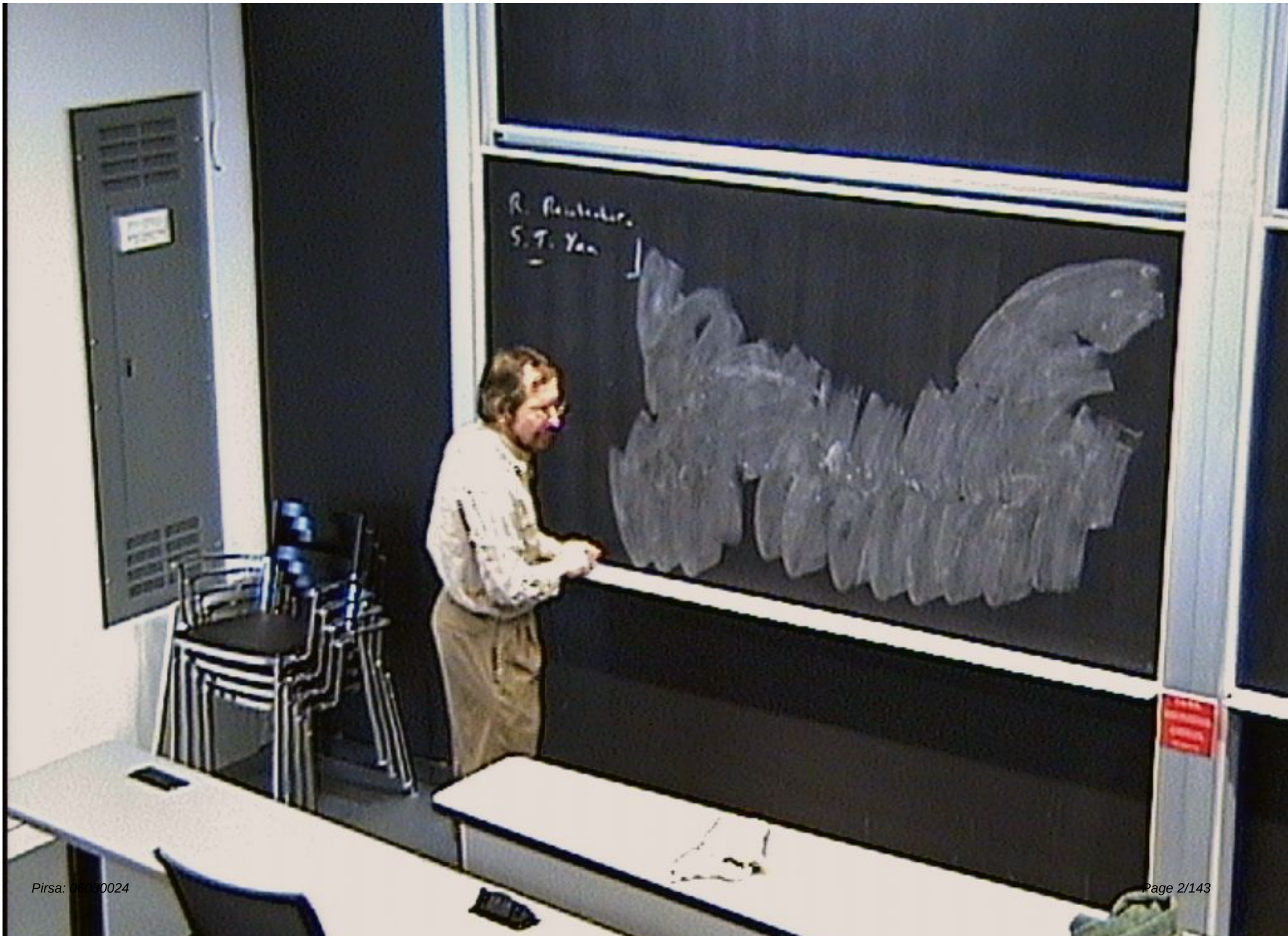


Title: Branes, Bundles and Attractors

Date: Mar 28, 2006 02:00 PM

URL: <http://pirsa.org/06030024>

Abstract:

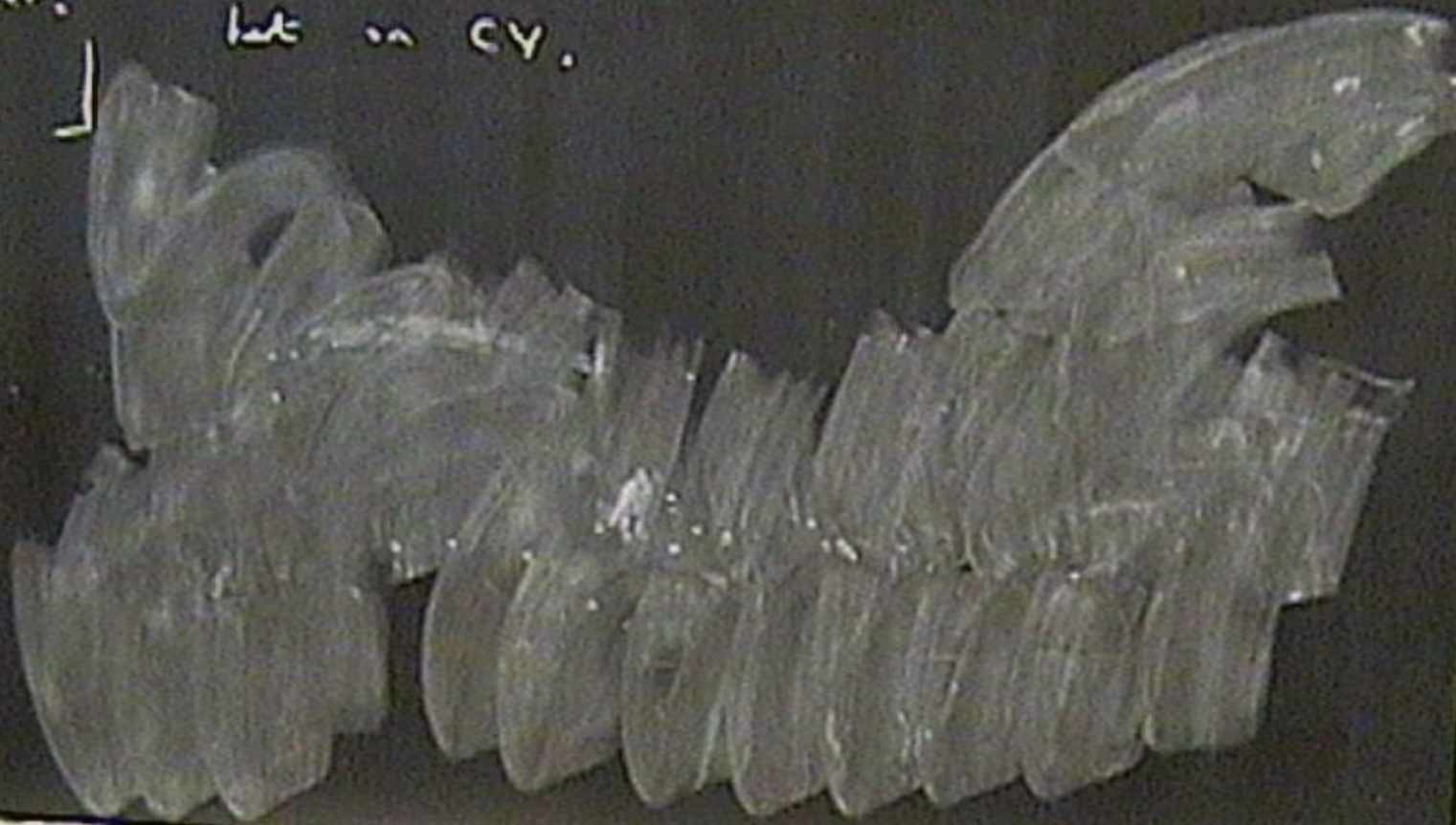


R. Reinbacher.
S.T. Yau
—



P. Reinbacher.
S.T. Yan

late on CY.



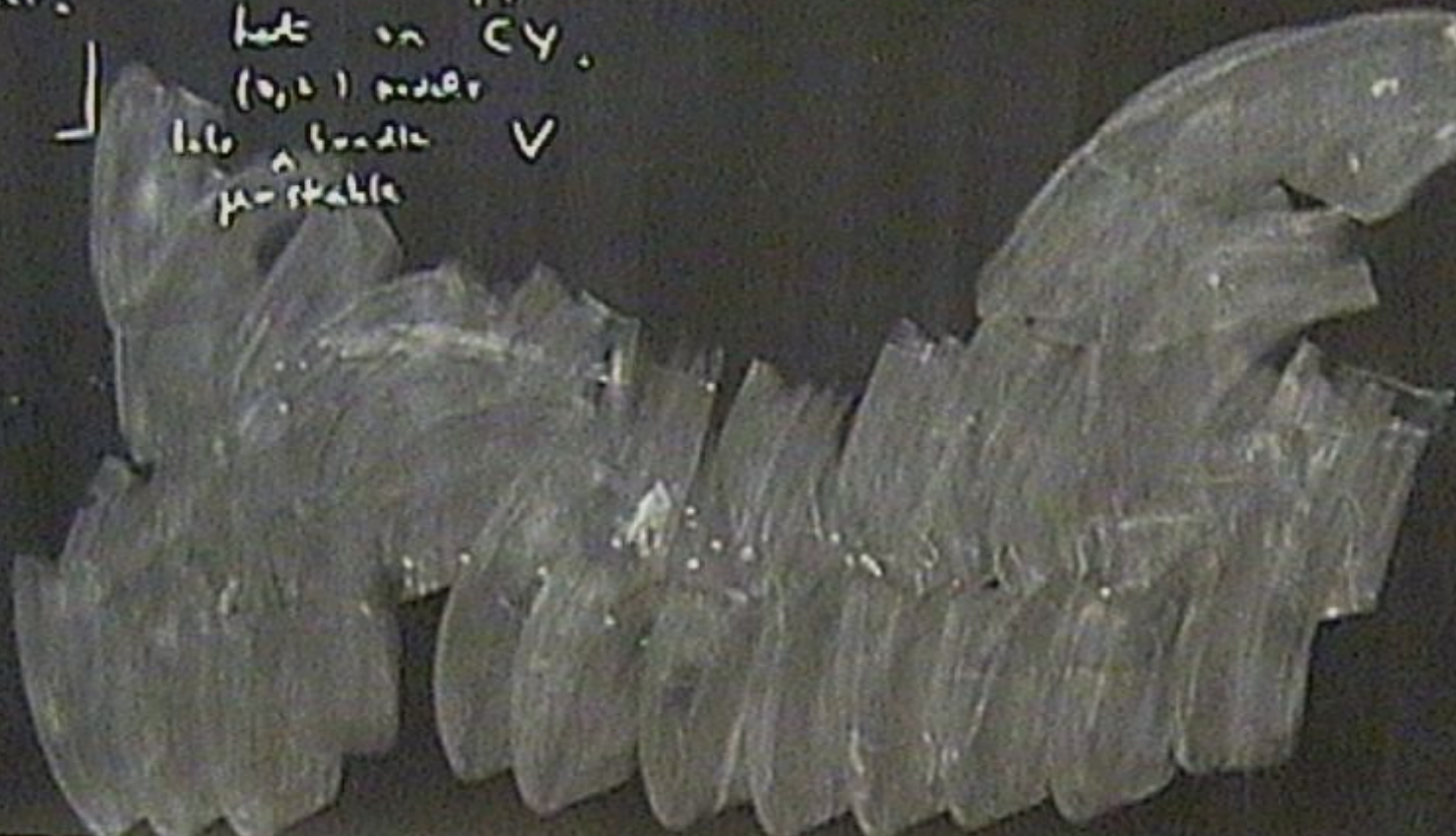
R. Reinbacher.
S.T. Yan
—

lets on M
CY.
(9, 10) under
lets bundle V



R. Reinbacher.
S. T. Yan
—

lets on M
CY.
(0,1) model
lets a bundle V
μ-stable



R. Reinbacher,

S.T. Yau

Let on M
CY.

(0,2) moduli

Let $\wedge$ bundle V
 μ -stable

Let M be a
Yang-Mills
 $F = 2, 0 = 0$
 $F^1, 1_2$

R. Reinbacher.

S.T. Yan

lets on M
(0,1) models
lets a bundle V
mu-stable

harmonic
map $\phi: M \rightarrow N$
 $\phi^* \omega = 0$
 $\phi^* \omega = \omega$

R. Reinbacher.

S.T. Yau

lets on M
CY.
(0,1) moduli
lets a bundle V
 μ -stable

$$\Delta H = \text{tr } F^2 - \text{tr } R^2$$

(v)

hermitian
Yang-Mills
 $F^2 = 0$
 $F^2 = 0 \Rightarrow \omega$

R. Reinbacher,

S.T. Yau

lets on M
CY.

(0,1) moduli

lets a bundle V
mu-stable

harmonic
map - Hilb

$$F^2 \cdot \partial = 0$$

$$F^1 \cdot \partial = 0$$

$$\alpha H = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TM)$$

R. Reinbacher.

S.T. Yau

lets on M
CY.

(0,1) mod 2

lets a bundle V

mu-stable

$$\det F^1 = \det F^2 = \det R^2$$

$$c_2(V) = c_2(TM)$$

harmonic
map $M \rightarrow N$

$$F^1 \oplus F^2 = \mathcal{O}$$
$$F^1 \oplus F^2 \subset \mathcal{O}$$

cut

rank V
2, 4, 5

R. Reinbacher.

S.T. Yau

lets on M
CY.
(0,1) moduli
lets a bundle V
 μ -stable

$$\det H = \text{tr } F^1 - \text{tr } R^2$$

$$c_2(V) = c_2(TM)$$

harmonic
" Yang-Mills
 $F^2 = 0 = 0$
 $F^1, F^2 \in W$

cut
rank V
2, 3, 4, 5
 $c_1(V) = 0$

R. Reinbacher.
S.T. Yau

lets on M
CY.
(0,1) mod 2
lets $\wedge$ bundle V
 μ -stable

$$\alpha(t) = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = \frac{1}{4\pi^2} (\text{tr } F^2 - \text{tr } R^2)$$

harmonic
map $\phi: M \rightarrow N$

$$F = \frac{1}{2} \phi^* \omega \in \Omega^2(M)$$

cut

rank V
3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda I)$$

R. Reinbacher,
S.T. Yau

Let M be a
CY.
(1,1) moduli
Let V be a bundle
on M .

Infinitesimal
deformation
of M is $H^1(M, T_M)$

$$F^1 H^1(M, T_M) = 0$$

Let

$$c_1(V) = 0$$

$$\alpha(V) = \int_M F^1 \cdot c_2(V)$$

$$c_2(V) = c_2(TM)$$

$$\frac{1}{2} c_2(V) = \frac{1}{2} c_2(TM)$$

$$\sum \lambda^k c_{2k}(V) \dim H^k(M, \mathbb{C})$$

$$N_{1,1} = N_{1,1}$$

R. Reinbacher.

S.T. Yau

lets in M
CY.

$(0,1)$ moduli

lets a bundle V
projectable

$$\det F^1 = \det F^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{gen}$$

harmonic
in Yang-Mills

$$F^1 = 0 \in W$$

for

rank $V = r$

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) \det(F^1)$$

$$N_{20} = N_n$$

R. Reinbacher.
S.T. Yau

lets on M
CY.
(0,1) mod 2
lets a bundle V
 μ -stable

harmonic
map $\phi: M \rightarrow N$

$$F = \phi^* \omega = 0$$

out
rank V
2, 3, 4, 5
 $c_1(V) = 0$

$$\alpha(F) = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) = c_2(TM)$$

$$\frac{1}{2} c_3(V) = 2\pi \chi(M)$$

$$\sum \lambda^k c_{2k}(V) \det(F - \lambda I)$$

$$N_{2,0} = N_{0,2}$$

R. Reinbacher.
S.T. Yau

Let M be a CY.
Let $(\mathcal{E}, \mathcal{F})$ be a bundle V
Let μ -stable on

Let $\mathcal{F}$ be a bundle
Let $\mathcal{F} = \mathcal{F}_1 \oplus \mathcal{F}_2$
Let $\mathcal{F}_1 \oplus \mathcal{F}_2 \in W$

Let $\mathcal{F}$ be a bundle
Let $\mathcal{F} = \mathcal{F}_1 \oplus \mathcal{F}_2$
Let $\mathcal{F}_1 \oplus \mathcal{F}_2 \in W$

$$\det F = \det E$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{gen}$$

$$N_{gen} = N_{21} - N_{20}$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

Let $\mathcal{F}$ be a bundle
Let $\mathcal{F} = \mathcal{F}_1 \oplus \mathcal{F}_2$
Let $\mathcal{F}_1 \oplus \mathcal{F}_2 \in W$

R. Reinbacher.

S.T. Yau

wrapping D_6

on CY

lets on M
CY.

(2,1) mod 2

lets bundle V

μ -stable $\Rightarrow$

$$\alpha H = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = \chi(N_{\text{gen}})$$

given $ch, c, c_2, \dots$

harmonic
map $\Rightarrow$ Hilb

$$F^2 = 0$$

$$F^2 = 0 \in W$$

cut

rank V
2, 3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

$$N_{2,1} = N_{2,1}$$

Pick up $c_1, c_2, \dots$ Manuana.

R. Reinbacher.

S.T. Yau

wrapping D_6
on CY
IIa.

lens in M
 CY .

$(0,1)$ mod 2

lens bundle V

mu-stable $\Rightarrow$

$$dH = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

gives c_1, c_2, c_3 in $[0, \infty)$

hermitian
" Yang-Mills

$$F = 0 \Rightarrow 0$$

$$F^2 \in W$$

Got

rank V
2, 3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

$$N_{20} = N_{21}$$

Pick up CS in Maruyama.

R. Reinbacher.

S.T. Yau

Mark
wrapping De
on CY

lets on M
CY.

(0,1) mod 2
lets a bundle V
mu-stable on

harmonic
map $\phi: M \rightarrow U$
 $F = \phi^* \omega = 0$
 $F^2 = 0 \in W$

cut
rank V
3, 4, 5
 $c_1(V) = 0$

$$\det F = \det R$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{gen}$$

given $ch, c_1, c_2 \in \mathbb{Z}$

$$\sum \lambda^k c_{2k}(V) \det(F - \lambda)$$

$$N_{2,1} = N_{2,0}$$

Pick ch in CS^* Manojana.

R. Reinbacher,
S.T. Yau

mark
wrapping D6
on CY

IIa.

$$\bigoplus_{V \in \mathcal{V}} \mathbb{Z} \langle \gamma_V \rangle$$

BPS particle

hole in M
CY.

$(0,1)$ mod 2

hole in bundle V
mu-stable

$$\det F = \det R$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = \chi(V)$$

given c_1, c_2, c_3

harmonic
map

$$F^2 = 0 \in \omega$$

Got

rank V
 $r_3, 4, 5$

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) \det(F - \lambda)$$

$$N_{20} = N_{21}$$

Pick up γ_V in $\mathcal{V}$ Maruyama.

R. Reinbacher,
S.T. Yau

mark
wrapping D6
on CY
IIa.

$\Rightarrow$ via
 V

$\rightarrow$ trace
 $\rightarrow$ electromagnetic
in d=4

lets in M
CY.

(0,1) mod 2

lets a bundle V
 μ -stable $\Rightarrow$

$$\det H = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

given ch. c. c. c. 27. 12. Pick up in CY's.

harmonic
" Yang-Mills

$$F^2 = 0 \quad F^2 = 0 \quad F^2 = 0$$

cut

rank V
2, 3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) \det(F - \lambda)$$

$$N_{20} = N_{21}$$

Manifolds.

R. Reinbacher.

S.T. Yau

mark
wrapping De
on CY
IIa

lets on M
CY.

(0,1) mod 2

lets a bundle V
mu-stable $\Rightarrow$

$$\alpha H = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2\pi \chi_{\text{gen}}$$

$$\text{or } c_1, c_2, c_3 \Rightarrow \chi$$

harmonic
map - Higgs

$$F = 0, \quad \omega = 0$$

$$F^2 = 0 \quad \in \omega$$

cut

rank V
2, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

$$N_{2,0} = N_{0,2}$$

Manu...

R. Reinbacher,
S.T. Yau

Mark
wrapping D6
on CY
IIa.

$$\bigoplus_{V \in \text{BPS}} \text{BPS particle}$$

BPS particle

$$\text{ch } V = \text{Tr } e^F$$

→ electromagnetic charge
of dir.

lets on M
CY.

lets bundle V
μ-stable on

$$\det F = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

given ch. c_1, c_2, c_3

harmonic
map

$$F^2 = 0$$

$$F^2 = 0 \in \omega$$

Got

rank V
3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) \det(F - \lambda)$$

$$N_{20} = N_{21}$$

Pick up a CY. Maruyama.

R. Reinbacher

S.T. Yau

mark
wrapping D_6
on CY

IIa.

$\bigoplus_{V \in \text{BPS}} \text{particle}$

BPS particle

$\text{ch } V = \text{Tr } e^F$

$\rightarrow$ electromagnetic charge
2 div

C_{2k} bundle with $\mathcal{D}(C_{2k})$

bundle on M
 CY .

(0,1) fields

bundle V

mu-stable $\Rightarrow$

$$\det H = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

given $ch(V), c_1, c_2, c_3$

harmonic
map $Hilb$

$$F^2 = 0 \Rightarrow F^2 = 0$$

C_0

rank V
2, 3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

$$N_{2,0} = N_{0,2}$$

Pick up $\mathcal{D}(C_{2k})$ Maruyama.

R. Reinbacher

S.T. Yau

make wrapping Do on CY

IIa

⊕

ch

lets on M
CY.

(u, v) mod 2

lets on bundle V
parallel on

$$\Delta H = \text{tr } F^1 - \text{tr } F^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = \chi(N)_{\text{gen}}$$

given c_1, c_2, c_3 in $\mathbb{Z}$

harmonic map
to Yang-Mills

$$F^1 = 0, F^2 = 0$$

get

rank V
is 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) \det(F_0)$$

$$N_{21} - N_{11}$$

Maruyama

A. Langer

P. Reinbacher,

S.T. Yau

no ch
wrapping De
on CY

IIa

$\bigoplus_{V \in \text{graph}}$

BFS particle

$\text{ch } V = \text{Tr } e^F$

$\rightarrow$ eigenvalues of F

C_{2k} bundle with $\mathcal{D}(C_{2k})$

lets on M
CY.

(0,1) mod 2

lets a bundle V
 μ -stable on

$$\text{ch } F = \text{tr } F^1 = \text{tr } R^1$$

$$C_2(V) = C_2(TN)$$

$$\frac{1}{2} C_3(V) = 2N_{\text{gen}}$$

given $\text{rk}, c_1, c_2, \dots$

harmonic
map $\rightarrow$ Higgs

$$F^1 = 0 = 0$$

cut

rank V
 $2, 3, 4, 5$

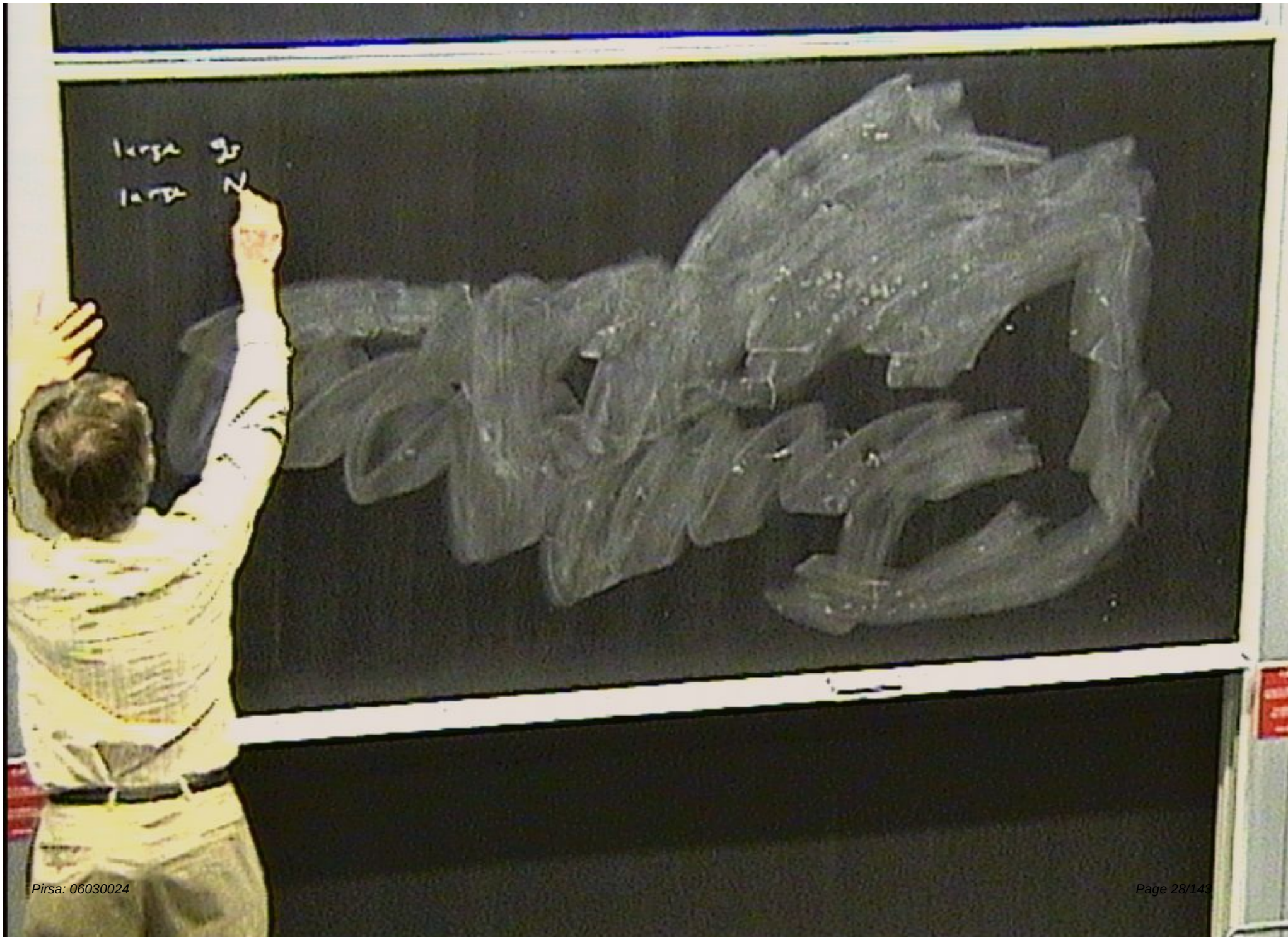
$$C_1(V) = 0$$

$$\sum \lambda^k C_{2k}(V) = \text{det}(F - \lambda)$$

$$N_{2,1} = N_{1,2}$$

Maruyama

A. Langer



larga 95
larga N
M = 95N



R. Reinbacher,
S.T. Yau

Let M on CV .
(0,1) mod 1
Let A bundle V
portable on

harmonic
in Yang-Mills

cot

rank V
 r_1, r_2, r_3, r_4, r_5
 $c_1(V) = 0$

vanishing of
 c_1
 Π_a

$$\bigoplus_{k=1}^n V_k$$

or 5 points

$$\text{ch } V = \text{Tr } cF$$

→ electric, magnetic charge
of div

c_2 bundle with csc $D(\text{csc})$

$$\det = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_2(V) = 2N_{\text{gen}}$$

given $ch, c_1, c_2, \dots$

$$F^2 = 0, \quad F^2 = 0, \quad F^2 = 0$$

$$\sum \lambda^k c_{2k}(V) / \Delta(F-2)$$

$$N_{2,3} = N_n$$

Dickson's c_2 in Macaulay's
A. Langer

large 32 $\Rightarrow$ black holes
large N in the
spectrum.
M = 9.5N



large $g_s \Rightarrow$ black holes
large N in Π e
supra.

$$M \sim g_s N$$

10d Π a sr

$\downarrow$ CY M

4d

wrapping D_6

on S^1

II_a



BPS particle

$$hV = Tr eF$$

→ electromagnetic charges
in d=4

C_k bound state with $D(6-2k)$

$$dH =$$

gives rk

large $g_s \Rightarrow$ black holes
 large N in IIa
 subtra.

$$M \sim g_s N$$

10d IIa so

$\downarrow$ CY M

4d $\int C^{(3)}$
 $C^{(1)}$

large $g_s \Rightarrow$ black holes
 large N in IIa
 supersym.

$$M \sim g_s N$$

low IIa so

$\downarrow$ CY M
 4d $\int C^{(3)}$ $\Rightarrow$ vector
 $\sum C^{(1)}$ $\mu \cdot 4$

large $g_s \Rightarrow$ black holes
large N in AdS
supergravity.

$$M \sim 95 N$$

102 IIa 55

$\downarrow$ CY M
 $\Rightarrow$ vector p.u.

R. Reinbacher.
S.T. Yau

lets on M
CY.
(0,1) mod 2
lets a bundle V
 μ -stable $\Rightarrow$

harmonic
in Yang-Mills

$$F^2 = 0 \Rightarrow \mathcal{O}$$

$$F^2 = 0 \Rightarrow \mathcal{O}$$

rank V
1, 2, 3, 4, 5
 $c_1(V) = 0$

wrapping D_6
on CY

II_a

$$\oplus_{V \in \text{rank } 1}$$

BPS particle

$$\text{ch } V = \text{Tr } e^F = Q$$

$\rightarrow$ electric, magnetic charge
in div

Set bundle with $D(6-2k)$

$$\det H = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

given (c_1, c_2, c_3)

$$\sum \lambda^k c_{2k}(V) = \Delta(F-2)$$

$$N_{\text{gen}} = N_{\text{gen}}$$

Manuscript
A. Lan

large 3d $\Rightarrow$ black holes
large N
M = gN

10d Σ 10

$\downarrow$ CY

4d

CY

$\Rightarrow$ vector
p.d.

large $2x \Rightarrow$ black holes
 large N in QCD
 $M = gN$ supra.

10d Σ in 10

$\downarrow$ CY M
 4d $\int C^{(1)} \Rightarrow$ 4d
 $\int C^{(2)}$ vector
 $\int C^{(3)}$ p.d.

large $2d \Rightarrow$ black holes
 large N in AdS
 $M = g N^2$

attractor
 mechanism. G. Horowitz

low AdS is

$\downarrow$ CY M
 $4d \int C^{(4)} \Rightarrow$ vector
 $C^{(2)}$ $C^{(1)}$ $C^{(0)}$ $g_{\mu\nu}$

large 25 $\Rightarrow$ black holes
 large N
 M = 9N

attractor
 Mechanism. E. M. ...

100 100 10

$\downarrow$ CY M
 40 $\left\{ \begin{array}{l} C \\ C \end{array} \right.$ $\Rightarrow$ vector
 10 $\left\{ \begin{array}{l} C \\ C \end{array} \right.$ $\Rightarrow$ vector

large g $\Rightarrow$ black holes
 large N in field
 $M = gN$ supra.

FKS
 attractor
 mechanism. G. Moore

10d $\Rightarrow$ 11

$\downarrow$ CY M
 4d $\left\{ \begin{array}{l} C \\ \epsilon \end{array} \right. \Rightarrow$ vector
 $\left. \begin{array}{l} C \\ C \end{array} \right\}$ p.d.

metric $\Rightarrow$ scalar
 field χ
 KK theory $\Rightarrow$ in metric
 field

large g $\Rightarrow$ black holes
 large N in field
 $M = g^2 N$ supra.

attractor
 mechanism. G. Moore

10d Σ 11

$\downarrow$ CY M
 4d $\int C^{(1)} \Rightarrow$ vector
 $\int C^{(2)}$ p.c.

metric moduli $\Rightarrow$ scalar fields

Kahler moduli $\Rightarrow$ in metric moduli

- Scalars at 0
- charge Q

$$\mathcal{L}_5 = e^{-A} \text{vol}_{10} \text{tr} F^2$$

large g $\Rightarrow$ black holes
 large N in field
 $M \sim g^2 N$ $\Rightarrow$ surface.

attractor
 mechanism. G. Horowitz

low Σ is

$\downarrow$ CY M
 $\Rightarrow$ $\int C^{(p)}$ $\Rightarrow$ vector
 $\in C^{(p)}$ $\Rightarrow$ p -form

metric $\Rightarrow$ scalar
 fields $\Rightarrow$ in metric
 fields $\Rightarrow$ in metric

- scalar $\Rightarrow$ Q

$$Q^2 = e^{-A} \text{vol} \cdot \text{vol}^2 \Rightarrow \text{area of horizon } A_H$$

large g $\Rightarrow$ black holes
 large N
 $M = gN$

attractor
 mechanism. G. Horowitz

$$S = \frac{A}{4}$$

low Σ in

$\downarrow$ CY M
 $\int C^{(p)}$ $\Rightarrow$ vector
 $\int C^{(q)}$ p.d.

metric $\Rightarrow$ scalar
 field ϕ

Kahler
 moduli $\Rightarrow$ in moduli

- Scalars ϕ^i
- charge Q

$$\mathcal{L}_5 = e^{-A} \text{vol} \cdot \text{tr} \dot{\phi}^i \dot{\phi}^i$$

$\Rightarrow$ area of horizon A_H (4m p1)

large g $\Rightarrow$ black holes
 large N in field
 $M \sim g^2 N$ supra.

attractor
 mechanism. G. Moore

$$S = \frac{A}{4}$$

low Σ in

$\downarrow$ CY M
 $\int C^{(1)} \Rightarrow$ vector
 $\int C^{(2)} \Rightarrow$ p.f.

metric $\Rightarrow$ scalar
 field χ

Kahler $\Rightarrow$ in metric
 moduli

- Scalars at U
- charge Q

$$\mathcal{L}_S = e^{-A_{\text{eff}}(x)} \mathcal{L}_S$$

$\Rightarrow$ area of horizon A_{horizon}

large g $\Rightarrow$ black holes
 large N
 $M \propto g^2 N$

10d Σ in 10

$\downarrow$ CY M
 4d $\int C^{(p)}$ $\Rightarrow$ vector
 $\int C^{(1)}$ p.c.

metric
 moduli
 Kähler
 moduli
 color
 fields
 in
 moduli
 space

attractor
 mechanism. G. Horowitz

$$S = \frac{A}{4}$$

$\frac{1}{4\pi G_5}$ black holes
 $\frac{2}{\partial r} \omega(r)$

$$\mathcal{Z}_5 = e^{-A_{\text{eff}}(r)} \mathcal{Z}_4$$

$\Rightarrow$ area of horizon $A(4m, q)$

large g $\Rightarrow$ black holes
 large N
 $M = gN$

low g $\Rightarrow$

$\downarrow$ CY M
 $\int C^{(1)} \Rightarrow$ vector
 $\int C^{(2)} \Rightarrow$ vector

metric $\Rightarrow$

Kähler
 moduli $\Rightarrow$

= scalar

= chiral

attractor
 mechanism. G. Horowitz

$$S = \frac{A}{4}$$

$$\frac{2}{\partial r} \omega(r) = -g \int \omega(r) \frac{1}{\partial r} \omega(r)$$

$$e^{-A} \omega(r) \frac{1}{\partial r} \omega(r)$$

large $2x \Rightarrow$ black holes
large N in slide
 $M \sim 9 \times N$ surface.

102 11

↓ C Y M
4.5 C 3.5 M → vector
C C

Meaning: 7 - 5 - 10

Kellie in 1964

Sentado at 60
a large Q

$$Q_s^2 = e^{-A_{coll} \cdot \ln(1 + \frac{Q^2}{Q_0^2})}$$

$\Rightarrow$ area of hadron (ASym 4)

attractor mechanism. G. Moore

5-15

٥٨٢

$$\frac{\partial}{\partial r} \psi(r) = -9 \sqrt{\frac{2}{a^3}}$$

$$f = (q, y)$$

$$= - \int_{\mathbb{R}^n} \frac{1}{Q} \Delta u \cdot e^{u^2} dx$$

large $N \Rightarrow$ black holes
 large N
 $M = g N$

attractor mechanism, G. Horowitz

$$S = A$$

10d theory
 $\downarrow$ CY M
 $\int C^{(1)} \rightarrow$ vector field
 $\int C^{(2)} \rightarrow$ scalar field

10d theory

$$\frac{d}{dr} \psi(r) = -g \psi(r) \frac{d}{dr} r$$

$$f = |\tilde{\chi}(\psi, Q)|$$

$$= - \int_{\mathcal{Q}} \text{tr} \mathcal{F} \wedge e^{B+2\sigma}$$

moduli $\rightarrow$ scalar fields
 Kähler moduli $\rightarrow$ in moduli space
 b.l. (BPS states)
 - scalar fields
 - charge Q

$$\mathcal{L}_S = e^{-A} \text{cdm} \cdot \text{rel} \cdot \dot{\psi}^2$$

$\Rightarrow$ new of black holes

large g $\Rightarrow$ black holes
 large N in AdS
 $M \sim g^2 N$

10d Σ in "

$\downarrow$ CY M
 $\Rightarrow$ vector
 $\{C^{(1)}, \dots, C^{(n)}\}$

metric $\Rightarrow$ scalar
 $g_{\mu\nu}$

Kaluza
 6d (Bosonic) in 10d

- scalars ϕ^i
- charge Q

$$Q^2 = e^{-A} \text{Vol}(S^1) \dots$$

$\Rightarrow$ mass of black hole

attractor
 mechanism, S. Murugan

$$S = \frac{A}{4}$$

DFS black holes

$$\frac{\partial}{\partial r} \omega^i(r) = -g^{ij}(u) \frac{\partial}{\partial u^j} F$$

$$F = |Z(\omega, Q)| \quad |1 - \frac{1}{2} \epsilon(r)|$$

$$= \left| \int_{\Sigma} e^{i\theta} \wedge e^{A + iJ} \sqrt{\det g} \right|^2$$

metric g

large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

low Σ in

$\downarrow$ CY M
 $\int C^{(1)} \Rightarrow$ vector
 $\int C^{(2)} \Rightarrow$ scalar

metric $\Rightarrow$ scalar field ϕ

Kaluza-Klein
 moduli $\Rightarrow$ in metric
 b.i. $(B + \sqrt{2} H^2 \wedge H^2)$

- scalars ϕ
- charge Q

$$\mathcal{L}_5^2 = e^{-A} \text{vol}_5 \wedge \frac{1}{2} \dot{\phi}^2$$

$\Rightarrow$ area of horizon $A(\phi_m, Q)$

attractor
 mechanism, G. Horowitz

$$S = \frac{A}{4} \sim \frac{A(\phi_{\text{attractor}}, Q)}{4}$$

DPS, black holes

$$\frac{1}{4\pi} \frac{\partial}{\partial r} \omega^i(r) = -g^{ij} \frac{\partial}{\partial u^j} F$$

$$F(u, r) = |\vec{z}(u, Q)| \cdot |1 - \dot{h}(u, r)|$$

$$= \left| \int_{\Sigma} e^{\beta} \wedge e^{\beta} \right| \sqrt{TdM}$$

vector $\int \Sigma$

large g $\Rightarrow$ black holes
 large N in field
 $M \sim g^2 N$ supra.

10d Σ in 10

$\downarrow$ CY M
 $\int_C C^{\text{odd}} \Rightarrow$ vector
 $\int_C C^{\text{even}}$ p.c.

metric $\Rightarrow$ scalar
 field χ

Kaluza
 moduli $\Rightarrow$ in moduli
 space (B + iF) $\sim$ $H^2(M, \mathbb{R})$

- scalars of $\mathcal{N}=2$
- charge Q

$$\mathcal{L}_5^2 = e^{-A} \text{cdm} \cdot \text{rt} \cdot \frac{1}{2} \dot{\chi}^2$$

$\Rightarrow$ area of horizon $A(\chi_m, Q)$

attractor
 mechanism. G. Horowitz

PKS

$$S = \frac{A}{4} \sim \frac{A(\chi_{\text{attractor}}, Q)}{4} \sim \frac{1}{4} |\vec{z}|^2$$

DPS black holes

$$\frac{1}{2} \frac{\partial}{\partial r} \omega^i(r) = -g^{\frac{1}{2}}(\omega) \frac{\partial}{\partial \omega^i} F$$

$$F(\omega, Q) = \left| \vec{z}(\omega, Q) \right| \cdot \left| \vec{H}(\omega, Q) \right|$$

$$= \left| \int_{\Sigma} e^{\frac{1}{2} A} \wedge e^{\frac{1}{2} i F} \sqrt{\det g} \right|^2$$

metric g



large g $\Rightarrow$ black holes
 large N in AdS
 $M \sim g^2 N$

10d Σ in 11

$\downarrow$ CY M
 $\mathcal{H} \subset C^{1,1} \Rightarrow$ vector
 $\mathcal{H} \subset C^{1,1}$ p.c.

metric $\Rightarrow$ scalar
 field ϕ

Kaluza
 model $\Rightarrow$ in 11d
 $(B + c_1 H^1 \wedge \phi)$

- scalars ϕ
- gauge Q

$$\mathcal{L}_5^2 = e^{-A} \text{vol}_5 \wedge \left(\frac{1}{2} F^2 + \frac{1}{2} d\phi^2 \right)$$

$\Rightarrow$ moduli space $\mathcal{M}(4n, 1)$

attractor
 mechanism. 6. M. M. M.

$$S = \frac{A}{4} \sim \frac{A(\mathcal{U}_{\text{attractor}})}{4} \sim \frac{1}{4} |\vec{z}|^2 > 0$$

DPS black holes

$$\frac{\partial}{\partial r} \omega^i(r) = -g^{ij}(u) \frac{\partial}{\partial u^j} F$$

$$F(u, \vec{z}) = |\vec{z}(u, Q)|^{20} |1 - \frac{1}{2} \ln(u)|$$



$$= \left| \int_{\mathcal{Q}} e^{i\phi} \wedge e^{B \wedge T} \sqrt{TdM} \right|^2$$

metric g_{ij}

given Q :
start with
the gradient



given Q :
start with
do gradient flow
at $z^0(x, Q)$

1. $|z_{\text{lin}}| > 0 \Rightarrow \text{BFI exists.}$

2. $|z| = 0$ at

gnd. Q =
part 200
to ground the
at 314.0)

1. 131.100 → 0.01
part 200

2. 131.100 → 0.01
part 200
to ground the
at 314.0)

→ 0.01 part 200
to ground the
at 314.0)

given Q :
start with
do gradient flow
at $\mathbb{R}^3(x, R)$

1. $|z_{\min}| > 0 \Rightarrow \text{BFI exists.}$

2. $|z| > 0$ at which
gives $\mathbb{R}^3(x, R)$
Ass. simplifies
 $\Rightarrow \text{BFI under this}$
error does not
exist.

given Q :
start with
do gradient flow
at $z^2(x, Q)$

1. $|z|_{\min} > 0 \Rightarrow$ BH exists.

2. $|z| = 0$ at which
on $g_{15}, z_{15}, \dots$
Analogous
 $\Rightarrow$ BH under this
conditions does not
exist.

3. $|z|_{\min} = 0$
and regular
?

R. Reinbacher,
S.T. Yau

Let M on CV ,
(0,1) mod 2
Let V be a bundle
 μ -stable $\Rightarrow$

harmonic
" Yang-Mills
 $F^2 = 0$
 $F^2 = 0$
rank V
 $r, s, t, 5$
 $c_1(V) = 0$

wrapping D_0
on CV
IIa.

$$\bigoplus_{i=1}^n V_i$$

or r points

$$\text{ch } V = \text{Tr } e^F = Q$$

$\rightarrow$ electric, mag. charge
in div

Set bundle mod with $D(\sigma_1)$

$$\alpha(t) = \text{tr } F^2 = \text{tr } R^2$$

$$c_2(V) \equiv c_2(TN)$$

$$\frac{1}{2} c_2(V) = 2N_{\text{gen}}$$

gen $ch \cdot c_1 \cdot c_2 \cdot \dots$

$$\sum \lambda^k c_{2k}(V) \Delta(F-2)$$

$$N_{\text{gen}} = N_{\text{D3}}$$

Pick up Δ ex. Manojima,
A. Langer

R. Reinbacher,
S.T. Yau

wrapping De
on CY
IIa
 $\bigoplus_{V \text{ rank } 1}$
if r
 $ch V =$
 $\rightarrow$
 c_2

let M on CY,
(0,1) stable
let V bundle
μ-stable $\Rightarrow$

$$\det = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TM)$$

$$\frac{1}{2} c_3(V) = N_{\text{gen}}$$

Q

$$ch(V_N) = N ch(V)$$

harmonic
map $M \rightarrow U$
 $F = 0, \omega = 0$
 $F^2 = 0$
 $F^2 \wedge \omega \in \omega$

cot
rank V
3, 4, 5
 $c_1(V) = 0$

$$\sum \lambda^k c_{2k}(V) = \det(F - \lambda)$$

$$N_{21} - N_{11}$$

Pick N & c_2 ... Maruyama.
A. Langer

given Q :
 plot when
 do gradient flow
 at $z(4,0)$

1. $12L_{in} = 0 \Rightarrow \text{RM}$

2. $12L = 0$ at which
 RM value the
 error is zero at
 RM value the
 error is zero at
 RM value the
 error is zero at

3. $12L = 0$
 ?

R. Reinbacher
S.T. Yau

Let M be a CY,
(0,1) mod 2
Let V be a bundle
possible on

For rank V
2, 3, 4, 5
 $c_1(V) = 0$
harmonic
in Yang-Mills
 $F^2 = 0$
 $F^2 = 0$

wrapping D_6
on CY
 Π_1
 $\bigoplus_{i=1}^n \mathbb{Z} \cdot v_i$
iff r
 $ch V =$

$$\begin{aligned} \det(F) &= \det F^1 - \det F^2 \\ c_2(V) &= c_2(TN) \\ \frac{1}{2} c_2(V) &= N_{gen} = N_{D1} - N_{D5} \\ \sum \lambda^k c_k(V) &= \Delta(F-2) \\ ch(V_N) &= N \cdot ch(V) \end{aligned}$$

gives $ch_1, c_2, c_3, \dots$ (D. P. ...)

$D(-2), \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Maruyama, A. Langer

large g $\Rightarrow$ black holes
 large N
 $M \propto N$

attractor mechanism G.M. in FKS

$$S = \frac{A}{4} = \frac{A(u_{\text{min}}, Q)}{4} \quad |z| > 0$$

CY M
 $\int C^{(1)} \Rightarrow$ vector p.c.
 $\int C^{(2)}$

metric $\Rightarrow$ scalar field

Kaluza-Klein models $\Rightarrow$ in extra dim.
 $(B + \sqrt{2} H^2) \wedge F$

- scalars at ∞
- charge Q

$$Q^2 = e^{-A} \omega_{\text{eff}}^2 |z|^2 \Rightarrow$$

area of horizon A_{horizon}

DPS black holes

$$F = \frac{2}{\partial r} \omega^i(r) = -g^{ij} \partial_i \omega^j \frac{2}{\partial r} F$$

$$F(u, h) = |Z(u, Q)|^2 \quad |1 - i \epsilon_1(r-1)|$$

$$= \left| \int_{\mathcal{Q}} \text{tr} e^{\frac{1}{2} \sigma} \wedge e^{\frac{1}{2} \sqrt{T} \Delta M} \right|^2$$

vector $\frac{1}{\partial r}$

given Q:
start with
do gradient flow
at $z^2(x, Q)$

1. $|z|_{\min} > 0 \Rightarrow$ BH exists.

2. $|z| = 0$ at which
gives $z^2(x, Q)$
Assumption
 $\Rightarrow$ BH under this
assumption does not
exist.

3. $|z|_{\min} = 0$ and regular
?

R. Reinbacher
S.T. Yau

wrapping D6
on CY

Ia.
vibrations
V

off-shell

lets on M
CY.
(0,1) mod 2
lets a bundle V
 μ -stable on

$$\det H = \text{tr } F^2 - \text{tr } R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_2(V) = 2N_{\text{gen}}$$

given ch, c_1, c_2 is $\mathbb{Q}$ -Picard

$$c_1 = Q$$

ch, deg slope
i div
it bound states with $D(E=0)$

harmonic
in Yang-Mills

$$F^2 = 0$$

$$F^2 = 0$$

GUT

rank V
3, 4, 5

$$c_1(V) = 0$$

$$\sum \lambda^k c_{2k}(V) = \Delta(F-2)$$

$$N_{\text{gen}} = N_{\text{gen}}$$

$$ch(V_{\text{eff}}) = N ch(V)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Maruyama.
A. Langer

given Q :
 start with
 do gradient flow
 at $z^2(x, Q)$

1. $|z|_{\min} > 0 \Rightarrow$ BH
 exists.

2. $|z| = 0$ at which
 $g_{\text{eff}} = 0$
 non-singular
 $\Rightarrow$ BH under this
 criteria does not
 exist.
 end regular

3. $|z|_{\min} = 0$
 ?

R. Reinbacher
S.T. Yan

Let M on CY .
Let V be a bundle
μ-stable on

Let H be a hyperplane
 $F = \mathcal{O}_H$
 $F^2 = \mathcal{O}_H$
rank V is 3, 4, 5
 $c_1(V) = 0$

wrapping D_6
on CY
 Π_a
 $\bigoplus_{i=1}^n \mathbb{C} \otimes \mathbb{C}^i$

$$\begin{aligned} \alpha(H) &= \text{tr } F^1 - \text{tr } F^2 \\ c_2(V) &= c_2(TH) \\ \frac{1}{2} c_3(V) &= \chi(V) \end{aligned}$$

$$\sum \lambda^k c_{2k}(V) = \chi(F-2)$$

ch $V = \text{Tr } e^F = Q$
→ characteristic class

ch $(V_n) = N \text{ ch}(V)$
 $\left(\begin{smallmatrix} n \\ 1 \end{smallmatrix} \right)$

c_{2k} bundle with $\mathcal{O}(c_{2k})$

pe - stable . 66

μ -stable. Let

$$c_1 \in H^2(M, \mathbb{Z})$$

μ -stable. ω

$$\exists \frac{\omega+1}{\omega} \in H^1(M, \mathbb{Z})$$

$$c_1(\mathcal{L}) = F$$

$$\sigma^n \bigoplus (z'_1, z''_1) \sim \lambda(z'_1, z''_1) / z'_1 \cdot 0$$

μ -stable. i.e.

$$\exists \pi \in H^1(M, \mathbb{C})$$

$$G(\mathcal{L}) = F$$

σ^n



$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) / z \cdot 0$$

for $z \neq 0$
for $z = 0$ then z



μ -stable. Let

$$\begin{aligned} & \text{Let } \mathcal{L} \subset H^k(M, \mathbb{R}) \\ \Rightarrow C(\mathcal{L}) &= F \end{aligned}$$

$$\begin{aligned} \text{can } (z_1, \dots, z_n) &\sim \lambda(z_1, \dots, z_n) / z_1 = 0 \\ &\text{for } z_1 \neq 0 \\ \text{for } \mathcal{L} \text{ of degree } k & \text{ is } \text{isomorphic to } \mathbb{C}^n \text{ for } k \leq J \end{aligned}$$

μ -stable. $\mathbb{A}^1$.

$r=1$

$$\mathbb{A}^1 \in H^{2,1}(M, \mathbb{Z})$$

$$\Rightarrow c_1(\mathcal{L}) = F$$

$\mathbb{C}P^n$



$$(z_1, \dots, z^{n+1}) \sim \lambda(z_1, \dots, z^{n+1})$$

for $z^i \neq 0$

$$z^i = 0$$

for $\mathbb{C}P^n$ degree k

$$\mathbb{C}P^n \cong \mathbb{C}P^k \cup \mathbb{C}P^{n-k-1}$$

μ -stable. L.L.

$$\exists \frac{1}{\epsilon} \in H^1(M, \mathbb{Z})$$

$$C(\mathcal{L}) = F$$

$$\bigoplus_{k=1}^{\infty} \mathcal{O}(k) \xrightarrow{f} \bigoplus_{k=1}^n \mathcal{O}(k)$$

σ^n



$$(z_1, \dots, z^n) \sim \lambda(z_1, \dots, z^n) / z^n = 0$$

for z^n/z^n
for of degree k : such as $\mathbb{C}[z]_{\leq k}$
 $\mathcal{O}(k)$

μ -stable. i.e.

$$\exists \frac{c_{i+1}}{c_i} \in H^1(M, \mathbb{C})$$

$$c_1(\mathcal{L}) = F$$

σ^n



$$(z_1, \dots, z^{n+1}) \sim \lambda(z_1, \dots, z^{n+1}) / z^{n+1} = 0$$

for $z^k \neq 0$
 for of degree k : $\frac{c_{i+1}}{c_i} \sim k \mathcal{J}$
 $\mathcal{O}(k)$

4.9.6

$$\bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i)$$

$$\sim \left(z^{\frac{n}{2}-4} \right)$$

μ -stable. u_0

$$u_0 \in H^0(M, \mathcal{L})$$

$$C(\mathcal{L}) = F$$

$$0 \rightarrow V \rightarrow \bigoplus_{k=0}^{\infty} \mathcal{O}(k) \xrightarrow{f} \bigoplus_{k=0}^{\infty} \mathcal{O}(k) \rightarrow 0$$

$$\left(\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \right) \sim \left(z^{\lambda-k} \right)$$

$k \in \mathbb{N}$

σ^n



$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) / z_1 \neq 0$$

for $z_1 \neq 0$

for $\mathcal{O}$ degree k : $\{C_i : \gamma_i = k\}$
 $\mathcal{O}(k)$

$\mu = \text{stable, loc.}$

$$\pi_1^{\text{loc}} \in H^1(M, \pi)$$

$$\exists c(\mathcal{L}) = F$$

$$0 \rightarrow V \rightarrow \bigoplus_{k=1}^n \mathcal{O}(k) \rightarrow \mathcal{O}(\mathcal{L}) \rightarrow 0$$

$\left(\begin{smallmatrix} \vdots \\ 1 \end{smallmatrix} \right)$
 $k \in \mathbb{Z}$



$$(z_1', z_2') \sim \lambda(z_1', z_2') \quad \text{for } z_1, z_2 \neq 0$$

for of degree k : $\text{rank } \mathcal{O}(k) = kJ$
 $\mathcal{O}(k)$

$$\text{ch}(V) = \text{trch } \mathcal{O}(e) = m \text{ch } \mathcal{O}(k)$$

$$\mu = \text{stable} \cdot \omega$$

$$\begin{aligned} & \text{then } \omega \in H^1(M, \mathbb{R}) \\ & \Rightarrow C(\mathcal{L}) = F \end{aligned}$$

$$\begin{aligned} & \text{then } (z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \\ & \text{for } z^k/z^l \\ & \text{for } \text{degree } k \text{ : } \text{such as } [C, \gamma, k, \mathcal{L}] \\ & \mathcal{O}(k) \end{aligned}$$

$$\begin{aligned} 0 \rightarrow V \rightarrow \bigoplus_{k=1}^n \mathcal{O}(k) \xrightarrow{F} \bigoplus_{k=1}^n \mathcal{O}(k) \rightarrow 0 \quad e^{V \cdot T} \\ \left(\begin{array}{c} \vdots \\ \vdots \end{array} \right) \sim \left(z^{k-k} \right) \quad \text{ch}(V) = \text{trch } \mathcal{O}(e) = m \text{ch } \mathcal{O}(k) \\ k \in F \sim V \end{aligned}$$

μ -stable. $\text{Gr}(n, r)$

$$\frac{r(n-r)}{n} \in H^4(M, \mathbb{Z})$$

$$\exists \mathcal{O}(\mathcal{L}) = F$$

$$\begin{aligned} \text{Gr}(n) \quad (z_1^1, \dots, z_n^1) &\sim \lambda (z_1^2, \dots, z_n^2) \\ &\text{for } z_i^1/z_i^2 \\ &\text{for } \text{if degree } k \text{, } \text{rank} \leq k \text{ } \mathcal{O}(k) \end{aligned}$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{F} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\left(\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \right) \sim \left(z^{l-k} \right)$$

$k_i \in F = V$

$$e^{V \cdot T}$$

$$\text{ch}(V) = \text{trch } \mathcal{O}(e) = m \text{ch } \mathcal{O}(k)$$

S. T. Yau

map from π to π map π to π
 $F^1 \cdot F^2 = 0$ $C_1(V) = 0$
 $F^1 \cdot F^2 = c w$

wrapping D6

on CY

IIa

$\bigoplus_{V \in \text{rank } 1}$

NS particles

$ch V = Tr e^F = Q$

$\rightarrow$ electric, mag charge
 $\&$ dir

c_{2k} brane mod class $D(2k)$

$$ch(V) = \frac{1}{2} F^2 - \frac{1}{2} R^2$$

$$c_2(V) = c_2(TN)$$

$$\frac{1}{2} c_3(V) = 2N_{\text{gen}}$$

$$\sum \lambda^k c_{2k}(V) = \chi(F - \lambda)$$

$$N_{2,1} - N_{2,3}$$

gives $ch(V)$ (E. Picard, M. Maruyama, A. Langer)

$$ch(V_N) = N ch(V)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

μ -stable, b6

$$\sigma^n \bigoplus (z^1, \dots, z^n) \sim \lambda(z^1, \dots, z^n) / z^1 = 0$$

S. T. Yau

map from $\mathbb{R}^4$ to $\mathbb{R}^4$
 $F^2 = 0$
 $F^2 \wedge \omega \in \omega$
 $c_1(V) = 0$

wrapping D6
 on CY
 IIa.

$$\bigoplus_{\mathbb{Z}} \mathbb{Z} \langle 1 \rangle$$

BPS particle

$$ch V = Tr e^F = Q$$

→ electromagnetic charge
 in d=4

c_{2k} linked with $\eta(6-2k)$

$$2H = Tr F^2 = Tr R^2$$

$$c_2(V) = c_2(TN)$$

$$\sum \lambda^k c_{2k}(V) = \Delta(P, \lambda)$$

$$\frac{1}{2} c_3(V) = 2N_{gen}$$

$$N_{2,1} - N_{1,2}$$

gives $ch, c_1, c_2, \dots$

Pick an $\omega \in \Omega^2$ Maniyappa.

A. Langer

$$ch(V_N) = N ch(V)$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

μ -stable loc.

$$H^1(M, \mathbb{Z}) \in H^1(M, \mathbb{Z})$$

$$(z^1, z^2) \sim \lambda(z^1, z^2)$$

$$\bigoplus$$

for z^1, z^2

for $\dim k = 1$ and $\dim k = 2$

pr = stable, bob.

$$\Rightarrow G(\mathcal{L}) = F$$

$$0 \rightarrow V \rightarrow \bigoplus_{k=1}^n \mathcal{O}(k) \xrightarrow{f} \bigoplus_{l=1}^n \mathcal{O}(l) \rightarrow 0$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} z^{1-k} \\ \vdots \\ z^{1-k} \end{pmatrix}$$

$\sigma^n \rightarrow (z^1, \dots, z^n) \sim \lambda(z^1, \dots, z^n) / z^1 = 0$
 for z^1/z^1
 for σ degree k : $\sigma \in \mathbb{C}^n$: $k \in \mathbb{Z}$
 $\mathcal{O}(k)$

$$\chi(V) = \frac{e^2 T}{4\pi} \chi(0) - m \frac{e^2 T}{4\pi} \chi(k)$$

μ -stable. let

$$\mathbb{C}P^n$$

$$(z_1, \dots, z^{n+1}) \sim \lambda(z_1, \dots, z^{n+1}) \quad z \neq 0$$

$$\text{for } z^k \neq 0$$

$$\text{for } \dim k = \dim \mathbb{C}P^k = k$$

$$\mathbb{C}P^1 \subset \mathbb{C}P^n \quad H^*(H, \mathbb{Z})$$

$$A = 5^2 3$$

$$A = 6$$

$$\mathcal{O}(k)$$

$$\mathcal{C}_1(\mathcal{L}) = F$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^m \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{j=1}^n \mathcal{O}(l_j) \rightarrow 0$$

$$\mathbb{C}^{n \times m}$$

$$\begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \in \mathbb{C}^{n \times m}$$

$$\text{ch}(V) = \sum_{i=1}^m e^{k_i T} - \sum_{j=1}^n e^{l_j T}$$

partially ...



$$(z_1, \dots, z^n) \sim \lambda(z_1, \dots, z^n) \quad \text{for } \lambda \in \mathbb{C}^*$$

$$\exists \mathcal{O}(L) = F$$

$$\Lambda = \mathbb{Z}^n$$

$$\text{For } \mathcal{O}(L) \text{ of degree } k, \text{ we have } \mathcal{O}(L) \cong \mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \rightarrow \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\begin{pmatrix} \vdots \\ \vdots \end{pmatrix}$$

$$\text{ch}(V) = \sum_{i=0}^n \text{ch}(\mathcal{O}(k_i)) = \sum_{i=0}^n \text{ch}(\mathcal{O}(k_i))$$

μ -stable - 66



$$(z_1', \dots, z_n') \sim \lambda (z_1', \dots, z_n') \quad \text{for } z_i' \neq 0$$

$$\exists \text{ } \mathcal{O}(L) = F$$

$$\Lambda = 5^{1/2} g$$

$$5\Lambda = \frac{1}{2} \mu$$

for σ degree k : roots $\in \mathbb{C}, \gamma \in k \mathbb{Z}$

$$\mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(L_i) \xrightarrow{F} \bigoplus_{i=1}^n \mathcal{O}(L_i) \rightarrow 0$$

$(\lambda_1, \dots, \lambda_n)$

$$e^{V \cdot T}$$

$$\text{ch}(V) = \text{trch } \mathcal{O}(L) = n \text{ch } \mathcal{O}(L)$$

stable $V \in \text{hyp}$

not μ -stable. $\hookrightarrow V' \rightarrow V$

$$\mu(V') \geq \mu(V)$$

$$\mu(V) = \frac{1}{\text{rk}(V)} \int \text{trch } \mathcal{O}(L) \cdot J^{n-1}$$

μ -stable. b.b.



$$(z^1, \dots, z^{n+1}) \sim \lambda(z^1, \dots, z^{n+1}) \quad \text{for } z^k \neq 0$$

$$\exists \quad \mathcal{H} \in H^1(M, \mathbb{C}) \quad c_1(\mathcal{L}) = F$$

$$\Lambda = \mathbb{Z}^2, \quad \delta \Lambda = \mathbb{Z}^2$$

$$\text{for } \mathcal{O} \text{ degree } k \quad \text{roots } \mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}^* \quad \mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{F} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$e^{i\pi/2}$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{Z}^{n-k}$$

$$\text{ch}(V) = \sum_{i=1}^n e^{i\pi/2} \text{ch}(\mathcal{O}(k_i)) - m \text{ch}(\mathcal{O}(k))$$

$$\text{not } \mu\text{-stable. } \text{on } V' \rightarrow V \quad \mu(V') \geq \mu(V)$$

given Q is
 start when
 do gradient flow
 at $z(x, Q)$

let $CH(\mathcal{D})$
 set of $ch(v)$ (internal
 s.t. $\exists$ postorder v)

1. $|z_{\min}| > 0 \Rightarrow BHI$ exists.
2. $|z| = 0$ at which
 $g_{15}, z_{11}, \dots$
 are singular
 $\Rightarrow BHI$ under this
 criteria does not
 exist.
 and regular
3. $|z|_{\min} = 0$
 ?

given Q :

start when
do gradient flow
at $\mathbb{R}^2(x, y)$

1. $|z|_{\min} = 0$ $\rightarrow$ ∇f exists at $\pi(\gamma)$ set of Q

2. $|z| = 0$ at which $\gamma_1, \gamma_2, \dots$ are singular
 $\rightarrow$ ∇f under this case does not exist.

3. $|z|_{\min} = 0$ and regular
?

let $CH(\gamma)$
set of $ch(v)$ (internal)
 $\{1, 2\}$ possible v

$$|\pi(\gamma)| \subset CH(\gamma)$$



μ -stabile. l.u.

$$\pi_1^* \in H^1(M, \mathbb{Z})$$

$$c_1(\mathcal{L}) = F$$



$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$5A = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \quad \lambda \in \mathbb{C}^*$$

Far of degree k : $\mathcal{O}(k)$

$$V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}$$

$$\text{ch}(V) = \sum_{i=0}^n \text{ch}(\mathcal{O}(k_i)) = \sum_{i=0}^n e^{k_i T}$$

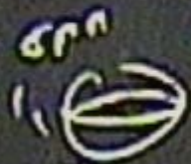
hypothesis: μ -stable $V \subset V'$

not possible: $\mu(V') < \mu(V)$

$$\mu(V') = \frac{1}{n+1} \sum_{i=1}^{n+1} k_i$$

μ -stable. b.

$$\exists \pi \in H^1(M, \pi) \quad G(\mathcal{L}) = F$$



$$(z^1, \dots, z^n) \sim \lambda(z^1, \dots, z^n) \quad \text{for } z^i \neq 0$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \Delta A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{for } \mathcal{O} \text{ degree } k \quad \mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \begin{pmatrix} z^{l_1-k_1} \\ \vdots \\ z^{l_n-k_n} \end{pmatrix}$$

$$\text{ch}(V) = \text{trch } \mathcal{O}(e) = n \text{ ch } \mathcal{O}(k)$$

$$\text{not } \mu\text{-stable. } 0 \rightarrow V' \rightarrow V \quad \mu(V') \geq \mu(V)$$

unipolar D₂

in C₁

IIa

unipolar V

RF particle

$$\text{ch } V = \text{Tr } e^F = Q$$

→ electrostatic steps
in det

C_{2k} bound state like D(0, n)

$$\chi(t) = \text{Tr } F^t = \text{tr } R^t$$

$$C_1(V) = C_2(TN)$$

$$\sum \lambda^k C_{2k}(V) = \text{Id}(F - \lambda)$$

$$\frac{1}{2} C_3(V) = 2N_{\text{gen}}$$

$$N_{11} = N_{22}$$

gives ch. C₁, C₂ etc.

Pick up x, y, z

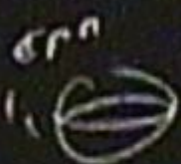
Manifolds

$$\text{ch}(V_n) = N \text{ch}(V)$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

A Langer

particle, but



$$(z_1^1, z_1^{n+1}) \sim \lambda(z_1^1, z_1^{n+1})$$

for z₁ = 0

given Q :
 start when
 do gradient flow
 if $z^2(u, Q)$

let $CH(\mathcal{T})$
 set of $ch(v)$ (internal
 nodes) $\{1, \dots, n\}$ possible v

$ATP(\mathcal{T})$
 set of Q

or

str.

$ATP(\mathcal{T}) \in CH(\mathcal{T})$

conjecture.

1. $|z|_{\min} > 0 \rightarrow BH$

2. $|z|_{\min} = 0$

at which
 $g(\mathcal{T}, z)$
 has singularities

$\rightarrow BH$ under this
 condition does not
 exist.
 each regular

3. $|z|_{\min} = 0$

?

given Q :

start with
do gradient flow
at $z(x, y)$

1. $|z|_{\min} = 0$ $\rightarrow$ $AP(1)$
set of Q
at

2. $|z| = 0$ at which
gives $z(x, y)$
Acquisition

$\rightarrow$ RH under the
control does not
exist
and regular

3. $|z|_{\min} = 0$
?

let $CH(1)$

set of $ch(v)$ (internal)
(1.2) postulate v

$AP(1) \subset CH(1)$ condition.

given Q :
 start when
 do gradient flow
 at $\nabla f(x, Q)$

let $CH(\mathcal{T})$
 set of $ch(v)$ (internal
 nodes) ∇

1. $|Z_{min}| = 0 \rightarrow$ BH exists at $\nabla f(x, Q)$

2. $|Z| = 0$ at which
 $\nabla f(x, Q) = 0$
 BH exists at $\nabla f(x, Q)$
 BH exists at $\nabla f(x, Q)$
 BH exists at $\nabla f(x, Q)$

3. $|Z| = 0$
 ?

$ATP(\mathcal{T}) \subset CH(\mathcal{T})$ condition.



large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

low Σ in

$\downarrow$ CY M
 $\mathcal{H} \subset C^{loc} \Rightarrow$ vector
 $\mathcal{H} \subset C^{in}$ p.m.

metric moduli $\Rightarrow$ scalar fields

Kahler moduli $\Rightarrow$ in moduli
 b.c. (B + iH) in c)

- radius of S^1
- charge Q

$$\mathcal{L}_5^2 = e^{-A_{eff} - \ln |z|} \Rightarrow$$

$\Rightarrow$ area of horizon $A(\mathcal{H}_\infty)$



attractor mechanism. 6 M. in

$$S = \frac{A}{4} + \frac{A(\mathcal{H}_{\infty})}{4} + \frac{1}{2} \ln |z| > 0$$

for $\frac{\partial}{\partial r} \ln |z| = -g^{ij} \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} F$

$$F(u, k, f) = \| \tilde{z}(u, Q) \|^2$$

$$= \left| \int_{\mathcal{H}} e^{i\theta} \wedge e^{B \frac{1}{2} \tilde{z}} \right|$$

vector

large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

low Σ in

$\downarrow$ CY M
 $\mathcal{H} \subset C^{loc} \Rightarrow$ vector
 $\mathcal{H} \subset C^{loc}$

metric $\Rightarrow$ scalar
 field $\mathcal{H}$

Klein
 model $\Rightarrow$ in $\mathcal{H}$
 $b^1, 1$ (B + c.c. $\mathcal{H}$ in $\mathcal{H}$)

- scalar $\mathcal{H}$
- charge Q

$\mathcal{L}^2 = e^{-A_{eff} \cdot \ln |z|}$
 $\Rightarrow$ area of horizon $A_{eff} \ln |z|$

attractor
 mechanism, G. Horowitz

$$S_1 = \frac{A}{4} + \frac{A(\mathcal{H}_{eff} \ln |z|)}{4} \quad \underline{\underline{e^{|z|^2} > 0}}$$

BPS black holes

$$F = \frac{\partial}{\partial r} \omega(r) = -g \frac{\partial}{\partial r} \left(\frac{1}{\sqrt{4\pi}} F \right)$$

$$F(0, k, F) = \| \mathcal{Z}(\omega, Q) \|^2 \left(1 - \frac{1}{2} \ln |z| \right)$$



$$= \left| \int_{\mathcal{H}} e^{i\theta} \wedge e^{\frac{1}{2} \sqrt{4\pi} M} \right|^2$$

$$e^{|z|^2}$$

large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

10d Σ $\rightarrow$ II 4d 11d
 $\downarrow$ CY M
 4d $\int C^{(2)} \rightarrow$ vector
 $\int C^{(1)}$ p.f.

metric $\Rightarrow$ scalar field ϕ

Kaluza-Klein $\Rightarrow$ in 4d
 b.v. $(B + i\sqrt{2}H) \rightarrow$

- scalar ϕ
- charge Q

$Q^2 = e^{-A} \text{vol}(\Sigma) \int \Sigma F^2$
 $\Rightarrow$ area of horizon A_H

attractor mechanism. FKS

$$S = \frac{A}{4} = \frac{A(\phi_{\text{min}}, Q)}{4} \quad \text{as } |z|^2 > 0$$

DPS black hole
 $\frac{d}{dr} \omega(r) = -g \sqrt{2} \frac{1}{\omega(r)} F$

$$F(\omega, F) = |\tilde{z}(\omega, Q)|^2 \quad || \cdot ||_{H^1(\Sigma)}$$



$$= \left| \int_{\Sigma} e^{\frac{1}{2} \sqrt{2} \phi} \wedge e^{\frac{1}{2} \sqrt{2} \phi} \sqrt{2} dM \right|^2$$

$\int_{\Sigma} \Omega \wedge F$ $\frac{1}{\sqrt{2}} \int_{\Sigma} F^2$ $22-6.412$ $21 \sqrt{2}$

$$e^{\frac{1}{2} \sqrt{2} \phi} = 0$$

large $g_s \Rightarrow$ black holes
 large N
 $M = g_s N$

low Λ or
 $\downarrow$ CY M
 $\mathcal{U} \subset \mathcal{C} \subset \dots$
 $\Rightarrow$ vector
 p.f.

metric $\Rightarrow$ scalar
 field ϕ

Kaluza
 model $\Rightarrow$ in
 5D $(B + i\sqrt{2}H) \Rightarrow$

- radius of S^1
- charge Q

$$Q^2 = e^{-A} \omega_{d-1} \cdot |F|^2$$

$\Rightarrow$ area of horizon $A(Y_{d-1})$

attractor
 mechanism, 6 M. mod

$$S = \frac{A}{4} + \frac{A(\omega_{d-1} \cdot |F|^2)}{4} \quad \text{with } |F|^2 > 0$$

DFS static limit

$$F = \frac{2}{\partial r} \omega^i(r) = -g^{ij} \omega_j \frac{2}{\partial r} F$$

$$F(\omega, F) = |\tilde{Z}(\omega, Q)|^2 \cdot (1 - \frac{1}{2} \omega^2(r))$$

$$= \left| \int_{\Sigma} e^{iQ} \wedge e^{B + i\sqrt{2}H} \sqrt{TdM} \right|^2$$

$$\int \Omega \wedge F$$

$$e^{\frac{1}{2} |F|^2} = 0$$

metric $\Rightarrow$ 2D $\Rightarrow$ 2D $\Rightarrow$ 2D $\Rightarrow$ 2D

large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

I & II $\Rightarrow$
 $\downarrow$ CY M
 $\mathcal{H} \subset \mathcal{C}^{(1)} \Rightarrow$ vector
 $\mathcal{C}^{(1)}$ field

metric $\Rightarrow$ scalar
 field $\mathcal{H}$

Kaluza
 moduli $\Rightarrow$ in theory
 $b^1, \dots, (B + i\sqrt{2}H)_{\text{in } \mathcal{H}}$

- scalar at $\mathcal{H}$
- charge Q

$\mathcal{L}_S = e^{-A \phi} \dots$
 $\Rightarrow$ area of horizon $A(\mathcal{H}_\infty)$

attractor
 mechanism. G. Horowitz

$$S = \frac{A}{4} \sim \frac{A(\mathcal{H}_\infty)}{4} \sim |z|^2 > 0$$

DPS black hole
 $\frac{\partial}{\partial r} \omega(r) = -g^2 |u|^2 \frac{\partial}{\partial r} F$

$$F(u, \mathcal{H}) = |\tilde{z}(u, Q)|^2 |1 - i\mathcal{H}(r)|$$



$$= \left| \int_{\mathcal{H}} e^{i\sqrt{2} \sqrt{TdM}} \right|^2$$

$$\int_{\mathcal{H}} \Omega \wedge F$$

$$e^{i\sqrt{2} \sqrt{TdM}} = 0$$

we can see $2\pi \cdot Q \cdot \mathcal{H}$
 $2\pi \cdot D \cdot \mathcal{H}$
 $2\pi \cdot \mathcal{H}$

given Q :

start when
do gradient flow
at $z^0(x, Q)$

1. $|z|_{\min} = 0$ $\rightarrow$ $ATP(\tau)$
set of Q
at

2. $|z| = 0$ at which
 $z^0, z^1, \dots$
converges
 $\rightarrow$ BFI under this
circumstance does not
exist.

3. $|z|_{\min} = 0$
?

let $CH(\tau)$

set of $ch(v)$ (normal)
 $(1, \exists)$ possible v

$ATP(\tau) \subset CH(\tau)$ condition.

attractor
mechanism. G. H. Moore

$$S_1 = \frac{A}{1} = \frac{A(1 + \tan^2 \theta)}{1} = |z|^2 > 0$$

$\frac{f}{g} = \frac{1}{x^2} \cdot \frac{x^2}{(x-1)^2}$

$$f(a, b) = | \tilde{c}(y, Q) |^{20} | \dots |$$



$$= \frac{1}{2} \int \text{Tr} e^{\frac{1}{2} \sqrt{\Lambda} M} \wedge e^{\frac{1}{2} \sqrt{\Lambda} M} \sqrt{\Lambda} M$$

$$Q_s^2 = e^{-A_{coll} \cdot \ln(1/x)}$$

$$\Rightarrow \text{more } F \text{ from } A \text{ (from } Q^2)$$

0 0 0 0
0 0 0 0
0 0 0 0

given Q:

start when
do gradient flow
if $z^2(u, Q)$

let $CH(\mathcal{T})$

set of $ch(u)$ (internal
s.t. $\exists$ postorder v

$ATP(\mathcal{T})$

set of Q

at

exists. ATP

$$ATP \subset CH(\mathcal{T})$$

conjecture.

$$ATP = \{ Re \bar{C} e^{D_{1,5}'} \}_{v \in \mathcal{T}, \text{ Kahl's use}}$$

CIC

1. $|z_{min}| > 0$

at which

$z_{15}, z_{11}, \dots$

Acceptor.

$\rightarrow$ BH under this
errors does not
exist.

and regular.

3. $|z_{min}| = 0$

?

large g $\Rightarrow$ black holes
 large N
 $M = g^2 N$

low Λ $\Rightarrow$

$\downarrow$ CY M
 $\downarrow$ $\{C^{(1)}, \dots, C^{(n)}\}$
 $\Rightarrow$ vector space

metric $\Rightarrow$ scalar product

Kähler metric $\Rightarrow$ in metric

- regular at ∂
- charge Q

$$Q^2 = e^{-A} \text{vol} \cdot |F|^2$$

$\Rightarrow$ area of horizon $A(\gamma_{\text{hor}})$

attractor mechanism G. Horowitz

$$S = \frac{A}{4} + \frac{A(\gamma_{\text{hor}})}{4} \quad \text{with } |F|^2 > 0$$

DPS black holes
 $\Rightarrow \frac{\partial}{\partial r} \ln(e^{A/4} \sqrt{|F|^2}) = -g^2 \frac{1}{4} \frac{1}{r^2} F$

$$F(r, \theta) = |Z(\omega, Q)|^2 \quad |F| = 1$$



$$= \left| \int_{\Sigma} e^{i\theta} \wedge e^{B/2} \sqrt{g} dM \right|^2$$

$$\Omega \wedge F \quad \text{where } \int \Omega \wedge F = 0$$

where $\int \Omega \wedge F = 0$
 $\Rightarrow$ $\int \Omega \wedge F = 0$
 $\Rightarrow$ $\int \Omega \wedge F = 0$

given Q :
 start when
 do gradient flow
 at $z^*(u, Q)$

let $CH(\mathcal{T})$
 set of $ch(v)$ (internal
 nodes) $\exists$ possible v

$ATP(\mathcal{T})$
 set of Q

1. $|z_{min}| > 0 \rightarrow$ BH exists at $ATP(\mathcal{T}) \subset CH(\mathcal{T})$ condition.

2. $|z| = 0$ at which
 $g(\mathcal{T}, z) = 0$
 no singularities
 $\rightarrow$ BH under this
 criteria does not
 exist.
 and regular

$$ATP = \{ Re \bar{C} e^{D_{i,j}^{(k)}} \}_{v \in \mathcal{T}, k \in \mathbb{N}}$$

3. $|z|_{min} = 0$
 ?

partielle Lb.

$$\exists \pi \in H^1(M, \mathbb{Z}) \quad c_1(\mathcal{L}) = F$$



$$(z_1^1, \dots, z_n^1) \sim \lambda (z_1^1, \dots, z_n^1) \quad \text{for } z^1/z^2$$

for $\mathcal{L}$ degree k : $\mathcal{O}(k)$

$$0 \rightarrow V \rightarrow \mathcal{O}(k) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \begin{pmatrix} z^1 \\ \vdots \\ z^k \end{pmatrix}$$

$f = \nu$

$$\text{ch}(V) = \text{trch } \mathcal{O}(e) = m \text{ ch } \mathcal{O}(k)$$

not possible. $\text{ch } V' \rightarrow V$

$$V \oplus \mathcal{O}$$

$$\mu(V) = \frac{1}{2\pi h_1} \int c_1(V) \cdot \omega$$

μ -stable. b.b.

$\pi_1^* \in H^{n-1}(H, \mathbb{Z})$
 $\Rightarrow c_1(\mathcal{L}) = F$



$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 $5A = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n)$
 for $z^h(z)$
 for $\mathcal{O}$ degree k : $\mathcal{O}(k)$
 $\mathcal{O}(k)$

$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$ $e^{h \cdot T}$

$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \begin{pmatrix} z^{k_1-k} \\ \vdots \\ z^{k_n-k} \end{pmatrix}$
 $k_1 \leq k \leq k_n$

$ch(V) = \text{trch } \mathcal{O}(e) = n \cdot ch \mathcal{O}(k)$
 $ch(V) = \sum_{i=0}^{\infty} \frac{ch_i(V)}{i!} e^i$
 $ch(\mathcal{O}(k)) = \sum_{i=0}^{\infty} \frac{ch_i(\mathcal{O}(k))}{i!} e^i$

$c(V \oplus W) = c(V) + c(W)$
 $ch(V \oplus W) = ch(V) + ch(W)$
 $ch(V) = \sum_{i=0}^{\infty} \frac{ch_i(V)}{i!} e^i$
 $ch(W) = \sum_{i=0}^{\infty} \frac{ch_i(W)}{i!} e^i$
 $ch(V \oplus W) = \sum_{i=0}^{\infty} \frac{ch_i(V \oplus W)}{i!} e^i$
 $ch(V \oplus W) = ch(V) + ch(W)$

given Q :
 start when
 do gradient flow
 at $z^0(x, Q)$

1. $|z_{\min}| = 0 \rightarrow$ BH exists at $ATP(\tau)$ set of Q

2. $|z| = 0$ at which $z(\tau) = z(0) \dots$
 assumption
 $\rightarrow$ BH under this
 errors does not
 exist.

3. $|z|_{\min} = 0$
 ?
 and regular

let $CH(\tau)$

set of $ch(v)$ (interval)
 $(1, \infty)$ possible v

$ATP \subset CH(\tau)$ conjecture
 $ATP = \{Re \bar{C} e^{i\theta} \mid \theta \in [0, 2\pi)\}$
 $C \in \mathbb{C}$ $\theta \in [0, 2\pi)$ $e^{i\theta}$
 Killing us.

μ -stable, l.b.

$$\exists \quad \text{with } \mathcal{L} \in H^1(M, \mathbb{C}) \quad c_1(\mathcal{L}) = F$$

$$\begin{aligned} & \text{for } \mathcal{L} \in H^1(M, \mathbb{C}) \\ & \Lambda = \mathbb{C}^* \times \mathbb{C}^* \\ & \text{for } \mathcal{L} \in H^1(M, \mathbb{C}) \end{aligned}$$

$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \quad \text{for } \lambda \in \mathbb{C}^*$$

Far of degree k : $\mathcal{O}(k)$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{F} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \in \mathbb{C}^{n \times (k_1 + \dots + k_n)}$$

ch(V) is not stable

$$c_1(V) \otimes \mathcal{O}(k)$$

not stable

$$c_1(V) \geq 0$$

$$\mu(V) + \frac{1}{n} \int c_1(V) \cdot \omega \geq \mu(k)$$

μ -stable. l.b.

$$\pi^* \in H^{n-1}(M, \mathbb{C})$$

$$c_1(\mathcal{L}) = F$$

$$\sigma^n$$

$$\Lambda = \mathbb{C}^n$$

$$\delta \Lambda = \mathbb{C}^n$$

$$(z_1, \dots, z^n) \sim \lambda(z_1, \dots, z^n) \quad \lambda \in \mathbb{C}^*$$

For of degree k : $\mathbb{C}^n \rightarrow \mathbb{C}^n$ $k \in \mathbb{Z}$

$$\mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$$c_1(V) \cdot J \geq 0$$

$$\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \in \mathbb{C}^{n-k}$$

$ch(V) \in H^*(M, \mathbb{R})$

$ch(\mathcal{O}(k)) = e^k$

$ch(V) = \sum_{i=1}^n e^{k_i}$

$$c_1(V) \in H^2(M, \mathbb{R})$$

not μ -stable. $\mathcal{O}(k) \rightarrow V \rightarrow \mathcal{O}(k)$

$\mu(V) \geq \mu(k)$

μ -stable. l.c.

$$\begin{aligned} & \text{rank } 1 \\ & \text{then } \pi \in H^1(M, \mathbb{C}) \\ & \Rightarrow c_1(\mathcal{L}) = F \end{aligned}$$

$$\begin{aligned} & \text{then } \pi \in H^1(M, \mathbb{C}) \\ & \Lambda = \sum \gamma_i^2 \\ & \text{then } \pi \in H^1(M, \mathbb{C}) \end{aligned}$$

$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \quad \text{for } \lambda \in \mathbb{C}^*$$

$$\text{For of degree } k \quad \text{then } \pi \in H^1(M, \mathbb{C})$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{j=1}^m \mathcal{O}(l_j) \rightarrow 0$$

$$\text{then } \pi \in H^1(M, \mathbb{C}) \quad \text{then } \pi \in H^1(M, \mathbb{C})$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{C}^n$$

$$\text{then } \pi \in H^1(M, \mathbb{C}) \quad \text{then } \pi \in H^1(M, \mathbb{C})$$

$$\text{then } \pi \in H^1(M, \mathbb{C})$$

$$\text{then } \pi \in H^1(M, \mathbb{C}) \quad \text{then } \pi \in H^1(M, \mathbb{C})$$

given Q :

start when
do gradient flow
if $z^2(u, Q)$

1. $|z|_{\min} > 0 \rightarrow$ ∇H exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which
if $z^2(u, Q) \dots$
non-singular
 $\rightarrow \nabla H$ under this
conditions does not
exist.

3. $|z|_{\min} = 0$
and regular
?

let $CH(T)$

set of $ch(u)$ (internal
(l.e.) postulates u

$ATP \subset CH(T)$ conjecture:
 $ATP = \{ R_e \bar{C} e^{D_{1,5,7,3}} \}$
CIC $\nabla H, T, 3$
Kahle u .

μ -stable. G.G.

$\pi \in H^1(M, \mathbb{Z})$
 $\exists \mathcal{L} \in \mathcal{L} = F$



$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n)$
 $\text{for } z_i \in \mathbb{C}^*$
 $z_i = 0$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
 $5A = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

for $\mathcal{L}$ degree k is $\mathcal{L} \in \mathcal{L}(k)$
 $\mathcal{L}(k)$

$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(k_i)$
 $\left(\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} \right) \in \left(\mathbb{Z}^{n-1} \right)$

$k_i \in f = V$

$c(V \oplus \dots)$
 $h(V) \in \mathbb{Z}$

$\text{ch } \mathcal{O}(k) = m \text{ ch } \mathcal{O}(k)$
 $\text{ch } \mathcal{O}(k) = m \text{ ch } \mathcal{O}(k)$
 $\text{ch } \mathcal{O}(k) = m \text{ ch } \mathcal{O}(k)$

$\mathcal{L}(V) \in \mathcal{L} \geq 0$
 $\mathcal{L}(V) = 0$

$\mu(V) = \frac{1}{\text{rk}(V)} \int c_1(V) \cdot \text{ch}(V)$
 $\mu(V) \geq \mu(V)$

μ -stabile, l.u.

$$\pi_1^* \in H^1(H, \mathbb{Z}) \quad \Rightarrow \quad c_1(\mathcal{L}) = F$$



$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \quad \text{for } z_i \neq 0$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \Delta A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{for } \mathcal{O} \text{ degree } k \quad \mathcal{O}(k)$$

$$0 \rightarrow \mathcal{O}(k) \xrightarrow{f} \mathcal{O}(k) \rightarrow 0$$

$$e^{kT}$$

$$c_1(V) \cdot T \geq 0$$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{Z}^n$$

$$\text{ch}(V) = \text{trch } \mathcal{O}(e) = m \cdot \text{ch } \mathcal{O}(k)$$

$$c_1(V \oplus \mathcal{O}(k))$$

$$\text{not positive, } G \rightarrow V' \rightarrow V \quad \mu(V') \geq \mu(V)$$

$$\mu(V) + \frac{1}{T} \int c_1(V) \cdot T$$

given Q :

start when
do gradient flow
if $\nabla^2 f(x, Q)$

1. $|\nabla^2 f| > 0 \rightarrow$ $\nabla^2 f$ exists. $\nabla^2 f$ set of Q $\nabla^2 f$ $\nabla^2 f$

2. $|\nabla^2 f| = 0$ at which
if $\nabla^2 f$ does not
exist.

3. $|\nabla^2 f| = 0$ and regular
?

let $CH(\mathcal{T})$

set of $ch(v)$ (internal
nodes) v

$\nabla^2 f \in CH(\mathcal{T})$ conjecture:
 $\nabla^2 f = \sum R_i \bar{C}_i e^{v_i}$
Kahle's

large g $\Rightarrow$ black holes
 large N
 $M \sim g^2 N$

attractor
 mechanism. G. Moore

10d $\mathcal{N}=2$
 $\downarrow$ CY M
 4d $\int C^{(1)} \rightarrow$ vector
 $\int C^{(2)} \rightarrow$ scalar
 $\int C^{(3)} \rightarrow$ scalar

$$S = \frac{A}{4} + \frac{A(\mathcal{U}_{\text{min}}(Q))}{4} \quad \text{with } |z| > 0$$

DPS black hole

$$r \frac{d}{dr} \omega(r) = -g \int \omega(r) \frac{d}{dr} F$$



$$F(\omega, r) = \left| \mathcal{Z}(\omega, Q) \right|^2 \left(1 - \frac{1}{4} \left(\frac{r}{r_0} \right)^2 \right)$$

$$\int \Omega \dots$$

metric model $\Rightarrow$ scalar fields
 Kaluza-Klein
 6d (B, C, H, I, J, K)
 - scalar ϕ
 - charge Q

$$\mathcal{L}_5 = e^{-A} \left(\frac{1}{2} \dot{\phi}^2 + \dots \right)$$

$\Rightarrow$ area of horizon $A(\mathcal{U}_{\text{min}}(Q))$

large g $\Rightarrow$ black holes
 large N
 $M = g^2 N$

low Π //

$\downarrow$ CY M
 $\mathcal{H} \subset C$
 $\mathcal{H} \subset C$

II $\Rightarrow$ 4d
 $\Rightarrow$ vector
 p.d.

metric $\Rightarrow$ scalar
 field ϕ

Kähler
 $(B = \frac{i}{2} \partial \bar{\partial} H^1)$

- scalar ϕ
 - charge Q

$Q^2 = e^{-A} \text{vol} \cdot |F|^2$
 $\Rightarrow$ area of horizon $A(\mathcal{H}_\infty)$

attractor
 mechanism. G.H. Horowitz

$$S = \frac{A}{4} = \frac{A(\mathcal{H}_\infty)}{4} \quad |z|^2 > 0$$

DPS static holes
 $\frac{1}{r} \frac{\partial}{\partial r} \omega(r) = -g \int \frac{1}{r} \frac{\partial}{\partial r} F$

$$F(\omega, F) = |Z(\omega, Q)|^2 \quad |F(\omega, F)|^2$$



$$= \left| \int_{\mathcal{H}_\infty} e^{i\pi} \wedge e^{i\pi} \sqrt{TdM} \right|$$

$$\int \Omega \wedge F$$

$$e^{|z|^2} > 0$$

provable, but

$$\exists \frac{c_{n+1}}{c_n} \in H^{1,1}(M, \mathbb{R}) \quad \Lambda = \sum_{i=1}^n \gamma_i \gamma_i^*$$

$$c_1(\mathcal{L}) = F$$



$$(z^1, \dots, z^{n+1}) \sim \lambda(z^1, \dots, z^{n+1}) \quad \text{for } z^i \neq 0$$

for of degree k : $\mathcal{O}(k)$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{j=1}^m \mathcal{O}(l_j) \rightarrow 0$$

$$\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} = \begin{pmatrix} z^{\lambda_1} \\ \vdots \\ z^{\lambda_n} \end{pmatrix}$$

$k_i = \deg V$

$ch(V) = \sum_{i=0}^n ch(\mathcal{O}(k_i)) = n \cdot ch(\mathcal{O}(k))$
 multiplicative $\chi \in H^2$
 not machine, $ch(V) \sim V$

$$cl(V) \otimes \mathbb{Q} \cong \mathbb{Q} \cdot cl(V)$$

not machine

$$\mu(V) = \frac{1}{n} \int_0^1 \chi(t) dt$$

given Q :

start when
do gradient flow
at $\tau^0(u, Q)$

1. $|z|_{\min} > 0$ $\rightarrow$ BH exists at $\tau^0(u, Q)$

2. $|z| = 0$ at which $\tau^0(u, Q) \dots$
BH under this
conditions does not
exist.
and regular.

3. $|z|_{\min} = 0$?

let $CH(\tau)$

set of $ch(v)$ (interval)
 $\{1, \dots, n\}$ possible v

$ATP \subset CH(\tau)$ conjecture: $ch = -ch$
 $ATP = \{R_e \bar{C} e^{i\theta} \mid \theta \in [0, 2\pi)\}$
CIC $\forall \theta \in [0, 2\pi)$ $\{ \tau, \tau' \mid \tau' > 0 \}$

given Q:

start when
do gradient flow
of $z^2(u, Q)$

1. $|z|_{\min} > 0$ $\rightarrow$ BH exists. $\leftarrow$ $\begin{matrix} \text{ATP}(\mathcal{T}) \\ \text{set of } Q \end{matrix}$ at

2. $|z| = 0$ at which
of $\mathcal{T}, \mathcal{T}' \dots$
Analogous
 $\rightarrow$ BH under this
criteria does not
exist.
and regular.

3. $|z|_{\min} = 0$
?

let $CH(\mathcal{T})$

set of $ch(v)$ (internal)
 $\{1, \dots, n\}$ possible v

$ATP \subset CH(\mathcal{T})$ conjecture: $ch_1 = -ch_2$
 $ATP = \{R_c \bar{C} e^{i\theta} \mid \theta \in \mathbb{R}\}$
CIC $\forall \theta \in \mathbb{R}$ stable one. $\mathcal{T}$ $(\mathcal{T}, \mathcal{T}' \mid \mathcal{T}' > 0.$

given Q :
 start when
 do gradient flow
 of $\mathcal{F}(u, Q)$

1. $|z|_{\min} > 0 \rightarrow \nabla H$ exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which ∇H does not exist.
 $\rightarrow \nabla H$ under this state does not exist.

3. $|z|_{\min} = 0$?

let $CH(T)$

set of $ch(v)$ (internal)
 $\{1, \dots, n\}$ must be v

$ATP \subset CH(T)$ conjecture: $ch_1 = -ch_2$
 $ATP = \{R_1 \bar{C} e^{i\theta} \mid \theta \in [0, 2\pi)\}$
 $C \in \mathbb{R}^n$ $\nabla H(T)$ $\nabla H(T)$ $\nabla H(T)$

$$|C_2| \leq |T|^{-1}$$



given Q :

start when
do gradient flow
if $z^2(u, Q)$

1. $|z|_{\min} > 0 \rightarrow \nabla H$ exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which ∇H does not exist.
at which ∇H does not exist.
at which ∇H does not exist.

3. $|z|_{\min} = 0$?

let $CH(T)$

set of $ch(v)$ (internal)
 $\{1, \dots, n\}$ possible v

$ATP = \{R_c \bar{C} e^{D_{H,T}}\}$ conjecture: $ch = -C_2$
 $C_1 C$ ∇H exists. $\{T\}$ (T, T) $\nabla H > 0$.

$|C_2| \leq \frac{1}{2} T$

given Q:
start when
do gradient flow
if $\nabla^2 f(u, Q)$

1. $|z_{\min}| > 0 \rightarrow \nabla^2 f$ exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which $\nabla^2 f$ does not exist.
Assumption:
 $\rightarrow \nabla^2 f$ under this
assumption does not
exist.

3. $|z|_{\min} = 0$
?

let $CH(T)$

set of $ch(v)$ (internal
nodes) $\nabla^2 f$ positive

$ATP \subset CH(T)$ conjugate: $ch = -c_2$

$$ATP = \{ R_c \bar{C} e^{i \theta} \mid \theta \in [0, 2\pi) \}$$

$C \in C$ Kähler cone. $\{T\}$ $(T, T) = 1$
 $\{T\} \{T\} > 0$

$$|C_2| \leq \frac{1}{2} \sqrt{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}$$

large g $\Rightarrow$ black holes
 large N
 $M = g \cdot N$

10d $\Rightarrow$ 11d

$\downarrow$ CY M
 $\int_C$ $\int_{C'}$
 $\int_C$ $\int_{C'}$

II $\Rightarrow$ 11d
 $\Rightarrow$ vector
 $\Rightarrow$ p.d.

metric $\Rightarrow$ scalar
 field $\Rightarrow$

Klein
 model $\Rightarrow$ in
 (B + iC) $\Rightarrow$ ψ

- scalars $\Rightarrow$ ψ
- charge Q

attractor
 mechanism. 6 M. mod

$$S = \frac{A}{4} + \frac{A(\log \det g)}{4} + \frac{1}{2} \epsilon |z|^2 > 0$$

DFS black holes

$$\frac{\partial}{\partial r} \psi(r) = -g \sqrt{1 + \frac{1}{4} \frac{\partial \psi}{\partial r}^2} F$$

$$f = |\tilde{z}(u, Q)|^2 > 0 \quad (1 - \frac{1}{2} \psi(r-1))$$

$$= \left| \int_{\Sigma} e^{i\theta} \wedge e^{i\theta} \sqrt{\det M} \right|^2$$

$$\int \Omega \wedge F$$

$$e^{i\theta} |z|^2 = 0$$

given Q :

start when
do gradient flow
if $z^2(u, Q)$

1. $|z|_{\min} > 0$ $\rightarrow$ ∇H exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which ∇H does not exist.

3. $|z|_{\min} = 0$?

let $CH(T)$

set of $ch(v)$ (normal)
 $\{1, \dots, n\}$ positive v

$ATP \subset CH(T)$ conjecture: $ch_1 = -c_2$
 $ATP = \{Re \bar{c} e^{i\theta} \mid \theta \in [0, 2\pi)\}$
 $c_1 c_2$ Kähler cone

$$|c_2| \leq |c_1|$$



given Q:

start when
do gradient flow
if $\nabla^2 f(x, Q)$

1. $|z|_{\min} > 0 \rightarrow$ BH exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which $\nabla^2 f(x, Q)$ is singular.
 $\rightarrow$ BH under this
assumption does not
exist.

3. $|z|_{\min} = 0$
?

let $CH(T)$

set of $ch(v)$ (normal)
s.t. $\exists$ μ -stable v

$ATP \subset CH(T)$ conjecture: $ch_1 = -c_2$
 $ATP = \{ R e^{\bar{c}} e^{i \theta} \}$
 $c \in \mathbb{C}$ $\nabla^2 f(x, Q)$ is positive definite
Kähler metric. $\{T\}$ (T, T) in $\mathbb{R}^n$
 $|c_2| \leq |T|$ $c_1 = 0$

$|c_2| \leq |T|$ $c_1 = 0$
 $c_2/ch \sim (c_1/ch)$

given Q :

start when
do gradient flow
at $z^0(x, Q)$

1. $|z|_{\min} > 0 \rightarrow$ BH exists. $\text{ATP}(T)$
 $\text{set of } Q$

2. $|z| = 0$ at which
 $z(T, z^0)$
 $\rightarrow$ BH under this
 exists does not
 exist.
 and regular

3. $|z|_{\min} = 0$
 ?

let $CH(T)$

set of $ch(v)$ (normal)
 $(1, \exists)$ μ -stable v

$c \in CH(T)$ conjugate, $ch_1 = -c_1$
 $ATP = \{ R_c \bar{C} e^{D_{\text{dist}}(T, z^0)} \}$
 CIC Kahle's $1/T$ (T, T) 1^{st}
 $|C_2| \leq 1/T$ $1 = \frac{1}{2} |C_1|^{1/2}$ $C_1 = 0$

$C_2/k \sim (C_1/k)^{1/2}$

given Q:

start with
do gradient flow
if $\mathbb{R}^2(u, Q)$

1. $|z|_{\min} > 0 \rightarrow$ BH exists. $ATP(T)$ set of Q

2. $|z| = 0$ at which $ATP(T) \subset CH(T)$
BH exists. $ATP(T) = \{Re \bar{C} e^{i\theta}\}$
BH exists. $CH(T)$ conjecture: $CH = -C_2$
BH exists. $CH(T)$ $\{T, T\}$ $T' > 0$

3. $|z|_{\min} = 0$?
BH exists. $CH(T)$ $|C_2| \leq T$ $C_1 = 0$
BH exists. $CH(T)$ $|C_2| \leq C_1$ $1, h$

let $CH(T)$
set of $ch(v)$ (internal)
s.t. $\exists$ μ -stable v

$ATP(T) \subset CH(T)$ conjecture: $CH = -C_2$
 $ATP(T) = \{Re \bar{C} e^{i\theta}\}$ $\{T, T\}$ $T' > 0$

$|C_2| \leq T$ $C_1 = 0$
 $|C_2| \leq C_1$ $1, h$

given Q :

start when
do gradient flow
at $z^2(u, Q)$

1. $|z|_{\min} > 0 \rightarrow$ BH exists. $ATP(T)$ set of Q at

2. $|z| = 0$ at which $z^2 = 0$.
Ass. singular.
 $\rightarrow$ BH under these
conditions does not
exist.

3. $|z|_{\min} = 0$?
with negative
EM - $A_4 C'$
 $\langle Q, Q' \rangle$

let $CH(T)$

set of $ch(v)$ (internal)
($\exists$ possible v)

$ATP \subset CH(T)$ conjecture: $ch_1 = -ch_2$
 $ATP = \{ Re \bar{C} e^{i \theta} \}$
 $C \in C$ where θ is the angle.

$|C_2| \leq |T| = \frac{1}{2} |C_1|$ $C_1 = 0$
 $|C_2| \sim (C_1/r)^2$
 $|C_2| \leq C_1^{1/2}$

given Q :
start when
do gradient flow
at $z^2(u, Q)$

let $CH(\mathcal{J})$

set of $ch(v)$ (internal)
(1.7) must be v

ATT(T)
cut : FQ

 $C(14(5))$

conjecture: $ch_2 = -C_2$

$$A\pi = \{R_c \bar{c} e^{i\theta_{c,T}}\}$$

CIC

$$(\mathbf{T}, \mathbf{T}) \geq 0.$$
$$|C_{jk}| \leq \frac{1}{T} \cdot \frac{C_1}{T_k} \quad C_1 = 0$$
$$C_{\text{eff}} \sim (C_{\text{eff}})^2$$
 $|c_3| \leq c_1, r_k.$

neg. $e m' - A m e'$
 $\langle Q, Q' \rangle > 0$
 and $\text{Im } z_{1/2}' > 0$

mod. $2\frac{1}{2}$ 70

given Q:

start when
do gradient flow
of $\mathcal{F}(u, Q)$

let $CH(\mathcal{T})$

set of $ch(v)$ (internal)
($\exists$ μ -stable v)

$ATP(\mathcal{T})$
set of Q

1. $|z_{\min}| > 0$

$\rightarrow$ BH exists. ATP

$C \in CH(\mathcal{T})$

conjugate: $ch_1 = -C_2$

$ATP = \{ R e \bar{C} e^{i \arg C} \}$

$C \in C$

$\{ \mathcal{T} \mid \mathcal{T} > 0 \}$

2. $|z| = 0$

at which
BH exists.

$\rightarrow$ BH under this
exists.

when
BH
can

$|C_2| \leq \mathcal{T} = \frac{C_1}{\mathcal{T}_k}$

$C_1 = 0$

3. $|z|_{\min} > 0$

?

and negative

$\langle Q, Q' \rangle > 0$

and $\text{Im } z/2 > 0$

$\exists$ $\langle \mathcal{T} \rangle$

$|C_2| \leq C_1$

$|C_2| \leq C_1$

given Q:

start when
do gradient flow
of $\mathbb{Z}^2(\mathbb{C}, Q)$

let $CH(\mathbb{T})$

set of $ch(v)$ (internal)
($\exists$ postulate v)

$ATP(\mathbb{T})$
set of Q

1. $|z_{min}| > 0 \rightarrow$ BH exists. ATP

2. $|z| = 0$ at which
if $\mathbb{T}$ is a
Assumption
 $\rightarrow$ BH under this
assumption does not
exist.

$ATP \subset CH(\mathbb{T})$

$ATP = \{ R e^{i\theta} e^{i\phi} \}$
where θ, ϕ are
variables.

conjecture: $ch_1 = -C_2$

$|C_2| \leq \mathbb{T} \cdot \mathbb{T} = \mathbb{T}^2$
if $\mathbb{T} > 0$.

3. $|z|_{min} = 0$
?

and negative
 $e^M = A e^C$
 $\langle Q, Q' \rangle > 0$
and $\text{Im } z/2 > 0$
p. 3 (1.1)

$|C_2| \leq C_1^2$
if $C_1 > 0$

μ -stable. Lb.

$$\begin{aligned} & \text{r.h.s.} \\ & \text{if } G/H^1(H, \mathbb{Z}) \\ & \Rightarrow C_1(\mathcal{L}) = F \end{aligned}$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(1) \rightarrow \mathcal{O}(2) \rightarrow 0$$



$$\begin{aligned} & \text{Lb. } \mathbb{P}^n \\ & \Lambda = \mathbb{Z}^n \\ & 5\Lambda = \mathbb{Z}^{5n} \end{aligned}$$

$$(z_1, \dots, z_n) \sim \lambda(z_1, \dots, z_n) \quad \text{for } z_i \neq 0$$

$$\text{For } \mathbb{C}^n \text{ define } k \text{ } \rightarrow \text{ } \mathbb{C}^n \text{ } \rightarrow \mathbb{C}^n \text{ } \rightarrow \mathbb{C}^n$$

$$V \xrightarrow{f} V$$

$$\rightarrow 0$$

$$e^{kT}$$

$$\begin{aligned} & \text{For } \mathbb{C}^n \text{ } \rightarrow \mathbb{C}^n \\ & C_1(V) = \mathbb{Z} \geq 0 \\ & C_1(V) = 0 \end{aligned}$$

$$\begin{aligned} & \text{ch } \mathcal{O}(1) = \text{ch } \mathcal{O}(k) \\ & \text{for } k \geq 1 \end{aligned}$$

$$\begin{aligned} & \text{For } \mathbb{C}^n \text{ } \rightarrow \mathbb{C}^n \\ & \mu(V) = \frac{1}{n} \int \text{ch}(V) \text{ch}(T^* \mathbb{C}^n) \end{aligned}$$

μ -stable. Lb.

$$\sigma^n \quad (z^1, \dots, z^n) \sim \lambda(z^1, \dots, z^n) \quad / \quad z^i = 0$$

for $z^i \neq 0$

$\pi_1 \in H^1(M, \mathbb{Z})$ $A = \mathbb{C}^n$ $\text{Far of degree } k$ $\text{such that } \mathbb{C}^n \ni k \mathbb{Z}$

$\Rightarrow C(\mathcal{L}) = F$ $5A = \mathbb{Z}^n$ $V' \rightarrow V \xrightarrow{f} V'$ $\mathcal{O}(k)$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{f} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

$\text{reg. } \mathcal{O}(V) \cap \mathbb{Z} \geq 0$
 $\mathcal{O}(V) = 0$

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \in \begin{pmatrix} z^1 \\ \vdots \\ z^n \end{pmatrix}$$

$k_i \in \mathbb{Z} \rightarrow V$ $\text{ch}(V) = \sum \text{ch } \mathcal{O}(k_i) = \sum \text{ch } \mathcal{O}(l_i)$

$$c(V \oplus V') = c(V) + c(V')$$

$\text{not } \mu$ -stable. $0 \rightarrow V' \rightarrow V \rightarrow V/V' \rightarrow 0$ $\mu(V) + \frac{1}{2} \mu(V/V') \geq \mu(V)$

given Q:
start when
do gradient flow
if $z^2(u, Q)$

let $CH(\mathcal{T})$

set of $ch(u)$ (internal)
($\exists$ possible u)

$ATP(\mathcal{T})$

set of Q

at

exists.

ATP

$\in CH(\mathcal{T})$

conjugate: $ch_1 = -ch_2$

$$ATP = \{ Re \bar{C} e^{i \theta} \}$$

$C \in C$

where $u, \mathcal{T}$

$(\mathcal{T}, \mathcal{T})$ in $\mathcal{T}' > 0$.

2. $|z| = 0$ at which
if $\mathcal{T}, \mathcal{T}'$
Ass. simple
 $\Rightarrow$ BM under this
exists does not
exist.

where
BM
can be

$$|C_2| \leq \frac{1}{\mathcal{T}} \cdot \frac{1}{\mathcal{T}'} = \frac{1}{\mathcal{T} \mathcal{T}'}$$

$C_1 = 0$

3. $|z| = 0$ with negative
EM - AM C'

$$\langle Q, Q' \rangle > 0$$

and $\Im m \mathcal{Z}/\mathcal{Z}' > 0$
p. 3 (1.1)

$$|C_2| \leq C_1, \text{ ch.}$$

μ -stable. b.

$$\sigma^n \left(\bigoplus_{i=1}^n (z'_1, \dots, z'_n) \right) \sim \lambda(z'_1, \dots, z'_n) / z'_1 \cdot 0$$

$\exists \pi \in H^1(M, \mathbb{Z})$ $\Lambda = \pi^* \mathbb{Z}$ far is degree k : $\pi^* \mathbb{Z} = k \mathbb{Z}$

$$C_1(\mathcal{L}) = F$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(1) \xrightarrow{F} \mathcal{O}(1) \rightarrow 0$$

$$\mathcal{O}(k)$$

degree $c_1(\mathcal{L})$

$$c_1(V) \wedge \mathbb{Z} \geq 0$$

$$c_1(V) = 0$$

$$k \in \mathbb{Z}$$

$ch(V) = \text{trch } \mathcal{O}(1) = n \cdot ch \mathcal{O}(1)$

not μ -stable. $0 \rightarrow V' \rightarrow V$ $\mu(V') > \mu(V)$

$$\mu(V) = \frac{1}{\text{rk}(V)} \int c_1(V) \wedge \omega$$

μ -stable. Lb.

$$\mathbb{C}P^n \quad (z_1, \dots, z^{n+1}) \sim \lambda(z_1, \dots, z^{n+1}) \quad / \quad z_i = 0$$

$$\pi_1 \in H^1(M, \pi)$$

$$\Lambda = \mathbb{Z}^2$$

for of degree k : $\pi_1 \in H^1(M, \pi)$

$$C(\mathcal{L}) = F$$

$$5A \cong \mathbb{Z}^2$$

$$\mathcal{O}(k)$$

$$0 \rightarrow V \rightarrow \bigoplus_{i=1}^n \mathcal{O}(k_i) \xrightarrow{F} \bigoplus_{i=1}^n \mathcal{O}(l_i) \rightarrow 0$$

entire

$$c(V) = T \geq 0$$

$$\left(\begin{smallmatrix} \vdots \\ \vdots \end{smallmatrix} \right) \in \mathbb{Z}^{l-k}$$

$$k \in \mathbb{Z} \rightarrow V$$

$ch(V) = e^{c_1(V)}$
 physical
 stability
 $ch(\mathcal{O}(k)) = e^{c_1(\mathcal{O}(k))}$
 not
 $ch(V) = e^{c_1(V)}$

$$c_1(V) \otimes c_1(K)$$

$$c_1(V) \otimes c_1(K) \rightarrow V' \rightarrow V \quad \mu(V) = \frac{1}{2} \int c_1(V) \wedge c_1(K)$$

given Q :
 start when
 do gradient flow
 at $z^2(u, Q)$

let $CH(\mathcal{T})$

set of $ch(v)$ (internal)
 $\{1, \dots, n\}$ possible v

$ATP(\mathcal{T})$

set of Q

exists. ATP

$c \in CH(\mathcal{T})$

conjecture: $ch_1 = -c_2$

$$ATP = \{Re \bar{c} e^{i\theta} \mid \theta \in [0, 2\pi)\}$$

$c \in C$

$(\mathcal{T}, \mathcal{T})$ in $\mathcal{T}' > 0$.

$$|c_2| \leq \frac{1}{\sqrt{2}} \sqrt{\sum_{k=1}^n |c_k|^2} \quad c_1 = 0$$

$$|c_2| \leq c_1, \text{ i.e.,}$$

2. $|z| = 0$ at which
 $z^2(u, Q)$ is a minimum.
 $\rightarrow$ BM under this
 exists does not
 exist.

3. $|z| = 0$?
 $\langle Q, Q' \rangle > 0$
 and $\Im m z^2(u, Q) > 0$
 $\exists \langle \mathcal{T}, \mathcal{T}' \rangle$

given Q :

start when
do gradient flow
at $z^2(u, Q)$

1. $|z|_{\min} > 0 \Rightarrow$ BH exists. $ATP(T)$ set of Q at

2. $|z| = 0$ at which $z(T, z)$ is singular.
 $\Rightarrow$ BH under these conditions does not exist.

3. $|z|_{\min} = 0$?
 $\langle Q, Q' \rangle > 0$
and $\text{Im } z/2' > 0$
p. 3 (p. 13)

let $CH(T)$

set of $ch(v)$ (internal)
($\exists$ possible v)

$ATP \subset CH(T)$ conjecture: $ch_1 = C_1$
 $ATP = \{ R e^{\bar{C}} e^{i \theta} \}$
 $C \in CH(T)$ stable in $CH(T)$ if $\theta' > 0$.

$|C_2| \leq \frac{1}{T} \sum_{k=1}^T |C_k|$ $C_1 = 0$

$|C_2| \leq C_1$ (ph)

given Q :
start with
the gradient flow
of $z^2(x, Q)$

let $CH(5)$

set of $ch(v)$ (internal)
s.t. $\exists$ possible v

$$ATP(T)$$
$$c \in (H(\mathcal{T}))$$

conjecture: $ch_2 = C_2$

$$A_{TT} = \{ R_L \bar{C} e^{D_{L,T}} \}$$

CIC

מ'תש"ח
ב' תשרי
ה'תש"ח

$$\begin{pmatrix} T & T & 1' \\ r' & & \end{pmatrix} \geq 0.$$
$$|C_3| \leq |J| = |C_1| \quad C_1 = 0$$
$$C_3/h = (C_3/h)^2$$
$$|c_i| \leq c_i, r_k.$$
$$\langle Q, Q' \rangle = 0$$

Q, Q
and $\lim_{t \rightarrow \infty} z(t) = 0$
p. 3 (p. 1)

given Q :
 start when
 do gradient flow
 of $z(u, Q)$

let $CH(\mathcal{T})$

set of $ch(v)$ (internal)
 $\{1, \dots, n\}$ possible v

$ATP(\mathcal{T})$
 set of Q

1. $|z_{\min}| > 0 \Rightarrow$ BH exists. ATP

$C \in CH(\mathcal{T})$ conjugate: $ch = -C$

$ATP = \{Re \bar{C} e^{i \arg C}\}$

2. $|z| = 0$ at which z is 0

$\Rightarrow$ BH under this
 exists does not exist.

$|C_k| \leq \frac{1}{\sqrt{2}} \sqrt{\frac{C_1^2}{C_k^2}}$ $C_1 = 0$

3. $|z|_{\min} = 0$

?

$\langle Q, Q' \rangle > 0$

and $\Im m z / z' > 0$
 $\exists \langle p, q \rangle$

$|C_k| \leq C_i$