Title: Detection of vacuum entanglement in an ion trap

Date: Mar 01, 2006 04:00 PM

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Abstract: Quantum information methods have been recently used for studying the properties of ground state entanglement in several many body and field theory systems. We will discuss a thought experiment wherein entanglement can be extracted from the vacuum of a relativistic field theory into a pair of arbitrarily spatially separated atoms. In order to simulate the detection process, we will consider the ground state of a linear chain of cooled trapped ions, and discuss a scheme for detecting the entanglement between the ion's motional degrees of freedom.

Vacuum Entanglement In an Ion Trap

A. Retzker

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J. Silman (*Tel-Aviv Univ.)*

Quantum Information
Seminar
Perimiter 1/03/2006

Vacuum Entanglement in an Ion Trap

➡ 1 Vacuum Entanglement

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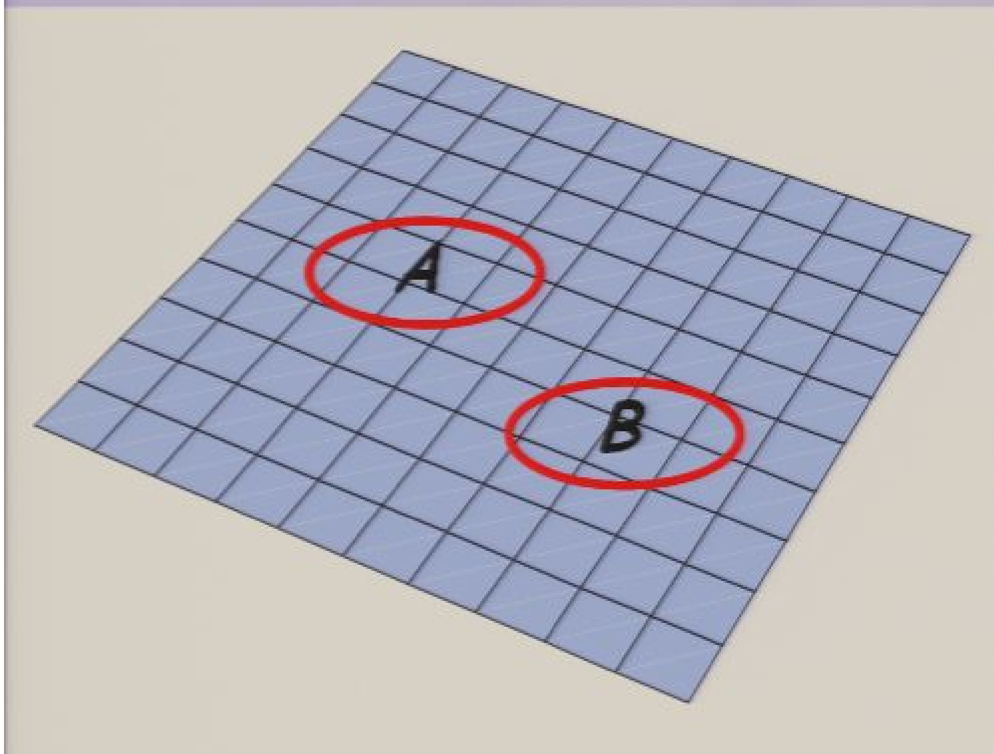
2 Introduction to Ion trap Quantum Computing

3 Ground state Entanglement of an Ion Trap

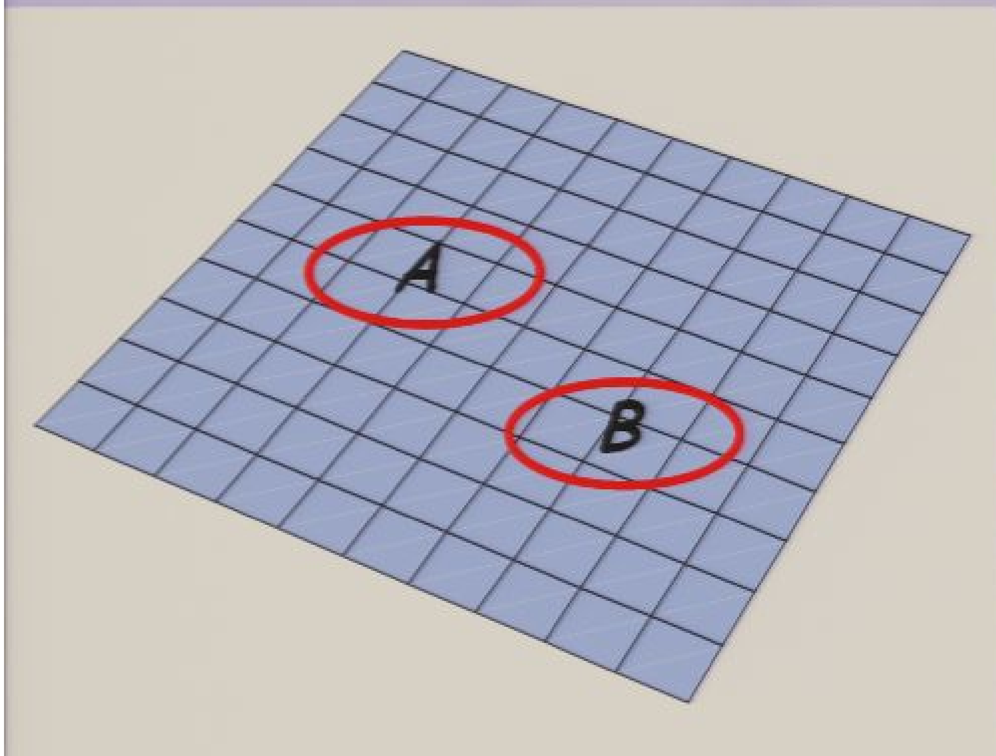
Vacuum Entanglement



Motivation:

Reznik, Found. Phys. 2003, quant-ph/0008006
Reznik, Retzker, Silman PRA 71, 042104

Vacuum Entanglement



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Quantum Information

natural set up to study Ent
causal structure L.O.

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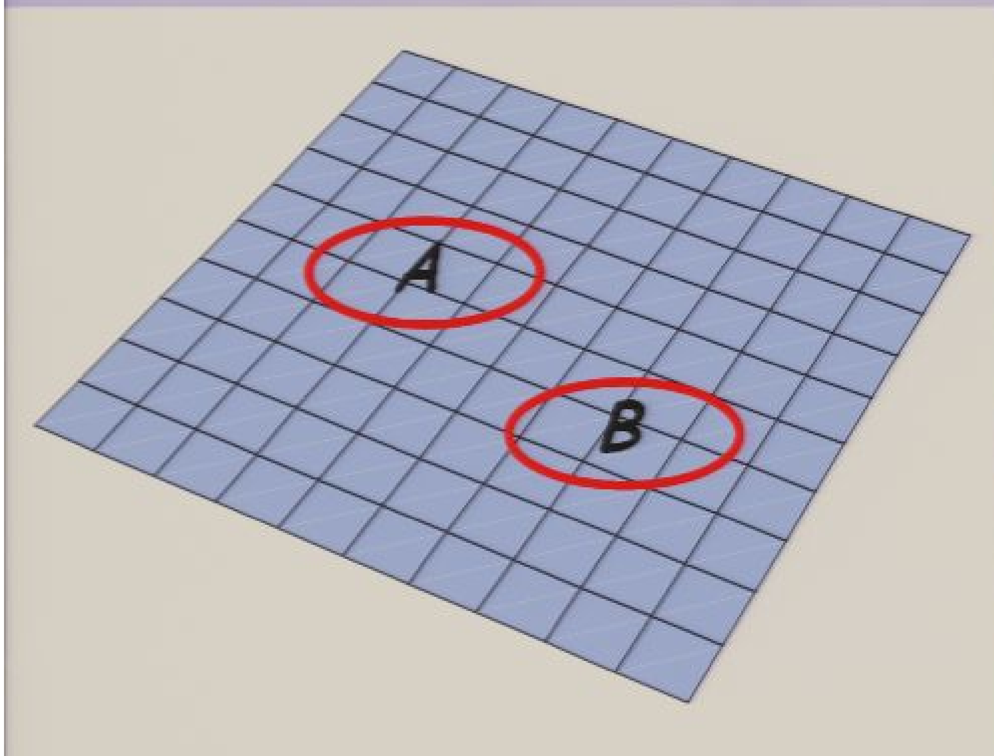
Vacuum Entanglement



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Quantum Physics

Can Ent. shed light on "quantum effects"? (Q. phase transitions, Entropy Area law.)

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Background

Continuum results:

BH Entanglement entropy:

Unruh (76), Bombelli et. Al. (86), Srednicki (93), Callan & Wilczek (94) , Terno(04).

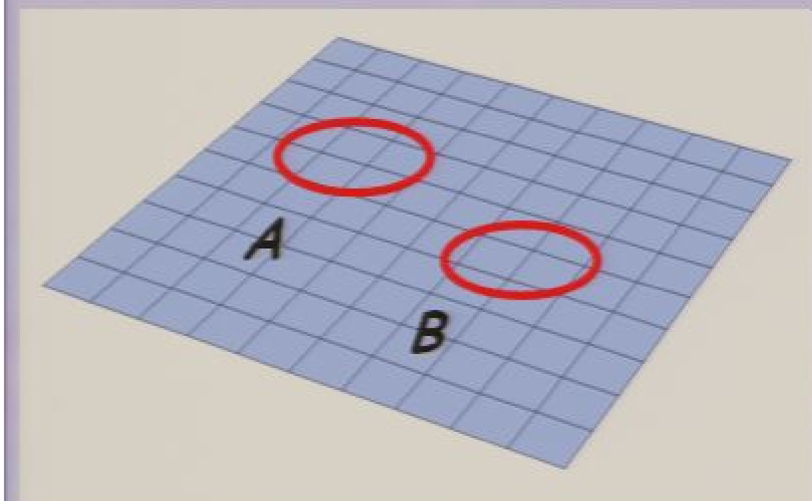
Algebraic Field Theory:

Summers & Werner (85), Halvarson & Clifton (00),Verch & Werner (2004).

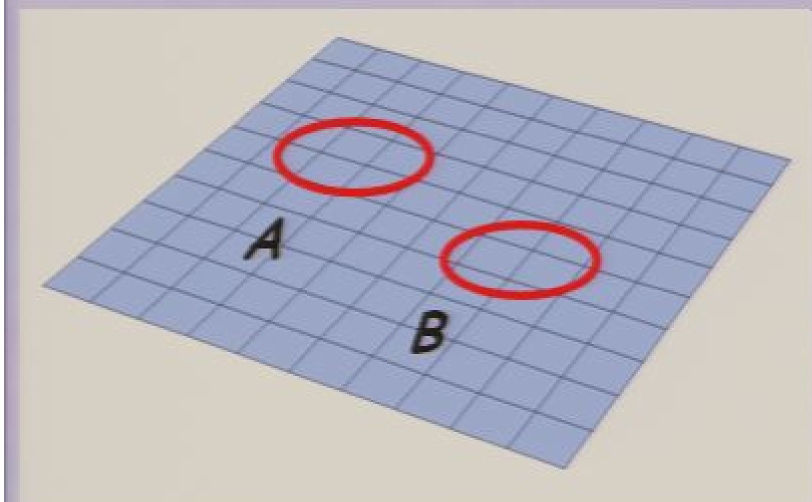
Discrete models:

Spin chains: *Wootters (01), Nielsen (02), Latorre et. al. (03).*

Harmonic chains: *Audenaert et. al (02), Botero & Reznik (04).*


A
B

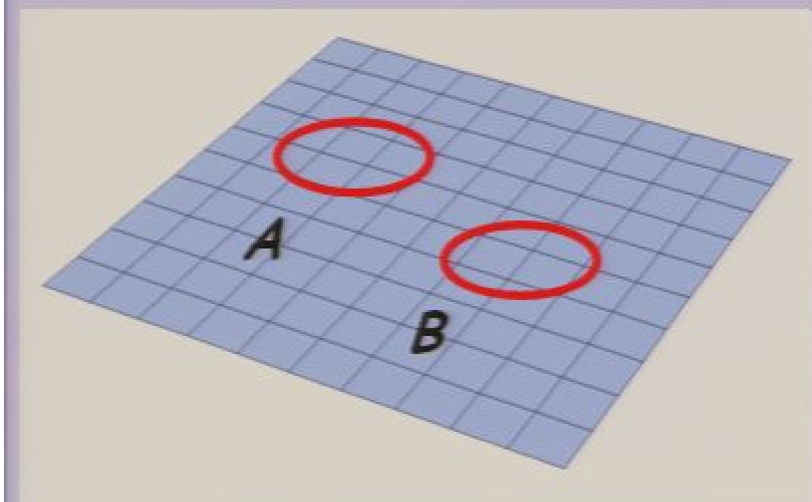
Are A and B entangled?



Are A and B entangled?

Yes, for arbitrary separation.

("Atom probes").



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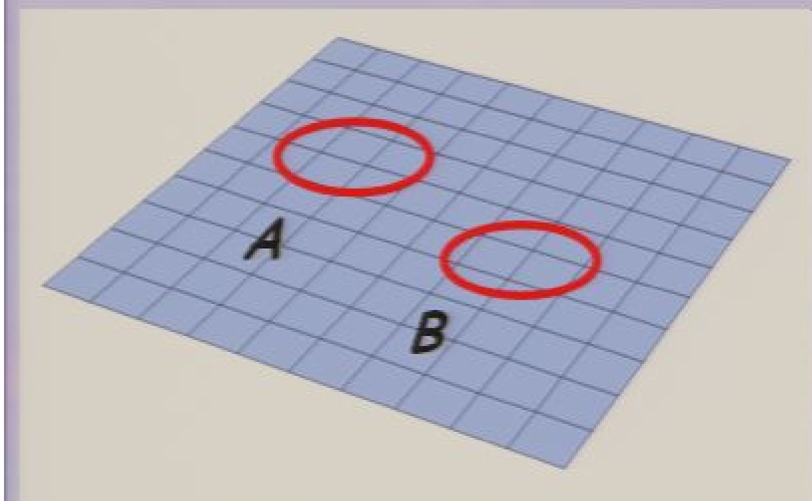

Are Bell's inequalities violated?

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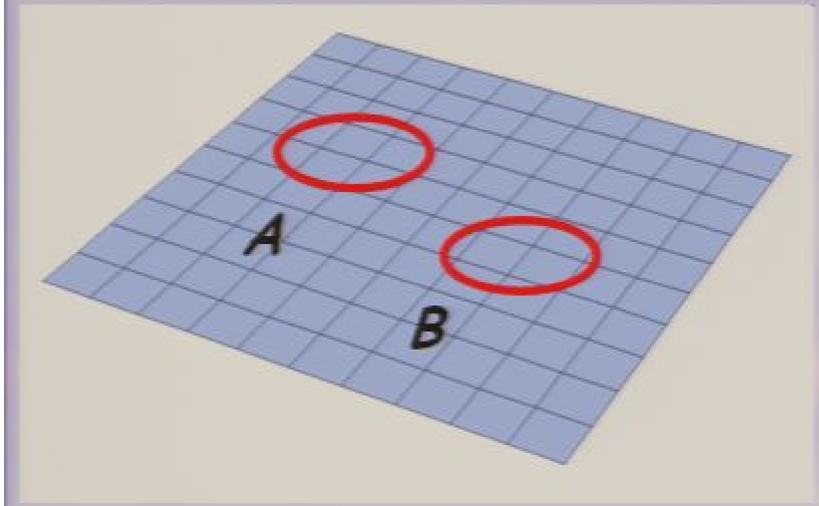

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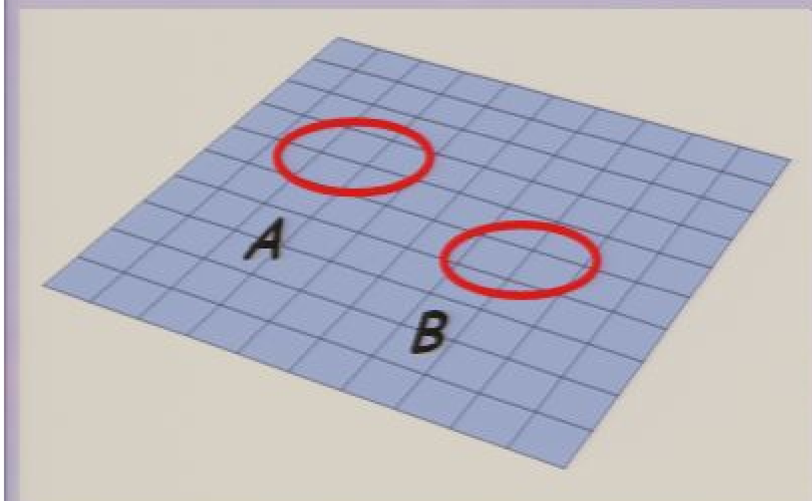

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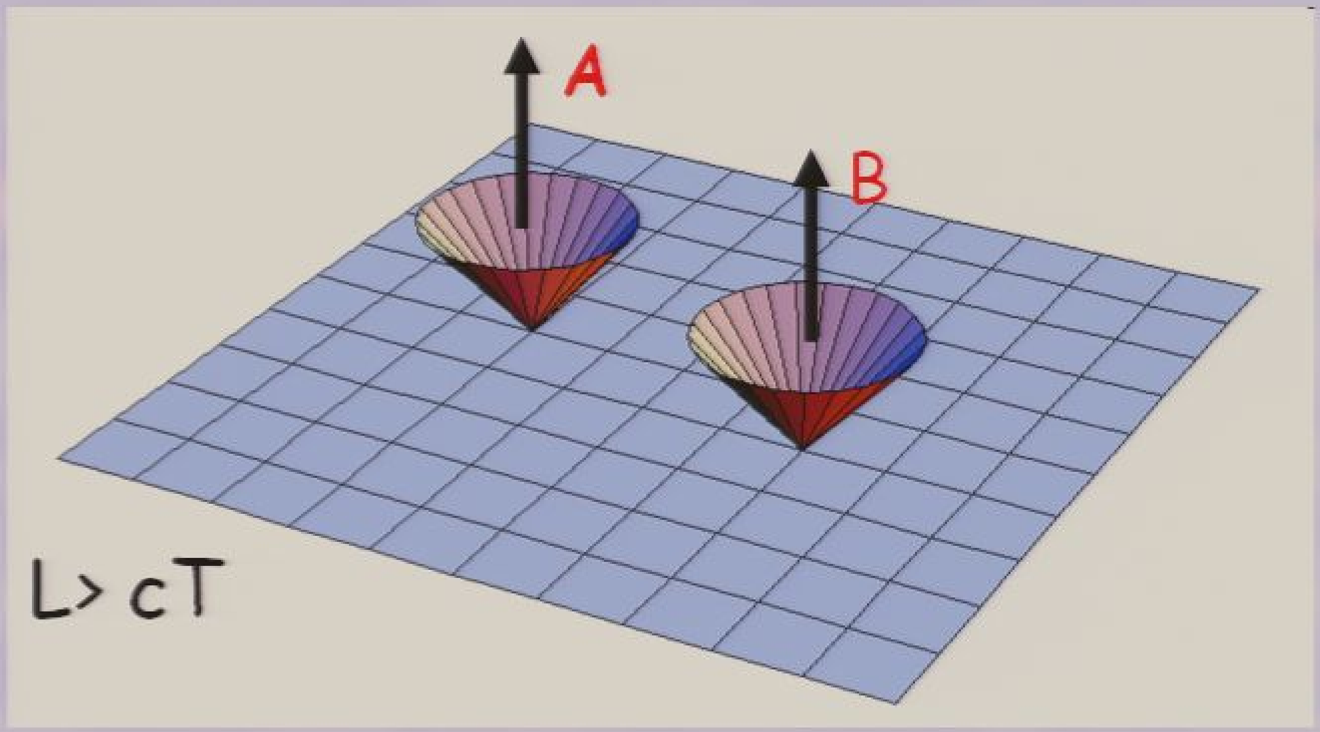
Can we detect it?

Entanglement Swapping.

(Linear Ion trap).

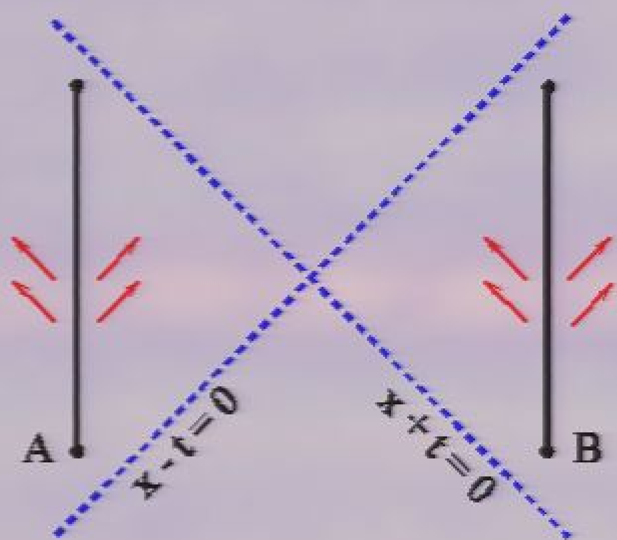
Probing Field Entanglement

RFT → Causal structure → LOCC → QI



A pair of causally disconnected localized detectors

Causal Structure + LO



For L>cT, we have $[\phi_A,\phi_B]=0$
Therefore $U_{INT}=U_A\ U_B$ →**LO**

$\Delta\ E_{Total}$ =0, but
$\Delta\ E_{AB}$ >0. (Ent. Swapping)

Vacuum ent ! Detectors' ent.
Lower bound.

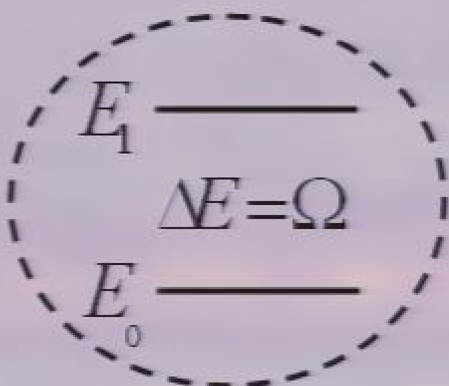
Field – Detectors Interaction

Interaction:

$$H_{\text{int}} = H_A + H_B$$

$$H_A = \varepsilon_A(t)\left(e^{+i\Omega t}\sigma_A^+ + e^{-i\Omega t}\sigma_A^-\right)\phi\left(x_A, t\right)$$

Window Function



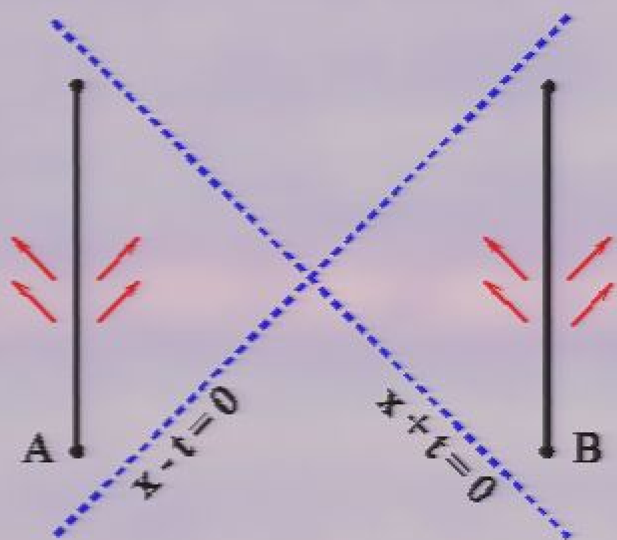
Two-level system

Initial state:

$$\left|\Psi(0)\right\rangle = \left|\downarrow_A\right\rangle\left|\downarrow_B\right\rangle\left|0\right\rangle$$

We do not use the rotating wave approximation

Causal Structure + LO



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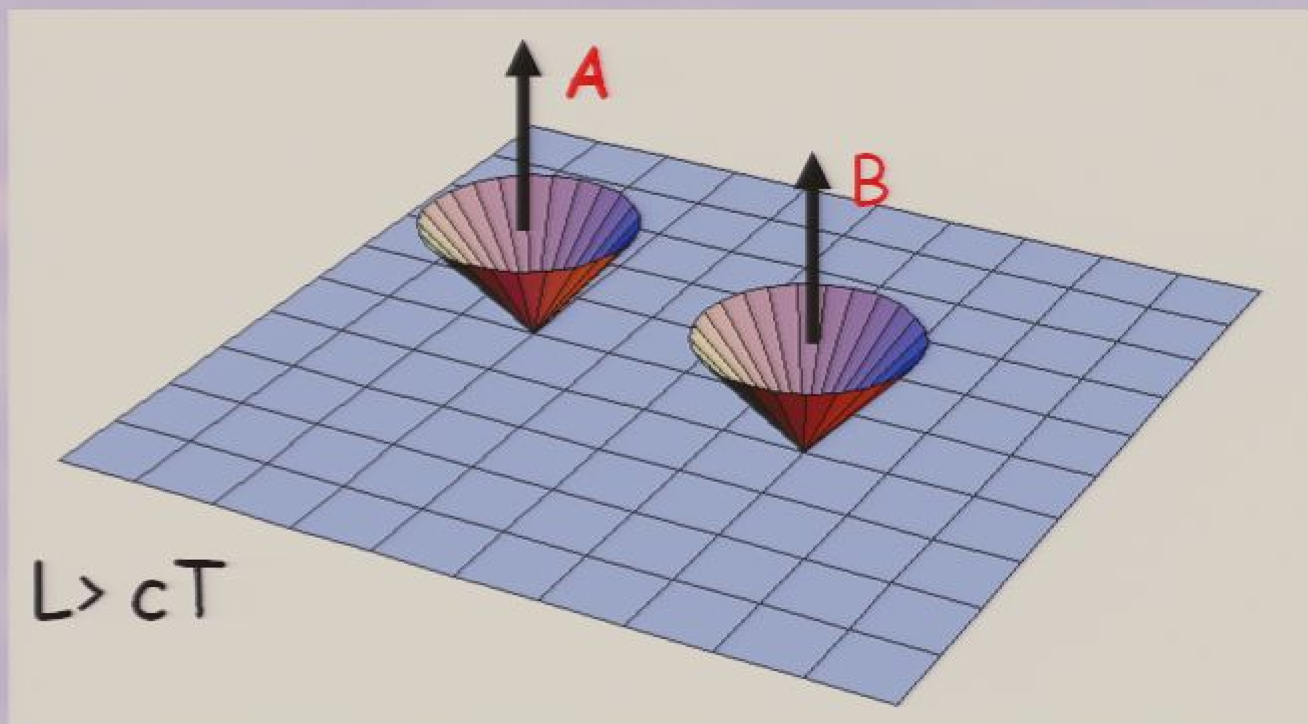

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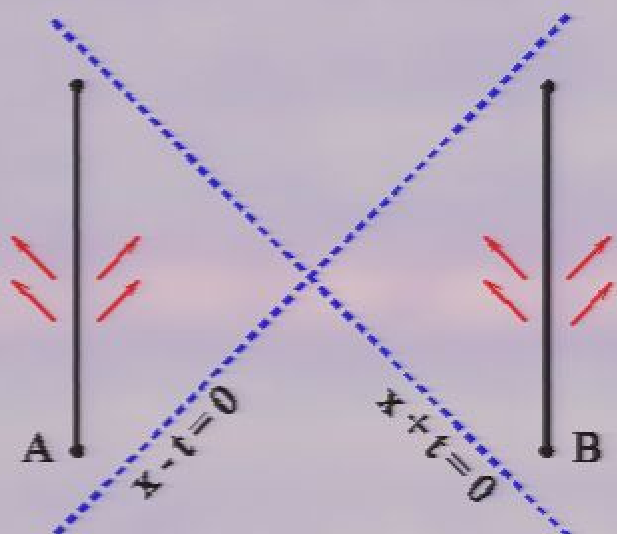

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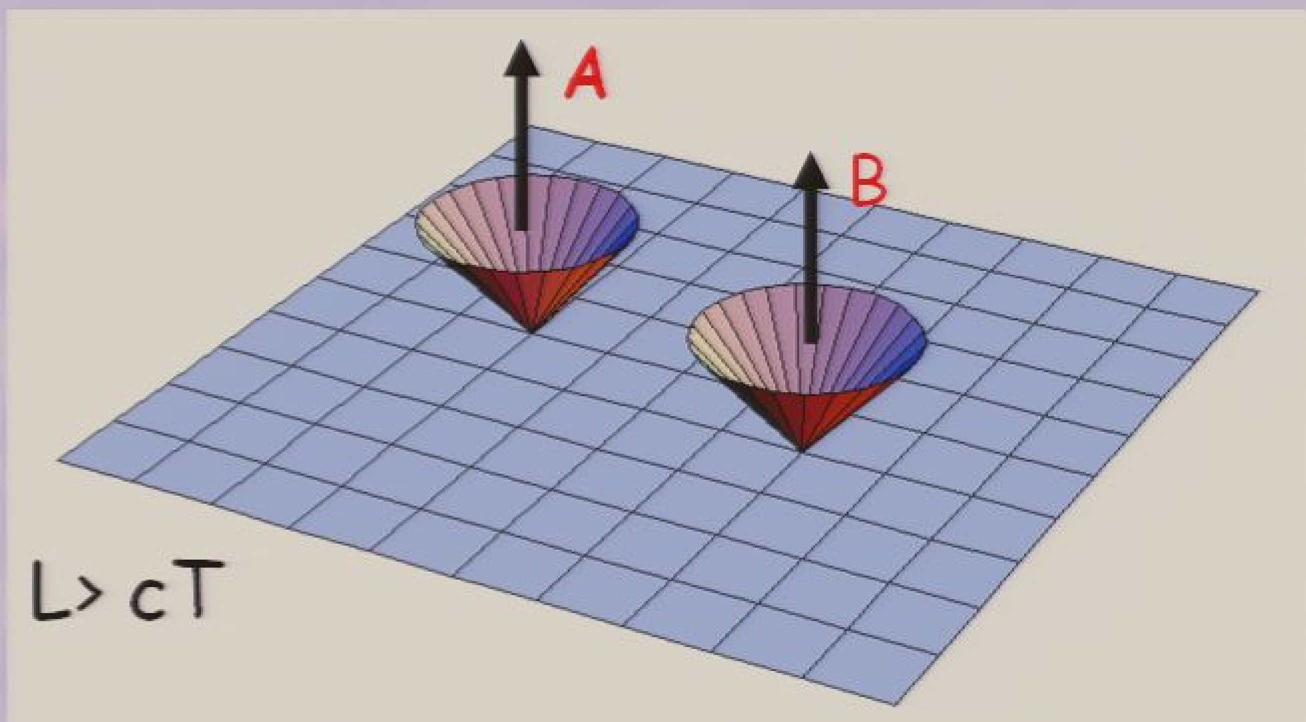
↓

QI ⟵ LOCC



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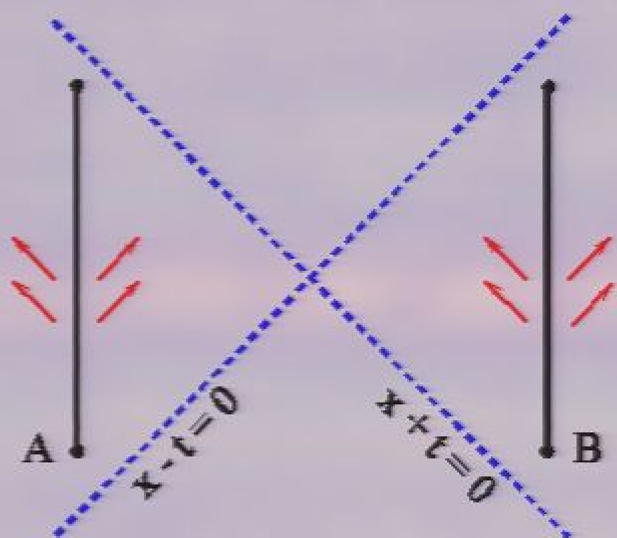

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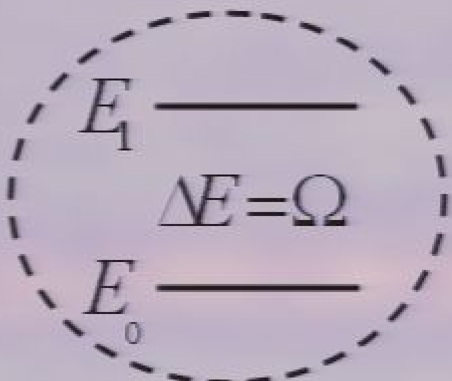

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Probe Entanglement

$$\rho_{AB}^{(4\times4)} = Tr_F \rho^{(total)}$$

$$? \neq \sum_i p_i \rho_A^{2\times2} \rho_B^{2\times2}$$

Calculate to the second order (in ε) the final state, and evaluate the reduced density matrix.

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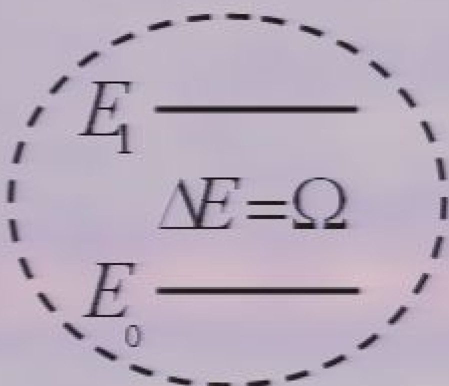

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$$\left|\Psi(T)\right\rangle = U_{Interaction} \left|\downarrow_A\right\rangle \left|\downarrow_B\right\rangle \left|0\right\rangle$$

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$$\downarrow\downarrow \qquad \uparrow\uparrow \qquad \downarrow\uparrow \qquad \uparrow\downarrow$$

$$\rho_{AB}(T) = \begin{bmatrix} 1 & \left\langle 0 \middle| X_{AB} \right\rangle & & \\ \left\langle X_{AB} \middle| 0 \right\rangle & \left\| X_{AB} \right\|^2 & & \\ & & \left\| E_A \right\|^2 & \left\langle E_A \middle| E_B \right\rangle \\ & & \left\langle E_B \middle| E_A \right\rangle & \left\| E_B \right\|^2 \end{bmatrix} + O(\varepsilon^5)$$

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Negativity

$$\text{Emission} \rightarrow \left\|E_A\right\|\left\|E_B\right\| < \left|\langle 0|X_{AB}\rangle\right| \leftarrow \text{Exchange}$$

Negativity

Emission → $\|E_A\|\|E_B\| < |\langle 0|X_{AB}\rangle|$ ← Exchange

$$\int_0^\infty \omega d\omega[\tilde{\varepsilon}(\Omega+\omega)]^2 < \int_0^\infty \frac{d\omega}{L} Sin(\omega L)\widetilde{\varepsilon_A}(\Omega+\omega)\widetilde{\varepsilon_B}(\Omega-\omega)$$

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Off resonance (↑ $\Omega+\omega$)

On resonance (↑ $\Omega-\omega$)

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Off resonance

Vacuum Window Function

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Off resonance — Vacuum Window Function — On resonance

$$\Downarrow$$

$$N \geq e^{-(L/T)^2}$$

Superocillatory functions, (Aharonov(88) Berry(94)).

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$$N \geq e^{-(L/T)^2}$$

Numerics on a Lattice

$$N = e^{-\alpha L/T}$$

Bell's Inequalities

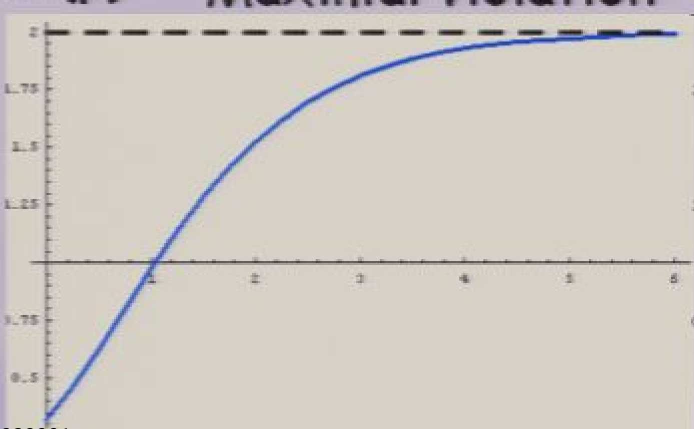
$N(\rho)$ Maximal Ent.

Ω

No violation of Bell's inequalities.

But, by applying local filters Bell's inequalities are violated

$M(\rho)$ Maximal violation



Ω

CHSH ineq. Violated iff
$M(\rho)>1$, (Horodecki (95).)

"Hidden" non-locality.
Popescu (95). Gisin (96).

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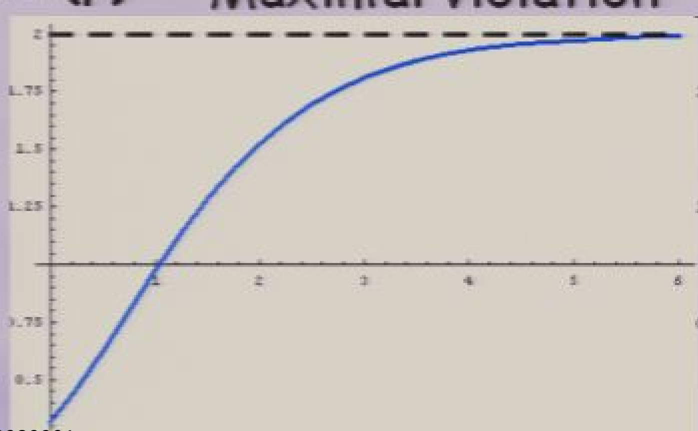

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Summary

1) Vacuum entanglement can be distilled!

2) Lower bound: E ¸ $e^{-(L/T)^2}$
(possibly $e^{-L/T}$)

3) High frequency (UV) effect: $\Omega = L^2$.

4) Bell inequalities violation for arbitrary separation(maximal “hidden” non-locality).

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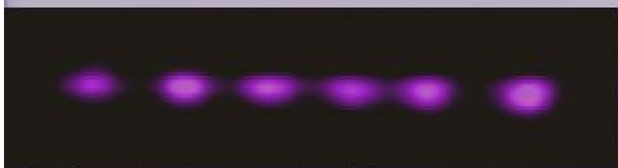
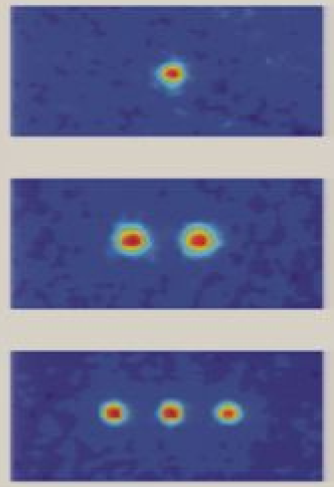
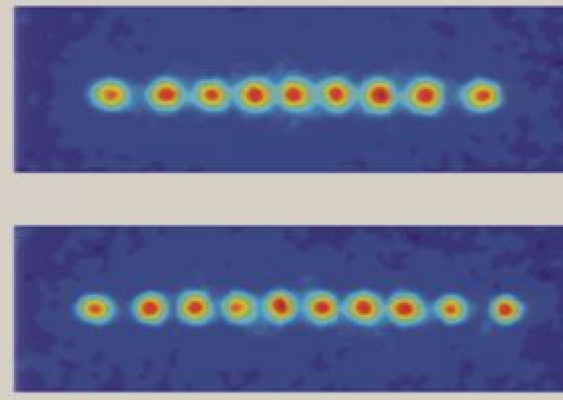
Cold ion crystals



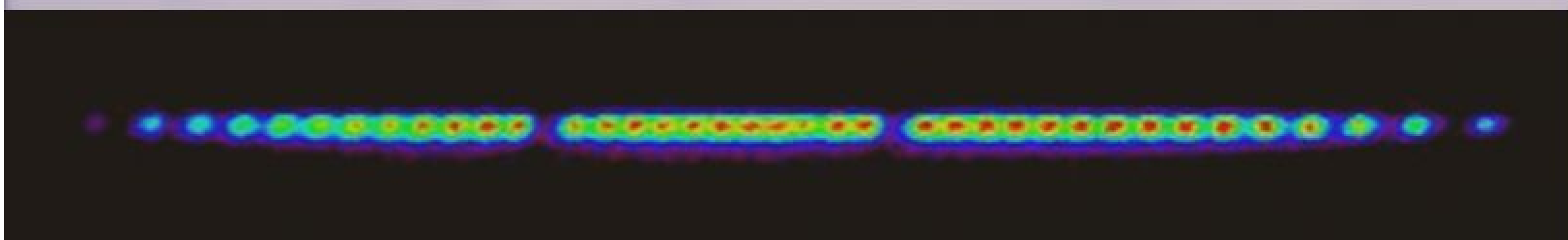
Oxford, England: $^{40}Ca^{+}$

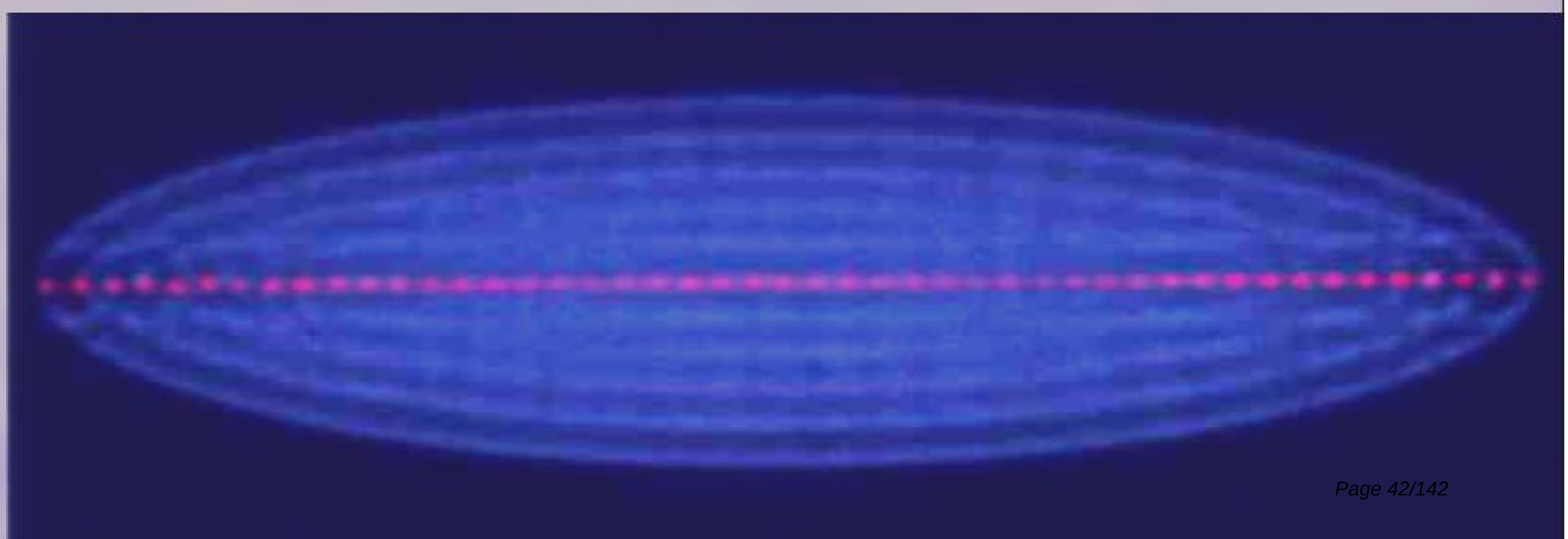




Innsbruck, Austria: $^{40}Ca^{+}$

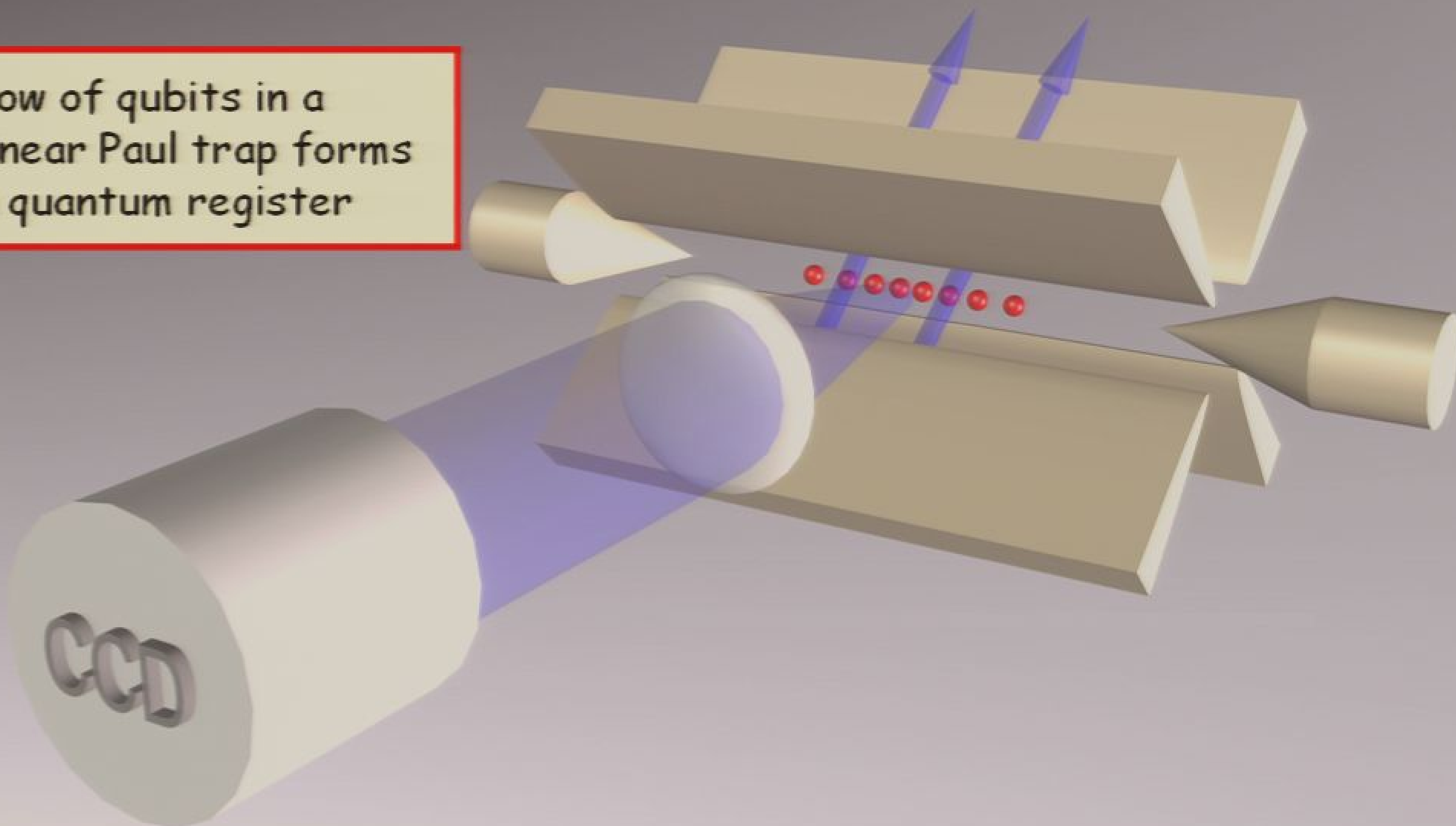
Boulder, USA: Hg^{+} (mercury)

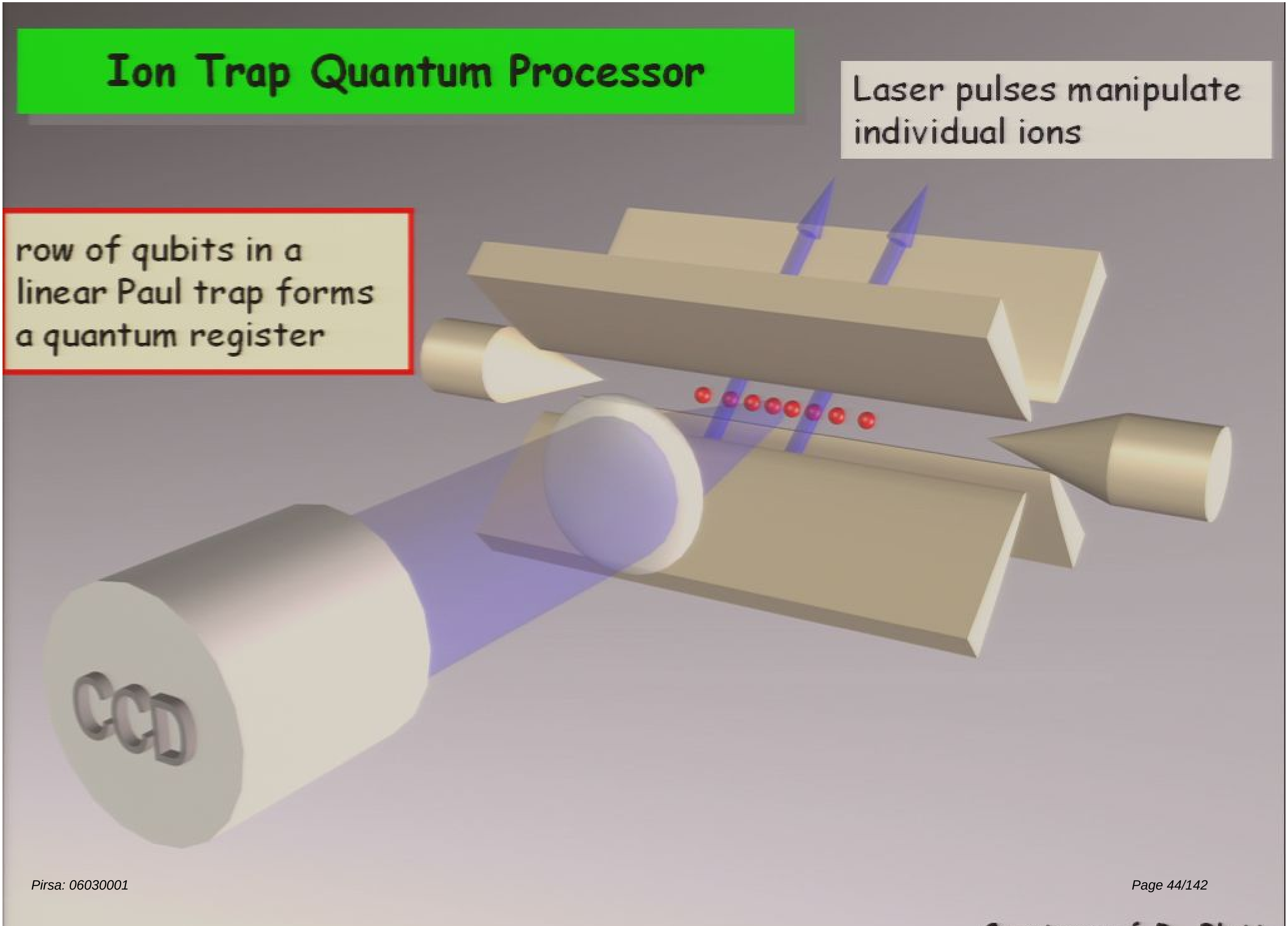


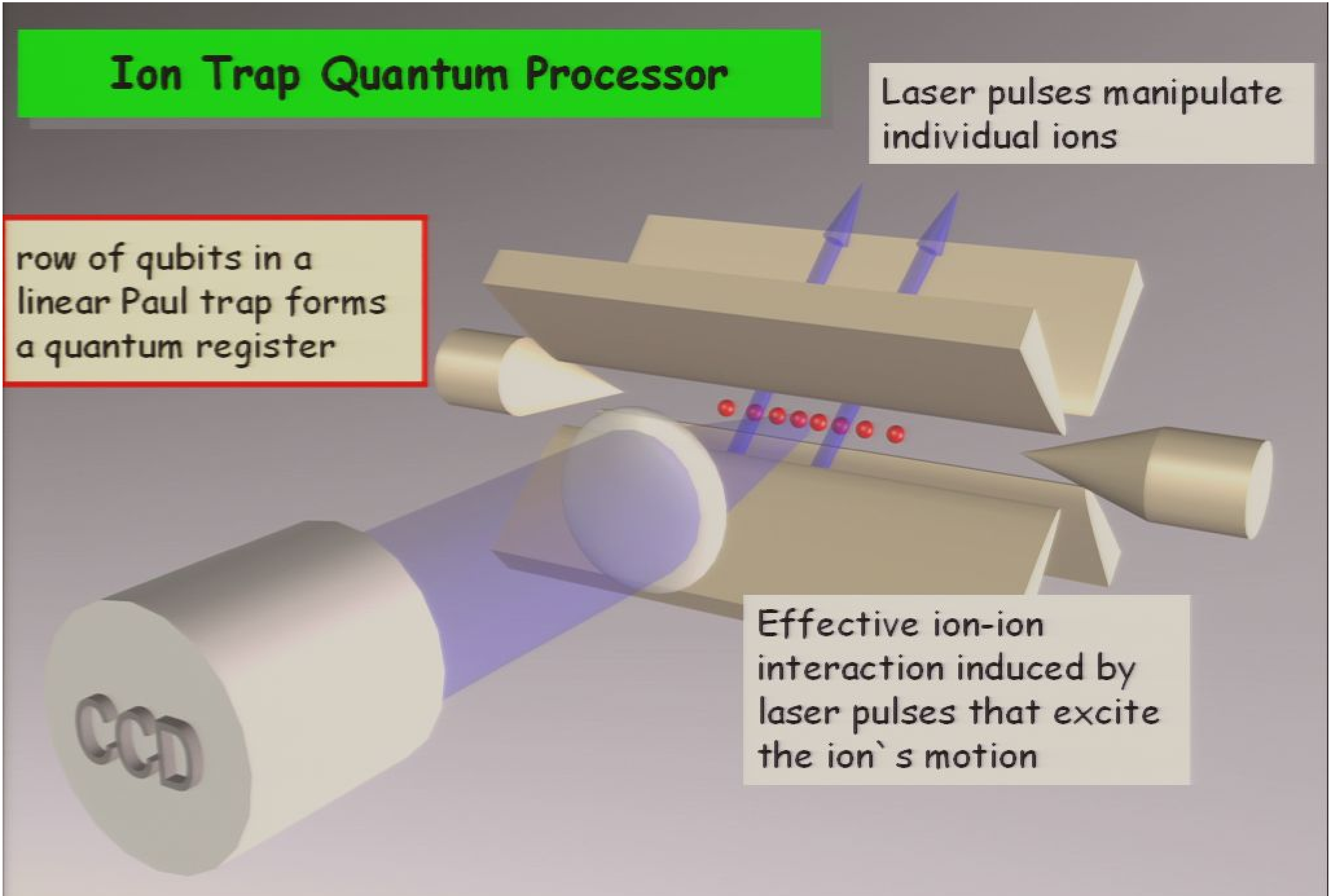


Ion Trap Quantum Processor

row of qubits in a linear Paul trap forms a quantum register




Ion Trap Quantum Processor
Laser pulses manipulate individual ions
row of qubits in a linear Paul trap forms a quantum register
CCD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Effective ion-ion interaction induced by laser pulses that excite the ion`s motion
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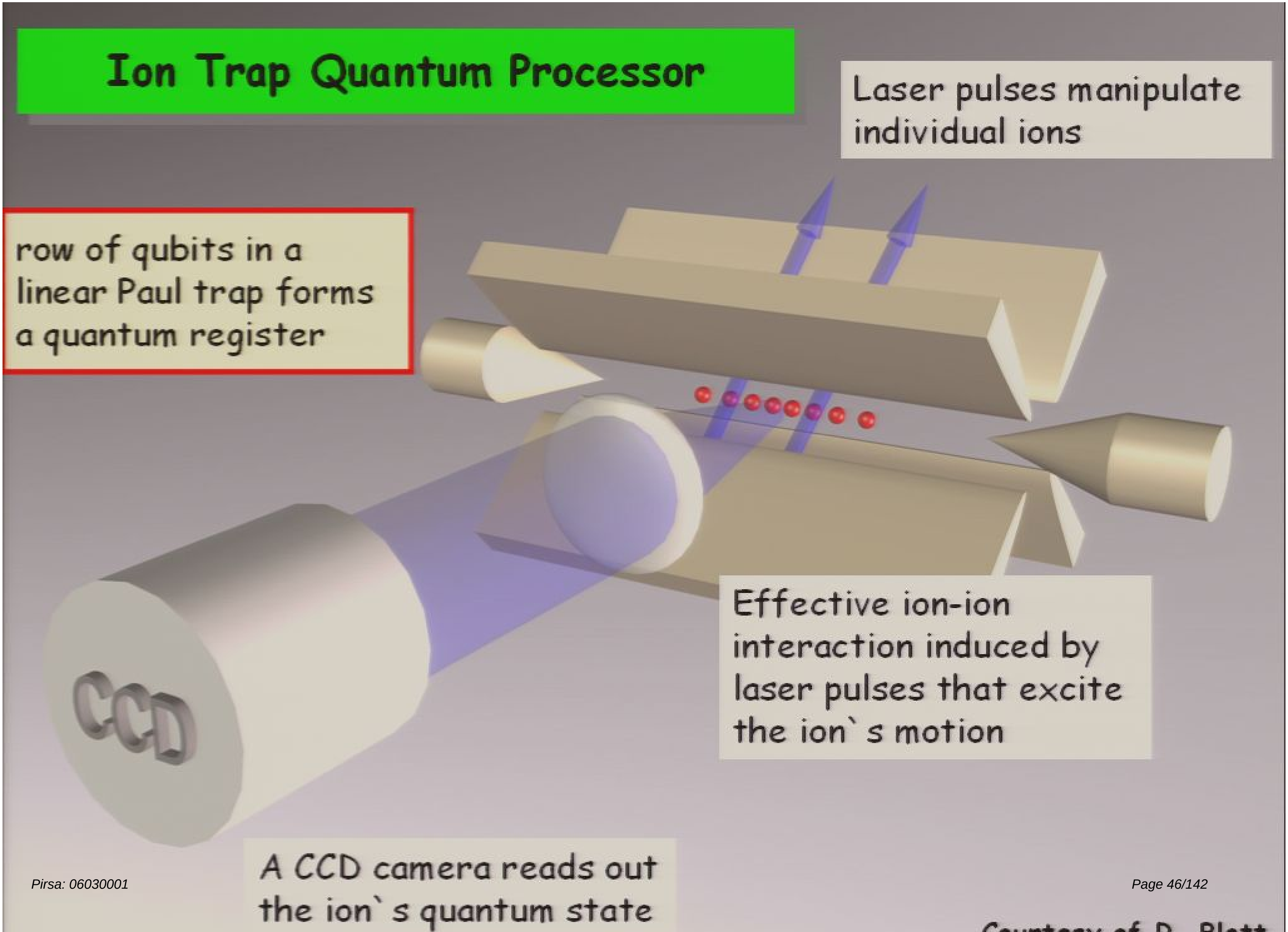
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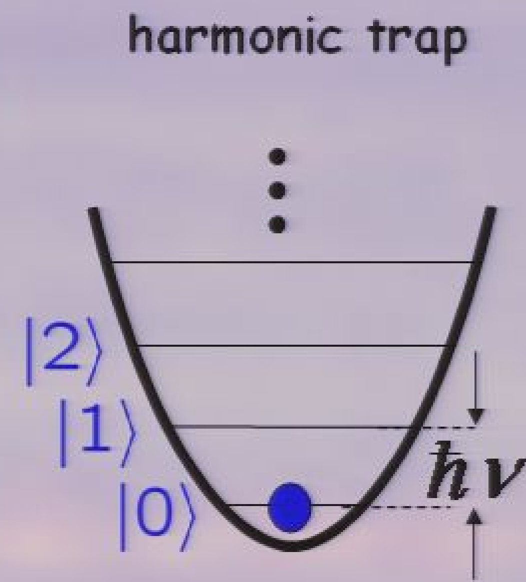
CCD
A CCD camera reads out the ion`s quantum state

Orders of Magnitude



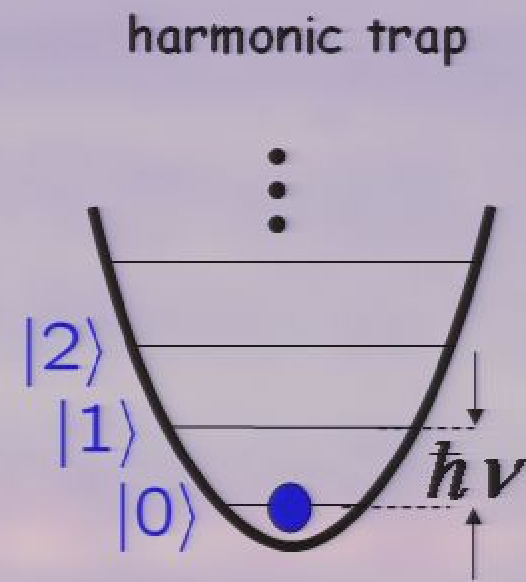
Orders of Magnitude

Physical size of the ground state:

$\nu = (2\pi)\,1\text{MHz}$
$m = 40\text{u}$

$$\left\langle x^2 \right\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11nm$$

Size of the wave packet << wavelength of visible light



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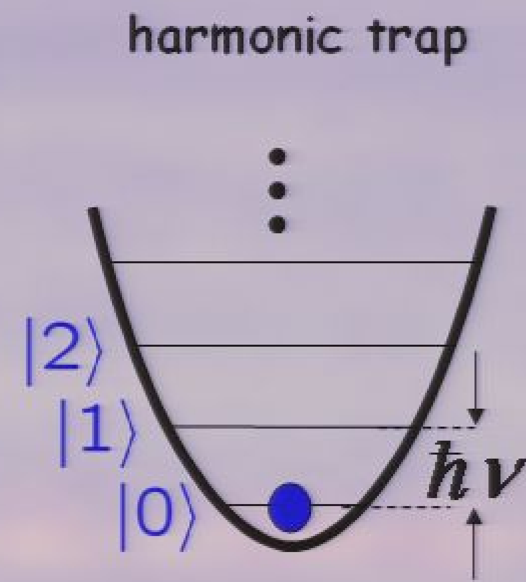
$$\left\langle x^2 \right\rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11nm$$



Size of the wave packet << wavelength of visible light

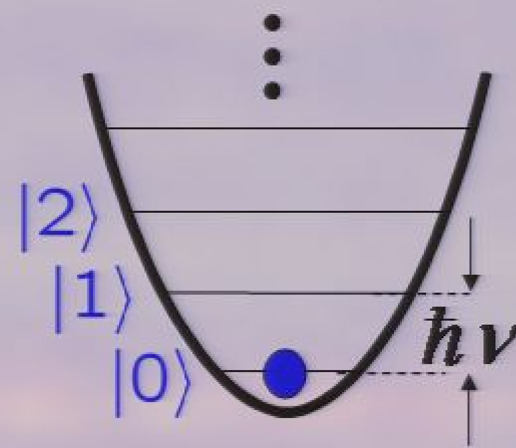
Energy scale of interest:

$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu K$$

Orders of Magnitude

harmonic trap

Physical size of the ground state:



$$\left.\begin{matrix} \nu = (2\pi)\,1\text{MHz} \\ m = 40\text{u} \end{matrix}\right\} \quad \langle x^2 \rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11nm$$

Size of the wave packet << wavelength of visible light

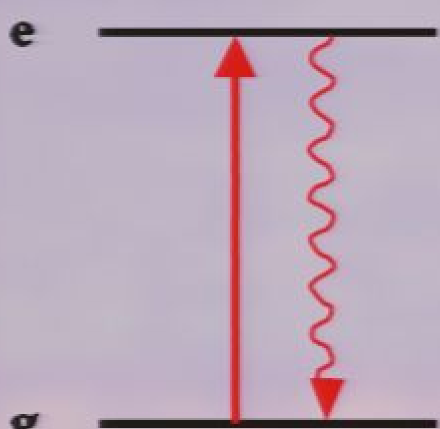
Energy scale of interest:

$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu K$$

Seperation between ions:

$$d \approx 5\mu m$$

Detection of Ions



Lifetime of excited state: $\tau \approx 10ns$

Maximum photon scattering rate: $r = \frac{1}{2\tau} \approx 50MHz$

$\eta \approx 10^{-3}$

Rate of detected photons: $R = \eta r \approx 50kHz$

$\longrightarrow$ **50 photons per ms**

Detection within 1 ms feasible provided that the background scattering rate is low.

$^{40}Ca^{+}$: Important energy levels

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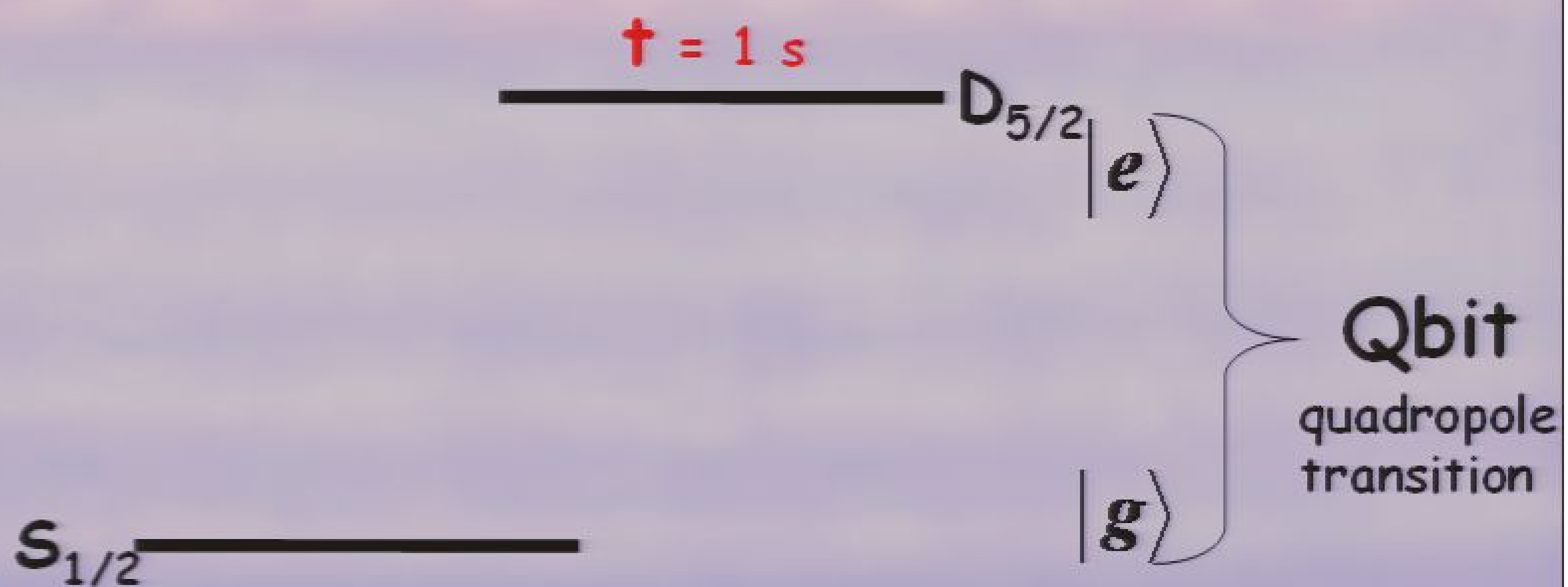

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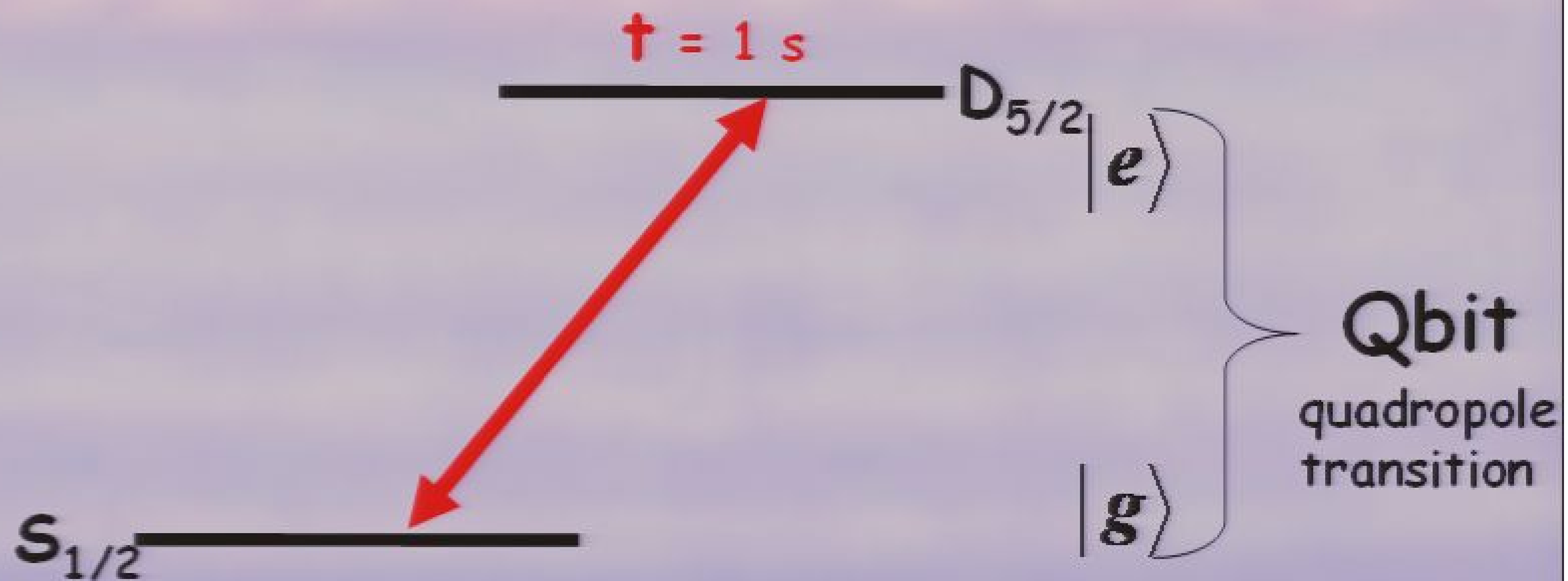

$$\phi = \alpha |g\rangle + \beta |e\rangle$$

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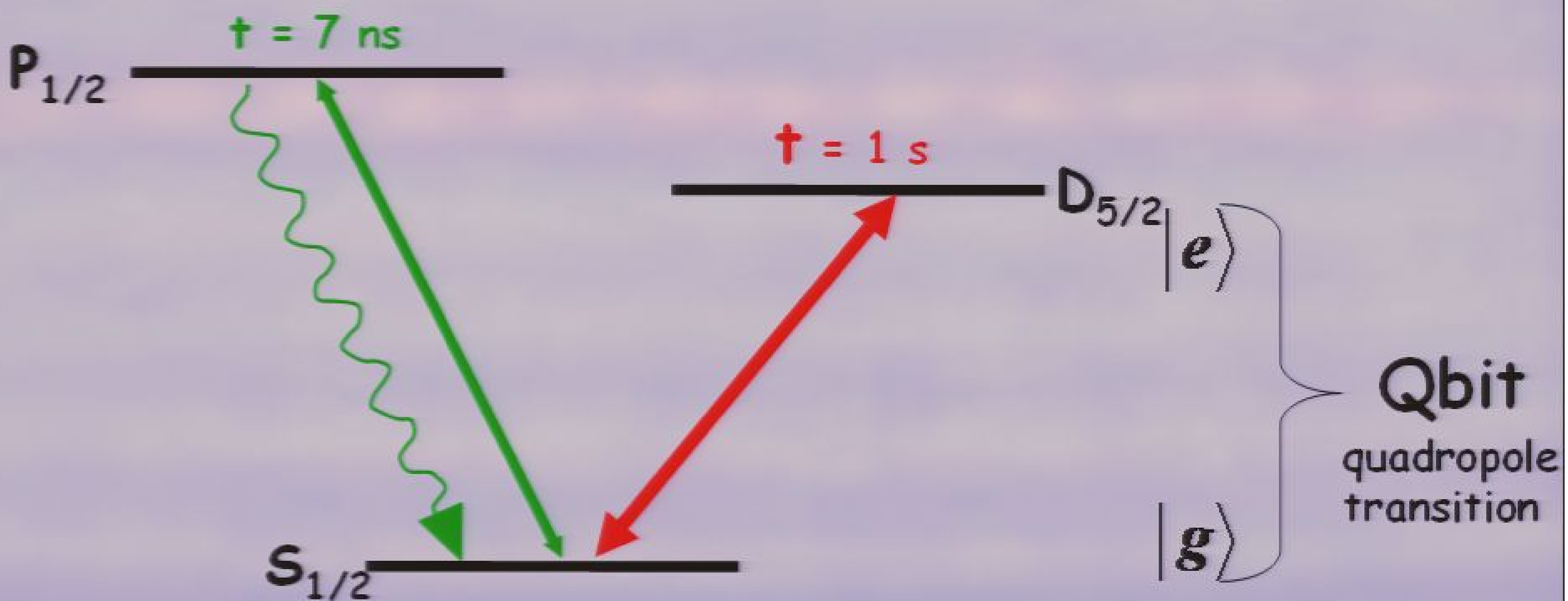
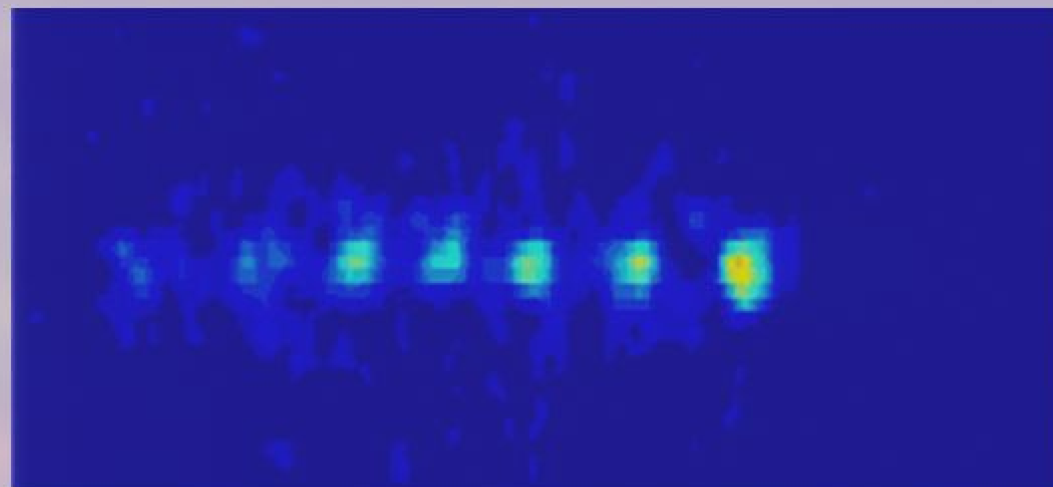


$$\phi = \alpha|g\rangle + \beta|e\rangle$$

Center-of-mass and breathing mode excitation

„center-of-mass mode"



Courtesy of R. Blatt

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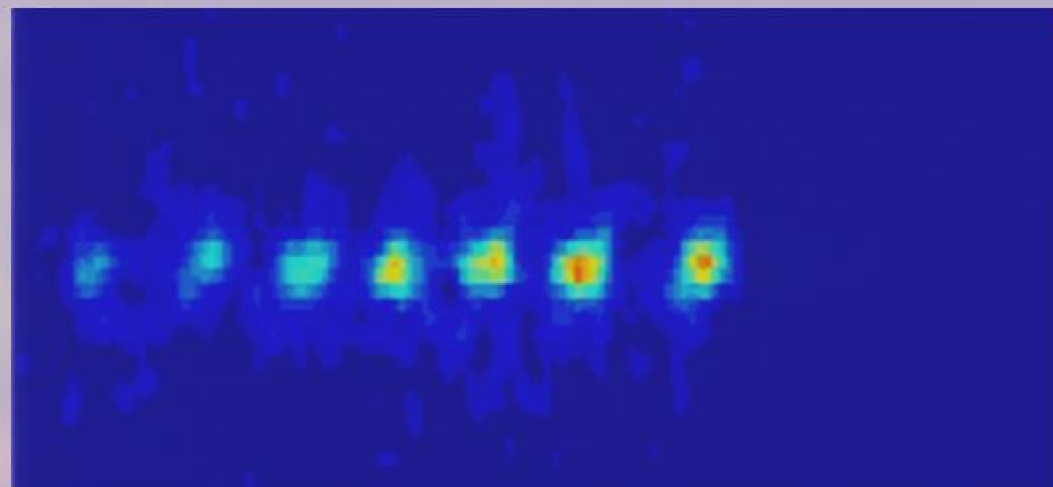

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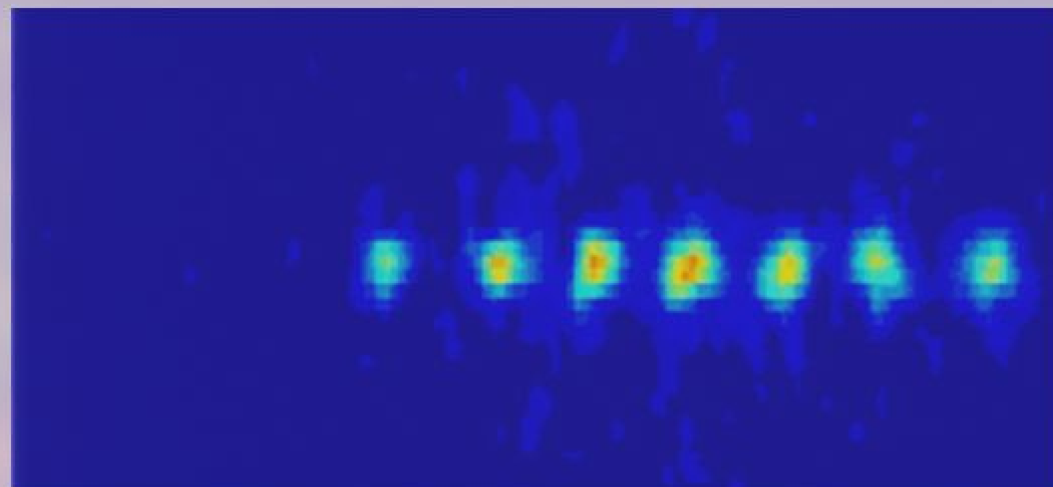

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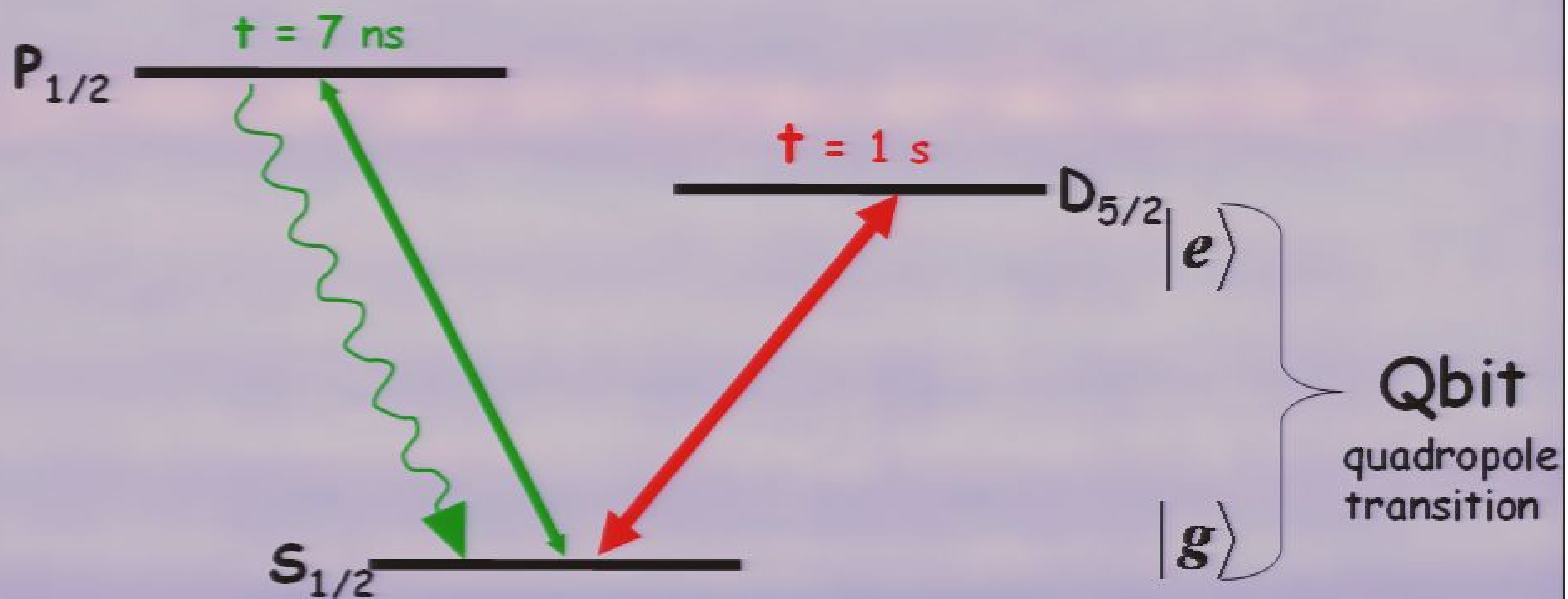

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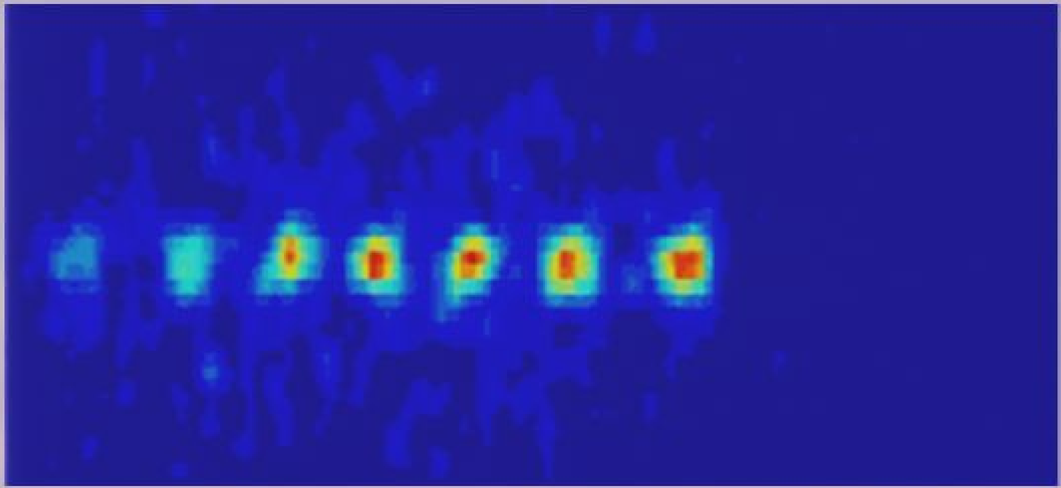

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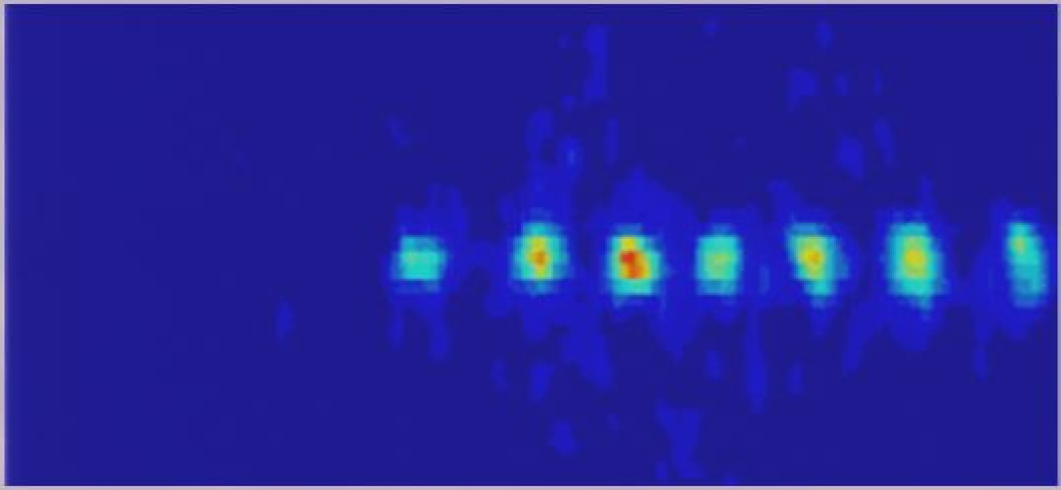

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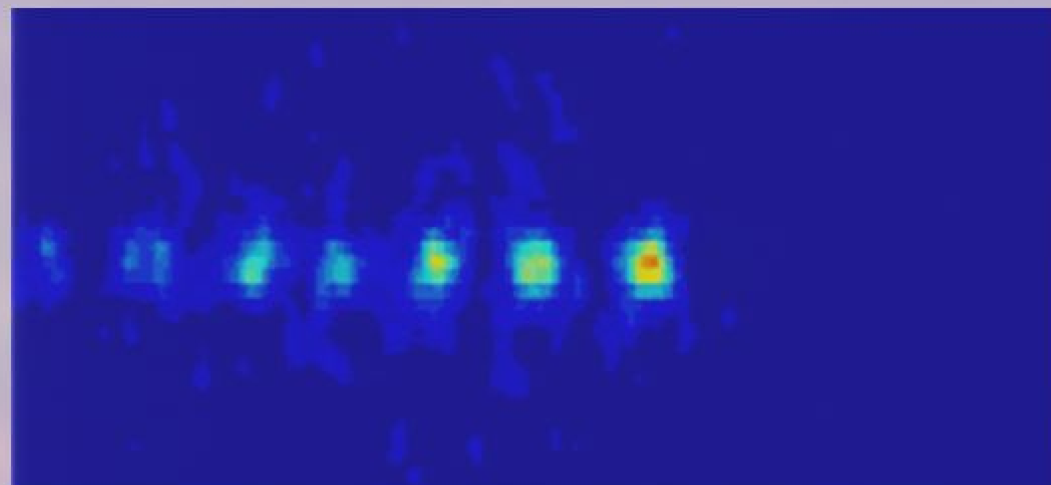

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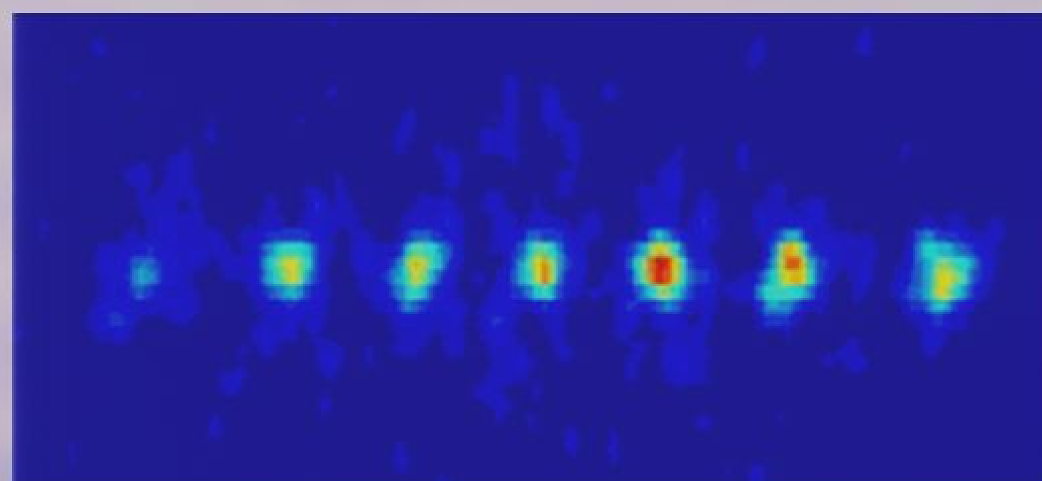

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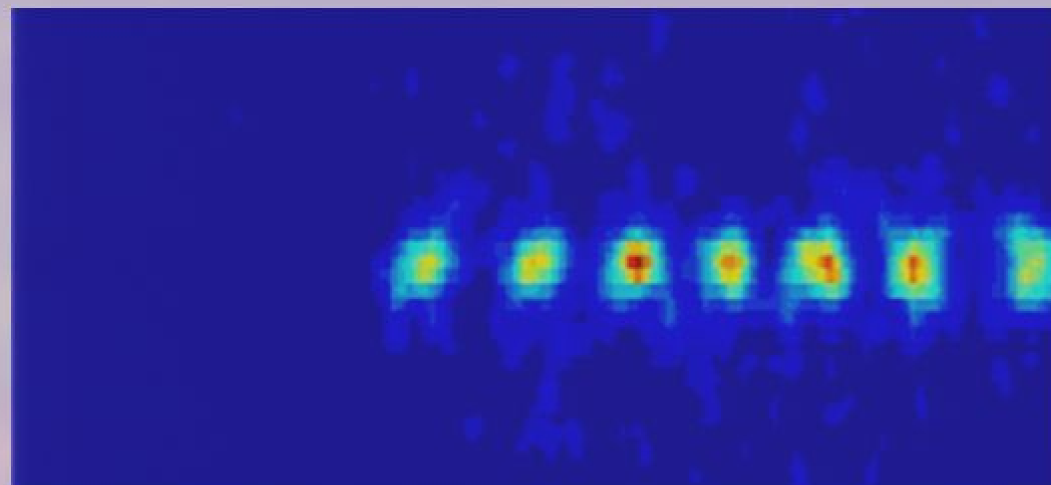

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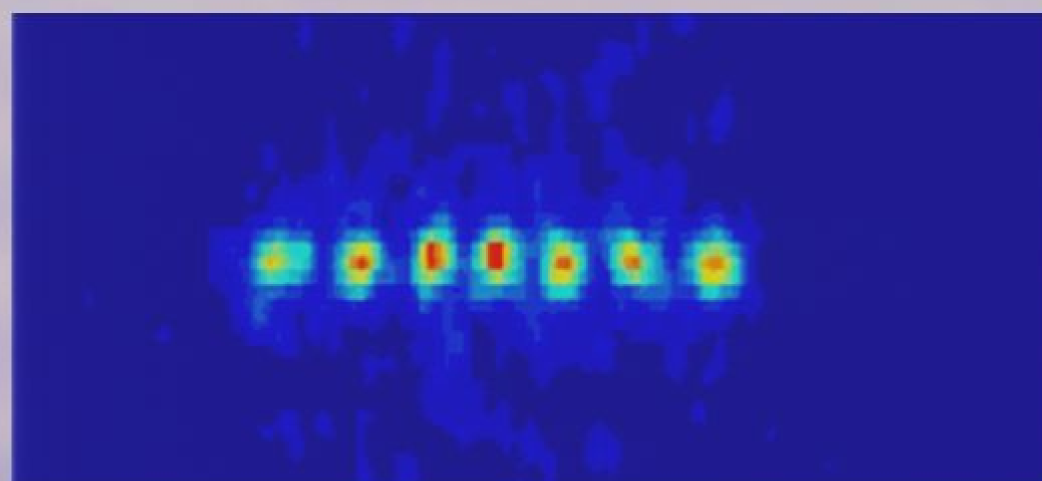

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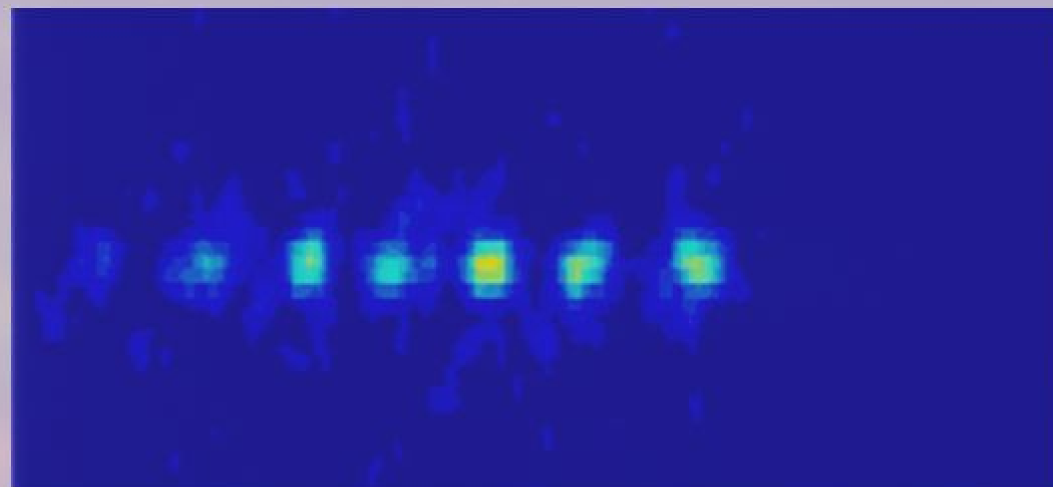

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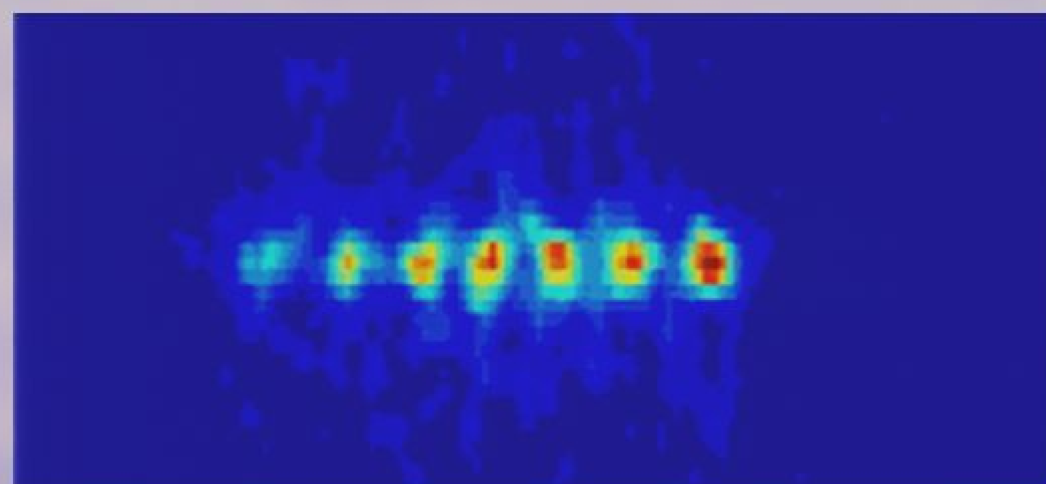

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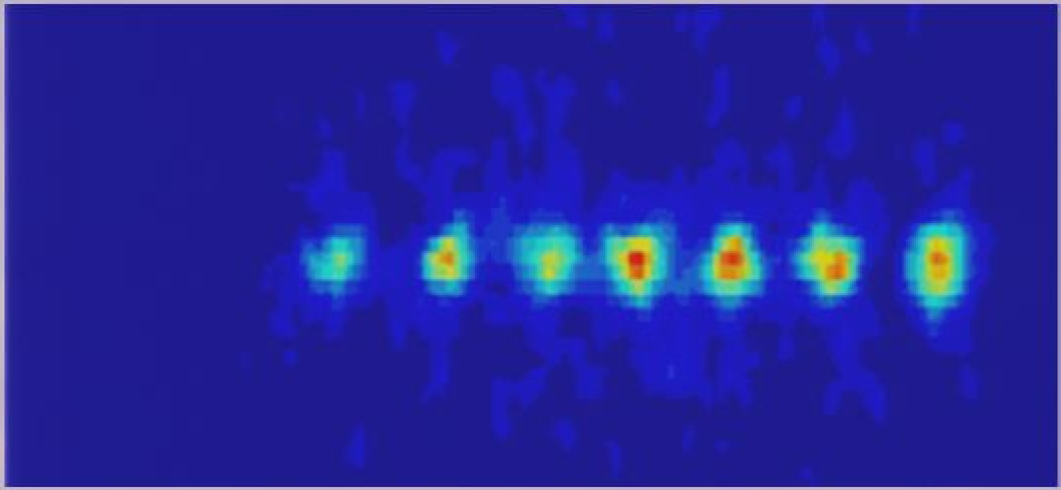

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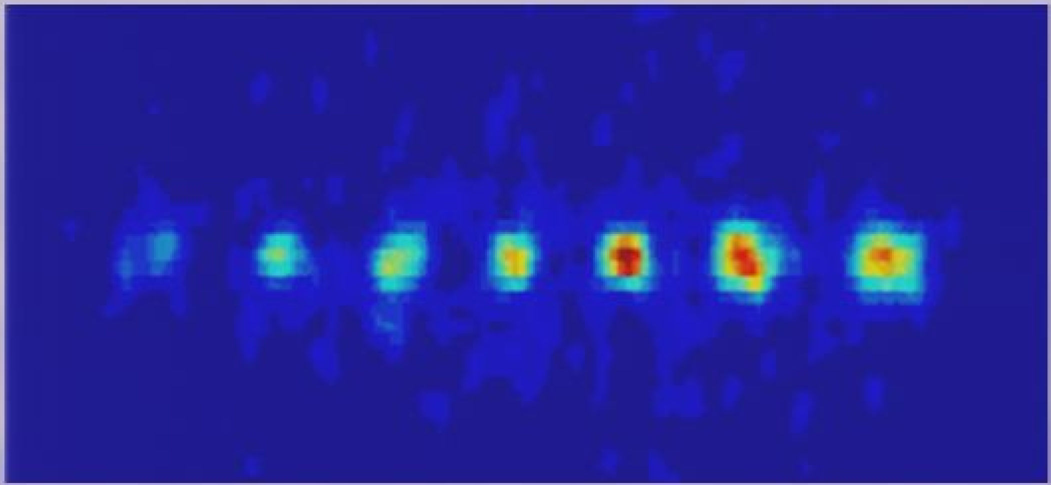

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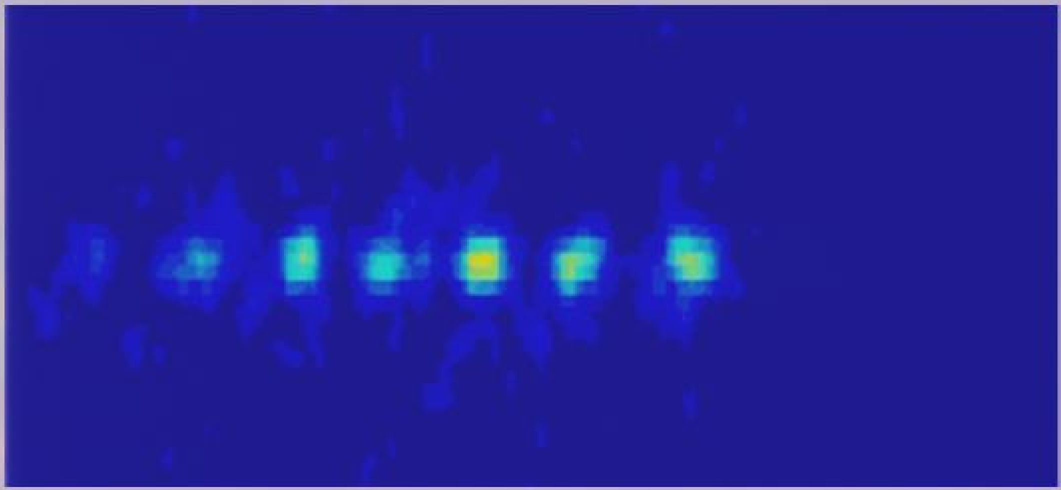

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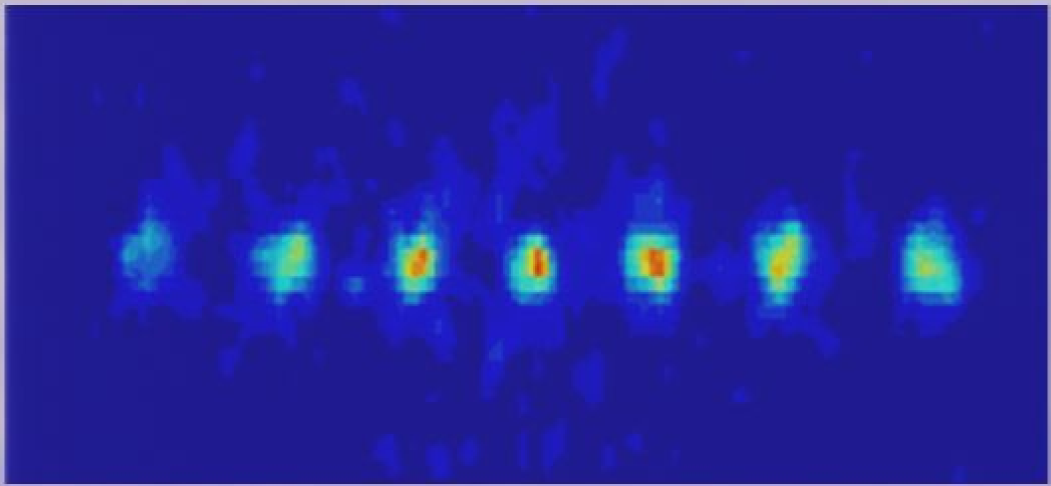

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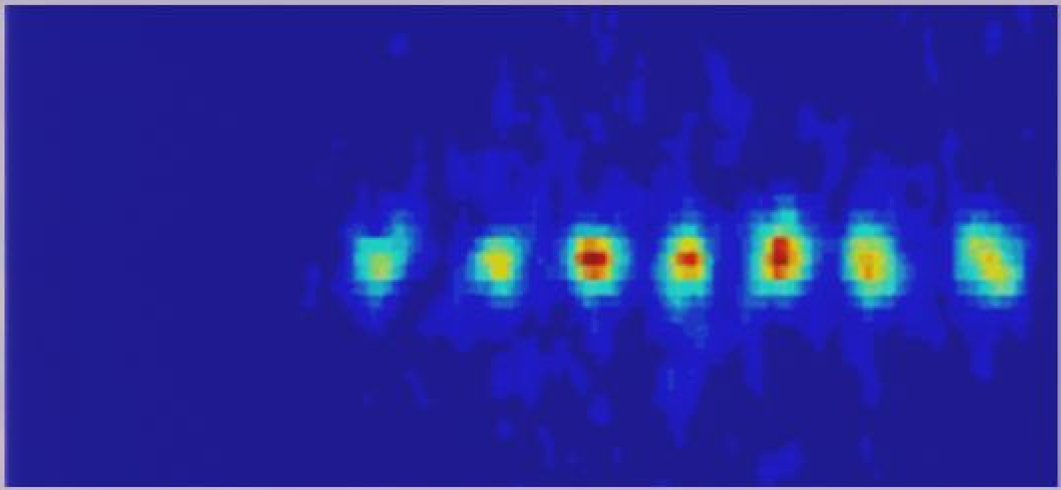

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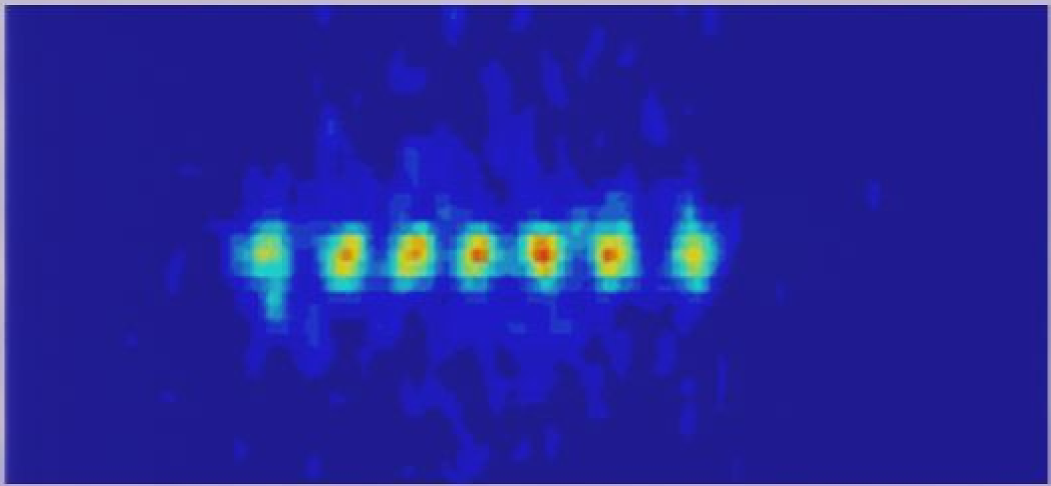

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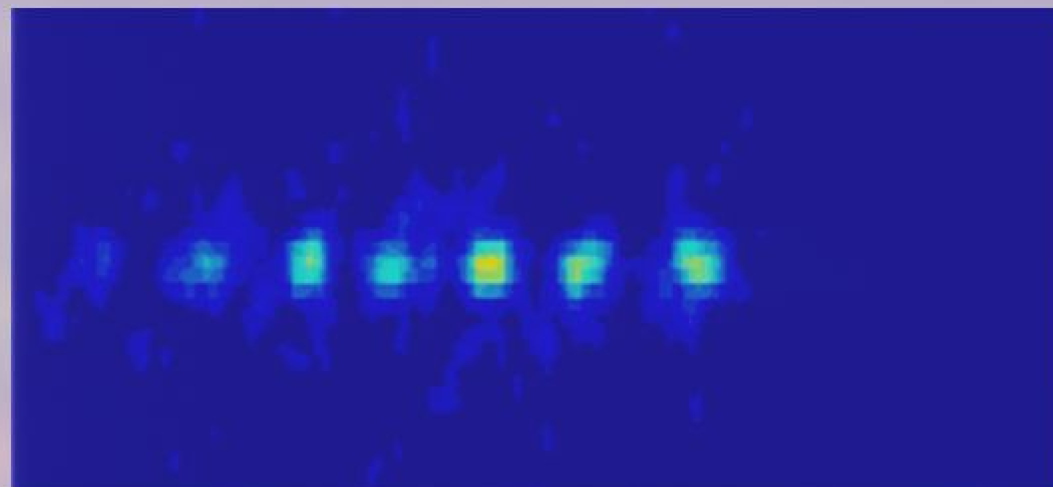

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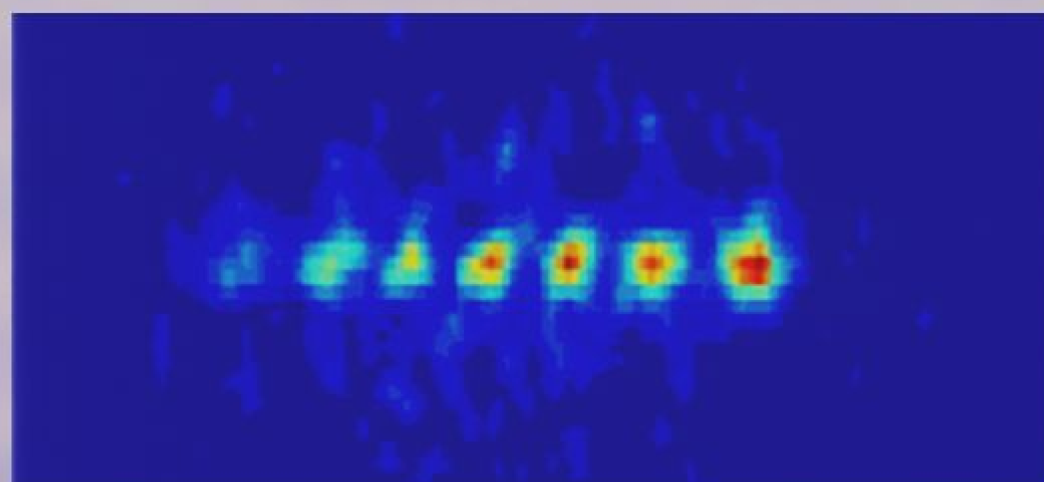

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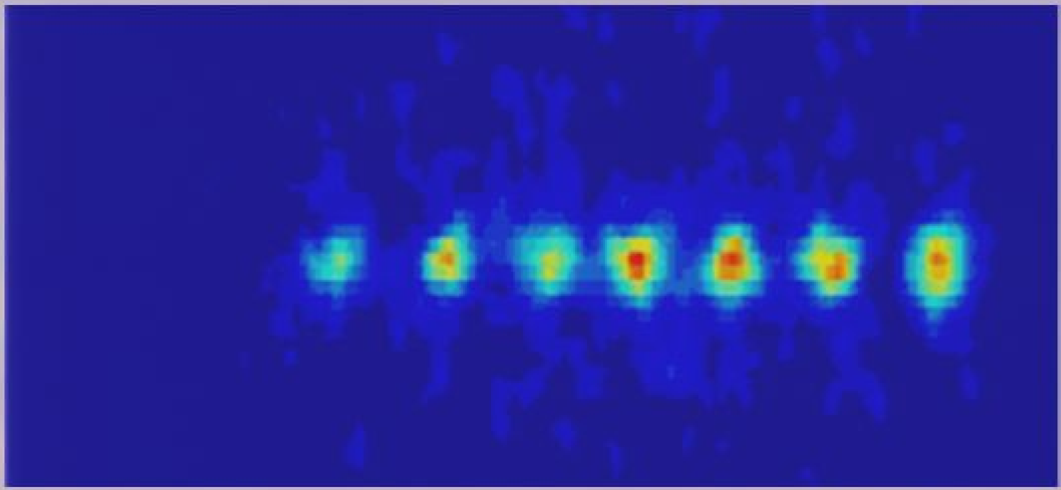

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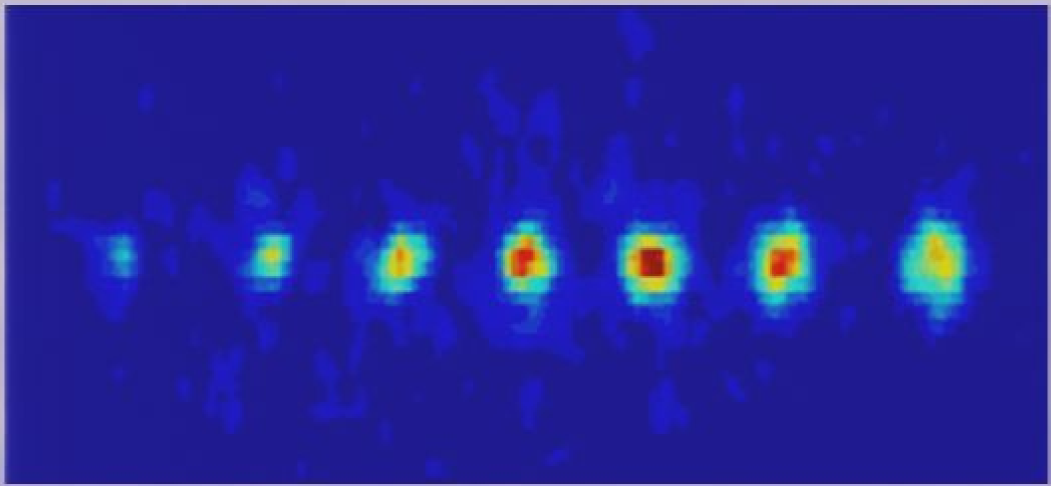

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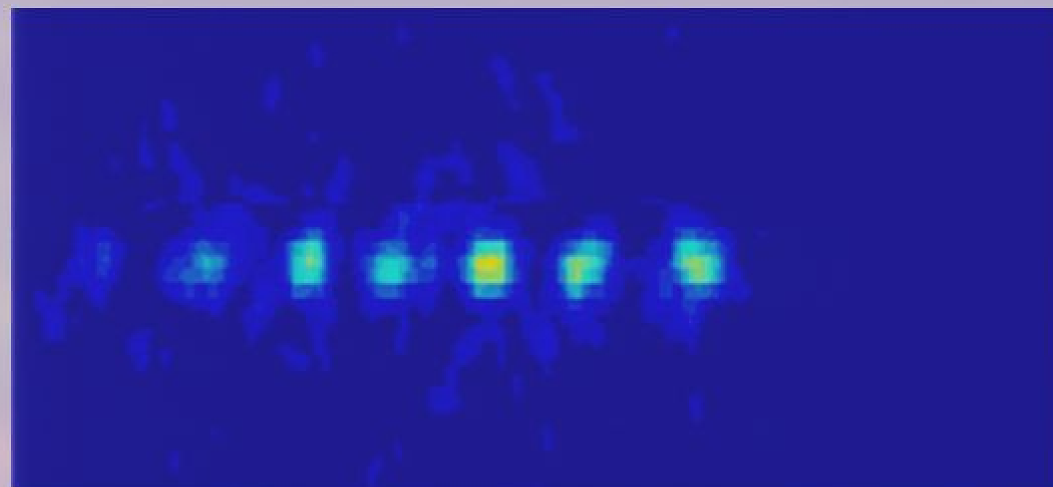

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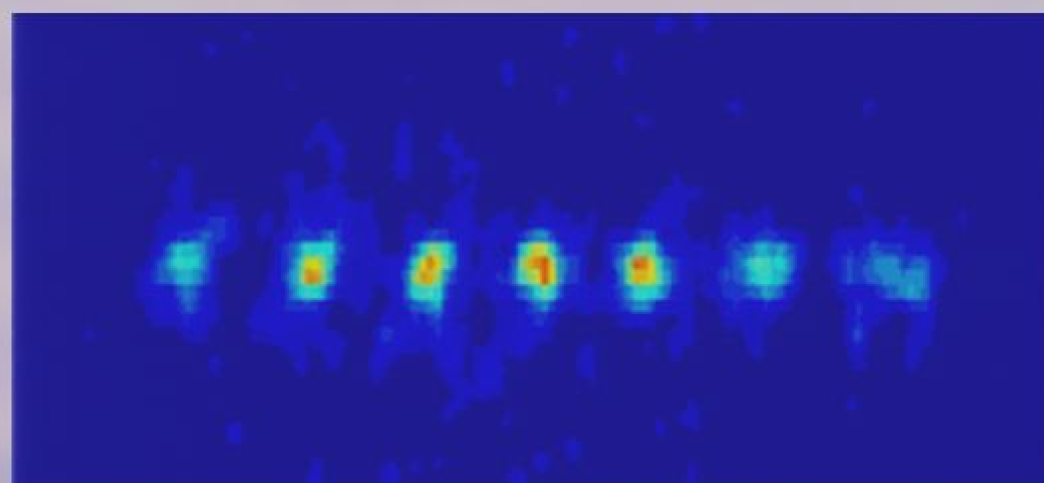

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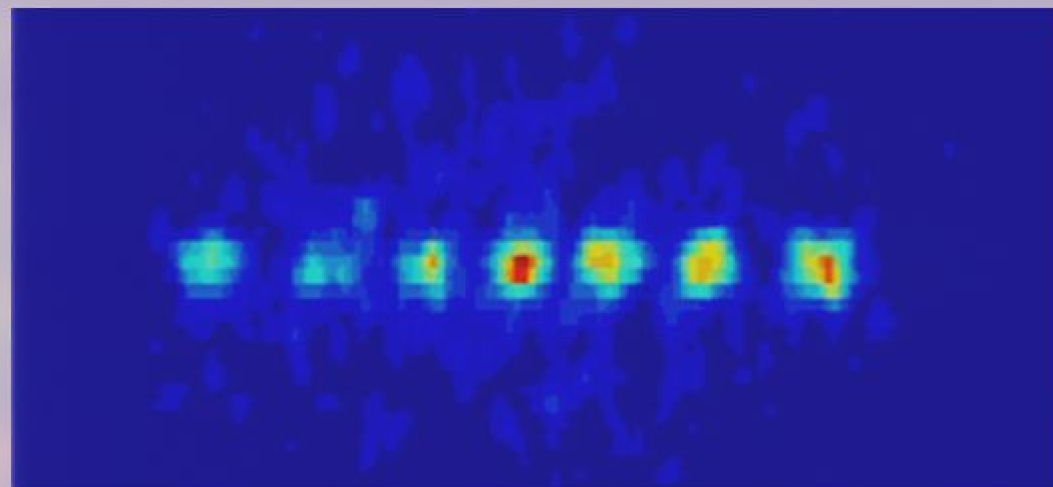

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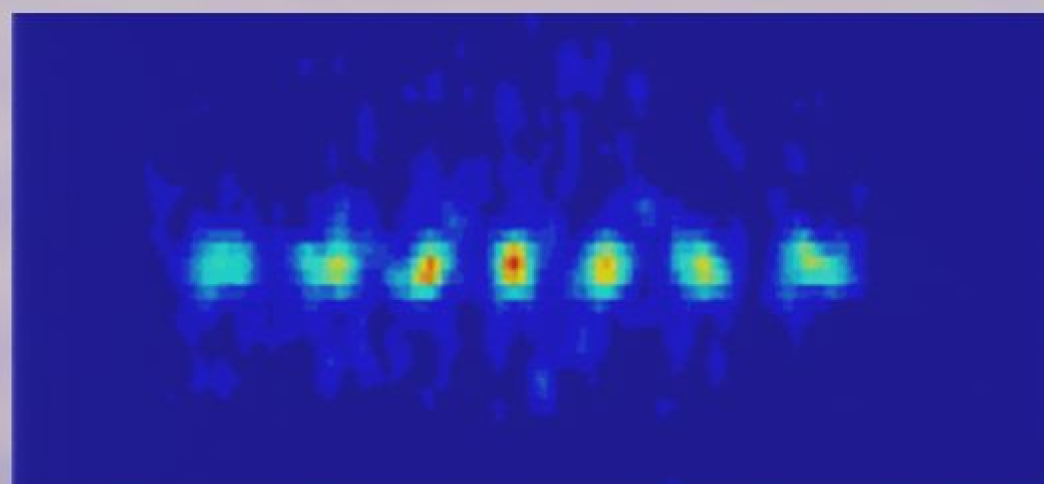

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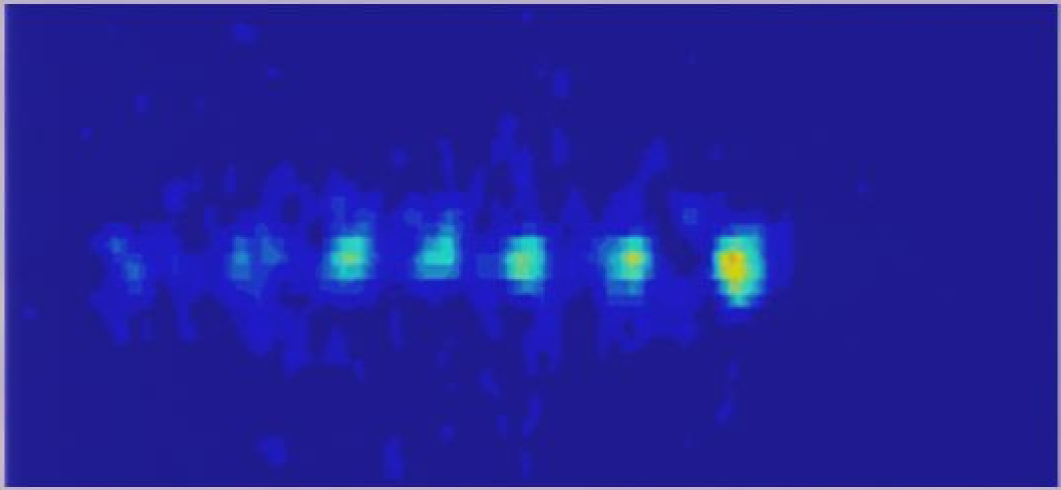

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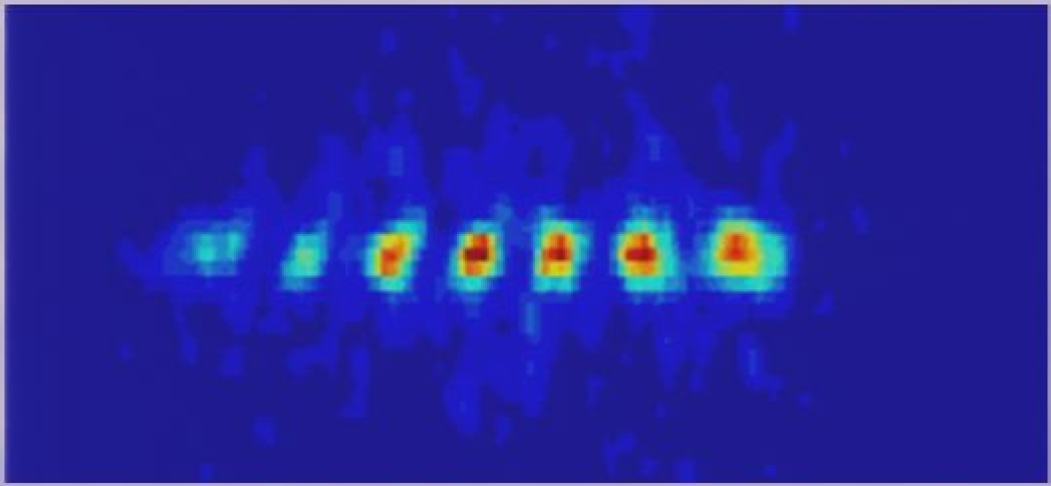

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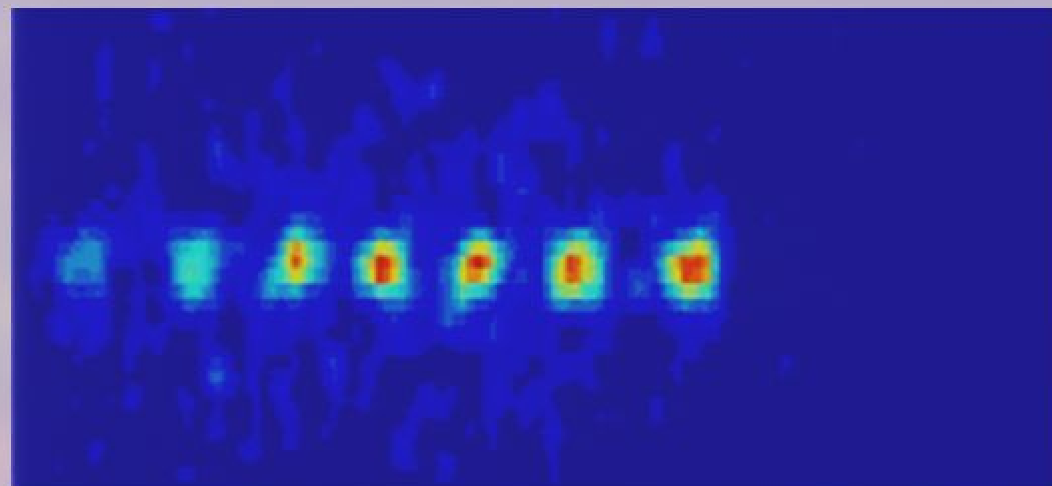

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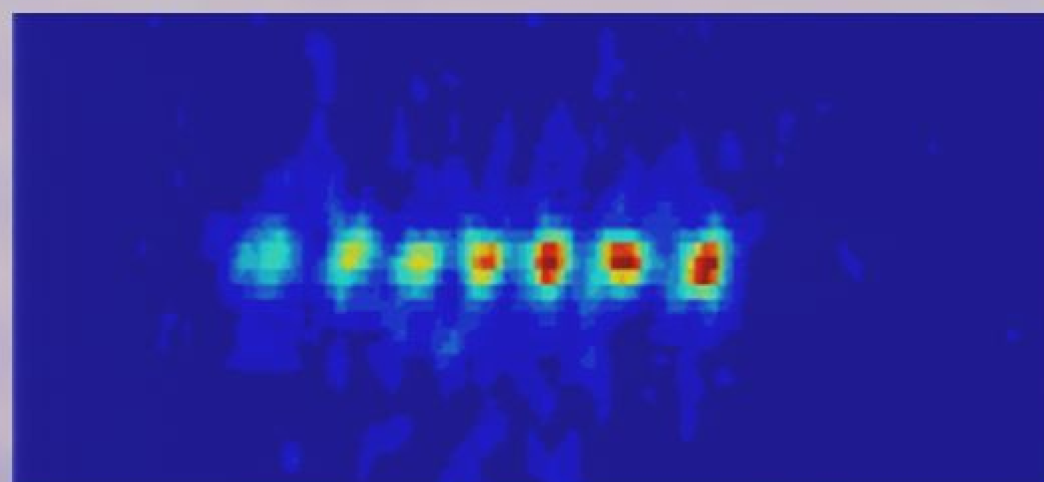

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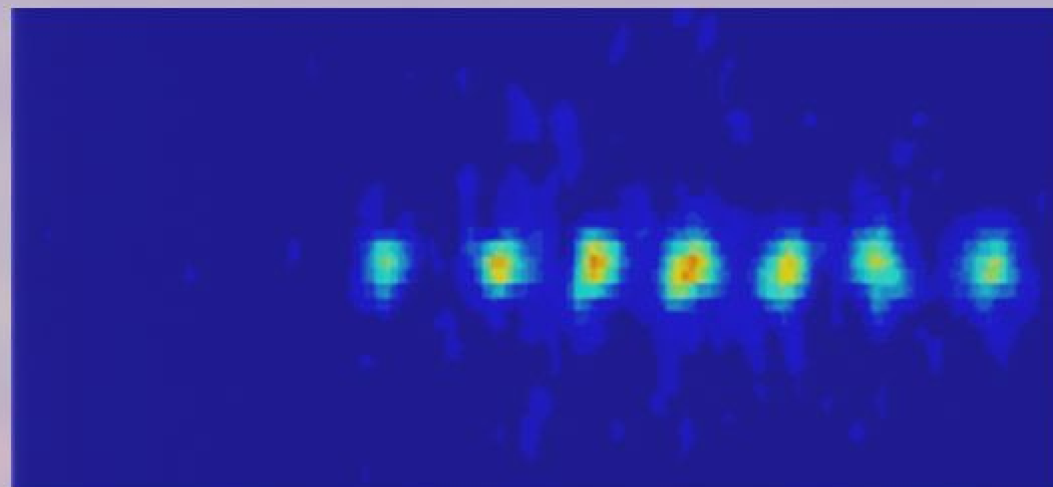

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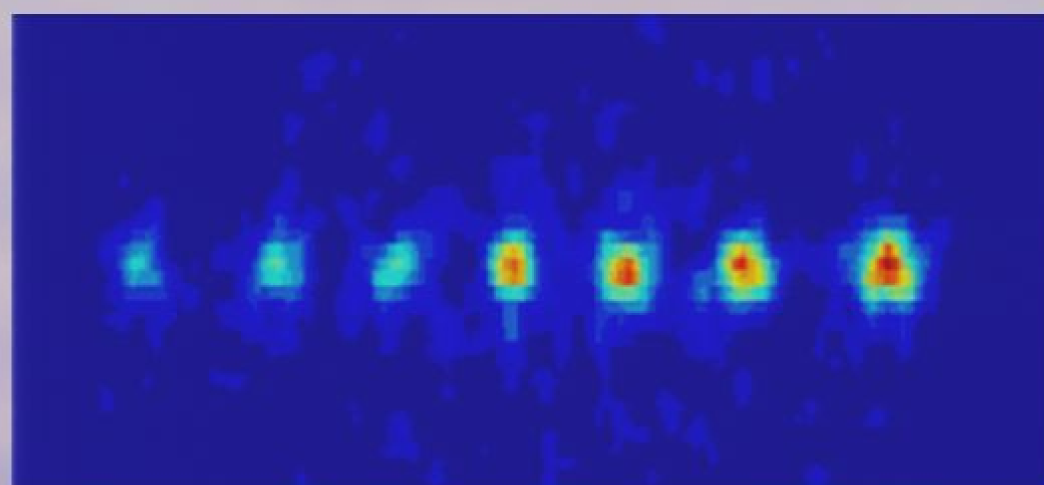

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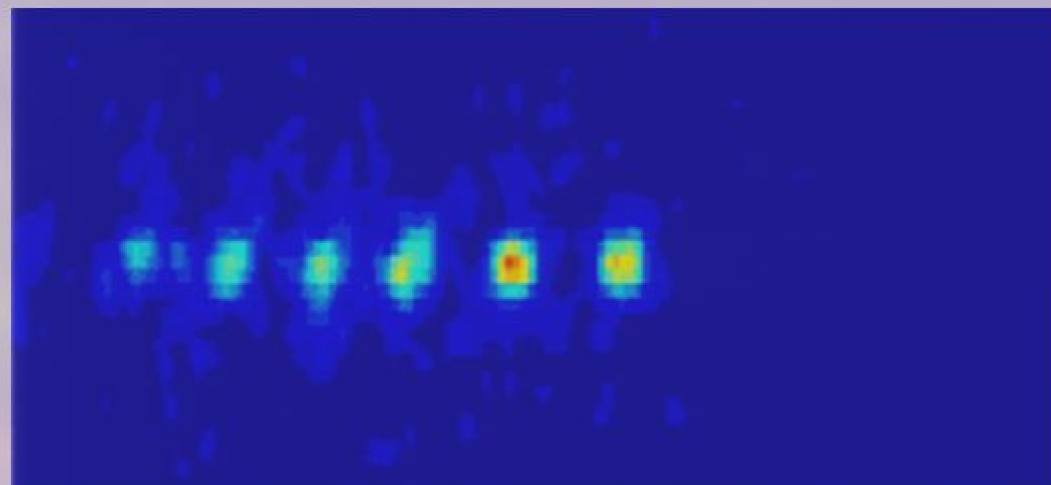

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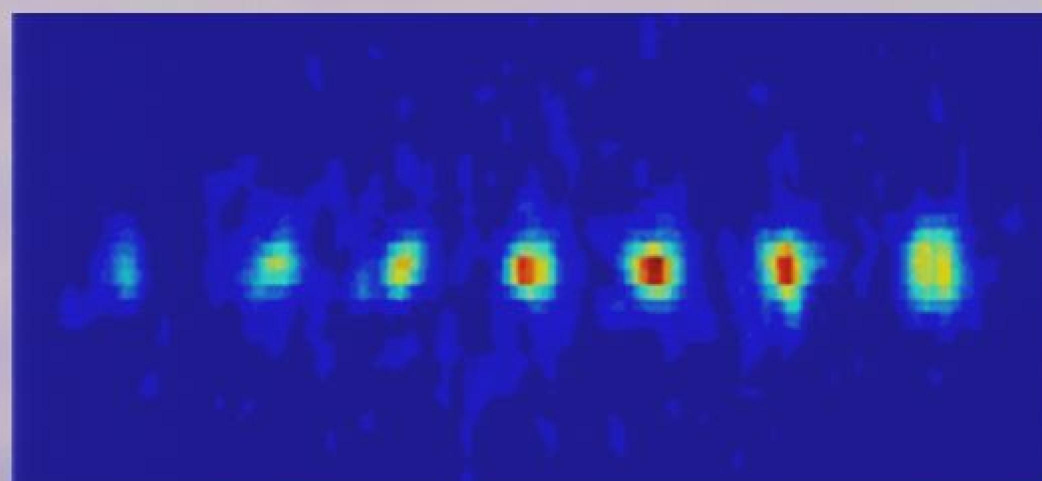

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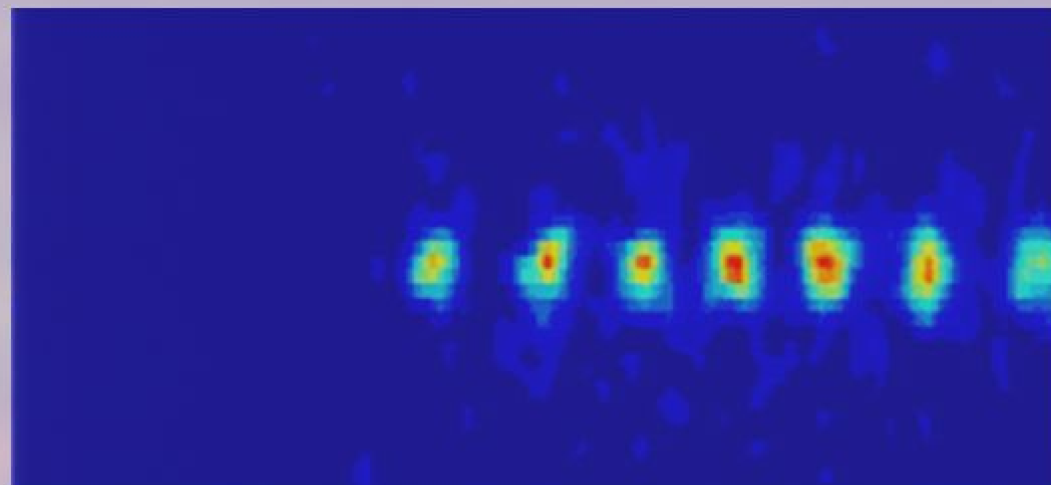

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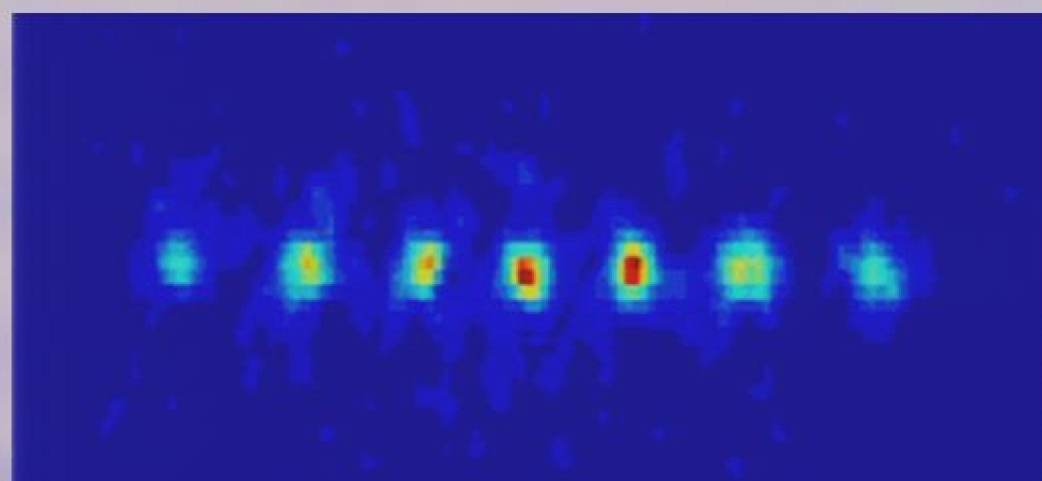

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Laser – Ion Interactions

Hamiltonian: $H = H^{(Internal)} + H^{(External)} + H^{(Interaction)}$

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mode frequencies

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mode frequencies

$$H^{(Interaction)} = \hbar \Omega \left(|g\rangle\langle e| + |e\rangle\langle g| \right) \cos(k\hat{x} - \omega t + \phi)$$

Rabi frequency

Laser frequency

Laser – Ion Interactions

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_{+}\exp\left\{i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^{\dagger}\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

Laser - Ion Interactions

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Lamb-Dicke parameter

$$\eta = kx_0 = k\sqrt{\frac{\hbar}{2m\nu}}$$

relates size of ground state to wave length of light

In ion trap experiments,

usually $\eta \ll 1$

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relates size of ground state to wave length of light

In ion trap experiments,

usually $\eta \ll 1$

$$\delta = \omega - \omega_0$$

Detuning of laser with respect to atomic transition

Lamb – Dicke regime

Taylor expansion of the exponentiel up to first order:

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_{+}\left\{1 + i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^{+}\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

Lamb – Dicke regime

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Carrier resonance:

$\delta = 0$ $$H_{\text{int}} = \frac{\hbar\Omega}{2}\left\{\sigma_+ e^{+i\phi} + \sigma_- e^{-i\phi}\right\}$$ $|g,n\rangle \leftrightarrow |e,n\rangle$

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Red sideband:

$\boldsymbol{\delta = -\nu}$ $$H_{\text{int}} = \frac{\hbar\Omega}{2}i\eta\left\{\sigma_+ a e^{+i\phi} - \sigma_- a^+ e^{-i\phi}\right\}$$ $|g,n\rangle \leftrightarrow |e,n-1\rangle$

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Taylor expansion of the exponentiel up to first order:

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Carrier resonance:

$\delta = 0$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\left\{\sigma_{+}e^{+i\phi} + \sigma_{-}e^{-i\phi}\right\}$$

$|g,n\rangle \leftrightarrow |e,n\rangle$

Red sideband:

$\delta = -\nu$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}i\eta\left\{\sigma_{+}ae^{+i\phi} - \sigma_{-}a^{+}e^{-i\phi}\right\}$$

$|g,n\rangle \leftrightarrow |e,n-1\rangle$

Blue sideband:

$\delta = +\nu$

$$H_{\text{int}} = \frac{\hbar\Omega}{2}i\eta\left\{\sigma_{+}a^{+}e^{+i\phi} - \sigma_{-}ae^{-i\phi}\right\}$$

$|g,n\rangle \leftrightarrow |e,n+1\rangle$

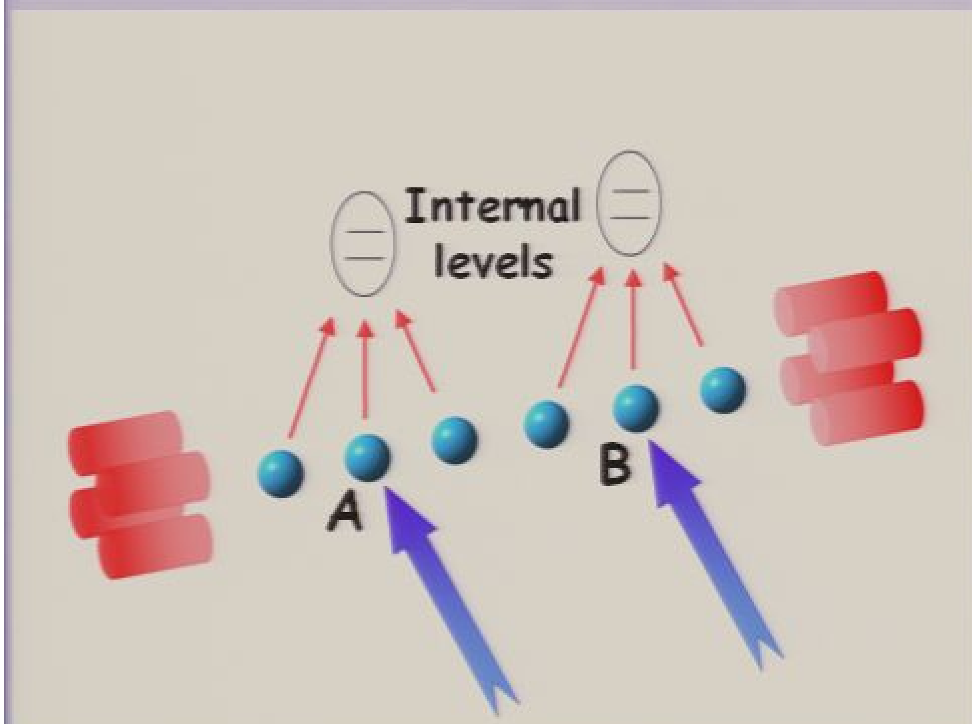
Vacuum Entanglement in an Ion Trap

1 Vacuum Entanglement

2 Introduction to Ion trap Quantum computing

3 Ground state Entanglement of an Ion Trap

Detection of Vacuum Entanglement in a Linear Ion Trap



$$H = H_0 + H_{int}$$

$$H_0 = \omega_0 \left(\sigma_Z^A + \sigma_Z^B \right) + \sum_i \nu_i a_i^+ a_i$$

$$H_{int} = \Omega(t) \left(e^{-i\phi} \sigma_+^{(k)} + e^{i\phi} \sigma_-^{(k)} \right) x_k$$

$$\underbrace{1/\omega_0 \ll T}_{\text{RWA}} \ll 1/\nu_0 \quad \text{(RWA struck out over } T \ll 1/\nu_0\text{)}$$

A. Retzker, J. I. Cirac, B. Reznik PRL 94, 050504

The Operators in an ion trap

"Field Operators":

$$\phi_i = \sum_n \frac{b_n^i}{\sqrt{2\nu_n}} \left(a_n e^{-i\nu_n t} + a_n^+ e^{i\nu_n t} \right)$$

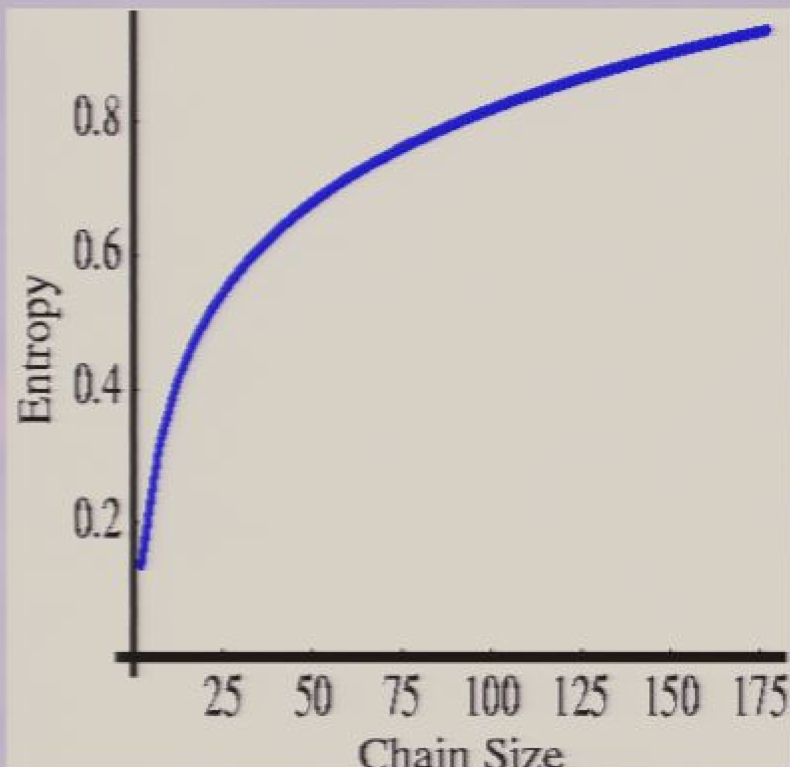
$$\pi_i = -i \sum_n b_n^i \sqrt{\frac{\nu_n}{2}} \left(a_n e^{-i\nu_n t} - a_n^+ e^{i\nu_n t} \right)$$

Correlation Function: $\left\langle \phi_i(t_1)\phi_j(t_2) \right\rangle = \sum_n \frac{b_n^i b_n^j}{2\nu_n} e^{i\nu_n (t_1 - t_2)}$

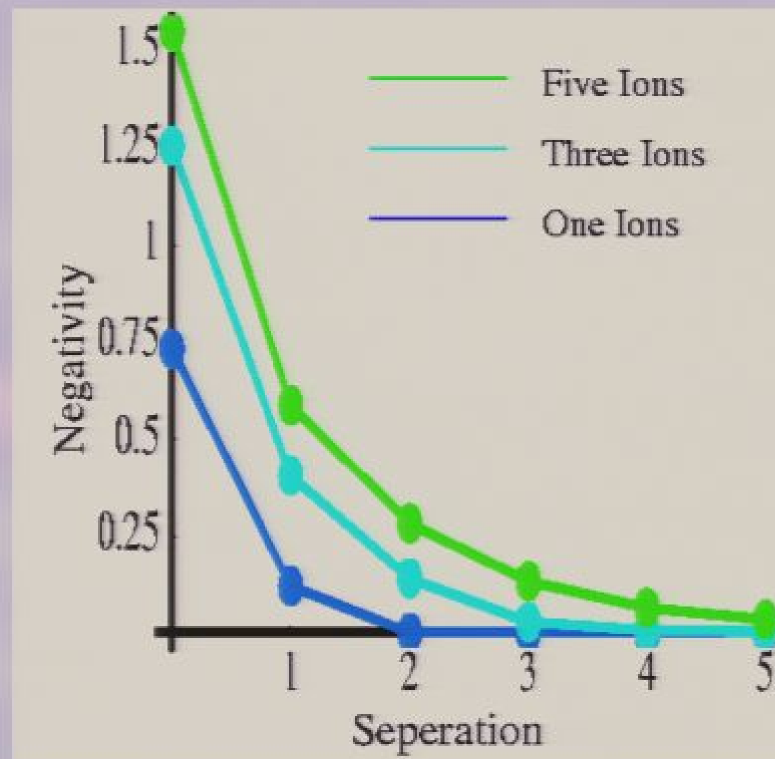
Commutator: $\left[\phi_i(t_1), \phi_j(t_2) \right] = i \sum_n \frac{b_n^i b_n^j}{\nu_n} Sin\left(\nu_n \left(t_2 - t_1\right)\right)$

Exchange: $\sum_n \frac{b_n^i b_n^j}{\nu_n} e^{-i\nu_n (t_1 - t_2)}$

Entanglement in a linear trap

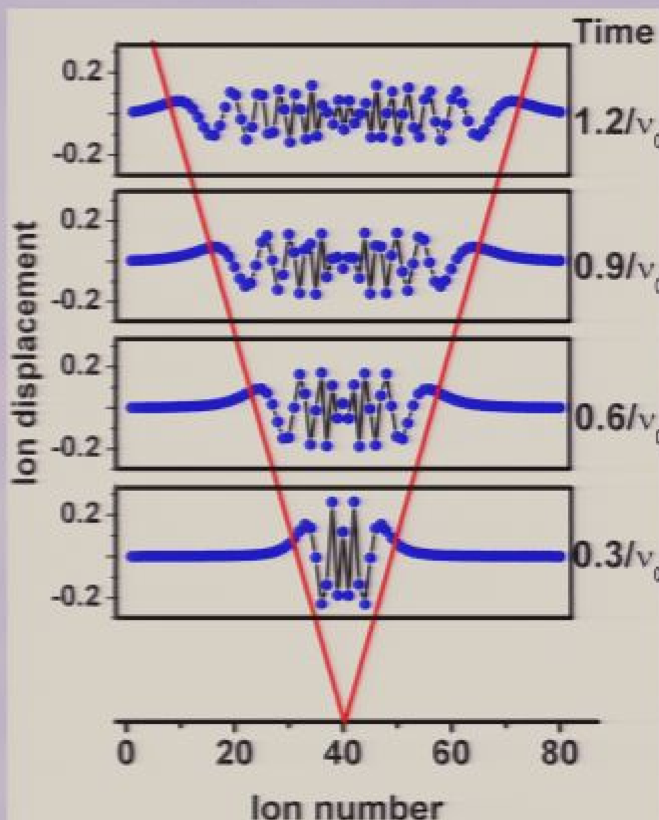


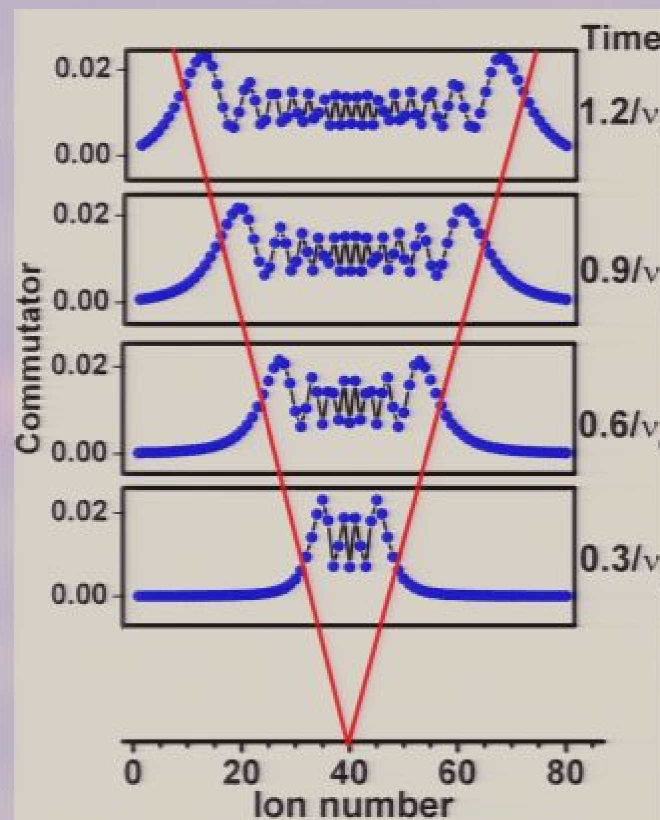
Entanglement between complementary symmetric groups of ions as a function of the total number



Entanglement between finite symmetric groups of ions as a function of the separation.

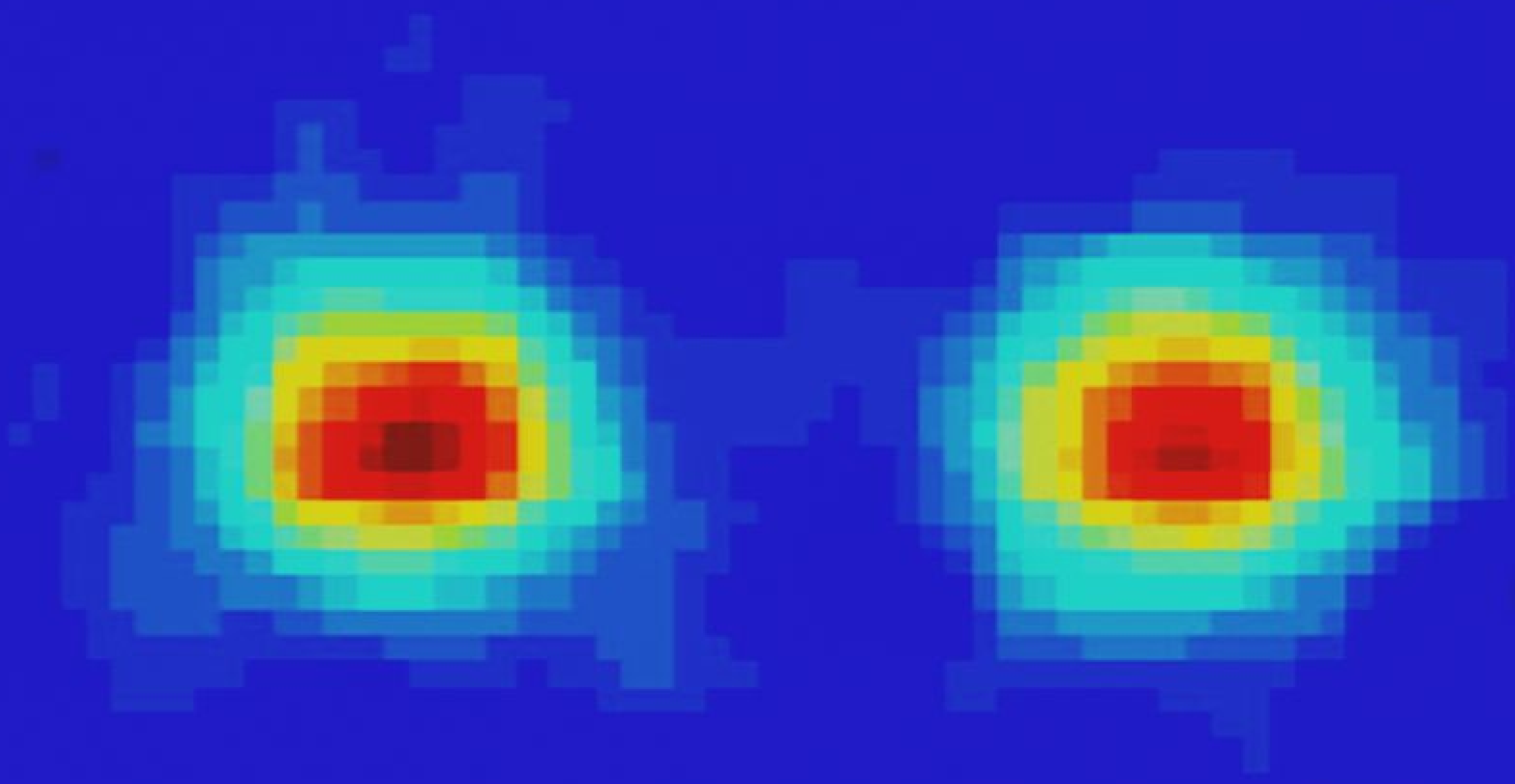
Causal Structure





$\rightarrow \quad U_{AB} = U_A \cdot U_B + O([x_A(0), x_B(T)])$

Two Trapped Ions

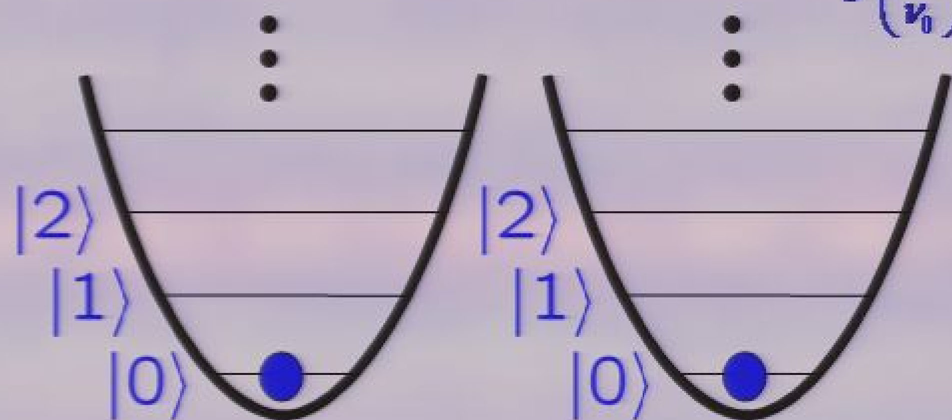


Two mode squeezed state

$$\left|0_c\right\rangle\left|0_r\right\rangle = \sqrt{1-e^{-2\beta}}\sum_n e^{-\beta n}\left|n\right\rangle_A\left|n\right\rangle_B$$

$f\left(\frac{\nu_1}{\nu_0}\right) = 1.99$

The population of the first two levels is 99%



The entanglement is 0.136 e-bits

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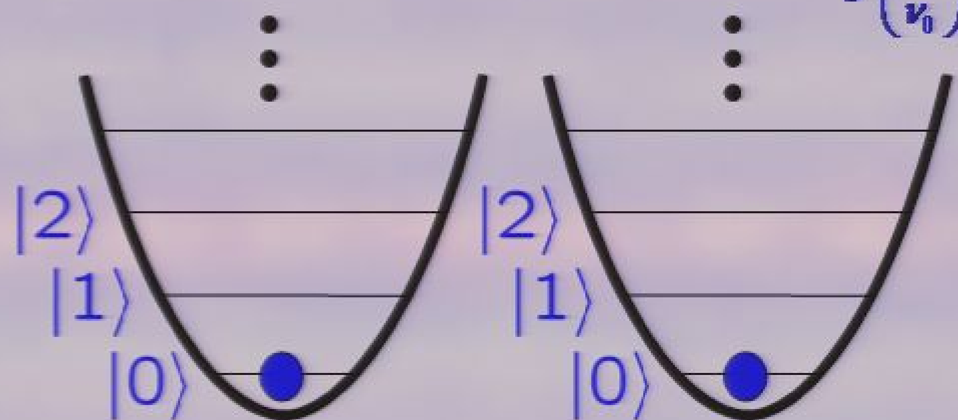


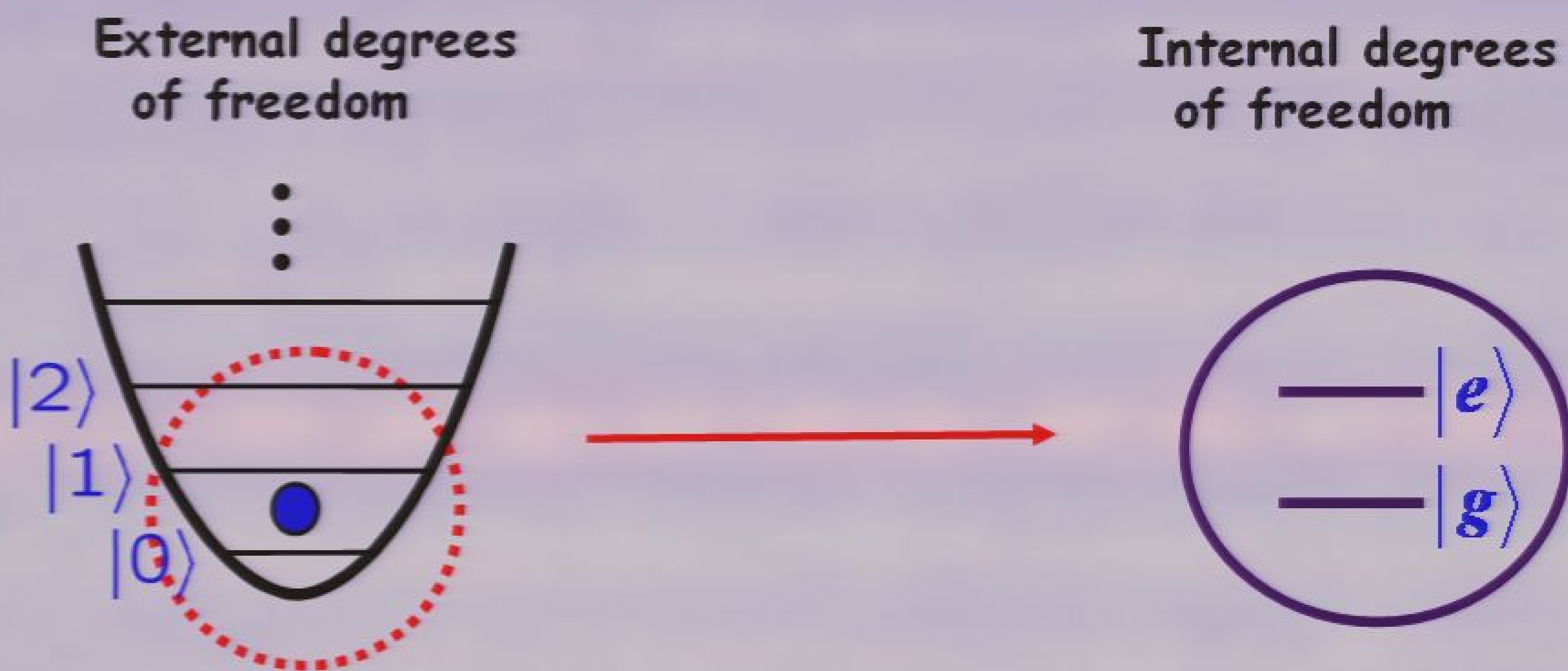
The entanglement is 0.136 e-bits

The available operations:

$$H_{\text{int}}^{(k)} = \Omega(t)\left(e^{-i\phi}\sigma_+^{(k)} + e^{i\phi}\sigma_-^{(k)}\right)x_k = \Omega(t)\sigma_{\hat{n}}x_k$$

$\hat{n}$ is a unit vector in the x-y plane

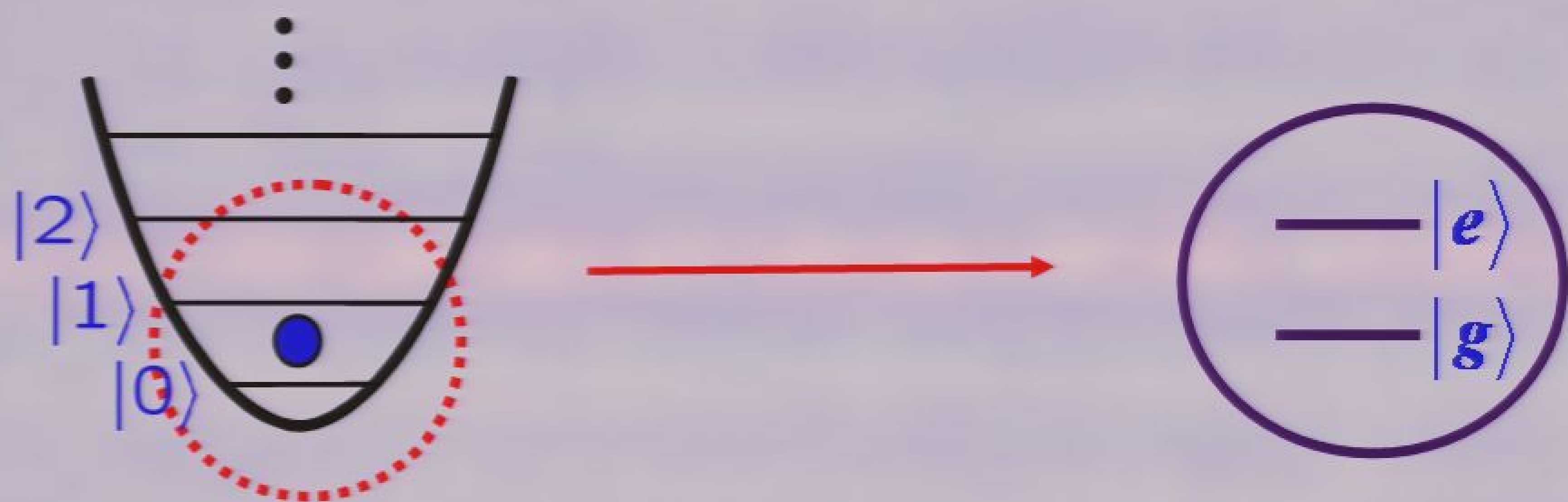
Swap Operation



Swap Operation

External degrees of freedom

Internal degrees of freedom



$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|g\rangle + \beta|e\rangle$$

Swap Operation

$$U_{Swap} = e^{i\frac{\pi}{4}(\sigma_x^1\sigma_x^2+\sigma_y^1\sigma_y^2+\sigma_z^1\sigma_z^2)}$$

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$$\tilde{\sigma}_x = |0\rangle\langle 1| + |1\rangle\langle 0| \qquad P_{n\geq 2} \ll 1 \qquad \tilde{\sigma}_x \approx x$$

$$\tilde{\sigma}_y = i|1\rangle\langle 0| - i|0\rangle\langle 1| \qquad \longrightarrow \qquad \tilde{\sigma}_y \approx p$$

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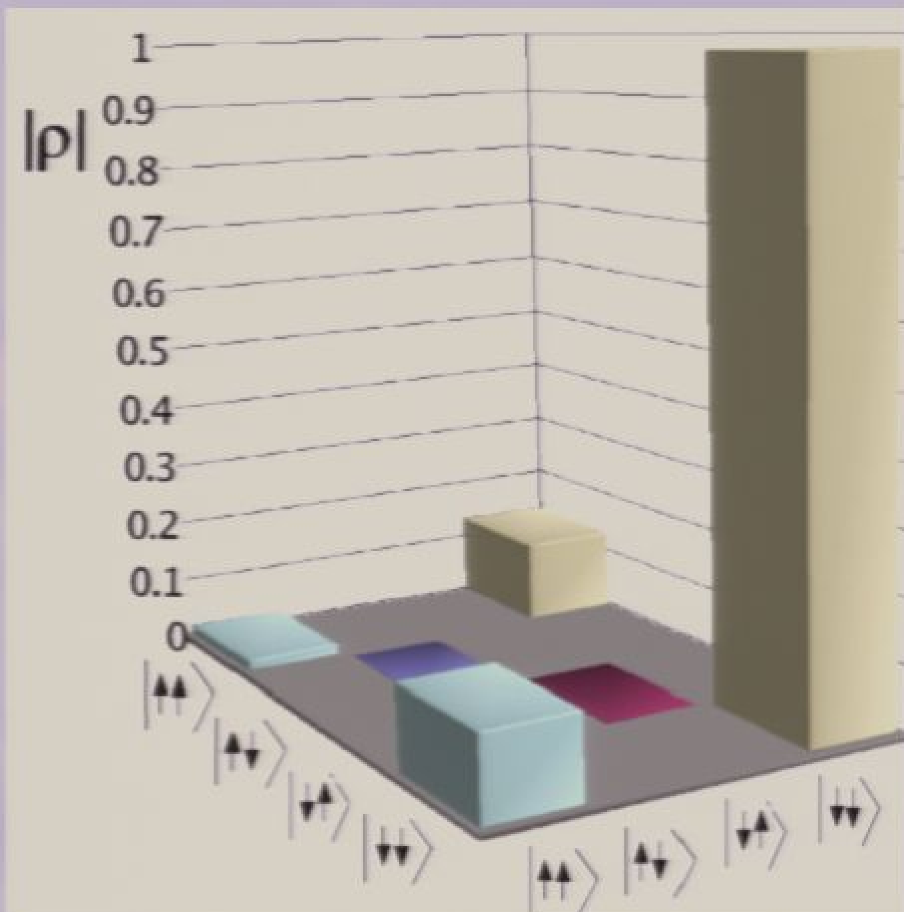
In order to realize the p coupling we use two kicks in opposite directions

$$e^{-i\beta\sigma_y x}\Big|_{t=\tau}\, e^{i\beta\sigma_y x}\Big|_{t=0} \approx e^{-i\beta\sigma_y\left(x+\frac{p\tau}{m}\right)} e^{i\beta\sigma_y x} = e^{-i\frac{\beta}{m}\sigma_y\left(p\tau+\frac{1}{2}\tau\beta\right)}$$

Two Trapped Ions



Final internal state

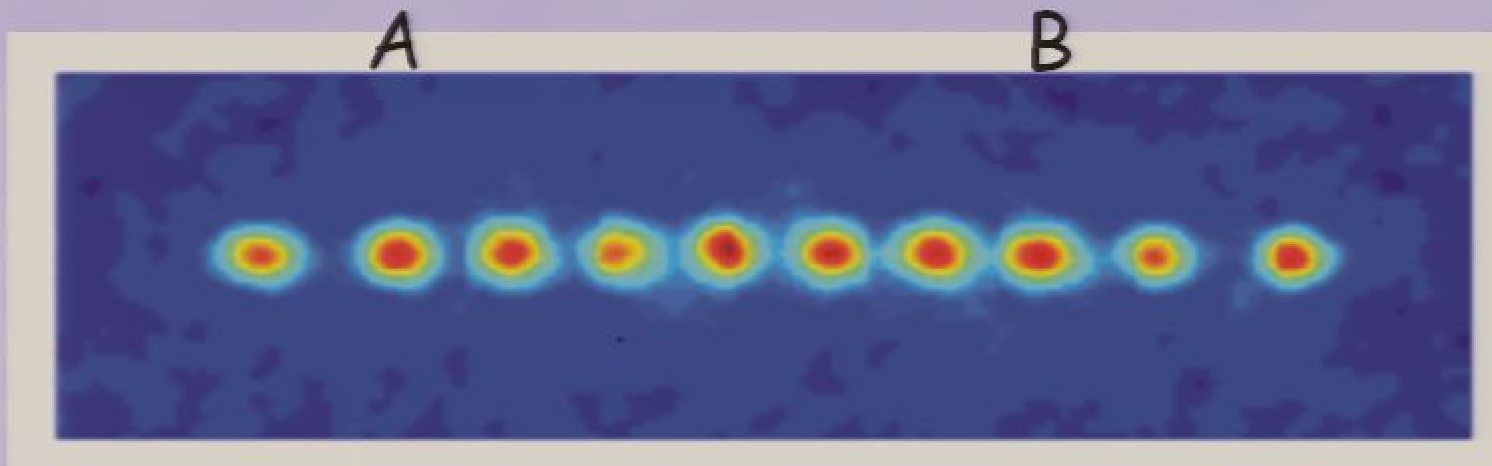
"Swapping" spatial internal states

$$|vac\rangle|\downarrow\downarrow\rangle \rightarrow |\chi\rangle\left(|\downarrow\downarrow\rangle + e^{-\beta}|\uparrow\uparrow\rangle\right)$$

$$U = e^{i\alpha x\sigma_x} \cdot e^{i\beta p\sigma_y} \cdots$$

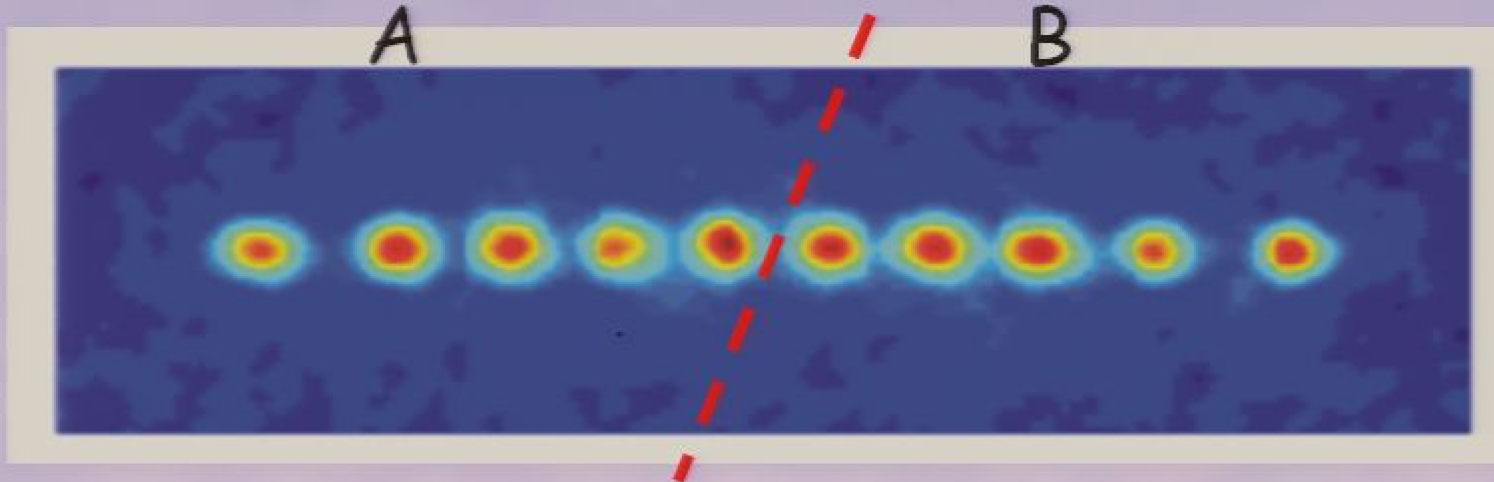
$E_{formation}(\rho_{final})$ accounts for 97% of the calculated Entangtlement: E(|vac>)=0.136 e-bits.

Long Ion Chain



But how do we check that ent. is not due to "non-local" interaction?

Long Ion Chain



But how do we check that ent. is not due to "non-local" interaction?

H_{AB} $H_{truncated}=H_A H_B$

We compare the cases with a truncated and free Hamiltonians

Negativity

Negativity

$$\text{Emission} \rightarrow \|E_A\| \|E_B\| < |\langle 0 | X_{AB} \rangle| \leftarrow \text{Exchange}$$

Negativity

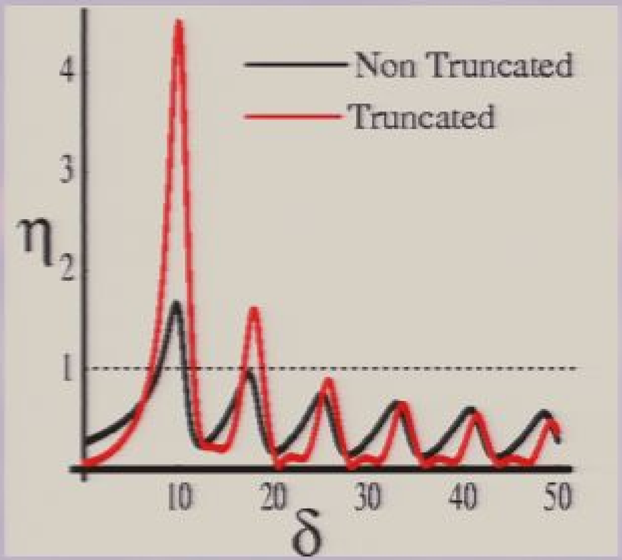
Emission → $\|E_A\| \|E_B\| < |\langle 0 | X_{AB} \rangle|$ ← Exchange

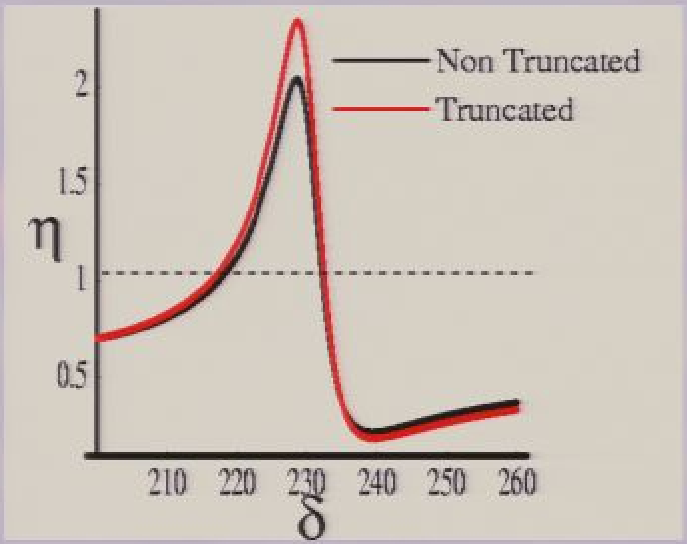
$$\eta = \frac{Exchange}{Emission}$$

Long Ion Chain

L=6,15, N=20

L=10,11 N=20





η=exchange/emission >1 , signifies entanglement.

δ denotes the detuning, L the locations of A and B.

Summary

Atom Probes:

Vacuum Entanglement can be "swapped" to detectors.

Bell's inequalities are violated.

Ent. reduces exponentially with the separation.

High probe frequencies are needed for large separation.

Linear ion trap:

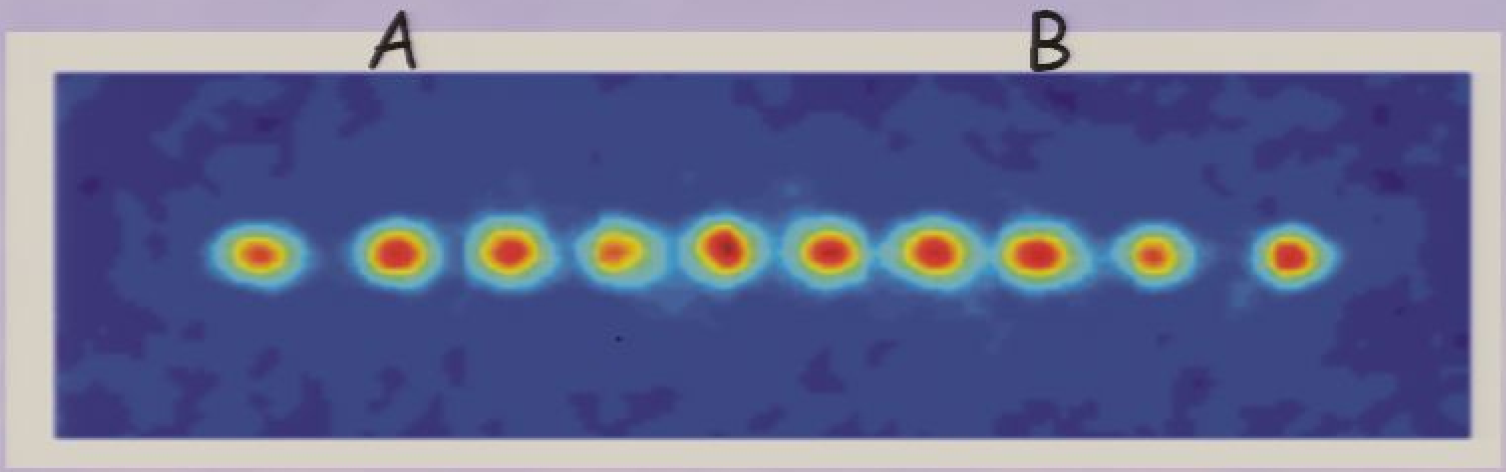
-A proof of principle of the general idea is experimentally feasible.

-One can entangle internal levels of two ions without performing gate operations.

Reznik, Retzker, Silman PRA 71, 042104

A. Retzker, J. I. Cirac, B. Reznik PRL 94, 050504

Long Ion Chain



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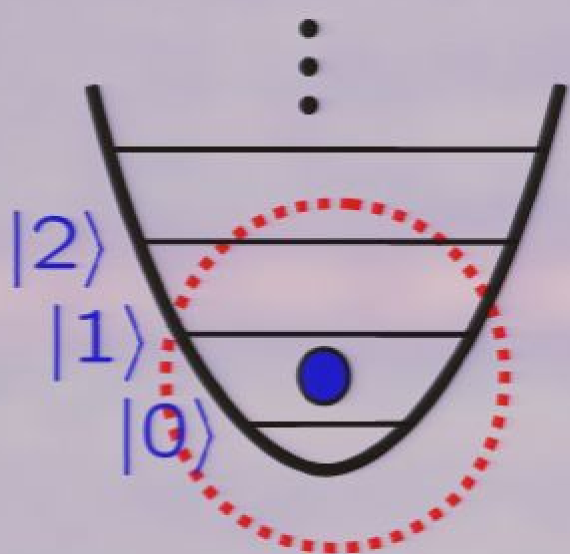
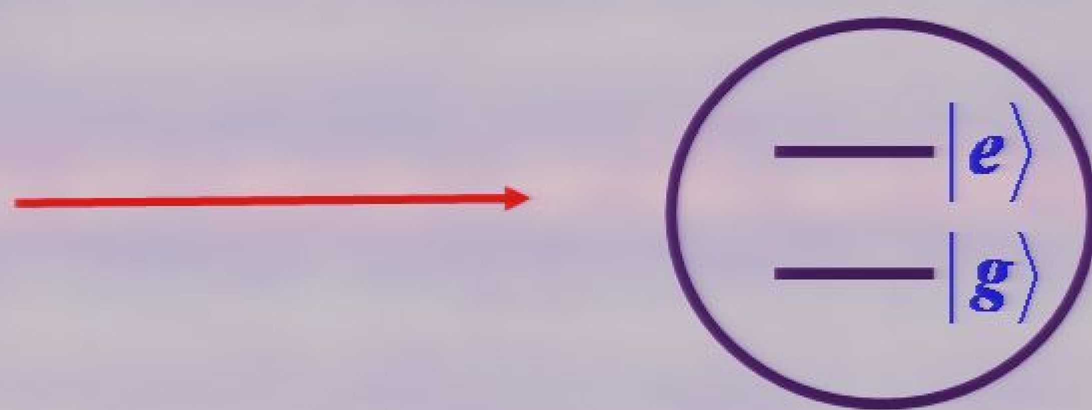
Swap Operation

External degrees of freedom

Internal degrees of freedom





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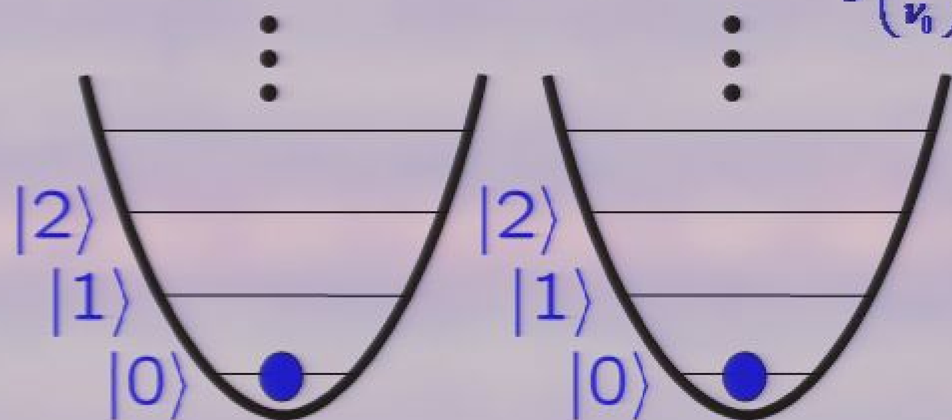

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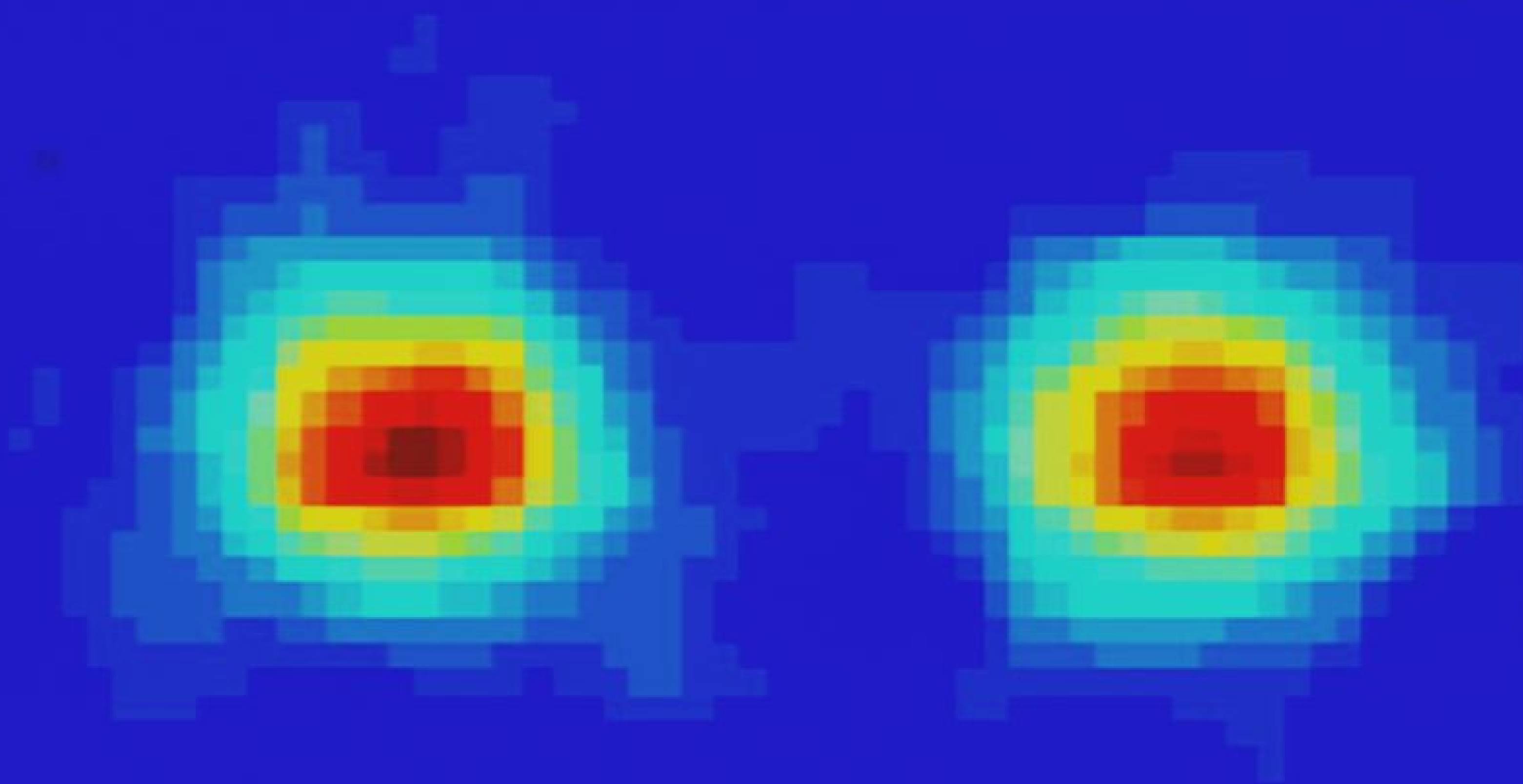
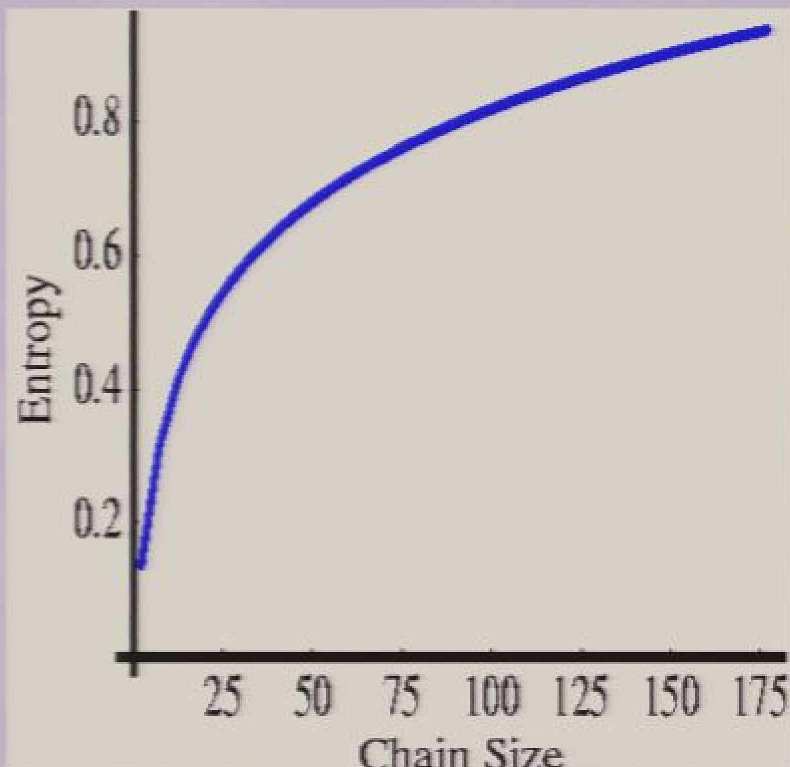
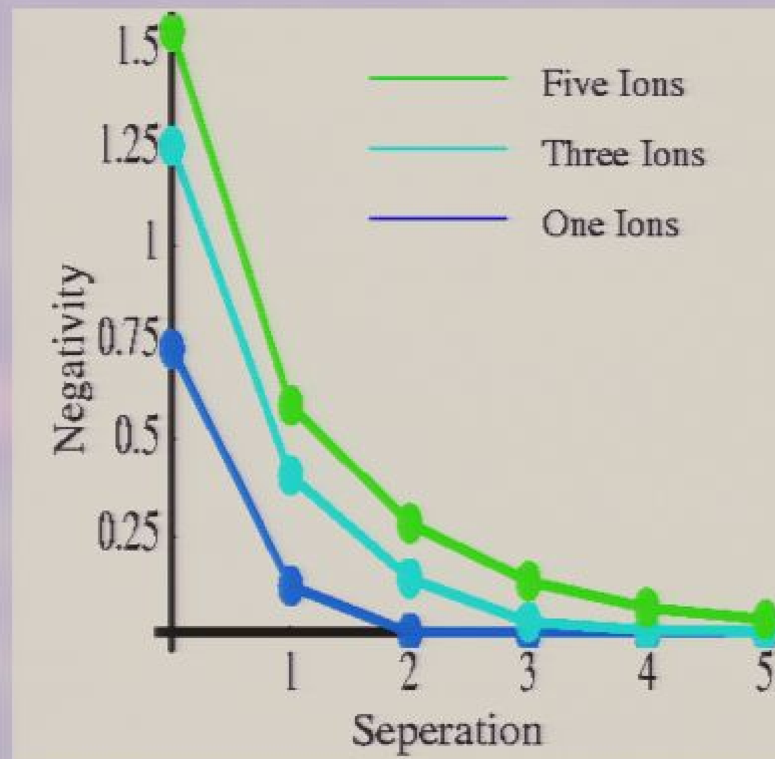


Entanglement in a linear trap



Entanglement between complementary symmetric groups of ions as a function of the total number



Entanglement between finite symmetric groups of ions as a function of the separation.

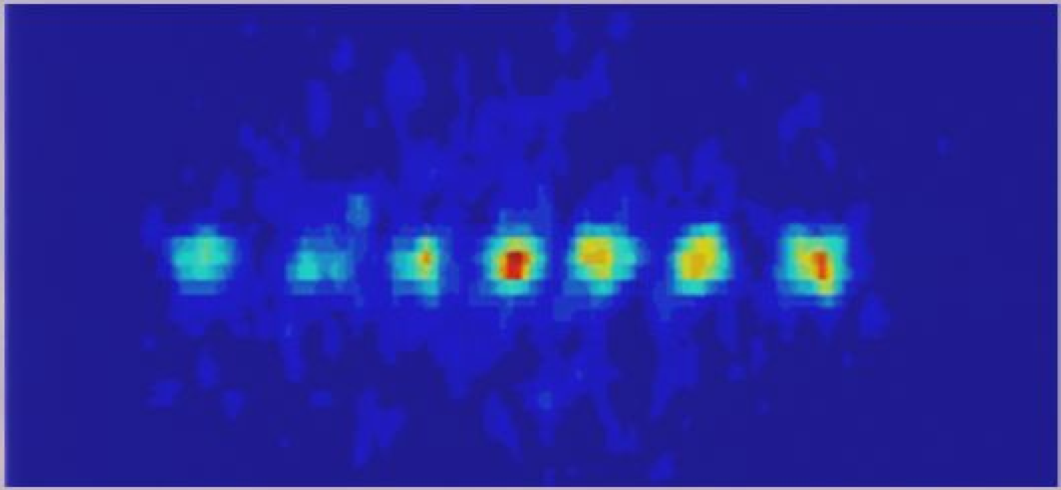
Lamb – Dicke regime

Taylor expansion of the exponentiel up to first order:

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_{+}\left\{1 + i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^{+}\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

Center-of-mass and breathing mode excitation

„center-of-mass mode“

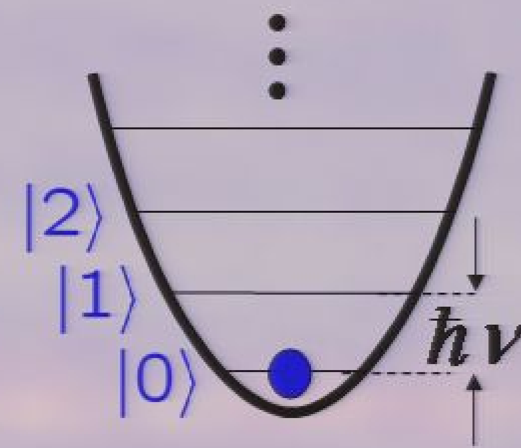


Courtesy of R. Blatt

Orders of Magnitude

harmonic trap

Physical size of the ground state:



$$\left.\begin{array}{l} \nu = (2\pi)\,1\text{MHz} \\ m = 40u \end{array}\right\} \quad \langle x^2 \rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11nm$$

Size of the wave packet << wavelength of visible light

Energy scale of interest:

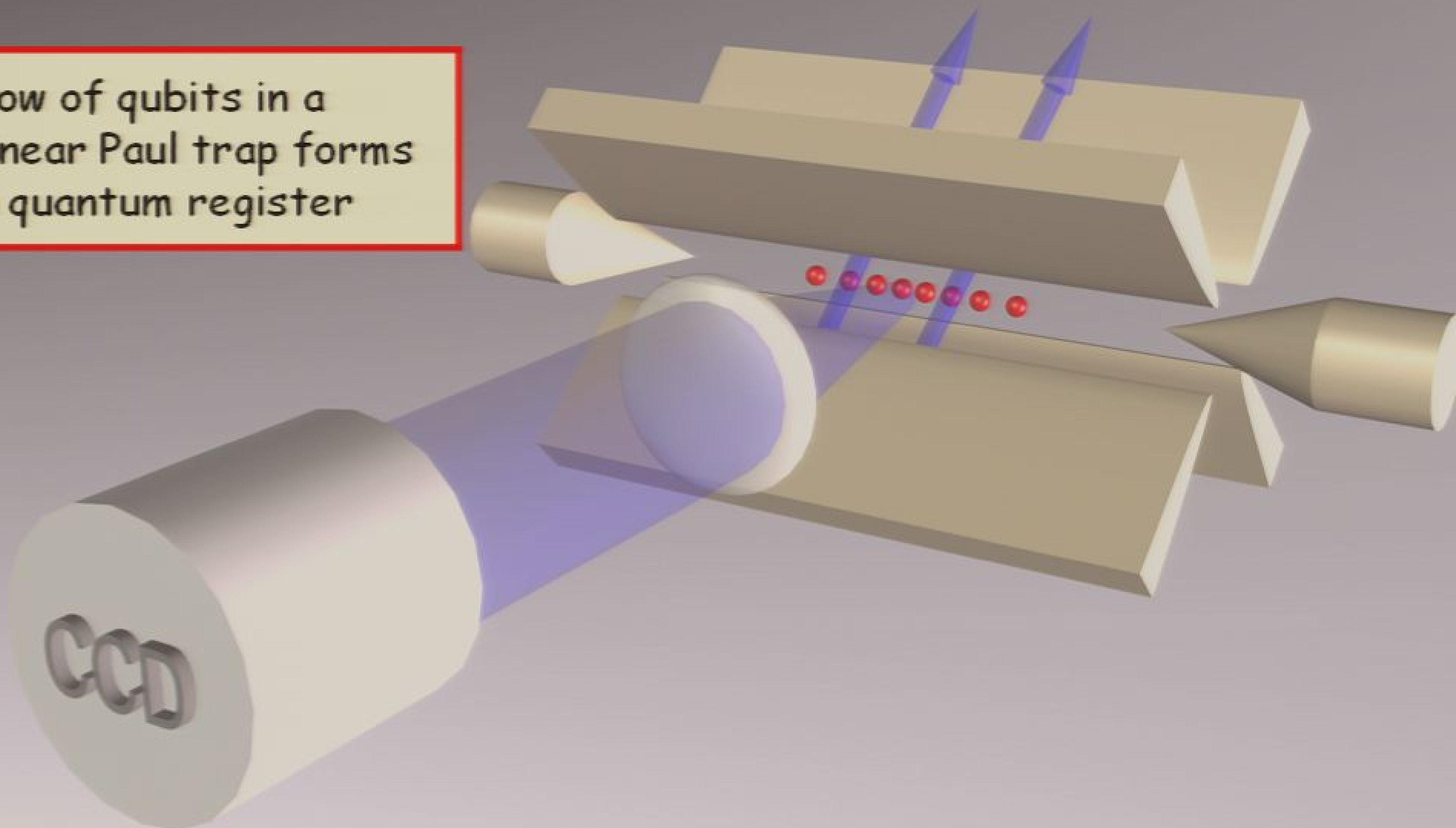
$$\hbar\nu = k_B T \longrightarrow T = \frac{\hbar\nu}{k_B} \approx 50\mu K$$

Seperation between ions:

$$d \approx 5\mu m$$

Ion Trap Quantum Processor

row of qubits in a linear Paul trap forms a quantum register



Negativity

Emission → $\|E_A\|\|E_B\| < |\langle 0|X_{AB}\rangle|$ ← Exchange

$$\int_0^\infty \omega d\omega[\tilde{\varepsilon}(\Omega+\omega)]^2 < \int_0^\infty \frac{d\omega}{L} Sin(\omega L)\widetilde{\varepsilon_A}(\Omega+\omega)\widetilde{\varepsilon_B}(\Omega-\omega)$$

Off resonance

On resonance

Negativity

$$\text{Emission} \rightarrow \left\| E_A \right\| \left\| E_B \right\| < \left| \langle 0 | X_{AB} \rangle \right| \leftarrow \text{Exchange}$$

$$\int_0^\infty \omega d\omega [\tilde{\varepsilon}(\Omega+\omega)]^2 < \int_0^\infty \frac{d\omega}{L} \mathit{Sin}(\omega L) \widetilde{\varepsilon_A}(\Omega+\omega) \widetilde{\varepsilon_B}(\Omega-\omega)$$

Off resonance

On resonance

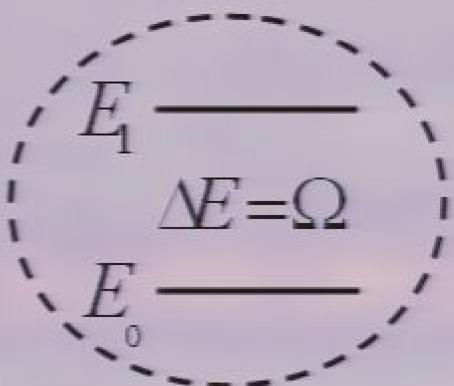
$$U_{Interaction} = U_A \otimes U_B = (1 - i\varepsilon \int dt H_A - \frac{1}{2}\varepsilon^2 T \iint dt dt' H_A H_A)(\ldots)$$

$$|\Psi(T)\rangle = U_{Interaction} |\downarrow_A\rangle |\downarrow_B\rangle |0\rangle$$

$$\downarrow\downarrow \qquad \uparrow\uparrow \qquad \downarrow\uparrow \qquad \uparrow\downarrow$$

$$\rho_{AB}(T) = \begin{bmatrix} 1 & \langle 0|X_{AB}\rangle & & \\ \langle X_{AB}|0\rangle & \|X_{AB}\|^2 & & \\ & & \|E_A\|^2 & \langle E_A|E_B\rangle \\ & & \langle E_B|E_A\rangle & \|E_B\|^2 \end{bmatrix} + O(\varepsilon^5)$$

Field – Detectors Interaction



Two-level system

Interaction:

$$H_{\text{int}} = H_A + H_B$$

$$H_A = \varepsilon_A(t)\left(e^{+i\Omega t}\sigma_A^+ + e^{-i\Omega t}\sigma_A^-\right)\phi\left(x_A, t\right)$$

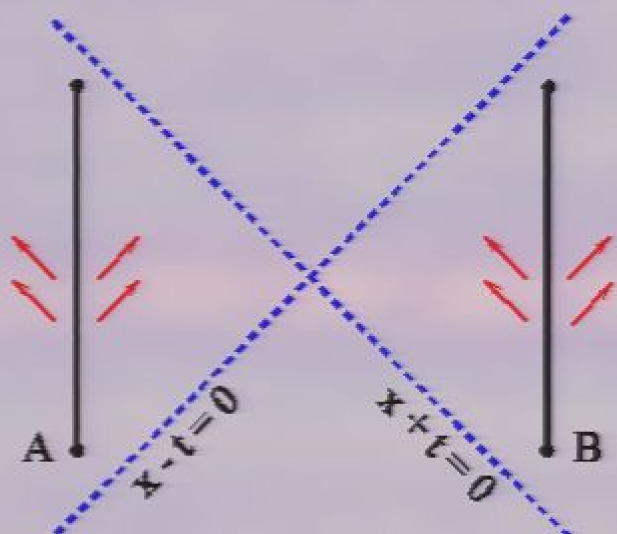
Window Function

Initial state:

$$\left|\Psi(0)\right\rangle = \left|\downarrow_A\right\rangle\left|\downarrow_B\right\rangle\left|0\right\rangle$$

We do not use the rotating wave approximation

Causal Structure + LO



For L>cT, we have $[\phi_A, \phi_B]=0$

Therefore $U_{INT}=U_A\ U_B$ →**LO**

$\Delta E_{Total}=0$, but

$\Delta E_{AB}>0$. (Ent. Swapping)

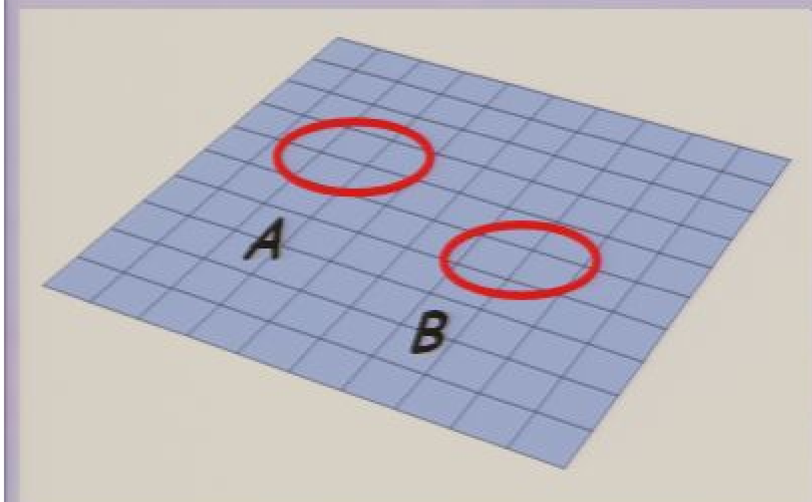
Vacuum ent ! Detectors' ent.

Lower bound.

Are A and B entangled?

Yes, for arbitrary separation.

("Atom probes").



Are Bell's inequalities violated?

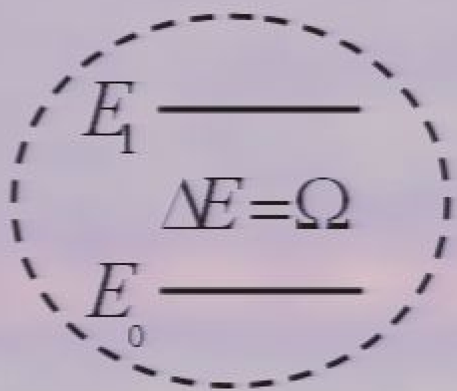
Yes, for arbitrary separation.

Can we detect it?

Entanglement Swapping.

(Linear Ion trap)

Field – Detectors Interaction



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We do not use the rotating wave approximation

Probe Entanglement

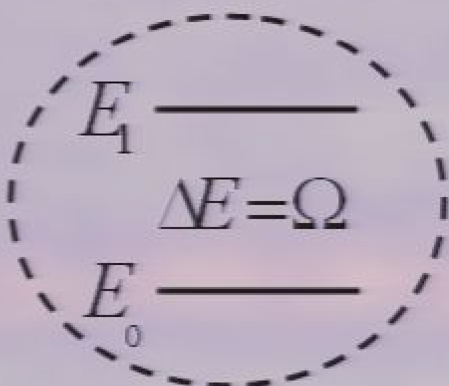
$$\rho_{AB}^{(4\times4)} = Tr_F \rho^{(total)}$$

$$? \neq \sum_i p_i \rho_A^{2\times2} \rho_B^{2\times2}$$

Calculate to the second order (in ε) the final state, and evaluate the reduced density matrix.

Finally, we use Peres (96) partial transposition criterion to check inseparability and use the Logarithmic Negativity as a measure.

Field – Detectors Interaction



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P.T.

P.T.

$$|X_{AB}\rangle = \Phi^+{}_A \Phi^+{}_B |0\rangle = |0 \text{ or } 2 \text{ photons}\rangle$$

$$|E_A\rangle = \Phi^+{}_A |0\rangle = |1 \text{ photon}\rangle$$

$$\Phi^\pm{}_i = \int \varepsilon_i(t) e^{\pm i\Omega t} \phi(x_i, t)$$

Vacuum Entanglement in an Ion Trap

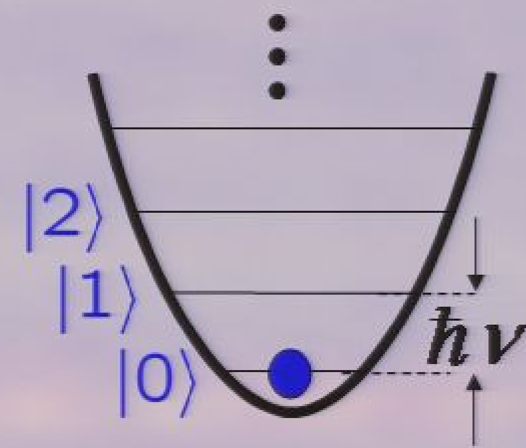
1 Vacuum Entanglement

2 Introduction to Ion trap Quantum computing

3 Ground state Entanglement of an Ion Trap

Orders of Magnitude

harmonic trap



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Size of the wave packet << wavelength of visible light

Energy scale of interest:

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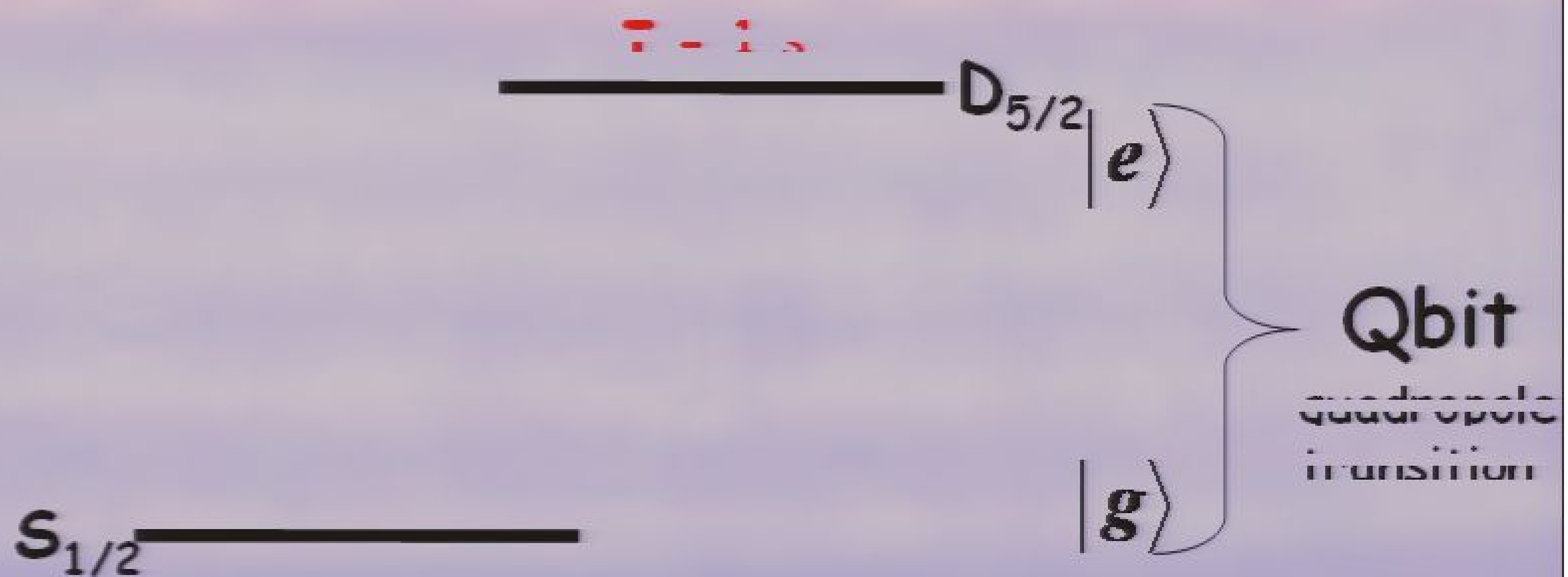
$^{40}Ca^{+}$: Important energy levels



Laser – Ion Interactions

Hamiltonian: $H = H^{(Internal)} + H^{(External)} + H^{(Interaction)}$

$$H^{(Internal)} = \hbar \frac{\omega_0}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$H^{(External)} = \sum_i \hbar \nu_i a_i^\dagger a_i$$

mode frequencies

Laser – Ion Interactions

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mode frequencies

$$H^{(Interaction)} = \hbar\Omega\left(|g\rangle\langle e| + |e\rangle\langle g|\right)\cos(k\hat{x} - \omega t + \phi)$$

Rabi frequency

Laser frequency

Laser – Ion Interactions

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mode frequencies

Laser – Ion Interactions

$$H_{\text{int}} = \frac{\hbar\Omega}{2}\sigma_{+}\exp\left\{i\eta\left(e^{-i\nu t}a + e^{i\nu t}a^{\dagger}\right)\right\}e^{-i\delta t + i\phi} + h.c.$$

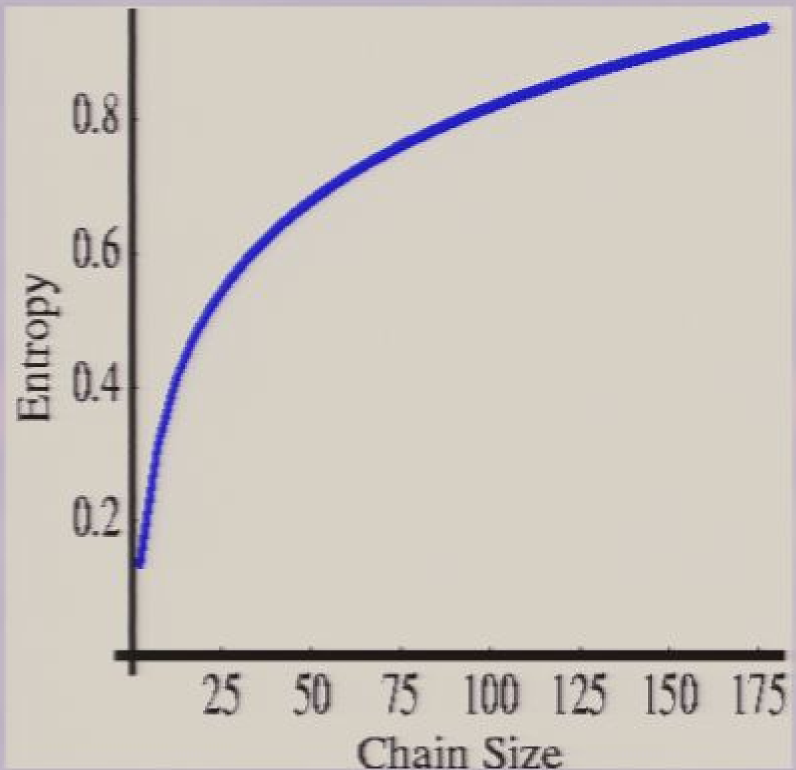
Lamb-Dicke parameter

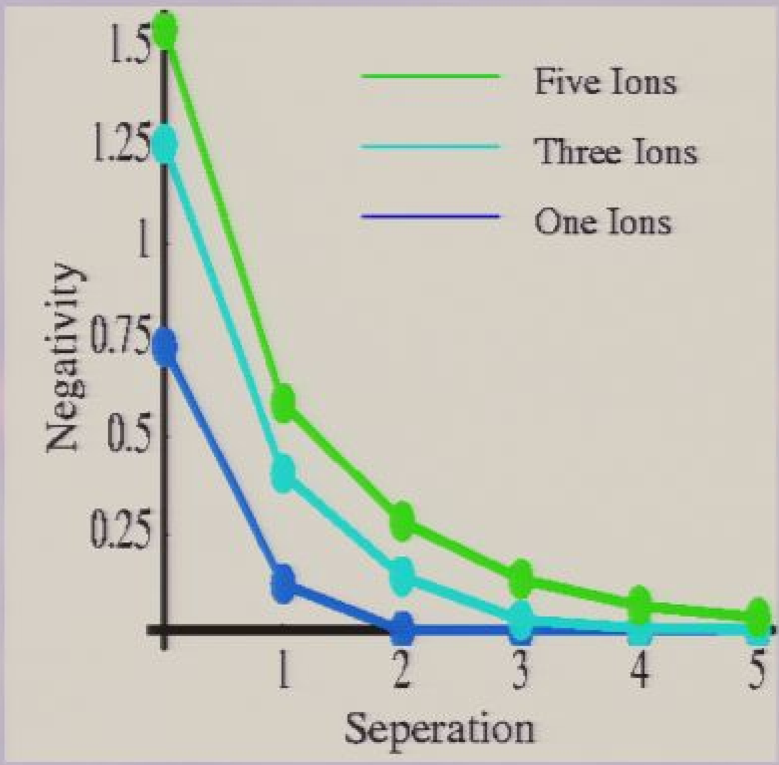
$$\eta = k x_0 = k\sqrt{\frac{\hbar}{2m\nu}}$$

relates size of ground state to wave length of light

In ion trap experiments, usually $\eta \ll 1$

Entanglement in a linear trap





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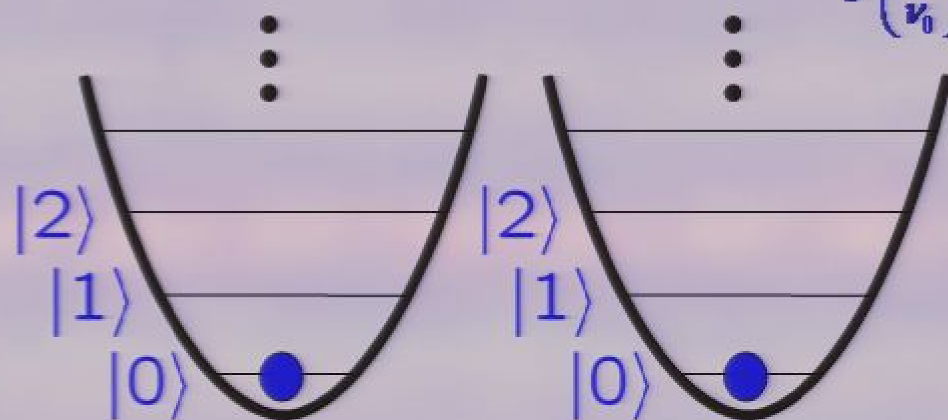

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