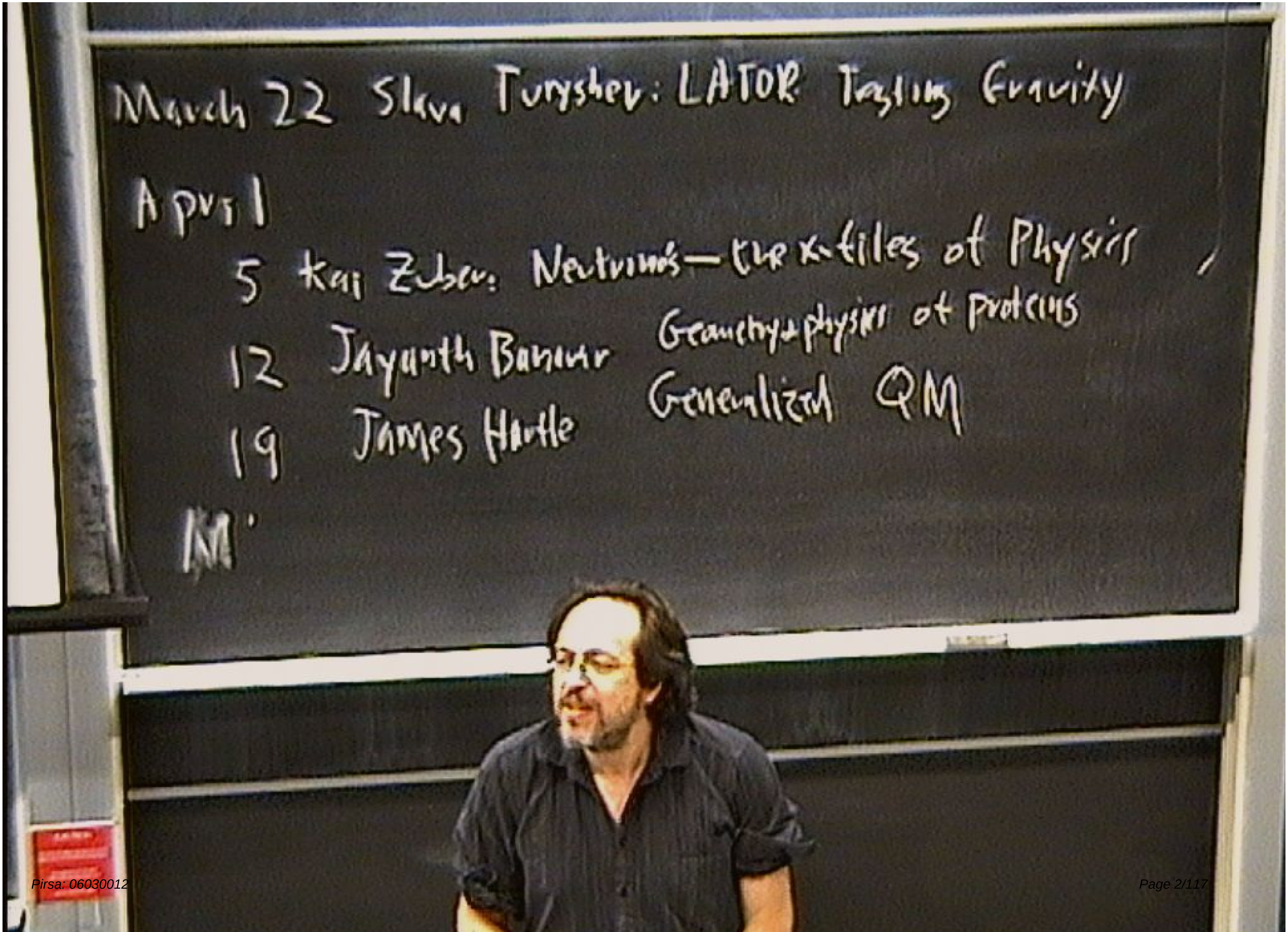
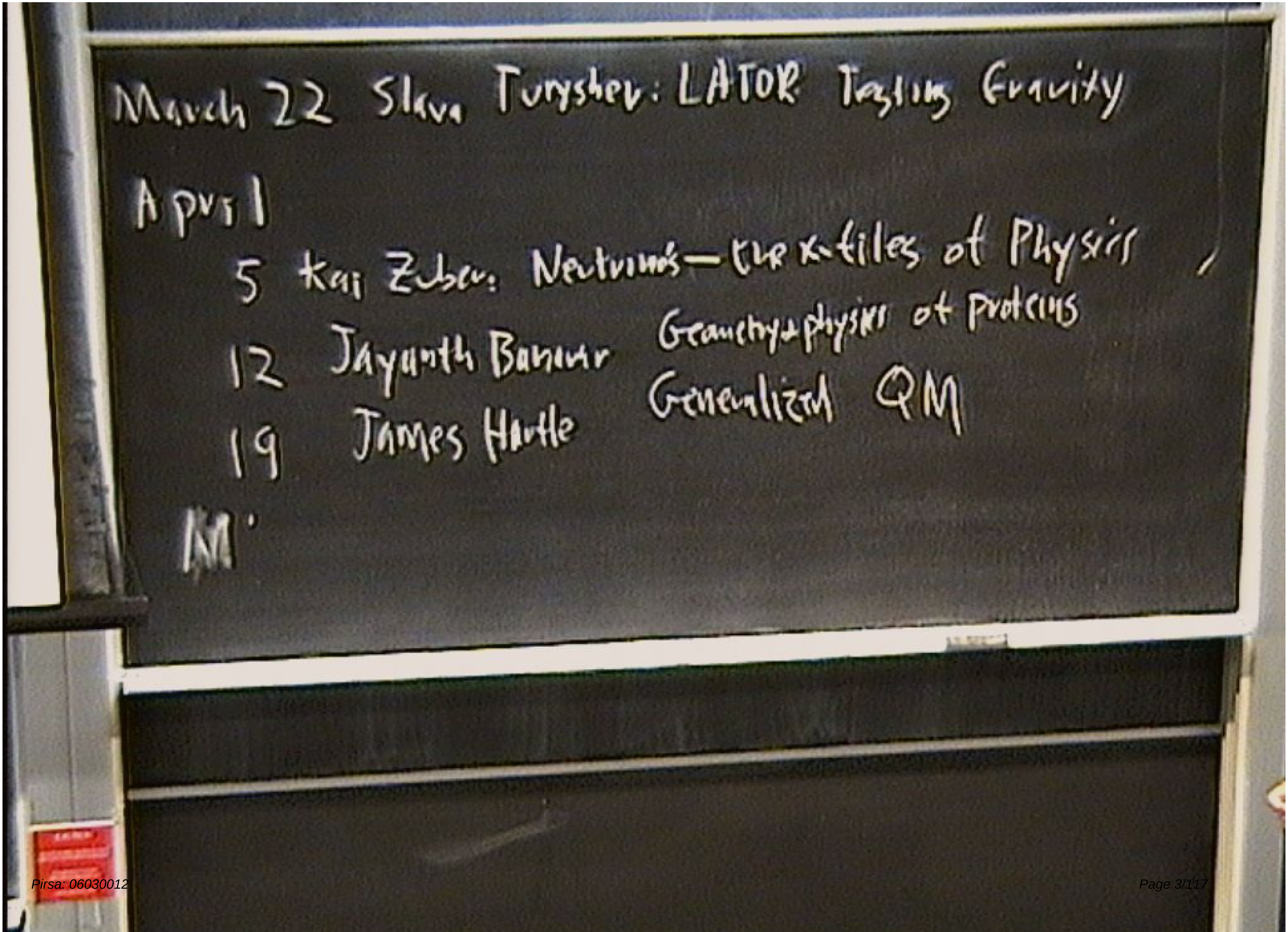
Title: Chaos from the Big Bang to Black Holes

Date: Mar 15, 2006 02:00 PM

URL: http://pirsa.org/06030012

Abstract:


March 22 Slava Turyshev: LATOR Testing Gravity
April
5 Kai Zuber: Neutrinos — the x-files of Physics
12 Jayanth Banavar Geometry + physics of proteins
19 James Hartle Generalized QM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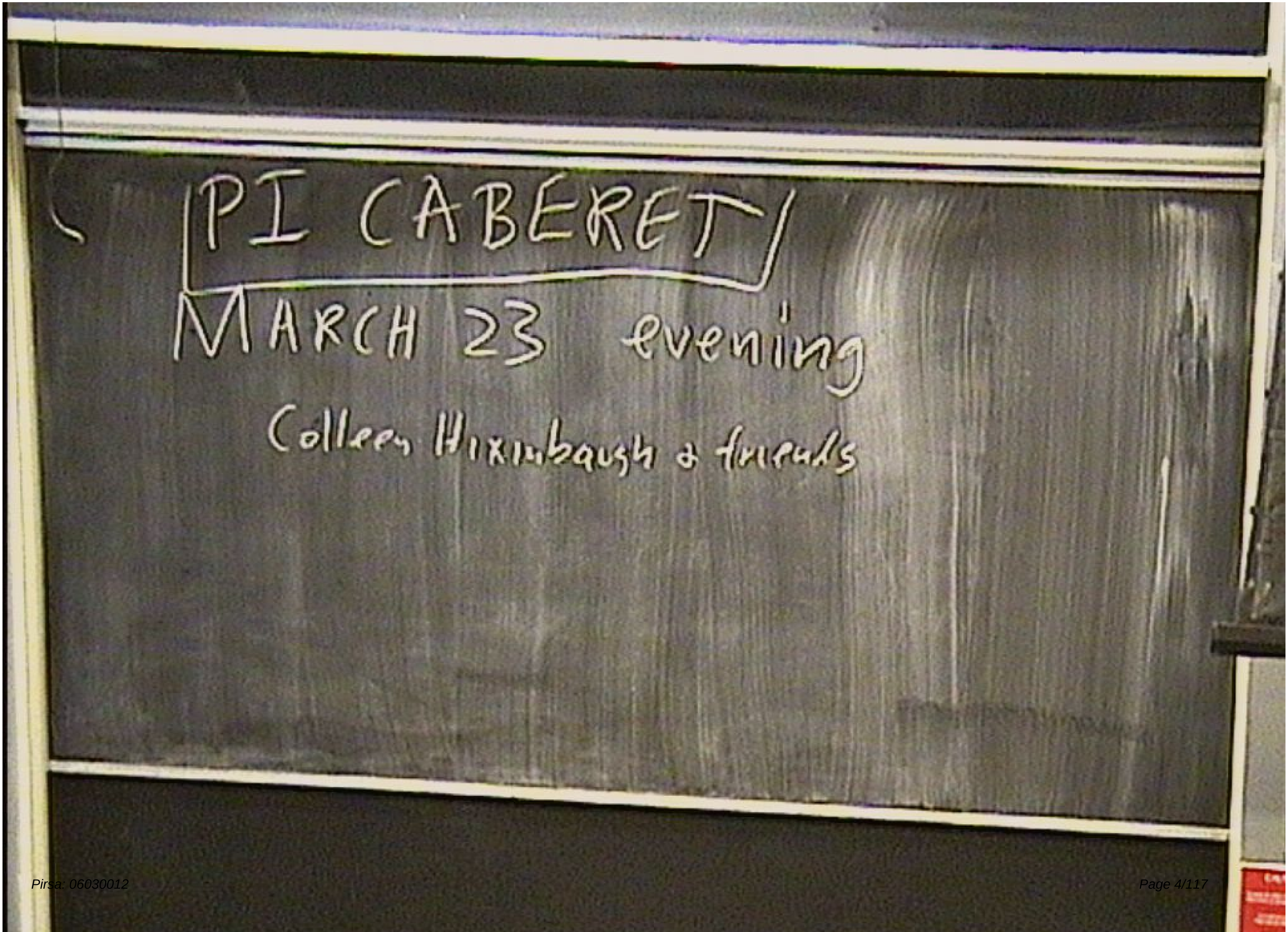
PI CABERET
MARCH 23 evening
Colleen Hiximbaugh & friends

Chaos from the
Big Bang to Black Holes

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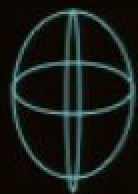
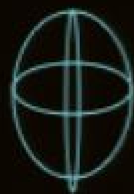


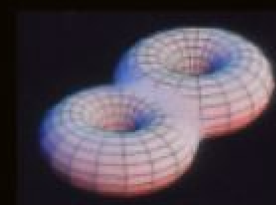
Anisotropic big bang (generic singularity)

Chaos from the Big Bang to Black Holes



Anisotropic big bang (generic singularity)

Geodesic flows on a compact hyperbolic space



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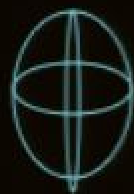

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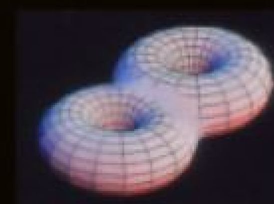
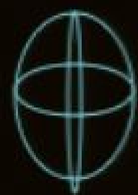


$V(\phi,\psi)$ Coupled scalar fields

Chaos from the Big Bang to Black Holes



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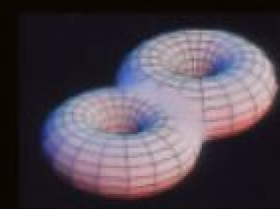
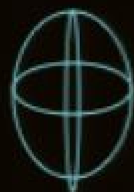


$V(\phi,\psi)$ Coupled scalar fields

Real astrophysical black hole pairs

Chaos from the Big Bang to Black Holes



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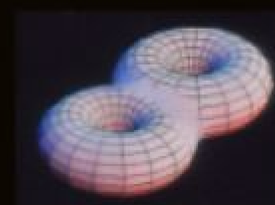

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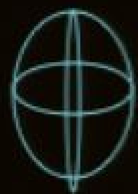
General Relativity vulnerable to chaos

Chaos from the Big Bang to Black Holes



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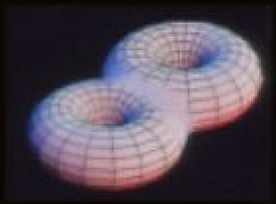

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General Relativity vulnerable to chaos

Could chaos be fundamental
not just an applied tool?
Quantum Gravity → Quantum Chaos

→ Secret agenda

Integrable versus Chaotic

Integrable versus Chaotic

symmetry ⇒ integrable

Hamiltonian System

$$\dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q}$$

N coordinates and N conjugate momenta

Integrable versus Chaotic

symmetry ⇒ integrable

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N coordinates and N conjugate momenta

If there are N constants of motion can always perform a canonical transformation to action-angle variables

$$I = N, \vartheta = \omega t$$

Integrable versus Chaotic

symmetry $\Rightarrow$ integrable

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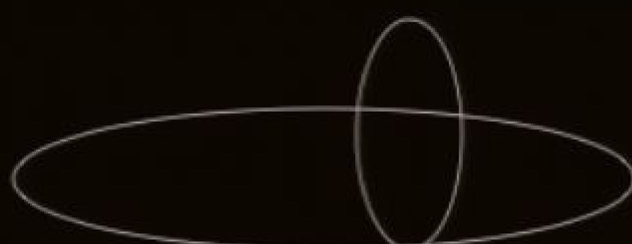
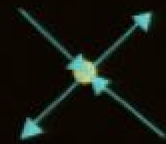


Motion on an N-torus

The periodic orbits

Elliptical/stable



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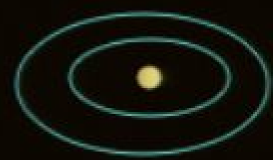

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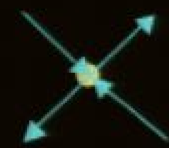
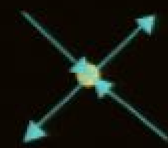


Hyperbolic/unstable

Periodic orbits are a set of measure 0 but
they define the phase space

The periodic orbits

Elliptical/stable



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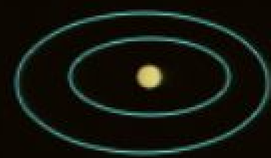
Under a small perturbation, most torii survive KAM theorem

The periodic orbits



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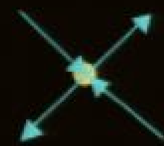

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Under a small perturbation, most torii survive KAM theorem

The complement set, those torii that are destroyed, correspond to the periodic orbits -> chaos

The periodic orbits

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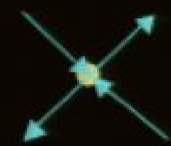

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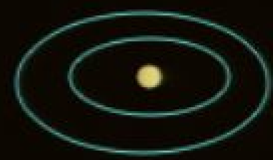

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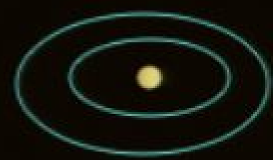
Proliferate into pairs of elliptical and hyperbolic

The periodic orbits



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Proliferate into pairs of elliptical and hyperbolic

Homoclinic tangle

Chaos

Chaos

symmetry ⇨ integrable

loss of symmetry ⇨ nonintegrable

Chaos

symmetry ⇨ integrable

loss of symmetry ⇨ nonintegrable

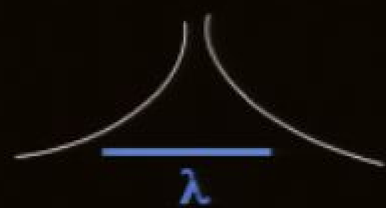
extreme sensitivity
to initial conditions

mixing

Chaos

symmetry ⇒ integrable

loss of symmetry ⇒ nonintegrable



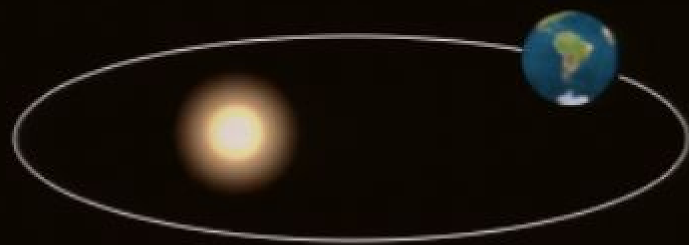
extreme sensitivity
to initial conditions

mixing

- blend of order and disorder
- loss of predictibility
- chaotic sets
- fractal dimension D, entropy H

2body problem is insoluble

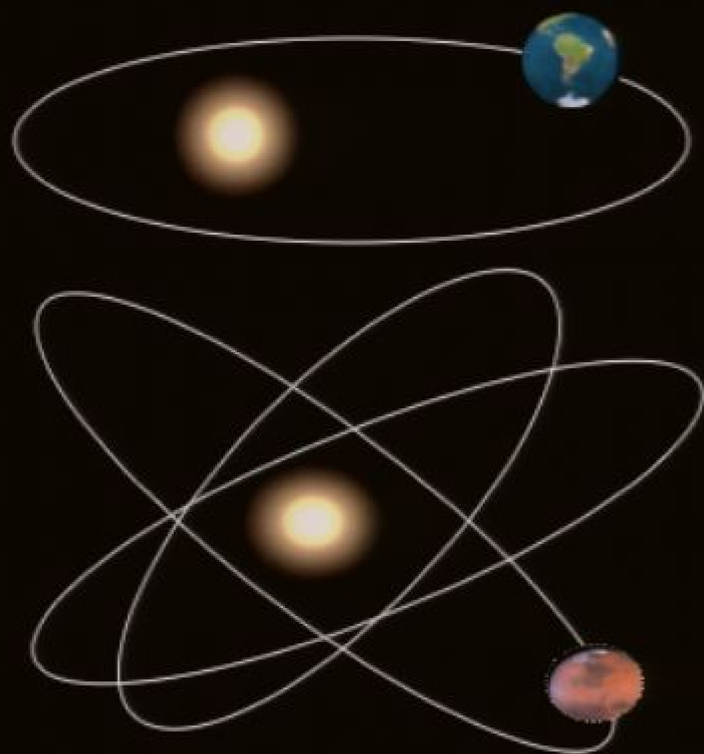
2body problem is insoluble



Newtonian
elliptical orbits

regular

2body problem is insoluble



Newtonian
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Relativistic
precession

test particle
regular

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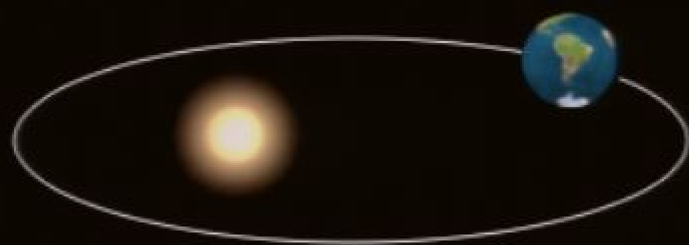

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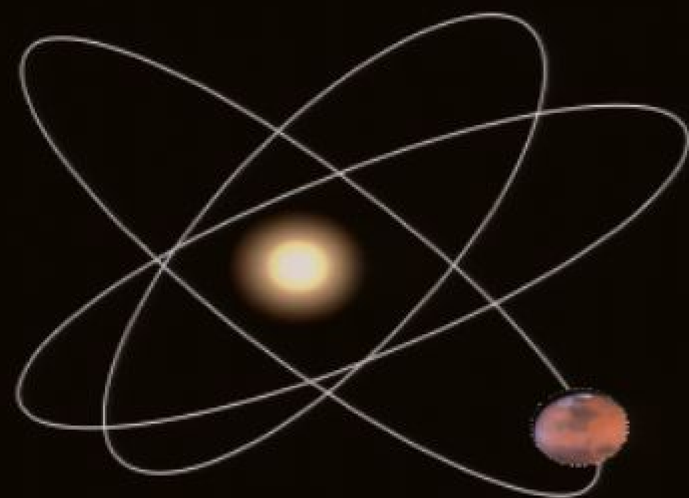

Relativistic
precession

test particle
regular

Relativistic
2body with
spins - chaotic

On the Verge of seeing the universe with new eyes and ears

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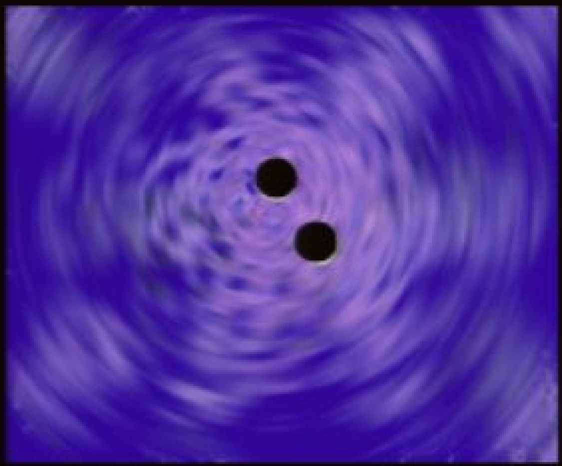
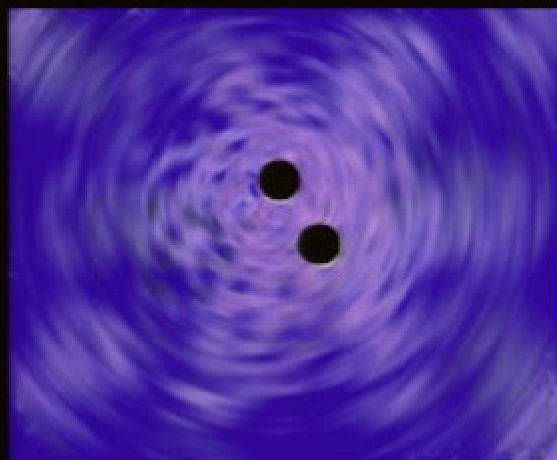


An orbiting pair of black holes will generate waves in the fabric of spacetime, Gavitational Waves

On the Verge of seeing the universe with new eyes and ears



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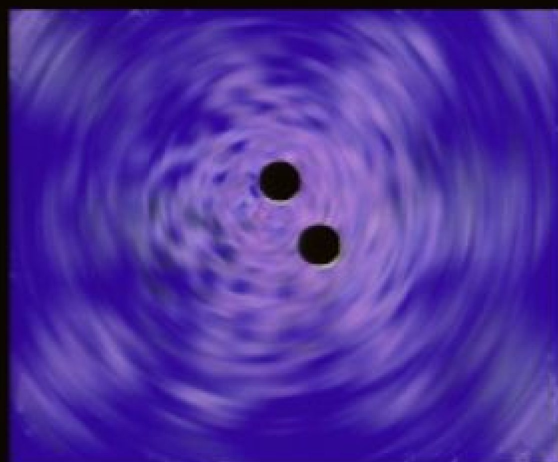

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LISA 2015

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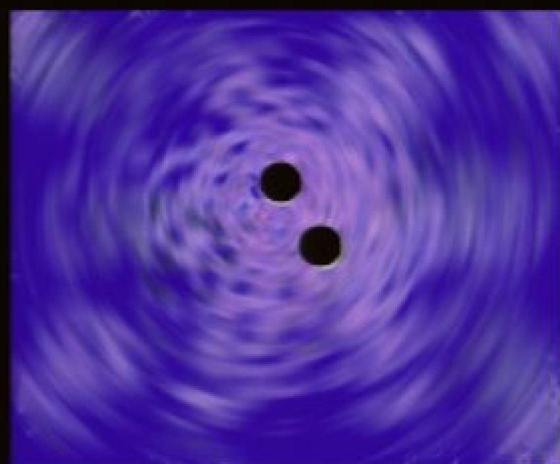

An orbiting pair of black holes will generate waves in the fabric of spacetime, Gavitational Waves

LISA 2015

Detection of Gravitational Waves Requires Knowledge of Dynamics

Stellar Mass BH/BH binaries

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Black holes form individually in clusters

M5 © Anglo-Australian Observatory
Photograph by David Malin

Stellar Mass BH/BH binaries

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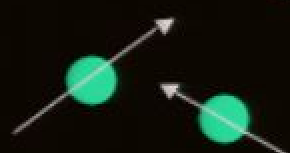
Tidally capture a companion on highly eccentric orbits



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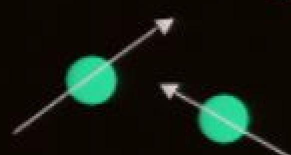

Spinning Pair can be ejected from the cluster

Portegeis Zwart and McMillan 1/day Advanced LIGO

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Photograph by David Malin

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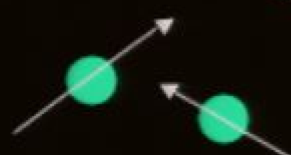

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Need an approach to 2body problem

"This symposium has gotten completely out of hand!"

What do we Know

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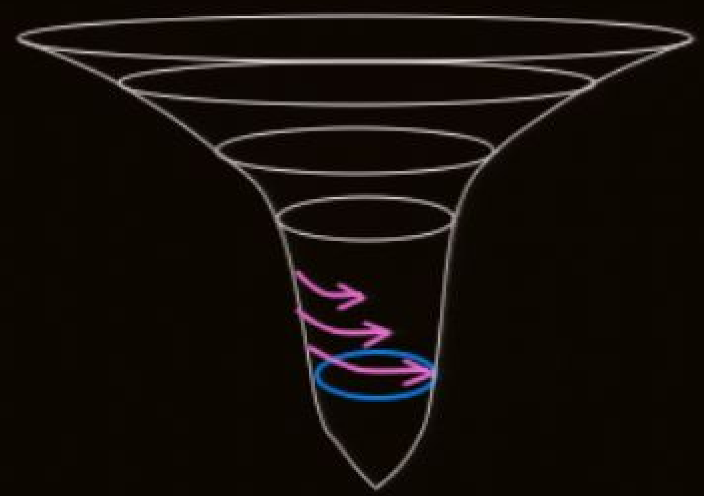
What do we Know

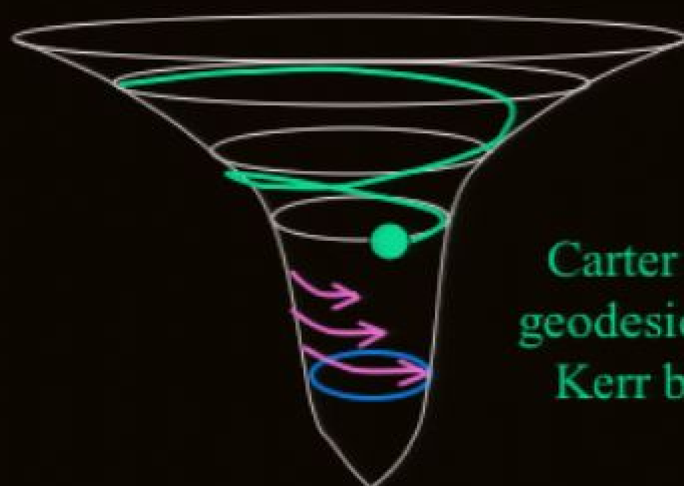
What do we Know



What do we Know

Carter found the geodesics around a Kerr black hole

What do we Know



Carter found the geodesics around a Kerr black hole

Therefore we know everything about the one-body problem

Orbits around a Schwarzschild Black Hole

Orbits around a Schwarzschild Black Hole

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1} dt^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

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Starting with the metric we can define a Lagrangian

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$$\mathcal{L} = -\frac{1}{2}\left(1-\frac{2M}{r}\right)\dot{t}^2 + \frac{1}{2}\left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2 + \frac{1}{2}r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right)$$

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Then the geodesic eqns come from Lagrange's eqns

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0$$

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Or using $p = \frac{\partial L}{\partial \dot{q}}$

$$H = -\left(1-\frac{2M}{r}\right)^{-1}\frac{p_t^{\,2}}{2} + \left(1-\frac{2M}{r}\right)\frac{p_r^{\,2}}{2} + \frac{1}{2r^2}\left(p_\theta^{\,2} + \sin^{-2}\theta p_\phi^{\,2}\right)$$

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Define H and find EOM $\dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q}$

4 q's and 4 p's

4 constants of motion:

E,L,θ=π/2,H=-1/2

Orbits around a Schwarzschild Black Hole

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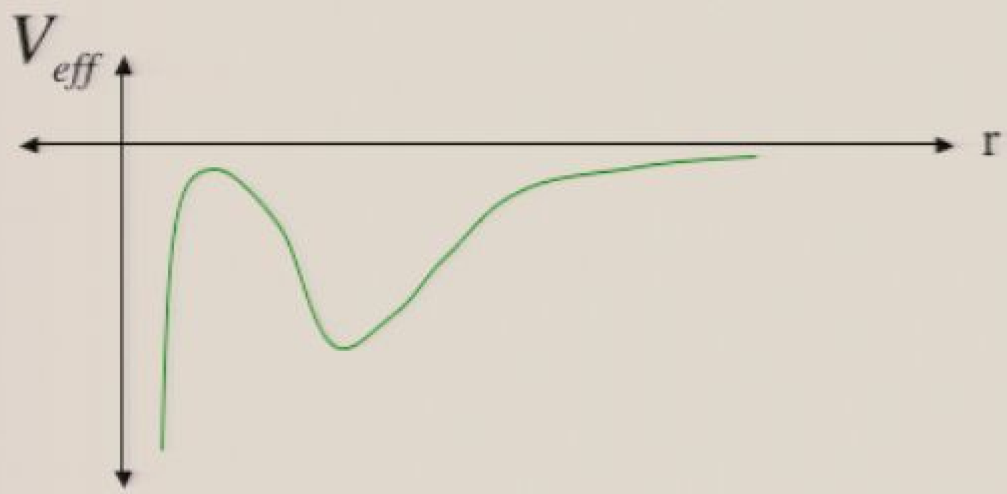
Motion is confined to 4-torus

Integrable

Reduces to motion in one dimension

$$\frac{1}{2}\dot{r}^2 + \underbrace{\frac{1}{2}\left(1-\frac{2M}{r}\right)\left(\frac{L^2}{r^2}+1\right)}_{V_{eff}} = \frac{E^2}{2}$$



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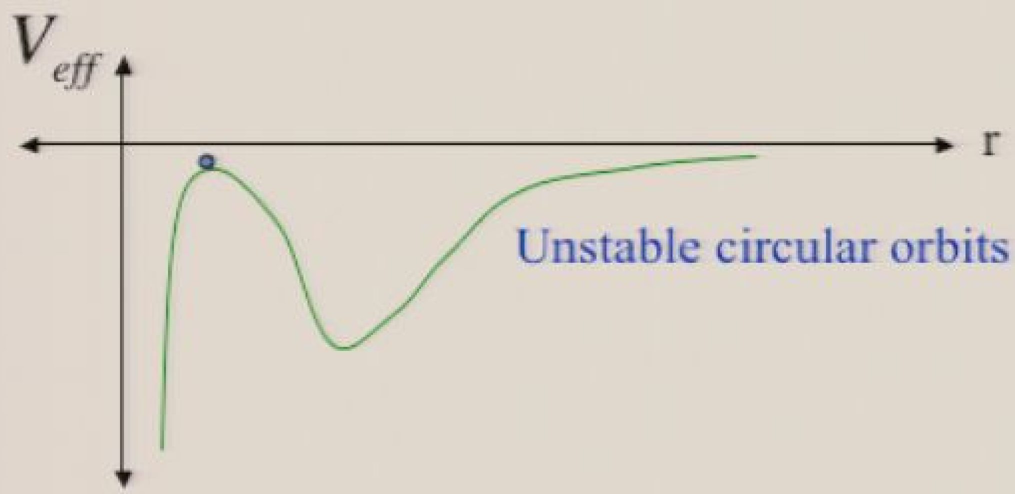

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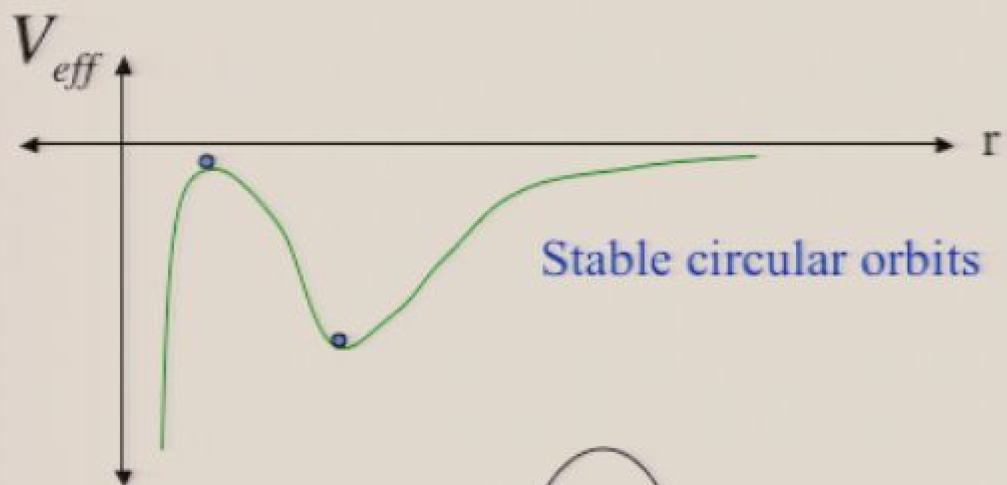
$$\frac{1}{2}\dot{r}^2 + \underbrace{\frac{1}{2}\left(1-\frac{2M}{r}\right)\left(\frac{L^2}{r^2}+1\right)}_{V_{eff}} = \frac{E^2}{2}$$



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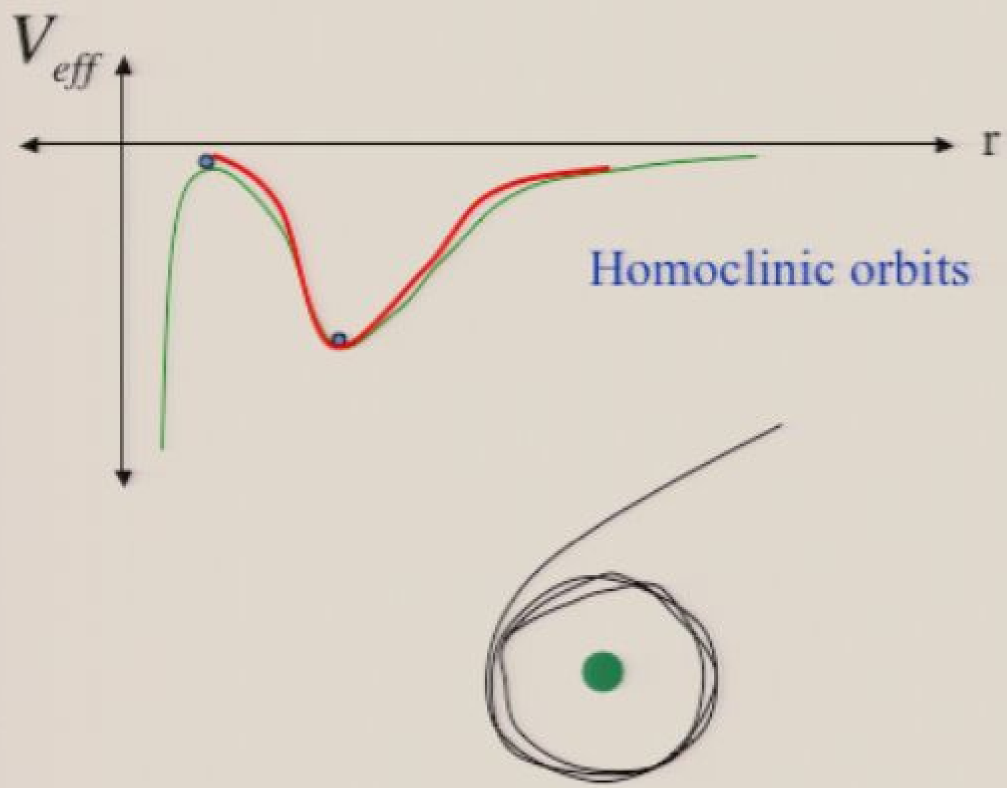

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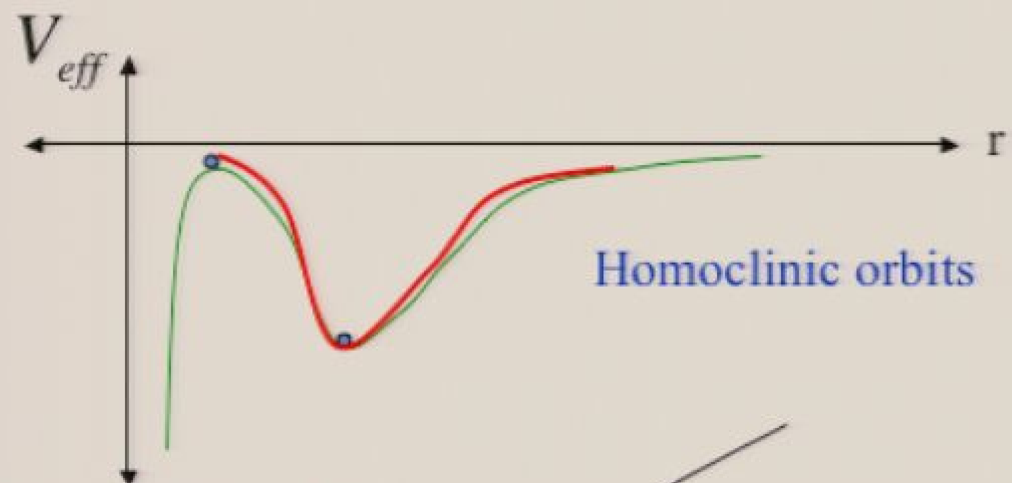
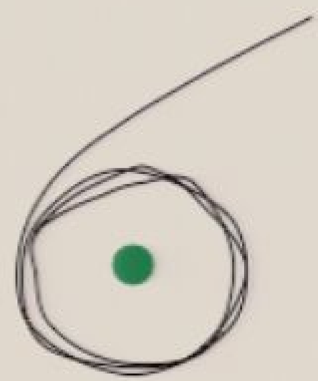


Homoclinic orbits approach the same fixed point in the infinite past & the infinite future: mark intersection of stable



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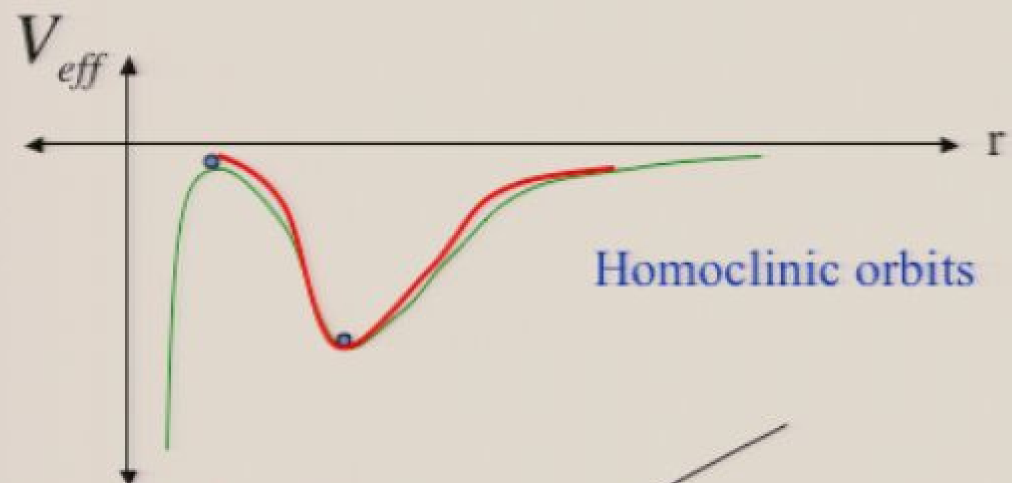

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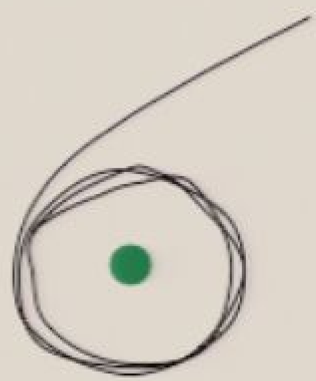

Simple structure of periodic orbits

We also know orbits around a Kerr black hole

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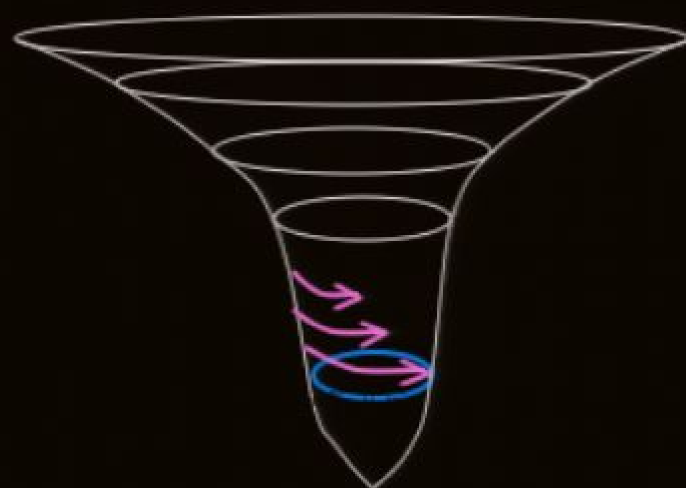

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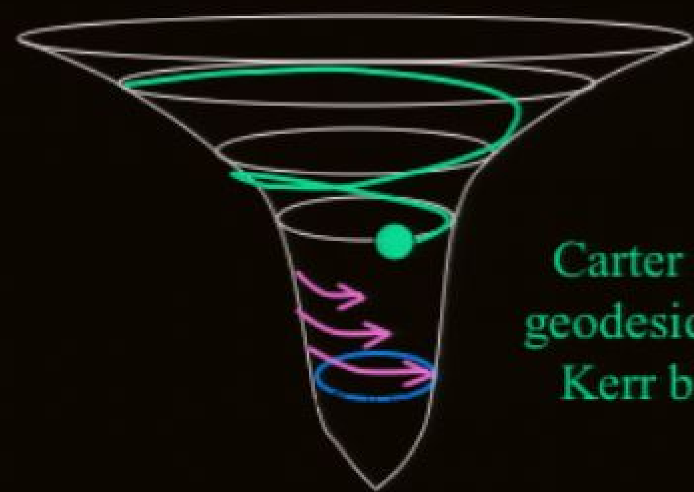

Carter found the geodesics around a Kerr black hole

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Carter found the geodesics around a Kerr black hole

Timelike Killing field, an axial Killing field, and orbits still timelike

E,L,H=-1/2,C

Where C is the Carter constant generated by a Killing tensor

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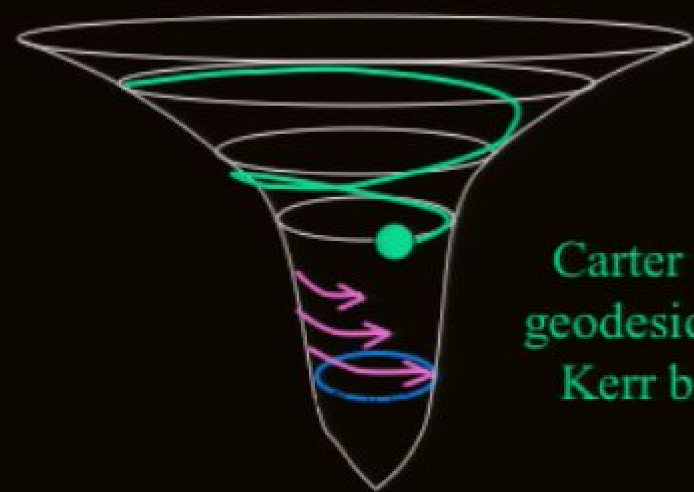

Carter found the geodesics around a Kerr black hole

Timelike Killing field, an axial Killing field, and orbits still timelike

E,L,H=-1/2,C

Where C is the Carter constant generated by a Killing tensor

Therefore motion lies on torii and integrable

What don't we Know

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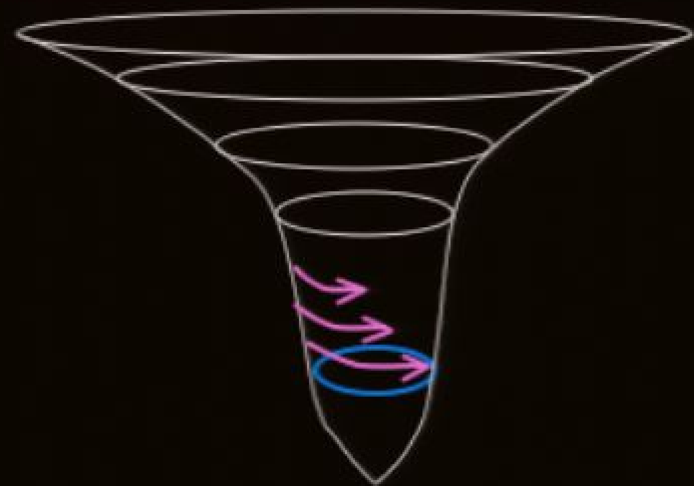

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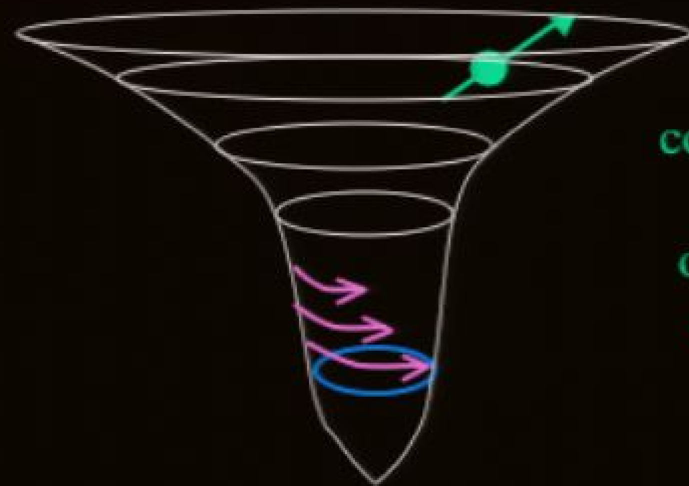

If the companion spins - orbits are strongly altered

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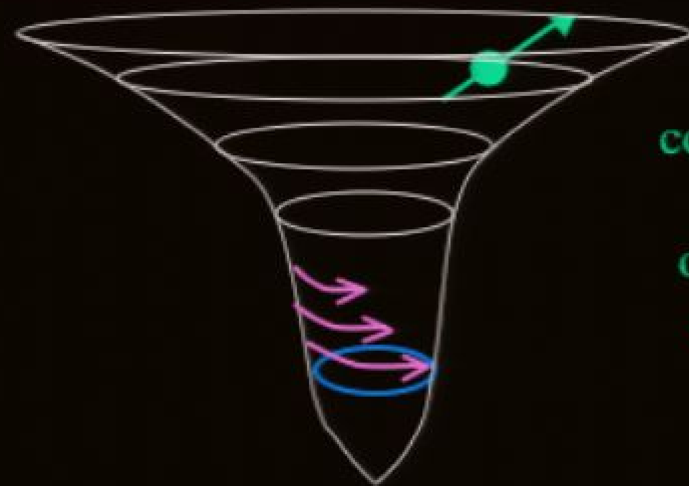

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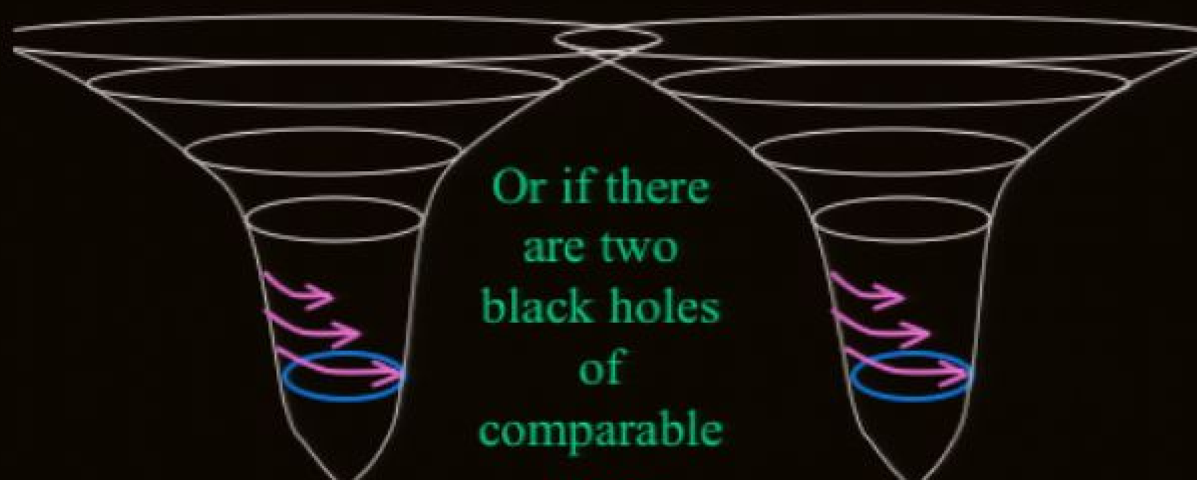
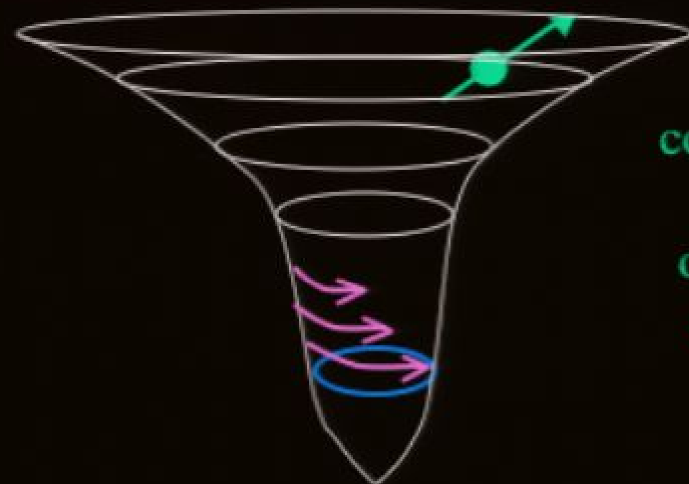


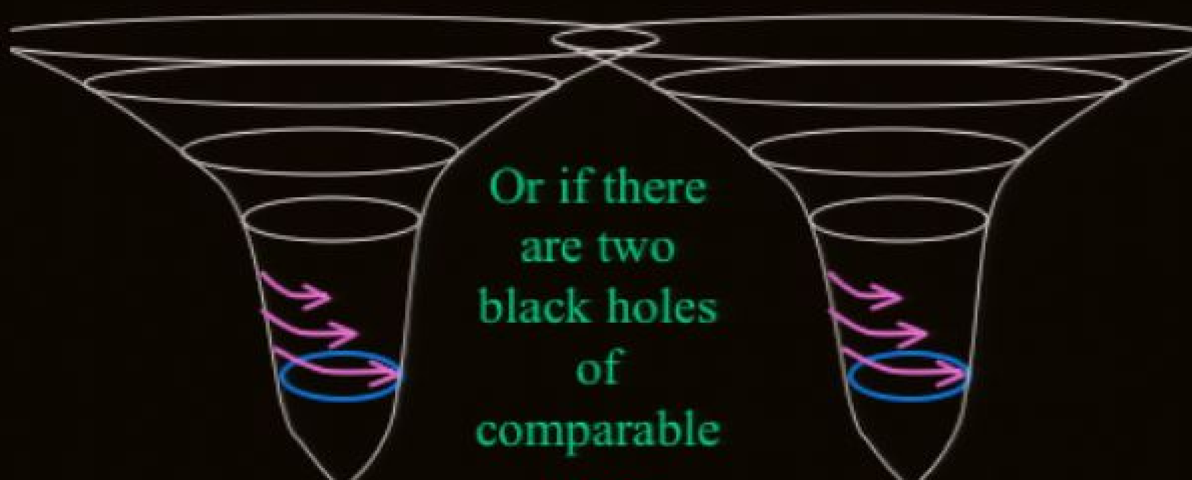
Or if there are two black holes of comparable

What don't we Know

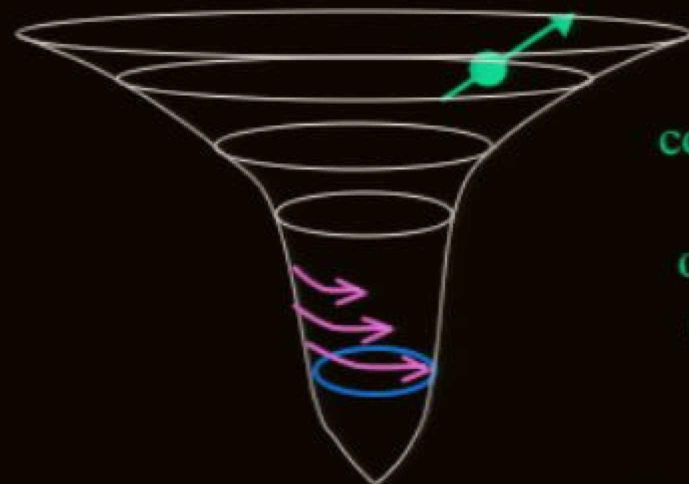


If the companion spins - orbits are strongly altered

There are no known exact solutions to the orbits of these BH binaries



Spinning test particle around a black hole



If the companion spins - orbits are strongly altered

$$\frac{dx^\alpha}{d\tau} = u^\alpha + \frac{1}{4\mu^2}\left(\varepsilon^{\alpha}{}_{\nu\rho\sigma} R^{\rho\sigma}{}_{\delta\gamma} \varepsilon^{\delta\gamma}{}_{\beta\phi}\right) S^\nu S^\beta u^\phi$$

$$\frac{Dp^\alpha}{d\tau} = \frac{1}{2\mu}\left(R^{\alpha}{}_{\nu\rho\sigma} \varepsilon^{\rho\sigma}{}_{\beta\delta}\right) v^\nu S^\beta p^\delta$$

$$\frac{DS^\alpha}{d\tau} = \frac{1}{2\mu^3}\left(R_{\nu\gamma\rho\sigma} \varepsilon^{\rho\sigma}{}_{\beta\delta}\right) S^\nu v^\gamma S^\beta p^\delta p^\alpha$$

$$\frac{dx^\alpha}{d\tau} = u^\alpha$$

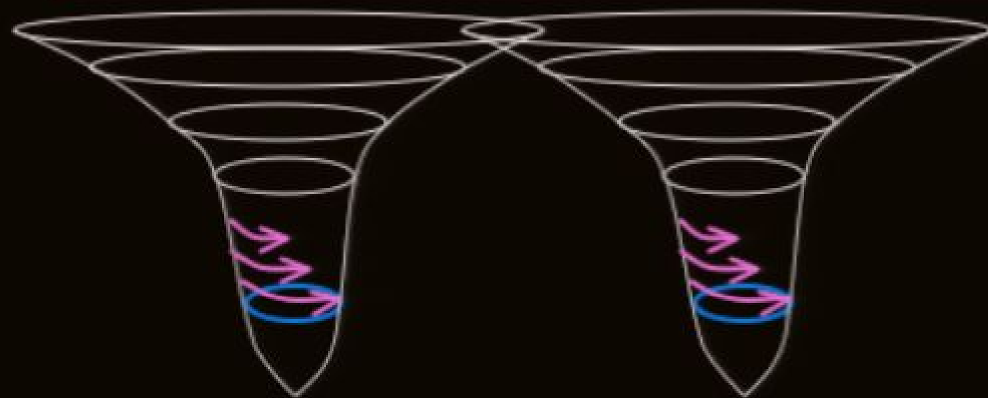
$$\frac{Dp^\alpha}{d\tau} = 0$$

$$S^\alpha = 0$$

Suzuki and Maeda found orbits chaotic but only for

$$S \geq 1\mu M \sim 1\mu^2\left(\frac{M}{\mu}\right) \sim 10^6 \mu^2$$

Comparable mass black hole pairs



$$H = H_N + H_{1PN} + H_{2PN} + H_{3PN} + H_{SO}$$

$$H_N = \frac{p^2}{2} - \frac{1}{r}$$

And PN corrections are a page long

Spin orbit coupling in H as well as spin spin coupling

$$H_{SO} = \frac{\vec{L} \times \vec{S}_{eff}}{r^3}$$

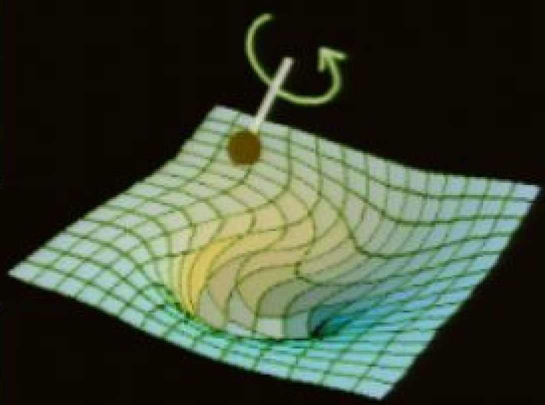
$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}$$

Not enough constants of motion

$$\dot{\vec{S}}_i = [\vec{S}_i, H]$$

Spin precession

•Geodetic precession
Thomas precession
Spin-Orbit



•Frame dragging
Lense-Thirring
Spin-Spin

Spin precession

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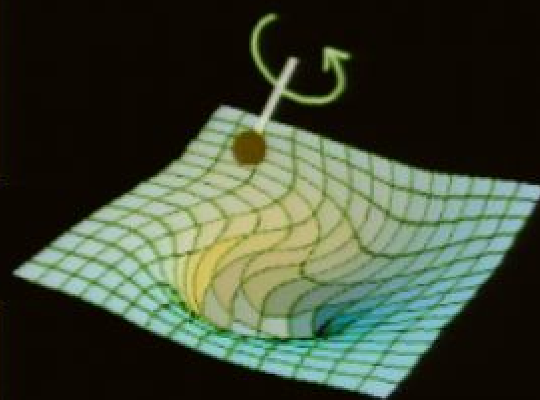

•Frame dragging
Lense-Thirring
Spin-Spin

The spin of the orbiting companion gets dragged around with the spinning center

Spin precession

$$\dot{\vec{S}} = \vec{\Omega} \times \vec{S}$$

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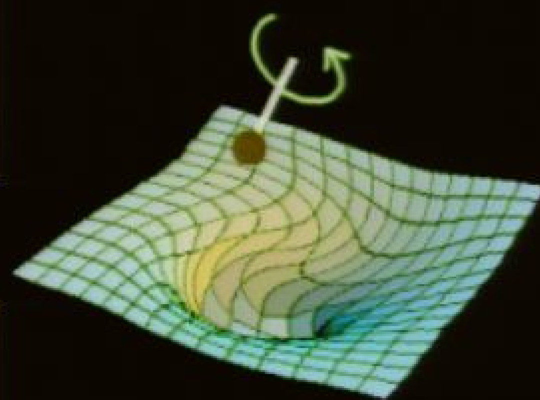

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Spin precession

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Spin precession can destabilize an orbit to the point of chaos

Orbital plane precesses

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Total angular momentum is the sum of orbital and spin

$$\vec{J} = \vec{L} + \vec{S}$$

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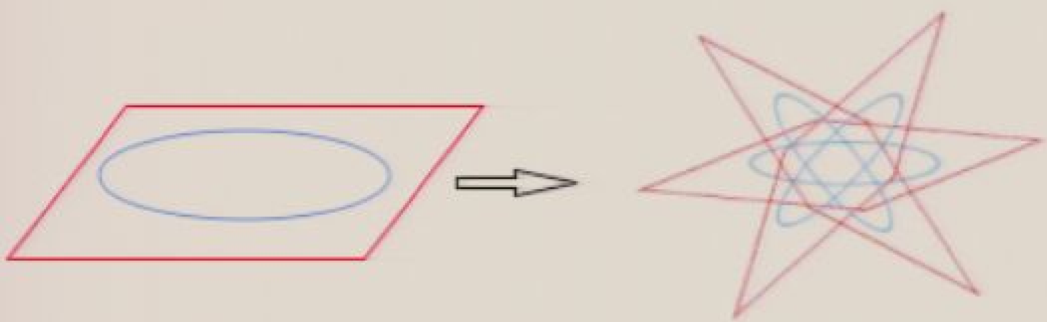
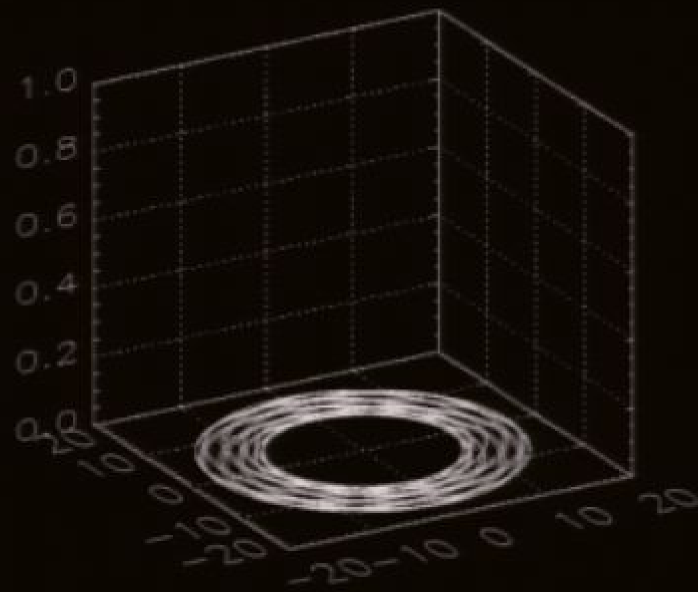
$$\dot{\vec{L}} = -\dot{\vec{S}}$$

In addition to the precession of the perihelion, the entire orbital plane precesses



No spin: the orbit is confined to a plane



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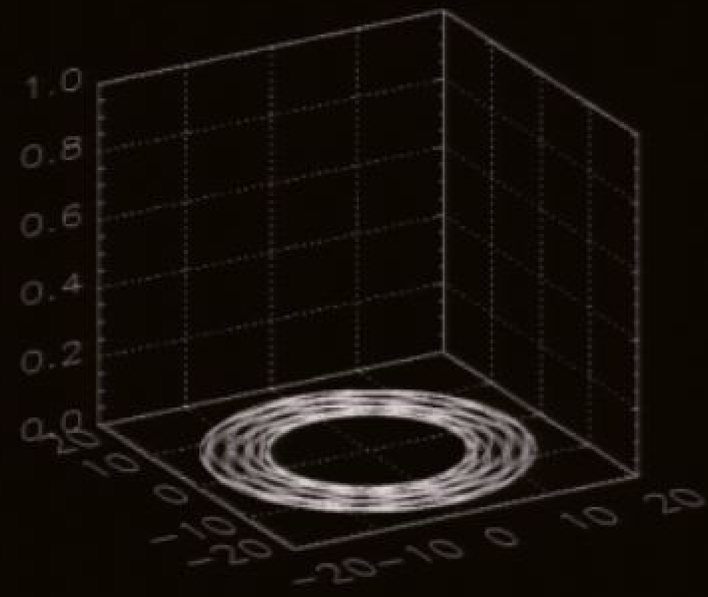
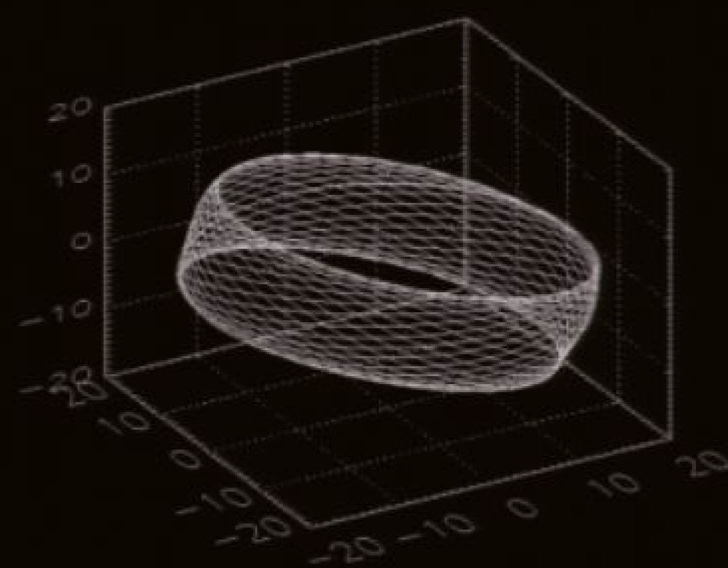


Spinning: the orbit is not confined to a plane



Chaos

symmetry ⇨ integrable

loss of symmetry ⇨ nonintegrable

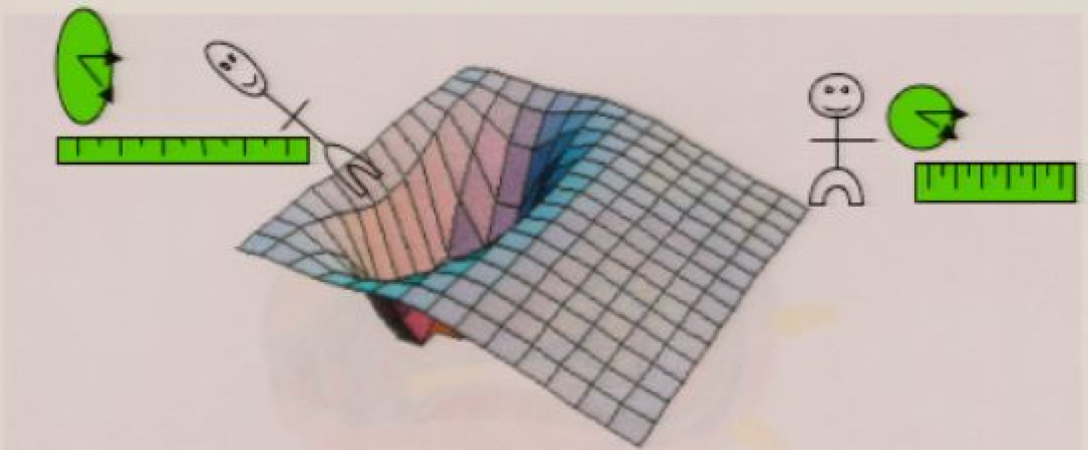
extreme sensitivity
to initial conditions

mixing

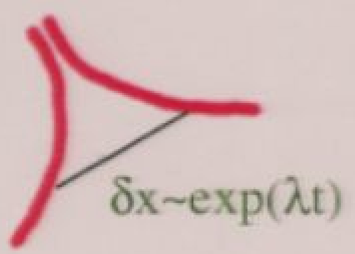
Measuring chaos in curved space

Space & time are relative:

Clocks run at different rates
Rulers shrink and stretch



the relativism of spacetime ->
standard indicators of chaos are relative as well

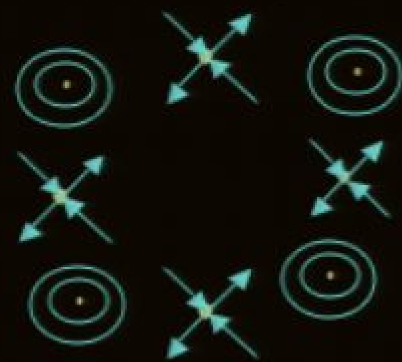


The Lyapunov exponent depends on the rate at which the observer's clock ticks

all observer's agree on the occurrence

The periodic orbits define skeleton of dynamics

In a chaotic region the periodic orbits proliferate



Proliferate into pairs of elliptical and hyperbolic

Homoclinic tangle

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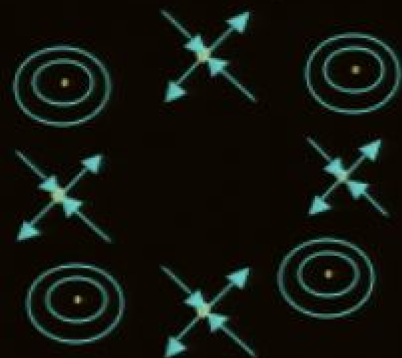

Proliferate into pairs of elliptical and hyperbolic

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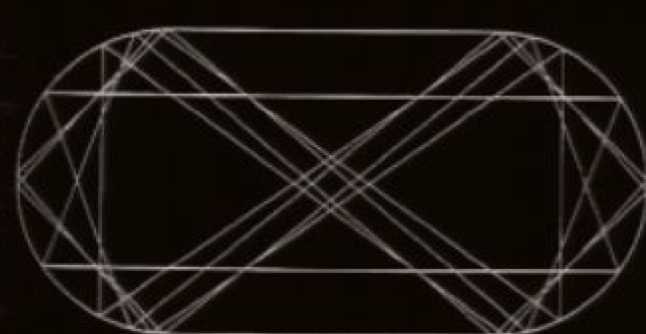
Like a game of chaotic billiards or pinball



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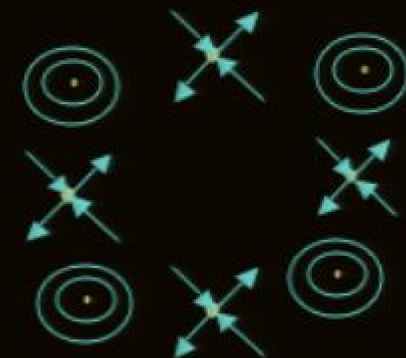

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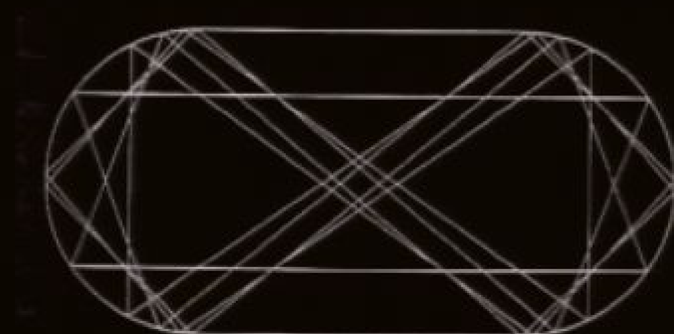
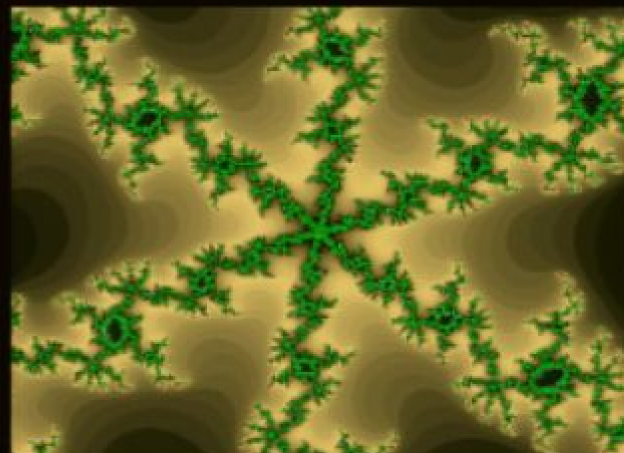


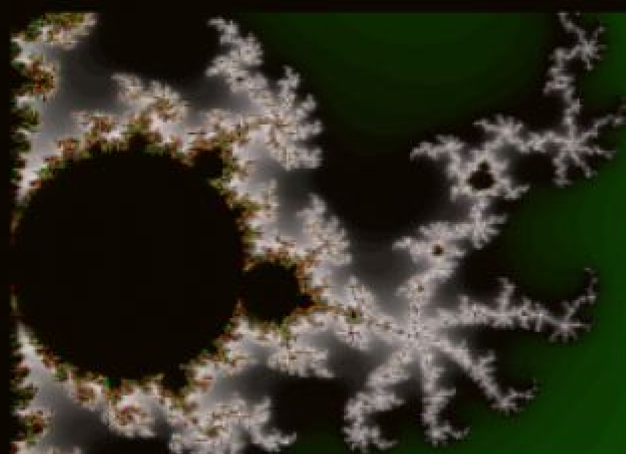
To pack a proliferating infinity of orbits
in a finite phase space form a
Fractal

Fractals are self-similar structures

Pack a lot of information in a bounded area



Pattern repeats on smaller and smaller scales



If you measured the length around the perimeter it would seem infinite $L= N(\varepsilon)\, \varepsilon$ though a finite area

Interpret as having fractional dimension $L=N(\varepsilon)\, \varepsilon^{D}$

Fractal Basin Boundaries

Post-Newtonian approximation to the 2body problem

$$\theta = \cos^{-1}\left(\vec{S}\cdot\hat{z}\right)$$

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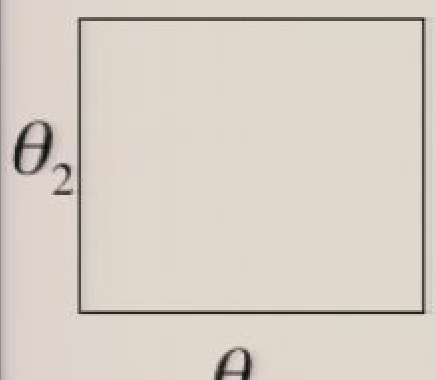
find they're fractal



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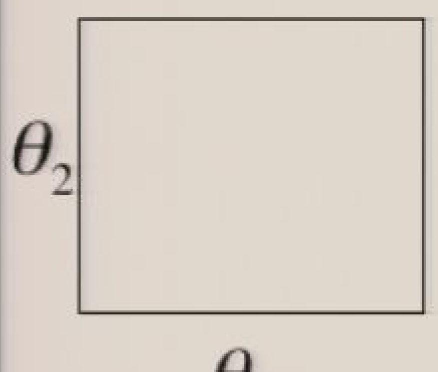

•Black ⟹ merger

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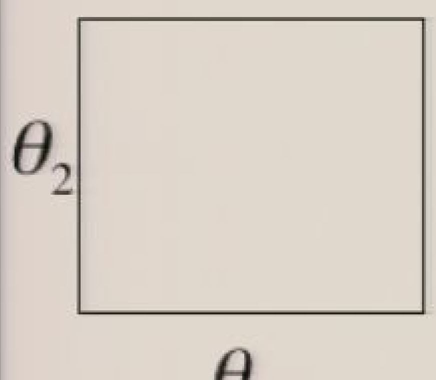

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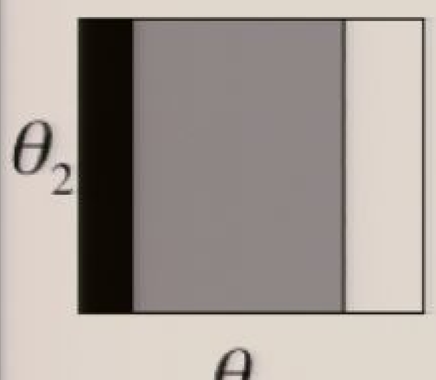

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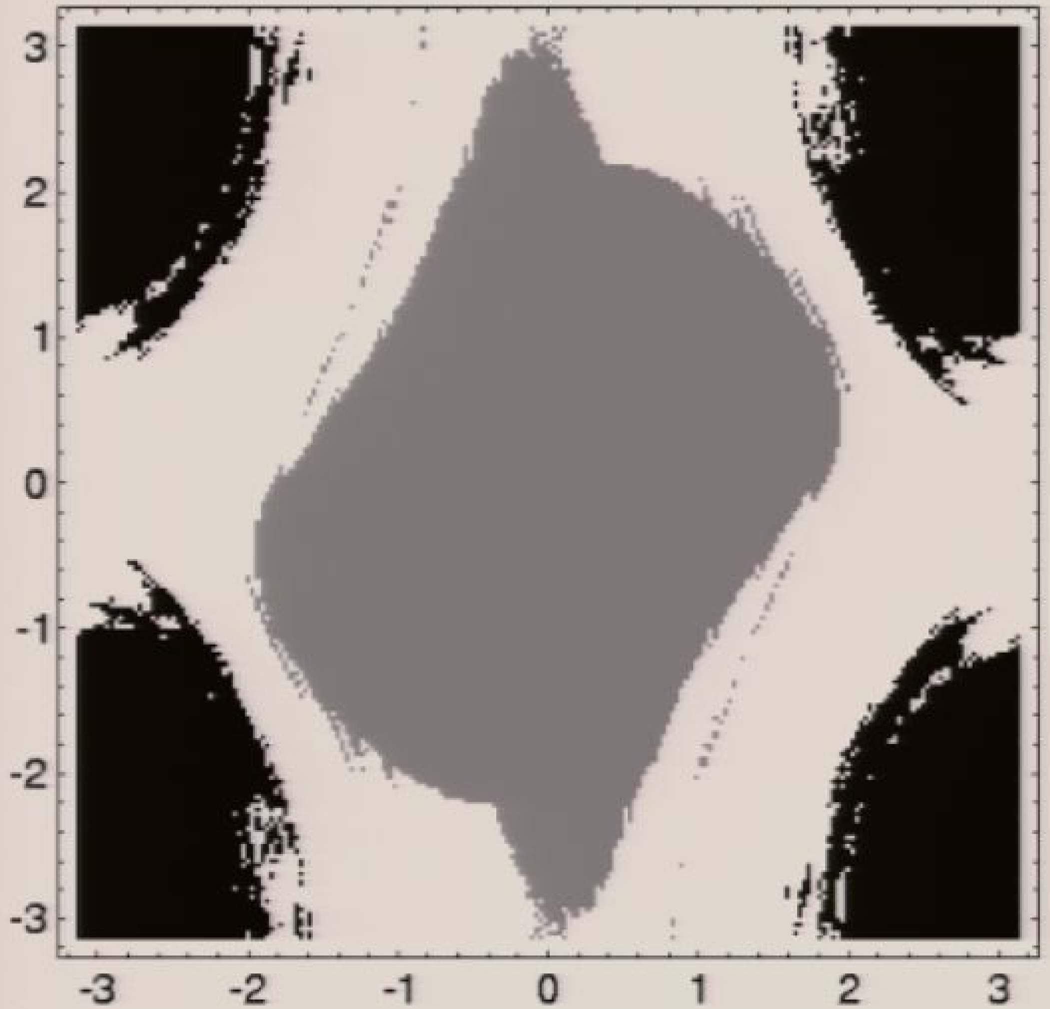
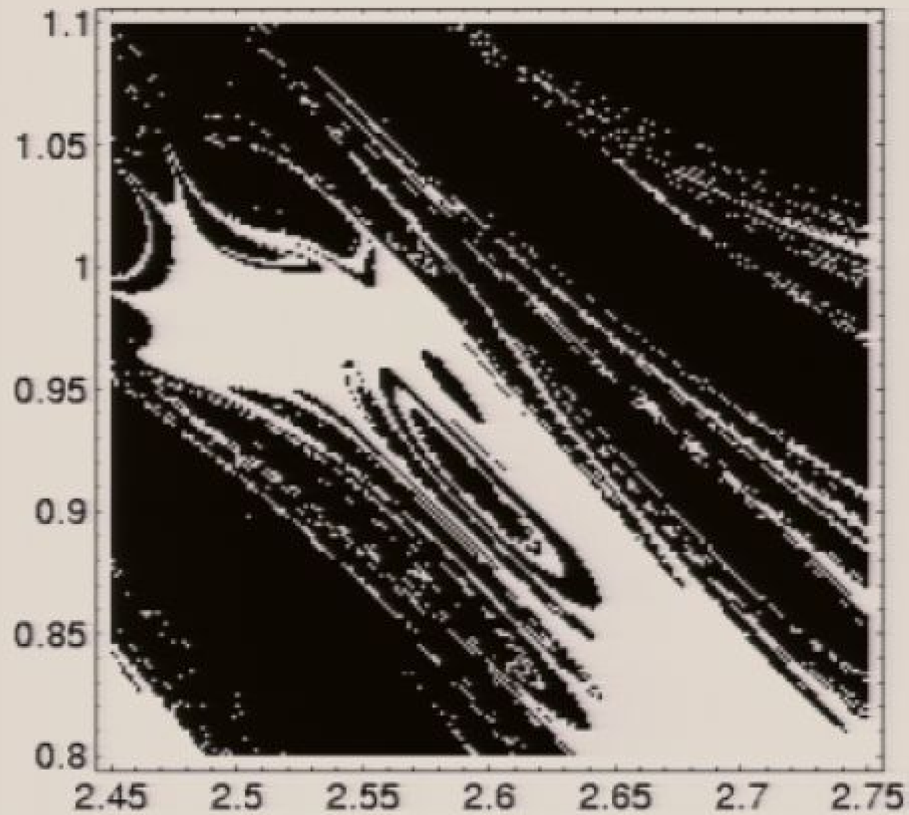
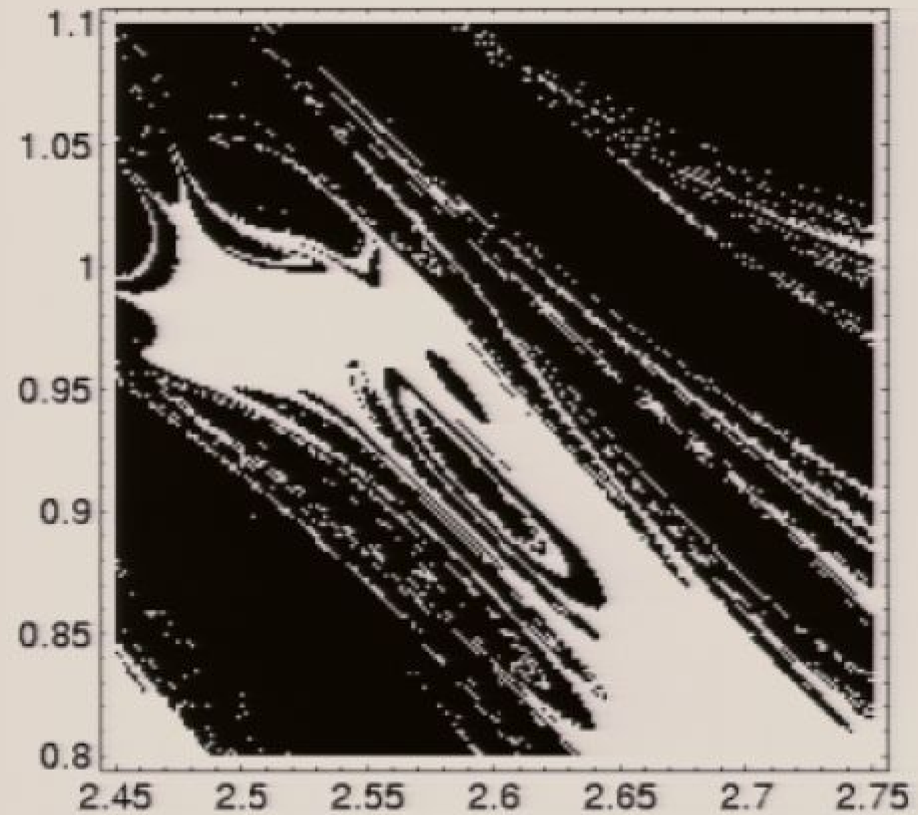



1.1
1.05
1
0.95
0.9
0.85
0.8
2.45
2.5
2.55
2.6
2.65
2.7
2.75


1.1
1.05
1
0.95
0.9
0.85
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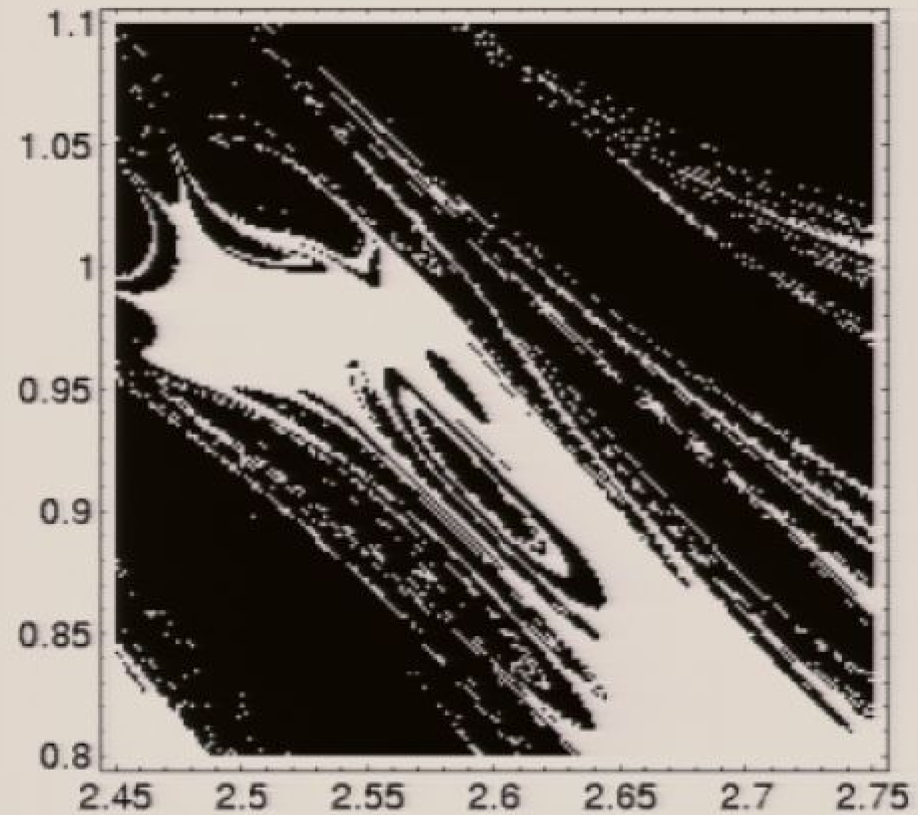
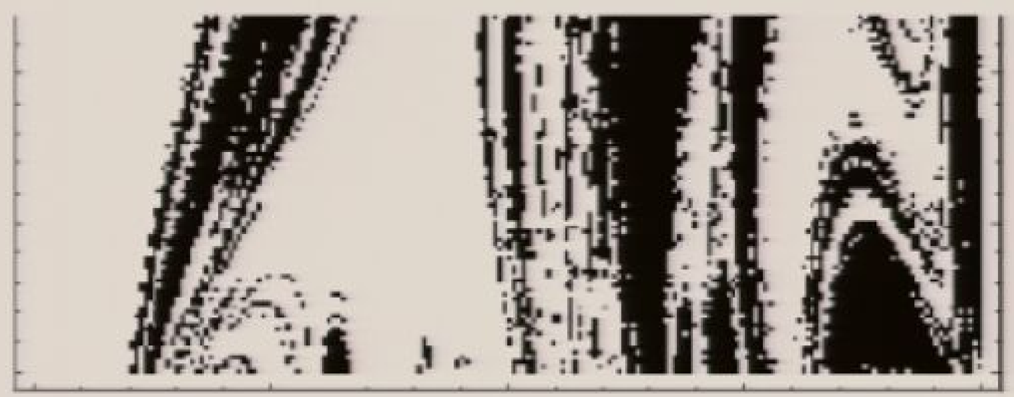
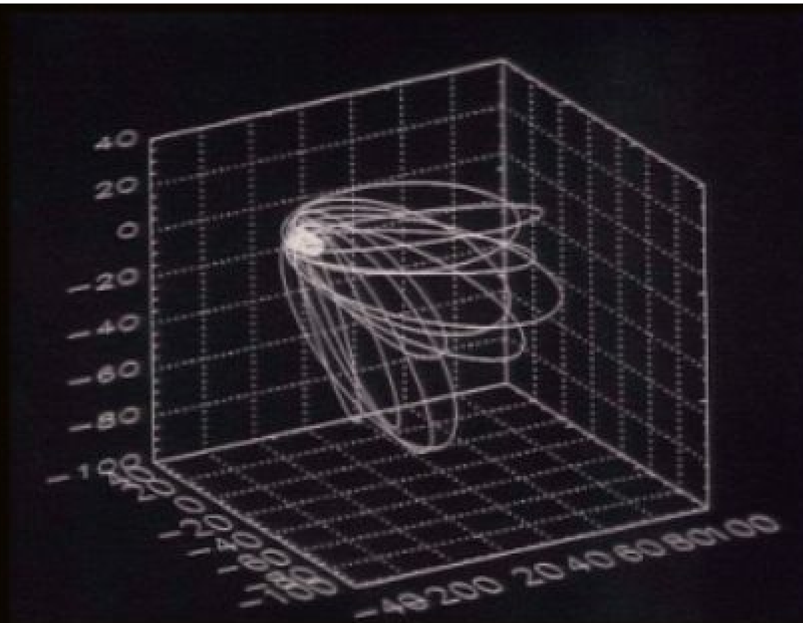
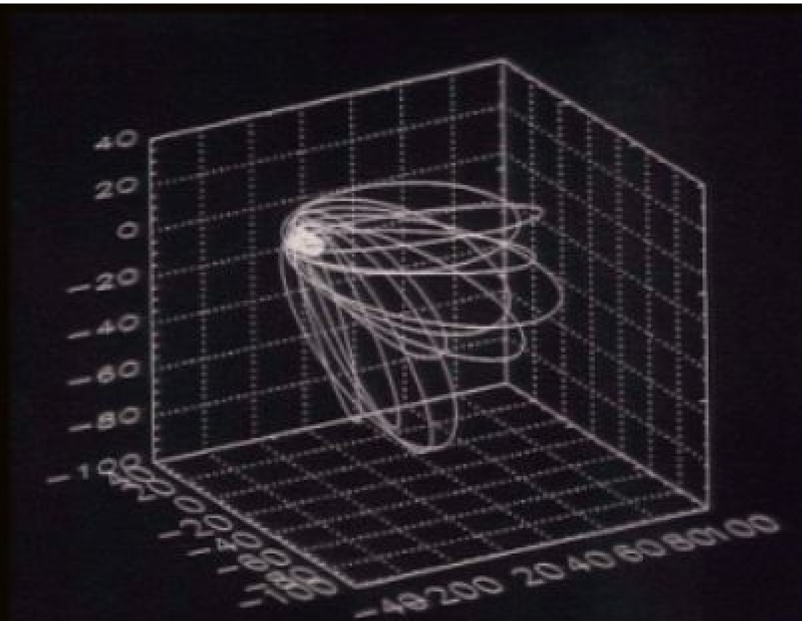
2.45
2.5
2.55
2.6
2.65
2.7
2.75





Chaotic scattering off a fractal set of periodic orbits


40
20
0
−20
−40
−60
−80


40
20
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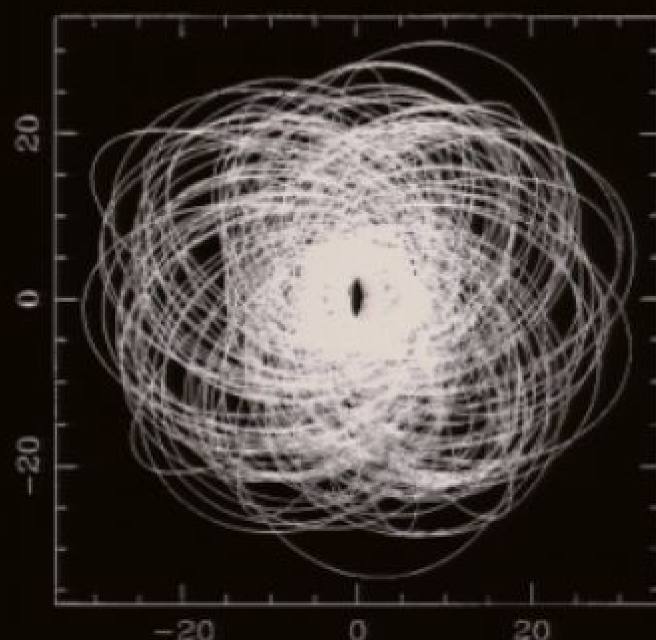
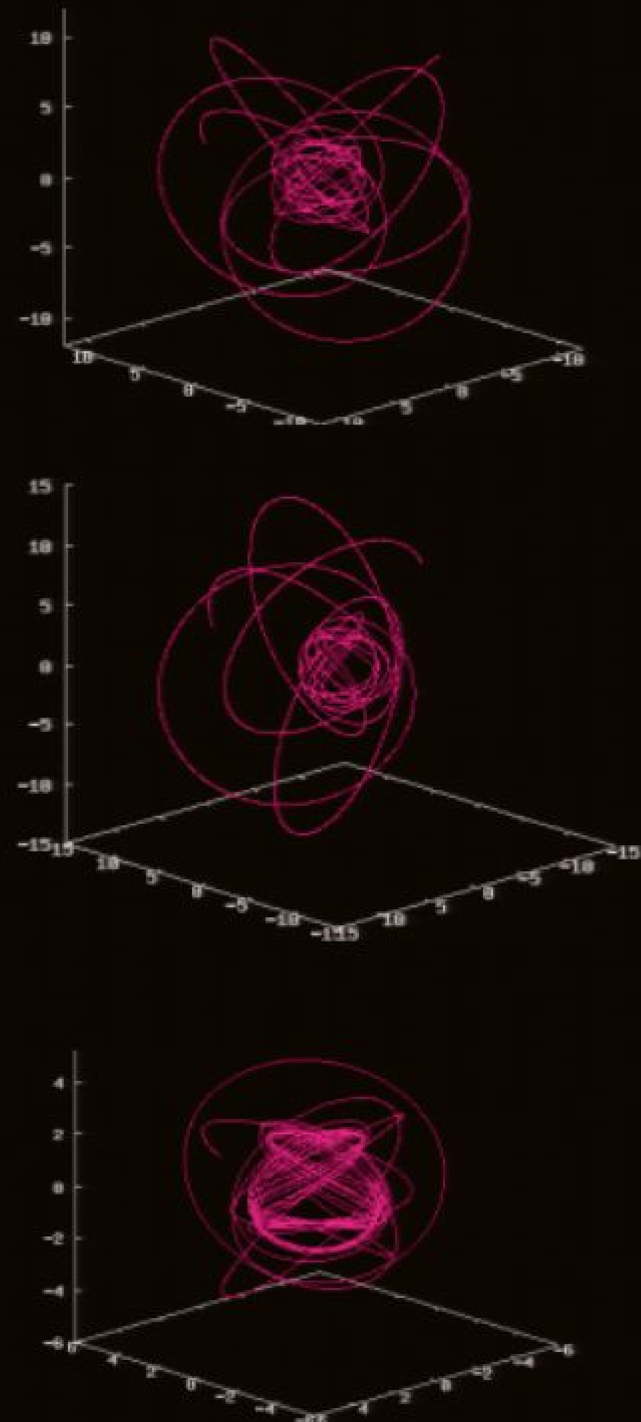

20
0
−20
−20
0
20



Enough Chaos for Now

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Difficult to Proceed:

1. Chaos in Conservative Dynamics

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2. Damped by Gravitational Waves

Enough Chaos for Now

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3. Whether completely damped or observable in grey area

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2PN not relativistic enough

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2PN not relativistic enough

Kerr with spinning test particle doesn't include mass of companion

In Brief

Black Holes are Cool

They are real astrophysical objects

And they exhibit chaos

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color oscillations, moduli fileds, inflatons

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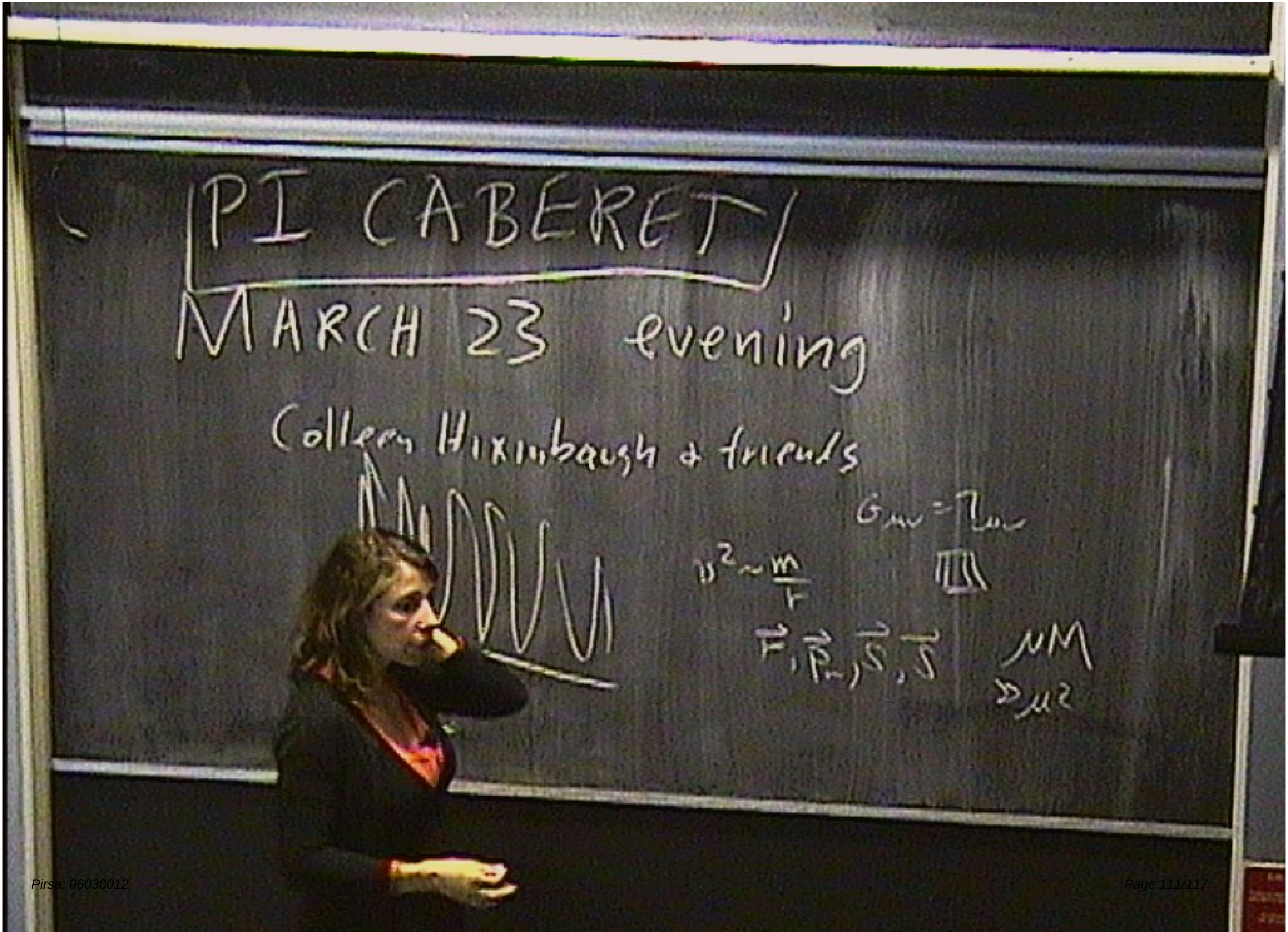
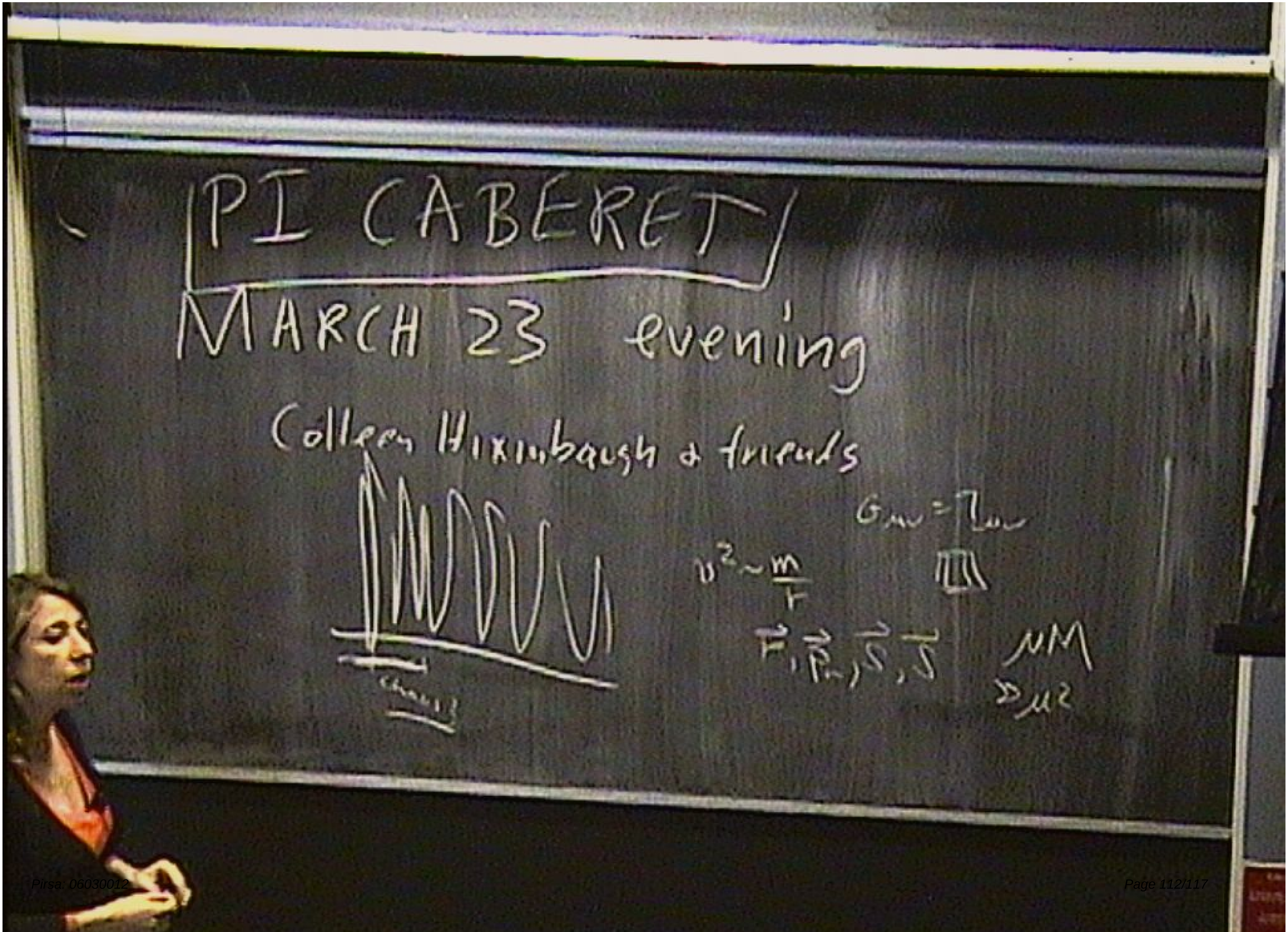
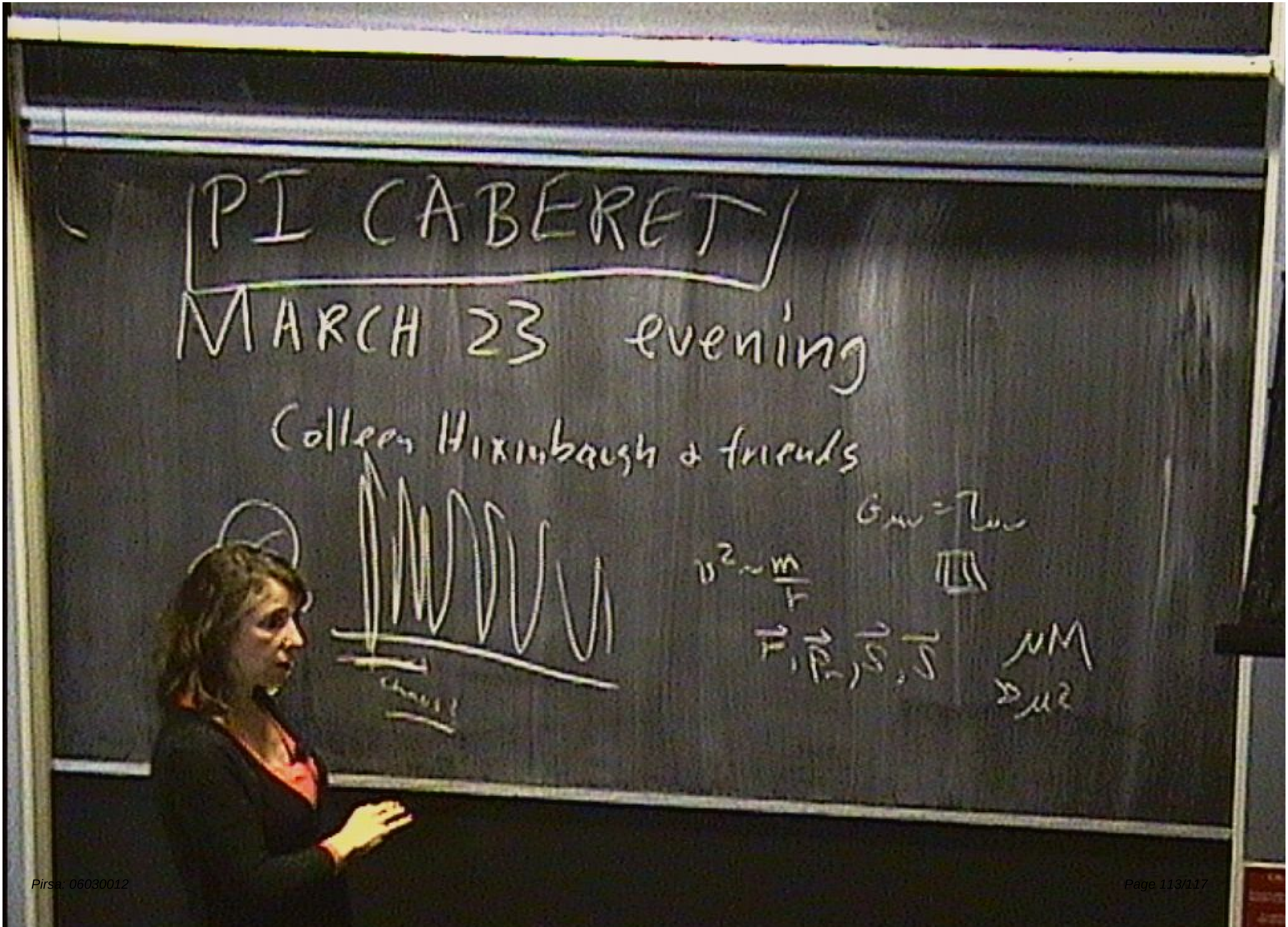
Theoretical Physics is completely reductionist

Could Chaos play a more fundamental role

Could we have fundamental physics without symmetries?


PI CABERET
MARCH 23 evening
Colleen Hixinbaugh & friends


PI CABERET
MARCH 23 evening
Colleen Hixinbaugh & friends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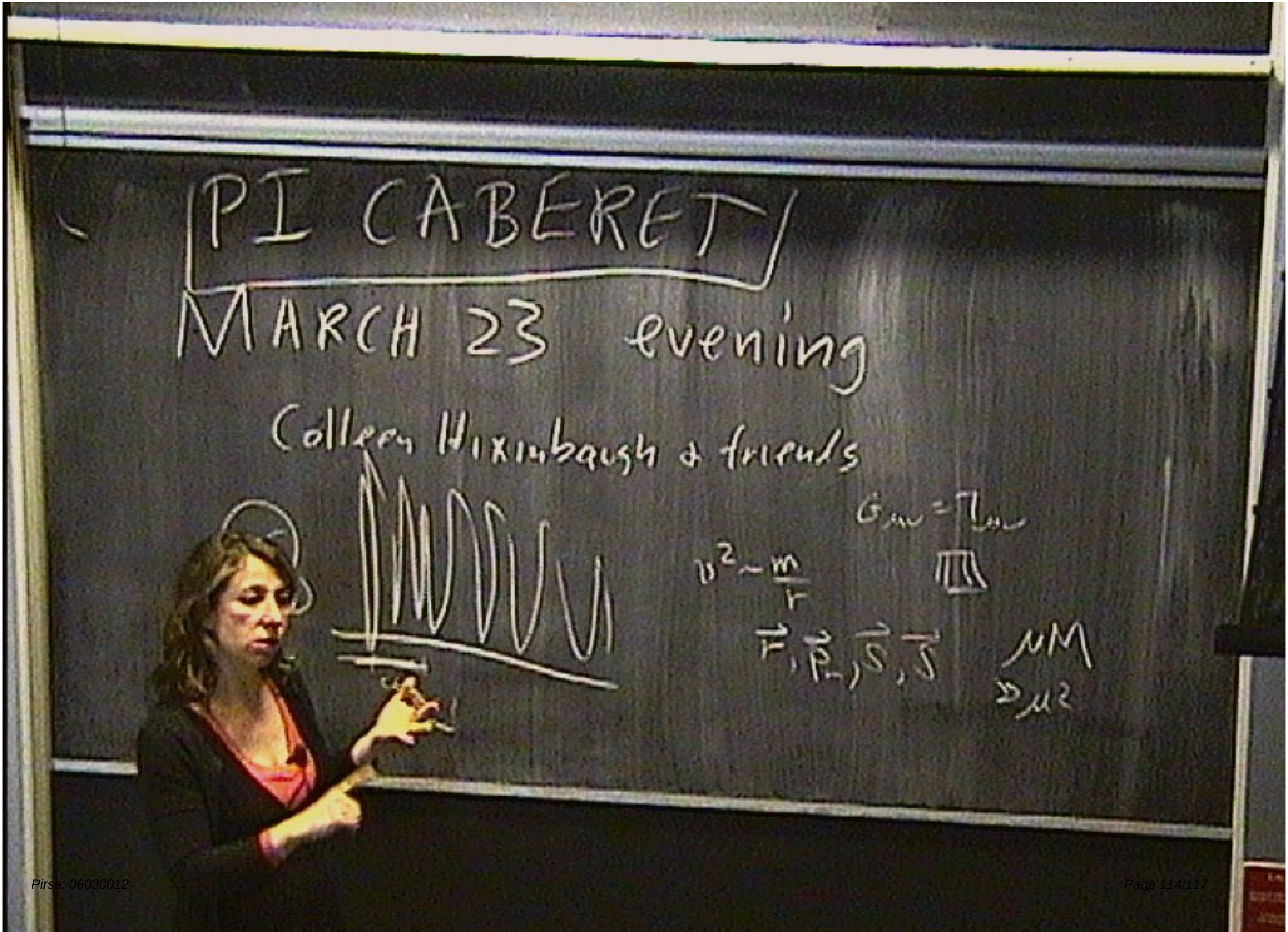
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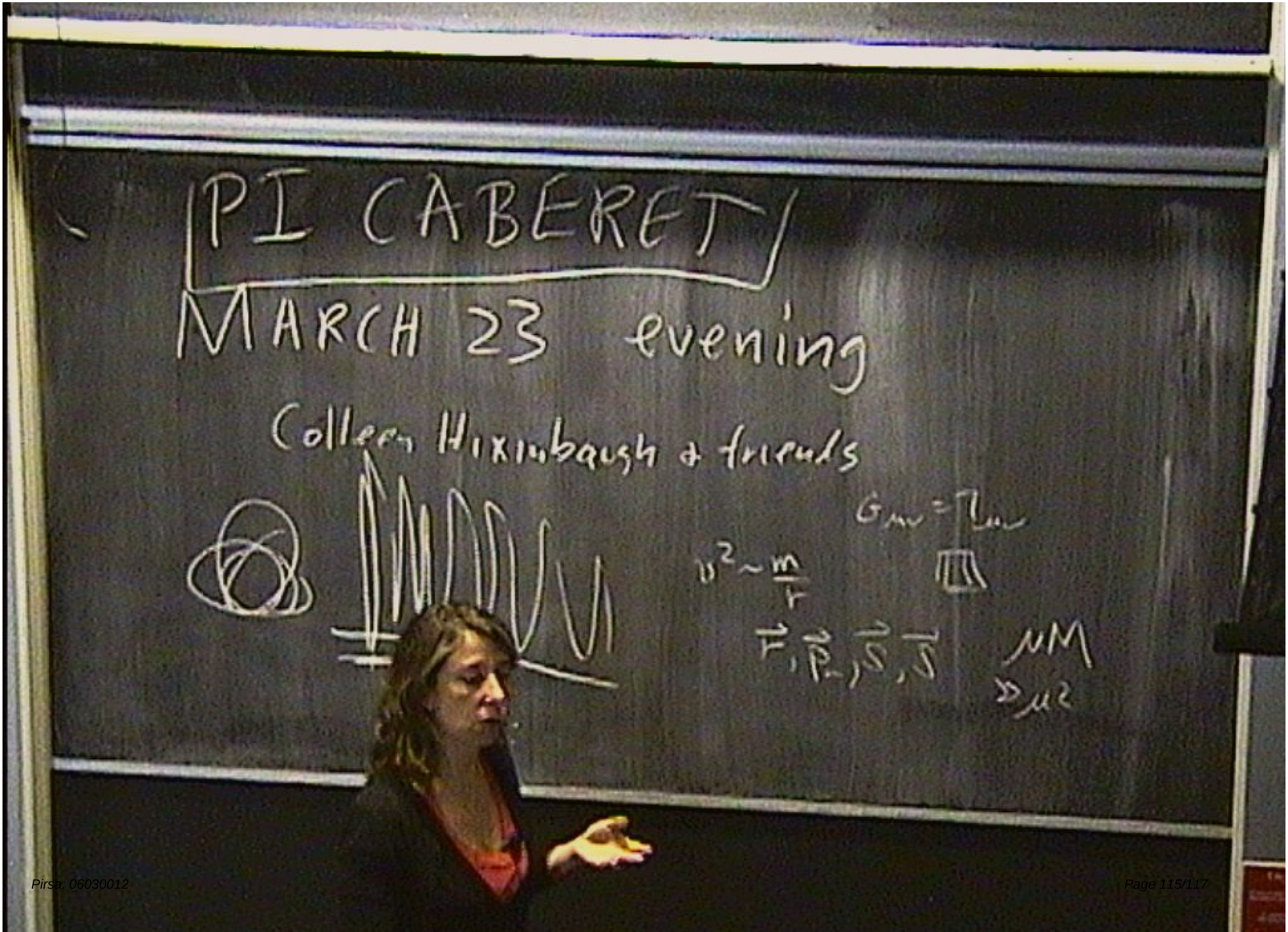
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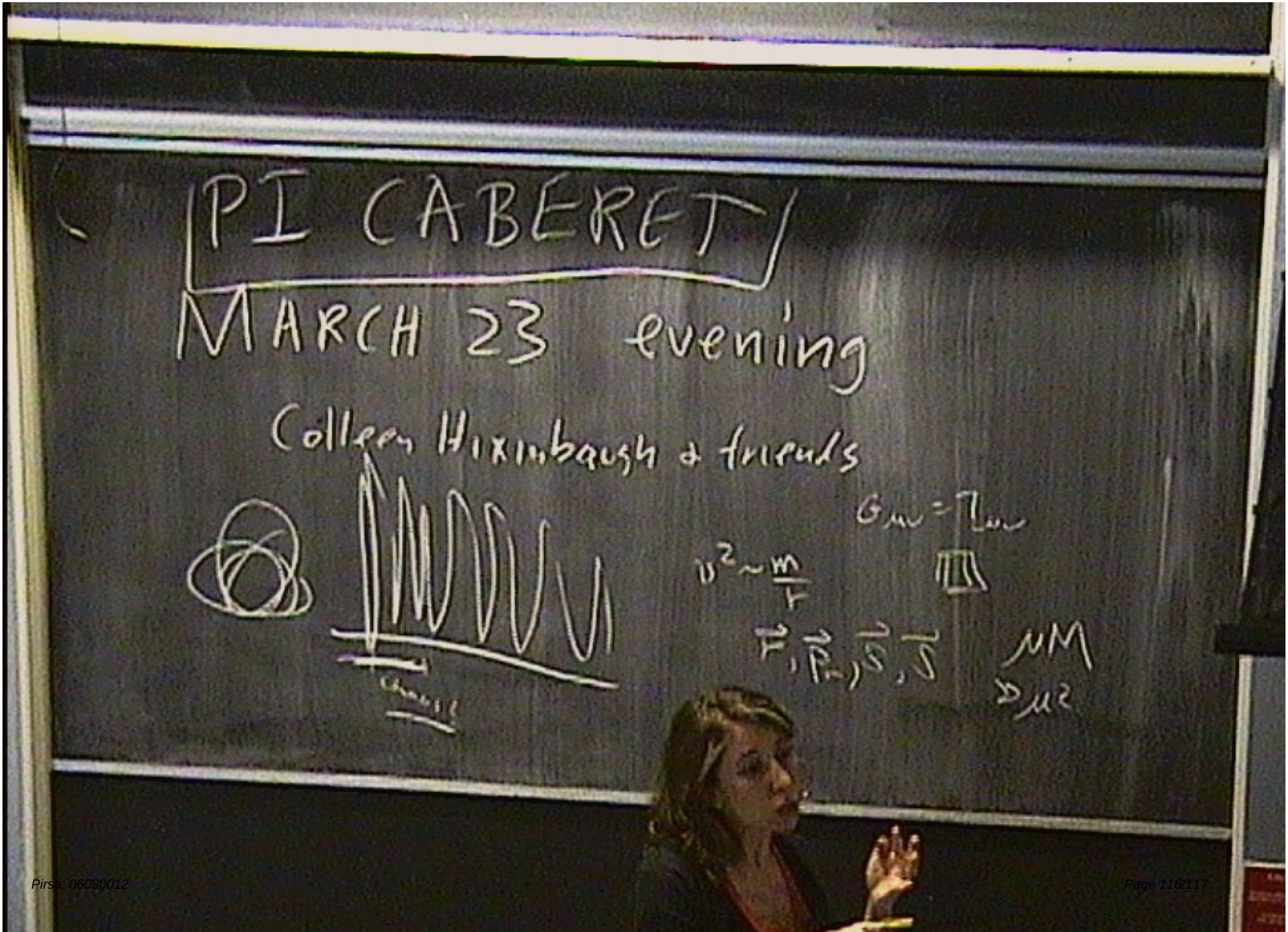
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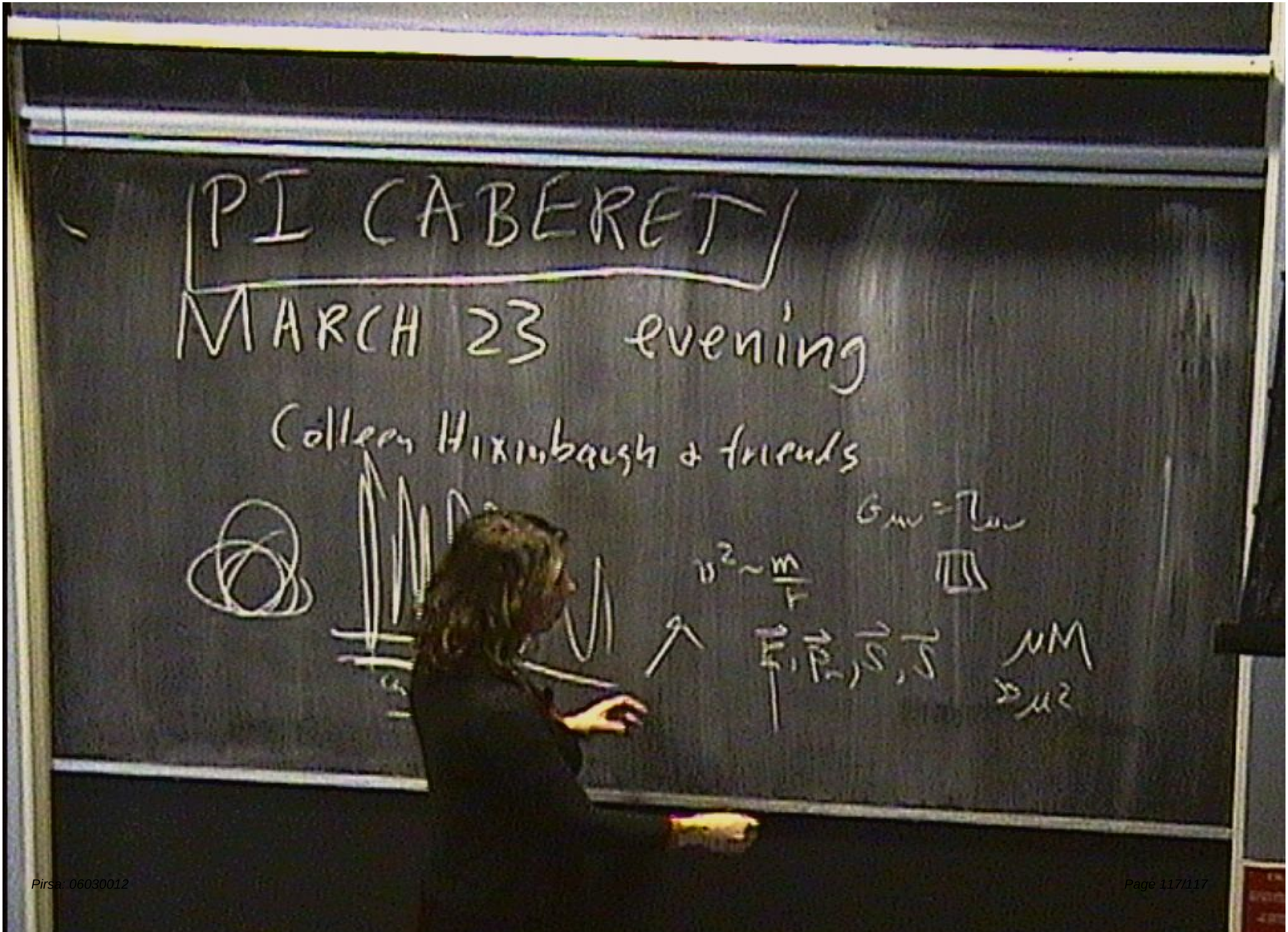
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