Title: The chaotic evolution of Newton's universe

Date: Mar 09, 2006 04:00 PM

URL: http://pirsa.org/06030010

Abstract: In this expository talk, I describe how "chaotic behavior" not only was discovered in the study of the Newtonian N-body problem, but also is responsible for several strange appearing motions. Then, a mathematical outline of the general evolution of the universe, under Newton's laws, is provided. No prior background in dynamics or the mathematics of the N-body problem is needed to follow this lecture

Chaotic Evolution of the Universe

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Chaotic Evolution of the Universe
Newtonian N-body problem

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Shepherds, ancient astronomers and astrologers

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To introduce a research problem

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To introduce a research problem

Does the Earth go about the Sun, or does the Sun go about the Earth?

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body problem

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research problem

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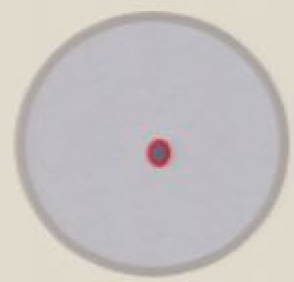
Geocentric vs. heliocentric approach

Geocentric vs. heliocentric approach

Geocentric vs. heliocentric approach

One year, distance AU



Geocentric vs. heliocentric approach

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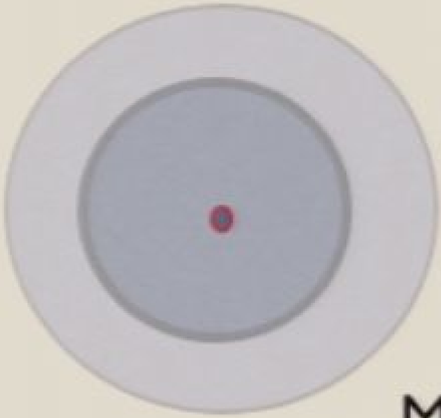

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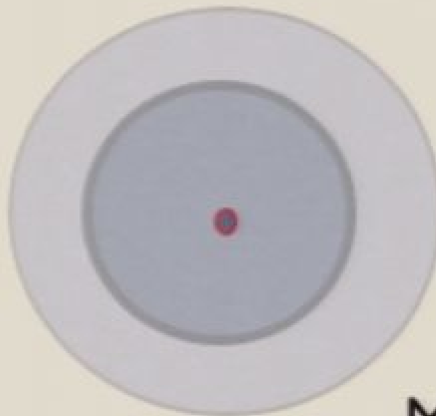
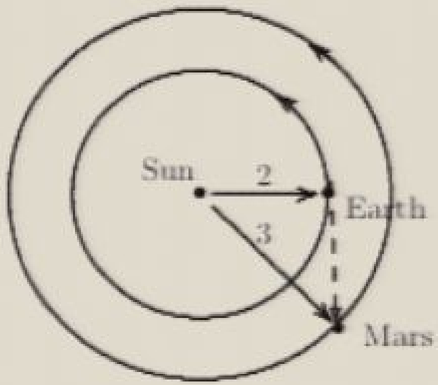


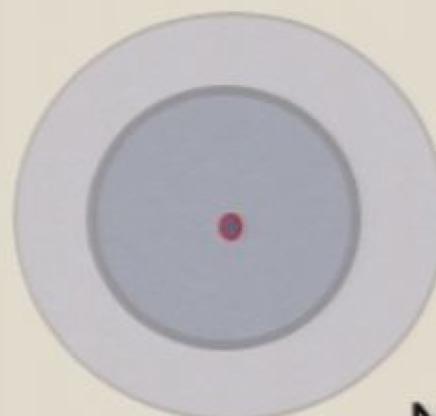
Two years, 1.5 AU

Geocentric vs. heliocentric approach



The Sun-Earth-Mars system in half-astronomical units

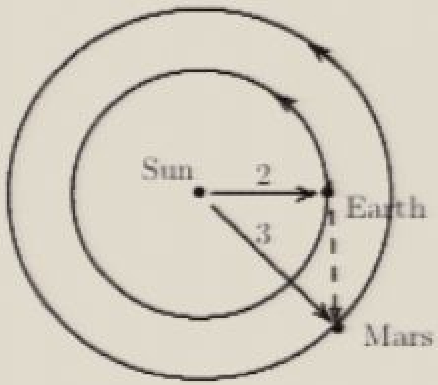
One year, distance AU



Mars

Two years, 1.5 AU

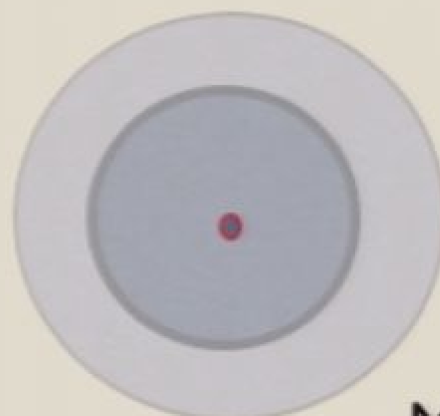
Geocentric vs. heliocentric approach



The Sun-Earth-Mars system in half-astronomical units

$$\mathbf{r}_E = 2e^{2\pi i t} \qquad \mathbf{r}_M = 3e^{\pi i t}$$

One year, distance AU



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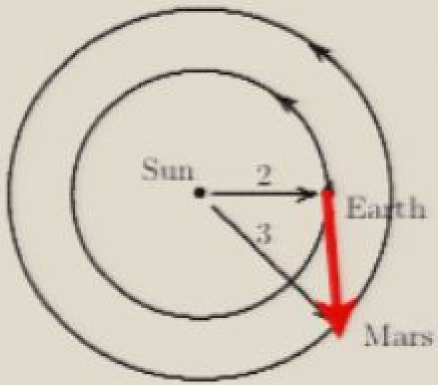

$$\mathbf{r}_E = 2e^{2\pi i t} \qquad \mathbf{r}_M = 3e^{\pi i t}$$

$$\mathbf{r}_M - \mathbf{r}_E = 2 + [3 - 4\cos(\pi t)]e^{\pi i t}$$

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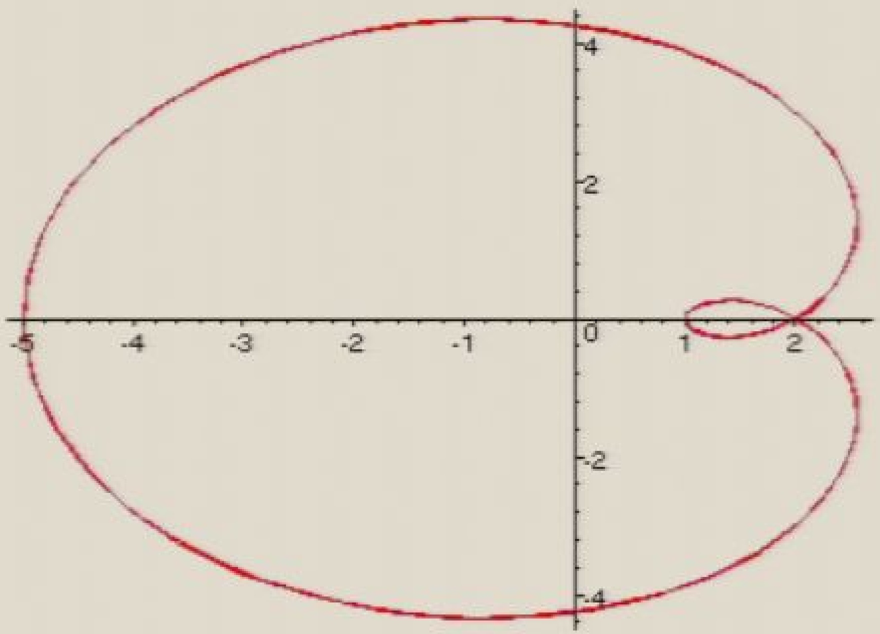
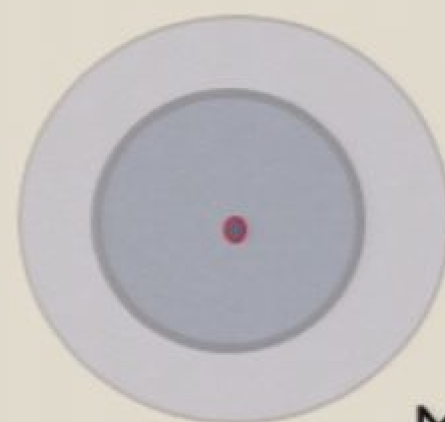


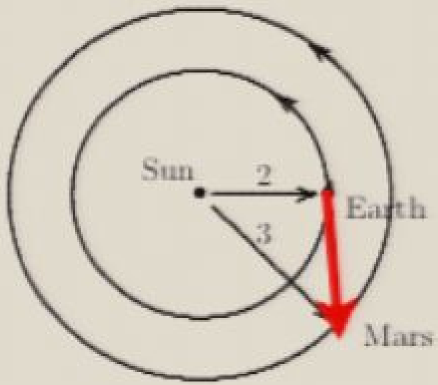
One year, distance AU



Mars

Two years, 1.5 AU

Geocentric vs. heliocentric approach



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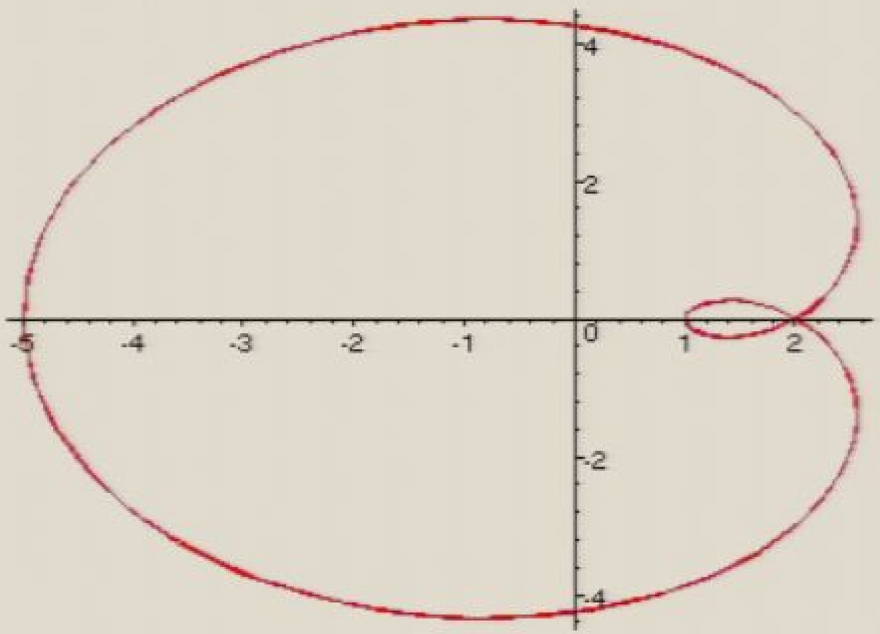

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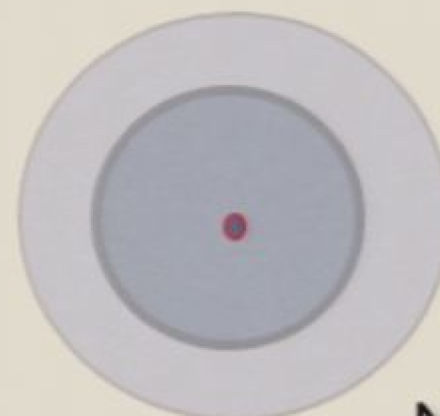


Mars

Two years, 1.5 AU

Classroom: Double angle equations

Ptolomy and his epicycles

Ptolomy and his epicycles

Aristotle:

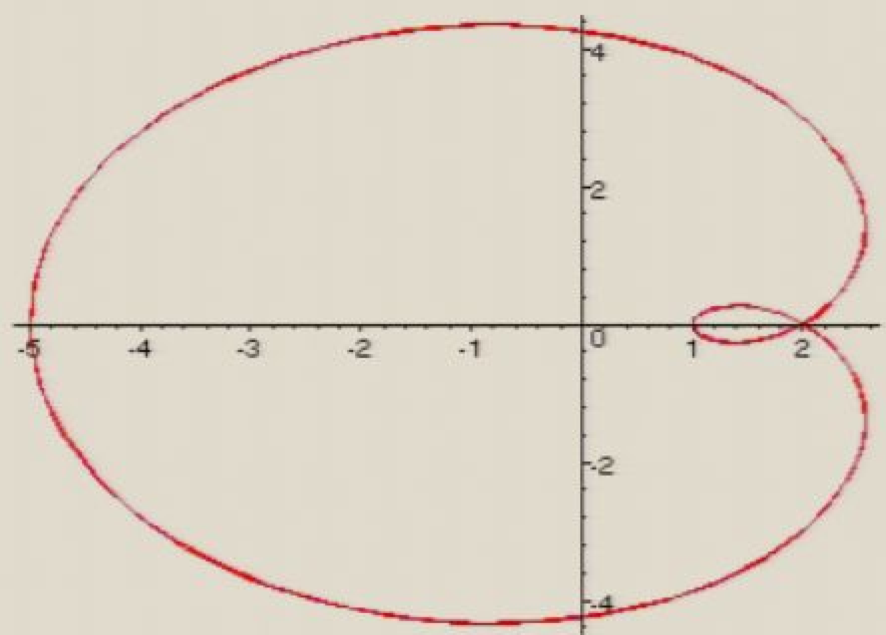
Ptolomy and his epicycles

Aristotle: Uniform motion

Ptolomy and his epicycles

Aristotle: Uniform motion

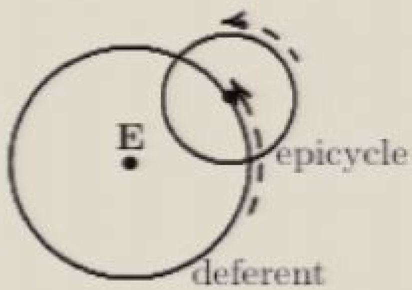
Circular



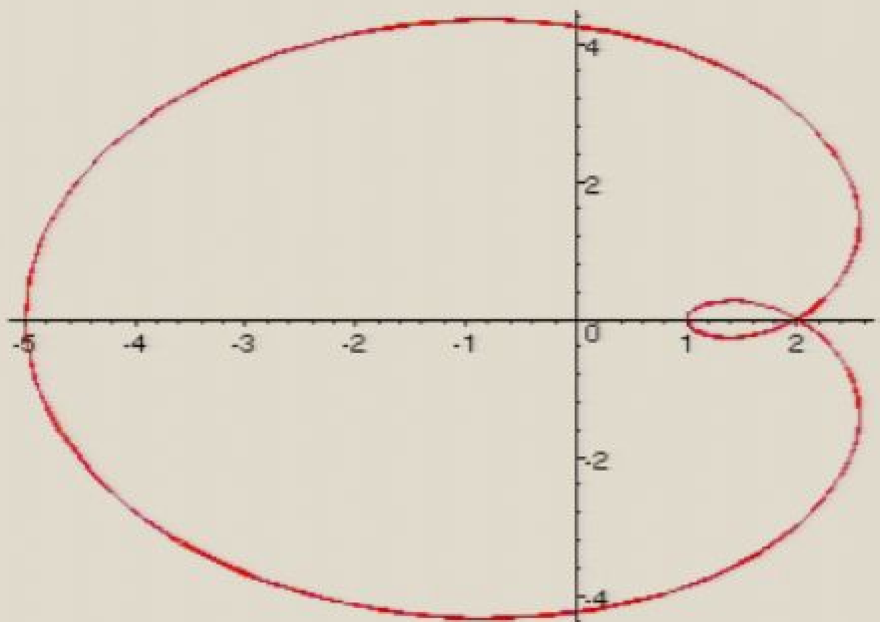
Ptolomy and his epicycles

Aristotle: Uniform motion

Circular



Epicycles



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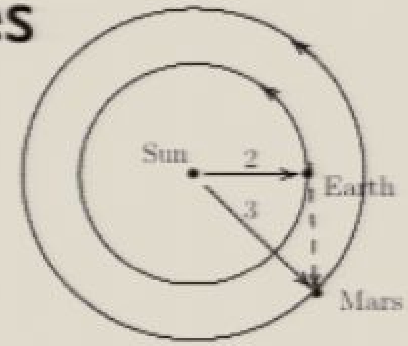
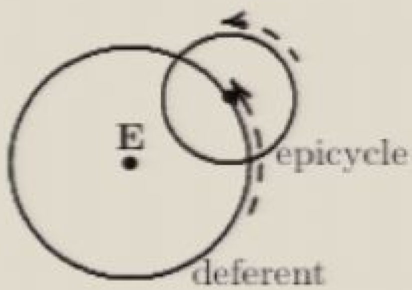


The Sun-Earth-Mars system in half-astronomical u



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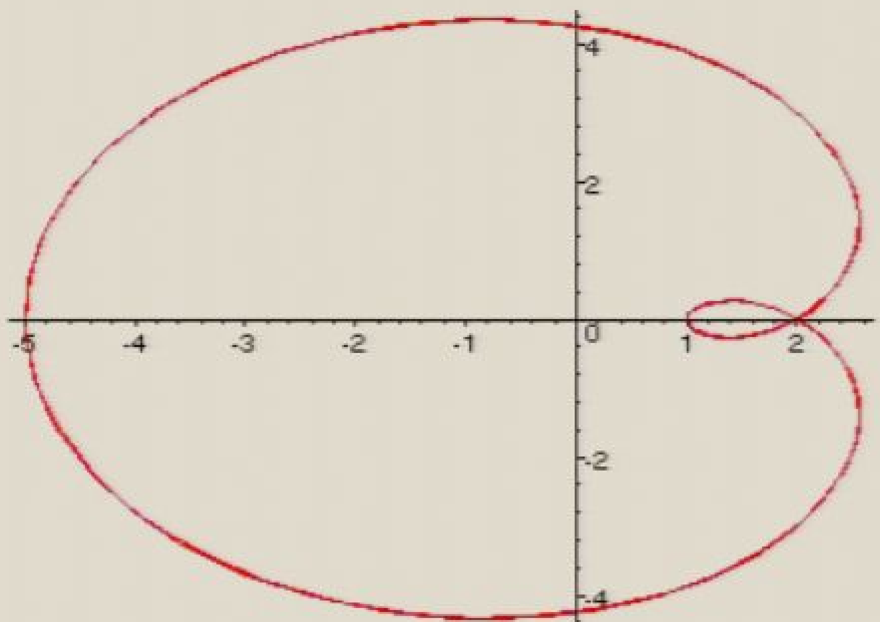

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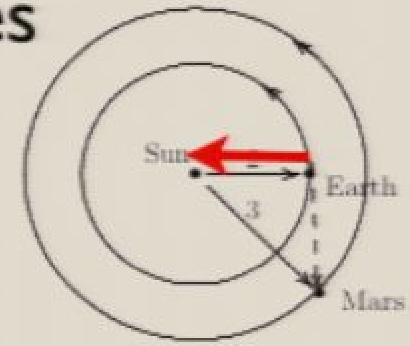

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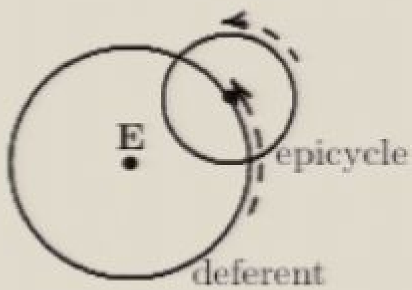
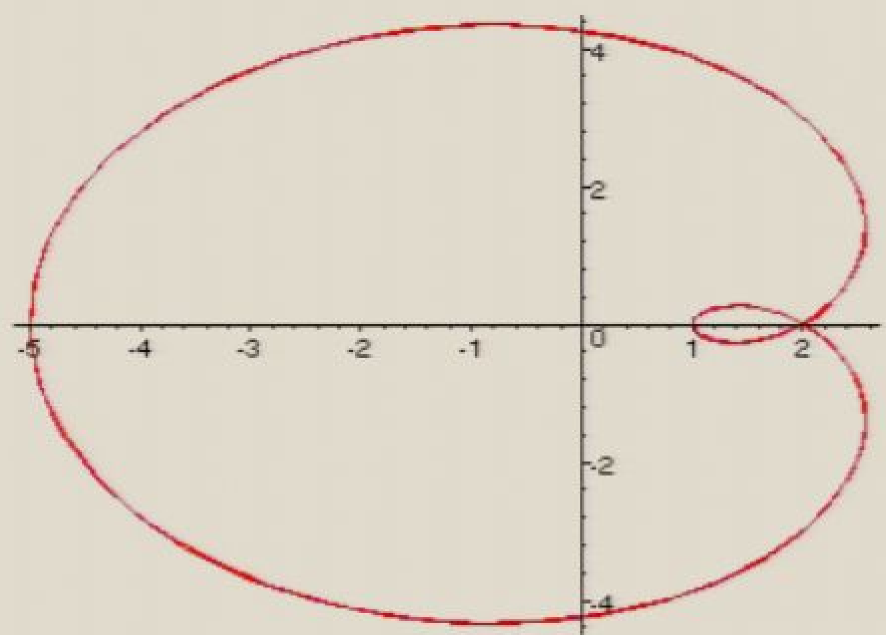


Epicycles



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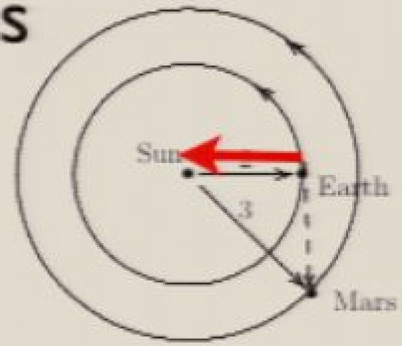

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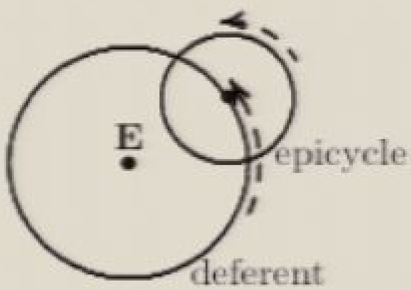
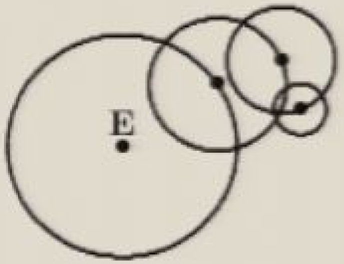


Epicycles



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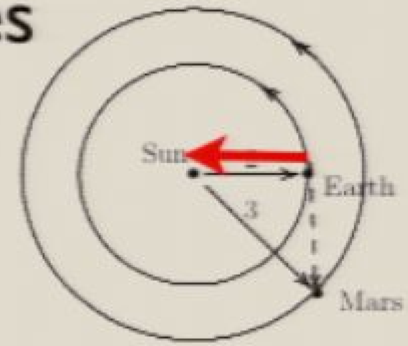

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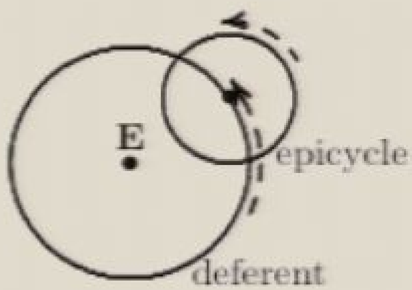
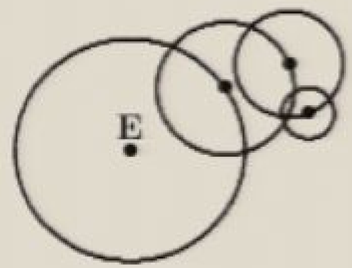
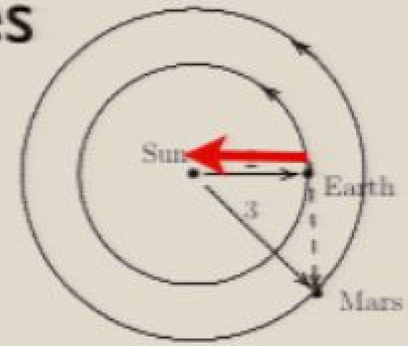


Epicycles



Quasi-periodic motion

Ptolomy and his epicycles



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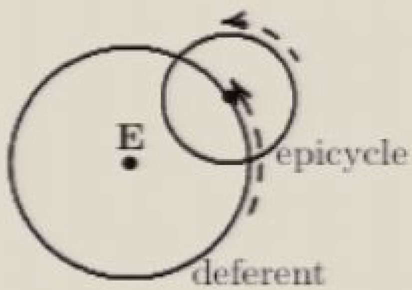
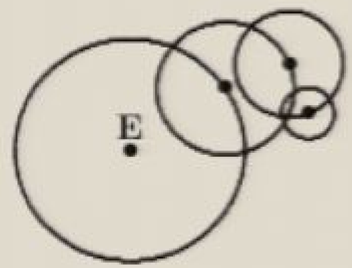


Epicycles



Quasi-periodic motion

$$z(t) = \sum_{j=0}^{\infty} a_j e^{b_j i t}$$

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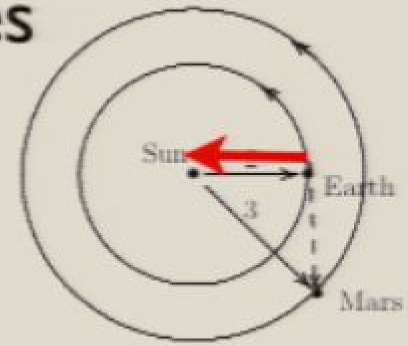

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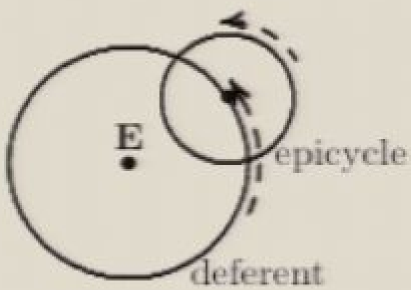

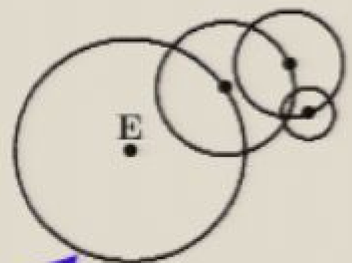

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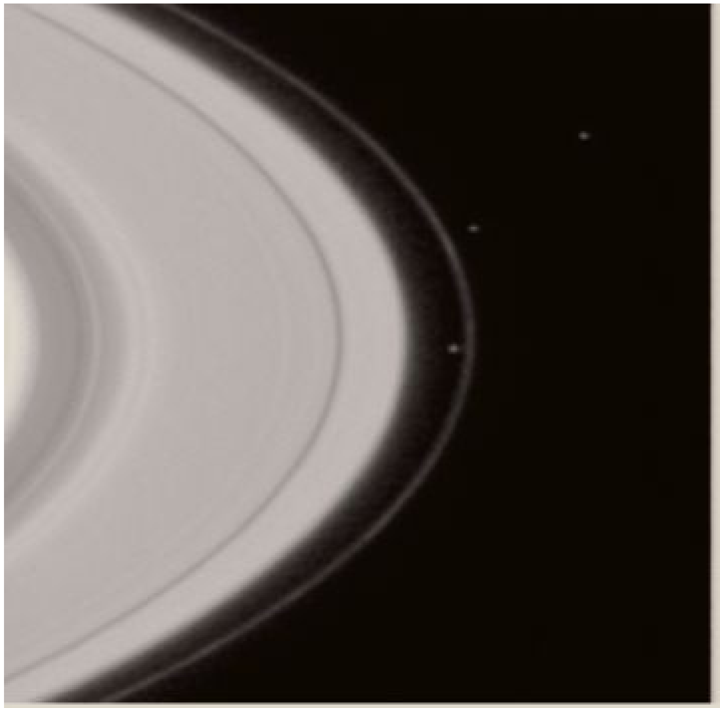
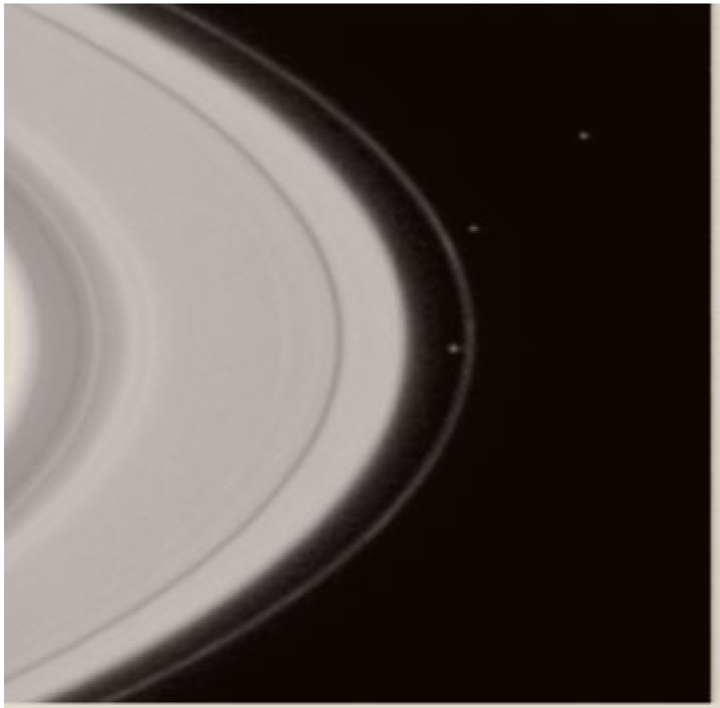
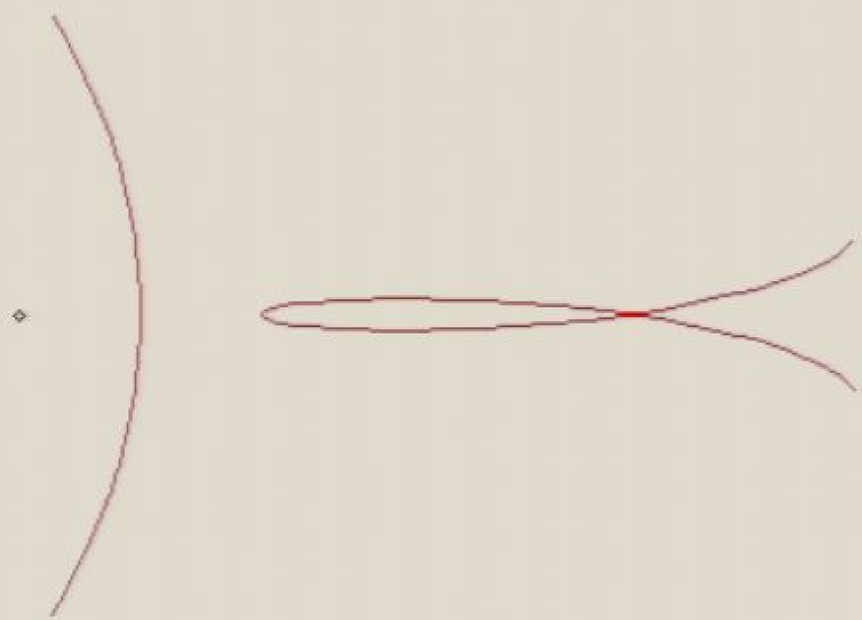
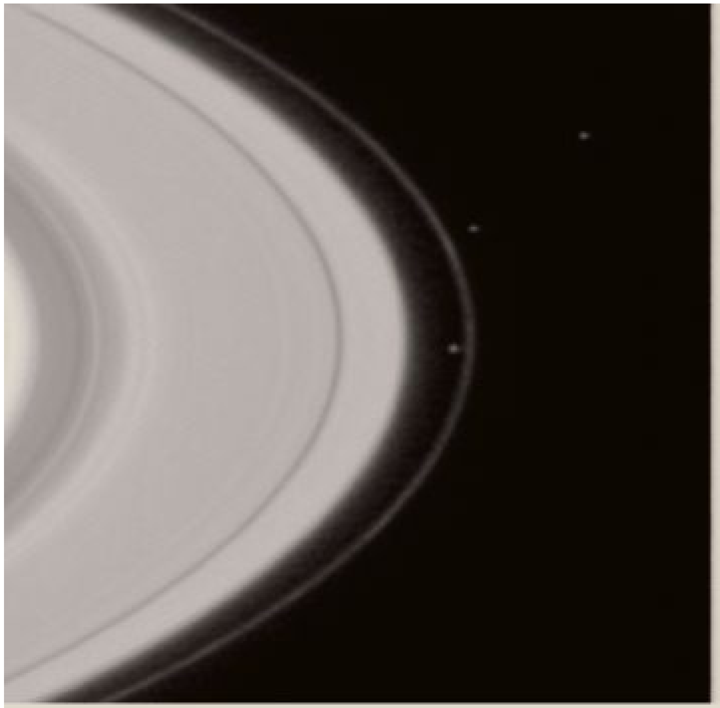
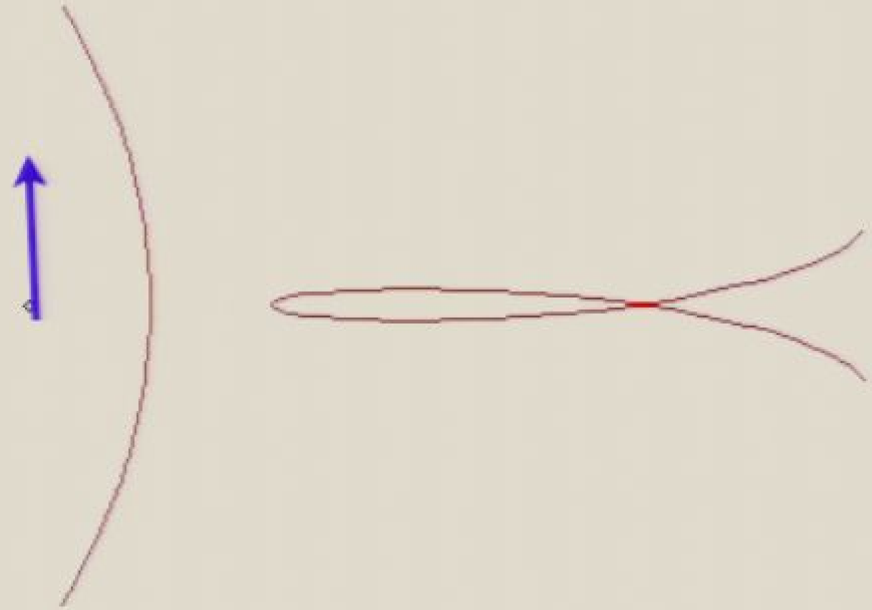
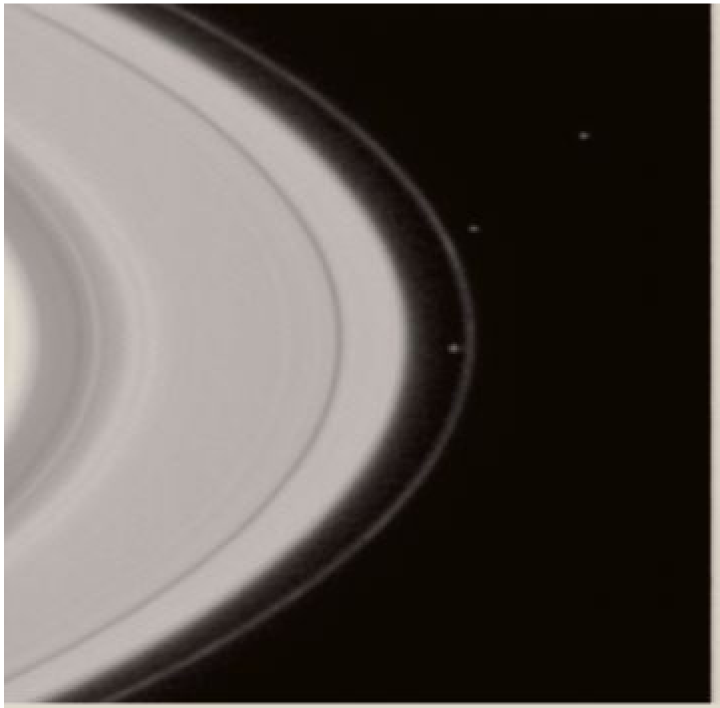


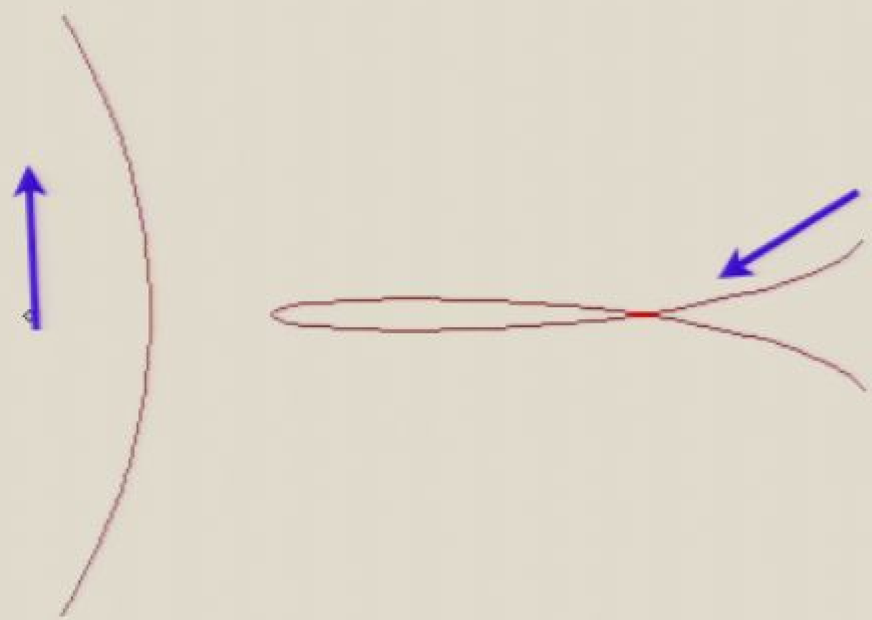


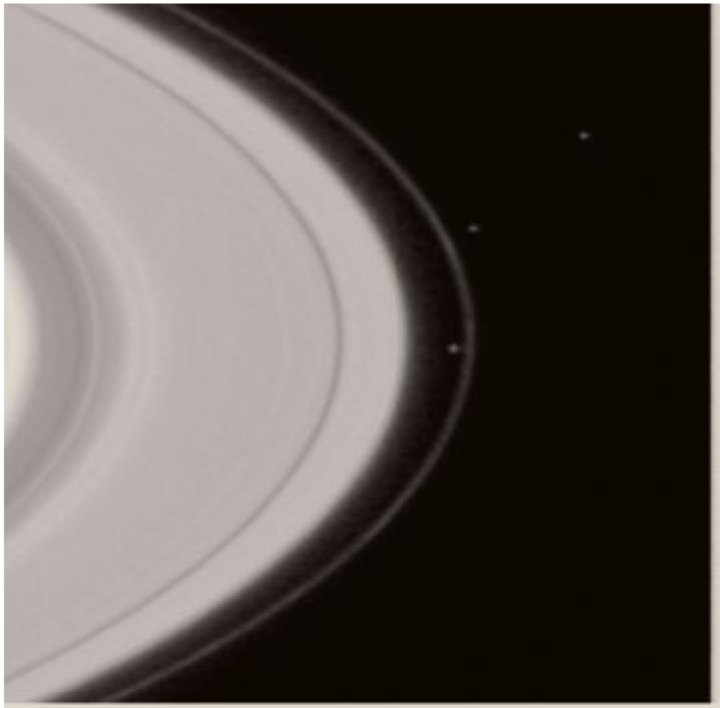


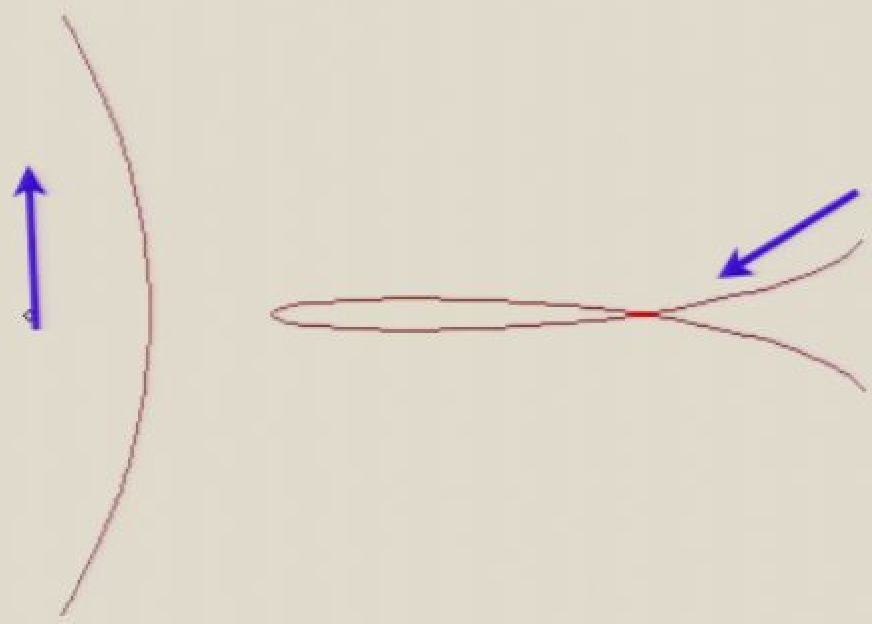


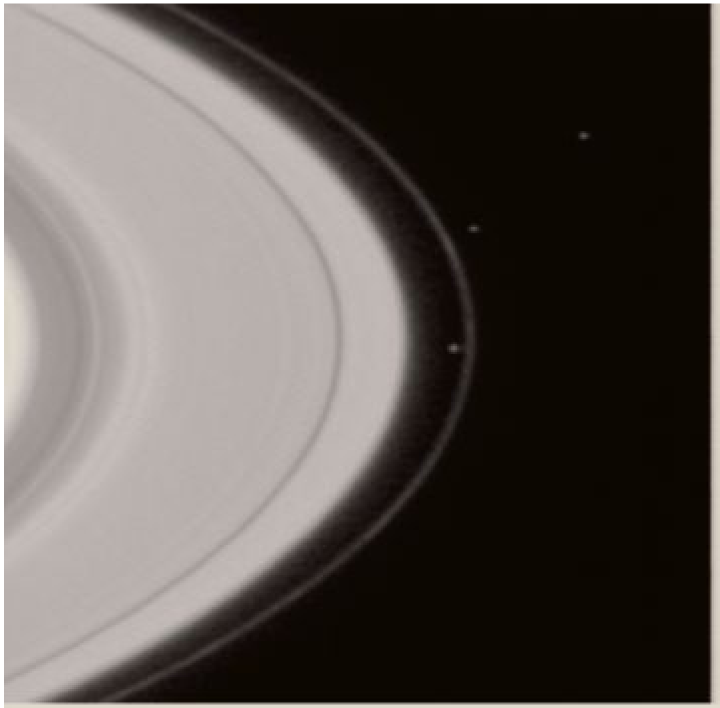


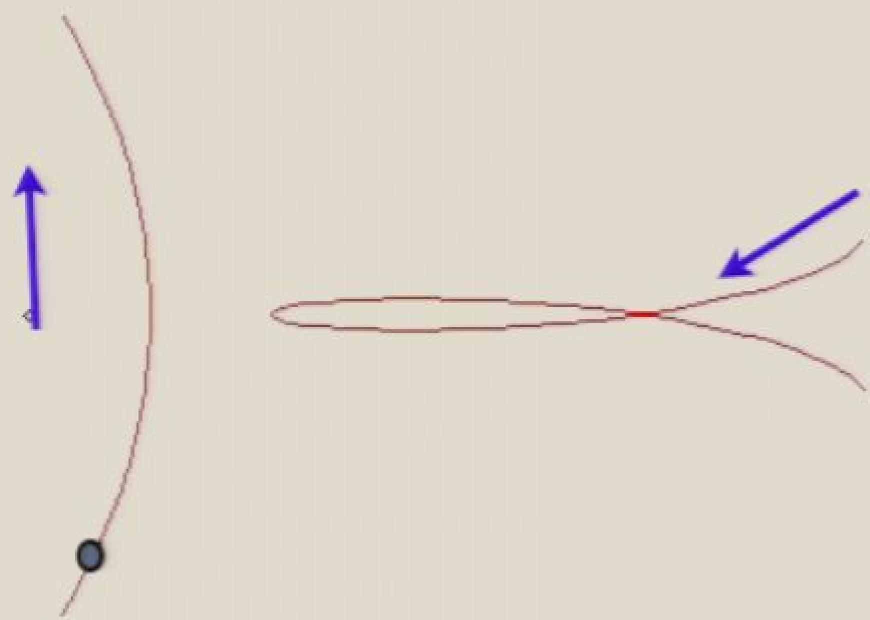


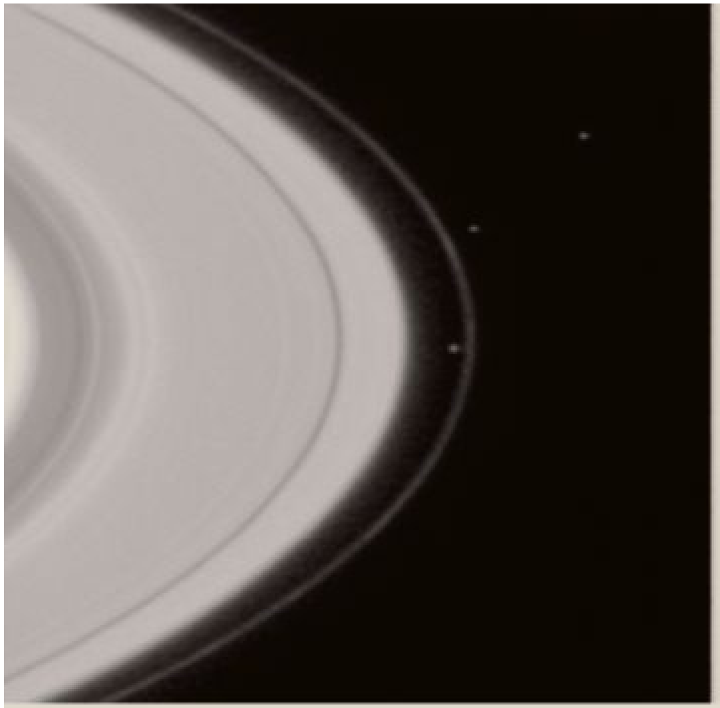


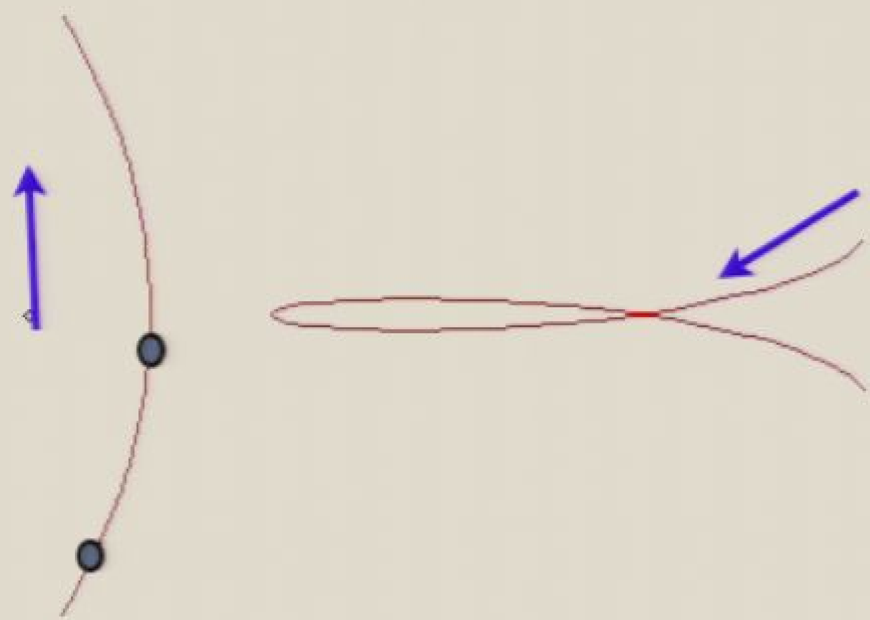


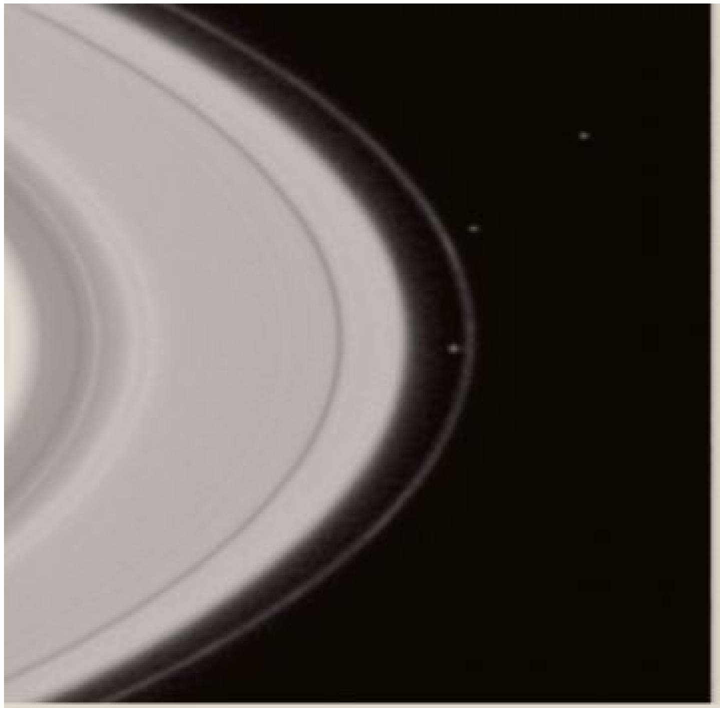


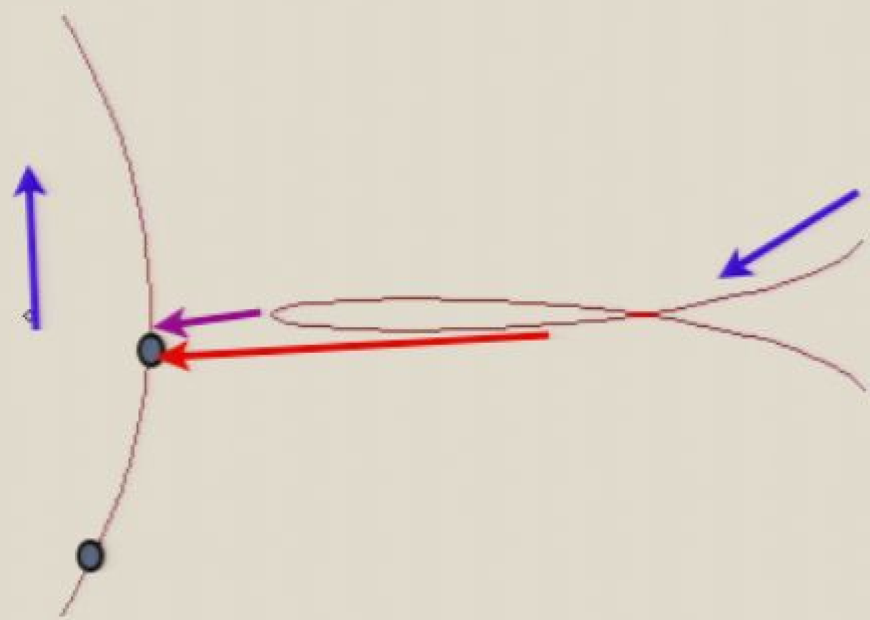


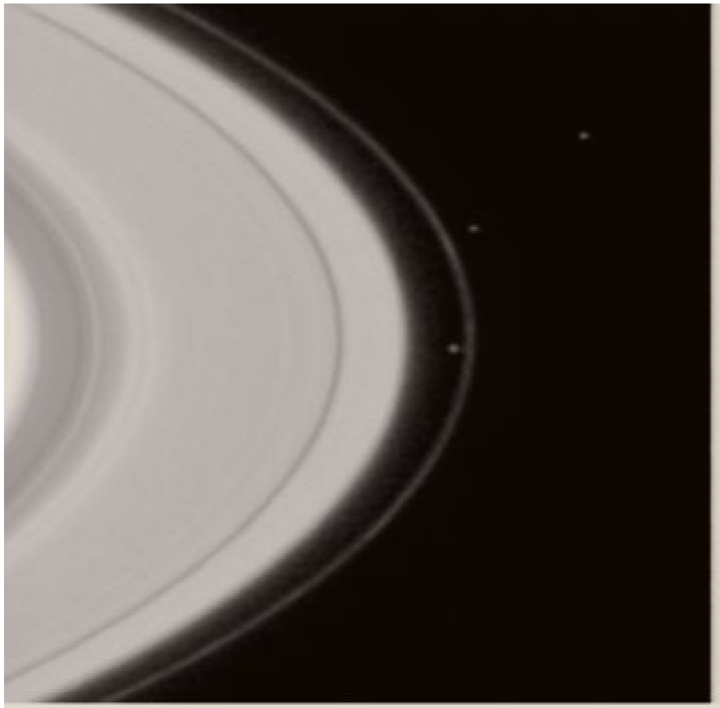




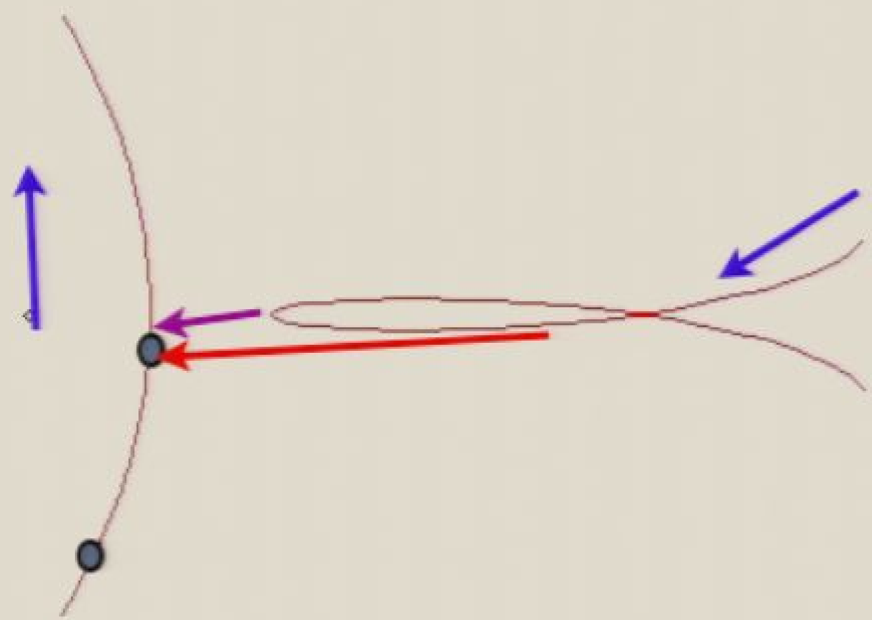


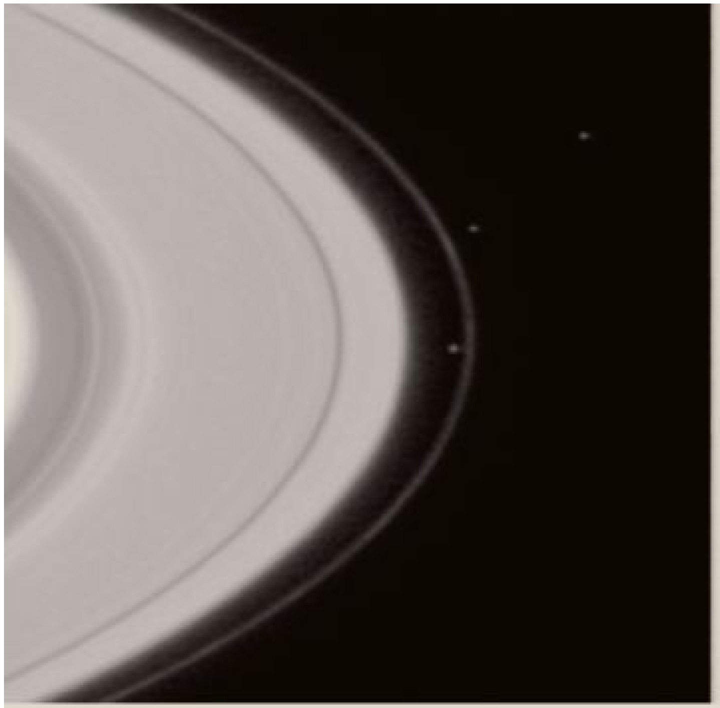


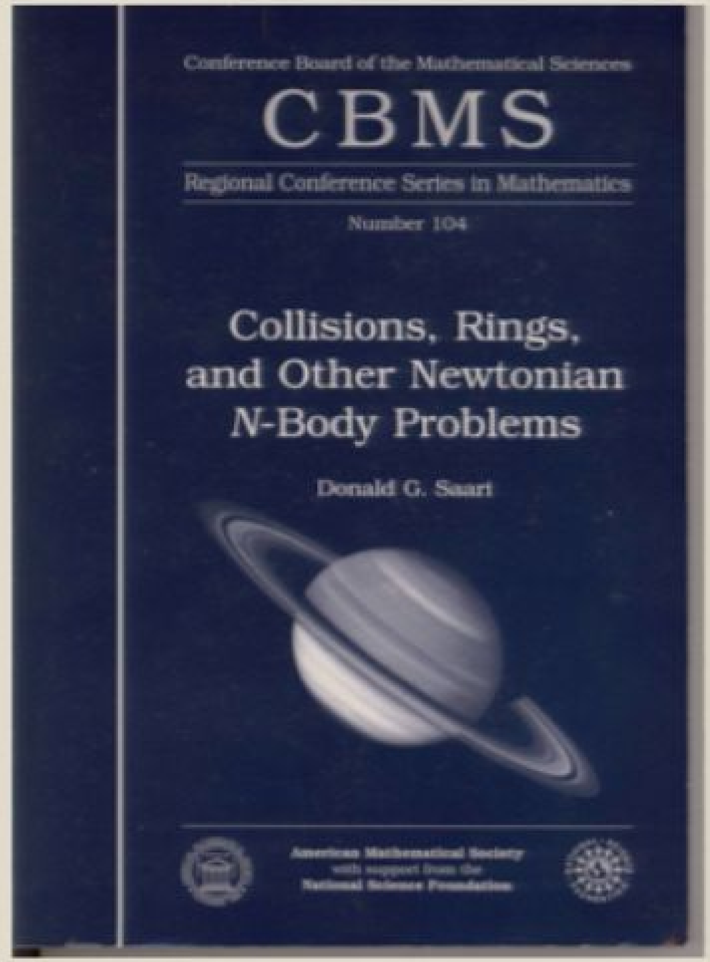




Chaos can appear in seemingly well-behaved settings







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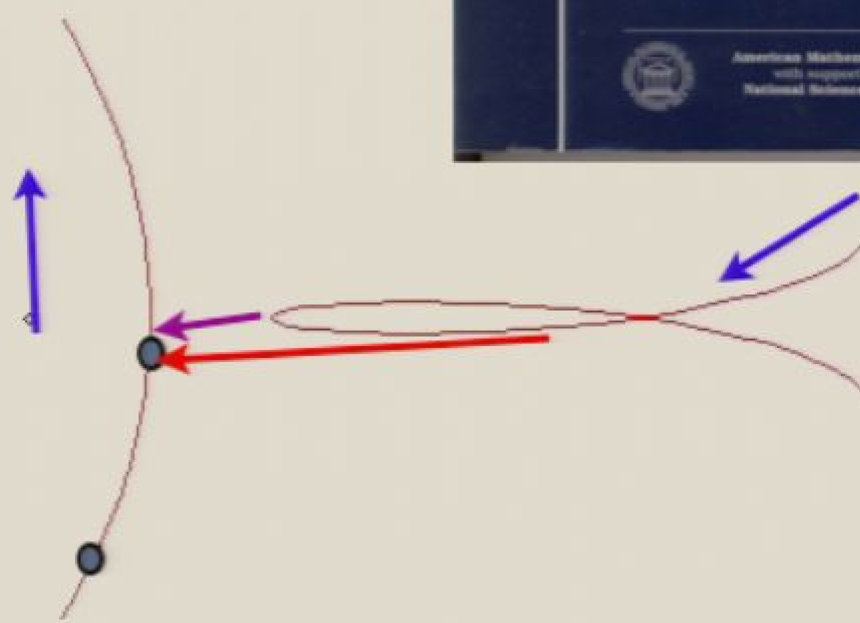

For the evolution of the universe, what do we think might happen?

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Mathematics for the evolution of the universe

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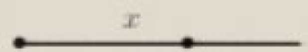
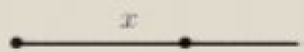
$$x'' = \frac{-1}{x^2}$$

Mathematics for the evolution of the universe



$$x' \, x'' = \frac{-1}{x^2} \, x'$$

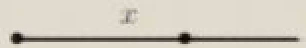
Mathematics for the evolution of the universe



$$x' \; x'' = \frac{-1}{x^2} \, x'$$

$$\frac{1}{2} x'^2 = \frac{1}{x} + h$$

Mathematics for the evolution of the universe



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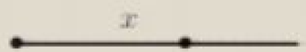
Mathematics for the evolution of the universe

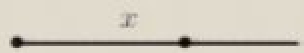


$$x' \, x'' = \frac{-1}{x^2} \, x'$$

$$\frac{1}{2}x'^2 = \frac{1}{x} + h$$

h sign	**x behavior**
$h < 0$	
$h = 0$	
$h > 0$	

Mathematics for the evolution of the universe



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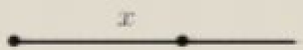
Mathematics for the evolution of the universe



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$h < 0$ $\qquad$ $x \leq \frac{1}{|h|}$

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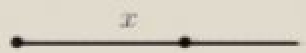

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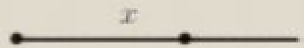

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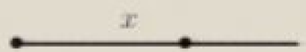

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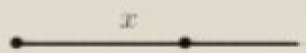

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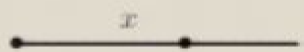

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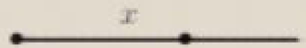

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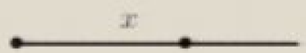

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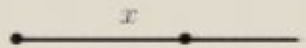

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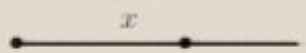

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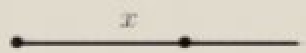

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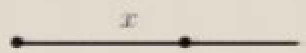

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Need analytic tricks, but same for general two body problem

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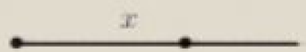

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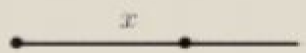
Three bodies?

Mathematics for the evolution of the universe



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Three bodies? **Newton's Headache**

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Acta Mathematica

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Solve the Newtonian
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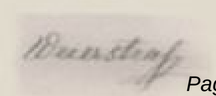
Errors



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Solve the Newtonian
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And they
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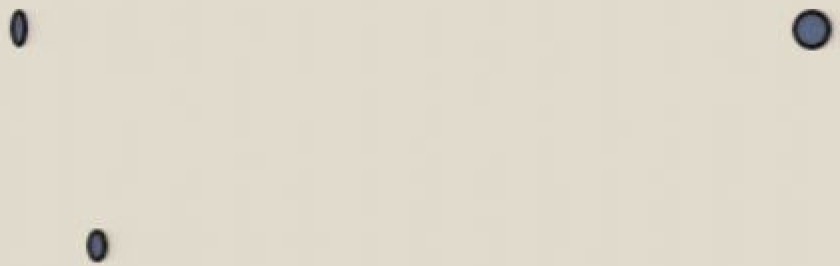
Three-body problem -- and Newton's headache



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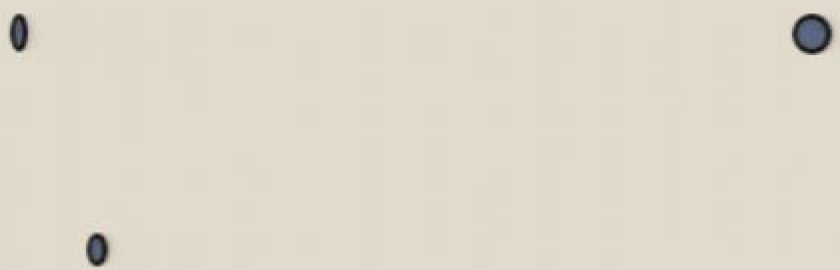

Jean Chazy, 1920s

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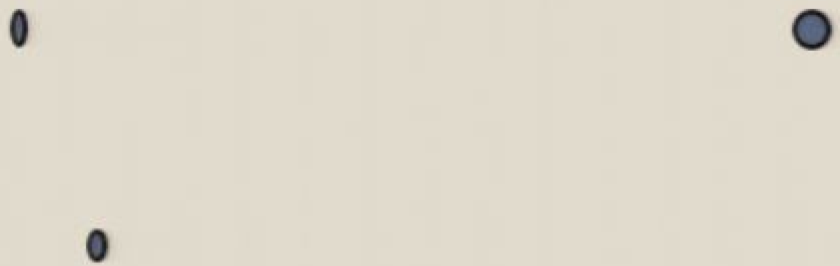

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Expanding Universe, Maximum distance goes to Infinity

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Triangle inequality
to make two two-body problems

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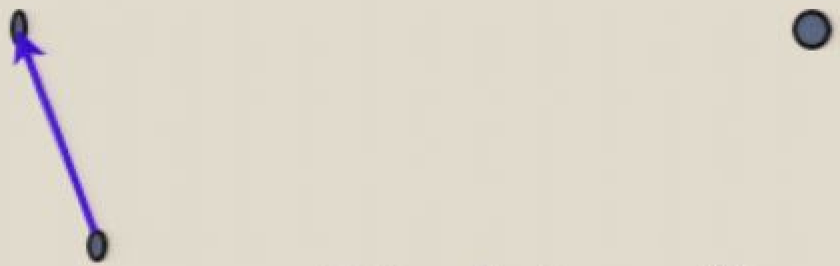

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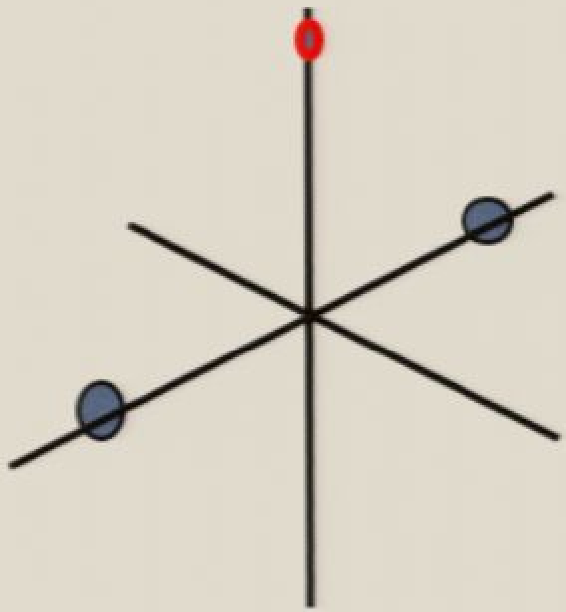
Oscillatory Motion

K. Sitnikov and Oscillatory Motion, 1960



Circular

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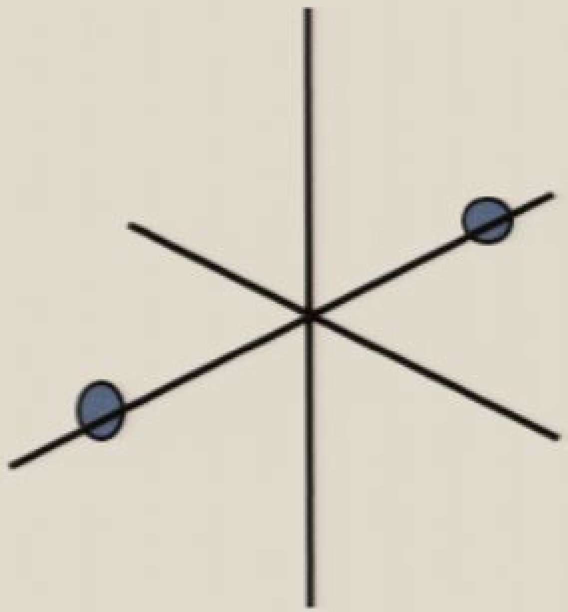

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Elliptic

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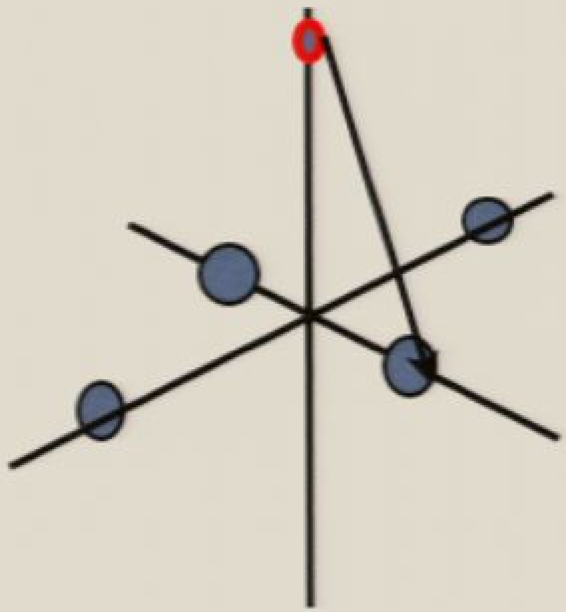

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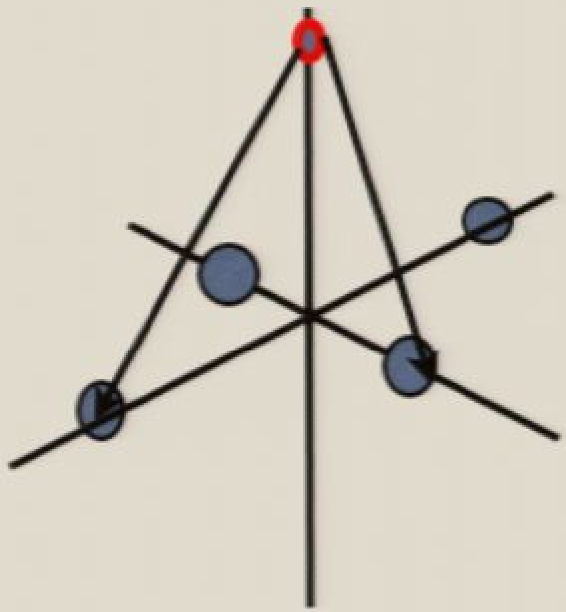

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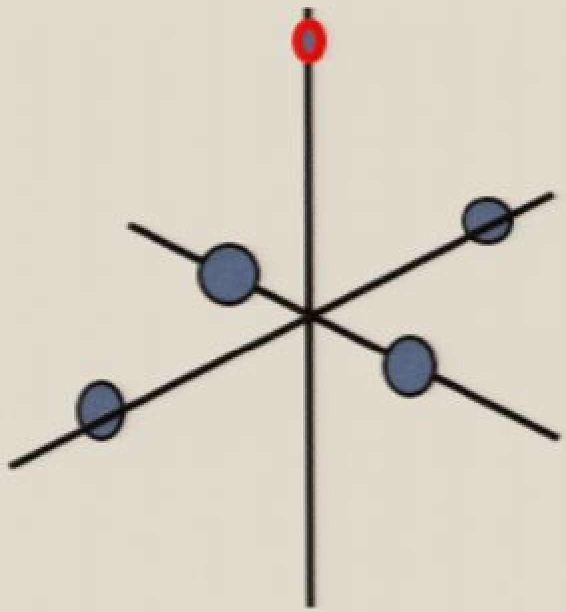

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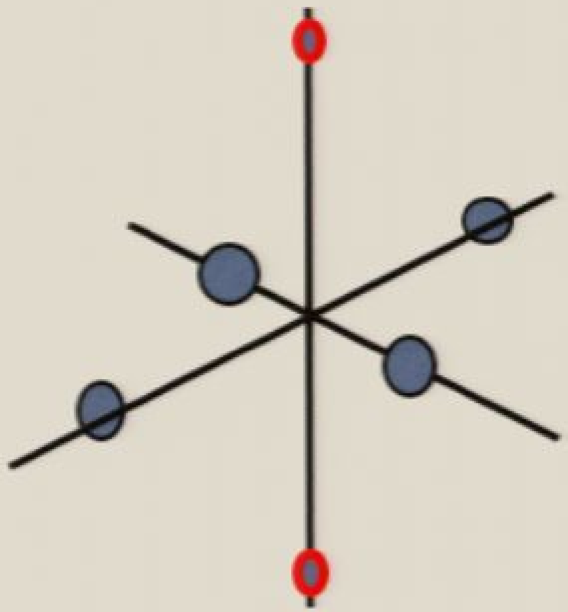

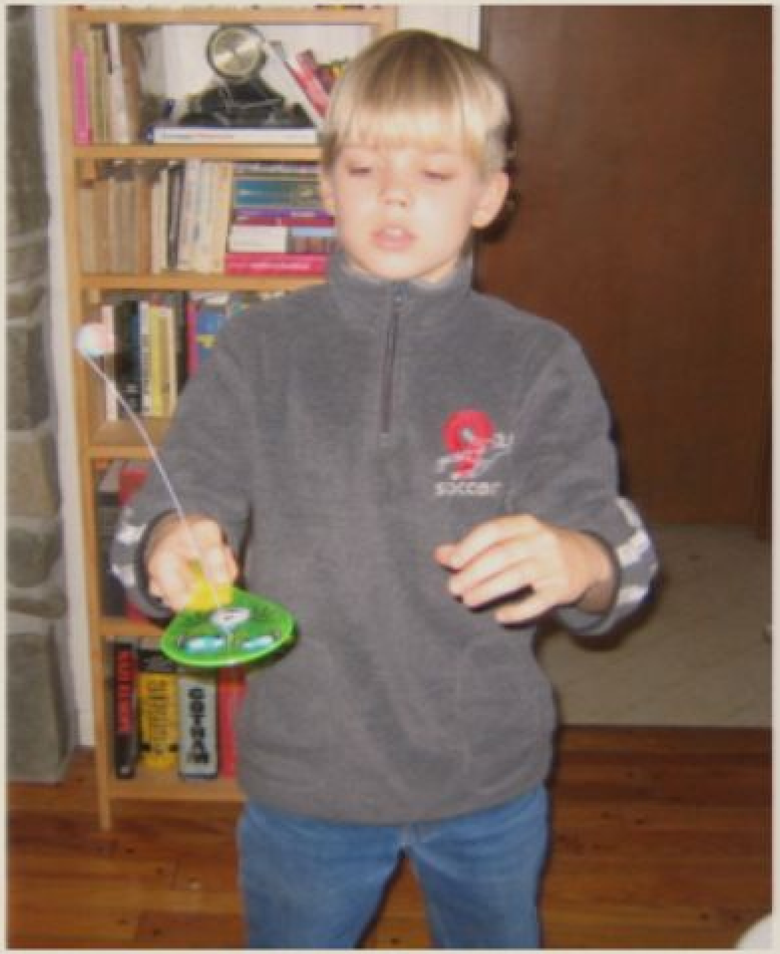

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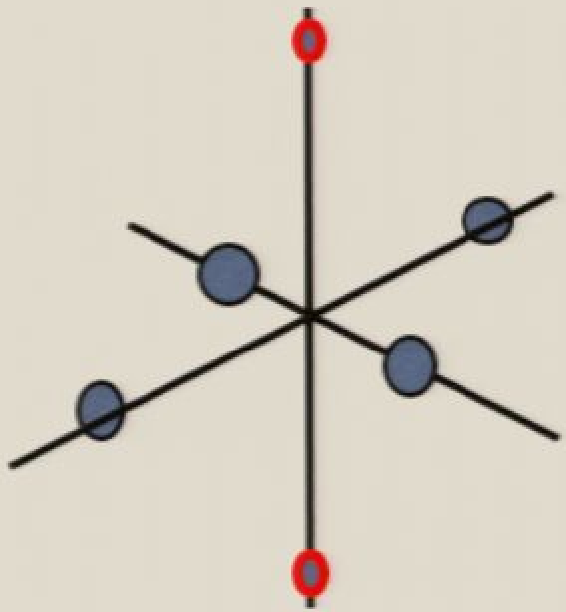

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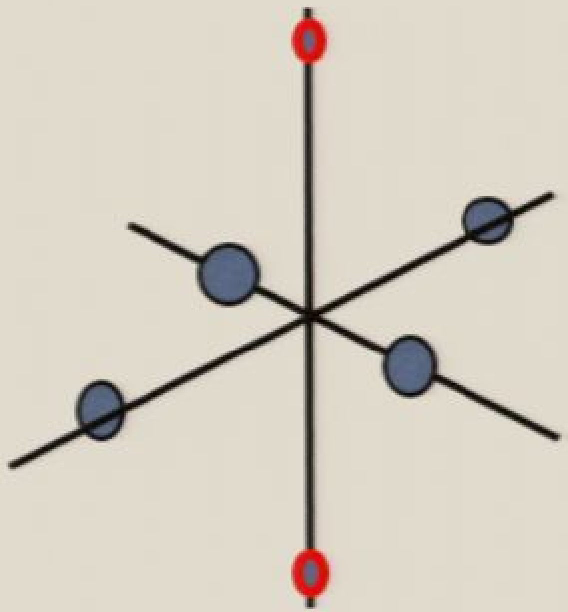

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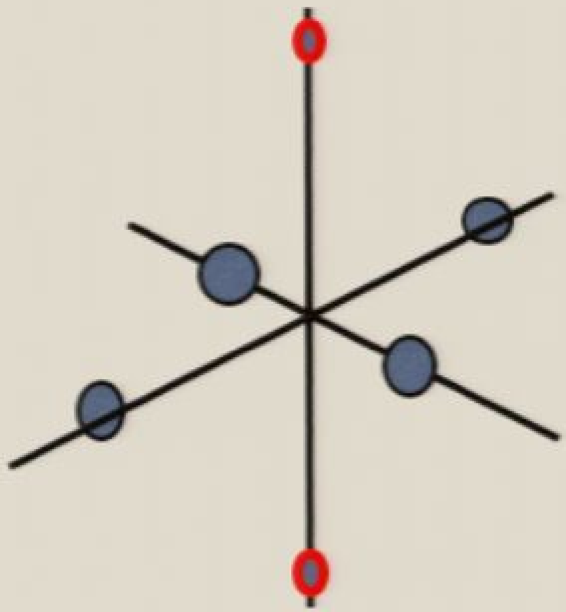

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Oscillatory motion and chaos elsewhere

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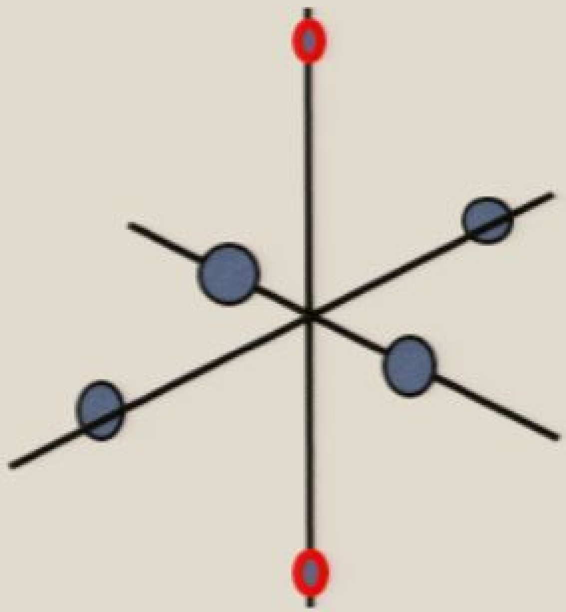

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Saari and Xia--collinear case

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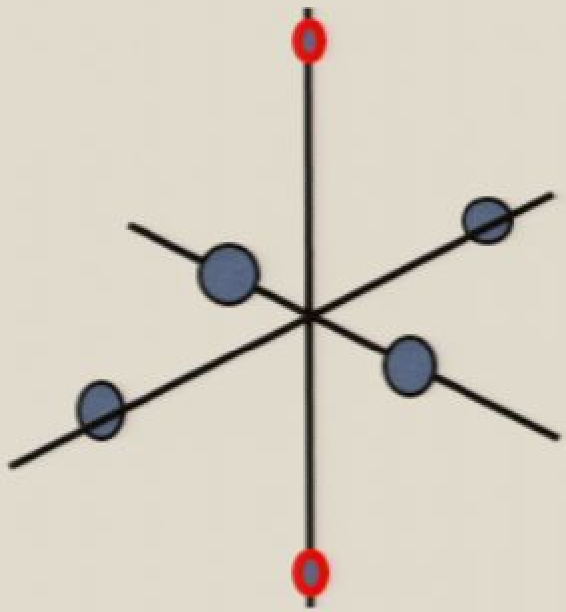

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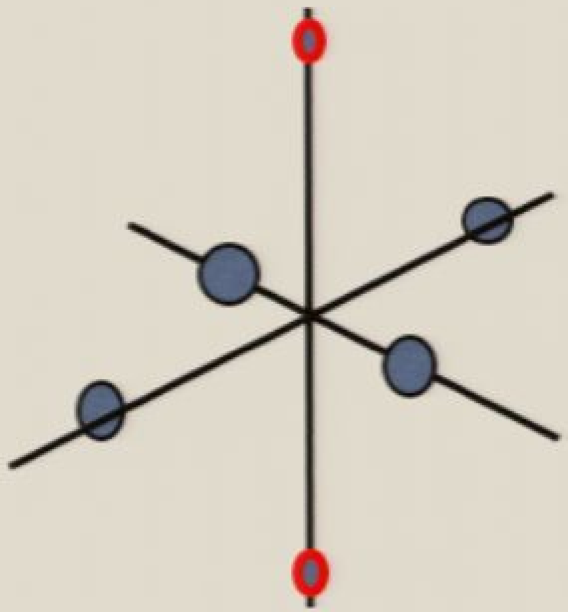

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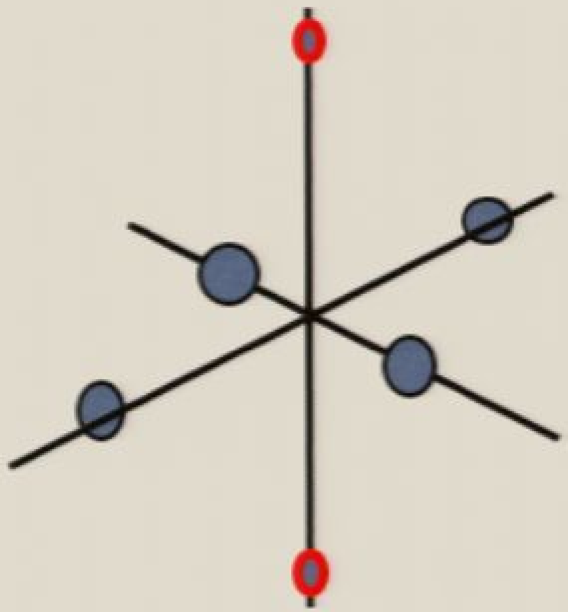

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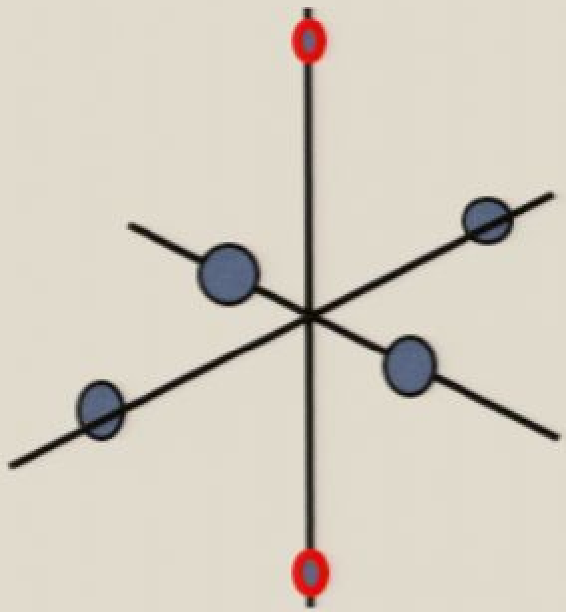

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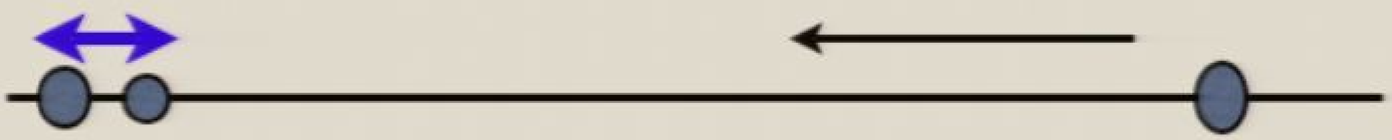
Osc ... ere



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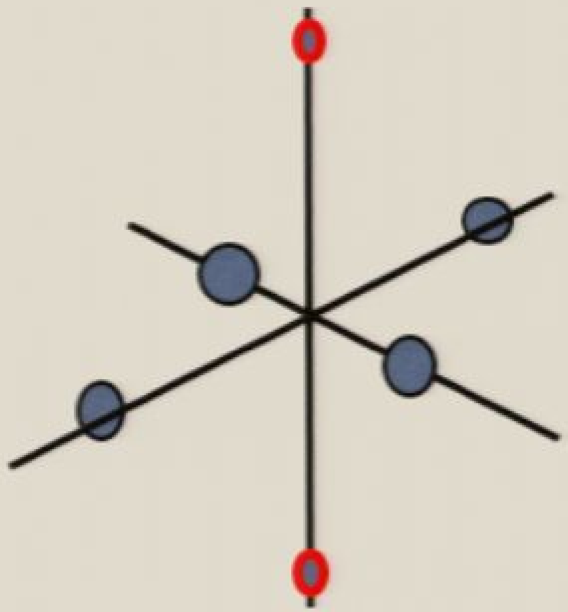

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GALEX

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Visible light image (John Gleason)

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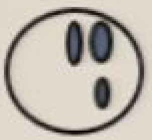

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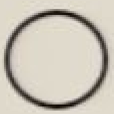

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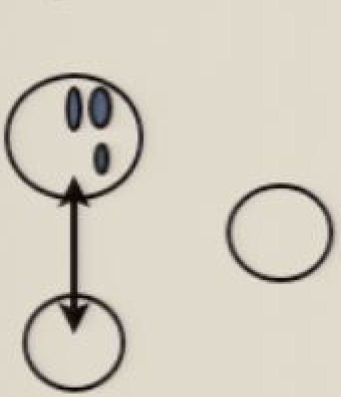

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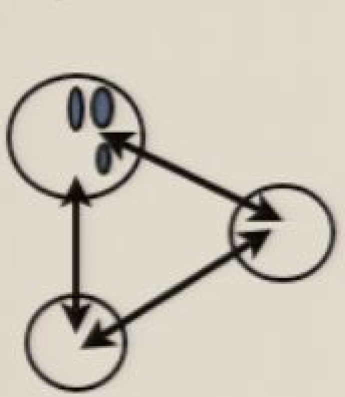

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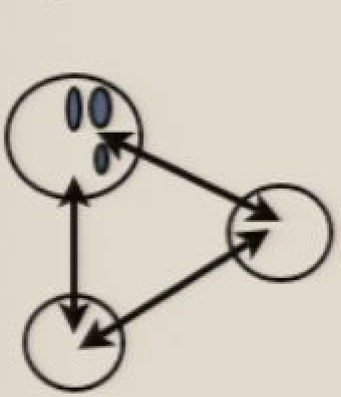

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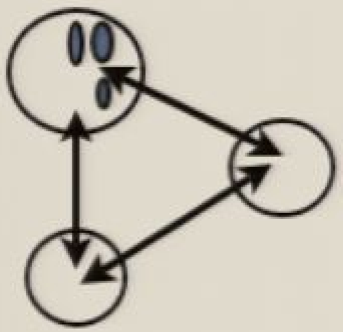

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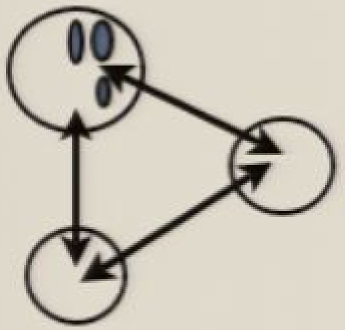

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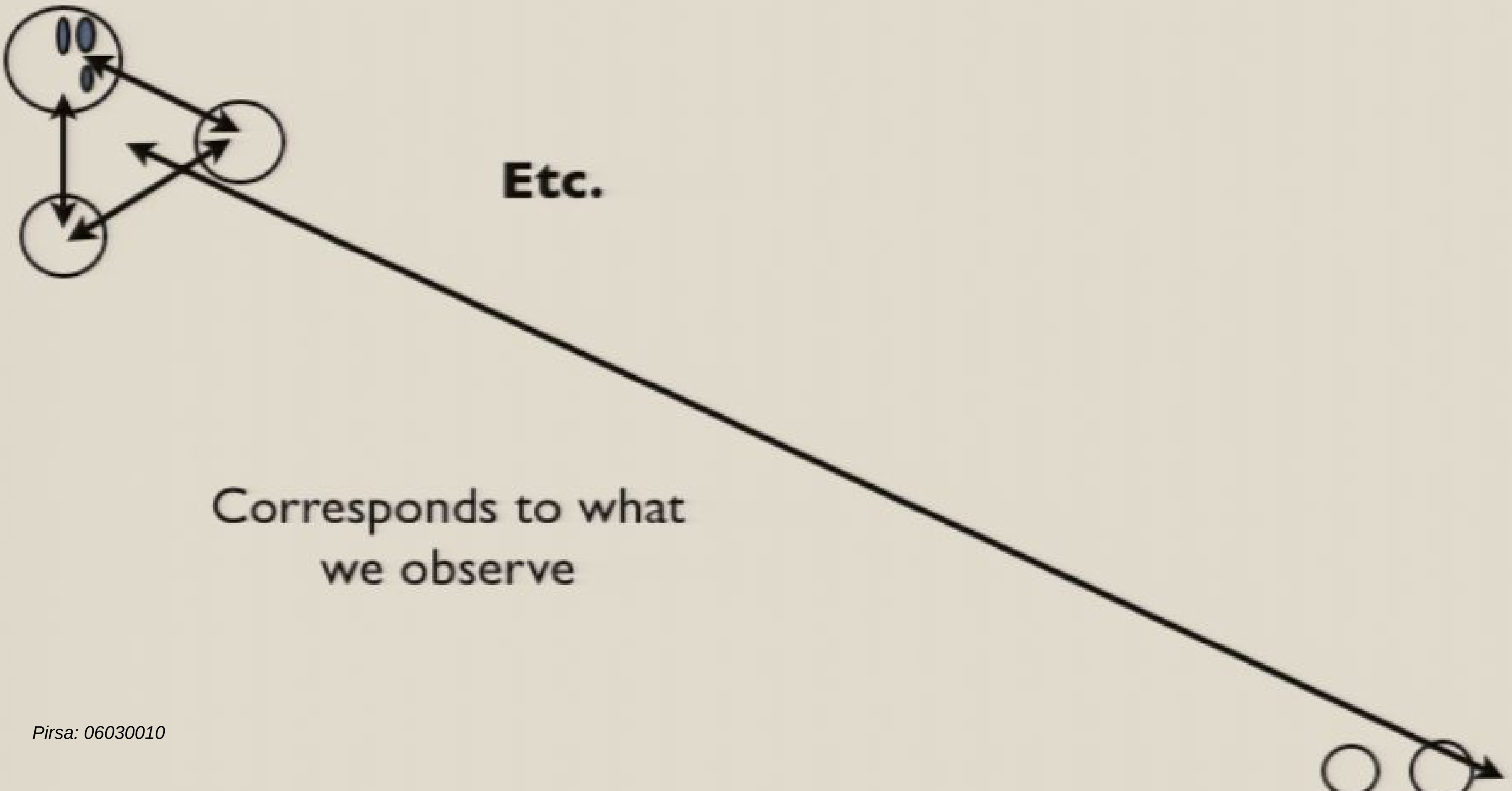

Etc.

Corresponds to what we observe

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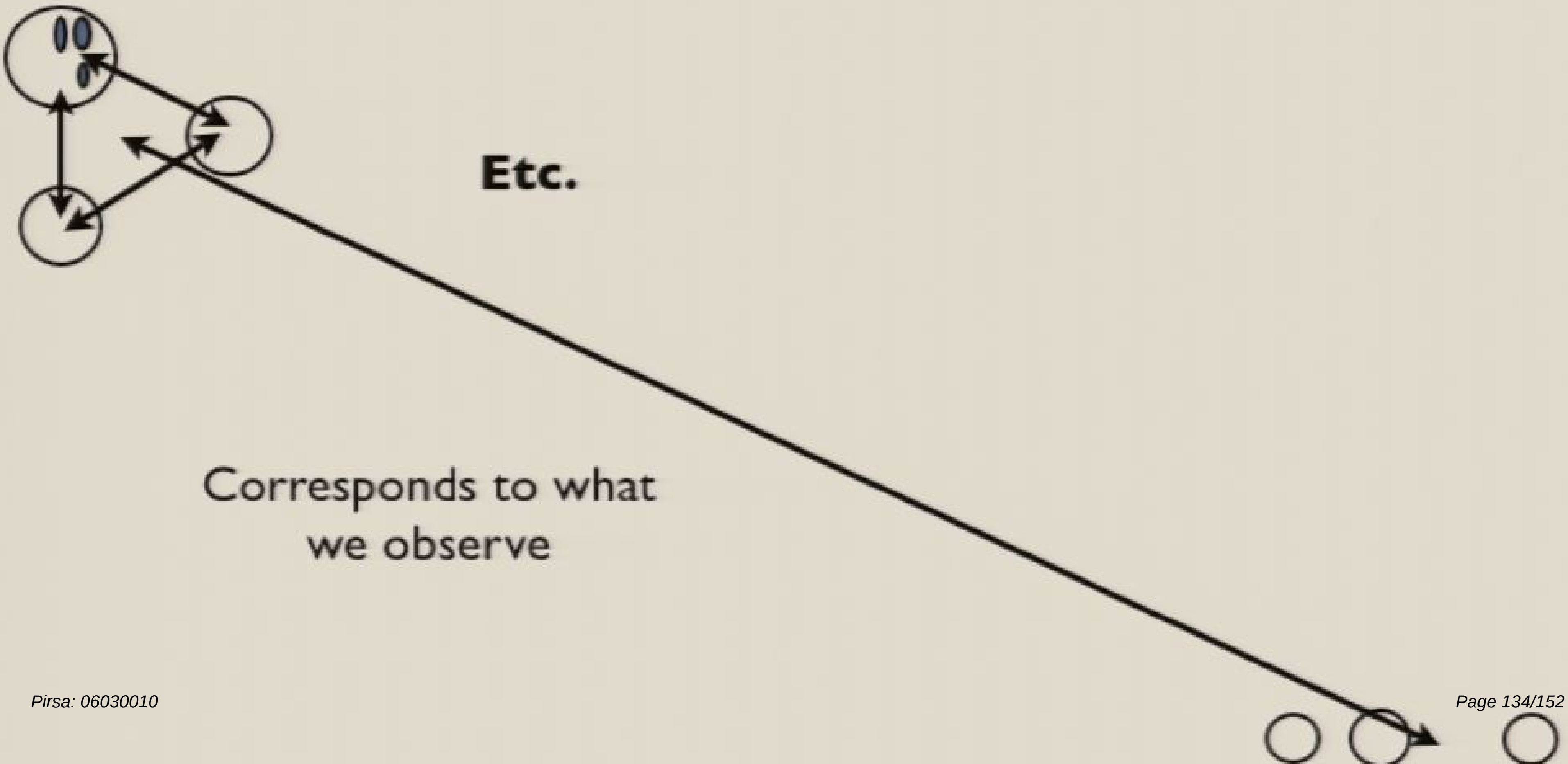

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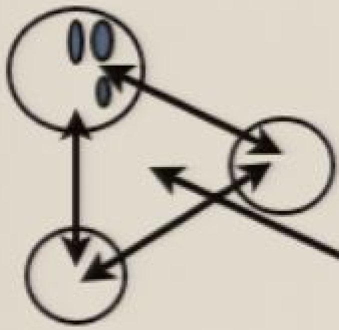

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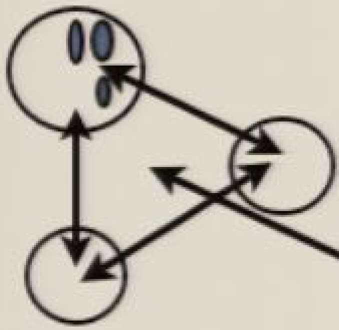

Etc.

The diameter of the universe goes to infinity faster than any multiple of t

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Theorem (Saari and Xia) For n > 3, there exists a cantor set of examples where

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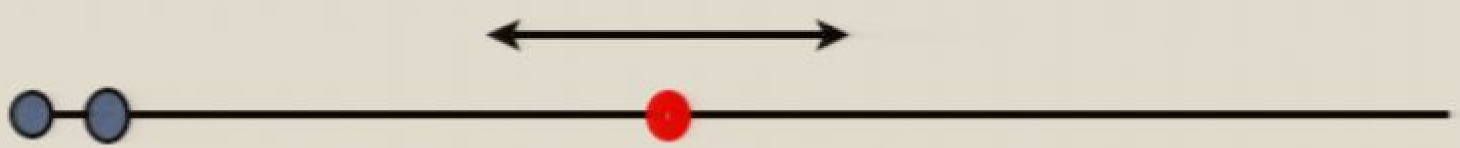

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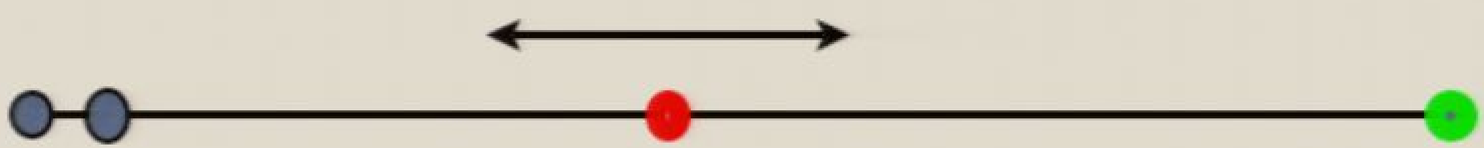

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Join us in investigating the N-body problem

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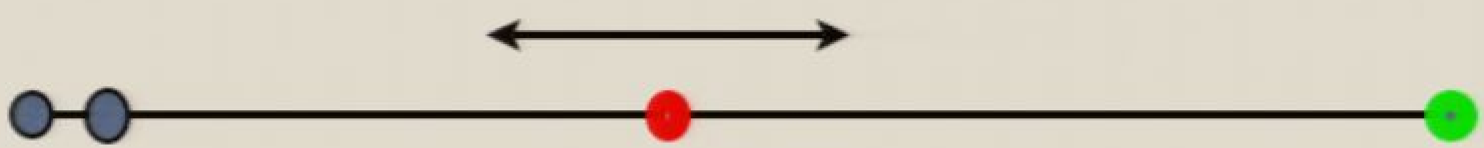

Thank you!

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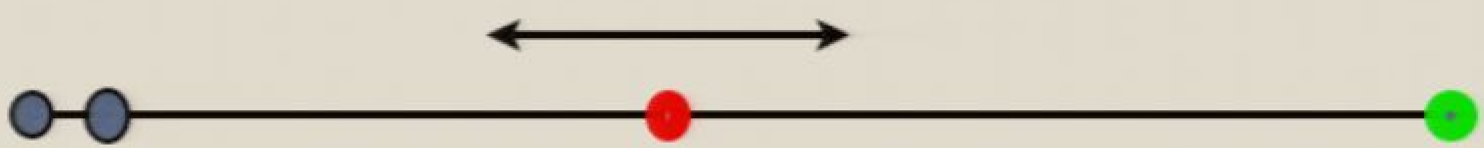

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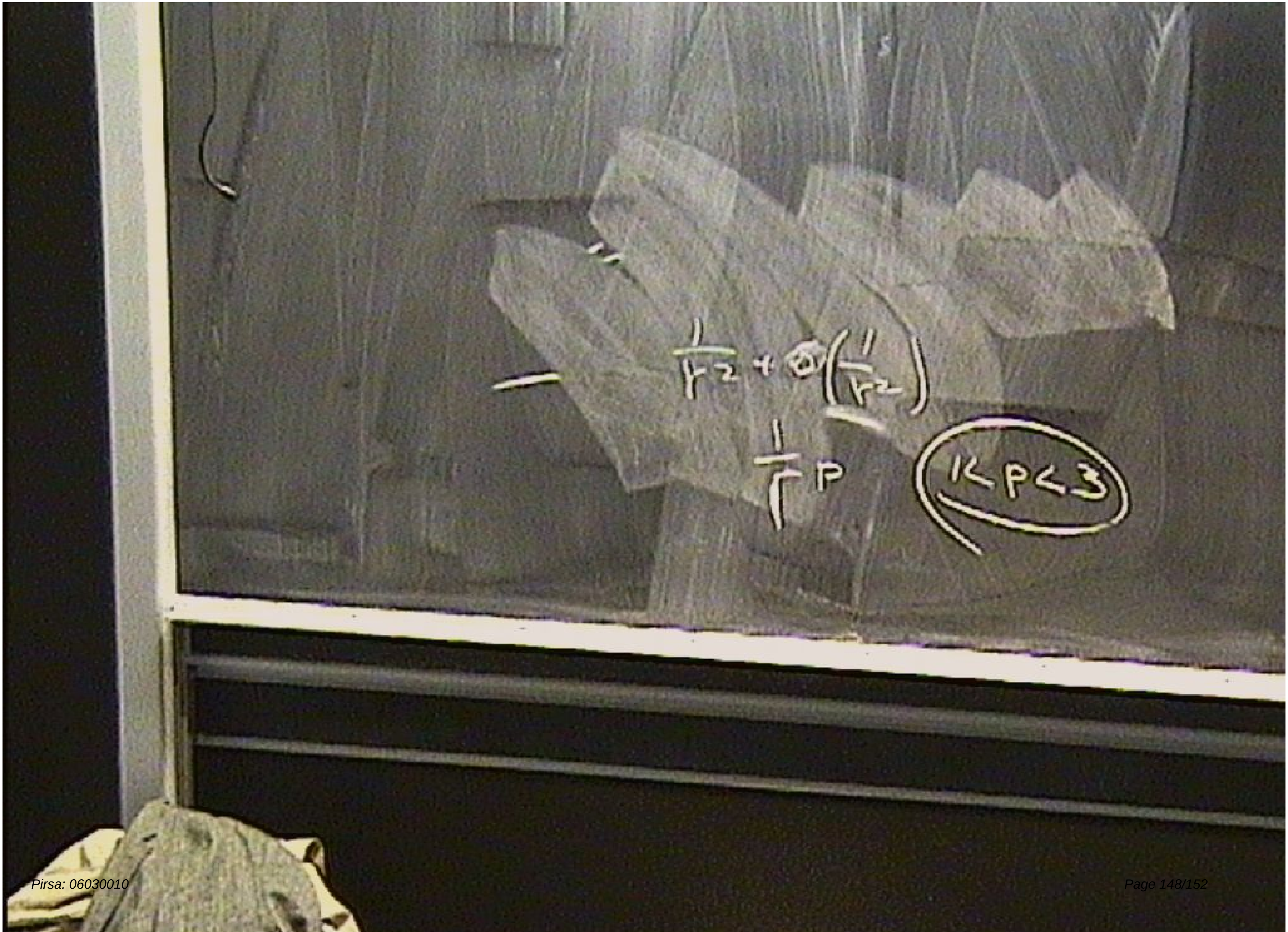
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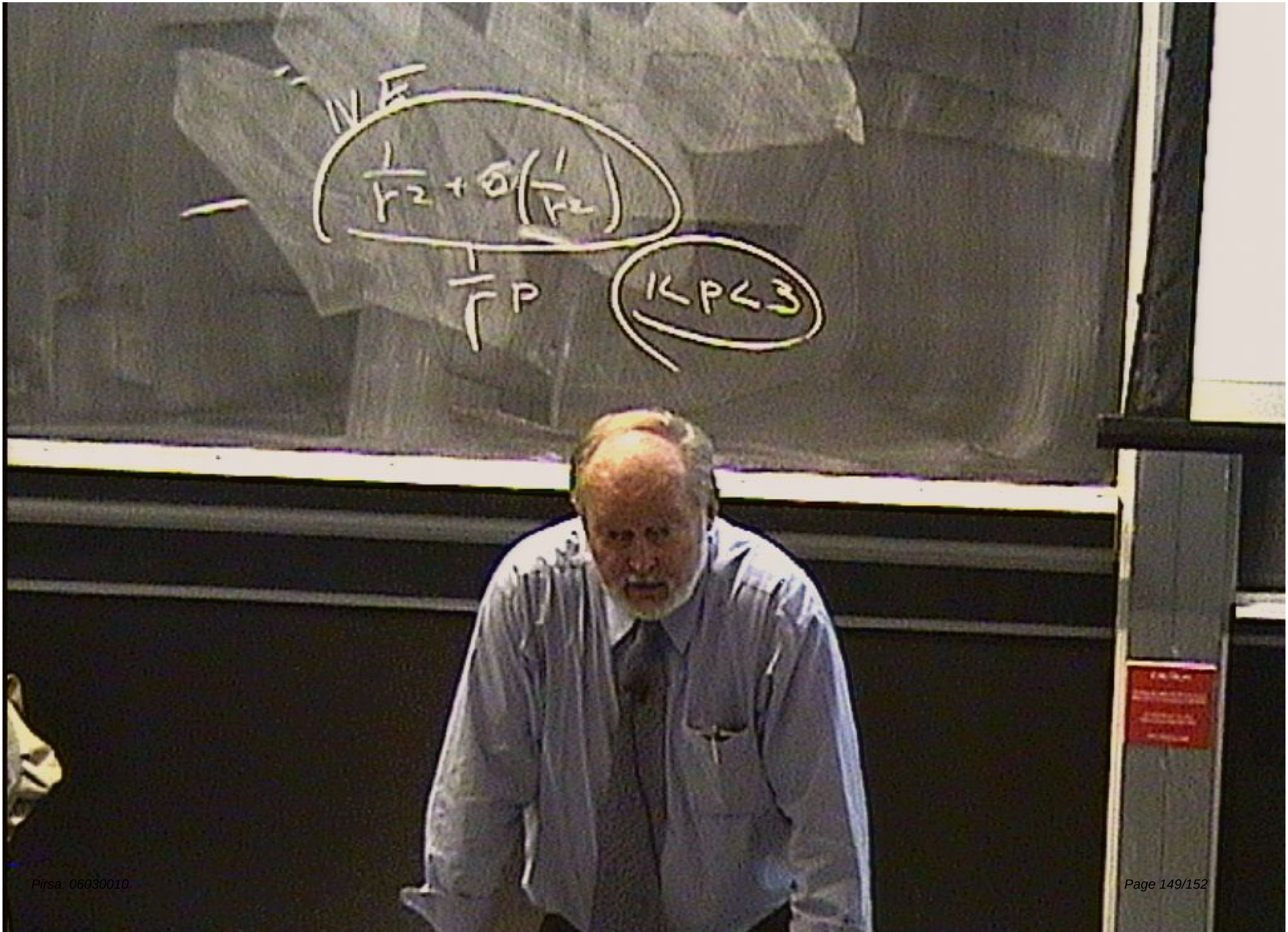
$1 < p < 3$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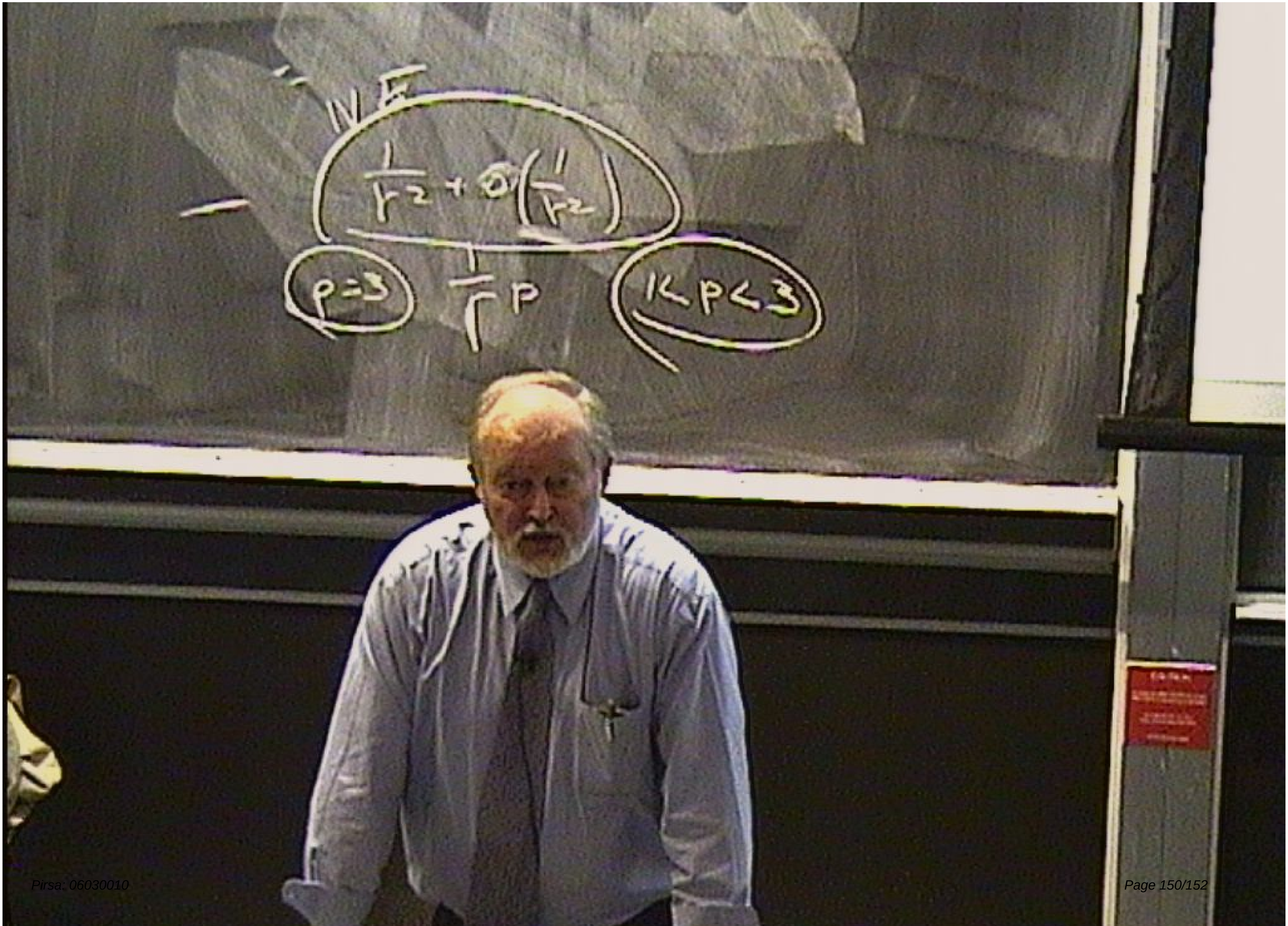
p=3
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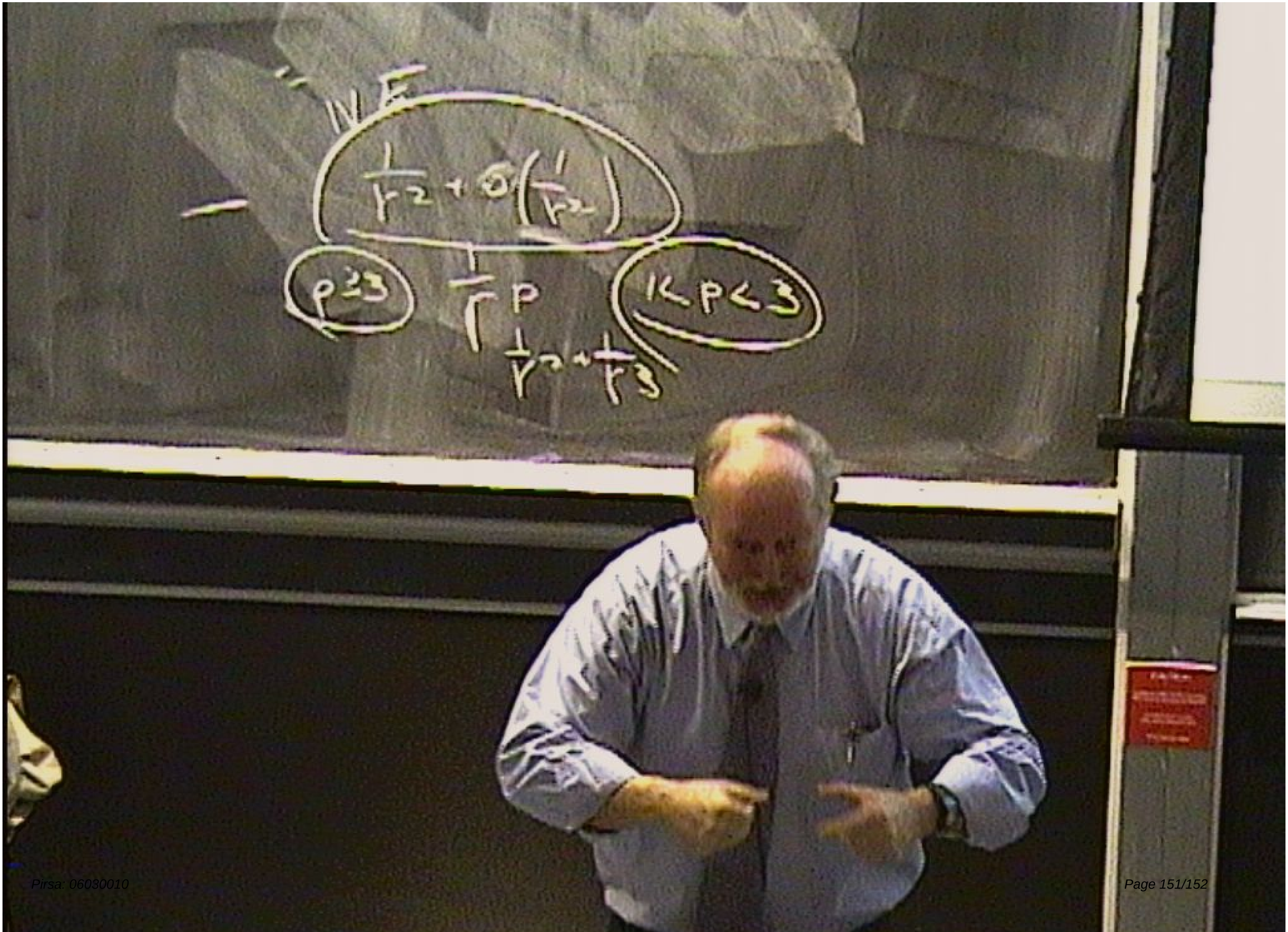
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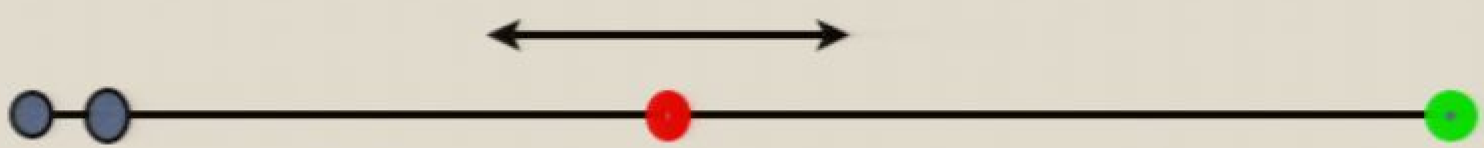

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