Title: GHZ correlations are just a bit nonlocal

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Abstract: The amount of nonlocality in the GHZ state can be quantified by determining how much classical communication is required to bring a local-hidden-variable model into agreement with the predictions of quantum mechanics. It turns out that one bit suffices, and, of course, nothing less will do. I will discuss generalizations of this result to graph states and its relation to the stabilizer formalism.

GHZ correlations are just a bit nonlocal

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Perimeter Institute
2006 March 8

APS Topical Group on Quantum Information, Concepts, and Computation

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Locality, realism, or nihilism

We consider the consequences of the observed violations of Bell's inequalities. Two common responses are (i) the rejection of realism and the retention of locality and (ii) the rejection of locality and the retention of realism. Here we critique response (i). We argue that locality contains an implicit form of realism, since in a worldview that embraces locality, spacetime, with its usual, fixed topology, has properties independent of measurement. Hence we argue that response (i) is incomplete, in that its rejection of realism is only partial.

R. Y. Chiao and J. C. Garrison
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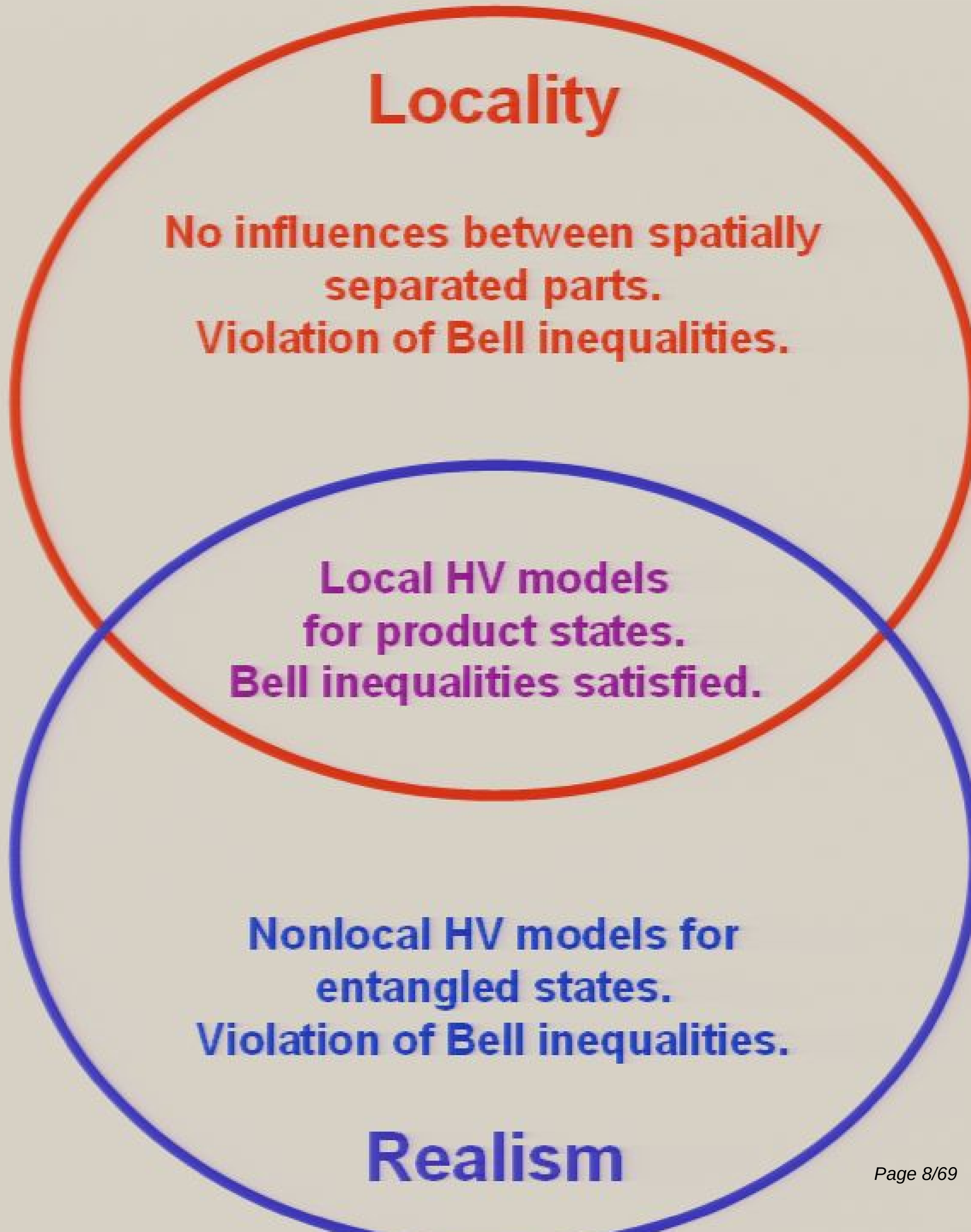
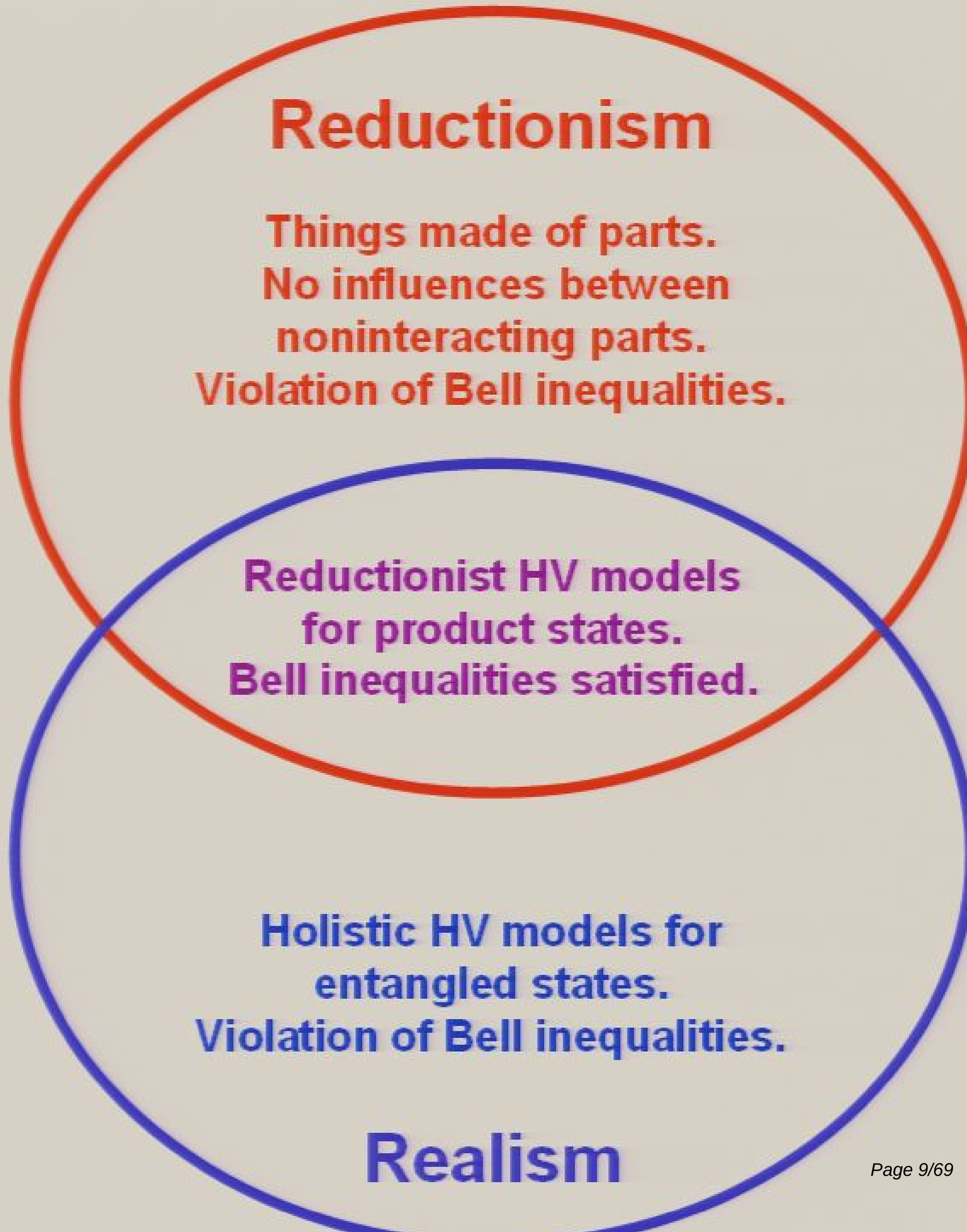
Nihilism



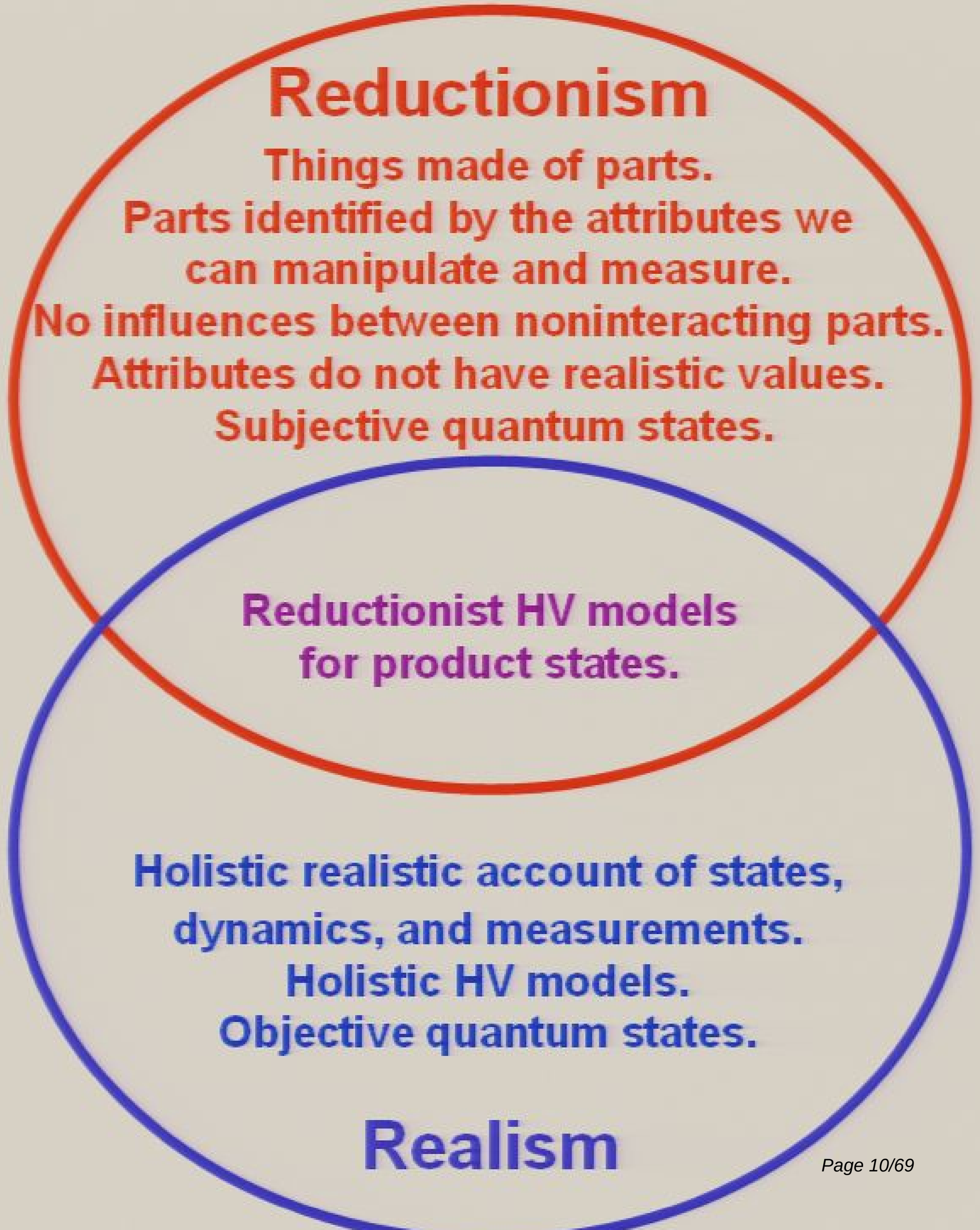
Reductionism or realism



Quantum mechanics

or

Soothing stories about a reality beneath quantum mechanics



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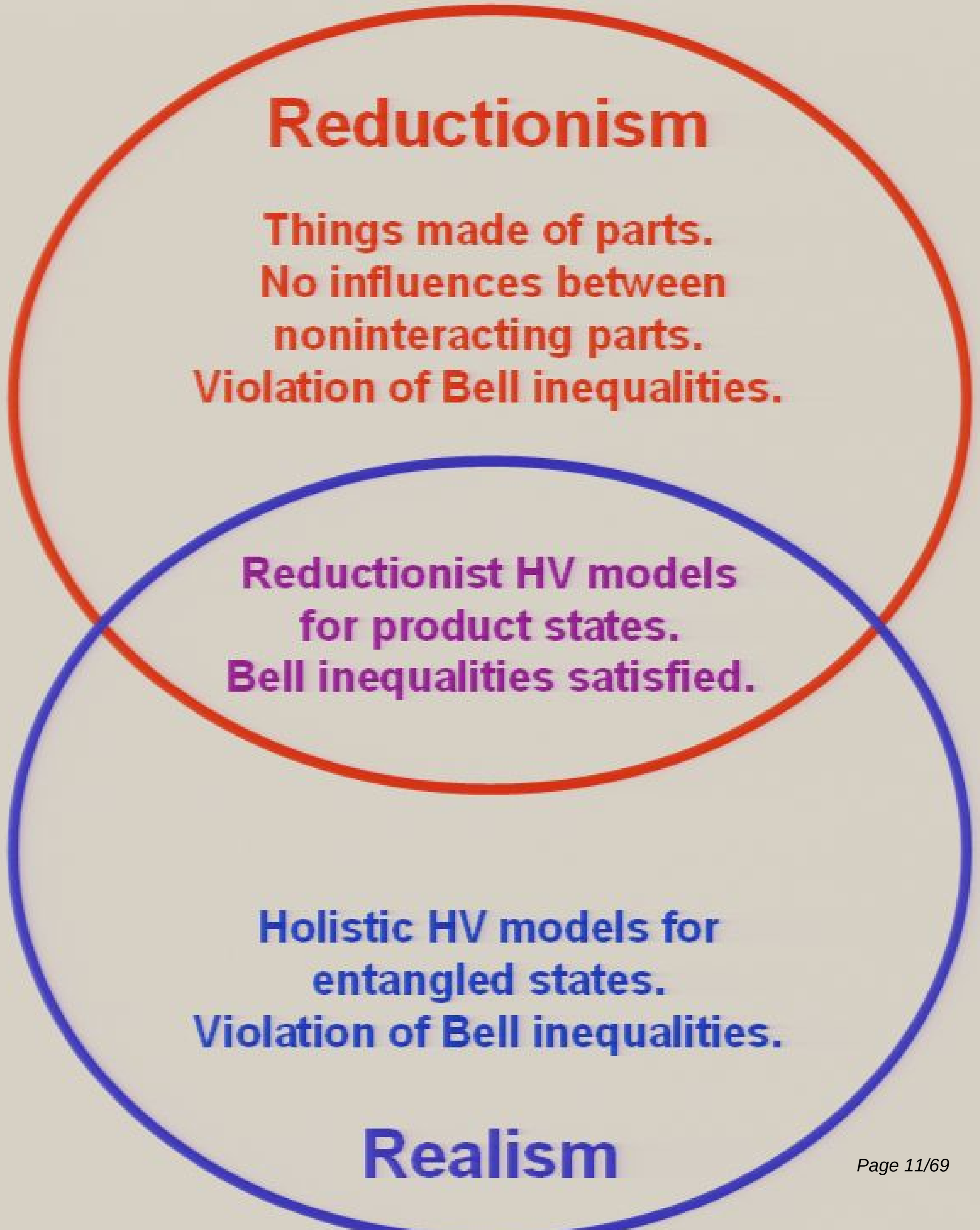

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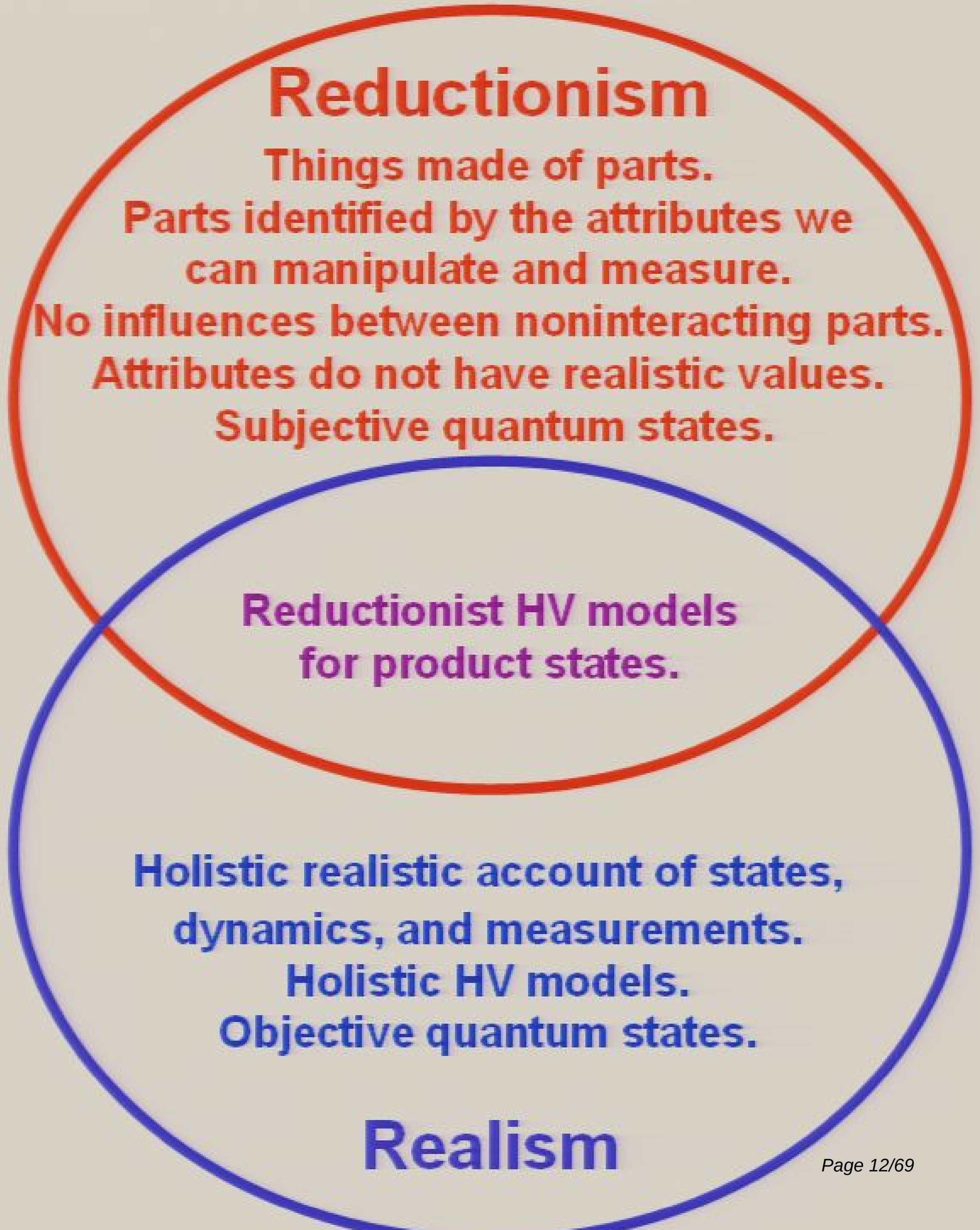
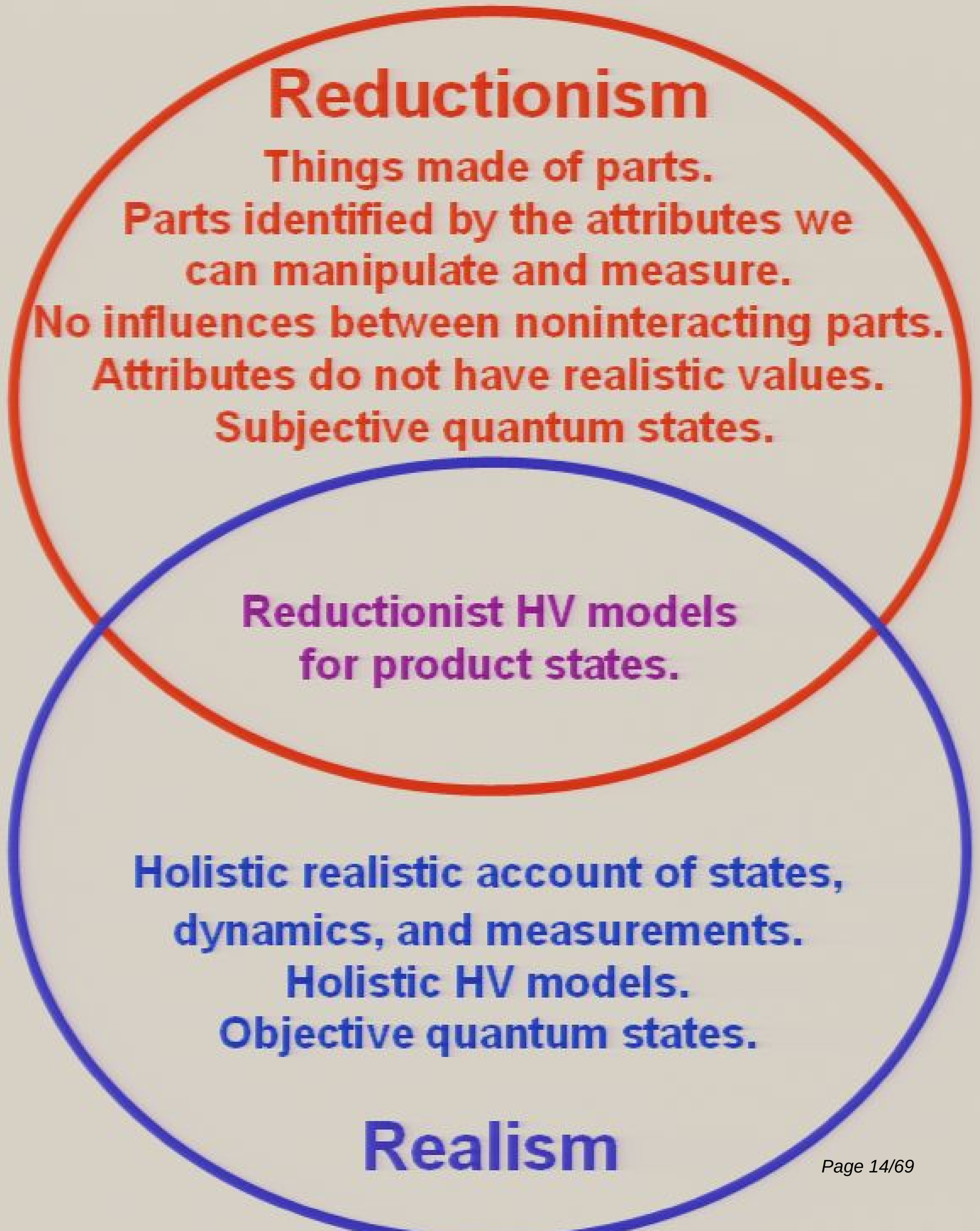


So how about a different soothing story, one that comes from quantum information science?

Quantum mechanics

or

Soothing stories about a reality beneath quantum mechanics



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The old story

A new story from quantum information?

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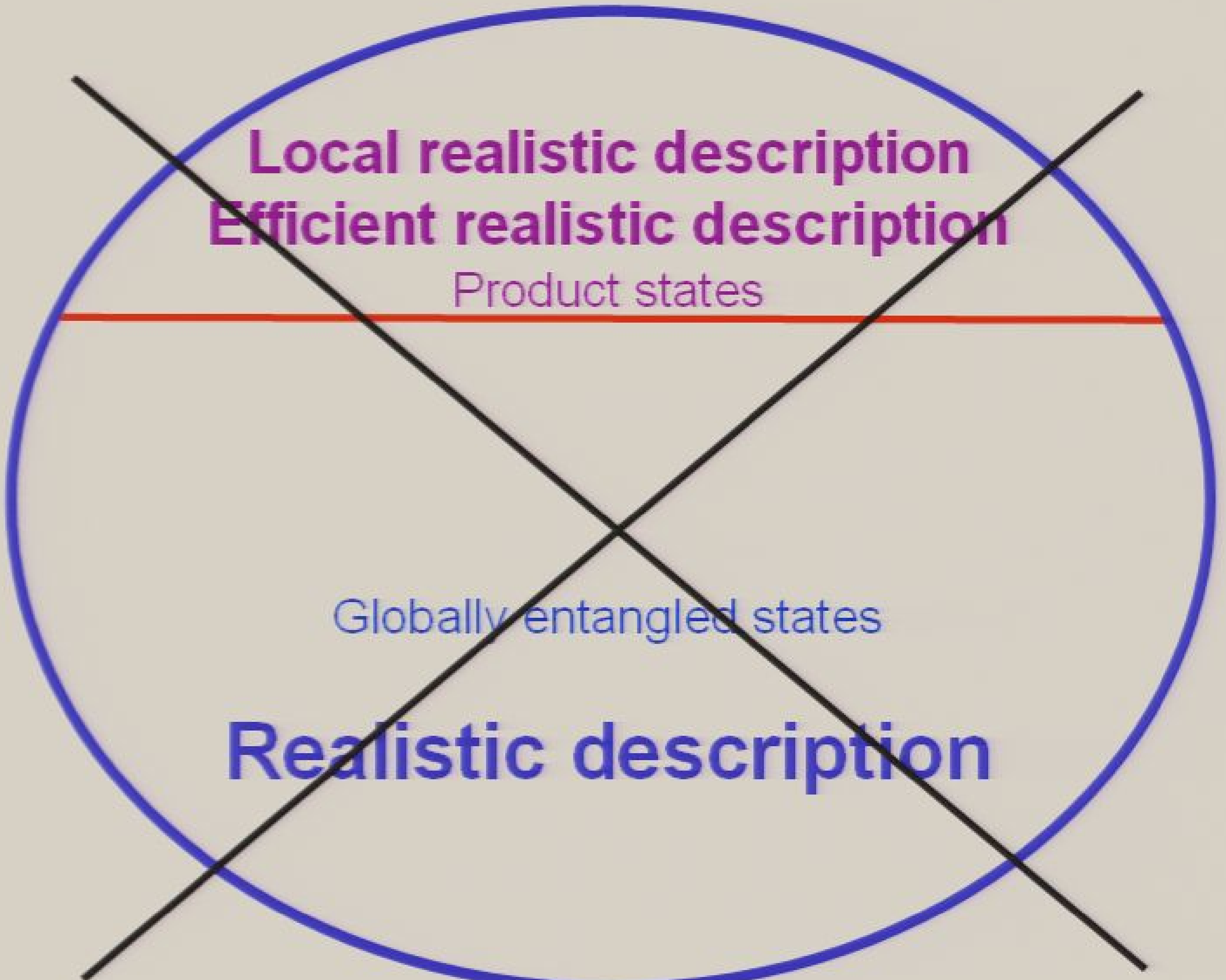
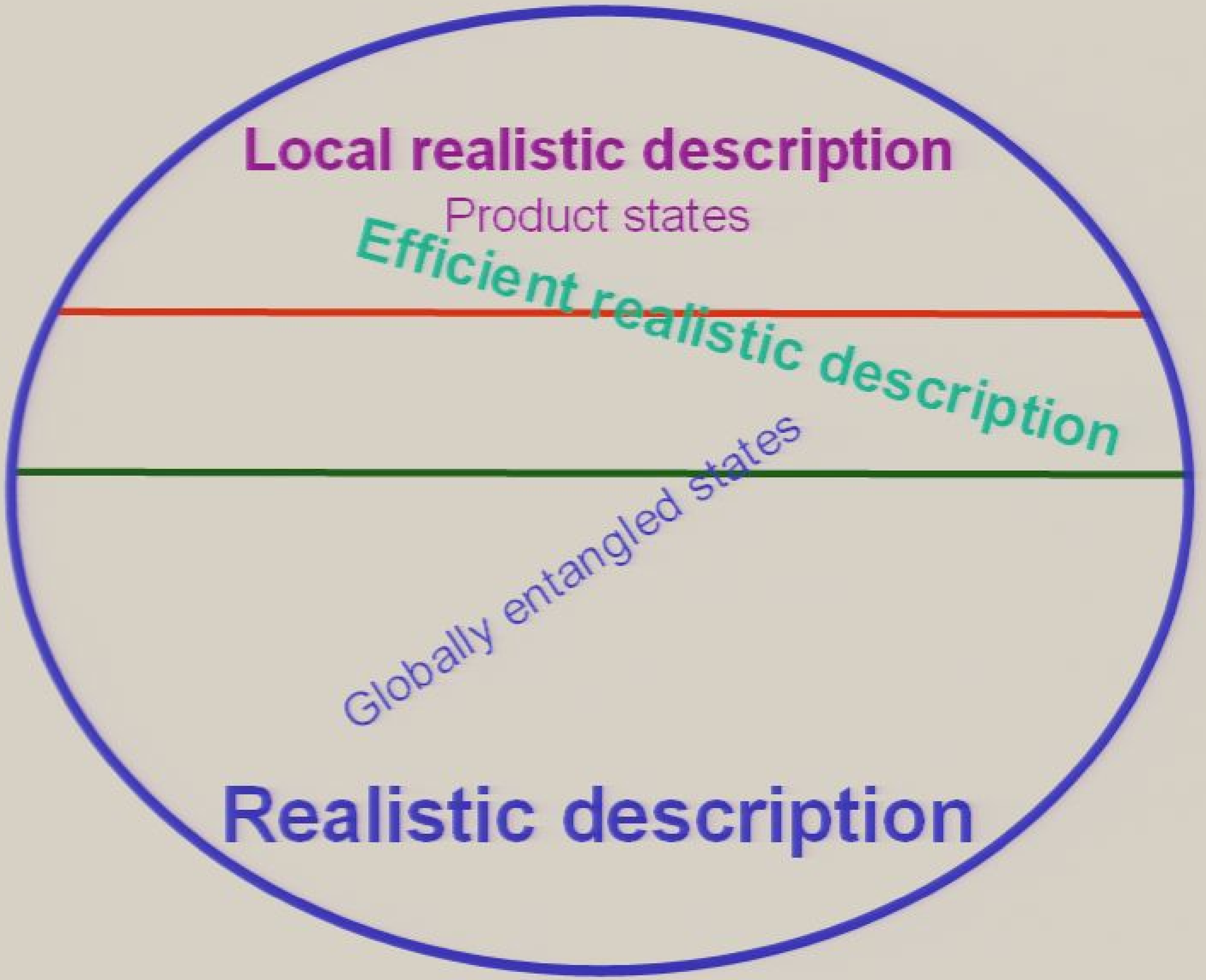


The new story from quantum information



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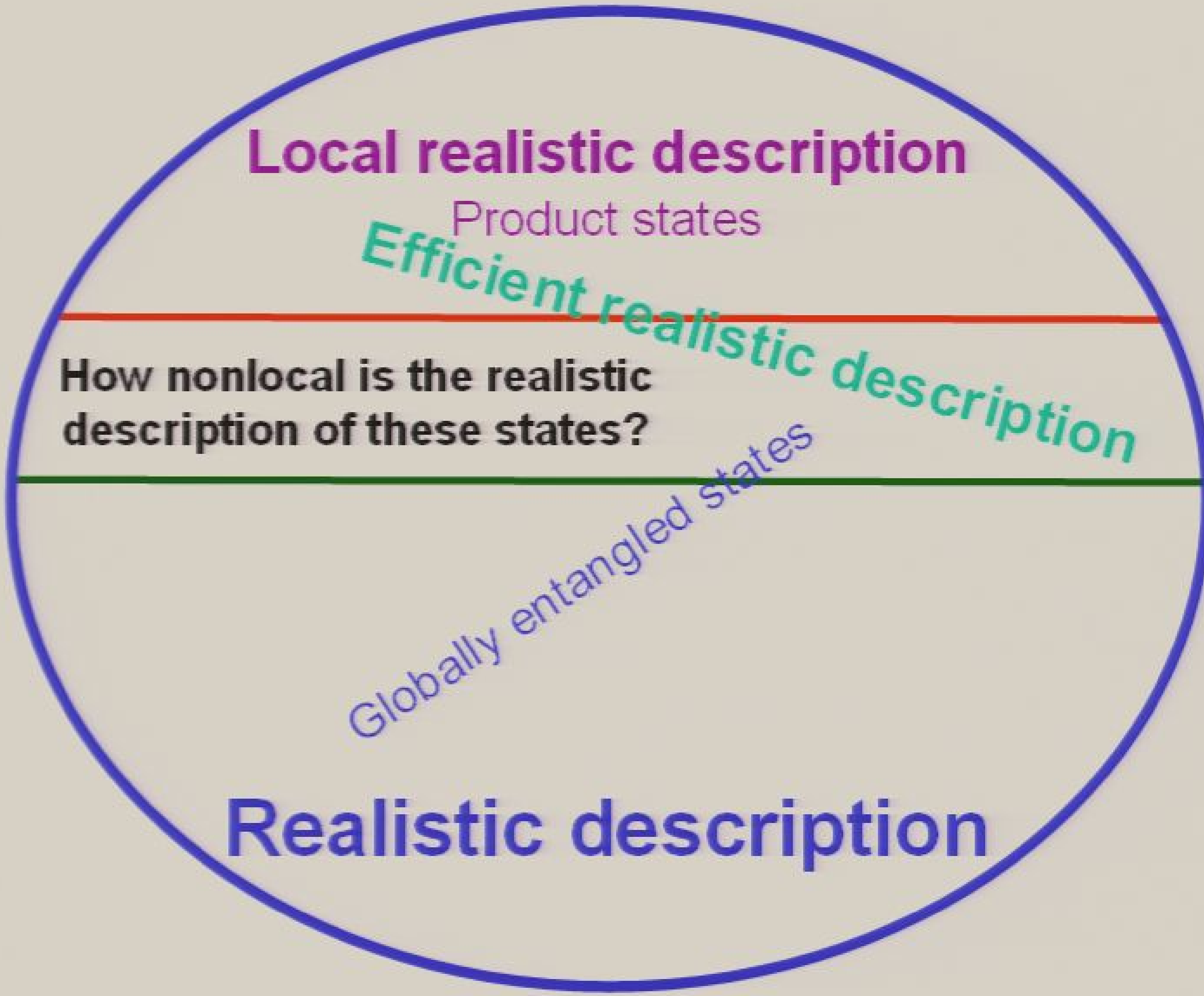
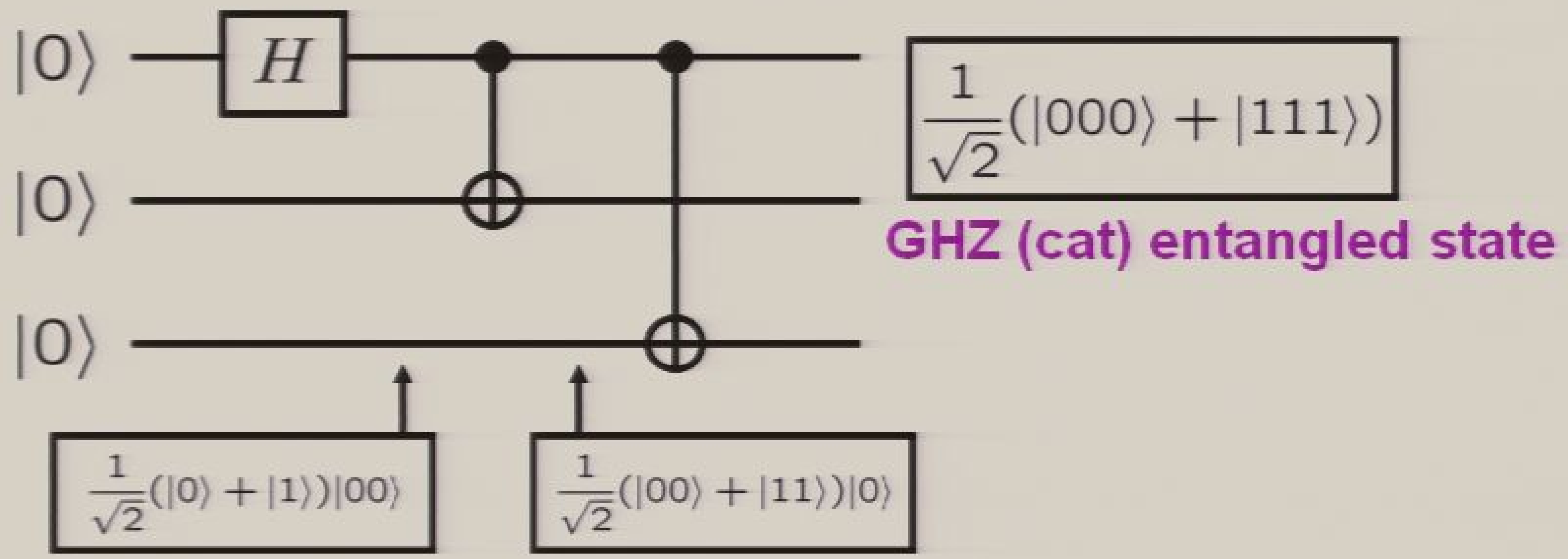
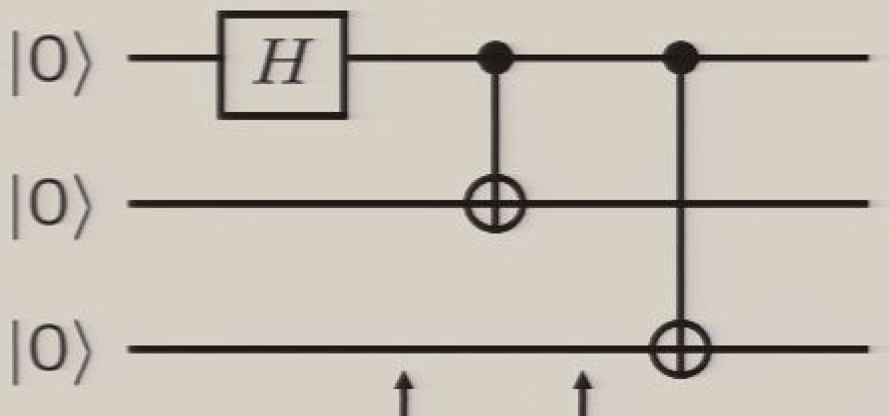


Modeling GHZ (cat) correlations



Modeling GHZ (cat) correlations

ZII
IZI
IIZ



$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$

GHZ (cat) entangled state

XXX, ZZI, ZIZ

$$S = \left\{ \begin{array}{c} III, ZZI, ZIZ, IZZ, \\ XXX, -XYY, -YXY, -YYX \end{array} \right\}$$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$

XII, IZI, IIZ

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$

XXI, ZZI, IIZ

Stabilizer formalism

Modeling GHZ (cat) correlations

Measure *XYY*, *YXY*, and *YYX*: All yield result -1.
Local realism implies *XXX* = -1.
Quantum mechanics says *XXX* = +1.

ZII
IZI
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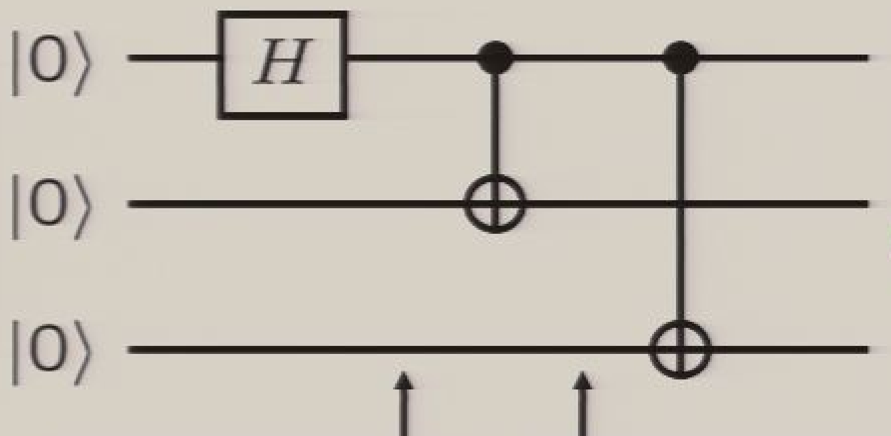

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

XII, IZI, IIZ

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

XXI, ZZI, IIZ

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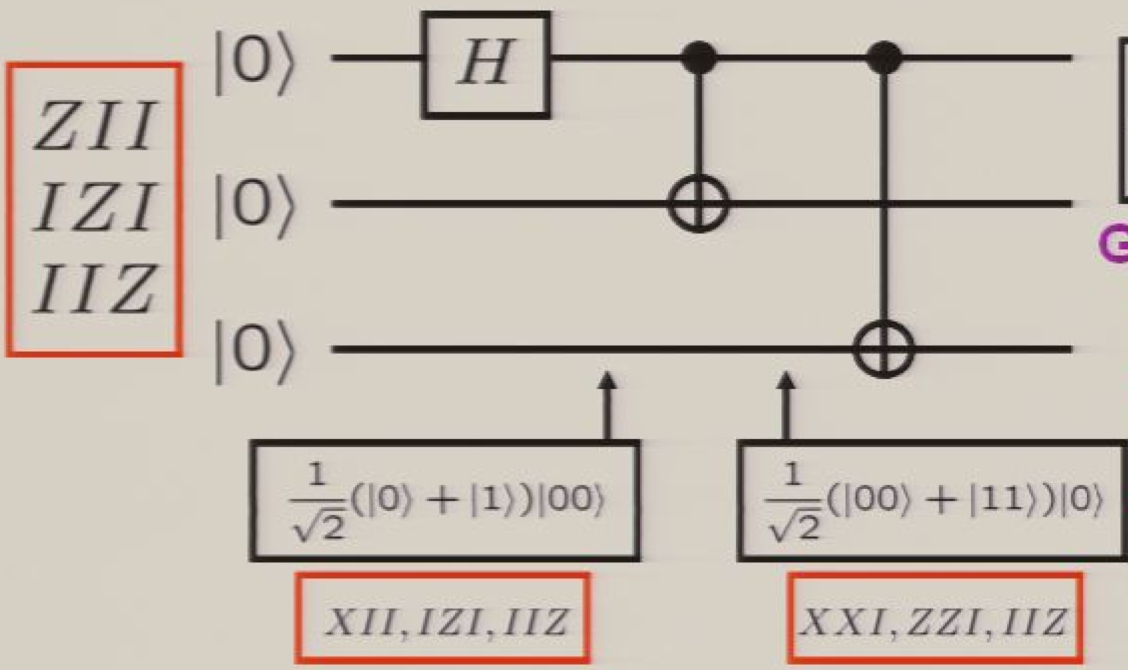

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

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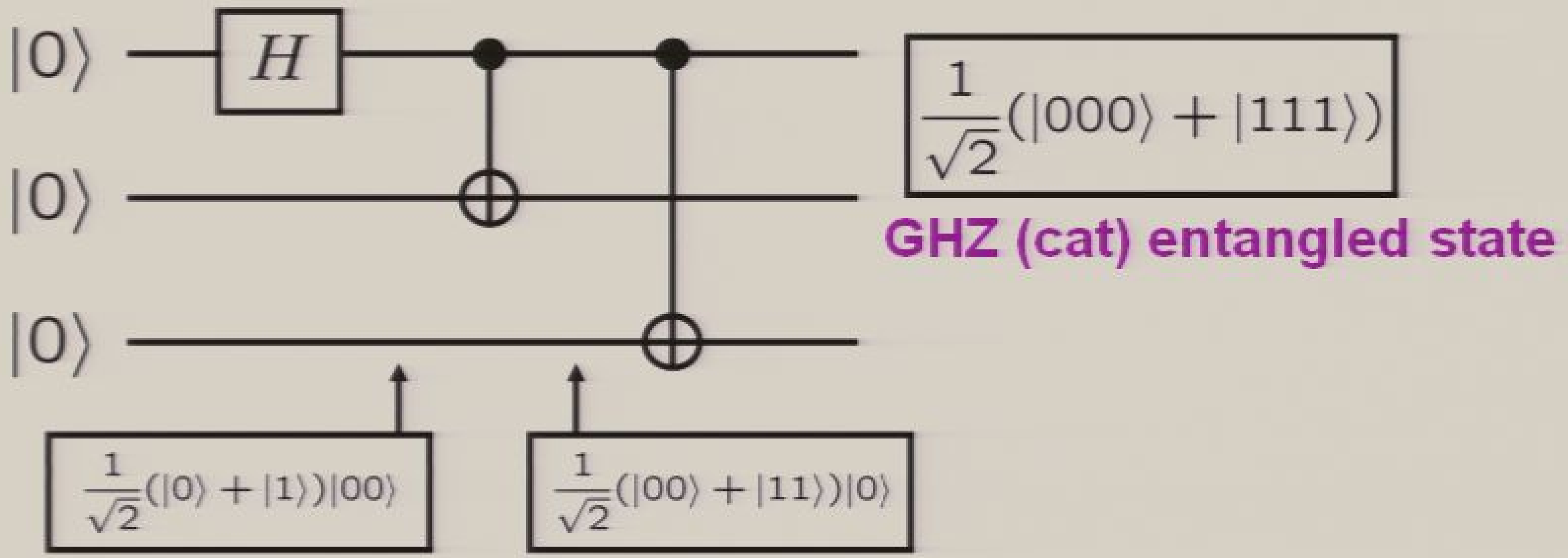
Stabilizer formalism

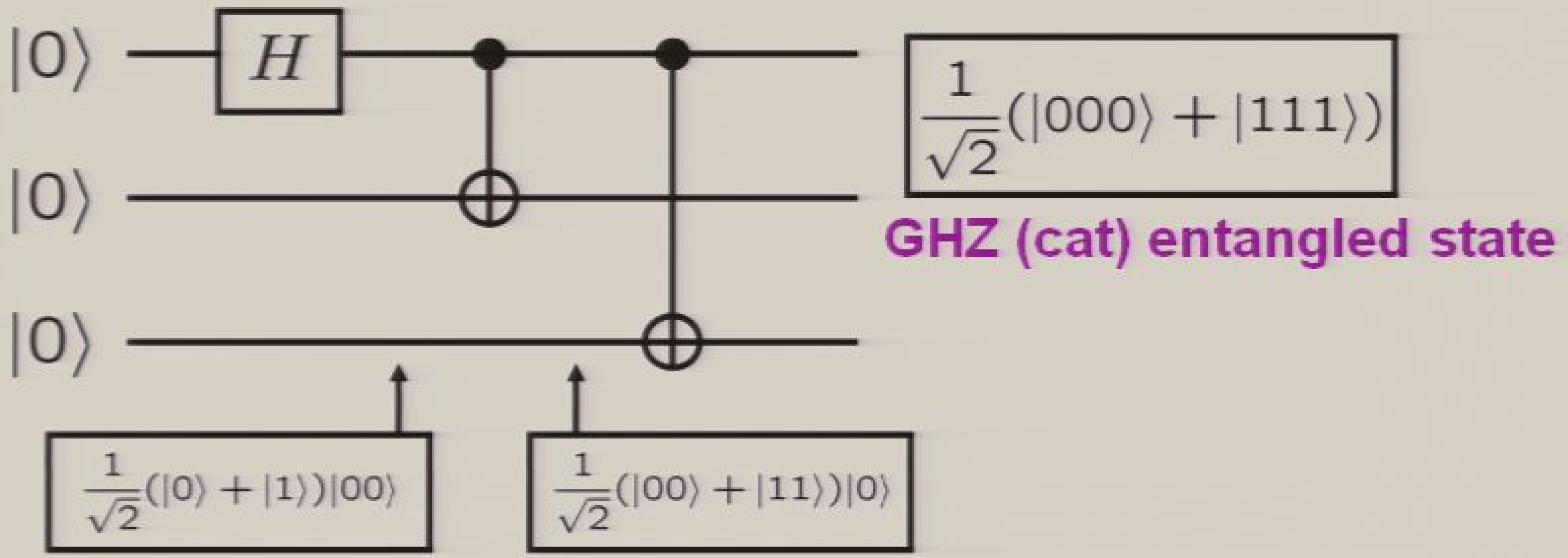
Efficient **(nonlocal) realistic description of** ***states, dynamics,*** **and** ***measurements***

Modeling GHZ (cat) correlations



Modeling GHZ (cat) correlations

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



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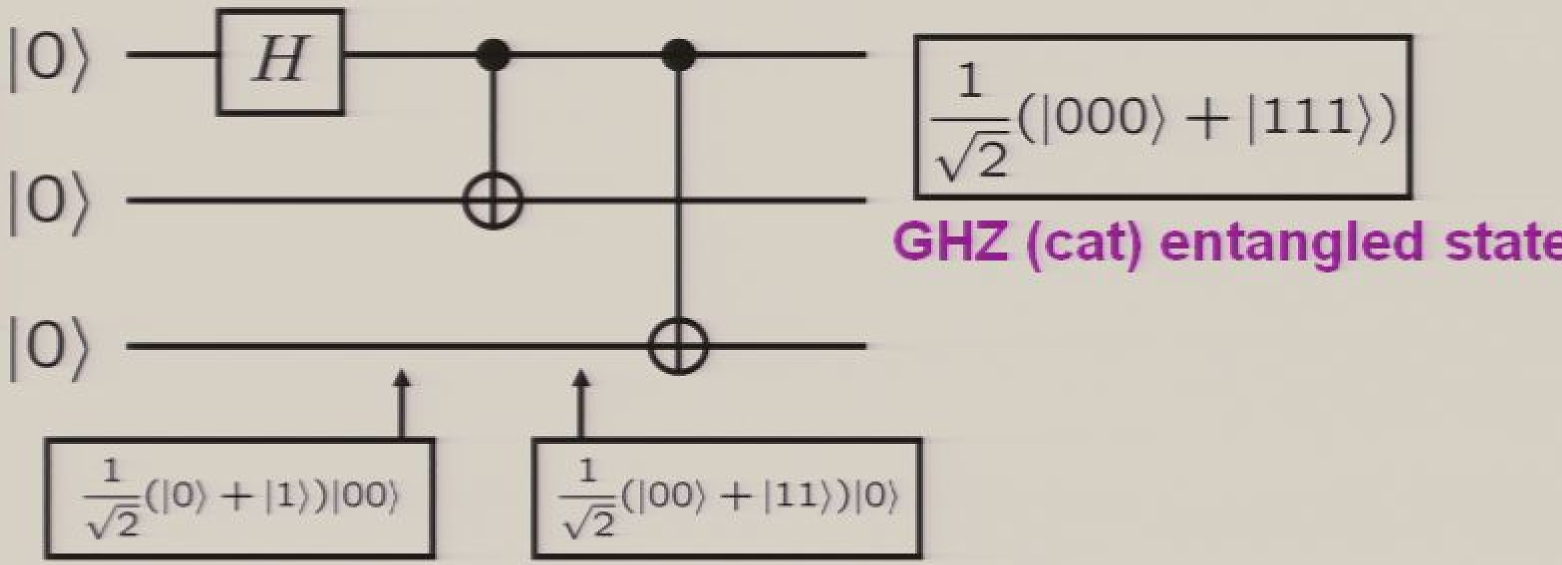

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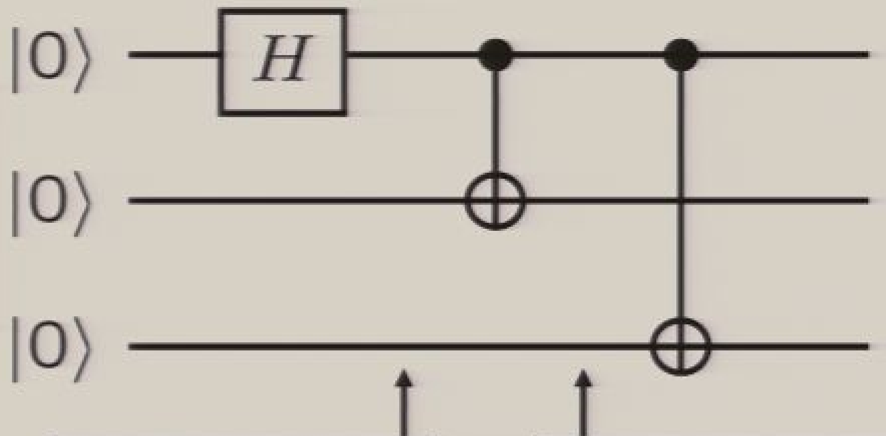


$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

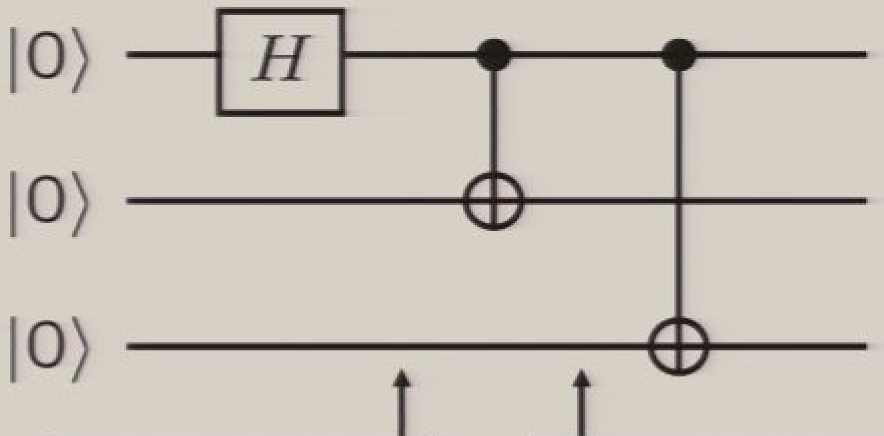
x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

Modeling GHZ (cat) correlations

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



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x	y	z
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r_2	r_2	1
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$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

Modeling GHZ (cat) correlations

ZZI = *ZIZ* = *IZZ* = *XXX* = +1; *XYY* = *YXY* = *YYX* = -1.
To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether *Y* is measured on qubit 2; party 1 flips her result if *Y* is measured on either 1 or 2.

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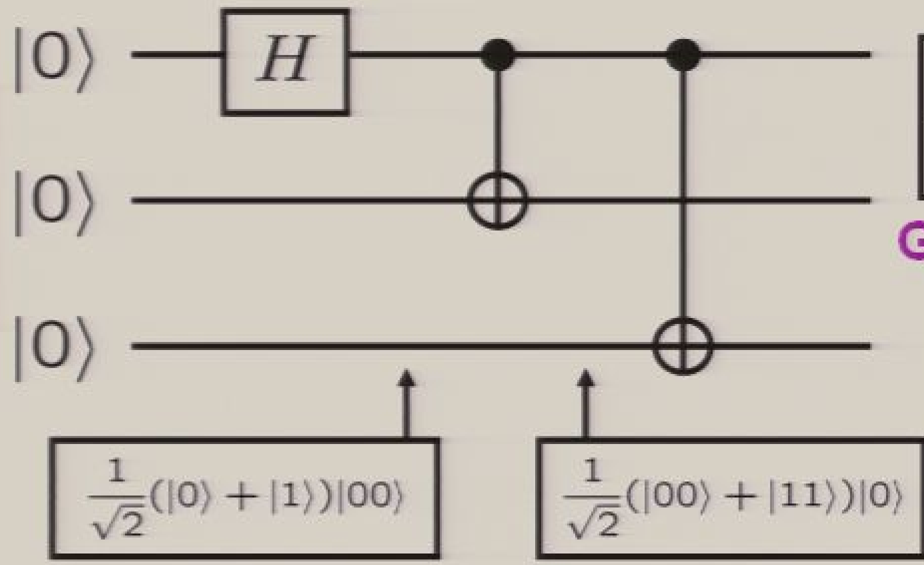
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

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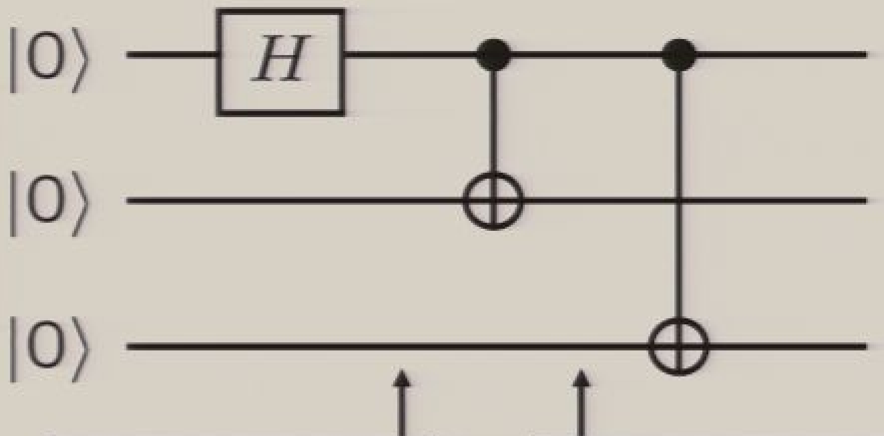
r_2	r_2	1
r_3	r_3	1



$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

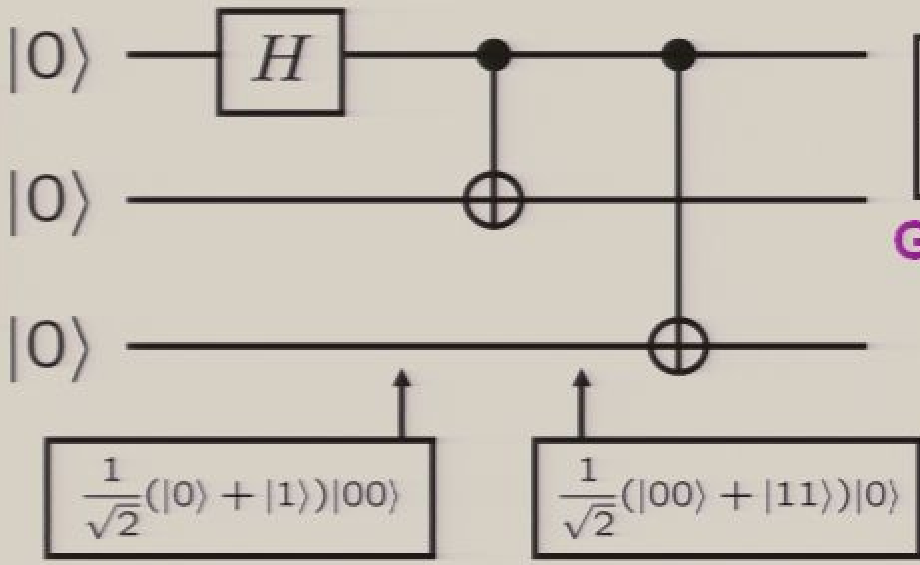
x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

When party 1 flips her result, this can be thought of as a nonlocal disturbance that passes from qubit 2 to qubit 1. The communication protocol quantifies the required amount of nonlocality.

Modeling GHZ (cat) correlations

$ZZI = ZIZ = IZZ = XXX = +1$; $XYY = YXY = YYX = -1$. To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether Y is measured on qubit 2; party 1 flips her result if Y is measured on either 1 or 2.

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

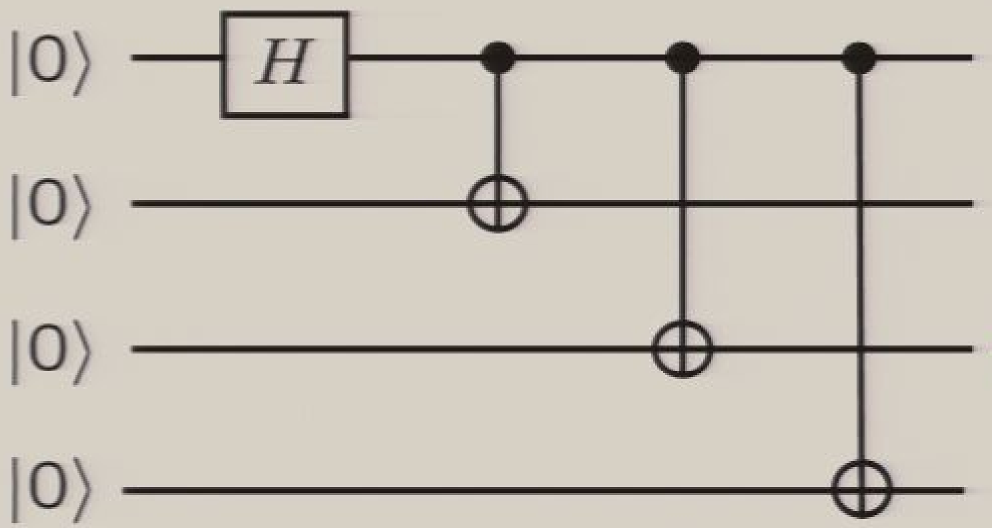
x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

T. E. Tessier, C. M. Caves, I. H. Deutsch, B. Eastin, and D. Bacon, ``Optimal classical-communication-assisted local model of n-qubit Greenberger-Horne-Zeilinger correlations,'' Phys. Rev. A **72**, 032305 (2005).

For N-qubit GHZ states, this same procedure gives a *local realistic* description, aided by N-2 bits of *classical communication* (provably minimal), of *states, dynamics*, and *measurements* (of Pauli products).

Communication-assisted LHV model

Modeling GHZ (cat) correlations



$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

Modeling GHZ (cat) correlations

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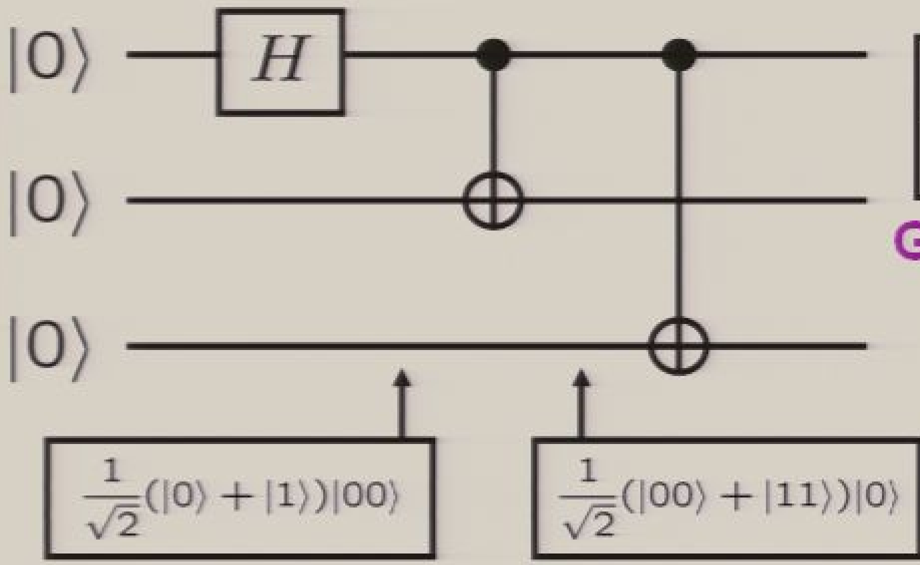
To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether *Y* is measured on qubit 2; party 1 flips her result if *Y* is measured on either 1 or 2.

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
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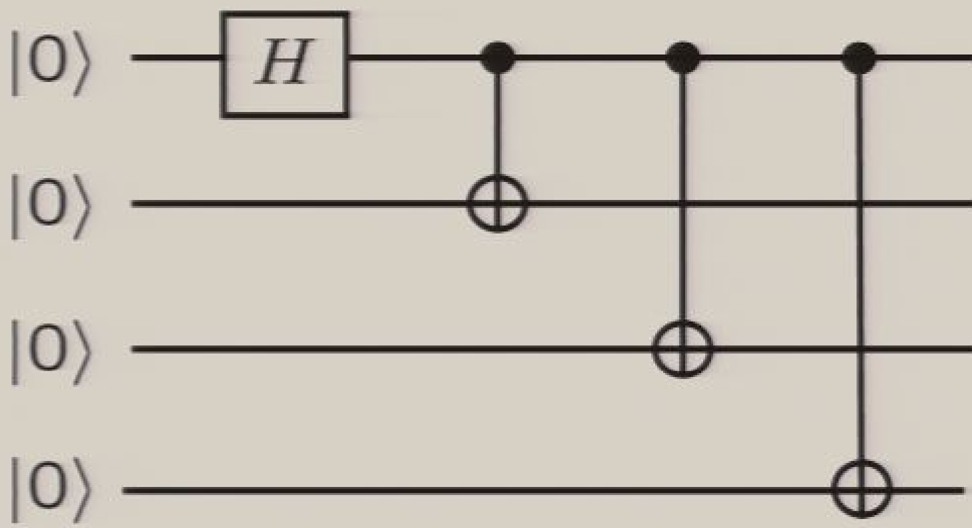
Communication-assisted LHV model

Modeling GHZ (cat) correlations



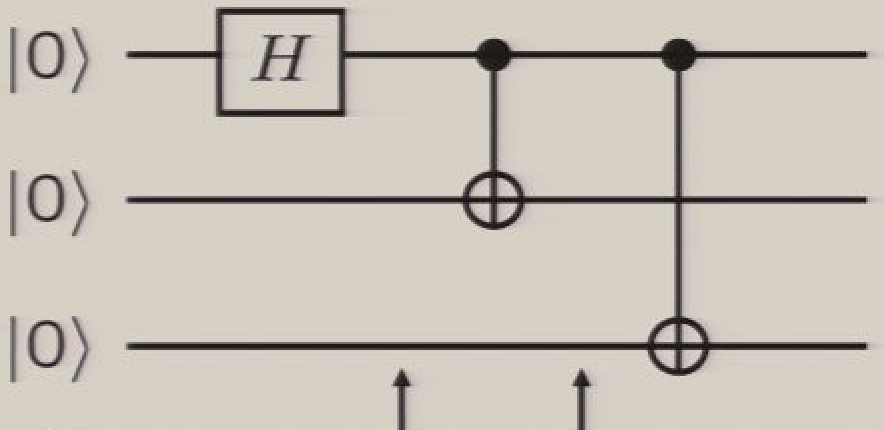
$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

Modeling GHZ (cat) correlations

***ZZI* = *ZIZ* = *IZZ* = *XXX*= +1; *XYY* = *YXY* = *YYX* = -1.**
To get correlations right requires 1 bit of classical communication: party 2 tells party 1 whether *Y* is measured on qubit 2; party 1 flips her result if *Y* is measured on either 1 or 2.

x	y	z
r_1	$-r_1$	1
r_2	r_2	1
r_3	r_3	1



$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

GHZ (cat) entangled state

x	y	z
$r_2 r_3$	$r_1 r_2 r_3$	r_1
r_2	$r_1 r_2$	r_1
r_3	$r_1 r_3$	r_1

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$$

x	y	z
1	r_1	r_1
r_2	r_2	1
r_3	r_3	1

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)|0\rangle$$

x	y	z
r_2	$r_1 r_2$	r_1
r_2	$r_1 r_2$	r_1
r_3	r_3	1

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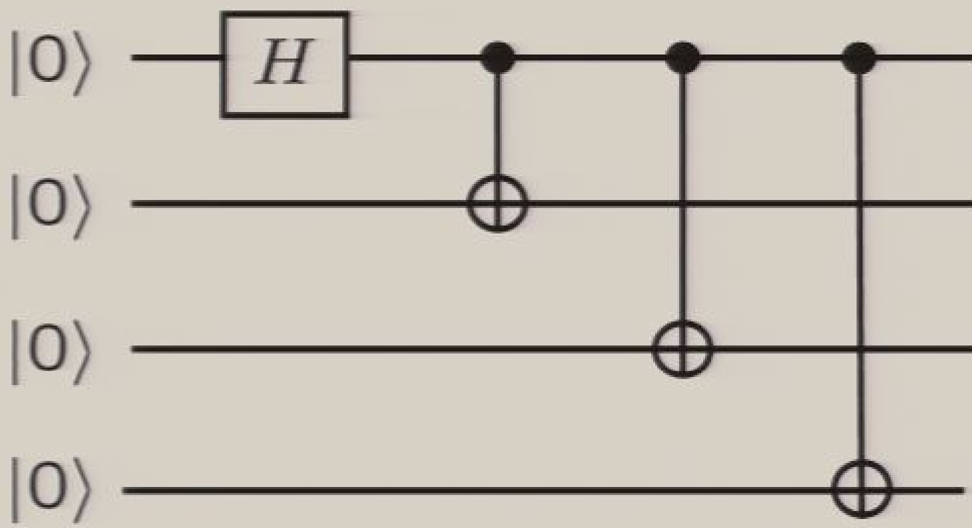
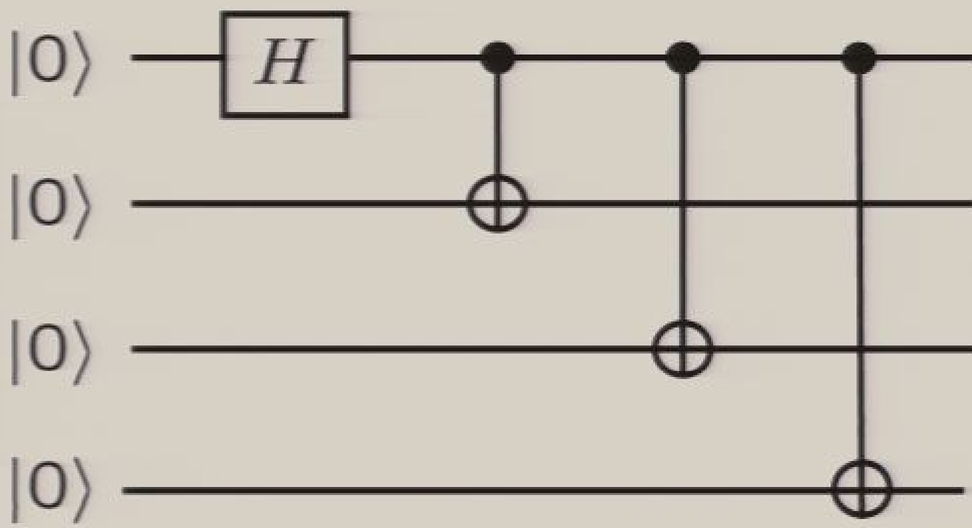


$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

Modeling GHZ (cat) correlations

$$\begin{matrix} ZIII \\ IZII \\ IIZI \\ IIIZ \end{matrix}$$



$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

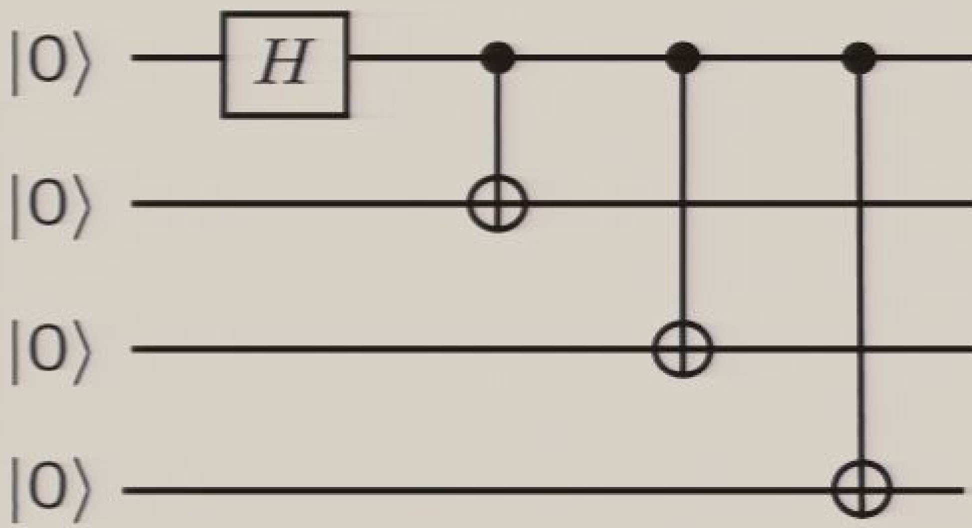
$$XXXX, ZZII, ZIZI, ZIIZ$$

$$S = \left\{ \begin{matrix} IIII, ZZII, ZIZI, ZIIZ, IZZI, IZIZ, IIZZ \\ XXXX, -XXYY, -XYXY, -XYYX, \\ -YXXY, -YXYX, -YYXX, YYYY \end{matrix} \right\}$$

Modeling GHZ (cat) correlations

Assume 1 bit of communication between qubits 1 and 2.
Let $S=XX$ and $T=XY$ be Pauli products for qubits 1 and 2; then we have $SYY=TXY=TYX = -1$.
Local realism implies $SXX = -1$.
Quantum mechanics says $SXX = +1$.

$$ZIII$$
$$IZII$$
$$IIZI$$
$$IIIZ$$



$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

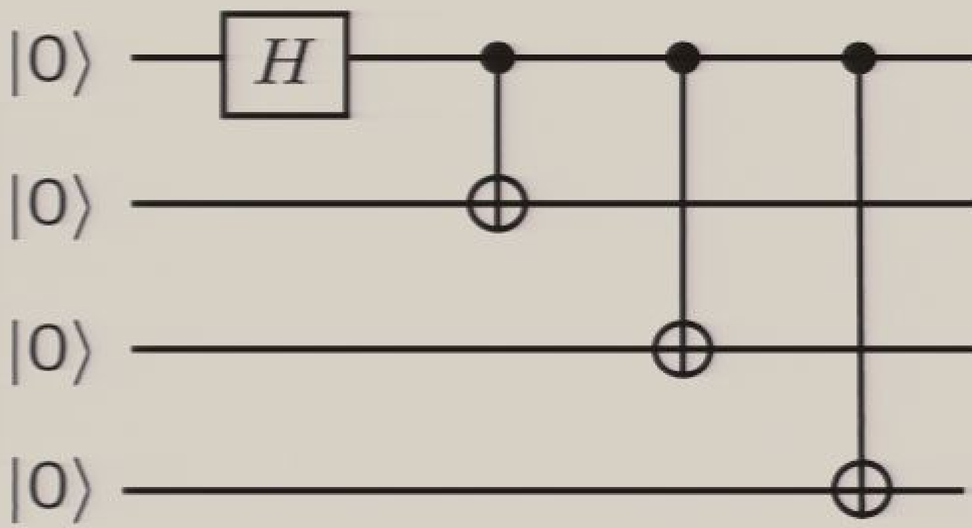
$$XXXX, ZZII, ZIZI, ZIIZ$$

$$S = \left\{ \begin{matrix} IIII, ZZII, ZIZI, ZIIZ, IZZI, IZIZ, IIZZ \\ XXXX, -XXYY, -XYXY, -XYYX, \\ -YXXY, -YXYX, -YYXX, YYYY \end{matrix} \right\}$$

Modeling GHZ (cat) correlations

Assume 1 bit of communication between qubits 1 and 2. Let *S*=*XX* and *T*=*XY* be Pauli products for qubits 1 and 2; then we have *SYY*=*TXY*=*TYX* = -1.
Local realism implies *SXX* = -1.
Quantum mechanics says *SXX* = +1.

$ZIII$
$IZII$
$IIZI$
$IIIZ$



$$\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$$

4-qubit GHZ entangled state

$XXXX, ZZII, ZIZI, ZIIZ$

$$S = \left\{ \begin{matrix} IIII, ZZII, ZIZI, ZIIZ, IZZI, IZIZ, IIZZ \\ XXXX, -XXYY, -XYXY, -XYYX, \\ -YXXY, -YXYX, -YYXX, YYYY \end{matrix} \right\}$$

For *N*-qubit GHZ states, a simple extension of this argument shows that *N-2* bits of *classical communication* is the minimum required to mimic the predictions of quantum mechanics for measurements of Pauli products.

Clifford circuits: Gottesman-Knill theorem

- N qubits in an initial product state in Z basis
- Allowed gates: Pauli operators X, Y, and Z, plus H, S, and C-NOT
- Allowed measurements: Products of Pauli operators

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Global entanglement

but

Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

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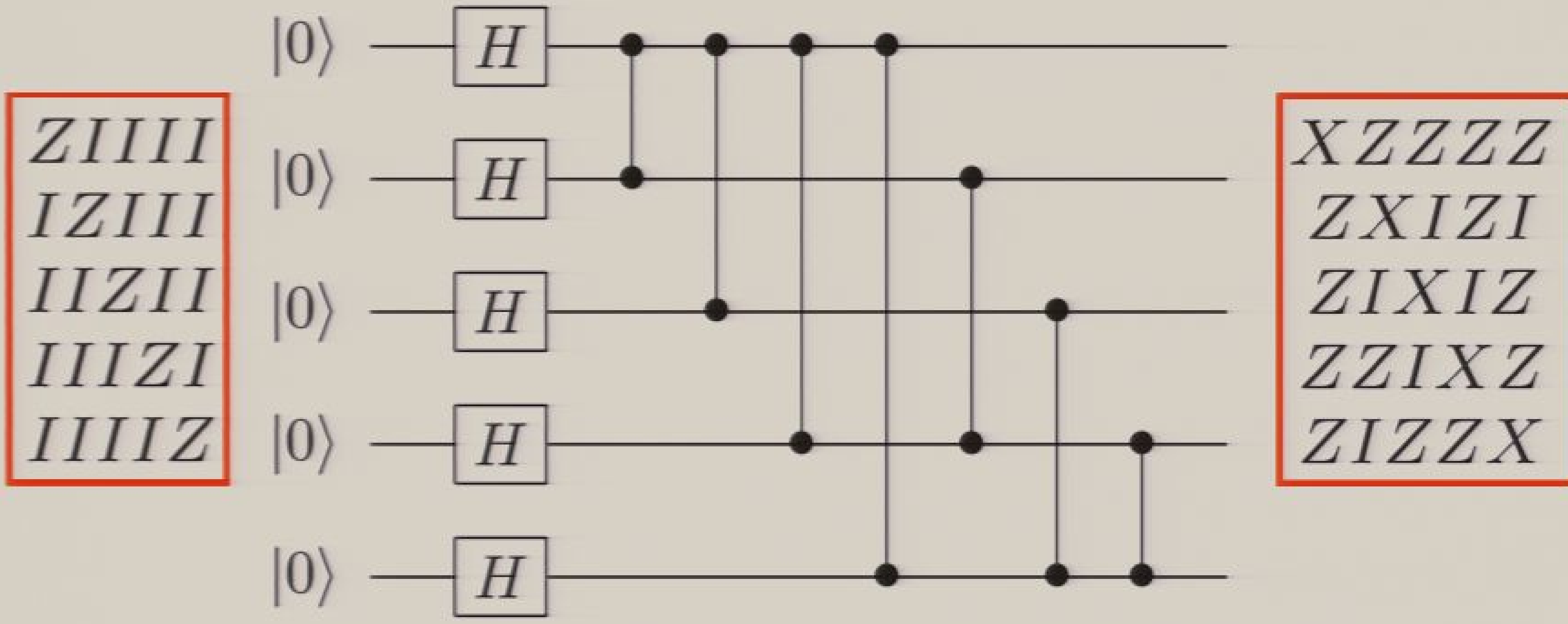
Global entanglement

but

Efficient (nonlocal) realistic description of states, dynamics, and measurements (in terms of stabilizer generators)

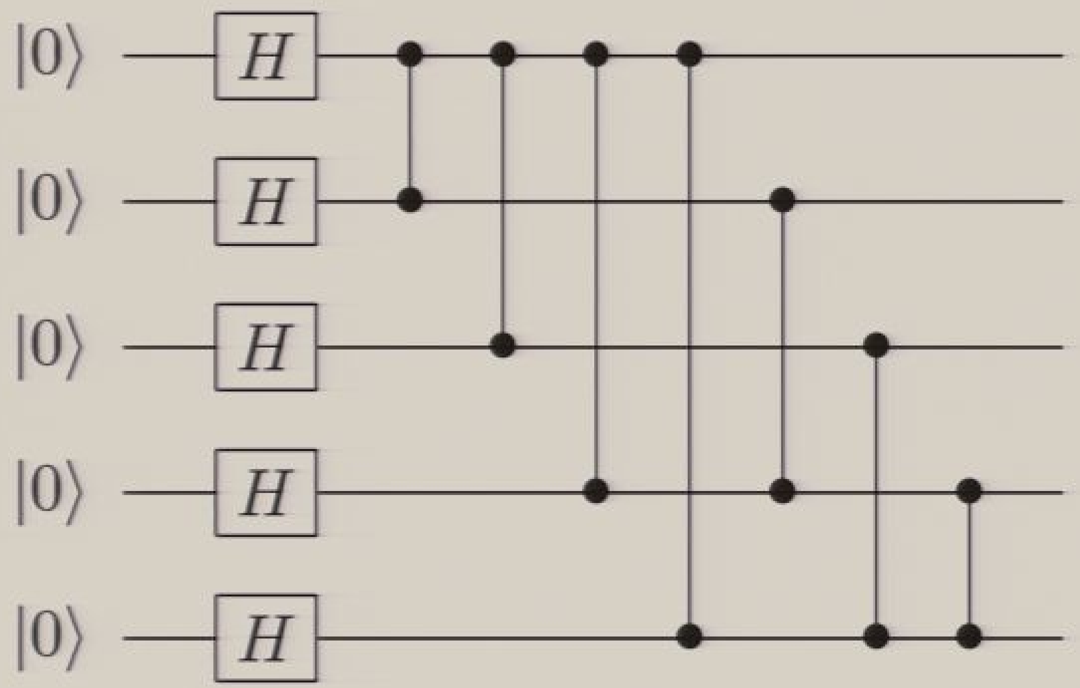
This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently and thus does not provide an exponential speedup for quantum computation.

Graph states

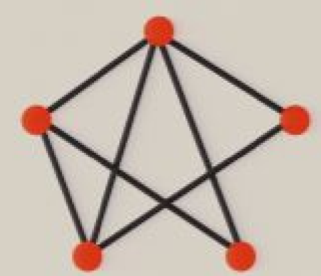


Graph states

$ZIIII$
$IZIII$
$IIZII$
$IIIZI$
$IIIIZ$

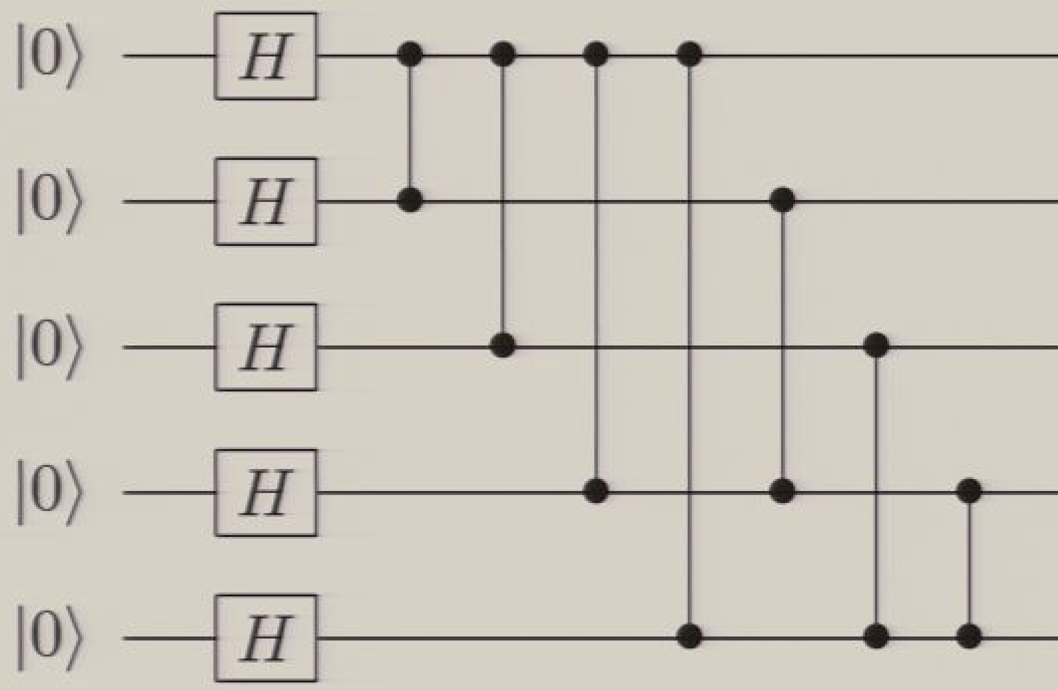


$XZZZZ$
$ZXIZI$
$ZIXIZ$
$ZZIXZ$
$ZIZZX$



Graph states

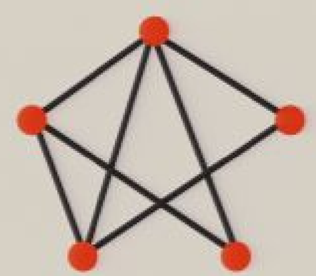
$$\begin{matrix} ZIIII \\ IZIII \\ IIZII \\ IIIZI \\ IIIIZ \end{matrix}$$



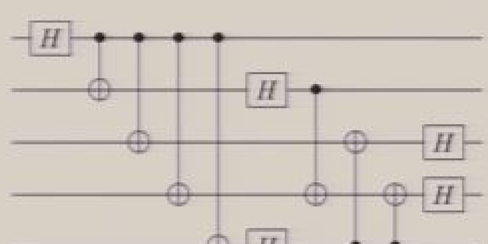
$$\begin{matrix} XZZZZ \\ ZXIZI \\ ZIXIZ \\ ZZIXZ \\ ZIZZX \end{matrix}$$

$$|\psi\rangle \;=\; \frac{1}{2\sqrt{2}}\big(|00\bar{0}\bar{0}0\rangle + |01\bar{0}\bar{1}0\rangle + |00\bar{1}\bar{1}1\rangle + |01\bar{1}\bar{0}1\rangle$$

$$+ |10\bar{1}\bar{1}0\rangle - |11\bar{1}\bar{0}0\rangle - |10\bar{0}\bar{0}1\rangle + |11\bar{0}\bar{1}0\rangle\big)$$



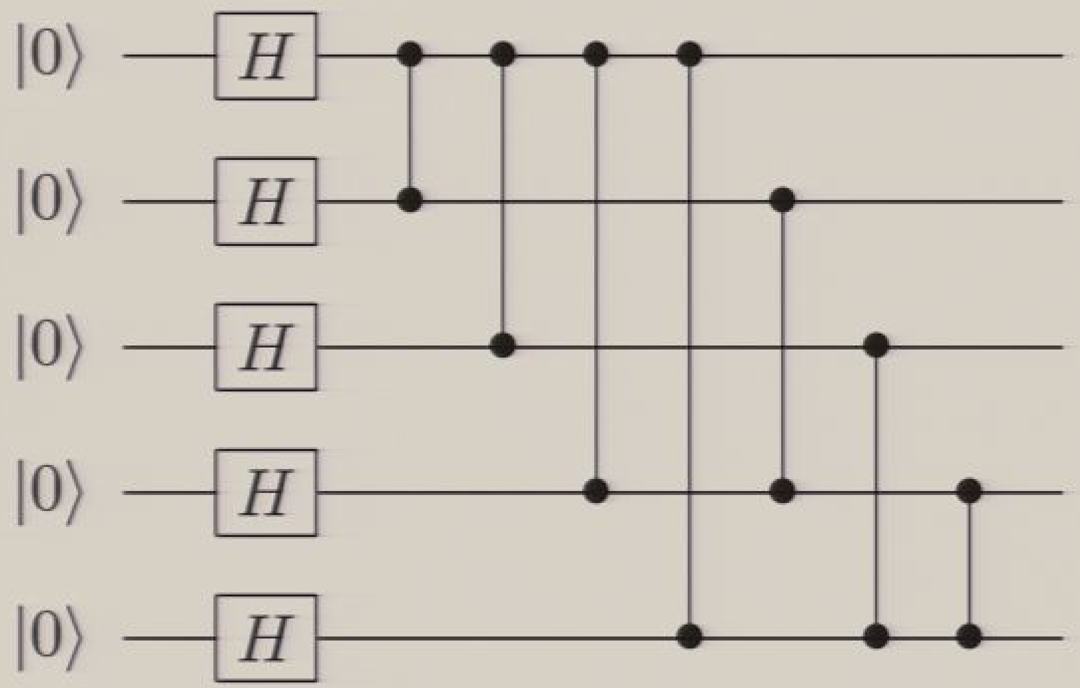
$$|\bar{a}\rangle \equiv H|\bar{a}\rangle$$



Graph states

All Clifford states are related to graph states by *Z*, Hadamard, and S gates.

$ZIIII$
$IZIII$
$IIZII$
$IIIZI$
$IIIIZ$



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$ZIXIZ$
$ZZIXZ$
$ZIZZX$

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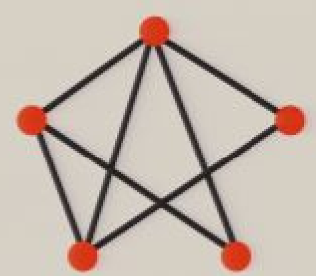
$$+\,|10\bar{1}\bar{1}0\rangle - |11\bar{1}\bar{0}0\rangle - |10\bar{0}\bar{0}1\rangle + |11\bar{0}\bar{1}0\rangle\big)$$



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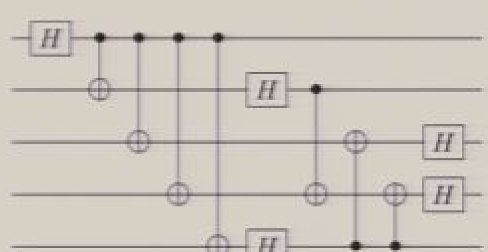
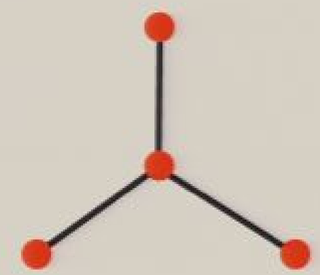


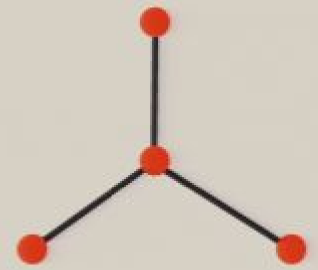
Graph states

4-qubit GHZ graph state

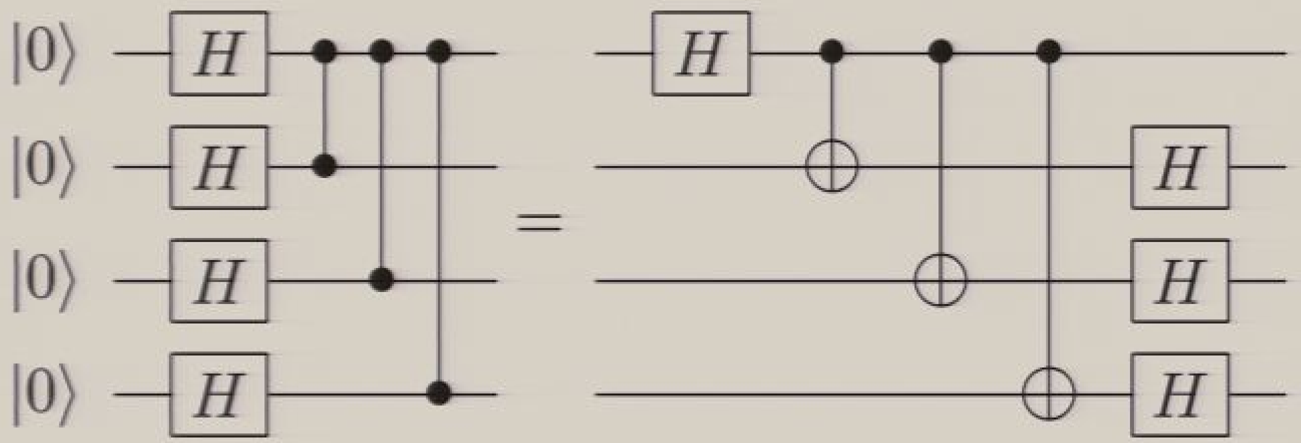


Graph states

4-qubit GHZ graph state



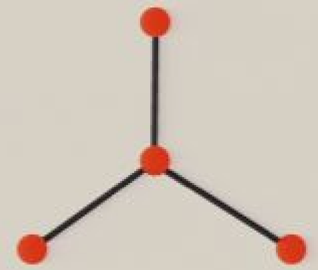
$ZIII$
$IZII$
$IIZI$
$IIIZ$



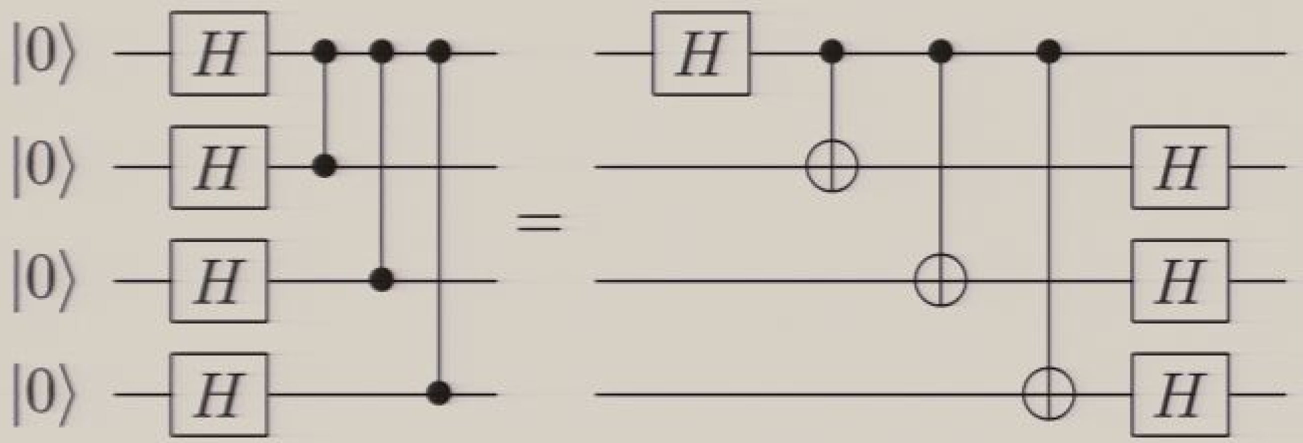
$XZZZ$
$ZXII$
$ZIXI$
$ZIIX$

Graph states

4-qubit GHZ graph state



$$\begin{matrix} ZIII \\ IZII \\ IIZI \\ IIIZ \end{matrix}$$

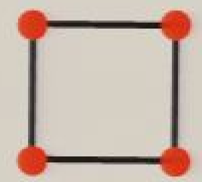


$$\begin{matrix} XZZZ \\ ZXII \\ ZIXI \\ ZIIX \end{matrix}$$

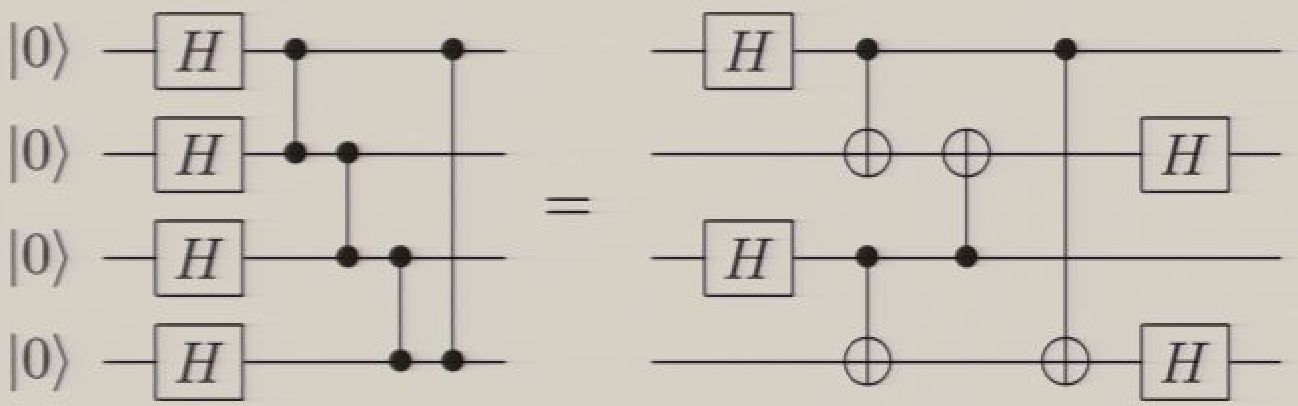
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\bar{0}\bar{0}\bar{0}\rangle + |1\bar{1}\bar{1}\bar{1}\rangle)$$

Graph states

2 x 2 cluster state



$$\begin{matrix} ZIII \\ IZII \\ IIZI \\ IIIZ \end{matrix}$$



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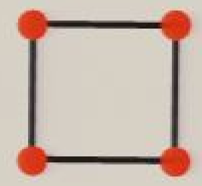
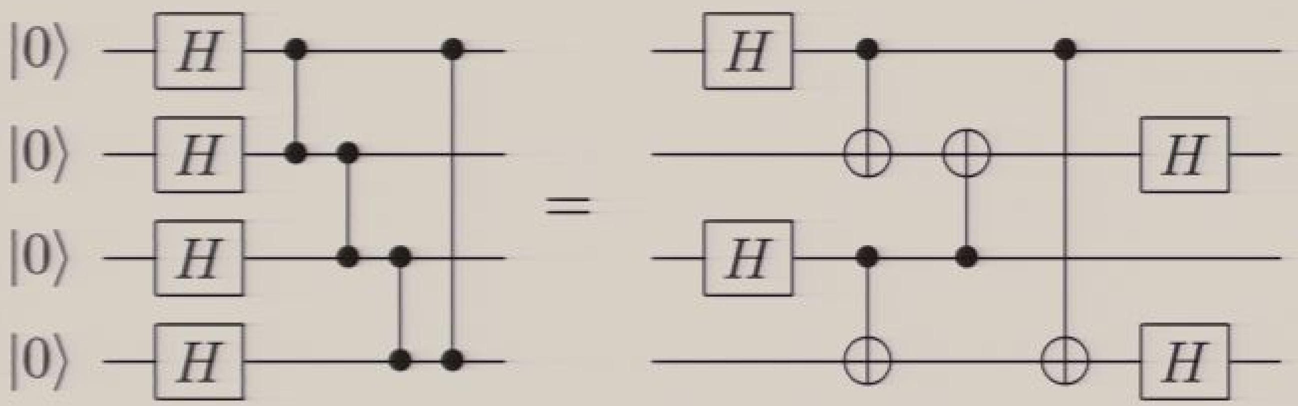


ZIII
IZII
IIZI
IIIZ



XZIZ
ZXZI
IZXZ
ZIZX

$$|\psi\rangle = \frac{1}{2}\left(|0\bar{0}0\bar{0}\rangle + |1\bar{1}0\bar{1}\rangle + |0\bar{1}1\bar{1}\rangle + |1\bar{0}1\bar{0}\rangle\right)$$

Graph states: LHV model

J. Barrett, C. M. Caves, B. Eastin, M. B. Elliott, and S. Pironio, "Modeling Pauli measurements on graph states with nearest-neighbor classical communication," submitted to PRA.

$ZIIII$
$IZIII$
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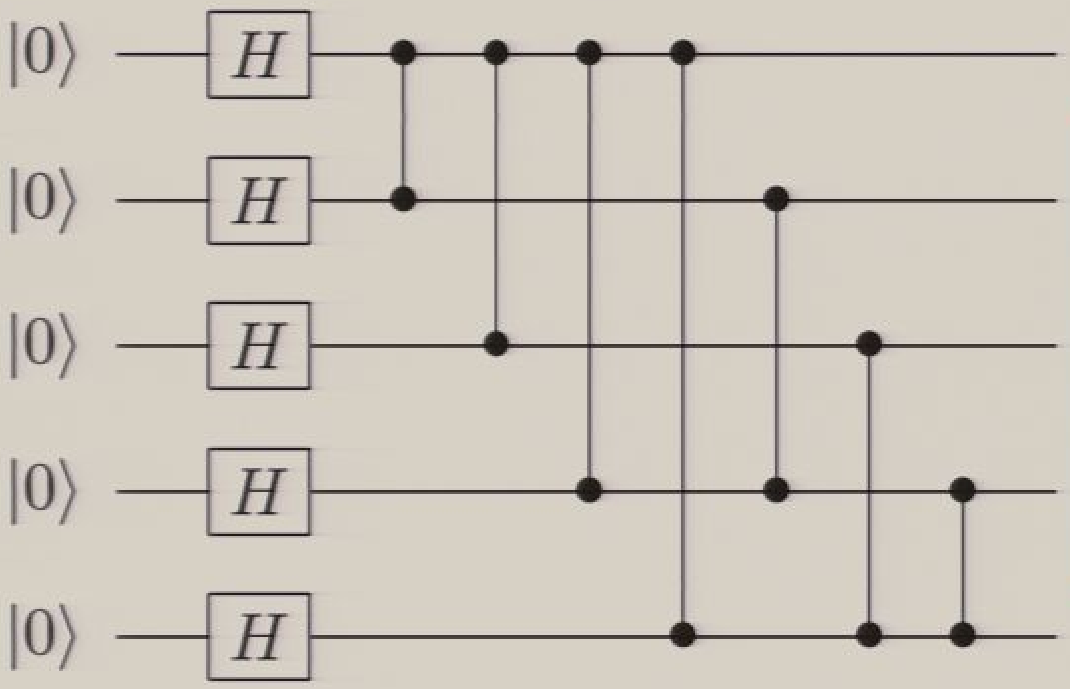
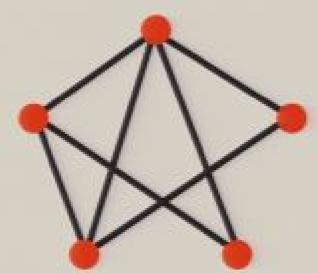
$IIIZI$
$IIIIZ$



$$
\begin{aligned}
g_1 &= XZZZZ \\
g_2 &= ZXIZI \\
g_3 &= ZIXIZ \\
g_4 &= ZZIXZ \\
g_5 &= ZIZZX
\end{aligned}
$$



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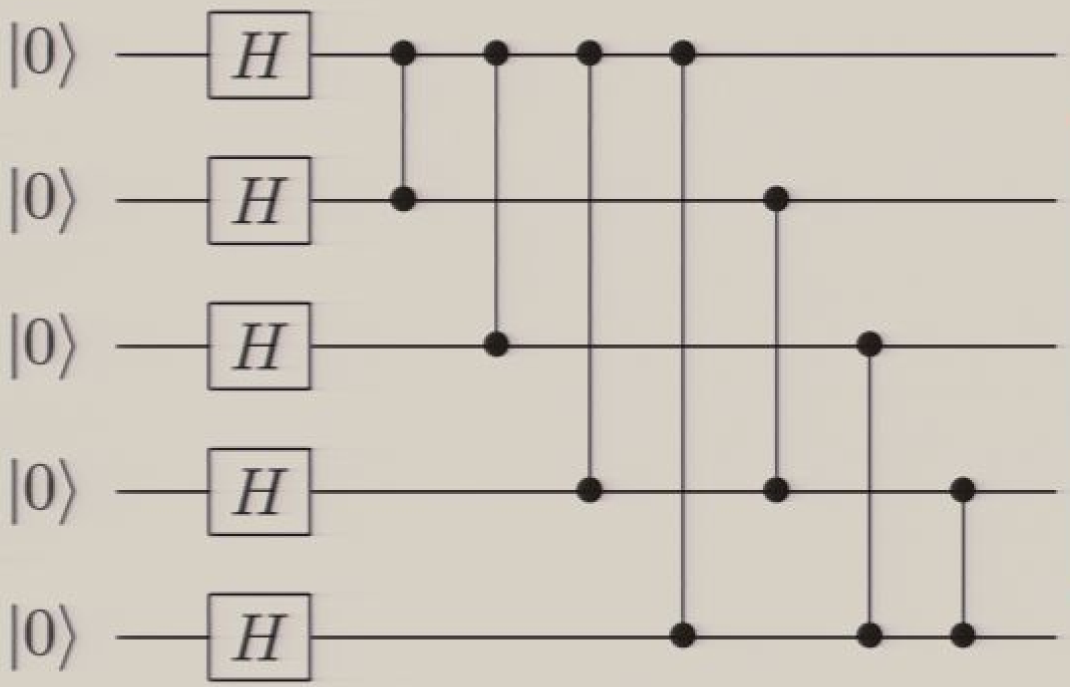

$g_1 = XZZZZ$
$g_2 = ZXIZI$
$g_3 = ZIXIZ$
$g_4 = ZZIXZ$
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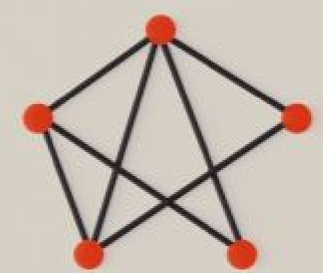

$$g_j = X_j \bigotimes_{k \in \mathcal{N}(j)} Z_j$$

$$x_j = \prod_{k \in \mathcal{N}(j)} z_j \, ,$$

$$x_j y_j z_j = \pm 1$$

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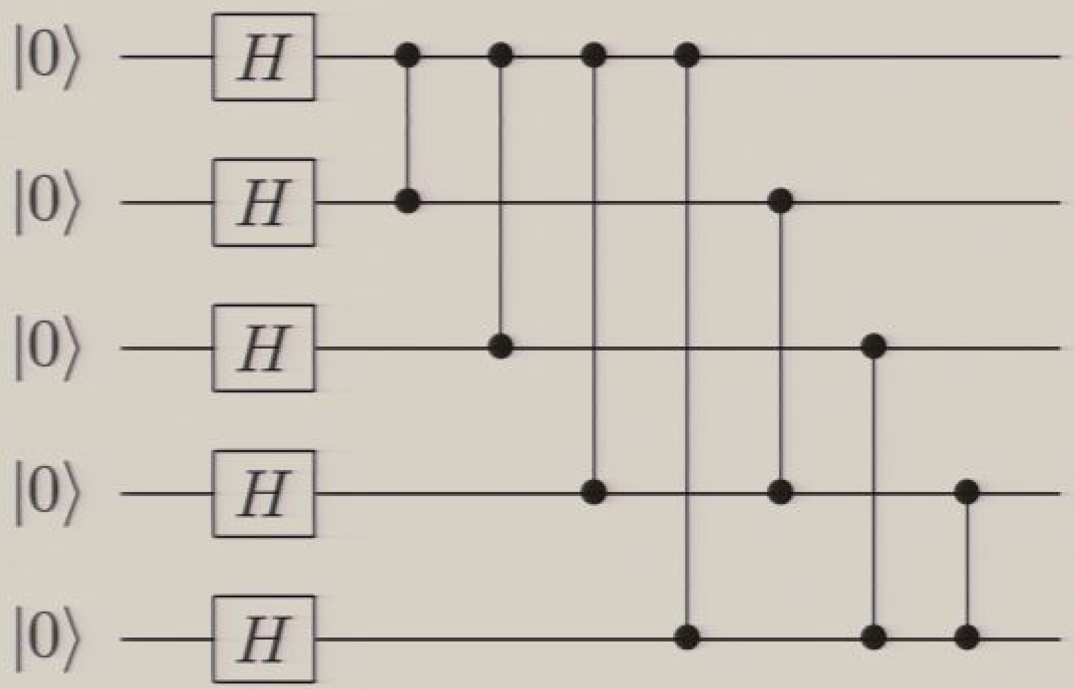

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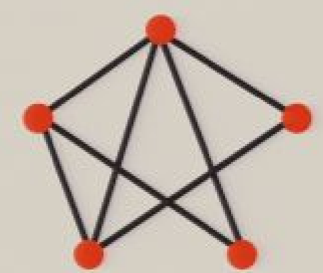

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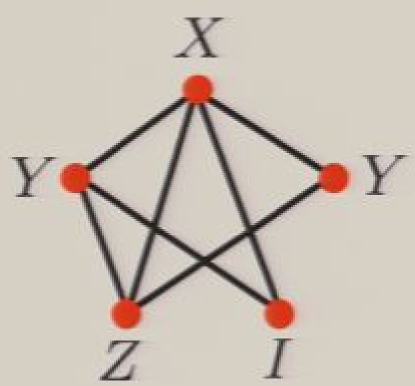
x	y	z
$z_2z_3z_4z_5$	$z_1z_2z_3z_4z_5$	z_1
z_1z_4	$z_1z_2z_4$	z_2
z_1z_5	$z_1z_3z_5$	z_3
$z_1z_2z_5$	$z_1z_2z_4z_5$	z_4
$z_1z_3z_4$	$z_1z_3z_4z_5$	z_5

Graph states: Communication protocol

For qubit j, let n_j be the number of neighbors that measure X or Y. Certainty (stabilizer element) requires

$$n_j = \begin{cases} 0 \bmod 2\,, & \text{if qubit } j \text{ measures } I \text{ or } X, \\ 1 \bmod 2\,, & \text{if qubit } j \text{ measures } Z \text{ or } Y. \end{cases}$$



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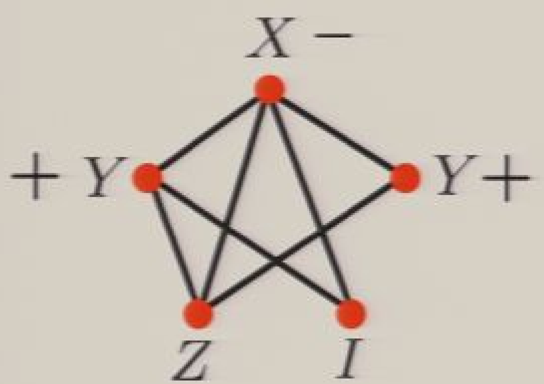

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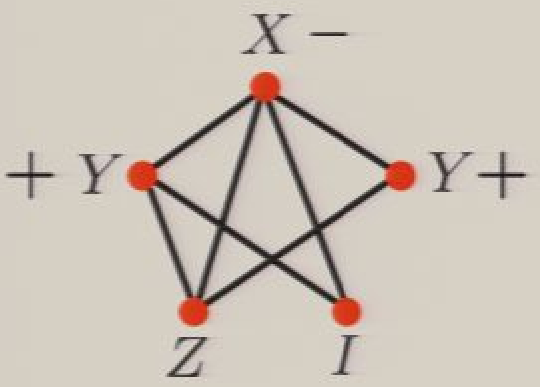
$$g_1 g_2 g_5 = -XYIZY$$

$$\left(\begin{matrix}\text{overall}\\ \text{sign}\end{matrix}\right) = (-1)^{(\#\text{ of } X \text{ qubits with } n = 2 \bmod 4)} \times (-1)^{(\#\text{ of } Y \text{ qubits with } n = 3 \bmod 4)}$$

Graph states: Communication protocol



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Each qubit tells its neighbors if it measures X or Y. A qubit flips its table entry if it measures X or Y and the number of neighbors measuring X or Y is $2, 3 \bmod 4$.

OR

Each qubit tells its neighboring qubits if it measures X or Y. A qubit flips its table entry if it measures X (Y) and the number of neighbors measuring X or Y is $2, 3 \bmod 4$ $(0, 3 \bmod 4)$.

Site-invariant nearest-neighbor communication protocols

Graph states: Subcorrelations

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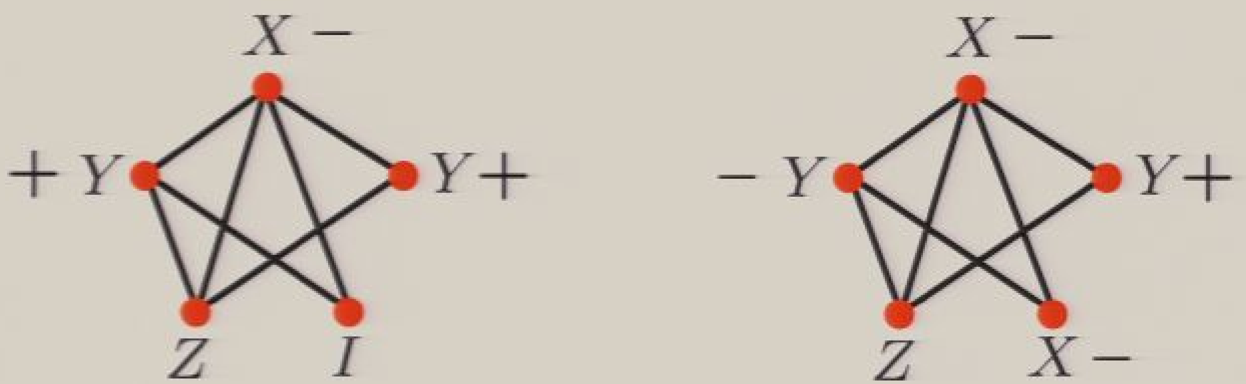

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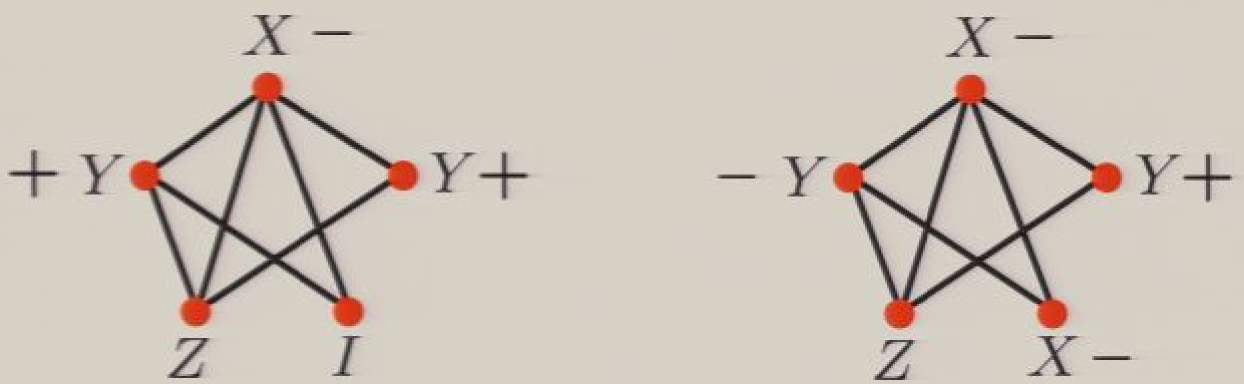

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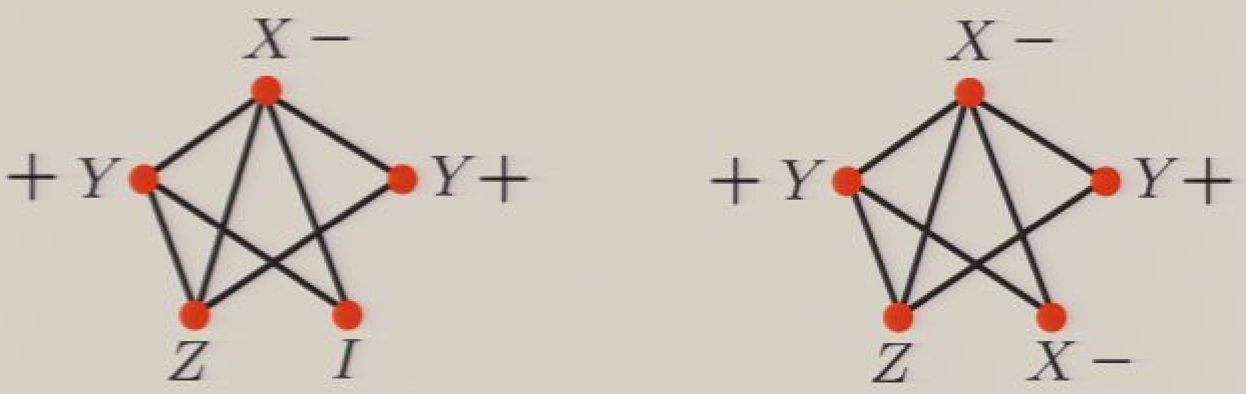
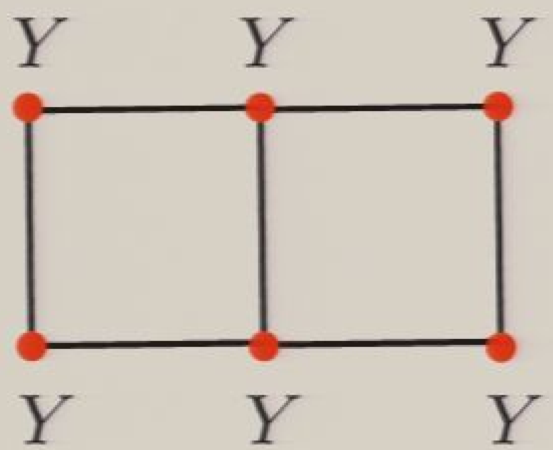
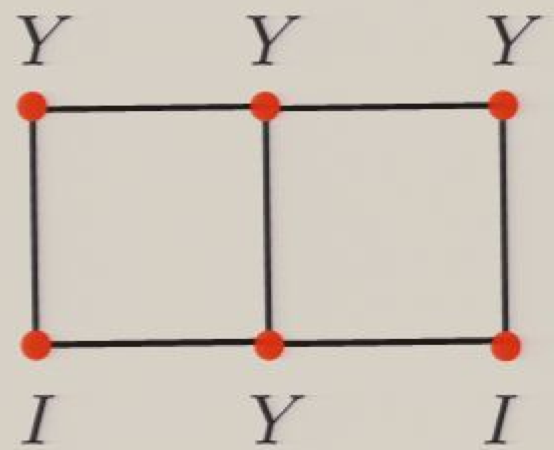


Graph states: Subcorrelations

Site-invariant protocols can get all correlations right, but any such protocol fails on some subcorrelations for some graphs.



Random result



Certain result -1

A site-invariant protocol cannot introduce a sign flip when this measurement is viewed as a submeasurement of the one on the left.

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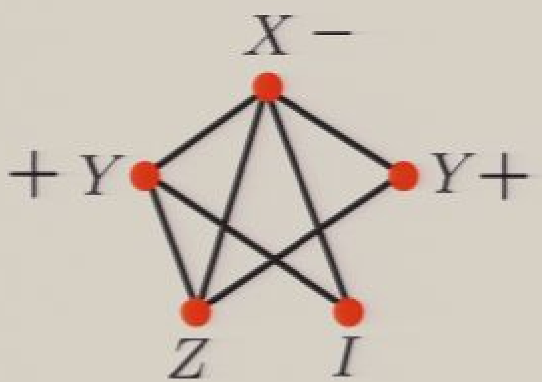

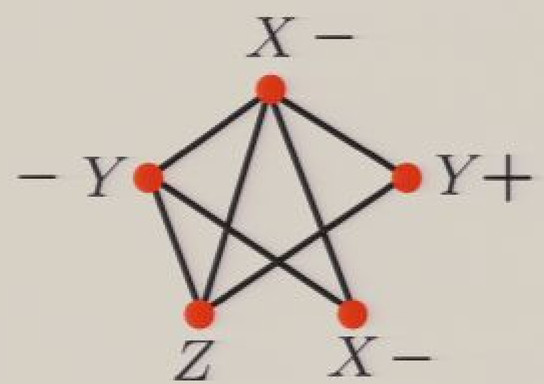

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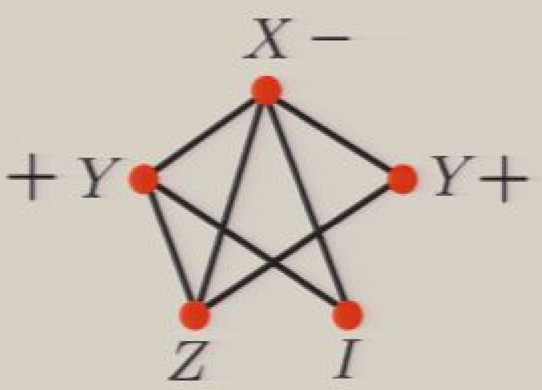

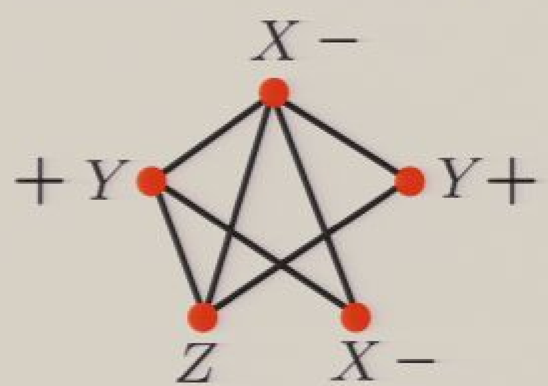

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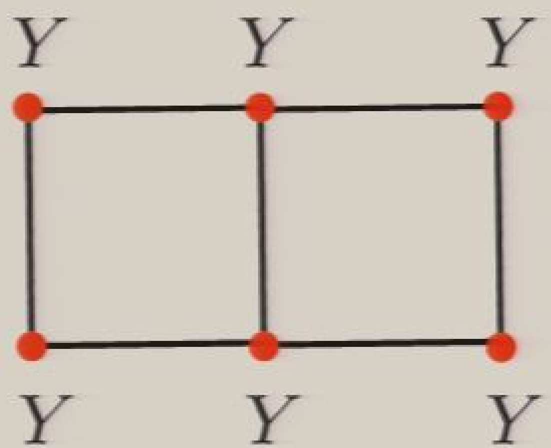
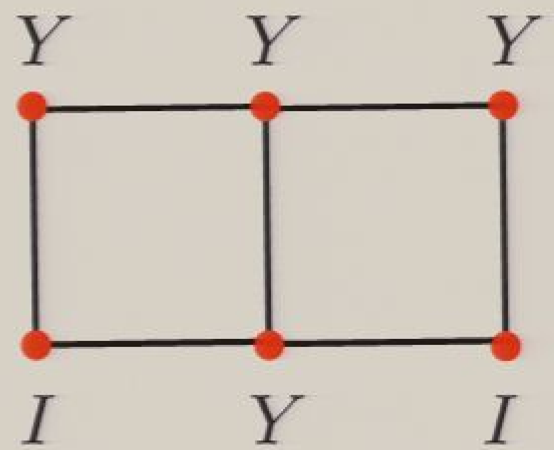


Random result



Certain result -1

A site-invariant protocol cannot introduce a sign flip when this measurement is viewed as a submeasurement of the one on the left.

Graph states: Getting it all right

1. Select a special qubit that knows the adjacency matrix of the graph.
2. Each qubit tells the special qubit if it measures *X* or *Y*.
3. From the adjacency matrix, the special qubit calculates a generating set of certain submeasurements (stabilizer elements) each of which has a representative qubit that participates in none of the other submeasurements. Since these submeasurements commute term by term, the overall sign for any certain submeasurement is a product of the signs for the participating submeasurements.
4. The special qubit tells each of the representative qubits whether to flip the sign of its table entry.

M. B. Elliott, B. Eastin, and C. M. Caves, B. Eastin, "Local-hidden-variables models assisted by classical communication for stabilizer states," in preparation.

Clifford circuits: Gottesman-Knill theorem

- N qubits in an initial product state in Z basis
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Global entanglement

but

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This kind of global entanglement, when measurements are restricted to the Pauli group, can be simulated efficiently because it can be described efficiently by local hidden variables assisted by classical communication.

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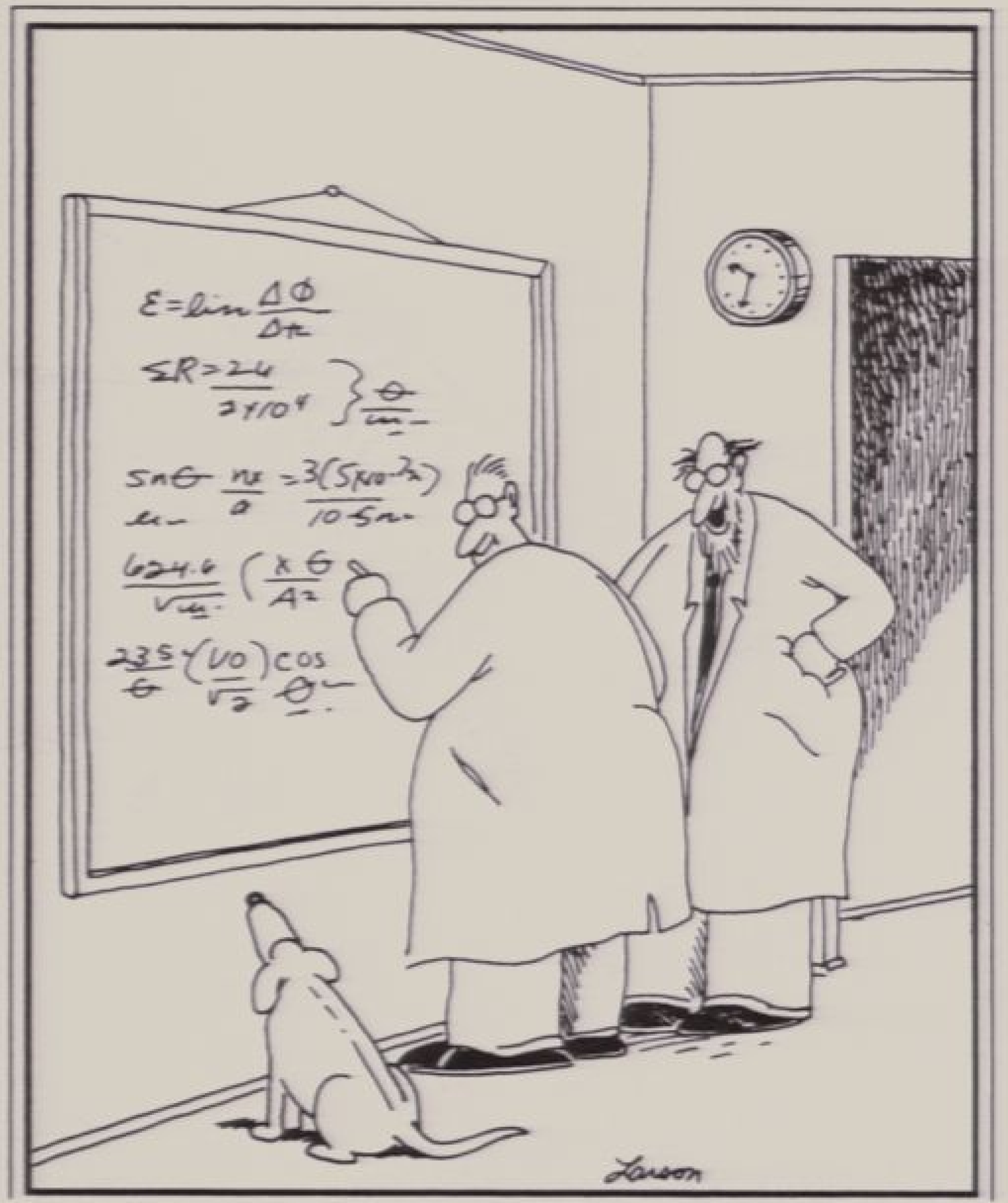
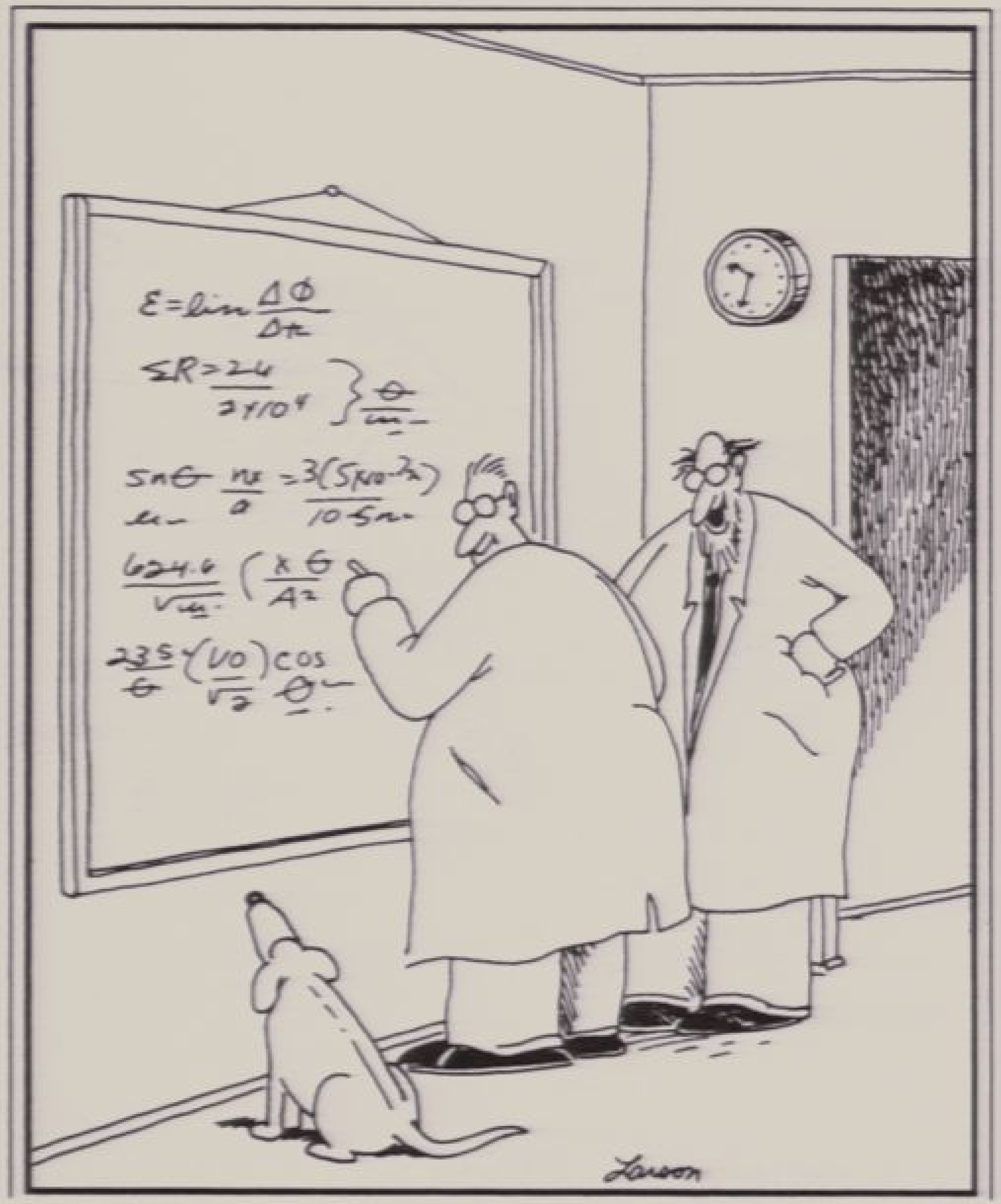


"Ohhhhhh…Look at that, Schuster…Dogs are so cute when they try to comprehend quantum mechanics."



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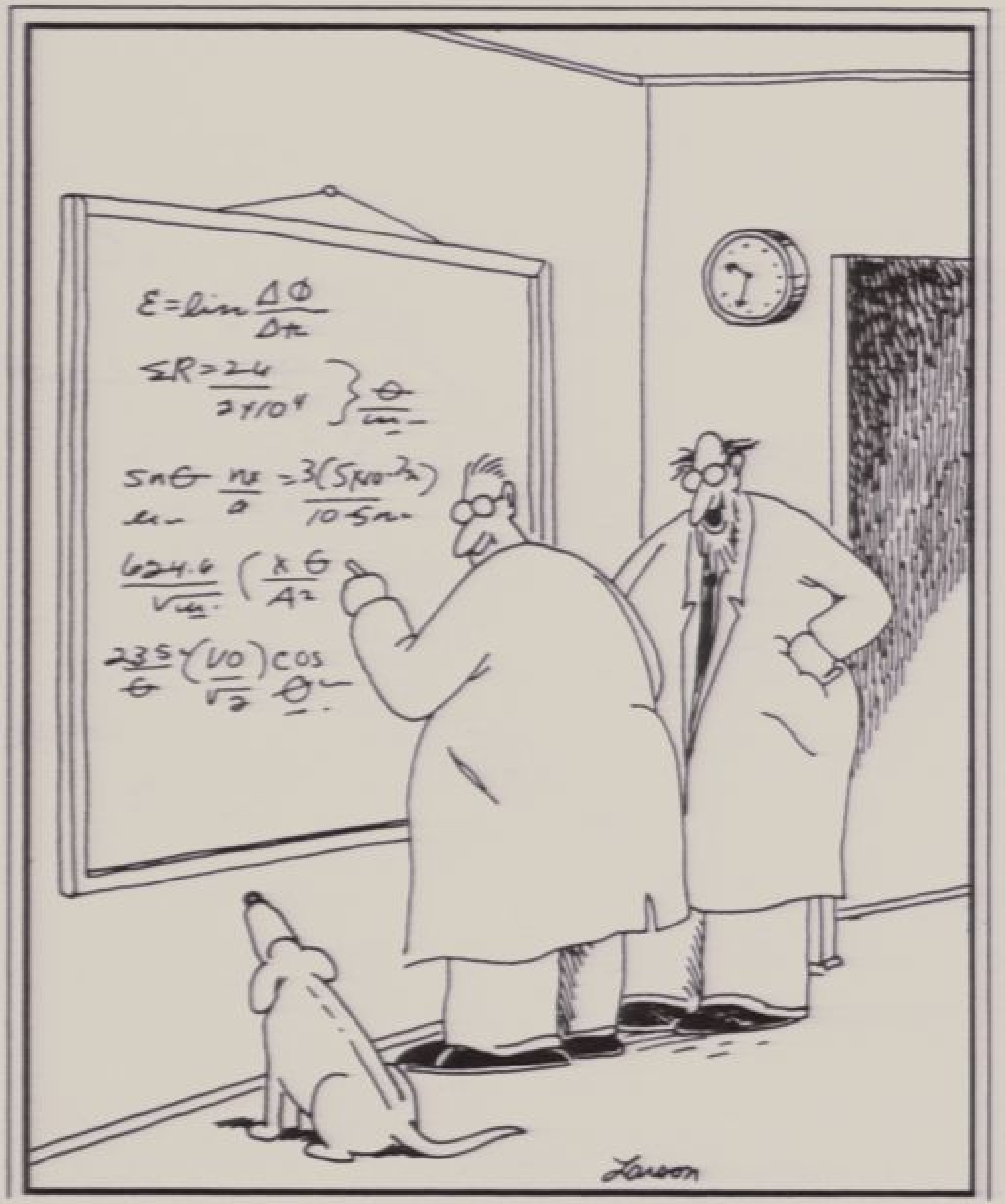
It's not only dogs that can't understand quantum mechanics, and …



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Quantum information science is the discipline that explores information processing within the quantum context where the mundane constraints of realism and determinism no longer apply.