

Title: Introduction to quantum gravity - Part 18

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

-undergraduate quantum mechanics

-basics of classical gauge field theories

-basic general relativity

-hamiltonian and lagrangian mechanics

-basics of lie algebras

N parts

$$\hat{A}(s) = \lim_{n \rightarrow \infty} \int \sqrt[n]{nE^2}$$

$$A(s) = \int u^2 \sqrt{h}$$

check

Volume



No better than...
 ...
 ...



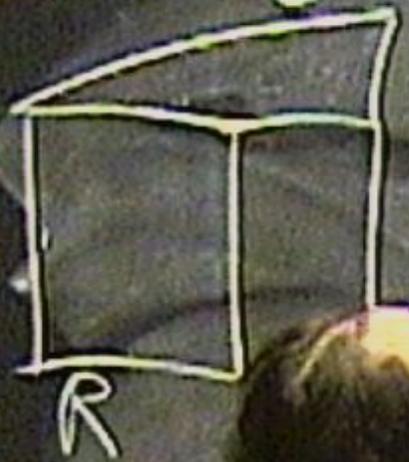
N.P. 143

$$\hat{A}(s) = \lim_{\Delta z \rightarrow 0} \left(\prod_{k=1}^n \left(1 + \frac{\Delta z}{h} \sqrt{h^2 E^2 - 1} \right) \right)$$

$$A(s) = \int_{-1}^1 \sqrt{1 - s^2} ds$$

check

Volume



$$V(R) = \int_{-R}^R \sqrt{1 - x^2} dx$$

No better way to compute
 the volume of a cylinder

$$\hat{A}(S) = \lim_{\rho \rightarrow \infty} \int_{\rho}^{\rho+d\rho} \sqrt{|h|} E^2$$

$$A(S) = \int d^2x \sqrt{h}$$

check

Volume

$$V(R) = \int \sqrt{|\det g|}$$

$$\det \tilde{E}_i = \det[\sqrt{2} e_i^a]$$

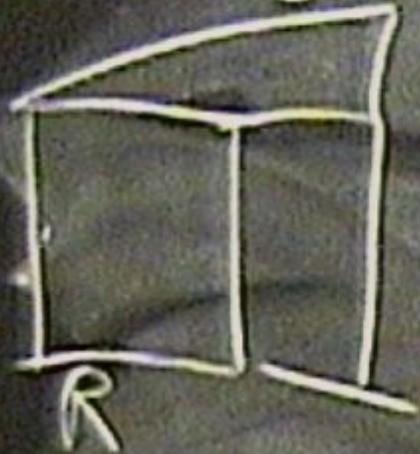
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$$\hat{A}(S) = \lim_{M \rightarrow \infty} \int \sqrt{|h_E|}$$

$$A(S) = \int \sqrt{|g|} \quad \text{check}$$

Volume



$$V(R) = \int \sqrt{|g|}$$

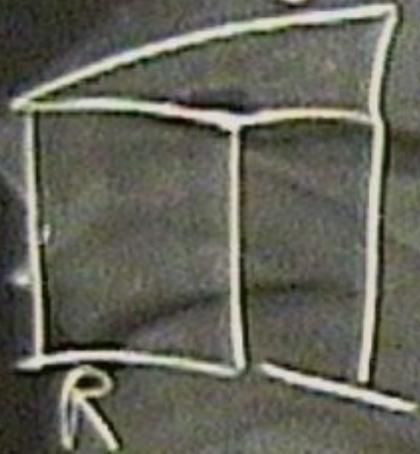
$$\det \tilde{E} = \det(\sqrt{g} e_i^a)$$

$$\frac{1}{3!} \epsilon_{ijk} \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$

$$\hat{A}(S) = \lim_{M \rightarrow \infty} \left(\frac{1}{M} \sum_{i=1}^M \sqrt{h_i^2 + z_i^2} \right)$$

$$A(S) = \int_{\text{check}} \sqrt{h^2 + z^2}$$

Volume



$$V(R) = \int \sqrt{h^2 + z^2}$$

$$M \cdot \sqrt{z^2} = |Q|$$

$$\det \tilde{E}_i = \det \begin{bmatrix} \sqrt{z} & z_i^2 \end{bmatrix}$$

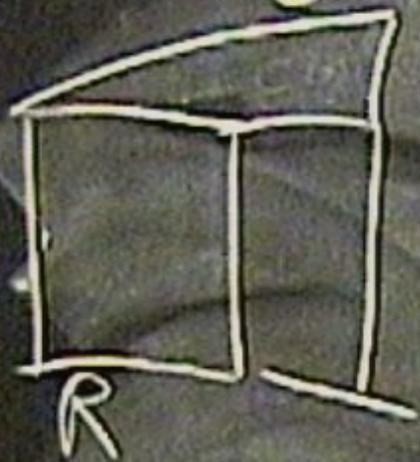
$$\tilde{E}_1, \tilde{E}_2, \tilde{E}_K$$

$$\hat{A}(S) = \lim_{\rho \rightarrow \infty} \int_{\rho}^{\rho+d\rho} \sqrt{|h|} dz$$

$$A(S) = \int_{\Sigma} \sqrt{|h|}$$

check

Volume



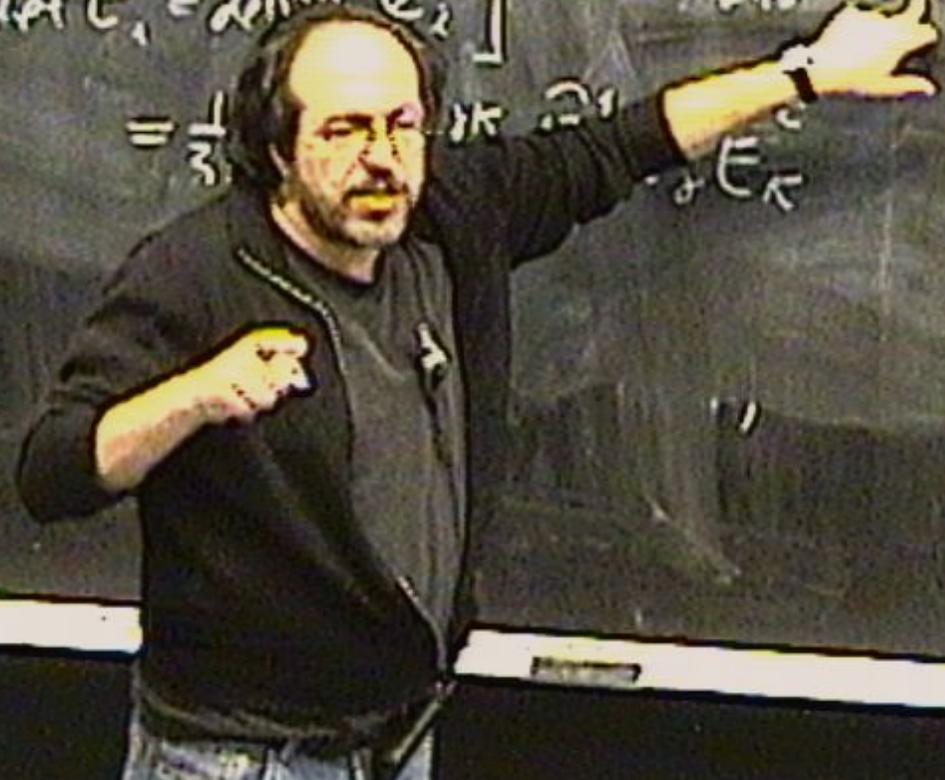
$$V(R) = \int \sqrt{|\det g|}$$

$$\det \tilde{E}_i = \det [e_1^a, e_2^a]$$

$$= \frac{1}{3!} \epsilon^{abc} e_1^a e_2^b e_3^c$$

$$\sqrt{|g|} = |\det g|$$

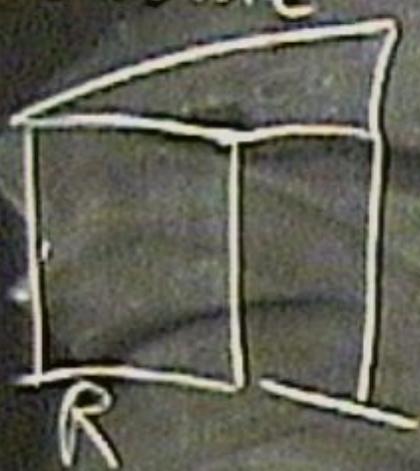
$$g_{ab} = e_i^a e_j^b$$



$$\hat{A}(S) = \lim_{\Delta \rightarrow 0} \sum \sqrt{h_{E^2}}$$

$$A(S) = \int \sqrt{h} \quad \text{check}$$

Volume



$$V(R) = \int \sqrt{\det g}$$

e_i

$$N = \sqrt{g} = |\det g|$$

$$g_{ab} = e_a^i e_b^j g_{ij}$$

$$\det \tilde{E}_i = \det[\sqrt{g} e_i^a]$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$



$$\hat{A}(S) = \lim_{\Delta \sigma \rightarrow 0} \sum \sqrt{\Delta \sigma^2}$$

$$A(S) = \int d^2x \sqrt{h}$$

check

Volume



$$V(R) = \int \sqrt{\det g}$$

e_i^a

$$N = \sqrt{g} = |\det e_i^a|$$

$$g_{ab} = e_i^a e_j^b$$

$$\det \tilde{E}_i^a = \det [\sqrt{g} e_i^a]$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$

$$= \frac{1}{3!} g^{\frac{3}{2}} \det e^a$$

$$\hat{A}(S) = \lim_{\rho \rightarrow \infty} \int_{\rho}^{\rho+d\rho} \sqrt{h} E^2$$

$$A(S) = \int d^2x \sqrt{h}$$

check

Volume



$$V(R) = \int \sqrt{\det g}$$

$$N \sqrt{g} = |\Omega|$$

$$g_{\mu\nu} = e_{\mu}^i e_{\nu}^j g_{ij}$$

$$\det \tilde{E}_i = \det [e_{\mu}^i e_{\nu}^j g_{ij}]$$

$$= \frac{1}{3!} \epsilon_{\mu\nu\kappa} \epsilon^{\alpha\beta\gamma} \tilde{E}_\mu^\alpha \tilde{E}_\nu^\beta \tilde{E}_\kappa^\gamma$$

$$= \epsilon^3 \det \Omega = \frac{1}{3!} \epsilon^{\alpha\beta\gamma} \epsilon_{\mu\nu\kappa} \tilde{E}_\mu^\alpha \tilde{E}_\nu^\beta \tilde{E}_\kappa^\gamma$$



$$\hat{A}(S) = \lim_{M \rightarrow \infty} \int \sqrt{|h_E|} \quad ||$$

$$A(S) = \int d^3x \sqrt{h}$$

check

Volume



$$V(R) = \int \sqrt{|det q|}$$

e_i^a

$$N \sqrt{|q|} = |Q|$$

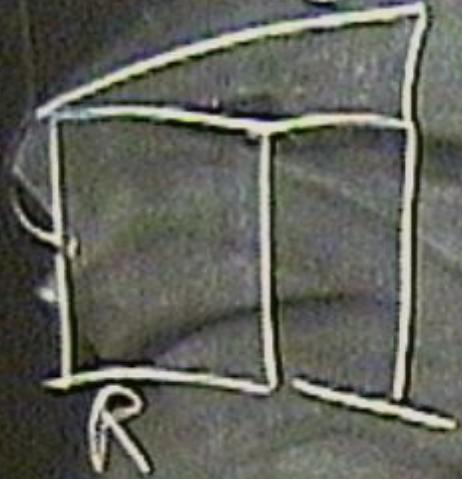
$$q_{ab} = e_a^i e_b^j g_{ij}$$

$$det \hat{E}_i = det[\sqrt{|q|} e_i^a]$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \hat{E}_i^a \hat{E}_j^b \hat{E}_k^c$$

$$= \epsilon^{abc} \det e_i^a = \det q_{ab}$$

Volume



$$V[R] = \sqrt{\det q}$$

$$N = \sqrt{q} = |\Omega|$$

$$q_{ab} = \Omega_a \cdot \Omega_b$$

$$\det \tilde{E}_i = \det[\Omega_i^a]$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$

$$= \Omega^3 \det \Omega' = \Omega^3 = \det q_{ab}$$

$$\Rightarrow V[R] = \sqrt{\det \tilde{E}_i} = \sqrt{\frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c}$$

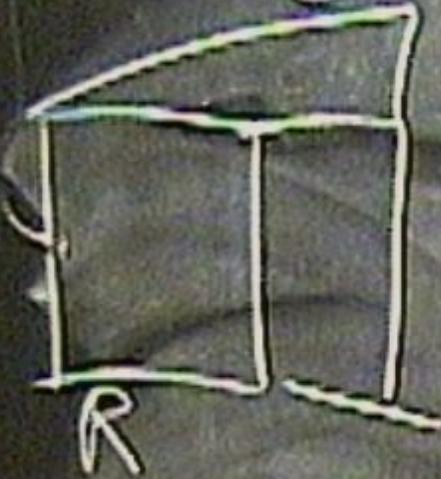


$$y = z = 0$$

$$\delta(5,0)$$

$$\delta(6,0) \quad \delta(6,5)$$

Volume



$$V(R) = \int \sqrt{\det g}$$

$$N = \sqrt{g} = |\Omega|$$

$$g_{ab} = \Omega_a^i \Omega_b^j$$

$$\det \tilde{E}_i = \det[\sqrt{g} \Omega_i^a]$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$

$$= \epsilon^{abc} \det \Omega^i = \frac{1}{3!} \epsilon^{abc} \det g_{ab}$$

$$V(R) = \int \sqrt{|\det \tilde{E}_i|} = \int \sqrt{\frac{1}{3!} \epsilon^{abc} \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c}$$

$$\begin{aligned} z=0 \\ y=0 \\ x=0 \end{aligned}$$

$$\begin{aligned} \delta(5,0) \\ \delta(6,0) \end{aligned}$$

$$\chi(\psi) = X$$



$$W_I \approx \text{det}$$

A I L

$$y(x) = x$$



$$W_I \approx \det E_I$$

$$\sqrt{\Delta}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{W_I}$$

$$\chi(x) = x$$



$$W_I \approx \det \tilde{E}_I$$

$$\hat{V}[R] \equiv \lim_{N \rightarrow \infty} \sum_I \sqrt{W_I}$$



$$\chi(x) = x$$



$$W_I \approx \det E_I$$

$$\hat{V}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{|W_I|}$$



$$W_I = \int d^3x \int d^3x' \int d^3x'' \begin{vmatrix} \tilde{E}^a(x) & & \\ & \tilde{E}^b(x') & \\ & & \tilde{E}^c(x'') \end{vmatrix}$$

$$Y(x) = X$$



$$W_I \approx \det E_I$$

$$\hat{V}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{|W_I|}$$



$$W_I = \int d^3x \int d^3x' \int d^3x'' \left(\det \left(\begin{matrix} E^1(x) u_{x^1} & E^2(x) u_{x^2} & E^3(x) u_{x^3} \\ E^1(x') u_{x^1} & E^2(x') u_{x^2} & E^3(x') u_{x^3} \\ E^1(x'') u_{x^1} & E^2(x'') u_{x^2} & E^3(x'') u_{x^3} \end{matrix} \right) \right)$$

$$\chi(x) = x$$



$$W_I \approx \text{det } E_{x'}^{x''}$$

$$\hat{V}[R] \equiv \lim_{N \rightarrow \infty} \sum_I \sqrt{|W_I|}$$

$$W_I = \int d^3x \int d^3x' \int d^3x'' \left(\text{det } E \left(E^A(x) U_{x''}^{x'}, E^B(x') U_{x''}^{x'}, E^C(x'') U_{x''}^{x'} \right) \right)$$





$$W_I \approx |\det E_I|$$

$$\hat{V}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{M V_I}$$



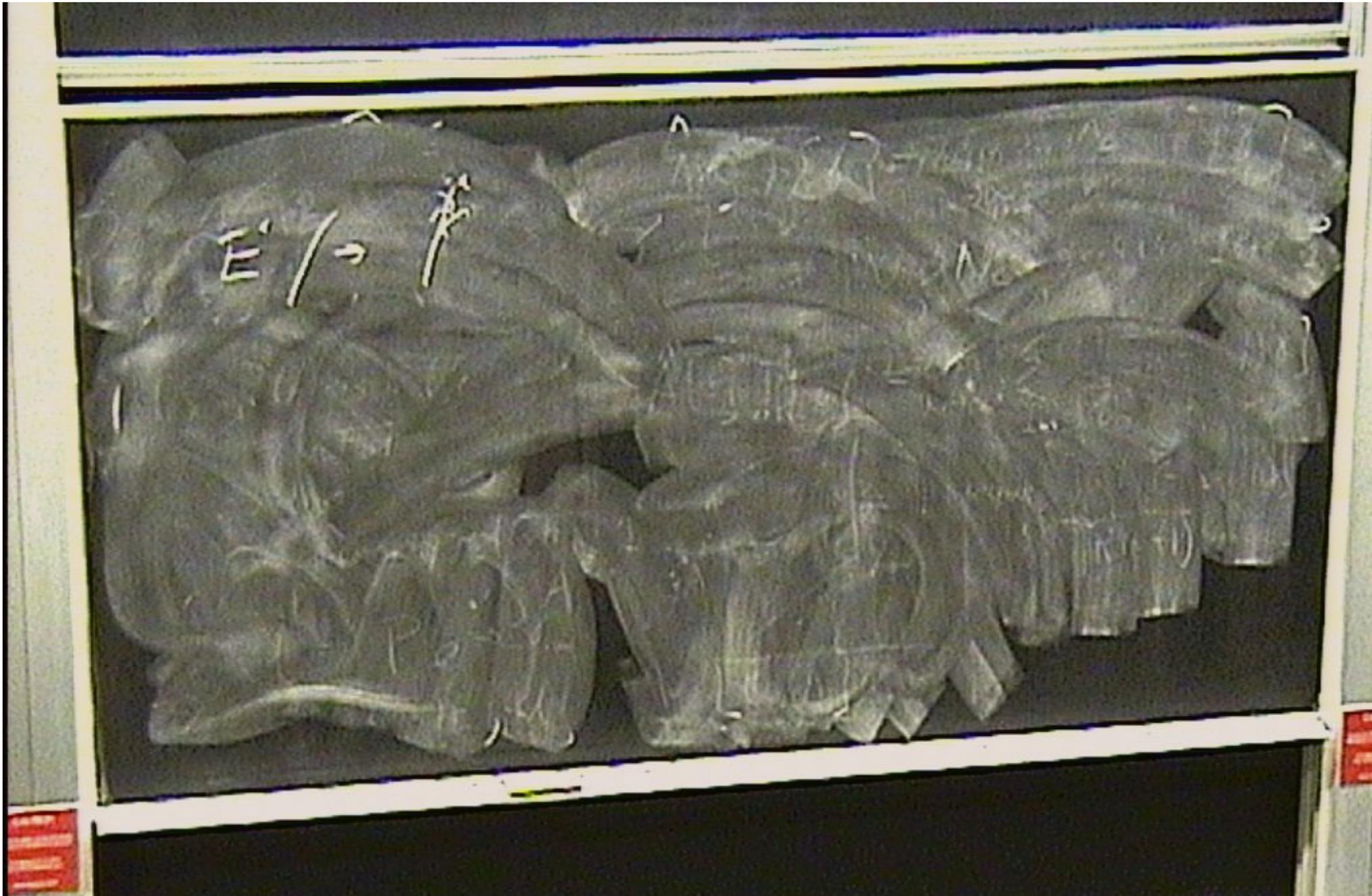
$$W_I = \int_{L^3} d^3x \int_{L^3} d^3x' \int_{L^3} d^3x'' \left(\det \left(\begin{matrix} E^a(x) U_{x^a} & E^b(x') U_{x'^b} & E^c(x'') U_{x''^c} \end{matrix} \right) \right)$$

$$|I| = 2=0$$

$$X = 4 \quad Y = 0$$

$$|I| = 2=0$$

$$S(0,0) \subset (0,2,0)$$



$$E' \rightarrow \text{graph}$$

$$E^a E^b(x)$$

$$\vec{E} \rightarrow \vec{F} = \int \vec{E} \cdot \vec{r} \, dV$$

$$| E^a E^b |$$

$$E' \rightarrow \vec{f} = \int \delta^3(\mathbf{r}-\mathbf{r}') \delta^3(\mathbf{r}-\mathbf{r}'')$$

$$| E^a E^b(x) | \rightarrow \delta^a \delta^b$$

$$E' \rightarrow \tilde{f} = \int \delta(x) \delta(y) \delta(z) \delta(t) \delta(x^2) \delta(y^2) \delta(z^2) \delta(t^2)$$

$$E \wedge E^b \rightarrow \tilde{\delta}^b \tilde{\gamma}^b$$

$$E' \rightarrow \vec{f} = \int \delta^3(\vec{r}-\vec{r}') \delta^3(\vec{r}-\vec{r}'')$$

$$E' \rightarrow \vec{f} = \int \delta^3(\vec{r}-\vec{r}') \delta^3(\vec{r}-\vec{r}'') (\vec{y}^L + \vec{y}^R) \delta^3(\vec{r}-\vec{r}')$$

$$E' \rightarrow \vec{f} = \int \delta^3(\vec{r}-\vec{r}') \delta^3(\vec{r}-\vec{r}'') \delta^3(\vec{r}-\vec{r}''')$$

$$E_{(6)}^{(4)} \rightarrow \delta^6(\vec{y}) / \delta^4(\vec{y} + \delta^4 \vec{y})$$

$$E' \rightarrow \mathbb{F} = \sum_{i=0}^{\infty} \mathbb{F}^i(x)$$

$$E \left(\begin{matrix} E^i(x) \\ E^j(x) \end{matrix} \right) \rightarrow \mathbb{F}^i \mathbb{F}^j = \mathbb{F}^{i+j}$$

$$E^i \left(\begin{matrix} E^j \\ E^k \end{matrix} \right) \rightarrow \mathbb{F}^i \mathbb{F}^j \mathbb{F}^k$$

$$E' \rightarrow \mathbb{F} = \sum_{i=1}^n \delta^i \omega^i(\rho, \gamma^i)$$

$$E \begin{matrix} \xrightarrow{\gamma^1} \\ \xrightarrow{\gamma^2} \\ \xrightarrow{\gamma^3} \\ \xrightarrow{\gamma^4} \end{matrix} \rightarrow \sum_{i=1}^n \delta^i \omega^i(\rho, \gamma^i)$$

$$E \begin{matrix} \xrightarrow{\gamma^1} \\ \xrightarrow{\gamma^2} \\ \xrightarrow{\gamma^3} \\ \xrightarrow{\gamma^4} \end{matrix} \rightarrow \sum_{i=1}^n \delta^i \omega^i(\rho, \gamma^i)$$

$$E' \rightarrow \mathbb{F} = \sum_{i=0}^{\infty} \delta^i \mathbb{F}^i(\mathbb{F})$$

$$E \begin{matrix} E^1 \\ E^2 \\ \vdots \\ E^i \end{matrix} \rightarrow \sum_{i=0}^{\infty} \delta^i \mathbb{F}^i(\mathbb{F})$$

$$E \begin{matrix} E^1 \\ E^2 \\ \vdots \\ E^i \end{matrix} \rightarrow \sum_{i=0}^{\infty} \delta^i \mathbb{F}^i(\mathbb{F})$$

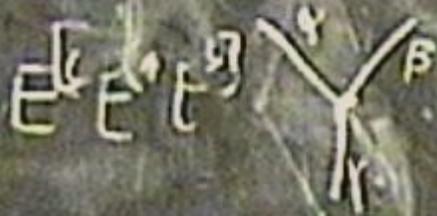
$$E' \rightarrow \tilde{f} = \sum_{i,j} \delta^i \delta^j \delta^k \delta^l \delta^m \delta^n$$

$$E^{(a)} E^{(b)} \rightarrow \delta^a \delta^b \delta^c \delta^d \delta^e \delta^f \delta^g \delta^h \delta^i \delta^j \delta^k \delta^l \delta^m \delta^n$$

$$E^{(a)} E^{(b)} \vee$$

$$E' \rightarrow \vec{f} = \int dS \delta^3(x) \delta^3(y) \delta^3(z)$$

$$E^{(a)} E^{(b)} \rightarrow \delta^6 \delta^6 \delta^6 \delta^6 (\delta^6 + \delta^6 + \delta^6 + \delta^6)$$



$$E' \rightarrow \mathbb{F} = \int \alpha \delta^{\mu\nu} \beta^{\rho\sigma} \gamma^{\lambda\tau} \delta^{\mu\nu\rho\sigma\lambda\tau}$$

$$E_{\alpha}^{\mu} E_{\beta}^{\nu} \rightarrow \delta^{\mu\nu} \gamma^{\rho\sigma} \delta^{\mu\nu\rho\sigma}$$

$$E_{\alpha}^{\mu} E_{\beta}^{\nu} E_{\gamma}^{\rho} \rightarrow \delta^{\mu\nu\rho} \alpha^{\sigma} \beta^{\lambda} \gamma^{\sigma}$$

$$E' \rightarrow \vec{f} = \int dS \delta^3(\mathbf{r} - \mathbf{r}(t)) \mathbf{W}_I \mathbf{T}(\mathbf{r}) = \sum_{\text{nodes}}$$

$$E \left(\begin{matrix} E^a \\ E^b \end{matrix} \right) \rightarrow \delta \left(\begin{matrix} E^a \\ E^b \end{matrix} \right) / \delta$$

$$E \left(\begin{matrix} E^a \\ E^b \\ E^c \end{matrix} \right) \left(\begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} \right)$$



$$E' \rightarrow \vec{k} = \int \delta^3(x) \delta^3(x-x') \delta^3(x-x'') \dots \delta^3(x-x^{(n)}) \left(W_I \Pi(\Omega) \right) = \sum_{\text{notes}}$$

$$E^{(a)} E^{(b)} \rightarrow \delta^{(a)} \delta^{(b)} \delta^{(c)}$$



$$E^{(a)} E^{(b)} E^{(c)} \left(\begin{array}{c} \text{Y} \\ \text{X} \end{array} \right) \alpha^{(a)} \beta^{(b)} \gamma^{(c)}$$



$$W_I \approx \det E_{ij}$$

$$\Delta[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{|W_I|}$$

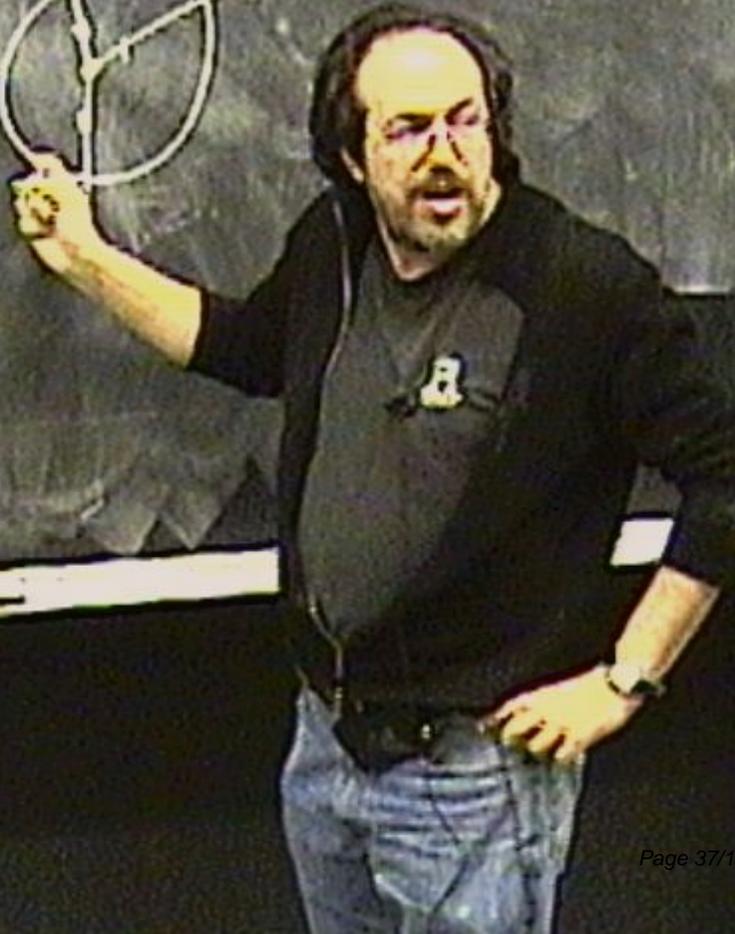
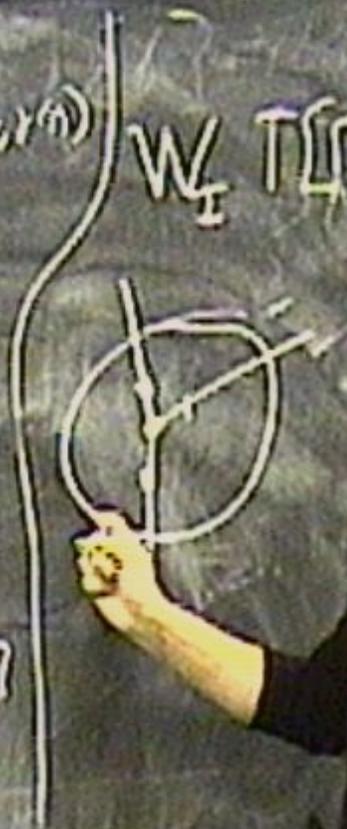


$$W_I = \int d^3x \int d^3x' \int d^3x'' \left[\det \left(E(x) U_{x'x} E(x') U_{x''x'} E(x'') U_{xx''} \right) \right]$$

$$E' / \rightarrow \vec{f} = \left(\int \delta^3(x) \delta^3(y) \delta^3(z) \delta^3(w) \right) W_E T(\Gamma) = \sum_{\text{nodes}}$$

$$E \begin{matrix} (a) \\ (b) \end{matrix} \left(\begin{matrix} (c) \\ (d) \end{matrix} \right) \rightarrow \begin{matrix} \delta^3(x) \\ \delta^3(y) \end{matrix} / \delta^3(z)$$

$$E \begin{matrix} (c) \\ (d) \end{matrix} \left(\begin{matrix} (a) \\ (b) \end{matrix} \right) \rightarrow \delta^3(x) \delta^3(y) \delta^3(z)$$



$$E' \rightarrow \vec{k} = \int \delta^3(\vec{r}) \delta^3(\vec{r}-\vec{r}') \delta^3(\vec{r}-\vec{r}'') \dots W_I \Pi(\vec{r}) = \sum_{\text{nodes}}$$

$$E_{(a)}^{(b)} E_{(c)}^{(d)} \rightarrow \delta^{(a)} \delta^{(b)} \delta^{(c)} \delta^{(d)}$$



$$E^{(a)} E^{(b)} E^{(c)} \rightarrow \delta^{(a)} \delta^{(b)} \delta^{(c)}$$

$$G = G' = G''$$

$$= \frac{6(h\nu) \pm h\nu c (2l+1)}{c}$$

$$hG = \lambda_{Pe}^2 = 10^{-66} \text{ cm}^2$$

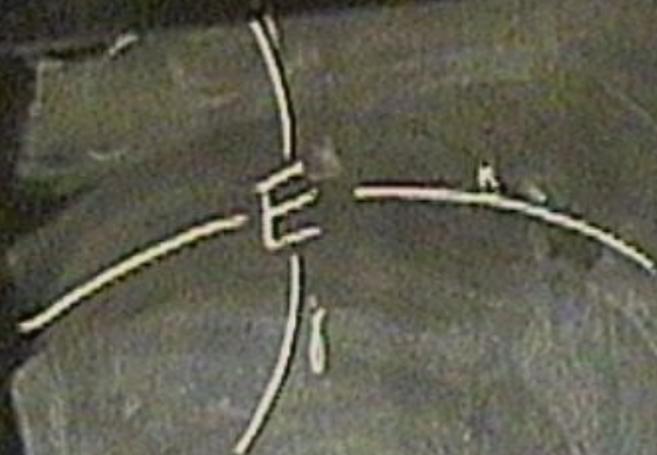
$$\delta_{\lambda\gamma} \tau(\tau_{\lambda}^i \tau_{\lambda}^j) = (1/2)K(K+1)$$



$$G = G = G$$

$$\hbar G = l_{Pl}^2 = 10^{-66} \text{ cm}^2$$

$$\delta_{xy} \Gamma(\gamma^x \gamma^y) = (1/2) k(k+1)$$



$$\hbar G k(k+1)$$

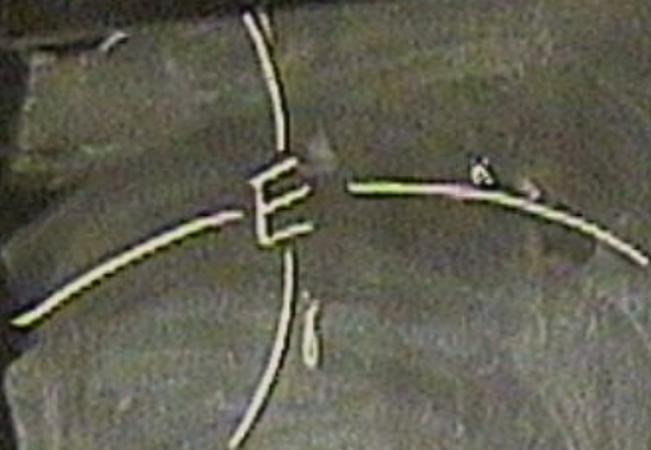
$$\equiv (i \gamma^x E U_y) U_k$$

$$G = G' = G''$$

$$= \frac{6(20) \pm \dots}{\dots}$$

$$\hbar G = l_{Pl}^2 = 10^{-66} \text{ cm}^2$$

$$\delta_{xy} \prod_{m=1}^k \prod_{n=1}^k = (1/k(k+1))$$



$$\hbar G = k(k+1)$$



$$\equiv \left(\int_{\mathcal{M}} \epsilon U_y \right) U_k$$

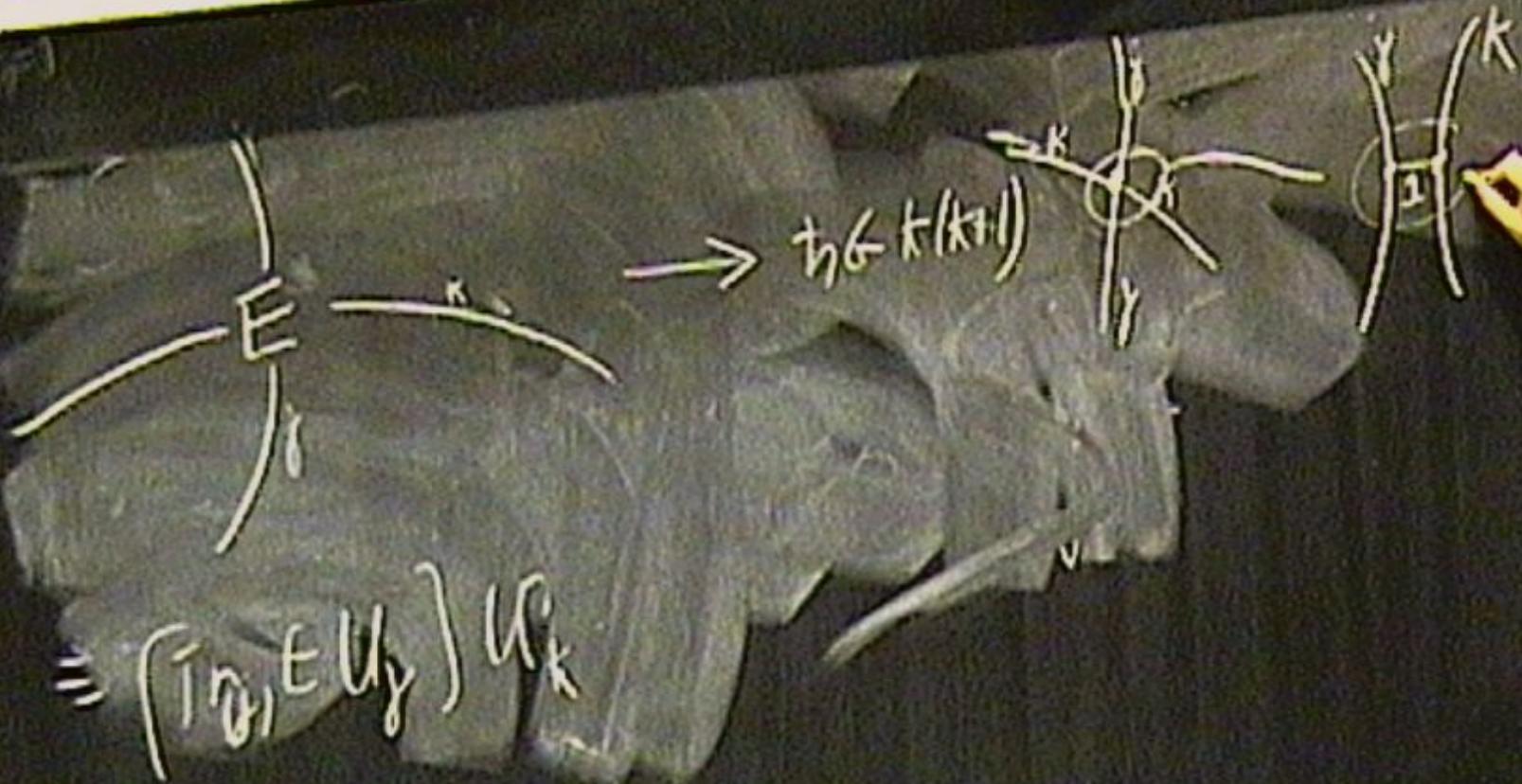
$$h \in \mathbb{R}^2 = 10^{-4} \text{ cm}^2 \quad \text{for } (0,0) = (0,0)$$



$$G = G' = G''$$

$$\hbar G = \lambda_{Pe}^2 = 10^{-66} \text{ cm}^2$$

$$\delta_{xy} \quad \Gamma \left(\begin{matrix} \gamma & \gamma \\ m & m \end{matrix} \right) = (1/2) K(K+1)$$

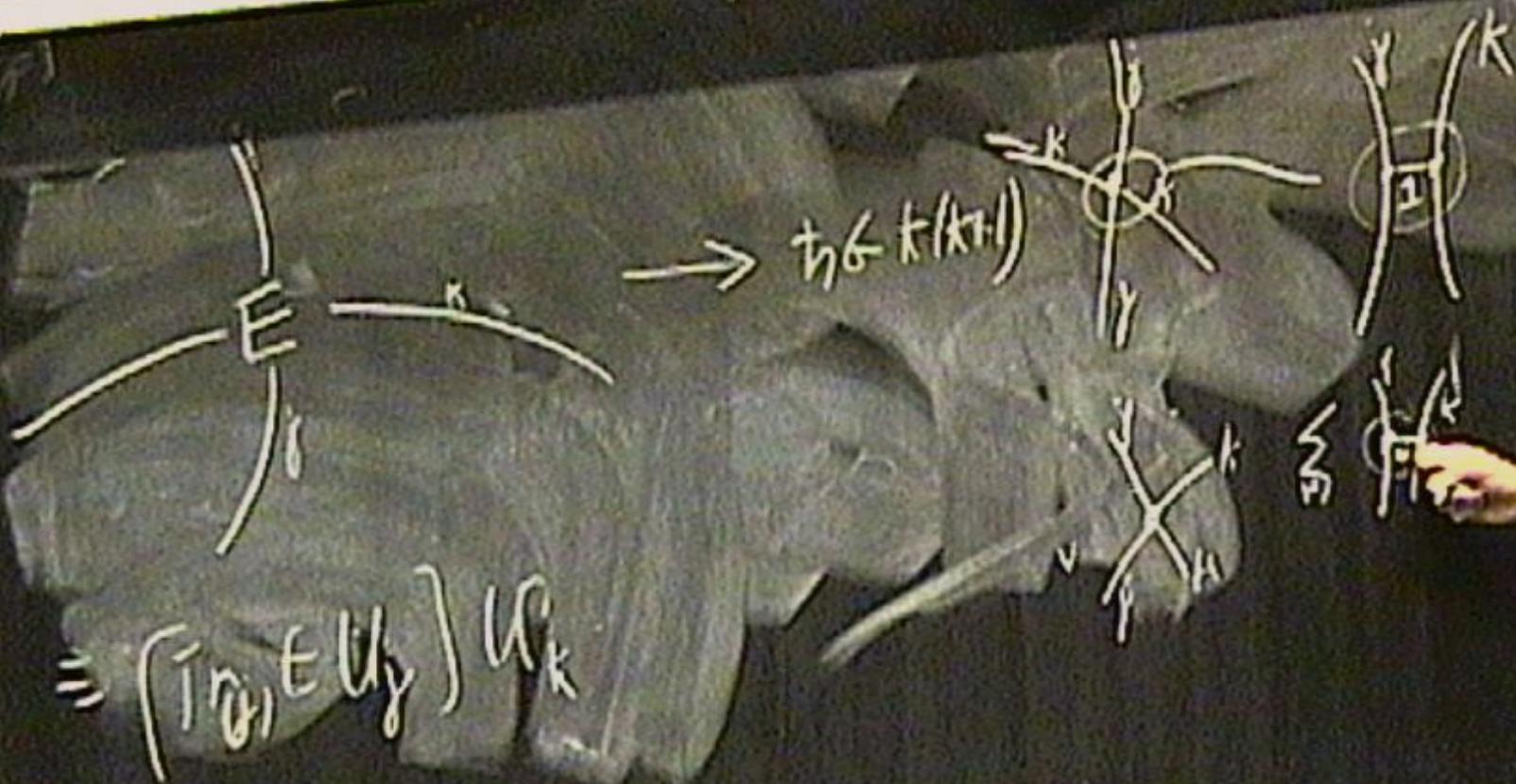


$$\equiv \left(\int \psi_y^\dagger E \psi_y \right) \psi_k$$

$$G = G' = G'' = \dots$$

$$\hbar G = \ell_{Pl}^2 = 10^{-66} \text{ cm}^2$$

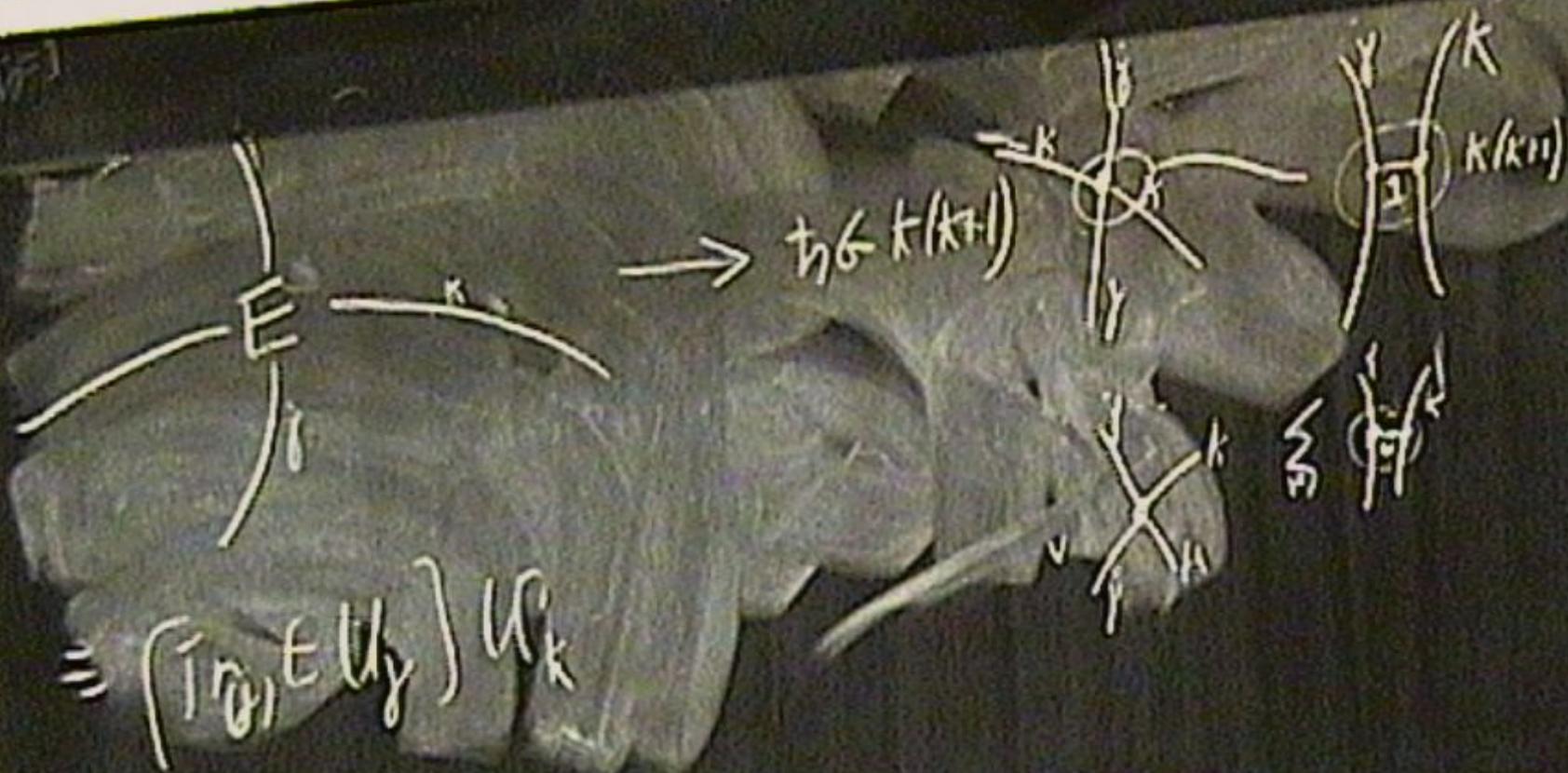
$$\delta_{xy} \prod_{i=1}^k \tau_i = (1/k(k+1))$$



$$G = G = G$$

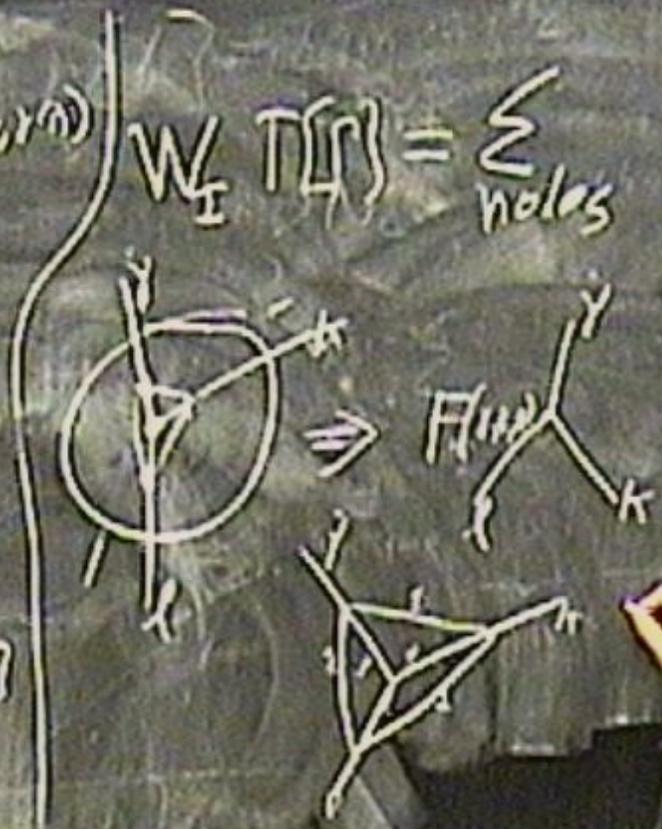
$$\hbar G = l_{Pl}^2 = 10^{-66} \text{ cm}^2$$

$$\delta_{xy} \prod_{i=1}^k \tau_i = (1/k(k+1))$$

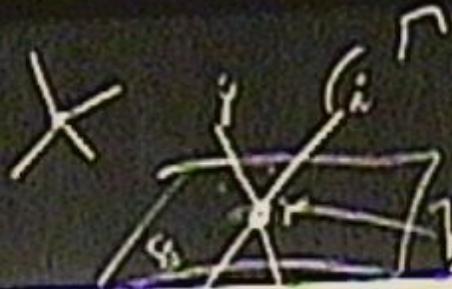
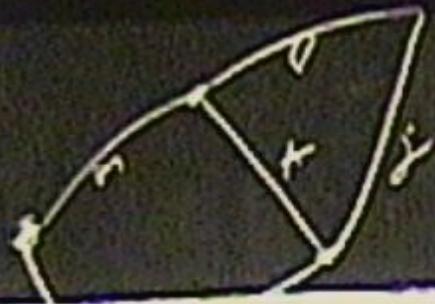


$$E' \rightarrow \vec{k} = \int d^3x' \delta^3(\vec{x} - \vec{x}') W_I T(\dots) = \sum_{\text{nodes}}$$

$$E_{(a)}^{(b)} E_{(c)}^{(d)} \rightarrow \delta^{(a)} \delta^{(b)} \delta^{(c)} \delta^{(d)}$$

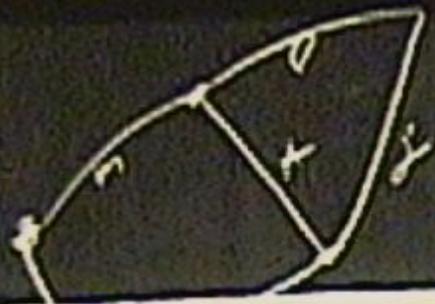


$$E^{(a)} E^{(b)} E^{(c)} \rightarrow \alpha^{(a)} \beta^{(b)} \gamma^{(c)}$$



$$V_{site} = \text{Inv} \left(\begin{matrix} \downarrow \\ V_1 \otimes V_2 \otimes V_3 \end{matrix} \right)$$

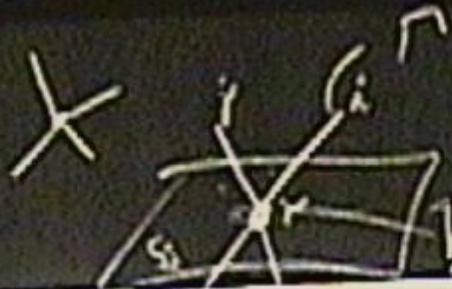
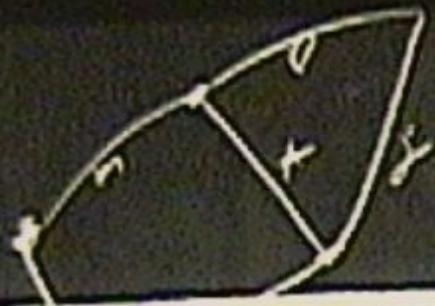
$$W_I = 0$$



$$V_{\text{cube}} = I_{\text{inv}}(V_1 \otimes V_2 \otimes V_3 \otimes V_4)$$

$$W_I \otimes Y = 0$$

$$W_I \otimes X = W_I \otimes Y$$

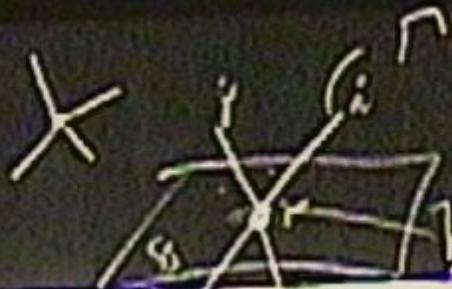
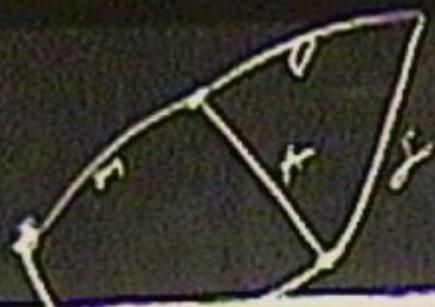


$$V_{site} = \int_{\text{site}} (V_A \otimes V_B \otimes V_C \otimes V_D)$$

$$W_I \text{ (diagram) } = 0$$

$$W_I \text{ (diagram with } \otimes \text{)} = W_I \text{ (diagram with } \oplus \text{)} = (hc)^3 W_n^m \text{ (diagram)}$$





$$V_{site} = I_{site} (V_{i1} \otimes V_{i2} \otimes V_{i3} \otimes V_{i4})$$

$$W_I \text{ (diagram) } = 0$$

$$W_I \text{ (diagram with } \otimes \text{)} = W_I \text{ (diagram with } \oplus \text{)} = (\hbar c)^3 \frac{W_I^m}{V_{site}^m} \text{ (diagram with } \otimes \text{)}$$

$\sum \epsilon_{ij}$ Hermitian ≥ 0

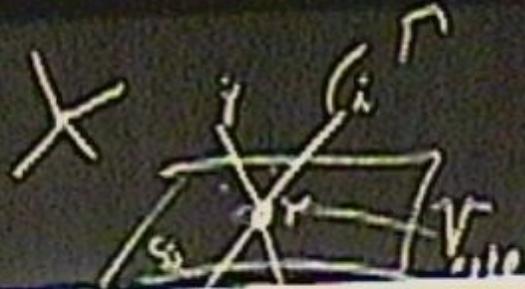
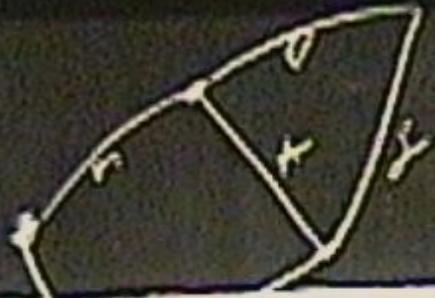


$$W_I \approx \det E_{ij}$$

$$\hat{V}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{M I}$$



$$W_I = \int d^3x \int d^3x' \int d^3x'' \int_{\text{paths}} \left(E(x) U_{x,x'} E(x') U_{x',x''} E(x'') U_{x'',x} \right)$$



$$V_{\text{eff}} = \text{Im} \left(V_1 \otimes V_2 \otimes V_3 \otimes V_4 \right)$$

$$W_{\text{I}} \text{ (diagram) } = 0$$

$$W_{\text{I}} \text{ (diagram)} = W_{\text{I}} \text{ (diagram)} = (\hbar G)^3 W_{\text{I}} \text{ (diagram)}$$

Σ G² Hamiltonian > 0



$$W_I \approx |\det E_x|$$

$$\hat{V}[R] = \lim_{N \rightarrow \infty} \sum_I \sqrt{M V_I}$$



$$\int dx^1 \int dx^2 \int dx^3 \dots \int dx^N \left(E(x^1) U_{x^1} E(x^2) U_{x^2} E(x^3) U_{x^3} \dots \right)$$

$$W_I \otimes \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ k \quad l \end{array} = W_I \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ k \quad l \end{array} = (hg)^3 \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ m \quad n \end{array}$$

$$\sqrt{R} T \{ \Gamma \} = \sum_{\text{nodes } PC R} \sqrt{W} \text{---} T$$



$$W_I \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} = W_I \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array} = (\text{hg})^3 \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$\sqrt{R} T \{ \Gamma \} = (\text{hg})^3 \sum_{\substack{\text{nodes} \\ \text{PC R}}} \sqrt{W} T \text{ node}$$

$$\mathbb{H} = \langle \hat{E}_\alpha, \hat{E}_\beta, F_{\alpha\beta}, \zeta_{\alpha\beta} \rangle$$

$$\tilde{H} = \underbrace{E_+ E_-}_{\text{Fock}} \zeta_{\text{Fock}}$$

$$= 10^2 \zeta_{\text{Fock}} =$$

$$\tilde{H} = \underbrace{E_i E_j}_{\text{antisym}} F_{ab} \xi^{ab}$$

$$= e^2 \epsilon_{ij} \epsilon^{ab} = \epsilon^{ab} \rho$$

$$\tilde{H} = \underbrace{E_i E_j}_{\text{EM}} F_{ab} \zeta^{ijk}$$

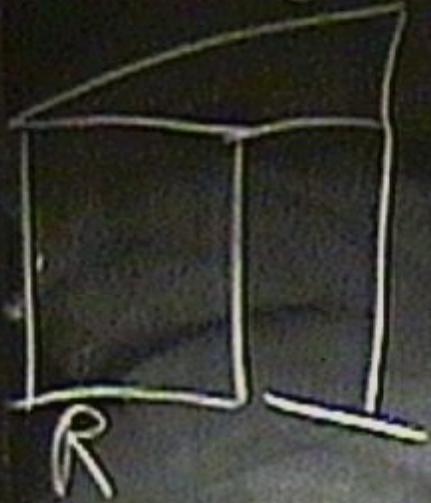
$$= 10^2 \underbrace{F_{0i} F_{0j}}_{\text{EM}} = \epsilon^{abc} Q_c \zeta^{ijk} E_{ijk}$$

$$\tilde{H} = \underbrace{E_i E_j}_{\text{antisym}} F_{ab} \xi^{ijk}$$

$$= 10^2 \epsilon_{ij}^{\alpha\beta} = \epsilon^{\alpha\beta\gamma} \epsilon_{\gamma}^{\alpha\beta} \epsilon_{ijk}$$

$$\tilde{H} = \frac{1}{2} \underbrace{E_i E_i}_{\text{}} F_{ab} \xi^{ab}$$

$$= \frac{1}{2} \epsilon_{ij} \epsilon_{kl} = \epsilon^{ijkl} \xi^{ab}$$



$$\det E_i = \det[\sqrt{g} e_i^a]$$

$$\sqrt{g} = |Q|$$

$$g_{ab} = e_a^i e_b^i$$

$$= \frac{1}{3!} \epsilon_{abc} \epsilon^{abc} \hat{E}_a^i \hat{E}_b^j \hat{E}_c^k$$

$$= Q^3 \det Q^{-1} = \frac{1}{Q^2} = \det g_{ab}$$

$$\Rightarrow V[R] = \int \sqrt{|\det \tilde{E}_i|} = \int \sqrt{\frac{1}{3!} \epsilon_{abc} \epsilon^{abc} \hat{E}_a^i \hat{E}_b^j \hat{E}_c^k}$$



$$W_I \approx \det E_i$$

$$\vec{H} = \frac{1}{c} \underbrace{\vec{E} \times \vec{E}}_{\vec{E} \times \vec{E}} F_{ab} \xi^{ab}$$

$$= \frac{1}{c} \int \rho_a \rho_b = \int \epsilon^{abc} \rho_c \xi^{ab}$$

$$\vec{H} = \frac{1}{\mu_0} \underbrace{(\vec{E}_1 + \vec{E}_2)}_{\vec{E}} F_{1234} \xi_{1234} = \epsilon^{abc} Q_a^i F_{bc} \xi$$

$$= \epsilon^{abc} Q_a^i Q_c^j \xi_{1234}$$

$$\int \mathcal{H} = \frac{1}{2} \underbrace{(\vec{E}_i^2 + \vec{E}_j^2)}_{F_{ab}^2} \xi^{ab} = \epsilon^{abc} Q_a^i F_{bc} \xi^i$$

$$= \epsilon^{abc} Q_c^i Q_a^j \xi^{ij} = \epsilon^{abc} Q_c^i \xi^{ij}$$



$$\vec{H} = \frac{1}{2} \underbrace{(\vec{E} \times \vec{E})}_{\vec{E} \times \vec{E}} F_{ab} \xi^{ab} = \epsilon^{abc} Q_a^L F_{bc} \xi$$

$$= \epsilon^{abc} Q_c^R \xi_{ab}$$

$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{\text{Lagrangian}} F_{ab} \xi^{ab} = \int \epsilon^{abc} \mathcal{L}_a F_{bc} dx$$

$$\Rightarrow \text{ie) } \mathcal{L}_a \mathcal{L}_b = \int \epsilon^{abc} \mathcal{L}_c \xi_{ab}$$

$$\mathcal{L}_a(x) = \int A_a(x)$$

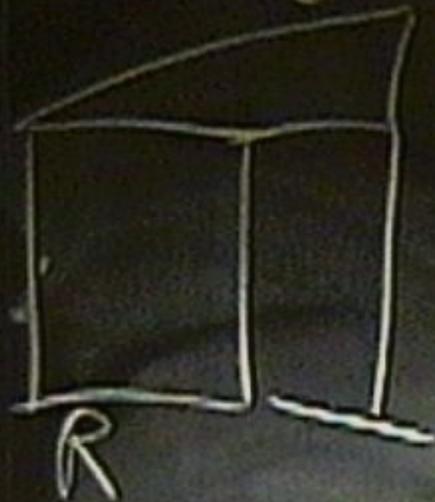


$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^a, \vec{E}^b)}_{\substack{\mathcal{L}_E^a \\ \mathcal{L}_E^b}} F_{ab\kappa} \xi^{a\kappa} = \epsilon^{abc} \mathcal{L}_A^a F_{bc\kappa}$$

$$= \frac{1}{2} \mathcal{L}_E^a \mathcal{L}_E^b = \epsilon^{abc} \mathcal{L}_c^{\kappa} \xi_{a\kappa}$$

$$\mathcal{L}_a^{\mu}(x) = \{A_a^{\mu}(x), V(\xi)\}$$





$$\det \tilde{E}_i = \det(\sqrt{g} \hat{e}_i)$$

$$\sqrt{g} = |\Omega_i|$$

$$g_{ab} = \hat{e}_a^i \hat{e}_b^i$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c$$

$$= \sqrt{g} \det \hat{e}_i^a = \sqrt{g} = \det \Omega_i$$

$$\Rightarrow V[R] = \int \sqrt{|\det \tilde{E}_i|} = \int \sqrt{|\frac{1}{3!} \epsilon_{ijk} \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c|}$$

$$\tilde{H} = \frac{1}{2} \underbrace{E_i^a E_i^b}_{\mathcal{L}_a^b} F_{abk} \xi^{abk} = \epsilon^{abc} \mathcal{L}_a^i F_{bc}^k$$

$$= \frac{1}{2} \mathcal{L}_a^i \mathcal{L}_j^b = \epsilon^{abc} \mathcal{L}_c^k \xi^{abk}$$

$$\mathcal{L}_a^i(x) = \{A_a^i(x), V(\xi)\}$$



$$\tilde{H} = \frac{1}{2} \underbrace{E_i^+ E_i^-}_{\text{}} F_{ab} \xi^{ab} = \epsilon^{abc} Q_a F_{bc}$$

$$= \frac{1}{2} Q_i^+ Q_i^- = \epsilon^{abc} Q_c \xi^{ab}$$

check

$$Q_a^\mu(x) = \{ A_a^\mu(x), V(\xi) \}$$

$$\tilde{H} = \frac{1}{c} \underbrace{(\vec{E} \times \vec{E})}_{\vec{E} \times \vec{E}} \cdot \vec{F}_{ab} \cdot \vec{z}^{ab} = \epsilon^{abc} \mathcal{L}_a \vec{F}_{bc} \quad \text{check}$$

$$= \frac{1}{c} \epsilon^{abc} \mathcal{L}_a \vec{F}_{bc} = \epsilon^{abc} \mathcal{L}_c \epsilon_{ijk}$$

$$\mathcal{L}_a(x) = \{A_a^i(x), V(x)\}$$

$\vec{E} \times \vec{E}$

$$\tilde{H} = \frac{1}{2} (\vec{E}^2 + \vec{B}^2) F_{ab} \dot{x}^a \dot{x}^b = \epsilon^{abc} \mathcal{L}_a \dot{x}^b \dot{x}^c \quad \text{check}$$

$$= \frac{1}{2} \dot{x}^i \dot{x}^j = \epsilon^{abc} \mathcal{L}_c \dot{x}^a \dot{x}^b$$

$$\mathcal{L}_a(x) = \{ A_a^i(x), V(x) \}$$

(2)



$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{\substack{= \epsilon^0 \epsilon_i \epsilon_j \epsilon_k \\ = \epsilon^{1bc} \epsilon_c \epsilon_{ijk}}} F_{ab} F^{ik} = \epsilon^{abc} \mathcal{L}_a F_{bc} \quad \text{check}$$

$$\mathcal{L}_a(x) = \{ A_a^i(x), V(x) \}$$

$$\frac{1}{2} \epsilon^{ijk} \epsilon_{ijk}$$

$$U_{T,m} \sim H(A(x))$$

$$\tilde{H} = \frac{1}{c} \underbrace{E_i^a E_j^b}_{\text{check}} F_{ab} \zeta^{ijk} = \epsilon^{abc} \mathcal{L}_a^i F_{bc} \zeta^i$$

$$= \frac{1}{c} \mathcal{L}_i^a \mathcal{L}_j^b = \epsilon^{abc} \mathcal{L}_c^k \zeta^{ijk}$$

$$\mathcal{L}_a^i(x) = \{A_a^i(x), V(\xi)\}$$

$$= U_{\gamma_{in}} \{U_{\xi_{in}}, V(\xi)\}$$



$U_{\gamma_{in}} \{U_{\xi_{in}}, V(\xi)\}$

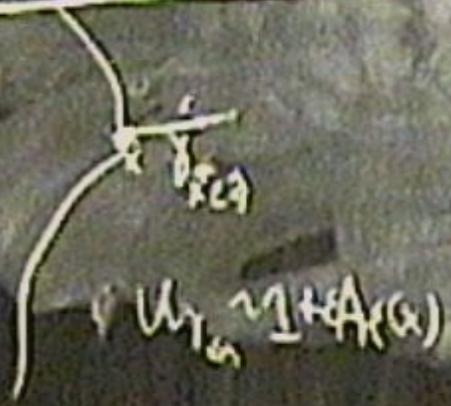
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}_i^+ \vec{E}_i^-)}_{\mathcal{L}_A} F_{ab} \dot{x}^a \dot{x}^b = \epsilon^{abc} \mathcal{L}_A F_{bc} \dot{x}^a \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_i \mathcal{L}_j^{\dot{a}b} = \epsilon^{abc} \mathcal{L}_c \dot{x}^a$$

$$\mathcal{L}_A(x) = \{ A_A^i(x), V(x) \}$$

$$\{ U_{im}^i, V(x) \}$$

(2)



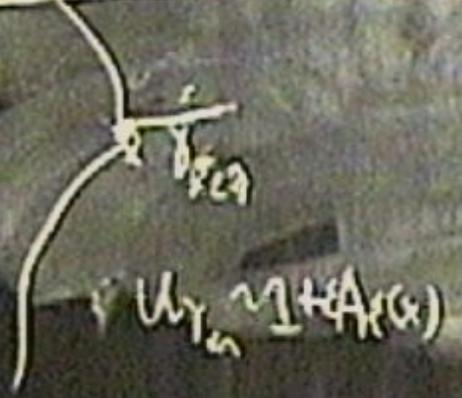
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{\substack{= \epsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2 \\ = \epsilon_0 \epsilon_{ijk} \epsilon_{lmn} \dots}} F_{ab} F^{cd} = \epsilon^{abc} \mathcal{L}_a \dot{F}_{bc}$$

check

$$\mathcal{L}_a(x) = \{A_A^i(x), V(\phi)\}$$

$$\lim_{\epsilon \rightarrow 0} \mathcal{L}_{\mu\nu}(\phi_{fin}, V(\phi))$$

(2)



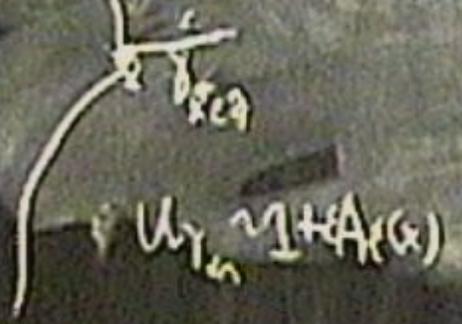
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{\text{check}} F_{ab} \zeta^{ab} = \epsilon^{abc} \mathcal{L}_a F_{bc} \zeta^c$$

$$= \frac{1}{2} \mathcal{L}_a \mathcal{L}_b \zeta^{ab} = \epsilon^{abc} \mathcal{L}_c \zeta^{ab}$$

$$\mathcal{L}_a(x) = \{A_a(x), V(x)\}$$

$$\mathcal{L}_a(x) = \{U_{im}, V(x)\}$$

(2)



$$\vec{H} = \frac{1}{2} (\vec{E} \cdot \vec{E} + \vec{B} \cdot \vec{B}) = \frac{1}{2} \sum_{i,j} (E_i^2 + B_i^2) = \frac{1}{2} \sum_{i,j} (\dot{A}_i^2 + (\nabla \times \vec{A})_i^2) = \frac{1}{2} \sum_{i,j} (\dot{A}_i^2 + \epsilon^{abc} \partial_a \partial_b A_c)^2$$

check

$$\mathcal{L}_A(x) = \left\{ A_A(x), V(x) \right\}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \mathcal{L}_{\text{reg}}(\epsilon) = \left\{ \mathcal{L}_{\text{reg}}^{\text{fin}}, V(x) \right\}$$

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + A(x)$$

$$\hat{\mathcal{H}} = \int d^3x (A(x) \epsilon^{\mu\nu\lambda} [\dot{A}_\mu, V(x)] F_{\nu\lambda})$$

$$\hat{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{\mathcal{L}_A} F_{ab} \zeta^{ab*} = \epsilon^{abc} \mathcal{L}_A^i F_{bc} \zeta^i \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_i^a \mathcal{L}_j^b = \epsilon^{abc} \mathcal{L}_c^T \zeta^{ab}$$

$$\mathcal{L}_A^i(x) = \{ A_A^i(x), V(\xi) \}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \mathcal{L}_{Y_{int}} [U_{int}^i, V(\xi)]$$

$$\hat{\mathcal{H}} = \int d^3x N(x) \{ T_{tr} U [U^i, V(\xi)] \} F_{bc}$$

(2)

$\frac{1}{\epsilon} \mathcal{L}_{Y_{int}} [U_{int}^i, V(\xi)]$

$$\vec{H} = \frac{1}{c} \underbrace{\vec{E} \times \vec{E}}_{\vec{E} \times \vec{E}} + \vec{F}_{ab} \times \vec{z}_{ab} = \epsilon^{abc} Q_a^i \vec{F}_{bc} \quad \text{check}$$

$$= \frac{1}{c} \underbrace{\vec{E} \times \vec{E}}_{\vec{E} \times \vec{E}} = \epsilon^{abc} Q_c^T \vec{E}_{ab}$$

$$Q_a^i(x) = \{A_a^i(x), V(x)\}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} U_{\gamma_{in}} \{U_{\gamma_{in}}^i, V(x)\}$$

(2)



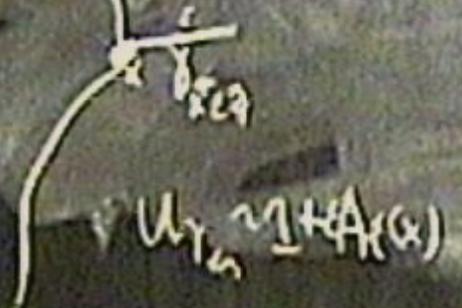
$$\hat{\Phi} = \int dx N(x) \epsilon^{\mu\nu} [U [U^i, V(x)] F_{\mu\nu}]$$

$$\hat{H} = \frac{1}{2} (\vec{E}^2 + \vec{B}^2) F_{ab} \xi^{ab} = \epsilon^{abc} \mathcal{L}_A^i F_{bc} \xi^i \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_i^a \mathcal{L}_j^b = \epsilon^{abc} \mathcal{L}_c^T \xi_{ab}$$

$$\mathcal{L}_a^i(x) = \{ A_A^i(x), V(\xi) \}$$

(2)



$$\hat{H} = \int dx N(x) \left[\frac{1}{2} \mathcal{L}_i^a [U, V(\xi)] F_{bc} \right]$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} U_{\gamma, \alpha} [U_{\gamma, \alpha}, V(\xi)]$$

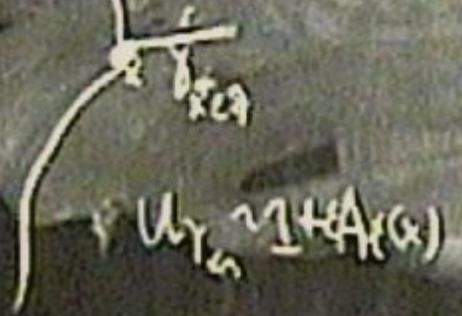
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^a \cdot \vec{E}_a + F_{ab} F^{ab})}_{\text{check}} = \epsilon^{abc} Q_a^i F_{bc} \dot{x}^i$$

$$= \frac{1}{2} \underbrace{Q_a^i Q_b^j}_{\epsilon^{abc} Q_c^k} \epsilon_{ijk} \dot{x}^k$$

$$Q_a^i(x) = \{A_a^i(x), V(x)\}$$

$$\lim_{\epsilon \rightarrow 0} \mathcal{L}_{\gamma_{\text{min}}}[\mathcal{L}_{\text{min}}, V(x)]$$

(2)



$$\hat{\tilde{H}} = \int dx N(x) \frac{1}{\epsilon} \mathcal{H}[\mathcal{L}_{\text{min}}, V(x)] F_{bc}$$

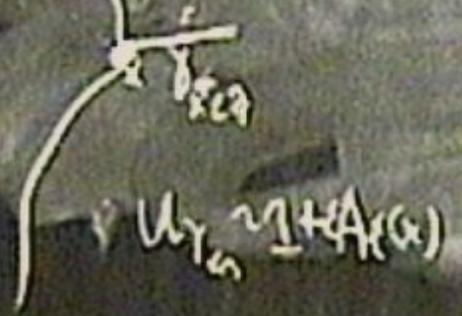
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E} + \vec{E}^*)}_{\vec{E}} \cdot \underbrace{(\vec{B} + \vec{B}^*)}_{\vec{B}} = \epsilon^{abc} \mathcal{L}_A^i F_{bc} \dot{x}^a \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_i \mathcal{L}_j^B = \epsilon^{abc} \mathcal{L}_c^A \dot{x}^a$$

$$\mathcal{L}_A^i(x) = \{A_A^i(x), V(x)\}$$

$$\lim_{\epsilon \rightarrow 0} \mathcal{L}_{\gamma_{in}}^i(\mathcal{L}_{in}^i, V(x))$$

(2)



$$\hat{\mathcal{H}} = \int dt \int dx \left[\frac{1}{2} \dot{\phi}^2 - \mathcal{H}[\phi, \pi] \right]$$

$F_{43}(x)$



$F_{45}(x)$



$$\left(U_{\gamma_{\text{clockwise}, x}} - 1 \right) \frac{1}{\alpha^2}$$

$F_{\alpha, \beta}(x)$



$$\left(U_{\gamma_{\alpha, \beta, x}} - 1 \right) \frac{1}{\alpha^2} \equiv \hat{F}_{\alpha, \beta}(x)$$

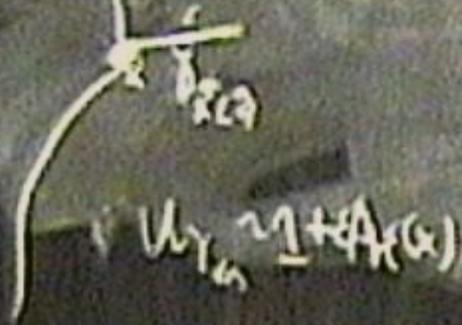
$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{F_{ab} F^{ab}} = \frac{1}{2} \epsilon^{abc} \mathcal{L}_a F_{bc} \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_a \mathcal{L}_b \mathcal{L}_c = \frac{1}{2} \epsilon^{abc} \mathcal{L}_c \mathcal{E}_{ab}$$

$$\mathcal{L}_a(x) = \{A_a(x), V(x)\}$$

$$\mathcal{L}_a(x) = \{U_{F_{ab}}, V(x)\}$$

(2)



$$\hat{\mathcal{H}} = \int d^3x N(x) \left[\frac{1}{2} \mathcal{E}_{ab}^2 + \mathcal{L}_a \right]$$

$$\tilde{H} = \frac{1}{2} \underbrace{\vec{E} \cdot \vec{E}}_{\vec{E}^2} + F_{ab} \xi^{ab} = \epsilon^{abc} \mathcal{L}_a F_{bc}$$

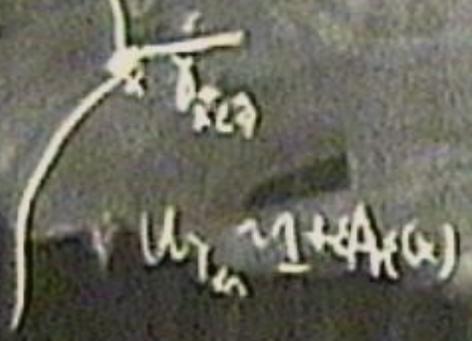
$$= \frac{1}{2} \mathcal{L}_a \mathcal{L}_b \mathcal{L}_c \xi^{abc} = \epsilon^{abc} \mathcal{L}_c \xi^{ab}$$

check

$$\mathcal{L}_a(x) = \{A_a^\mu(x), V(\xi)\}$$

$$\lim_{\xi \rightarrow 0} \frac{1}{\xi} \mathcal{L}_{\xi^a} \{U_{\xi^a}, V(\xi)\}$$

(2)



$$\hat{\mathcal{L}}_{\xi^a} = \int d^3x N(x) \frac{1}{\xi} \left[\mathcal{L}_{\xi^a} [U_{\xi^a}, V(\xi)] F_{bc} \right]$$

$F_{45}(x)$



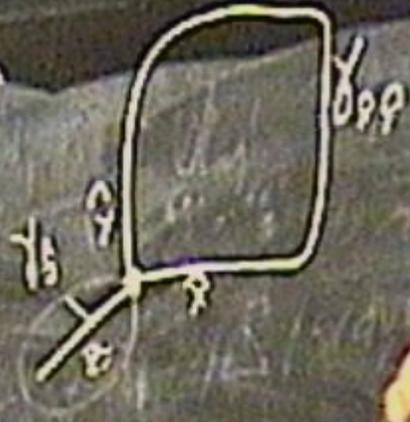
$$\left(U_{\gamma_{\alpha, \beta, x}} - 1 \right) \frac{1}{\alpha^2} \equiv \hat{F}_{45}(k)$$



$F_{43}(x)$



$$\left(U_{\gamma_{\text{clockwise}}} - 1 \right) \frac{1}{\alpha^2} \equiv \hat{F}_{41}(x)$$

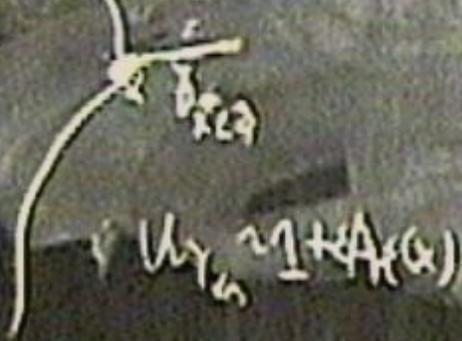


$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^2 + \vec{B}^2)}_{F_{ab} F^{ab}} = \epsilon^{abc} \mathcal{L}_a F_{bc} \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_i \mathcal{L}_j = \epsilon^{abc} \mathcal{L}_c \epsilon_{ijk}$$

$$\mathcal{L}_a(x) = \{A_a^\mu(x), V(x)\}$$

(2)



$$\hat{\mathcal{H}}_\epsilon = \int d^3x N(x) \left[\frac{1}{\epsilon} \text{Tr} [U, [U, V(x)]] F \right]$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} U_{cl} \{U_{cl}^{-1}, \dots\}$$

$F_{45}(x)$



F_{ab}



$$\left(U_{\gamma_{\text{clockwise}, x}} - 1 \right) \frac{1}{\alpha^2}$$



$$= \frac{1}{\alpha^2 \epsilon} \left(U_{\text{clockwise}} - 1 \right)$$

$$\left[\frac{1}{\epsilon}, \nabla(\epsilon) \right]$$

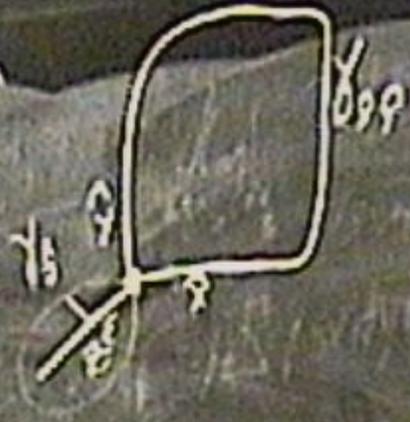
$F_{43}(x)$



F_{ab}

$$\left(U_{\gamma_{\alpha, \beta, x}} - 1 \right) \frac{1}{\alpha^2}$$

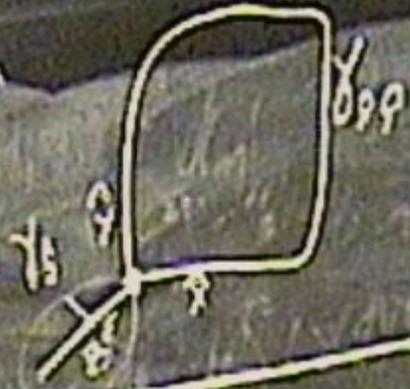
$$\hat{F}_{\alpha, \beta} = \sum_{\gamma} \frac{1}{\alpha^2 \epsilon} \left(U_{\gamma, \beta} - 1 \right) U_{\gamma} \left[U_{\gamma}^{-1}, V(\epsilon) \right]$$



$F_{45}(x)$



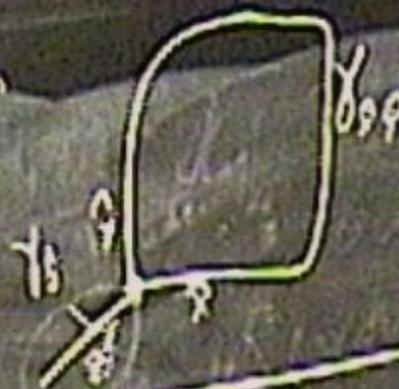
F_{ab}



$$\left(U_{\gamma_{\alpha, \beta, x}} - 1 \right) \frac{1}{\alpha^2}$$

$$\hat{\mathcal{H}}_{\alpha, \beta} = \sum_{\gamma} \frac{1}{\alpha^2 \beta} \left(U_{\gamma, \beta} - 1 \right) U_{\gamma} \left[U_{\gamma}^{-1}, V(\epsilon) \right]$$

$F_{ab}(x)$



$$\left((U_{\gamma_{\alpha, \alpha, x}} - 1) \frac{1}{\alpha^2} \right)$$

F_{ab}

$$\hat{H}_{\alpha, \epsilon} = \sum_{\substack{U_k \\ \gamma \neq \alpha}} \frac{1}{\alpha^2 \epsilon} (U_{\gamma \alpha} - 1) U_{\alpha} [U_{\alpha}^{\prime}, V(\epsilon)]$$

$F_{ab}(x)$

$$(u_{\gamma_{max}} - 1) \frac{1}{\alpha^2}$$

F_{ab}

$$\hat{H}_{\epsilon} = \sum_{j=1}^3 \frac{1}{d \epsilon} (u_{ij} - 1) u_{ij} [u_{ij}', \sqrt{1/\epsilon}]$$

$F_{43}(x)$



F_{46}



$$\left(\frac{U_{\alpha, \beta}(x) - 1}{\alpha^2} \right)$$

$$\hat{F}_{\alpha, \beta} = \sum_{k=1}^{\infty} \frac{1}{\alpha^2 \epsilon} (U_{\alpha, \beta} - 1) U_k [U_k', V(\epsilon)]$$



$F_{ab}(x)$



F_{ab}



$$(U_{\gamma} - 1) \frac{1}{\alpha^2}$$

$$\hat{F}_{\alpha, \beta} = \int_{\gamma} \frac{1}{\alpha^2} (U_{\gamma} - 1) U_{\beta} [U_{\beta}', V(\epsilon)]$$



$F_{ab}(x)$



F_{ab}

$$(U_{\gamma_{ab}(x)} - 1) \frac{1}{\alpha^2}$$

$$\hat{\Phi}_{\alpha/\epsilon} = \sum_{\gamma \in \Gamma} \frac{1}{\alpha^2} (U_{\gamma} - 1) U_{\epsilon} [U_{\epsilon}', V(\epsilon)]$$



$F_{45}(x)$



F_{ab}

$$(U_{\gamma} - 1) \frac{1}{\alpha^2}$$

$$\hat{F}_{\alpha, \epsilon} = \sum_{\gamma \in \Gamma} \frac{1}{\alpha^2 \epsilon} (U_{\gamma} - 1) U_{\epsilon} [U_{\epsilon}^{-1}, V(\epsilon)]$$



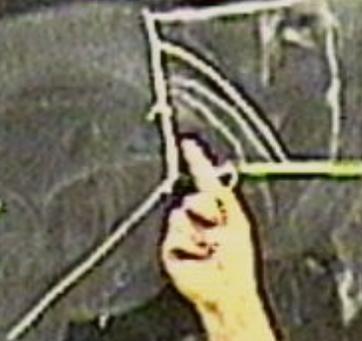
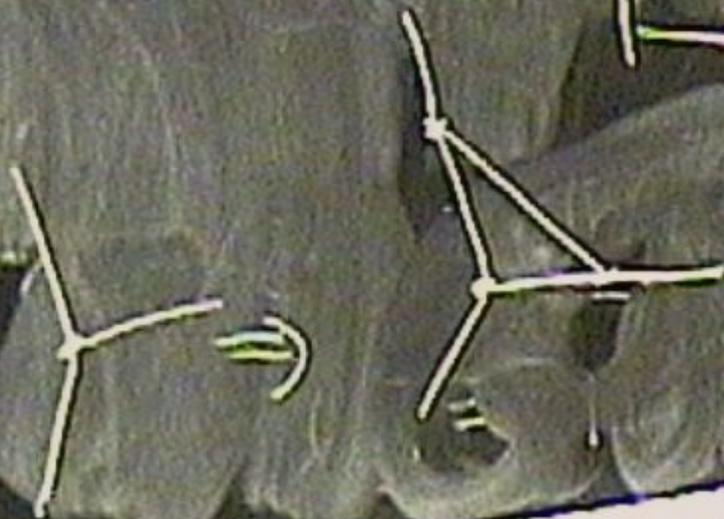
$F_{ab}(x)$



F_{ab}

$$(U_{\gamma} - 1) \frac{1}{\alpha^2}$$

$$\hat{\Phi}_{\alpha, \epsilon} = \sum_{\gamma \in \Gamma} \frac{1}{\alpha^2} (U_{\gamma} - 1) U_{\gamma} [U_{\gamma}^{-1}, V(\epsilon)]$$



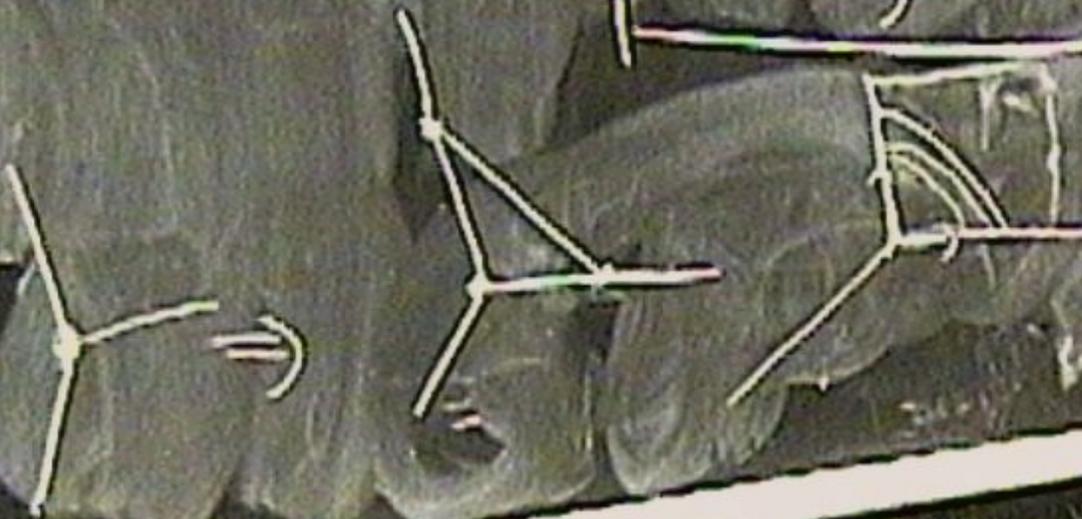
$F_{ab}(x)$



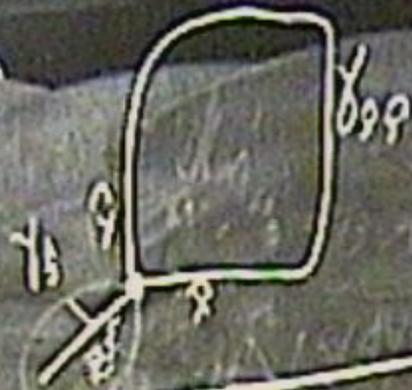
F_{ab}

$$(U_{\gamma_{a,b}, x} - 1) \frac{1}{\alpha^2}$$

$$\hat{F}_{\alpha, \epsilon} = \sum_{\substack{1 \leq k \leq n \\ \gamma_{a,b}^k}} \frac{1}{\alpha^2 \epsilon} (U_{\gamma_{a,b}^k} - 1) U_{\gamma_{a,b}^k} [U_{\gamma_{a,b}^k}, V(\epsilon)]$$



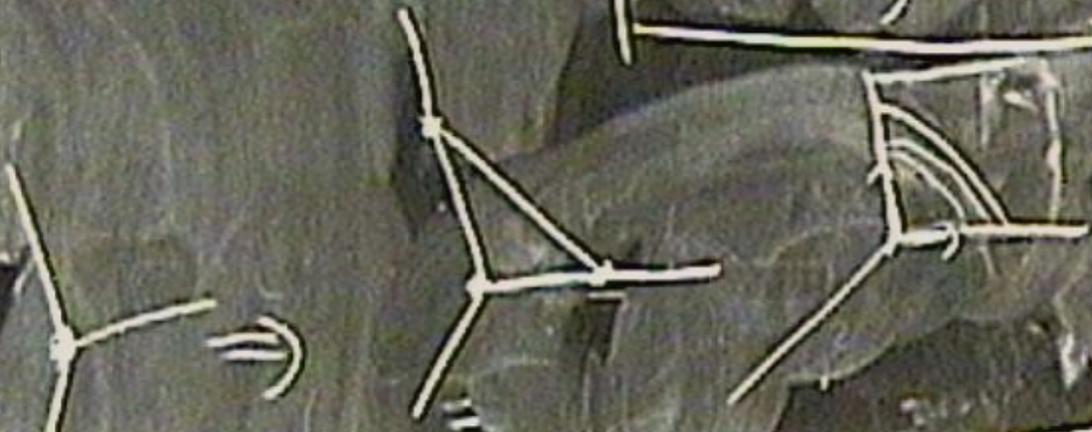
$F_{45}(x)$



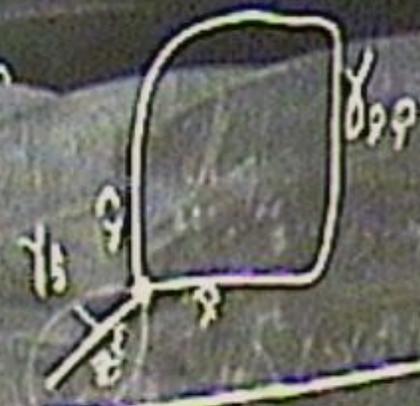
F_{45}

$$\left(U_{\gamma_{\alpha, \beta}, x} - 1 \right) \frac{1}{\alpha^2}$$

$$\hat{H}_{\alpha, \beta} = \sum_{\substack{z \in \mathbb{Z} \\ \alpha \leq z \leq \beta}} \frac{1}{\alpha^2} \left(U_{\gamma_{\alpha, \beta}} - 1 \right) U_z \left[U_z^{-1}, \sqrt{\epsilon} \right]$$

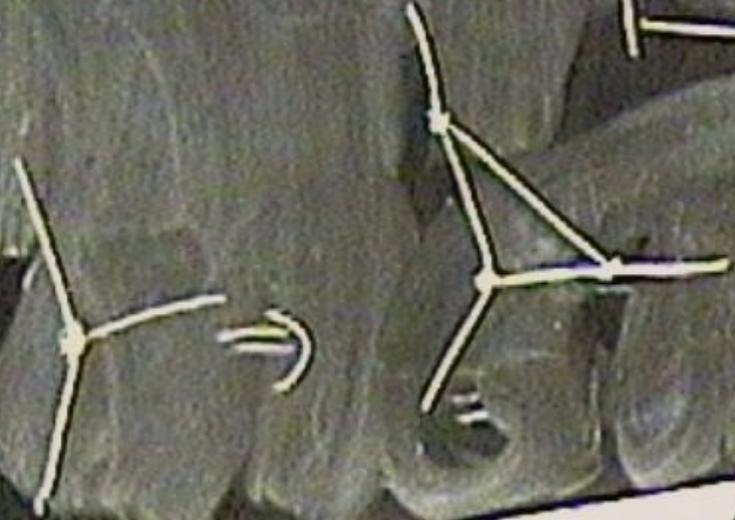


$F_{45}(X)$



$$\left((U_{\gamma_{x,y,z,\alpha}} - 1) \right) \frac{1}{\alpha^2}$$

$$\hat{\Phi}_{\alpha, \epsilon} = \sum_{\gamma \in \Gamma(\alpha, \epsilon)} \frac{1}{\alpha^2 \epsilon} \left((U_{\gamma} - 1) U_{\alpha} \left[U_{\alpha}^{-1}, \sqrt{\epsilon} \right] \right)$$



$$\hat{H} = \frac{1}{2} \hat{E}_a^T \hat{E}_a + F_{ab} \xi^{ab} = \epsilon^{abc} Q_a^L F_{bc} \quad \text{check}$$

$$= \frac{1}{2} \hat{E}_a^T \hat{E}_a + \epsilon^{abc} Q_c^T \xi^{ab}$$

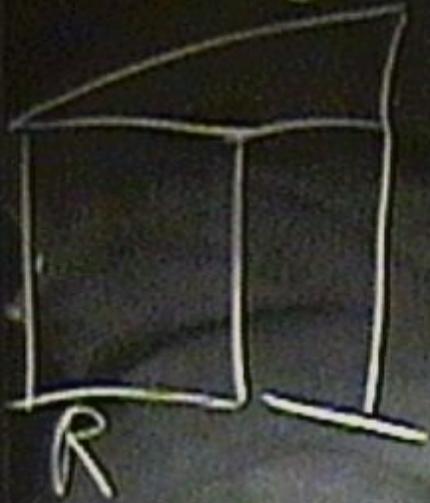
$$Q_a^L(x) = \{A_a^L(x), V(\xi)\}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} U_{\epsilon, \text{int}} \{U_{\epsilon, \text{int}}, V(\xi)\}$$

ξ^{ab}

$U_{\epsilon, \text{int}} \sim 1 + \epsilon A(x)$

$$\hat{H} = \int dx N(x) \left[\frac{1}{2} \hat{E}_a^T \hat{E}_a + \epsilon^{abc} [U, [U, V(\xi)]] F_{bc} \right]$$



$$\det E_i = \det[\sqrt{g} Q_i^a]$$

$$\sqrt{g} = |Q|$$

$$g_{ab} = Q_a^i Q_b^i$$

$$= \frac{1}{3!} \epsilon_{ijk} \epsilon^{ijk} \hat{E}_i^a \hat{E}_j^b \hat{E}_k^c$$

$$= Q^3 \det Q^i = \frac{1}{3!} \epsilon^{ijk} \epsilon_{ijk} = \det g_{ab}$$

$$\Rightarrow V[R] = \int \sqrt{|\det \hat{E}_i|} = \int \sqrt{\frac{1}{3!} \epsilon_{ijk} \epsilon^{ijk} \hat{E}_i^a \hat{E}_j^b \hat{E}_k^c}$$

$$\tilde{H} = \frac{1}{2} \mathbf{E}^a \mathbf{E}_a + F_{ab} \dot{x}^b = \epsilon^{abc} \mathcal{L}_a \dot{x}^b$$

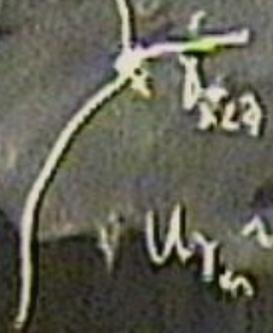
check

$$= \frac{1}{2} \mathcal{L}_a \mathcal{L}_a \dot{x}^b = \epsilon^{abc} \mathcal{L}_c \dot{x}^b$$

$$\mathcal{L}_a(x) = \{A_a^i(x), V(x)\}$$

$$\lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} U_{\epsilon, t_0} \{U_{\epsilon, t_1}, V(x)\}$$

(2)



$$\hat{H} = \dots$$

$$\{U_{\epsilon, t_1}, V(x)\} F_{bc}$$

$$\tilde{H} = \frac{1}{2} \underbrace{(\vec{E}^a \cdot \vec{E}_a)}_{\text{check}} F_{ab} \xi^{ab} = \epsilon^{abc} Q_a^L F_{bc} \xi$$

check

$$= \frac{1}{2} \epsilon^{abc} Q_a^L \xi^{bc} = \epsilon^{abc} Q_c^T \xi_{ab}$$

$$Q_a^L(x) = \{A_a^L(x), V(\xi)\}$$

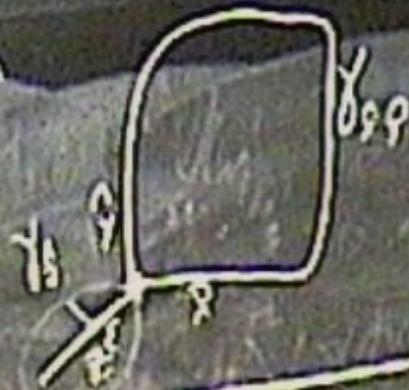
$$\lim_{\xi \rightarrow 0} \frac{1}{\xi} U_{\xi, \text{int}} \{U_{\xi, \text{in}}, V(\xi)\}$$



$$U_{\xi, \text{in}} \sim 1 + \xi A(x)$$

$$\hat{\tilde{H}} = \int dx N(x) \frac{1}{\xi} \left[\frac{1}{\xi} U_{\xi, \text{in}} [U_{\xi, \text{in}}, V(\xi)] F_{bc} \right]$$

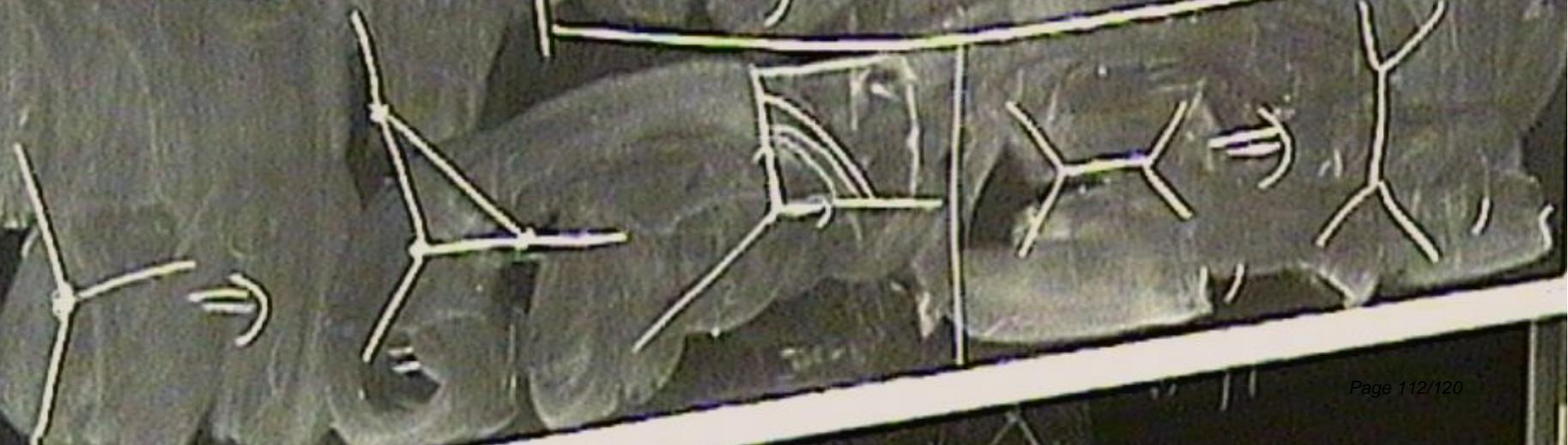
$F_{43}(x)$

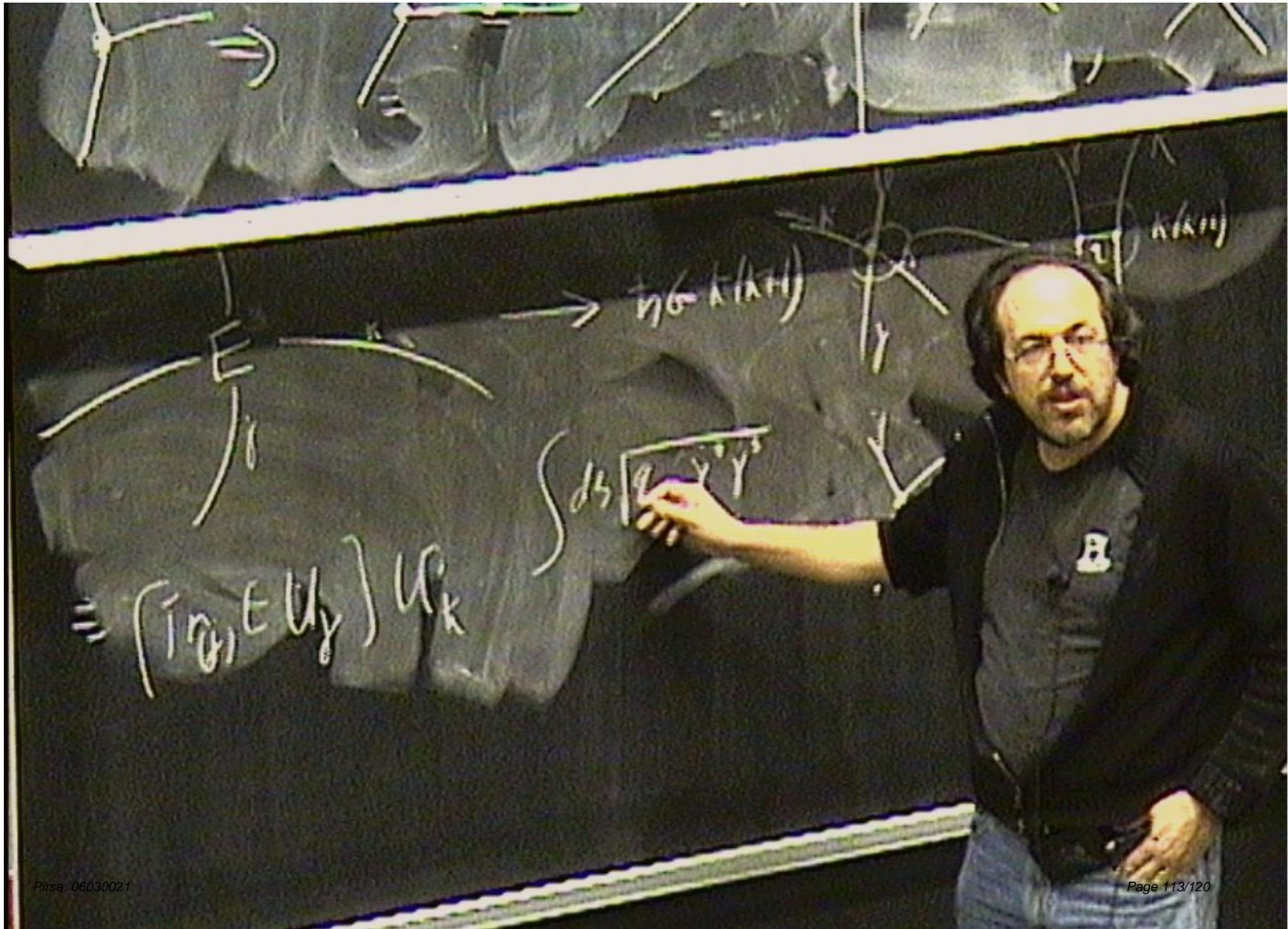


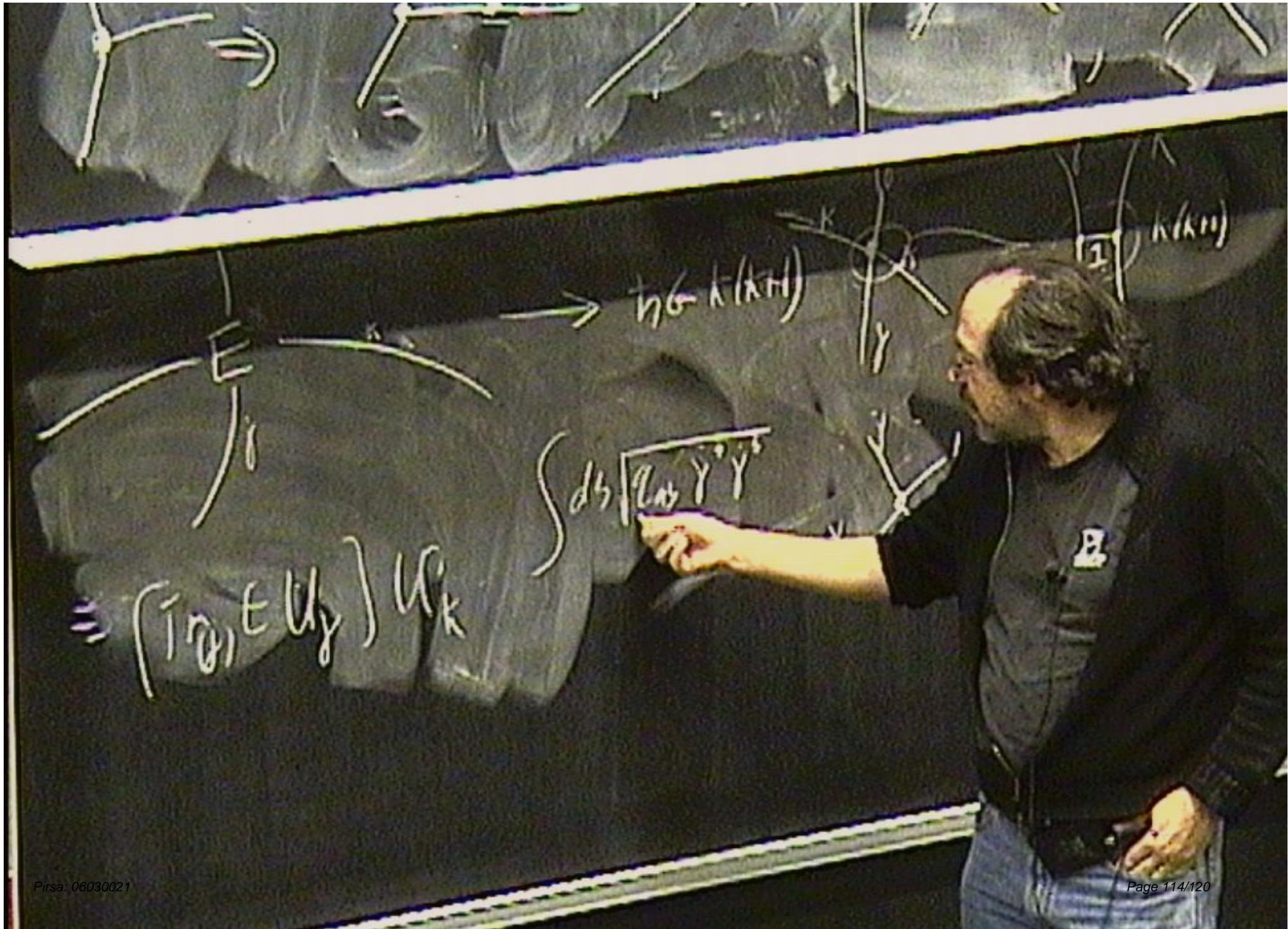
F_{ab}

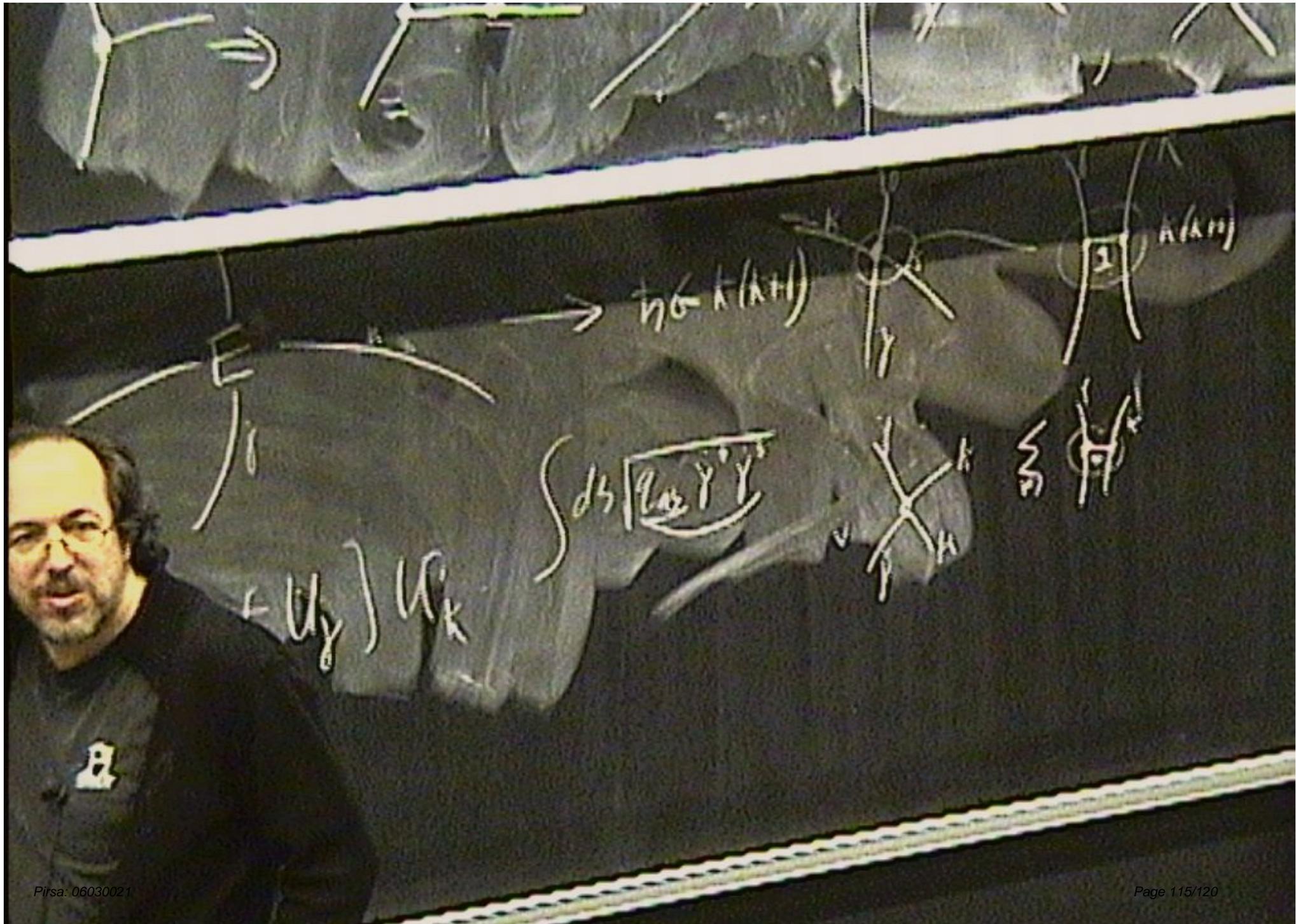
$$\left((U_{\gamma_{\delta+\delta, x}} - 1) \frac{L}{\alpha^2} \right)$$

$$\hat{\Phi}_{\alpha, \epsilon} = \int_{\mathbb{R}^3} \frac{1}{\alpha^2 \epsilon} \left((U_{\delta \epsilon} - 1) U_2 [U_2^{-1}, \sqrt{\epsilon}] \right)$$





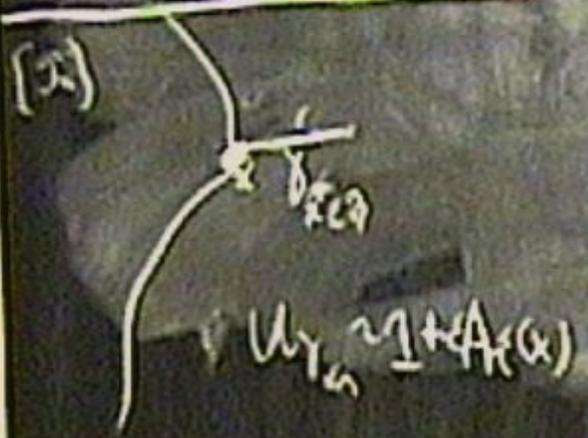




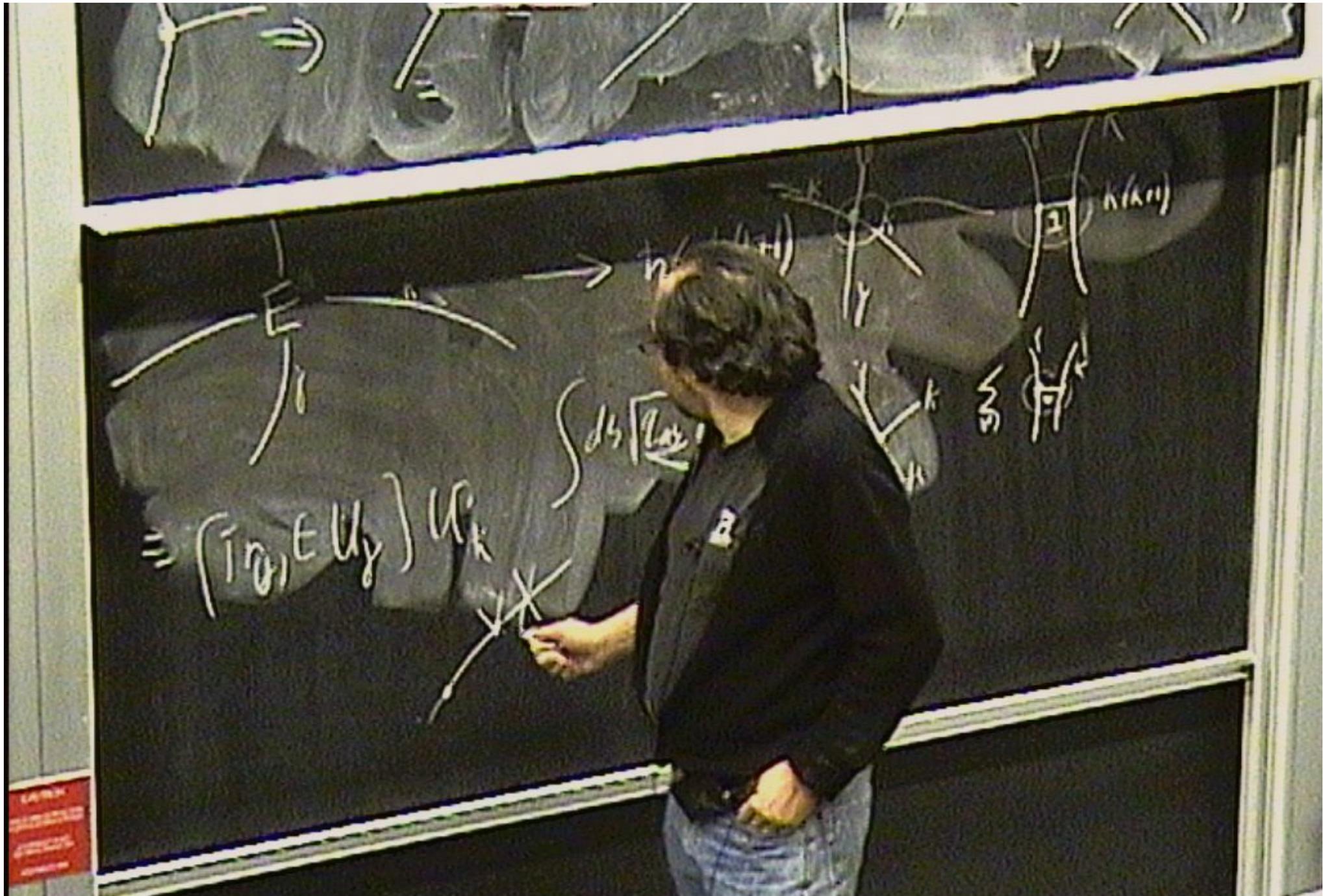
$$\tilde{H} = \frac{1}{2} \mathbf{E}^a \mathbf{E}_a + F_{ab} \dot{x}^b = \epsilon^{abc} \mathcal{L}_a \dot{x}^b \quad \text{check}$$

$$= \frac{1}{2} \mathcal{L}_a \mathcal{L}_a = \epsilon^{abc} \mathcal{L}_c \dot{x}^b$$

$$\mathcal{L}_a(x) = \{A_a^i(x), V(x)\}$$

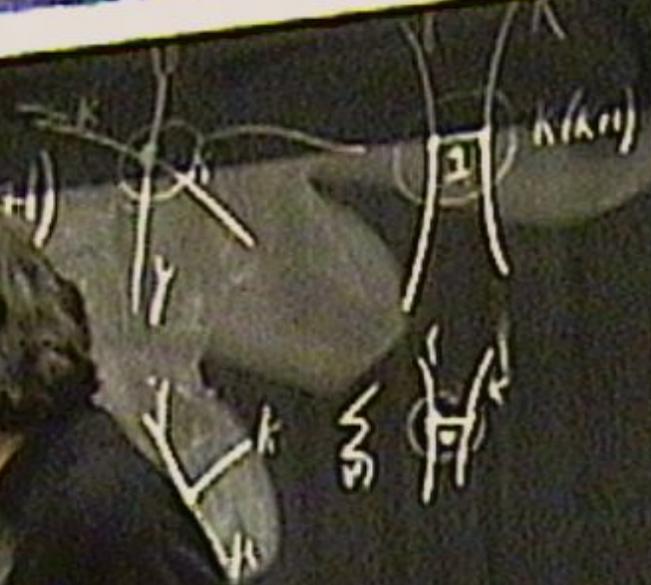


$$\hat{H} = \int dx^0 dx^1 \frac{1}{\epsilon} \left[\mathcal{L}_a [u_{x^a}, V(x)] F_{bc} \right]$$

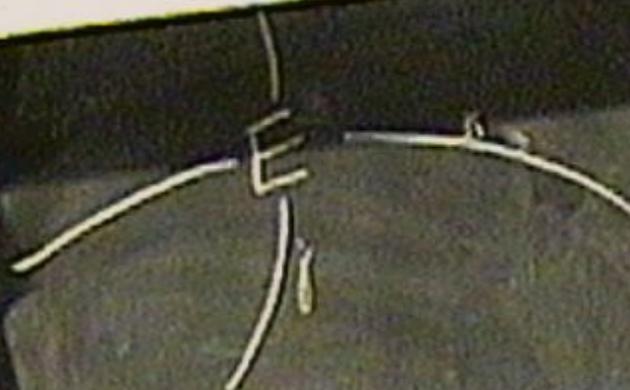
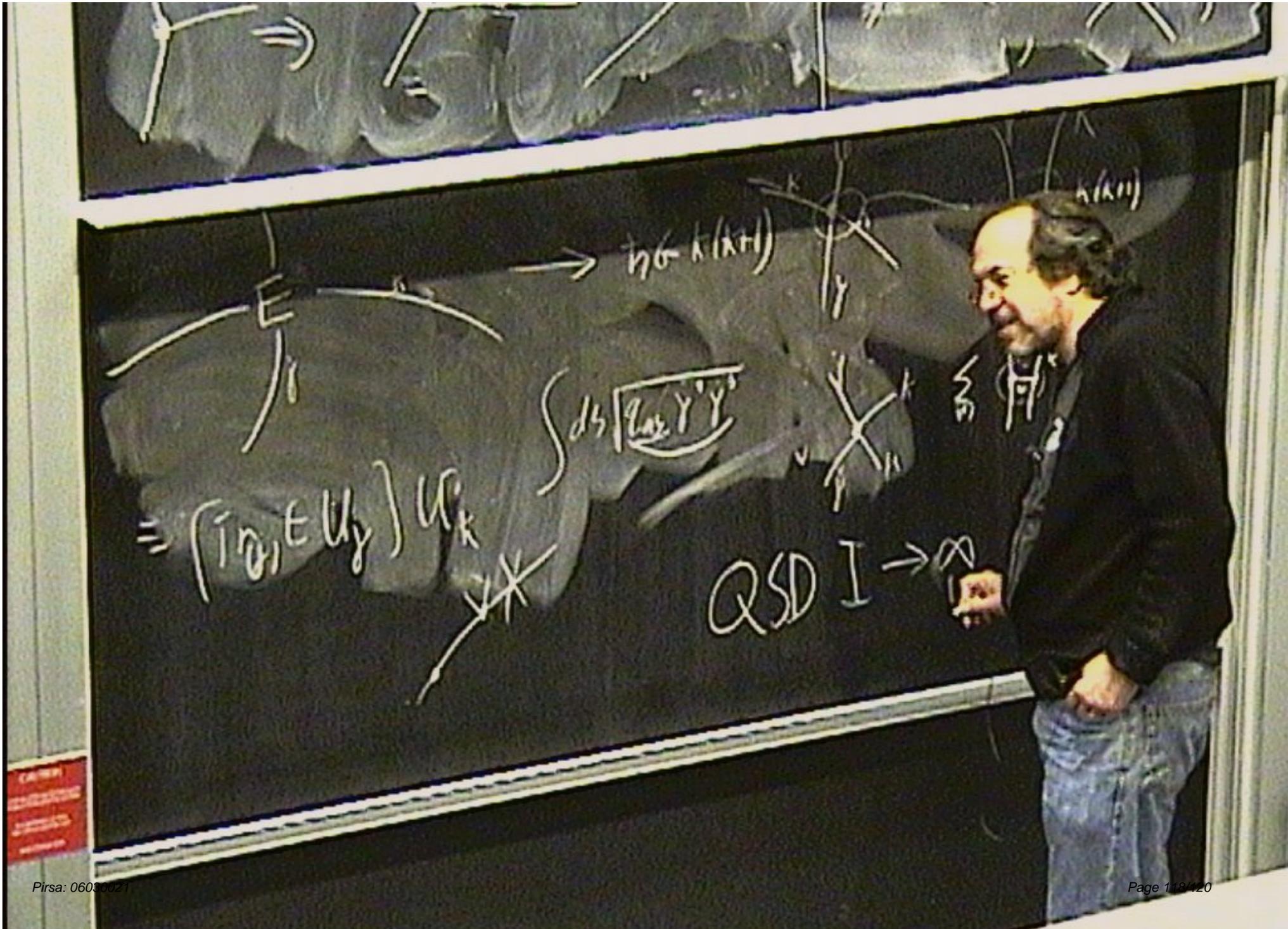


$$\int ds \sqrt{g_{\mu\nu}} \dot{x}^\mu \dot{x}^\nu$$

$$\equiv \left(\int_{\partial U} E U_y \right) U_k$$



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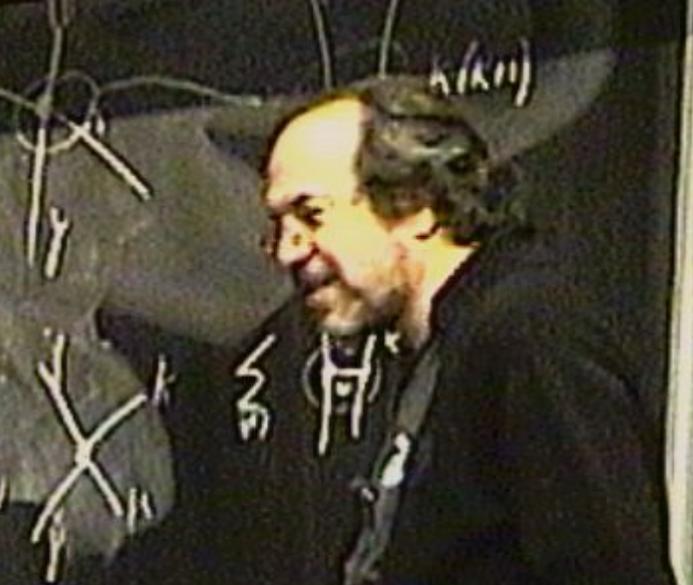


$$\rightarrow \ln k(k+1)$$

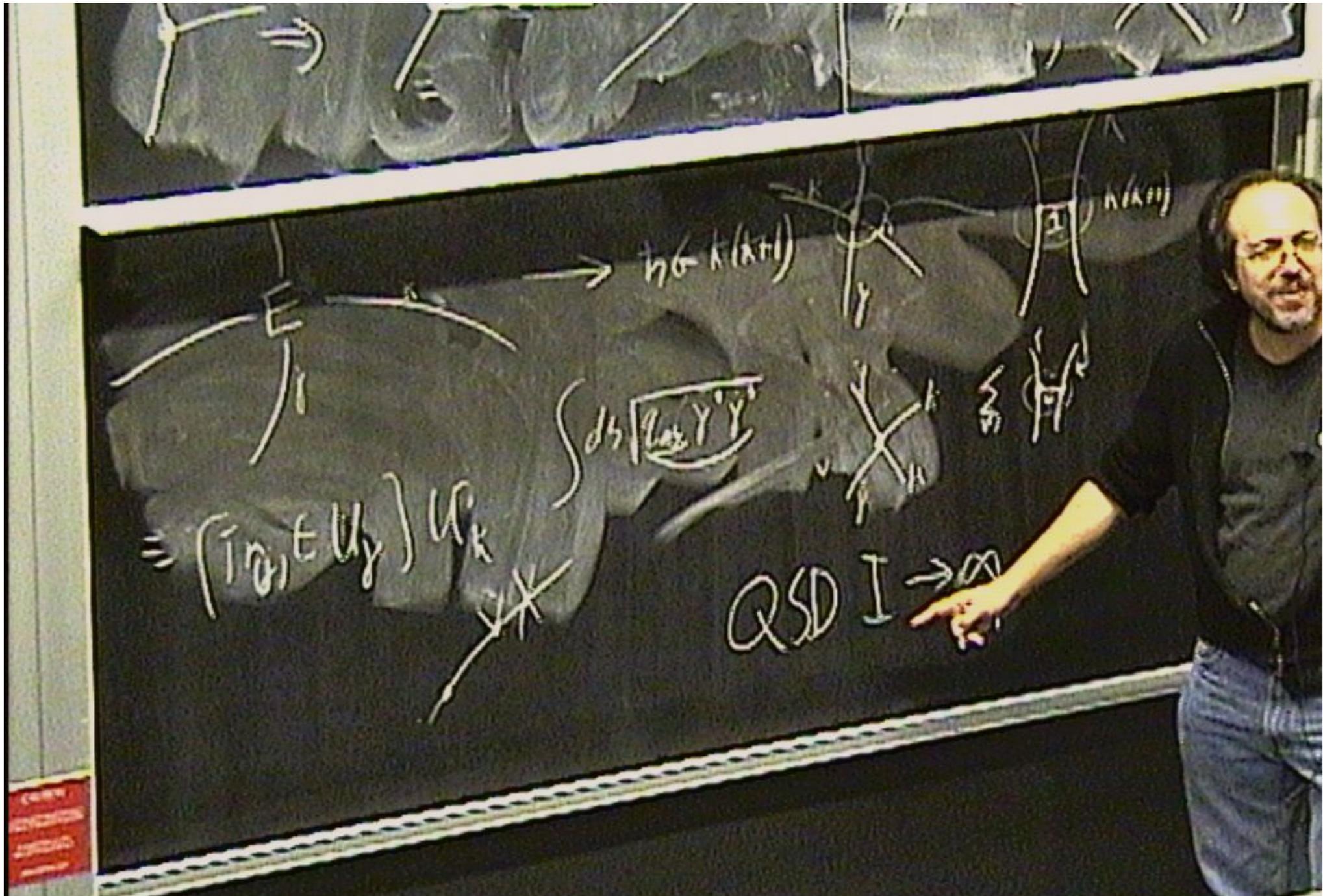
$$\int ds \sqrt{g_{ab} \dot{y}^a \dot{y}^b}$$

$$\equiv (\int_{\mathcal{C}_y} U_y) U_k$$

QSD I $\rightarrow \infty$



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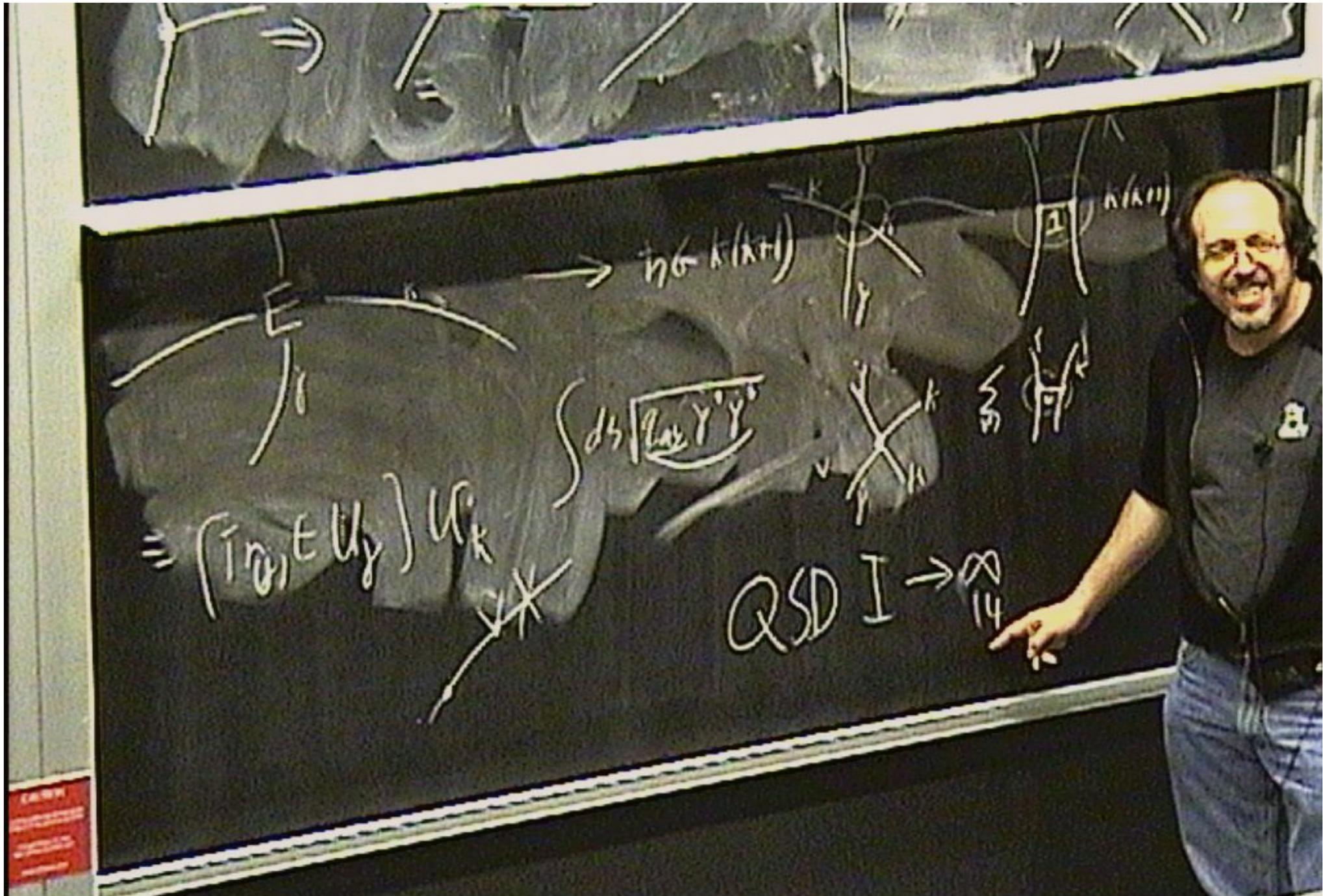


$$\rightarrow \int \delta k(k+1)$$

$$\int ds \sqrt{g_{ab} \dot{y}^a \dot{y}^b}$$

$$\equiv (\int_{\mathcal{U}_y} \mathcal{U}_k)$$

QSD I $\rightarrow \infty$



$$E \rightarrow \int \dots$$

$$\int d^3y \sqrt{g_{ab} \dot{y}^a \dot{y}^b}$$

$$\equiv \left(\int \dots \right) U_k$$

$$QSD I \rightarrow \infty$$