

Title: Introduction to quantum gravity - Part 16

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

-undergraduate quantum mechanics

-basics of classical gauge field theories

-basic general relativity

-hamiltonian and lagrangian mechanics

-basics of lie algebras

Boundary condition

$$B = S^2$$

$U \vee \text{rep}$
 $H^1(U)$

Secondary to ...
 $\int du$

$A + D$
 $\pi \ll$



Boundary condition $\Gamma = \{u, \partial u\}$ $B = S^2$

$\int_{\Gamma} u \nu \text{tr} \rho$
 $\int_{\Gamma} H^2(u)$

$\int_{\Gamma} d\mu H^2(u) = 0$

$\int_{\Gamma} d\mu H^2(u) H^2(u) = C_2 \delta_{ij}$

$A \neq D$
 ν
 K

$\int_{\Gamma} \psi_{\nu}^{\text{rep}}$
 $H^{\nu}(\mu)$

$$\int d\mu H^{\nu}(\mu) = 0$$

$$\int d\mu H^{\nu}(\mu) H^{\nu'}(\mu) = c_{\nu} \delta_{\nu\nu'}$$

$\Rightarrow \Gamma \neq \Gamma'$ are different

$$\bigcirc \Rightarrow \langle \psi_{\Gamma} | \psi_{\Gamma'} \rangle = 0$$

$\int_{\Gamma} \psi_{\mu}^{\nu} \text{rep}$
 $H^1(\Gamma)$

$$\int d\mu H^1(\mu) = 0$$

$$\int d\mu H^1(\mu) H^2(\mu) = C_2 \delta_{ij}$$

$\Rightarrow \Gamma \neq \Gamma'$ are different

$$\mathbb{D} \Rightarrow \langle \psi_{\Gamma} | \psi_{\Gamma'} \rangle = \delta_{\Gamma \Gamma'}$$

$\int_{\Gamma} \psi_{\Gamma}^* \psi_{\Gamma'} \text{ rep}$
 $H^1(\mathbb{R})$

$$\int d\mu H^1(\mathbb{R}) = 0$$

$$\int d\mu H^1(\mu) H^1(\mu) = C_1 \delta_{\Gamma\Gamma'}$$

$\Rightarrow \Gamma \neq \Gamma'$ are different

$$\bigcirc \Rightarrow \langle \psi_{\Gamma} | \psi_{\Gamma'} \rangle = \delta_{\Gamma\Gamma'}$$

$\Rightarrow \psi_{\Gamma}(A)$ are an orthonormal basis

\Rightarrow not a countable basis \Rightarrow NON-SEPARABLE

$$\phi \in \text{Diff}(\Sigma)$$

-2

$$\phi \in \text{Diff}(\Sigma) \quad \phi: \Gamma \rightarrow \Gamma' = \phi \circ \Gamma$$

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$$[\mathcal{U}(\phi) \Psi_\Gamma](A) = \Psi_{\phi \circ \Gamma}(A) = \Psi_\Gamma[\phi \circ A]$$



$$\phi \in \text{Diff}(\Sigma) \quad \phi: \Gamma \rightarrow \Gamma' = \phi \circ \Gamma$$

$$\Rightarrow \text{UNITARY} \quad [\hat{U}(\phi) \Psi_\Gamma](A) = \Psi_{\phi \circ \Gamma}(A) = \Psi_\Gamma[\phi \circ A]$$

$$\phi \in \text{Diff}(\Sigma) \quad \phi: \Sigma \rightarrow \Sigma = \phi \circ \text{id}$$

$$\Rightarrow \text{UNITARY} \quad [\hat{U}(\phi) \Psi]_A = \Psi_{\phi^{-1}A} = \Psi_{\phi \circ A}$$

$$\hat{U}(\phi) \hat{U}(\phi') = \hat{U}[\phi \circ \phi']$$

$$\langle \Psi | \Phi \rangle = \langle \hat{U}(\phi) \Psi | \hat{U}(\phi) \Phi \rangle$$

$$\phi \in \text{Diff}(E) \quad \phi: \Gamma \rightarrow \Gamma' = \phi \circ \Gamma$$

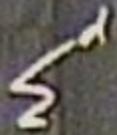
$$\Rightarrow \text{UNITARY} \quad [\hat{U}(\phi) \Psi]_{\Gamma}(A) = \Psi_{\phi \circ \Gamma}(A) = \Psi_{\Gamma}[\phi \circ A]$$

$$\hat{U}[\phi] \hat{U}[\phi'] = \hat{U}[\phi \circ \phi']$$

$$\langle \Psi | \Phi \rangle = \langle \hat{U}(\phi) \Psi | \hat{U}(\phi) \Phi \rangle$$

$$[\hat{T}[\Gamma] \Psi](A) = T[\Gamma, A] \Psi[T_{\Gamma}]$$

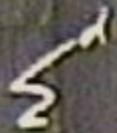
$$[\hat{T}[\Gamma], \hat{T}[\Gamma']] = 0$$



$$\Psi_0\{A\} = 1$$

$$\Psi_p(A) = \hat{T}[\hat{H}] \Psi_0$$

$$\hat{T}^g[\chi, s]$$



$$\Psi_0\{A\} = 1$$

$$\Psi_p(A) = \hat{T}[\hat{H}] \Psi_0$$

$$\hat{T}^a[\chi, s] \Psi_p[A] \equiv \{T[\chi, s], T_p[A]\} \Psi_0$$



$$\Psi_0\{A\} = 1$$

$$\Psi_P(A) = \hat{T}[\rho] \Psi_0$$

$$\hat{T}^\alpha[\gamma, s] \Psi_P[A] = \{T[\gamma, s], T_P[A]\} \Psi_0$$

$$\{T[\gamma], T^\alpha[\alpha, s]\} = \int dt \, S^3(\gamma(t), \alpha(s)) \dot{\gamma}^\alpha(t) \left([T[\alpha \circ \gamma] - T[\alpha \circ \gamma^{-1}]] \right)$$





$$\Psi_0\{A\} = 1$$

$$\Psi_P(A) = \hat{T}[\Gamma] \Psi_0$$

$$\hat{T}^\alpha[\gamma, s] \Psi_P[A] = \{T[\gamma, s], T_P[A]\} \Psi_0$$

$$\{T[\gamma], T^\alpha[\alpha, s]\} = \int dt \delta^3(\gamma(t), \alpha(s)) \dot{Y}^\alpha(t) \left([T[\alpha \circ \gamma] - T[\alpha \circ \gamma^{-1}]] \right)$$



$$\hat{T}^\alpha[\alpha, s] \Psi_\gamma[A] \rightarrow \int dt \delta^3(\alpha(t), \gamma(s)) \dot{Y}^\alpha(t) \left[\Psi_{\alpha \circ \gamma} - \Psi_{\alpha \circ \gamma^{-1}} \right]$$



$$T^a[\gamma, s] = \text{Tr}[U_{\gamma, s} E^a(\gamma(s))]$$

$T^a[\gamma, s]$ & $T[\gamma]$ form a Lie algebra

$$\{T, T^a\} = 0 \quad \{T, T^a\} = (\) T^a \quad \{T^a, T^b\} = T^c \dots$$

little loop algebra



'U(1) e a H(a, a)



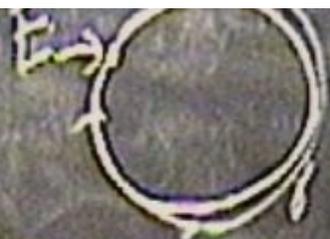
$$T^a[\gamma, s] = \text{Tr} \left[\prod_{\gamma, s} E^a(\gamma(s)) \right]$$

$T^a[\gamma, s]$ & $T[\gamma]$ form a Lie algebra

$$\{T, T^a\} = 0 \quad \{T, T^b\} = (\) T^a \quad \{T^a, T^b\} = T^c \quad \text{CHECK}$$

little loop algebra

$$C = \int EEF + \Lambda EEE$$



$$T^a[\gamma, s] = \text{Tr}[U_{\gamma, s} \hat{E}^a(\gamma(s))]$$

$T^a[\gamma, s]$ & $T[\gamma]$ form a Lie algebra

$$\{T, T^a\} = 0 \quad \{T, T^b\} = (\) T^a$$

little loop algebra

$$\{T^a, T^b\} = T^c \quad \text{CHECK}$$

$$\hat{E}^a(x) \Psi(A) = \frac{\delta \Psi(A)}{\delta A(x)} = \delta^a(x)$$

$$C = \int E E F + \Lambda E E E \quad \hat{E}(x) \hat{F}(x) =$$



$$T^a[\gamma, s] = \text{Tr}[U_{\gamma, s} \hat{E}^a(\gamma(s))]$$

$T^a[\gamma, s]$ & $T[\gamma]$ form a Lie algebra

$$\{T, T\} = 0 \quad \{T, T^a\} = (\) T^a$$

little loop algebra

$$\{T^a, T^b\} = T^c \quad \text{CHECK}$$

$$\hat{E}^a(x) \Psi(A) = \frac{\delta \Psi(A)}{\delta A(x)} = \delta^a(x)$$

$$\hat{E}^a(x) \hat{F}(x) = \frac{\delta \hat{F}(x)}{\delta A(x)} = [\delta^a(x), \hat{F}(x)]$$

$$C = \int EEF + \Lambda EEE$$

$$\{T, T^2\} = Q \quad \{T, T^3\} = (\dots) \quad \{T, T^4\} = (\dots)$$

little loop algebra

$$C = \underbrace{E E F + \lambda E E E E}_{\text{little loop algebra}}$$

$$\hat{E}(x) \Psi(A) = \pi \int \frac{\Psi(x)}{\delta A(x)} = \delta(x)$$

$$\hat{E}(x) \hat{F}(x) = \frac{\Psi(x)}{\delta A(x)} = [\delta(x, 1)]^2$$

define $E(x)^2 = \lim_{y \rightarrow x} (E(y) E(x)) \Psi$

$$|\Psi(y)|^2 = \prod_{\text{edges}} \int dU(u) \uparrow \text{Haar measure} \quad |\Psi(y)|^2 = 1 \quad \int dU(u) = \int dU = \int dU(u)$$

$$T^*[\gamma, s] = \text{Tr} \left[U_{\gamma, s} \hat{E}^*(\gamma(s)) \right]$$



$T^*[\delta, s]$ & $T[\gamma]$ form a Lie algebra

$$\{T, T^*\} = 0 \quad \{T, T^*\} = (\) T^*$$

little loop algebra

$$\{T^*, T^*\} = T^* \quad \text{CHECK}$$

$$C = \int EEF + \int \Lambda EEE$$

$$\hat{E}(x) \hat{\Psi}(A) = \frac{\delta \Psi(A)}{\delta A(x)} = \delta^d(x, \dots)$$

$$\hat{E}(x) \hat{F}(x) = \frac{\delta^2 \Psi(A)}{\delta A(x) \delta A(x)} = [\delta^d(x, \dots)]^2$$

$$= \lim_{y \rightarrow x} \left(\int d^d y \delta^d(y, z) \delta^d(x, z) f(\dots) \right)$$

define $E(x)^2 = \lim_{y \rightarrow x} \left(\int E(y) E(x) \Psi \right)$



$$|\Psi(y)|^2 = \int_{\text{d.o.f.}} \int dU(y) |\Psi(y)|^2 = 1 \quad \int d(\dots) = \int dU = \int d(\dots)$$

↑
measure

Task 1 $\text{Tr } E^a(x) E^b(x) = g^a_b$

Trick 1

$$\begin{array}{c} m_{xy} \\ \downarrow \\ E^a(x) E^b(y) \end{array}$$

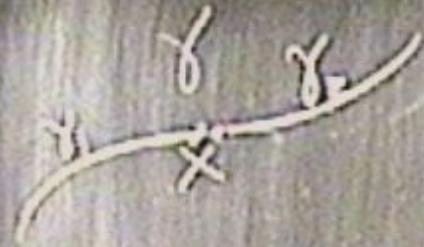
$$\text{Tr } E^a(x) E^b(x) = q q^{ab}(x)$$

$$\text{Tr} [E^a(x) U_{m_{xy}} E^b(y) U_{m_{xy}}]$$

Trick 1 $\text{Tr } E^a(x) E^b(x) = g^{ab}(x)$

$\begin{matrix} \eta_{\mu\nu} \\ \downarrow \\ E^a(x) E^b(y) \end{matrix}$

$\text{Tr} [E^a(x) U_{\mu\nu}(x) E^b(y) U_{\mu\nu}(x)]$



$$\hat{E}_{\lambda}^a(x) \mathcal{D}^{(A)}[\gamma] = \int ds \dot{\gamma}^\mu(s) \dot{\gamma}^\nu(x, \gamma(s)) U[\gamma] \gamma_{\mu}^{(A)}(y) U[\gamma_2]$$

$$= -i \hbar \int \frac{\delta U^{(A)}[\gamma]}{\delta A_{\mu}^a(x)} = \Delta^a[x, \gamma] U[\gamma_1] \gamma_{\mu}^{(A)}(y) U[\gamma_2]$$

$$\hat{E}^a(x_1) \dots \hat{E}^a(x_n) \cup_{\text{set}} \{ \delta \} = \prod_i \Delta^a[x_i, \delta]$$

all points distinct

$$\hat{E}^a(x_1) \dots \hat{E}^a(x_n) \cup_{\mathcal{G}} \{\gamma\} = \prod_i \Delta^a[x_i, \gamma]$$

all points declared

$$\hat{E}_{x_1}^n(x_1) \dots \hat{E}_{x_n}^n(x_n) \mathcal{U}_{\mathcal{X}}[\gamma] = \prod_i \Delta[x_i, \gamma] \mathcal{U}(\gamma) \gamma^{\mathcal{U}} \mathcal{U}(\gamma)$$

all points distinct

$$\hat{E}_n^a(x_1) \dots \hat{E}_n^a(x_n) U_{\text{reg}}[\gamma] = \prod_i \Delta^a[x_i, \gamma] u(\gamma) \gamma_{\mu}^{\nu} u(\gamma_{\mu}) \gamma_{\nu}^{\mu} u(\gamma_{\nu})$$

all points distinct

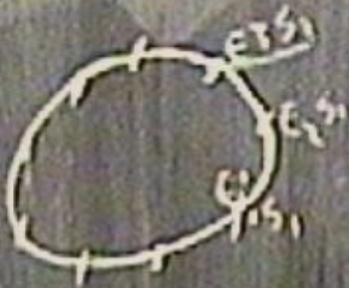
$$\text{Tr} U E^{\mu}(x) U E^{\nu}(x) U$$

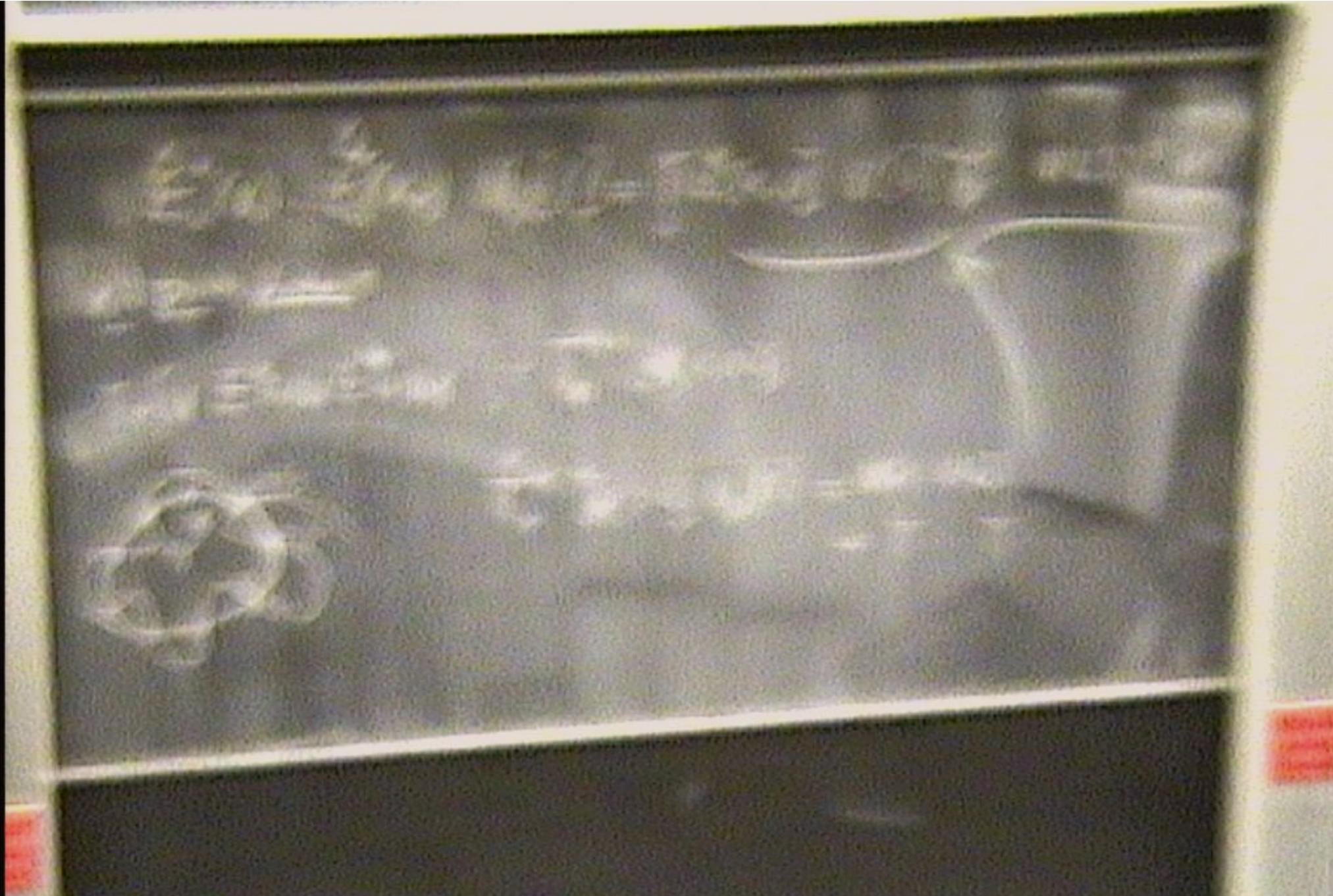


$$\hat{E}_n^a(x_1) \dots \hat{E}_n^a(x_n) \mathcal{U}_\alpha[\gamma] = \prod_i \Delta^a[x_i, \gamma] \mathcal{U}(\gamma) \gamma_n^a \mathcal{U}(L) \gamma_n^a \mathcal{U}(L)$$

all points distinct

$$\text{Tr}(\mathcal{U} E_n^a(x_1) \mathcal{U} E_n^a(x_2) \dots \mathcal{U} E_n^a(x_n) \mathcal{U}) = \text{Tr}_n^a[\gamma, s, s, s]$$

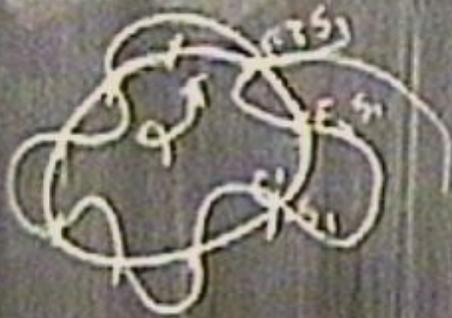




$$\hat{E}_n(x) \dots \hat{E}_n(x_n) U_{\text{op}}[\gamma] = \prod_i \Delta[x_i, \gamma] U(\gamma) T_{\text{op}} \dots U(x_i) T_{\text{op}} U(x_i)$$

all points distinct

$$\text{Tr}(U E(x_1) U E(x_2) U) = T_n[x_1, x_2, x_3]$$



$$T_n[x_1, x_2, x_3] U_{\text{op}}[\gamma] = U(x_1) U(x_2) \dots = T \dots T \dots$$

$$\hat{E}_{x_1}^{\alpha_1} \dots \hat{E}_{x_n}^{\alpha_n} U_{\text{cl}}[\gamma] = \prod_i \Delta^{\alpha_i}[x_i, \gamma] U(\gamma) T_{\text{cl}}^{\alpha_i} U(\gamma) T_{\text{cl}}^{\alpha_i} U(\gamma)$$

all points distinct

$$T_n[U E^{\alpha_1} U E^{\alpha_2} U] = T_n^{\text{cl}}[\alpha_1, \alpha_2, \alpha_3]$$



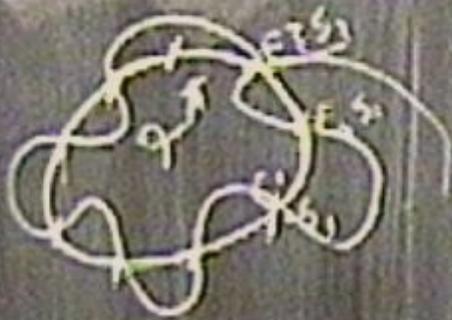
$$\hat{T}_n^{\text{cl}}[\alpha_1, \alpha_2, \alpha_3] U_{\text{cl}}[\gamma] = U(\gamma) U(\gamma) = T \dots T \dots$$

$$T[\phi, \alpha] = \int \mathcal{D}\alpha_i \hat{T}[\alpha_i, \gamma]$$

$$\hat{E}_n(x_1) \dots \hat{E}_n(x_n) U_{\text{eff}}[\gamma] = \prod_i \Delta[x_i, \gamma] U[\gamma] T_{\text{eff}} U(x_1) T_{\text{eff}}^{-1} U(x_n)$$

all points distinct

$$\text{Tr}(U E^{\text{eff}}(x_1) U E^{\text{eff}}(x_2) \dots U) = T_n^{\text{eff}}[\alpha, s, \sigma, \tau]$$



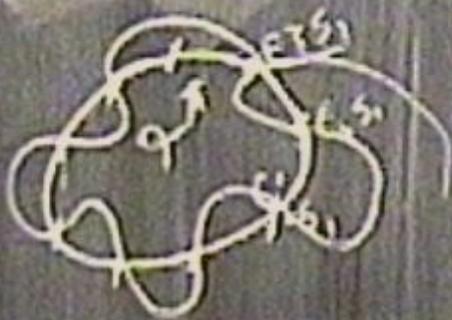
$$T_n^{\text{eff}}[\alpha, s, \sigma, \tau] U_{\text{eff}}[\gamma] = U(x_1) U(x_2) \dots = T \dots T \dots$$

$$T[\phi, \alpha] = \int \mathcal{D}x \hat{T}[\alpha, s]$$

$$\hat{E}_n^{\alpha}(x_1) \dots \hat{E}_n^{\alpha}(x_n) U_{\text{path}}[\gamma] = \prod_i \Delta^{\alpha}[x_i, \gamma] U(\gamma) T_{\text{tr}} U(\gamma) T_{\text{tr}} U(\gamma)$$

all points distinct

$$\text{Tr} [U E^{\alpha}(x_1) U E^{\alpha}(x_2) U] = T_n^{\alpha}[\alpha, s, s, s]$$



$$\hat{T}_n^{\alpha}[\alpha, s, s, s] U_{\text{path}}[\gamma] = U(\gamma) U(\gamma)$$

$$= T \dots T \dots$$

$$T[\phi, \alpha] = \int \phi_{\mu}^{\alpha}[\alpha, s] ds$$