Title: Time as ignorance, algebraically

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Abstract:

Time as ignorance,
algebraically

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Outline

1. Where to start?
   - Information-theoretic approach
   - Measurement order is irrelevant
   - Phenomenal time
   - Mathematical idealization and physical interpretation

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   - States
   - Types of algebras
   - Modular automorphisms

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   - Topologies and algebras
   - States
   - Types of algebras
   - Modular automorphisms
3. Time in the algebraic formalism
   - Operational approach is atemporary
   - Modular time
   - An Interpretation

Reconstruct QM as information theory

Quantum theory is a general theory of information constrained by several information-theoretic principles.

Quantum logical formulation

Table: Quantum logical language

Basic notion	**Formal representation**
System	Physical system S
Information	Yes-no question
Fact	Answer to yes-no question at a given time t

Algebraic formulation

Table: C^*-algebraic language

Basic notion	**Formal representation**
System	C^*-algebra $\mathfrak{A}$
Information	State over algebra
Fact	Change of state over algebra

Order of measurements says nothing about time

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> *Exact time points are not given in intuition in any manner, and so are not absolute, but are concepts (products of reason), attaining full definiteness only in the purely formal arithmetico-analytic concept of the real number.*

Weyl on method

- "In order to have some hope of connecting phenomenal time with the world of mathematical concepts, let us grant the ideal possibility that time-points can be exhibited."

Dirac on method

- "The most powerful advance would be to perfect and generalize the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities."

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Topologies

- In the linear space $\mathfrak{B}(\mathcal{H})$ of bounded operators on a Hilbert space $\mathcal{H}$ consider a system of ε-neighbourhoods of operator A defined by $||A - B|| < \varepsilon$. The topology defined by this system of neighbourhoods is called the norm topology in $\mathfrak{B}(\mathcal{H})$.

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- Topology provided by the system of seminorms $|\,\mathrm{Tr}\,(A\omega)\,|$ is called the weak $*$-topology on $\mathfrak{B}(\mathcal{H})$ induced by the set of states ω.

C^*-algebra

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- A vector x belonging to the Hilbert space $\mathcal{H}$ on which acts a C^*-algebra $\mathfrak{U}(\mathcal{H})$ is called separating if $Ax = 0$ only if $A = 0$ for all $A \in \mathfrak{U}$.

GNS construction

Given a faithful separating state ω over an abstract C^*-algebra $\mathfrak{U}$, the Gelfand-Naimark-Segal construction provides a Hilbert space $\mathcal{H}$ with a preferred state $|\Psi_0\rangle$ and a representation π of $\mathfrak{U}$ as a concrete C^*-algebra of operators on $\mathcal{H}$, such that

$$\omega(A) = \langle \Psi_0 | \pi(A) | \Psi_0 \rangle.$$

Folium of the state

- Given a state ω on $\mathfrak{U}$ and the corresponding GNS representation of $\mathfrak{U}$ in $\mathcal{H}$, a folium determined by ω is a set of all states ρ over $\mathfrak{U}$ that can be represented as

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  where $\hat{\rho}$ is a positive trace-class operator in $\mathcal{H}$.
- Consider an abstract C^*-algebra $\mathfrak{A}$ and a preferred state ω. Via the GNS construction one obtains a representation of $\mathfrak{A}$ in a Hilbert space $\mathcal{H}$. The folium of ω then determines a weak topology on $\mathfrak{A}$. By closing $\mathfrak{A}$ under this weak topology we obtain a von Neumann algebra $\mathfrak{R}$.

Characterization theorem

- $P(\mathfrak{R})$ is the lattice of all self-adjoint, idempotent operators in a von Neumann factor $\mathfrak{R}$.

Classification of von Neumann factors

Table: Classification of von Neumann factors

Range of d	**Type of $\mathfrak{R}$**	**Lattice P$(\mathfrak{R})$**
$\{0, 1, 2, \ldots n\}$	I_n	modular, atomic, non-distributive if $n > 2$
$\{0, 1, 2, \ldots \infty\}$	I_∞	orthomodular, non-modular, atomic
$[0, 1]$	II_1	modular, non-atomic
$[0, \infty]$	II_∞	non-modular, non-atomic
$\{0, \infty\}$	III	non-modular, non-atomic

Characterization theorem

- $P(\mathfrak{R})$ is the lattice of all self-adjoint, idempotent operators in a von Neumann factor $\mathfrak{R}$.
- There exists a unique (up to multiplication by a constant) map $d : P(\mathfrak{R}) \mapsto [0, \infty]$ such that
  - (i) $d(A) = 0$ if and only if $A = 0$
  - (ii) If $A \perp B$, then $d(A + B) = d(A) + d(B)$
  - (iii) $d(A) \leq d(B)$ if and only if $A \preceq B$
  - (iv) $d(A) < \infty$ if and only if A is a finite projection
  - (v) $d(A) = d(B)$ if and only if $A \sim B$
  - (vi) $d(A) + d(B) = d(A \wedge B) + d(A \vee B)$

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  defines a 1-parameter group of automorphisms of $\mathfrak{R}$.
- This group is called the group of modular automorphisms, or the modular group, of the state ω over the algebra $\mathfrak{R}$.

Inner and outer automorphisms

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  for every A and some unitary U in $\mathfrak{R}$.
- Resulting classes of automorphisms are called outer automorphisms and they form $\mathrm{Out}\,\mathfrak{R}$.

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- Therefore, all states of the von Neumann algebra $\mathfrak{R}$, or of the folium of the C^*-algebra $\mathfrak{U}$ that has defined $\mathfrak{R}$, lead to the same 1-parameter group in $\mathrm{Out}\,\mathfrak{R}$.
- In other words $\tilde{\alpha}_t$ does not depend on the normal state ω. The von Neumann algebra possesses a canonical 1-parameter group of outer automorphisms.

Invariant set $\mathcal{T}$

- From the Cocycle Radon-Nikodym theorem follows the intertwining property

$$(D\omega_1 : D\omega_2)(t)\,(\alpha_t^{\omega_2}) = (\alpha_t^{\omega_1})\,(D\omega_1 : D\omega_2)(t),$$

where $(D\omega_1 : D\omega_2)(t)$ is the Radon-Nikodym cocycle.

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- Therefore the set of t-values

$$\mathcal{T} = \{t : \alpha_t^{\omega} \text{ is inner}\}$$

is a property of $\mathfrak{R}$ independent of the choice of ω.

$\mathcal{T}$ and spectrum of modular operators

- Notice that $0 \in \mathcal{T}$ and, if $t_1, t_2 \in \mathcal{T}$, then $t_1 \pm t_2 \in \mathcal{T}$. So $\mathcal{T}$ is a subgroup of $\mathbb{R}$.

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- Connes showed that $\mathcal{T}$ is related to the spectrum of the modular operators Δ_ω.
- Define the spectral invariant $S(\mathfrak{R}) = \bigcap_\omega \mathrm{Spect}\, \Delta_\omega$, where ω ranges over all normal states of $\mathfrak{R}$, and the set $\Gamma(\mathfrak{R}) = \{\lambda \in \mathbb{R} \,:\, e^{i\lambda t} = 1 \quad \forall\, t \in \mathcal{T}\}$.

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Alain Connes's classification

Type *III* von Neumann algebras are classified according to the value of $S(\mathfrak{R})$.

Table: Connes's classification of von Neumann factors

Range of $S(\mathfrak{R})$	**Type of factor $\mathfrak{R}$**
$\{1\}$	*I* and *II*
$\{0 \cup \lambda^n,\ n \in \mathbb{Z}\}$	$III_\lambda \quad (0 < \lambda < 1)$
$\mathbb{R}_+$	III_1
$\{0, 1\}$	III_0

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- Consider the set $\mathfrak{S}$ of states over $\mathfrak{U}$ and the complex linear span of $\mathfrak{S}$, denoted as Σ.
- The dual of Σ is a W^*-algebra $\mathfrak{R}$ which is closed in the weak topology induced by Σ, and $\mathfrak{U}$ is weakly dense in $\mathfrak{R}$.

How things are seen usually

- $\mathfrak{U}$ is the total algebra, i.e. an inductive limit of the algebras of a net of local algebras

$$\mathfrak{U} = \overline{\bigcup_{\mathcal{O}} \mathfrak{U}(\mathcal{O})},$$

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- $\mathfrak{S} \rightarrow \mathfrak{S}(\mathcal{O}), \quad \Sigma \rightarrow \Sigma(\mathcal{O}), \quad \mathfrak{R} \rightarrow \mathfrak{R}(\mathcal{O}).$

Lessons from Haag

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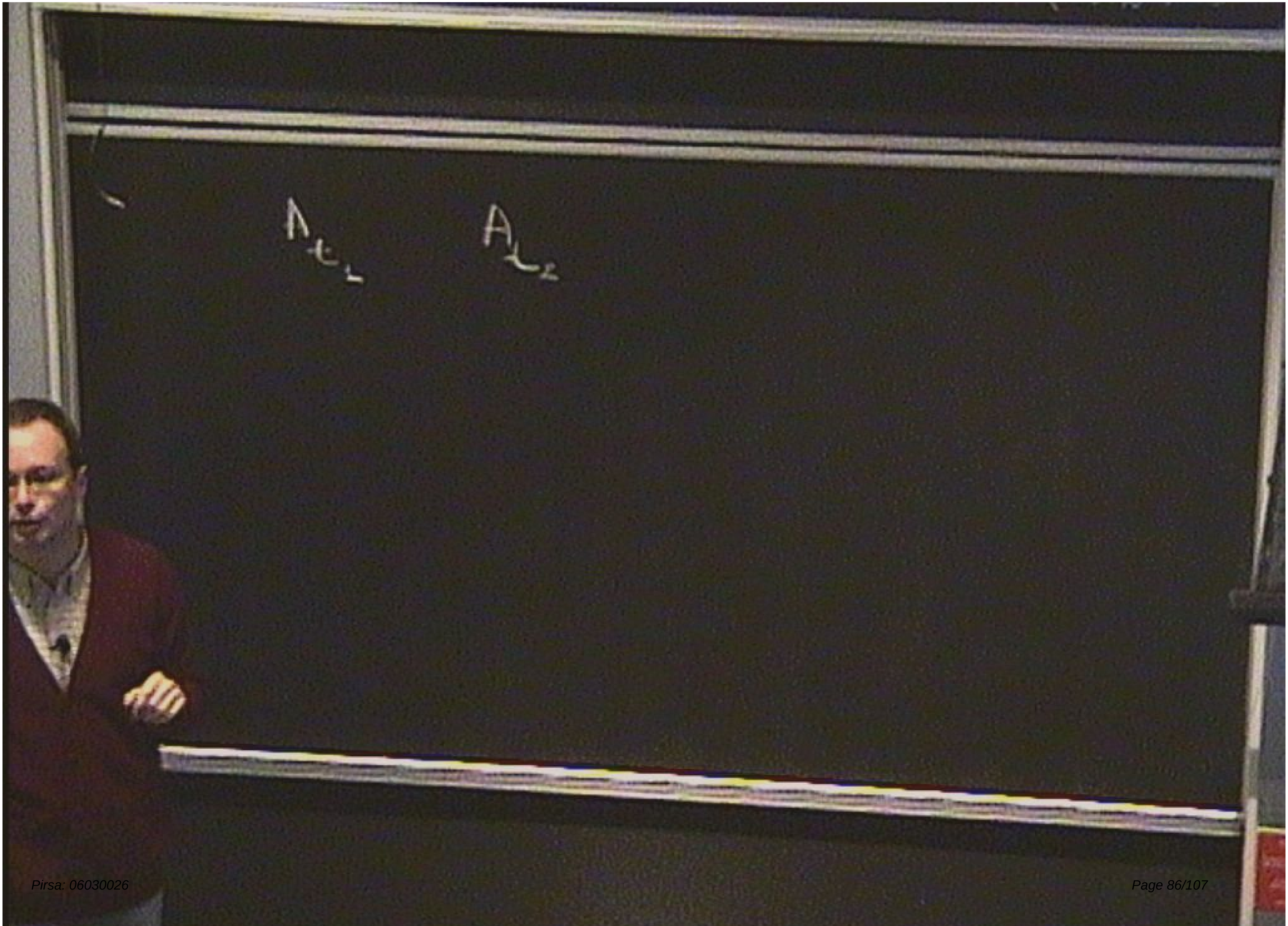
- "If we look at the efforts to relate the Hilbert space formalism of quantum mechanics to natural operational principles it appears that one central assumption is that any observable can be measured in any state."
- "Since, clearly, we cannot make a measurement before the state is prepared, this assumption is reasonable only if we distinguish between observables and observation procedures and if we can choose for each observable an observation procedure at an arbitrary time."



Need to single out an observable

- We are looking for an identification procedure that will allow one to single out an observable $\widetilde{A}$ such that it can be instantiated at different time moments:

$$\widetilde{A}_{t_1}, \widetilde{A}_{t_2}, \widetilde{A}_{t_3}, \ldots$$

More lessons from Haag

- "This means that one needs a dynamical law which identifies procedures at different times as the same observable, and this law should not depend on the state."
- "General relativity indicates that the dynamical law must involve the state to some extent."
- "The relevance of this within the context of special relativistic quantum physics is open, but we may take it as an indication that the dynamical law cannot be regarded as an algebraic relation in $\mathfrak{A}$ but arises on the level of von Neumann algebra $\mathfrak{R}$ and therefore needs at least the weak topology induced by states $\mathfrak{S}$."

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- Instances of $\widetilde{A}$ are observation procedures at particular time moments and belong to the local $\mathfrak{R}(\mathcal{O})$.
- So we need to forget the individual differences between $\widetilde{A}_{t_i}$!

Connes-Rovelli thermodynamic time hypothesis

- In nature, there is no preferred physical time variable t. There are no equilibrium states preferred *a priori*. Rather, all variables are equivalent; we can find the system in an arbitrary state. If the system is in state ω, then a preferred variable is singled out by the state of the system. This variable is what we call time.

KMS condition

- Consider a system with a finite number of the degrees of freedom. In thermodynamics, one uses the Gibbs condition

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- For a system with infinitely many degrees of freedom, call a state ω over $\mathfrak{U}$ a KMS state at inverse temperature β, with respect to γ_t, if, for all $A, B \in \mathfrak{U}$

$$f(t) = \omega(B(\gamma_t A))$$

is analytic in the strip $0 < \mathrm{Im}\, t < \beta$ and

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- KMS condition generalizes both the Gibbs condition and the Wick rotation.

KMS and the Tomita-Takesaki theorem

This is arguably one of the most important and profound theorems in all 20th century physics!

- Any faithful state over a C^*-algebra is a KMS state at the inverse temperature $\beta = 1$ with respect to the modular automorphism γ_t that it itself generates.

State-dependent time

- Using KMS formalism via Tomita-Takesaki theorem, define time as modular flow. It is state-dependent. Unless the state is changed, time does not change. A change in the state means a change in information. If the change of state takes one out the folium of the previous state, then state-dependent time "restarts."

Correct time spectrum

- If the algebra is a type III_1 factor, the spectrum of t is from 0 to $+\infty$. Internal, state-dependent time behaves "correctly": it is a real positive one-dimensional parameter.

State-independent time is ignorance

- Factorization by inner automorphisms leads to the state-independent notion of time. This factorization corresponds to neglecting the difference between states. It is also a condition of possibility of the notion of observable. Thus, the concept of time arises due to the possibility to ignore certain information.

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Outline

1. Where to start?
   - Information-theoretic approach
   - Measurement order is irrelevant
   - Phenomenal time
   - Mathematical idealization and physical interpretation
2. Algebraic formalism
   - Topologies and algebras
   - States
   - Types of algebras
   - Modular automorphisms

Where to start? Try phenomenal time

- Phenomenal time is not sharply defined. It comes to us strangely by way of words with fuzzy meanings, and with a non-linear structure.
- Compare with Husserl, *Ideen* I.
- Compare with Weyl as rephrased by Ryckman:

> *Exact time points are not given in intuition in any manner, and so are not absolute, but are concepts (products of reason), attaining full definiteness only in the purely formal arithmetico-analytic concept of the real number.*

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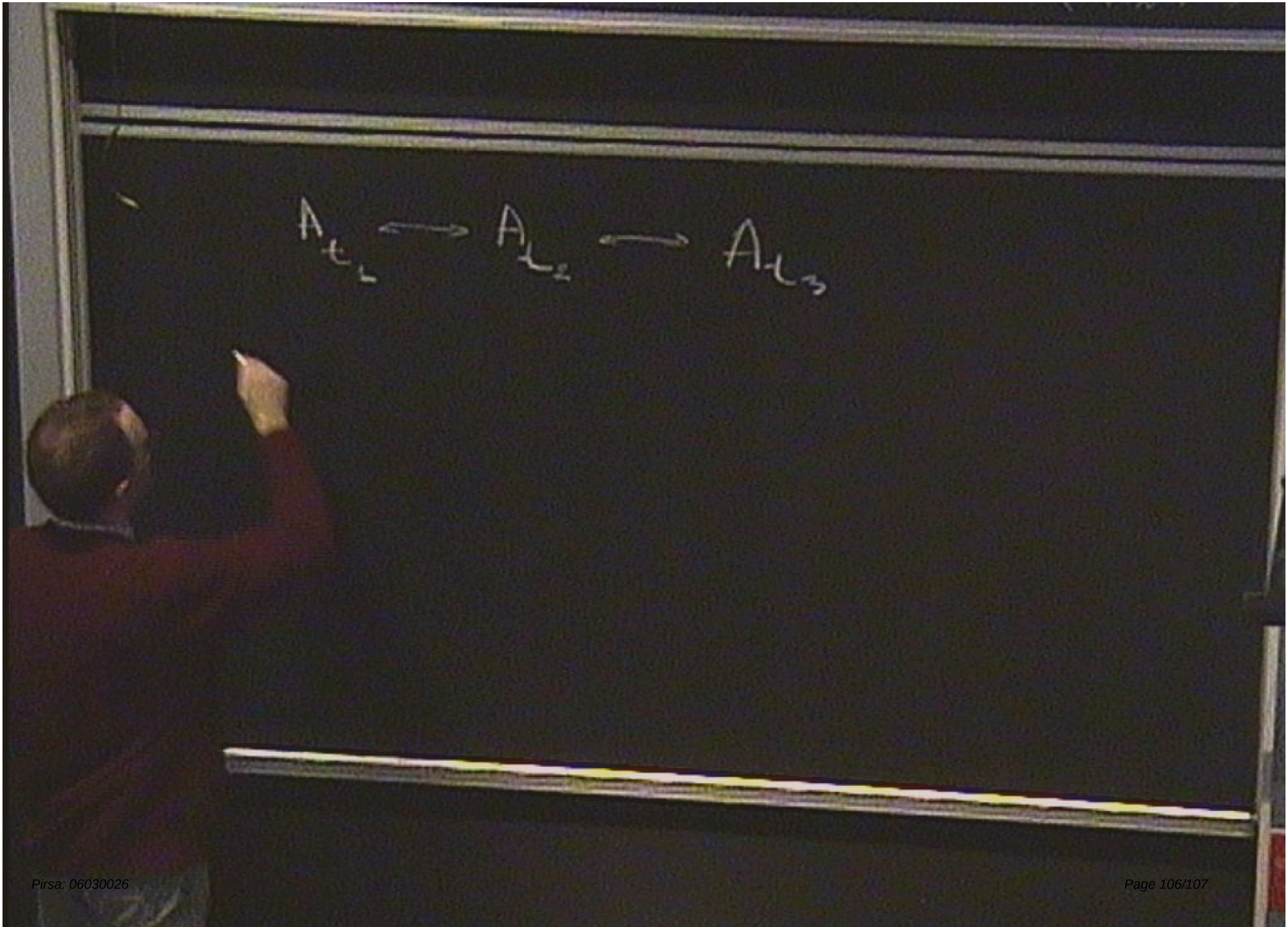
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