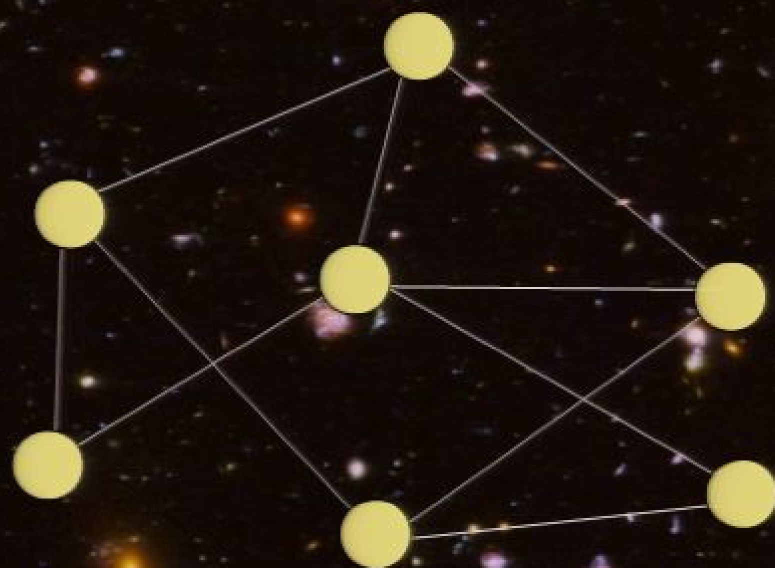
Title: Limits on efficient computation in the physics world

Date: Mar 15, 2006 04:00 PM

URL: http://pirsa.org/06030013

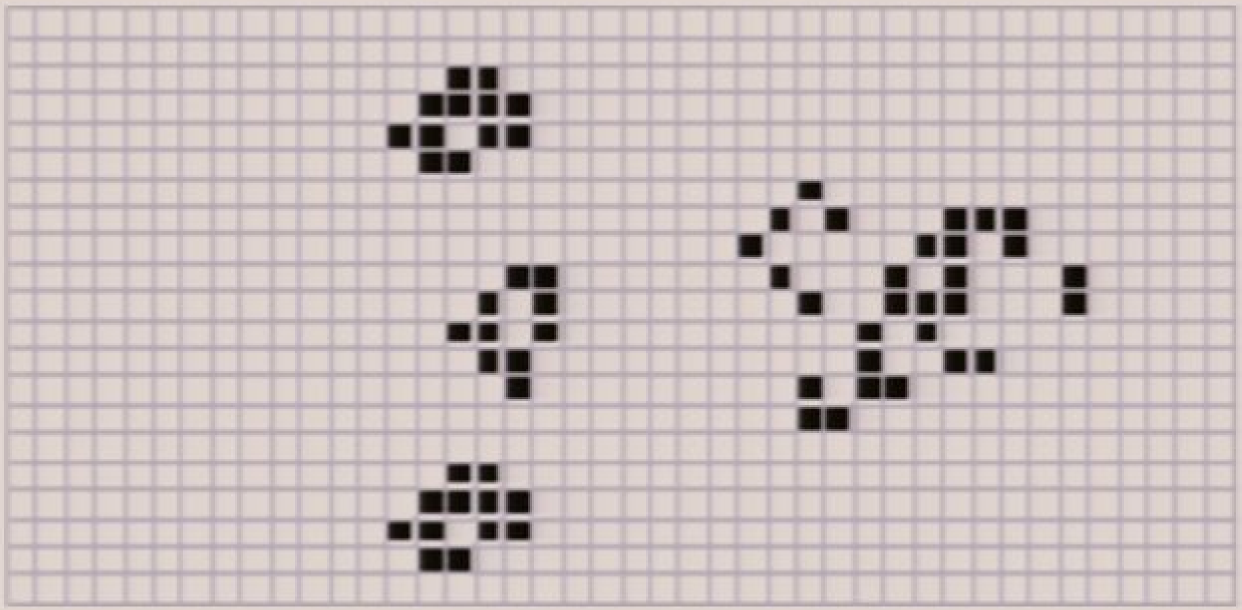
Abstract:

Limits on Efficient Computation in the Physical World



Scott Aaronson
University of Waterloo

The Computer Science Picture of Reality



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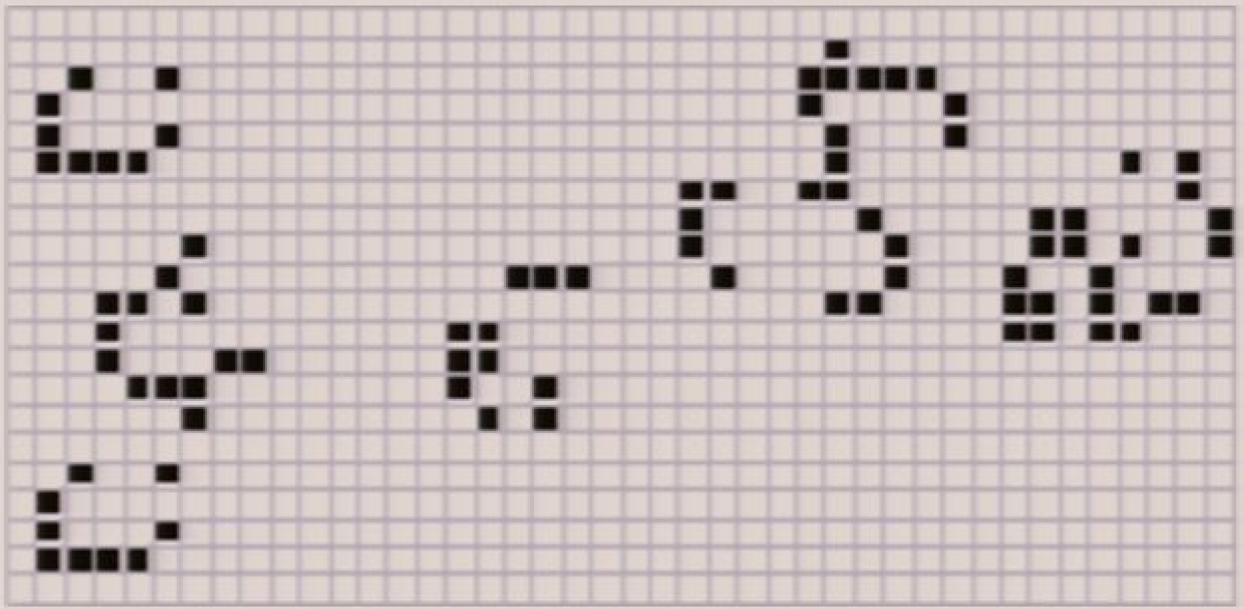

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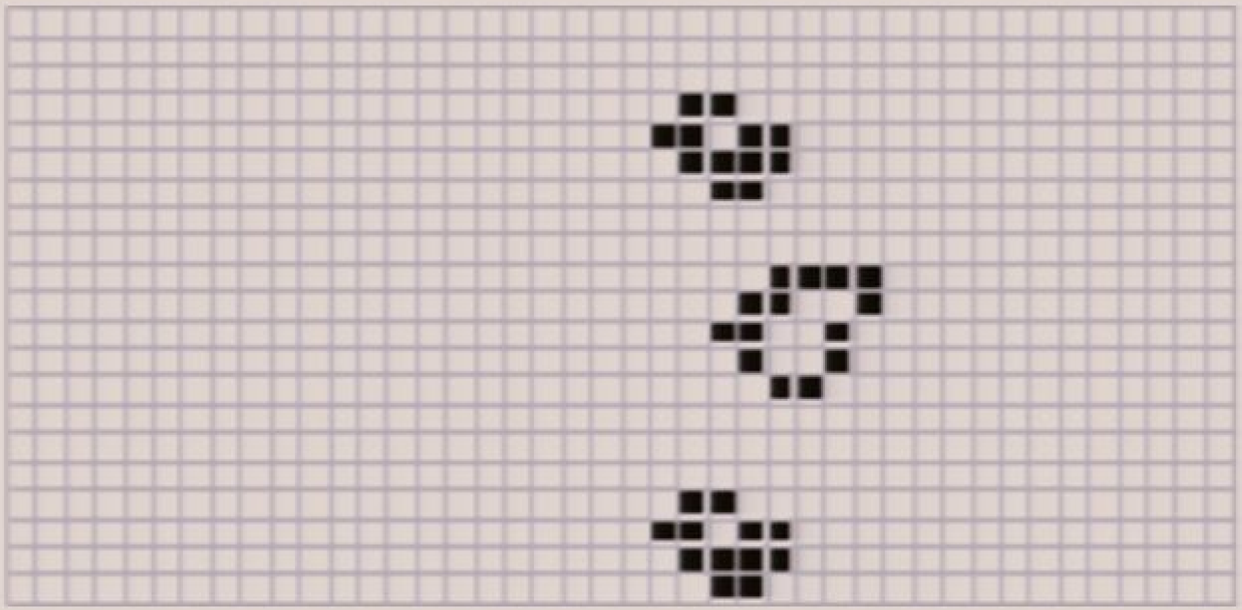

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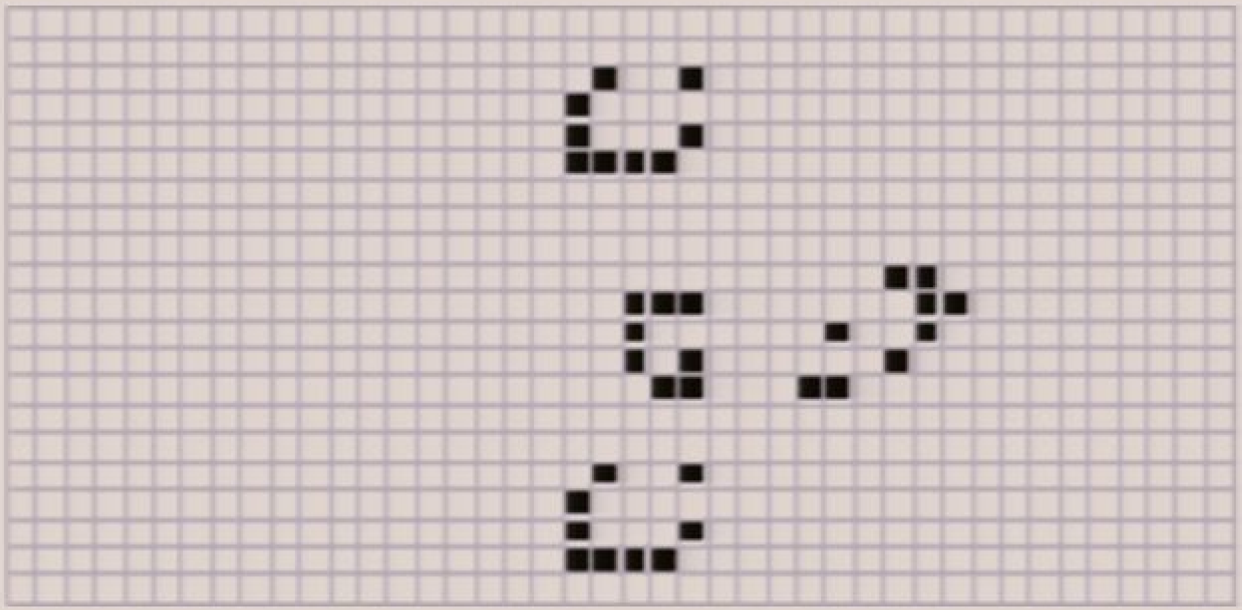

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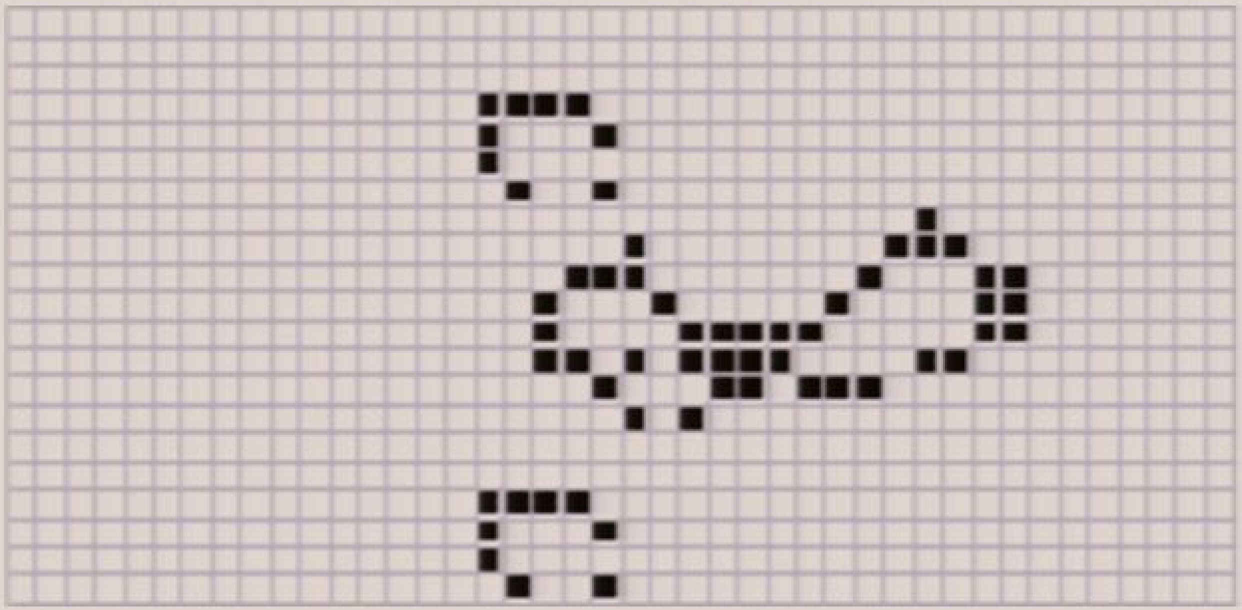

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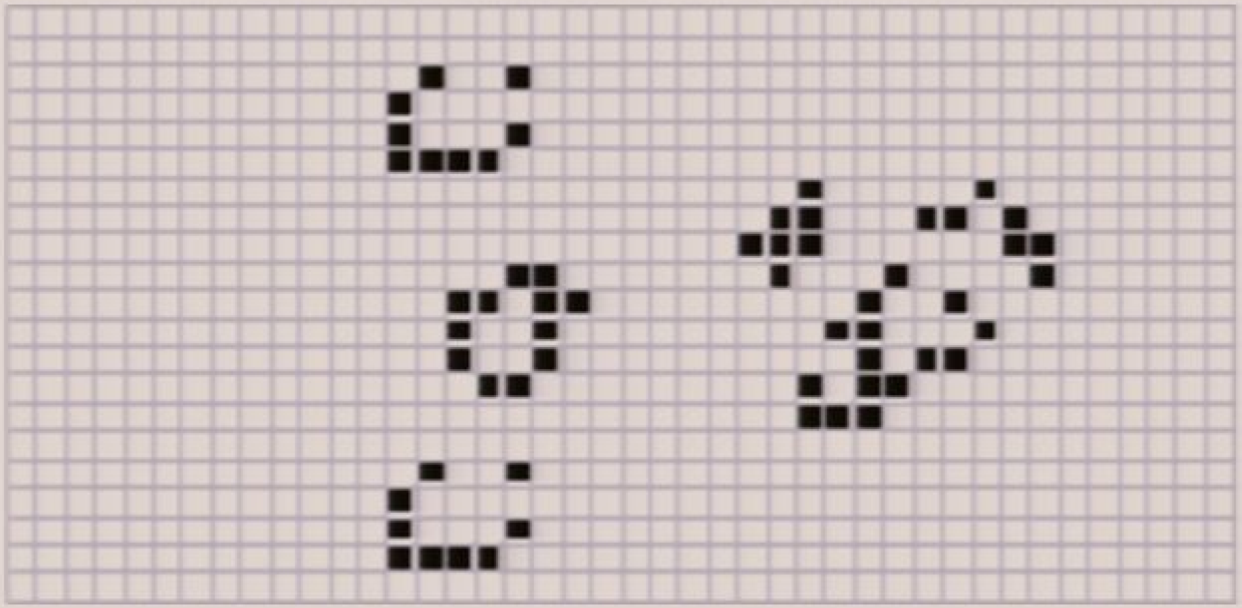

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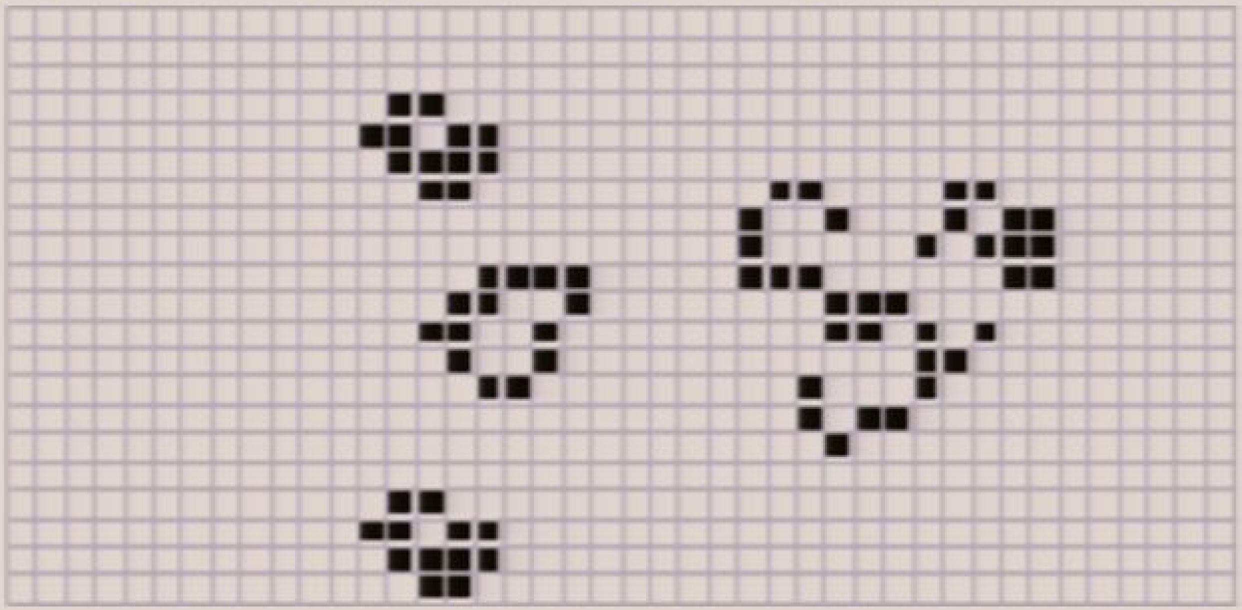

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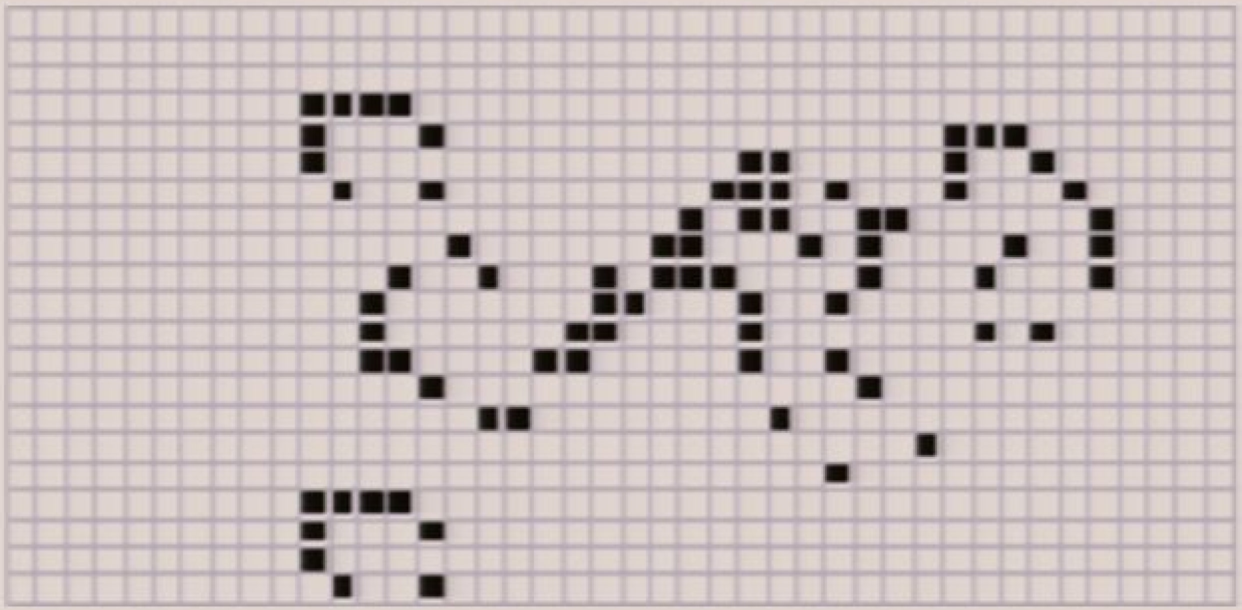

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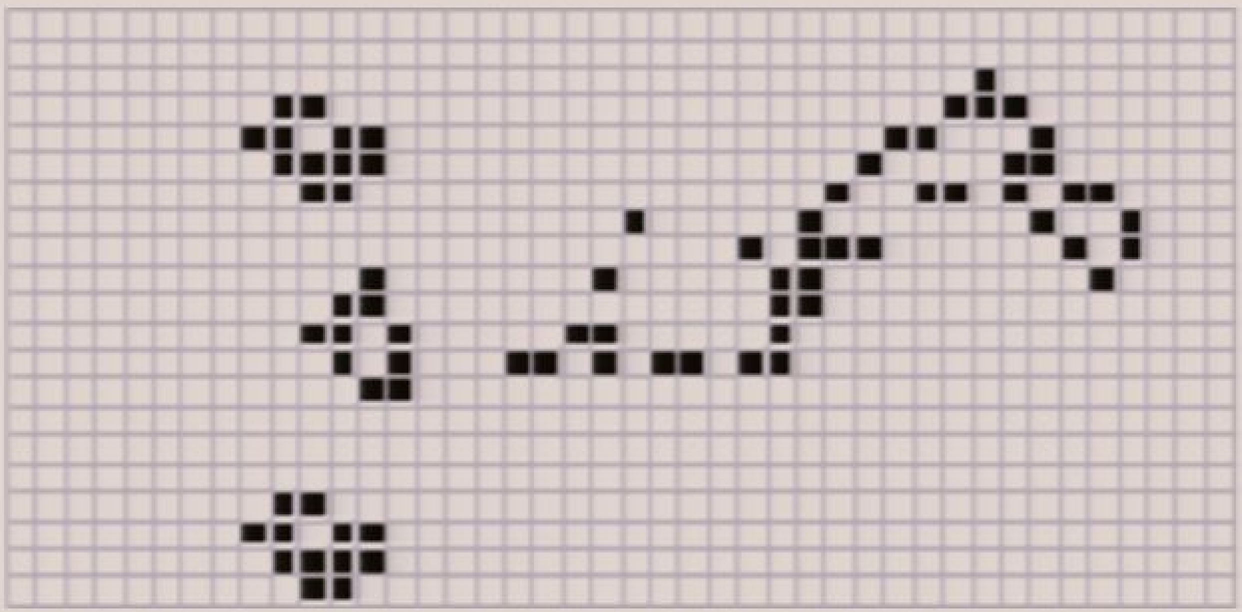

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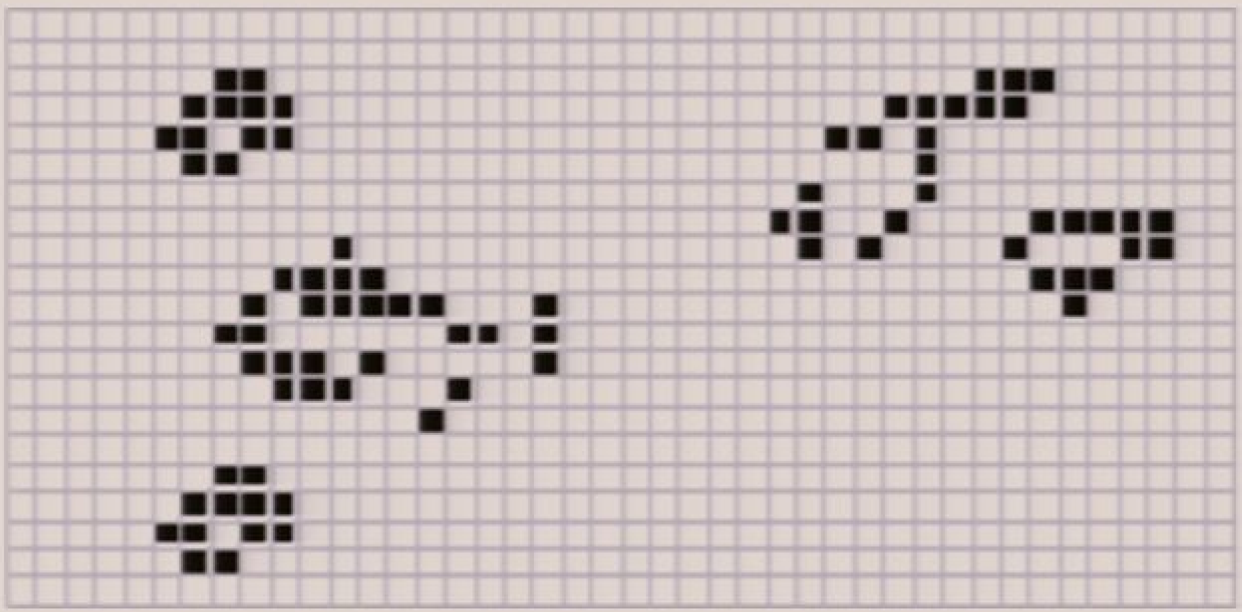

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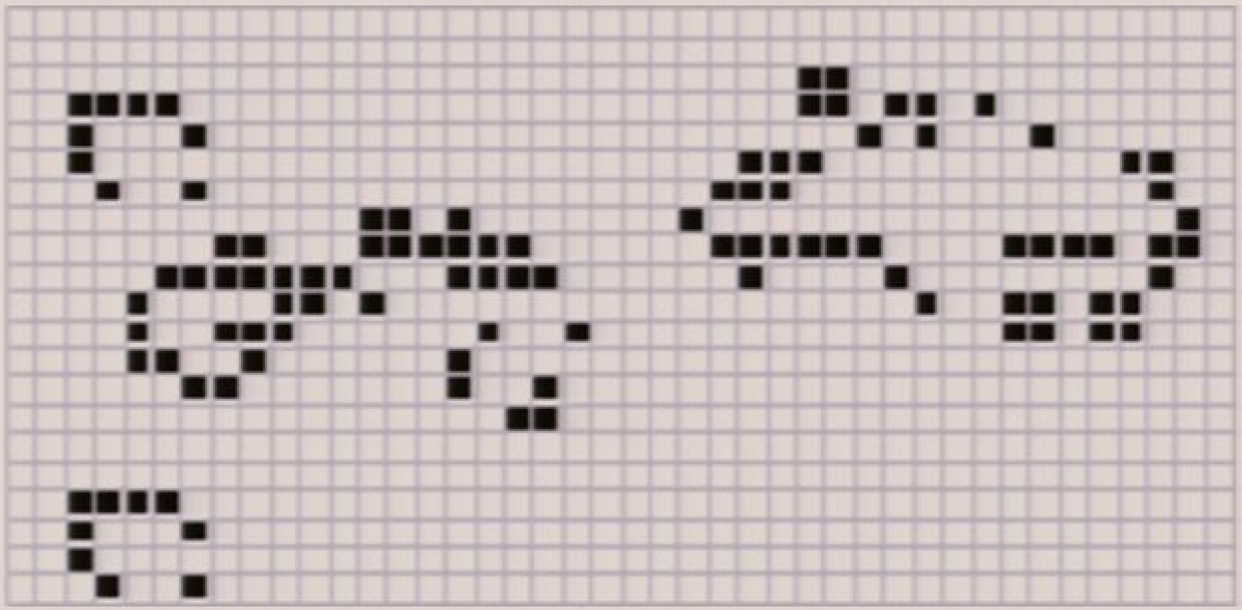

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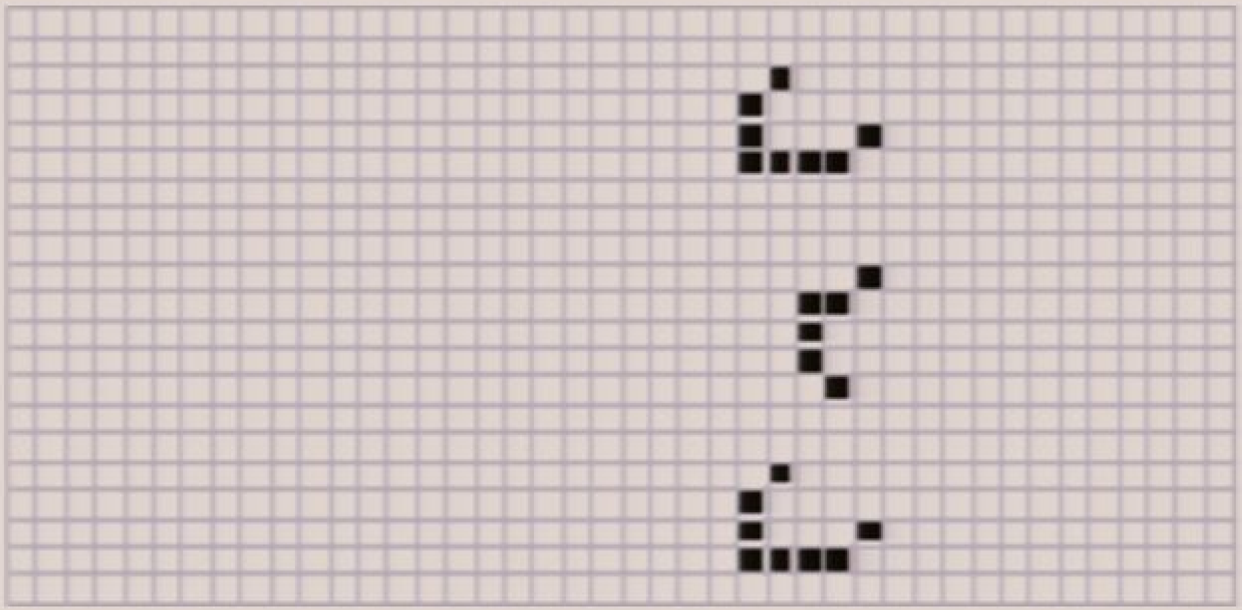

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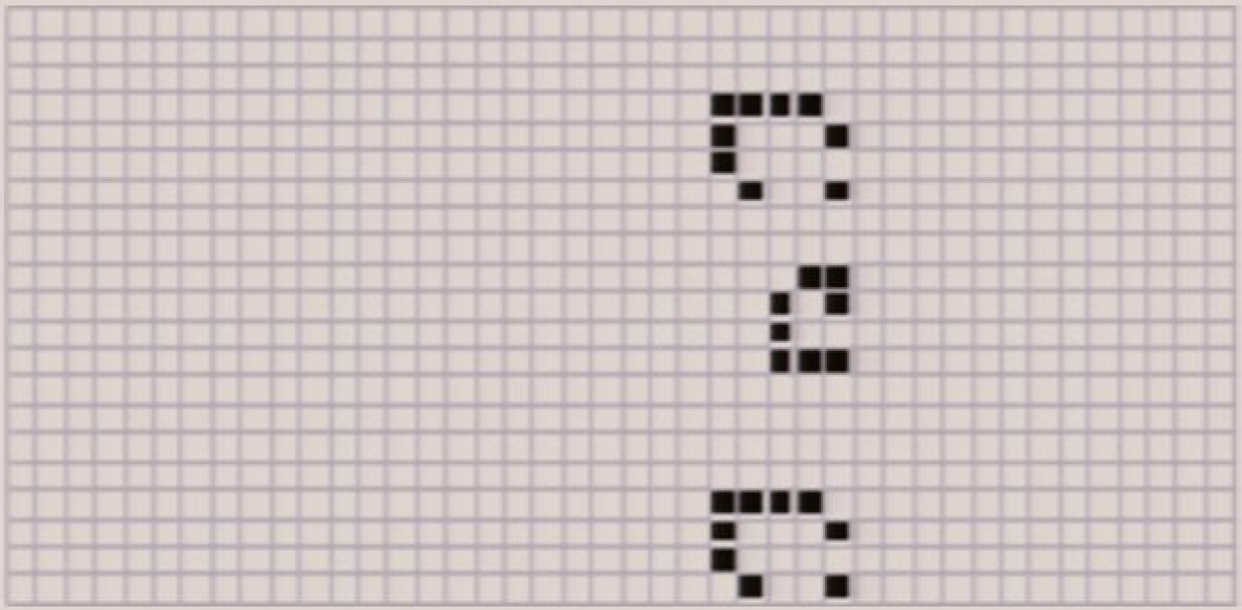

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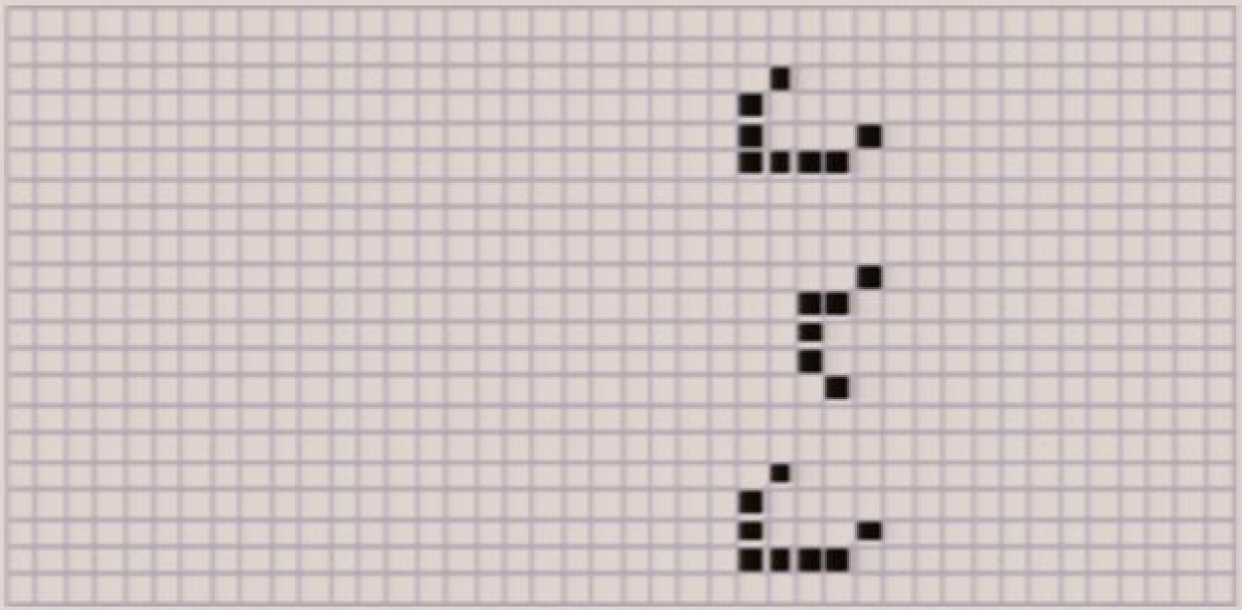

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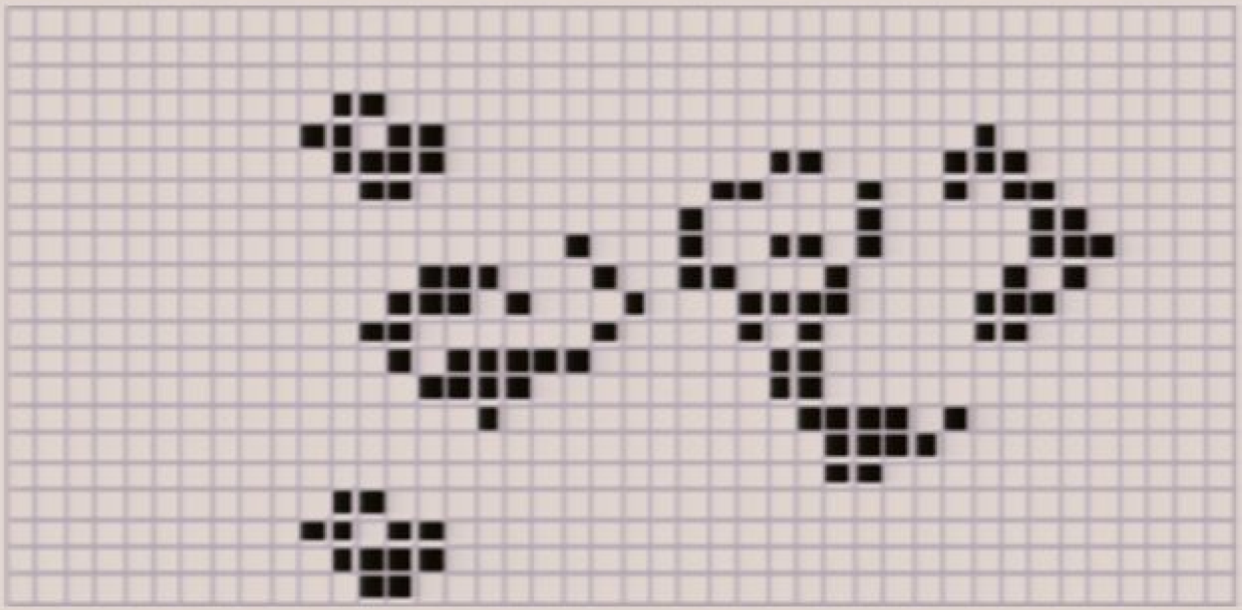

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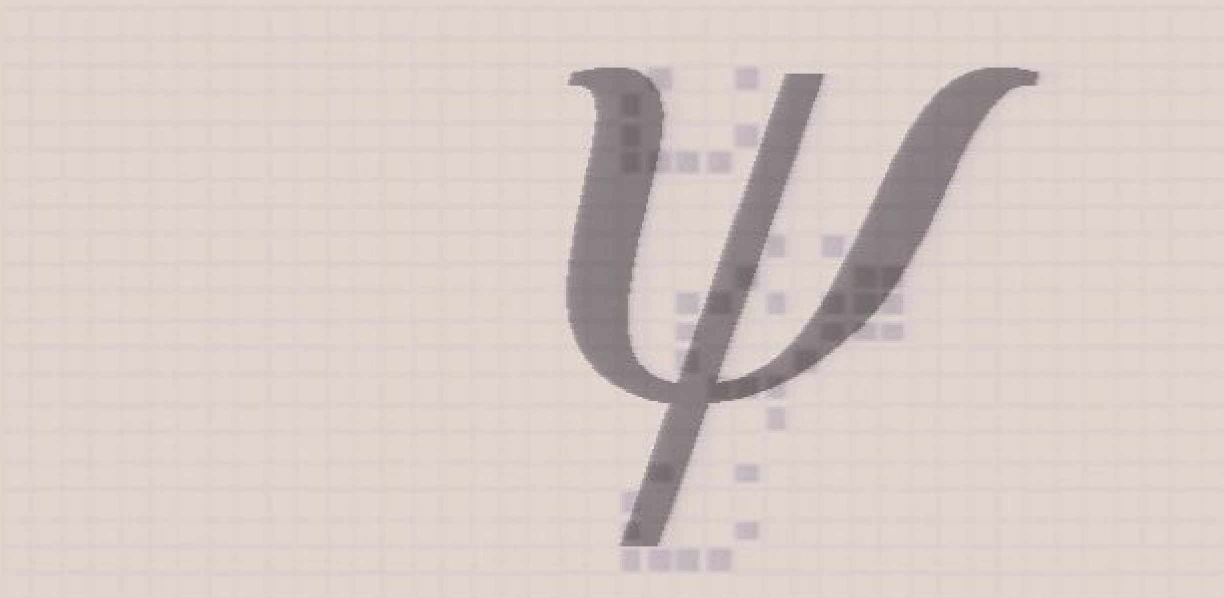

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The Computer Science Picture of Reality

$$\psi$$

Quantum computing challenges this picture

That's why everyone should care about it, whether or not quantum factoring machines are ever built

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PLAN OF TALK

Background

The gospel according to Shor

Part I: Limitations of Quantum Computers

A lower bound extravaganza

Part II: Models and Reality

Is the quantum computing model too powerful? Or not powerful enough?

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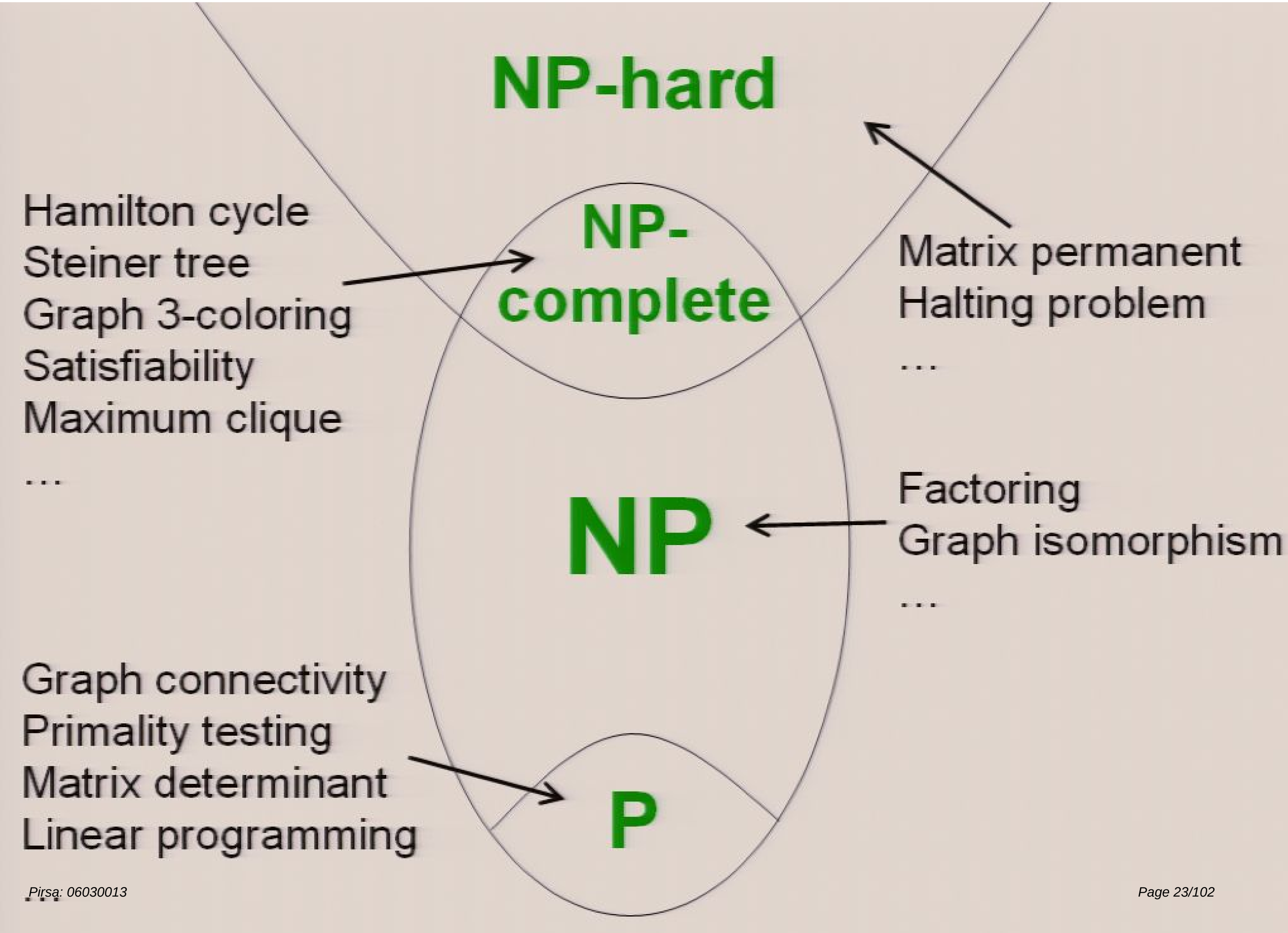
NP-hard
Hamilton cycle
Steiner tree
Graph 3-coloring
Satisfiability
Maximum clique
…
NP-
complete
Matrix permanent
Halting problem
…
NP
Factoring
Graph isomorphism
…
Graph connectivity
Primality testing
Matrix determinant
Linear programming
P
…

Quantum Computing

A quantum state of n "qubits" takes 2^n complex numbers to describe:

$$\sum_{x \in \{0,1\}^n} \alpha_x |x\rangle$$

The goal of quantum computing is to exploit this exponentiality in Nature.

BQP: Bounded-Error Quantum Polynomial-Time

Class of problems solvable efficiently using a quantum computer

Bernstein-Vazirani 1993:

$$P \subseteq BQP \subseteq PSPACE$$

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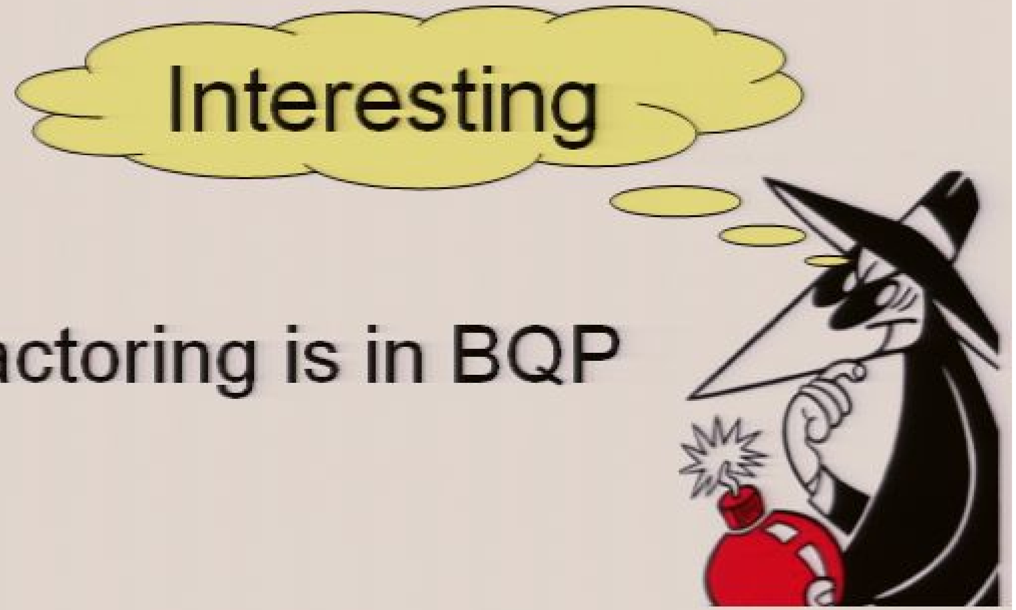

Shor 1994: Factoring is in BQP

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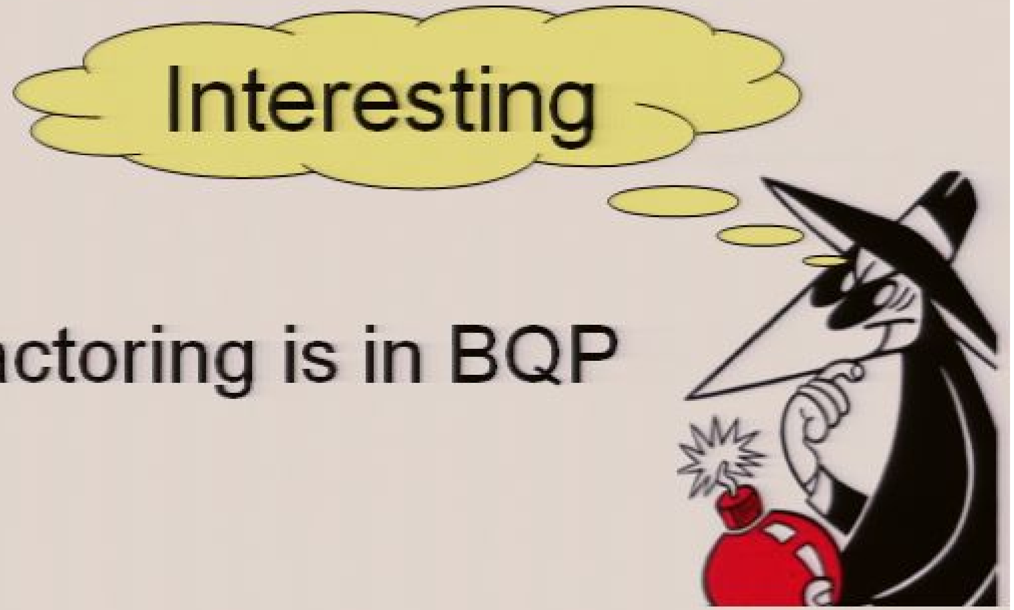

Shor 1994: Factoring is in BQP

Grover 1996: Quantum algorithm to search an N-element array in $\sqrt{N}$ steps

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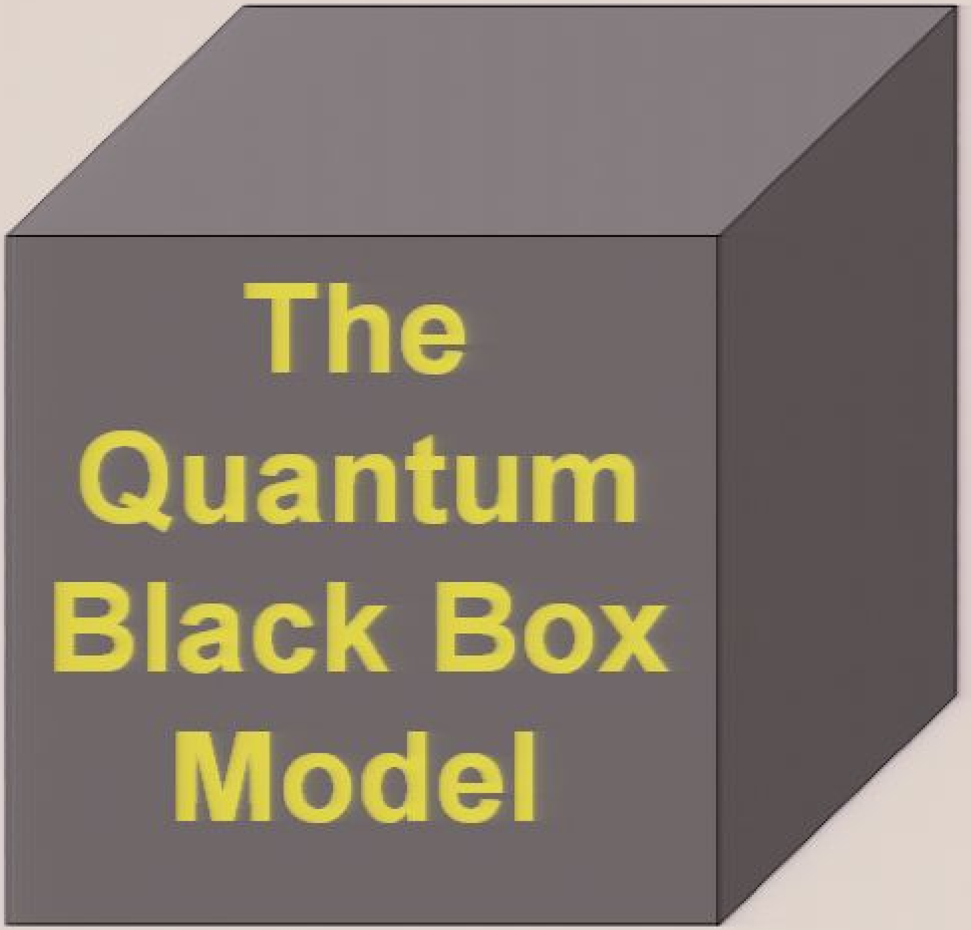

I do believe it
Against an oracle.
—Shakespeare, *The Tempest*

We count only the number of **queries** to an **oracle**, not the number of computational steps

Example: Given a function $f:\{0,1\}^n \rightarrow \{0,1\}$, decide if there's an x such that $f(x)=1$

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- Like solving an NP-complete problem by brute force
- Classically, $\sim 2^n$ queries to f needed
- Grover's algorithm uses only $\sim 2^{n/2}$
- **BBBV 1997:** Grover is optimal
- Provides evidence that $NP \not\subset BQP$

Algorithm's state:

$$\sum_{x,w} \alpha_{x,w} |x,w\rangle$$

x: location to query
w: "workspace" qubits

After a query transformation:

$$\sum_{x,w} \alpha_{x,w} |x,w \oplus f(x)\rangle$$

Between two queries, we can apply an arbitrary unitary matrix that doesn't depend on f

Complexity = minimum number of queries needed to achieve

$$\sum_{\substack{|x,w\rangle \\ \text{corresponding to} \\ \text{right answer}}} |\alpha_{x,w}|^2 \geq \frac{2}{3}$$

for all oracles f

Problem: Find 2 numbers that are the same (each number appears twice)

28	12	18	76	96	82	94	99	21	78	88	93	39	44	64
32	99	70	18	94	82	92	64	95	46	53	16	35	42	72
31	40	75	71	93	32	47	11	70	37	78	79	36	63	40
69	92	10	28	85	41	80	10	52	63	88	65	43	84	67
57	31	98	39	65	74	24	90	26	83	60	91	27	96	35
20	26	52	95	57	66	97	54	30	62	79	33	84	50	38
49	17	47	24	54	48	98	23	41	16	66	75	38	13	58
56	86	34	73	61	73	21	44	62	34	14	51	74	76	83
37	90	58	13	71	25	29	25	56	68	12	11	51	23	77
68	72	43	69	46	87	97	45	59	14	30	19	81	81	49
60	85	80	50	61	59	89	67	89	29	86	48	22	15	17
55	36	27	42	55	77	19	45	15	53	22	91	87	20	33

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By **"birthday paradox"**, a randomized algorithm must examine $\sqrt{N}$ of the N numbers

28	12	18	76	96	82	94	99	21	78	88	93	39	44	64
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	53	16	35	42	72
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	78	79	36	63	40
69	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	88	65	43	84	67
57	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60	91	27	96	35
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Problem: Find 2 numbers that are the same (each number appears twice)

28 12 18 76 96 82 94 99 21 78 88 93 39 44 64
32 … 53 16 35 42 72
31 … 78 79 36 63 40
6 … 88 65 43 84 67
57 … 60 91 27 96 35
20 26 52 95 57 66 97 54 30 62 79 33 84 50 38
4 … 41 16 66 75 38 13 58
5 … 62 34 14 51 74 76 83
3 … 56 68 12 11 51 23 77
6 … 59 14 30 19 81 81 49
6 … 89 29 86 48 22 15 17
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By **"birthday paradox"**, a randomized algorithm must examine $\sqrt{N}$ of the N numbers

Brassard, Høyer, Tapp: quantum algorithm (based on Grover) that makes $N^{1/3}$ queries

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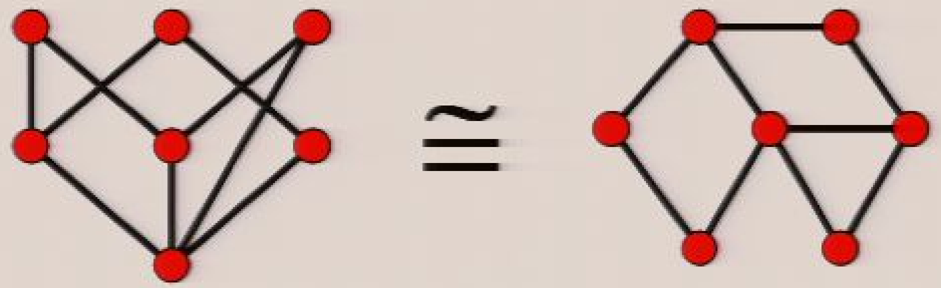
Is that optimal? Proving a lower bound better than constant was open for 5 years

55 36 27 42 55 77 19 45 15 53 22 91 87 20 33

Motivation for the Collision Problem

Cryptographic Hash Functions



Graph Isomorphism:
find a collision in
$\sigma_1(G),\ldots,\sigma_{n!}(G),\sigma_1(H),\ldots,\sigma_{n!}(H)$

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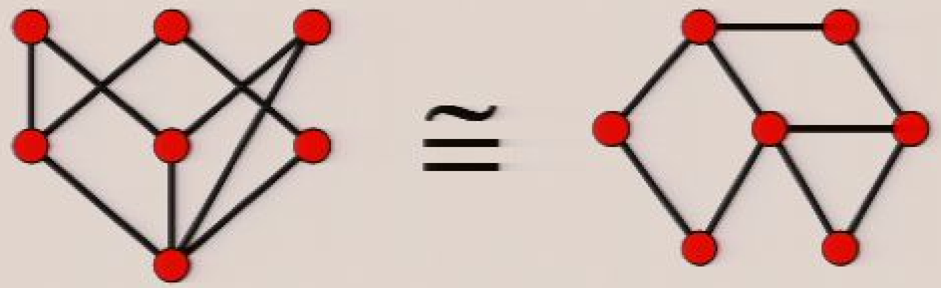

Graph Isomorphism:

find a collision in

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What makes proving a lower bound hard is that a quantum computer can **almost** find a collision in 1 query:

$$\frac{1}{\sqrt{N}}\sum_{x=1}^{N}|x\rangle|f(x)\rangle \xrightarrow{\text{Measure 2}^{\text{nd}}\text{ register}} \frac{|x\rangle+|y\rangle}{\sqrt{2}}|f(x)\rangle$$

A. 2002: $N^{1/5}$ lower bound on quantum query complexity of the collision problem

Improved to $N^{1/3}$ and generalized by Shi, Kutin, Ambainis, and Midrijanis

Proof Sketch (only one in the talk)

T-query quantum algorithm that finds collisions in 2-to-1 functions

→

T-query algorithm that distinguishes 1-to-1 from 2-to-1 functions

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T-query quantum algorithm that finds collisions in 2-to-1 functions

→ T-query algorithm that distinguishes 1-to-1 from 2-to-1 functions

→ **Beals et al. 1998:** Multilinear polynomial p of degree ≤ 2T, such that p(X) = probability algorithm says X is 2-to-1

$p(X) \in [0, 1/3]$ if X is 1-to-1

$p(X) \in [2/3, 1]$ if X is 2-to-1

Key insight: $p(X) \in [0, 1]$ even if X is 3-to-1, 4-to-1, etc.

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↓

Univariate polynomial q such that deg(q)≤deg(p), and q(g) = average of p(X) over all g-to-1 functions X

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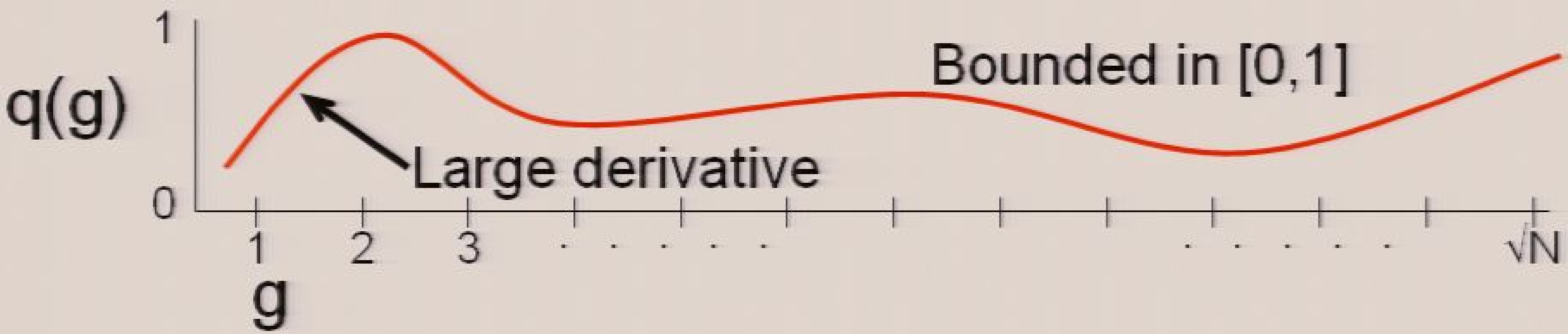
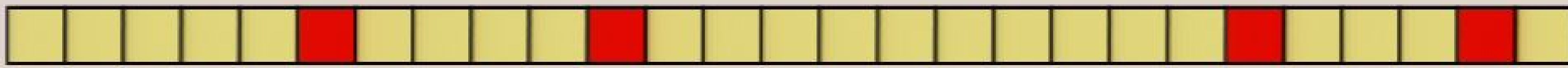


Markov's Inequality implies such a polynomial must have large degree

Direct Product Theorem for Quantum Search



N items, K of them marked

A. 2004: With few ($\ll\sqrt{N}$) queries, the probability of finding all K marked items is $2^{-\Omega(K)}$

Proof uses polynomial method

Corollary 1: Exists oracle relative to which

$$NP \not\subset BQP/qpoly$$

(BQP/qpoly = BQP with polynomial-size "quantum advice")

Corollary 2: Fixes flawed result of Klauck on quantum time-space tradeoffs for sorting

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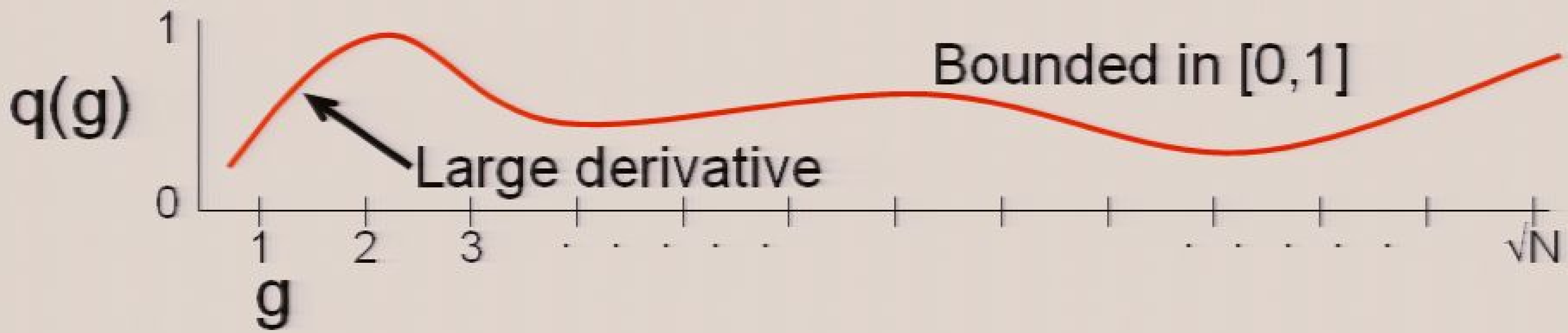

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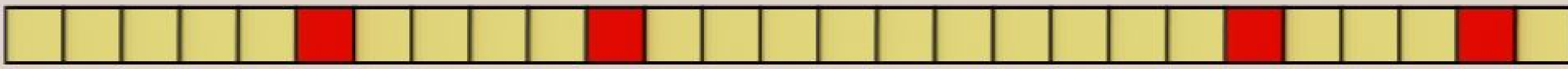

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Can quantum ideas help us prove *classical* lower bounds?

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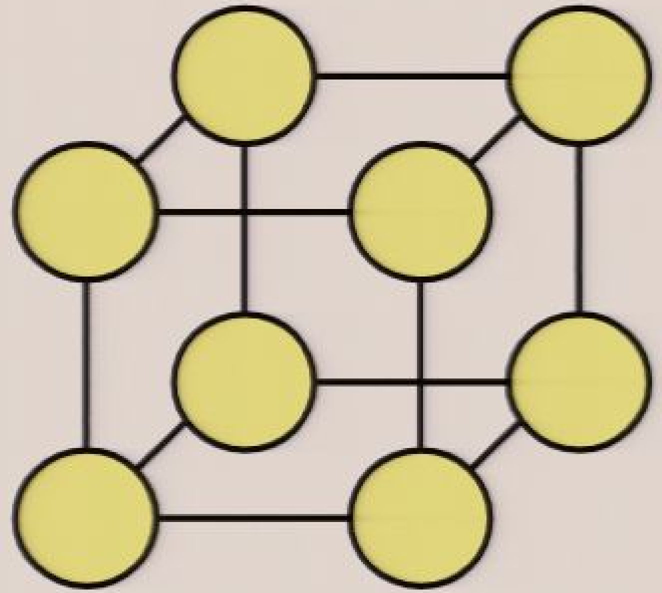
Quantum Generosity … Giving back because we care™

Examples: Kerenidis & de Wolf 2003, Aharonov & Regev 2004

Can quantum ideas help us prove *classical* lower bounds?



Local Search: Given oracle access to $f:\{0,1\}^n \rightarrow \mathbb{Z}$, find a local minimum of f using as few queries as possible

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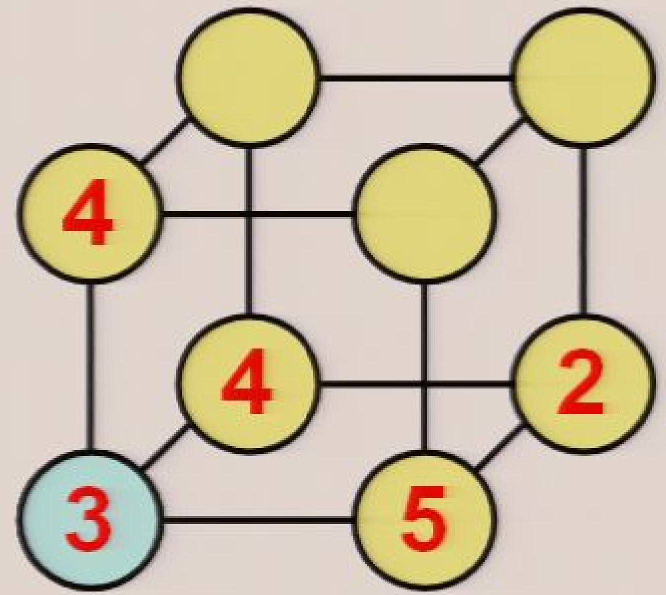

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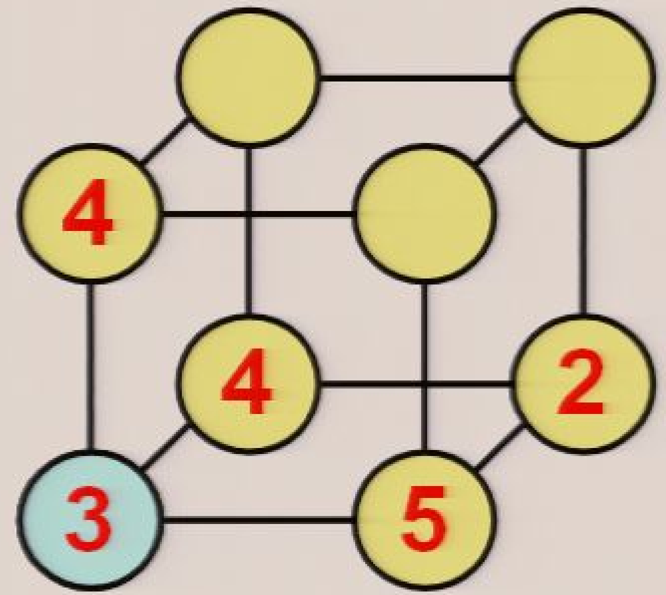


Aldous 1983: Randomized algorithm needs $2^{n/2-o(n)}$ queries

A. 2004: Quantum algorithm needs $2^{n/4}/n$ queries

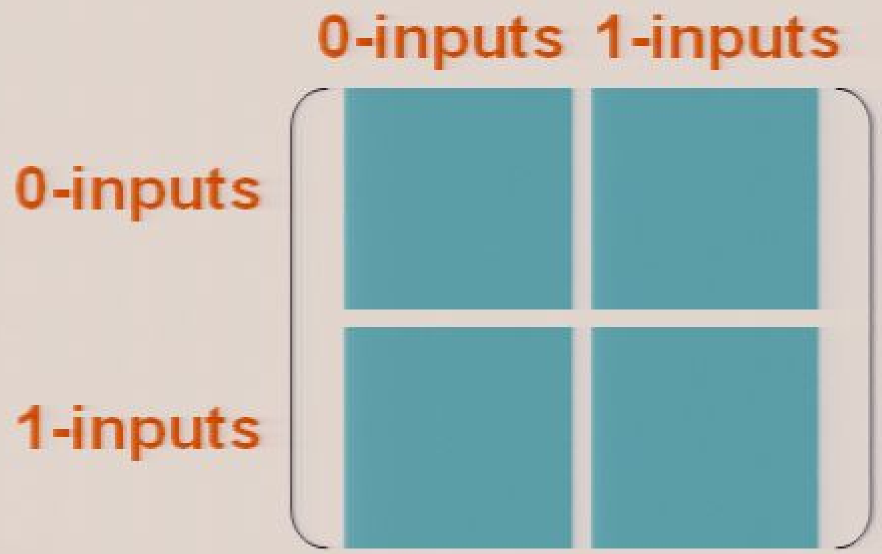
$\Rightarrow$ PLS (Polynomial Local Search) is hard for BQP relative to oracle

Upper bounds:
$2^{n/2}\sqrt{n}$ randomized,
$2^{n/3}n^{1/6}$ quantum

Can quantum ideas help us prove *classical* lower bounds?

Proof technique based on **Ambainis' quantum adversary method**

Each query only separates 0-inputs from 1-inputs by so much



Technique also yields

- $2^{n/2}/n^2$ randomized lower bound
- First lower bounds (randomized or quantum) for constant-dimensional grid graphs

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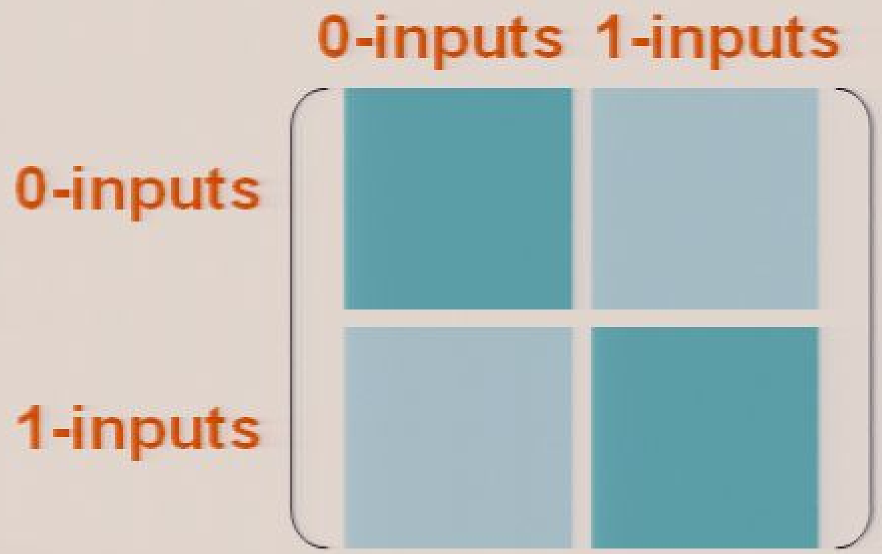

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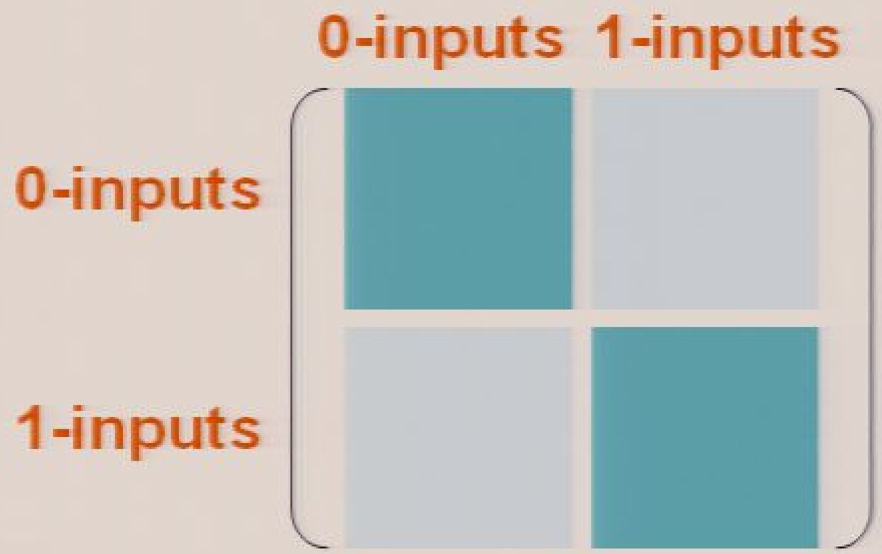

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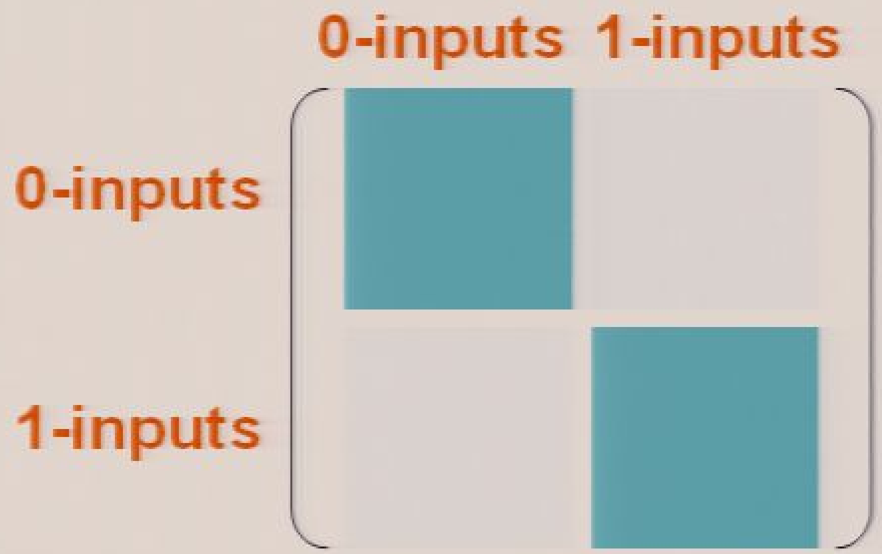
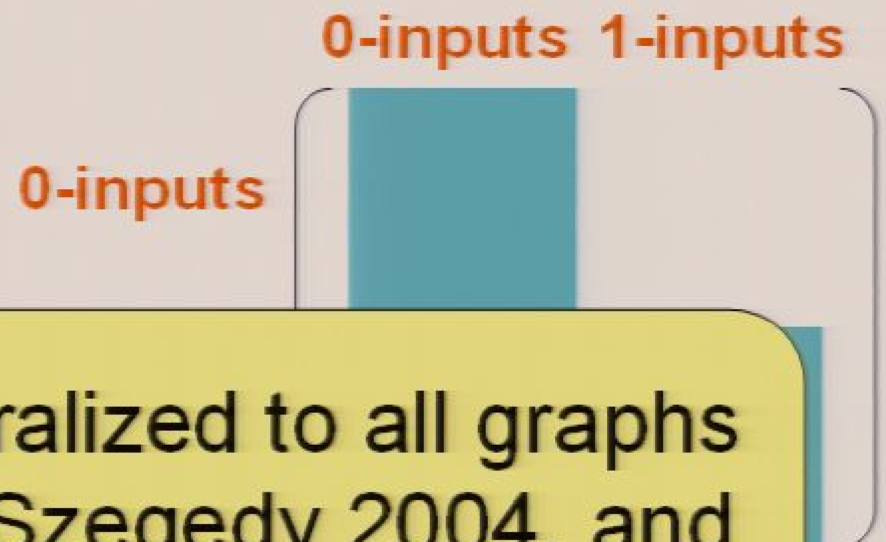


Technique also yields

- $2^{n/2}/n^2$ randomized lower bound
- First lower bounds (randomized or quantum) for constant-dimensional grid graphs

Can quantum ideas help us prove *classical* lower bounds?

Proof technique based on **Ambainis' quantum adversary method**



Each query only separates 0-inputs from 1 inputs by so

Results generalized to all graphs by Santha & Szegedy 2004, and tightened by Zhang 2006

Techn

• $2^{n/2}/$r

• First lower bounds (randomized or quantum) for constant-dimensional grid graphs

Summary

- The Art of the Quantum Lower Bound
  - –Polynomials and adversaries—the dynamic duo
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  - –Polynomials and adversaries—the dynamic duo
  - –Techniques even applied classically
- Quantum computing is not a panacea
  - –Many problems still intractable: NP, collision-finding, local search…
  - –Even with quantum advice
- Quantum computing $\neq$ exponential parallelism
  - –Popular articles get this wrong
  - –Because of linearity, one "parallel universe" can't shout above the others

Background

The gospel according to Shor

Part I: Limitations of Quantum Computers

A lower bound extravaganza

Part II: Models and Reality

Is the quantum computing model too powerful? Or not powerful enough?

Is quantum computing just *obvious* baloney?

Leonid Levin:
"We have never seen a physical law valid to over a dozen decimals"

Oded Goldreich:
Exponentially long vectors $\Rightarrow$ exponential time to manipulate

Sure/Shor separators

My response: What criterion separates the quantum states that suffice for factoring from the states we've already seen?

DIVIDING LINE

Sure/Shor separators

My response: What criterion separates the quantum states that suffice for factoring from the states we've already seen?

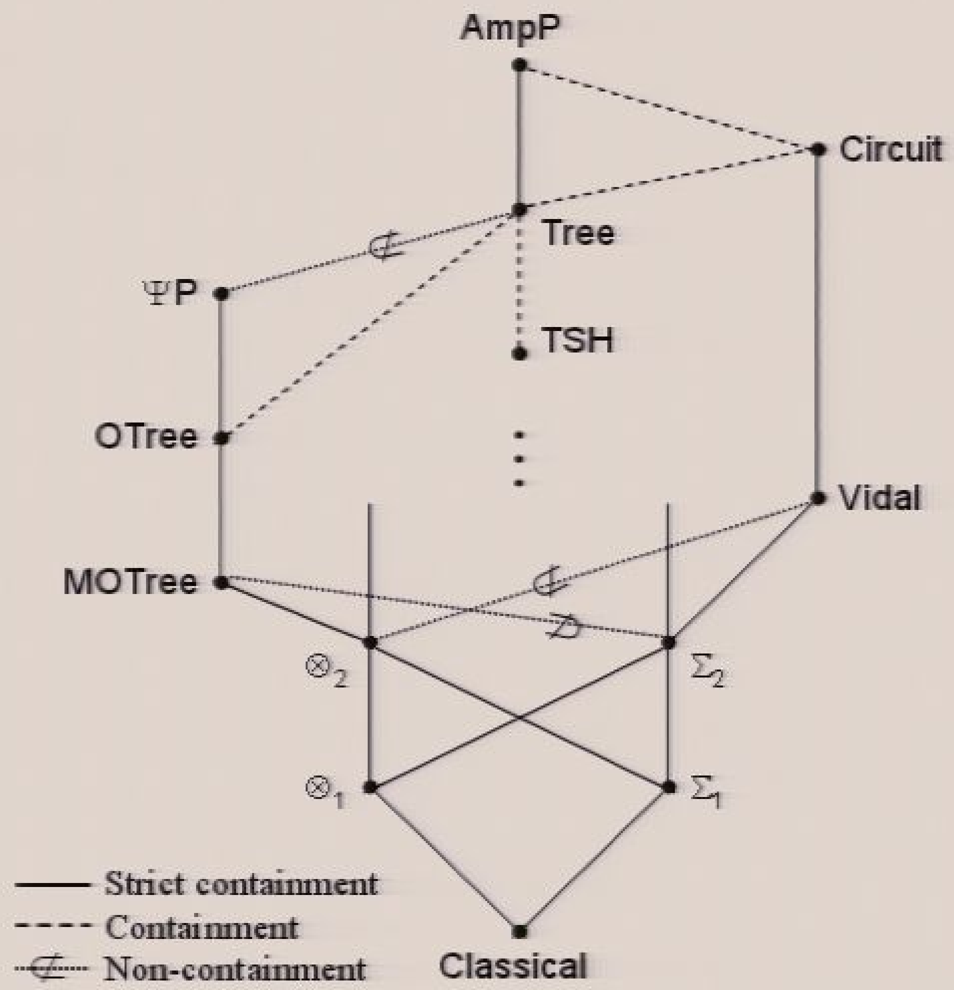
Not exponentially small amplitudes or thousands of coherent qubits

$$\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)^{\otimes 10000} \qquad \frac{|0\rangle^{\otimes 10000}+|1\rangle^{\otimes 10000}}{\sqrt{2}}$$

A. 2004 proposes a complexity classification of quantum states to help answer this question

Main result: States arising in quantum error-correction take $n^{\Omega(\log n)}$ additions and tensor products to express

Proof applies Ran Raz's breakthrough lower bound on multilinear formula size



Are quantum states *really* "exponential-sized objects"?

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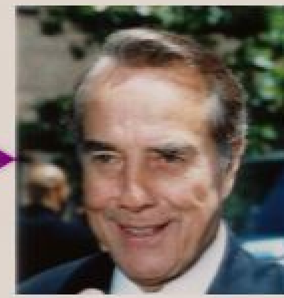
x

Alice

$|\psi_x\rangle$

1-way communication



y

Bob

f(x,y)

Are quantum states *really* "exponential-sized objects"?

A., CCC'04: Given $f:\{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}$ (partial or total),

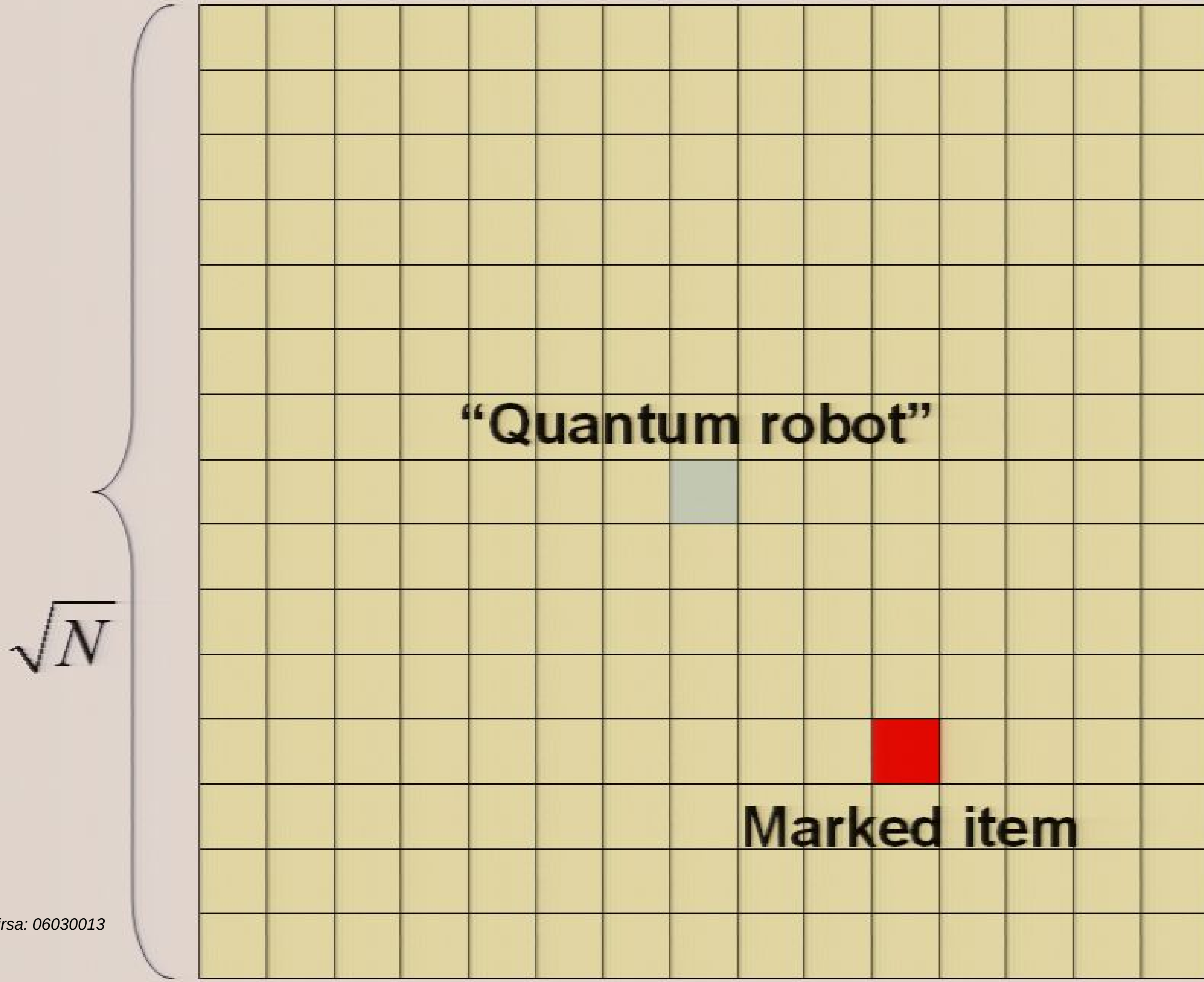
$$D^1(f) = O(m\, Q^1(f) \log Q^1(f))$$

$D^1(f)$ = deterministic 1-way communication complexity

$Q^1(f)$ = bounded-error quantum 1-way complexity

Corollary: **BQP/qpoly $\subseteq$ PostBQP/poly**

Grover Search of a Physical Region



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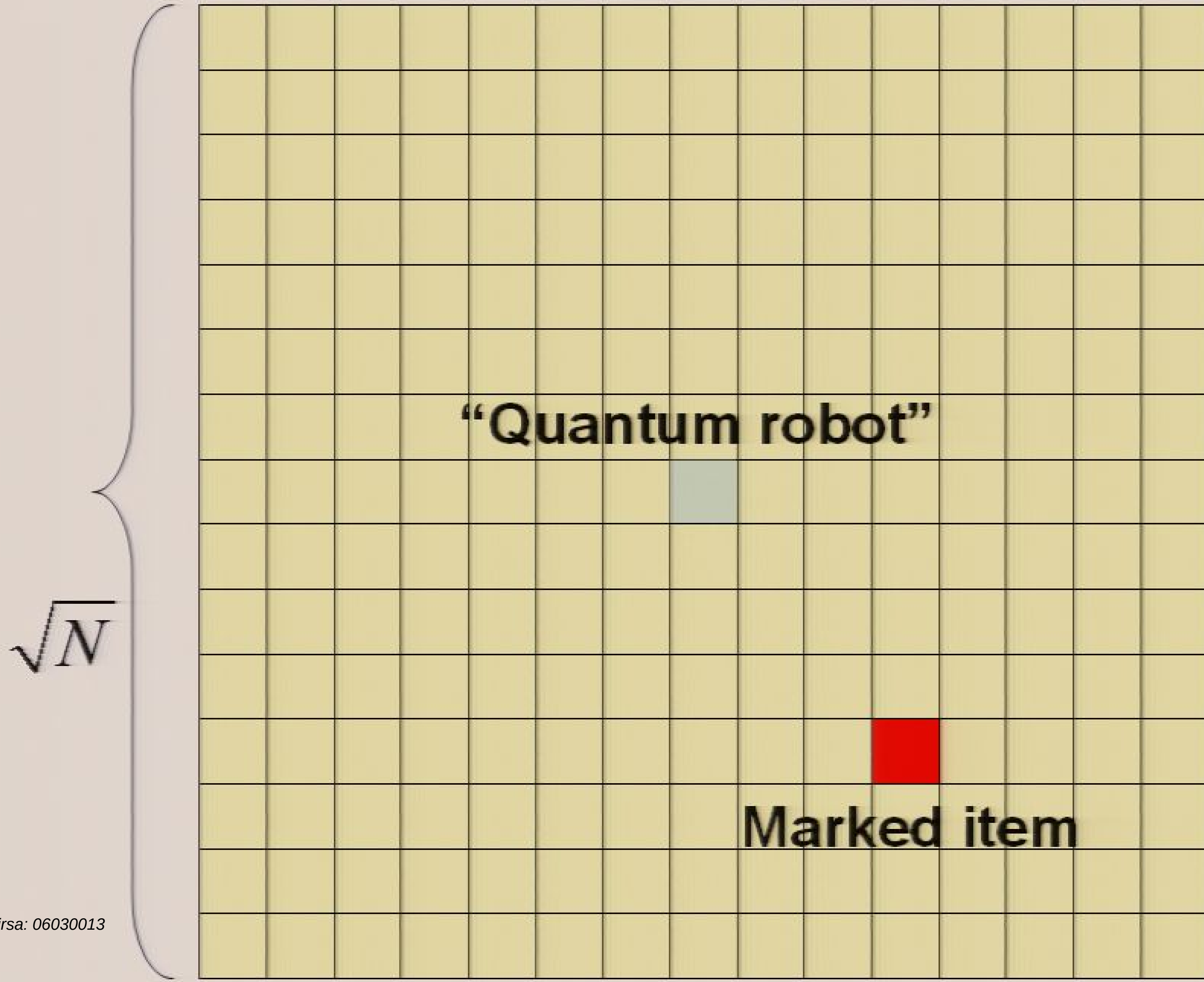

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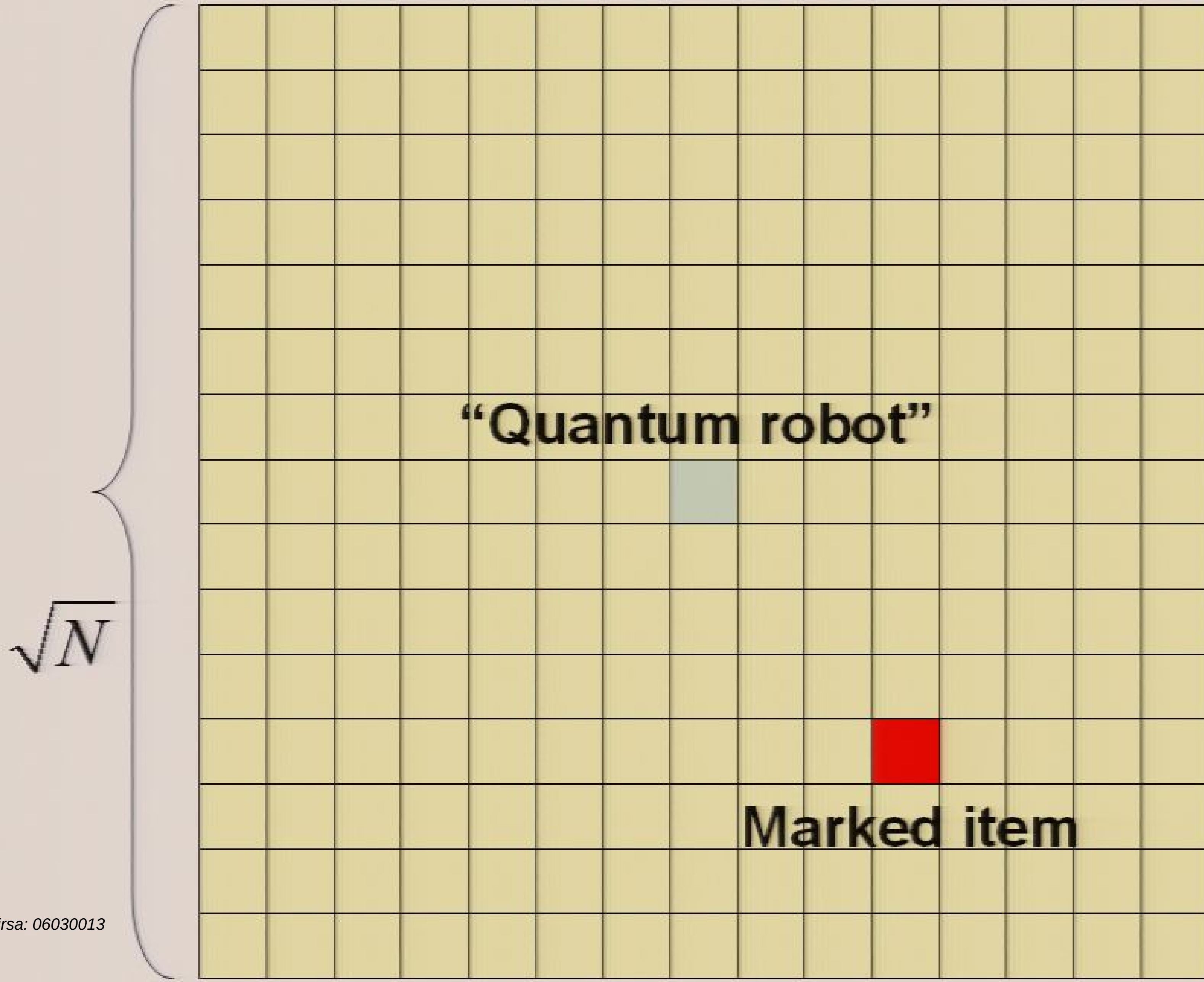

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Corollary: $O(\sqrt{N})$-qubit disjointness protocol

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My motivation: What computational limitations are imposed by the speed of light being finite?

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Guess a random solution by measuring electron spins. If solution is wrong, kill yourself

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Let PostBQP (Postselected Bounded-Error Quantum Polynomial-Time) be class of problems solvable this way

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Corollary: Numerous "small" changes to quantum mechanics would let us solve PP-complete problems—nonunitary matrices, $|\alpha|^p$ for $p\neq 2$, …

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Coro ntum mechanics te problems— p for 2, …

Immediately implies **Beigel-Reingold-Spielman Theorem** from classical CS:

PP is closed under intersection


Quantum mechanics
What we experience

Stochastic Hidden-Variable Theories

Time ↓

$$\alpha_1^{(1)}|1\rangle + \alpha_2^{(1)}|2\rangle + \alpha_3^{(1)}|3\rangle + \alpha_4^{(1)}|4\rangle + \alpha_5^{(1)}|5\rangle$$

$$\alpha_1^{(2)}|1\rangle + \alpha_2^{(2)}|2\rangle + \alpha_3^{(2)}|3\rangle + \alpha_4^{(2)}|4\rangle + \alpha_5^{(2)}|5\rangle$$

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Quantum state of the universe

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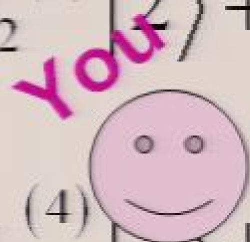

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Quantum state of the universe

Suppose your whole life history flashed before you in an instant

Let DQP (Dynamical Quantum Polynomial-Time) be the class of problems you could then solve efficiently (assuming transition probabilities satisfy two reasonable axioms—symmetry and locality)

A. 2002: DQP contains Graph Isomorphism (indeed all of Statistical Zero Knowledge)

$$\frac{1}{\sqrt{2}}\left(|\sigma\rangle+|\tau\rangle\right)|\sigma(G)\rangle$$

$$\frac{1}{\sqrt{2}}\left(|\sigma\rangle+|\tau\rangle\right)|\sigma(G)\rangle$$

Together with collision lower bound, strong evidence that BQP $\subset$ DQP

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QCMA: Same as QMA, but with quantum verification of *classical* proofs

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Current Work

- Quantum copy-protection and quantum software obfuscation
- Learning of quantum states / quantum Occam's Razor theorem
  AvgBQP/qpoly $\subseteq$ AvgQMA/poly
- BQP with closed timelike curves = PSPACE (with John Watrous)

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- Computer science and fundamental physics: a match made in Hilbert space
  - New perspective forces us to take QM seriously
  - Insights into hidden variables, postselection, holographic entropy bound, …
  - Computational input to quantum gravity?
  - Intractability as a physical axiom?

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