Title: Liouville mechanics with an epistemic restriction and Bohr's response to EPR

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Abstract: I will discuss a toy theory that reproduces a wide variety of qualitative features of quantum theory for degrees of freedom that are continuous. The ontology of the theory is that of classical particle mechanics, but it is assumed that there is a constraint on the amount of knowledge that an observer may have about the motional state of any collection of particles -- Liouville mechanics with an epistemic restriction. The formalism of the theory is determined by examining the consequences of this "classical uncertainty principle" on state preparations, measurements, and dynamics. The result is a theory of hidden variables, although it is not a hidden variable model of quantum theory because it is both local and noncontextual. Despite admitting a simple classical interpretation, the theory also exhibits the operational features of Bohr's notion of complementarity. In fact, it includes all of the features of quantum mechanics to which Bohr appeals in his response to EPR. This theory demonstrates, therefore, that Bohr's arguments fail as a defense of the completeness of quantum mechanics. Joint work with Stephen Bartlett and Terry Rudolph

...present quantum theory not only does not use -- it does not even dare to mention -- the notion of a "real physical situation." Defenders of the theory say that this notion is philosophically naive, a throwback to outmoded ways of thinking, and that recognition of this constitutes deep new wisdom about the nature of human knowledge. I say that it constitutes a violent irrationality, that somewhere in this theory the distinction between reality and our knowledge of reality has become lost, and the result has more the character of medieval necromancy than of science.

--E.T. Jaynes

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Liouville mechanics with an epistemic restriction and Bohr's response to EPR

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March 22, 2006,
Piqudos seminar

Joint work with:
Stephen Bartlett (University of Sydney, Australia
Terry Rudolph (Imperial College, London)

Funding by:
The Royal Society

But this is not enough to derive quantum theory!

Much recent foundations work suggests (to me at least) that

Maximal information about reality is incomplete information

is a foundational principle for quantum theory

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Caves and Fuchs, quant-ph/9601025
Rovelli, quant-ph/9609002
Hardy, quant-ph/9906123
Brukner and Zeilinger, quant-ph/0005084
Hardy, quant-ph/0101012
Kirkpatrick, quant-ph/0106072
Fuchs, quant-ph/0205039
Spekkens, quant-ph/0401052

But this is not enough to derive quantum theory!

Previous work:

"In defence of the epistemic view of quantum states: a toy theory"
quant-ph/0401052

Start with an epistemic constraint:

Every system has an internal degree of freedom about which # questions answered = # questions unanswered

Derive a toy theory

Quantum phenomena that have analogues in the toy theory

- Nonorthogonality
- ambiguity of decomposition of mixed states
- Noncommutativity
- Coherent superposition
- Interference
- Projection postulate
- Distinction between product and entangled pure states
- Distinction between separable and nonseparable mixed states
- Ambiguity of decomposition of a CP map
- Multiple purifications of mixed states
- The Jamiolkowski isomorphism
- Multiple unitary extensions of CP maps
- Multiple Neumark extensions of measurements
- …

- No-cloning
- Teleportation
- Dense-coding
- No information gain without disturbance
- Secure key distribution
- No perfectly secure bit commitment
- Partially secure coin flipping

- No universal state inverter
- EPR-type steering
- Mutually unbiased bases
- tri-partite entanglement
- The monogamy of entanglement
- Locally immeasurable product bases (nonlocality without entanglement)
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The diversity and quality of the analogy and the fact that the toy theory is essentially derived from a single principle provides strong evidence (in my view) that quantum states are states of incomplete knowledge.

Quantum phenomena that do not arise in the toy theory

The answer is not a local noncontextual hidden variable theory.

What then is the knowledge about?

Identifying and studying the missing phenomena provides the best clues for answering this question

Quantum phenomena that do not arise in the toy theory

- Nonlocality (Violations of Bell inequalities)
- Contextuality (the Kochen-Specker theorem)

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- The fact that two levels of a qutrit behave like a qubit

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- The continuum of quantum states, measurements, and transformations
- The fact that convex combination and coherent superposition are full rather than partial binary operations
- The fact that two levels of a qutrit behave like a qubit
- The possibility of an exponential speed-up relative to classical computation

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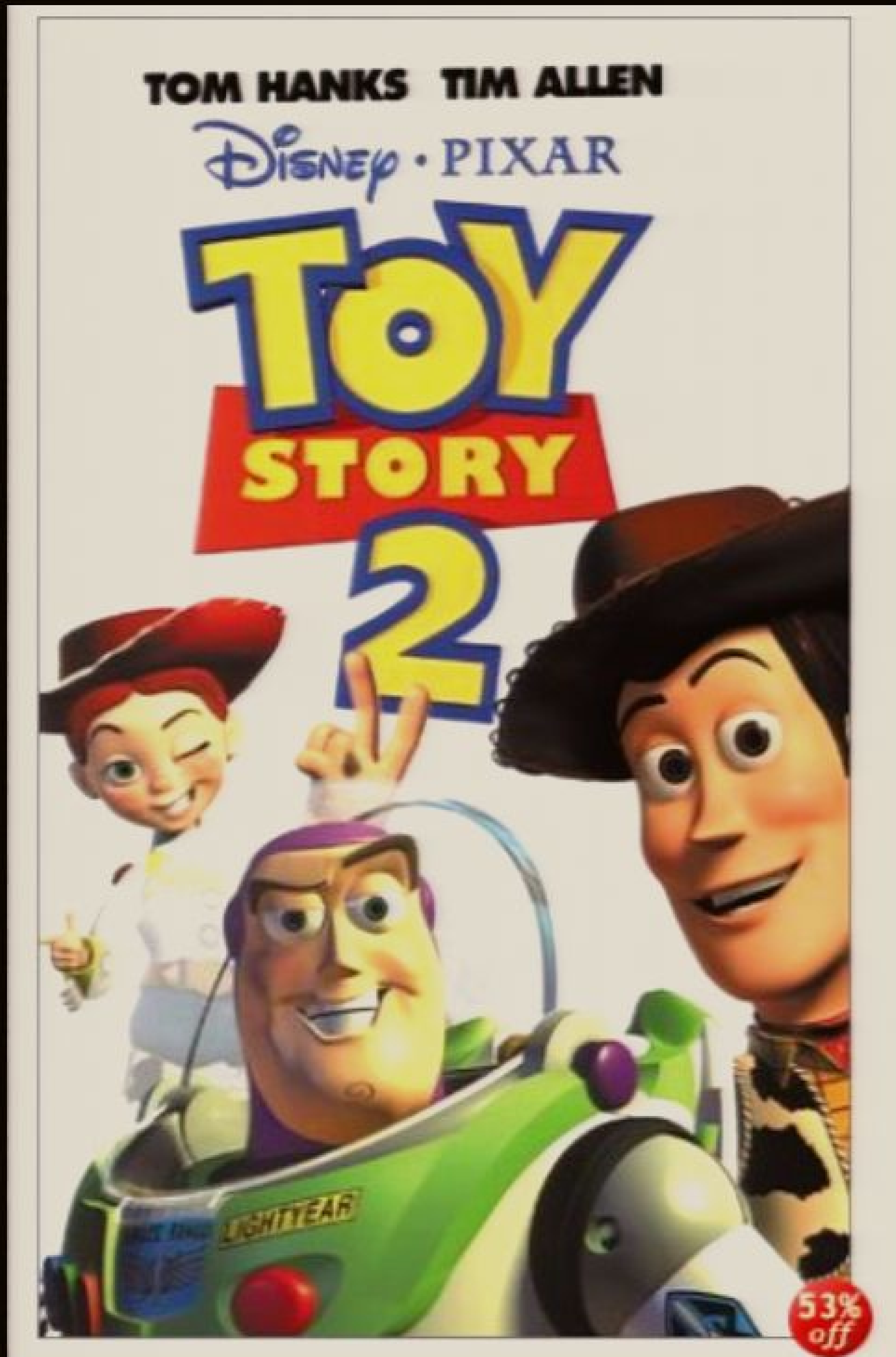
In this toy theory, the motional degree of freedom was treated classically

Can we do something similar for the motional degree of freedom?

Yes.



Coming soon to a quant-ph arxiv near you!

Outline

- A quantum uncertainty principle
- A classical uncertainty principle
- Epistemically restricted Liouville mechanics
- Bohr’s response to EPR
- Discussion

What is a good epistemic restriction to apply to a continuous degree of freedom?

-- look to quantum theory

Quantum particle mechanics

Consider *n* canonical degrees of freedom $\mathcal{H} = (\mathcal{L}^2)^{\otimes n}$

ex: n particles in 1d, n/3 particles in 3d

$$[\hat{x}_k, \hat{p}_l] = i\hbar \hat{I} \delta_{kl} \qquad k, l \in \{1, ..., n\}$$

$$\hat{z} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2, \ldots, \hat{x}_n, \hat{p}_n)$$

$$[\hat{z}_i, \hat{z}_j] = i\hbar \hat{I} \Sigma_{ij} \qquad i, j \in \{1, ..., 2n\}$$

$$\Sigma = \begin{pmatrix} \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} & & & \mathbf{0} \\ & \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} & & \\ & & \cdots & \\ \mathbf{0} & & & \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} \end{pmatrix}$$

Define

$$\langle \hat{f} \rangle_\rho \equiv \mathrm{Tr}(\hat{\rho}\hat{f})$$

The "covariance matrix" γ is defined by

$$\gamma_{ij}(\hat{\rho}) = 2\mathrm{Re}\langle(\hat{z}_i - \langle\hat{z}_i\rangle_\rho)(\hat{z}_j - \langle\hat{z}_j\rangle_\rho)\rangle_\rho$$

General form of the uncertainty principle is:

$$\boxed{\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0}$$

For a single canonical degree of freedom

$$\gamma(\hat{\rho}) = \begin{pmatrix} 2(\Delta x)^2 & \langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle - 2\langle \hat{x}\rangle\langle \hat{p}\rangle \\ \langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle - 2\langle \hat{x}\rangle\langle \hat{p}\rangle & 2(\Delta p)^2 \end{pmatrix}$$

$$(\Delta z)^2 = \langle(\hat{z} - \langle\hat{z}\rangle)^2\rangle$$

The condition $\gamma(\hat{\rho}) + i\hbar\Sigma \geq 0$ for a 2×2 matrix is equivalent to $\det(\gamma(\hat{\rho}) + i\hbar\Sigma) \geq 0$

$$4(\Delta x)^2(\Delta p)^2 \geq (\langle \hat{x}\hat{p} + \hat{p}\hat{x}\rangle - 2\langle \hat{x}\rangle\langle \hat{p}\rangle)^2 + \hbar^2$$

$$\Delta x \Delta p \geq \hbar/2$$

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$$\Delta x \Delta p \geq \hbar/2$$

Liouville mechanics

Consider *n* canonical degrees of freedom $\mathcal{M} = \mathbb{R}^{2n}$

ex: n particles in 1d, n/3 particles in 3d

$$z \equiv (x_1, p_1, x_2, p_2, \ldots, x_n, p_n)$$

Denote a probability distribution over $\mathcal{M}$ by $\mu(z)$

Define

$$\langle f \rangle_\mu \equiv \int_{\mathcal{M}} f(z)\mu(z)dz$$

The "covariance matrix" γ is defined by

$$\gamma_{ij}(\mu) = 2\langle (z_i - \langle z_i \rangle_\mu)(z_j - \langle z_j \rangle_\mu) \rangle_\mu$$

It satisfies

$$\gamma(\mu) \geq 0$$

Liouville mechanics with an epistemic constraint

Assume:

The classical uncertainty principle (CUP):

Liouville distributions describing an observer's knowledge must satisfy

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$

For a single canonical degree of freedom

$$\gamma(\mu) = 2\begin{pmatrix} (\Delta x)^2 & \langle xp\rangle - \langle x\rangle\langle p\rangle \\ \langle xp\rangle - \langle x\rangle\langle p\rangle & (\Delta p)^2 \end{pmatrix}$$

$$(\Delta z)^2 = \langle (z - \langle z\rangle)^2\rangle$$

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Some valid epistemic states

Wigner representation of quantum states

$$W_\rho(z) = \mathrm{Tr}(\rho \hat{A}_z)$$

where $\hat{A}_z = \bigotimes_{i=1}^n \hat{A}_{z_i}$

$$\hat{A}_{z_i} = \frac{1}{\pi\hbar}\int e^{-ip_i y/\hbar}\left|x_i - \tfrac{1}{2}y\right\rangle\left\langle x_i + \tfrac{1}{2}y\right| dy$$

Theorem (Hudson,Soto,Claverie):
The only quantum states with positive Wigner representation are the Gaussian states

"Gaussian state" means Gaussian characteristic function

$$\chi_\rho(z) = \frac{1}{\pi^n} e^{-(1/4)z^T\gamma z - i d^T z}$$

where $\chi_\rho(z) = \mathrm{Tr}(\rho \hat{D}_z)$

where $\hat{D}_z = e^{iz^T\Sigma\hat{z}}$

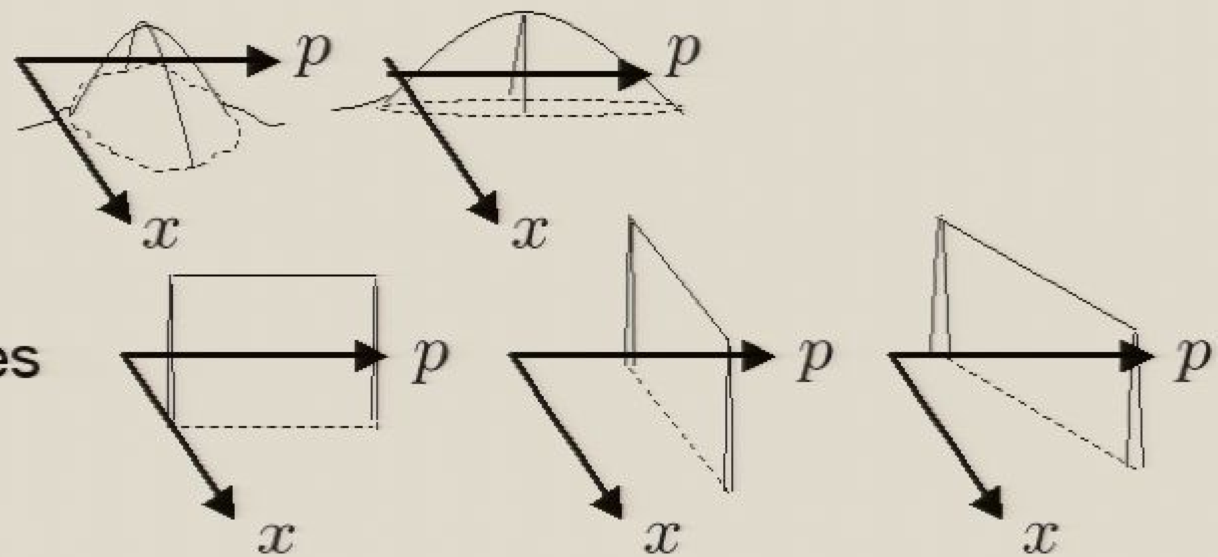
Wigner fn' is the symplectic Fourier transform of characteristic fn'

$$W_\rho(z) = \frac{1}{(2\pi)^{2n}}\int e^{iz^T\Sigma z'}\chi_\rho(z')dz'$$

If a quantum state satisfies the quantum uncertainty principle
Its Wigner function satisfies the classical uncertainty principle

Thus, the Wigner functions for Gaussian quantum states are valid epistemic states

Examples:
coherent states
Squeezed states
quadrature eigenstates
EPR state



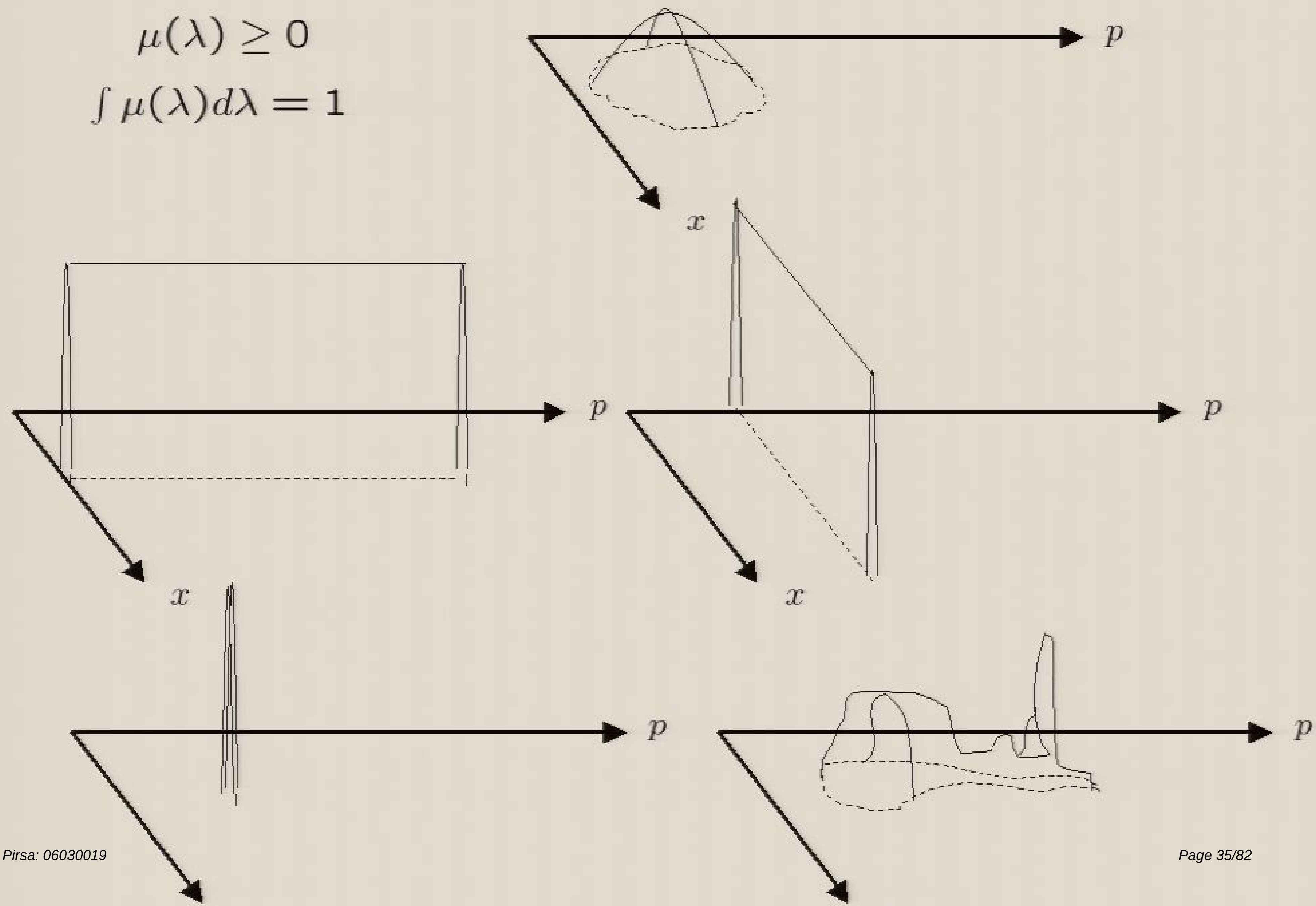
$$|EPR\rangle = \int dq_1\, dq_2\, \delta(q_1 - q_2)|q_1\rangle|q_2\rangle$$

$$W_{EPR}(q_1, p_1; q_2, p_2) = \frac{1}{N}\delta(q_1 - q_2)\delta(p_1 + p_2)$$

Epistemic states

$$\mu(\lambda) \geq 0$$

$$\int \mu(\lambda) d\lambda = 1$$

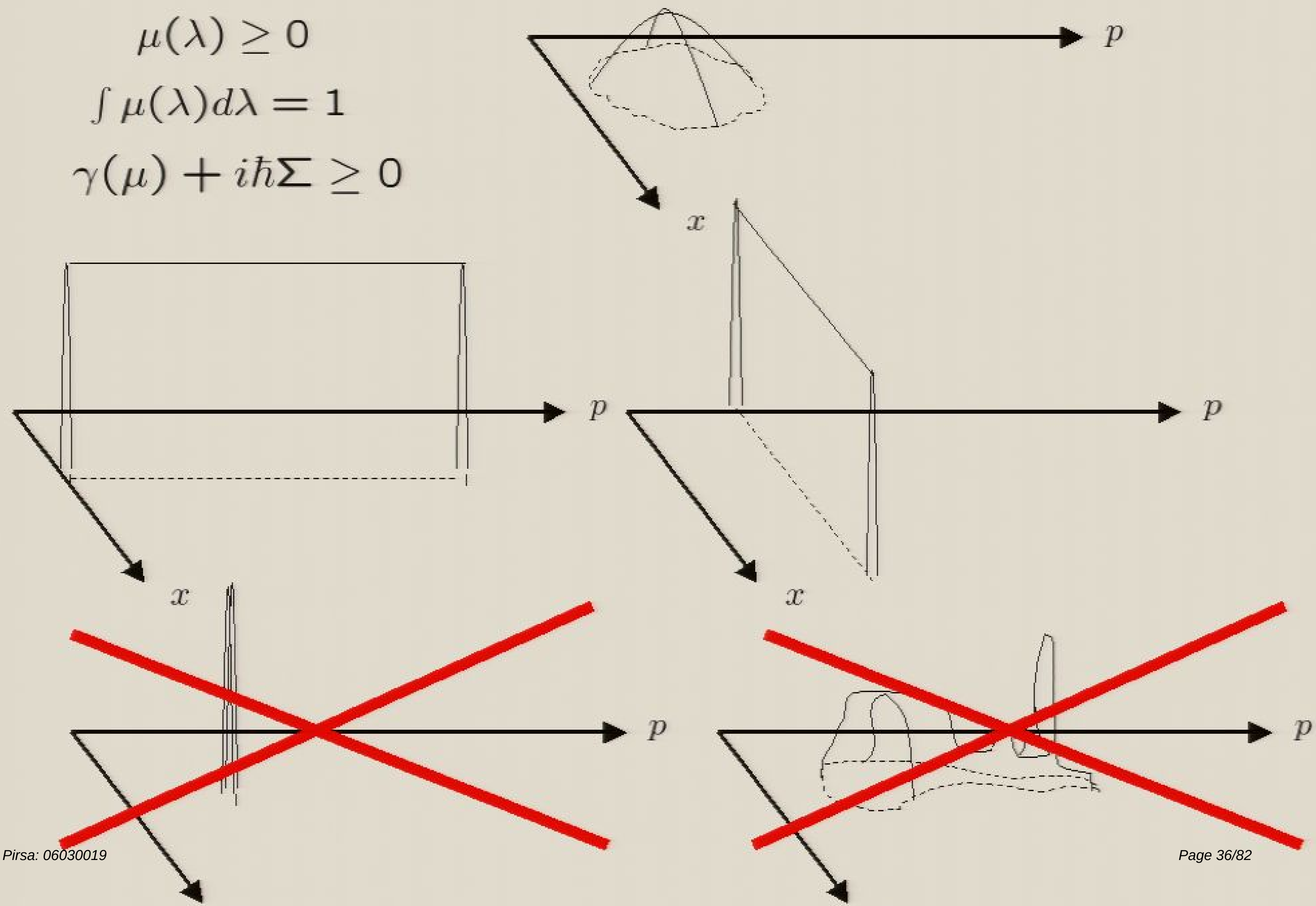


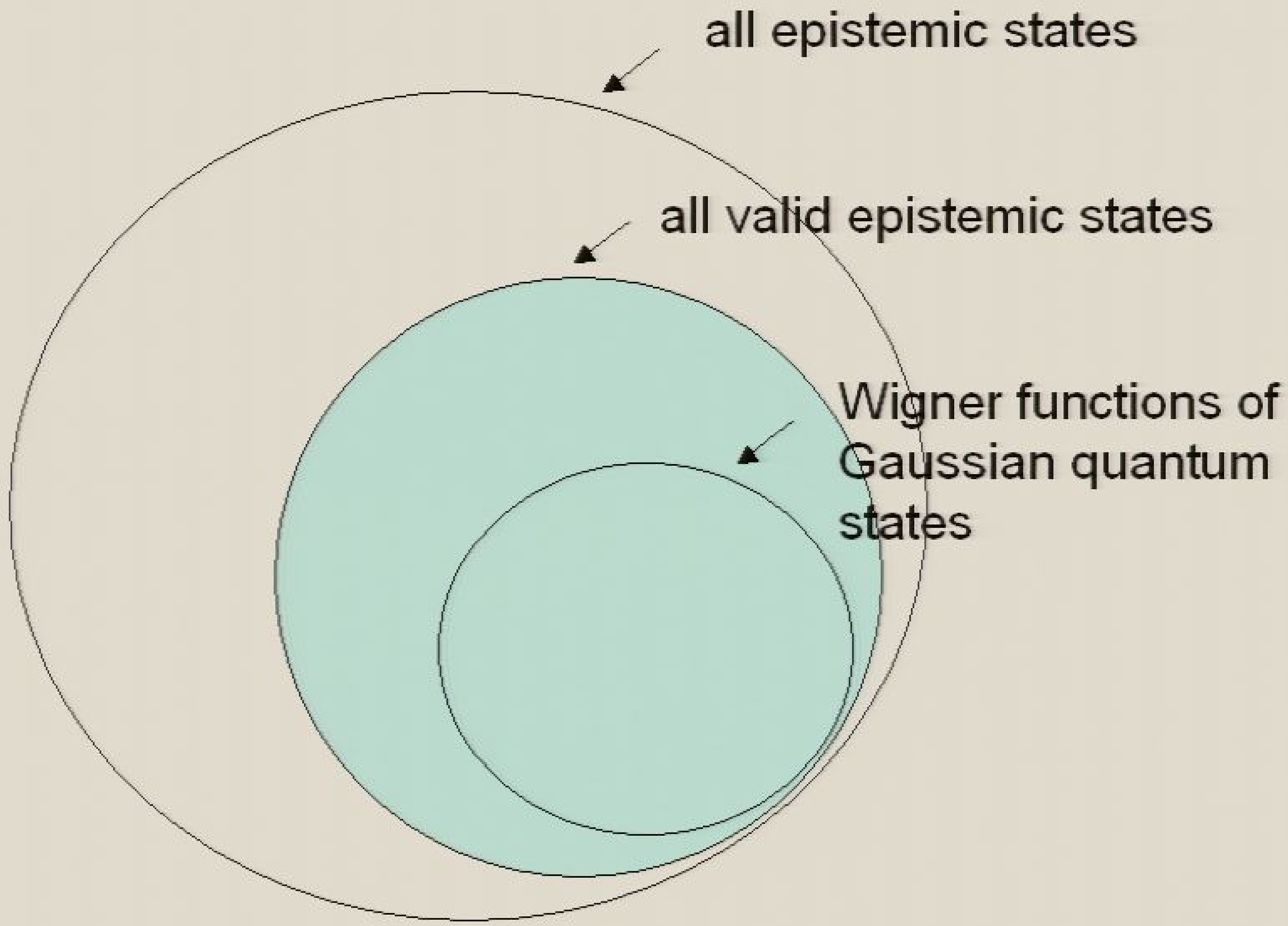
Valid epistemic states

$$\mu(\lambda) \geq 0$$

$$\int \mu(\lambda) d\lambda = 1$$

$$\gamma(\mu) + i\hbar\Sigma \geq 0$$




all epistemic states
all valid epistemic states
Wigner functions of Gaussian quantum states

measurements and transformations

Most general measurements

Quantum

POVMs $\{E_k\}$ Such that

$$E_k \geq 0$$

$$\sum_k E_k = I$$

Probability of outcome k

$$p_k = \mathrm{Tr}(E_k \rho)$$

Liouville

Sets of indicator functions $\{\xi_k\}$ s.t.

$$\xi_k(z) \geq 0 \text{ for all } z$$

$$\sum_k \xi_k(z) = 1 \text{ for all } z$$

Probability of outcome k

$$p_k = \int \xi_k(z) \mu(z) dz$$

Sets of Indicator functions

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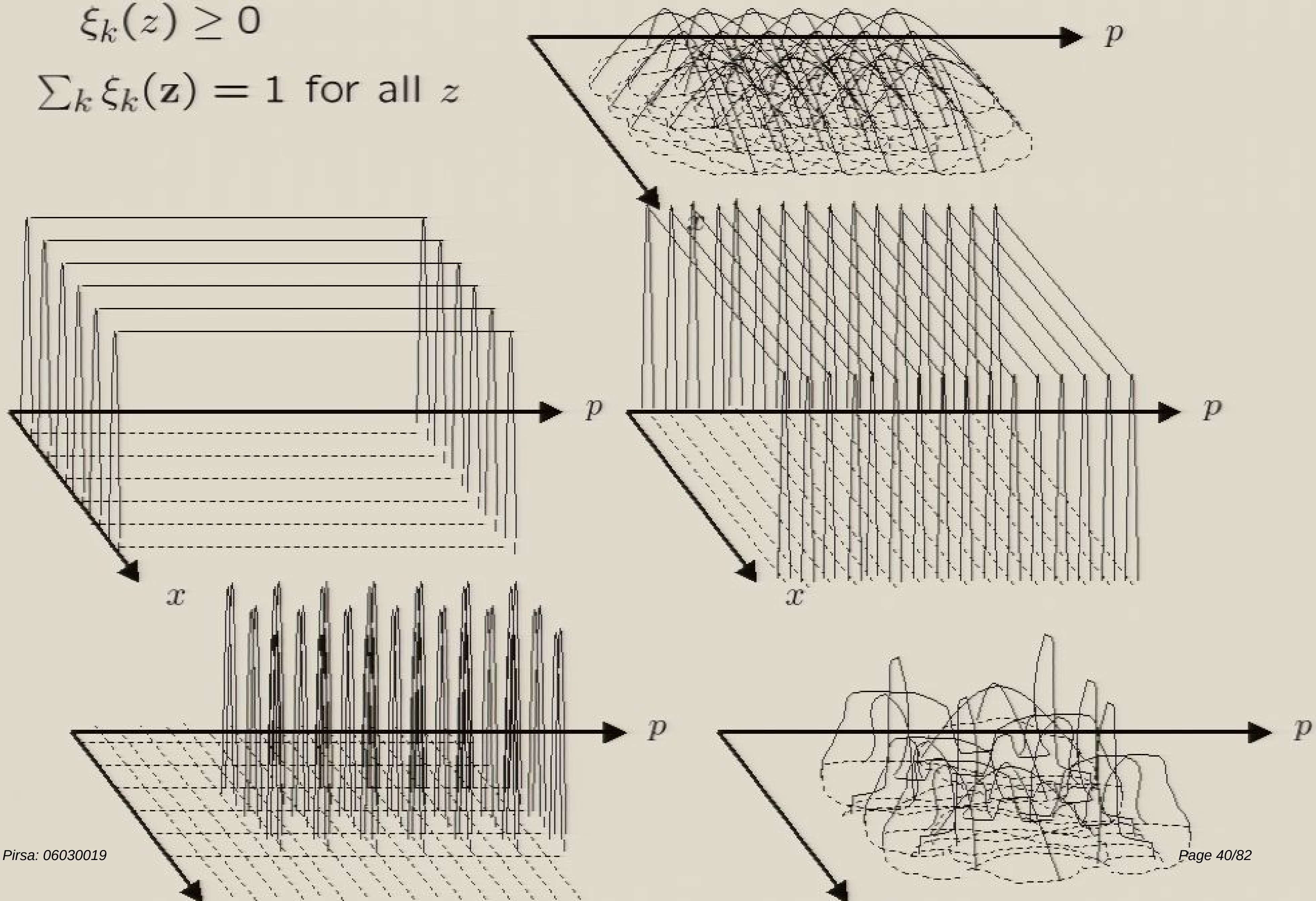

Epistemically restricted Liouville mechanics

Theorem: the valid indicator functions are the

$\xi(z)$ such that $\mu^{\xi}(z) \equiv \frac{\xi(z)}{|\xi(z)|}$

satisfies $\gamma(\mu^{\xi}) + i\hbar\Sigma \geq 0$

Valid sets of Indicator functions

$$\xi_k(z) \geq 0$$

$$\sum_k \xi_k(z) = 1 \text{ for all } z$$

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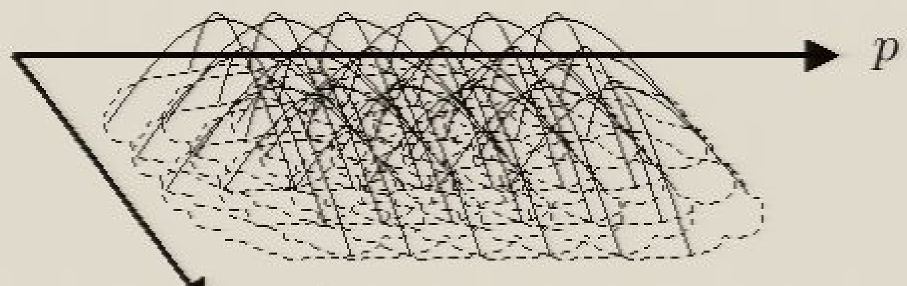
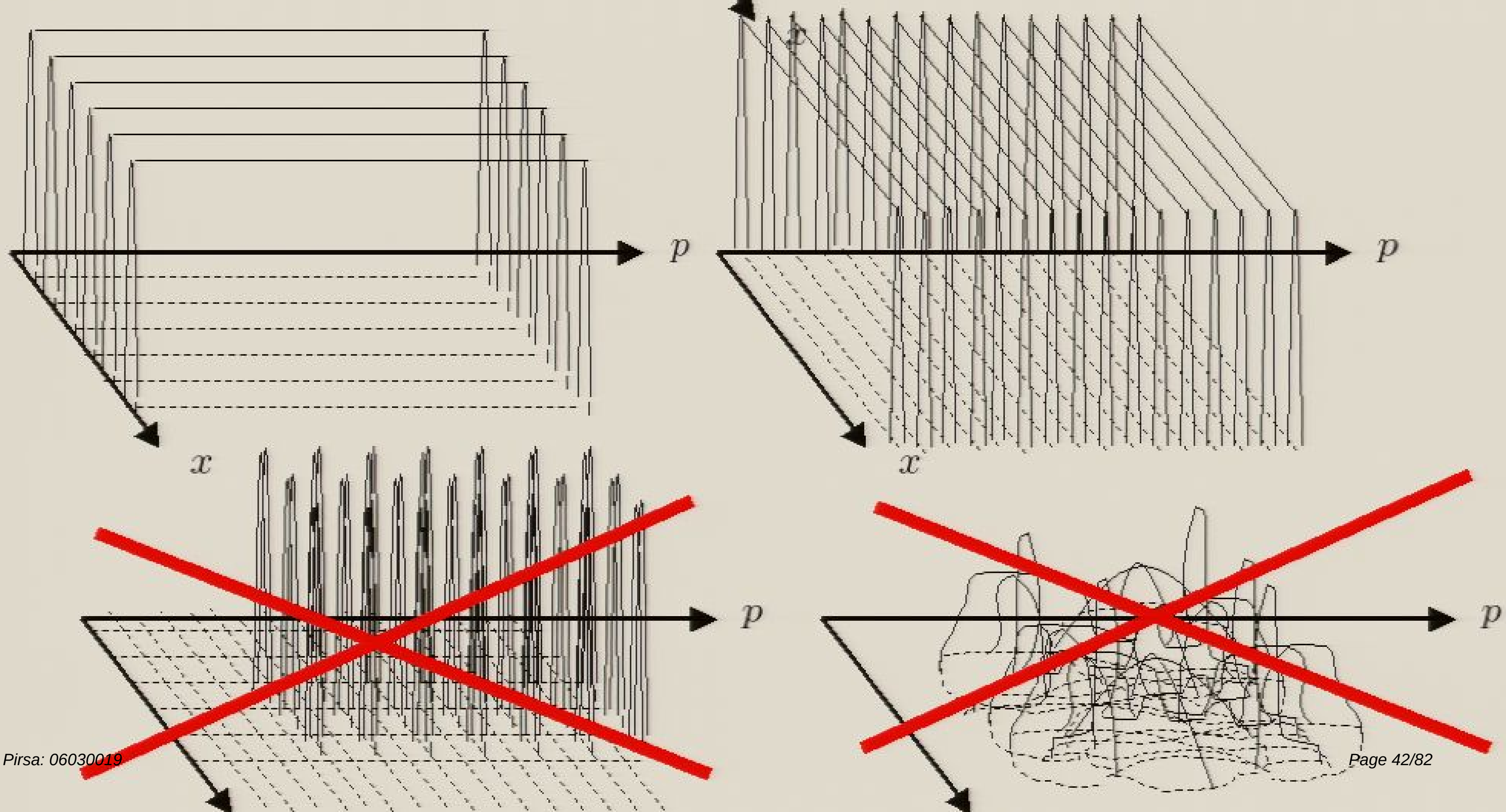




Transformations

Quantum

A map $\mathcal{E}$ such that

The state updates to

$$\rho' = \mathcal{E}[\rho]$$

trace-preserving

$$\mathrm{Tr}(\rho') = 1$$

Completely positive

$$\mathcal{E}^A \otimes \mathcal{I}^B[\rho^{AB}]) \geq 0$$

Liouville

A map Γ such that

The epistemic state updates to

$$\mu'(z) = \int \Gamma(z, z')\mu(z')dz'$$

normalized

$$\int \Gamma(z, z')dz = 1$$

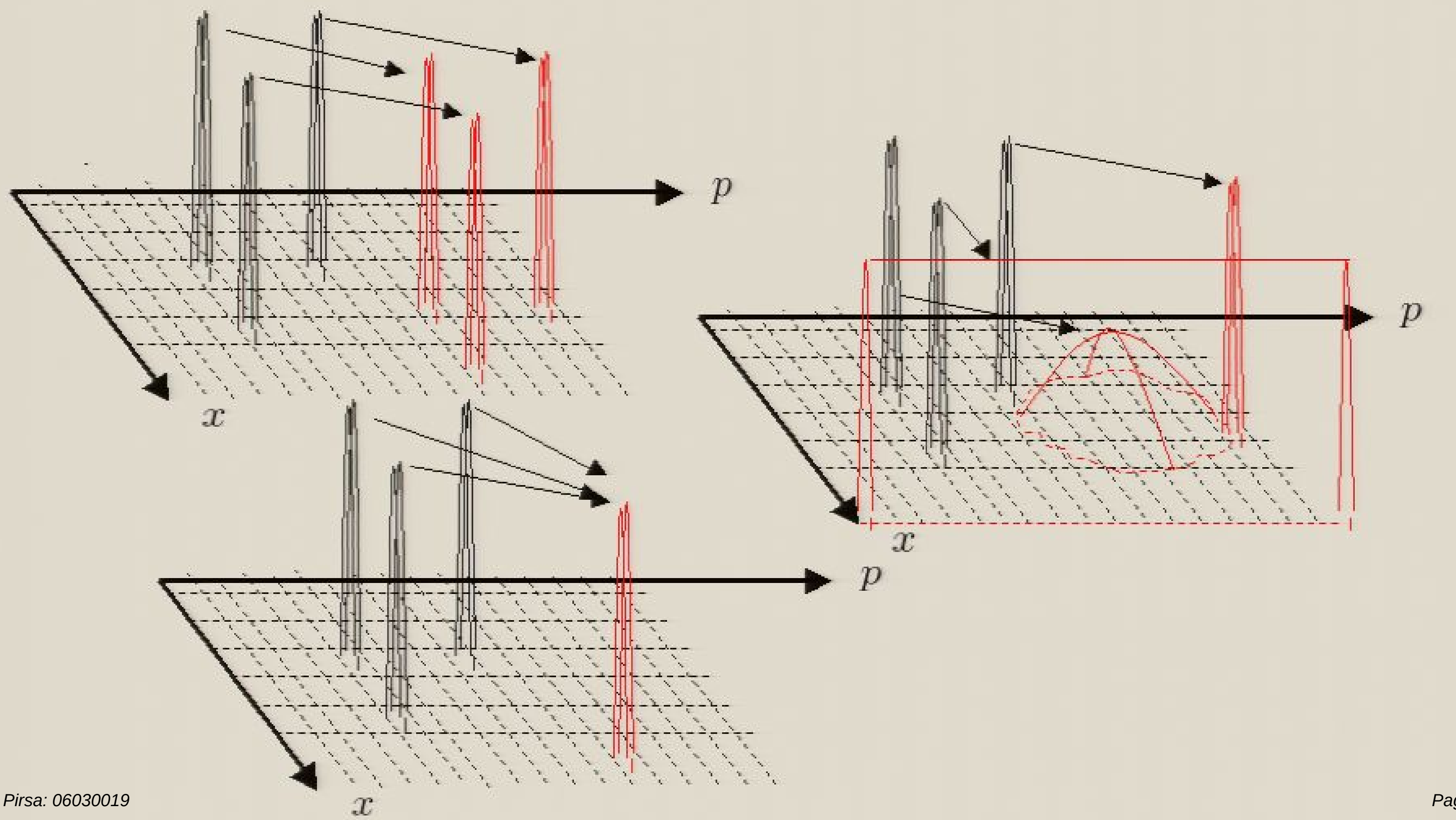
positive

$$\Gamma_k(z, z') \geq 0$$

Transfer functions

$$\Gamma(z', z) \geq 0$$

$$\int \Gamma(z', z) dz' = 1 \text{ for all } z$$



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Theorem: the valid transfer functions are the

$\Gamma(z', z)$ such that $\mu^{\Gamma}(z', z) \equiv \frac{\Gamma(z', z^C)}{|\Gamma(z', z^C)|}$

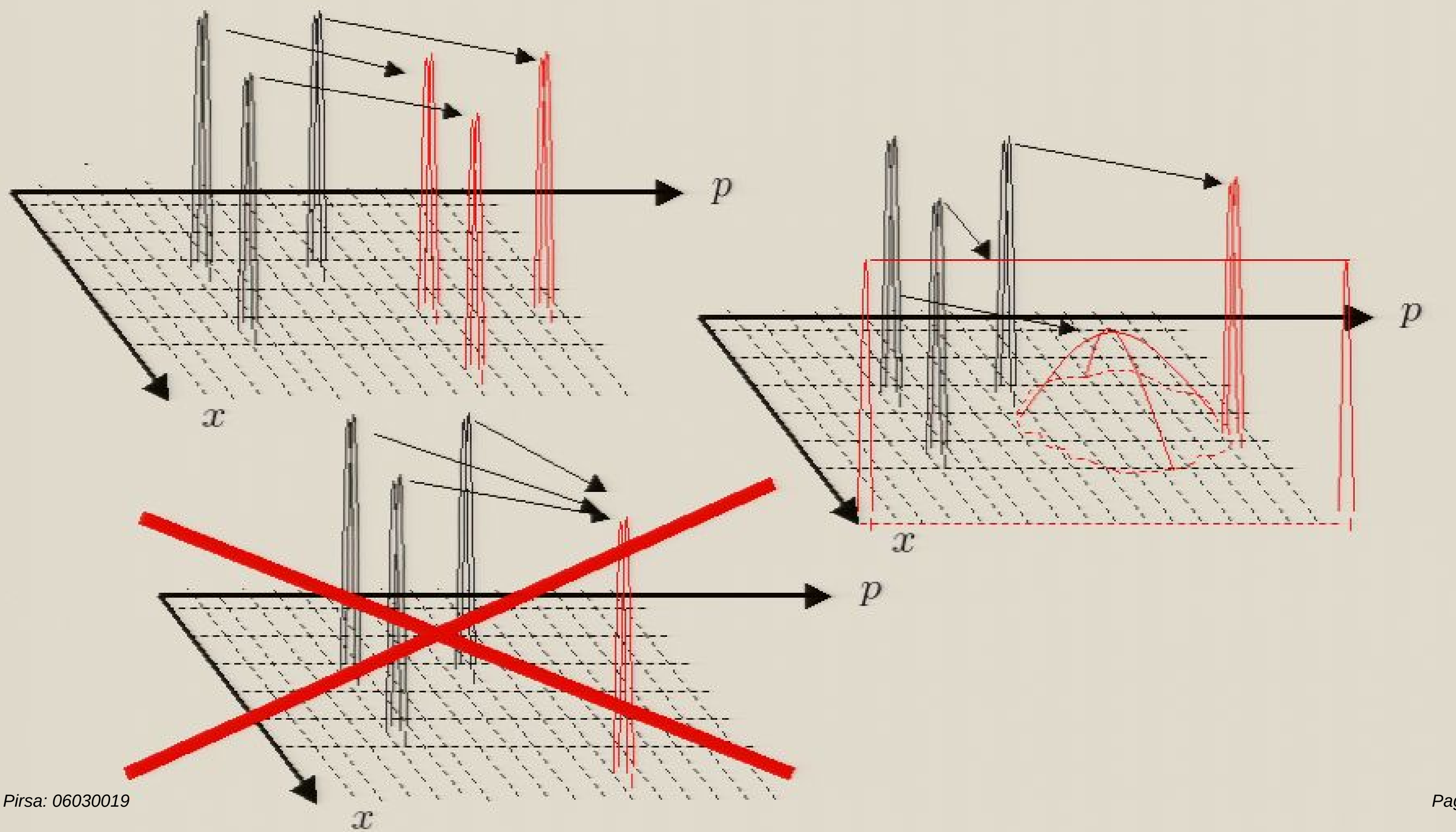
satisfies $\gamma(\mu^{\Gamma}) + i\hbar\Sigma \geq 0$

Valid transfer functions

$$\Gamma(z', z) \geq 0$$

$$\int \Gamma(z', z) dz' = 1 \text{ for all } z$$

$$\gamma(\Gamma/|\Gamma|) + i\hbar\Sigma \geq 0$$



Epistemically restricted Liouville mechanics

Valid epistemic states

$\mu(z)$ satisfying $\gamma(\mu) + i\hbar\Sigma \geq 0$

Valid transfer matrices:

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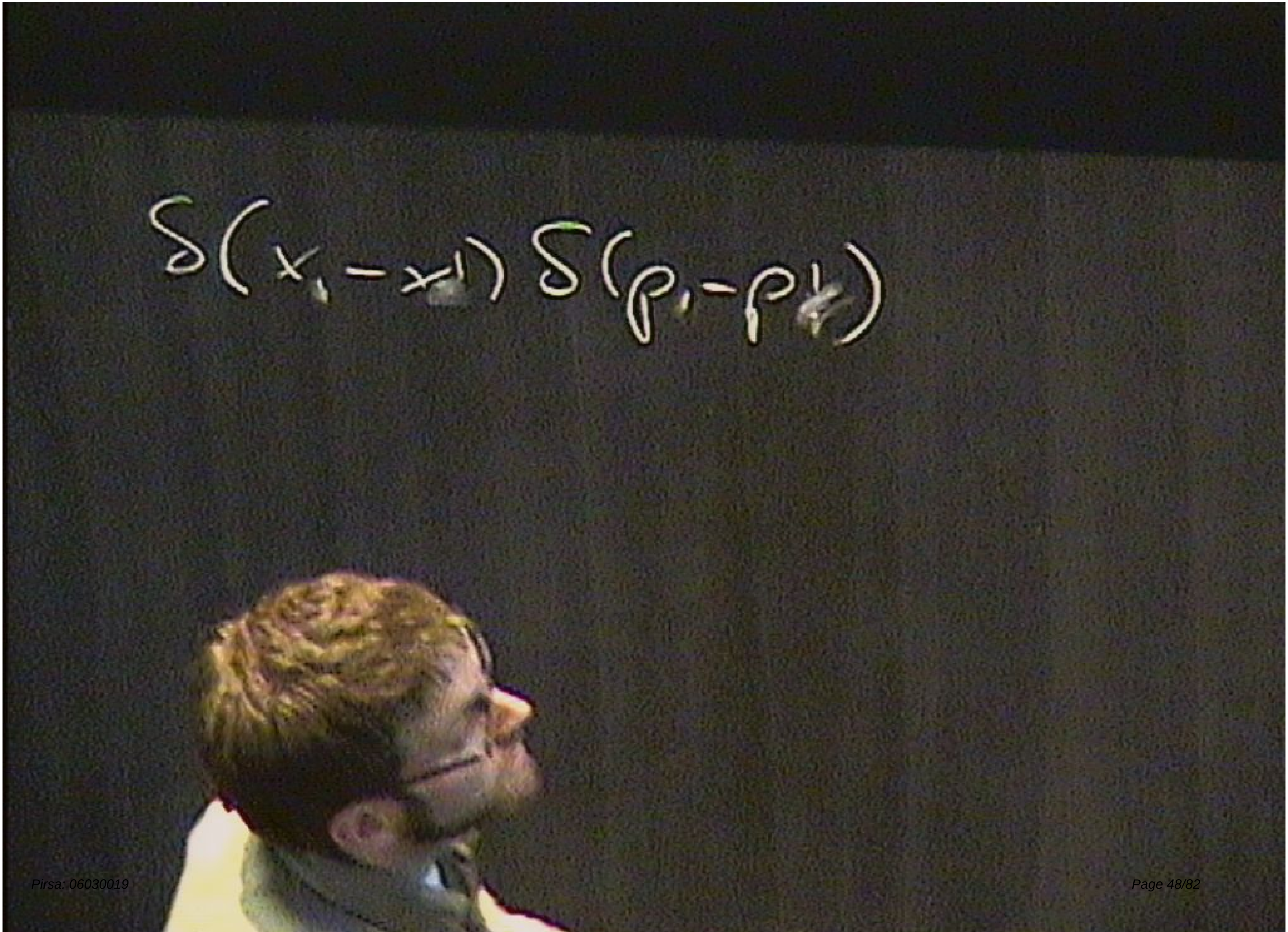
satisfies $\gamma(\mu^{\Gamma}) + i\hbar\Sigma \geq 0$

Valid indicator functions

$\xi(z)$ such that $\mu^{\xi}(z) \equiv \frac{\xi(z)}{|\xi(z)|}$

satisfies $\gamma(\mu^{\xi}) + i\hbar\Sigma \geq 0$


$\delta(x_1 - x_1')\,\delta(p_1 - p_1')$

$$\delta(x_1 - x')\,\delta(p_1 - p')$$

$$\delta(x - x')\,\delta(p + p')$$

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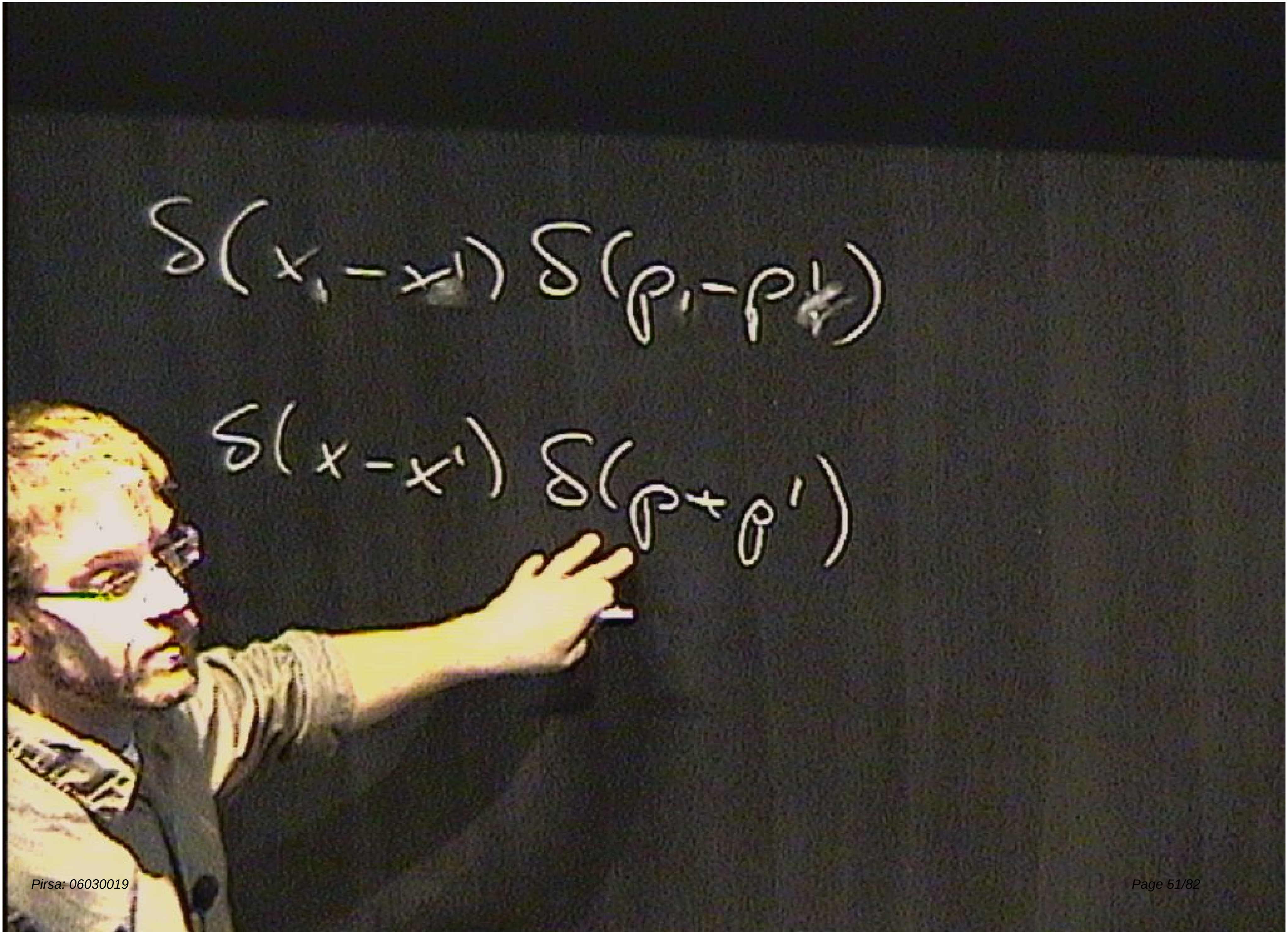
$\delta(x_1 - x'_1)\,\delta(p_1 - p'_1)$
$\delta(x - x')\,\delta(p + p')$

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Some important theorems

A change of canonical coordinates

$$z \equiv (x_1, p_1, x_2, p_2, \ldots, x_n, p_n)$$

$z' = Az$ where A is a symplectic transformation

Theorem: If the covariance matrix defined w.r.t. one choice of canonical coordinates satisfies the inequality, it does so w.r.t. all choices $\gamma(\mu) + i\hbar\Sigma \geq 0 \longrightarrow \gamma'(\mu) + i\hbar\Sigma \geq 0$

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Proof: The covariance matrix transforms as

$$\gamma_{ij}(\mu) = 2\langle (z_i - \langle z_i \rangle_\mu)(z_j - \langle z_j \rangle_\mu) \rangle_\mu$$

$$\Longrightarrow \quad \gamma'_{ij}(\mu) = 2\langle (z'_i - \langle z'_i \rangle_\mu)(z'_j - \langle z'_j \rangle_\mu) \rangle_\mu$$

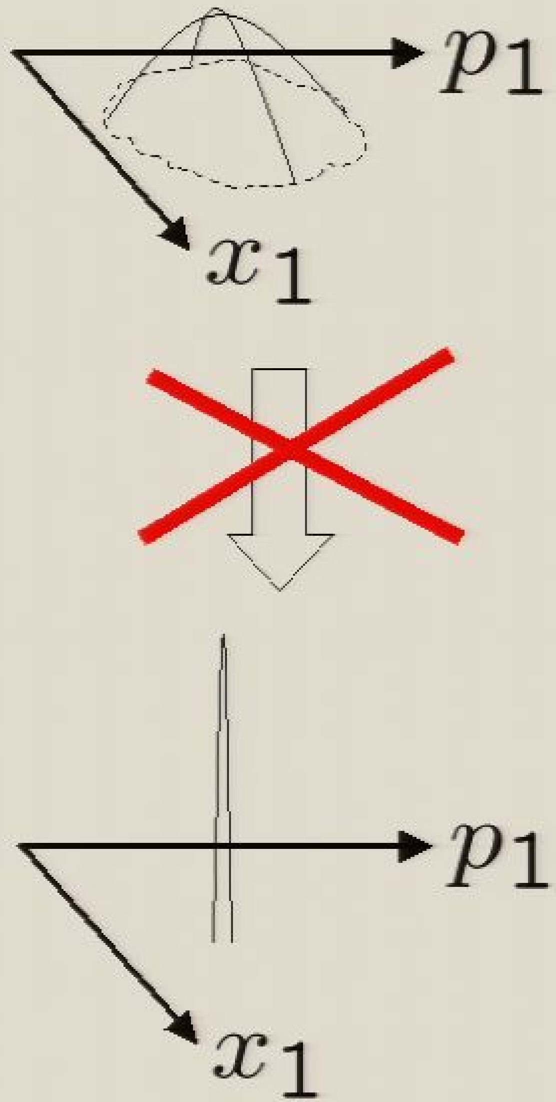
Thus $\gamma' = A\gamma A^\dagger$

so $\gamma(\mu) + i\hbar\Sigma \geq 0$ implies $\gamma'(\mu) + i\hbar A\Sigma A^\dagger \geq 0$

But $A\Sigma A^\dagger = \Sigma$ Q.E.D.

Hamiltonian evolution
is a canonical
transformation

Therefore, the CUP is
preserved under
Hamiltonian evolution



This is simply Liouville's theorem

Theorem: the CUP is satisfied by a distribution μ over a phase space $\mathcal{M}$
if and only if
it is satisfied for the marginal of μ on any canonical subspace $\mathcal{N} \subset \mathcal{M}$

$$\gamma(\mu) + i\hbar\Sigma \geq 0 \quad \longleftrightarrow \quad \text{For all } \mathcal{N} \subset \mathcal{M} \quad \gamma(\mu|_{\mathcal{N}}) + i\hbar\Sigma|_{\mathcal{N}} \geq 0$$

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if and only if
it is satisfied for the marginal of μ on any canonical subspace $\mathcal{N} \subset \mathcal{M}$

$$\gamma(\mu) + i\hbar\Sigma \geq 0 \quad \longleftrightarrow \quad \text{For all } \mathcal{N} \subset \mathcal{M} \quad \gamma(\mu|_{\mathcal{N}}) + i\hbar\Sigma|_{\mathcal{N}} \geq 0$$

Proof ($\longrightarrow$):

$$\gamma(\mu) = \begin{pmatrix} \gamma(\mu_{|\mathcal{N}}) & B \\ B^\dagger & \gamma(\mu_{|\mathcal{N}^\perp}) \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \Sigma_{|\mathcal{N}} & 0 \\ 0 & \Sigma_{|\mathcal{N}^\perp} \end{pmatrix}$$

$$\gamma(\mu)+i\hbar\Sigma = \begin{pmatrix} \gamma(\mu_{|\mathcal{N}}) + i\hbar\Sigma_{|\mathcal{N}} & B \\ B^\dagger & \gamma(\mu_{|\mathcal{N}^\perp}) + i\hbar\Sigma_{|\mathcal{N}^\perp} \end{pmatrix} \geq 0$$

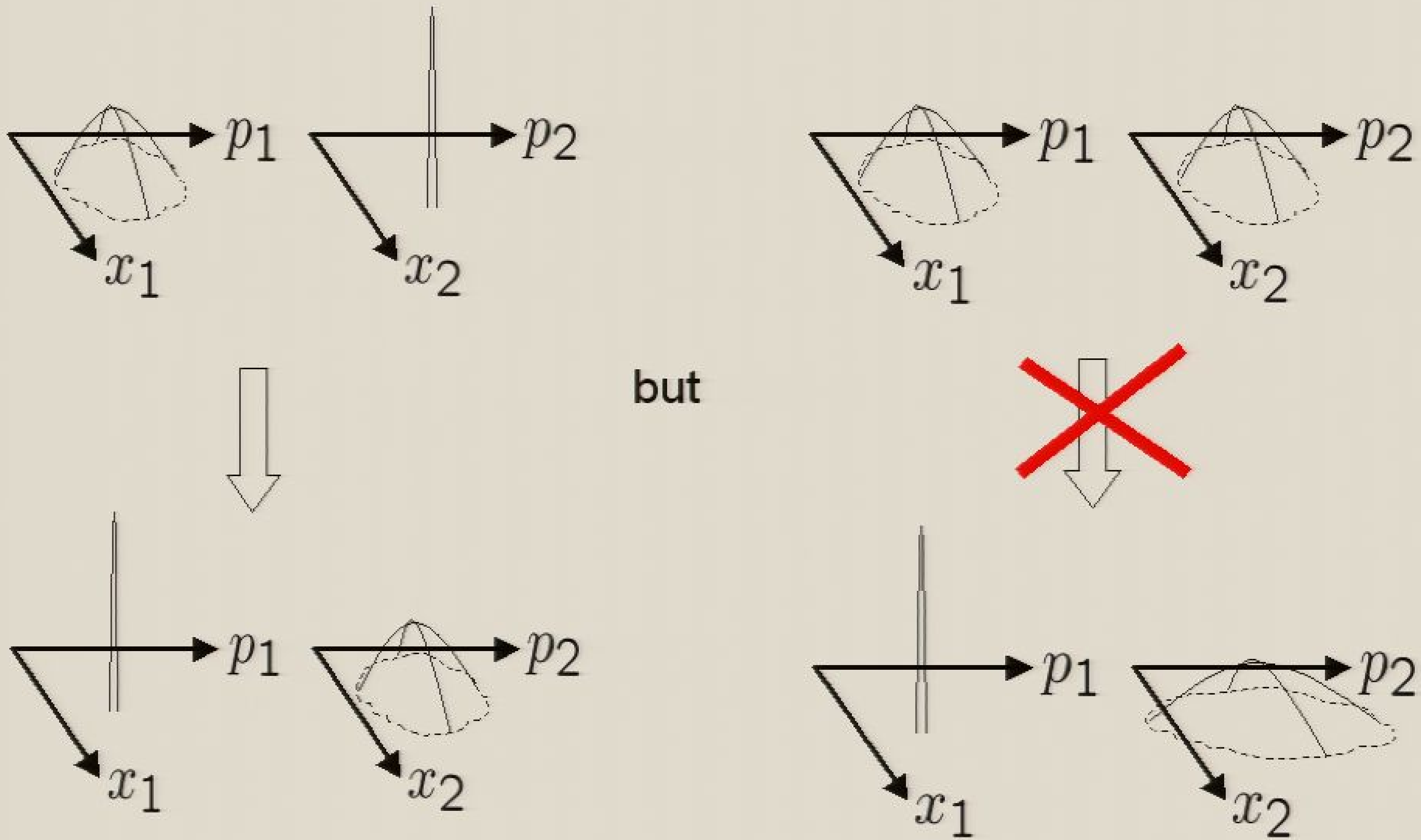
But $\begin{pmatrix} A & B \\ B^\dagger & C \end{pmatrix} \geq 0 \longrightarrow A \geq 0$

Proof ($\longleftarrow$):

By Williamson's theorem, there is a symplectic transformation that diagonalizes the covariance matrix

$$\gamma(\mu) = \begin{pmatrix} \gamma(\mu_{|\mathcal{N}_1}) & & & 0 \\ & \gamma(\mu_{|\mathcal{N}_2}) & & \\ & & \cdots & \\ 0 & & & \gamma(\mu_{|\mathcal{N}_n}) \end{pmatrix} \qquad \Sigma = \begin{pmatrix} \begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix} & & & 0 \\ & \begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix} & & \\ & & \cdots & \\ 0 & & & \begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix} \end{pmatrix}$$

$$\gamma(\mu|_{\mathcal{N}_i}) + i\hbar\Sigma|_{\mathcal{N}_i} \geq 0 \longrightarrow \gamma(\mu) + i\hbar\Sigma \geq 0$$ Q.E.D.

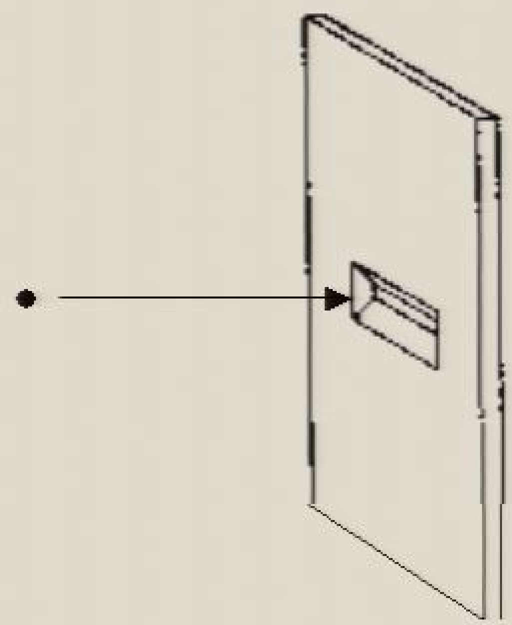


The minimal uncertainty can be moved around but cannot be decreased

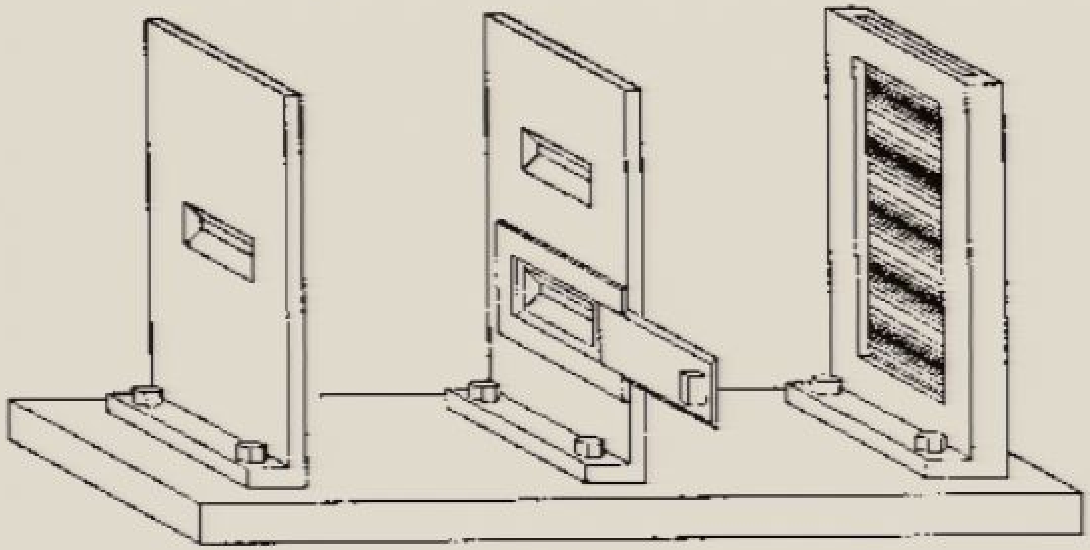
Bohr’s defence of the Heisenberg uncertainty principle

The challenge:

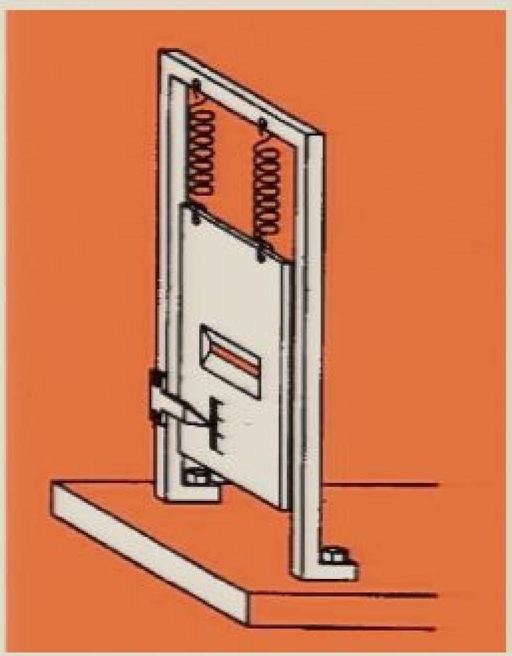
By passing a particle through a slit, we learn its position
By taking account of the momentum transferred, we learn its momentum



Bohr's response



"… [when] the knowledge of relative positions of the diaphragms and the photographic plate is secured by a rigid connection, it is obviously impossible to control the momentum exchanged between the particle and the separate parts of the apparatus."



Measure: momentum in vertical direction w.r.t. support

$p_d(t_i)$ Initial *p* of diaphragm

$p_d(t_f)$ Final *p* of diaphragm

$p_s(t_i)$ Initial *p* of particle

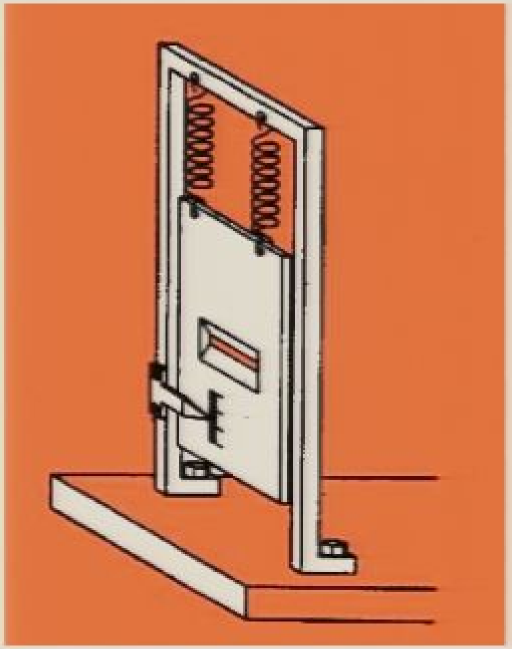
Infer:

$p_s(t_f)$ final *p* of particle

$$p_s(t_f) = p_s(t_i) + p_d(t_i) - p_d(t_f)$$

It *appears* as if one thereby comes to learn:

both $p_s(t_f)$ and $x_s(t_f)$



But in fact, one only learns

$$x_s(t_f) - x_d(t_f)$$

So to infer $x_s(t_f)$ we need to know $x_d(t_f)$

However, assuming that the UP applies to the diaphragm

We cannot know $p_d(t_f)$ and $x_d(t_f)$

So we cannot infer both $p_s(t_f)$ and $x_s(t_f)$

"... [the diaphragm] can no longer be used as a measuring instrument for the same purpose as in the previous case, but must, as regards its position relative to the rest of the apparatus, be treated, like the particle traversing the slit, as an object of investigation, in the sense that the quantum-mechanical uncertainty relations regarding its position and momentum must be taken explicitly into account."

"In fact, even if we knew the position of the diaphragm relative to the space frame before the first measurement of its momentum, and even though its position after the last measurement can be accurately fixed, we lose, on account of the uncontrollable displacement of the diaphragm during each collision process with the test bodies, the knowledge of its position when the particle passed through the slit."

Bohr speaks of:

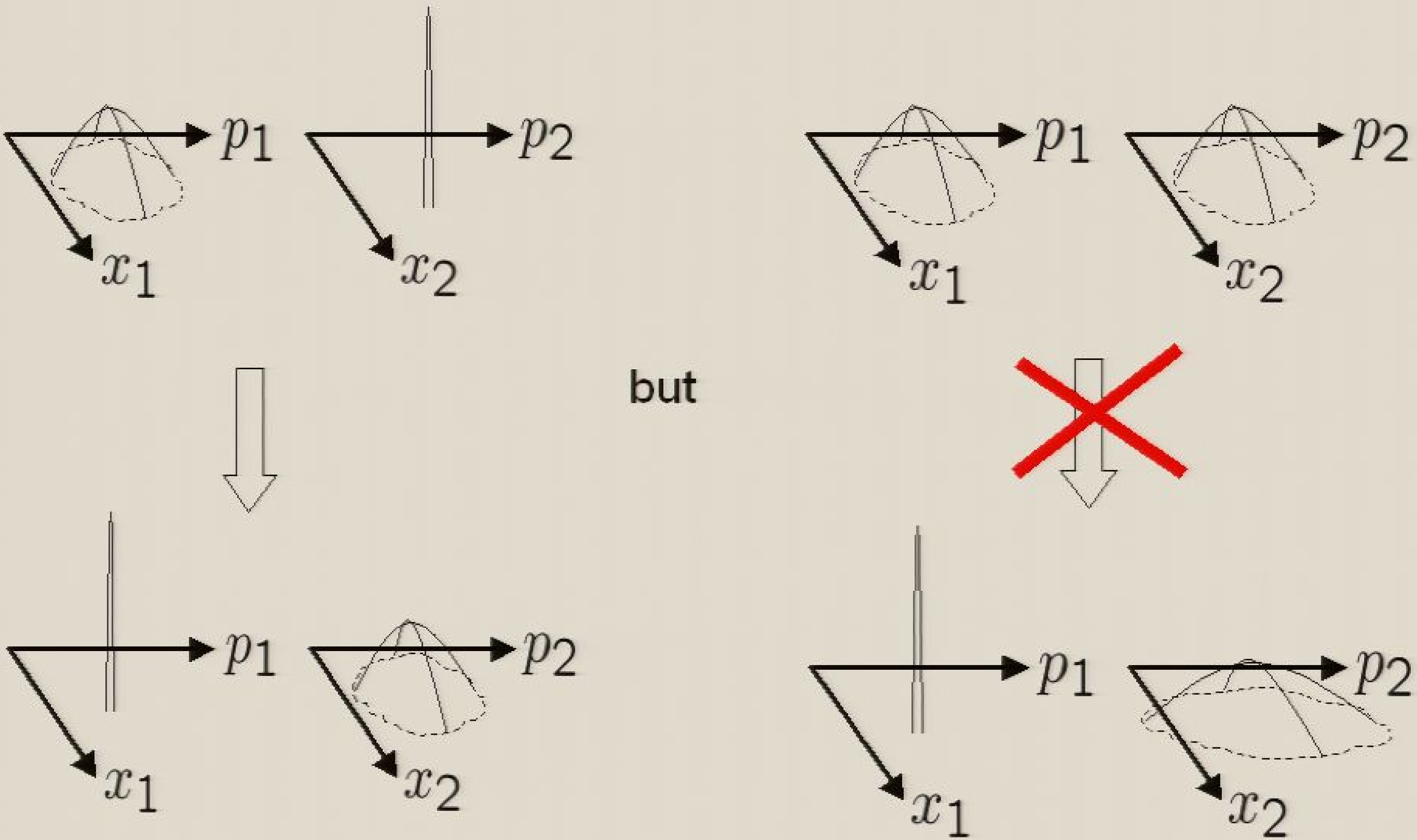
"the position of the diaphragm"

"its momentum"

He never makes use of Hilbert space

His argument goes through verbatim as an argument for the consistency of the CUP in ERL mechanics

More generally

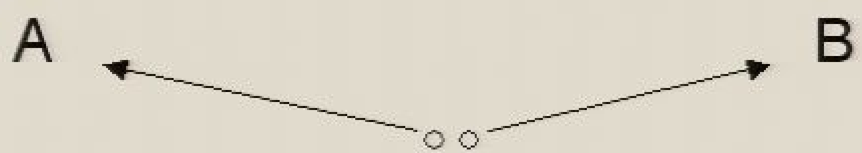


The minimal uncertainty can be moved around but cannot be decreased

The EPR argument and Bohr's reply

EPR criterion of reality:

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity



$$
\begin{aligned}
|EPR\rangle &= \int dx_1\, dx_2\, \delta(x_1 - x_2)|x_1\rangle|x_2\rangle \\
&= \int dp_1\, dp_2\, \delta(p_1 + p_2)|p_1\rangle|p_2\rangle
\end{aligned}
$$

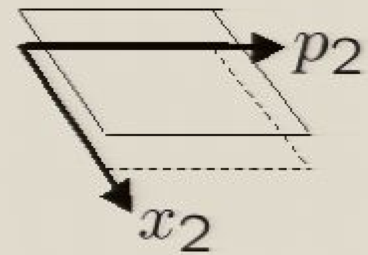
Either measure x or p on particle 1

This the experiment has an exact analogue in ERL mechanics

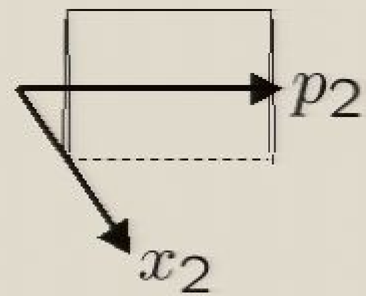
$$W_{EPR}(q_1, p_1; q_2, p_2) = \frac{1}{N}\delta(q_1 - q_2)\delta(p_1 + p_2)$$

$$\mu(q_2, p_2) = \frac{1}{N}$$

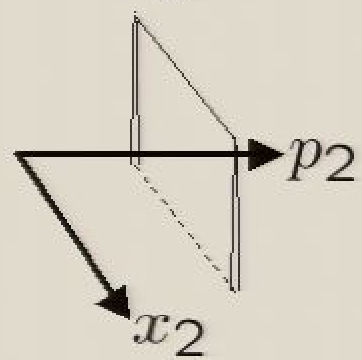
Initially A is completely ignorant of 2



If A measures x on 1, she infers x of 2



If A measures p on 1, she infers p of 2



A's decision does not affect the reality at 2,
the x and p were already elements of reality

Bohr's response:

By learning x_1, you disturb p_1
So p_1 is no longer correlated with p_2
So a *subsequent* measurement of p_1
does not allow one to infer p_2

"By allowing an essentially uncontrollable momentum to pass from the first particle into the mentioned support, however, we have by this procedure cut ourselves off from any future possibility of applying the law of conservation of momentum to the system consisting of the diaphragm and the two particles and therefore have lost our only basis for an unambiguous application of the idea of momentum in predictions regarding the behavior of the second particle."

Essentially: You can't come to know both x_2 and p_2
But this is just another defence of the uncertainty principle!

A tension in Bohr's response to EPR

"In fact, even if we knew the position of the diaphragm relative to the space frame before the first measurement of its momentum, and even though its position after the last measurement can be accurately fixed, we lose, on account of the uncontrollable displacement of the diaphragm during each collision process with the test bodies, the knowledge of its position when the particle passed through the slit."

Just in this last respect any comparison between quantum mechanics and ordinary statistical mechanics,---however useful it may be for the formal presentation of the theory,---is essentially irrelevant. Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way.

My conclusion:

Bohr must believe that two quantities can be jointly well-defined only if they can be jointly measured

Otherwise, why from the impossibility of two quantities being jointly measured would he infer the impossibility of their being jointly well-defined, as opposed to merely inferring the impossibility of their being jointly known

But none of Bohr's arguments are at odds with a hidden variable interpretation of the EPR experiment
In fact, they resonate nicely with such an interpretation

Bohr's operationalism is assumed in his analysis of EPR
His analysis provides no new argument for his operationalism

Discussion

Quantum phenomena that are reproduced in ERL mechanics

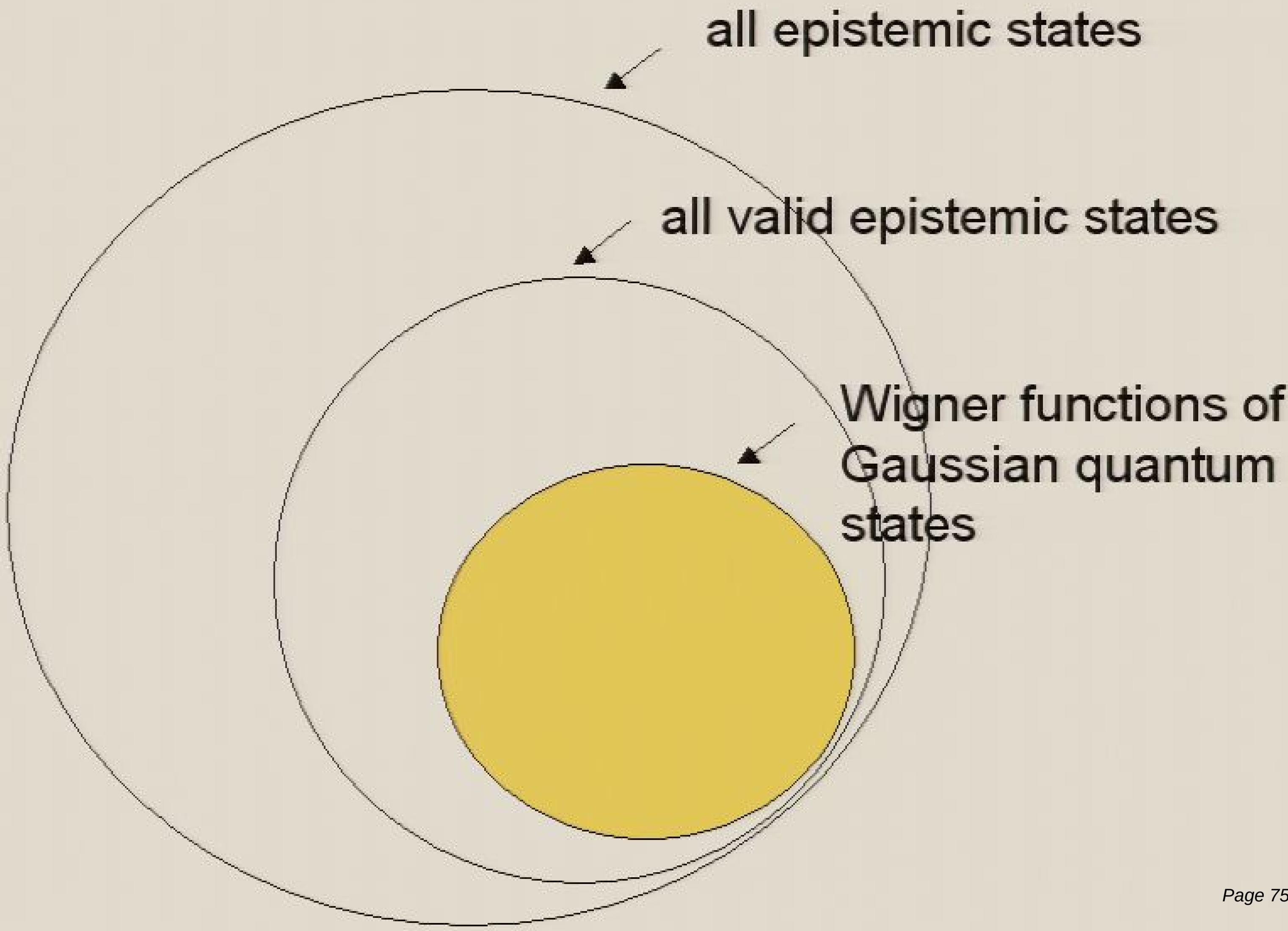
- Jamiolkowski isomorphism
- CV Teleportation
- No information gain without disturbance
- Poisson bracket of two functions determines whether they are simultaneously measurable (with J. Emerson and F. Girelli)

Presumably, most of what can be done with Gaussian states and operations alone

- Key distribution
- A large part of entanglement theory
- Bound entanglement
- …

Improvements to the theory

There is likely to be a better epistemic constraint!



Phenomena that are not reproduced

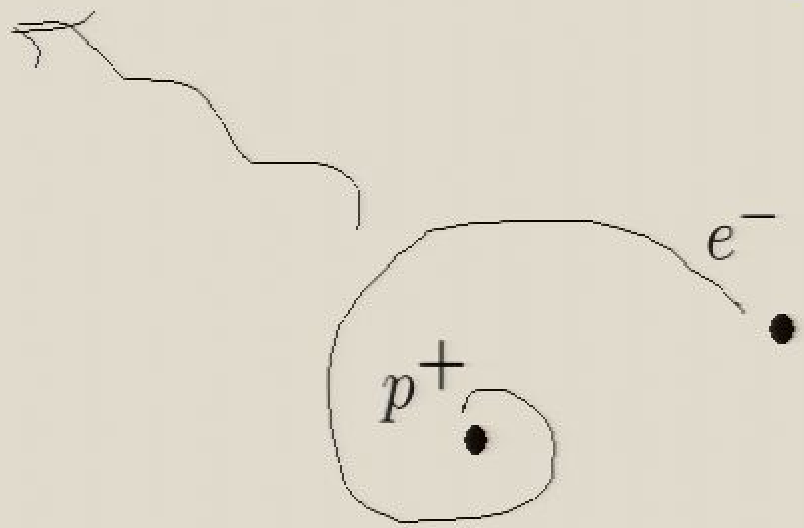
- Nonlocality
- Contextuality
- Exponential speed-up in computation (if it exists)
- many others…

- Quantization??

These phenomena may teach us the way…

Beyond particle mechanics…

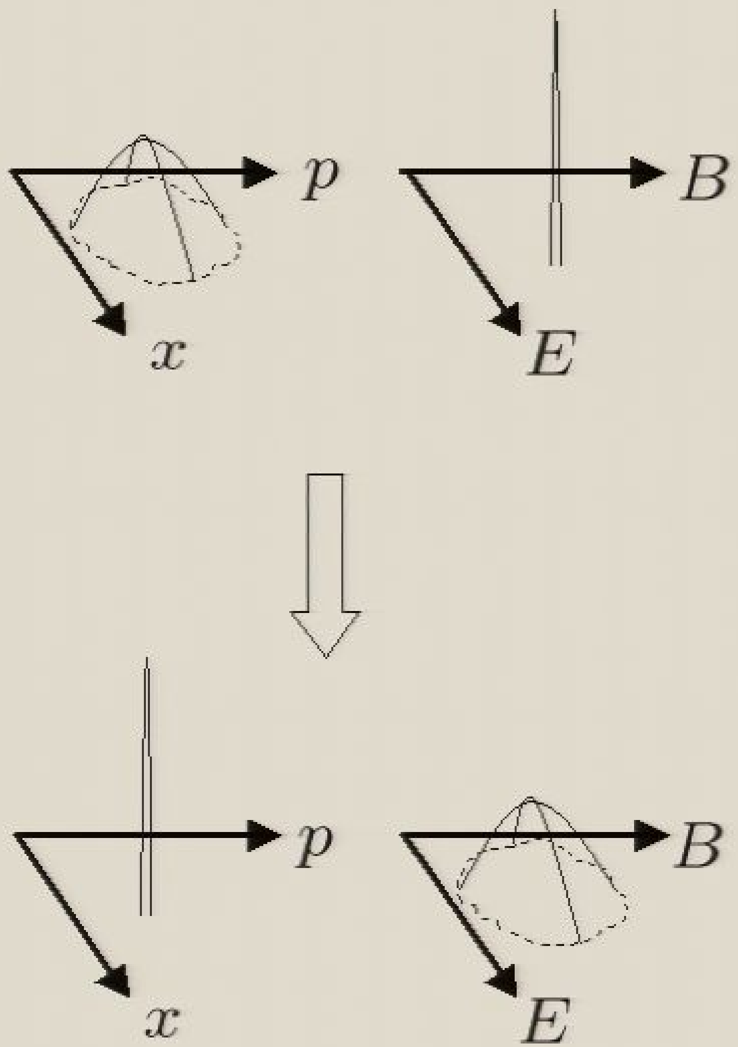
Atomic stability



Standard argument: electron will spiral into nucleus and radiate

But: this would imply certainty about relative x and relative p!

Conclusion: If CUP is satisfied for the particles, then for consistency, we must demand that CUP is satisfied for the EM field as well!



Thus the quantum vacuum state is not emptiness

– it is an epistemic state

This is reminiscent of the theory of stochastic electrodynamics

Stochastic electrodynamics

- L. de la Pena and A. M. Cetto, "The Quantum Dice: An Introduction to Stochastic Electrodynamics", Kluwer (1996)

- T. H. Boyer, "A Brief Survey of Stochastic Electrodynamics", in Foundations of Radiation Theory and Quantum Electrodynamics, edited by A. O. Barut, Plenum (1980)

- T. W. Marshall and E. Santos, Found. Phys. 18, 185 (1988); Phys. Rev. A39, 6271 (1989)
- T. H. Boyer, Found. Phys. 19, 1371 (1989)
- D. C. Cole, Phys. Rev. A42, 1847 (1990); Phys. Rev. A42, 7006, (1990)

Quantum phenomena which SED reproduces

- Stability of atomic ground states
- Certain features of atomic ground states
- Planck blackbody spectrum
- Einstein A and B coefficients
- Lamb shift
- Casimir effect
- Unruh effect
- various quantum optical phenomena
- …

Further evidence for vacuum substructure (together with Elliot Martin)

Toy field theory has analogues of:

- Spatial interference (of Mach-Zehnder variety)
- Interaction-free measurement
- quantum eraser
- Hardy's "reality of the empty wave" experiment

Next stop: epistemically restricted electrodynamics (fields and particles)

"To infinity … and beyond!"