Title: String Loops vs. AdS/CFT

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Abstract:

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References:

G.DeRisi, G.Grignani, M.Orselli, G.W.Semenoff: **JHEP 0411:053,2004** e-Print Archive: **hep-th/0409315**

G.Grignani, M.Orselli, B.Ramadanovic, G.W.Semenoff and D.Young: **JHEP 0512:017,2005** e-Print Archive: **hep-th/0508126**

G.Grignani, M.Orselli, B.Ramadanovic, G.W.Semenoff and D.Young, **hep-th/0603???**

Testing AdS/CFT duality for interacting strings and non-planar Yang-Mills theory.

AdS/CFT

Maximally super-symmetric Yang-Mills theory on 4D flat space with gauge group SU(N) and coupling constant g_{YM}

is exactly equivalent to

Type IIB superstring theory on background $\mathrm{AdS}_5 \otimes \mathrm{S}^5$

with N units of 4-form flux through S^5

Radius of curvature $R = \left(4\pi g_s N\right)^{1/4}\sqrt{\alpha'}$

and $4\pi g_s = g_{YM}^2$ $\qquad \lambda = g_{YM}^2 N$

String state $\leftrightarrow$ gauge invariant composite operator

Energy $\leftrightarrow$ conformal dimension of composite operator

This is a weak coupling – strong coupling duality.

Gauge fields are weakly coupled when λ is small.

Strings are classical when $g_s \to 0$ holding geometry, R, fixed

This coincides with the $g_{YM} \to 0, N \to \infty$,λ = constant 'tHooft limit

Coupling constant of string sigma model ~ $1/\sqrt{\lambda}$

Strings are solvable when λ is large – supergravity.

•Penrose limit of $AdS_5 \otimes S^5$ geometry gives a pp-wave space.

$$ds^2 = -4dx^+dx^- - \mu^2 \sum_{i=1}^{8} (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2$$

•Free strings are exactly solvable in the light-cone gauge (R.Metsaev,hep-th/ 0112044)

•The analogous limit of super Yang-Mills can be taken (D. Berenstein, J. Maldacena and I. Nastase, hep-th/0202021) → detailed matching of Yang-Mills theory states (BMN operators) in the large N, planar limit and free string states

•Non-planar corrections in Yang-Mills = string interactions (C.Kristjansen,J.Plefka,G.Semenoff,M.Staudacher,hep-th/0205033; N.Constable,D.Freedman,M.Headrick,S.Minwalla,L.Motl,A.Postnikov, W.Skiba,hep-th/0205089) → (ongoing) matching interactions in light-cone string field theory on a pp-wave background and Yang-Mills theory computations.

Free string on pp-wave:

$$ds^2 = -4\,dx^+dx^- - \mu^2\left(\vec{x}^2\right)dx^+dx^+ + d\vec{x}\cdot d\vec{x}$$

$$F_{+2345} = F_{+6789} = \mu$$

light cone gauge $X^+ = x^+ + \alpha' p^+ \tau$

$$2p^- = \mu \sum_{n=-\infty}^{\infty} \left(a_n^i * a_n^i + b_n^a * b_n^a\right)\sqrt{1 + \left(\mu\alpha' p^+\right)^{-2} n^2}$$

level matching $\sum_{n=-\infty}^{\infty} n\left(a_n^i * a_n^i + b_n^a * b_n^a\right) = 0$

$$a_n^i \left| p^+ \right\rangle = 0 \quad b_n^i \left| p^+ \right\rangle = 0 \quad \forall n$$

$\left| p^+ \right\rangle$ $\quad p^- = 0$

$a_0^i * \left| p^+ \right\rangle$ $\quad b_0^a * \left| p^+ \right\rangle$ $\quad 2p^- = \mu$

$a_n^{i_1} * a_{-n}^{i_2} * \left| p^+ \right\rangle \quad a_n^i * b_{-n}^a * \left| p^+ \right\rangle \quad 2p^- = 2\mu\sqrt{1 + \left(\mu\alpha' p^+\right)^{-2} n^2}$

$\mathcal{N}$=4 Supersymmetric Yang-Mills Theory

$$S = \int d^4x \mathrm{TR}\left(\frac{1}{2} F_{\mu\nu}{}^2 + (D_\mu \phi)^2 + \overline{\psi}\left(\gamma \cdot D + \Gamma^i \phi^i\right)\psi - g_{YM}^2 \sum_{i<j=1}^{6} [\phi, \phi]^2 \right)$$

-gauge group is SU(N)

-16 supersymmetries+16 conformal SUSY

-conformal field theory – beta function vanishes

→ S(5,1) conformal symmetry (~AdS_5)

- SO(6)~SU(4) R-symmetry (~S^5)

-planar limit $N \to \infty$ $\lambda = g_{YM}^2 N$ fixed is (probably) integrable

-protected operators – chiral primaries

$\mathrm{TR}\left(\phi^{i_1}(x)\phi^{i_2}(x)\ldots\phi^{i_k}(x)\right)$ symmetrized, traceless, ½ BPS

$\Delta = k$

State with large momentum on equator of 5-sphere = $\mathrm{TR}\left(\left(\phi^5(x) + i\phi^6(x)\right)^J\right)$

$\Delta - J = 0$

Free string on pp-wave:

$$ds^2 = -4\,dx^+dx^- - \mu^2\left(\vec{x}^{\,2}\right)dx^+dx^+ + d\vec{x}\cdot d\vec{x}$$

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$$\left| p^+ \right\rangle \qquad\qquad p^- = 0$$

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$$a_n^{i_1} * a_{-n}^{i_2} * \left| p^+ \right\rangle \quad a_n^i * b_{-n}^a * \left| p^+ \right\rangle \quad 2p^- = 2\mu\sqrt{1+\left(\mu\alpha' p^+\right)^{-2} n^2}$$

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$\Delta - J = 0$

"Holographic dictionary" $\quad 2p^- = \mu(\Delta - J) \quad 2p^+ = \frac{\Delta + J}{\mu R^2} = \frac{\Delta + J}{\mu\alpha'\sqrt{g_{YM}^2 N}}$

$$z = \phi^5 + i\phi^6 \qquad (\phi^1,\phi^2,\phi^3,\phi^4),(D_\mu z) \qquad SO(4)\times SO(4)$$

$$\frac{1}{\sqrt{JN^J}}\mathrm{TR}\left(z^J(0)\right) \quad \leftrightarrow \quad \left|p^+\right\rangle \qquad \mu\alpha' p^+ = \sqrt{1/\lambda'} \qquad 2p^- = 0$$

$$\frac{1}{\sqrt{N^{J+1}}}\mathrm{TR}\left(\phi^i(0) z^J(0)\right) \quad \leftrightarrow \quad a_0^i\left|p^+\right\rangle \qquad \Delta - J = 1 \qquad 2p^- = \mu$$

$$O_n = \sum_K \cos\left(2\pi n\frac{K}{J+1}\right)\mathrm{TR}\left(z^K(0)\phi^i(0)z^{J-K}(0)\phi^j(0)\right) \quad \leftrightarrow \quad a_n^i a_{-n}^j\left|p^+\right\rangle$$

$$2p^- = 2\mu\sqrt{1 + n^2/\left(\mu\alpha' p^+\right)^2} = 2\mu\sqrt{1+\lambda' n^2}$$

$$\lambda' = \frac{g_{YM}^2 N}{J^2} = \frac{1}{\left(\mu\alpha' p^+\right)^2} \qquad g_2 = \frac{J^2}{N} = 4\pi g_s\left(\mu\alpha' p^+\right)^2$$

This string spectrum was computed using Yang-Mills theory by BMN hep-th/0202021 to one loop, D.Gross, A.Mihailov and R.Roiban hep-th/0205066 to 2 loops. → ***beautiful agreement***!

A rough guide:

- When the string coupling is turned off, the IIB superstring can be solved on a pp-wave background. The solution agrees beautifully with the BMN limit of N=4 Super Yang-Mills theory in the large N 't Hooft (planar) limit.
- However, string interactions have never been successfully matched to non-planar, next-to-leading order in 1/N Yang-Mills theory.
- IIB superstring on pp-wave background is only solvable in the light-cone gauge.
- Conformal field theory is unavailable as a tool to analyze string interactions (higher genus Riemann surfaces)
- Only tool is light-cone string field theory. String vertex is built by requiring locality and consistency with the spacetime supersymmetry algebra → ambiguity in pre-factor
- There are higher order contact terms in the string hamiltonian
- There are competing vertices:

  Spradlin, Volovich

  Di Vecchia et.al.

  Yoneya

  Maybe resolved by matching with supergravity (Yoneya).
- Existing computations use an (unjustified) truncation to two-impurity intermediate states, and the Spradlin-Volovich version of the vertex – come tantalizingly close to matching the gauge theory result.
- It was noted by Roiban, Spradlin and Volovich that a 4-impurity intermediate state seems to contribute a non-perturbative $\sqrt{\lambda'}$ order and divergences appear?

Non-planar corrections in Yang-Mills Theory

String loop corrections to the energy of the 2-impurity (2-oscillator) (9,1)-state were computed in Yang-Mills theory:

- Ch.Kristjansen, J.Plefka, G.W.Semenoff, M.Staudacher hep-th/0205033
- N. Constable, D. Freedman, M. Headrick, S. Minwalla, L. Motl, A. Postnikov, W.Skiba, hep-th/0205089
- N.Beisert, Ch.Kristjansen, J.Plefka, G.W.Semenoff, M.Staudacher, hep-th/0208178
- N. Constable, D. Freedman, M. Headrick, S. Minwalla, hep-th/0209002
- N.Beisert, C.Kristjansen, J.Plefka, M.Staudacher, hep-th/0212269

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\lambda' - \frac{1}{4}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)^2\lambda'^2 + \ldots\right) + \ldots$$

$$\frac{2p^-}{\mu} \approx 2\sqrt{1+(\mu\alpha' p^+)^{-2} n^2} + \boxed{\frac{g_2^2}{4\pi^2}\left(\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\frac{1}{(\mu\alpha' p^+)^2} - \frac{1}{4}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)^2\frac{1}{(\mu\alpha' p^+)^4} + \ldots\right)} + \ldots$$

for $n \neq 0$

↑

Has not been computed using string theory!

Because of the presence of the RR-field, the only available formulation of string theory in which string interactions can be computed in **light-cone string field theory**.

M.Spradlin and A.Volovich, hep-th/0204146, hep-th/0206073, hep-th/0310033 (review)
C.S.Chu, V.Khoze, M.Petrini, R.Russo and A.Tanzini, hep-th/0208146
Y.He, J.Schwarz, M.Spradlin, A.Volovich, hep-th/0211198
S.Dobashi, H.Shimada, T.Yoneya, hep-th/0209251
P.DiVecchia, J.L.Petersen, M.Petrini, R.Russo and A.Tanzini, hep-th/0304025
J.Gomis, S.Moriyana, J.W.Park, hep-th/0301250
A.Pankiewicz, hep-th/0304232
P.Gutjahr, A.Pankiewicz, hep-th/0407098
S.Dobashi and T.Yoneya, hep-th/0409058, hep-th/0406225
G.Grignani, M.Orselli, B.Ramadanovic, G.W.Semenoff, D.Young, hep-th/0508126

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\lambda' - \frac{1}{4}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)^2 \lambda'^2 + \ldots\right) + \ldots$$

Most advanced string computation in previous literature:
P.Gutjahr and A.Pankiewicz hep-th/0407098

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots$$

$$+ \frac{g_2^2}{4\pi^2}\left(\left(\frac{1}{12} + \frac{65}{64\pi^2 n^2}\right)\lambda' - \left(\frac{3\lambda'^{3/2}}{16\pi^2}\right) + \frac{n^2}{4}\left(\frac{1}{24} + \frac{89}{64\pi^2 n^2}\right)\lambda'^2 + \left(\frac{9n^2\lambda'^{5/2}}{32\pi^2}\right) + \ldots\right) + \ldots$$

- energy of (9,1) state $\alpha_n^i * \alpha_{-n}^j * \left| p^+ \right\rangle$ symmetric, traceless

- uses truncation to 2-impurities
- uses Spradlin-Volovich vertex
- fixes the pre-factor f=1

This apparently corrects previous work
R.Roiban, M.Spradlin and A.Volovich hep-th/0211220

$$H = H_2 + g_2 H_3 + g_2^2 H_4 + \ldots$$

$$\delta E = -\frac{1}{2}\langle e | g_2 H_3 \frac{1}{H_2 - E_0} g_2 H_3 | e\rangle + \langle e | g_2^2 H_4 | e\rangle$$

????
truncated to two impurity intermediate states
always assumed that term with Q_4 doesn't contribute

$$Q = Q_2 + g_2 Q_3 + g_2^2 Q_4 + \ldots$$

$$\delta E = -\frac{1}{2}\langle e | g_2 H_3 \frac{1}{H_2 - E_0} g_2 H_3 | e\rangle + \langle e | g_2^2 Q_3 Q_3 | e\rangle + \langle e | g_2^2 \{Q_2, Q_4\} | e\rangle$$

$$= \frac{g_2^2}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\lambda'$$

Results:

- Study light-cone string field theory on plane-wave background

- Natural expansion parameter is $\sqrt{\lambda'}$

- If supersymmetry is carefully taken into account, divergences which are encountered in computations of corrections to energies of 2-oscillator states cancel.
- At the same time the terms of order $\sqrt{\lambda'}$ in the perturbation theory also seem to cancel.
- The leading string loop correction is then indeed finite and of order λ' which is expected from perturbative Yang-Mills theory.
- Cancellation of singular terms requires the correct relative factor of 2.

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R.Roiban, M.Spradlin and A.Volovich hep-th/0211220

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$$= \frac{g_2^2}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\lambda'$$

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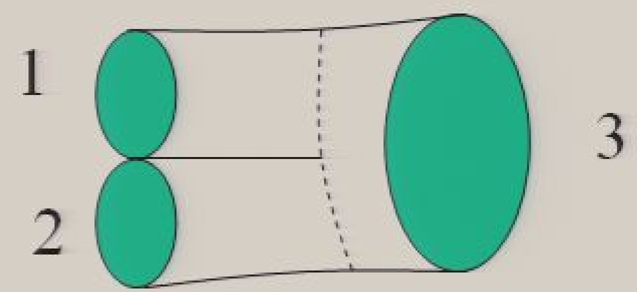
- Cancellation of singular terms requires the correct relative factor of 2.

Light-cone string field theory on pp-wave background:

Interactions are determined by the locality of vertex $(H \equiv P^-)$

$$H_3 \sim \delta\left(X_1(\sigma)+X_2(\sigma)-X_3(\sigma)\right)$$



and by the supersymmetry algebra:

$$Q = Q_2 + g_2 Q_3 + g_2^2 Q_4 + \ldots$$

$$H = \{Q, Q\} = H_2 + g_2 H_3 + g_2^2 H_4 + \ldots$$

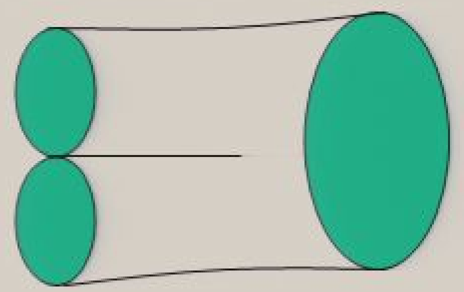
$$H_2 = \{Q_2, Q_2\} \qquad H_3 = 2\{Q_3, Q_2\} \qquad [Q_2, H_3] + [Q_3, H_2] = 0$$

$$H_4 = \{Q_3, Q_3\} + 2\{Q_2, Q_4\} \qquad \ldots$$

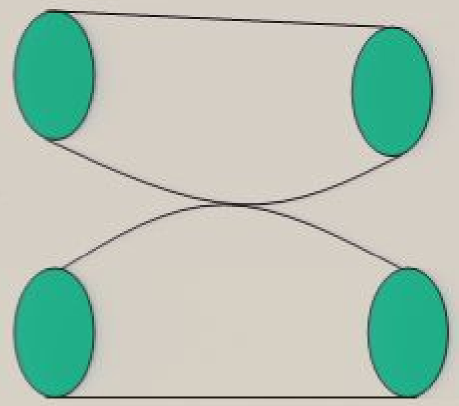
H_2 is known from quantization of the free theory. H_3, H_4 etc. must be consistent with the spacetime supersymmetry algebra.

This does not fix the Hamiltonian uniquely, only up to an overall $f(p^+)$

$$H_3 = g_s \cdot$$



$$H_4 = g_s^2 \cdot$$



In IIA/B superstrings on Minkowski space-time, it is known that the contact terms are necessary to cancel certain singularities in the integrations over parameters of light-cone string field theory diagrams. In the conformal field theory, they are also seen to arise as additional contributions needed to cancel certain singular surface terms which arise in the integration of correlators of vertex operators over the modular parameters of Riemann surfaces.

The oscillator representation of the delta function vertex is a squeezed 3-string state:

$$H_3 = \Pi e^{\frac{1}{2}\sum_{mnrs} a_m^{i(r)*} N_{mn}^{rs} a_n^{i(s)*} + \frac{1}{2}\sum_{mnrs} b_m^{a(r)*} Q_{mn}^{rs} b_n^{a(s)*}} \left|(1-r)p^+\right\rangle^{(1)} \otimes \left|rp^+\right\rangle^{(2)} \otimes \left|-p^+\right\rangle^{(3)}$$

$$Q_3 = \Phi e^{\frac{1}{2}\sum_{mnrs} a_m^{i(r)*} N_{mn}^{rs} a_n^{i(s)*} + \frac{1}{2}\sum_{mnrs} b_m^{a(r)*} Q_{mn}^{rs} b_n^{a(s)*}} \left|(1-r)p^+\right\rangle^{(1)} \otimes \left|rp^+\right\rangle^{(2)} \otimes \left|-p^+\right\rangle^{(3)}$$

$$N_{np}^{rs} = \frac{\sqrt{\beta_r\beta_s}}{4\pi} \frac{\sqrt{\omega_n^{(r)} - \beta_r\mu\alpha' p^+}\sqrt{\omega_p^{(s)} - \beta_s\mu\alpha' p^+} - e(n)e(p)\sqrt{\omega_n^{(r)} + \beta_r\mu\alpha' p^+}\sqrt{\omega_p^{(s)} + \beta_s\mu\alpha' p^+}}{\sqrt{\omega_n^{(r)}\omega_p^{(s)}}\left(\beta_s\omega_n^{(r)} + \beta_r\omega_p^{(s)}\right)}$$

$$Q_{np}^{rs} = \frac{i}{4\pi} \frac{\beta_s n - \beta_r p}{\sqrt{\omega_n^{(r)}\omega_p^{(s)}}\left(\beta_s\omega_n^{(r)} + \beta_r\omega_p^{(s)}\right)}$$

$$\omega_n^{(r)} = \sqrt{n^2 + \left(\beta_r\mu\alpha' p^+\right)^2} \qquad r,s = 1,2,3 \qquad \beta = 1-r, r, 1$$

Where the pre-factors Π, Φ and the Neumann matrices N,Q are known in the large μ limit.

Light-cone string field perturbation theory: we begin with A single string in a 2-oscillator state, $\left|\varphi_n\right\rangle$, and use quantum mechanical perturbation theory to compute the corrections to its light-cone energy

$$H_2\left|\varphi_n\right\rangle = E_n^{(0)}\left|\varphi_n\right\rangle$$

$$\left(\delta E_n^{(1)}\right) = g_2\left\langle\varphi_n\right|H_3\left|\varphi_n\right\rangle = 0$$

$$\left(\delta E_n^{(2)}\right) = g_2^2\left(\left\langle\varphi_n\right|H_3\frac{P}{E_n^{(0)}-H_2}H_3\left|\varphi_n\right\rangle + \left\langle\varphi_n\right|H_4\left|\varphi_n\right\rangle\right)$$

We can use the supersymmetry algebra to re-write the second order energy shift in a simpler form:

$$\delta E_n^{(2)} = g_2^2\left(\langle \varphi_n | H_3 \frac{P}{E_n^{(0)} - H_2} H_3 | \varphi_n \rangle + \langle \varphi_n | \{Q_3, Q_3\} | \varphi_n \rangle + \langle \varphi_n | 2\{Q_2 Q_4\} | \varphi_n \rangle \right)$$

$$H_3 = 2\{Q_2, Q_3\} \qquad H_4 = \{Q_3, Q_3\} + 2\{Q_2, Q_4\}$$

$$\delta E_n^{(2)} = 2g_2^2\left(\langle \varphi_n | Q_2 H_3 \frac{P}{E_n^{(0)} - H_2} Q_3 | \varphi_n \rangle + \langle \varphi_n | H_3 \frac{P}{E_n^{(0)} - H_2} Q_3 Q_2 | \varphi_n \rangle \right)$$
$$+ 2g_2^2\left(\langle \varphi_n | Q_2 Q_4 | \varphi_n \rangle + \langle \varphi_n | Q_4 Q_2 | \varphi_n \rangle \right)$$

Here, Q_2 operates on an external state once in each term.

$$\delta E_n^{(2)} = 2g_2^2\left(\langle \varphi_n | Q_2 H_3 \frac{P}{E_n^{(0)} - H_2} Q_3 | \varphi_n \rangle + \langle \varphi_n | H_3 \frac{P}{E_n^{(0)} - H_2} Q_3 Q_2 | \varphi_n \rangle \right)$$
$$+ 2g_2^2\left(\langle \varphi_n | Q_2 Q_4 | \varphi_n \rangle + \langle \varphi_n | Q_4 Q_2 | \varphi_n \rangle\right)$$

Here, Q_2 operates on an external state once in each term.

- this produces a single state, the supersymmetry transform of the external state, rather than a superposition of states that has to be summed. It extracts a quantity of order square root of energy (and $\sim 1/\sqrt{\lambda'}$) which reduces the dimension of the summations over intermediate states so that they converge.
- we see that if $|\varphi_n\rangle$ is annihilated by the dynamical supercharges Q_2, the energy shift vanishes.
- Q_4 is unknown at this point.
- counting of orders shows that the leading order is now $\lambda' \sim 1/\left(\mu\alpha' p^+\right)^2$

More results

We consider the Yoneya vertex = half of the Spradlin-Volovich plus Di Vecchia vertex.

$$|H_3\rangle = \sum_{r=1}^{3} H_2^{(r)} |V\rangle \qquad |Q_3\rangle = \sum_{r=1}^{3} Q_2^{(r)} |V\rangle$$

Truncated to 2-impurity intermediate states, the energy is

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$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 n^2}\right)\lambda'$$

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$$\left|H_3\right\rangle = \sum_{r=1}^{3} H_2^{(r)}\left|V\right\rangle \qquad \left|Q_3\right\rangle = \sum_{r=1}^{3} Q_2^{(r)}\left|V\right\rangle$$

Truncated to 2-impurity intermediate states, the energy is

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\lambda' - \frac{g_2^2}{4\pi^2}\frac{1}{8}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)^2\lambda'^2 + \ldots$$

To be compared with the prediction of Yang-Mills theory

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\lambda' - \frac{1}{4}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)^2\lambda'^2 + \ldots\right) + \ldots$$

More results

We consider the Yoneya vertex = half of the Spradlin-Volovich plus Di Vecchia vertex.

$$|H_3\rangle = \sum_{r=1}^{3} H_2^{(r)}|V\rangle \qquad |Q_3\rangle = \sum_{r=1}^{3} Q_2^{(r)}|V\rangle$$

Truncated to 2-impurity intermediate states, the energy is

$$\Delta - J = 2 + n^2\lambda' - \frac{1}{4}n^4\lambda'^2 + \ldots + \frac{g_2^2}{4\pi^2}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)\lambda' - \frac{g_2^2}{4\pi^2}\frac{1}{8}\left(\frac{1}{12}+\frac{35}{32\pi^2 n^2}\right)^2\lambda'^2 + \ldots$$

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Again, adding a relative factor of 2 between the two contributions of perturbation theory would repair the result to 2-loops

More results:

•Orbifold of S^5 has less supersymmetry and is dual to an *N*=2 superconformal Yang-Mills theory.

•Penrose limit of the orbifold is pp-wave with compact null coordinate. (S.Mukhi,M.Rangamani and E.Verlinde, hep-th/0204147)

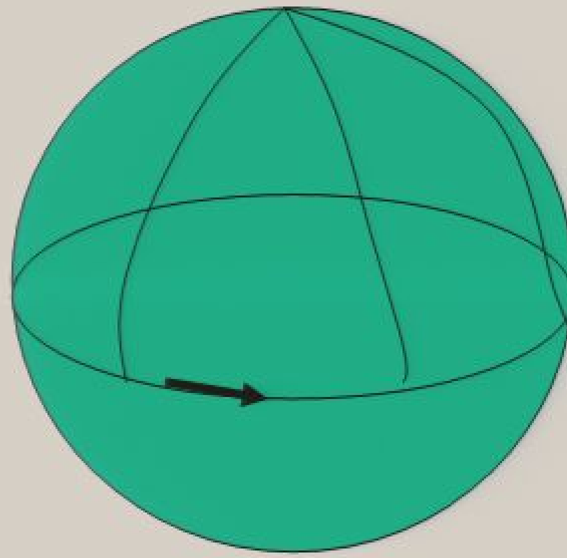
•Detailed matching of spectrum of gauge theory in large N planar limit and free DLCQ strings. There are wrapped states and discrete light-cone momentum.

String theory – get a DLCQ string on pp-wave by simultaneous Penrose plus of large order of orbifold group limit

Type IIB superstring on orbifold $AdS_5 \otimes S^5 / Z_M$ with MN units of RR 5-form flux

$$R = \left(4\pi g_s MN\right)^{1/4} \sqrt{\alpha'}$$

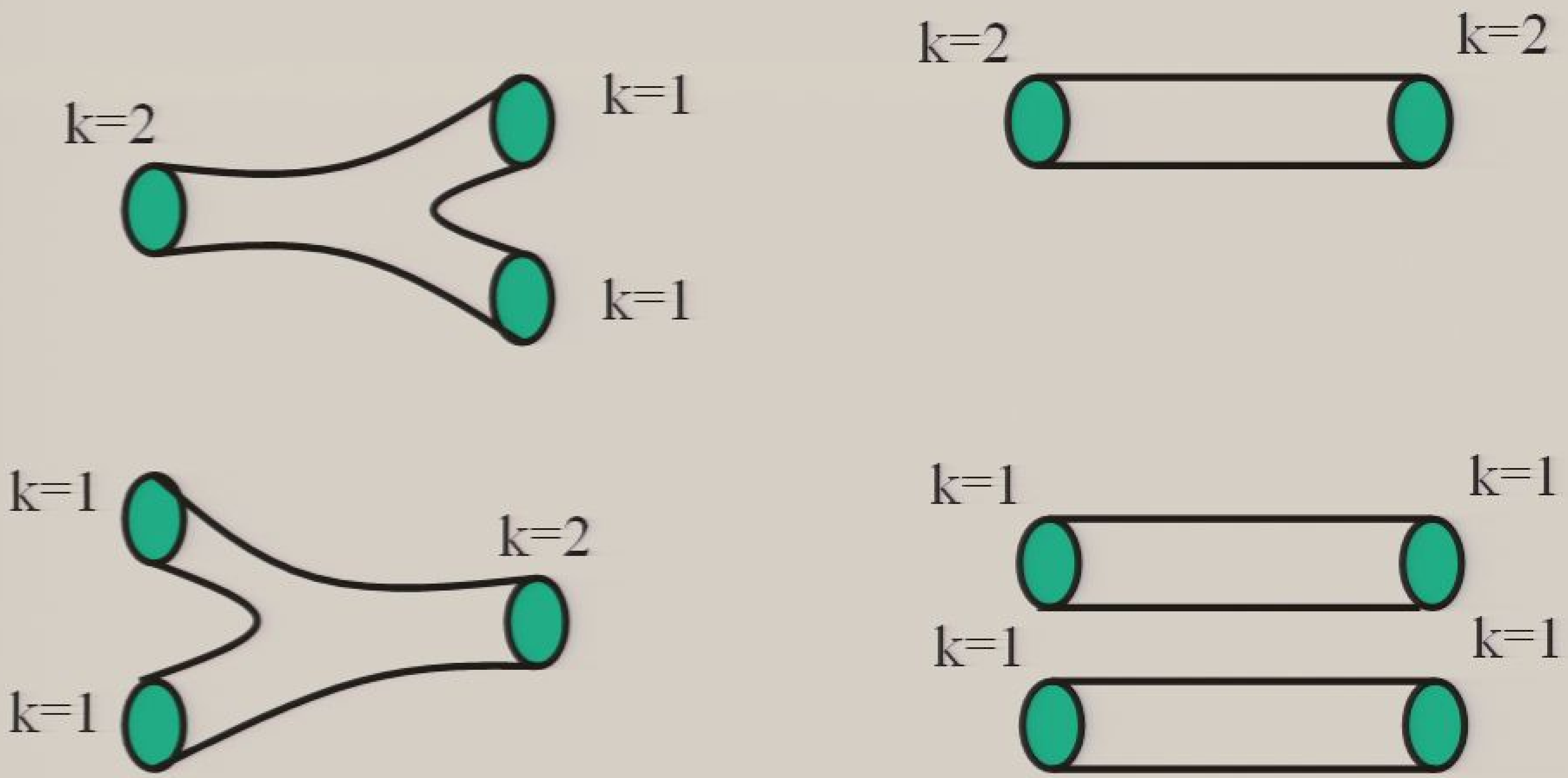
$$4\pi g_s = g_{YM}^2$$



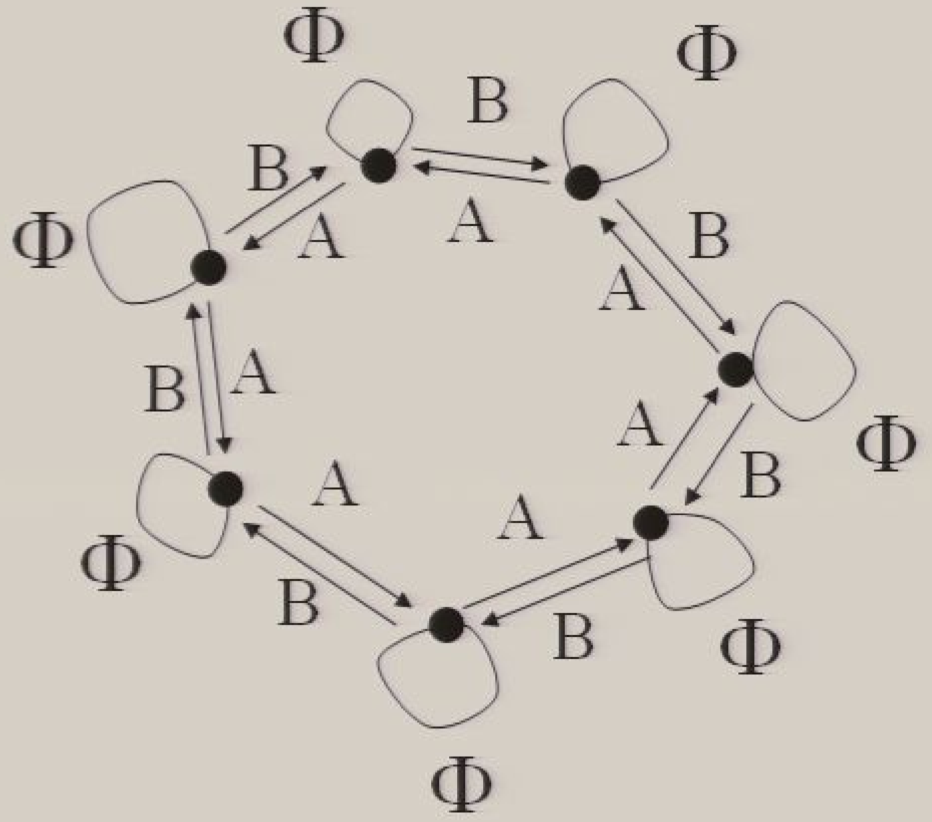
Take the Penrose limit holding $R^2 / \alpha' M = \sqrt{g_{YM}^2 NM} / \alpha' M$ fixed in gauge theory →M and N are taken to infinity with M/N fixed.

Since discrete light-cone momentum is positive, k=1,2,…, a string with k units can split into at most k strings.

e.g. when k=2 :



Orbifold of N=4 SYM results in an N=2 supersymmetric quiver gauge theory:



$$\mathrm{TR}\left(A_1 \ldots A_M A_1 \ldots A_M A_1 \ldots A_M \ldots\right)$$

Take the Penrose limit holding $R^2 / \alpha' M$ fixed – gauge theory limit
Where M and N are taken to infinity with M/N fixed.

$$ds^2 = -4dx^+dx^- - \mu^2 \sum_{i=1}^{8} (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2 \qquad x^\pm = \frac{1}{2}(x^0 \pm x^9)$$

$$F_{+1234} = F_{+5678} = \mu$$

$$x^- \sim x^- + 2\pi R^- \qquad R^- = \sqrt{\pi g_s N / M}\,\alpha' \mu$$

Compact null coordinate. DLCQ of IIB string theory.

$$X^+ = x^+ + \alpha' p^+ \tau \qquad p^+ = \frac{k}{2R^-} \qquad k = 1,2,\ldots$$

$$X^- = x^- + \alpha' p^- \tau + mR^- \sigma + \text{oscillator s}$$

$$X^i = \sqrt{\frac{\alpha'}{2}} \sum_{n=-\infty}^{\infty} \frac{a_n^i}{\sqrt{\omega_n}} e^{i(n\sigma + \omega_n \tau)} + \text{c.c.} \qquad \omega_n = \sqrt{(\mu p^+ \alpha')^2 + n^2}$$

String theory states

$$H = 2\frac{p^-}{\mu} = \sum_{i=1}^{8}\sqrt{1+\left(\frac{R^-}{\mu\alpha' k}\right)^2 n_i^{\,2}}\left(a_n^{i+}a_n^i + \text{ferm.}\right)$$

Vacuum with k units of light-cone momentum

$$|0,k,m=0\rangle$$

$$a_n^i|0,k,m\rangle \qquad km = n \qquad n = 0,\pm 1,\pm 2,\ldots$$

$$a_{n_1}^{i_1+}a_{n_2}^{i_2+}\ldots a_{n_p}^{i_p+}|0,k,m\rangle \qquad km = (n_1 + n_2 + \ldots + n_p)$$

Holographic duality

$J:\left(A,\overline{B}\right)\rightarrow\left(e^{i\zeta}A,e^{i\zeta}\overline{B}\right)$ $\quad J':\left(A,\overline{B}\right)\rightarrow\left(e^{i\vartheta}A,e^{-i\vartheta}\overline{B}\right)$

$H=2p^{-}=\left(\Delta-MJ-J'\right)$ $\quad 2p^{+}=\frac{\left(\Delta+MJ+J'\right)}{R^2}=\frac{\left(\Delta+MJ+J'\right)}{2MR^{-}}$

$R^2=\sqrt{4\pi g_s MN}\alpha'$

	Δ	MJ	J'	H		Δ	MJ	J'	H
A_I	1	$1/2$	$1/2$	0	$\overline{A}_I$	1	$-1/2$	$-1/2$	2
B_I	1	$-1/2$	$1/2$	1	$\overline{B}_I$	1	$1/2$	$-1/2$	1
Φ_I	1	0	0	1	$\overline{\Phi}_I$	1	0	0	1
χ_{AI}	$3/2$	$1/2$	0	1	$\overline{\chi}_{AI}$	$3/2$	$-1/2$	0	2
χ_{BI}	$3/2$	$-1/2$	0	2	$\overline{\chi}_{BI}$	$3/2$	$1/2$	0	1
$\psi_{\Phi I}$	$3/2$	0	$-1/2$	2	$\overline{\psi}_{\Phi I}$	$3/2$	0	$1/2$	1
ψ_I	$3/2$	0	$-1/2$	2	$\overline{\psi}_I$	$3/2$	0	$1/2$	1

$|0,k,m=0>\Leftrightarrow TR\left(\left(A_1A_2\ldots A_M\right)^k\right)$ $\quad M,N\rightarrow\infty$

Holographic dictionary

$$|0,k,m=0> \quad \Leftrightarrow \quad TR\left((A_1A_2\ldots A_M)^k\right) \quad M,N\to\infty$$

$$\left(a_n^{1+}+ia_n^{2+}\right)|0,k,m> \quad \Leftrightarrow \quad \frac{1}{M}\sum_{I=1}^{M}e^{2\pi i m/M}TR\left(A_1\ldots\Phi_I\ldots A_M(A_1\ldots A_M)^{k-1}\right)$$

$$\left(a_n^{5+}+ia_n^{6+}\right)|0,k,m> \quad \Leftrightarrow \quad \frac{1}{M}\sum_{I=1}^{M}e^{2\pi i m/M}TR\left(A_1\ldots B_I\ldots A_M(A_1\ldots A_M)^{k-1}\right)$$

$$\left(a_{n_1}^{1+}+ia_{n_1}^{2+}\right)\left(a_{n_2}^{1+}+ia_{n_2}^{2+}\right)|0,k,m> \quad \Leftrightarrow$$

$$\sum_{I,J=1}^{kM}e^{2\pi i(n_1I+n_2J)/kM}TR\left(A_1\ldots\Phi_I\ldots A_M(A_1\ldots A_M)^{k'}A_1\ldots\Phi_J\ldots A_M(A_1\ldots A_M)^{(k)-k'-2}\right)$$

level matching $\quad n_1+n_2=km$

spectrum $\quad \sum_{\mathrm{i}}\sqrt{1+\frac{4\pi g_sN}{M}\frac{n_i^2}{k^2}}=\sum_i\left(1+\frac{1}{2}\frac{4\pi g_sN}{M}\frac{n_i^2}{k^2}+\ldots\right)$

string loop $\quad \left(\frac{4\pi g_sN}{M}\right)\cdot\left(\frac{M}{N}\right)^2$

Example One impurity states:

$$H : TR\left(A_1 \ldots \overline{\Phi}_I \ldots A_M\right) = -\frac{g_s MN}{2\pi} TR\left(A_1 \ldots \overline{\Phi}_{I+1} \ldots A_M + A_1 \ldots \overline{\Phi}_{I-1} \ldots A_M - 2A_1 \ldots \overline{\Phi}_I \ldots A_M\right)$$

$$H : \sum_{i=1}^{M} e^{2\pi i n/M} TR\left(A_1 \ldots \overline{\Phi}_I \ldots A_M\right) = \frac{g_s MN}{2\pi} 4\sin^2\left(\frac{\pi n}{M}\right) \sum_{i=1}^{M} e^{2\pi i n/M} TR\left(A_1 \ldots \overline{\Phi}_I \ldots A_M\right)$$

eigenvalue $\xrightarrow[\mathrm{M,N}\to\infty]{} \frac{1}{2}\frac{4\pi g_s N}{M} n^2$ compare with $\sqrt{1+\frac{4\pi g_s N}{M}\frac{n^2}{k^2}}$

<u>Example</u>

$$\sum_{I=1}^{M} e^{2\pi i m/kM} TR\left(A_1 \ldots A_{I-1} \Phi_I A_I \ldots A_M \left(A_1 A_2 \ldots A_M\right)^{k-1}\right) +$$

$$+ \sum_{l=1}^{k-1} c_l \sum_{I=1}^{M} e^{2\pi i m/kM} TR\left(A_1 \ldots A_{I-1} \Phi_I A_I \ldots A_M \left(A_1 A_2 \ldots A_M\right)^{k-l-1}\right) TR\left(\left(A_1 A_2 \ldots A_M\right)^l\right)$$

eigenvalue of $2p^- = 1 + \frac{1}{2}\frac{4\pi g_s N}{M}\frac{n^2}{k^2} + O(g_s/M)$ $\quad n = mk$

no string loop corrections

Example: k=2 We can find exact eigenstates of dilatation operator

$$TR(A_1 \ldots \Phi_I \ldots \Phi_J \ldots A_M A_1 \ldots A_M) + TR(A_1 \ldots \Phi_I \ldots A_M A_1 \ldots \Phi_J \ldots A_M)$$
$$\pm TR(A_1 \ldots \Phi_I \ldots \Phi_J \ldots A_M) TR(A_1 \ldots A_M) \pm TR(A_1 \ldots \Phi_I \ldots A_M) TR(A_1 \ldots \Phi_J \ldots A_M)$$

Are eigenstates of H with eigenvalues

$$2p^- = 2 + \frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\left(\left(\frac{n_1}{2}\right)^2 + \left(\frac{n_2}{2}\right)^2\right)$$

$$TR(A_1 \ldots \Phi_I \ldots \Phi_J \ldots A_M A_1 \ldots A_M) - TR(A_1 \ldots \Phi_I \ldots A_M A_1 \ldots \Phi_J \ldots A_M)$$
$$\mp TR(A_1 \ldots \Phi_I \ldots \Phi_J \ldots A_M) TR(A_1 \ldots A_M) \pm TR(A_1 \ldots \Phi_I \ldots A_M) TR(A_1 \ldots \Phi_J \ldots A_M)$$

Are eigenstates of H with eigenvalues

$$2p^- = 2 + \frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\left(\left(\frac{n_1}{2}\right)^2 + \left(\frac{n_2}{2}\right)^2\right) \pm g_s^2 \frac{1}{2}\left(\frac{M}{4\pi g_s N}\right)$$

Example: k=3 with two impurities

$$\frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\left(\frac{n}{3}\right)^2$$

$$\frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\left(\frac{n}{3}\right)^2 + g_s^2 \cdot 9\left(\frac{M}{4\pi g_s N}\right)$$

$$\frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\left(\frac{n}{3}\right)^2 + g_s^2 \cdot 9\left(\frac{M}{4\pi g_s N}\right)\left(\frac{2\pi n \sin(2\pi n/3) - 9}{2\pi n \sin(2\pi n/3) - 9 + 12\sin^2(2\pi n/3)}\right) + \ldots$$

$$2p^- = 2 + \frac{1}{2}\left(\frac{4\pi g_s N}{M}\right)\lambda$$

$$4\lambda \cosh\sqrt{9M^2/N^2 - 2\lambda} + 2\lambda - 27M^2/N^2 = 0$$

Using DLCQ string field theory, we have computed the string loop correction to the k=2, two impurity string state in the two impurity limit. It agrees precisely with the prediction of Yang-Mills!

We are presently trying to assess the validity of this result by computing the 4-impurity channel.

Conclusion: The problem of matching non-planar Yang-Mills and string interactions in the context of AdS/CFT is still open.