

Title: Newton-Hooke algebras, non-relativistic branes and generalized pp-wave metrics

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URL: <http://pirsa.org/06030006>

Abstract:

- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

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- Non-Relativistic Branes
- pp-waves

Jan Brugies
Kiyohi Kamimura

- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

Jan Brüggen
Kiyohi Kimura

AdS/CFT correspondence

- Newton, Hooke al pibras
- Non-Relativistic Branes
- pp-waves

Jan Brügge
Kiyohi Kumamura

AdS/CFT correspondence

$N=4\text{SYM} \longleftrightarrow \text{strings in } \text{AdS}_5 \times S^5$

- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

Jan Breugels
Kiyohi Kamimura

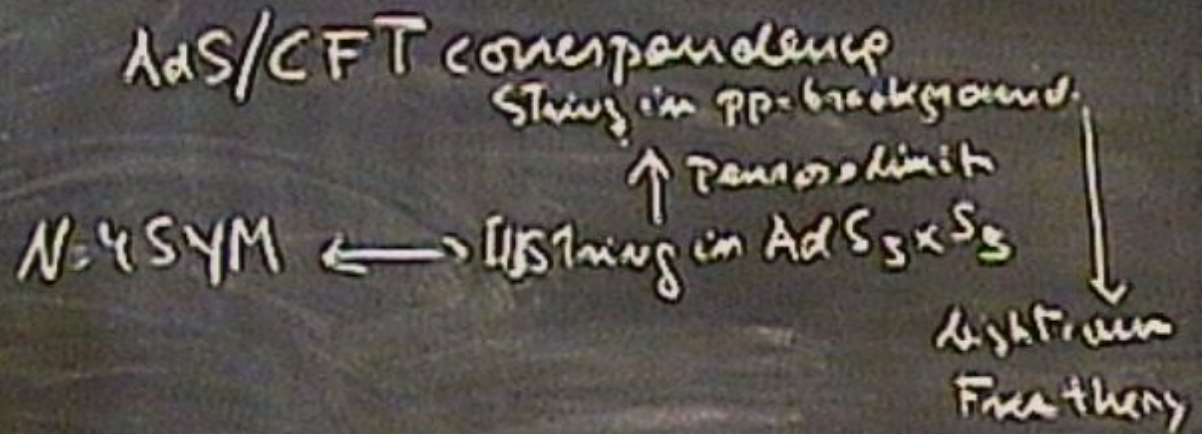
AdS/CFT correspondence
String in pp-background.

↑ Penrose limit

$N=4$ SYM $\longleftrightarrow$ String in AdS $\times$ S⁵

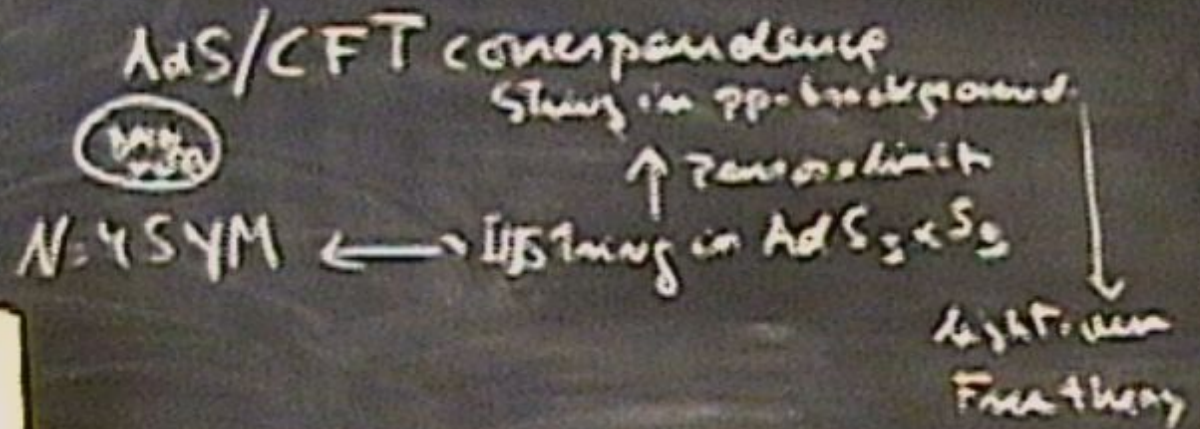
- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

Jan Bagger
Kiyohi Kurayama



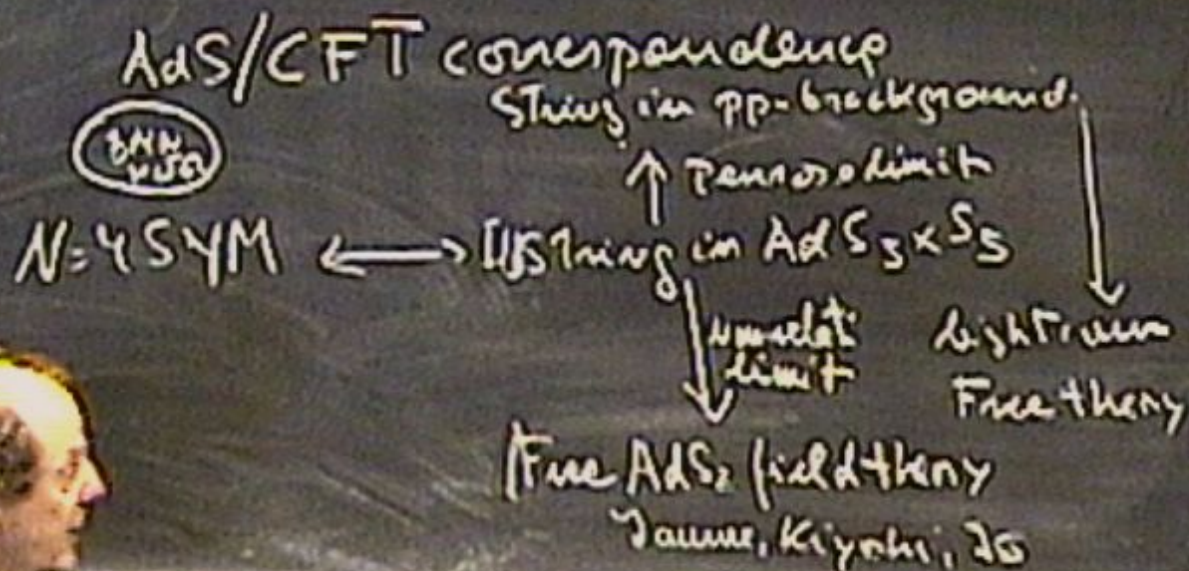
- Newton - Hooke al fibras
- Non-Relativistic Potatoes
- PP - Waves

Jan Brüggen
Keynote Kusanawa



- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

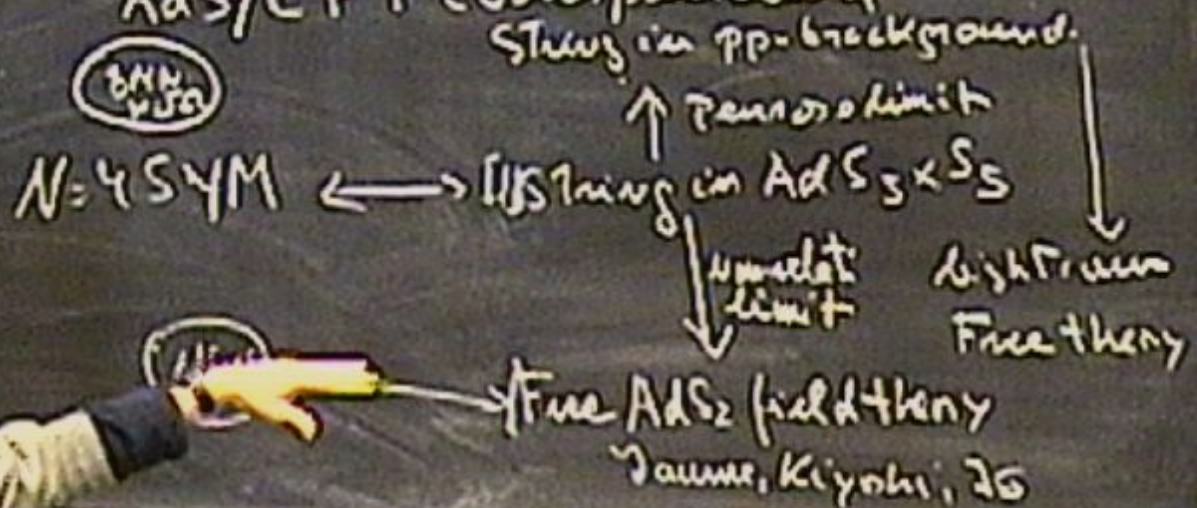
Jan Brugies
Kiyohi Kurosawa



- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

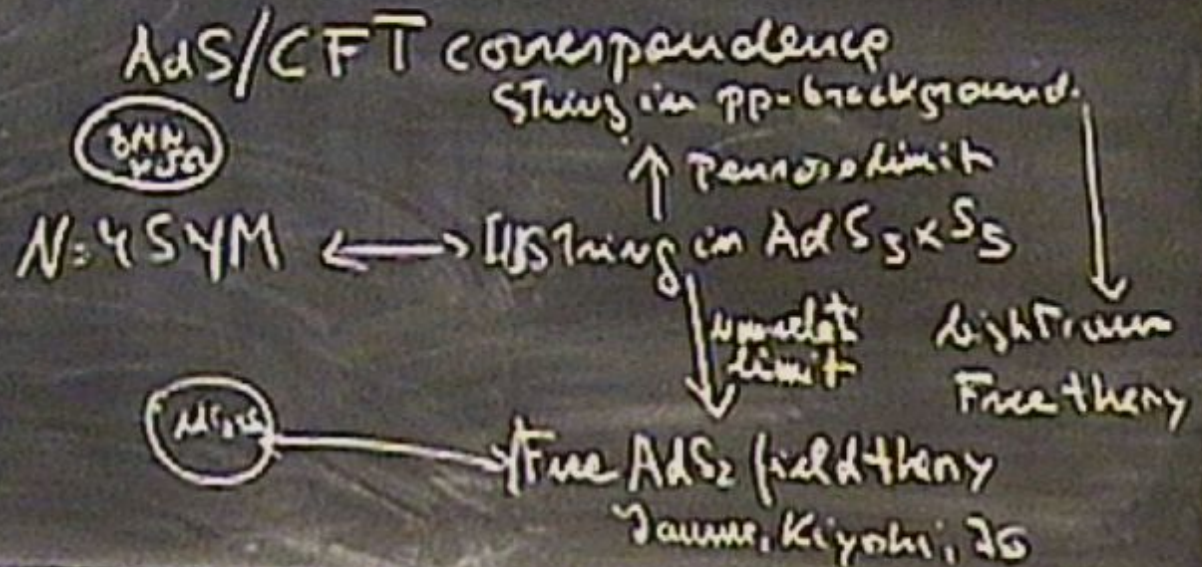
Jan Brüggen
Kiyohi Kurayama

AdS/CFT correspondence



- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

Jan Bagger
Kiyohi Kimura



$N=4$ SYM $\longleftrightarrow$ strings in $AdS_5 \times S^5$
 ↑ Penrose limit
 ↓ Unimodular limit
 Light cone Free theory
 (AdS) → Free AdS_2 field theory
 Taronna, Kiyoshi, 36

Bosonic Non-Rel. Strings

{ Taronna (Gouni)
 { Hiroshi Ogiue 2000

$N=4$ SYM $\longleftrightarrow$ strings in $AdS_5 \times S^5$
 $\uparrow$ Penrose limit $\downarrow$ Light cone
 $\downarrow$ Unfolded limit $\downarrow$ Free theory
 AdS_5 $\rightarrow$ Free AdS_2 field theory
 Taronne, Kiyoshi, 20

Bosonic Non-Rel Strings

Taronne Gaiotto 2000
 Hiroshi Ooguri
 Danilchev, Gaiotto, Kravtsov

$N=4$ SYM $\longleftrightarrow$ IIS strings in $AdS_5 \times S^5$
 ↑ Penrose limit
 ↓ Nonrelativistic limit
 Light cone Free theory
 (AdS₂) → Free AdS_2 field theory
 Taronna, Kiyoshi, 20

Bosonic Non-Rel Strings

{ Taronna, Gaiotto }
 { Hironaka, Ooguri } 2000
 Danilchev, Gaiotto, Kravtsov



$N=4$ SYM $\longleftrightarrow$ IIS strings in $AdS_5 \times S^5$

↑ Penrose limit
 ↓ Unfolded limit
 Light cone Free theory

(MSS) $\rightarrow$ Free AdS_2 field theory
 Taronne, Kiyoshi, 20

Bosonic Non-Rel Strings

{ Taronne Gaiotto
 Hiroshi Ooguri 2000

Danielsson, Gaiotto, Kraus, Zarembo



$$\begin{cases}
 X^\mu = \omega x^\mu \\
 T_P = \hat{T}_P \omega^{1-P}
 \end{cases}$$

$N=4$ SYM $\longleftrightarrow$ strings in $AdS_5 \times S^5$ $\xrightarrow{\text{Penrose limit}}$ AdS_2 $\xrightarrow{\text{unwrapping limit}}$ $Free theory$
 (M52) $\rightarrow$ Free AdS_2 field theory
 Taronne, Kiyoshi, 20

Bosonic Non-Rel Strings



$$\begin{cases} X^\mu = w x^\mu \\ T_p = \bar{T}_p w^{1-p} \end{cases}$$

w dimensionless
 $w \rightarrow \infty$

Taronne Gounis 2000
 Hiroshi Ogiue
 Daisuke Gaijima, Kazuo Sakai

$$\begin{cases} X^0 = w x^0 \\ w \rightarrow \infty \end{cases}$$

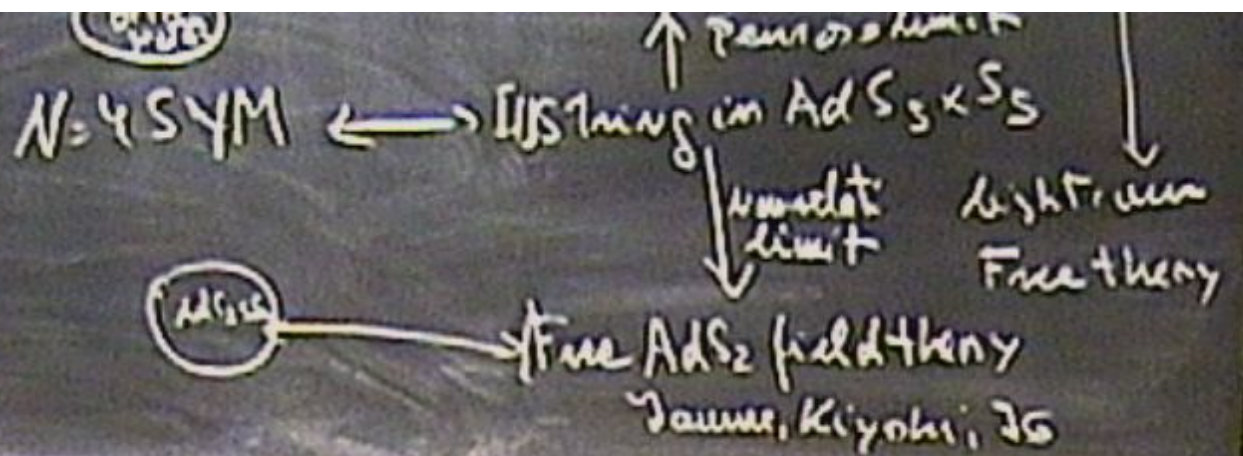
Non-dynamical
 non-transverse oscillations

$$\int d^4x \sqrt{-\det G_{ij}}$$

$$G_{ij} =$$

$$\int d^4x \sqrt{-\det G_{ij}}$$

$$G_{ij} = \partial_i X^a \partial_j X^a$$



Bosonic Non-Rel Strings



$$\begin{cases}
 X^\mu = w x^\mu \\
 \tilde{T}_P = \tilde{T}_P w^{1-P}
 \end{cases}$$

w dimensionless
 $w \rightarrow \infty$

(x^μ, x^9)
 x^μ

Taronna, Gaiotto, 2000
 Hironaka, Oguchi
 Danilchev, Gaiotto, Kraus, Sen

$$X^0 = w x^0$$

$w \rightarrow \infty$

Non-dynamical
 non-transverse oscillations

$$\int d^4x \sqrt{-\det G_{\mu\nu}}$$

$$G_{ij} = \partial_i X^m \partial_j X^n h_{mn}$$

$$\int d^4\sigma \sqrt{-\det G_{\alpha\beta}}$$

$$G_{ij} = \partial_i X^m \partial_j X^m h_{mn}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{\mu\nu} = \partial_\mu x^\mu \partial_\nu x^\nu h_{\mu\nu}$$

$$T \int d\sigma \sqrt{-\det G_{\alpha\beta}}$$

$$G_{ij} = \partial_i X^m \partial_j X^m h_{mn}$$

$$g_{ij} = \partial_i X^a \partial_j X^b \delta_{ab} \quad , \quad g_{ij} = \partial_i X^\mu \partial_j X^\nu h_{\mu\nu}$$

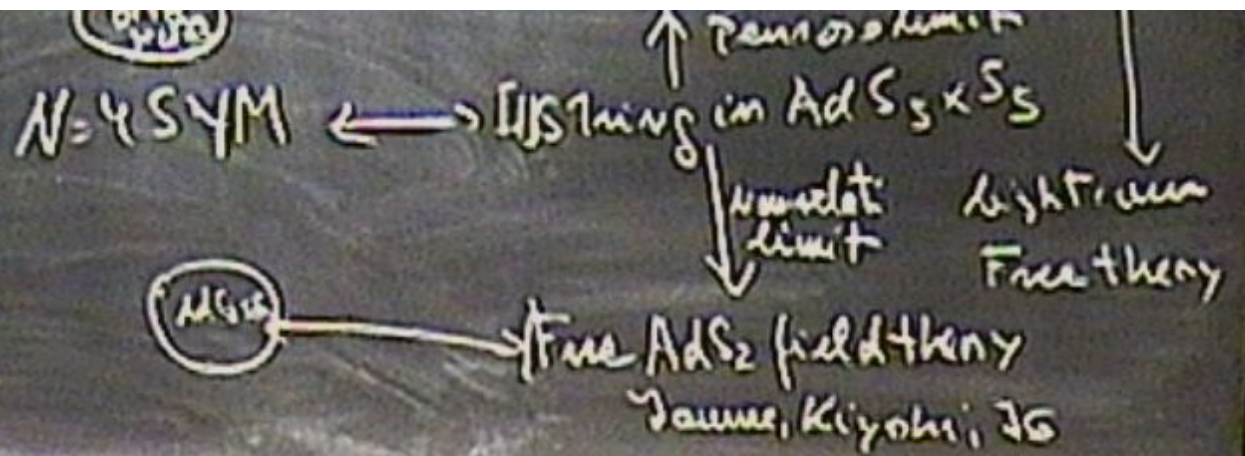
$$+ \left\{ \omega^2 \left[\sqrt{-\det g} + \sqrt{g} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right] \right\}$$

$$T \int d^4\sigma \sqrt{-\det G_{\alpha\beta}}$$

$$G_{ij} = \partial_i X^\mu \partial_j X^\nu h_{\mu\nu}$$

$$= \omega^2 g_{ij} + \partial_i X^\mu \partial_j X^\nu \delta_{\mu\nu} \quad , \quad g_{ij} = \partial_i x^\mu \partial_j x^\nu h_{\mu\nu}$$

$$= T \int d^4\sigma \left\{ \underbrace{\omega^2 \sqrt{-\det g}}_{\text{divergent}} + \sqrt{-g} g^{ij} \partial_i X^\mu \partial_j X^\nu \delta_{\mu\nu} + O\left(\frac{1}{\omega^2}\right) \right\}$$



Bosonic Non-Rel Strings



$$\begin{cases}
 X^p = w x^p \\
 \tilde{T}_p = \tilde{T}_p w^{1-p}
 \end{cases}$$

w dimensions
 (x^M, x^9)
 x^M

$w \rightarrow \infty$

Taronna, Gaiotto, 2000
 Hironaka, Oguchi
 Danielsson, Geilker, Krauss, 2001

$$\begin{aligned}
 X^0 &= w x^0 \\
 w &\rightarrow \infty
 \end{aligned}$$

Non-dynamical
 non-transverse oscillations

$$T \int d^2\sigma \sqrt{-\det G_{\alpha\beta}}$$

$$G_{ij} = \partial_i X^\mu \partial_j X^\mu h_{\mu\nu}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i x^\mu \partial_j x^\nu h_{\mu\nu}$$

$$= T \int d^2\sigma \left\{ \underbrace{\omega^2 \sqrt{-\det g}}_{\text{divergent}} + \sqrt{-g} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right\}$$

$$T \int d^2\sigma \sqrt{-\det G_{\alpha\beta}}$$

$$G_{ij} = \partial_i X^\mu \partial_j X^\mu h_{\mu\nu}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i x^\mu \partial_j x^\nu h_{\mu\nu}$$

$$= T \int d^2\sigma \left\{ \omega^2 \underbrace{\sqrt{-\det g}}_{\text{divergent}} + \sqrt{-g} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right\} + \int d^2\sigma B^{\alpha\beta}$$

$$T \int d^4\sigma \sqrt{-\det G_{ij}}$$

$$G_{ij} = \partial_i X^m \partial_j X^m h_{mn}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i x^\mu \partial_j x^\nu h_{\mu\nu}$$

$$= T \int d^4\sigma \left\{ \underbrace{\omega^2 \sqrt{-\det g}}_{\text{divergent}} + \sqrt{-g} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right\} + \int d^4\sigma B^{\mu\nu} \quad \text{,,} \quad H = dB = 0$$

$$T \int d^4\sigma \sqrt{-\det G_{ij}}$$

$$G_{ij} = \partial_i X^\mu \partial_j X^\mu h_{\mu\nu}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i x^\mu \partial_j x^\nu h_{\mu\nu}$$

$$= T \int d^4\sigma \left\{ \omega^2 \left[\sqrt{-\det g} + \frac{1}{\sqrt{-\det g}} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right] \right.$$

divergent

$$+ \int d^4\sigma B^{\mu\nu} \quad \text{,,} \quad H = dB = 0$$

$$B_{\mu\nu} = \epsilon_{\mu\nu} dx^\mu dx^\nu$$

$$= T \int d^4x \left\{ \underbrace{w^2}_{\text{divergent}} + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu \partial_a b^a \partial_c w^c \right\} + \int d^4x B^{\mu\nu} \quad \parallel \quad H = dB = 0$$

$$B_{\mu\nu} = \epsilon_{\mu\nu} dx^\mu dx^\nu$$



$$= T \int d^4x \left\{ \underbrace{w^2 (V - d\epsilon f g)}_{\text{divergent}} + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu \partial a b^{\mu\nu} \mathcal{O}(w^2) \right\} + \int d^4x B^{\mu\nu} \quad \parallel \quad H = dB = 0$$

$$B_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} dx^\rho dx^\sigma$$



$$= T \int d^4x \left\{ \underbrace{w^a}_{\text{divergent}} \left[\sqrt{-\det g} + \frac{1}{2} g^{\mu\nu} \partial_\mu X \partial_\nu \partial_a b^a \right] \right\} + \int d^3x B^x \quad \parallel \quad H = dB = 0$$

$$B_{\mu\nu} = \epsilon_{\mu\nu} dx^\mu dx^\nu$$



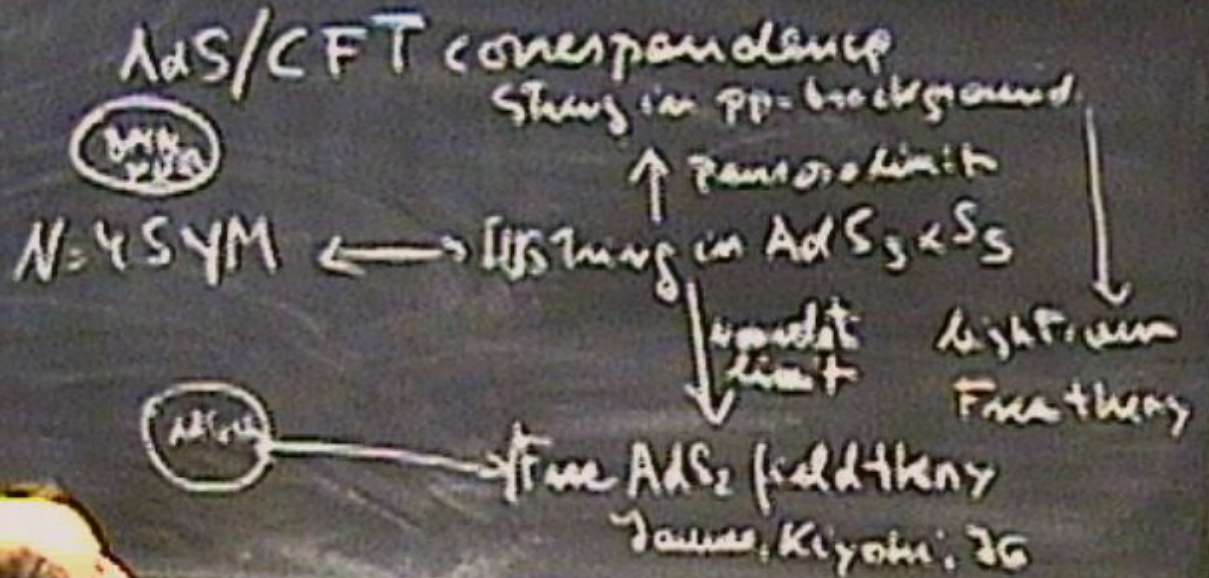
unmapping > 0

$\begin{matrix} = 0 \\ < 0 \end{matrix} \left\{ \begin{matrix} \text{inf.} \\ \text{maximizing} \end{matrix} \right.$



- Newton-Hooke algebras
- Non-Relativistic Branes
- pp-waves

Jan Bagger
Kiyohito Kurayama





Mapping

$\sum \omega$ inf. mapping

$$\int d^2 \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

$X^M = 0^M$ static gauge





unap

$\frac{1}{2} \{ \text{inf. maxing} \}$

$$\int d\alpha \sqrt{g} g^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b$$

Diff inv

$X^M = \sigma^M$ static gauge



unapplying

$= 0$ } inf. machine

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv $X^M = \sigma^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b \delta_{ab}$$

AdS₂

upward limit

light cone
Free theory

Free AdS₂ field theory

Tamura, Kiyohi, 25

Bosonic Non-Pol. Strass



$$\begin{cases} X^{\mu} = w x^{\mu} \\ \Pi_P = \tilde{T}_P w^{1-P} \end{cases}$$

w dimensions

$$(x^{\mu}, x^9) \quad w \rightarrow \infty$$

Tamura Gomi's 2000
Hiroshi Ogiue
Danchev, Gejorgi, Kraus, et al.

$$X^0 = w x^0$$

$w \rightarrow \infty$

Non-dynamical
non-transverse oscillations

$$\int d\sigma \sqrt{g} g^{\omega\lambda} \partial_\lambda X^a \partial_\omega X^b$$

Diff inv $X^M = \sigma^M$ static gauge.

$$\int d^2x h^{\omega\lambda} \partial_\lambda X^a \partial_\omega X^b S_{ab}$$

$$\theta =$$

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv. $X^M = \sigma^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b S_{ab}$$

$$\theta = \sqrt{w} \theta$$

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv $X^M = \sigma^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b \zeta_{ab}$$

$$\sqrt{w} \partial_+ \theta = \sqrt{w} \partial_- \theta + \frac{1}{\sqrt{w}} \partial_+ \theta$$

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv $X^M = 0^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b S_{ab}$$

$$\theta = \sqrt{w} \theta_+ \quad \theta = \sqrt{w} \theta_- + \frac{1}{\sqrt{w}} \theta_+$$

π_*

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv $X^M = \sigma^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b S_{ab}$$

$$= \sqrt{\omega} \theta_+ \quad \theta = \sqrt{\omega} \theta_- + \frac{1}{\sqrt{\omega}} \theta_+$$

$$P_* = P_0 \beta_2 \tau_3 \quad P \theta_\pm = \pm \theta_\pm$$

- pp-waves

AdS/CFT correspondence

String in pp-background

↑ tension limit

$N=4$ SYM

↓ tension limit

light-cone
Free theory

AdS

Free AdS₂ field theory

Tamara, Kiryushin, 36



$$\begin{cases} X^r = \omega x^M \\ \boxed{I_P = \tilde{T}_P \omega^{1-P}} \end{cases}$$

ω dimensionless

$$\begin{pmatrix} X^M, X^9 \\ X^M \end{pmatrix} \quad \omega \rightarrow \infty$$

Danielsson, Geijon, Krauss

$$X^0 = \omega x^0$$

$\omega \rightarrow \infty$

Non-dynamical

non-transverse

$$\int d^2 \sigma \sqrt{g} g^{\omega\bar{\omega}} \partial_i X^a \partial_{\bar{i}} X^b$$

Diff inv $X^M = \sigma^M$ static gauge.

$$\int d^2 \sigma h^{ij} \partial_i X^a \partial_j X^b \zeta_{ab}$$

$$\theta = \sqrt{\omega} \theta_- + \frac{1}{\sqrt{\omega}} \theta_+$$

11/15

$$\Pi_* = \Pi_0 \Pi_2 \quad \theta_{\pm} = \pm \theta_{\mp}$$

$$\int d^2x \sqrt{g} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^b$$

Diff inv $X^M = 0^M$ static gauge.

$$\int d^2x h^{\mu\nu} \partial_\mu X^a \partial_\nu X^b \zeta_{ab}$$

$$\theta = \sqrt{\omega} \theta_{\parallel} \quad \theta = \sqrt{\omega} \theta_{-} + \frac{1}{\sqrt{\omega}} \theta_{+}$$

11B

$$\pi_{*} = \pi_0 \pi_1 \pi_2 \quad \pi_{\pm} \theta_{\pm} = \pm \theta_{\pm}$$

$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{2w_2} \right]$$

IIA

$$\Pi^m = dx^m + i \bar{\Theta} \Gamma^m d\Theta$$

$$G_{ij} = \Pi_i^m \Pi_j^m h_{mm}$$

$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{2W_2} \right]$$

IIA

$$\Pi^m = dx^m + i \bar{\Theta} \Gamma^m d\Theta$$

$$G_{ij} = \Pi_i^m \Pi_j^m h_{mm}$$

same Diff
Kappa symmetry

6051

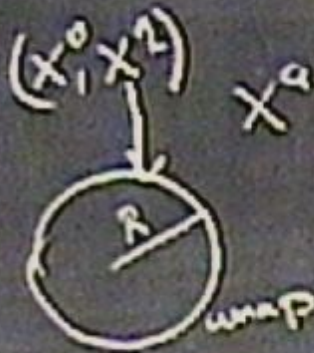
$$G_{ij} = \partial_i X^\mu \partial_j X^\mu h_{\mu\nu}$$

$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i X^\mu \partial_j X^\nu h_{\mu\nu}$$

$$= T \int d\tau \left\{ \omega^2 \left[\sqrt{-\det g} + \frac{1}{\omega^2} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right] \right.$$

$$\left. + \int d\sigma B^{\mu\nu} \right\} \quad \text{,,} \quad \underline{H = dB = 0}$$

$$B_{\mu\nu} = E_{\mu\nu} dx^\mu dx^\nu$$



mapping $\gamma \in$

$= \cup$
 < 0

amine



$$\int d^4x \sqrt{-\det g_{ij}} \mathcal{L}(w_i)$$

114

$$\Pi^m = dx^m + i \bar{\Theta} \Gamma^m d\Theta$$

$$g_{ij} = \eta_{ij} + h_{ij}$$

gauge Diffeo
Kappa symmetry

SUSY

$$\int d^4x \sqrt{-\det g_{\mu\nu}} + \textcircled{LWZ}$$

$$G_{\mu\nu} = \eta_{\mu}^{\alpha} \eta_{\nu}^{\beta} h_{\alpha\beta}$$

gauge $\left\{ \begin{array}{l} \text{Dil} \\ \text{Kappe symmetrie} \end{array} \right.$

SUSY

$$\mathbb{A} + \int \mathcal{B}$$

$$\Pi^{\mu\nu} = dx^{\mu} + i \bar{\Theta} \Gamma^{\mu} d\Theta$$

$$\int d^4x \left[\sqrt{-\det G_{ij}} + \mathcal{L}(W_2) \right]$$

$$G_{ij} = \eta_{ij} + \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$$

source $\rightarrow$ Diffeo
Kappa asymmetric τ_1

SUSY

$$\mathcal{H}A \pm \int \mathcal{B}^{\mu}$$

$$\Pi^\mu = dx^\mu + i \bar{\Theta} \Gamma^\mu d\Theta$$



$$G_{ij} = \partial_i X^m \partial_j X^m h_{mn}$$

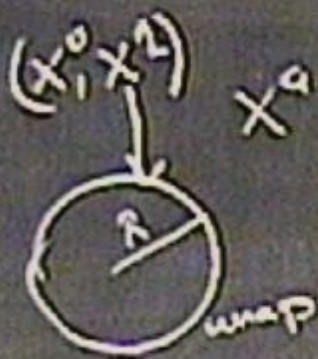
$$= \omega^2 g_{ij} + \partial_i X^a \partial_j X^b \delta_{ab} \quad \text{,,} \quad g_{ij} = \partial_i X^\mu \partial_j X^\nu h_{\mu\nu}$$

$$= T \int d^2\sigma \left\{ \omega^2 \left[\sqrt{-\det g} \right] + \frac{1}{\omega^2} g^{ij} \partial_i X^a \partial_j X^b \delta_{ab} + O\left(\frac{1}{\omega^2}\right) \right\}$$

divergent

$$+ \int d^2\sigma B^{\mu\nu} \left\{ \text{,,} \quad \underline{H = dB = 0} \right.$$

$$B_{\mu\nu} = \epsilon_{\mu\nu} dx^\mu dx^\nu$$



wrapping > 0

$\begin{matrix} \approx 0 \\ < 0 \end{matrix} \left\{ \begin{matrix} \text{inf.} \\ \text{meaning} \end{matrix} \right.$

115

$$V_x = V_0 V_2 \tau_3$$

$$P_\pm \theta_\pm = \pm \theta_\pm$$

$$\int d^4x \left[\sqrt{-\det g_{\mu\nu}} + \textcircled{LWZ} \right]$$

$$G_{\mu\nu} = \pi_{\mu}^{\alpha} \pi_{\nu}^{\beta} h_{\alpha\beta}$$

gauge $\rightarrow$ Diffeo
 Kappa symmetry τ_1

SUSY

$$\mathbb{A}^4 \oplus \mathbb{R}^4$$

$$\pi^m = \pi^{\mu\nu} + i \bar{\theta} \pi^m \theta$$

$$G_{12} = 1/6$$

jaar Diff
Kappe Symmetri

GUSY

Newton.

$$\int d^4x \sqrt{-\det G_{ij}} + \text{LW}$$

$$\mathbb{A}^4 \oplus \mathbb{S}^4$$

$$\Pi^m = dx^m + i \bar{\theta} \Gamma^m \theta$$

$$G_{ij} = \Pi_i^m \Pi_j^m \eta_{mn}$$

year Diff
Klein symmetry

SUSY

Newton-Hooke algebras



$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{2W_2} \right]$$

$$G_{ij} = \eta_{\mu\nu} \partial_i^\mu \partial_j^\nu h_{\mu\nu}$$

$$\mathbb{A} \oplus \mathbb{B}$$

$$\Pi^m = dx^m + i \bar{\theta} \Gamma^m d\theta$$

gauge $\frac{D}{dt}$
 Kappe symmetric τ

SUSY

Newton-Hooke algebras

1967 Bary-Levy-Leblond

$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{2W_2} \right]$$

$$\mathbb{A} \oplus \mathbb{B}$$

$$\Pi^m = dx^m + i \bar{\theta} \Gamma^m \theta$$

$$G_{ij} = \Pi_i^m \Pi_j^m h_{mm}$$

gauge $\mathbb{D}||b$
Kappa symmetry

SUSY

Newton-Hooke algebras

$$\psi'(x') = e^{ib} \psi(x)$$

1967 Barry-Levy-Leblond

$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{2W_2} \right]$$

$$\mathbb{A} \oplus \mathbb{B}$$

$$\Pi^m = dx^m + i \bar{\theta} \Gamma^m \theta$$

$$G_{ij} = \Pi_i^m \Pi_j^m h_{mm}$$

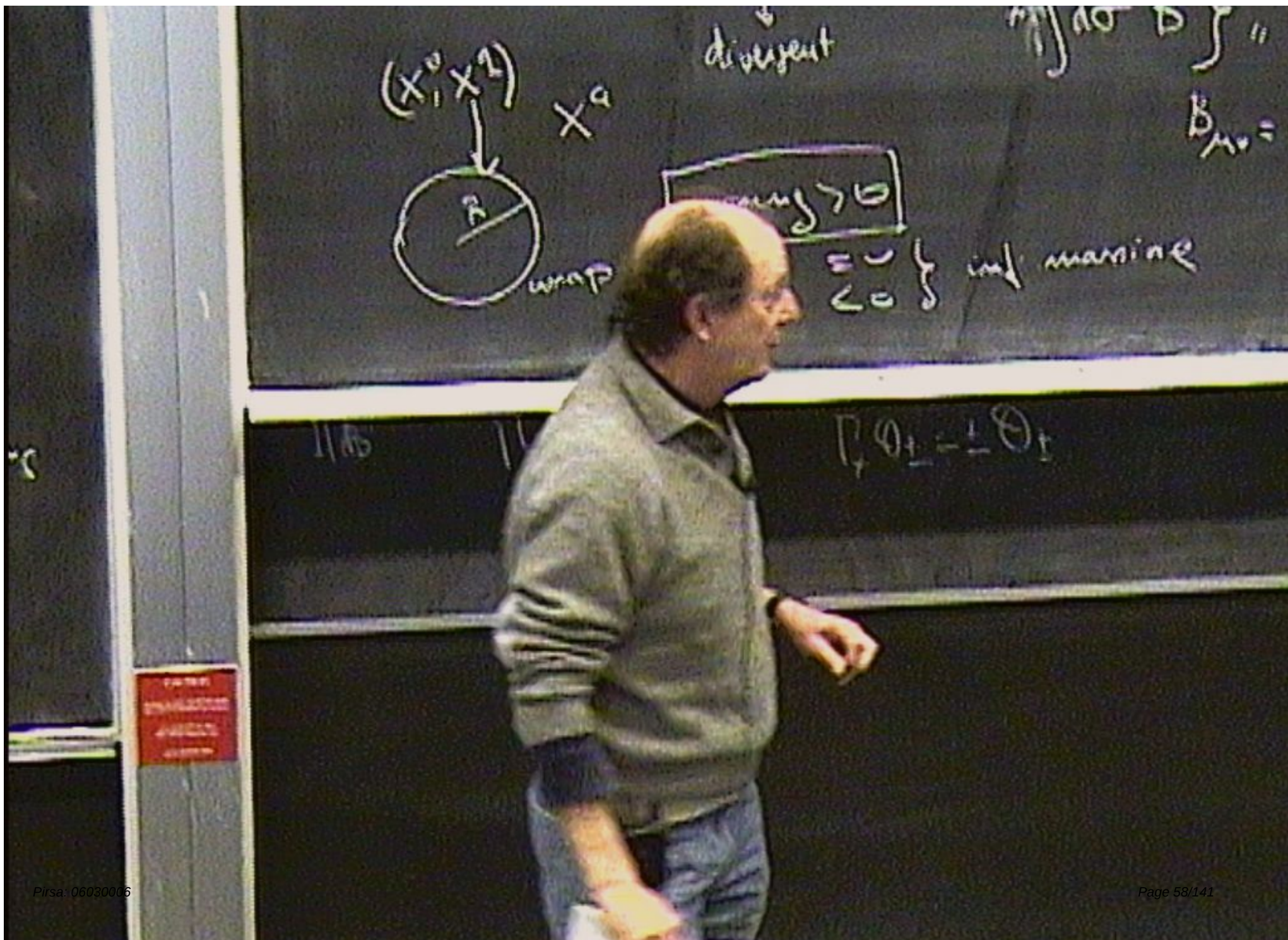
same $\frac{D}{dt}$
Kappe Symmetrie

SUSY

Newton-Hooke algebras

$$\psi'(x') = \textcircled{e^{i\phi}} \psi(x)$$

1967 Barry-Levy-Leblond



$$\int d^2\sigma \left[\sqrt{-\det G_{ij}} + \textcircled{LWZ} \right]$$

$$\mathbb{A} \oplus \mathbb{B}$$

$$\Pi^m = dx^m + i \bar{\theta} \Gamma^m \theta$$

$$G_{ij} = \Pi_i^m \Pi_j^n h_{mn}$$

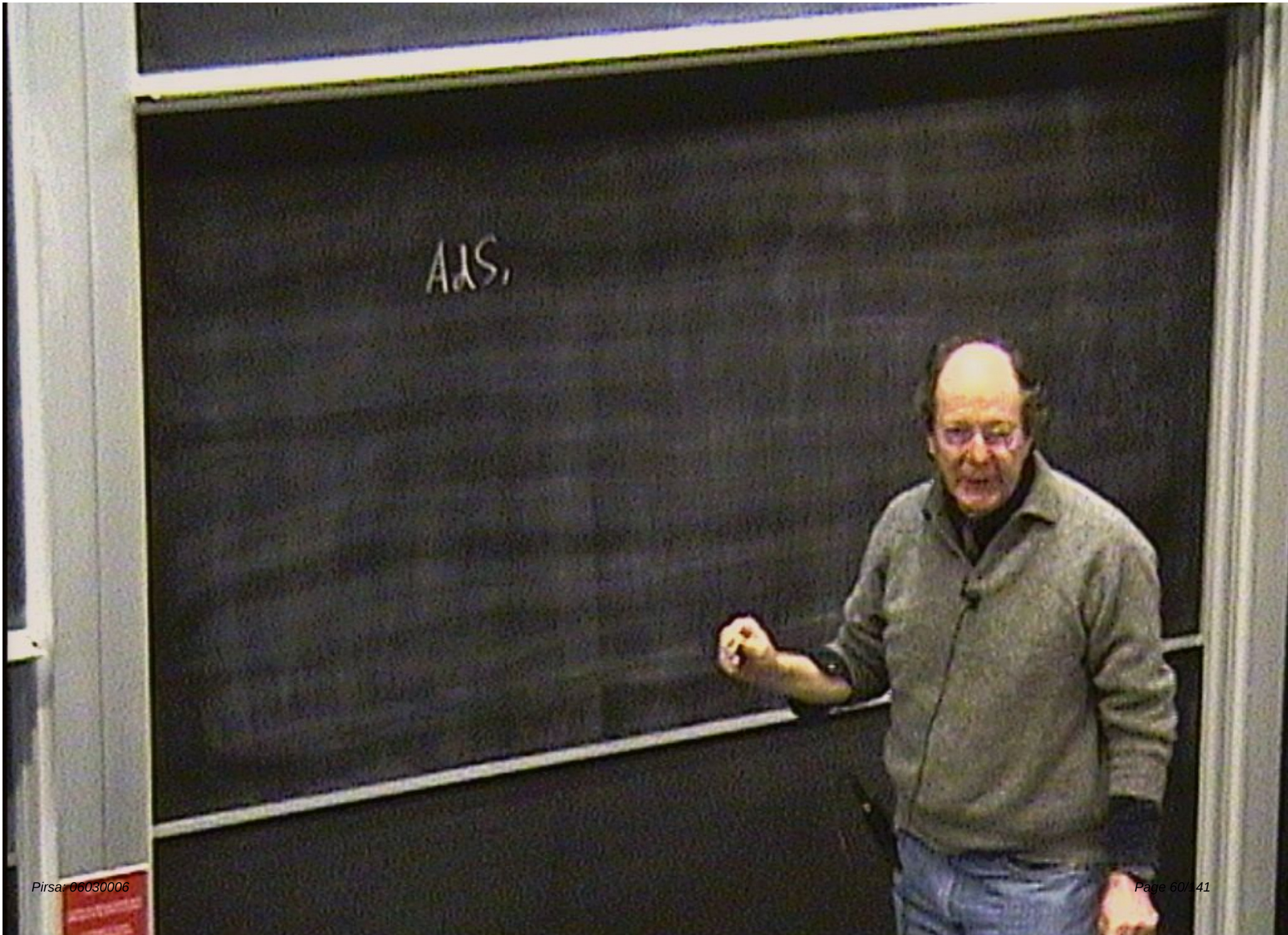
same Diffe
Kanpur

GUSY

Newton-Hooke algebras

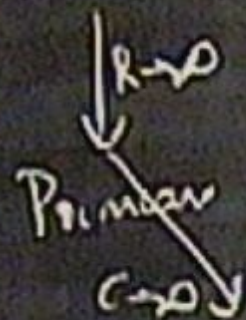
$$\psi'(x') = \textcircled{e^{\psi(x)}} \psi(x)$$

1967 Barry-Levy-Leblond



AdS, dS
↓
R=0
Primaries

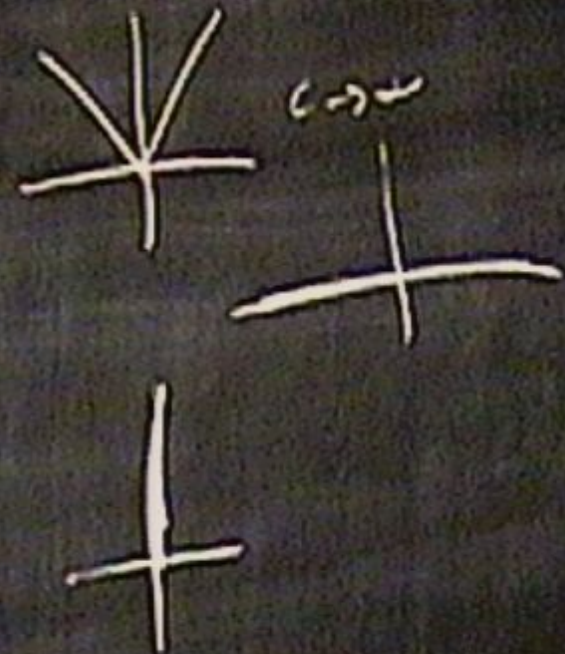
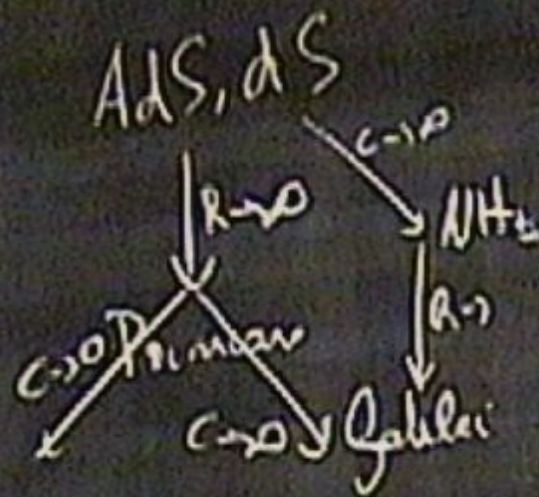
AdS, dS

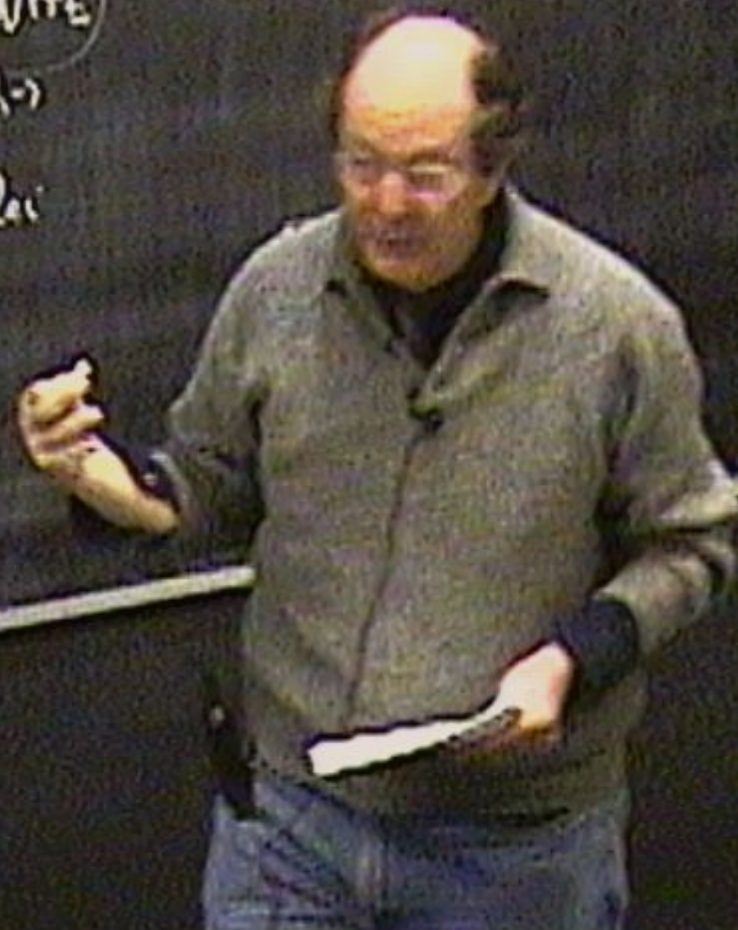
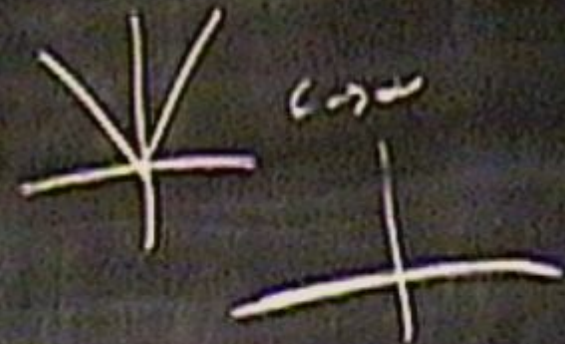
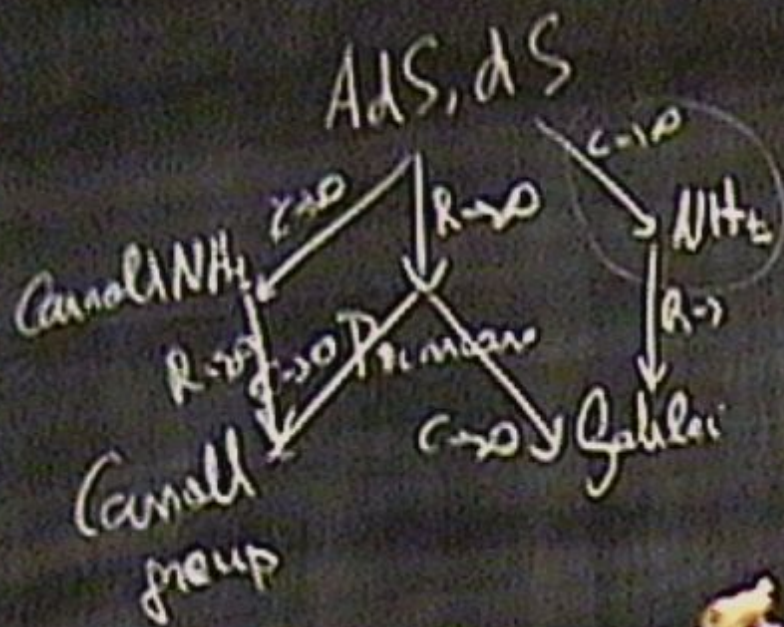


AdS, dS
↓ R=∞
Principles
C=∞ → Galilei

C=10 → NH₂

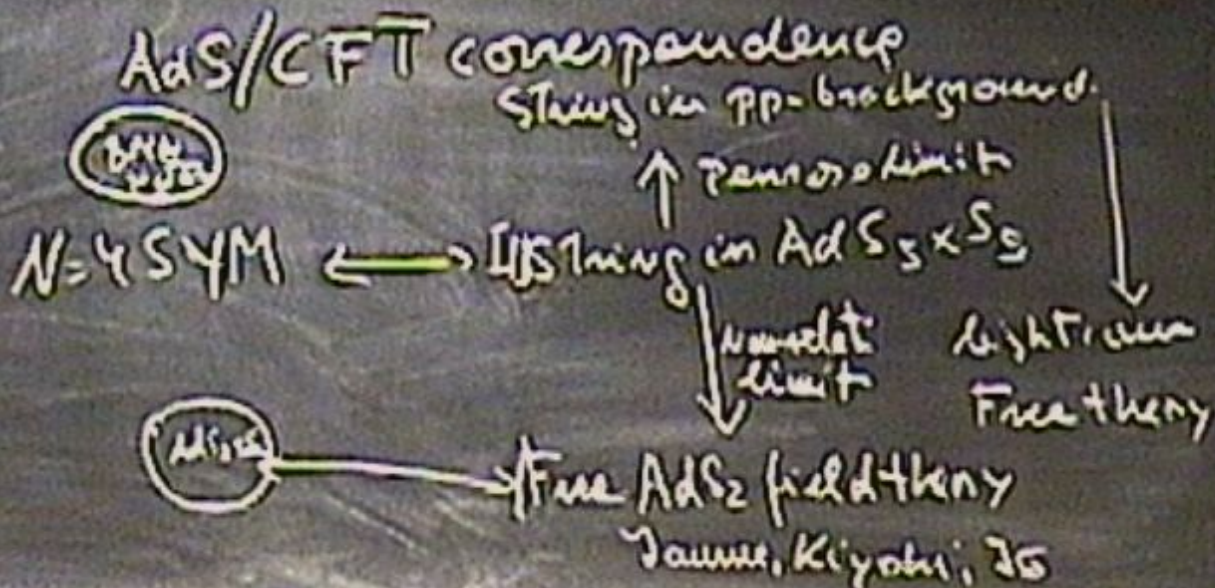






- Newton - Hooke al fibres
- Non-Relativistic Branes
- pp-waves

Jan Bruguis
Kiyohi Kameiura



Bosonic Non-Rel. Strings

{ Tauber, Gaiotto, 2000
Hiroshi Oginaka

Danielsson, Geilker, Krauss, Krieger



$$\left\{ \begin{array}{l} X^\mu = w x^\mu \\ \Pi_P = \hat{T} \dot{X}^\mu - P \end{array} \right.$$

$$X^0 = w x^0$$

$w \rightarrow \infty$

$$[P_m, P_n] = \pm \frac{i}{\hbar} H_{mn}$$

$$[M, M] = 0$$

$$[M, P] = P$$

$$[P_m, P_n] = \pm i \hbar m n$$

$$[M, M] = -M$$

$$[M, P] = P$$

$$\begin{cases} P_0 = \frac{\hat{P}_0}{\omega} \\ H_{01} = \omega \hat{H}_{01} \\ R = \omega \hat{R} \end{cases}$$



$$[P_m, P_n] = \pm \frac{i}{R^2} M_{mn}$$

$$[M, M] = 0$$

$$[M, P] = P$$

$$\begin{cases} P_0 = \frac{P_0}{\omega} \\ H_{0i} = \omega H_{0i} \\ R = \omega R \end{cases}$$

$$[P_0, P_i] = \pm \frac{i}{R^2} H_{0i}$$

$$[\frac{P_0}{\omega}, P_i] = \pm \frac{1}{\omega R^2} H_{0i}$$

$$[P_0, P_i] =$$

linear, isotropic
(LH, v.g.)

Non-dynamical
non-transverse oscillation

$$[P_m, P_n] = \pm \frac{1}{R^2} M_{mn}$$

$$[M, M] = M$$

$$[M, P] = P$$

$$P_0 = \frac{R_0}{\omega}$$

$$M_{0i} = \omega M_{0i}$$

$$R = \omega \hat{R}$$

$$[P_0, P_i] = \pm \frac{1}{R^2} M_{0i}$$

$$[\frac{P_0}{\omega}, P_i] = \pm \frac{1}{\omega R^2} M_{0i}$$

$$[P_0, P_i] = \pm \frac{1}{R^2} M_{0i}$$

$$[P_i, P_0] = 0$$

$$[M_{0i}, M_{0j}] = 0$$

non-dimensional

Non-dynamical

$$[P_m, P_n] = \pm \frac{L}{R^2} M_{mn}$$

$$[M, M] = 0$$

$$[M, P] = P$$

$$P_0 = \frac{\tilde{P}_0}{\omega}$$

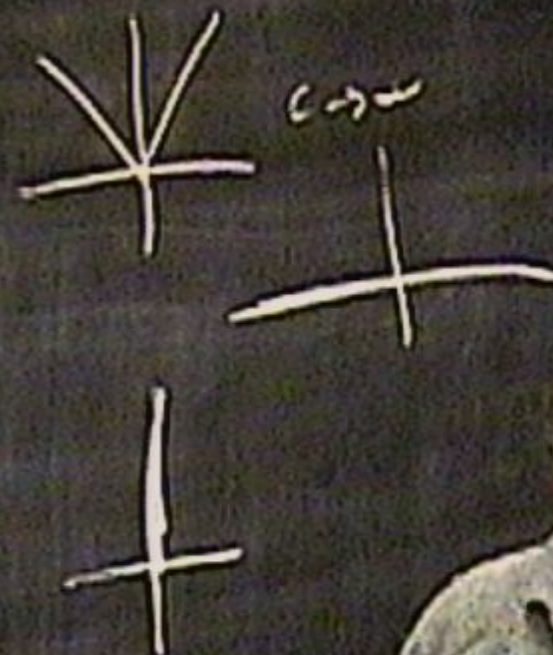
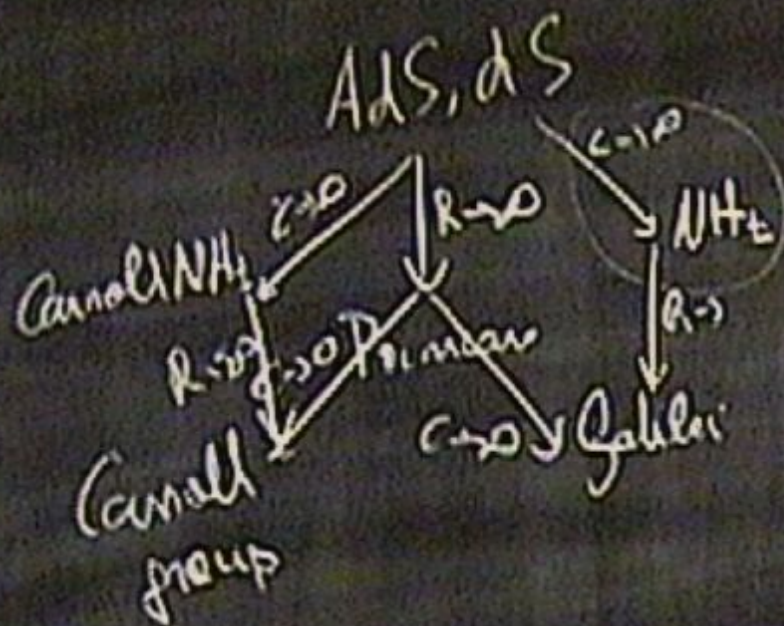
$$H_{0i} = \omega \tilde{H}_{0i}$$

$$R = \omega \tilde{R}$$

$$[P_0, P_i] = \pm \frac{L}{R^2} H_{0i}$$

$$[\frac{P_0}{\omega}, P_i] = \pm \frac{1}{\omega R^2} H_{0i}$$





$$[H_0, H_0] = -\frac{1}{R^2} H_0$$

$$[\frac{p_0}{\sqrt{R}}, p_i] = \pm \frac{1}{\sqrt{R}} \omega H_0$$

$$[p_0, p_i] = \pm \frac{i}{R} H_0$$

$$[p_i, p_0] = 0$$

$$[H_0, H_0] = 0$$

harmonic oscillator!

$$[H_0, H_0] = -\frac{1}{R^2} H_0$$

$$[\frac{P_0}{\sqrt{R}}, P_L] = \pm \frac{1}{\sqrt{R^2}} \omega H_0$$

$$[\vec{P}_0, P_L] = \pm \frac{c}{R_L} H_0$$

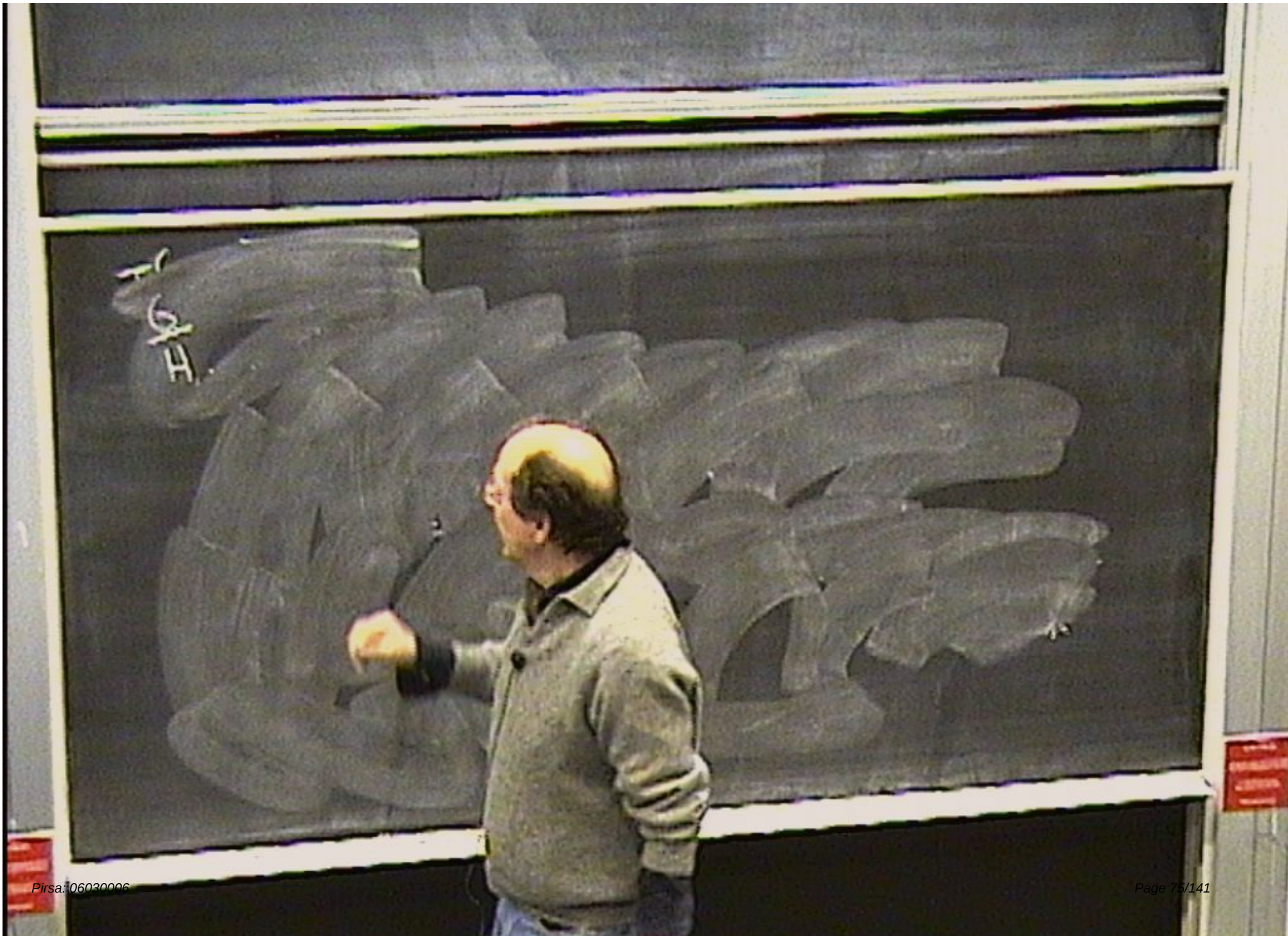
$$[P_L, P_0] = 0$$

$$[H_0, H_0] = 0$$

harmonic oscillator!

$$\begin{cases} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS₄ NH₋



$\frac{G}{H}$

• broken gen. $\mathbb{P}^1$ Mod
• unbroken gen. Rotations,



$\frac{G}{H}$

- broken gen. $\overline{P}$, H_u
- unbroken gen. Rotations, H

$\frac{Tr}{G/H}$

- broken gen $\overline{P}$, M_{br}
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

Π

$\frac{G}{H}$

- broken gen. $\overline{P}$, M
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X^0, \vec{x})$$

$$\frac{\Gamma}{G/H}$$

- broken gen. $\overline{P}$, M_0
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \tilde{X})$$

• Unbroken Translations $\simeq$ Espace

$$\frac{G}{H}$$

- broken gen. $\overline{P}$, M_{H}
- unbroken gen. Rotations, H

$$g = e^{i\alpha^0 P_0} e^{i\vec{\alpha} \cdot \vec{X}^0} e^{i\vec{V}^0 \cdot \vec{G}^0} e^{i\vec{V}^0 \cdot \vec{M}^0}$$

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \tilde{X})$$

• Unbroken Translations $\simeq$ Espace

$$\frac{G}{H}$$

- broken gen $\overline{P}$, M_{H}
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \tilde{X})$$

$$g = e^{i\alpha^0 T_0} e^{i\vec{\alpha} \cdot \vec{T}} e^{iV^a(x) T_a}$$

$$\eta = -i \dot{S}^\dagger dg = \underbrace{e^{i\alpha^0 T_0}}_{\text{rotations}} + \text{translations} \quad \text{Unbroken Translations} \simeq \text{Espace}$$

$\frac{d}{dt}$

unbroken sym. Rotations, H

$\langle 0 | 0 \rangle$

$$g = e^{i\alpha^0 P_0} e^{i\vec{\alpha} \cdot \vec{P}} e^{i\alpha^i \langle 0 | P_i | 0 \rangle}$$

$\tilde{H}(X^0, \vec{X})$

$$\Omega = -i \dot{g} g^{-1} = \underbrace{\frac{1}{2} \dot{\alpha}^0 P_0}_{\text{rotations}} + \text{translations} \quad \text{Unbroken Translations} \simeq \text{Espace}$$

$$L^0 = dx^0$$

$$L^i = dx^i - v^i dx^0$$

$$P_{\pm} \theta_{\pm} = \pm \theta_{\pm}$$

$\frac{v}{H}$

unbroken gen. Rotations, π

$\zeta(0)$

$$g = e^{i x^0 P_0} e^{i \vec{x}(x^0) \vec{P}} e^{i v^i(x^0) N^i}$$

$$\pi(x^0, \vec{x})$$

$$\Omega = -i \dot{\vec{x}} d\vec{g} = \underbrace{L^{\text{NMP}}}_{\text{rotations}} + \text{translations} \quad \text{Unbroken Translations} \simeq \text{Espace}$$

$$\begin{cases} L^0 = dx^0 \\ L^i = dx^i - v^i dx^0 \\ L^{0i} = dx^0 dx^i + dx^0 \frac{x^i}{R_2} \\ L^{ij} = 0 \end{cases}$$

$$\Pi_+ = \Pi_0 \Pi_1 \tau_3 \quad \Pi_+ \Theta_{\pm} = \pm \Theta_{\pm}$$

$\frac{1}{H}$

unbroken gen. Rotations, π

$\zeta(0)$

$$g = e^{i x^0 P_0} e^{i \vec{x}(\vec{x}^0) \vec{P}} e^{i v^i(\vec{x}^0) N^i}$$

$$\pi(x^0, \vec{x})$$

$$\Omega = -i \dot{g} dg = \underbrace{\left(\frac{1}{2} \text{Tr} P_{\mu\nu} \right)}_{\text{rotations}} + \underbrace{\text{Unbroken Translations}}_{\approx \text{Espace}} \leftarrow P_0 \leftarrow \text{Space}$$

$$\begin{cases} L^0 = \frac{1}{2} \dot{x}^2 \\ L^i = \dot{x}^i x^0 - \dot{x}^0 x^i \\ L^{0i} = \dot{x}^0 x^i - \dot{x}^i x^0 \\ L^{ij} = 0 \end{cases}$$

$$\Pi_5 \quad \Pi_* = P_0 P_2 P_3 \quad P_4 \Theta_{\pm} = \pm \Theta_{\pm}$$

$\frac{1}{H}$

unbroken gen. Rotations, π

$\zeta(0)$

$$g = e^{i x^0 P_0} e^{i \vec{x}(\vec{x}^0) \vec{P}} e^{i V^0(\vec{x}^0) N^0}$$

$$\pi(x^0, \vec{x})$$

$$\Omega = -i \oint dg = \underbrace{\int_{\text{rotations}}}_{\text{rotations}} + \underbrace{\int_{\text{translations}}}_{\text{translations}} \approx \text{Espai}$$

$$\begin{cases} L^0 = \frac{1}{2} p^2 \\ L^i = dx^i - v^i dx^0 \\ L^{0i} = dx^0 v^i + dx^i \frac{x^0}{R_2} \\ L^{ij} = 0 \end{cases}$$

$$P_0 \leftarrow \int dx^0 \rightarrow \text{surface term}$$

$$\Pi_5 \quad \Pi_* = P_0 P_2 \tau_3 \quad P_4 \theta_{\pm} = \pm \theta_{\pm}$$



$$\frac{G}{H}$$

- broken gen $\overline{P}$, H
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X^0, \vec{X})$$

$$g = e^{i x^0 P_0} e^{i \vec{X}(x^0) \vec{P}} e^{i v^i(x^0) N^i}$$

$$\Omega = -i \dot{g} dg = \underbrace{L^{\text{rot}}}_{\text{rotations}} + \underbrace{L^{\text{trans}}}_{\text{Unbroken Translations}} \approx \text{Espace}$$

$$P_0 \leftarrow \int dx^0 \rightarrow \text{surface term}$$

$$\Omega_2 = L^1 L^0$$

$$\begin{cases} L^0 = dx^0 \\ L^1 = dx^1 - v^1 dx^0 \\ L^{0i} = dv^i + dx^0 \frac{x^i}{R^2} \\ L^i = 0 \end{cases}$$

$$\Pi_x = P_0 P_2 \tau_3 \quad P_+ \theta_{\pm} = \pm \theta_{\pm}$$

$\frac{b}{H}$

- broken gen $\overline{P}$, H
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X^0, \vec{x})$$

$$g = e^{i x^0 P_0} e^{i \vec{x} \cdot \vec{P}} e^{i v^i(x^0) N^i}$$

$\Omega = -i \dot{g} dg = \underbrace{L^{\text{non}}}_{\text{rotations}} + \text{translations}$ Under Translations $\approx$ Espace

$P_0 \leftarrow \int dx^0 \rightarrow \text{surface term}$

$$\begin{cases} L^0 = \dot{x}^0 \\ L^i = \dot{x}^i - v^i \dot{x}^0 \\ L^{0i} = \dot{x}^0 \dot{x}^i + \dot{x}^0 \frac{x^i}{R_2} \\ L^{ij} = 0 \end{cases}$$

$$\Omega_2 = L^i L^{0i}, \quad d\Omega_2 = 0$$

$$P_* = P_0 P_1 P_2 \quad P_i \theta_i = \pm \theta_i$$

$$\frac{G}{H}$$

- broken gen $\overline{P}$, H
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(x^0, \vec{x})$$

$$g = e^{i x^0 P_0} e^{i \vec{x} \cdot \vec{P}} e^{i v^i(x^0) N^i}$$

$$\eta = -i \dot{g} dg = \underbrace{L^{NP}}_{\text{rotations}} + \text{translations} \quad \text{Unbroken Translations} \simeq \text{Espace}$$

$$P_0 \leftarrow \int dx^0 \rightarrow \text{surface term}$$

$$\begin{cases} L^0 = dx^0 \\ L^i = dx^i - v^i dx^0 \\ L^{0i} = dv^i + dx^0 \frac{x^i}{R^2} \\ L^{ij} = 0 \end{cases}$$

$$\eta_2 = L^i L^{0i}, \quad d\eta_2 = 0$$

$$\eta_2 = d\left(-v^i dx^i + dx^0 \frac{v^2}{2} + dx^0 \frac{\vec{x}^2}{R^2}\right)$$

$$\Pi_x = P_0 P_2 \tau_3 \quad \Pi_\pm \theta_\pm = \pm \theta_\pm$$

$$\left\{ \begin{array}{l} L^0 = dx^0 \\ L^i = dx^i - v^{0i} dx^0 \\ L^{0i} = dv^{0i} + dx^0 \frac{x^i}{R^2} \\ L^{ij} = 0 \end{array} \right.$$

$$\Omega_2 = L^i L^{0i}, \quad d\Omega_2 = 0$$

$$\Omega_2 = d \left(\underbrace{-v^{0i} dx^i + dx^0 \frac{v^2}{2} + dx^0 \frac{x^i}{R^2}}_{\Omega_2} \right)$$

$H_2 \neq 0$ Euler-Charakteristik

$\frac{G}{H}$

- broken gen $\overline{P}$, H
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$g = e^{i x^0 P_0} e^{i \vec{x} \cdot \vec{P}} e^{i \vec{v} \cdot \vec{L}} e^{i \vec{v} \cdot \vec{S}}$$

$$\pi(x^0, \vec{x})$$

$\Omega = -i \int d^3x \text{Tr} \{ \dot{A} \wedge A \}$ rotations Unter translations $\simeq$ Espace
 $T_0 \leftarrow \int dx^0 \rightarrow \text{surface}$

$$\begin{cases} L^0 = dx^0 \\ L^i = dx^i - v^i dx^0 \\ L^{0i} = dv^i + dx^0 \frac{x^i}{R^2} \\ L^{ij} = 0 \end{cases}$$

$$\Omega_2 = L^0 L^{0i}, \quad d\Omega_2 = 0$$

$$\Omega_2 = d \left(-v^i dx^i + dx^0 \frac{v^2}{2} + dx^0 \frac{\vec{x}^2}{R^2} \right)$$

Ω_2

$H_2 \neq 0$ Euler-Charakteristik

$$\int \Omega_2$$

$$\begin{cases} L = dx - n^0 dx^0 \\ L^{0i} = dx^0 \dot{x}^i + dx^i \frac{x^0}{R^2} \\ L^{ij} = 0 \end{cases} \quad \Omega_2 = d \left(\frac{-n^0 dx^0 + dx^i \frac{x^i}{2} + \dots}{\Omega_2} \right)$$

$H_2 \neq 0$ Eilenberg-Chern class

$$S_{H_2} = \int \Omega_2^* = \int dx \left(-n^0 \dot{x}^i + \dot{x}^0 \frac{x^i}{2} + \dot{x}^i \frac{x^j}{R^2} \right)$$

$$\begin{cases} L^i = dx^i - v^i dx^0 \\ L^{0i} = dv^i + dx^0 \frac{x^i}{R^2} \\ L^{ii} = 0 \end{cases}$$

$$\Omega_2 = d \left(\frac{-v^0 dx^0 + dx^0 \frac{v^0}{2} + \dots}{R^2} \right)$$

$H_2 \neq 0$ Eilenberg-Chernoff

$$S_{W2} = \int \Omega_2^* = \int dx^0 \left(-v^0 \dot{x}^i + \dot{x}^0 \frac{v^i}{2} + \dot{x}^0 \frac{x^i}{R^2} \right)$$

$$\dot{x}^i = v^i \dot{x}^0$$

$$\begin{cases} L^i = dx^i - v^i dx^0 \\ L^{0i} = dv^{0i} + dx^0 \frac{x^i}{R^2} \\ L^{ii} = 0 \end{cases}$$

$$\Omega_2 = d \left(\frac{-v^{0i} dx^i + dx^0 \frac{v^2}{2} + \dots}{R^2} \right)$$

$H_2 \neq 0$ Euler-Lagrange

$$S_{N2} = \int \Omega_2^* = \int dx^0 \left(-v^{0i} \dot{x}^i + \dot{x}^0 \frac{v^2}{2} + \dot{x}^0 \frac{x^i}{R^2} \right)$$

$$\dot{x}^i = v^{0i} \dot{x}^0$$

$$\mathcal{L} = \frac{1}{2} M \frac{\ddot{x}^2}{\dot{x}^0} - \frac{M}{2} \frac{1}{R^2} \dot{x}^2 \dot{x}^0$$

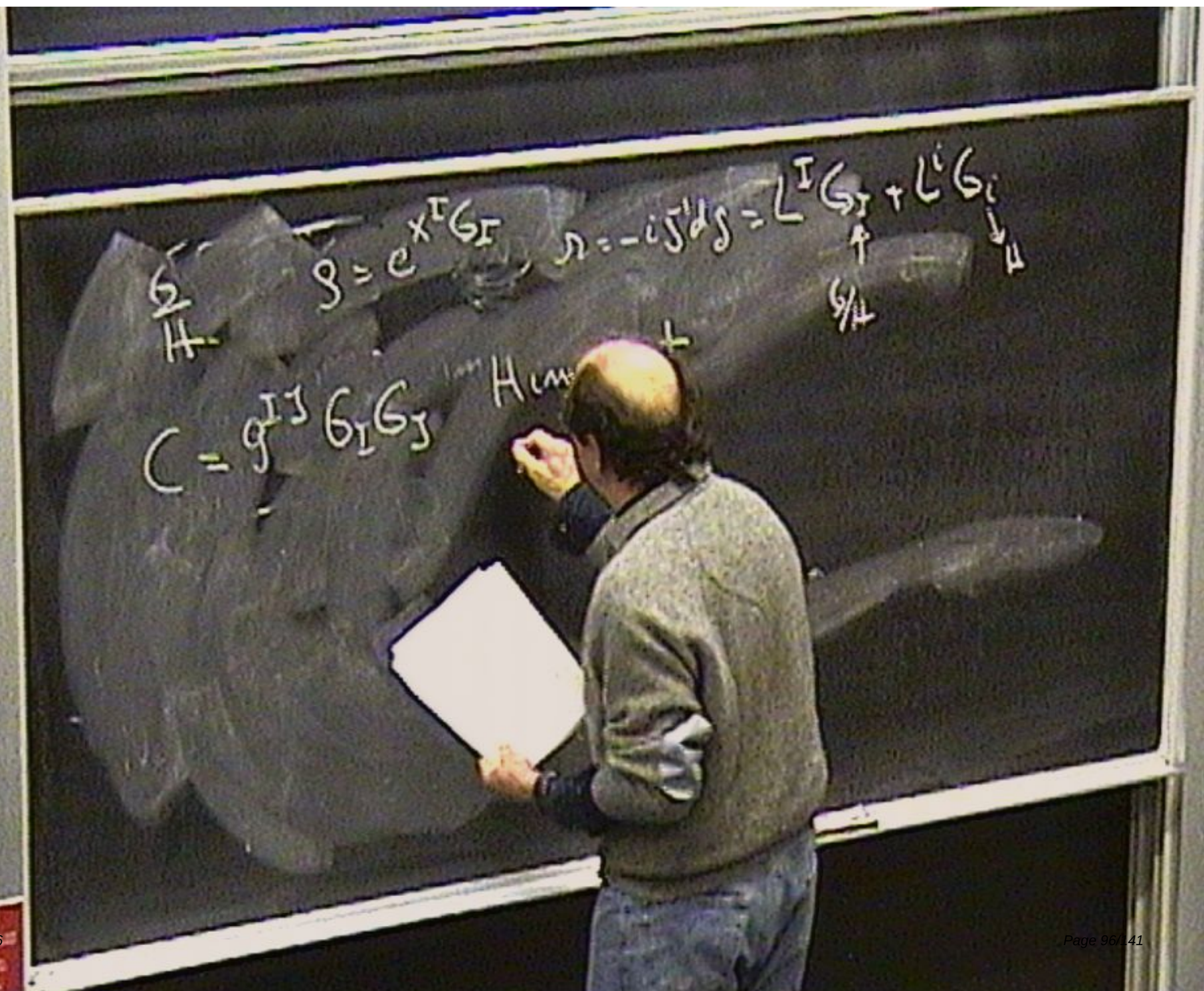
$$\begin{cases} L = dx - n^0 dx^0 \\ L^{0i} = dv^{0i} + dx^0 \frac{x^i}{R^2} \\ L^{ij} = 0 \end{cases} \quad \Omega_2 = d \left(\frac{-n^0 dx^0 + dx^0 \frac{x^i}{2} + \dots}{R^2} \right)$$

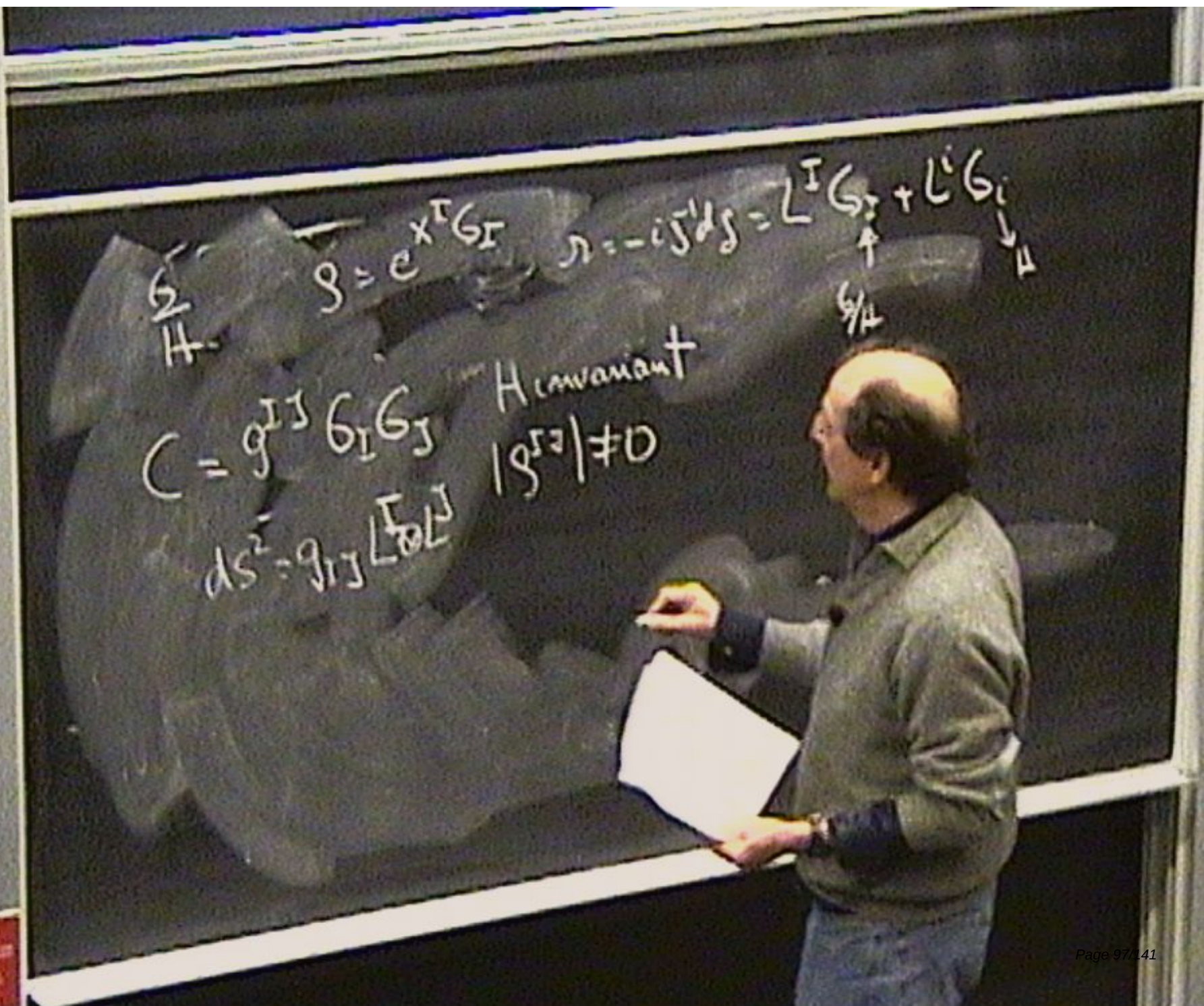
$H_2 \neq 0$ Euler-Lagrange

$$S_{W2} = \int \Omega_2^* = \int dx \left(-\dot{n}^0 \dot{x}^i + \dot{x}^0 \frac{x^i}{2} + \dot{x}^0 \frac{x^i}{R^2} \right)$$

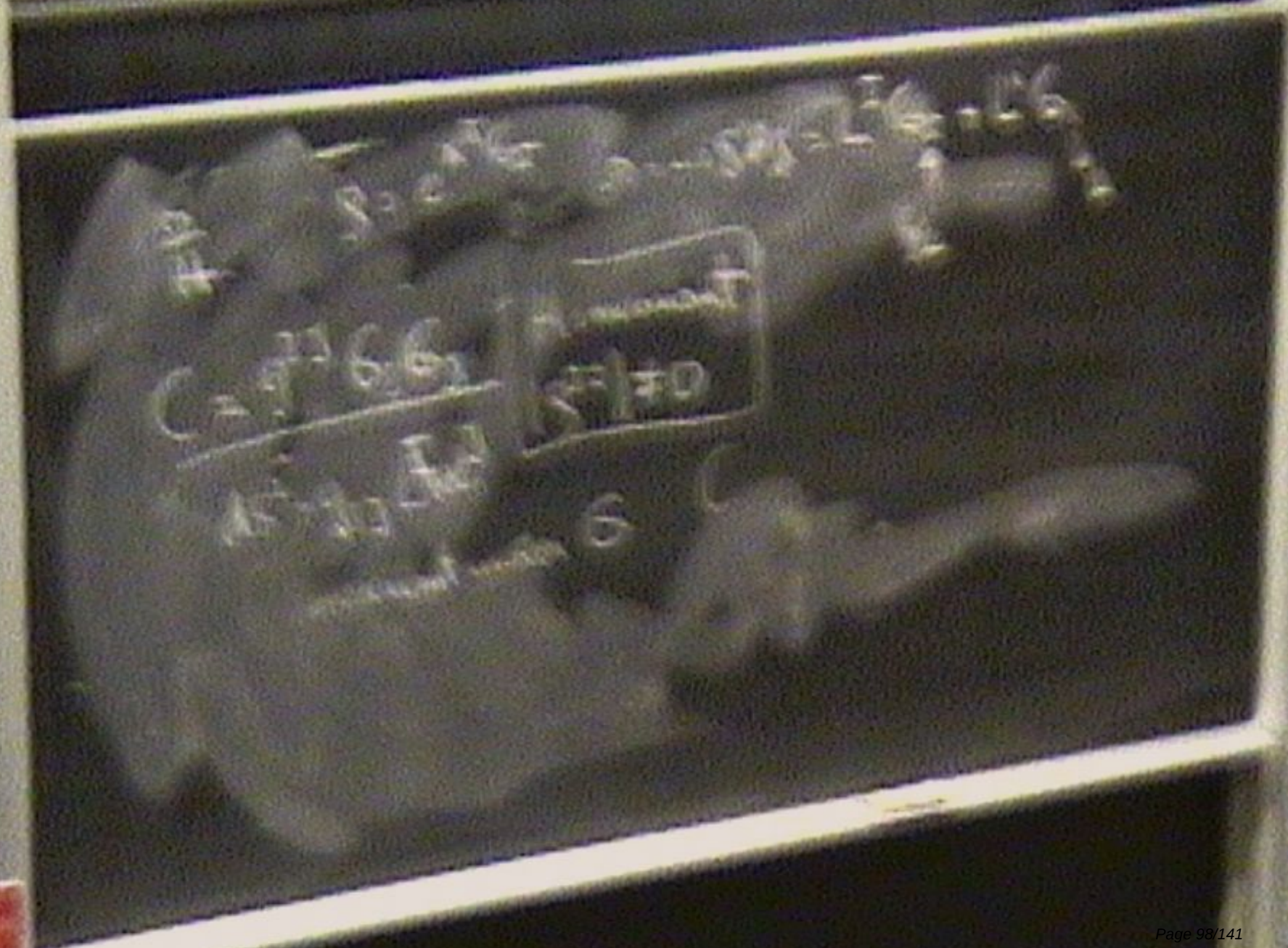
$$\dot{x}^i = v^{0i} \dot{x}^0$$

$$\mathcal{L} = \frac{1}{2} M \dot{\vec{x}}^2 - \frac{M}{2} \frac{1}{R^2} \vec{x}^2 \Rightarrow \text{D.K. } \dot{x}^0 = c$$





$$\sum_{i,j} g_{ij} x^i x^j$$
$$g = e^{x^T G x} \quad \eta = -i \sqrt{g} ds = L^I G_I + L^I G_I$$
$$C = g^{IJ} G_I G_J \quad \text{Invariant}$$
$$ds^2 = g_{IJ} L^I L^J$$
$$|g^{IJ}| \neq 0$$



$$\frac{\delta}{\delta} \quad g = e^{x^I G_I}, \quad \eta = -i \delta^I \delta g = L^I G_I + L^i G_i$$

$$C = \frac{g^{IJ} G_I G_J}{|g^{IJ}| \neq 0} \quad \left\{ \begin{array}{l} \text{H invariant} \\ |g^{IJ}| \neq 0 \end{array} \right.$$

$$ds^2 = g_{IJ} L^I L^J$$

invariant under G

$$\begin{cases} L = \dots \\ L_{0i} = dv_{0i} + dx^0 \frac{x^i}{R^2} \\ L_{ii} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-N}{2} dx^0 \frac{x^i}{R^2}\right)$$

$H_2 \neq 0$ Euler-Lagrange

$$S_{W2} = H \int \Omega_2^* = H \int dz \left(-\dot{x}^0 \dot{x}^i + \dot{x}^0 \frac{x^i}{2} + \dot{x}^0 \frac{x^i}{R^2} \right)$$

$\dot{x}^i = v_{0i} \dot{x}^0$

$$\mathcal{L} = \frac{1}{2} M \dot{\bar{x}}^2 - \frac{M}{2} \frac{1}{R^2} \bar{x}^2 \Rightarrow \text{D.H. } \dot{x}^0 = c$$

$$[H_0, H_1] = -\frac{1}{R^2} H_0$$

$$[\frac{P_0}{\sqrt{R}}, P_i] = \pm \frac{1}{\sqrt{R}} \omega H_0$$

$$[\vec{P}_0, P_i] = \pm \frac{i}{R} H_0$$

$$[P_i, P_0] = 0$$

$$[H_0, H_0] = 0$$

$$[P_i, H_0] = 0$$

harmonic oscillator

$$\begin{cases} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS₂ NH₂

$$[H_0, H_0] = -\frac{1}{R^2} H_0$$

$$\left[\frac{P_0}{\sqrt{R}}, P_i\right] = \pm \frac{1}{\sqrt{R}} M_{0i}$$

$$[P_0, P_i] = \pm \frac{i}{R} M_{0i}$$

$$[P_i, P_0] = 0$$

$$[M_{0i}, M_{0j}] = 0$$

$$[P_i, M_{0j}] = i \delta_{ij} \vec{z}$$

central extension

harmonic oscillator!

$$\begin{cases} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS, NH-

$$[H_0, H_1] = -\frac{1}{R^2} M_0$$

$$\left[\frac{P_0}{\sqrt{R}}, P_1\right] = \pm \frac{1}{\sqrt{R^2}} M_0$$

$$\left\{ \begin{array}{l} [\vec{P}_0, P_1] = \pm \frac{i}{R^2} M_0 \\ [P_1, P_1] = 0 \\ [P_1, P_1] = 0 \\ [P_1, P_1] = 0 \end{array} \right.$$

harmonic oscillator!

$$\left\{ \begin{array}{l} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{array} \right.$$

AdS, NH-

$$NH_{\neq 10}$$

central
exchange

$$[P_0, H_0] = -\frac{1}{R^2} H_0$$

$$[\frac{P_0}{\sqrt{R}}, P_L] = \pm \frac{1}{\sqrt{R}} H_0$$

$$[P_0, P_L] = \pm \frac{1}{R} H_0$$

$$[P_L, P_0] = 0$$

$$[H_0, H_0] = 0$$

$$[P_L, H_0] = i \left(\frac{1}{R} \right)$$

central
extension

harmonic oscillator!

$$\begin{cases} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS, NH-

$z \rightarrow c$

$\{ NH_2 \}$

10
(11)

$\{ e^{ic} \}$

$$\frac{G}{H} \quad g = e^{x^T G x}, \quad \eta = -i \dot{x}^T dx = \underbrace{L^I G_I}_{\frac{G}{H}} + \underbrace{L^i G_i}_{\frac{G}{H}}$$

$$C = \frac{g^{IJ} G_I G_J}{|g^{IJ}| \neq 0} \quad \boxed{\text{H invariant}}$$

$$ds^2 = g_{IJ} L^I L^J$$

invariant under G

$$C = -2z\vec{p}_0 + \vec{p}^2 + \frac{1}{4z} \vec{p}_0^2$$

$$\frac{G}{H} \quad g = e^{x^I G_I}, \quad \eta = -i \dot{g}^\dagger dg = \underbrace{L^I G_I}_{\frac{G}{H}} + \underbrace{L^i G_i}_{\frac{G}{H}}$$

$$C = g^{IJ} G_I G_J \quad \left\{ \begin{array}{l} \text{H invariant} \\ |g^{IJ}| \neq 0 \end{array} \right.$$

$$ds^2 = g_{IJ} L^I L^J$$

invariant under G

$$C = -2zP_0 + \vec{P}^2 + \frac{1}{R^2} \Pi a^2$$

$$ds^2 = \left(-2dx^0 dc - (dx^0)^2 \frac{\vec{x}^2}{R^2} + d\vec{x}^2 \right)$$

$$\frac{S}{H} \quad g = e^{x^I G_I}, \quad \eta = -i \dot{S} dS = \underbrace{L^I G_I}_{\frac{S}{4}} + \underbrace{L^i G_i}_{\frac{S}{4}}$$

$$C = \frac{g^{IJ} G_I G_J}{|g^{IJ}| \neq 0} \quad \left\{ \begin{array}{l} H \text{ invariant} \\ |g^{IJ}| \neq 0 \end{array} \right.$$

$$dS^2 = g_{IJ} L^I L^J$$

invariant under G

$$C = -2z\vec{p}_0 + \vec{p}^2 + \frac{1}{R^2} \pi \alpha^2$$

$$dS^2 = \underbrace{\left(-2dx^0 dc - \left(dx^0 \right)^2 \frac{x^2}{R^2} + \left(dx^1 \right)^2 \right)}_{\text{primär}} + \underbrace{\left(d(RN^{\alpha\alpha}) + dx^0 \frac{y^i}{R} \right)^2}_{\text{sekundär}}$$

$$\frac{G}{H} \quad g = e^{x^I G_I}, \quad \eta = -i \dot{g} dg = \underbrace{L^I G_I}_{\frac{G}{H}} + \underbrace{L^I G_I}_{\frac{G}{H}}$$

$$C = g^{IJ} G_I G_J \quad \left\{ \begin{array}{l} \text{H invariant} \\ |g^{IJ}| \neq 0 \end{array} \right.$$

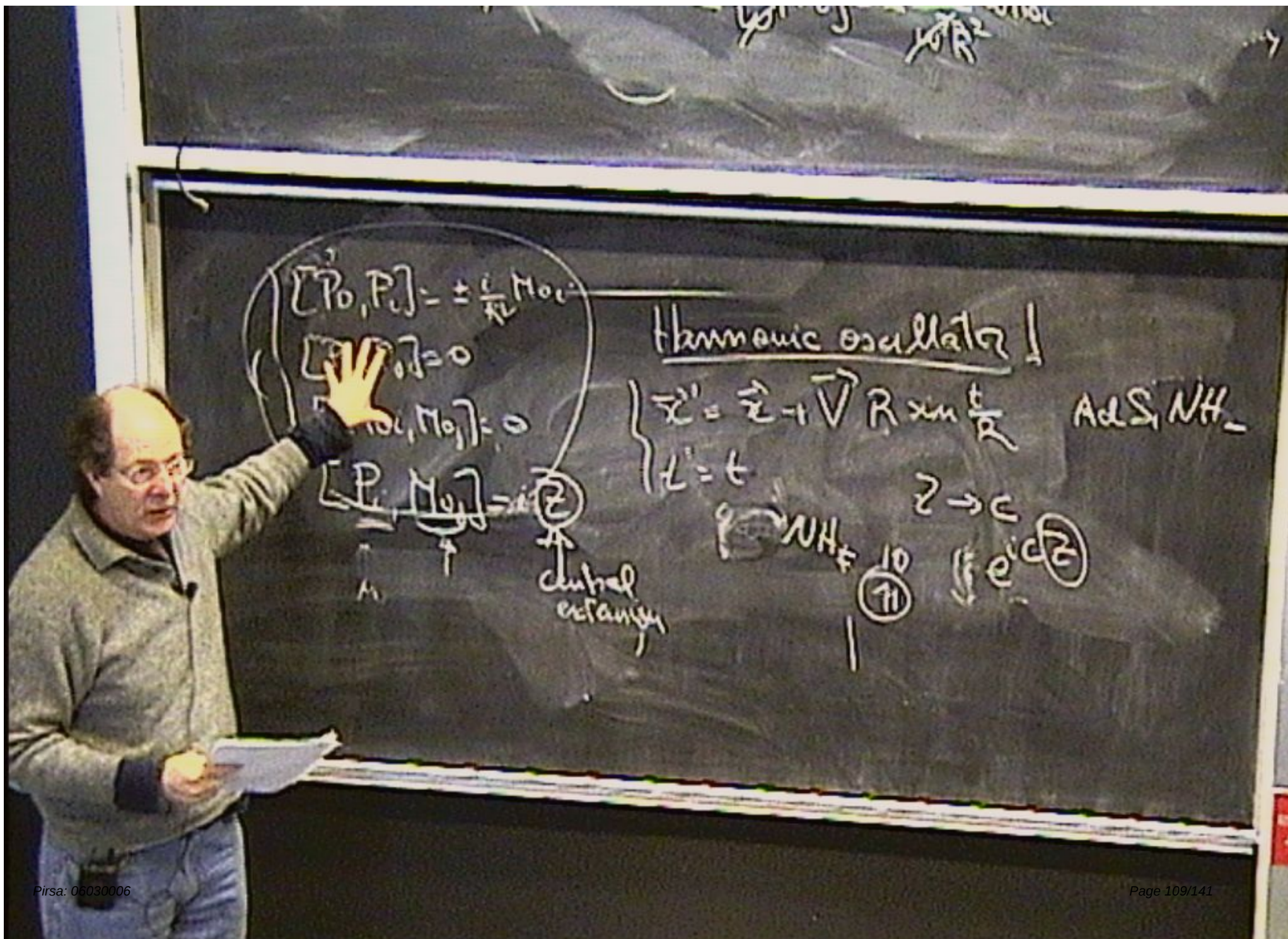
$$ds^2 = g_{IJ} L^I L^J$$

invariant under G

$$C = -2z\bar{z}P_0 + \vec{P}^2 + \frac{1}{R^2} \Pi_0^2$$

(x^μ, R, ψ)

$$ds^2 = \underbrace{\left(-2dx^0 dc - \left(dx^0 \right)^2 \frac{R^2}{R^2} + (dx^i)^2 \right)}_{\text{Minkowski}} + \underbrace{\left(d(RN^0 x^i) + dx^0 \frac{x^i}{R} \right)^2}_{\text{Sph.}}^2$$



$$[P_0, P_i] = \pm \frac{i}{\hbar} M_{0i}$$

$$[P_i, P_j] = 0$$

$$[M_{0i}, M_{0j}] = 0$$

$$[P_i, M_{0j}] = i\hbar \delta_{ij}$$

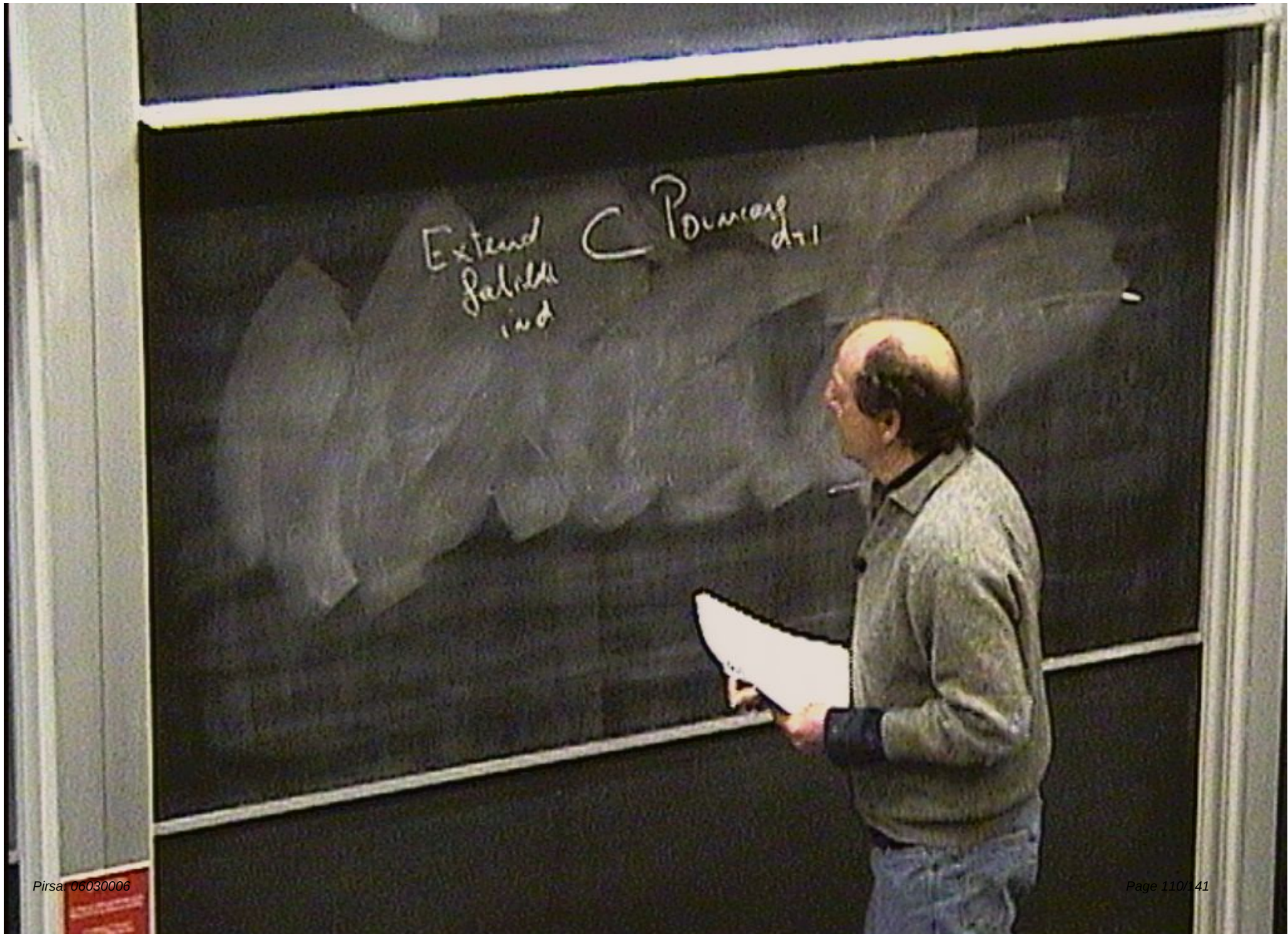
central extension

Harmonic oscillator!

$$\begin{cases} x' = x + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS, NH₋

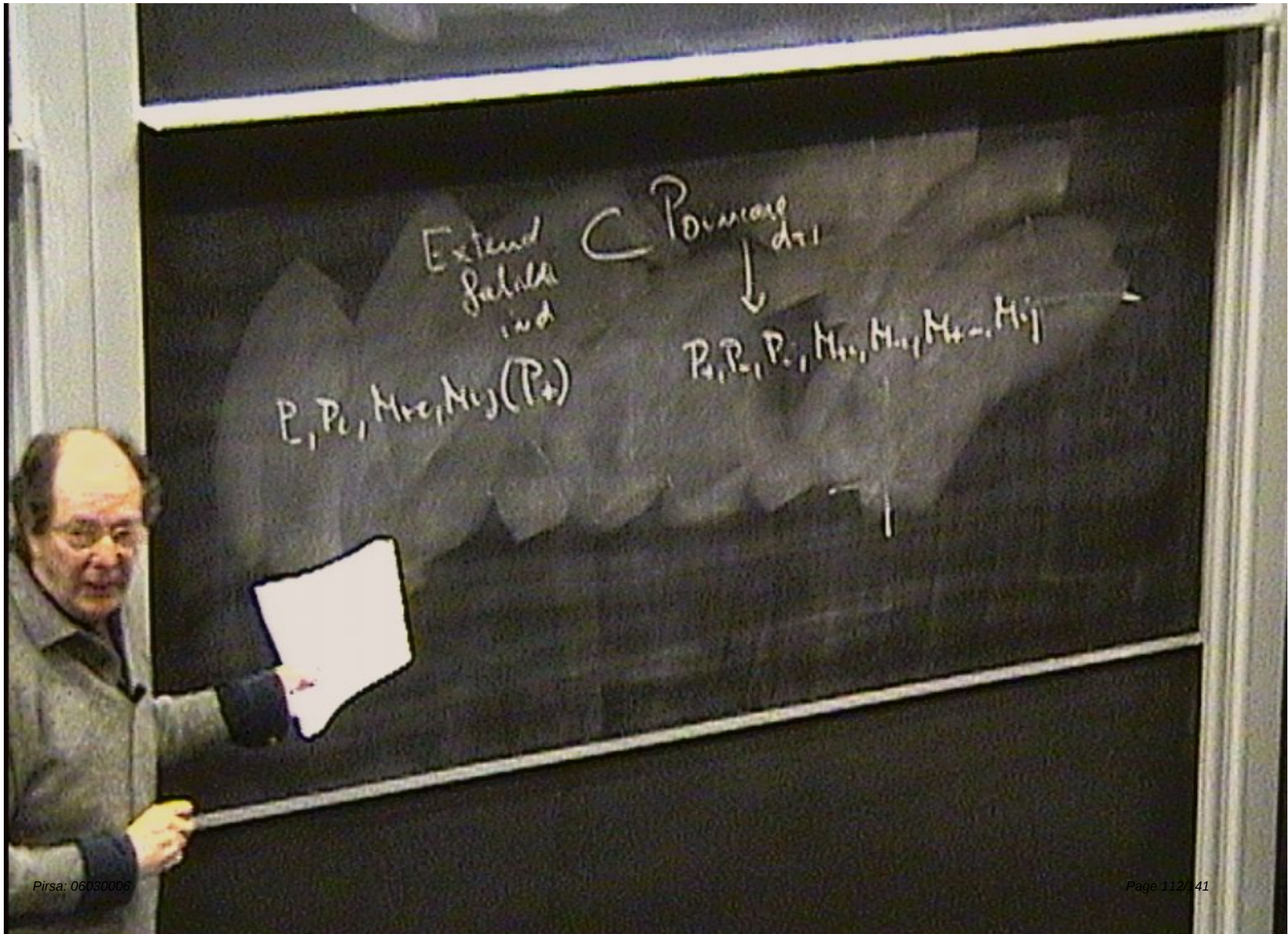
$$\begin{aligned} & \text{NH}_+ \xrightarrow{z \rightarrow c} \text{NH}_- \\ & \text{NH}_+ \xrightarrow{z \rightarrow c} \text{NH}_- \end{aligned}$$



Extend Saltillo ind C Poincaré d71

Extend
Salmon
ind

C Poincaré
↓
 P, P, P, M, M, M, M, M

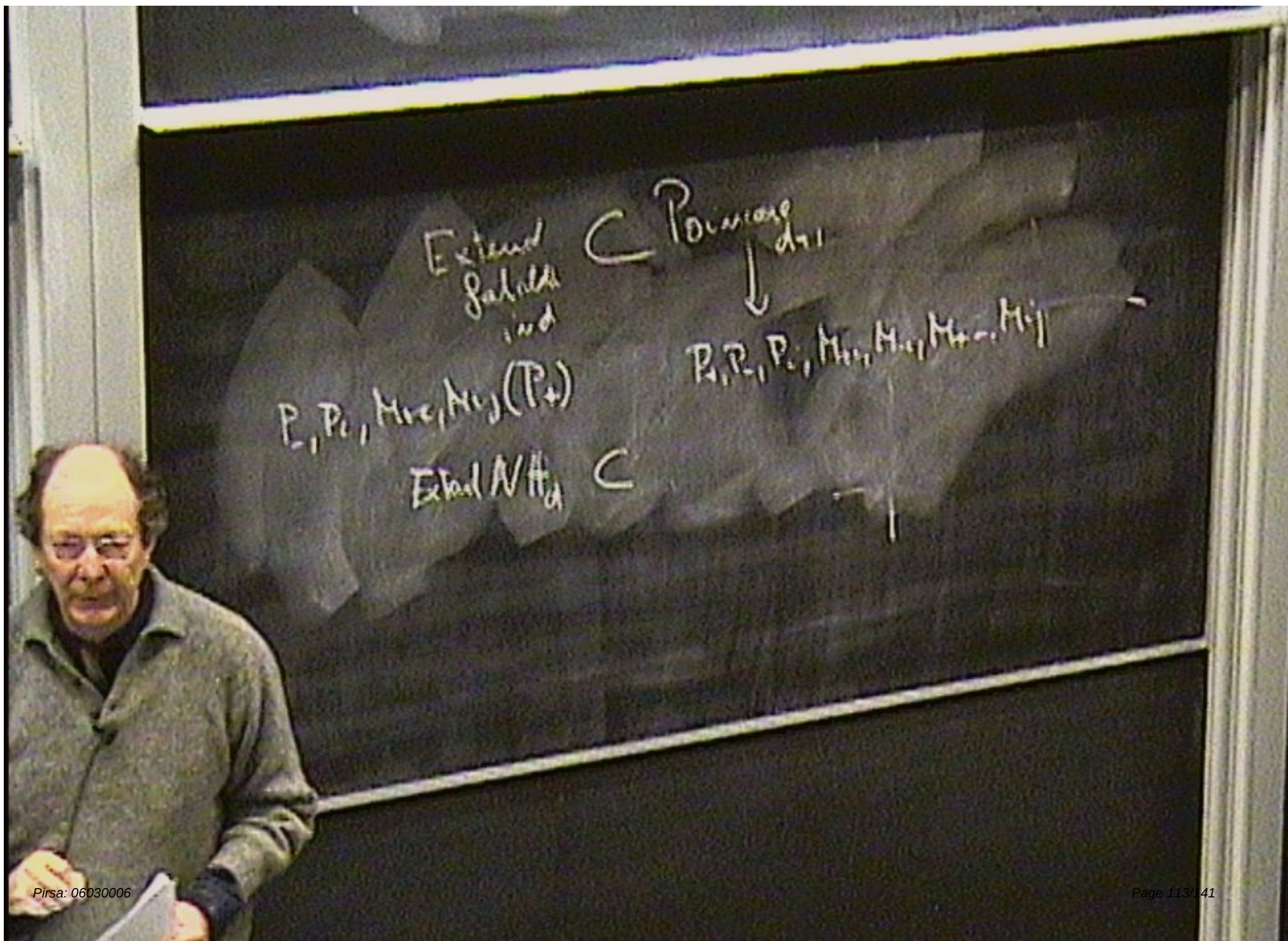


Extend
Salah
ind

$P, P_i, M_{i-1}, M_i, (P_i)$

$\subset$ Poincaré
 $d=1$
 $\downarrow$

$P, P_i, P_i, M_{i-1}, M_i, M_{i-1}, M_i$

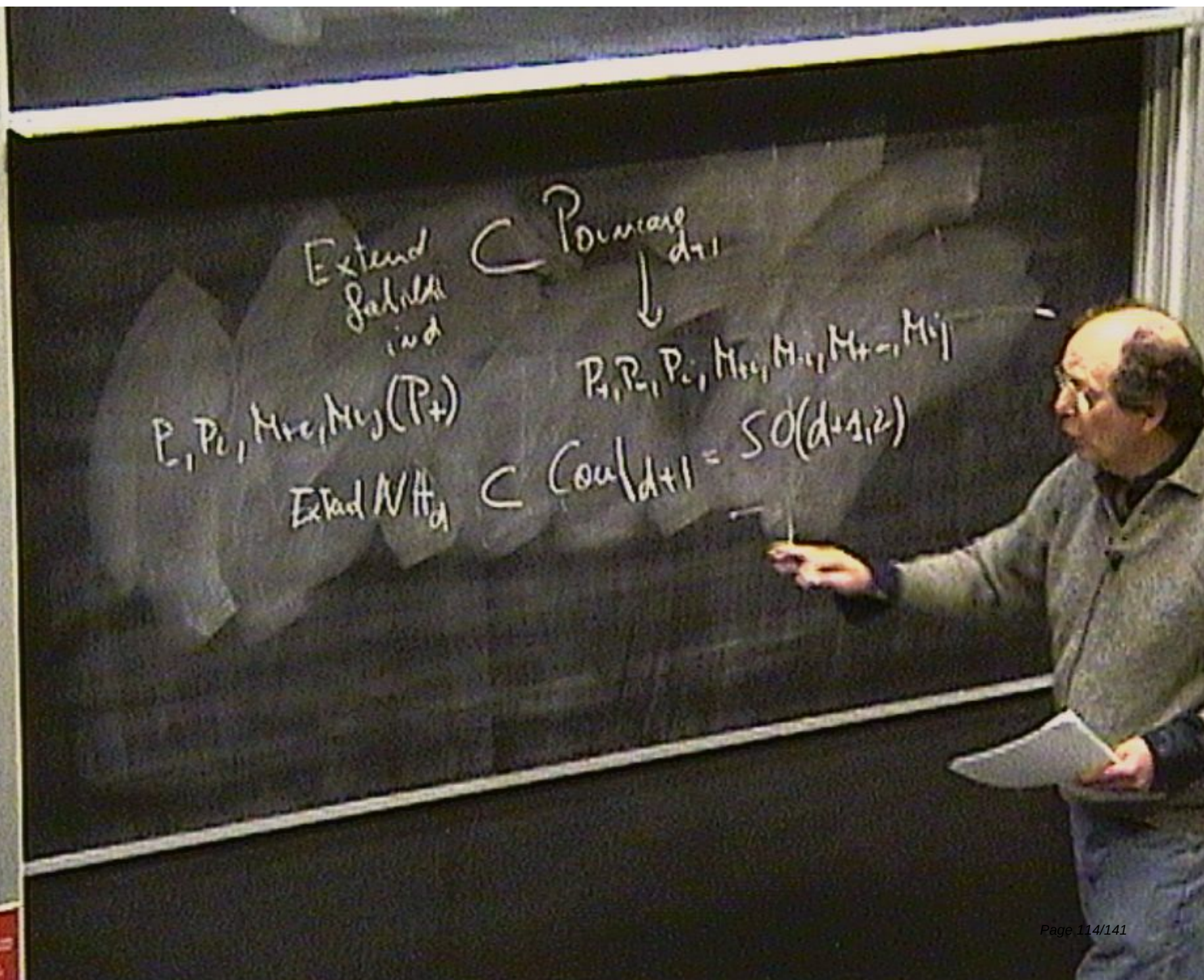


Extend Galois ind C Poincare d_1

$\downarrow$
 $P, P_1, P_2, H_1, H_2, H_3, H_4$

$P, P_1, H_1, H_2, (P_+)$

Ext H_1 C



Extend
Poincare
 $\subset$ $d+1$

$P_i, P_{i+1}, M_{i+1}, M_{i+2}, M_{i+3}$

Extad N/H_d

$P_i, P_{i+1}, P_{i+2}, M_{i+1}, M_{i+2}, M_{i+3}, M_{i+4}$

$\subset$ $Coul_{d+1} = SO(d+1, 2)$

$\text{Extend } \text{Subalgebra}_{ind} \subset \text{Poincaré}_{d+1}$
 $\downarrow$
 $P_1, P_2, P_3, M_1, M_2, M_3, \dots, M_j$
 $P_1, P_2, M_1, M_2, M_3(P_+)$
 $\text{Extend } N/A_d \subset \text{Conf}_{d+1} = \text{SO}(d+1, 2)$

Extend
Sahlin
ind

$\subset$ Poincaré
 $d+1$
 $\downarrow$

$P_1, P_2, P_3, M_1, M_2, M_3, \dots, M_j$

$P_1, P_2, M_1, M_2, (P_+)$

Extend N/H_d

$\subset$ Coul $d+1 = SO(d+1, 2)$

$-++++-$ | $A_{\pm} = \frac{A_1 \pm A_0}{2}$

$A_{\pm}' = \frac{A_{\pm 1} \pm A_{\pm 2}}{2}$

$$\begin{cases} L = dx - v dt \\ L_{oi} = dv_{oi} + dx_{oi} \frac{x_{oi}}{R^2} \\ L_{ii} = 0 \end{cases} \quad \Omega_2 = d\left(\frac{-v_{oi} dx + dx \frac{x}{2} + R^2}{\Omega_2} \right)$$

$$\begin{cases} P_o = \frac{\tilde{P}_o}{\omega} \\ M_{oi} = \omega \tilde{M}_{oi} \\ R = \omega \tilde{R} \end{cases}$$

$$\begin{cases} L = dx - v^i dt \\ L^{(i)} = dv^i + dx^0 \frac{x^i}{R^2} \\ L^{(i)} = 0 \end{cases} \quad \Omega_2 = d\left(\frac{-v^i dx^0 + dx^0 \frac{x^i}{2}}{R^2}\right)$$

$$\begin{cases} P_0 = \frac{\tilde{P}_0}{w} \\ M = w \tilde{M} \\ \tilde{R} \end{cases} \longrightarrow P_M$$

$$\begin{cases} L = dx - v dt \\ L_{oi} = dx v_{oi} + dx^0 \frac{x}{R^2} \\ L_{ii} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-v^0 dx + dx^0 \frac{x}{2} + \dots}{\Omega_2}\right) R^2$$

$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_{oi} = \omega \tilde{M}_{oi} \\ \omega \tilde{R} \end{cases}$$

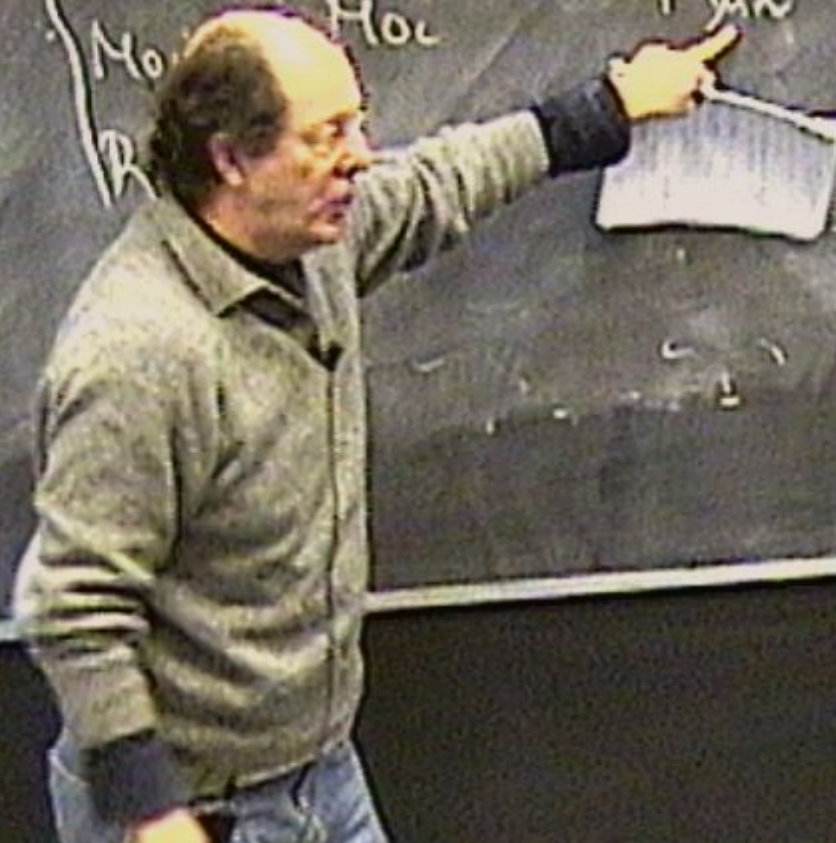
$$P_M = \frac{P_{\tilde{M}}}{\omega}$$

$$\begin{cases} L = dx - v dt \\ L_{0i} = dx^0 + dx^i \frac{x^0}{R_2} \\ L_{ii} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-v^0 dx + dx^0 \frac{x^0}{2} + \dots}{\Omega_2}\right) R^2$$

$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_0 \end{cases} \xrightarrow{\sim} P_M = \frac{P_M}{\omega}$$

Max



$$\begin{cases} L = dx - v dt \\ L_{oi} = dx v_{oi} + dx^0 \frac{x^1}{R^2} \\ L^i = 0 \end{cases}$$

$$\Omega_2 = d \left(\frac{-v^0 dx + dx^0 \frac{x^1}{2}}{\Omega_2} \right) R^2$$

$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_{oi} = \omega \tilde{M}_{oi} \\ R = \omega \tilde{R} \end{cases}$$

$$\begin{cases} P_M = \frac{\tilde{P}_M}{\omega} \\ M_{Mi} = \omega \tilde{M}_{Mi} \\ R = \omega \tilde{R} \end{cases}$$

$$[M, M] = M$$

$$[M, P] = P$$

$$\begin{cases} H_{01} = \omega \tilde{H}_{01} \\ R = \omega \tilde{R} \end{cases}$$

$$[P_0, P_i] = \pm \frac{i}{R^2} H_{0i}$$

$$[\frac{P_0}{\omega}, P_i] = \pm \frac{1}{\omega R^2} \omega H_{0i}$$

$$[P_0, P_i] = \pm \frac{i}{R^2} H_{0i}$$

$$[P_i, P_0] = 0$$

$$[H_{0i}, H_{0j}] = 0$$

$$[P_i, H_{0j}] = i \delta_{ij} \tilde{Z}$$

Harmonic oscillator!

$$\begin{cases} \vec{x}' = \vec{x} + \vec{V} R \sin \frac{t}{R} \\ t' = t \end{cases}$$

AdS, NH-

$z \rightarrow c$

$$\{NH_{\pm} = \frac{10}{11} \} e^{i\phi(z)}$$

central extension

$$\begin{cases} L_{oi} = d v_{oi} + d v_{oi} \frac{x}{R_2} \\ L_{vi} = 0 \end{cases}$$

$$\Omega_2 = \Omega_1 \left(\frac{R_1}{R_2} \right)$$



$$\begin{cases} P_o = \frac{\tilde{P}_o}{\omega} \\ M_{oi} = \omega \tilde{M}_{oi} \\ R = \omega \tilde{R} \end{cases}$$



$$\begin{cases} P_M = \frac{\tilde{P}_M}{\omega} \\ M_{Mi} = \omega \tilde{M}_{Mi} \\ R = \omega \tilde{R} \end{cases}$$

$$\begin{cases} L_{0i} = dx^0 dx^i + dx^0 \frac{x^i}{R^2} \\ L_{ij} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-1}{R^2} dx^0 dx^1 dx^2\right)$$

$$\left\{ \begin{array}{l} P_0 = \frac{\hat{P}_0}{\omega} \\ M_{0i} = \omega \hat{M}_{0i} \\ R = \omega \hat{R} \end{array} \right. \rightarrow \left\{ \begin{array}{l} P_\mu = \frac{\hat{P}_\mu}{\omega} \\ M_{\mu\nu} = \omega \hat{M}_{\mu\nu} \\ R = \omega \hat{R} \end{array} \right.$$

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu X^0 \partial_\nu X^0 \pm \frac{(P_{11})}{R^2} X^\mu X^\mu \right]$$

$$g_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}^{Mink} = \dots \quad \text{Diff inv. 1st} \\ X^\mu = 0^M$$

$$\begin{cases} L_{0i} = dx^0 dx^i + dx^0 \frac{x^i}{R^2} \\ L_{ij} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-1}{R^2} dx^0 dx^1 dx^2\right)$$

$H_2 \neq 0$



$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_{0i} = \omega \tilde{M}_{0i} \\ R = \omega \tilde{R} \end{cases}$$

$$\begin{cases} P_M = \frac{\tilde{P}_M}{\omega} \\ M_{M\alpha} = \omega \tilde{M}_{M\alpha} \\ R = \omega \tilde{R} \end{cases}$$

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu X^0 \partial_\nu X^0 \pm \frac{(P_{11})}{R^2} X^\mu X^\mu \right]$$

$$g_{\mu\nu} = \partial_\mu X^\alpha \partial_\nu X^\beta \quad \boxed{\text{GALC Div. 2}} = \text{Diff inv. 1st} \quad X^\mu = 0^\mu$$

$$\begin{cases} L = dx \\ L^{0i} = dt v^i + dx^0 \frac{x^i}{R^2} \\ L^i = 0 \end{cases} \quad \Omega_2 = d\left(\frac{-v^i dx^0 + dx^i \frac{x^0}{R^2}}{R^2}\right)$$

$$(H_2 \neq 0)$$



$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_{0i} = \omega \tilde{M}_{0i} \\ R = \omega \tilde{R} \end{cases} \rightarrow \begin{cases} P_M = \frac{\tilde{P}_M}{\omega} \\ M_{Mi} = \omega \tilde{M}_{Mi} \\ R = \omega \tilde{R} \end{cases}$$

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu X^0 \partial_\nu X^0 + \frac{(P^i)^2}{R^2} X^i X^i \right]$$

$$g_{ij} = \partial_i X^\mu \partial_j X^\nu \quad \boxed{\text{GALILEAN}} \quad \text{Diff inv. 1st} \quad + \text{translations}$$

$$X^M = 0^M$$

3 massive
5 massless
16 massless

Extend
Sakharov
ind

$\subset$ Poincaré $d+1$

$P_0, P_1, P_i, M_{01}, M_{0i}, M_{1i}, M_{ij}$

$P \rightarrow M_{0i}, M_{ij}(P)$

extend NH_d

$\subset$ Conformal $d+1 = SO(d+1, 2)$

$-++++-$ | $A_{\pm} = \frac{A_1 \pm A_0}{2}$

$A_{\pm}' = \frac{A_{d+1} \pm A_{d+2}}{2}$

$$\begin{cases} L = dx - v^i dx^i \\ L_{oi} = dv^i + dx^i \frac{x^i}{R^2} \\ L_{\psi} = 0 \end{cases}$$

$$\Omega_2 = d\left(\frac{-v^i dx^i + dx^i \frac{x^i}{2}}{R^2}\right)$$

$$H_2 \neq 0$$



$$\begin{cases} P_0 = \frac{\tilde{P}_0}{\omega} \\ M_{oi} = \omega \tilde{M}_{oi} \\ R = \omega \tilde{R} \end{cases} \rightarrow$$

$$\begin{cases} P_M = \frac{\tilde{P}_M}{\omega} \\ M_{Mc} = \omega \tilde{M}_{Mc} \\ R = \omega \tilde{R} \end{cases}$$

SUSY AdS₂

$$S = \int d\tau \int \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu X^a \partial_\nu X^a \pm \frac{(P^a)^2}{R^2} X^a X^a \right]$$

$$g_{ij} = \partial_i X^a \partial_j X^a \quad \boxed{g_{\text{AdS}_2}^{\text{inv.}}} = \text{Diff inv.} + \text{fermions}$$

$$X^a = 0^a$$

$\frac{G}{H}$

- broken gen $\overline{P}$, H_{br}
- unbroken gen. Rotations, H

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \tilde{X})$$

$$g = e^{i(x^0 P_0 + \vec{X}(x^i) \vec{P} + iV(x^0) H)}$$

Unbroken Translations $\approx$ Espace

$$\Omega = -i \oint d\theta = \oint \frac{1}{2\pi} \text{Tr} \left(\frac{1}{i} dU U^{-1} \right)$$

$P_0 \leftarrow \int dx^0 \rho \rightarrow \text{surface term}$

$$\Omega_2 = \frac{1}{2} \oint dx^0 \left(\frac{1}{2} \frac{d\vec{U}}{dx^0} \cdot \frac{d\vec{U}}{dx^0} + \frac{1}{2} \frac{d\vec{U}}{dx^0} \cdot \frac{d\vec{U}}{dx^0} \right)$$

$$L^2 = 0$$

$\frac{G}{H}$

- broken gen. $\overline{P}$, H
- unbroken gen. Rotations, H

$$g = \frac{e^{i x^0 P_0} e^{i \vec{X}(x^0) \vec{P}} e^{i v^0(x^0) N^0}}{e}$$

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \tilde{X})$$

$\Omega = -i \bar{S} d g = \frac{1}{4} \text{Ad} + \text{rotations}$ • Unbroken Translations $\simeq$ Espace

$P_0 \leftrightarrow \int dx^0 \rightarrow \text{surface term}$

$$AdS_5 \times S^5$$

$$\Omega_2 = L_1 L_0, \quad d\Omega_2 = 0$$

$$= d \left(-v^0 dx^1 + dx^0 \frac{v^2}{2} + dx^0 \frac{\vec{X}^2}{R^2} \right)$$

Ω_2

$$L^2 u = 0$$

$\frac{G}{H}$

- broken gen. $\overline{P}$, H
- unbroken gen. Rotations, H

$$g = e^{i x^0 P_0} e^{i \vec{x}(x^0) \vec{P}} e^{i V(x^0) N^i}$$

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \vec{x})$$

$\Omega = -i \bar{S} dg = \frac{1}{4} \text{Ad} + \text{rotations}$ Unbroken Translations $\simeq$ Espace

$P_0 \leftarrow \int dx^0 \rightarrow \text{surface term}$

$$AdS_5 \times S^5$$

$$\sqrt{-g} \frac{1}{2} \dot{x}^\mu \dot{x}_\mu - g \frac{1}{2} \dot{x}^i \dot{x}_i$$

$$\Omega_2 = \frac{d\Omega_1}{2} = 0$$

$$\Omega_2 = \left(dx^1 + dx^0 \frac{v}{2} + dx^0 \frac{\vec{x}^2}{R^2} \right)$$

$\frac{S}{H}$

- broken gen. $\overline{P}$, H
- unbroken gen. Rotations, H

$$g = \frac{e^{i x^0 P_0} e^{i \vec{X}(x^0) \vec{P}} e^{i v^i(x^0) M^i}}{e}$$

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X, \vec{X})$$

$\Omega = -i \bar{S} d g = L^{AdS_5} + \text{rotations}$ • Unbroken Translations $\simeq$ Espace

$$\begin{matrix} \uparrow \\ \text{AdS}_5 \times S^5 \end{matrix} \quad \begin{matrix} \uparrow \\ \text{SU}(2,2|4) \end{matrix} \quad P_0 \leftrightarrow \int dx^0 \rightarrow \text{surface term}$$

$$\Omega_2 = L^{1000} \quad d\Omega_2 = 0$$

$$\Omega_2 = 0 \quad \left(x^1 + dx^0 \frac{\sqrt{2}}{2} + dx^0 \frac{\vec{X}}{R^L} \right)$$

$$\sqrt{-g_{\mu\nu}} \dot{X}^\mu \dot{X}^\nu = g_{\mu\nu}^{\text{AdS}_5} \dot{X}^\mu \dot{X}^\nu$$

$\frac{G}{H}$

- broken gen. $\overline{P}$, H
- unbroken gen. Rotations, H

$$g = \frac{e^{i x^0 P_0} e^{i \vec{X}(x^0) \vec{P}} e^{i V(x^0) H}}{e}$$

$$\frac{SU(2) \times SU(2)}{SU(2)}$$

$$\pi(X^0, \vec{X})$$

$\Omega = -i \bar{S} d g = \underbrace{L_{\text{AdS}}}_{\text{AdS} + \text{rotations}} + \underbrace{L_{\text{trans}}}_{\text{Unbroken Translations} \simeq \text{Espace}}$

$$\boxed{AdS_5 \times S^5}$$

$$\boxed{SU(2,2|4)}$$

$$P_0 \leftarrow$$

$$\int dx^0 \rightarrow \text{surface term}$$

$$\Omega_2 = L_1 L_2$$

$$d\Omega_2 = 0$$

$$\Omega_2 = d \left(-v^0 dx^1 + dx^0 \frac{v^2}{2} + dx^0 \frac{\vec{X}^2}{R^2} \right)$$

$$\Omega_2$$

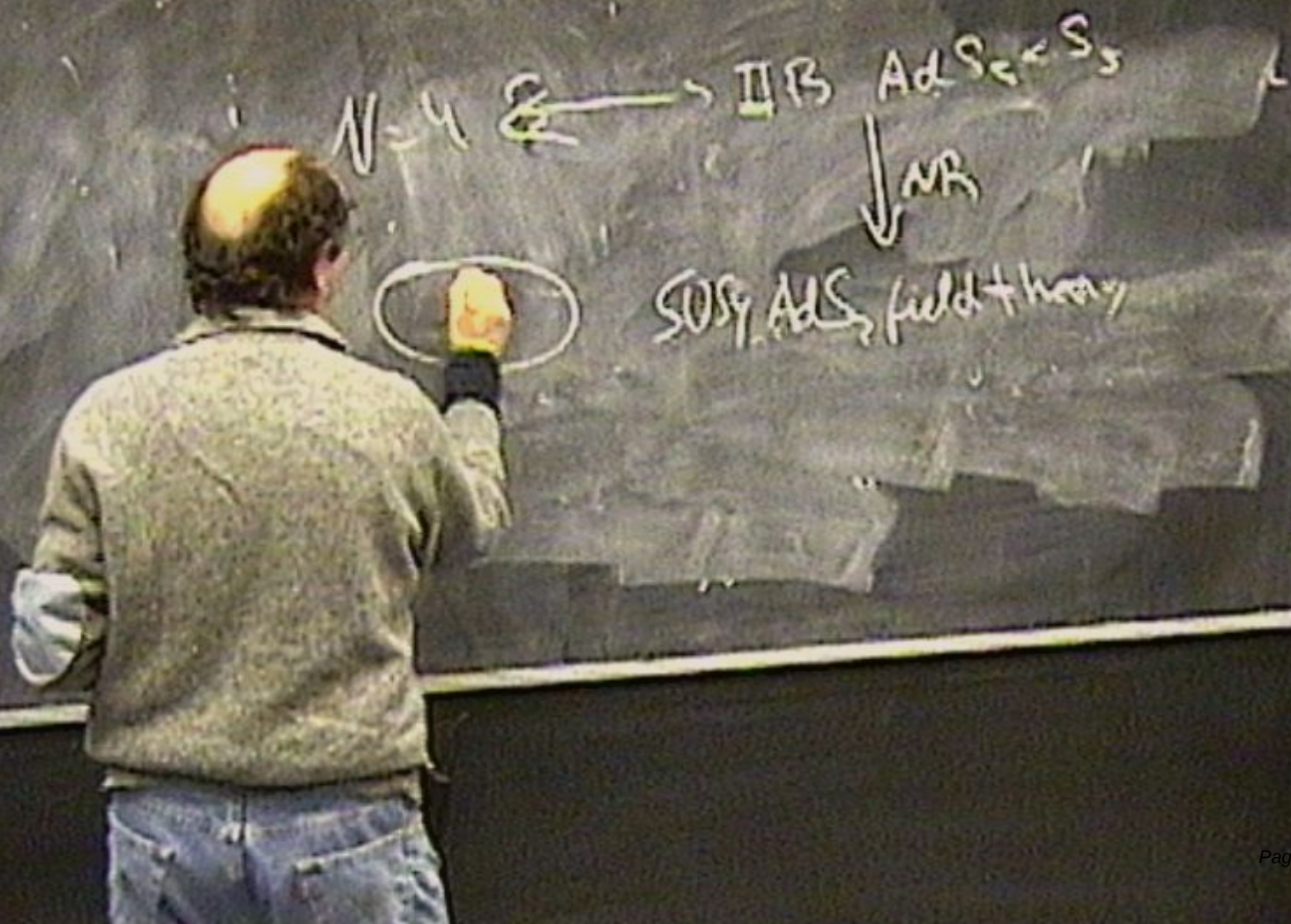
$$\sqrt{-g_{\mu\nu}} \dot{X}^\mu \dot{X}^\nu = g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu$$

AdS, $\mathcal{N}=4$



Poincare





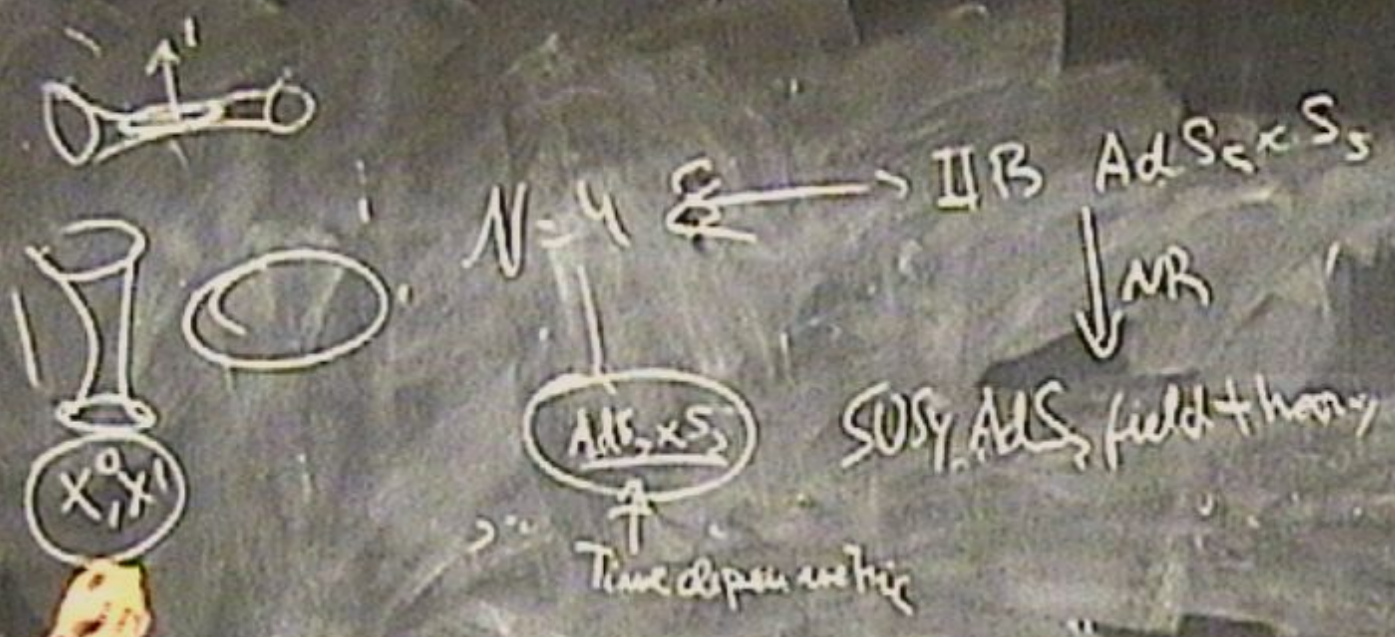
$N=4$ $\xrightarrow{\quad} \text{IR } \text{AdS}_5 \times S^5$

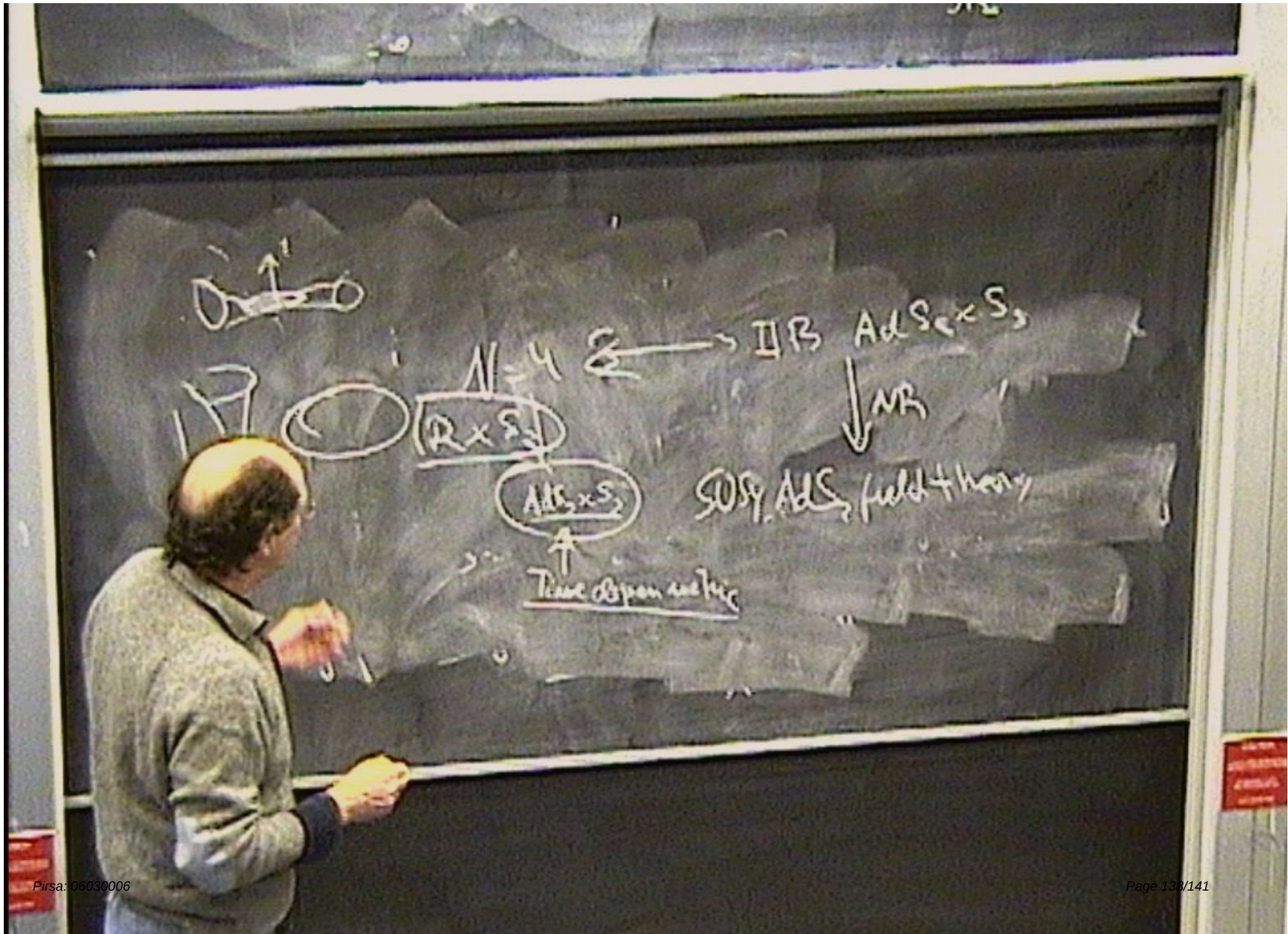
$\downarrow \text{NR}$

$\text{AdS}_5 \times S^5$

SUSY AdS, field + heavy

$\uparrow$
Time dependent





17



$N=4$
 $R \times S^3$

$AdS_2 \times S_2$

Time dependent

IR $AdS_2 \times S_2$

$\downarrow$ NR

SUSY AdS_2 field theory

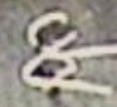


$(R \times S^1)$

$(AdS_2 \times S^2)$

Time dependent metric

$N=4$



IR $AdS_5 \times S^5$



SUSY AdS_5 field theory

