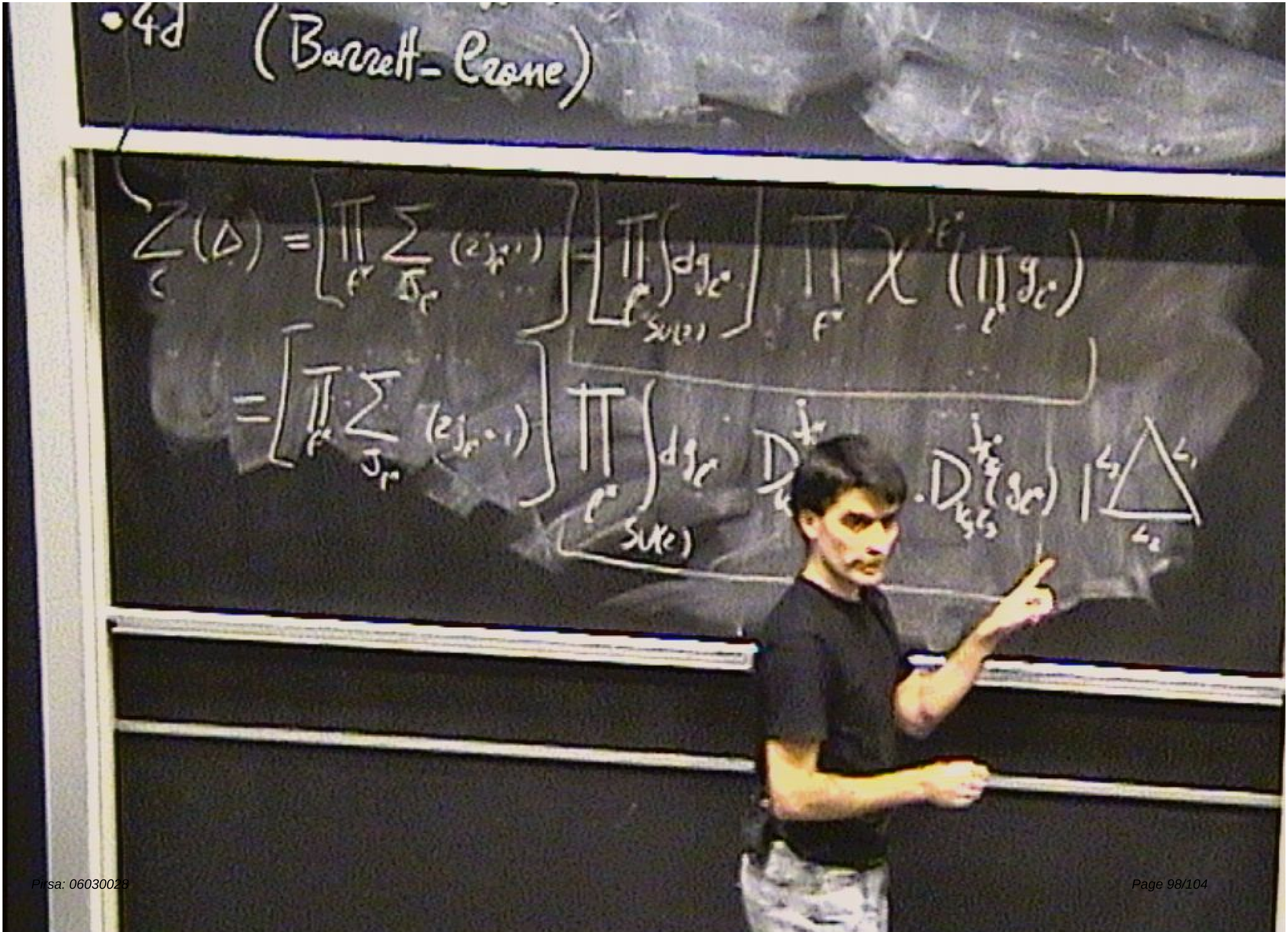

D-DIMENSIONAL COMPLEX
$Z = \int \mathcal{D}[g] \, e^{iS_M(g)}$
$\Sigma \times [0,1]$
F. Dowker, gr-qc/0508109

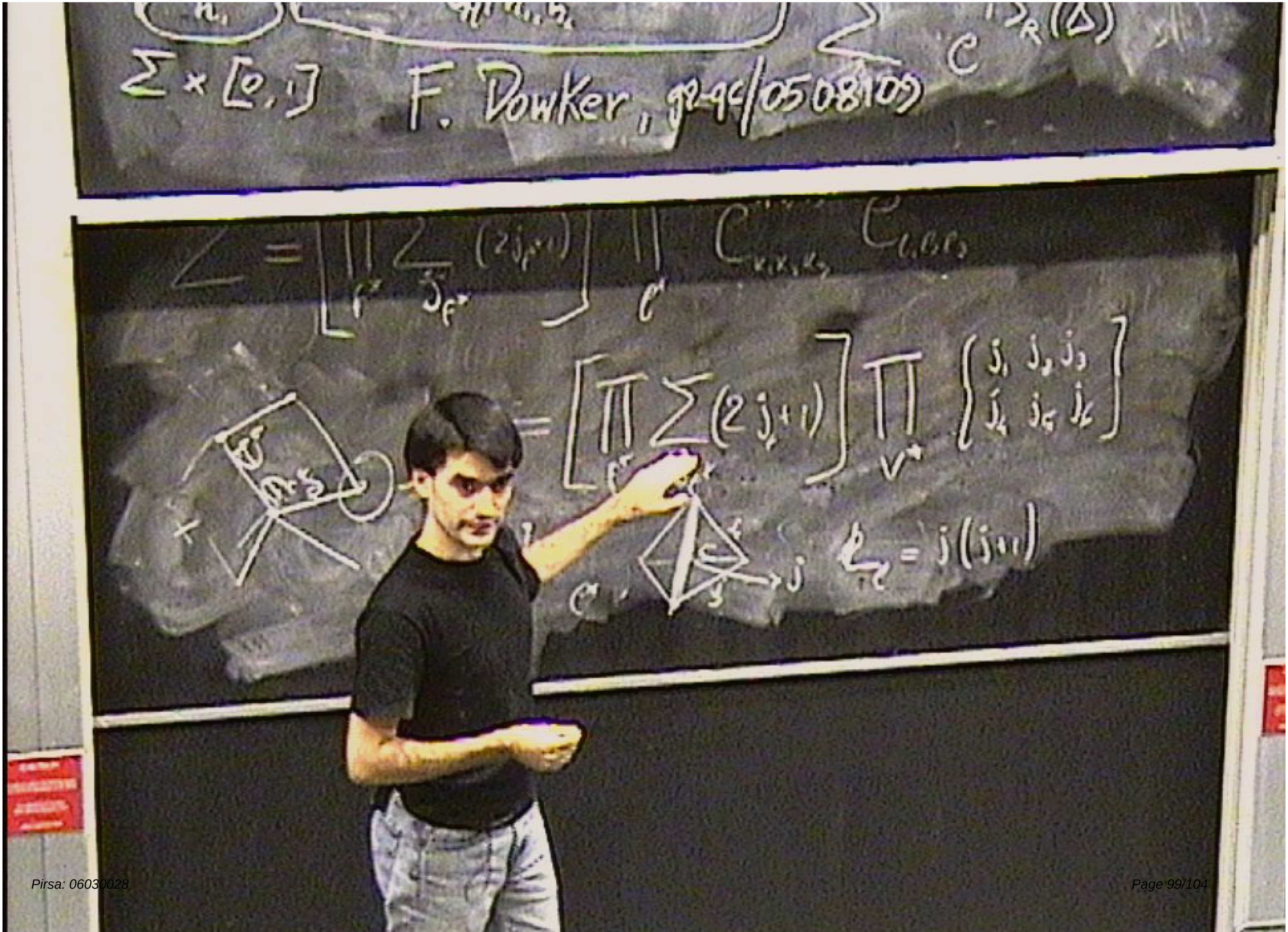


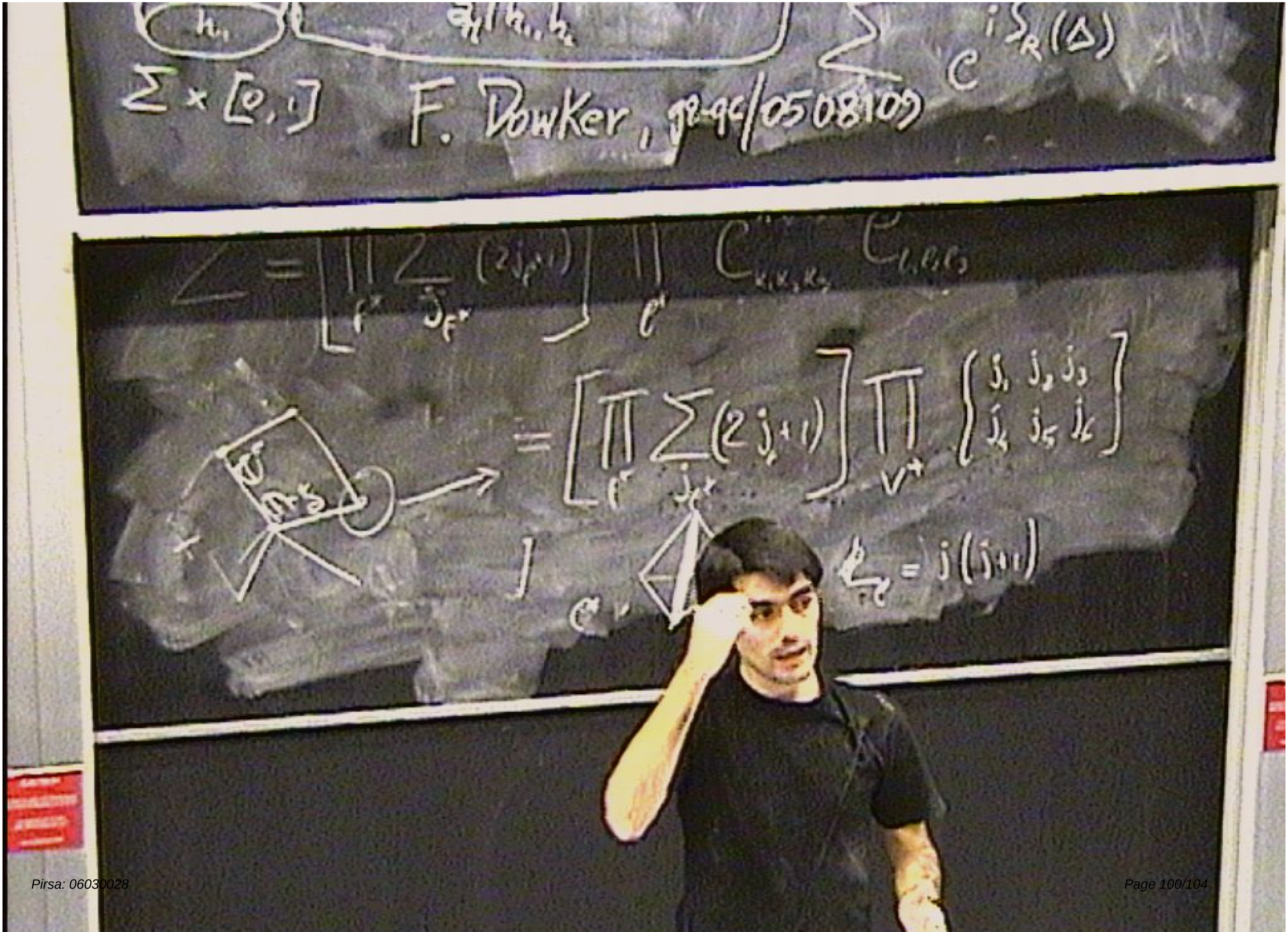
$U_{N,0} \to U(T) \to U = \int dT\, U(T) \cdots$
$Z_\sigma = \sum_{j_{f_\sigma}} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{fev})$
D. ORITI, gr-qc/0311066

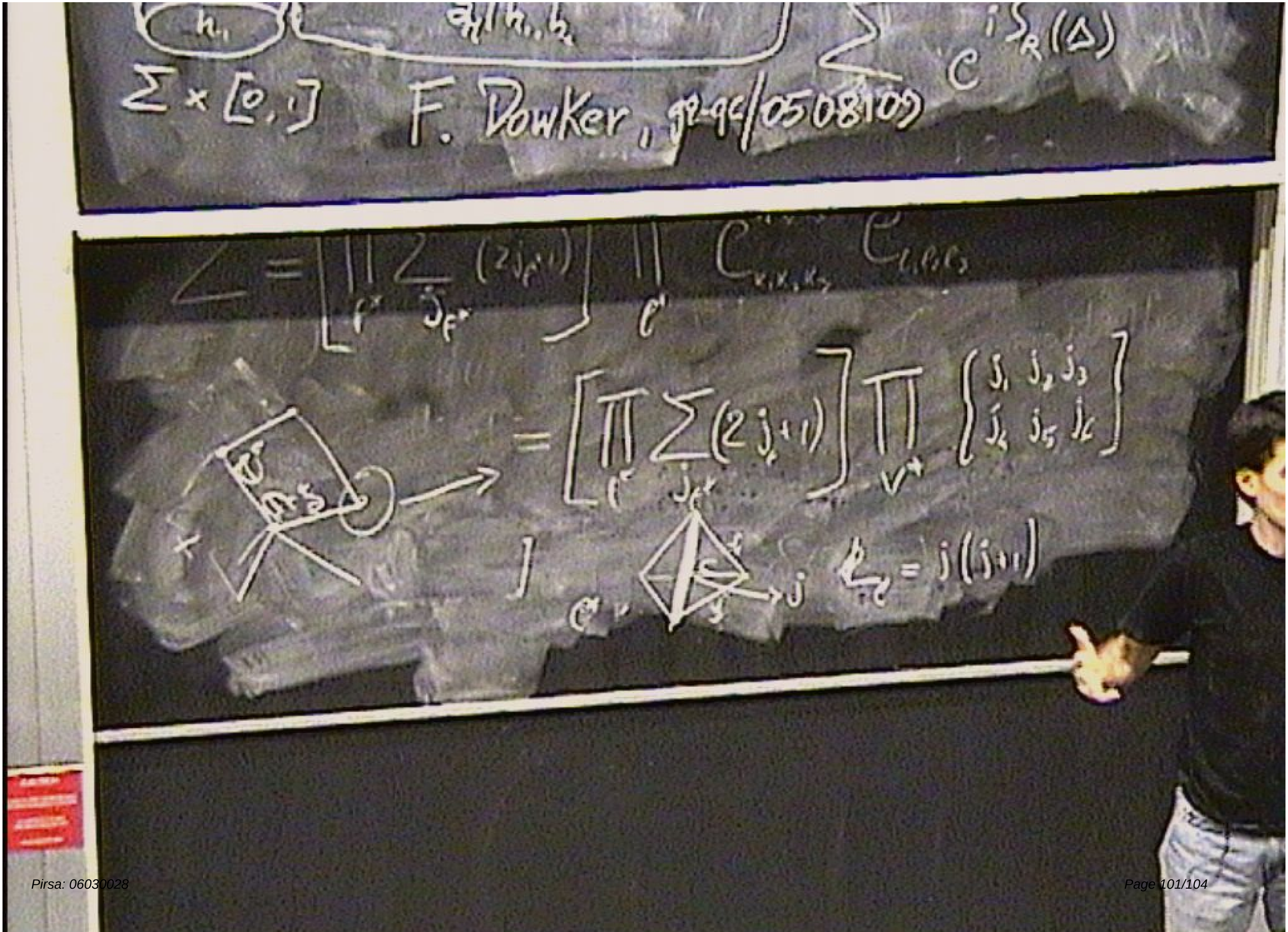

$U_{N,0} \rightarrow U(T) \rightarrow U = \int dT\, U(T) \cdots$
$Z_\sigma = \sum_{j_f} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{f \in v})$
D. ORITI, gr-qc/0311066
A. PEREZ, gr-qc/0301113

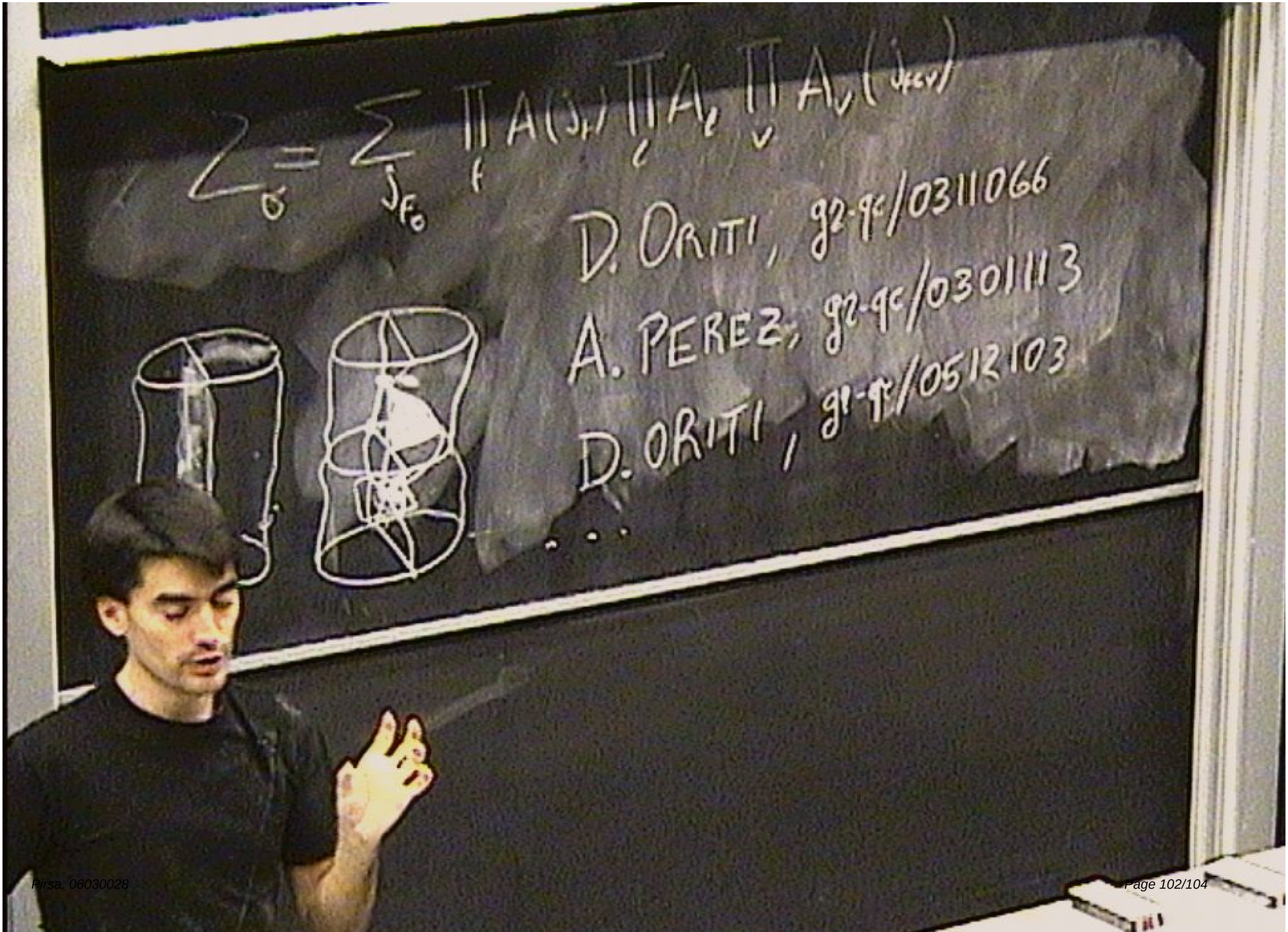

U_{N,0} → U(T) → U = ∫dT U(T) . . .
Z_σ = Σ_{j_{f_σ}} Π_f A(j_f) Π_e A_e Π_v A_v(j_{f⊂v})
D. ORITI, gr-qc/0311066
A. PEREZ, gr-qc/0301113
D. ORITI, gr-qc/0512103
. . .


• 4d (Barrett – Crane)
$Z(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$
$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \prod_e \int_{SU(2)} dg_e$


F. Dowker, gr-qc/0508109


Σ × [0,1]
F. Dowker, gr-qc/0508109

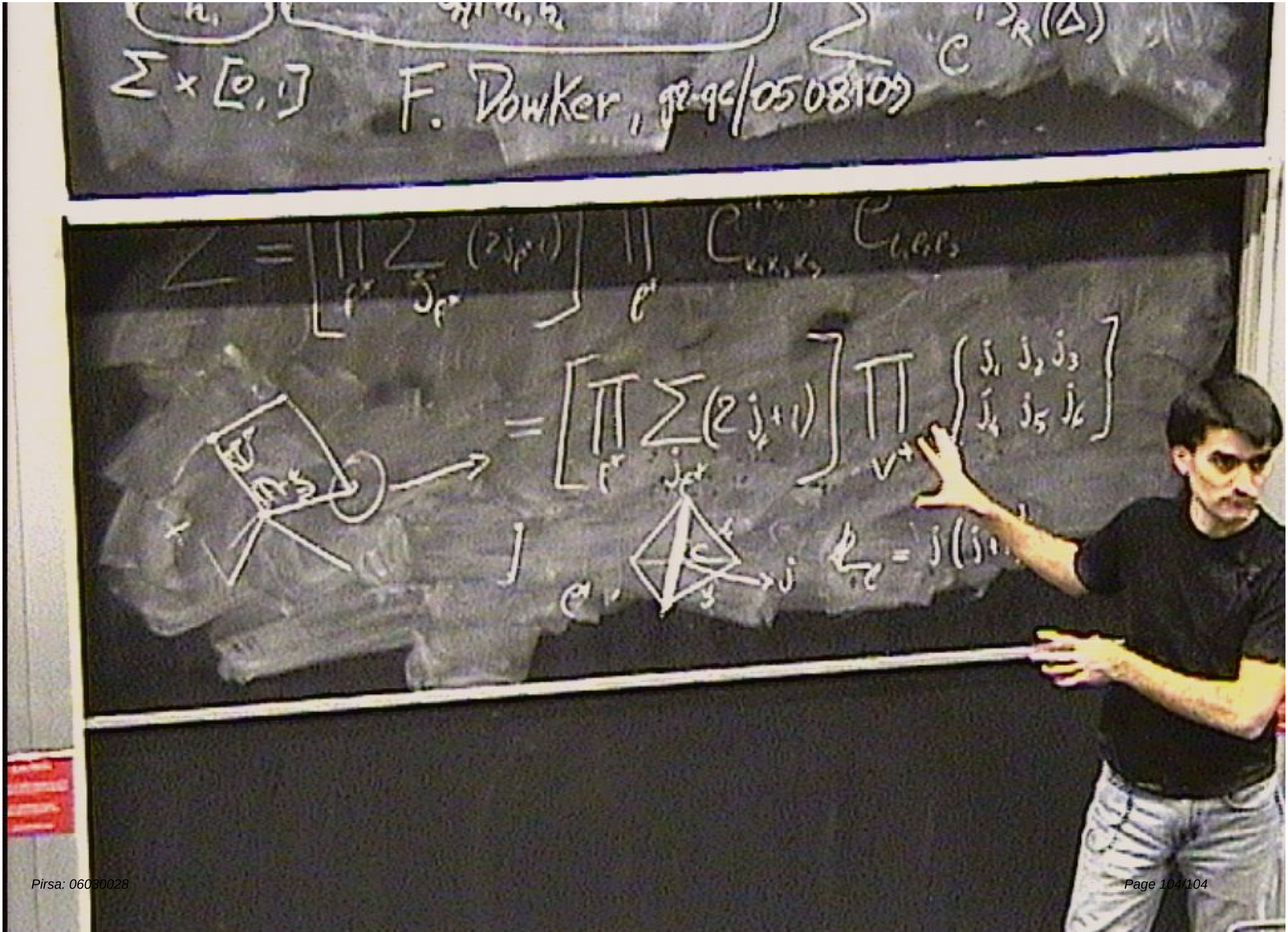

Σ × [0,1]
F. Dowker, gr-qc/0508109


$Z_\sigma = \sum_{j_{f_0}} \prod_f A(j_f) \prod_e A_e \prod_v A_v(j_{fev})$
D. ORITI, gr-qc/0311066
A. PEREZ, gr-qc/0301113
D. ORITI, gr-qc/0512103
. . .

• 4d (Barrett–Crane)

$$Z(\Delta) = \left[\prod_f \sum_{j_f} (2j_f+1)\right] \left[\prod_e \int_{SU(2)} dg_e\right] \prod_f \chi^{j_f}\left(\prod_e g_e\right)$$

$$= \left[\prod_f \sum_{j_f} (2j_f+1)\right] \underbrace{\prod_e \int_{SU(2)} dg_e \, D^{j_1}_{k_1 l_1}(g_e) \ldots D^{j_3}_{k_3 l_3}(g_e)}$$


F. Dowker, gr-qc/0508109