Title: Introduction to quantum gravity - Part 11

Date: Feb 28, 2006 06:30 PM

URL: http://pirsa.org/06020038

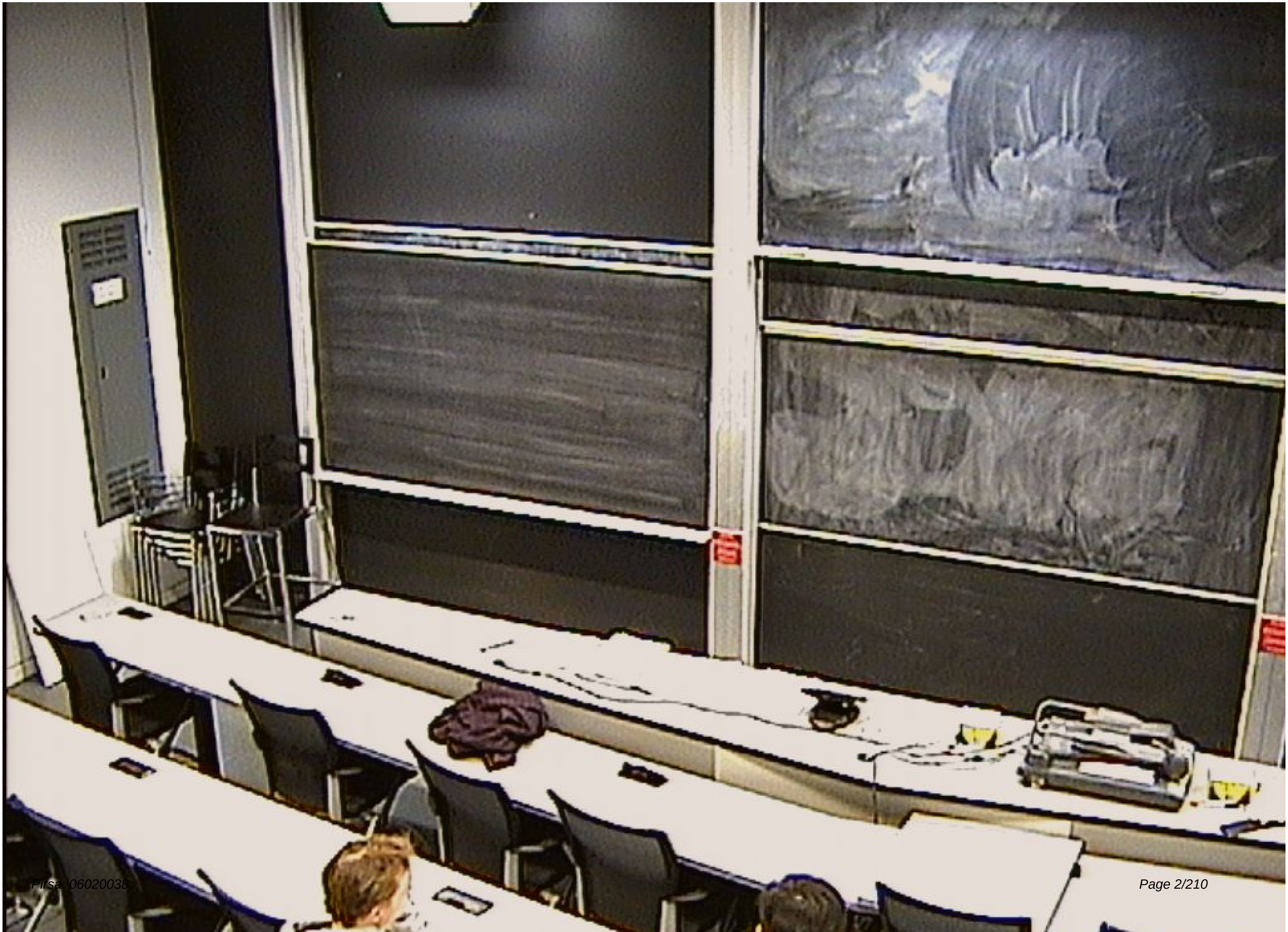
Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

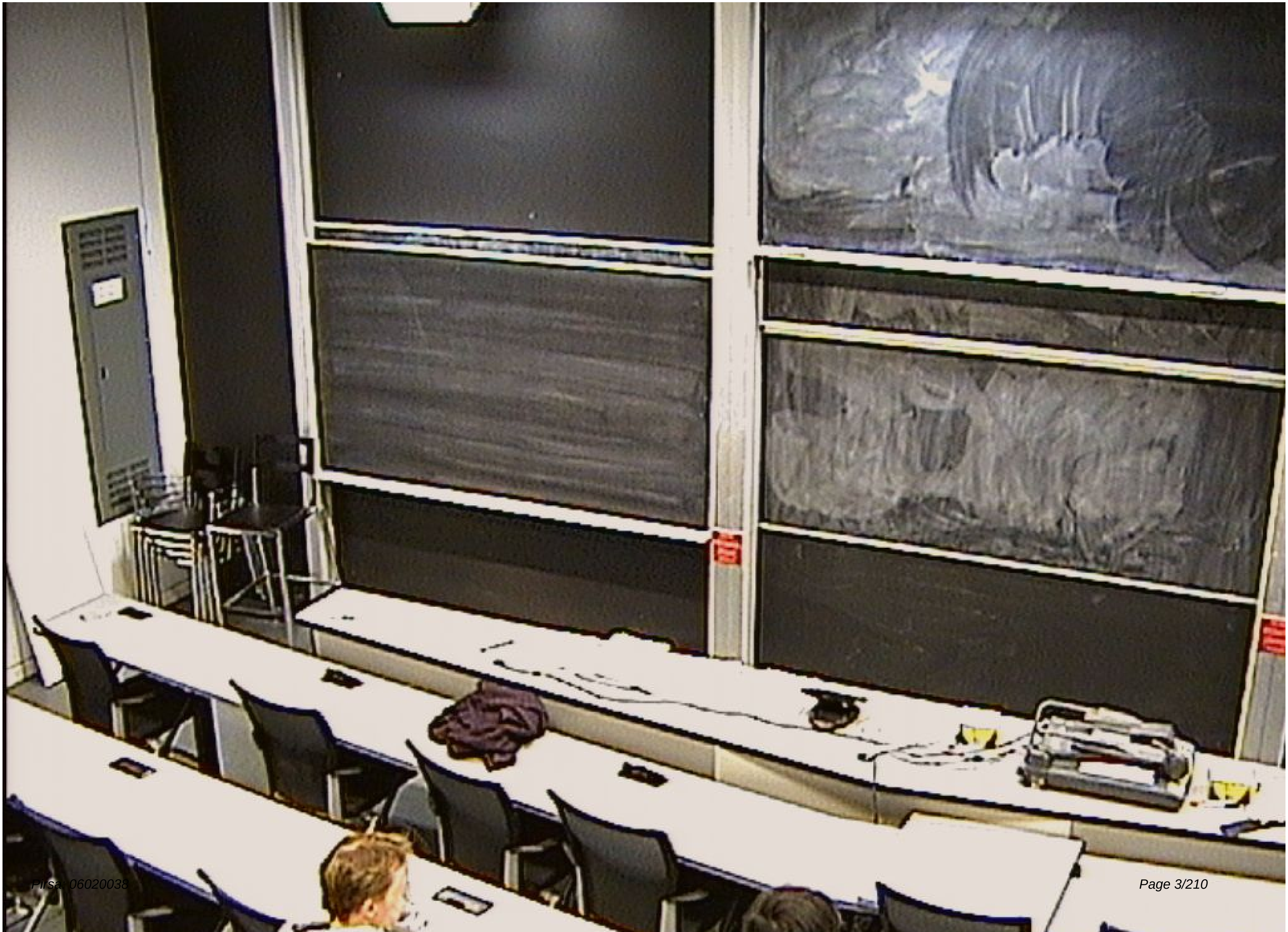
Basic texts:

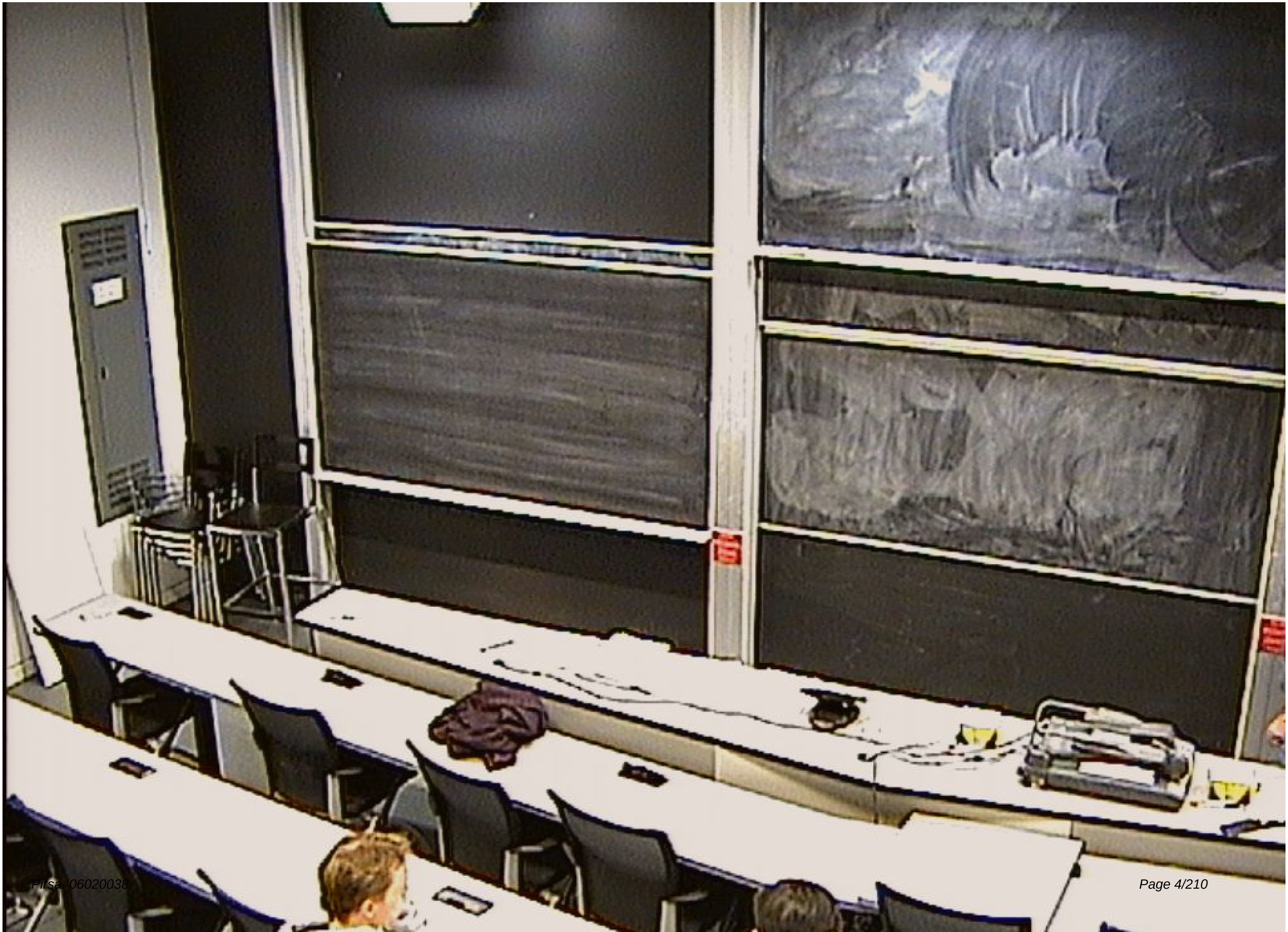
-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravityy with a positive cosmological constant, Lee Smolin, hep-th/0209079
-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

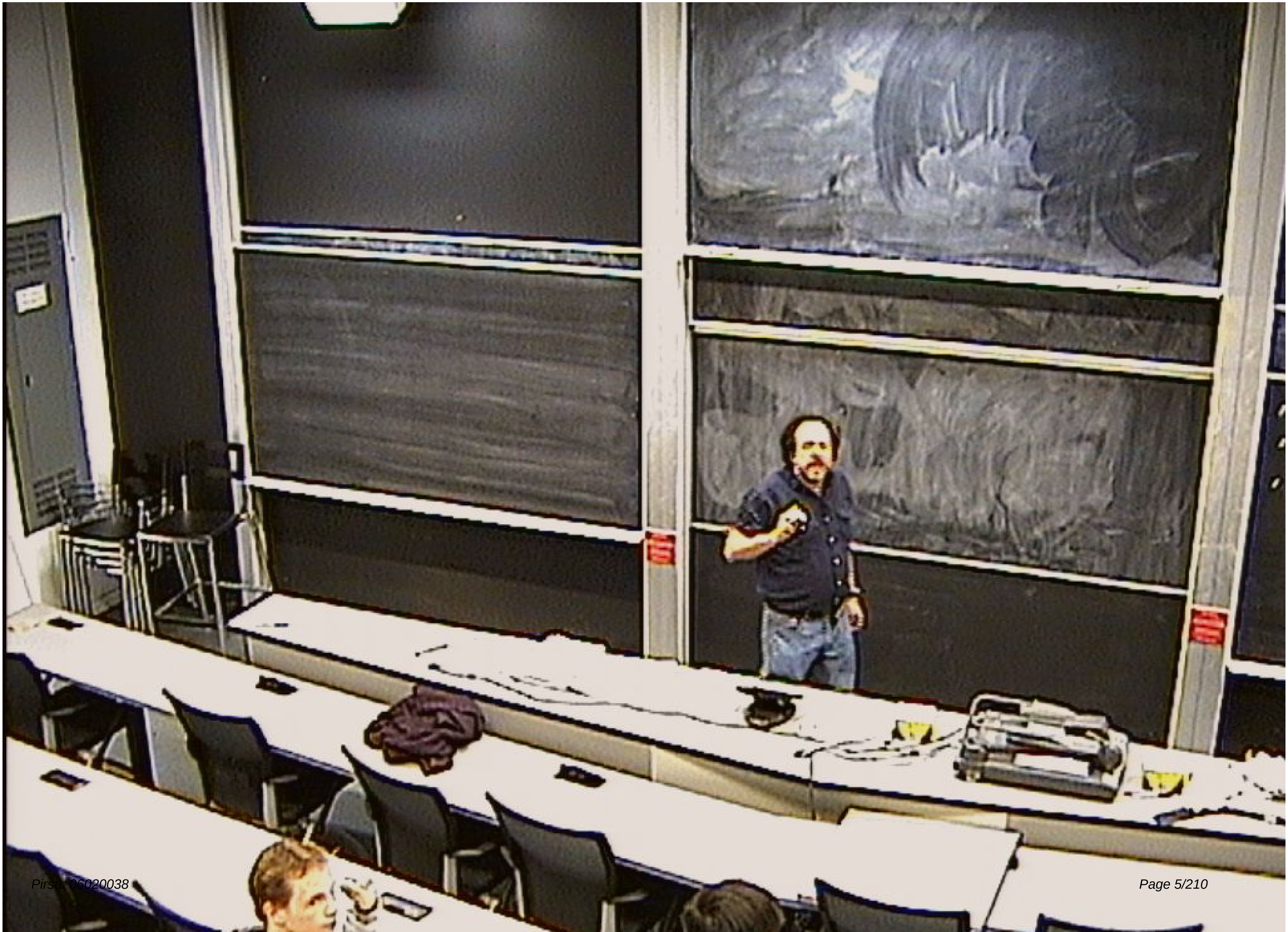
Prerequisites:

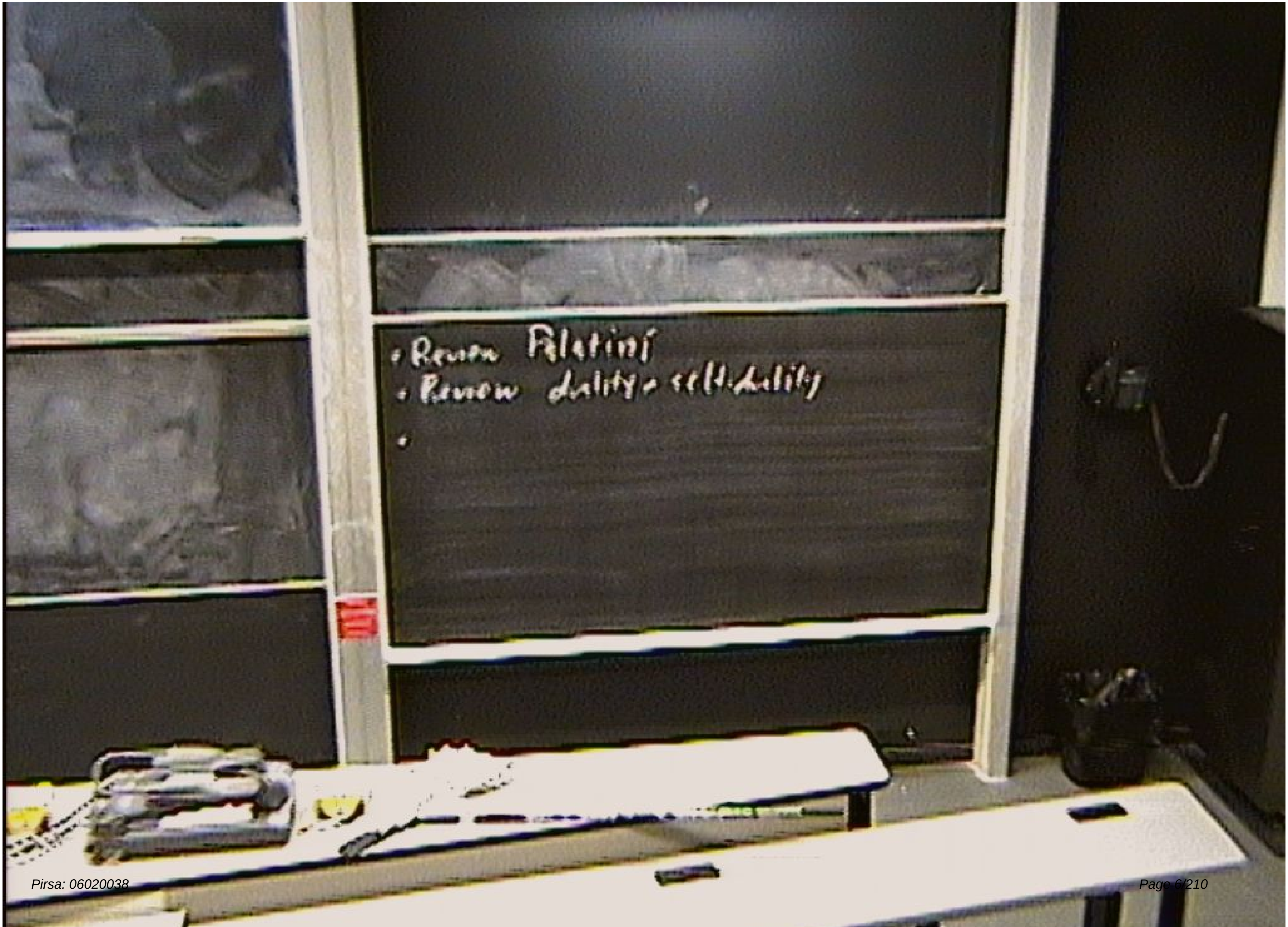
-undergraduate quantum mechanics
-basics of classical gauge field theories
-basic general relativity
-hamiltonian and lagrangian mechanics
-basics of lie algebras










• Review Palatini
• Review duality + self-duality
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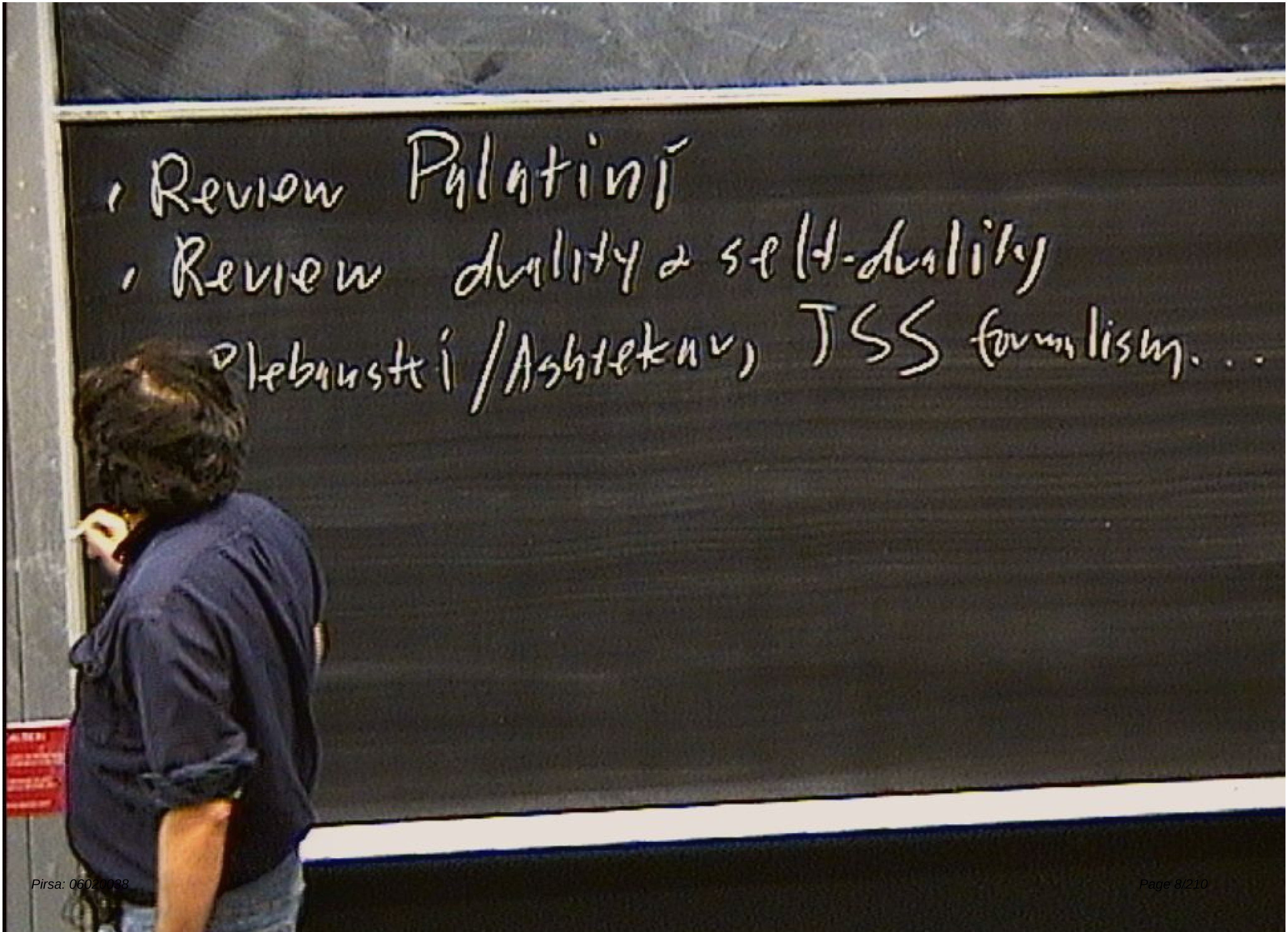
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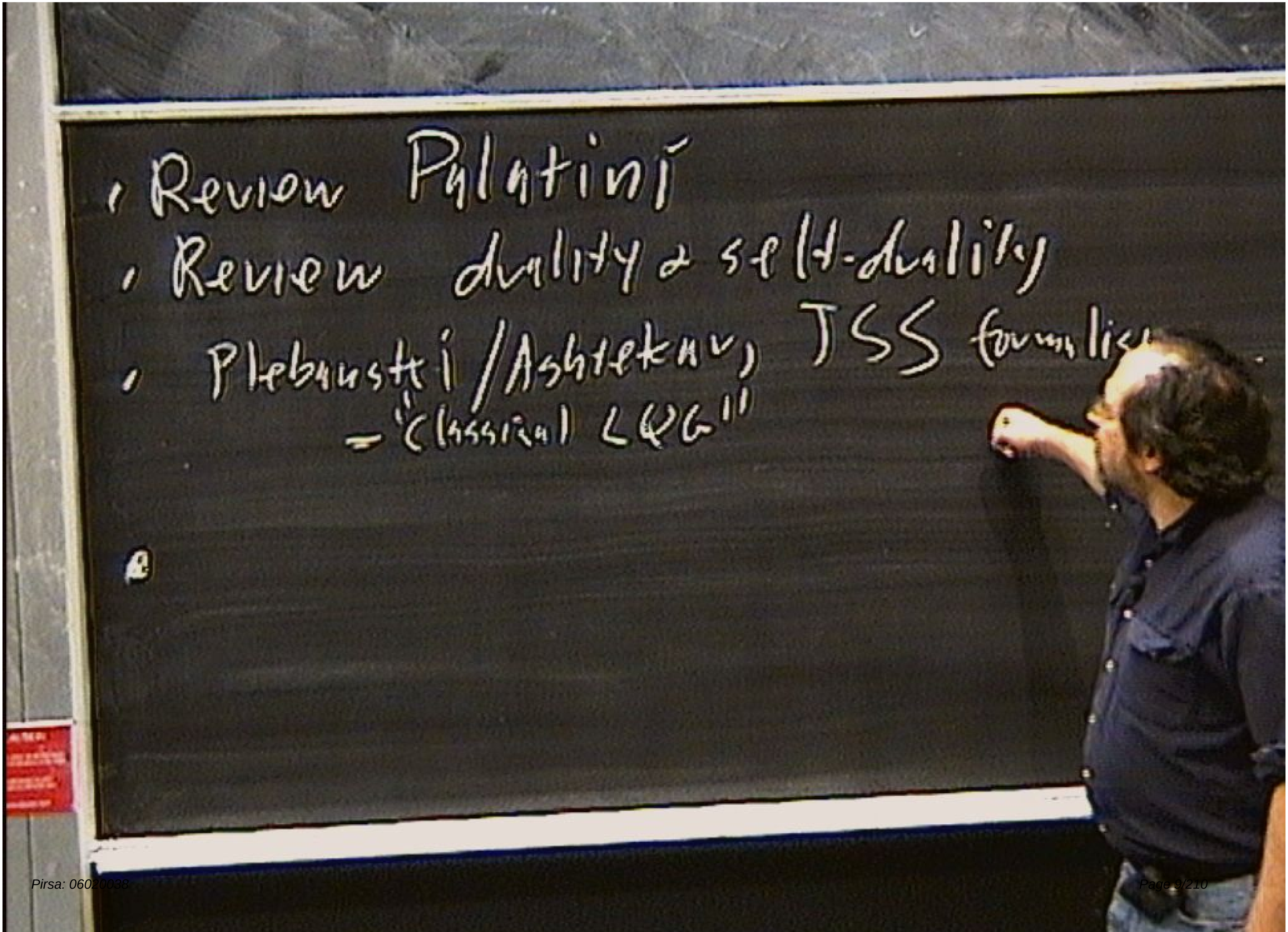
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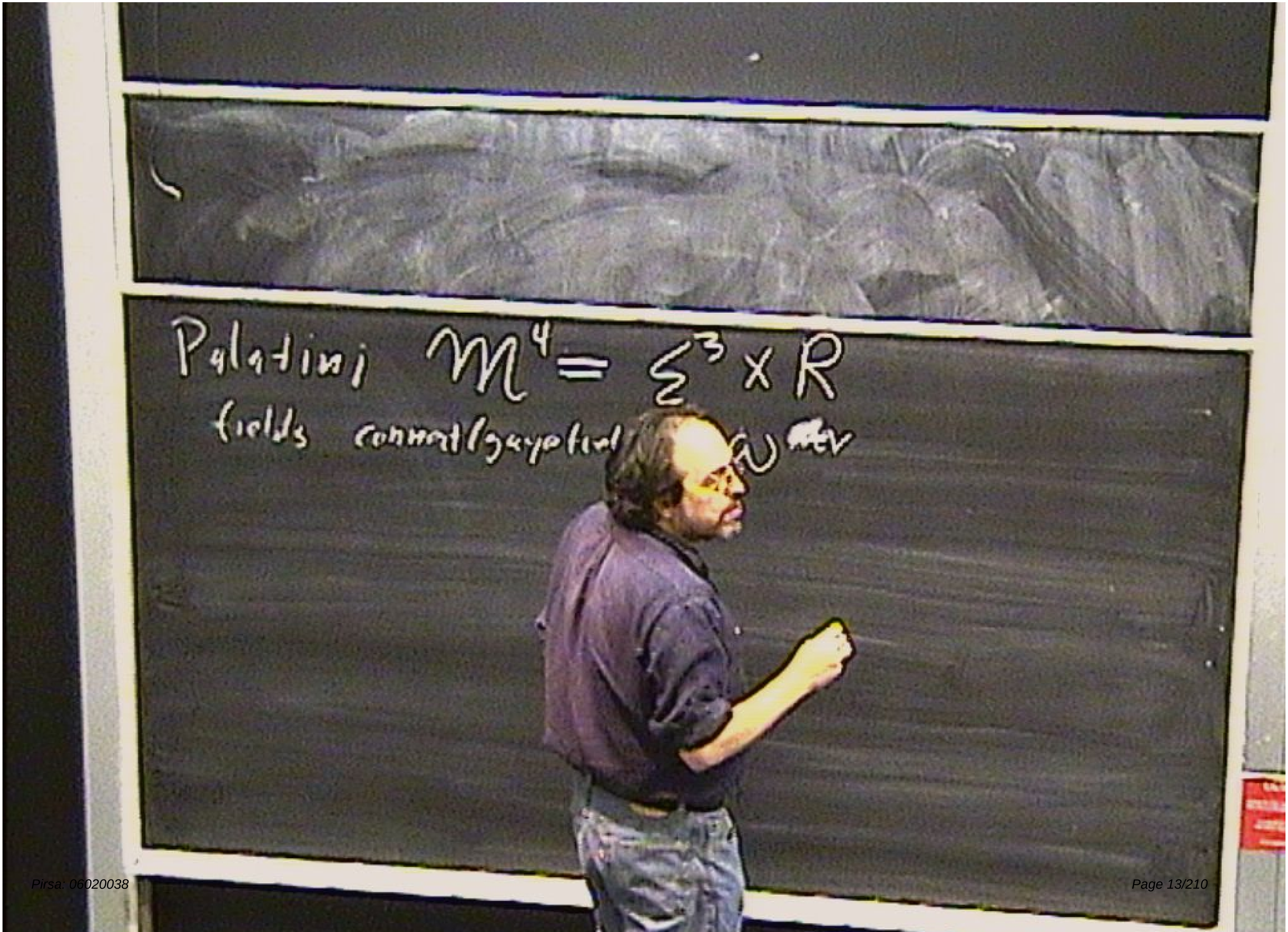
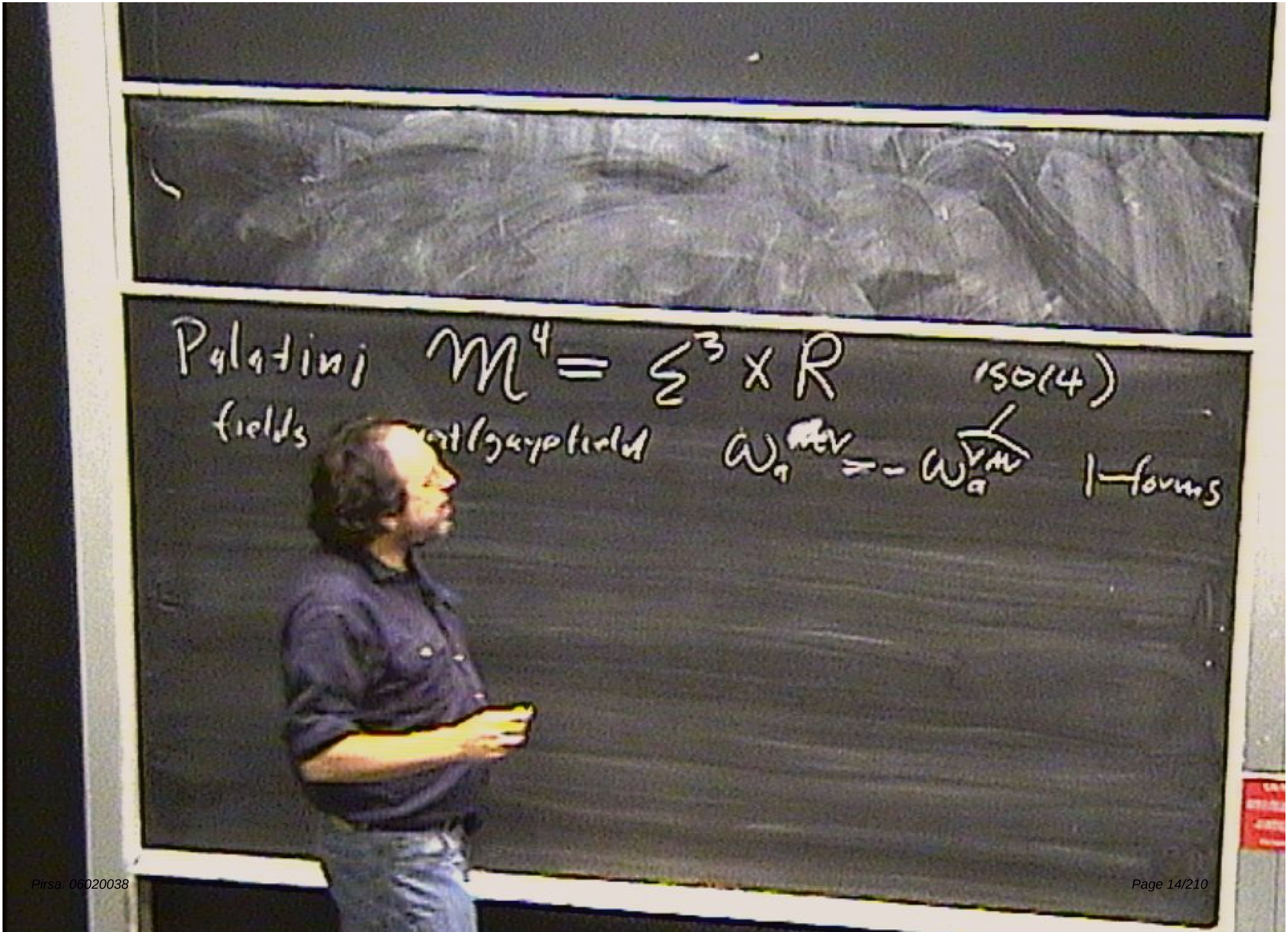
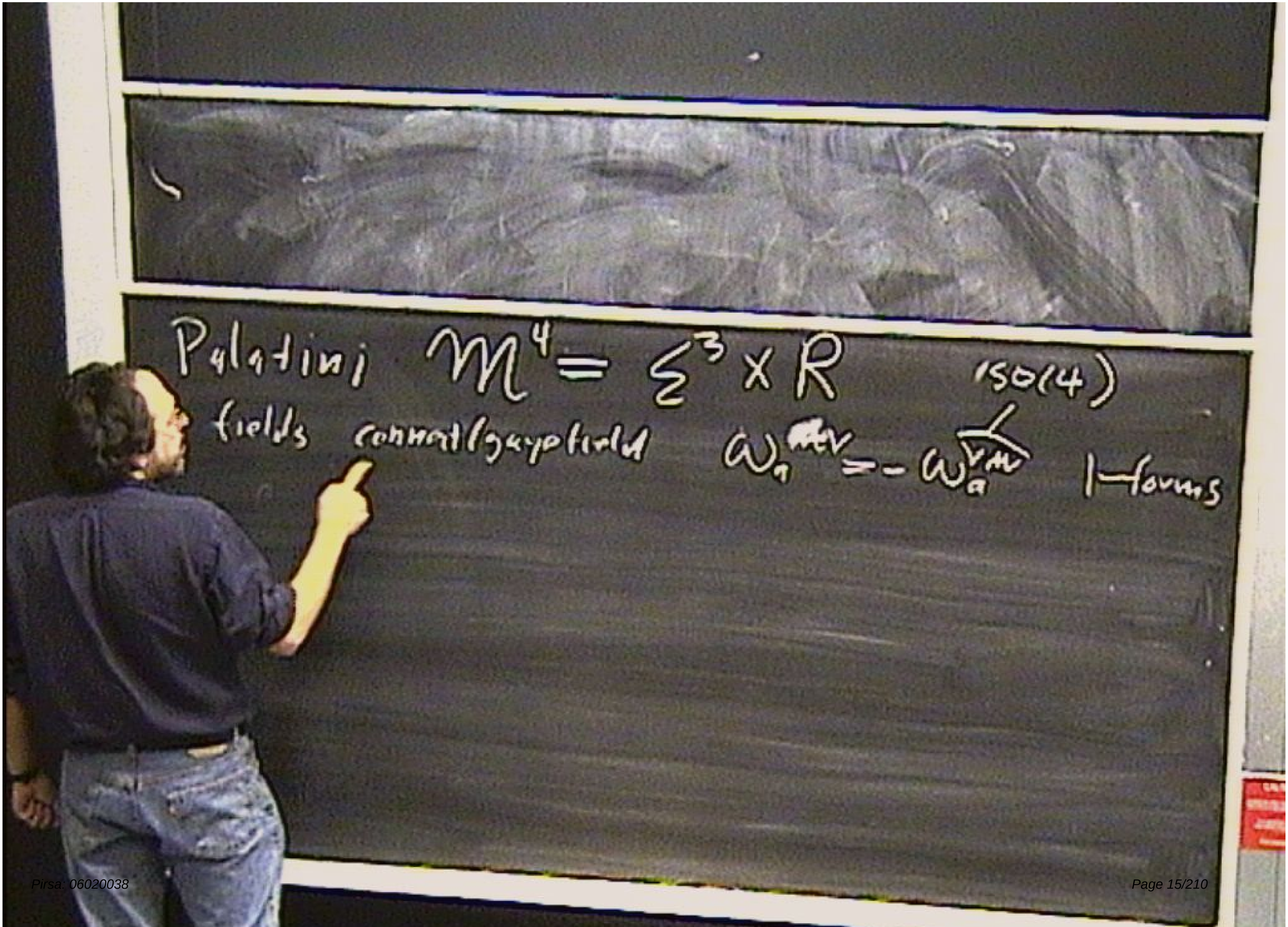

Palatini $\mathcal{M}^4 = \Sigma^3 \times R$
fields


Palatini
M⁴ = Σ³ × R
SO(4)
fields
ω_a^μν = - ω_a^νμ
1-forms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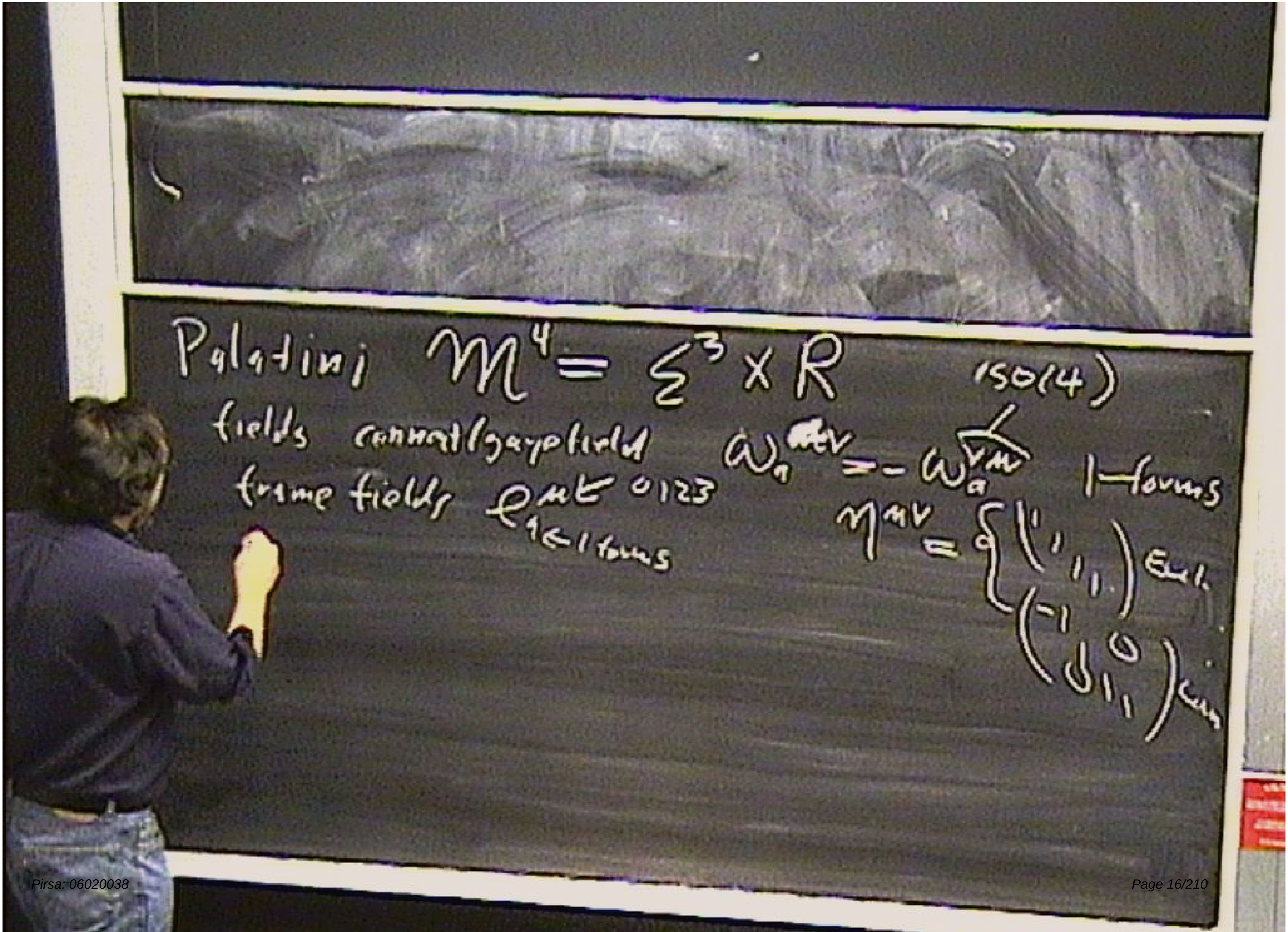
Palatini $M^4 = \Sigma^3 \times R$
$SO(4)$
fields connect/gauge field $\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ 1-forms
frame fields e_a^{μ} 0123
1-forms
$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Eucl.} \\ \begin{pmatrix} -1 & & & \\ & 1 & & 0 \\ & 0 & 1 & \\ & & & 1 \end{pmatrix} \text{Lorentz} \end{cases}$

Palatini $\mathcal{M}^4 = \Sigma^3 \times R$ $SO(4)$

fields connect/gauge field $\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ 1-forms

frame fields e_a^{μ} $\mu \in 0123$, $a \leftarrow$ 1 forms

$g_{ab} = e_a^{\mu} e_b^{\nu} \eta_{\mu\nu}$

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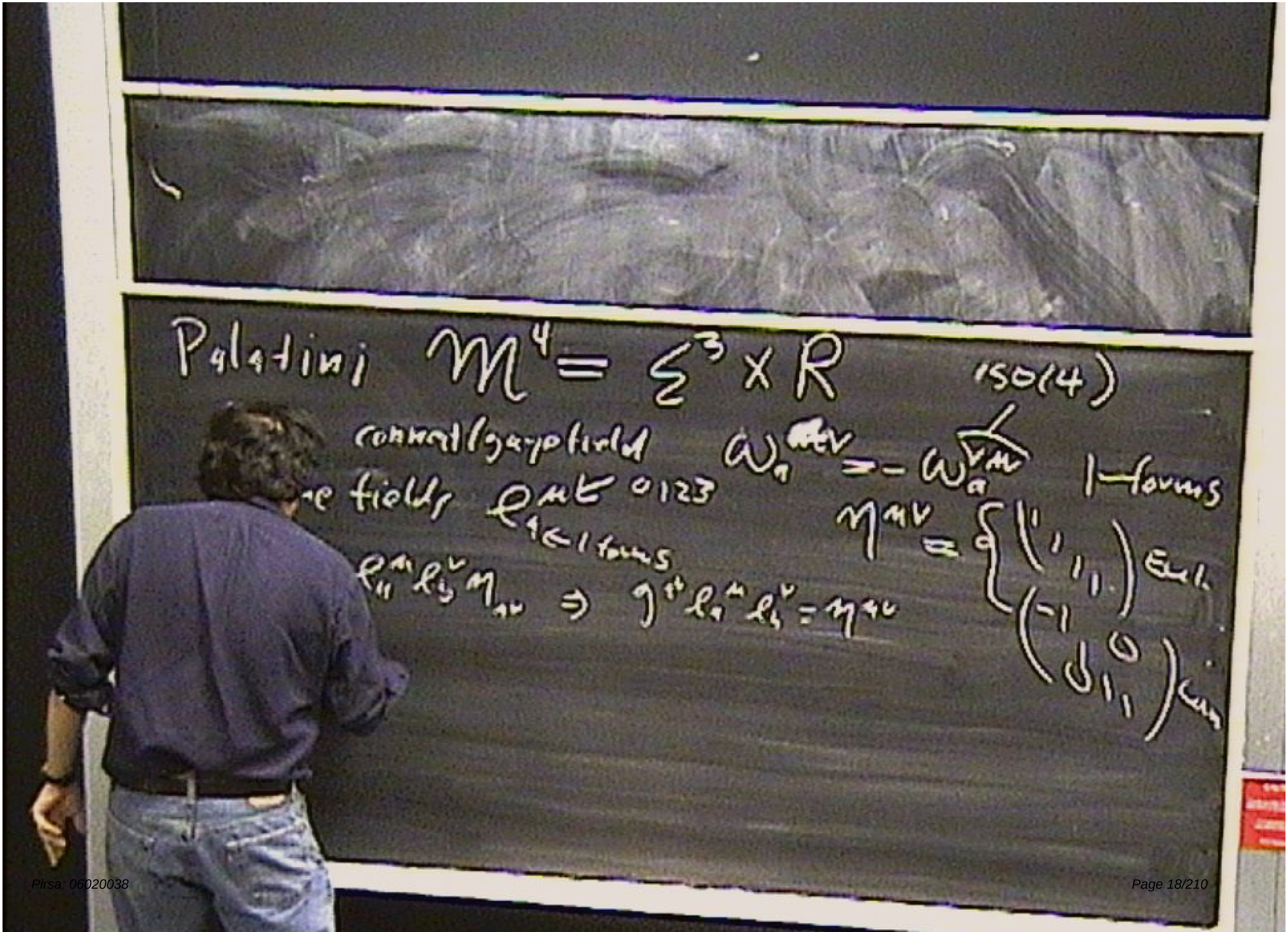

Palatini $M^4 = \Sigma^3 \times R$
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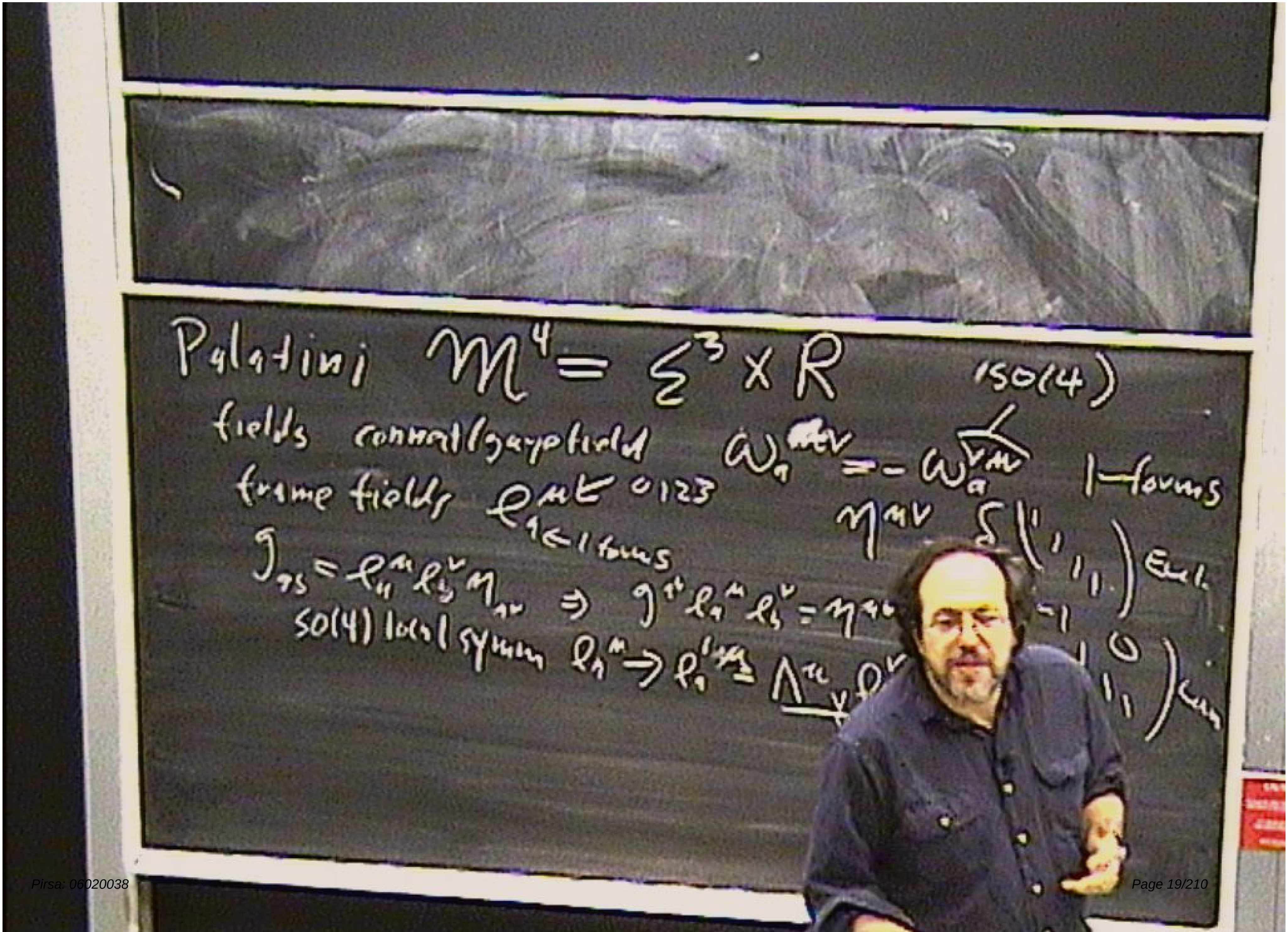
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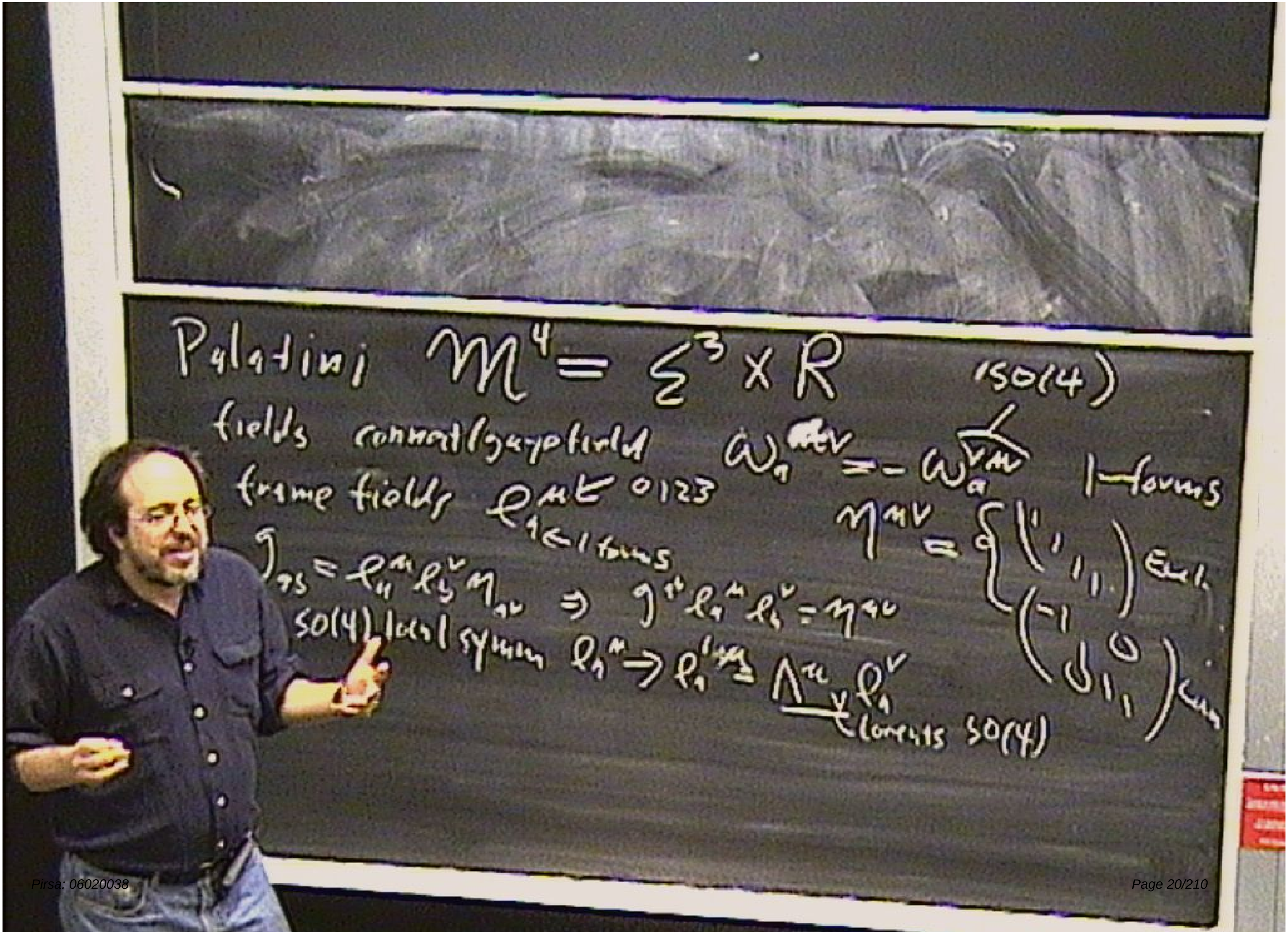
a ← 1 forms
η^μν
Eucl.
g_ab = e_a^μ e_b^ν η_μν ⇒ g^ab e_a^μ e_b^ν = η^μν
SO(4) local symm e_a^μ → e'_a^μ = Λ^μ_ν e^ν



Palatini M4 = Σ3 x R
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Eucl.
SO(4) local symm
Lorentz SO(4)

Euclidean 0209079

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hep-th/0501191

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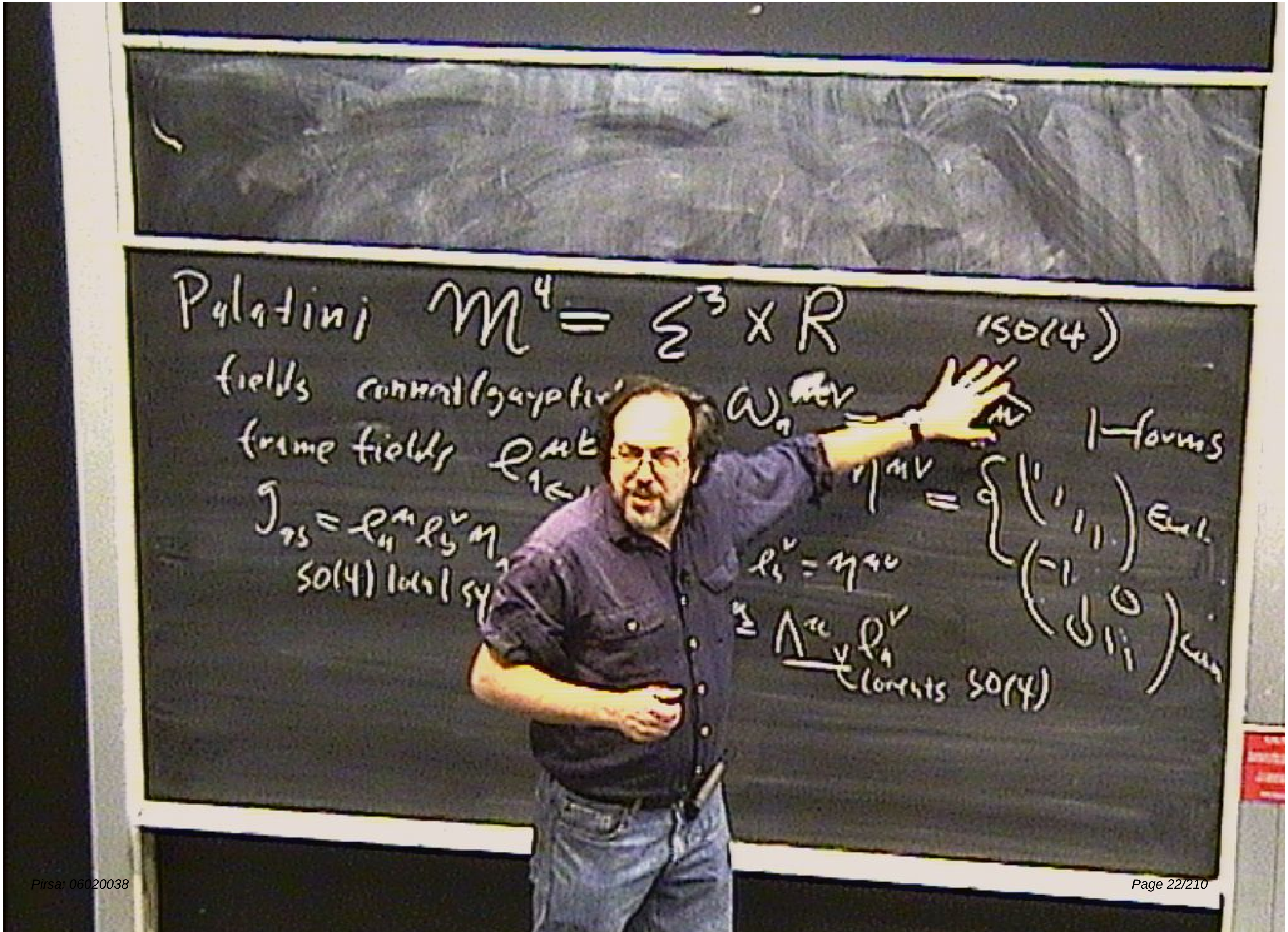
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frame fields $e_a^{\mu} \leftarrow 0123$, $\leftarrow$ 1 forms

$$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} & \text{Eucl.} \\ \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} \end{pmatrix} & \text{Lor.} \end{cases}$$

$g_{ab} = e_a^\mu e_b^\nu \eta_{\mu\nu} \Rightarrow g^{ab} e_a^\mu e_b^\nu = \eta^{\mu\nu}$

SO(4) local symm $e_a^\mu \to e_a'^\mu = \Lambda^\mu{}_\nu e_a^\nu$ $\leftarrow$ Lorentz SO(4)

Palatini $\mathcal{M}^4 = \Sigma^3 \times R$ $SO(4)$

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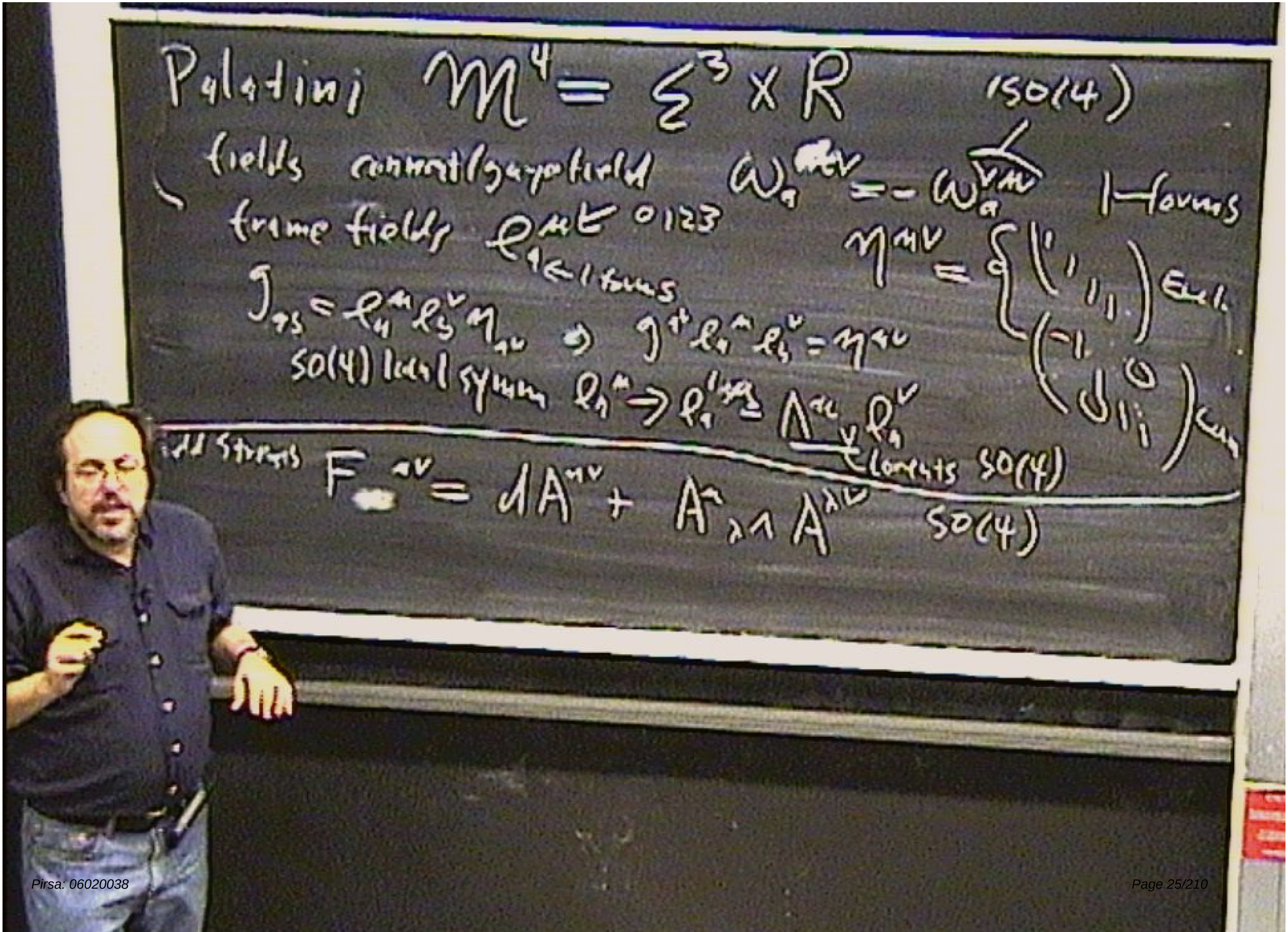
$a \leftarrow$ 1 forms

$g_{ab} = e_a^\mu e_b^\nu \eta_{\mu\nu} \Rightarrow g^{ab} e_a^\mu e_b^\nu = \eta^{\mu\nu}$

SO(4) local symm $e_a^\mu \to e_a'^\mu = \underbrace{\Lambda^\mu{}_\nu}_{\text{Lorentz } SO(4)} e_a^\nu$

$F_{ab}^{\mu\nu} =$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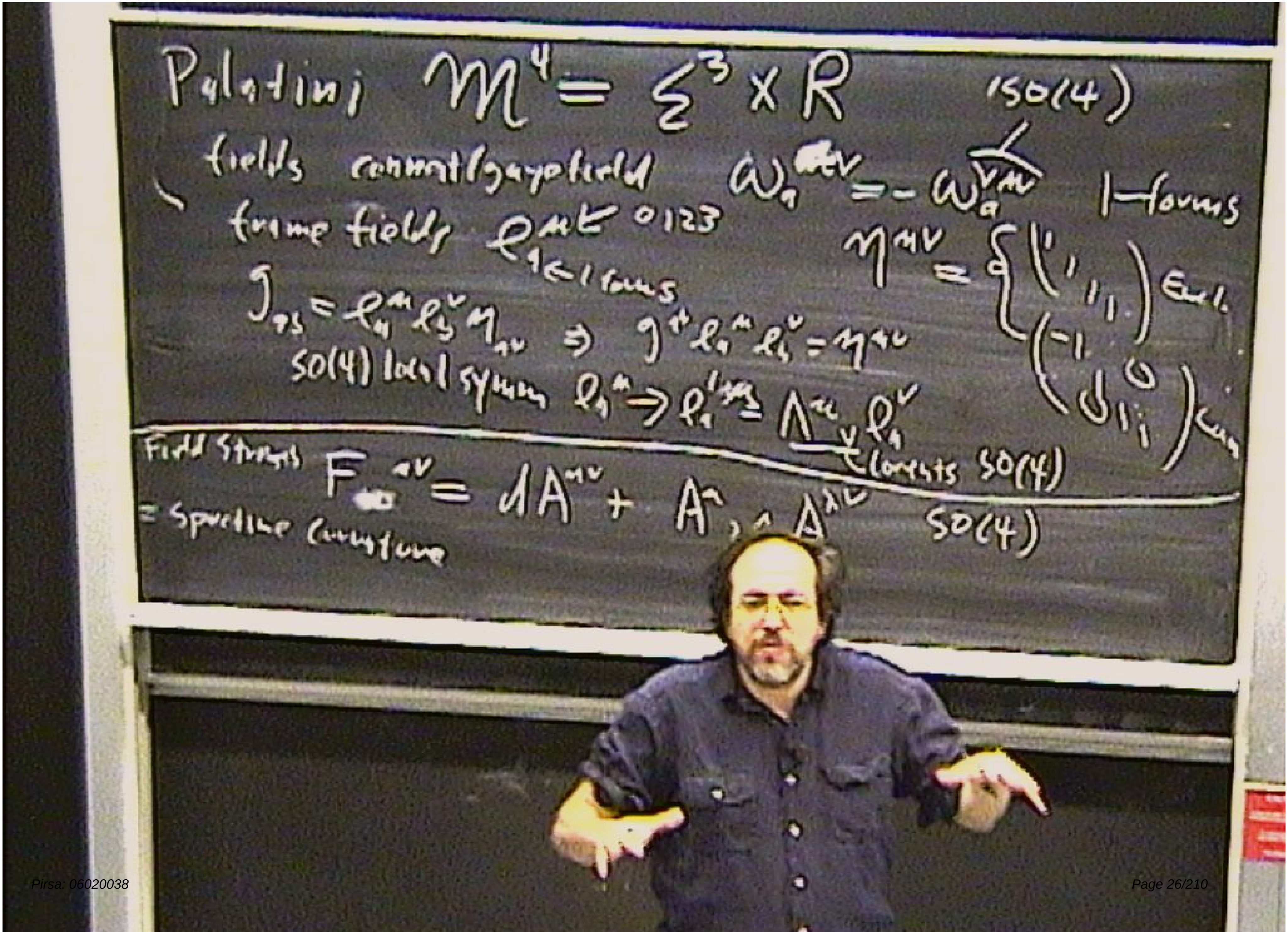
Field Stress F^μν = dA^μν + A^μ_λ ∧ A^λν
SO(4)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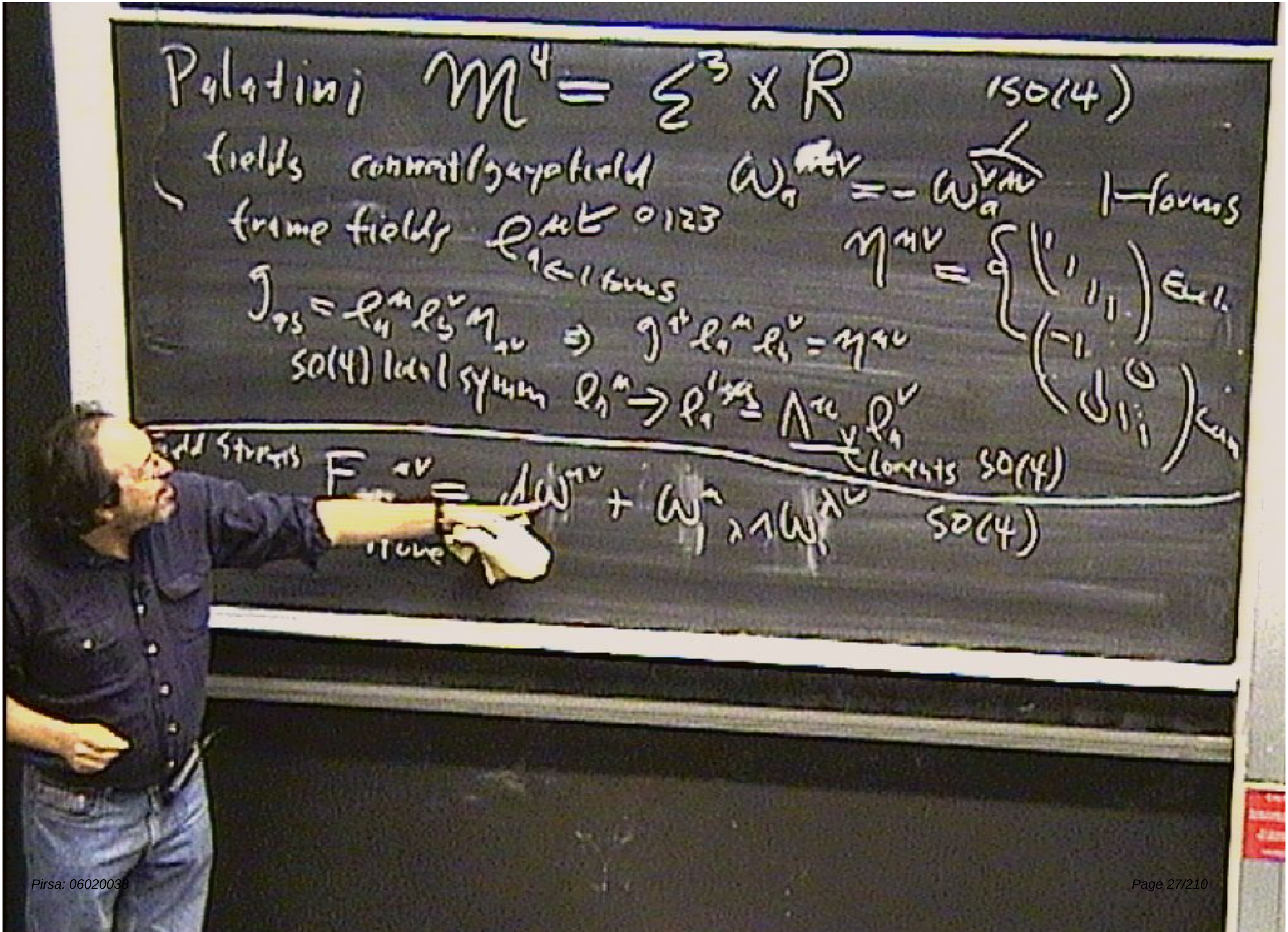
Field Strength F^{μν} = dA^{μν} + A^μ_λ ∧ A^{λν} SO(4)
= Spacetime Curvature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Palatini ... R

$SO(4)$

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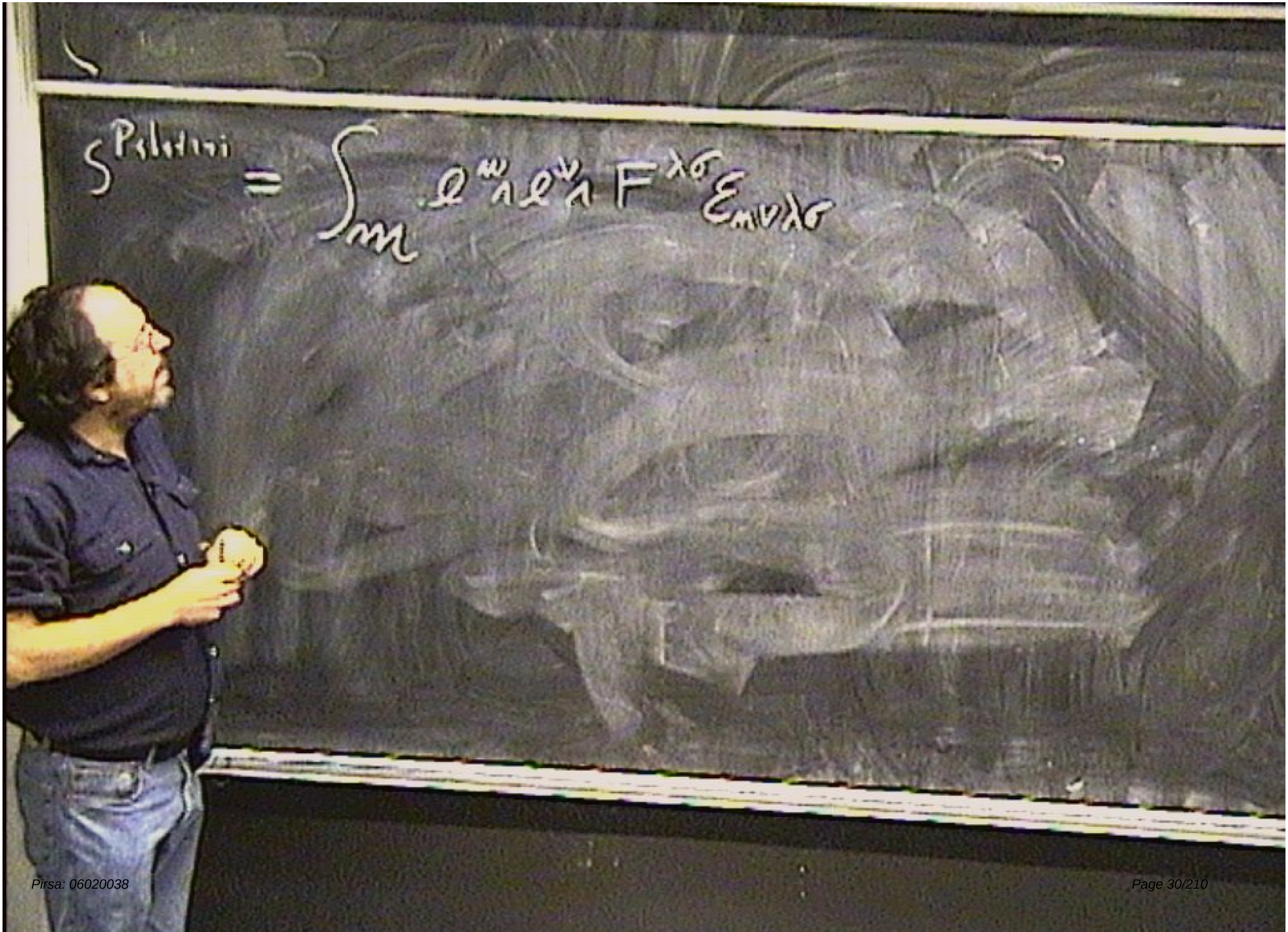
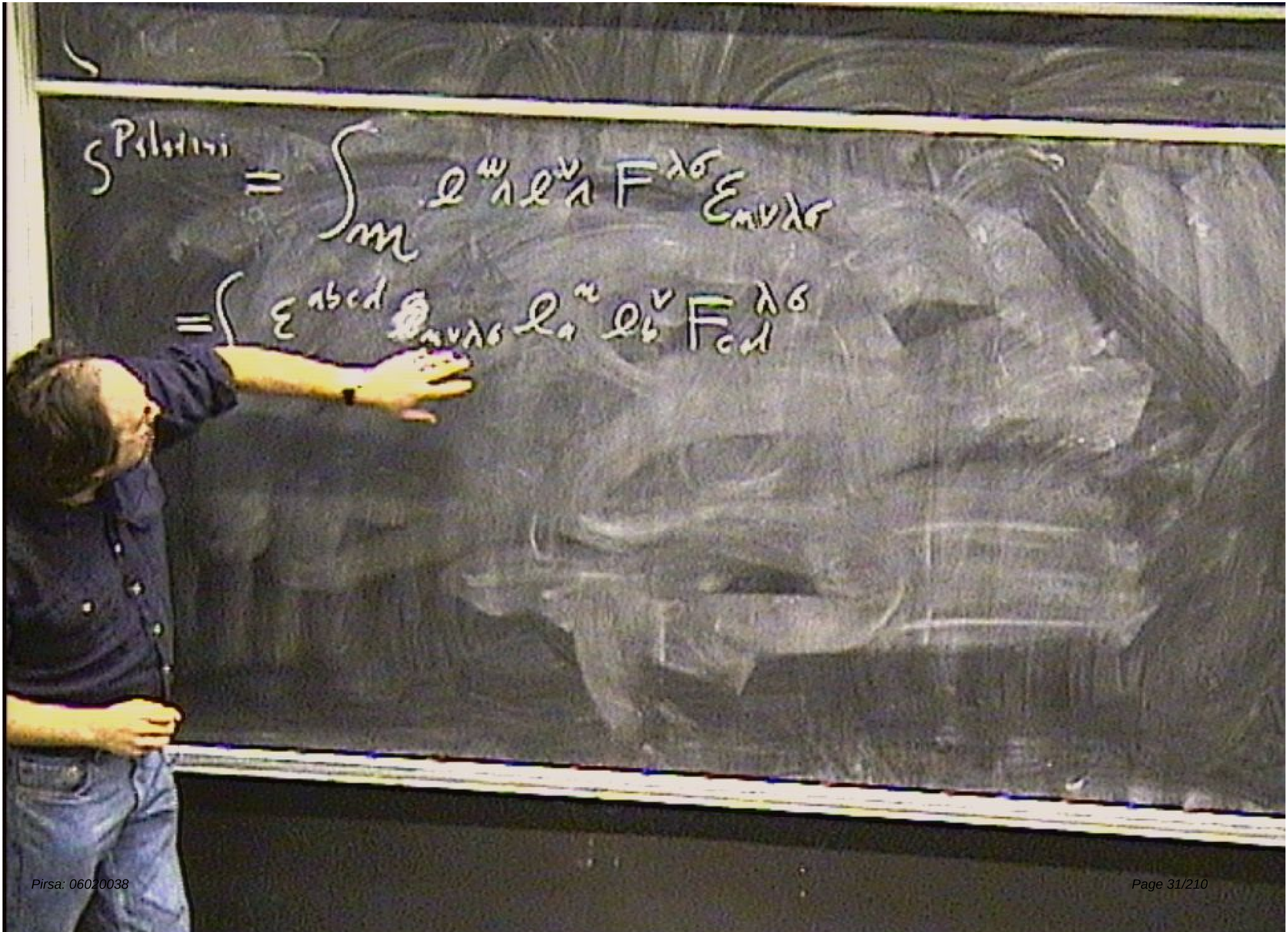
Field Strength $F_{ab}{}^{\mu\nu} = d\omega^{\mu\nu} + \omega_{\lambda}{}^{\mu} \wedge \omega^{\lambda\nu}$ $SO(4)$

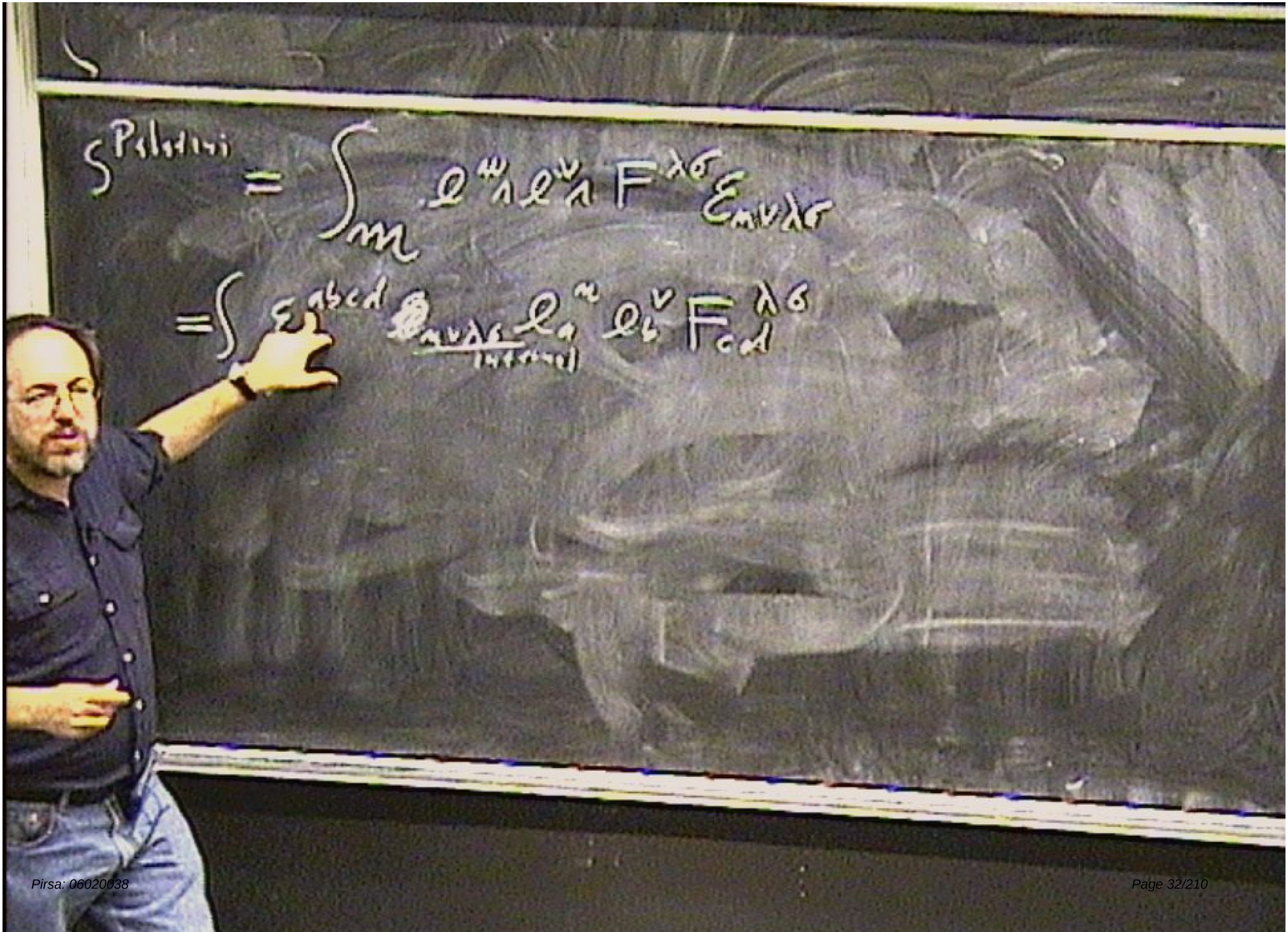
= Spacetime Curvature

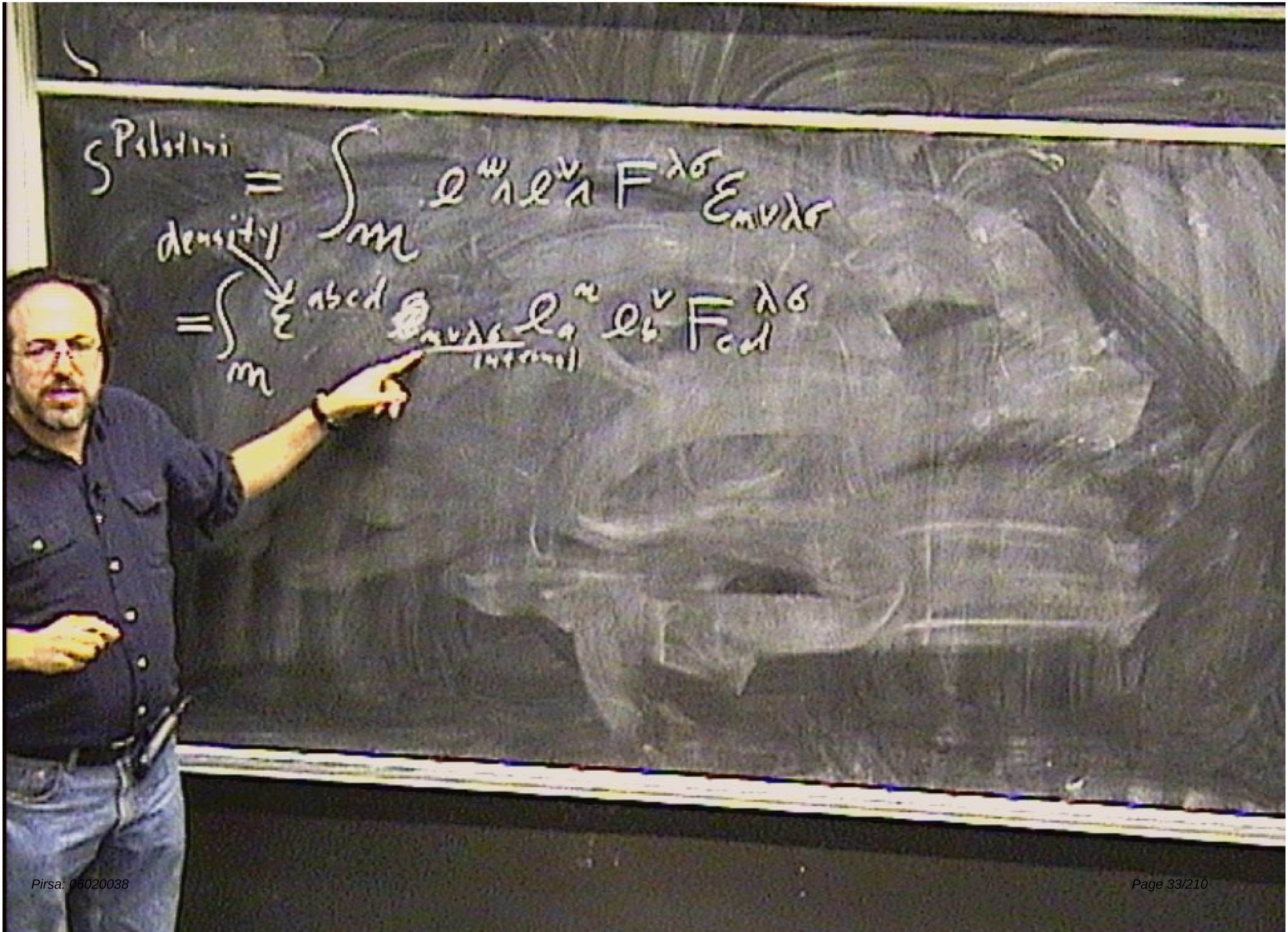
$S^{\text{Palatini}} = \int_{M}$

$$S^{\text{Palatini}} = \int_{M} e^{\mu}_{A}\, e^{\nu}_{A}\, F^{\lambda\sigma}$$

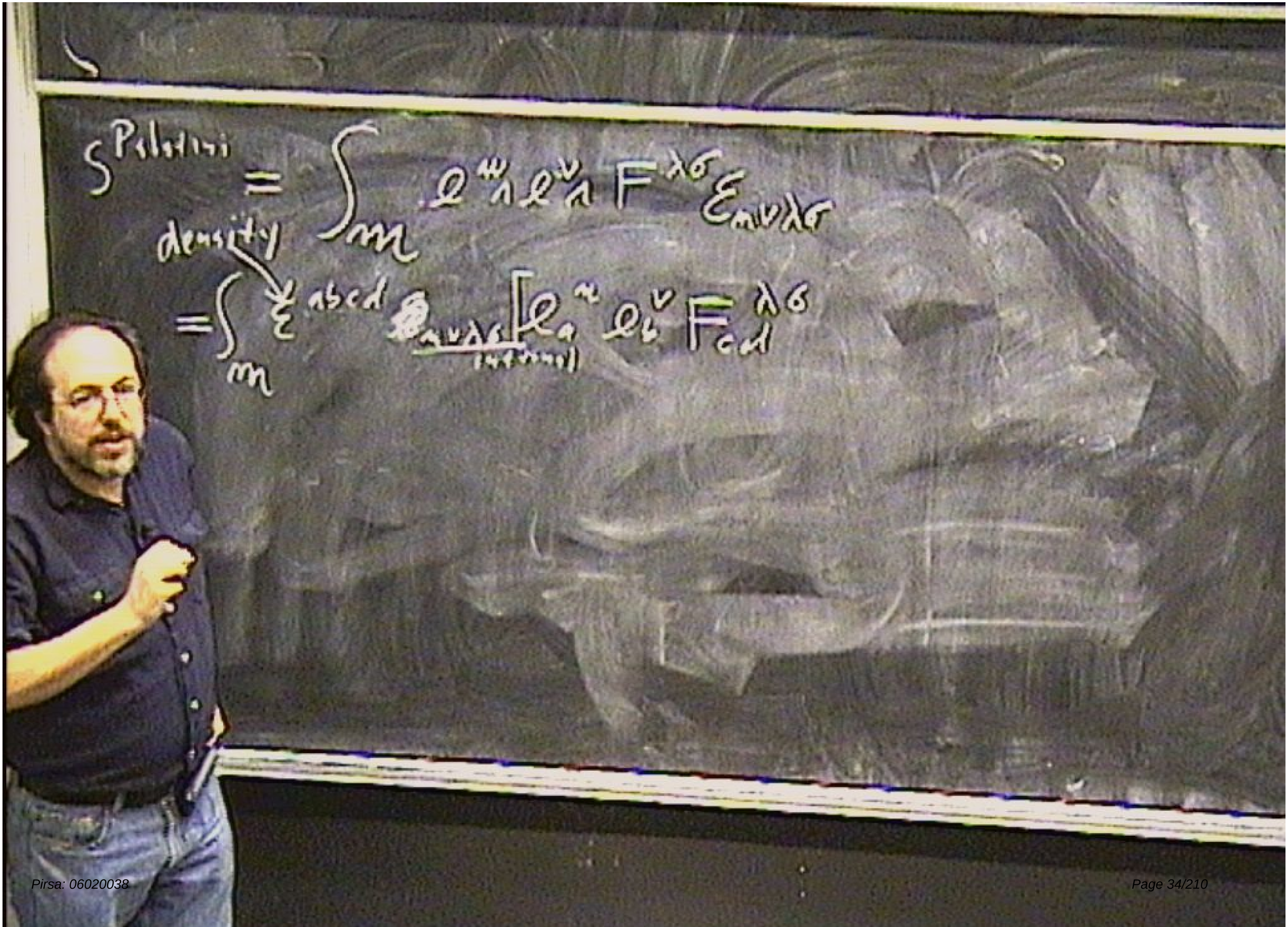

$S^{\text{Palatini}} = \int_M e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} \varepsilon_{\mu\nu\lambda\sigma}$

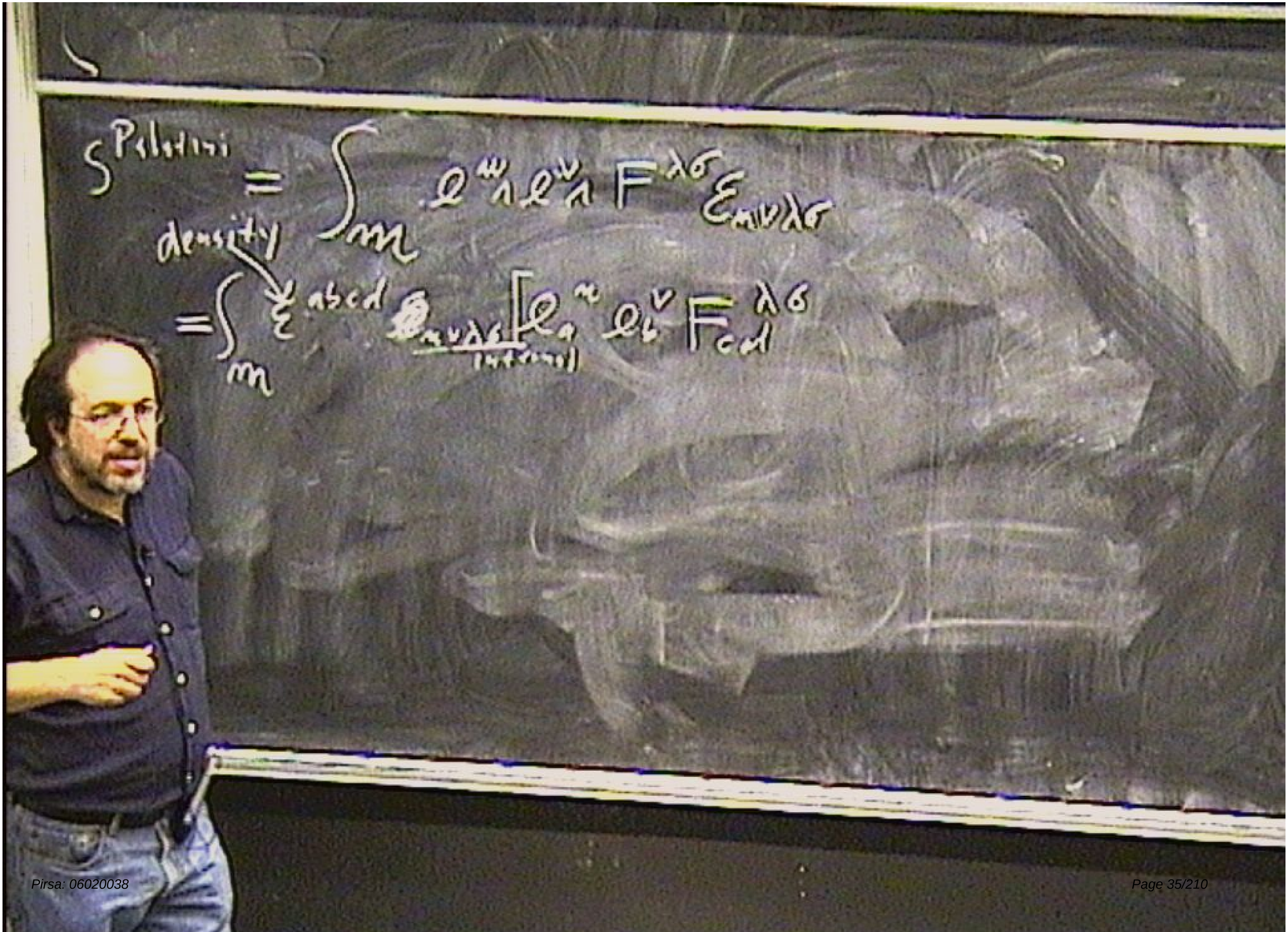

$S^{\text{Palatini}} = \int_M e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} \varepsilon_{\mu\nu\lambda\sigma}$
$= \int \varepsilon^{abcd} \varepsilon_{\mu\nu\lambda\sigma} e_a^{\ \mu} e_b^{\ \nu} F_{cd}^{\ \ \lambda\sigma}$


$S^{Palatini} = \int_{M} e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma}$
$= \int \epsilon^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} e_a^{\mu} e_b^{\nu} F_{cd}^{\lambda\sigma}$



$S^{\text{Palatini}} = \int_M e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma}$
density
$= \int_M \tilde{\epsilon}^{abcd} \epsilon_{\mu\nu\lambda\sigma} e_a^{\mu} e_b^{\nu} F_{cd}^{\lambda\sigma}$
(internal)


S^Palatini = ∫_M e^μ ∧ e^ν ∧ F^λσ ε_μνλσ
density
= ∫_M ε̃^abcd ε_μνλσ [e_a^μ e_b^ν F_cd^λσ
(internal)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density
= ∫_M ε^abcd ε_μνλσ [e_a^μ e_b^ν F_cd^λσ]
(internal)

Euclidean 0209079

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· Review duality & self-duality

· Review JSS formalism.

· Plebanski / A... ...ekar, "... ..."

– "Class...

hep-th/0501191

· Laurent-Artem

$g \eta \bar{g}^{-1} = \eta$

Euclidean 0209079

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  = "Classical LQG"
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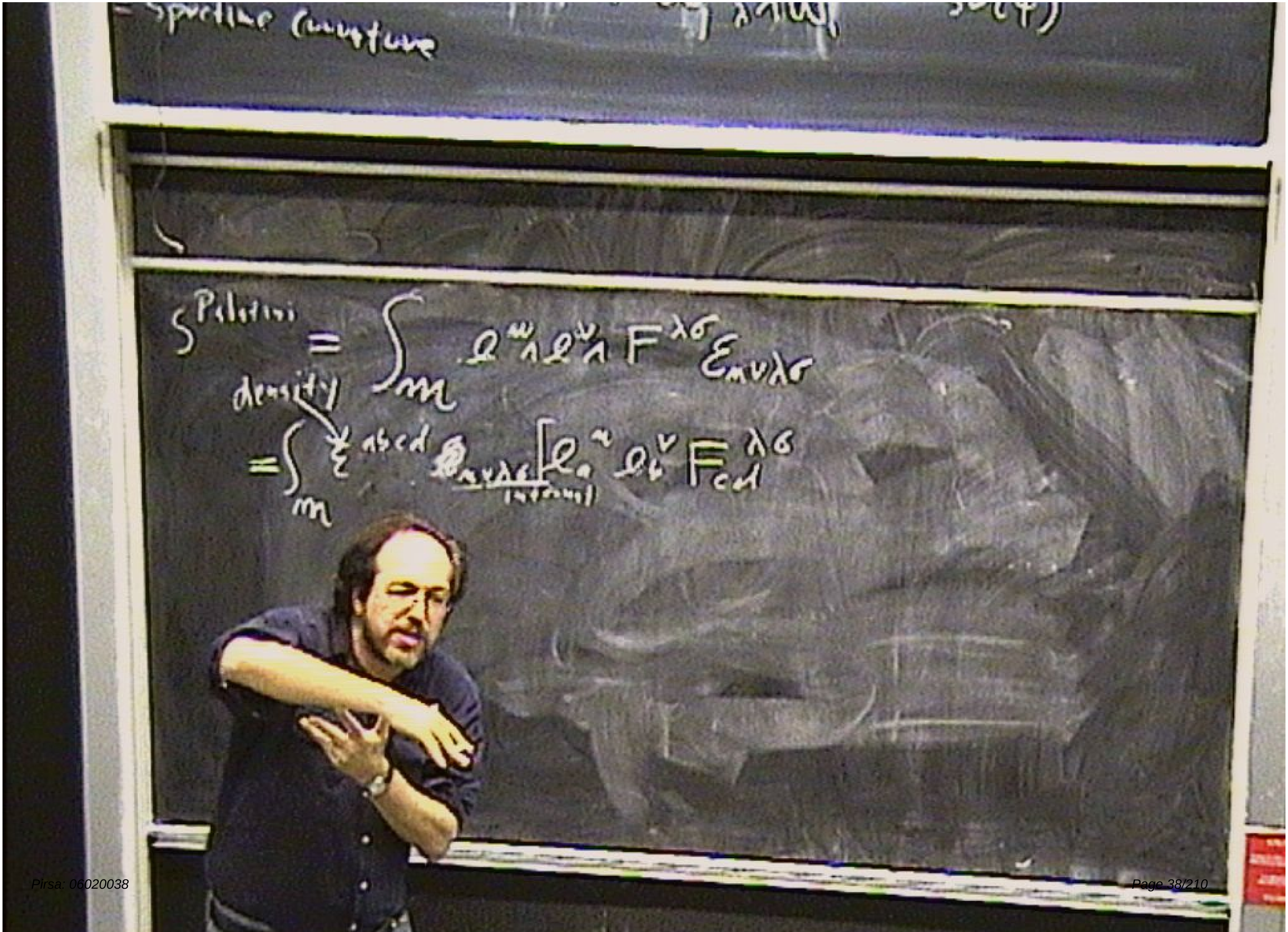
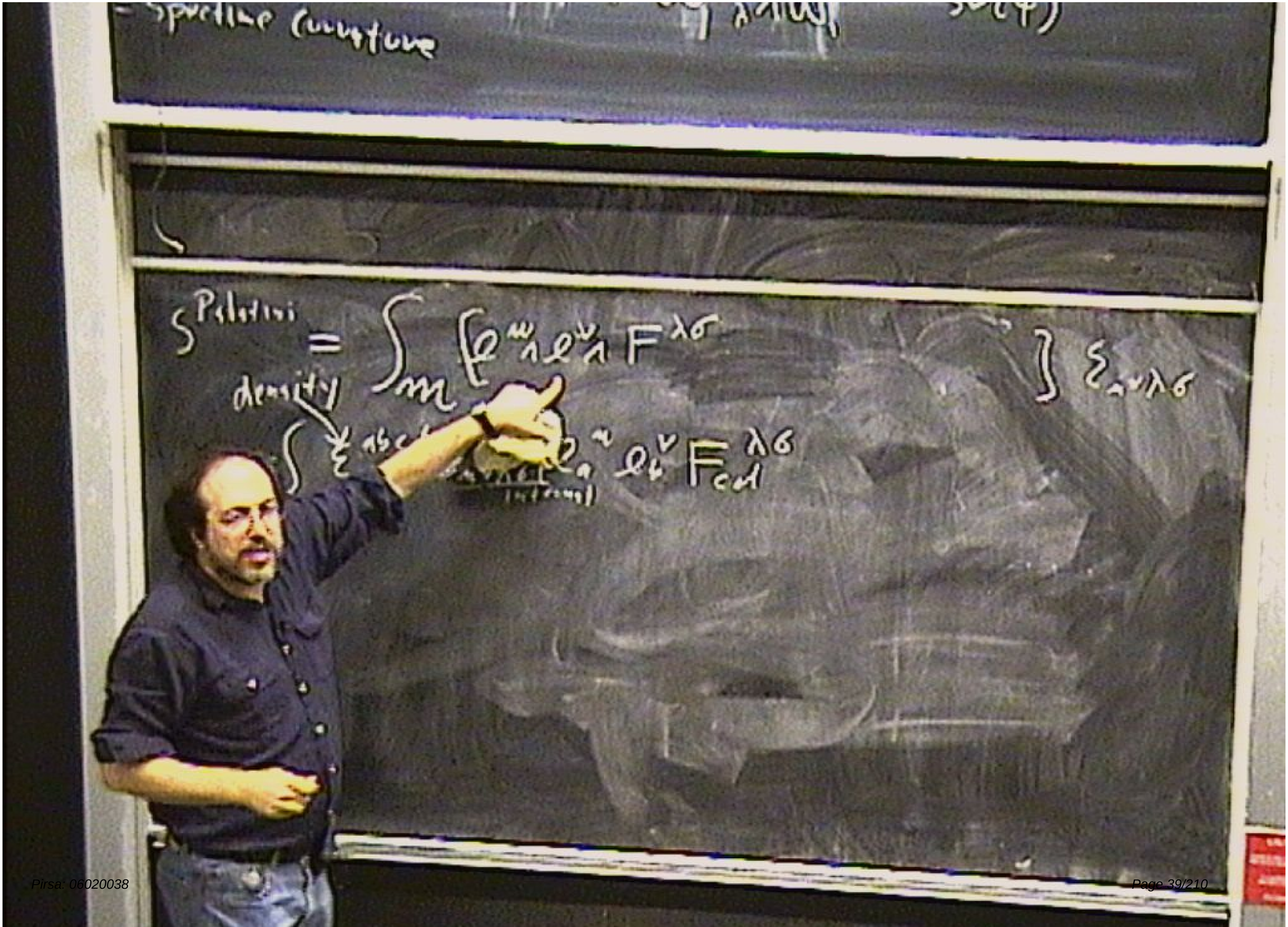
- $g \eta g^{-1} = \eta$

$g \to a\psi$


spacetime curvature
$S^{\text{Palatini}} = \int_M e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} \varepsilon_{\mu\nu\lambda\sigma}$
density
$= \int_M \tilde{\varepsilon}^{abcd} \varepsilon_{\mu\nu\lambda\sigma} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma}$
(internal)



S Palatini
density
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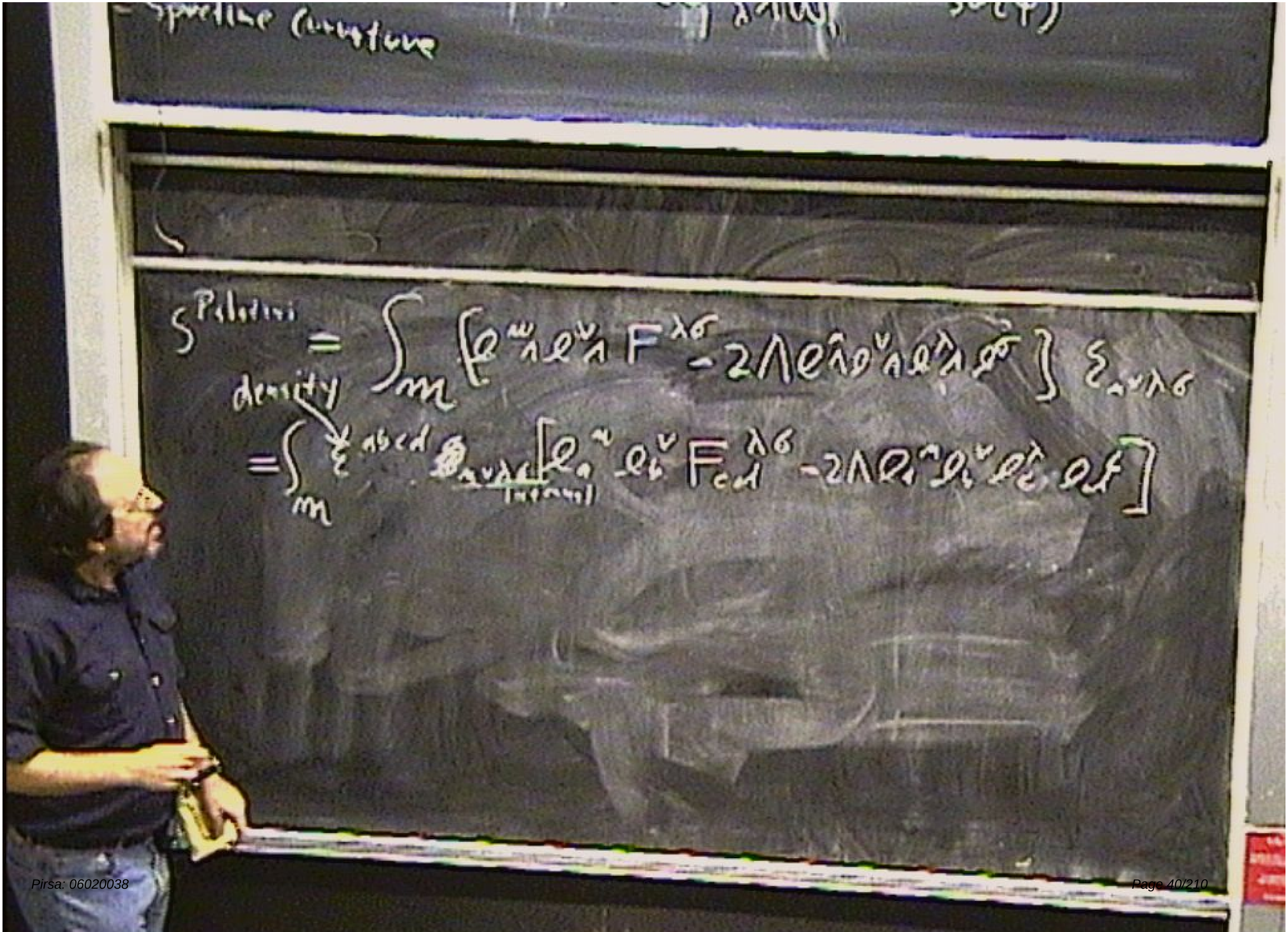
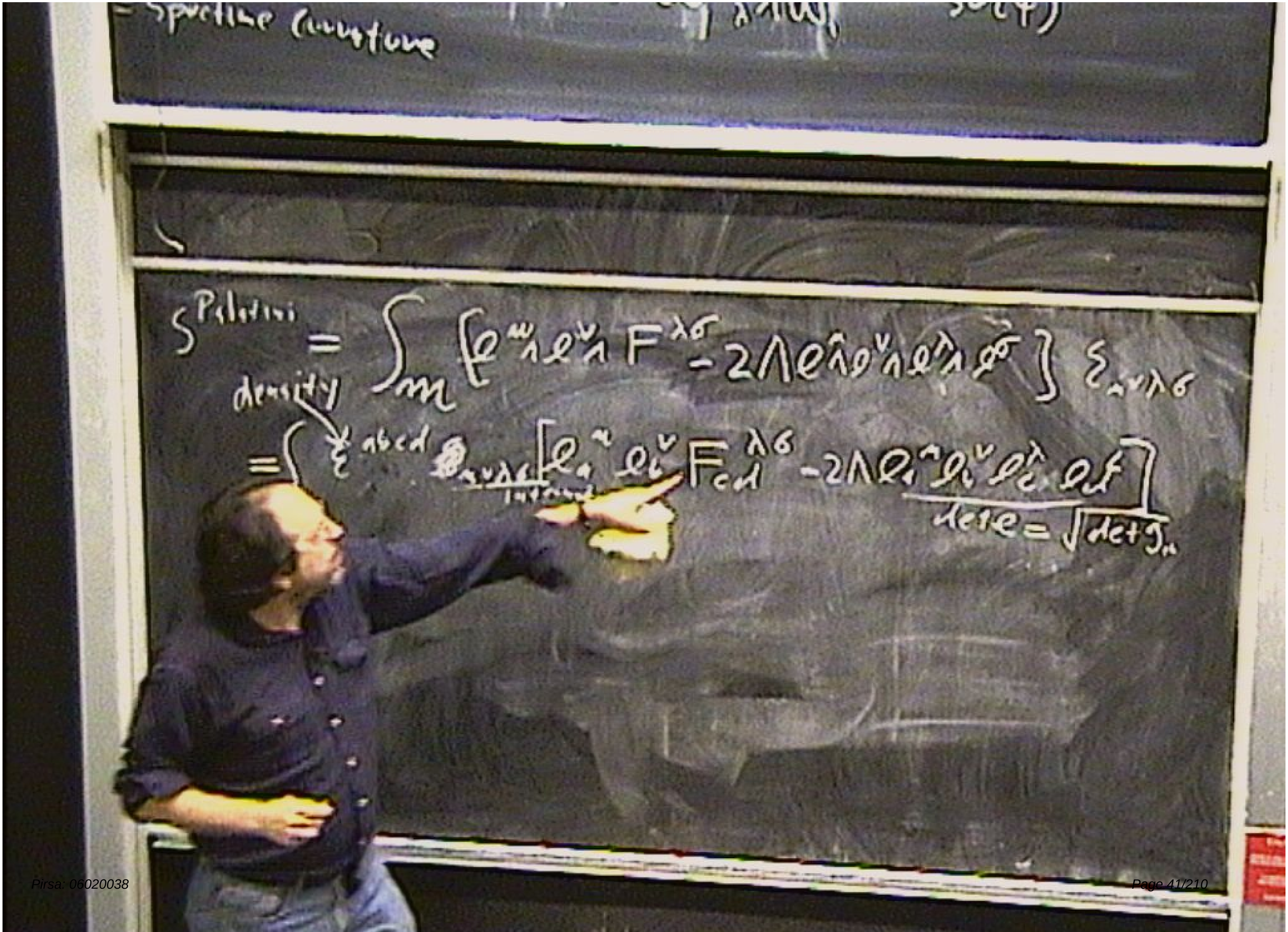
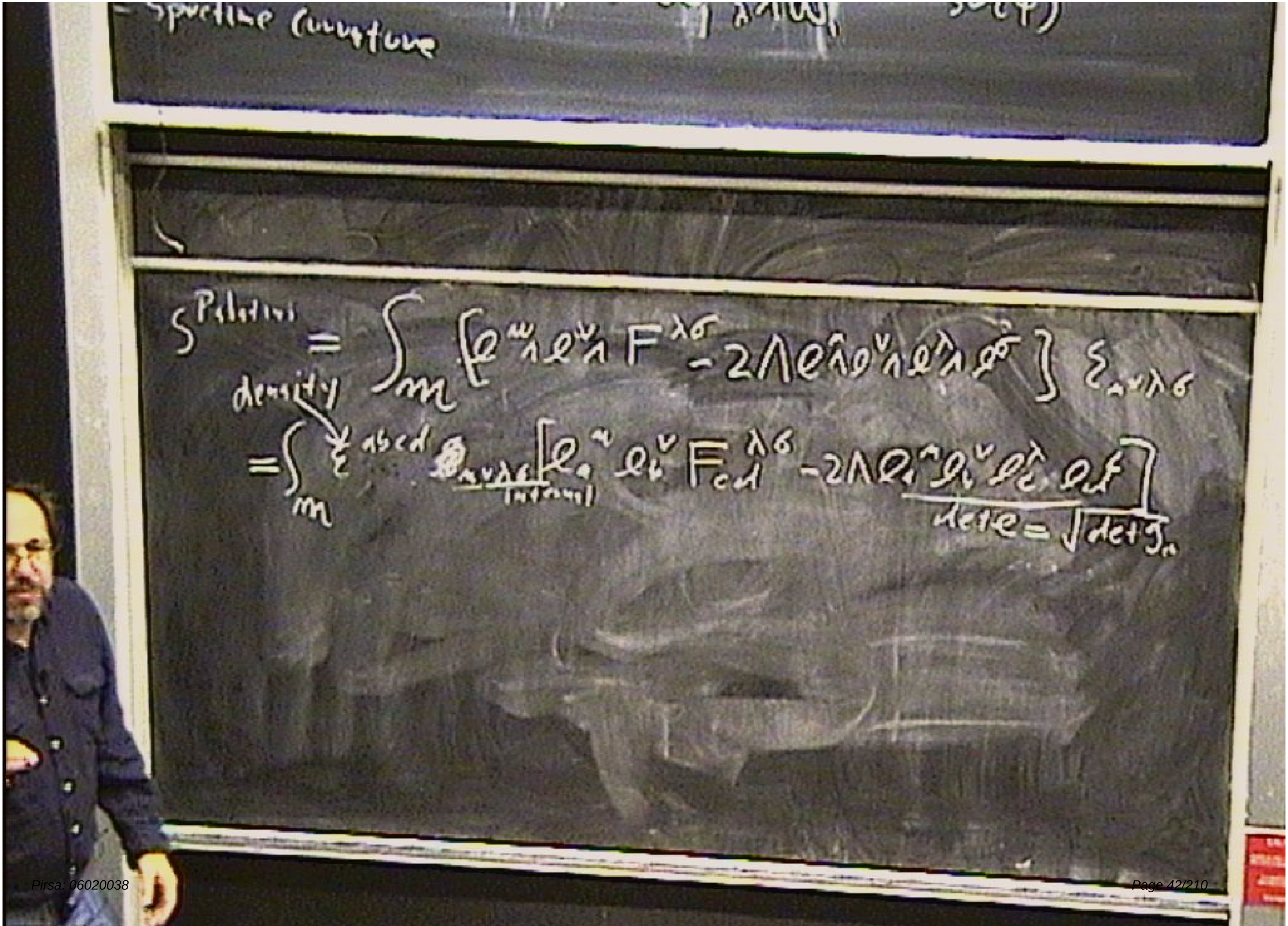
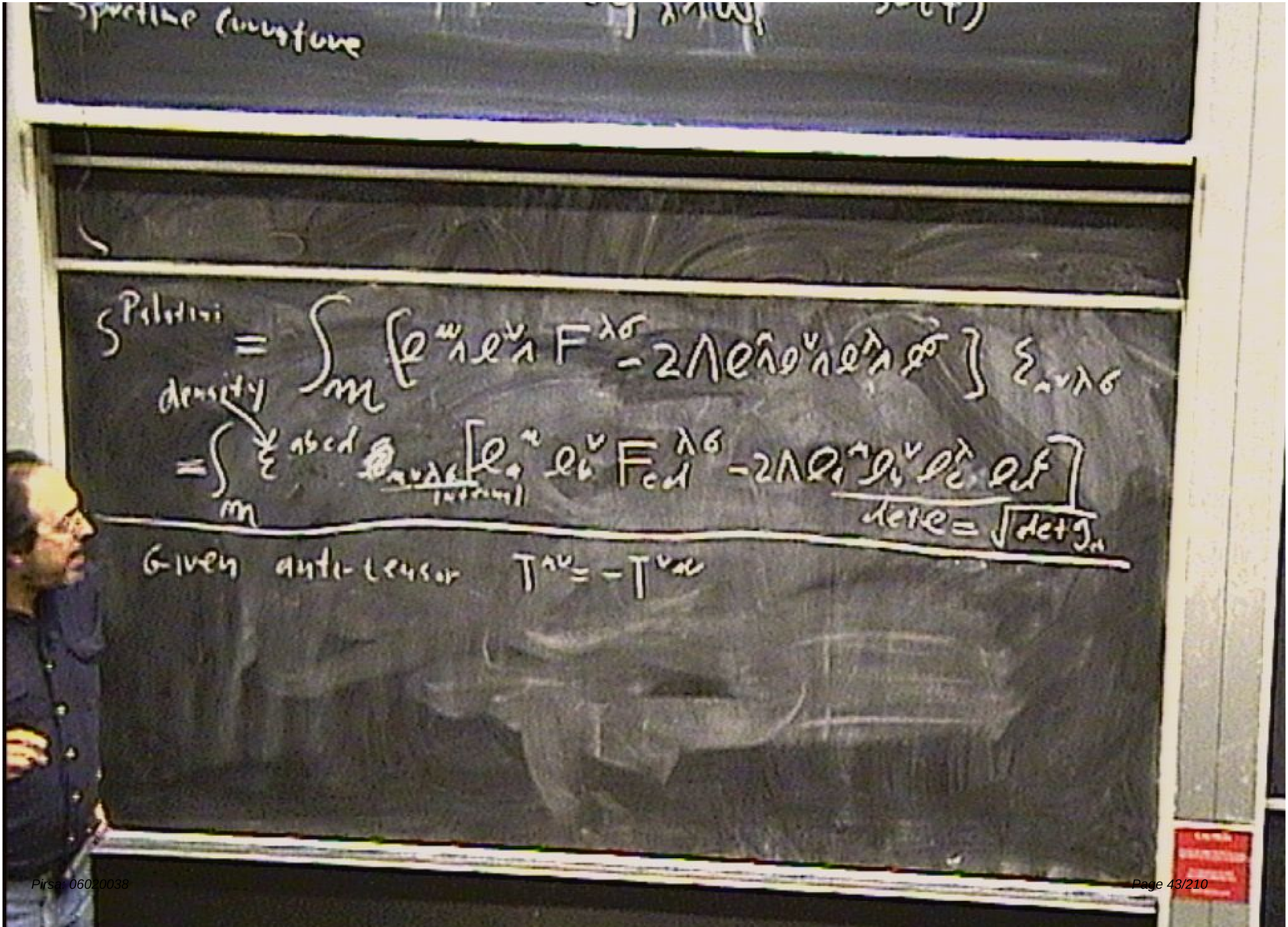


spacetime curvature
$S^{Palatini} = \int_M [e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma]\, \epsilon_{\mu\nu\lambda\sigma}$
density
$= \int_M \tilde{\epsilon}^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma]$


Spacetime curvature
Palatini
density
det e = √det g


Spacetime curvature
S^Palatini = ∫_M [e^μ ∧ e^ν ∧ F^λσ − 2Λ e^μ ∧ e^ν ∧ e^λ ∧ e^σ] ε_μνλσ
density
= ∫_M ε^abcd ε_μνλσ [e_a^μ e_b^ν F_cd^λσ − 2Λ e_a^μ e_b^ν e_c^λ e_d^σ]
internal
det e = √det g


Spacetime curvature
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density
$= \int_M \tilde{\epsilon}^{abcd}\, \epsilon_{\mu\nu\lambda\sigma} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma]$
(internal)
$\det e = \sqrt{\det g_{ab}}$
Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$

spacetime curvature

$$S^{\text{Palatini}} = \int_{M} \left[e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} - 2\Lambda e^{\mu} \wedge e^{\nu} \wedge e^{\lambda} \wedge e^{\sigma} \right] \epsilon_{\mu\nu\lambda\sigma}$$

density

$$= \int_{M} \tilde{\epsilon}^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{(internal)}} \left[e_a{}^{\mu} e_b{}^{\nu} F_{cd}{}^{\lambda\sigma} - 2\Lambda e_a{}^{\mu} e_b{}^{\nu} e_c{}^{\lambda} e_d{}^{\sigma} \right]$$

$\det e = \sqrt{\det g_{ab}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{*\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$

Palatini ... R SO(4)

fields connection $\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ 1-forms

frame fields e_a^{μ} $\mu = 0123$ 1-forms

$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} & \text{Eucl.} \\ \begin{pmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{pmatrix} & \text{Lorentz} \end{cases}$

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Field Strength $F^{\mu\nu} = d\omega^{\mu\nu} + \omega^\mu{}_\lambda \wedge \omega^{\lambda\nu}$ SO(4)

= Spacetime curvature

$S^{\text{Palatini}} = \int_M [e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma] \epsilon_{\mu\nu\lambda\sigma}$

density

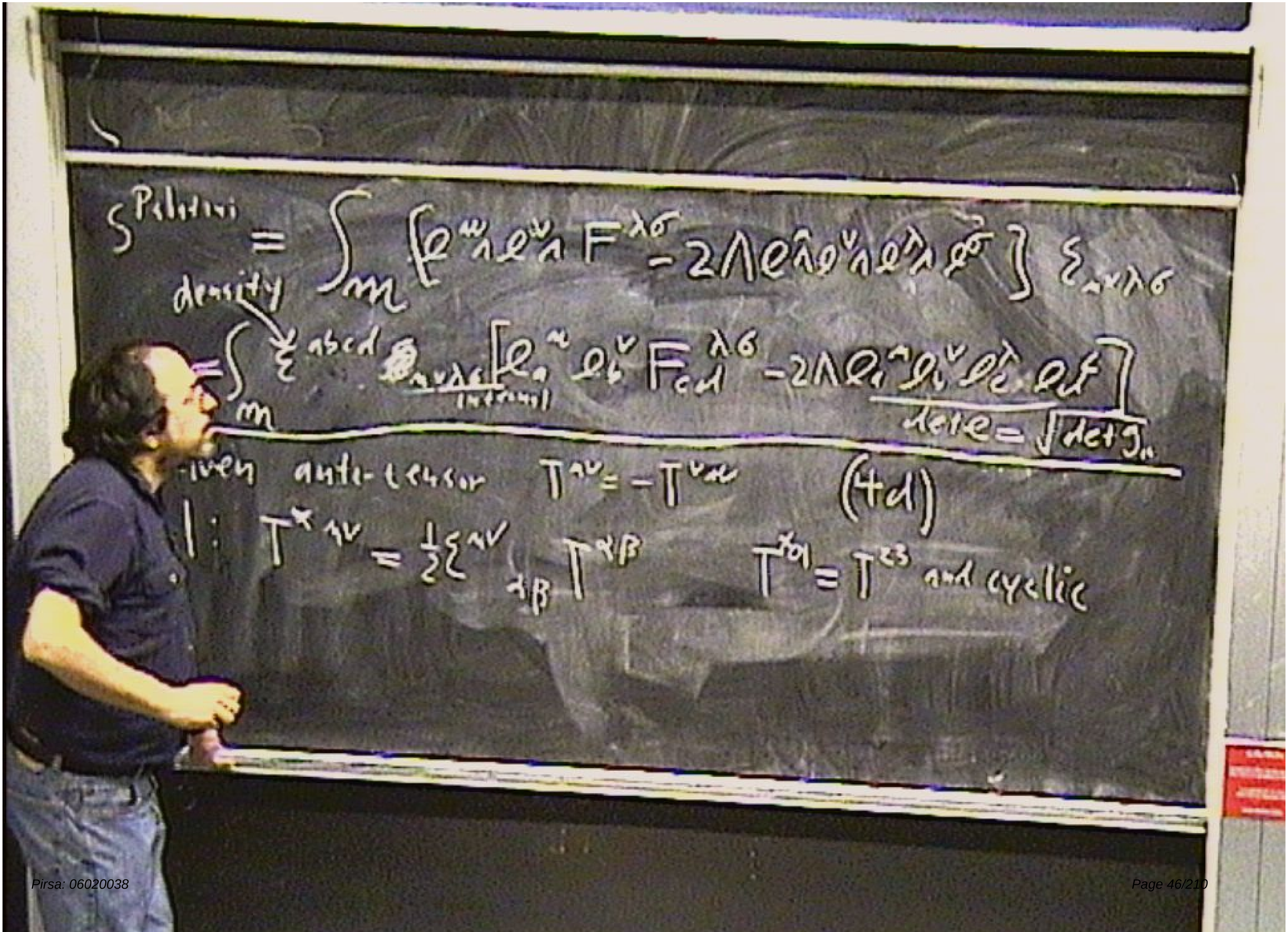
$= \int_M \epsilon^{abcd} \epsilon_{\mu\nu\lambda\sigma} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma]$

$\det e = \sqrt{\det g_{ab}}$

Given anti-tensor ... $= -T^{\nu\mu}$ (4d)

dual: $T^{*\mu\nu} =$

R R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density
= ∫_M ε^abcd ε_μνλσ [e_a^μ e_b^ν F_cd^λσ − 2Λ e_a^μ e_b^ν e_c^λ e_d^σ]
internal
det e = √(det g_μν)
given anti-tensor T^μν = −T^νμ
(4d)
*: T^*μν = ½ ε^μν_αβ T^αβ
T^*01 = T^23 and cyclic

$$S^{\text{Palatini}} = \int_M \left[e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda\, e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma \right] \epsilon_{\mu\nu\lambda\sigma}$$

density

$$= \int_M \tilde{\epsilon}^{abcd}\, \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} \left[e_a^{\ \mu} e_b^{\ \nu} F_{cd}^{\ \ \lambda\sigma} - 2\Lambda\, \underline{e_a^{\ \mu} e_b^{\ \nu} e_c^{\ \lambda} e_d^{\ \sigma}} \right]$$

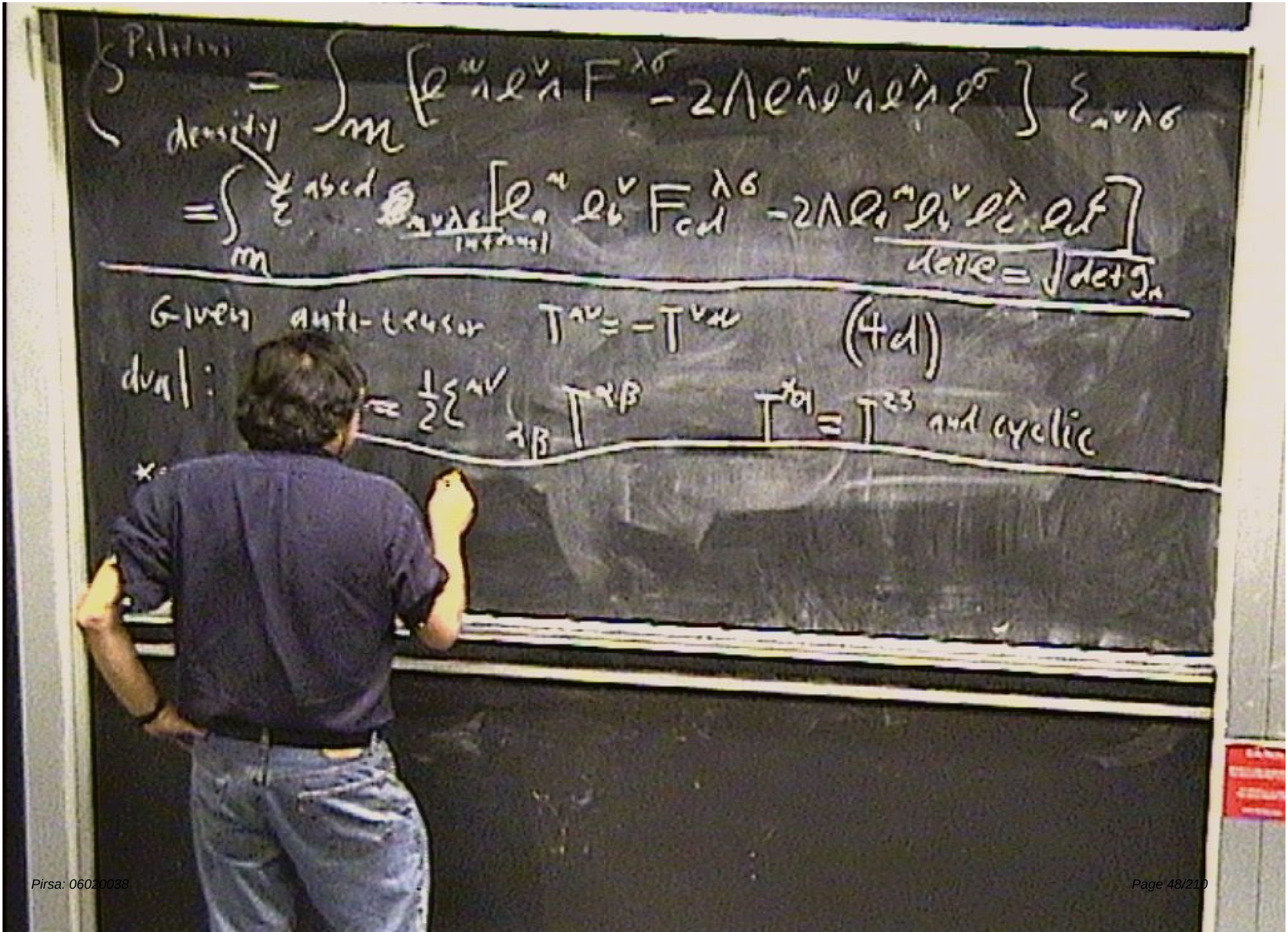
$\det e = \sqrt{\det g_{\mu\nu}}$

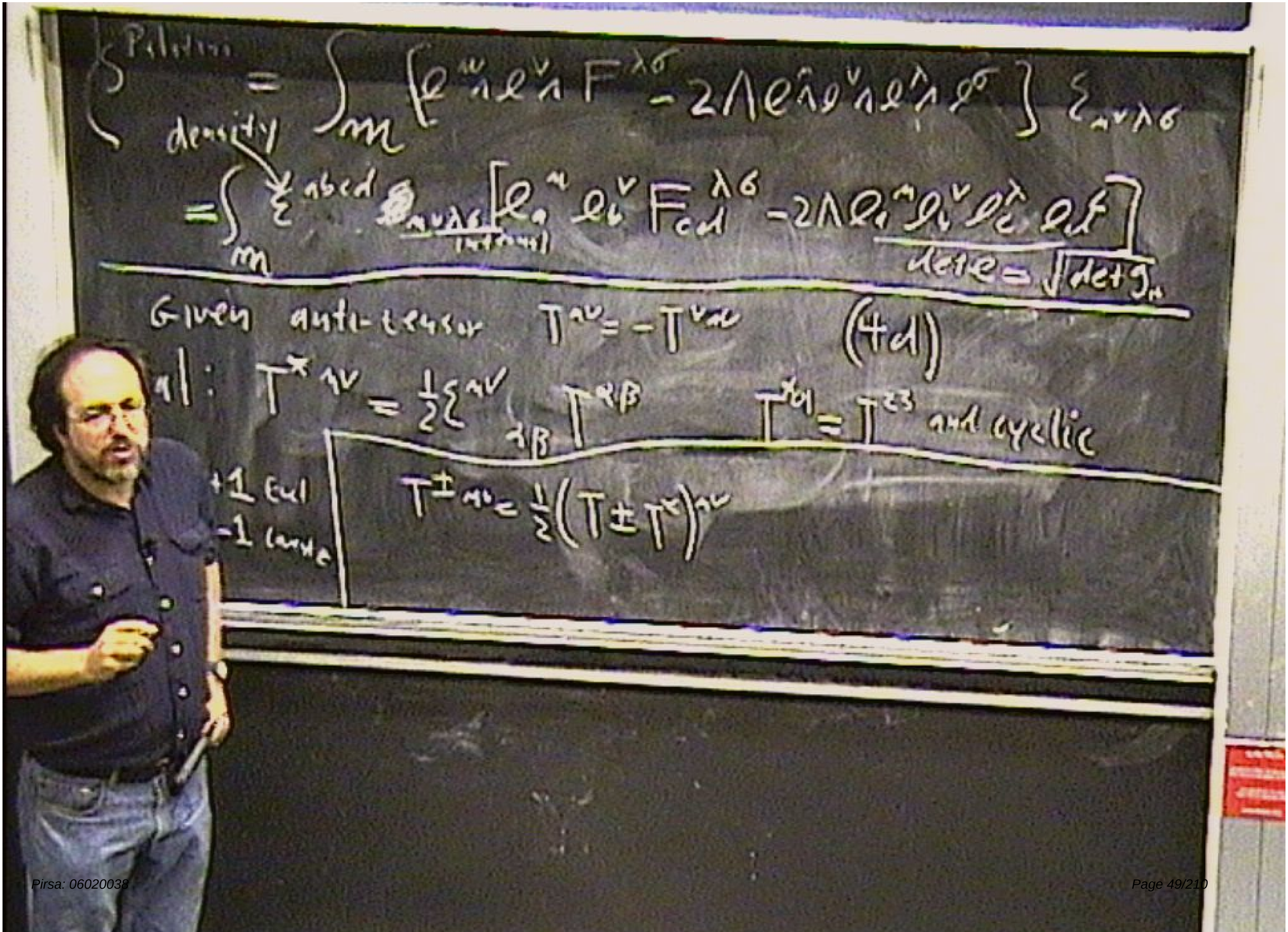
Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

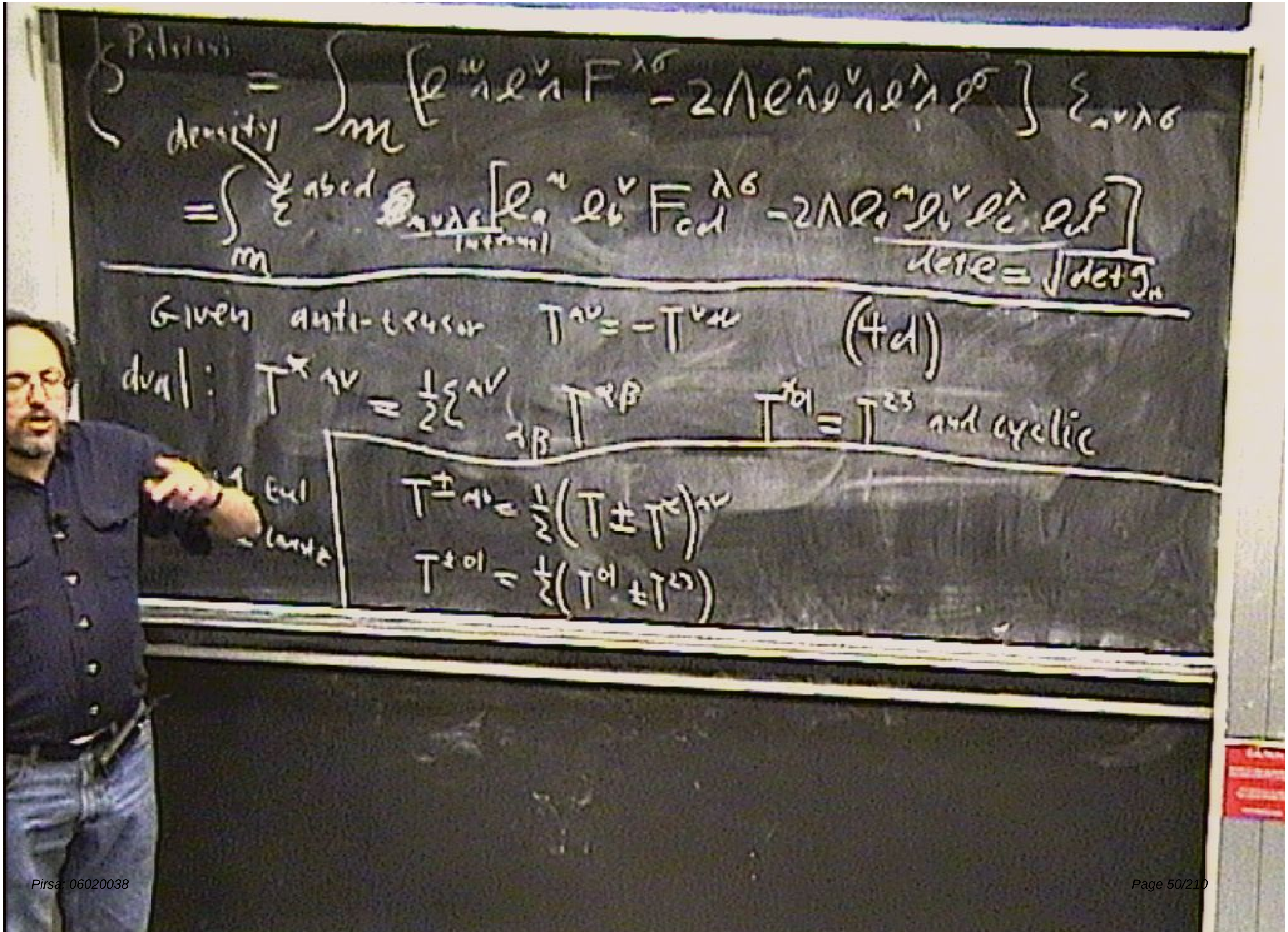
dual: $T^{*\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu}{}_{\alpha\beta}\, T^{\alpha\beta}$ $\quad T^{*01} = T^{23}$ and cyclic

$** = +1$ Eucl

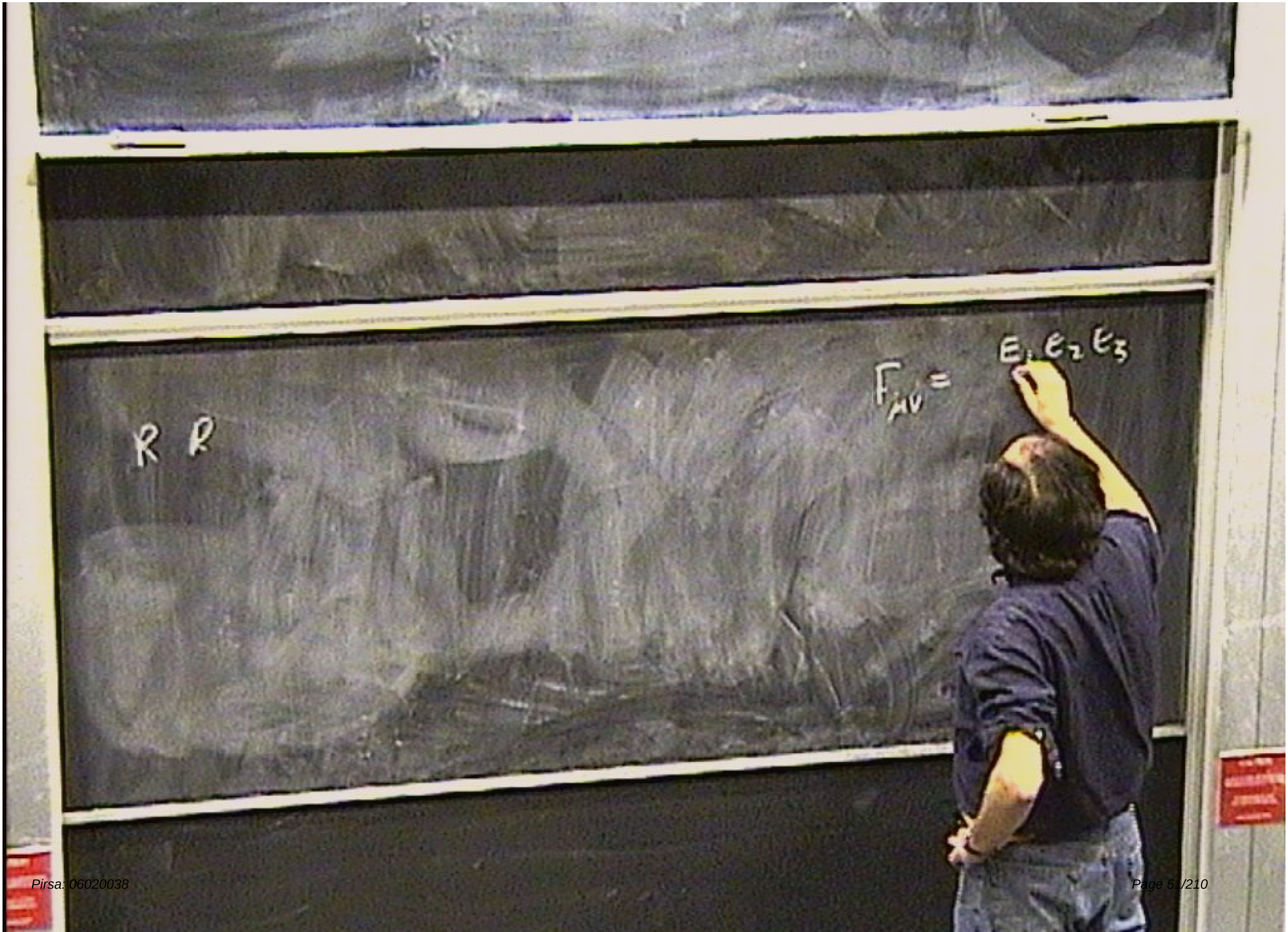
-1 Lorentz

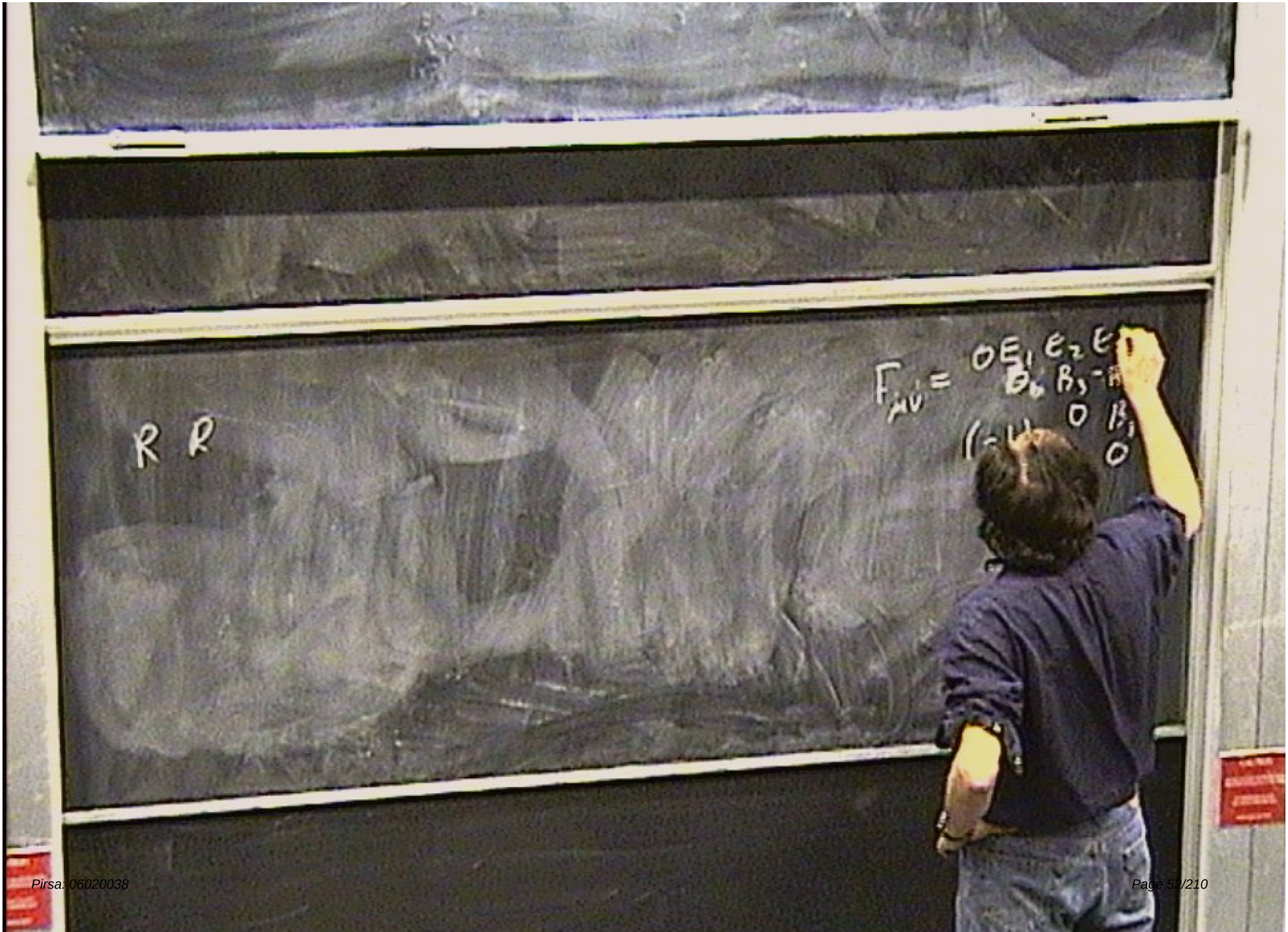

Palatini
density
internal
det e = √det g
Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$
(4d)
dual:
and cyclic

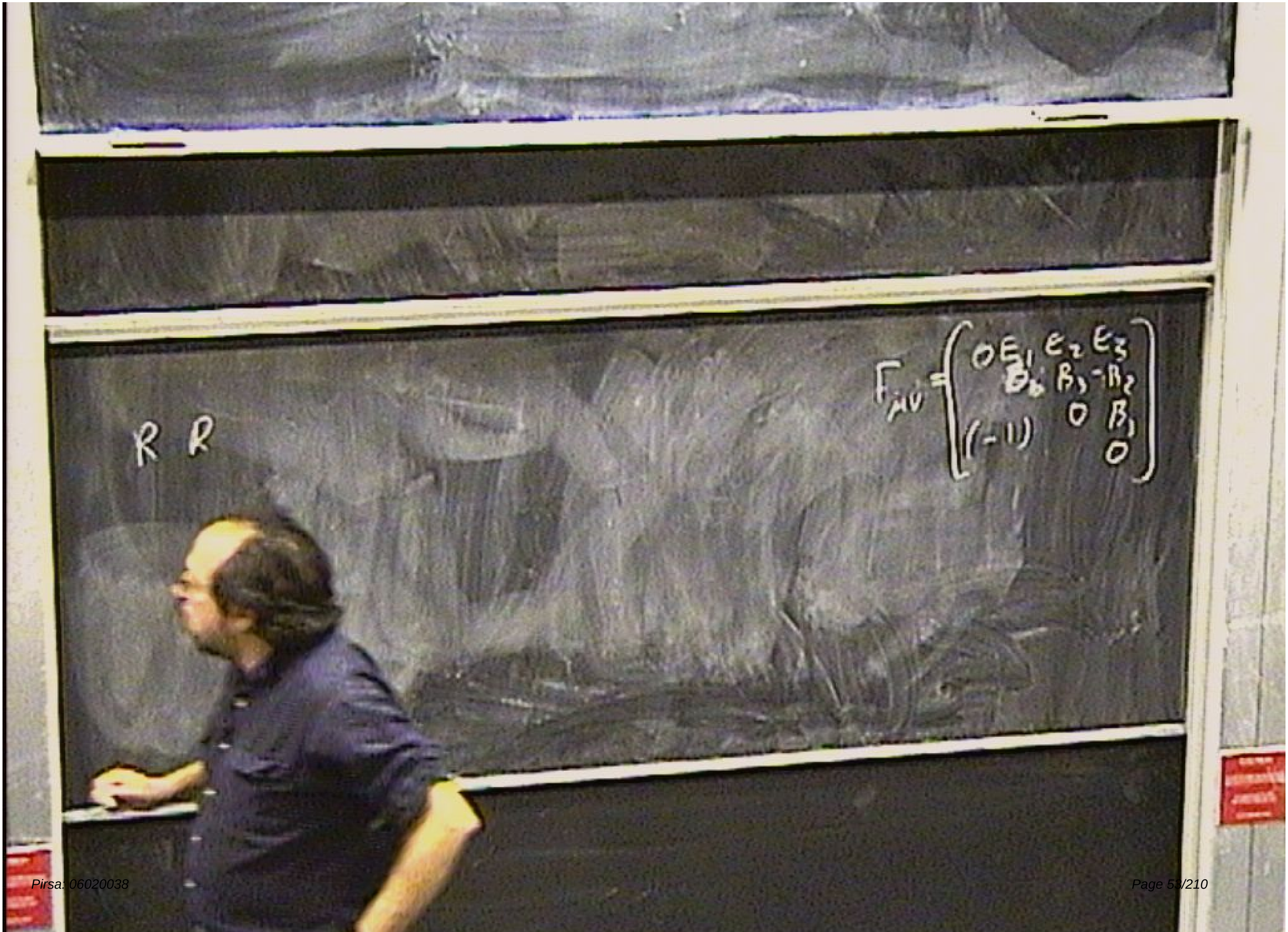

Palatini
density
Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$
(4d)
$T^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$
$T^{*01} = T^{23}$ and cyclic
+1 Eucl
−1 Lorentz
$T^{\pm\mu\nu} = \frac{1}{2}(T \pm T^{*})^{\mu\nu}$

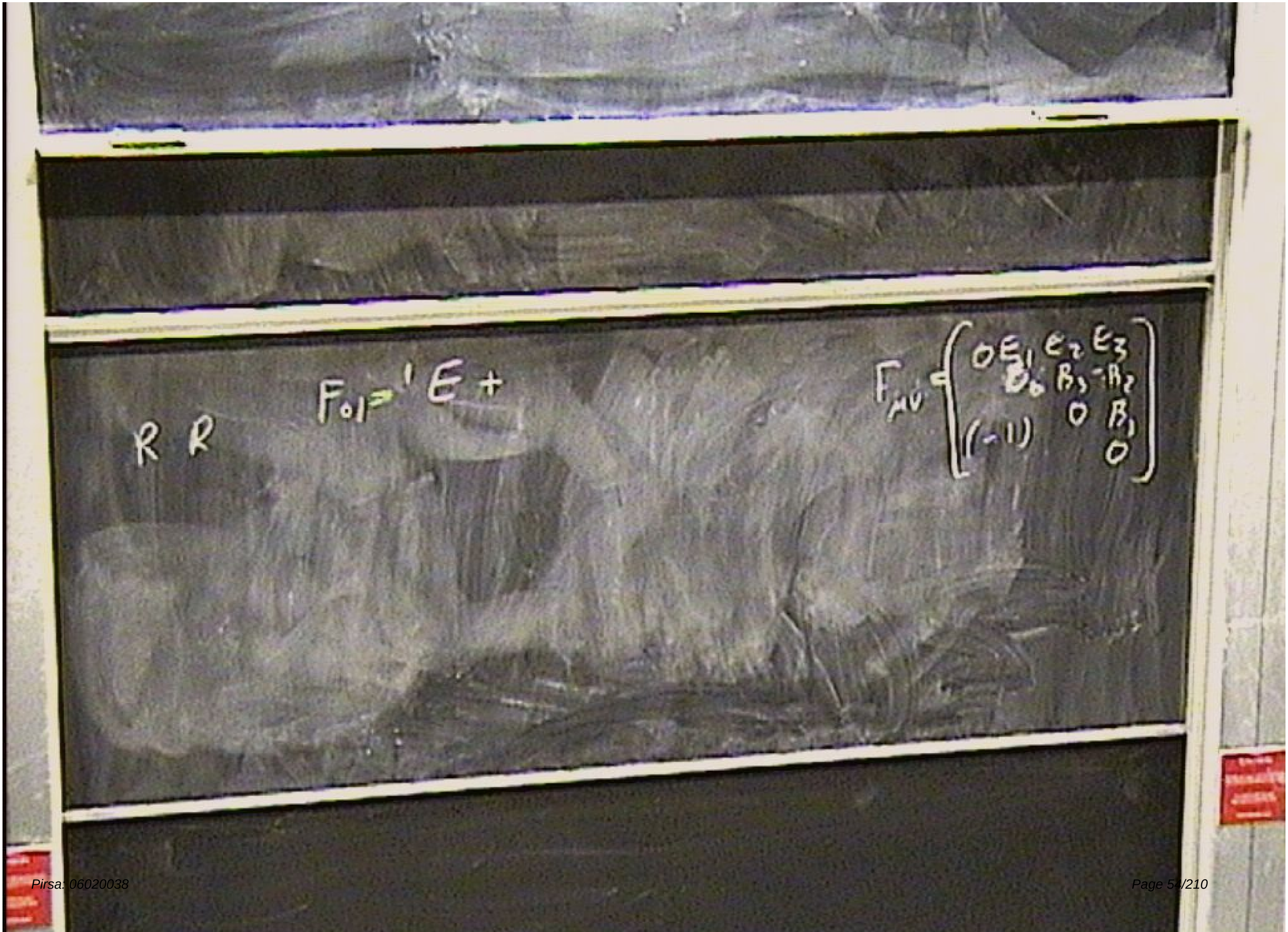


S^Palatini
density
= ∫_M [e^μ ∧ e^ν ∧ F^λσ − 2Λ e^μ ∧ e^ν ∧ e^λ ∧ e^σ] ε_μνλσ
= ∫_M ε̃^abcd ε_μνλσ [e_a^μ e_b^ν F_cd^λσ − 2Λ e_a^μ e_b^ν e_c^λ e_d^σ]
internal
det e = √|det g_μν|
Given anti-tensor T^μν = −T^νμ
(4d)
dual: T^*μν = ½ ε^μν_αβ T^αβ
T^*01 = T^23 and cyclic
T^±μν = ½(T ± T^*)^μν
T^±01 = ½(T^01 ± T^23)


$F_{\mu\nu} = E_1 E_2 E_3$
R R


R R
$F_{\mu\nu} =$


R R
F_{μν} =
0 E1 E2 E3
B3 -B2
(-1) 0 B1
0


R R
$F_{01} = {}^{!}E +$
$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$

R R

$$F^{\pm}_{01} = {}' E' \pm B'$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

Field strengths $F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$ $SO(4)$

= Spacetime curvature

$S^{\text{Palatini}} = \int_{M} \{ e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} - 2\Lambda\, e^{\mu} \wedge e^{\nu} \wedge e^{\lambda} \wedge e^{\sigma} \} \, \epsilon_{\mu\nu\lambda\sigma}$

density

$= \int_{M} \tilde{\epsilon}^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} [e_a{}^{\mu} e_b{}^{\nu} F_{cd}{}^{\lambda\sigma} - 2\Lambda\, e_a{}^{\mu} e_b{}^{\nu} e_c^{\lambda} e_d^{\sigma}]$

$\det e = \sqrt{\det g_{ab}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

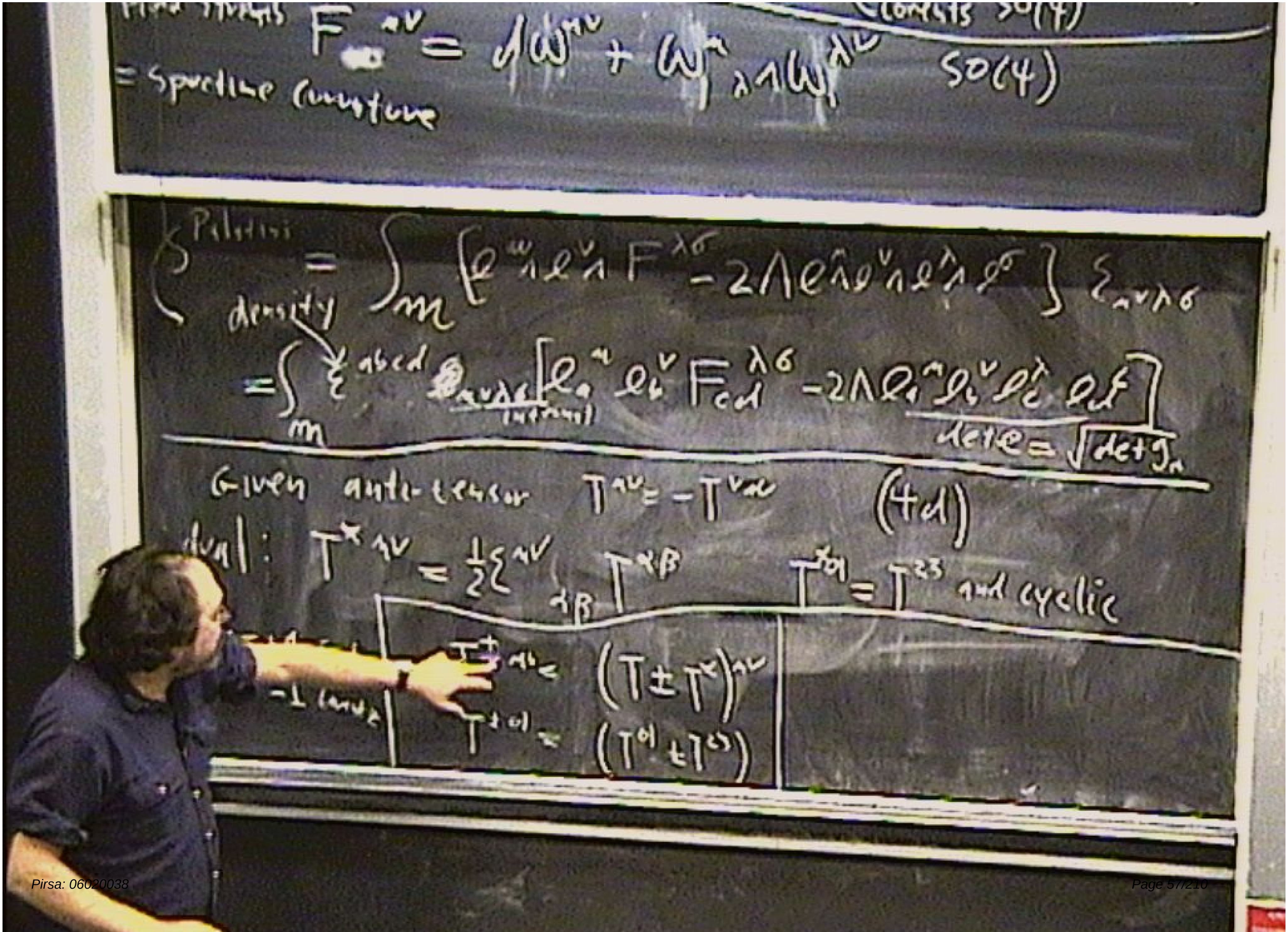
dual: $T^{*\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$ $T^{*01} = T^{23}$ and cyclic

$\epsilon = +1$ Eucl

-1 Lorentz

$T^{\pm\,\mu\nu} = (T \pm T^{*})^{\mu\nu}$

$T^{\pm\,01} = (T^{01} \pm T^{23})$


$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$
= Spacetime curvature
$SO(4)$
$S^{Palatini} = \int_M \{ e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma \} \varepsilon_{\mu\nu\lambda\sigma}$
density
$= \int_M \varepsilon^{abcd} \varepsilon_{\mu\nu\lambda\sigma} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma]$
internal
$\det e = \sqrt{\det g_{\mu\nu}}$
Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$
(4d)
dual: $T^{*\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$
$T^{*01} = T^{23}$ and cyclic
$T^{\pm\mu\nu} = (T \pm T^*)^{\mu\nu}$
$T^{\pm 01} = (T^{01} \pm T^{23})$

$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$ $SO(4)$

= Spacetime Curvature

$S^{\text{Palatini}} = \int_M \{ e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} - 2\Lambda e^{\mu} \wedge e^{\nu} \wedge e^{\lambda} \wedge e^{\sigma} \} \epsilon_{\mu\nu\lambda\sigma}$

density

$= \int_M \epsilon^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} [e_a{}^{\mu} e_b{}^{\nu} F_{cd}{}^{\lambda\sigma} - 2\Lambda e_a{}^{\mu} e_b{}^{\nu} e_c^{\lambda} e_d^{\sigma}]$

$\det e = \sqrt{\det g_{\mu\nu}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$ $T^{*01} = T^{23}$ and cyclic

$** = +1$ Eucl, -1 Lorentz

$T^{\pm\mu\nu} = (T \pm T^*)^{\mu\nu}$

$T^{\pm 01} = (T^{01} \pm T^{23})$

$(T^{\pm\mu\nu})^* = \pm T^{\mu\nu\pm}$ check

$F^{\pm}_{01} = {}' E' \pm B'$

$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B' & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$

R R

Lie algebra $so(4)$

$F^{\pm}_{01} = E^1 \pm B^1$

R R

Lie algebra $SO(4)$

$T^{\mu\nu} = -T^{\nu\mu}$ transform

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_1 & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$$

$F^{\pm}_{0i} = iE^i \pm B^i$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$$

R R

Lie algebra $SO(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$ tracefree

$[T_1, T_2] \in$ anti tracefree

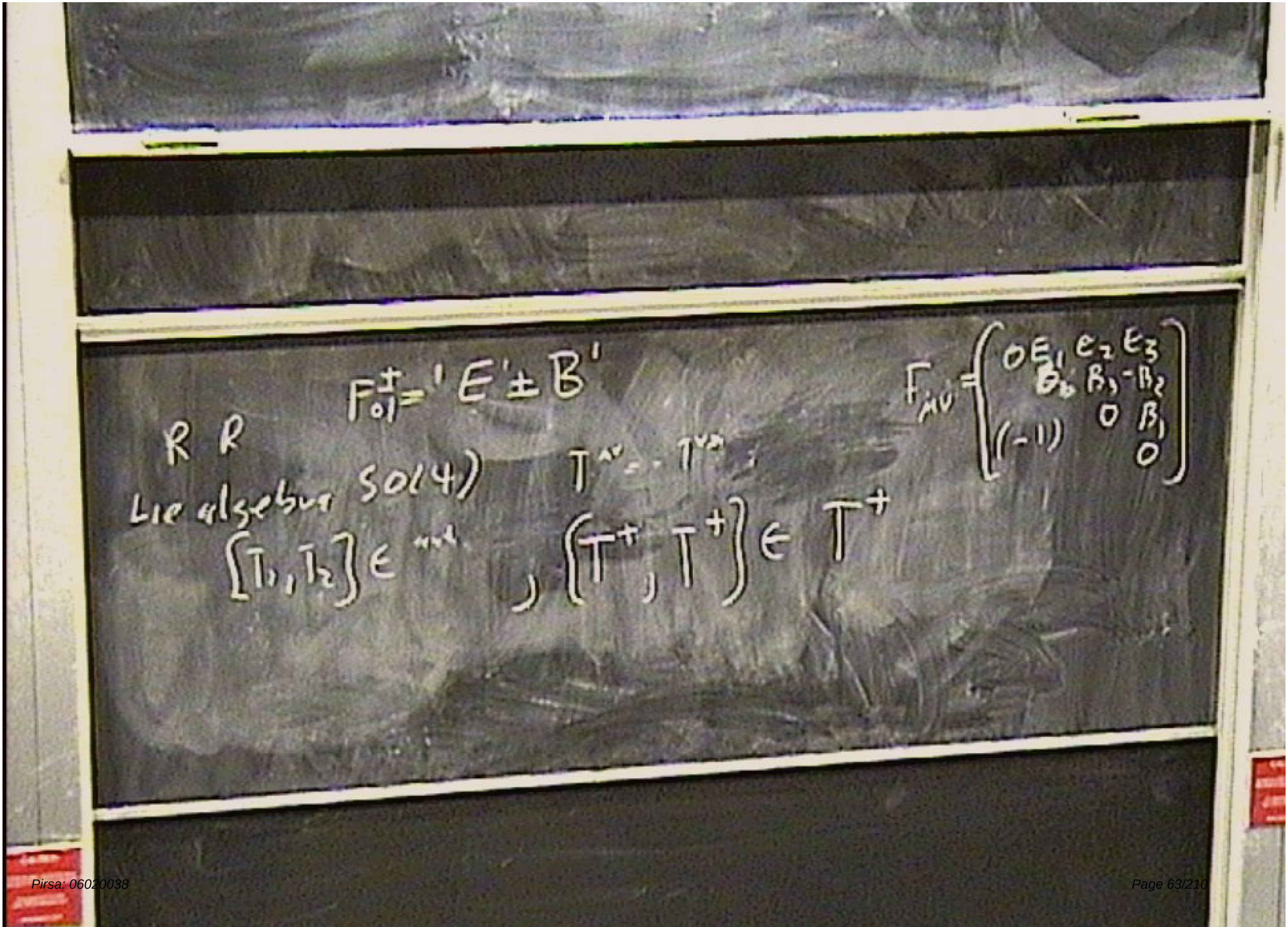
$F^{\pm}_{01} = E^{1} \pm B^{1}$

R R

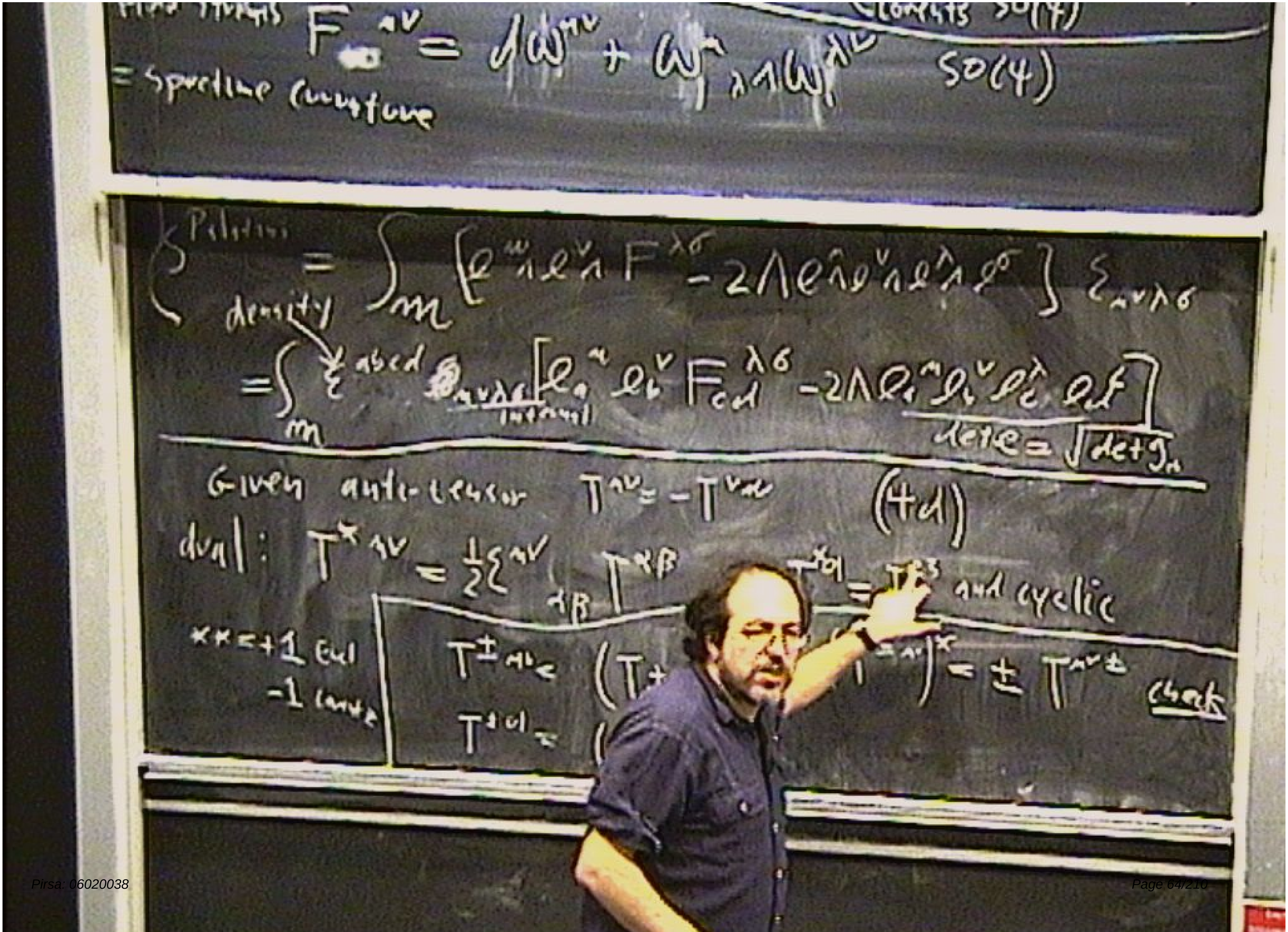
$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

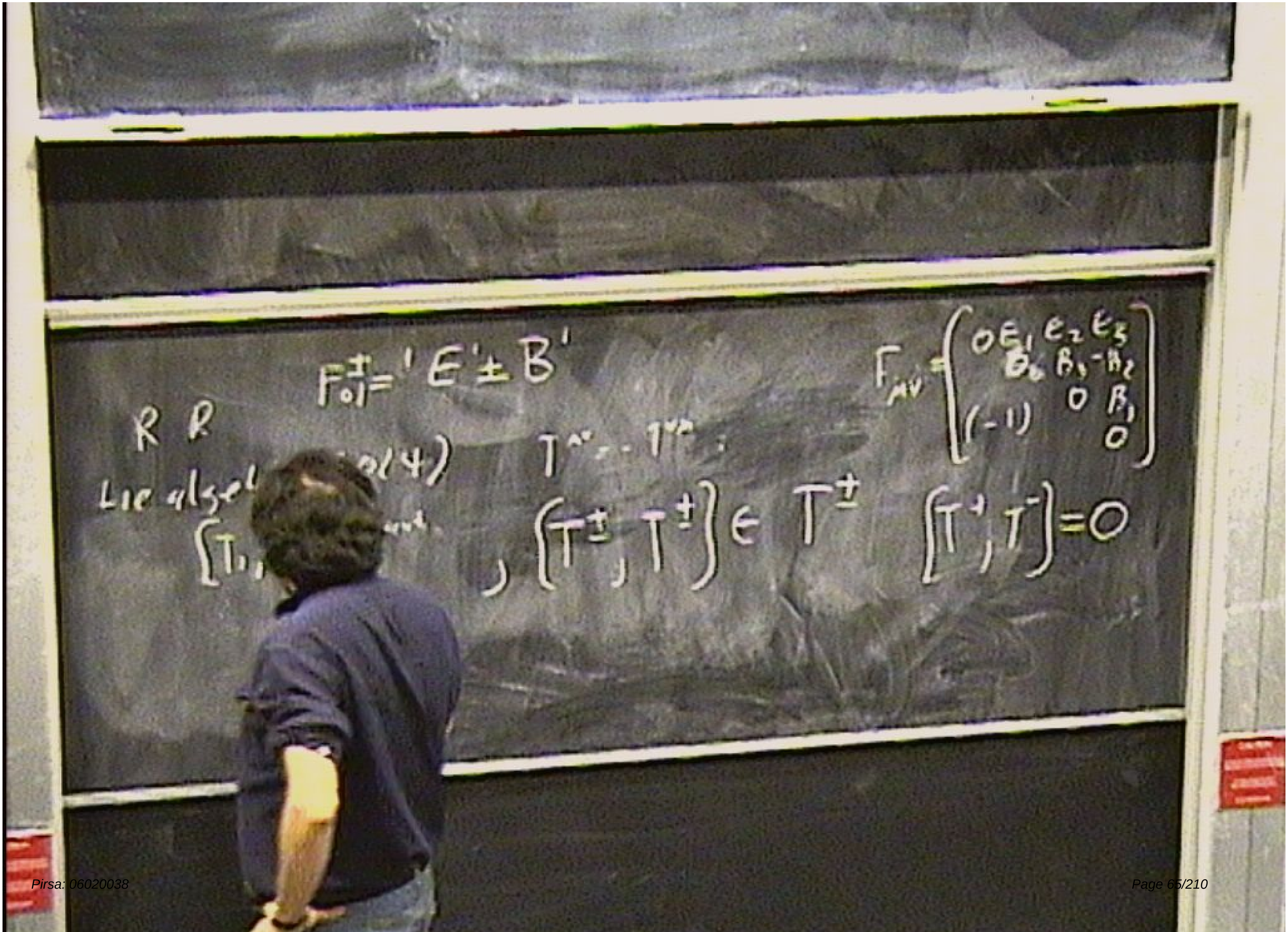
Lie algebra $SO(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ ...



$F^{\pm}_{01} = E' \pm B'$
R R
Lie algebra so(4)
$T^{\mu\nu} = -T^{\nu\mu}$
$[T_1, T_2] \in$
$[T^+, T^+] \in T^+$
$F_{\mu\nu} =$
0 E_1 E_2 E_3
B_3 $-B_2$
0 B_1
0
(-1)


F^{μν} = dω^{μν} + ω^μ_λ ∧ ω^{λν}
SO(4)
= spacetime curvature
Palatini
density
= ∫_M [e^μ ∧ e^ν ∧ F^{λσ} − 2Λ e^μ ∧ e^ν ∧ e^λ ∧ e^σ] ε_{μνλσ}
internal
det e = √(det g)
Given anti-tensor T^{μν} = −T^{νμ}
(4d)
dual: T^{*μν} = ½ ε^{μν}_{αβ} T^{αβ}
and cyclic
** = +1 Eucl
−1 Lorentz
check



F±01 = E1 ± B1
R R
Lie algeb
o(4)
Tμν = - Tνμ
[T±, T±] ∈ T±
[T+, T-] = 0
Fμν =
0 E1 E2 E3
B3 -B2
0 B1
(-1)
0

$$F^{\pm}_{0i} = E^i \pm B^i$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

R R

Lie algebra $so(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ and, $\quad [T^{\pm}, T^{\pm}] \in T^{\pm} \quad [T^{+}, T^{-}] = 0$

$$6 \to 3 + 3$$

$F^{\pm}_{01} = E^1 \pm B^1$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

R R

Lie algebra $so(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ ..., $\quad [T^{\pm}, T^{\pm}] \in T^{\pm} \quad [T^{+}, T^{-}] = 0$

$6 \to 3 + 3$

$SO(4)$

$$F^{\pm}_{0i} = iE^i \pm B^i$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$$

R R

Lie algebra $SO(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ ..., $\quad [T^{\pm}, T^{\pm}] \in T^{\pm} \quad [T^{+}, T^{-}] = 0$

$6 \rightarrow 3 + 3$

$SO(4) \quad SO(3) + SO(3)$

$\quad SU(2) + SU(2)$

[illegible] $F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$ (contains SO(4)) SO(4)

= Spacetime curvature

$$S^{\text{Palatini}} = \int_M \left[e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda\, e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma \right] \epsilon_{\mu\nu\lambda\sigma}$$

density

$$= \int_M \tilde{\epsilon}^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} \left[e_a^{\mu} e_b^{\nu} F_{cd}^{\lambda\sigma} - 2\Lambda\, \underbrace{e_a^{\mu} e_b^{\nu} e_c^{\lambda} e_d^{\sigma}}_{\det e = \sqrt{\det g_{ab}}} \right]$$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$ $T^{*01} = T^{23}$ and cyclic

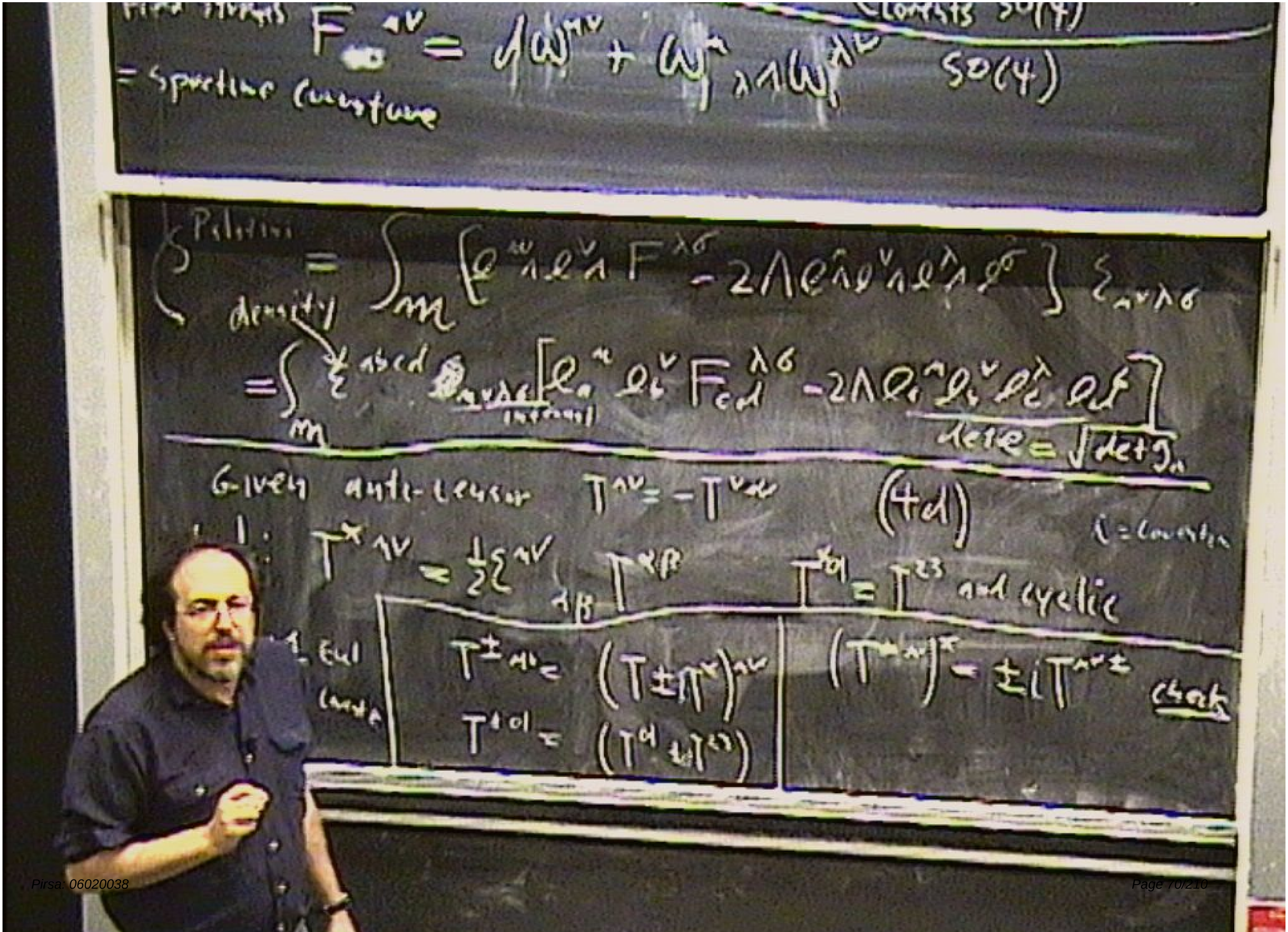
$** = +1$ Eucl

-1 Lorentz

$T^{\pm\mu\nu} = (T \pm T^*)^{\mu\nu}$

$T^{\pm 01} = (T^{01} \pm T^{23})$

$(T^{\pm\mu\nu})^* = \pm T^{\mu\nu\pm}$ check


= Spacetime Curvature
SO(4)
Palatini
density
internal
Given anti-tensor
(4d)
and cyclic
check

$F^{\pm}_{01} = E^1 \pm B^1$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

$R \quad R$

Lie algebra $SO(4)$ $\quad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ ..., $\quad [T^{\pm}, T^{\pm}] \in T^{\pm} \quad [T^+, T^-] = 0$

$6 \rightarrow 3 + 3$

$SO(4) \quad SO(3) + SO(3)$

$SU(2) + SU(2)$

$F^{\pm}_{01} = E^1 \pm i B^1$ $\qquad T^{+}_{\mu\nu} = (T_{\mu\nu})^{-}$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$$

R R

Lie algebra $SO(4)$ $\qquad T^{\mu\nu} = -T^{\nu\mu}$

$[T_1, T_2] \in$ $\qquad [T^{\pm}, T^{\pm}] \in T^{\pm}$ $\qquad [T^{+}, T^{-}] = 0$

$6 \rightarrow 3 + 3$

$SO(4)$ $\qquad SO(3) + SO(3)$

$\qquad SU(2) + SU(2)$

$$S^{\text{Palatini}}_{\text{density}} = \int_M \left[e^\mu \wedge e^\nu \wedge F^{\lambda\sigma} - 2\Lambda e^\mu \wedge e^\nu \wedge e^\lambda \wedge e^\sigma \right] \epsilon_{\mu\nu\lambda\sigma}$$

$$= \int_M \epsilon^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{internal}} \left[e_a{}^\mu e_b{}^\nu F_{cd}{}^{\lambda\sigma} - 2\Lambda e_a{}^\mu e_b{}^\nu e_c^\lambda e_d^\sigma \right]$$

$$\det e = \sqrt{\det g_{\mu\nu}}$$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

Λ = Constant

dual: $T^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta} T^{\alpha\beta}$ $\quad T^{*01} = T^{23}$ and cyclic

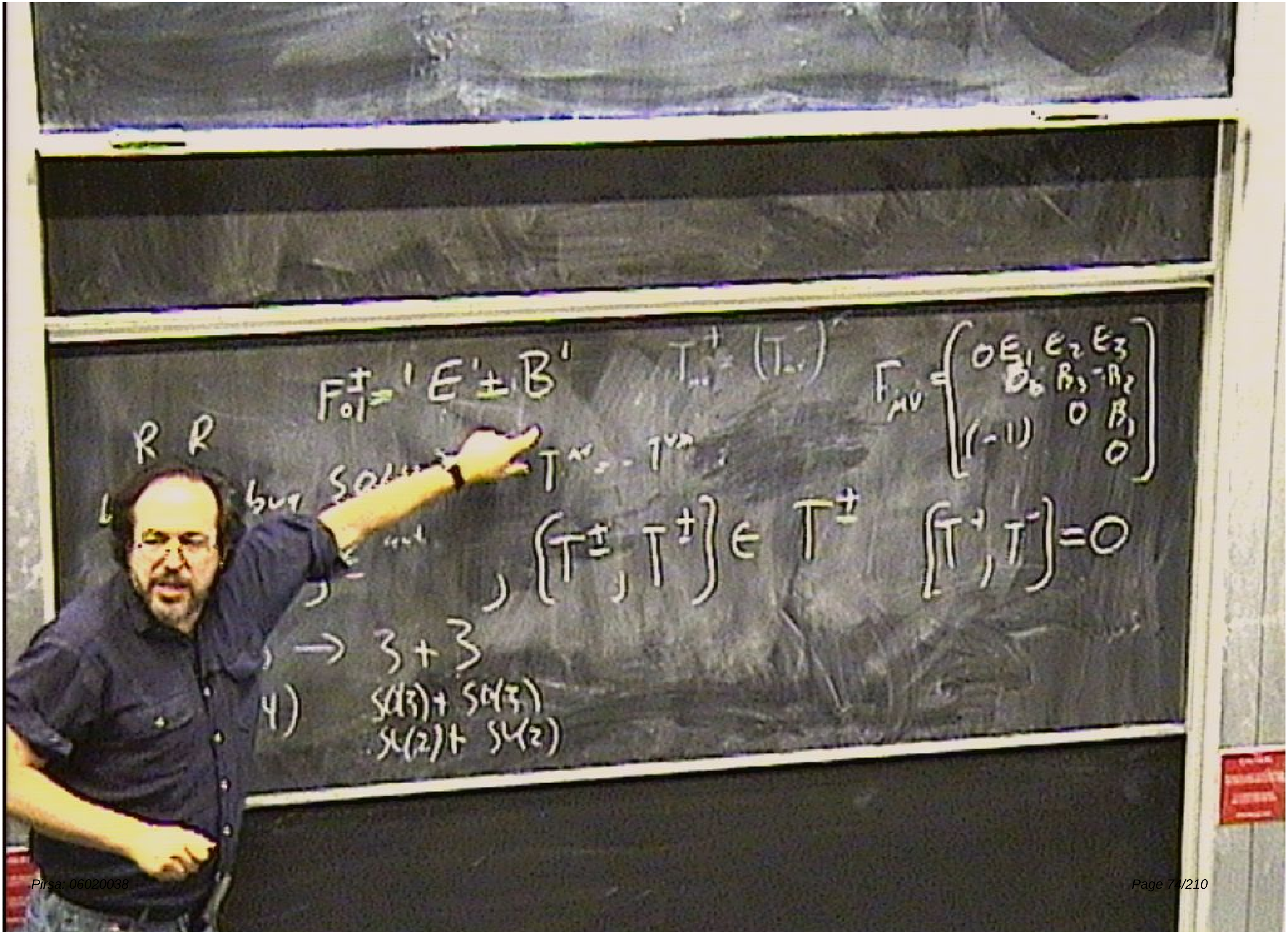
$** = +1$ Eucl

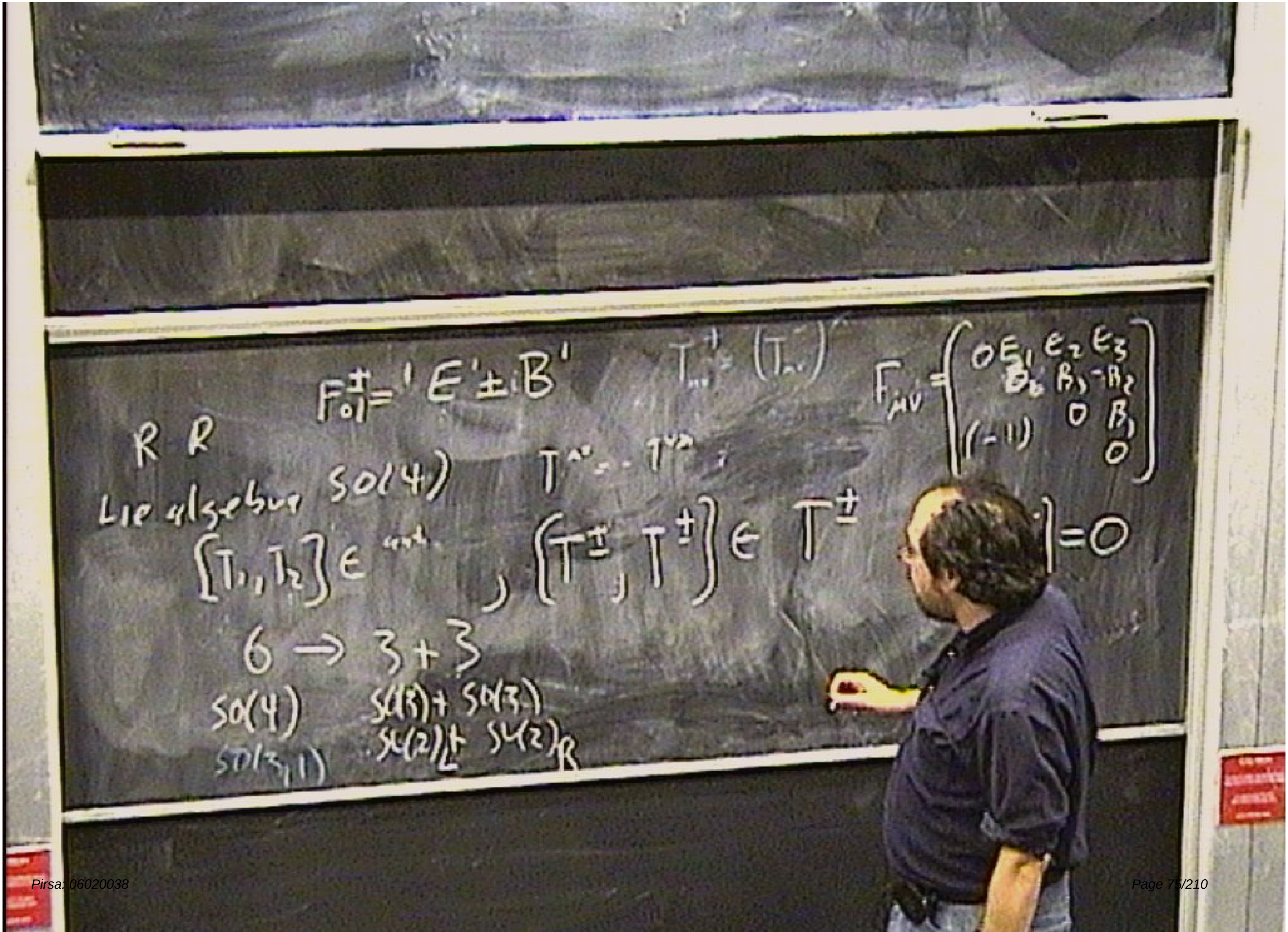
-1 Lorentz

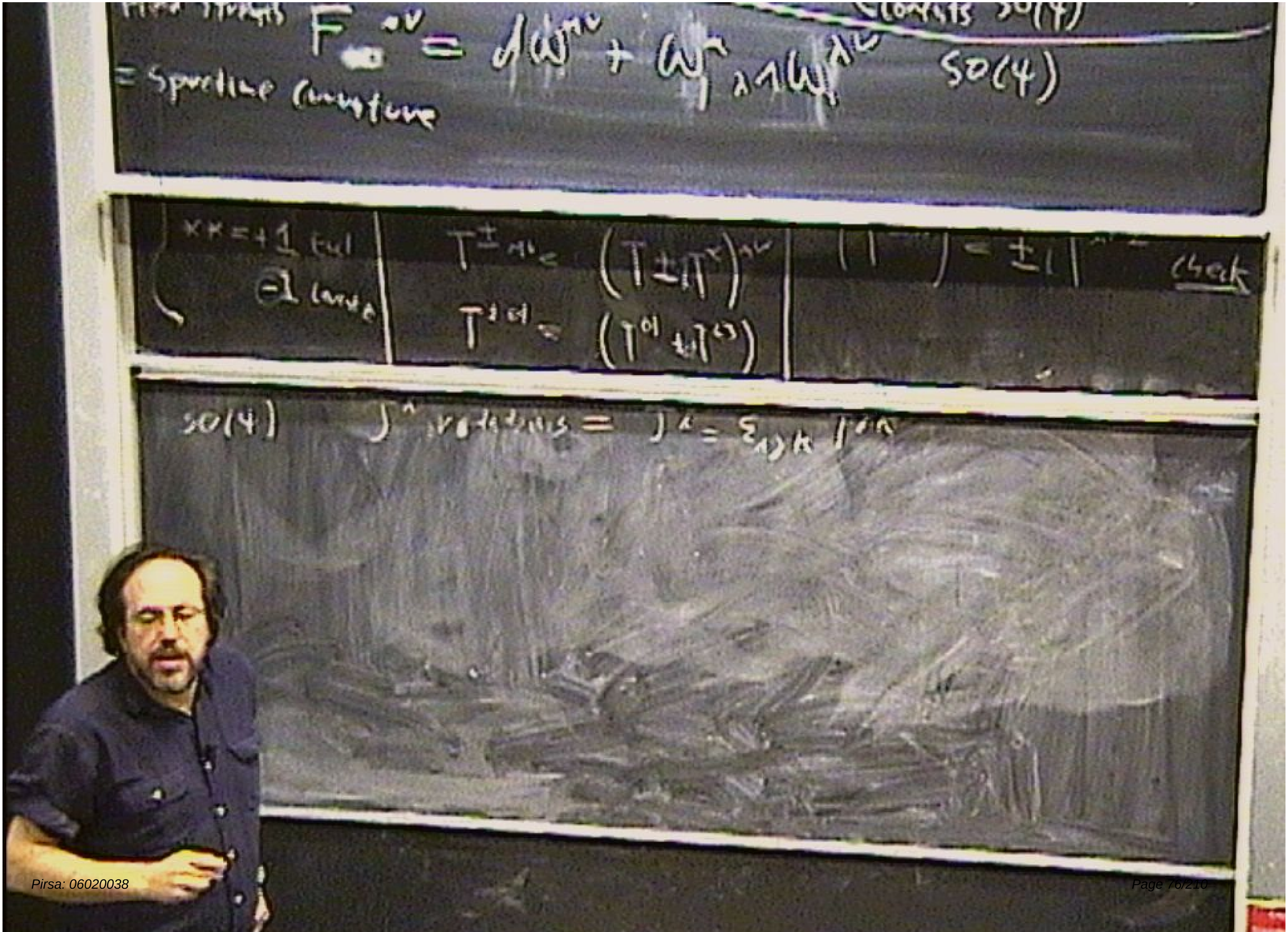
$T^{\pm\mu\nu} = (T \pm i T^*)^{\mu\nu}$

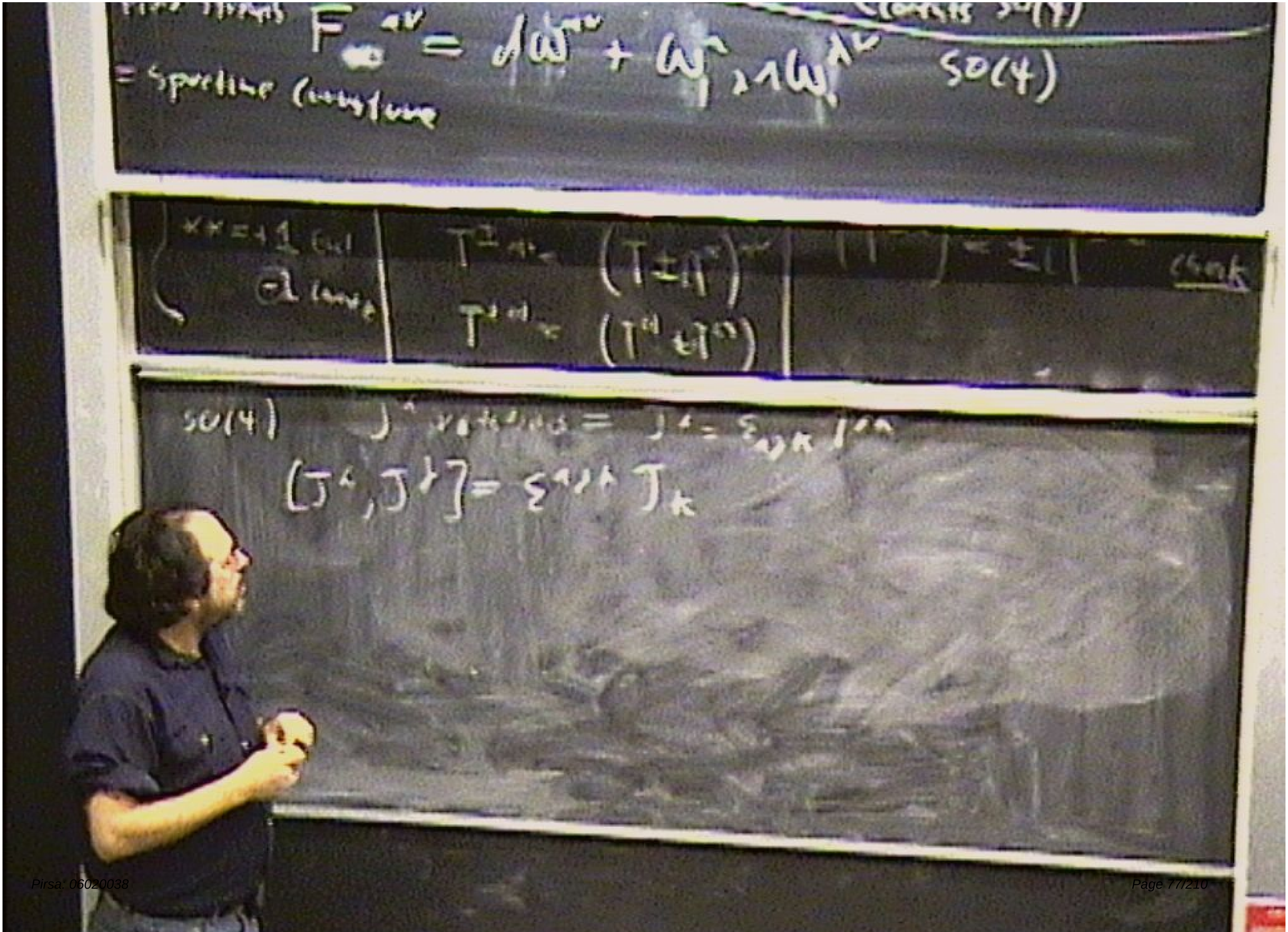
$T^{\pm 01} = (T^{01} \pm i T^{23})$

$(T^{\pm\mu\nu})^* = \pm i T^{\mu\nu\pm}$ check


R R
$F^{\pm}_{0i} = E^i \pm B^i$
$T^{\mu\nu} = -T^{\nu\mu}$
$[T^{\pm}, T^{\pm}] \in T^{\pm}$
$[T^+, T^-] = 0$
$\rightarrow 3 + 3$
SU(3) + SU(3)
SU(2) + SU(2)


$F^{\pm}_{0i} = E^i \pm iB^i$
$T^{\pm}_{\mu\nu} = (T_{\mu\nu})$
$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & 0 & B_3 & -B_2 \\ & & 0 & B_1 \\ (-1) & & & 0 \end{pmatrix}$
R R
Lie algebra SO(4)
$T^{\mu\nu} = -T^{\nu\mu}$
$[T_1, T_2] \in$
$[T^{\pm}, T^{\pm}] \in T^{\pm}$
$] = 0$
$6 \to 3 + 3$
SO(4) SO(3) + SO(3)
SO(3,1) $SU(2)_L + SU(2)_R$


SO(4)
= Spacetime Curvature
check
SO(4)



= Spacetime curvature
SO(4)
SO(4)
[J^i, J^j] = ε^{ijk} J_k

field strengths $F^{ab} = d\omega^{ab} + \omega^{a}{}_{\lambda} \wedge \omega^{\lambda b}$ (contains $SO(4)$)

= Spacetime curvature $SO(4)$

$** = +1$ Eucl

-1 Lorentz

$T^{\pm\,ab} = \left(T \pm i T^{*}\right)^{ab}$

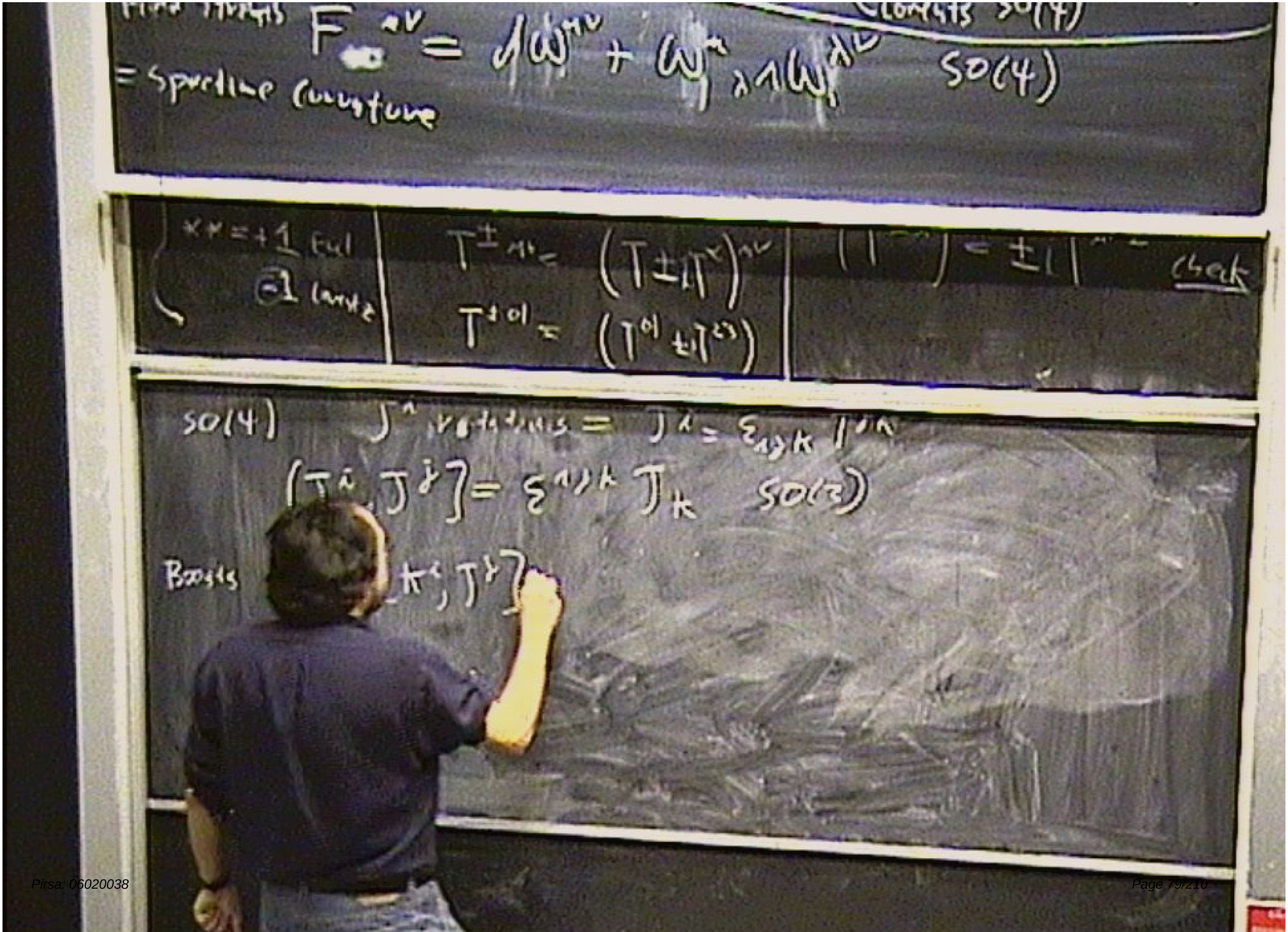
$T^{\pm\,01} = \left(T^{01} \pm i T^{23}\right)$

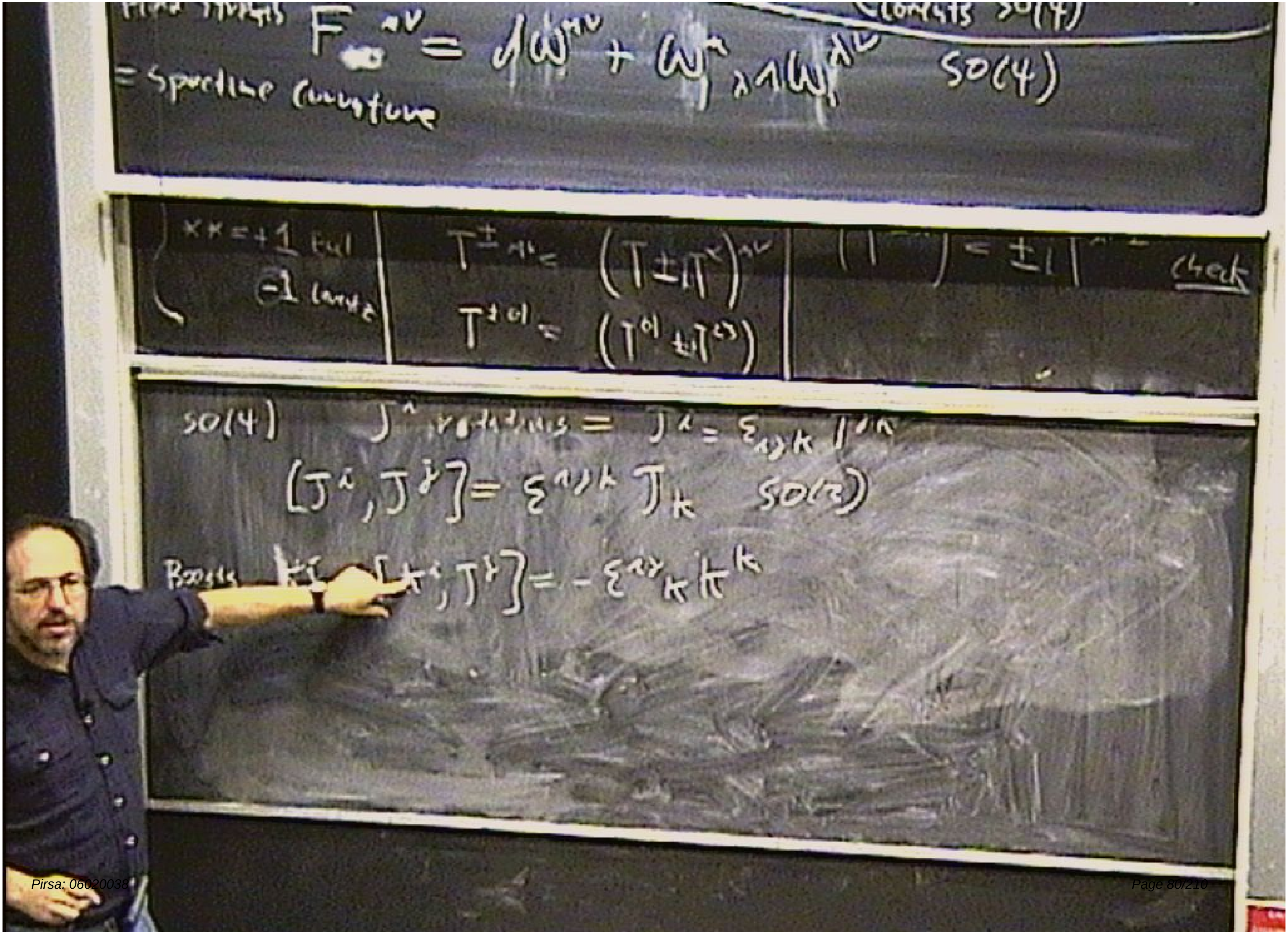
check

$SO(4)$ J^{i} rotations $= J^{i} = \varepsilon_{ijk} J^{jk}$

$[J^{i}, J^{j}] = \varepsilon^{ijk} J_{k}$ $SO(3)$

B


F^{μν} = dω^{μν} + ω^μ_λ ∧ ω^{λν}
= Spacetime Curvature
SO(4)
T^±μν = (T ± i*T)^μν
T^±01 = (T^01 ± iT^23)
check
SO(4)
[J^i, J^j] = ε^ijk J_k SO(3)
Boosts
[K^i, J^j]


= Spacetime curvature
SO(4)
check
SO(4)
SO(3)
Boosts

$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$ $SO(4)$

= Spacetime curvature

$** = +1$ Eucl

-1 Lorentz

$T^{\pm\,\mu\nu} = (T \pm i T^{*})^{\mu\nu}$

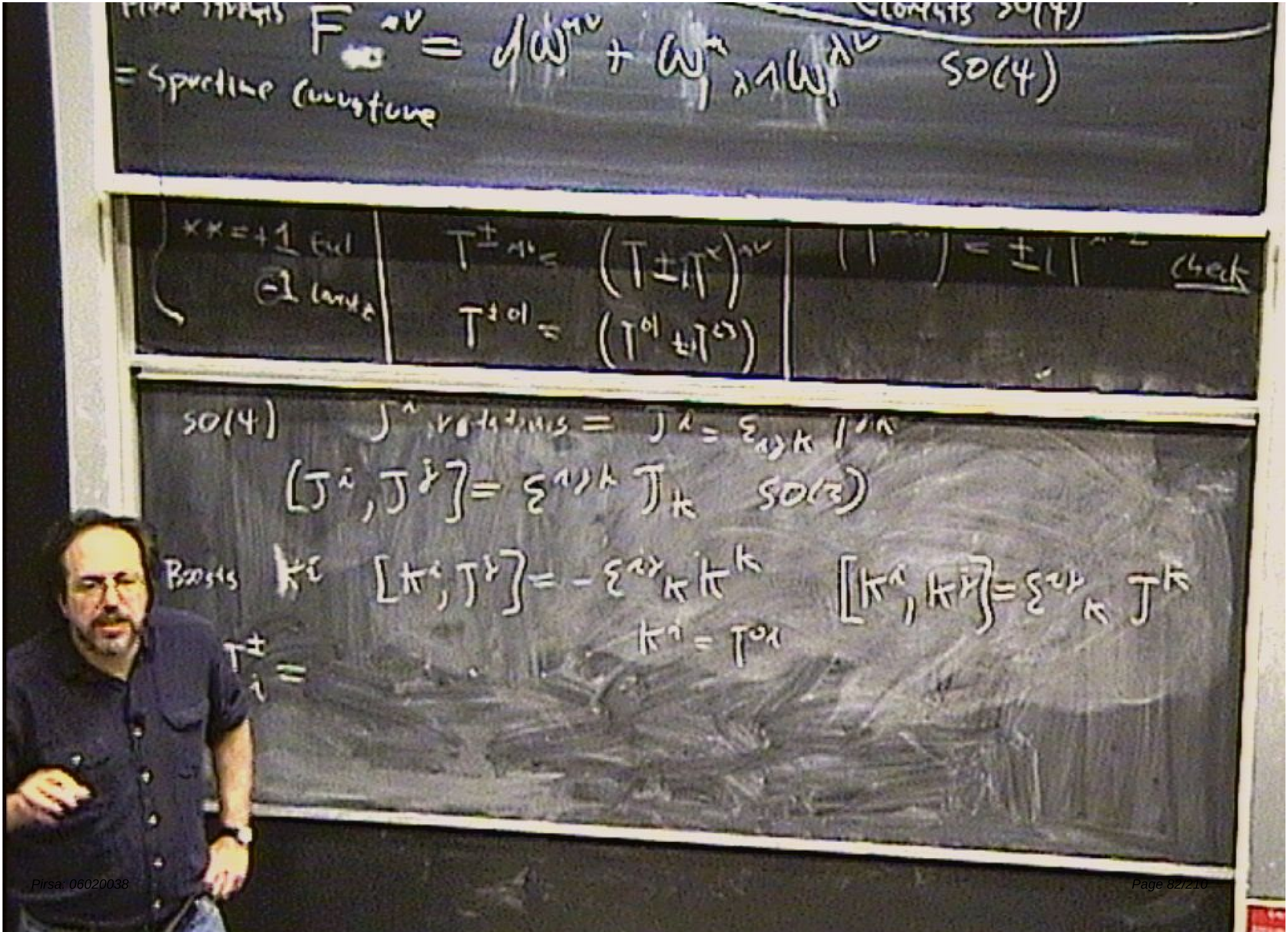
$T^{\pm\,01} = (T^{01} \pm i T^{23})$

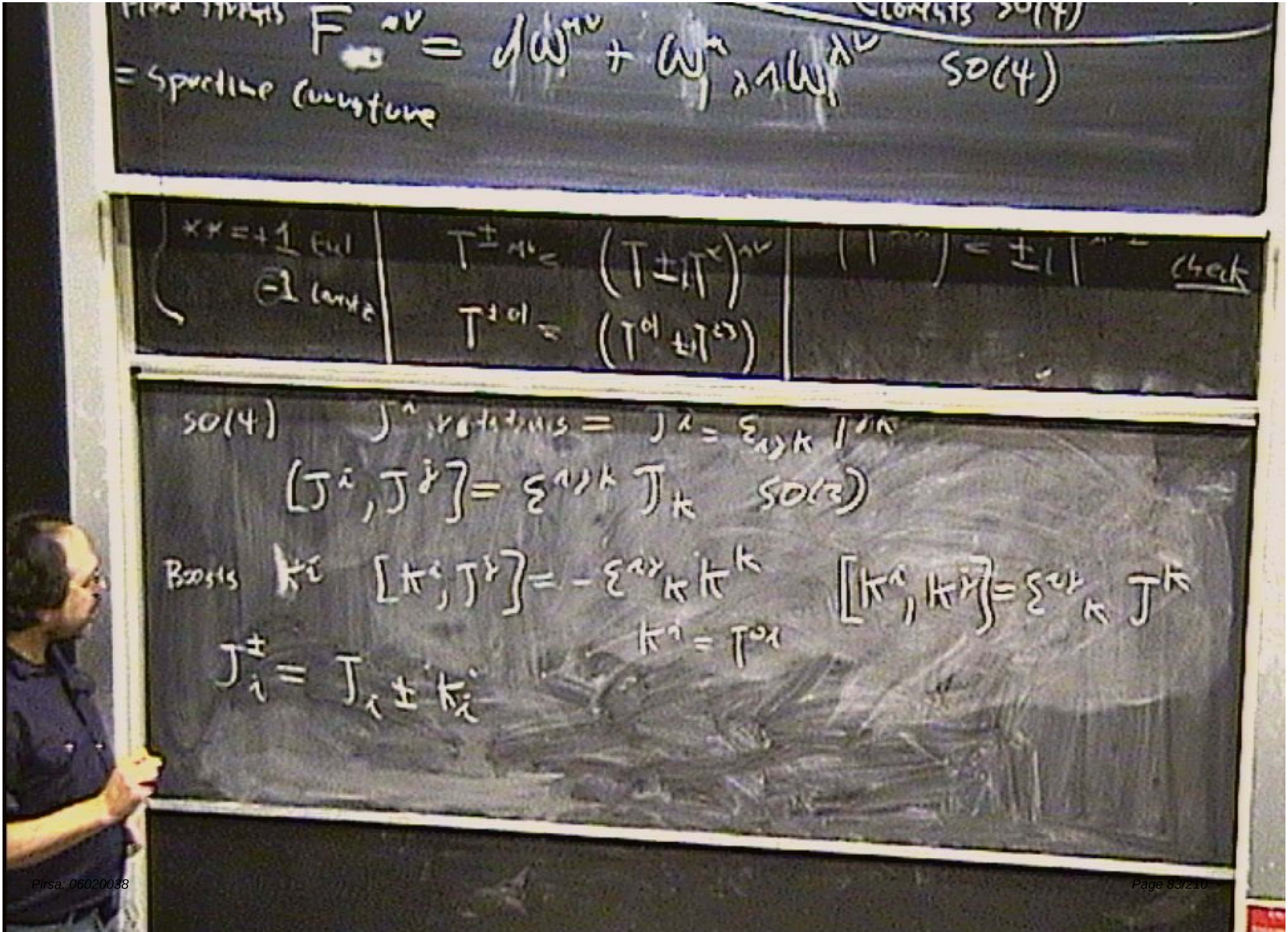
check

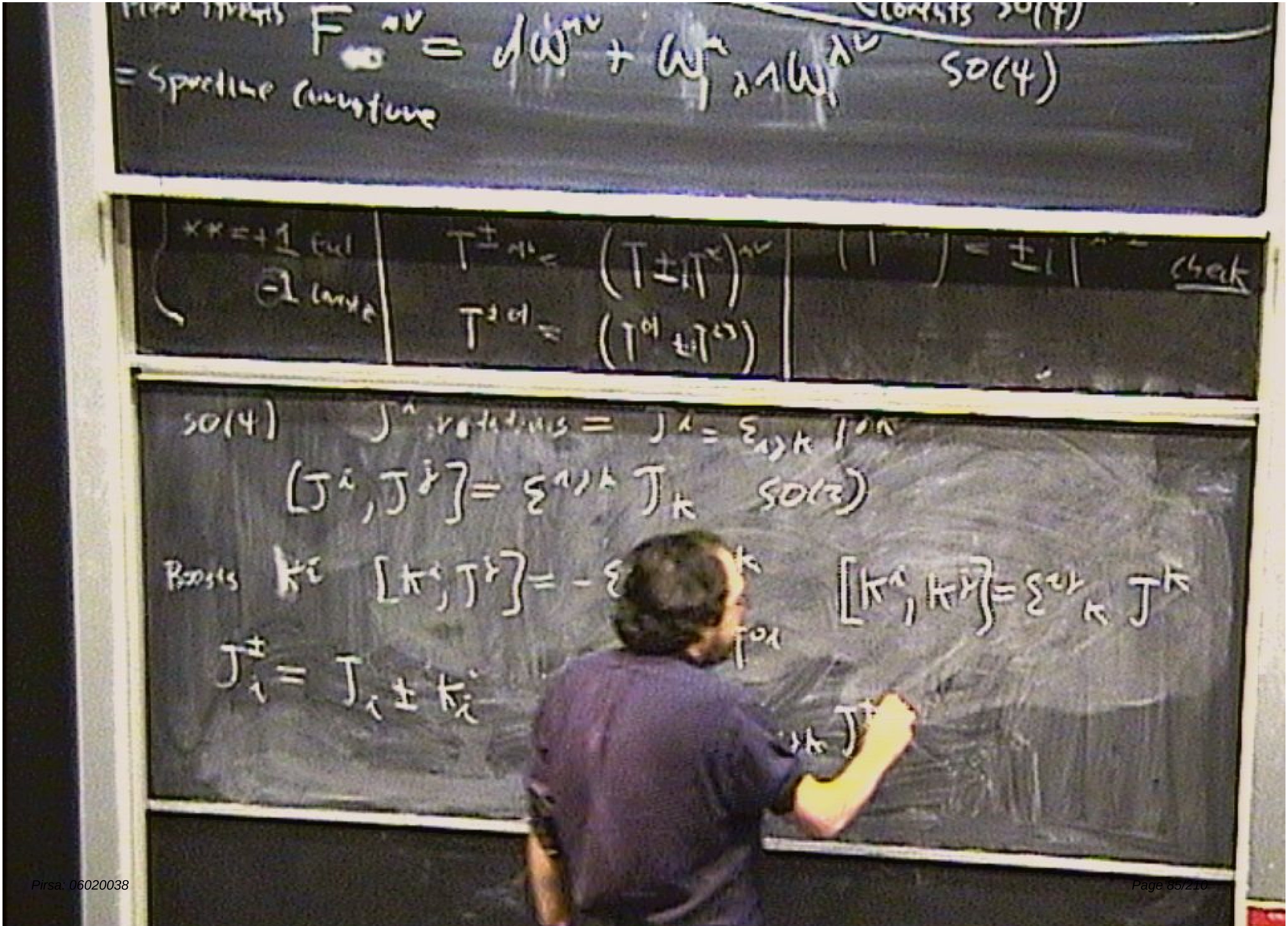
$SO(4)$ J^{i} rotations $= J^{i} = \varepsilon_{ijk} J^{jk}$

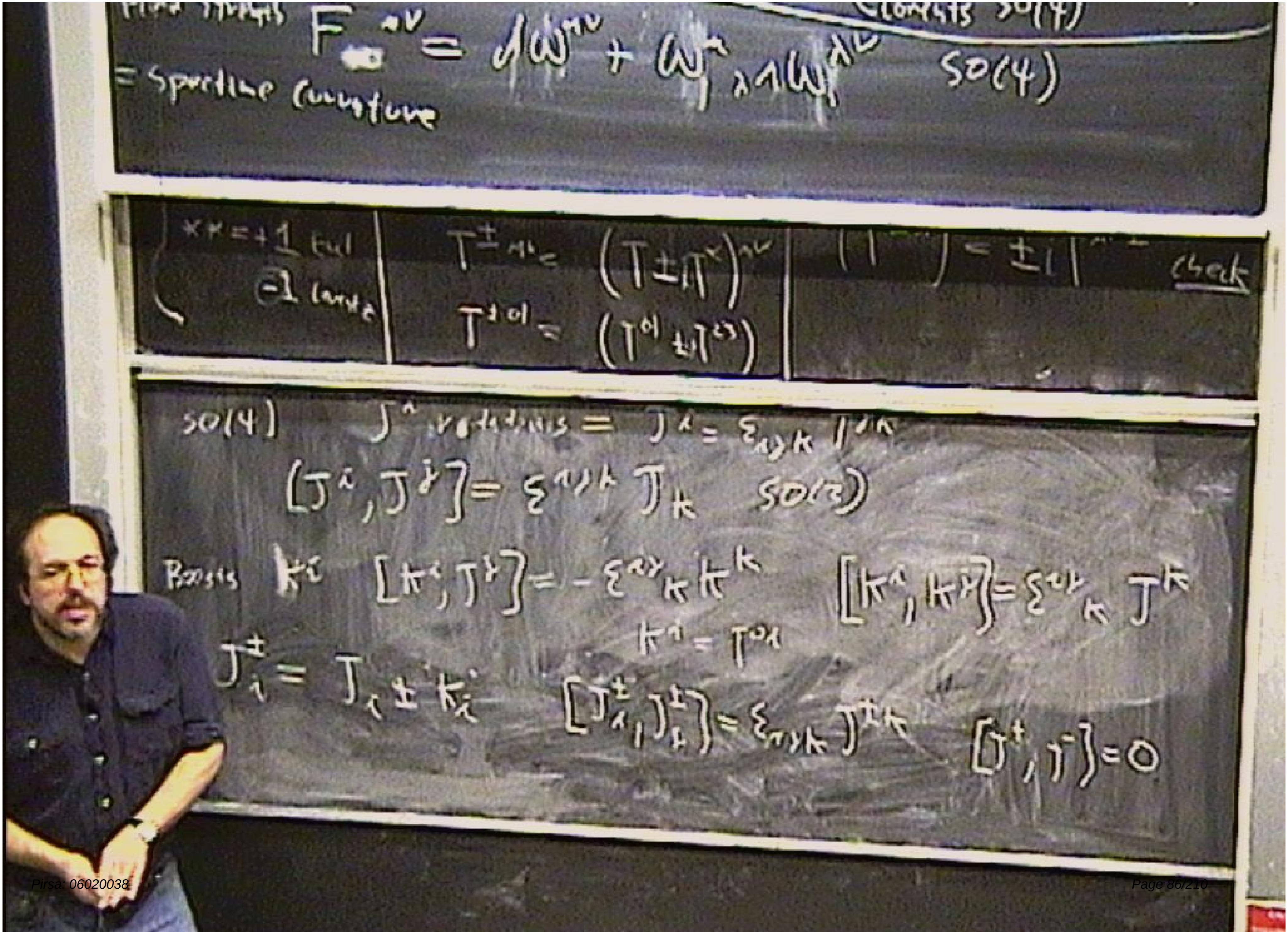
$[J^{i}, J^{j}] = \varepsilon^{ijk} J_{k}$ $SO(3)$

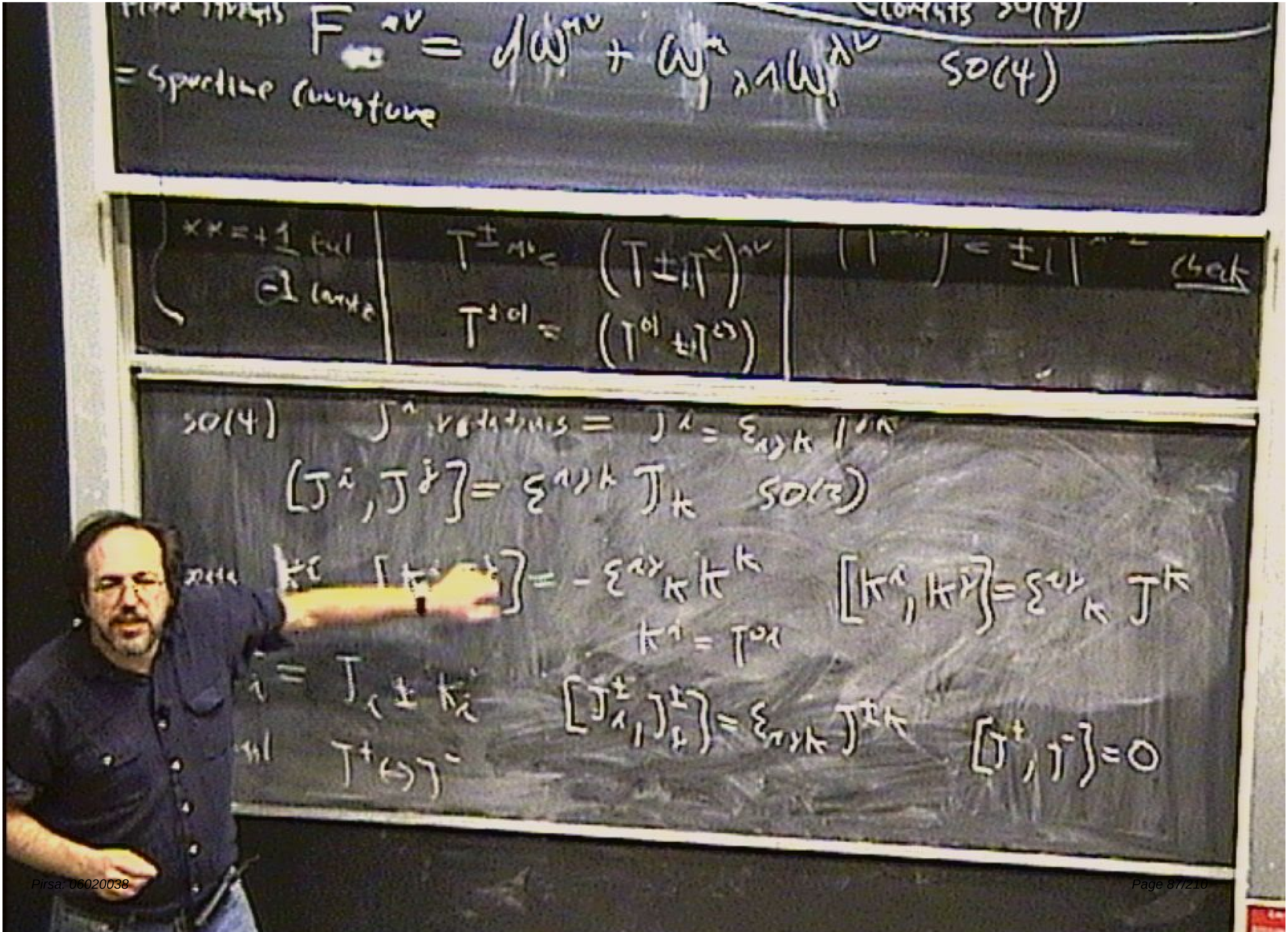
Boosts K^{i} $[K^{i}, J^{j}] = -\varepsilon^{ij}{}_{k} K^{k}$ $[K^{i}, K^{j}] = \varepsilon^{ij}{}_{k} J^{k}$


= Spacetime curvature
SO(4)
Check
SO(4)
Boosts
SO(3)


F = dω + ω ∧ ω
= Spacetime curvature
SO(4)
∗∗ = +1 Eucl
-1 Lorentz
T± = (T ± ∗T)
Check
SO(4)
rotations
[J, J] = ε J SO(3)
Boosts
[K, J] = − ε K
[K, K] = ε J
J± = J ± K


$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$
SO(4)
= Spacetime curvature
$** = +1$ Eucl
-1 Lorentz
$T^{\pm\,\mu\nu} = (T \pm i\,{}^*T)^{\mu\nu}$
$T^{\pm\,01} = (T^{01} \pm i T^{23})$
check
SO(4)
$J^i = \epsilon_{ijk} J^{jk}$
$[J^i, J^j] = \epsilon^{ijk} J_k$ SO(3)
Boosts K^i
$[K^i, K^j] = \epsilon^{ij}{}_k J^k$
$J^\pm_i = J_i \pm K_i$


= Spacetime curvature
SO(4)
check
SO(4)
SO(2)
Basis


= Spacetime curvature
SO(4)
SO(4)
check
SO(4)
SO(3)
[J^+, J^-] = 0

$$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$$

= Spacetime Curvature

SO(4)

$\epsilon\epsilon = +1$ full
-1 [illegible]

$$T^{\pm\mu\nu} = \left(T \pm T^{*}\right)^{\mu\nu}$$

$$T^{\pm 0i} = \left(T^{0i} \pm T^{*0i}\right)$$

$(\;\;) = \pm(\;\;)$

Check

SO(4) J^i rotations $= J^i = \epsilon_{ijk} J^{jk}$

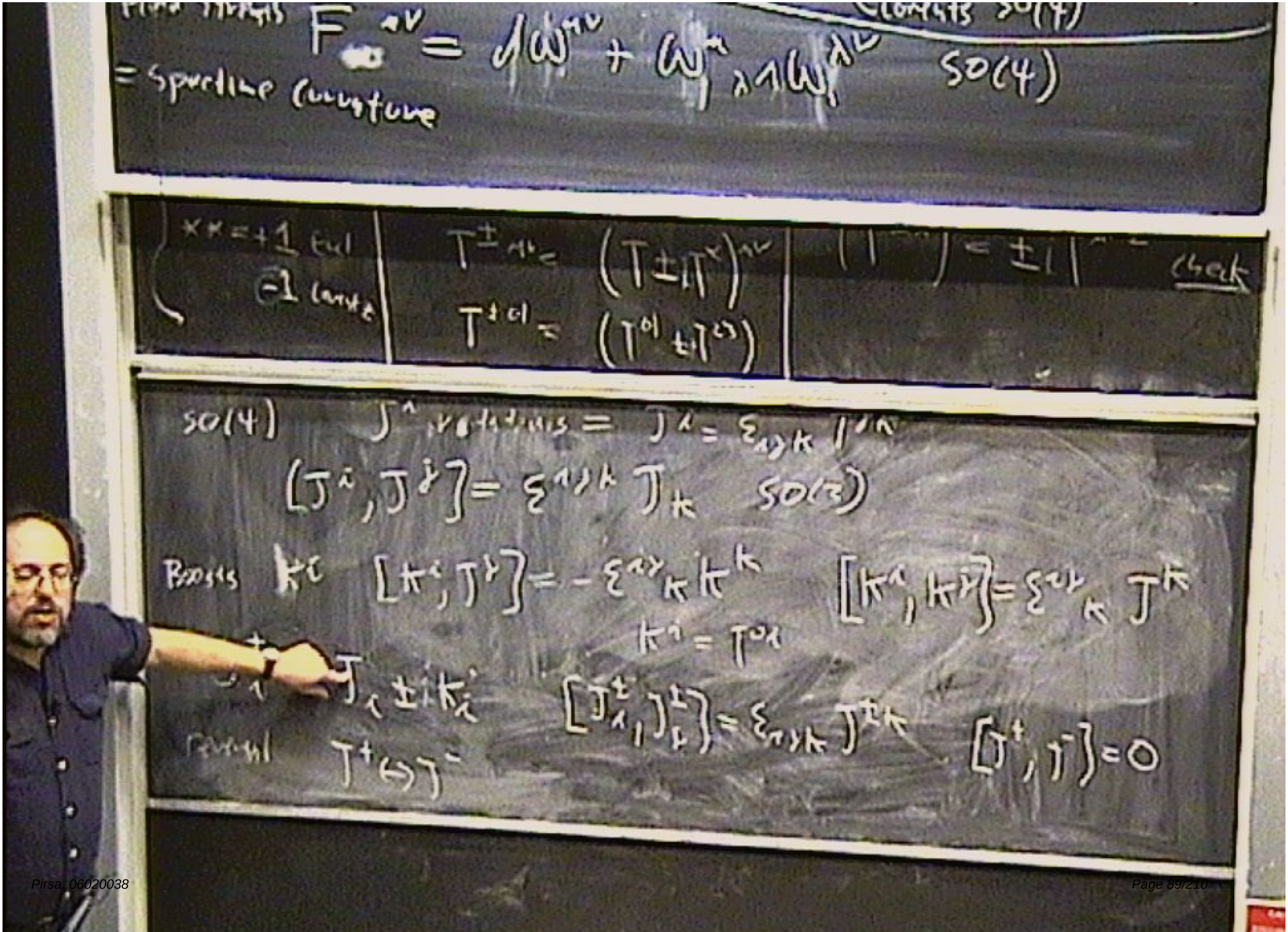
$$[J^i, J^j] = \epsilon^{ijk} J_k \quad SO(3)$$

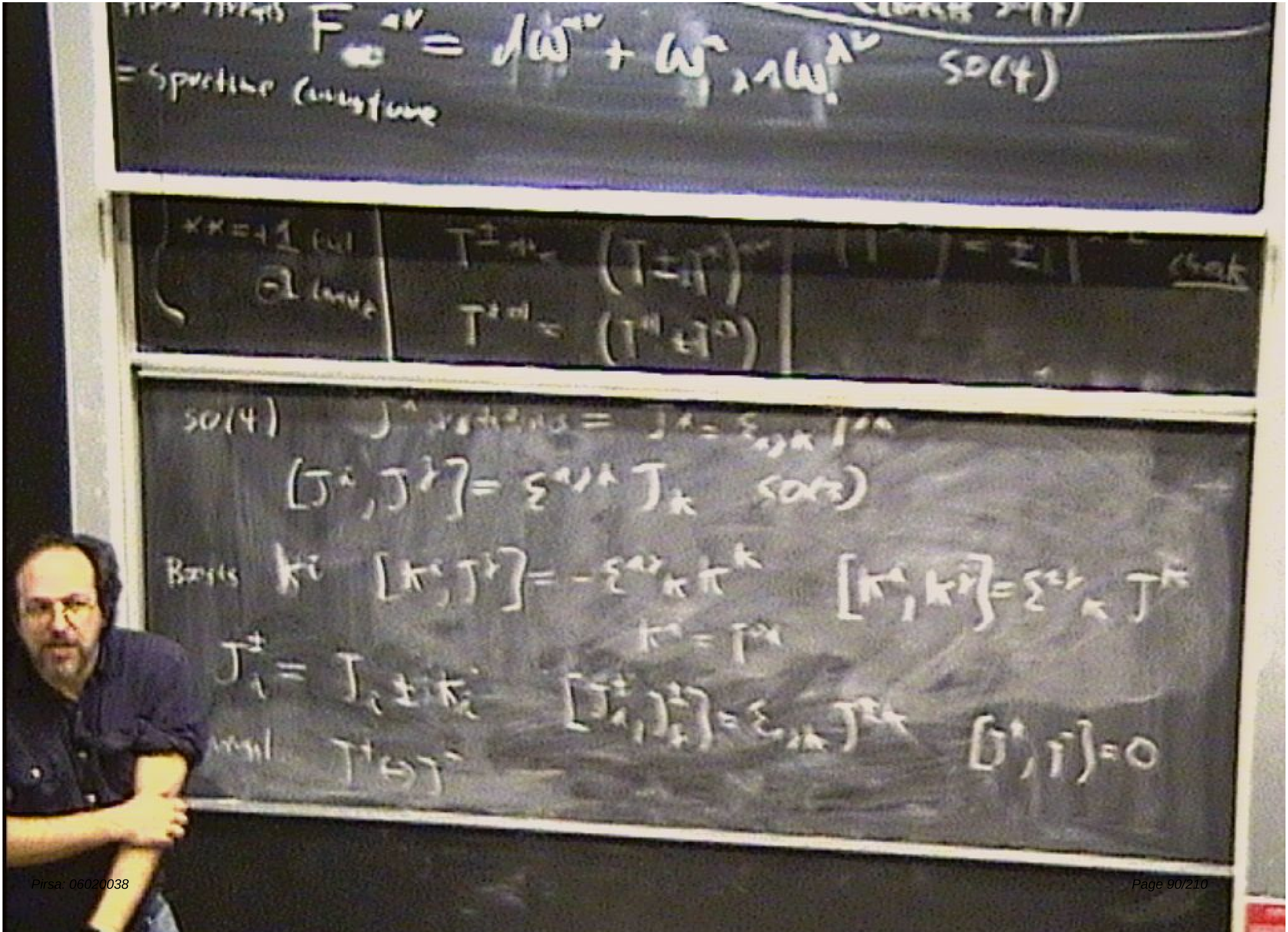
Boosts K^i $\quad [K^i, J^j] = -\epsilon^{ij}{}_{k} K^k \quad [K^i, K^j] = \epsilon^{ij}{}_{k} J^k$

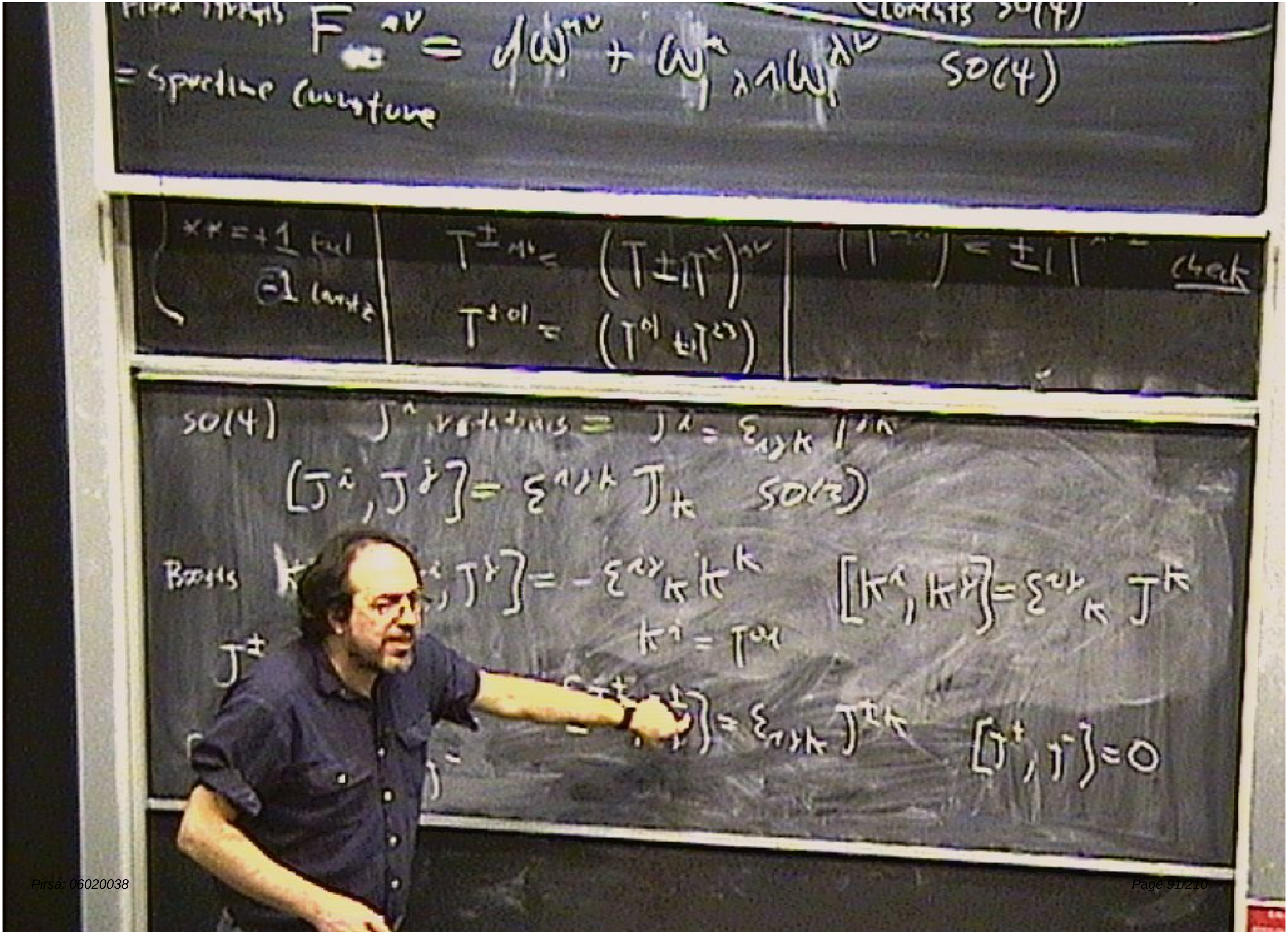
$$K^i = T^{0i}$$

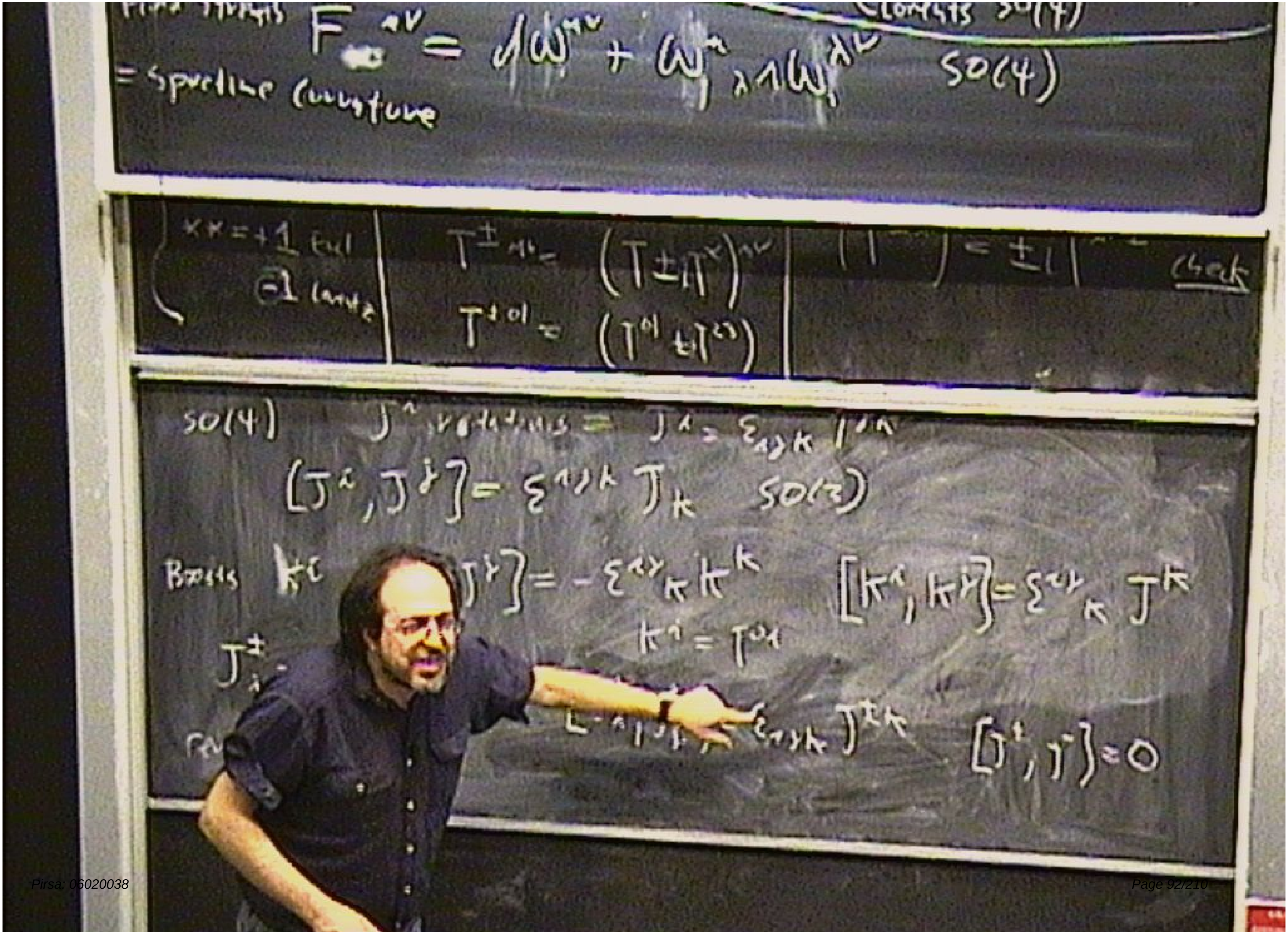
$$J^{\pm}_i = J_i \pm i K_i \quad [J^{\pm}_i, J^{\pm}_j] = \epsilon_{ijk} J^{\pm k}$$

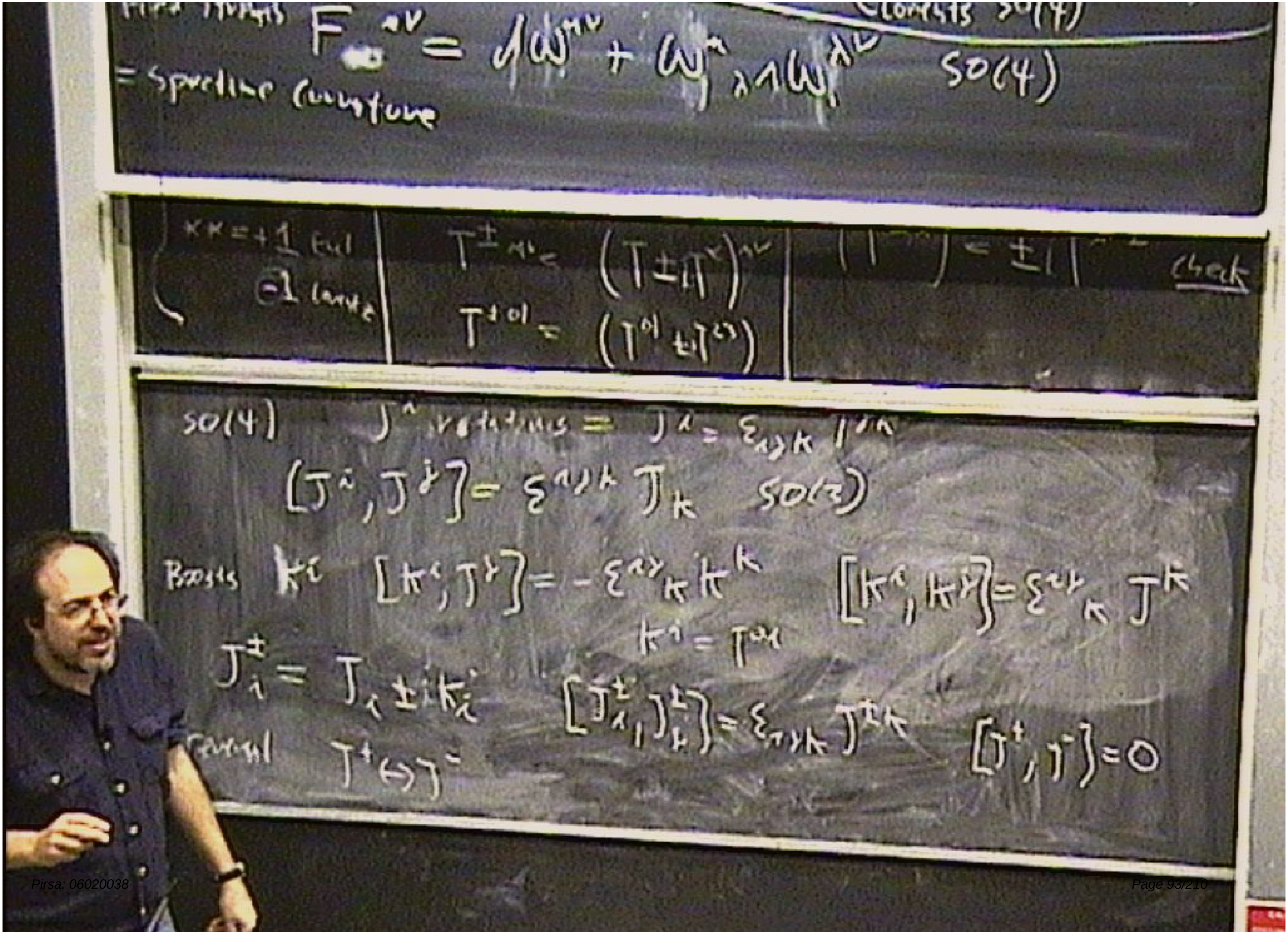
Parity $\quad J^+ \leftrightarrow J^- \quad [J^+, J^-] = 0$


F^μν = dω^μν + ω^μ_λ ∧ ω^λν
= Spacetime curvature
SO(4)
** = +1 Eucl
-1 Lorentz
T^± μν = (T ± i *T)^μν
T^±0i = (T^0i ± i T^jk)
check
SO(4)
J^i rotations = J^i = ε_ijk T^jk
[J^i, J^j] = ε^ijk J_k SO(3)
Boosts K^i [K^i, J^j] = -ε^ij_k K^k
[K^i, K^j] = ε^ij_k J^k
K^i = T^0i
J_i ± i K_i
[J^±_i, J^±_j] = ε_ijk J^±k
[J^+, J^-] = 0
Parity T^+ ↔ T^-

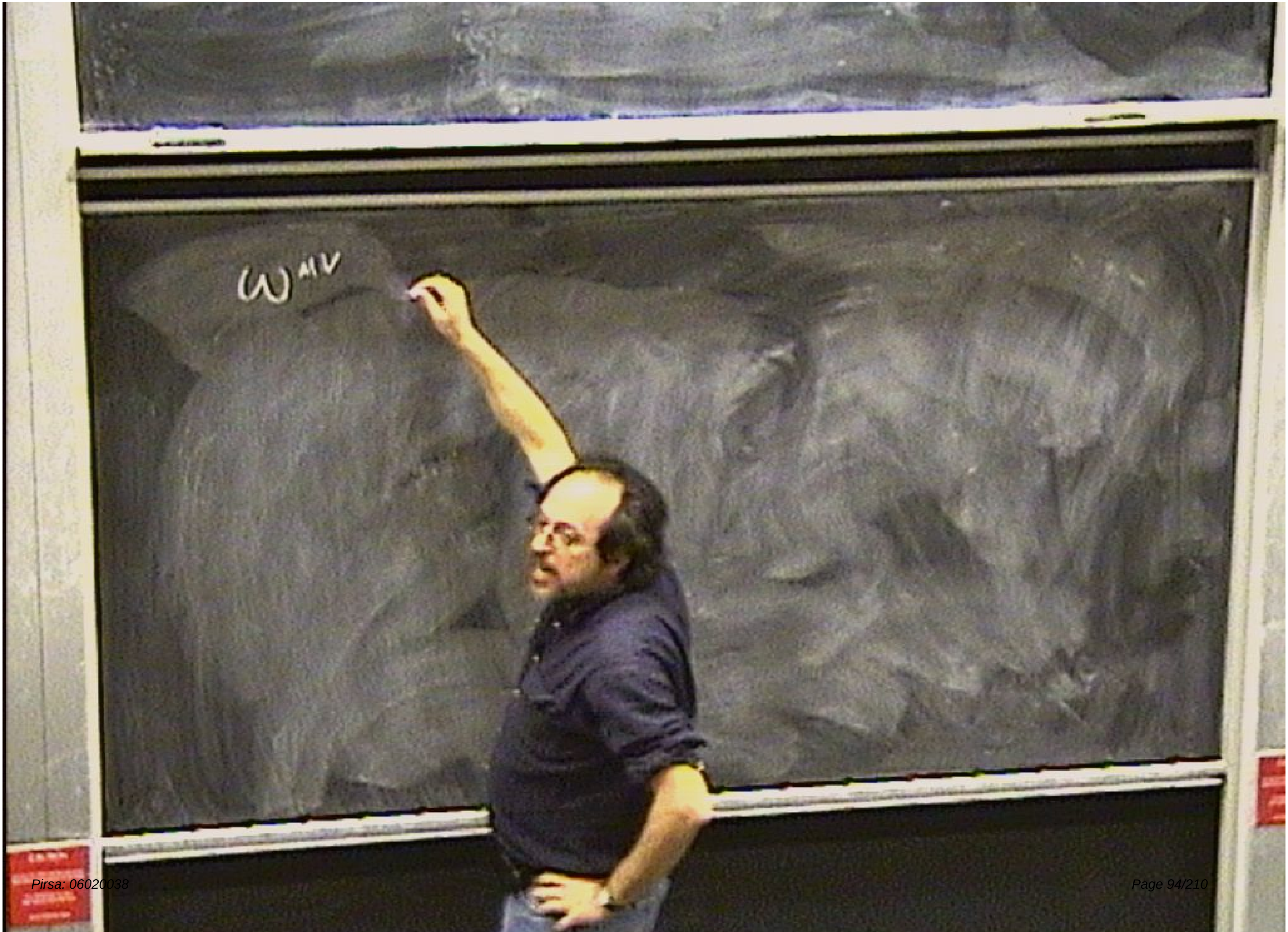

= Spacetime curvature
SO(4)
SO(4)
Basis
check

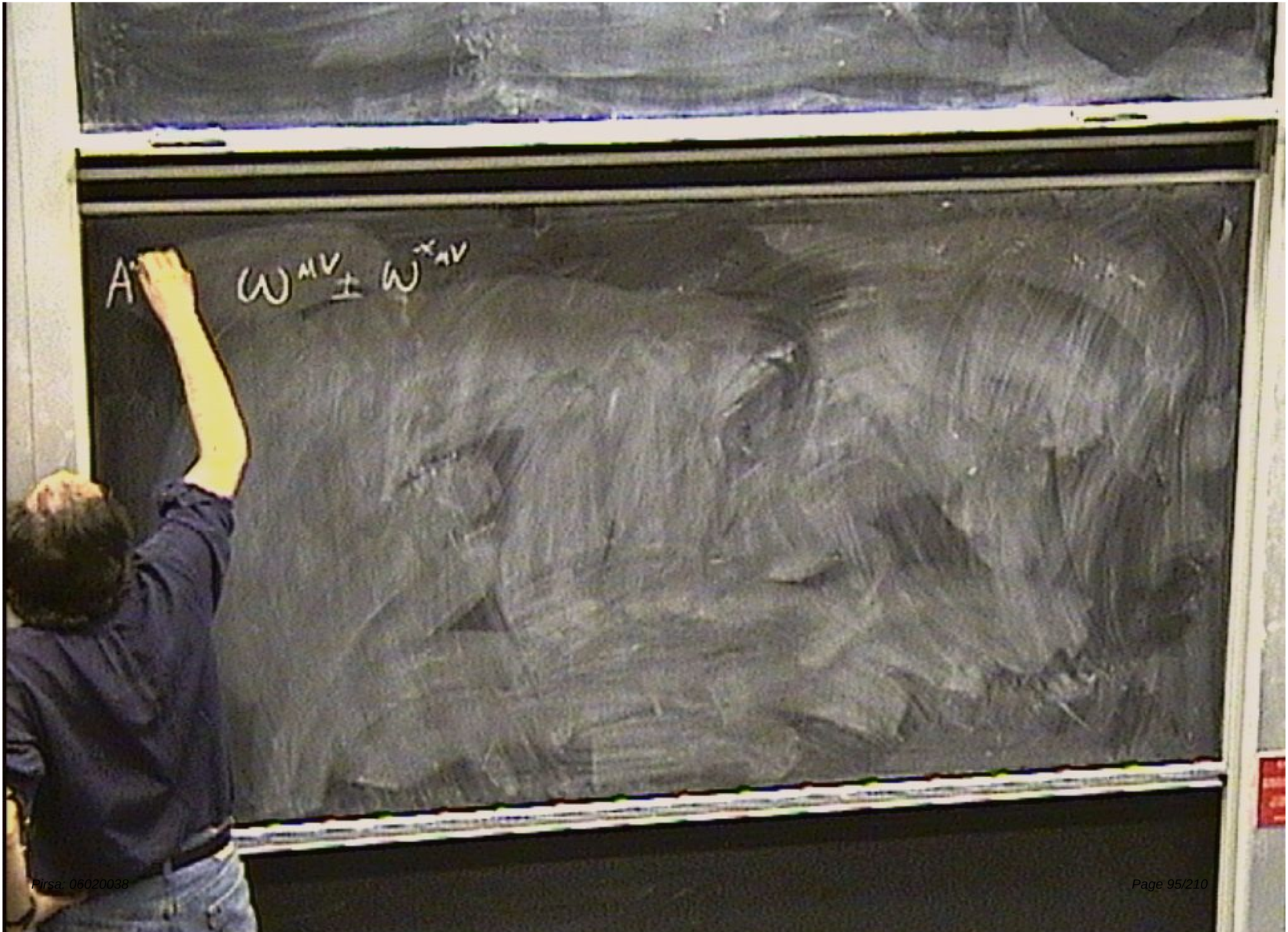

SO(4)
check
SO(4)
SO(3)
Boosts
[J+, J-]=0

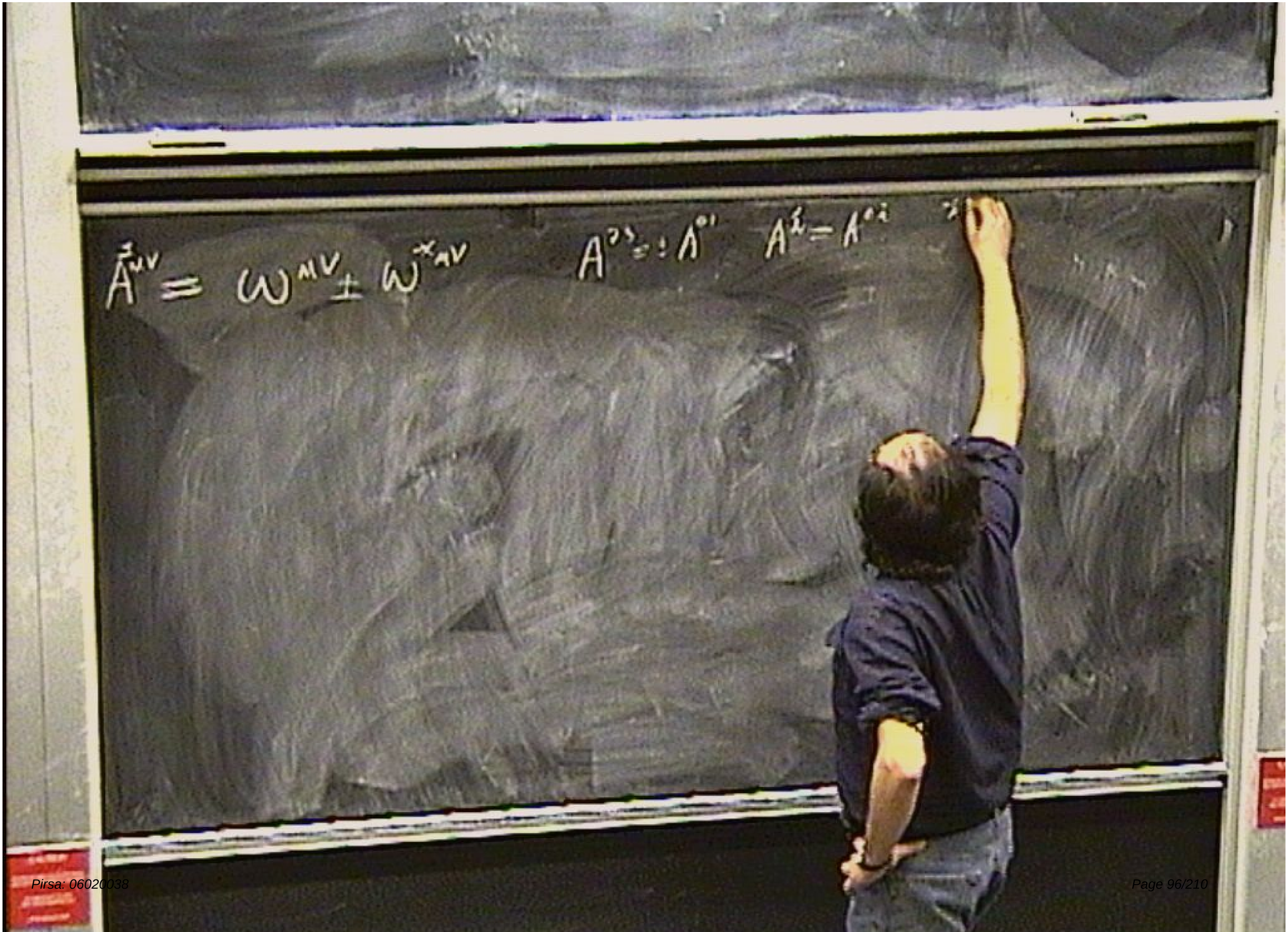

= Spacetime curvature
SO(4)
check
SO(3)
Basis
$[J^+, J^-] = 0$

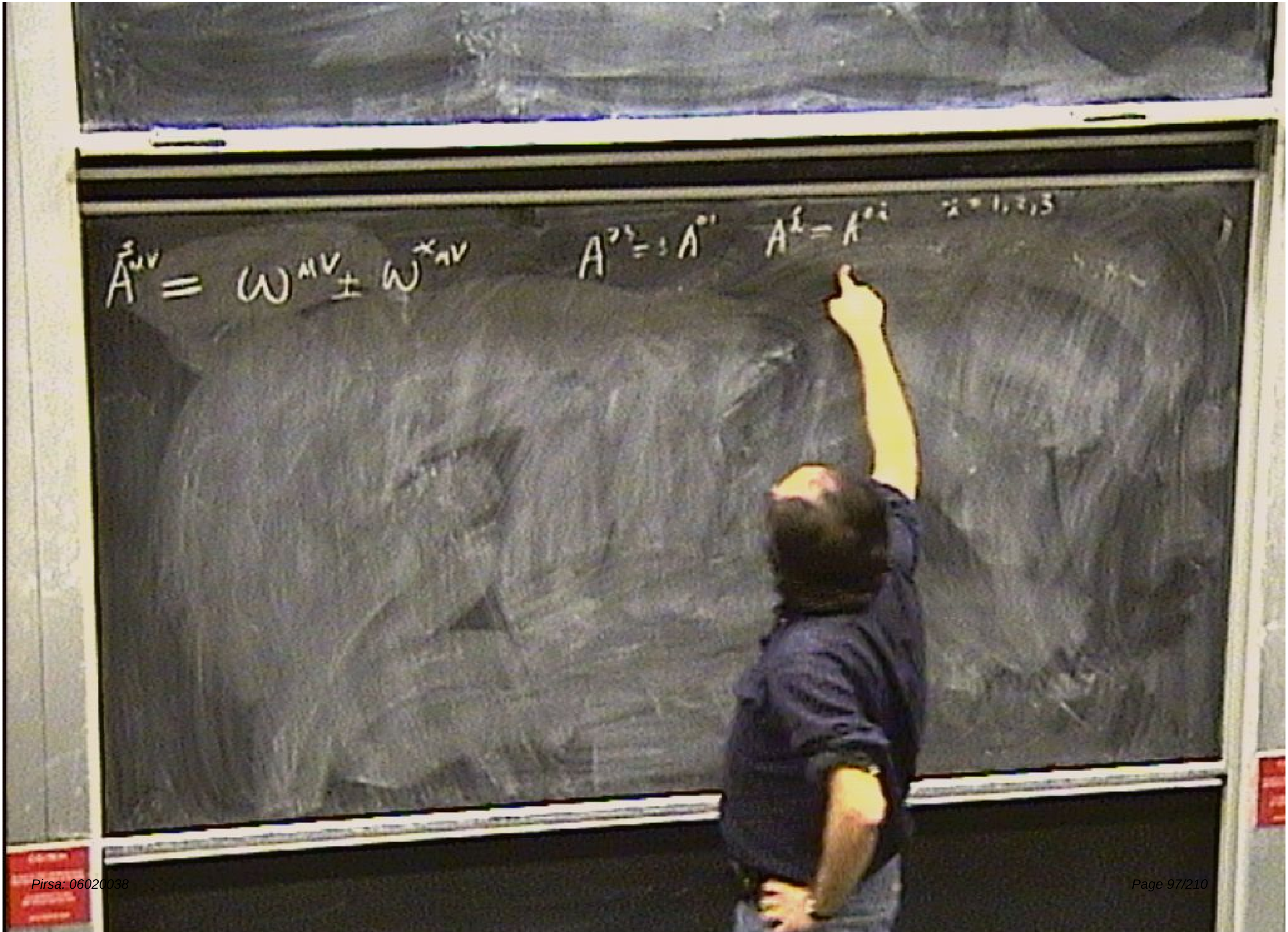


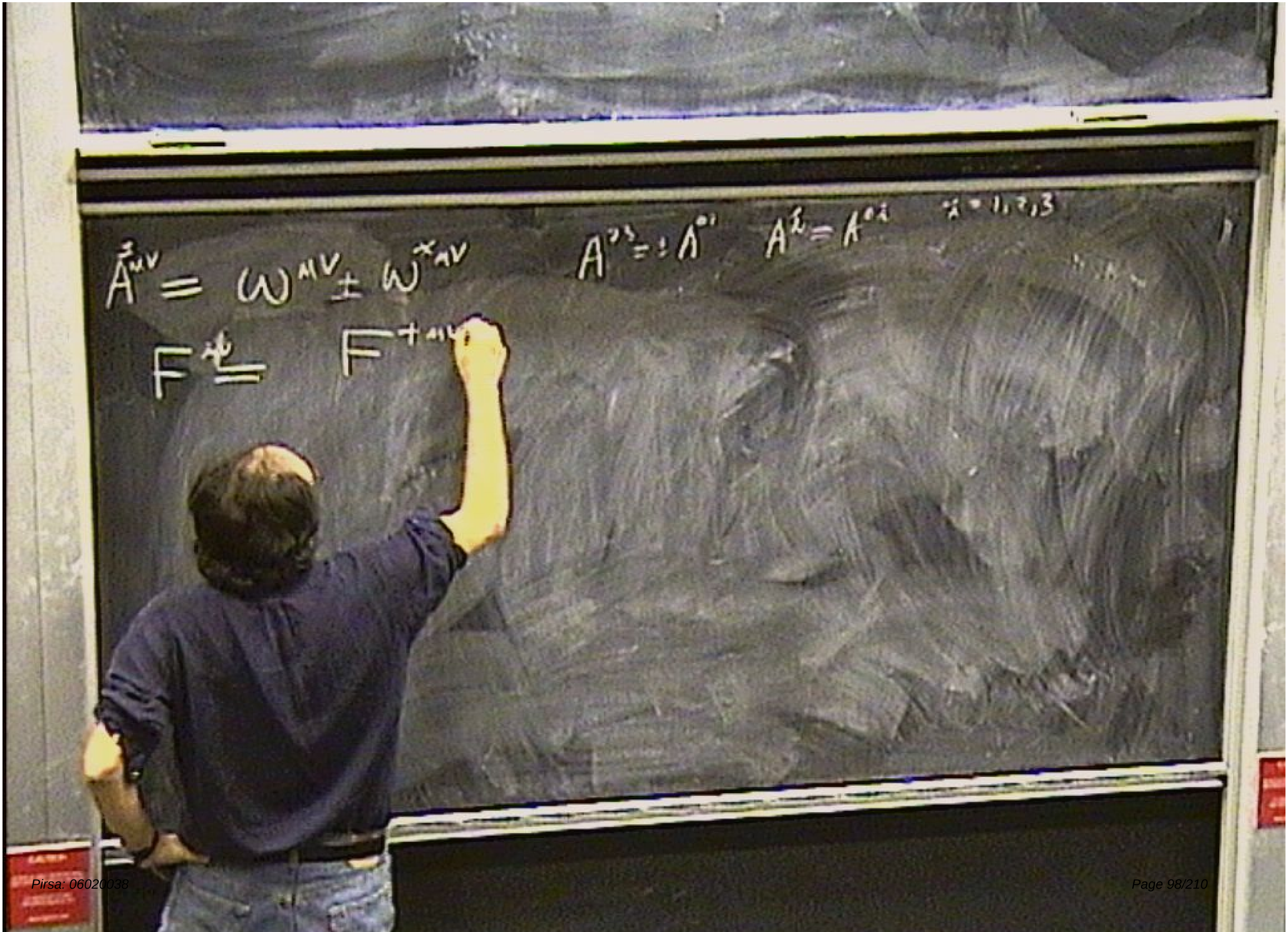
$F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}{}_{\lambda} \wedge \omega^{\lambda\nu}$
SO(4)
= Spacetime curvature
Check
SO(4)
$[J^i, J^j] = \varepsilon^{ijk} J_k$ SO(3)
Boosts K^i
$[K^i, J^j] = -\varepsilon^{ij}{}_k K^k$
$[K^i, K^j] = \varepsilon^{ij}{}_k J^k$
$J^{\pm}_i = J_i \pm i K_i$
$[J^{\pm}_i, J^{\pm}_j] = \varepsilon_{ijk} J^{\pm k}$
$[J^+, J^-] = 0$
$J^+ \leftrightarrow J^-$




A
$\omega^{\mu\nu} \pm \omega^{*\mu\nu}$




$\tilde{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$
$i = 1, 2, 3$



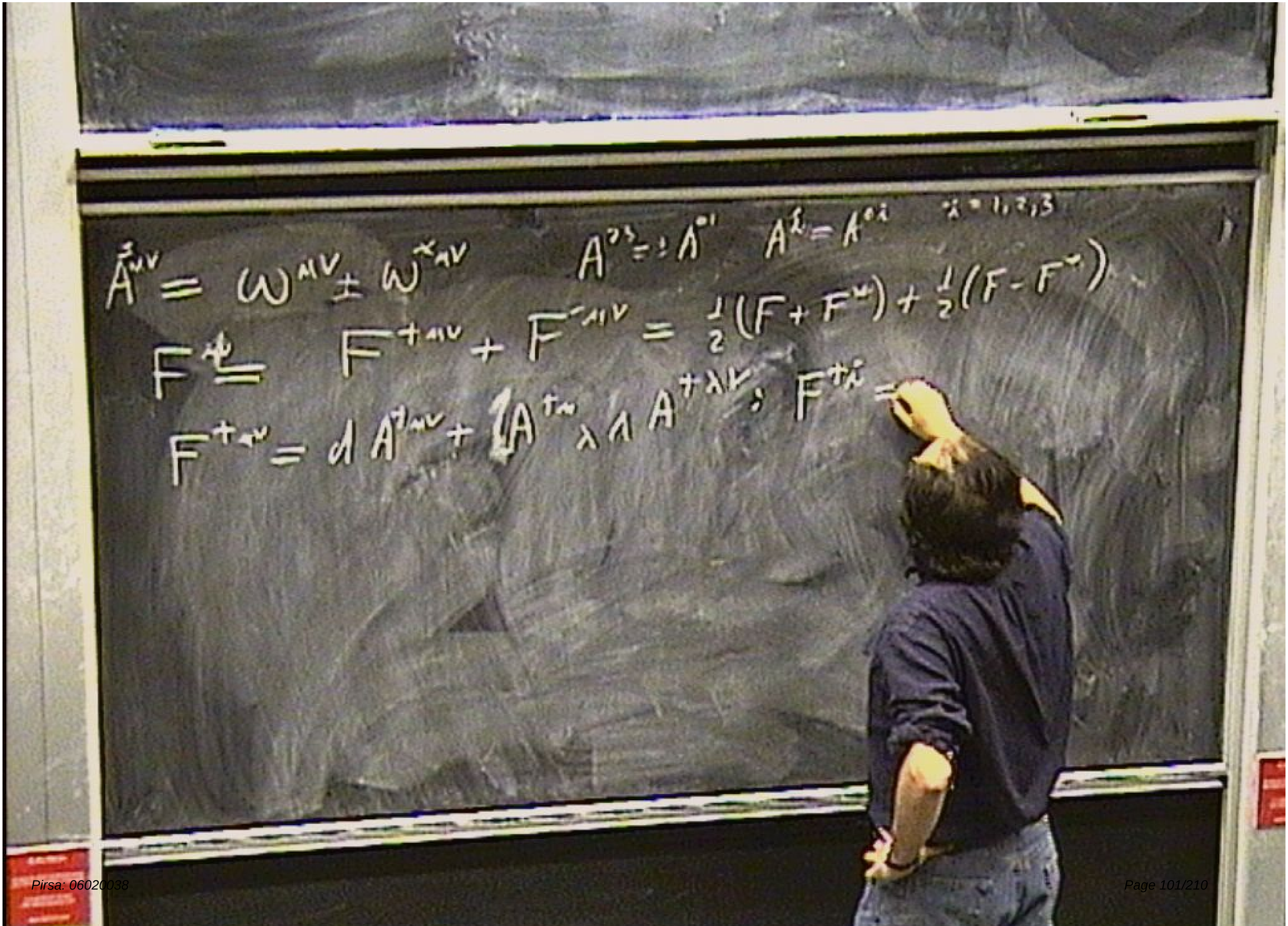
$$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu} \qquad A^{23} = \pm A^{01} \qquad A^{i} = A^{0i} \qquad i = 1,2,3$$

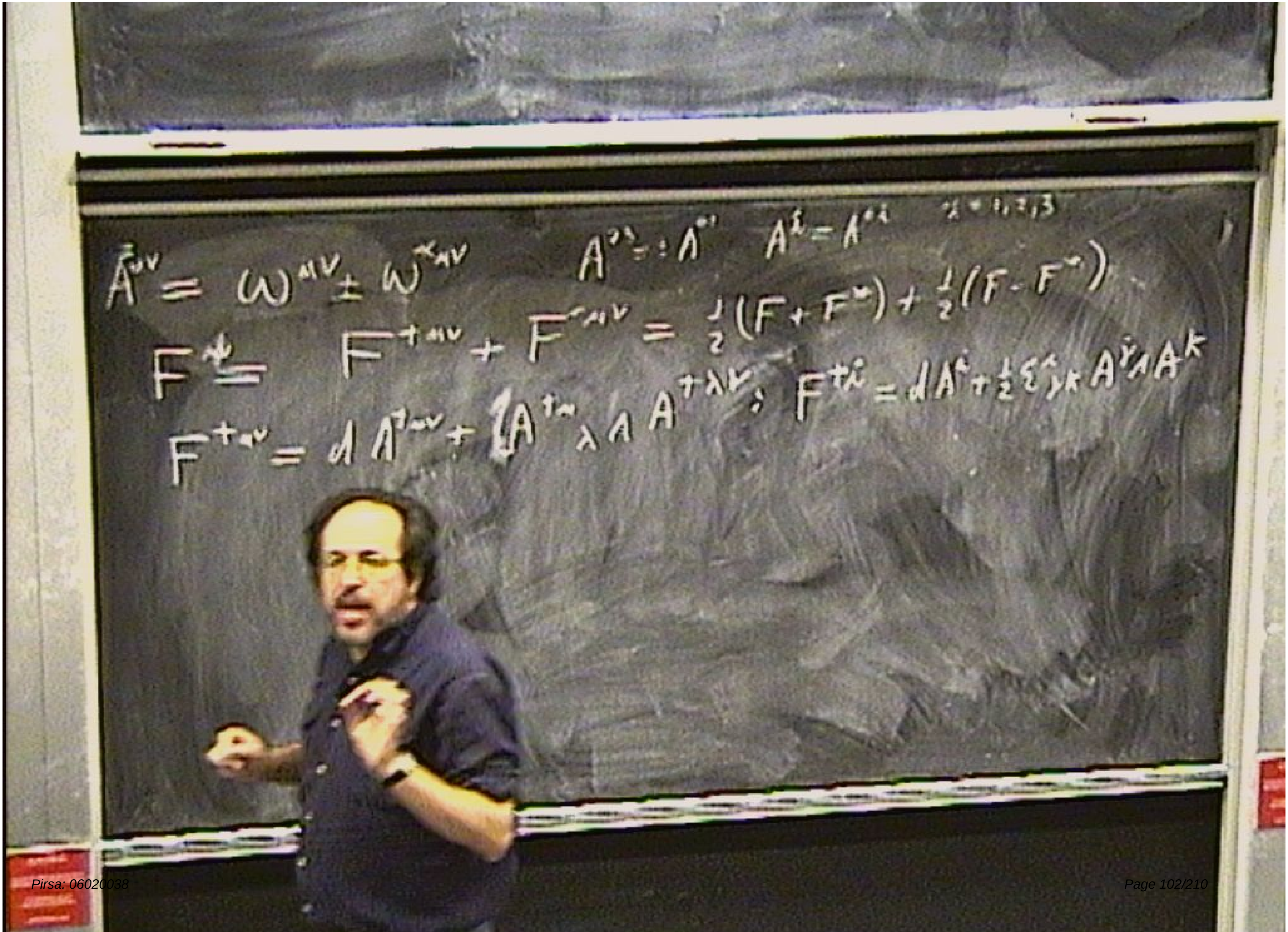
$$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu}$$

$A^{23} = \pm A^{01}$ $\quad A^{i} = A^{0i}$ $\quad i = 1,2,3$

$$\tilde{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$$

$$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$$


A^{±μν} = ω^{μν} ± ω^{*μν}
A^{0i} = ± A^{0i}   A^{i} = A^{0i}   i = 1,2,3
F^{μν} = F^{+μν} + F^{-μν} = 1/2(F + F^*) + 1/2(F - F^*)
F^{+μν} = dA^{+μν} + A^{+μλ} ∧ A^{+λν} ; F^{+i} =

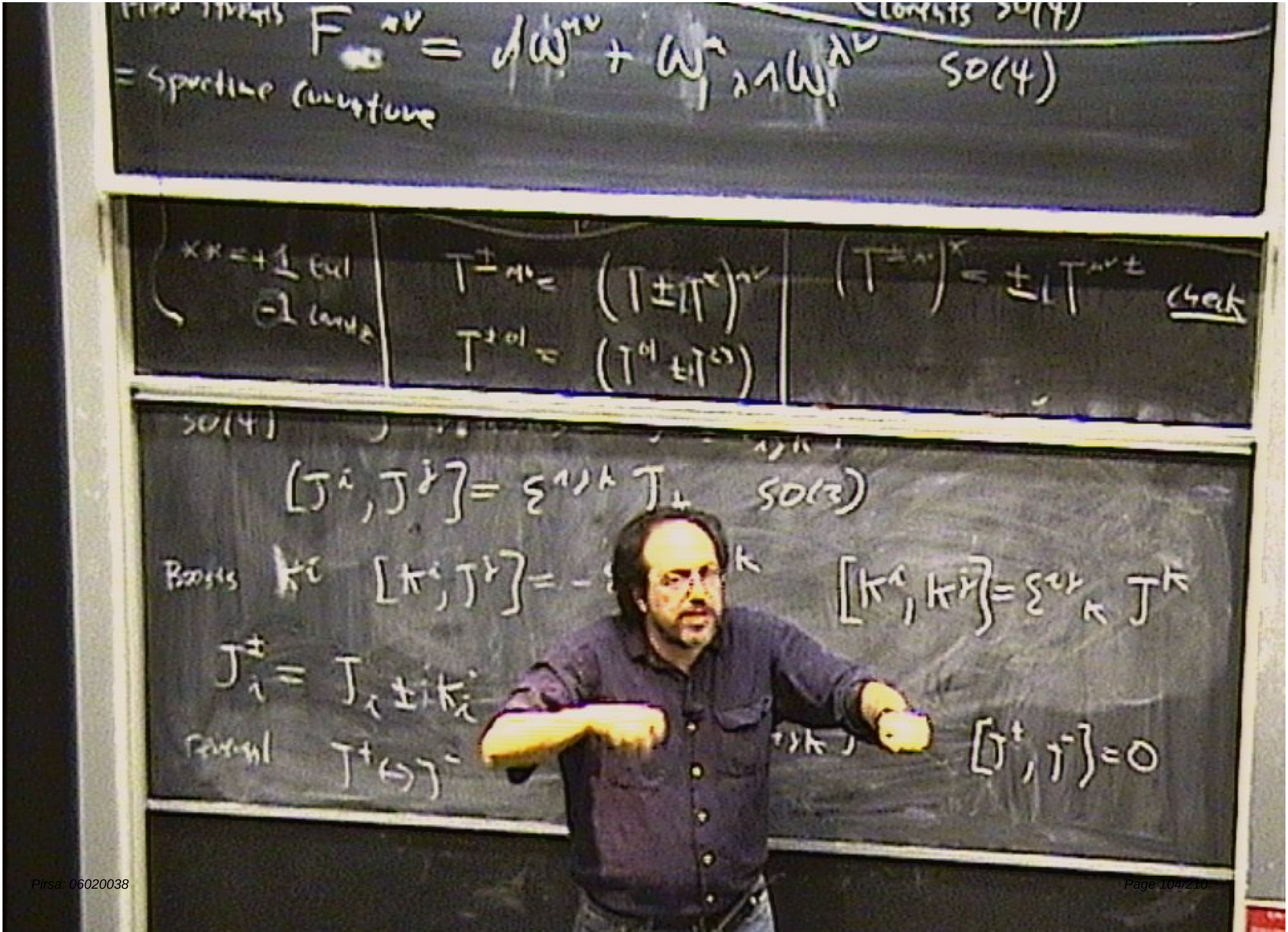

$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$
$i = 1,2,3$
$F = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$
$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu\lambda} \wedge A^{+\lambda\nu}$; $F^{+i} = dA^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$

$A^{0i} = i A^{0i} \qquad A^{\dot{i}} = A^{0i} \qquad i = 1,2,3$

$$\tilde{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$$

$$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$$

$$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}, \quad F^{+i} = dA^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$$


SO(4)
= Spacetime curvature
check
Boosts

Euclidean 0209079

Palatini

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekar, JSS formalism..

  = "Classical LQG"

hep-th/0501191

- Laurent-Artem formalism

$g\,\eta\,g^{-1} = \eta$

$g \downarrow$

$g(\psi)$

$A^{23} = \pm A^{01}$ $\quad A^{i} = A^{0i}$ $\quad i = 1,2,3$

$\vec{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$

$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$

$F^{+\mu\nu} = d A^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}, \quad F^{+i} = d A^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$

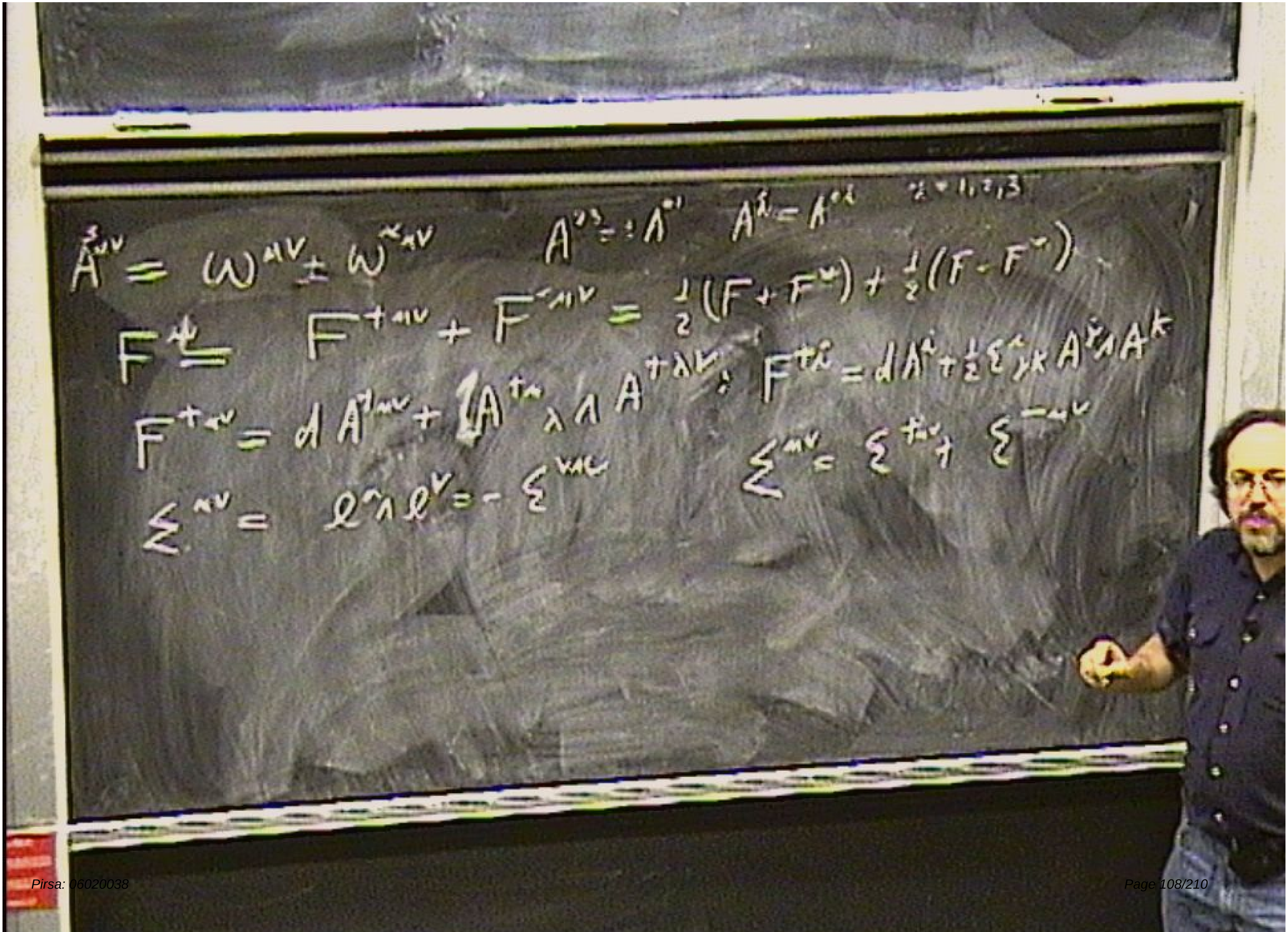
$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu}$

$$\overset{\pm}{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu} \qquad A^{ij} = \pm A^{0i} \qquad A^{i} = A^{\pm i} \quad i = 1,2,3$$

$$F^{\pm} = F^{+\mu\nu} + F^{-\mu\nu} = \tfrac{1}{2}(F + F^{*}) + \tfrac{1}{2}(F - F^{*})$$

$$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}, \qquad F^{+i} = dA^{i} + \tfrac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$$

$$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$$


$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$
$A^{i} = A^{0i}$ $i = 1, 2, 3$
$F^{\pm} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$
$F^{+i} = dA^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$
$F^{+\mu\nu} = d A^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}$
$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$
$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$

$i=1,2,3$

$A^{0i} = i A^{0i} \qquad A^{i} = A^{0i}$

$$\overset{\pm}{A}{}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$$

$$F \stackrel{\pm}{=} F^{+\mu\nu} + F^{-\mu\nu} = \tfrac{1}{2}(F + F^{*}) + \tfrac{1}{2}(F - F^{*})$$

$$F^{+\mu\nu} = d A^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}; \quad F^{+i} = dA^{i} + \tfrac{1}{2}\varepsilon^{i}{}_{jk} A^{j} \wedge A^{k}$$

$$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$$

$$S^{Pl.} = \int$$

$A^{23} = i A^{01} \qquad A^{i} = A^{0i} \qquad i = 1,2,3$

$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm {}^{*}\omega^{\mu\nu}$

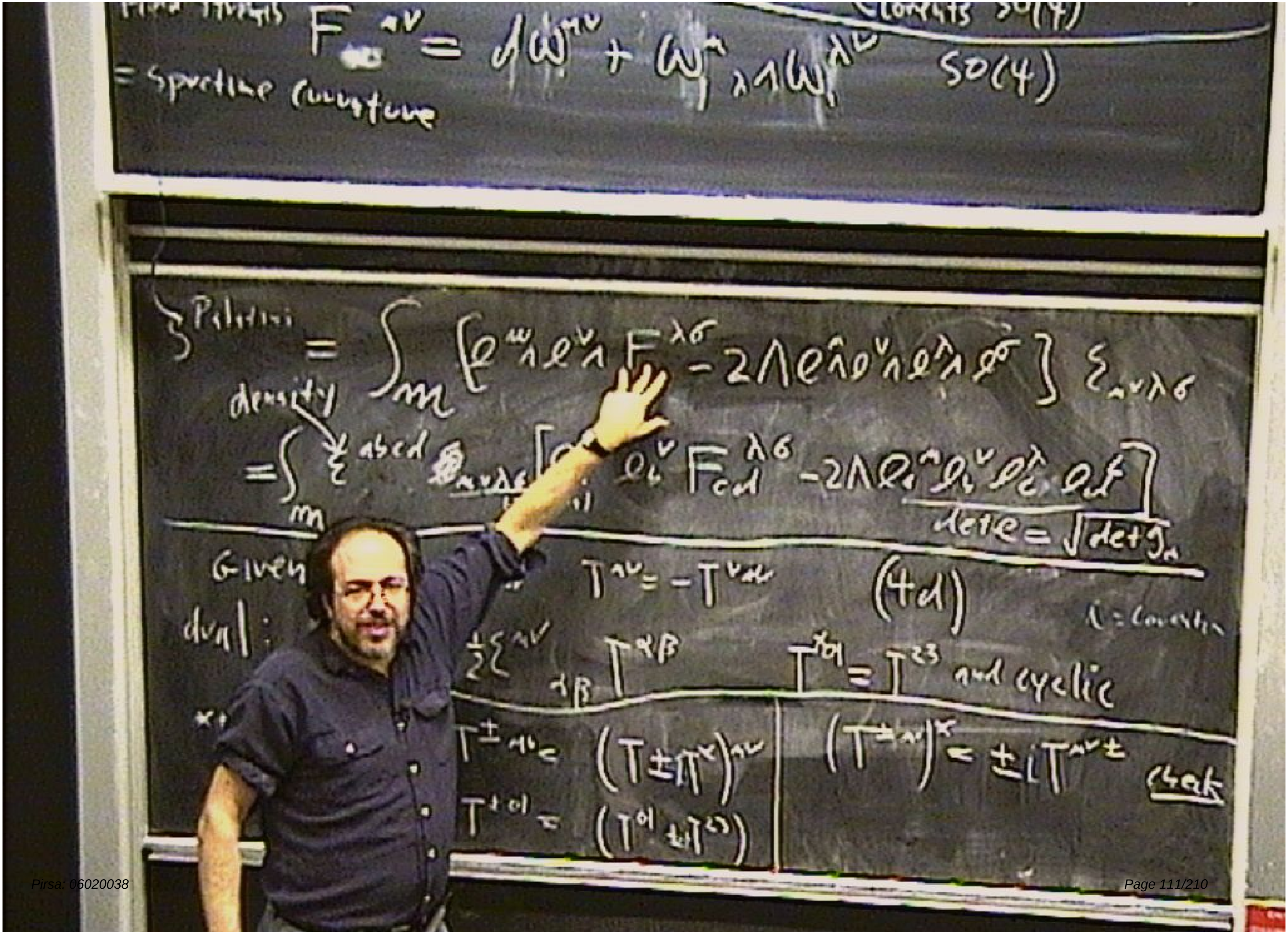
$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$

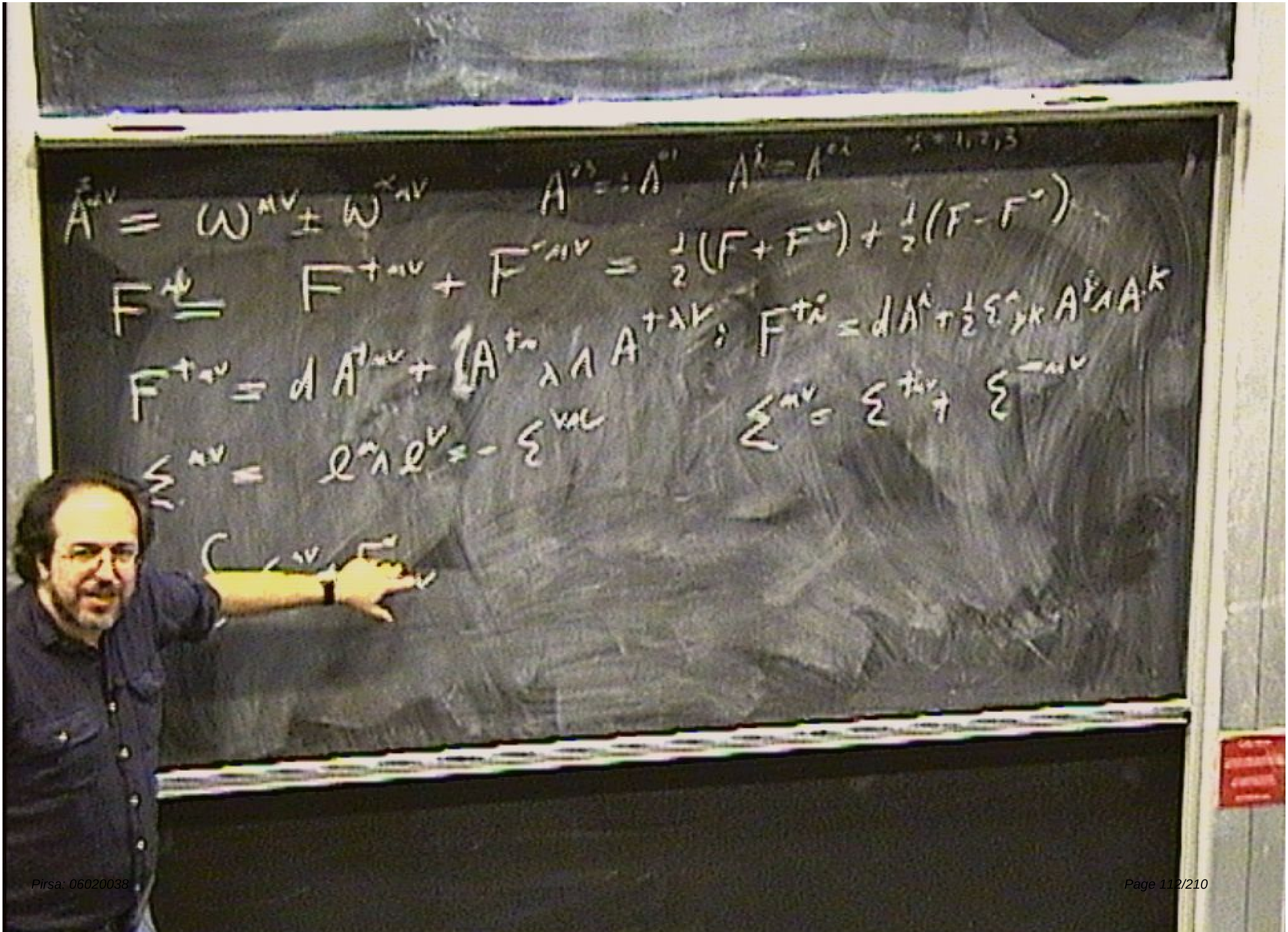
$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}; \quad F^{+i} = dA^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$

$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$

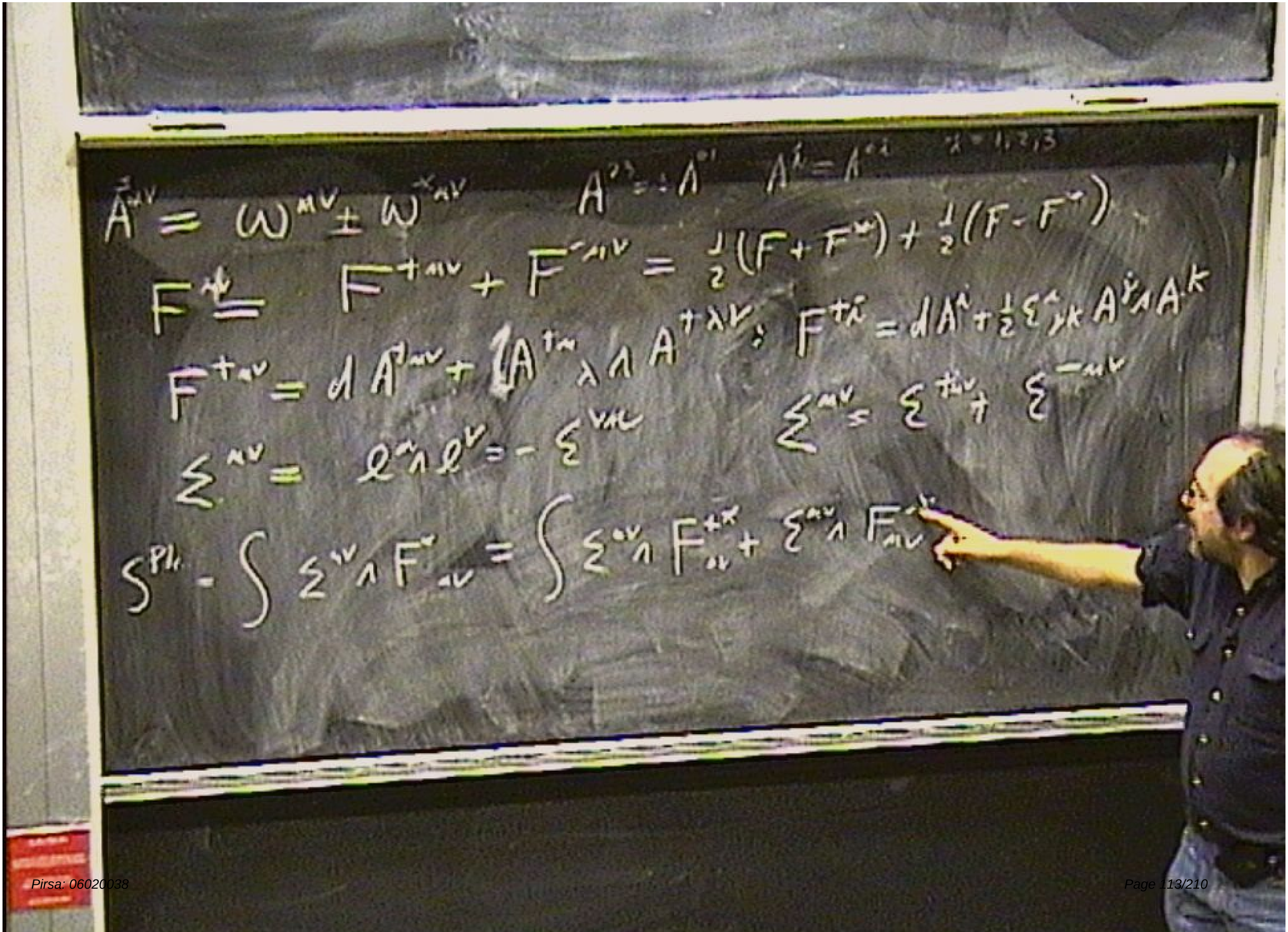
$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$

$S^{Pl.} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu}$

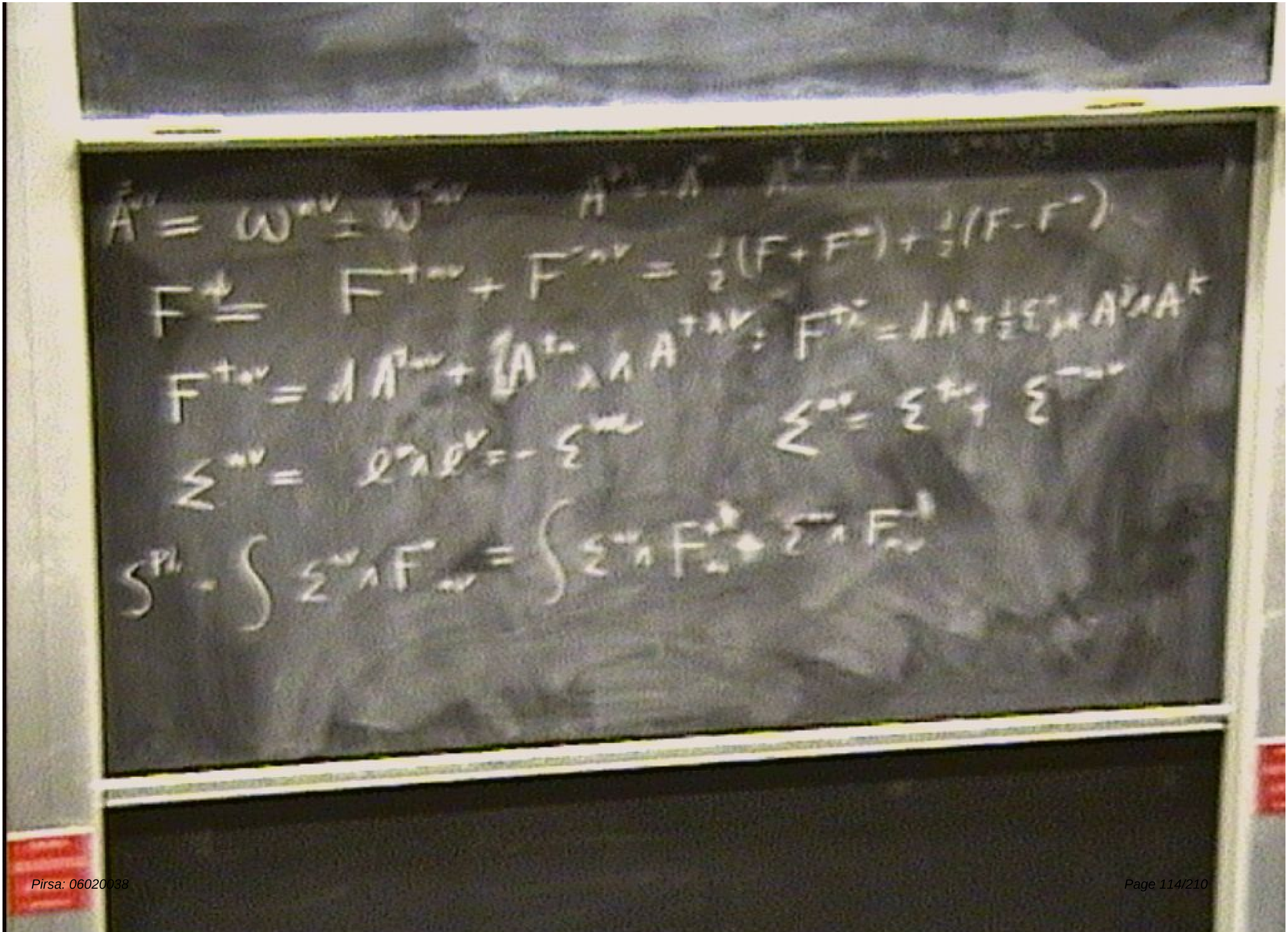

= Spacetime curvature
SO(4)
Palatini
density
Given
dual:
(4d)
and cyclic
Check

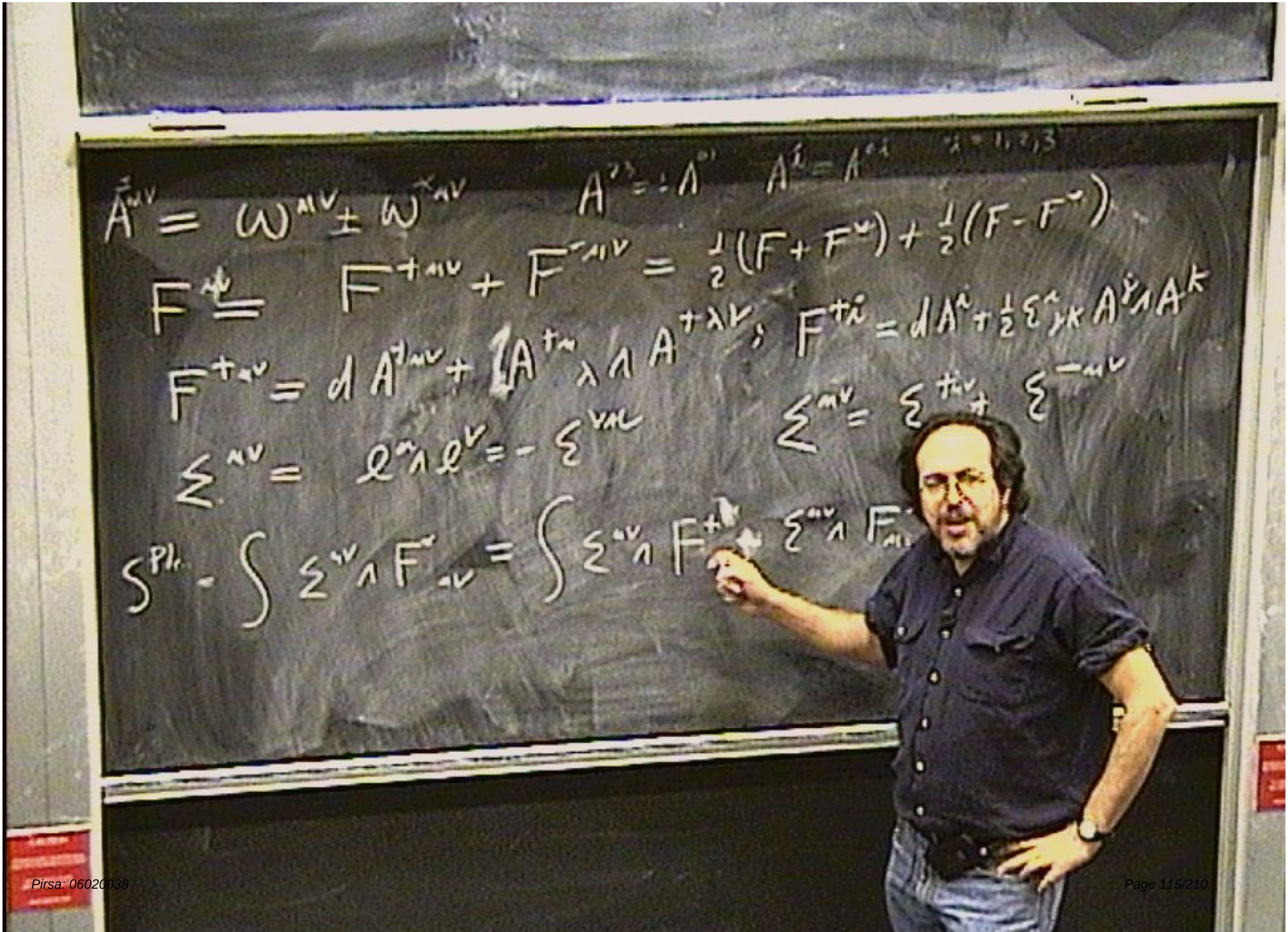


A = ω^{μν} ± ω^{*μν}
F^{+μν} + F^{-μν} = ½(F + F*) + ½(F − F*)
F^{+i} = dA^i + ½ ε^i_{jk} A^j ∧ A^k
Σ^{μν} = e^μ ∧ e^ν = − Σ^{νμ}
Σ^{μν} = Σ^{+μν} + Σ^{−μν}

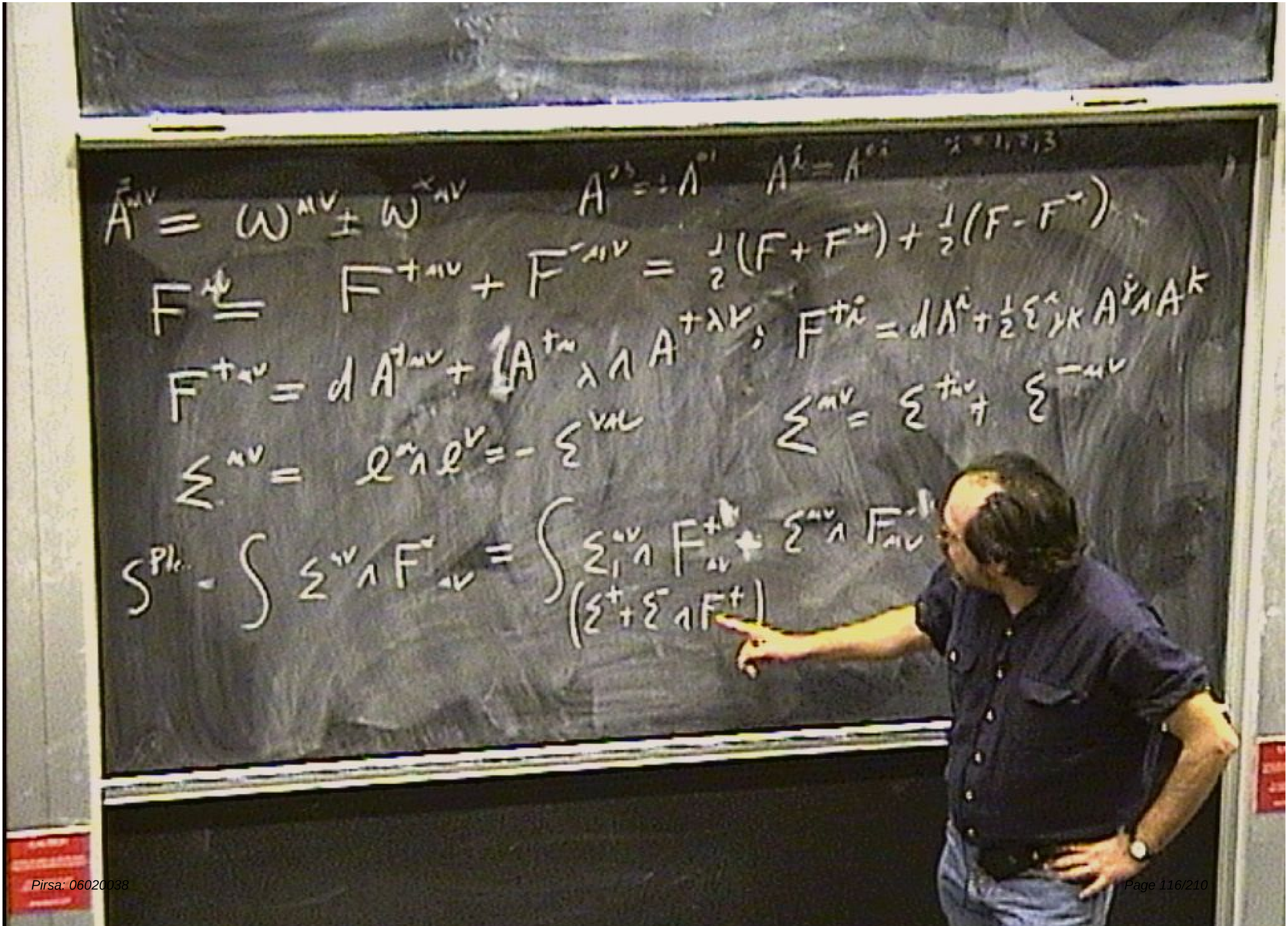


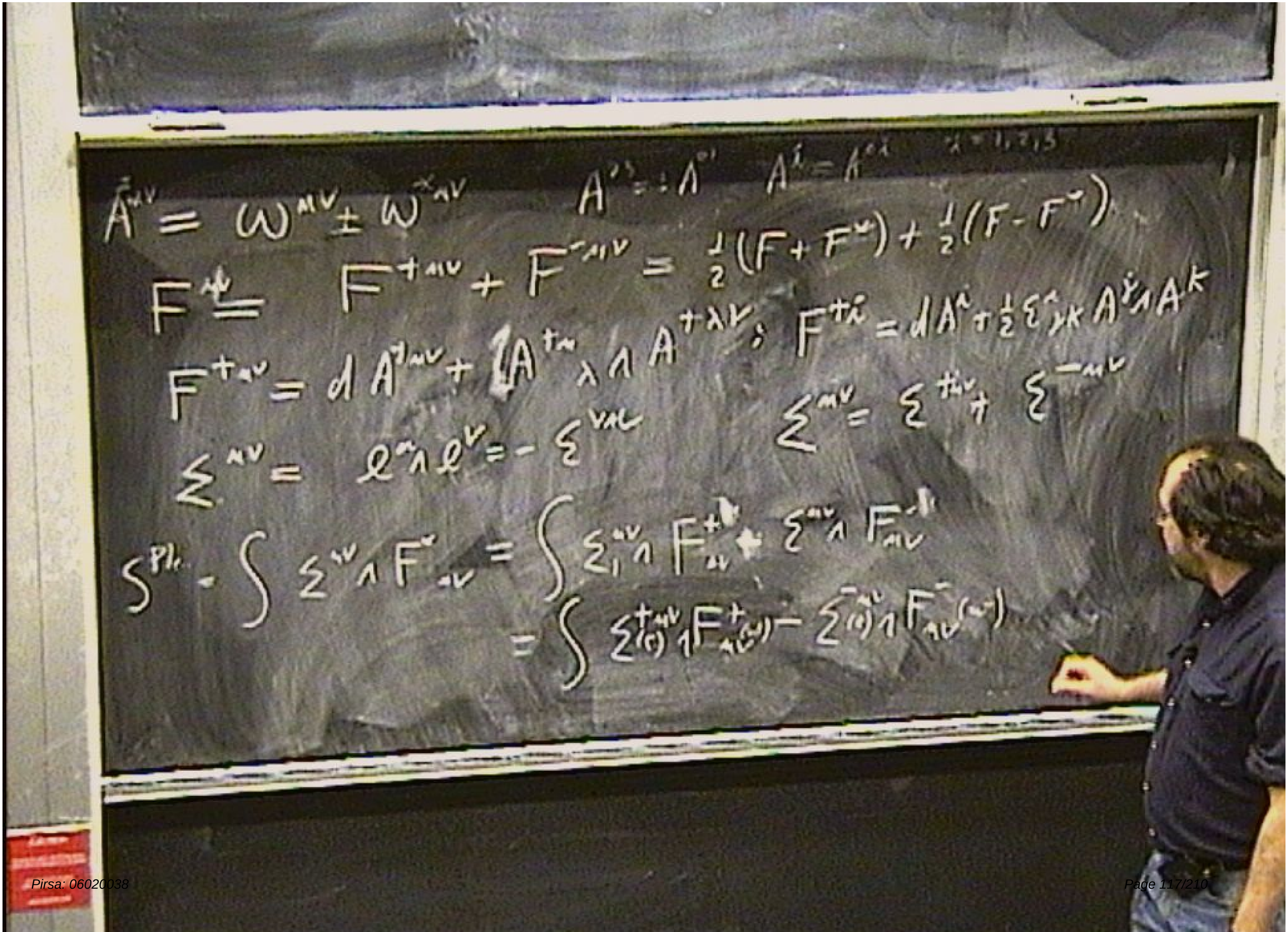
A^{0i} = ±A^{0i}   A^i = A^{0i}   i = 1,2,3
A^{±μν} = ω^{μν} ± ω^{*μν}
F^{μν} = F^{+μν} + F^{−μν} = ½(F + F*) + ½(F − F*)
F^{+μν} = dA^{+μν} + A^{+μ}_λ ∧ A^{+λν} ; F^{+i} = dA^i + ½ ε^i_{jk} A^j ∧ A^k
Σ^{μν} = e^μ ∧ e^ν = −Σ^{νμ}   Σ^{μν} = Σ^{+μν} + Σ^{−μν}
S^{Pl} = ∫ Σ^{μν} ∧ F_{μν} = ∫ Σ^{+μν} ∧ F^+_{μν} + Σ^{−μν} ∧ F^−_{μν}


$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm W^{*\mu\nu}$
$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F+F^*) + \frac{1}{2}(F-F^*)$
$F^{+i} = dA^i + \frac{1}{2}\epsilon^i{}_{jk} A^j \wedge A^k$
$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_\lambda \wedge A^{+\lambda\nu}$
$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$
$\Sigma^{\mu\nu} = e^\mu \wedge e^\nu = -\Sigma^{\nu\mu}$
$S^{Pl} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{+\mu\nu} \wedge F^{+}_{\mu\nu} + \Sigma^{-} \wedge F^{-}_{\mu\nu}$

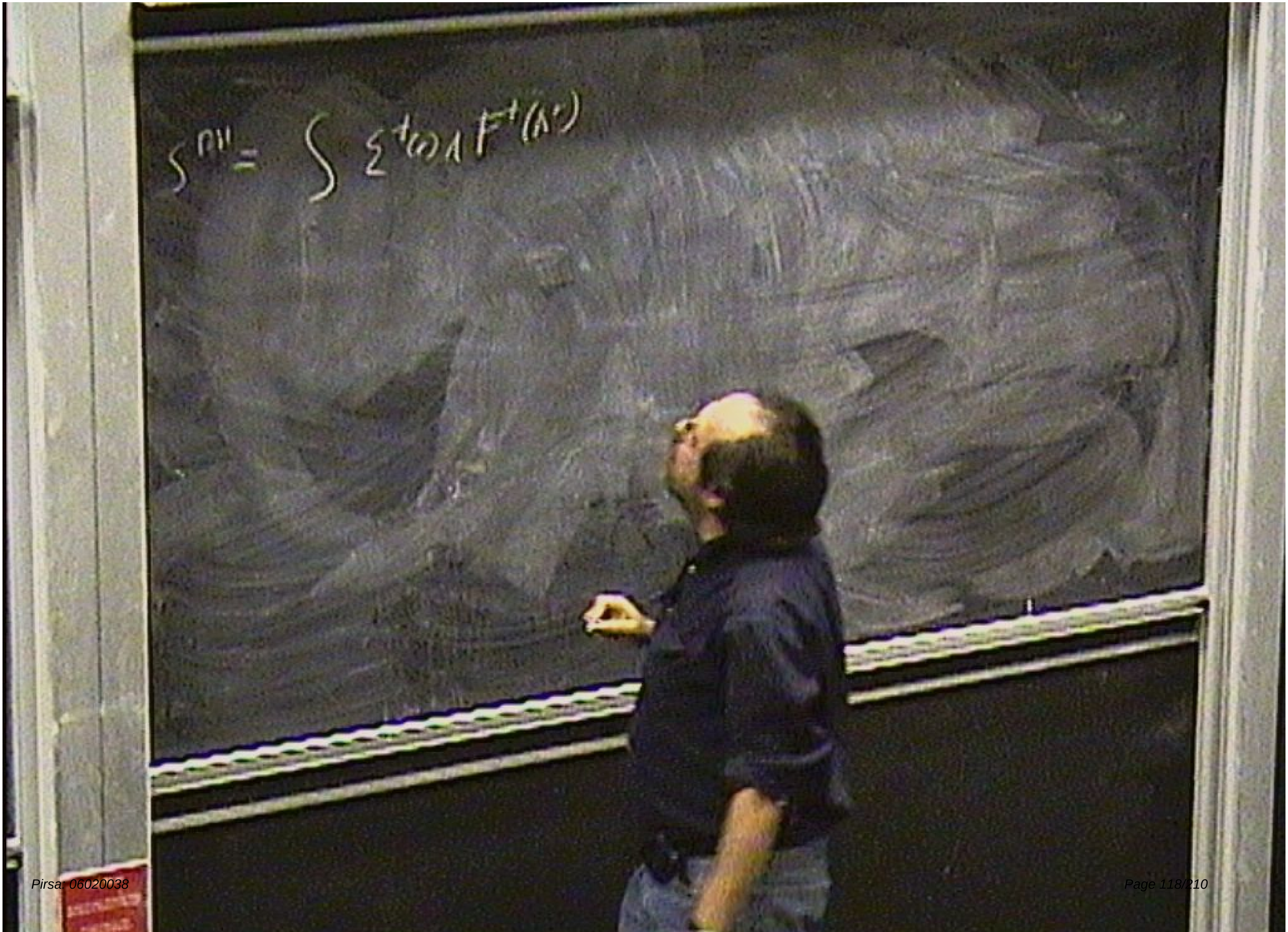


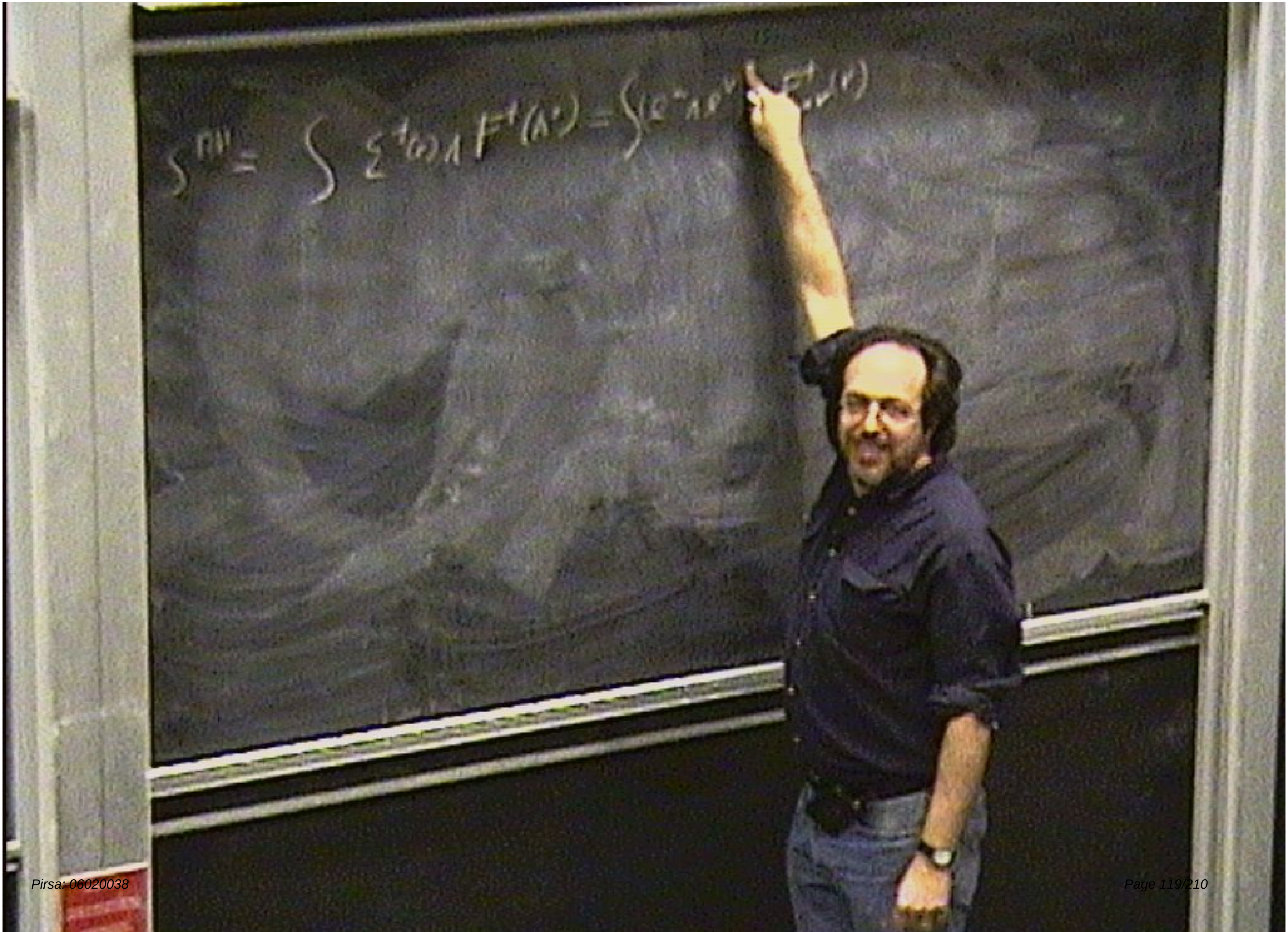
A^μν = ω^μν ± ω^*μν
F^± = F^+μν + F^-μν = ½(F + F*) + ½(F - F*)
F^+i = dA^i + ½ ε^i_jk A^j ∧ A^k
F^+μν = dA^+μν + A^+μ_λ ∧ A^+λν
Σ^μν = Σ^+μν + Σ^-μν
Σ^μν = e^μ ∧ e^ν = - Σ^νμ
S^Pl. = ∫ Σ^μν ∧ F_μν = ∫ Σ^μν ∧ F^+

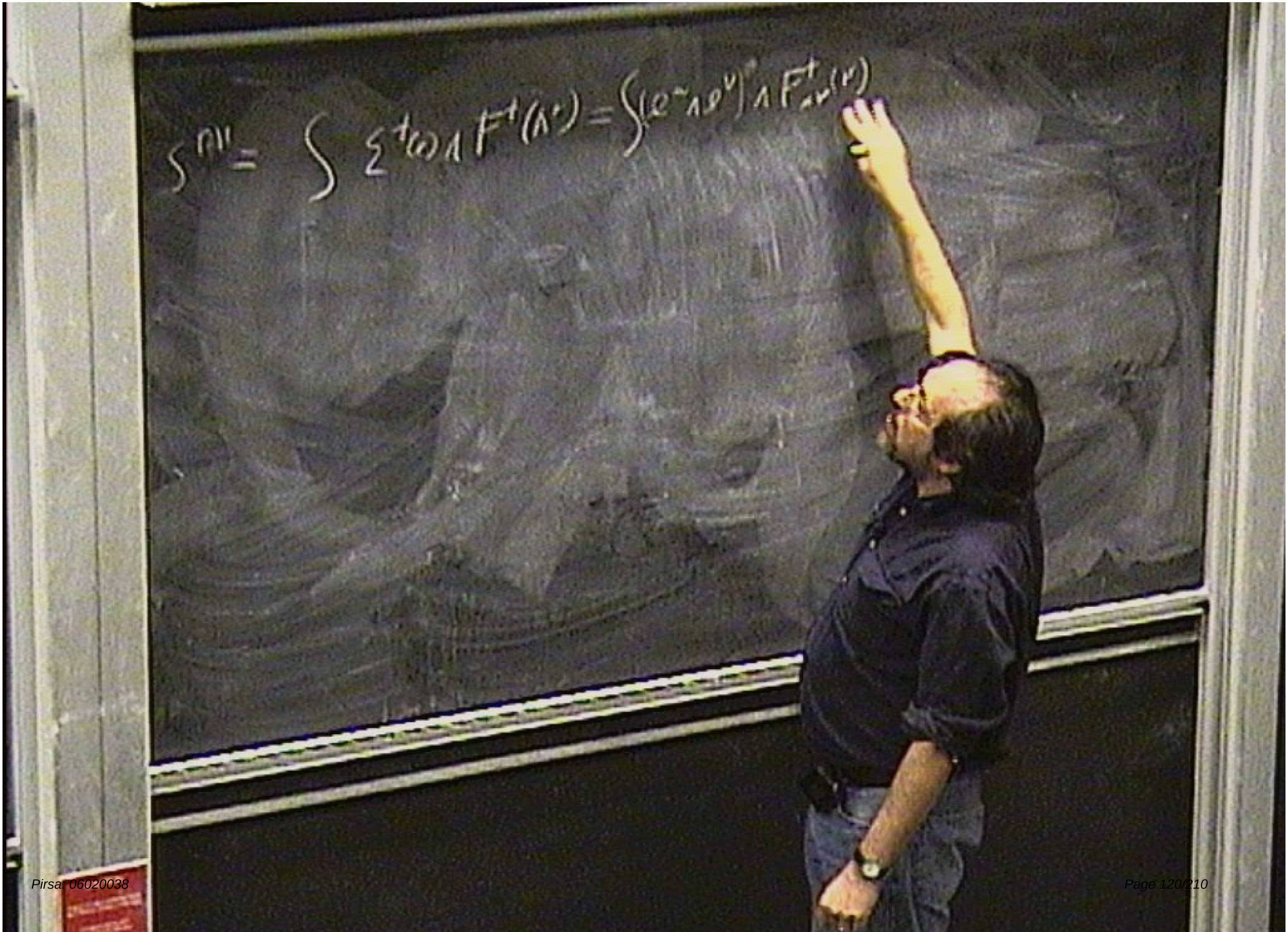

$i = 1,2,3$
$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$ $A^{03} = \pm A^{01}$ $A^{i} = A^{0i}$
$F \stackrel{\pm}{=} F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$
$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}$; $F^{+i} = dA^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$
$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$
$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$
$S^{Pl.} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{\mu\nu} \wedge F^{+}_{\mu\nu} + \Sigma^{\mu\nu} \wedge F^{-}_{\mu\nu}$
$(\Sigma^{+} + \Sigma^{-} \wedge F^{+})$

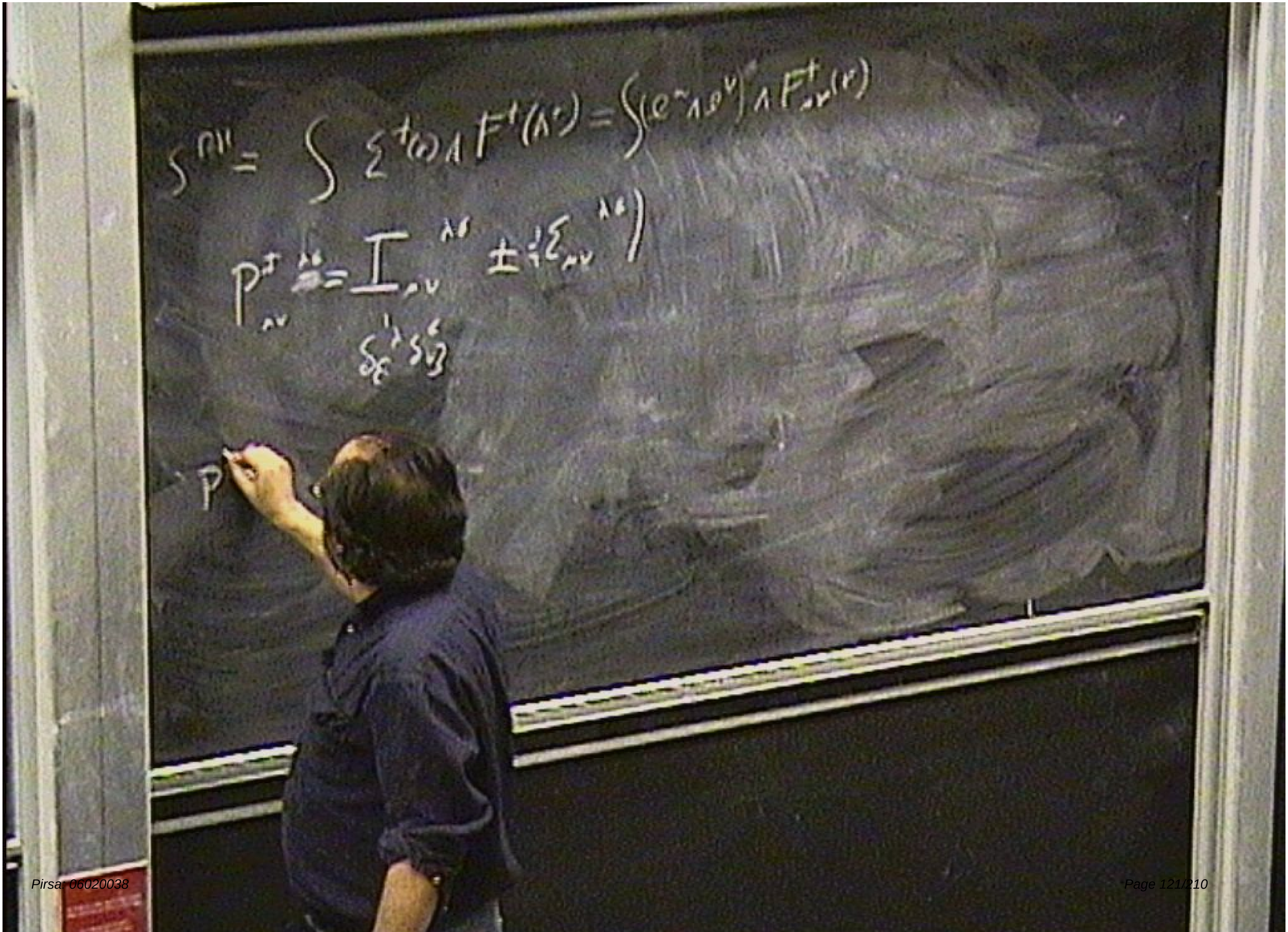


$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$
$A^{i} = A^{0i}$ $i = 1, 2, 3$
$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$
$F^{+\mu\nu} = d A^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}$; $F^{+i} = d A^{i} + \frac{1}{2} \epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$
$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = - \Sigma^{\nu\mu}$
$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$
$S^{Pl} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{+\mu\nu} \wedge F^{+}_{\mu\nu} + \Sigma^{-\mu\nu} \wedge F^{-}_{\mu\nu}$
$= \int \Sigma^{+\mu\nu}_{(e)} \wedge F^{+}_{\mu\nu}(\omega) - \Sigma^{-\mu\nu}_{(e)} \wedge F^{-}_{\mu\nu}(\omega)$


$S^{Pl} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+})$




$S^{new} = \int \Sigma^{i}(\omega) \wedge F^{i}(A^{+}) = \int (e^{\mu} \wedge e^{\nu})^{+} \wedge F^{+}_{\mu\nu}(A)$



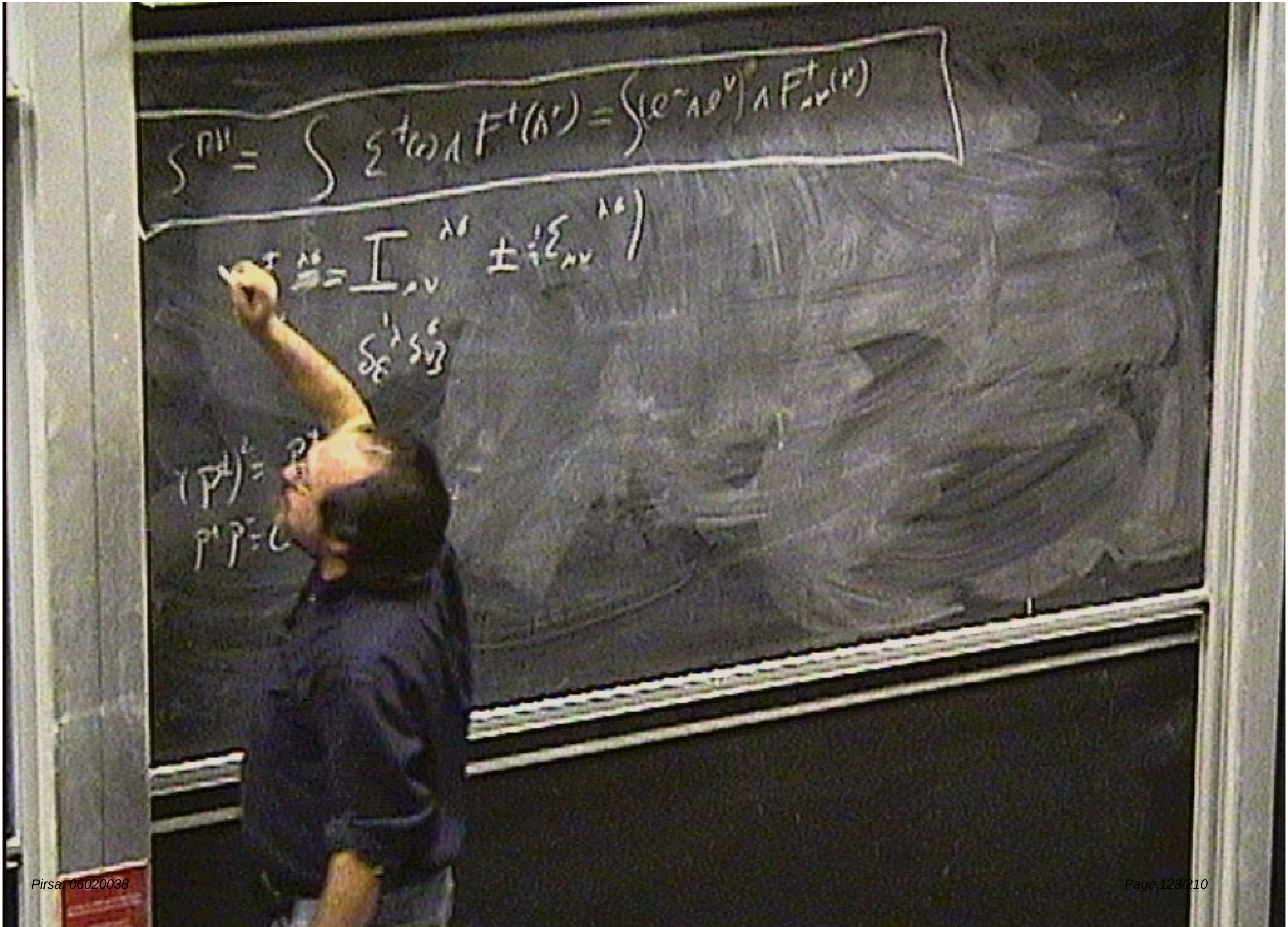
$$S^{\rm new} = \int \Sigma^+(\omega)\wedge F^+(A^+) = \int (e^\mu \wedge e^\nu)^+ \wedge F^+_{\mu\nu}(A^+)$$

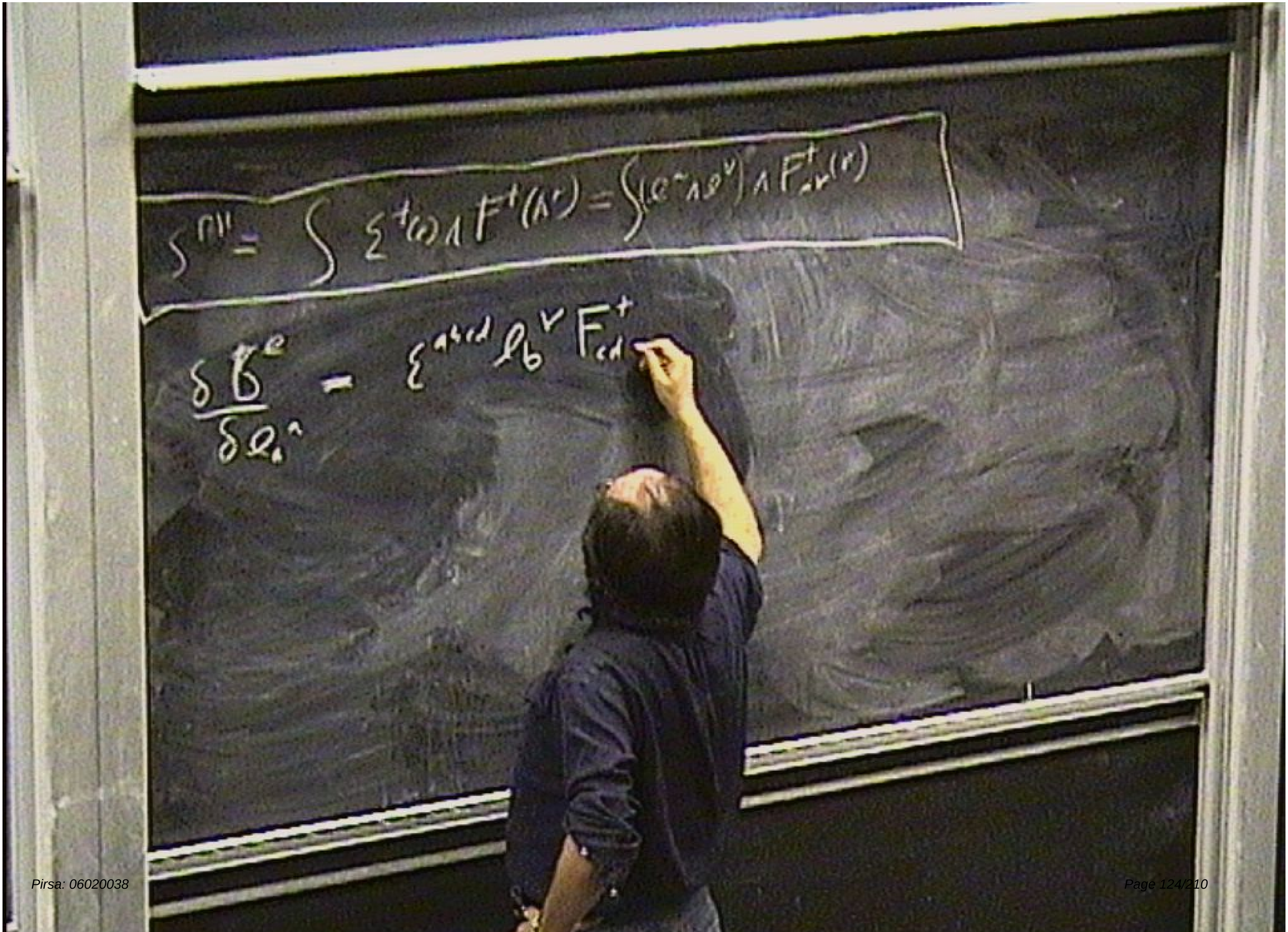
$$P^{\pm}{}_{\mu\nu}{}^{\lambda\sigma} = I_{\mu\nu}{}^{\lambda\sigma} \pm \tfrac{i}{2}\epsilon_{\mu\nu}{}^{\lambda\sigma})$$

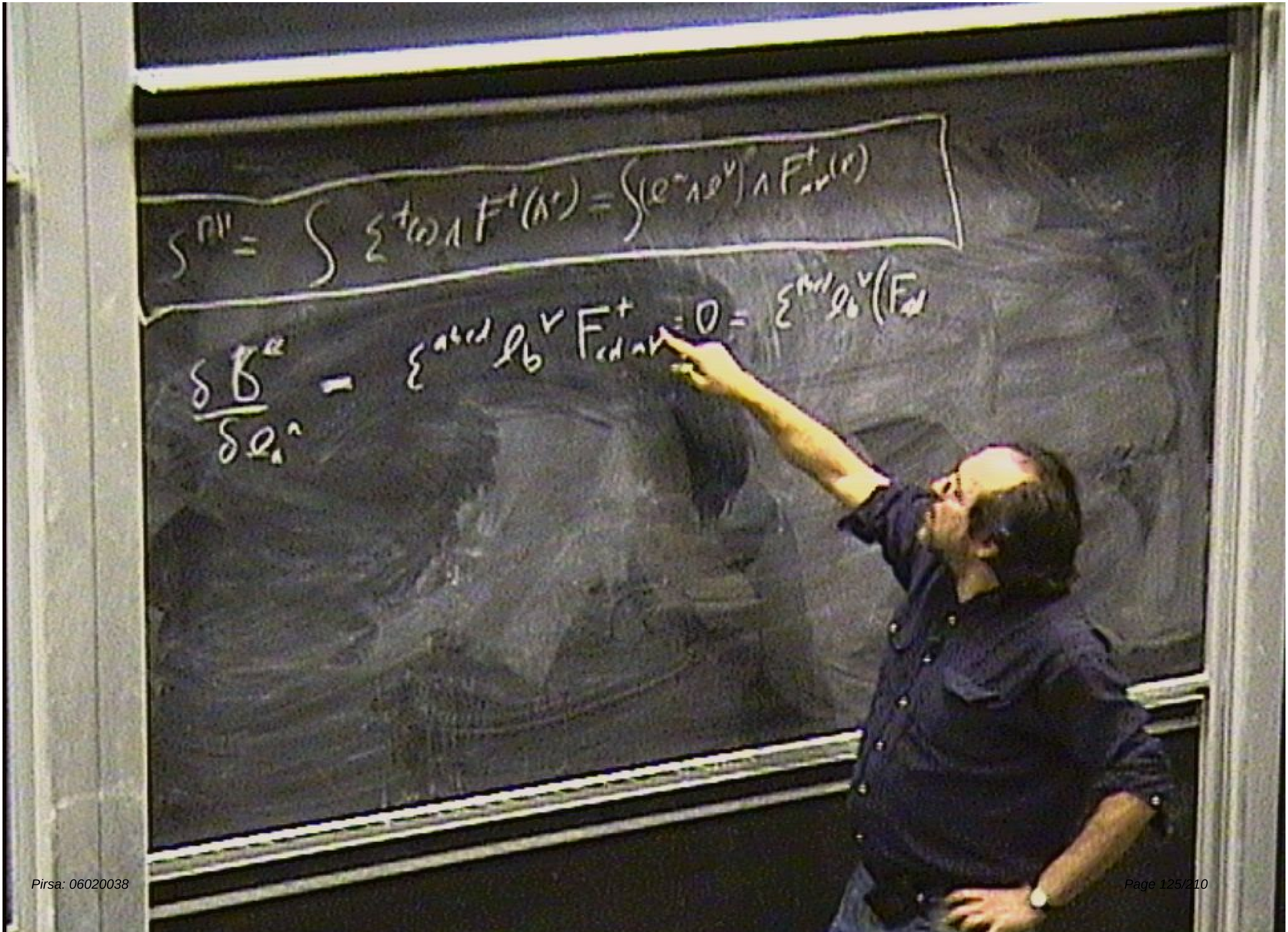
$$\delta_\mu^{[\lambda}\delta_\nu^{\sigma]}$$

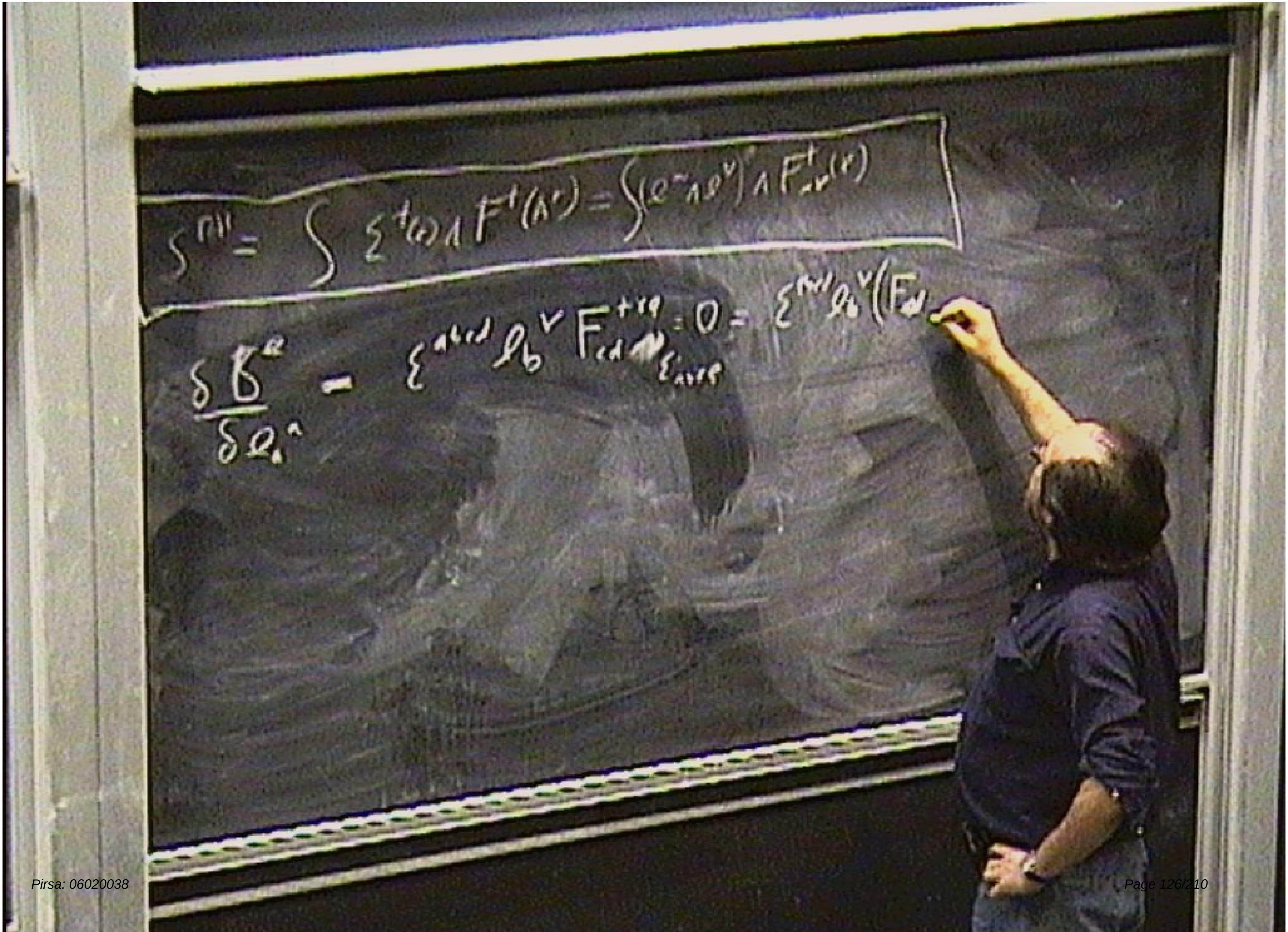
$$(P^\pm)^2 = P^\pm$$

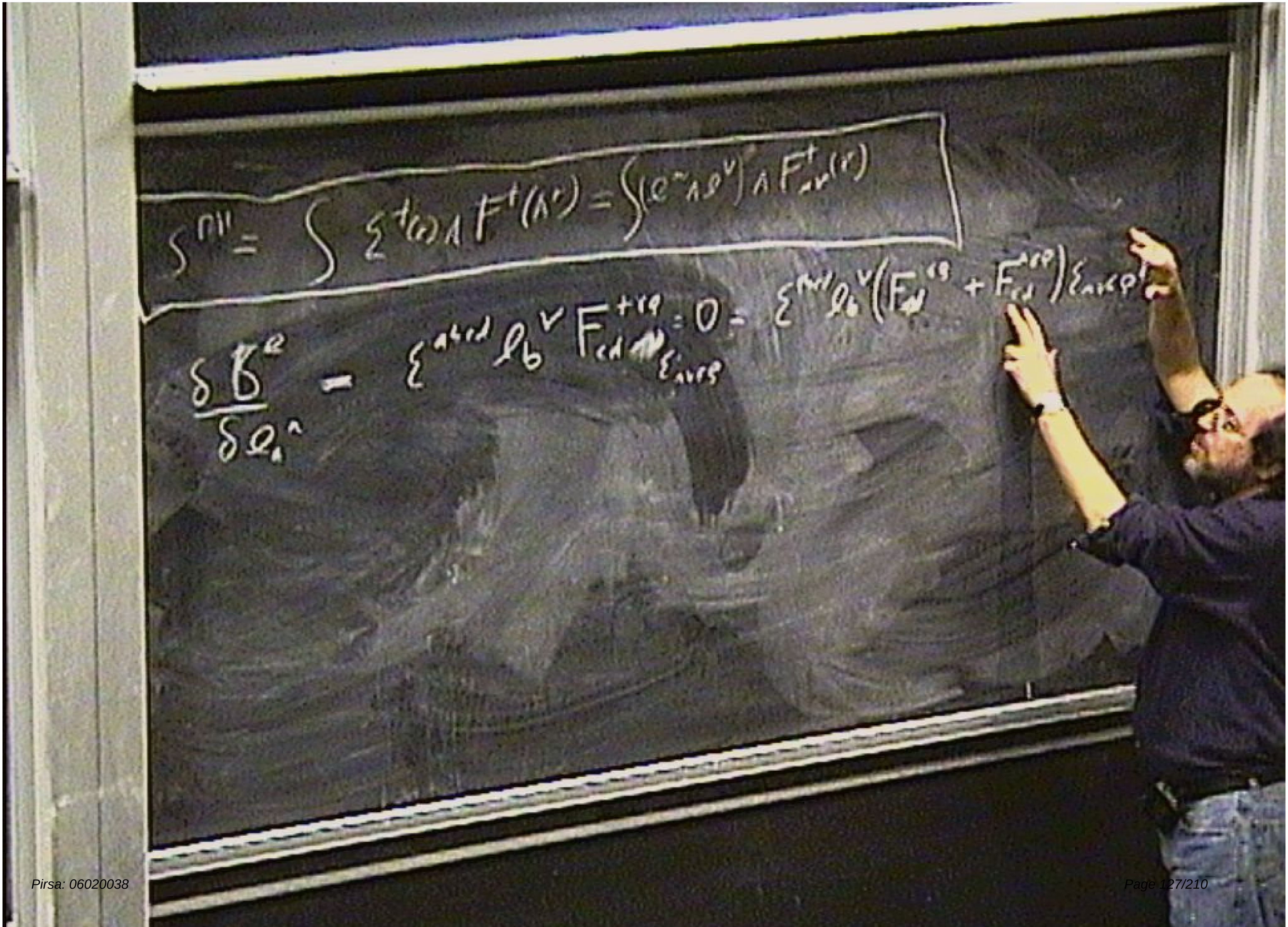
$$P^+P^- = 0$$

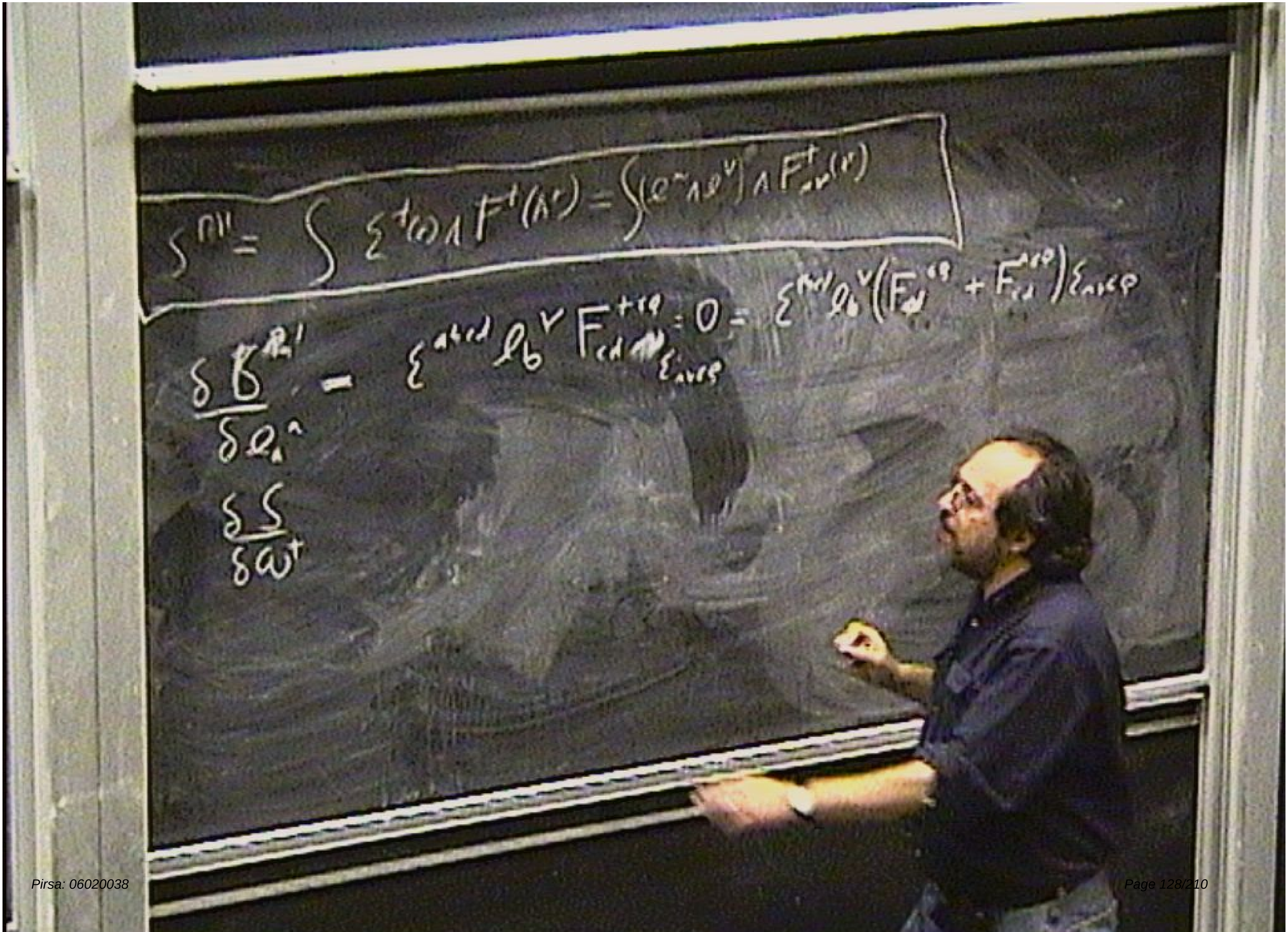











$S^{\text{new}} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})$
$\frac{\delta S^{\text{new}}}{\delta e_{a}{}^{\mu}} = \epsilon^{abcd} e_{b}{}^{\nu} F_{cd}{}^{+\rho\sigma} \epsilon_{\mu\nu\rho\sigma} = 0 = \epsilon^{abcd} e_{b}{}^{\nu} (F_{cd}{}^{\rho\sigma} + F_{cd}{}^{*\rho\sigma}) \epsilon_{\mu\nu\rho\sigma}$
$\frac{\delta S}{\delta \omega^{+}}$

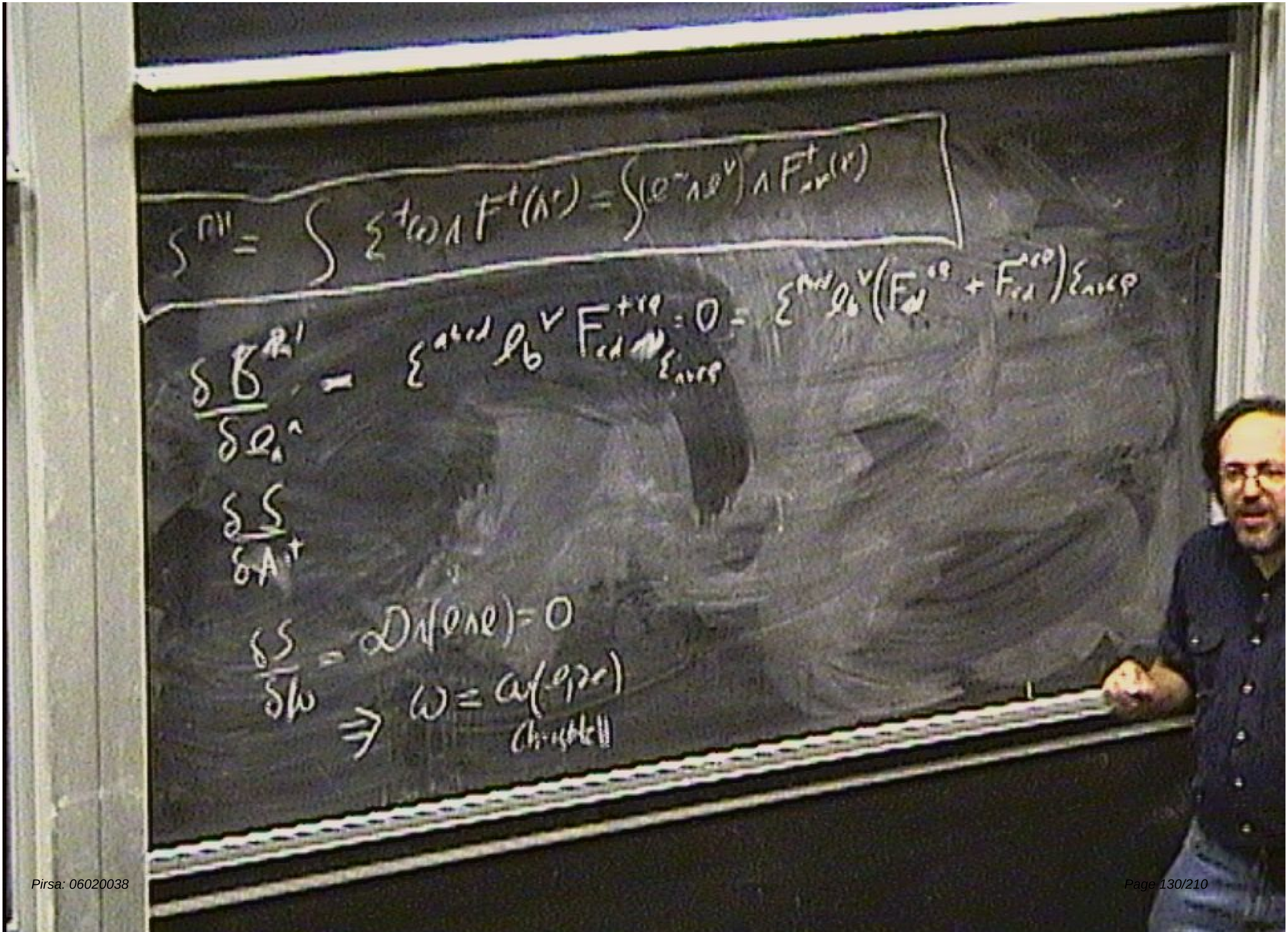
$$\boxed{S^{pl} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A)}$$

$$\frac{\delta S^{pl}}{\delta e_a^{\mu}} = \epsilon^{abcd} e_b^{\nu} F_{cd}^{+\alpha\beta} \Sigma_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd} e_b^{\nu} \left(F_{cd}^{\alpha\beta} + F_{cd}^{*\alpha\beta} \right) \Sigma_{\mu\nu\alpha\beta}$$

$$\frac{\delta S}{\delta A^{+}}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow$$


$S^{\Pi'} = \int \Sigma^+(\omega) \wedge F^+(A^+) = \int (e^\mu \wedge e^\nu) \wedge F^+_{\mu\nu}(A)$
$\epsilon^{abcd} e_b{}^\nu F_{cd}^{+\alpha\beta} = 0 = \epsilon^{abcd} e_b{}^\nu (F_{cd}{}^{\alpha\beta} + F_{cd}^{*\alpha\beta}) \epsilon_{\mu\nu\alpha\beta}$
$\frac{\delta S^{\Pi'}}{\delta e_a{}^\mu} = \epsilon^{abcd} e_b{}^\nu F_{cd}{}^{\alpha\beta} \epsilon_{\mu\nu\alpha\beta}$
$\frac{\delta S}{\delta A^+}$
$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$
$\Rightarrow \omega = \omega(e, \partial e)$
Christoffel

$$\boxed{S^{Pl} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A)}$$

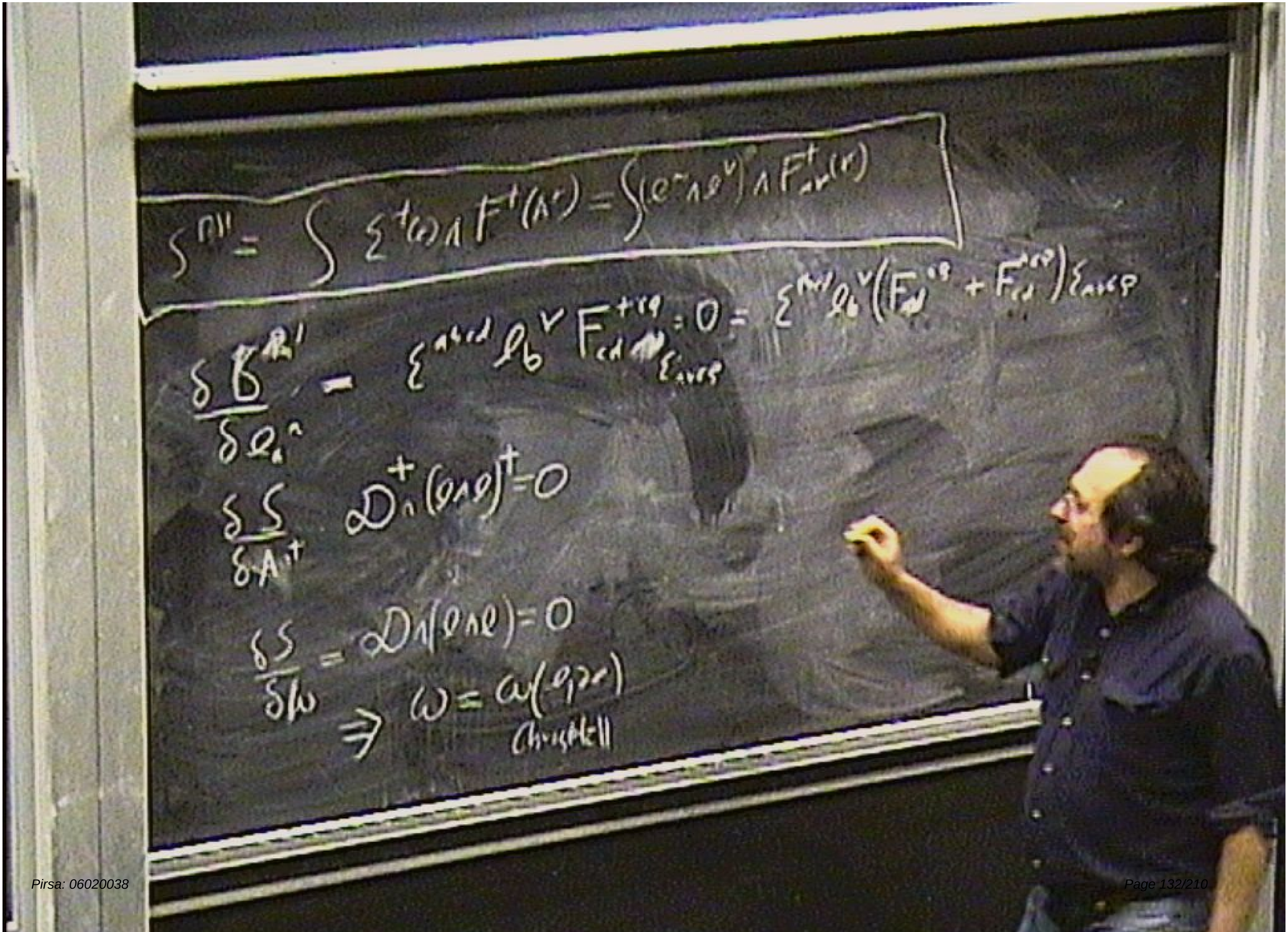
$$\frac{\delta S^{Pl}}{\delta e_a{}^{\mu}} = \epsilon^{abcd} e_b{}^{\nu} F_{cd}^{+\alpha\beta} \epsilon_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd} e_b{}^{\nu} \left(F_{cd}{}^{\alpha\beta} + F_{cd}^{*\alpha\beta} \right) \epsilon_{\mu\nu\alpha\beta}$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega = \omega(e)$$

Christoffel



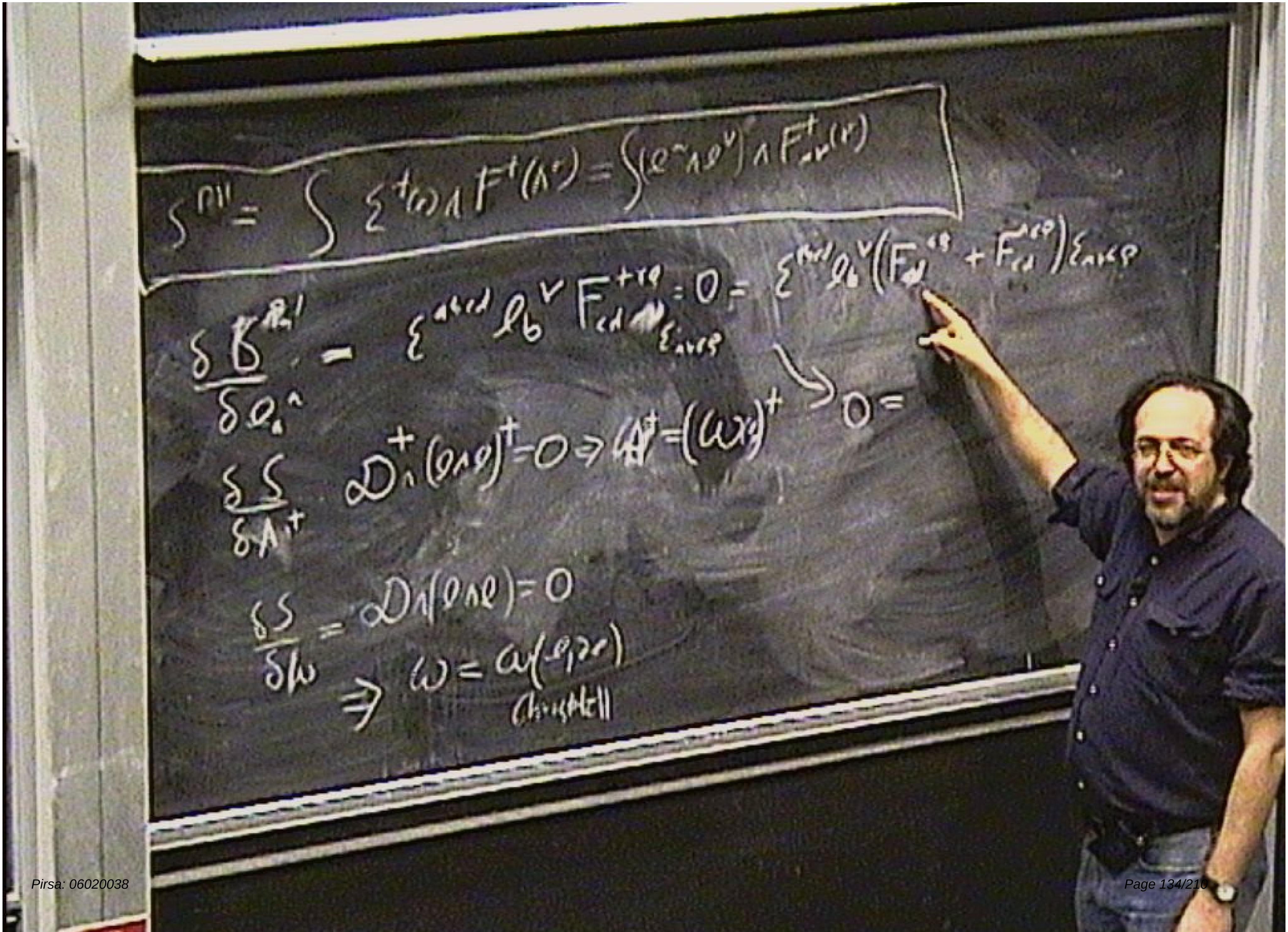
$$\boxed{S^{\Pi'} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A)}$$

$$\frac{\delta S^{\Pi'}}{\delta e_a{}^{\mu}} = \varepsilon^{abcd} e_b{}^{\nu} F_{cd}{}^{+\kappa\rho} \varepsilon_{\mu\nu\kappa\rho} = 0 = \varepsilon^{abcd} e_b{}^{\nu} \left(F_{cd}{}^{\kappa\rho} + F_{cd}{}^{*\kappa\rho} \right) \varepsilon_{\mu\nu\kappa\rho}$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+} \wedge (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega = \underset{\text{Christoffel}}{\omega(e, \partial e)}$$



$$S^{P\mathrm{II}} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A)$$

$$\frac{\delta S^{P\mathrm{II}}}{\delta e_a^{\mu}} = \epsilon^{abcd} e_b^{\nu} F_{cd}^{+\alpha\beta} \epsilon_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd} e_b^{\nu} \left(F_{cd}^{\alpha\beta} + F_{cd}^{*\alpha\beta}\right) \epsilon_{\mu\nu\alpha\beta}$$

$$\rightarrow 0 = \epsilon^{abcd} e_b^{\nu} \left[F_{cd}^{\alpha\beta}(\omega) + \quad\right] \epsilon_{\mu\nu\alpha\beta}$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+} \wedge (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega)^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega = \omega(e, \partial e)$$

Christoffel

$$\boxed{S^{PII} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu}\wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})}$$

$$\frac{\delta S^{PII}}{\delta e_a^{\mu}} = \epsilon^{abcd} e_b^{\nu} F^{+\alpha\beta}_{cd}\,\epsilon_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd} e_b^{\nu}\left(F_{cd}^{\alpha\beta} + F_{cd}^{*\alpha\beta}\right)\epsilon_{\mu\nu\alpha\beta}$$

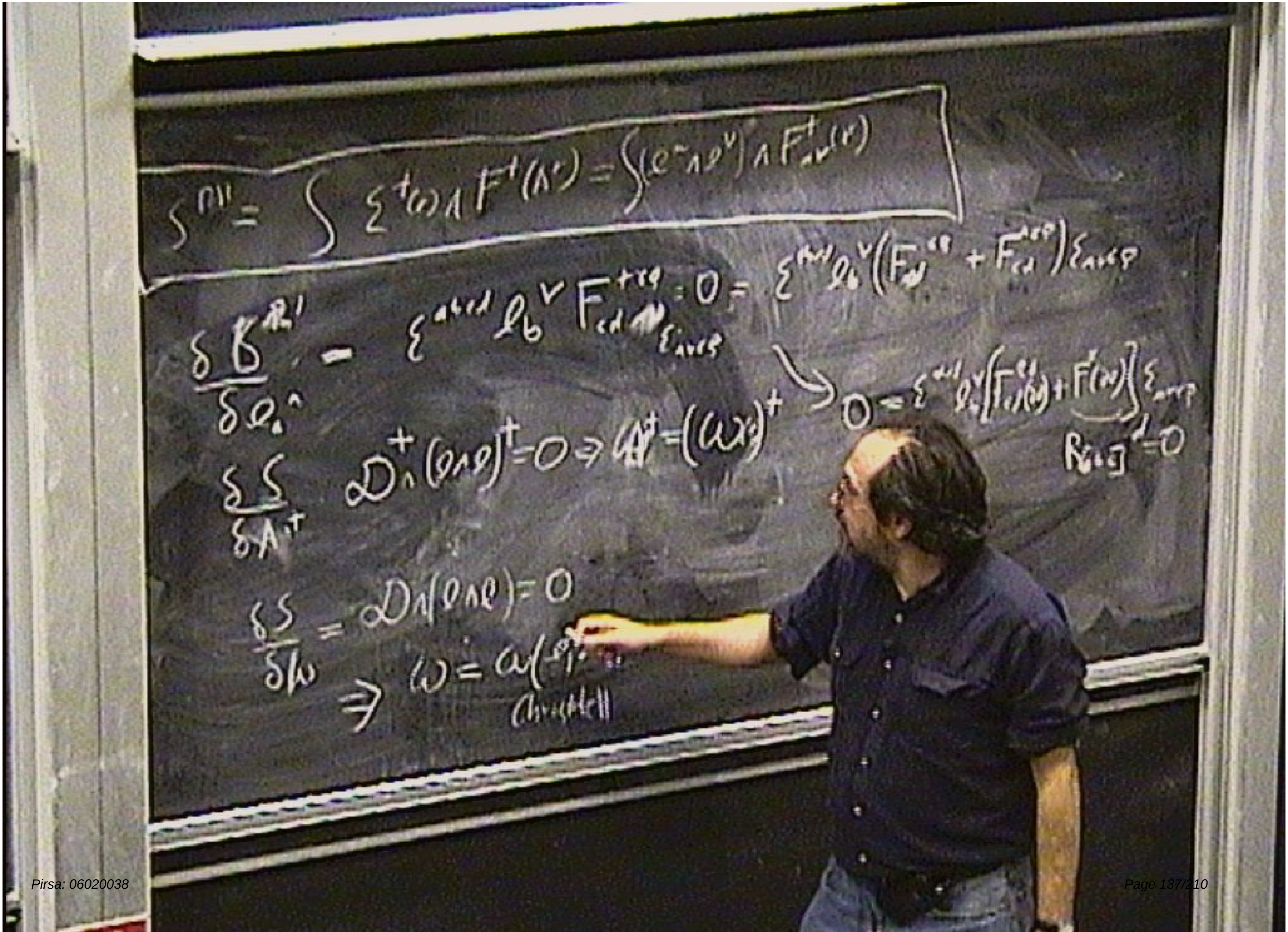
$$\Rightarrow 0 = \epsilon^{abcd} e_b^{\nu}\left[F_{cd}^{\alpha\beta}(\omega) + F(\omega)\right]\epsilon_{\mu\nu\alpha\beta}$$

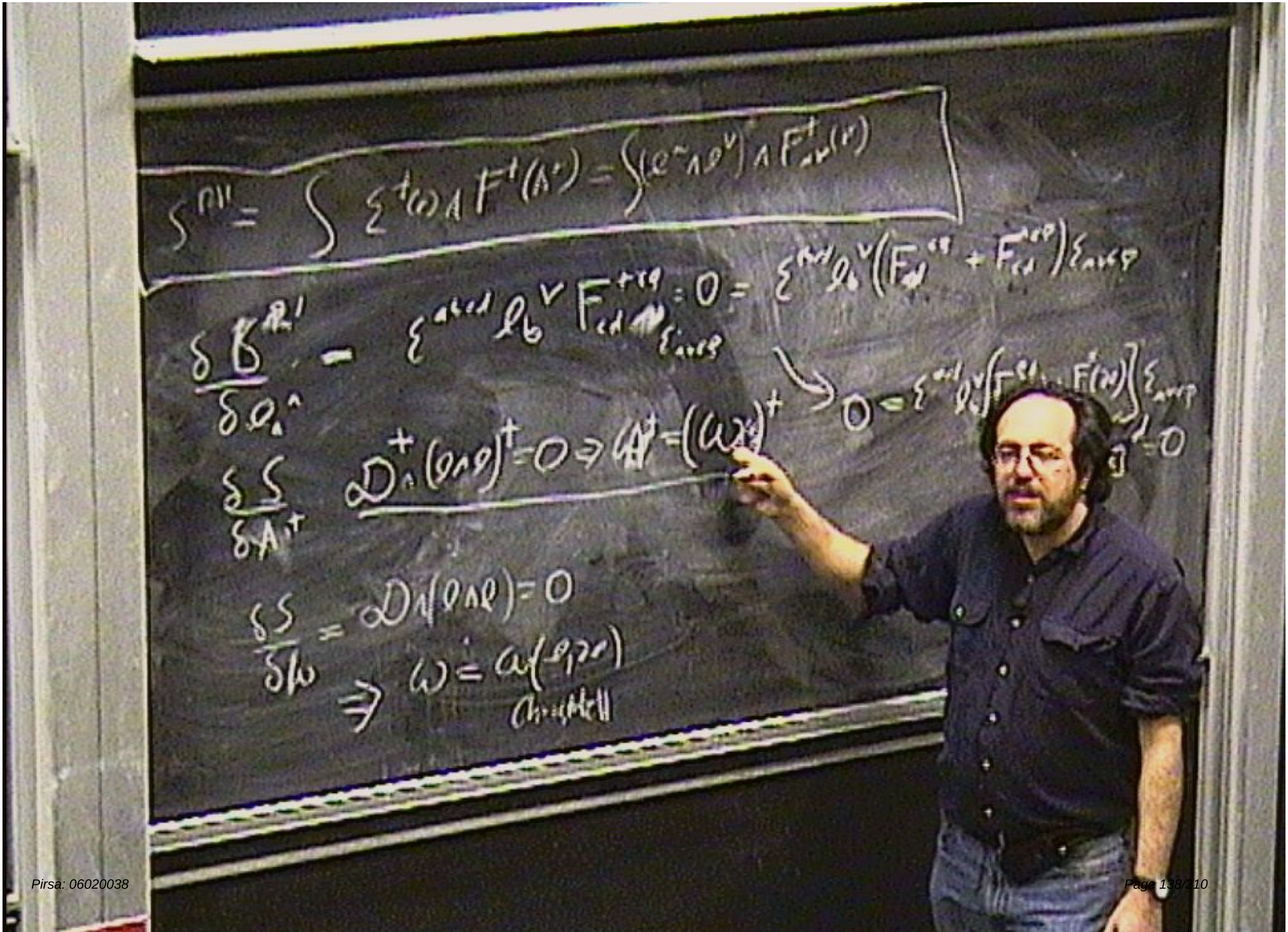
$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+}\wedge(e\wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}\wedge(e\wedge e) = 0$$

$$\Rightarrow \omega = \omega(e, \partial e)$$

Christoffel





$$\boxed{S^{\text{new}} = \int \Sigma^+(\omega) \wedge F^+(A^+) = \int (e^\mu \wedge e^\nu) \wedge F^+_{\mu\nu}(A^+)}$$

$$\frac{\delta S^{\text{new}}}{\delta e_a{}^\mu} = \epsilon^{abcd}\, e_b{}^\nu F^{+\alpha\beta}_{cd}\, \epsilon_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd}\, e_b{}^\nu \left(F_{cd}{}^{\alpha\beta} + F_{cd}{}^{\alpha\beta}\right)\epsilon_{\mu\nu\alpha\beta}$$

$$\to 0 = \epsilon^{abcd}\, e_b{}^\nu \left(\Gamma_{cd}(\omega) + F(\omega)\right)\epsilon_{\mu\nu\alpha\beta}$$

$$R = 0$$

$$\frac{\delta S}{\delta A^+} : \underline{\mathcal{D}^+_A (e \wedge e)^+ = 0 \Rightarrow \omega^+ = (\omega(e))^+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_\omega (e \wedge e) = 0$$

$$\Rightarrow \omega = \omega(e)$$

Christoffel

$$\boxed{S^{n''} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})}$$

$$\frac{\delta S^{n'}}{\delta e_a^{\mu}} = \epsilon^{abcd}\, e_b^{\nu}\, F_{cd}^{+\rho\sigma}\, \epsilon_{\mu\nu\rho\sigma} = 0 = \epsilon^{abcd}\, e_b^{\nu} \left(F_{cd}^{\rho\sigma} + F_{cd}^{*\rho\sigma}\right) \epsilon_{\mu\nu\rho\sigma}$$

$$\to 0 = \epsilon^{abcd}\, e_b^{\nu} \left[F_{cd}^{\rho\sigma}(\omega) + F(\omega)\right] \epsilon_{\mu\nu\rho\sigma}$$

$$R_{[abc]}{}^{d} = 0$$

$$\frac{\delta S}{\delta A^{+}} \quad \underline{\mathcal{D}^{+} \wedge (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega \doteq \omega(e, \partial e)$$

Christoffel

$$\boxed{S^{\Pi\Pi} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})}$$

$$\frac{\delta S}{\delta e_a{}^{\mu}} = \epsilon^{abcd}\, e_b{}^{\nu}\, F_{cd}^{+\alpha\beta}\, \epsilon_{\mu\nu\alpha\beta} = 0 = \epsilon^{abcd}\, e_b{}^{\nu} \left(F_{cd}{}^{\alpha\beta} + F_{cd}^{*\alpha\beta}\right) \epsilon_{\mu\nu\alpha\beta}$$

$$\rightarrow 0 = \epsilon^{abcd}\, e_b{}^{\nu} \Big[\underbrace{F_{cd}{}^{\alpha\beta}(\omega) + F^{*}(\omega)}\Big] \epsilon_{\mu\nu\alpha\beta}$$

$$R_{[abc]}{}^{d} = 0$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+}_{\wedge} (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega \doteq \omega(e, \partial e)$$

Christoffel

$A^{23} = \pm A^{01} \quad A^{i} = A^{0i} \quad i = 1,2,3$

$$A^{\pm\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$$

$$F = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{*}) + \frac{1}{2}(F - F^{*})$$

$$F^{+\mu\nu} = d A^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu} \; ; \; F^{+i} = d A^{i} + \frac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$$

$$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu} \qquad \Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$S^{Pl.} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{+\mu\nu} \wedge F^{+}_{\mu\nu} + \Sigma^{-\mu\nu} \wedge F^{-}_{\mu\nu}$$

$$= \int \left[\Sigma^{+\mu\nu}_{(e)} \wedge F^{+}_{\mu\nu}(\omega) - \Sigma^{-\mu\nu}_{(e)} \wedge F^{-}_{\mu\nu}(\omega) \right]$$

$$\boxed{S^{\mathrm{Pl}} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})}$$

$$\frac{\delta S^{\mathrm{Pl}}}{\delta e_a^{\mu}} = \epsilon^{abcd}\, e_b^{\nu}\, F^{+\rho\sigma}_{cd}\,\epsilon_{\mu\nu\rho\sigma} = 0 = \epsilon^{abcd}\, e_b^{\nu}\left(F_{cd}^{\rho\sigma} + F_{cd}^{*\rho\sigma}\right)\epsilon_{\mu\nu\rho\sigma}$$

$$\to 0 = \epsilon^{abcd}\, e_b^{\nu}\left[F_{cd}^{\rho\sigma}(\omega) + F(\lambda\omega)\right]\epsilon_{\mu\nu\rho\sigma}$$

$$R_{[\mu\nu]} = 0$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+}\wedge (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}\wedge(e \wedge e) = 0$$

$$\Rightarrow \omega \doteq \omega(e, \partial e)$$

Christoffel

$$\boxed{S^{Pl} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(A^{+})}$$

$\boxed{\text{Check}}$

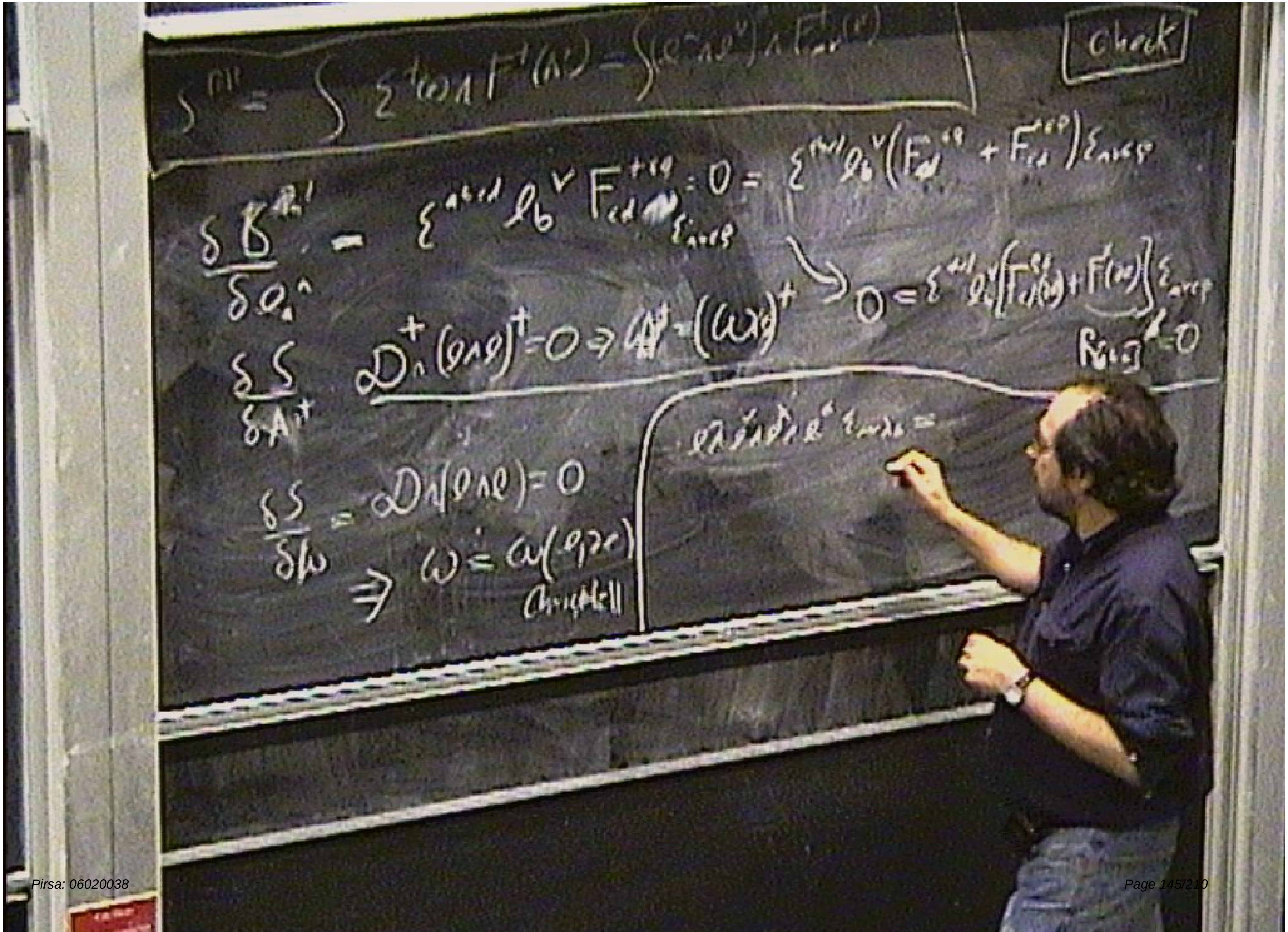
$$\frac{\delta S^{Pl}}{\delta e_a{}^{\mu}} = \epsilon^{abcd} e_b{}^{\nu} F_{cd}^{+\rho\sigma} \epsilon_{\mu\nu\rho\sigma} = 0 = \epsilon^{abcd} e_b{}^{\nu} \left(F_{cd}^{\rho\sigma} + F_{cd}^{*\rho\sigma}\right) \epsilon_{\mu\nu\rho\sigma}$$

$$\to 0 = \epsilon^{abcd} e_b{}^{\nu} \left[F_{cd}^{\rho\sigma}(\omega) + \underbrace{F(\omega)}_{R_{[abc]}{}^{d} = 0} \right] \epsilon_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta A^{+}} \quad \underline{D^{+} \wedge (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}}$$

$$\frac{\delta S}{\delta \omega} = D \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega \doteq \underbrace{\omega(e, \partial e)}_{\text{Christoffel}}$$


Check

$$S^{PU} = \int \Sigma^{+}(e) \wedge F^{+}(A^{+}) - \int (e^{\mu} \wedge e^{\nu}) \wedge F^{+}_{\mu\nu}(\omega)$$

Check

$$\varepsilon^{abcd} e_b{}^{\nu} F_{cd}{}^{+\kappa\rho} = 0 = \varepsilon^{abcd} e_b{}^{\nu} (F_{cd}{}^{\kappa\rho} + F_{cd}{}^{\kappa\rho}) \varepsilon_{\nu\lambda\kappa\rho}$$

$$\frac{\delta S}{\delta e_a{}^{\nu}} = \varepsilon^{abcd} e_b{}^{\nu} F_{cd} \varepsilon_{\nu\lambda\kappa\rho}$$

$$\Rightarrow 0 = \varepsilon^{abcd} e_b{}^{\nu} [F_{cd}(\omega) + F(\omega)] \varepsilon_{\nu\lambda\kappa\rho}$$

$$R_{[abc]}{}^{d} = 0$$

$$\frac{\delta S}{\delta A^{+}} \quad \mathcal{D}^{+}_{\wedge} (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

$$\Rightarrow \omega \doteq \omega(e_{\mu}{}^{a})$$

Christoffell

$$e \wedge e^{\nu} \wedge e^{\lambda} \wedge e^{\rho} \varepsilon_{\mu\nu\lambda\rho} =$$

$$\Sigma^{+\mu\nu} \wedge \Sigma^{+}_{\mu\nu} - \Sigma^{-\mu\nu} \wedge \Sigma^{-}_{\mu\nu}$$

$A^{23} = \pm A^{01} \qquad A^{i} = A^{0i} \qquad i = 1,2,3$

$$\vec{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{*\mu\nu}$$

$$F^{\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \tfrac{1}{2}\left((F + F^{*}) + \tfrac{1}{2}(F - F^{*})\right)$$

$$F^{+\mu\nu} = dA^{+\mu\nu} + A^{+\mu}{}_{\lambda} \wedge A^{+\lambda\nu}; \quad F^{+i} = dA^{i} + \tfrac{1}{2}\epsilon^{i}{}_{jk} A^{j} \wedge A^{k}$$

$$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$\Sigma^{\mu\nu} = e^{\mu} \wedge e^{\nu} = -\Sigma^{\nu\mu}$$

$$S^{Pl} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{\mu\nu}_{i} \wedge F^{+}_{\mu\nu} + \Sigma^{\mu\nu} \wedge F^{-}_{\mu\nu}$$

$$= \int \left[\Sigma^{+\mu\nu}_{(i)} \wedge F^{+}_{\mu\nu(i)} - \Sigma^{-\mu\nu}_{(i)} \wedge F^{-}_{\mu\nu(-i)} + 2\Lambda\left[\Sigma^{+} \wedge \Sigma^{+} - \Sigma^{-} \wedge \Sigma^{-}\right] \right]$$

$$S^{\Omega'} = \int \Sigma^{+}(\omega) \wedge F^{+}(A^{+}) = \int (e^{\mu} \wedge e^{\nu})^{+} \wedge F^{+}_{\mu\nu}(A^{+})$$

$\boxed{\text{Check}}$

$$\frac{\delta S^{\Omega'}}{\delta e_a^{\mu}} = \epsilon^{abcd} e_b^{\nu} F_{cd}^{+\kappa\rho} \epsilon_{\mu\nu\kappa\rho} = 0 = \epsilon^{abcd} e_b^{\nu} \left(F_{cd}^{\kappa\rho} + F_{cd}^{*\kappa\rho}\right) \epsilon_{\mu\nu\kappa\rho}$$

$$\Rightarrow 0 = \epsilon^{abcd} e_b^{\nu} \left[F_{cd}^{\kappa\rho}(\omega) + F^{*}(\omega)\right] \epsilon_{\mu\nu\kappa\rho}$$

$$R_{[abc]}{}^{d} = 0$$

$$\frac{\delta S}{\delta A^{+}} \qquad \mathcal{D}^{+}_{\wedge} (e \wedge e)^{+} = 0 \Rightarrow A^{+} = (\omega(e))^{+}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D} \wedge (e \wedge e) = 0$$

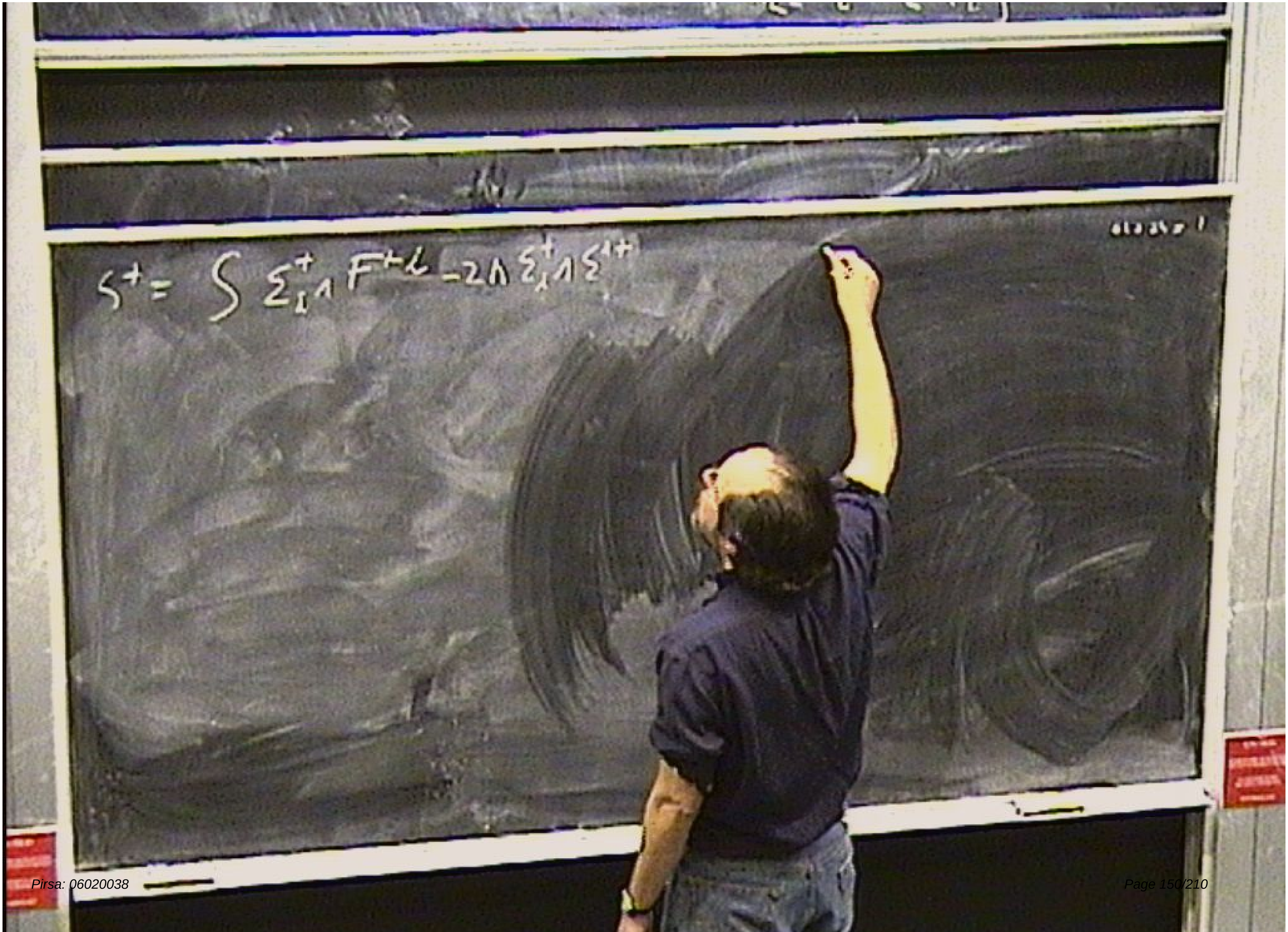
$$\Rightarrow \omega \doteq \omega(e, \partial e)$$

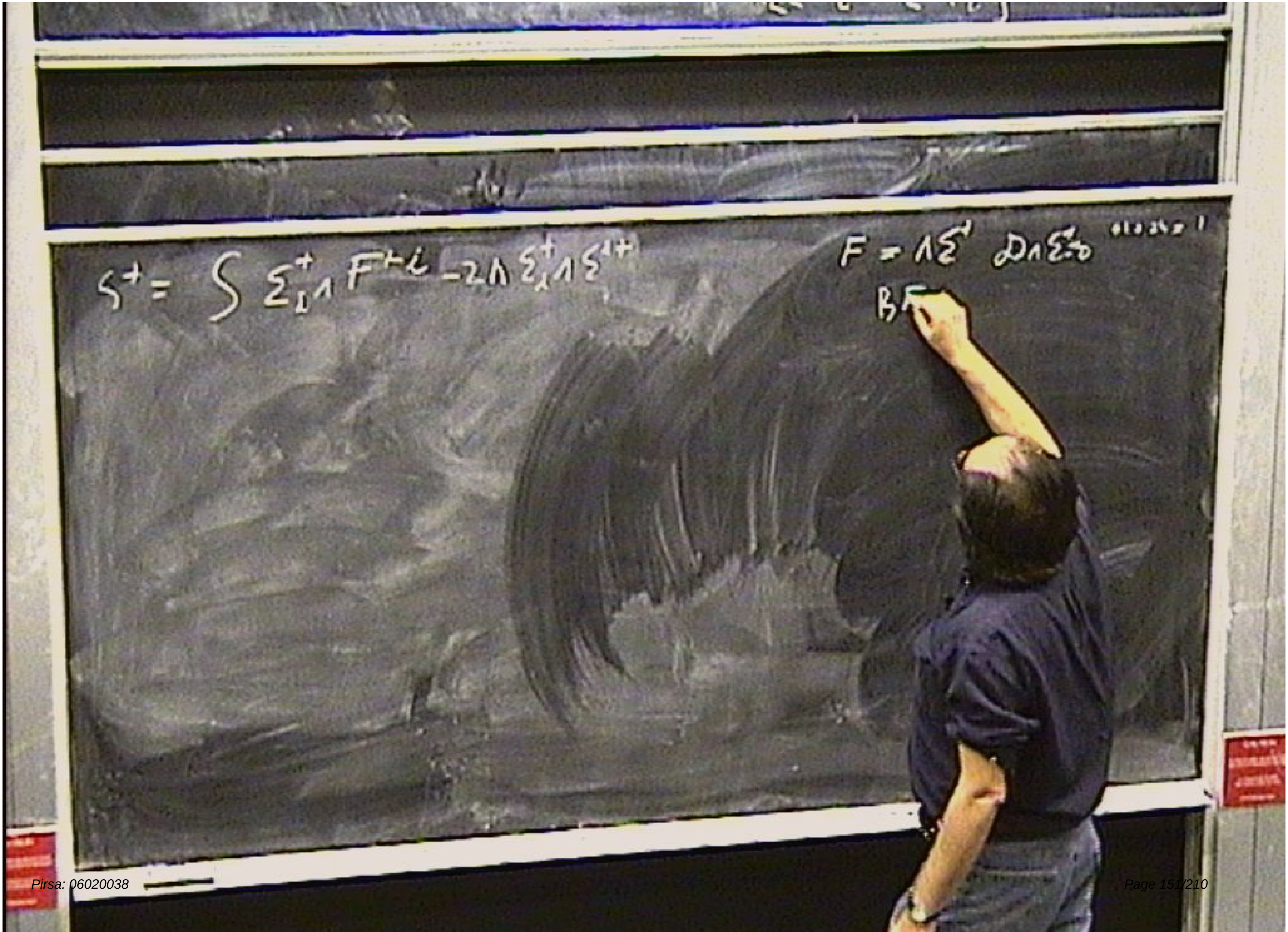
Christoffel

$$e \wedge e \wedge e \wedge e \, \epsilon_{\mu\nu\lambda\sigma} =$$

$$\Sigma^{+\mu\nu} \wedge \Sigma^{+}_{\mu\nu} - \Sigma^{-\mu\nu} \wedge \Sigma^{-}_{\mu\nu}$$

$$S^+ = \int \Sigma^+_i \wedge F^{+i}$$

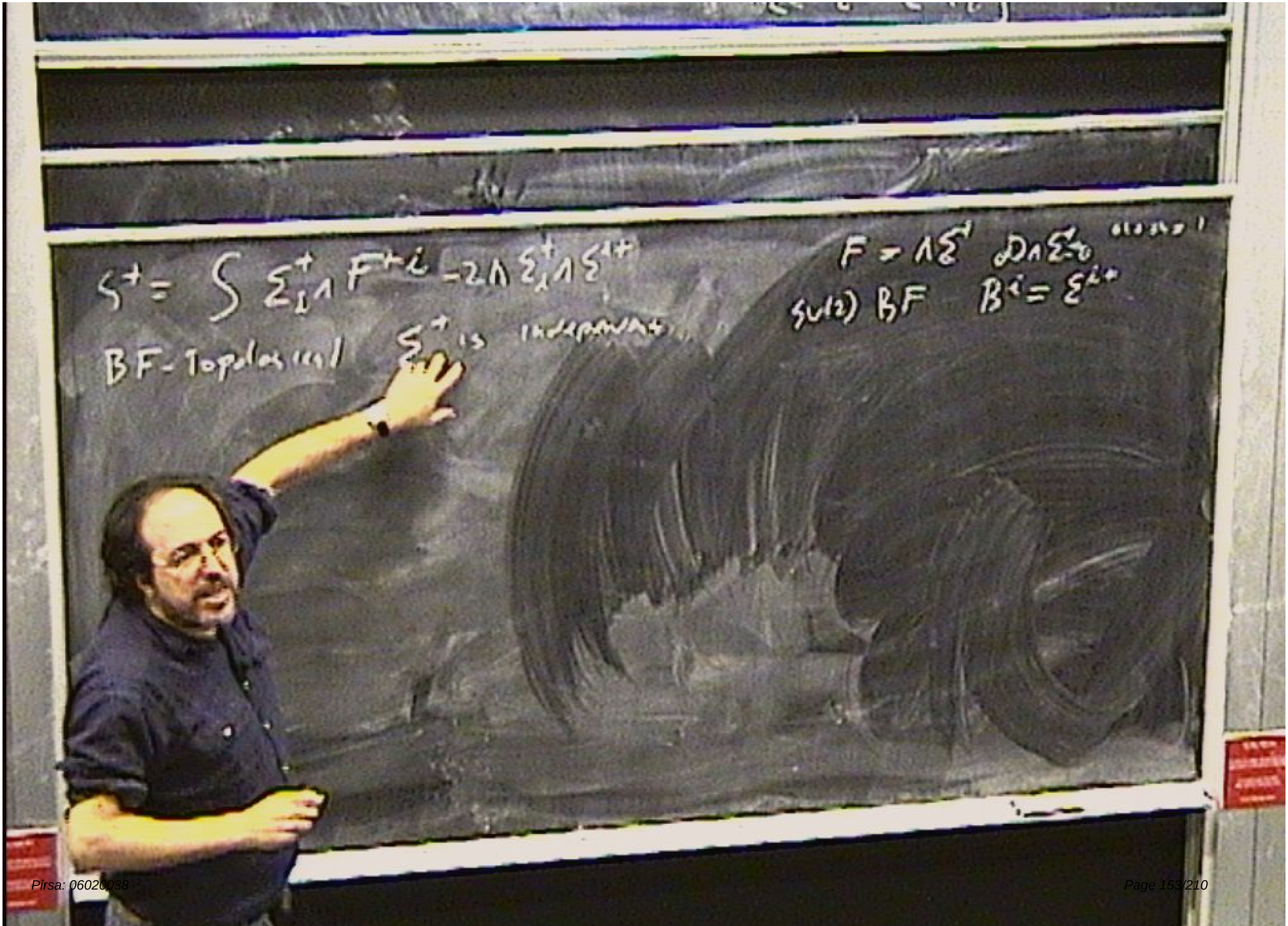

$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda \Sigma^+_i \wedge \Sigma^{i+}$

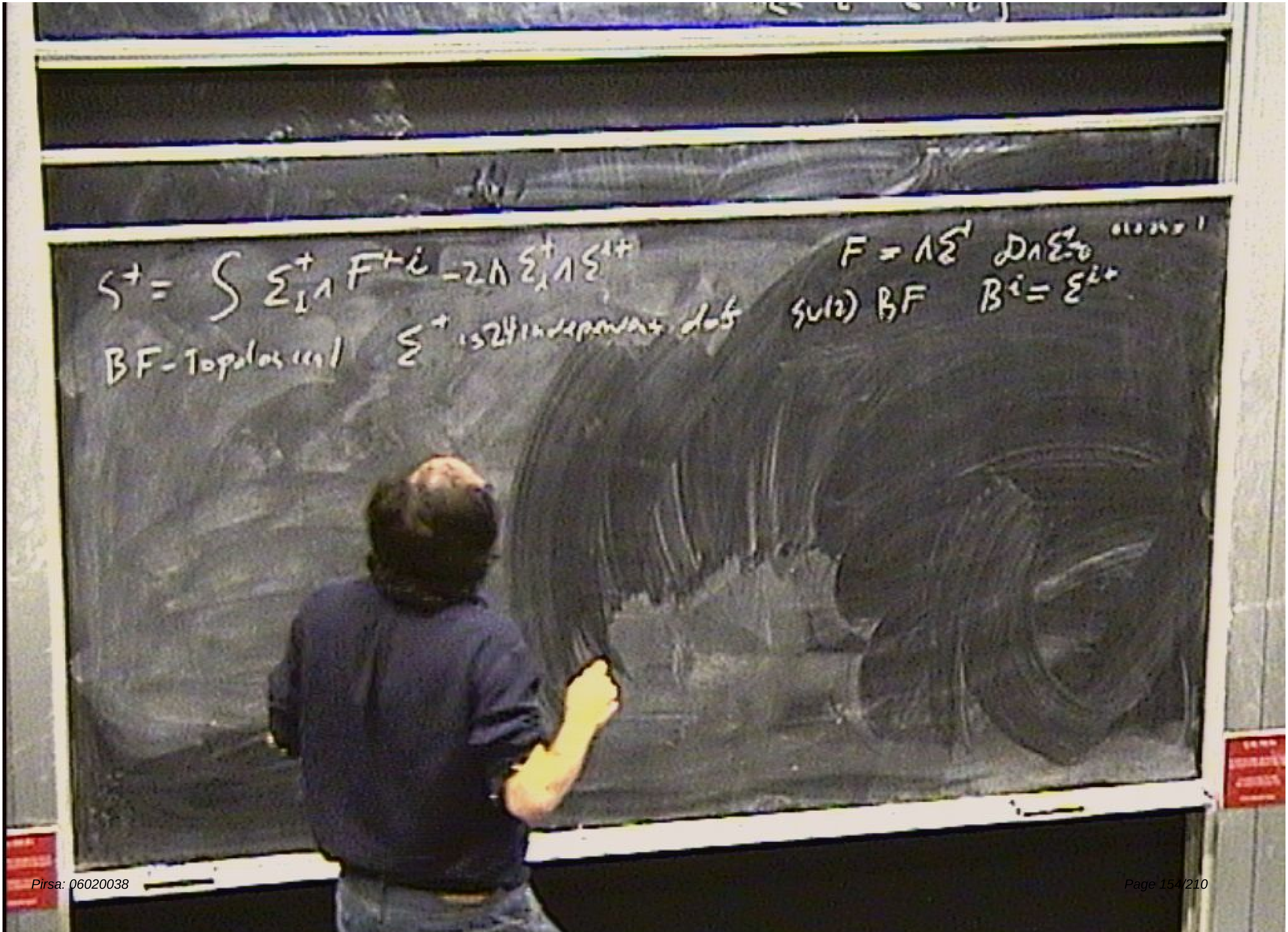


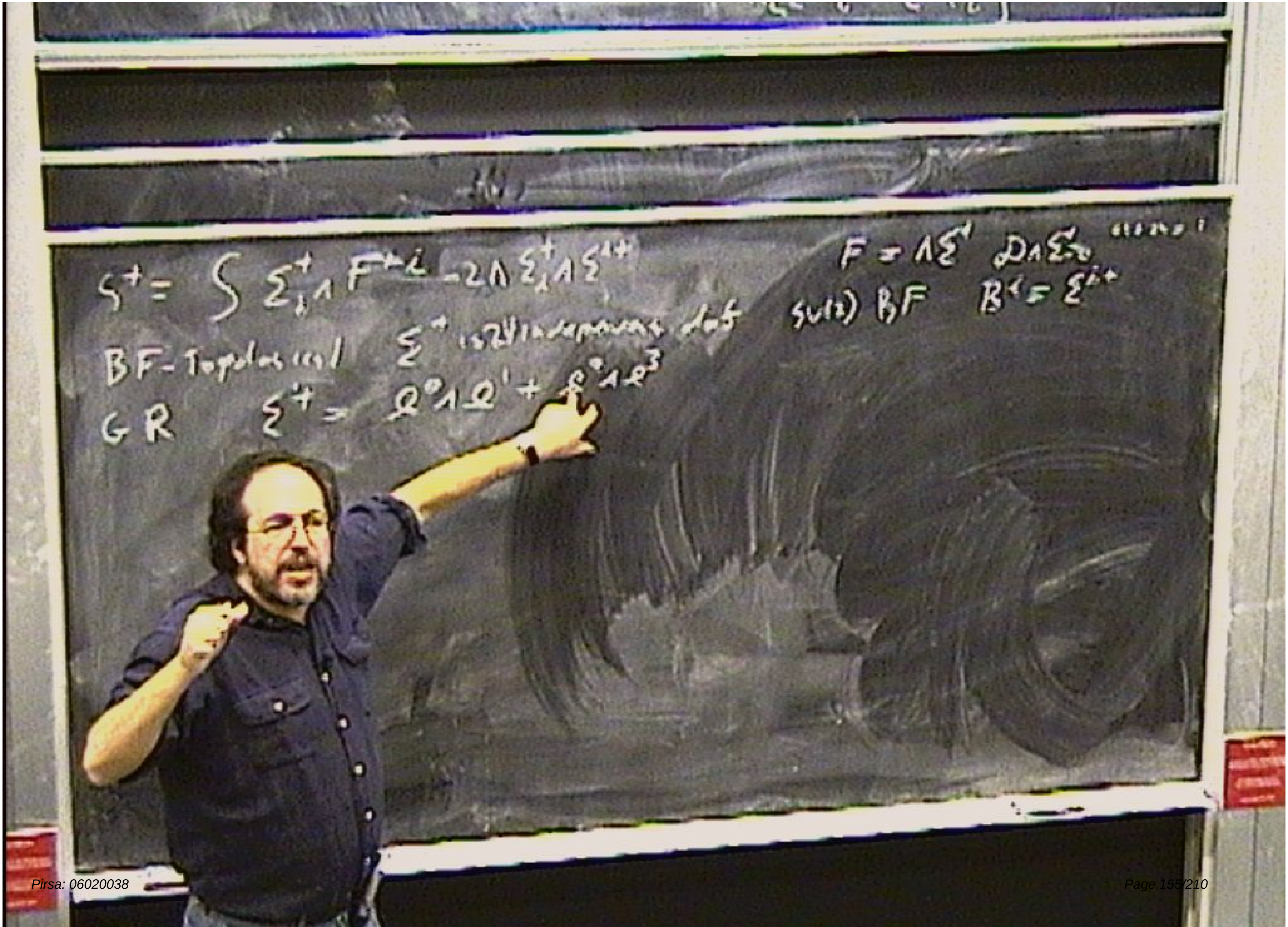
$$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda\, \Sigma_i^+ \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma^i \quad D \wedge \Sigma^i = 0$

$su(2)$ $BF \quad B^i = \Sigma^{i+}$


BF - Topological
is independent
su(2) BF

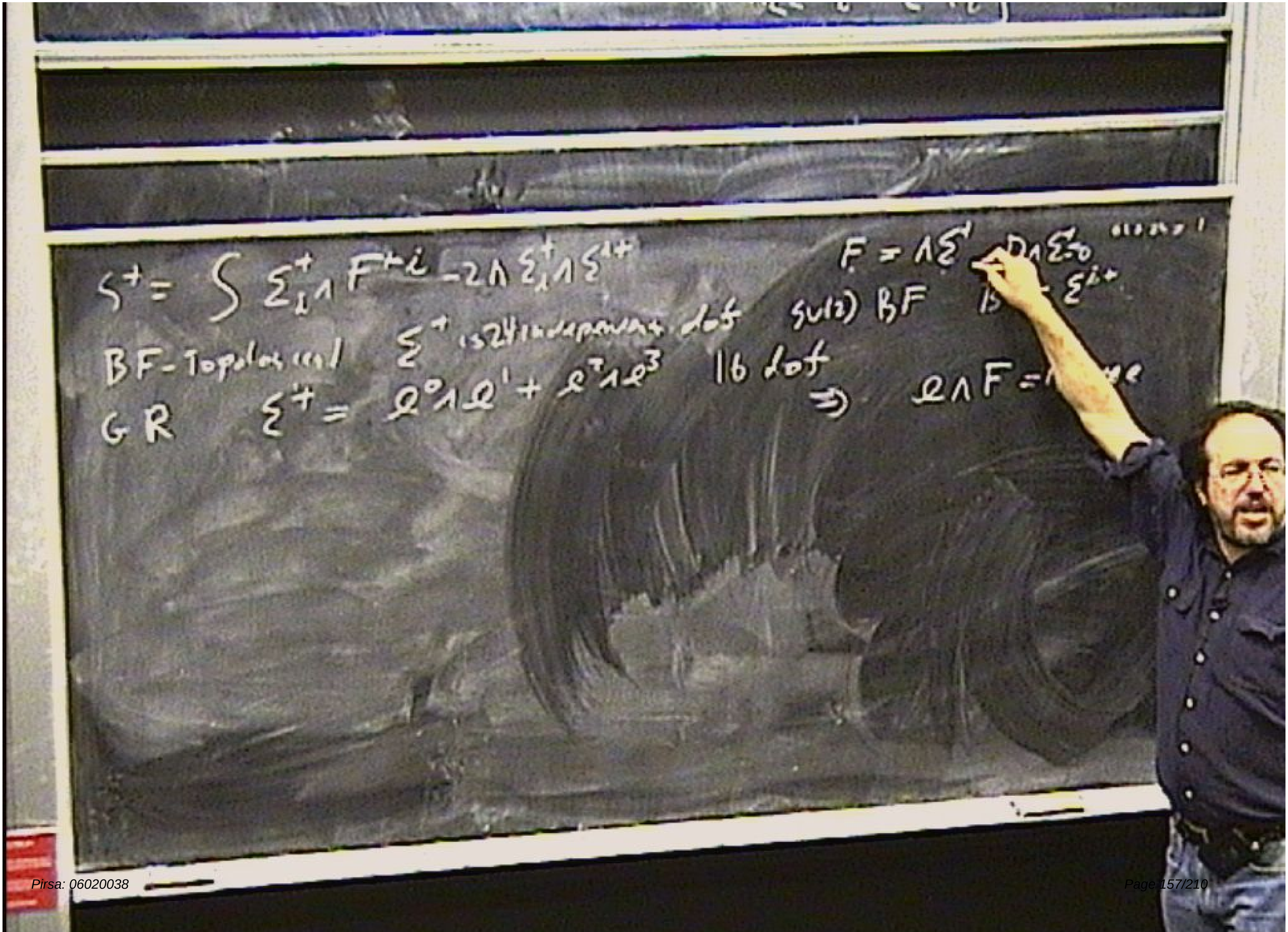

BF-Topological
su(2) BF


$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda \Sigma^+_i \wedge \Sigma^{i+}$
$F = \Lambda \Sigma^i$
$D \wedge \Sigma = 0$
su(2) BF
$B^i = \Sigma^{i+}$
BF - Topological
GR
$\Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3$

$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda\, \Sigma_i^+ \wedge \Sigma^{i+}$ $\qquad F = \Lambda \Sigma^i \quad D \wedge \Sigma^i = 0$

BF - Topological $\qquad \Sigma^+$ is 24 independent dof $\qquad su(2)$ BF $\qquad B^i = \Sigma^{i+}$

GR $\qquad \Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3 \qquad$ 16 dof

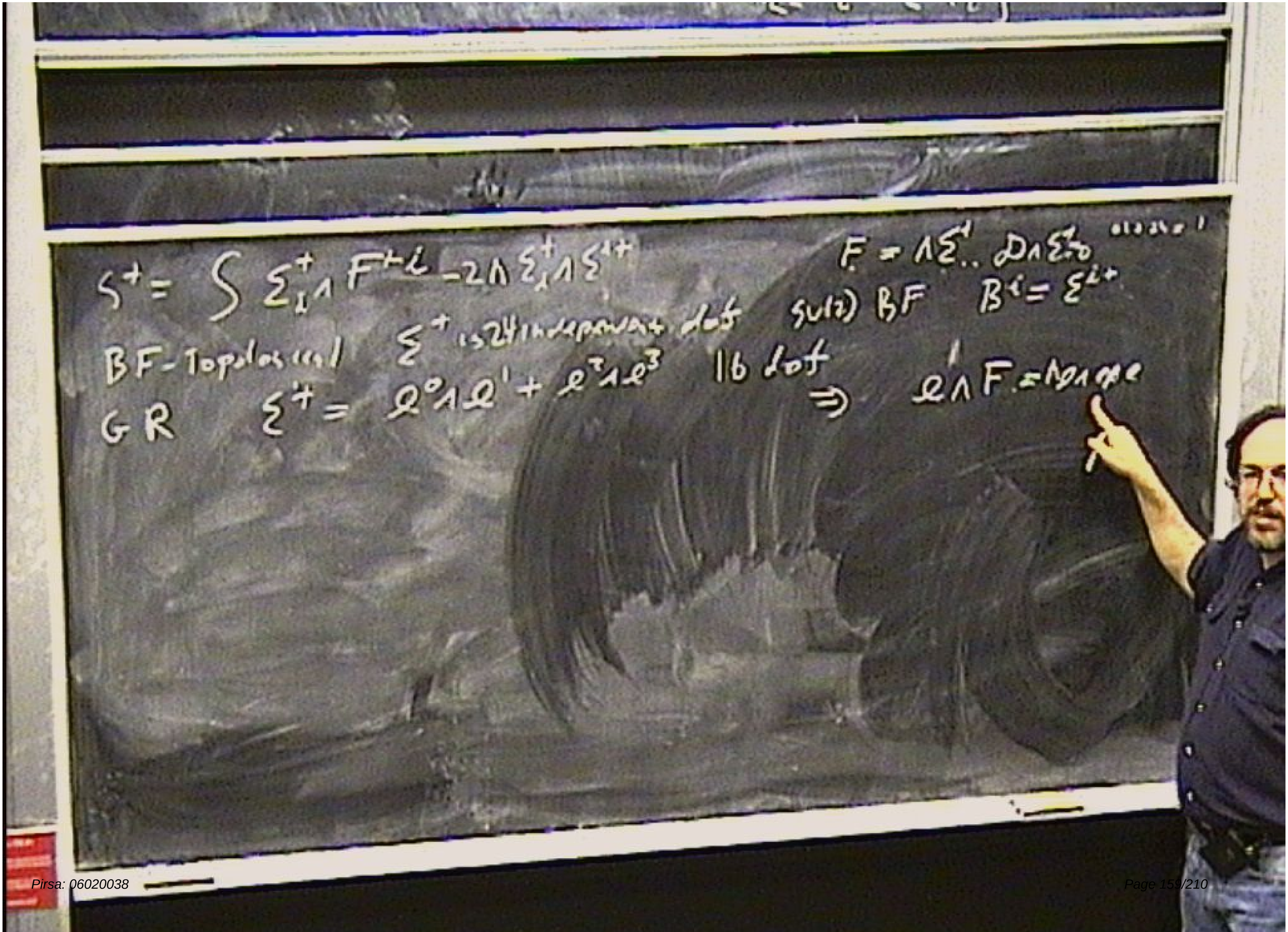

$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda \Sigma^+_i \wedge \Sigma^{i+}$
$F = \Lambda \Sigma^i$ $D \wedge \Sigma = 0$
$su(2)$ BF $B = \Sigma^{i+}$
Σ^+ is 24 independent dof
BF - Topological
16 dof
GR $\Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3$
$\Rightarrow$ $e \wedge F =$

$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda\, \Sigma_i^+ \wedge \Sigma^{i+}$ $\qquad F = \Lambda \Sigma_{..} \quad D \wedge \Sigma = 0$

Σ^+ is 24 independent dof $\qquad$ su(2) BF $\qquad B^i = \Sigma^{i+}$

BF - Topological $\qquad$ Σ $\qquad$ 16 dof

GR $\qquad \Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3 \qquad \Rightarrow \qquad e \wedge \dots \wedge e$


$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda \Sigma_i^+ \wedge \Sigma^{i+}$
su(2) BF $B^i = \Sigma^{i+}$
BF - Topological
Σ^+ is 24 independent dof
16 dof
GR $\Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3$
$\Rightarrow$

$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda\, \Sigma_i^+ \wedge \Sigma^{i+}$

$F = \Lambda \Sigma\ldots \quad D \wedge \Sigma = 0$

su(2) BF $\quad B^i = \Sigma^{i+}$

BF - Topological $\quad \Sigma^+$ is 24 independent dof

GR $\quad \Sigma^+ = e^0 \wedge e^1 + e^2 \wedge e^3 \quad$ 16 dof

$\Rightarrow \quad e \wedge F = \Lambda e \wedge e \wedge e$

$$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda\, \Sigma_i^+ \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma \quad D\wedge\Sigma = 0$

Σ^+ is 24 independent dof $\quad SU(2)$ BF $\quad B^i = \Sigma^{i+}$

BF - Topological

GR $\quad \Sigma^{i+} = e^0 \wedge e^1 + e^2 \wedge e^3 \quad$ 16 dof $\quad \Rightarrow \quad e^1 \wedge F = \Lambda\, e \wedge e$

$$\Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$

$F = \Lambda \Sigma$.. $D \wedge \Sigma = 0$

$su(2)$ BF $\quad B^i = \Sigma^{i+}$

<u>BF - Topological</u> $\quad \Sigma^+$ is 24 independent dof

<u>GR</u> $\quad \Sigma^+ = i(e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof $\quad \Rightarrow \quad e \wedge F = \Lambda e \wedge e \wedge e$

$$\Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

$$S^{+} = \int \Sigma^{+}_{i} \wedge F^{+i} - 2\Lambda\, \Sigma^{+}_{i} \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma^{i}$ .. $D \wedge \Sigma^{i} = 0$

$su(2)$ BF $\quad B^{i} = \Sigma^{i+}$

BF - Topological $\quad \Sigma^{+}$ is 18 independent dof

GR $\quad \Sigma^{i+} = i(e^{0} \wedge e^{1} + e^{2} \wedge e^{3})$ 16 dof $\Rightarrow$ $e^{1} \wedge F = \Lambda e \wedge e \wedge e$

$$\Sigma^{i} \wedge \Sigma^{j} - \frac{1}{3} \delta^{ij} \Sigma^{k} \wedge \Sigma^{k} = 0$$

$S^{+} = \int \Sigma^{+}_{i} \wedge F^{+i} - 2\Lambda \Sigma^{+}_{i} \wedge \Sigma^{+i}$

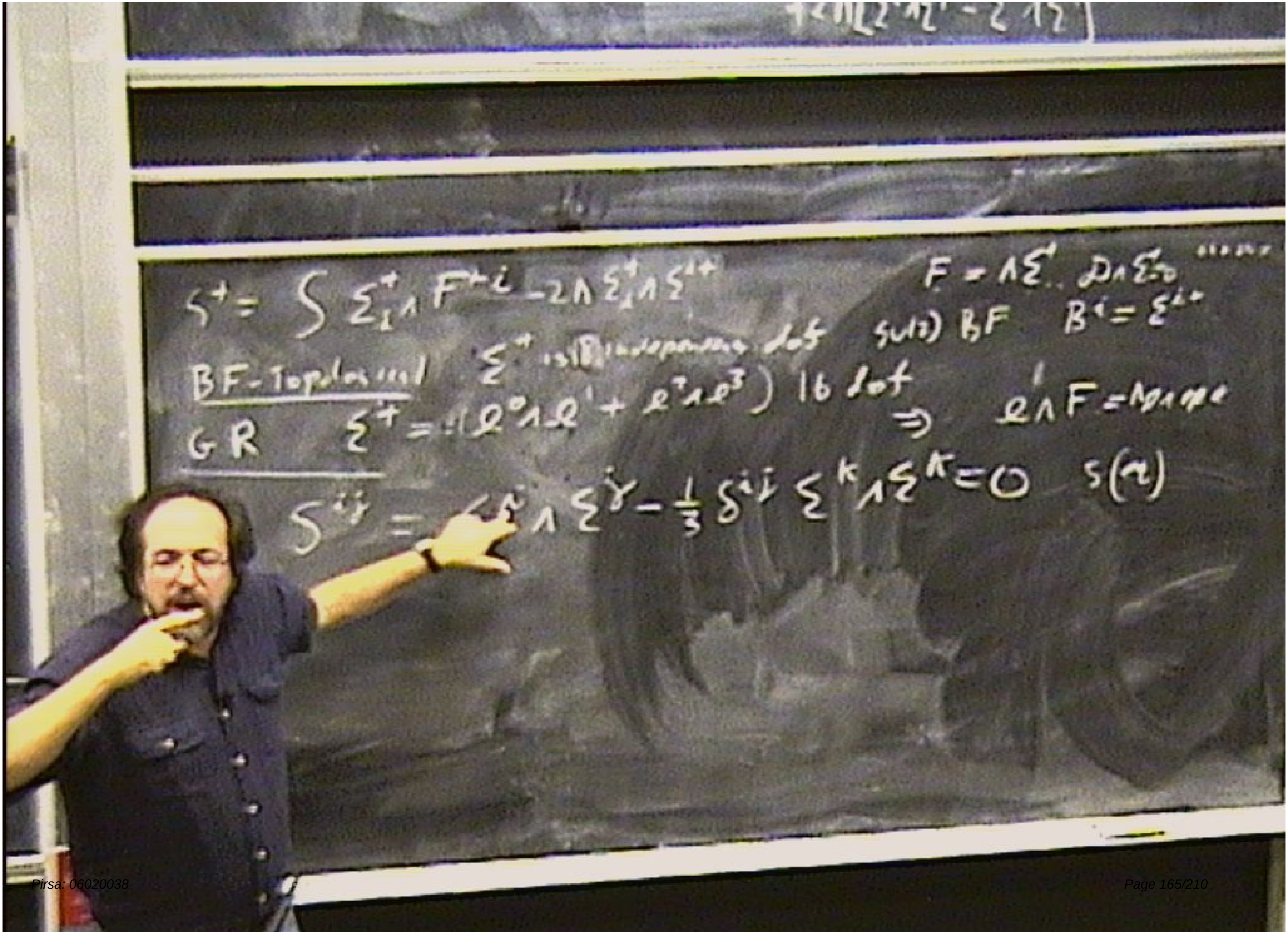
$\Sigma = \Lambda \Sigma^{+}$ $D_\omega \Sigma_+$

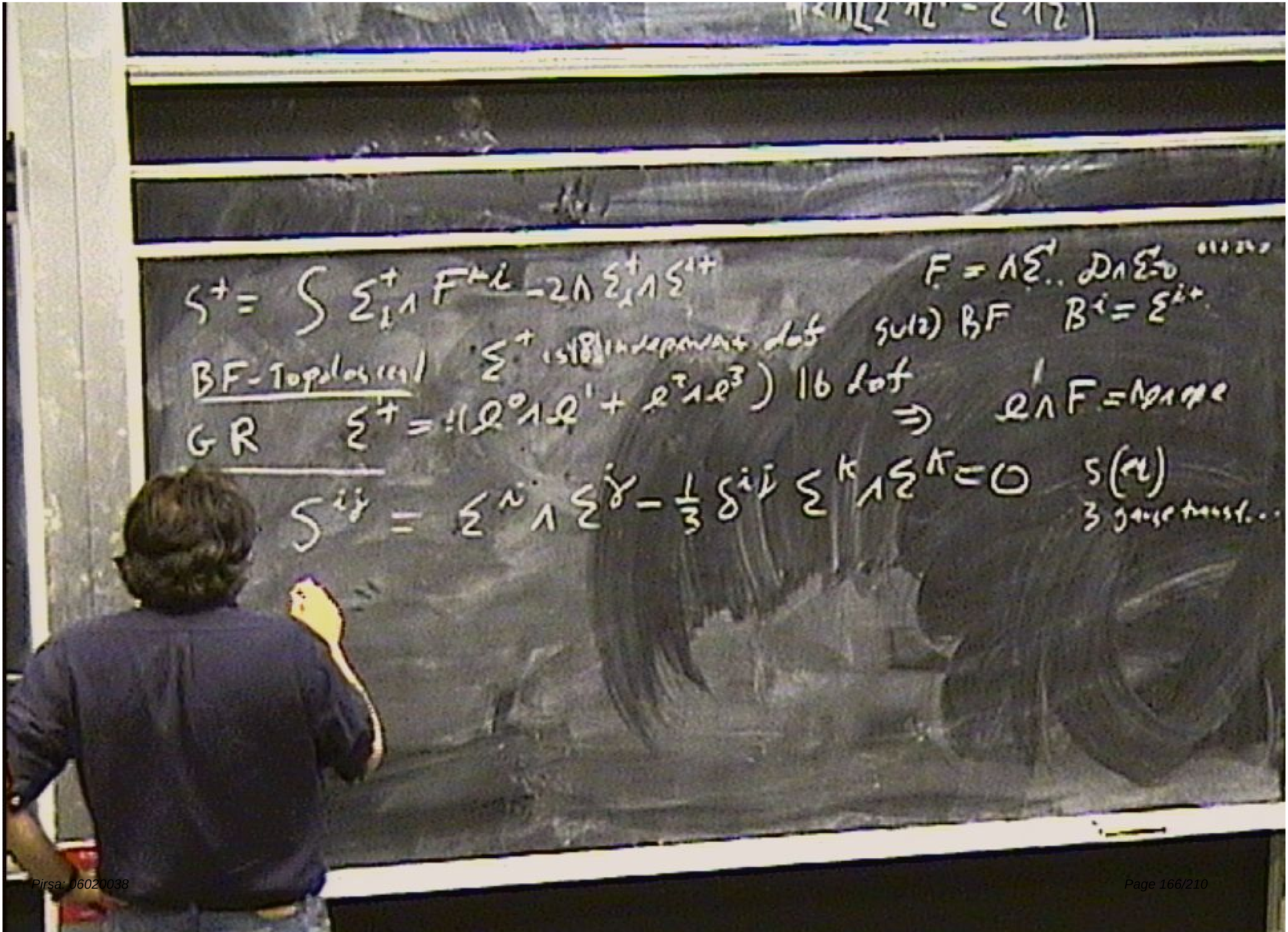
$s(2)$ BF $B^{+} = \Sigma^{+}$

BF-Topological Σ^{+} arbitrary 18 dof

GR $\Sigma^{+} = (e^{0} \wedge e + e^{i} \wedge e^{j})$ 16 dof $\Rightarrow$ $e \wedge F = \Lambda e \wedge e$

$S^{ij} = \Sigma^{i} \wedge \Sigma^{j} - \frac{1}{3} \delta^{ij} \Sigma^{k} \wedge \Sigma^{k} = 0$ $s(n)$


BF-Topological
GR
16 dof
su(2) BF


S+ = ∫ Σ+i ∧ F+i − 2Λ Σ+i ∧ Σi+
F = ΛΣ.. DΛΣ=0
su(2) BF   Bi = Σi+
BF-Topological   Σ+ is 18 independent dof
GR   Σ+ = i(e0∧e1 + e2∧e3) 16 dof
⇒ e∧F = ...
Sij = Σi ∧ Σj − 1/3 δij Σk ∧ Σk = 0
3 gauge transf...

$$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$$

$F = \Lambda\Sigma$.. $D\wedge\Sigma$ ...

$su(2)$ BF $\quad B^i = \Sigma^{i+}$

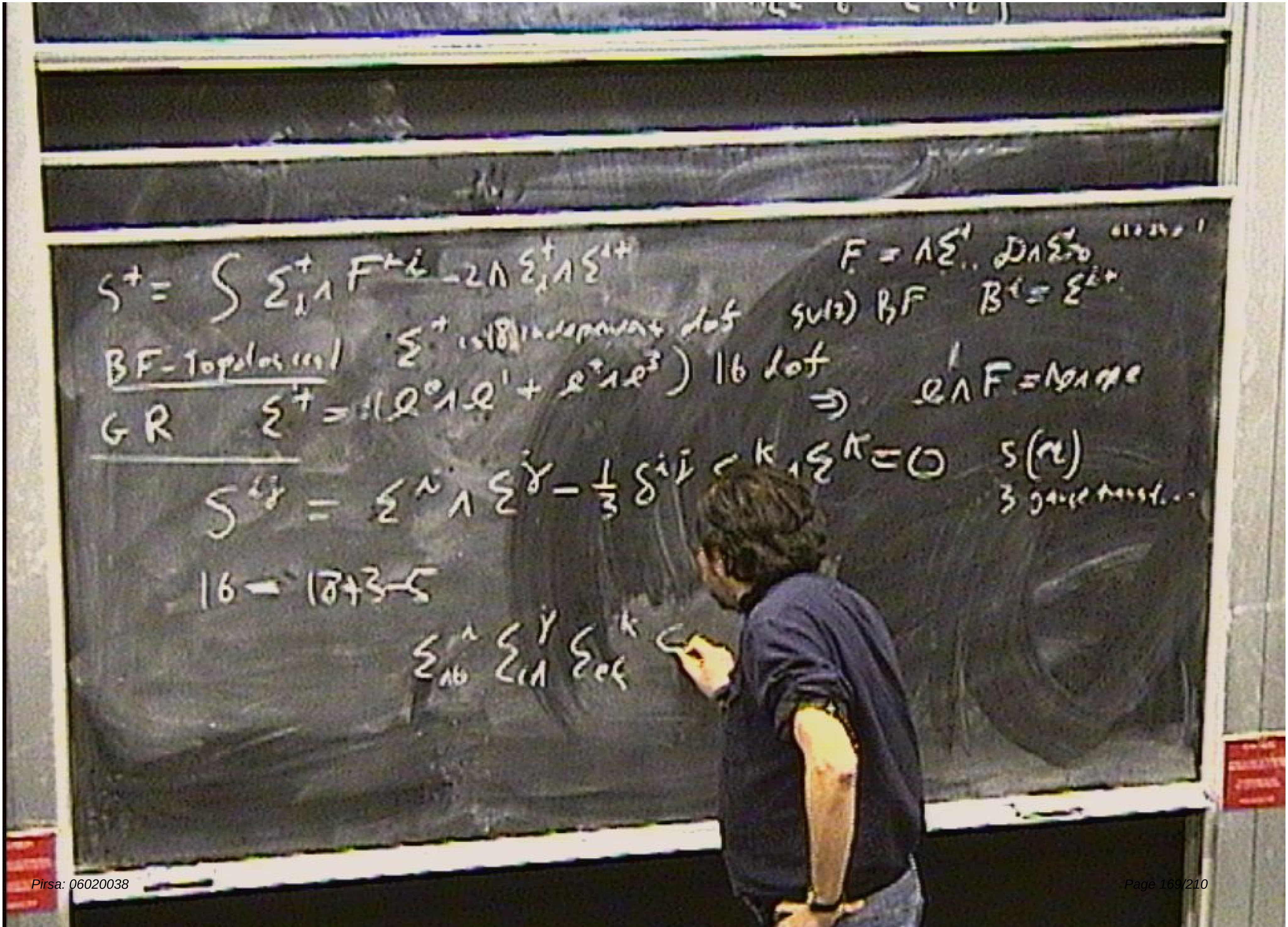
BF - Topological $\quad \Sigma^+$ is 18 independent dof

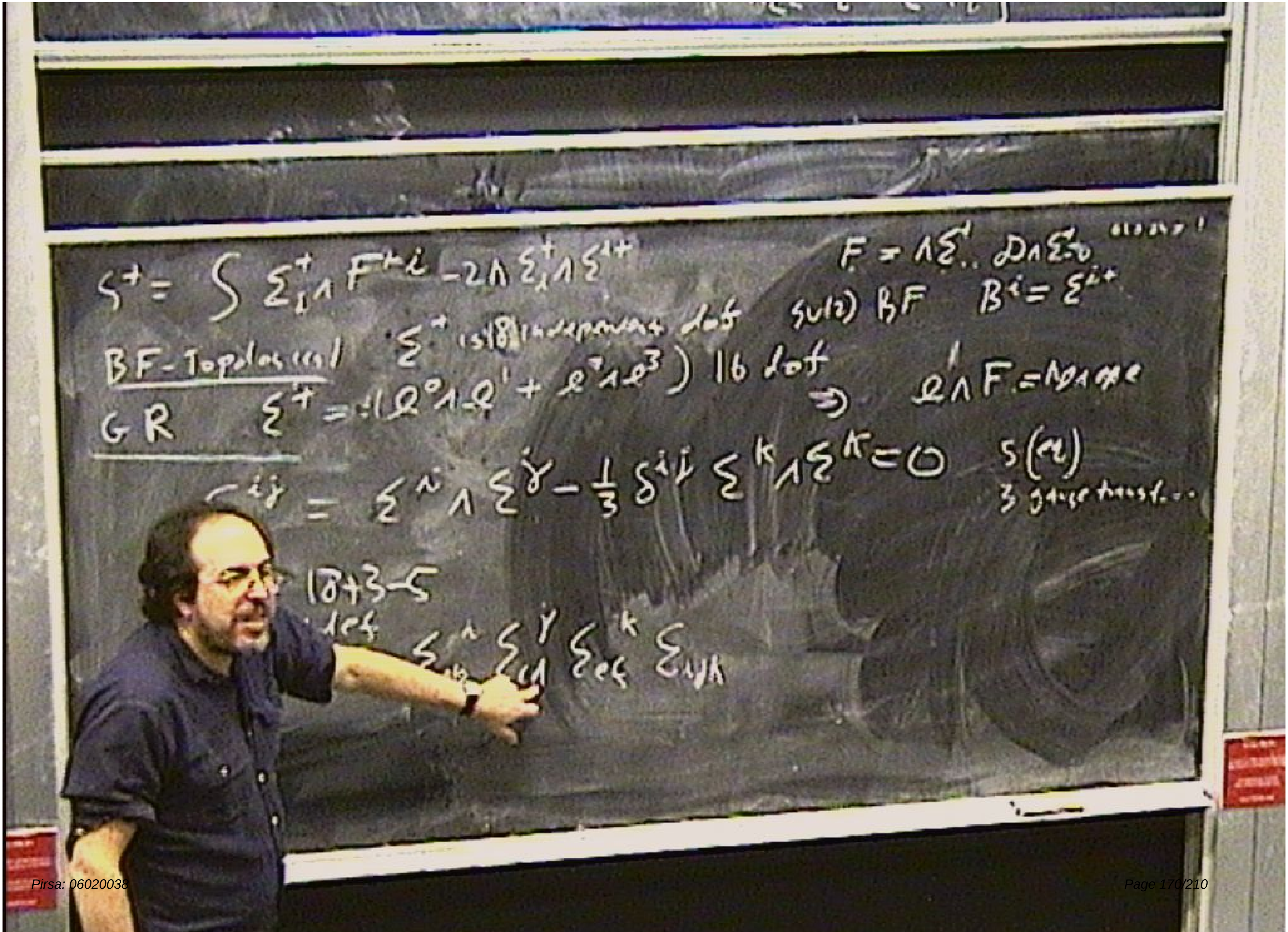
GR $\quad \Sigma^+ = 1(e^0\wedge e^1 + e^2\wedge e^3)$ 16 dof $\Rightarrow$ $e^1 \wedge F$ ...

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0 \quad S(\eta)$$

5 gauge transf. ..

$16 = 18 + 3 - 5$


S+ = ∫ Σ+i ∧ F+i − 2Λ Σ+i ∧ Σi+
F = Λ Σi
su(2) BF
Bi = Σi+
Σ+ is 18 independent dof
BF-Topological
(e0 ∧ e1 + e2 ∧ e3) 16 dof
GR
Σ+ =
e ∧ F = ...
Sij = Σi ∧ Σj − 1/3 δij Σk ∧ Σk = 0
S(e)
3 gauge transf...
16 = 18 + 3 − 5



BF - Topological
GR
su(2) BF
18 independent dof
16 dof
3 gauge transf...
18+3-5

$$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma .. \quad D \wedge \Sigma = 0$

Σ^+ is 18 independent dof $\quad SU(2)$ BF $\quad B^i = \Sigma^{i+}$

BF-Topological

GR $\quad \Sigma^+ = i\, e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof $\Rightarrow$ $e^1 \wedge F = \ldots$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

$S(\ldots)$

3 gauge transf...

$16 = 18 + 3 - 5$

$$\tilde{h}_{ac} = \epsilon^{bdef}\, \Sigma_{ab}^{\;i}\, \Sigma_{cd}^{\;j}\, \Sigma_{ef}^{\;k}\, \Sigma_{ijk}$$

$$S^{+} = \int \Sigma^{+}_{i} \wedge F^{+i} - 2\lambda\, \Sigma^{+}_{i} \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma \ldots \quad D \wedge \Sigma = 0$

$su(2)$ BF $\quad B^{i} = \Sigma^{i+}$

Σ^{+} is 18 independent dof

BF - topological

$$\Sigma^{+} = i(e^{0} \wedge e^{1} + e^{2} \wedge e^{3})$$ 16 dof $\Rightarrow e^{1} \wedge F = \ldots$

GR

$$S^{ij} = \Sigma^{i} \wedge \Sigma^{j} - \frac{1}{3}\delta^{ij}\, \Sigma^{k} \wedge \Sigma^{k} = 0 \quad S(\ldots)$$

5 ...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{ac} = \epsilon^{bdef}\, \Sigma_{ab}{}^{i}\, \Sigma_{cd}{}^{j}\, \Sigma_{ef}{}^{k}\, \epsilon_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{g}\, g_{ac}$$

$$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma \ldots \quad D_A \Sigma = 0$

su(2) BF $\quad B^i = \Sigma^{i+}$

BF-Topological $\quad \Sigma^+$ is 18 independent dof

GR $\quad \Sigma^+ = i(e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof $\Rightarrow \quad e^1 \wedge F = \ldots$

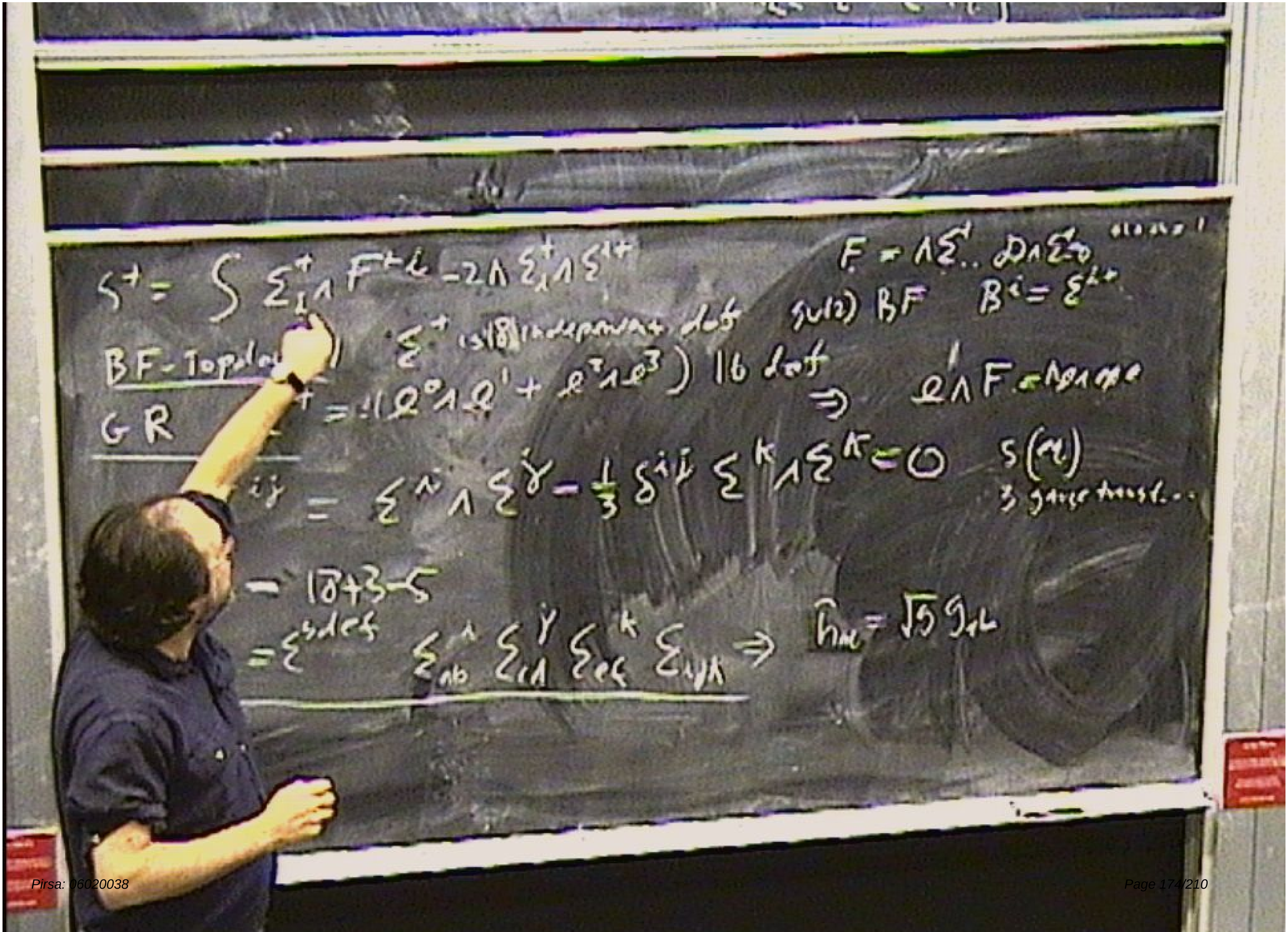
$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

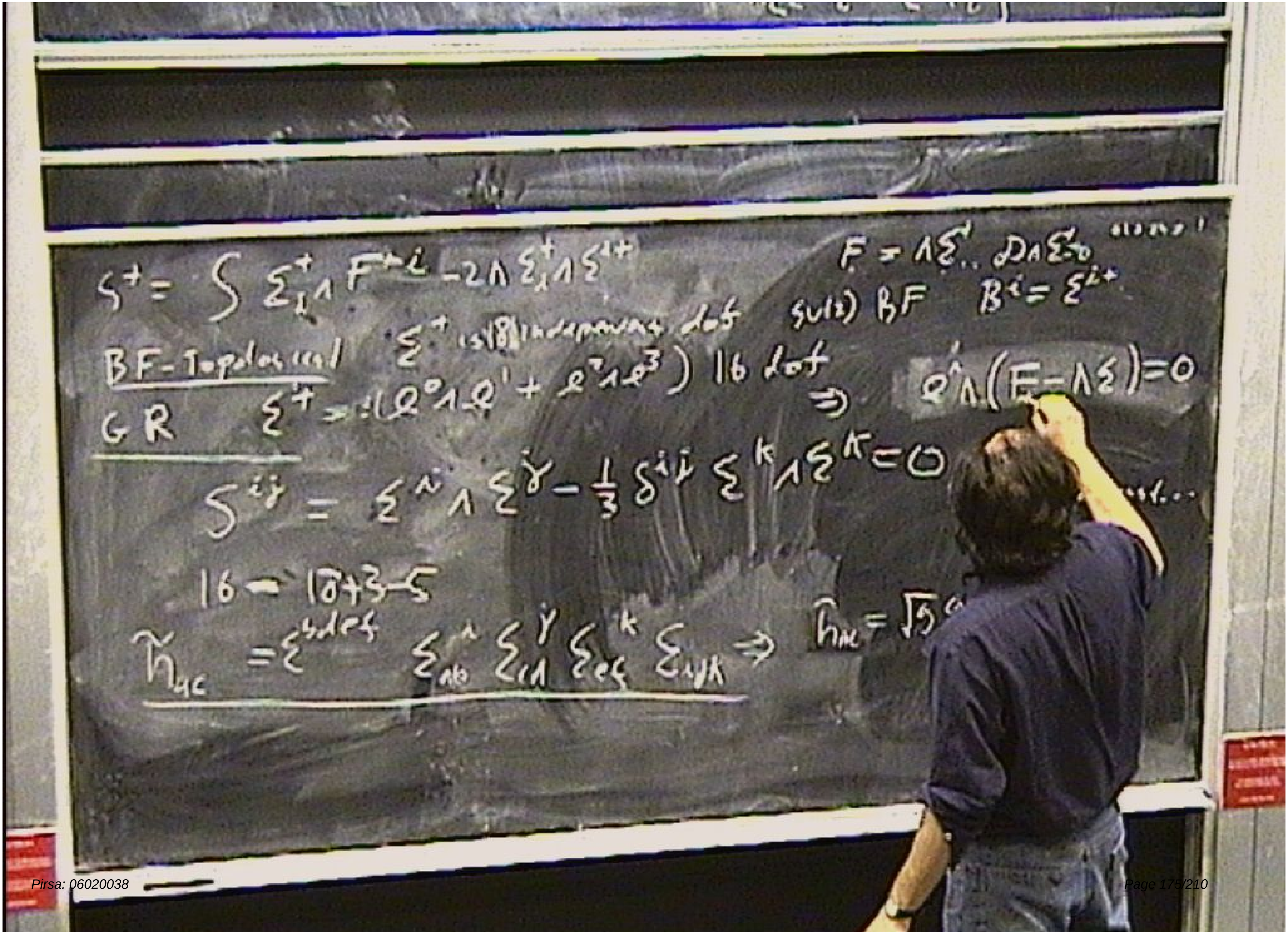
5 (eq)

3 gauge transf...

$16 = 18 + 3 - 5$

$$\tilde{h}_{ac} = \epsilon^{bdef}\, \Sigma^i_{ab}\, \Sigma^j_{cd}\, \Sigma^k_{ef}\, \epsilon_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{h}\, g_{ac}$$


S+ = ∫ Σ+i ∧ F+i − 2Λ Σ+i ∧ Σi+
F = ΛΣ
su(2) BF   Bi = Σi+
Σ+ is 18 independent dof
BF-Topol
16 dof
GR
= 18+3−5
3 gauge transf...
ĥab = √5 gab


S+ = ∫ Σ+ ∧ F+ − 2Λ Σ+ ∧ Σ+
F = ΛΣ.. DΛΣ
su(2) BF   B = Σ+
Σ+ is 18 independent dof
BF-Topological
GR   Σ+ = i(e0∧e1 + e2∧e3) 16 dof
⇒ e ∧ (F − ΛΣ) = 0
Σ ∧ Σ − 1/3 δ Σ ∧ Σ = 0
S =
16 = 18 + 3 − 5
h = ε Σ Σ Σ Σ ⇒ h = √5

$$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma \ldots \quad D \wedge \Sigma^i = 0$

$su(2)$ BF $\quad B^i = \Sigma^{i+}$

BF - Topological $\quad \Sigma^+$ is 18 independent dof

GR $\quad \Sigma^+ = i(e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof $\Rightarrow e^i \wedge (F_{ij} - \Lambda \Sigma_{ij}) = 0$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$S(ij)$

3 gauge transf...

$16 = 18 + 3 - 5$

$$\tilde{h}_{ac} = \epsilon^{bdef} \Sigma_{ab}{}^i \Sigma_{cd}{}^j \Sigma_{ef}{}^k \epsilon_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{g}\, g_{ac}$$

$$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$$

$F = \Lambda \Sigma \ldots \quad D\wedge\Sigma = 0$

$SU(2)$ BF $\quad B^i = \Sigma^{i+}$

Σ^+ is 18 independent dof

BF-Topological

GR $\quad \Sigma^+ = i(e^0\wedge e^1 + e^2\wedge e^3)$ 16 dof $\Rightarrow$ $e^i \wedge (F_i - \Lambda \Sigma_i) = 0$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

$S(ij)$

3 gauge transf...

$16 = 18 + 3 - 5$

$$\tilde{h}_{ac} = \epsilon^{bdef}\, \Sigma_{ab}^i\, \Sigma_{cd}^j\, \Sigma_{ef}^k\, \Sigma_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{g}\, g_{ac}$$

$\Sigma^i = F^i$

$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$

$F = \Lambda \Sigma \ldots \quad D \wedge \Sigma = 0$

$su(2)$ BF $\quad B^i = \Sigma^{i+}$

BF - Topological $\quad \Sigma^+$ is 18 independent dof

GR $\quad \Sigma^+ = i(e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof

$\Rightarrow \quad e^i \wedge (E - \Lambda \Sigma) = 0$

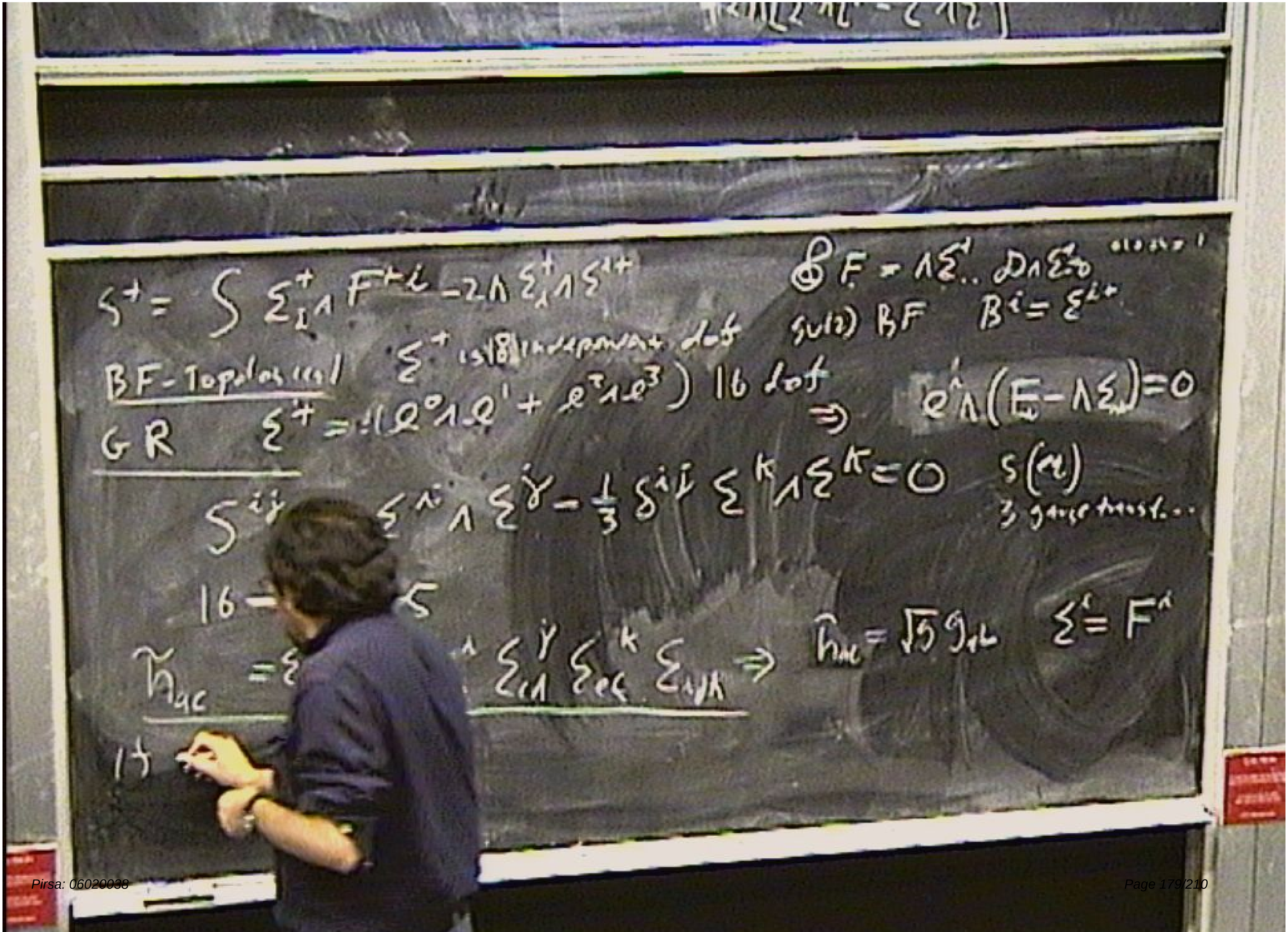
$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0$$

$S(ij)$

3 gauge transf...

$16 = 18 + 3 - 5$

$$\tilde{g}_{ac} = \epsilon^{bdef}\, \Sigma_{ab}^i\, \Sigma_{cd}^j\, \Sigma_{ef}^k\, \Sigma_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{g}\, g_{ab} \quad \Sigma^i = F^i$$


BF-Topological
GR
16 dof
su(2) BF
3 gauge transf...

$S^+ = \int \Sigma^+_i \wedge F^{+i} - 2\Lambda\, \Sigma^+_i \wedge \Sigma^{i+}$

Σ^+ is 18 independent dof

BF-Topological

GR $\quad \Sigma^+ = i(e^0 \wedge e^1 + e^2 \wedge e^3)$ 16 dof

$F = \Lambda \Sigma \ldots \; D \wedge \Sigma = 0$

$su(2)$ BF $\quad B^i = \Sigma^{i+}$

$\Rightarrow \quad e^i \wedge (F - \Lambda \Sigma) = 0$

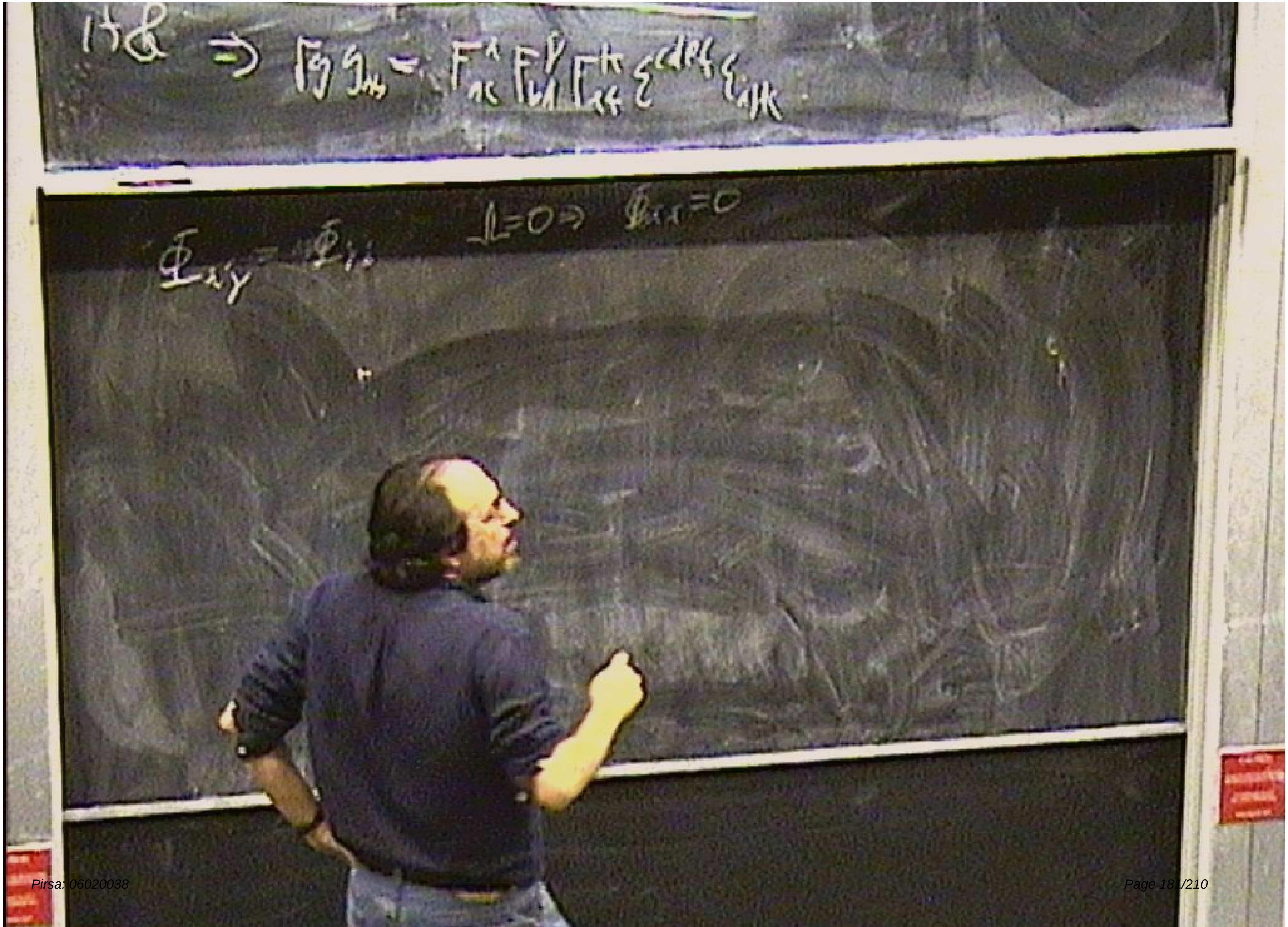
$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3}\delta^{ij}\, \Sigma^k \wedge \Sigma^k = 0 \qquad S(ij)$

3 gauge transf...

$16 = 18 + 3 - 5$

$\tilde{h}_{ac} = \epsilon^{bdef}\, \Sigma^i_{ab}\, \Sigma^j_{cd}\, \Sigma^k_{ef}\, \epsilon_{ijk} \Rightarrow \tilde{h}_{ac} = \sqrt{g}\, g_{ac} \qquad \Sigma^i = F^i$

$1 + 2 \Rightarrow \sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef}\, \epsilon^{cdef}\, \epsilon_{ijk}$

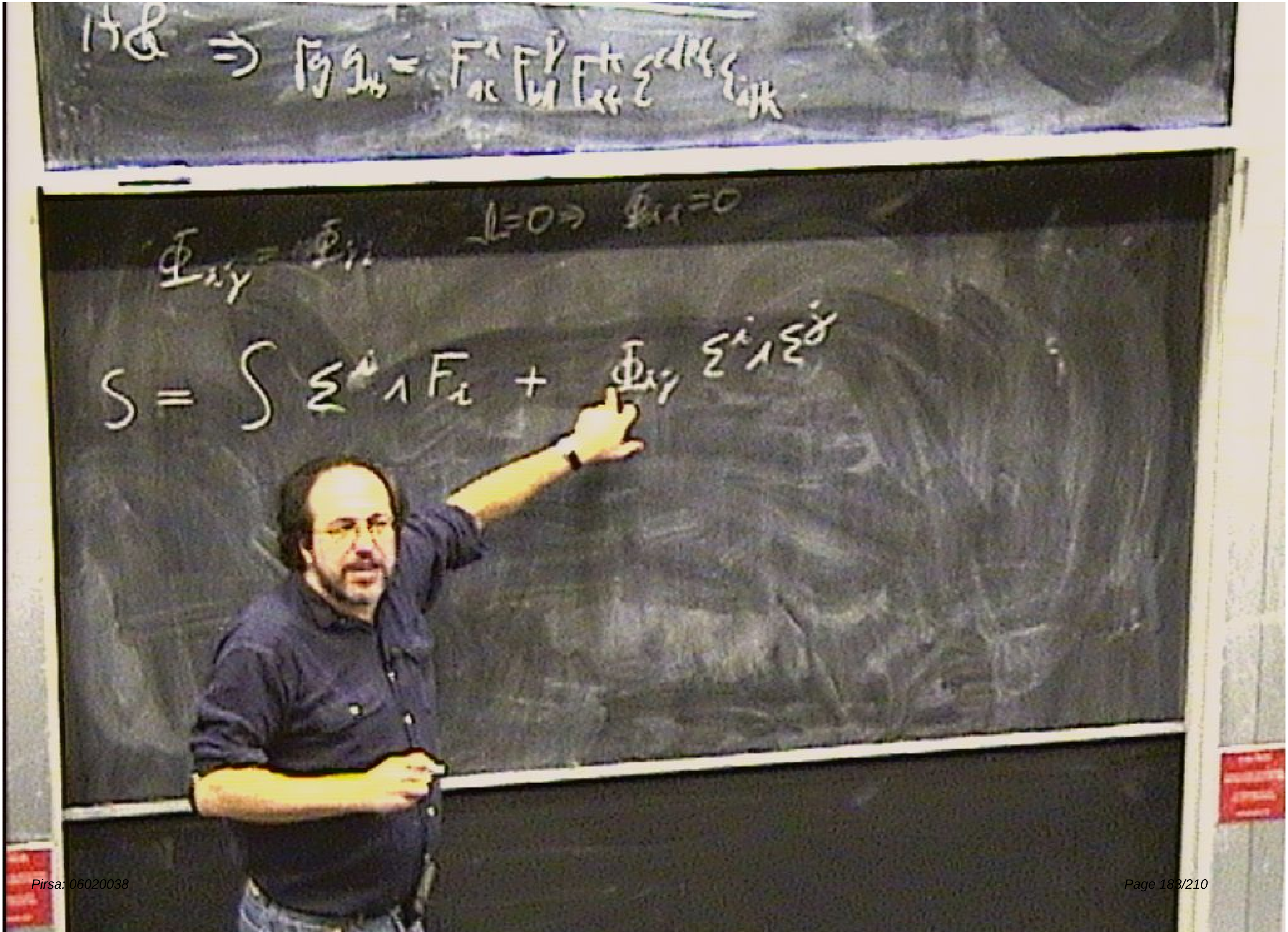


$\Rightarrow$ $F_{ac}^{i} F_{bd}^{j} F_{ef}^{k} \varepsilon^{cdef} \varepsilon_{ijk}$

$\perp = 0 \Rightarrow \Phi_{ij} = 0$

$\Phi_{xy} = \Phi_{ij}$

$S = \int \Sigma^{i} \wedge F_{i} +$


$S = \int \Sigma^i \wedge F_i + \Phi_{ij} \Sigma^i \wedge \Sigma^j$

$1 + \&$ $\Rightarrow$ $\sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef}\, \epsilon^{cdef} \epsilon_{ijk}$

$\Phi_{ij} = \Phi_{ji}$ $\qquad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$

$$S = \int \Sigma^i \wedge F_i + \Phi_{ij}\, \Sigma^i \wedge \Sigma^j$$

If $\mathcal{E} \Rightarrow \sqrt{g}\, g_{ab} = \epsilon\, F^i_{ac} F^j_{bd} F^k_{ef}\, \epsilon^{cdef} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

(sym) $\Phi_{ij} = \Phi_{ji}$

$$S = \int \Sigma^i \wedge F_i + \Phi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$1 + \& \Rightarrow \sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef} \varepsilon^{cdef} \varepsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

$(\mathrm{sym})\ \Phi_{ij} = \Phi_{ji}$

$$S = \int \Sigma^i \wedge F_i + \Phi_{ij} \Sigma^i \wedge \Sigma^j$$

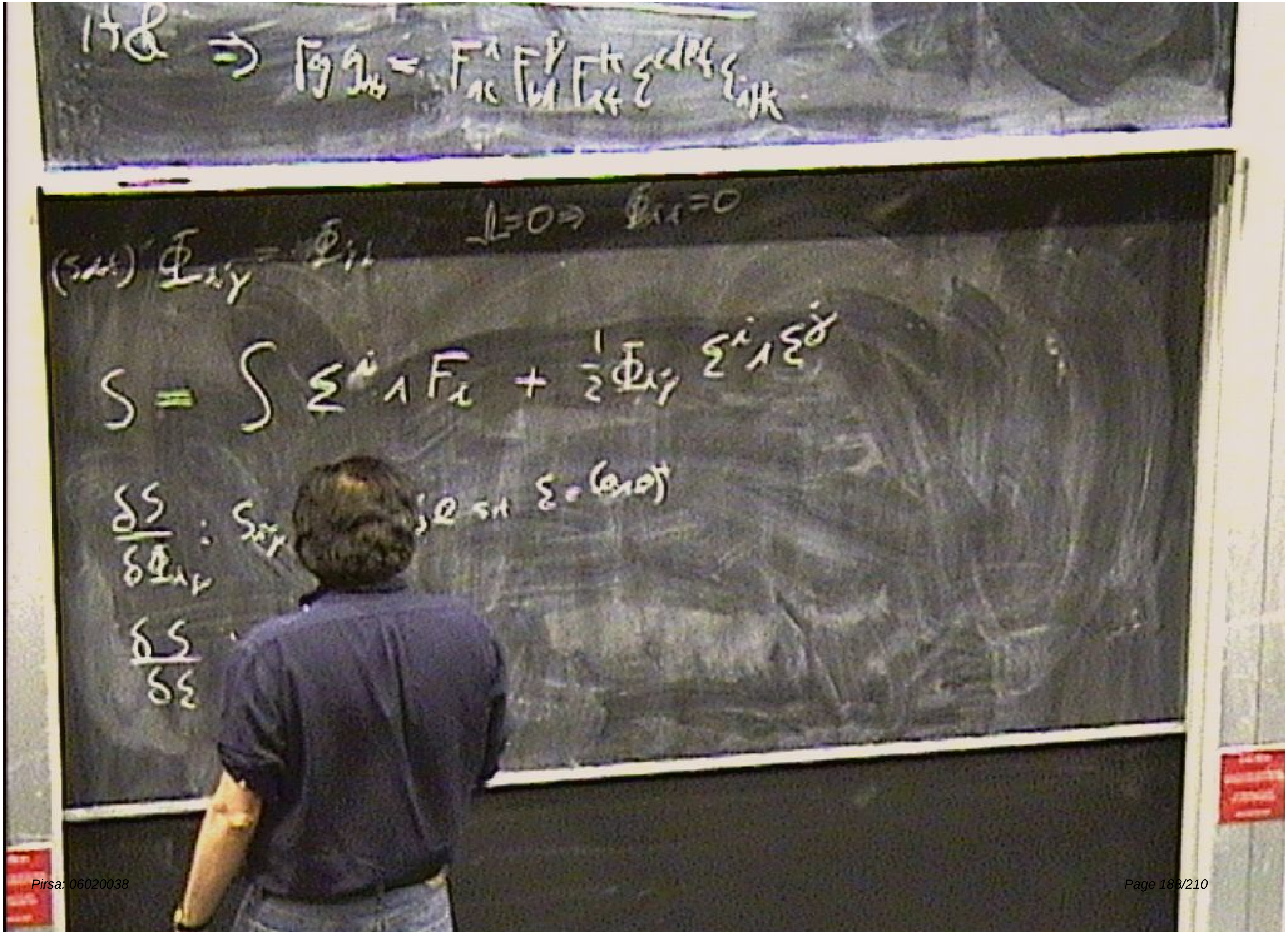
$\mathbb{H}\text{-}\mathbb{C} \Rightarrow \sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef}\, \epsilon^{cdef} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

$(\text{sym})\ \Phi_{ij} = \Phi_{ji}$

$$S = \int \Sigma^i \wedge F_i + \Phi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$\frac{\delta S}{\delta \Phi_{ij}} : S_{ij} = 0 \Rightarrow \exists e \text{ st } \Sigma = (e \wedge e)^+$


$S = \int \Sigma^i \wedge F_i + \frac{1}{2} \Phi_{ij} \Sigma^i \wedge \Sigma^j$

$1+2 \Rightarrow \sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef} \epsilon^{cdef} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

(sym) $\Phi_{ij} = \Phi_{ji}$

$$S = \int \Sigma^i \wedge F_i + \frac{1}{2} \Phi_{ij} \Sigma^i \wedge \Sigma^j$$

$$\frac{\delta S}{\delta \Phi_{ij}} : S_{ij} = 0 \Rightarrow \exists\, e \text{ st } \Sigma = (e \wedge e)^+$$

$$\frac{\delta S}{\delta \Sigma} : F^i = \Phi^{ij} \Sigma_j$$

$$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$$

$$1 \not\in \mathcal{G} \Rightarrow \sqrt{g}\, g_{\mu\nu} = F^i_{\mu\alpha} F^j_{\nu\beta} F^k_{\gamma\delta} \epsilon^{\alpha\beta\gamma\delta} \epsilon_{ijk}$$

$$(\text{sym})\ \Phi_{ij} = \Phi_{ji} \qquad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$$

$$S = \int \Sigma^i \wedge F_i + \frac{1}{2} \Phi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$$\frac{\delta S}{\delta \Phi_{ij}} : S_{ij} = 0 \Rightarrow \exists e \text{ st } \Sigma = (e \wedge e)^+$$

$$\frac{\delta S}{\delta \Sigma} : \quad F^i = \Phi^{ij} \Sigma_j$$

$$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$$

$$1+\& \Rightarrow \sqrt{g}\, g_{ab} = F^{i}_{ac} F^{j}_{bd} F^{k}_{ef}\, \epsilon^{cdef}\, \epsilon_{ijk}$$

$$\text{(sol.)}\quad \Phi_{ij} = \Phi_{ji} \qquad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$$

$$S = \int \Sigma^{i} \wedge F_{i} + \frac{1}{2} \Phi_{ij}\, \Sigma^{i} \wedge \Sigma^{j}$$

$$\boxed{\begin{array}{l} \dfrac{\delta S}{\delta \Phi_{ij}} : S_{ij} = 0 \Rightarrow \exists\, e \text{ s.t. } \Sigma = (e \wedge e)^{+} \\ \dfrac{\delta S}{\delta \Sigma} : \quad F^{i} = \Phi^{ij} \Sigma_{j} \\ \dfrac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0 \end{array}} \Rightarrow R_{ab} = 0$$

$\Rightarrow \sqrt{g}\, g_{ab} = F^{i}_{ac} F^{j}_{bd} F^{k}_{ef} \epsilon^{cdef} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

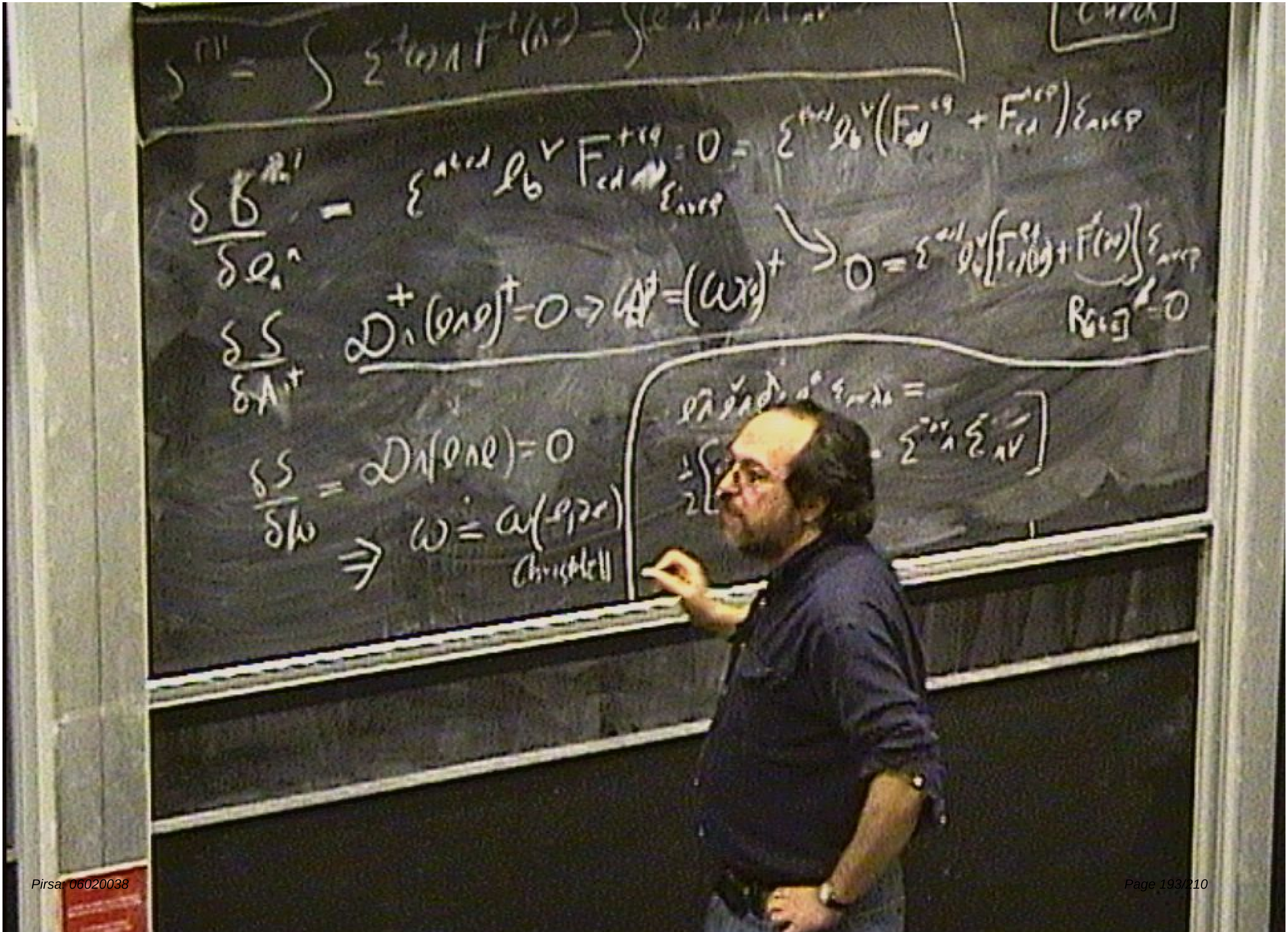
(sym) $\Phi_{ij} = \Phi_{ji}$

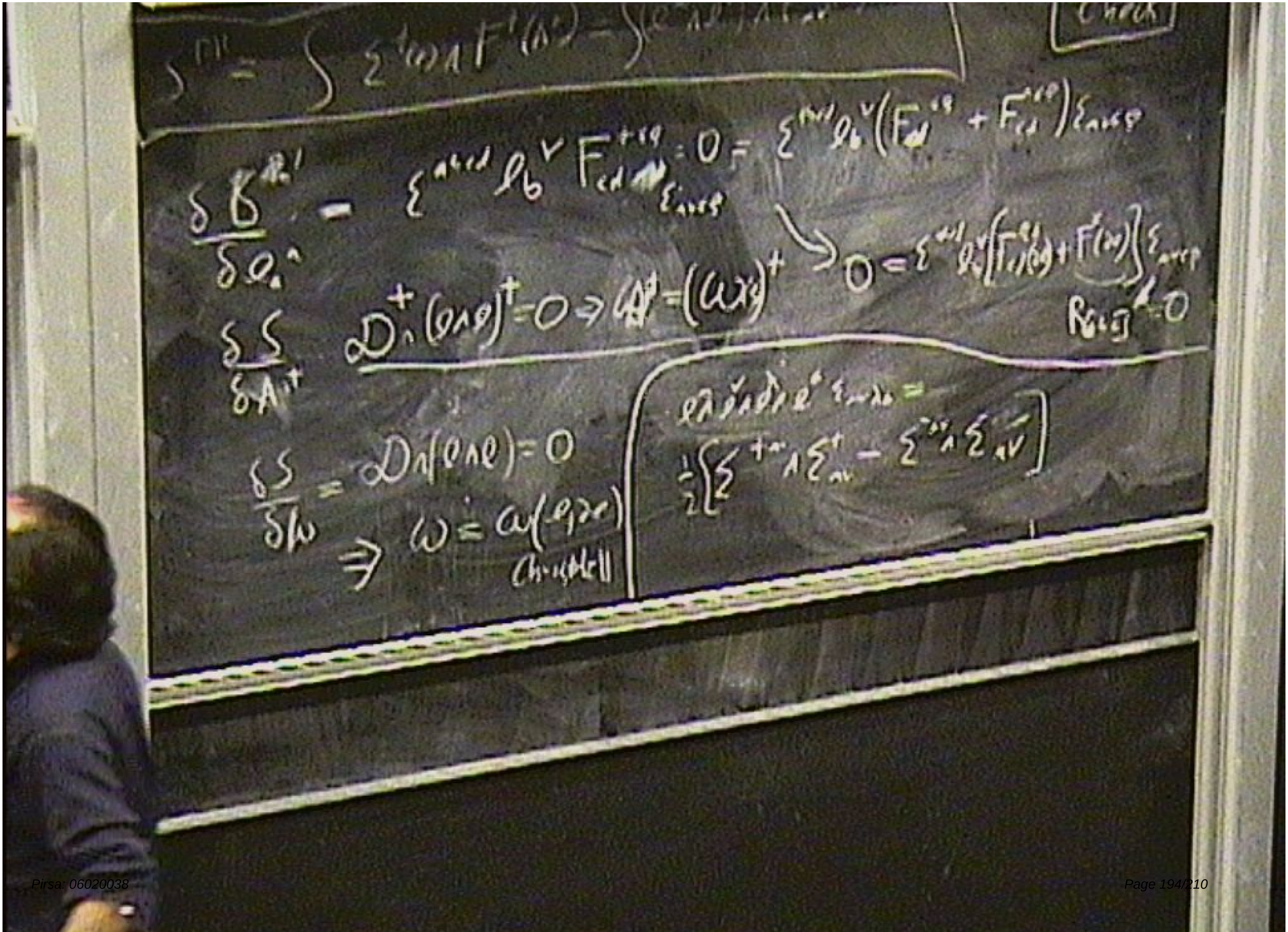
$$S = \int \Sigma^{i} \wedge F_{i} + \frac{1}{2} \Phi_{ij} \Sigma^{i} \wedge \Sigma^{j}$$

$\frac{\delta S}{\delta \Phi_{ij}}$: $S_{ij} = 0 \Rightarrow \exists e$ st $\Sigma = (e \wedge e)^{+}$

$\frac{\delta S}{\delta \Sigma}$: $F^{i} = \Phi^{ij} \Sigma_{j} \Rightarrow R_{ab} = 0$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$


Check


Check

$\Rightarrow \sqrt{g}\, g_{\mu\nu} = F^{i}_{\mu\alpha} F^{j}_{\nu\beta} F^{k}_{\gamma\delta}\, \epsilon^{\alpha\beta\gamma\delta} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

(sym) $\Phi_{ij} = \Phi_{ji}$

$$S = \int \Sigma^{i} \wedge F_{i} + \frac{1}{2} \Phi_{ij}\, \Sigma^{i} \wedge \Sigma^{j} - \Lambda\, \Sigma^{i} \wedge \Sigma_{i}$$

$\frac{\delta S}{\delta \Phi_{ij}}$: $S_{ij} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e \wedge e)^{+}$

$\frac{\delta S}{\delta \Sigma}$: $F^{i} = \Phi^{ij} \Sigma_{j} \Rightarrow R_{\mu\nu\alpha\beta} = 0$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$

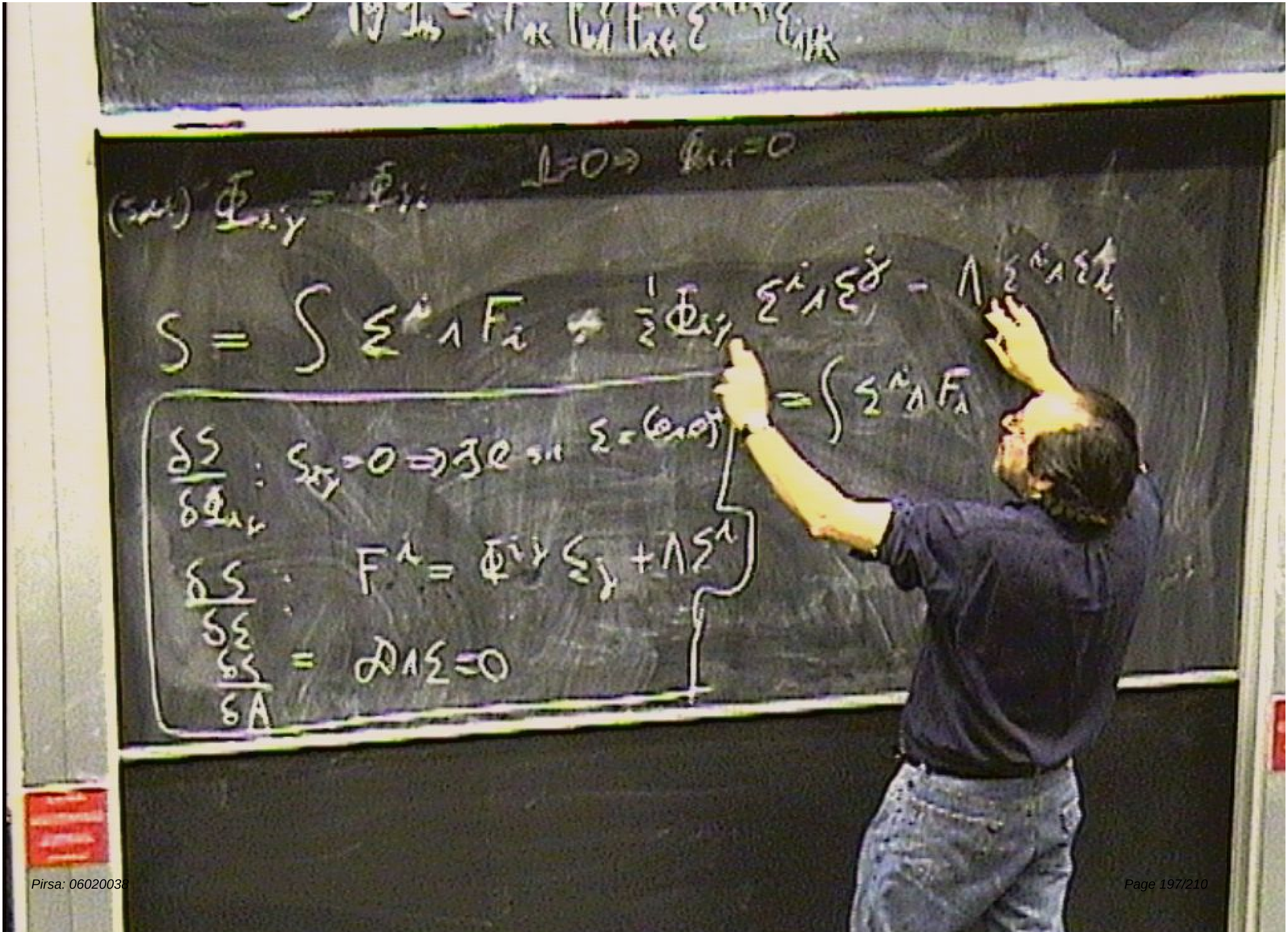
(Sdt) $\Phi_{ij} = \Phi_{ji}$ $\quad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$

$$S = \int \Sigma^i \wedge F_i + \frac{1}{2}\Phi_{ij}\, \Sigma^i \wedge \Sigma^j - \Lambda\, \Sigma^i \wedge \Sigma_i$$

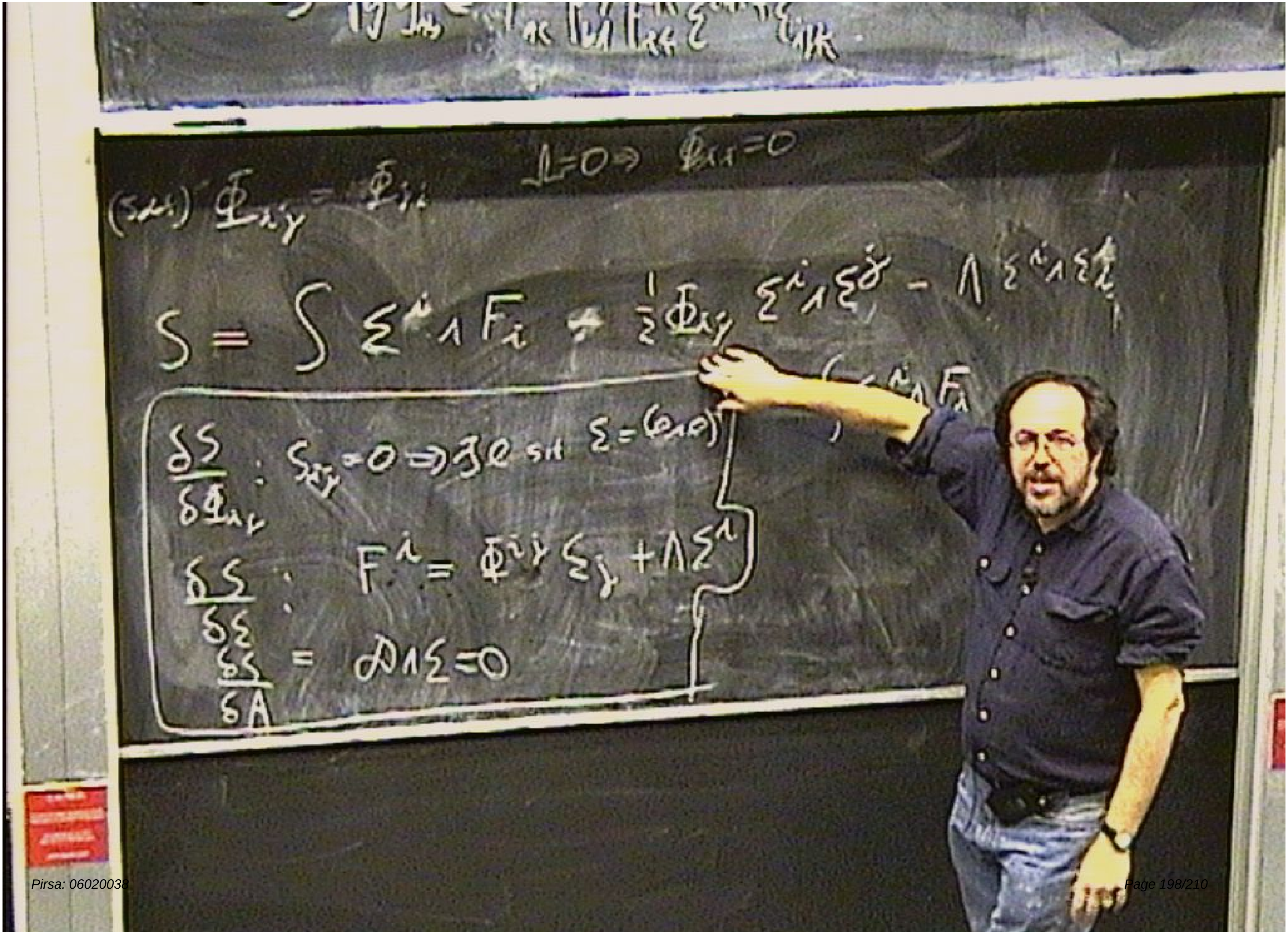
$\frac{\delta S}{\delta \Phi_{ij}}$: $S_{ij} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e \wedge e)^+$

$\frac{\delta S}{\delta \Sigma}$: $F^i = \Phi^{ij}\Sigma_j + \Lambda \Sigma^i$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$



$S = \int \Sigma^i \wedge F_i + \frac{1}{2}\Phi_{ij}\Sigma^i \wedge \Sigma^j - \Lambda \Sigma^i \wedge \Sigma_i$
$= \int \Sigma^i \wedge F_i$
$\frac{\delta S}{\delta \Phi_{ij}} : \Sigma_{ij} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e \wedge e)$
$\frac{\delta S}{\delta \Sigma} : F^i = \Phi^{ij}\Sigma_j + \Lambda \Sigma^i$
$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$



(Sdu) $\Phi_{ij} = \Phi_{ji}$ $\quad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$ $\quad \Psi_{ij} = \Phi_{ij} + \Lambda \delta_{ij}$

$\Psi_{ii} = 3\Lambda$

$$S = \int \Sigma^i \wedge F_i - \frac{1}{2}\Phi_{ij}\, \Sigma^i \wedge \Sigma^j - \Lambda\, \Sigma^i \wedge \Sigma_i$$

$$= \int \Sigma^i \wedge F_i - \frac{1}{2}\Psi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$\frac{\delta S}{\delta \Phi_{ij}}$: $\Sigma_{ij} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e \wedge e)^+$

$\frac{\delta S}{\delta \Sigma}$: $F^i = \Phi^{ij}\Sigma_j + \Lambda \Sigma^i$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

(sym) $\Phi_{ij} = \Phi_{ji}$

$\Psi_{ij} = \Phi_{ij} + \Lambda \delta_{ij}$

$\boxed{\Psi_{ii} = 3\Lambda}$

$$S = \int \Sigma^i \wedge F_i + \frac{1}{2}\Phi_{ij}\, \Sigma^i \wedge \Sigma^j - \Lambda\, \Sigma^i \wedge \Sigma_i$$

$$= \int \Sigma^i \wedge F_i - \frac{1}{2}\Psi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$$F^i = \Psi^i{}_j\, \Sigma^j$$

$\frac{\delta S}{\delta \Phi_{ij}}$: $S_{ij} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e \wedge e)^+$

$\frac{\delta S}{\delta \Sigma}$: $F^i = \Phi^{ij}\Sigma_j + \Lambda \Sigma^i$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$

$(\text{sym})\ \Phi_{ij} = \Phi_{ji}$ $\quad \Lambda = 0 \Rightarrow \Phi_{ii} = 0$ $\quad \Psi_{ij} = \Phi_{ij} + \Lambda\delta_{ij}$

$\boxed{\Psi_{ii} = 3\Lambda}$

$$S = \int \Sigma^i \wedge F_i + \frac{1}{2}\Phi_{ij}\, \Sigma^i \wedge \Sigma^j - \Lambda\, \Sigma^i \wedge \Sigma_i$$

$$= \int \Sigma^i \wedge F_i - \frac{1}{2}\Psi_{ij}\, \Sigma^i \wedge \Sigma^j$$

$F^i = \Psi^i{}_j\, \Sigma^j$ $\quad$ def

$\frac{\delta S}{\delta \Phi_{ij}} : \Sigma_{ij} = 0 \Rightarrow \exists e \text{ s.t. } \Sigma = (e \wedge e)^+$

$\frac{\delta S}{\delta \Sigma} : \quad F^i = \Phi^{ij}\Sigma_j + \Lambda \Sigma^i$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$

(self) $\Phi_{\lambda\gamma} = \Phi_{\gamma\lambda}$ $\quad \Lambda = 0 \Rightarrow \Phi_{\lambda\lambda} = 0$ $\quad \Psi_{\lambda\gamma} = \Phi_{\lambda\gamma} + \Lambda\delta_{\lambda\gamma}$

$\boxed{\Psi_{\lambda\lambda} = 3\Lambda}$

$$S = \int \Sigma^{\lambda} \wedge F_{\lambda} + \frac{1}{2}\Phi_{\lambda\gamma}\,\Sigma^{\lambda}\wedge\Sigma^{\gamma} - \Lambda\,\Sigma^{\lambda}\wedge\Sigma_{\lambda}$$

$$= \int \Sigma^{\lambda}\wedge F_{\lambda} - \frac{1}{2}\Psi_{\lambda\gamma}\,\Sigma^{\lambda}\wedge\Sigma^{\gamma}$$

$\frac{\delta S}{\delta \Phi_{\lambda\gamma}}$: $S_{\lambda\gamma} = 0 \Rightarrow \exists e$ s.t. $\Sigma = (e\wedge e)^{+}$

$\frac{\delta S}{\delta \Sigma}$: $F^{\lambda} = \Phi^{\lambda\gamma}\Sigma_{\gamma} + \Lambda\Sigma^{\lambda}$

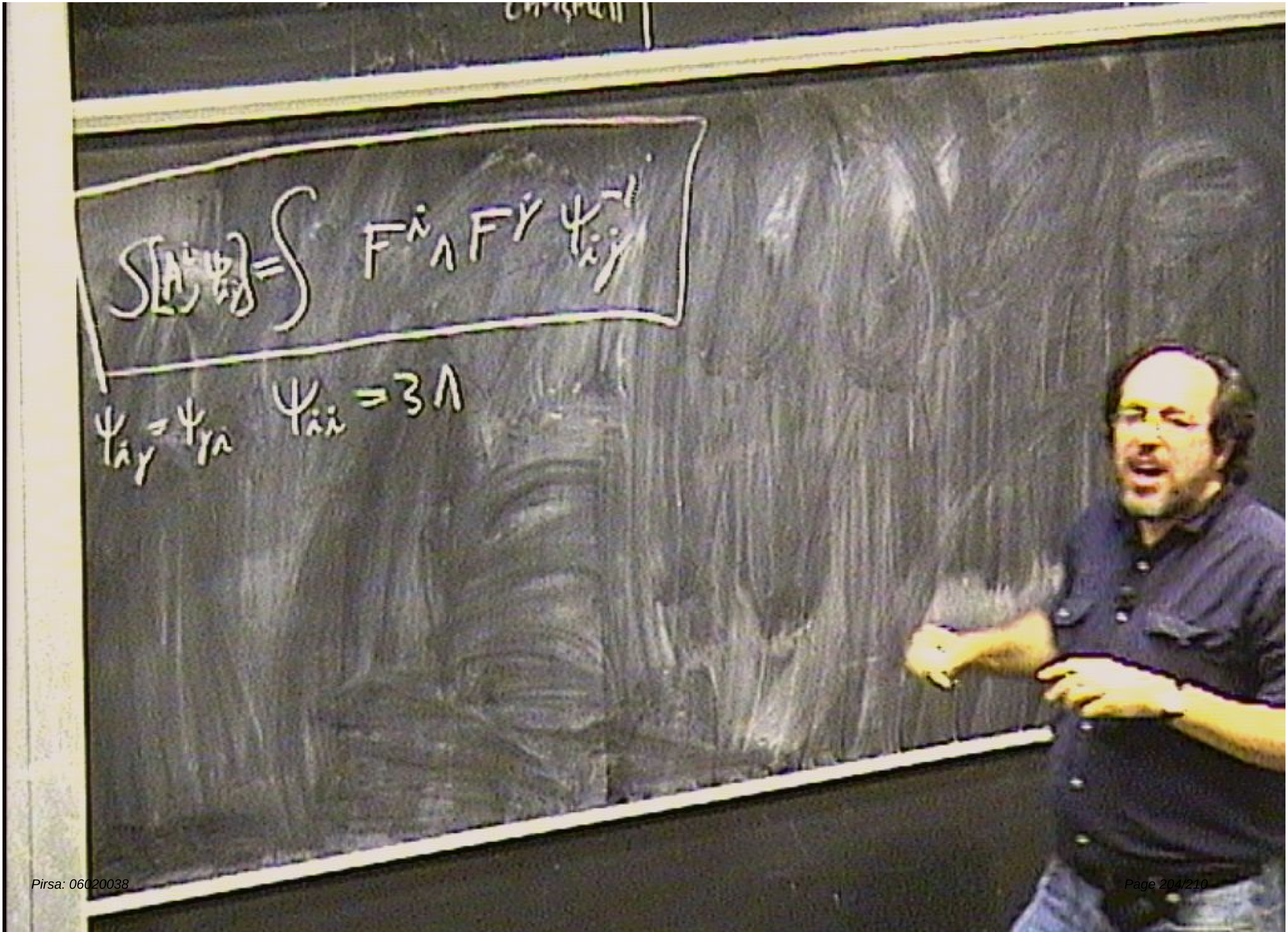
$\frac{\delta S}{\delta A} = \mathcal{D}\wedge\Sigma = 0$

$F^{\lambda} = \Psi^{\lambda}{}_{\gamma}\,\Sigma^{\gamma}$ $\quad \det\Psi \neq 0$

$\Rightarrow \Sigma^{\gamma} = \Psi^{-1\,\gamma}{}_{\kappa}\,F^{\kappa}$

$$\boxed{S = \int F^{i} \wedge F^{j} \, \psi^{-1}_{ij}}$$

$$\psi_{ij} = \psi_{ji} \qquad \psi_{ii} = 3\Lambda$$


$S[A^i, \psi_{ij}] = \int F^i \wedge F^j \psi^{-1}_{ij}$
$\psi_{ij} = \psi_{ji}$
$\psi_{ii} = 3\Lambda$

$$\boxed{S[A^i, \psi_{ij}] = \int F^i \wedge F^j \, \psi^{-1}_{ij}}$$

$\psi_{ij} = \psi_{ji}$ $\psi_{ii} = 3\Lambda$

$$\boxed{S[A^i, \Psi_{ij}] = \int F^i \wedge F^j \, \Psi^{-1}_{ij}}$$

$\Psi_{ij} = \Psi_{ji}$ $\quad \Psi_{ii} = 3\Lambda$

$\Rightarrow \sqrt{g}\, g_{ab} = F^i_{ac} F^j_{bd} F^k_{ef} \epsilon^{cdef} \epsilon_{ijk}$

$\Lambda = 0 \Rightarrow \Phi_{ii} = 0$

$\Phi_{ij} = \Phi_{ji}$

$\Psi_{ij} = \Phi_{ij} + \Lambda \delta_{ij}$

$\boxed{\Psi_{ii} = 3\Lambda}$

$$= \int \Sigma^i \wedge F_i + \frac{1}{2} \Phi_{ij} \Sigma^i \wedge \Sigma^j - \Lambda \Sigma^i \wedge \Sigma_i$$

$$= \int \Sigma^i \wedge F_i - \frac{1}{2} \Psi_{ij} \Sigma^i \wedge \Sigma^j$$

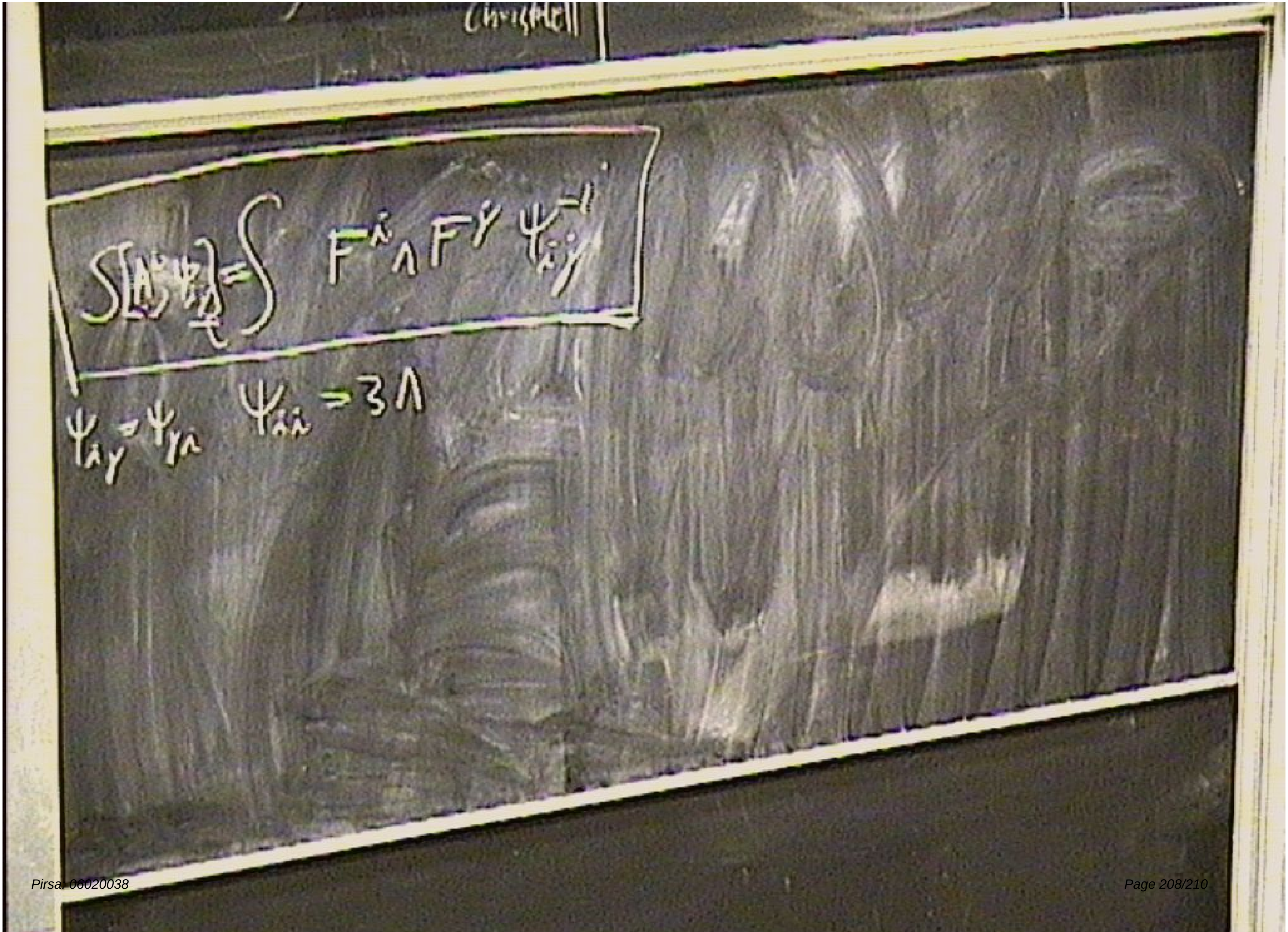
$\frac{\delta S}{\delta \Phi_{ij}} : \Sigma_{(ij)} = 0 \Rightarrow \exists e \text{ st } \Sigma = (e \wedge e)^+$

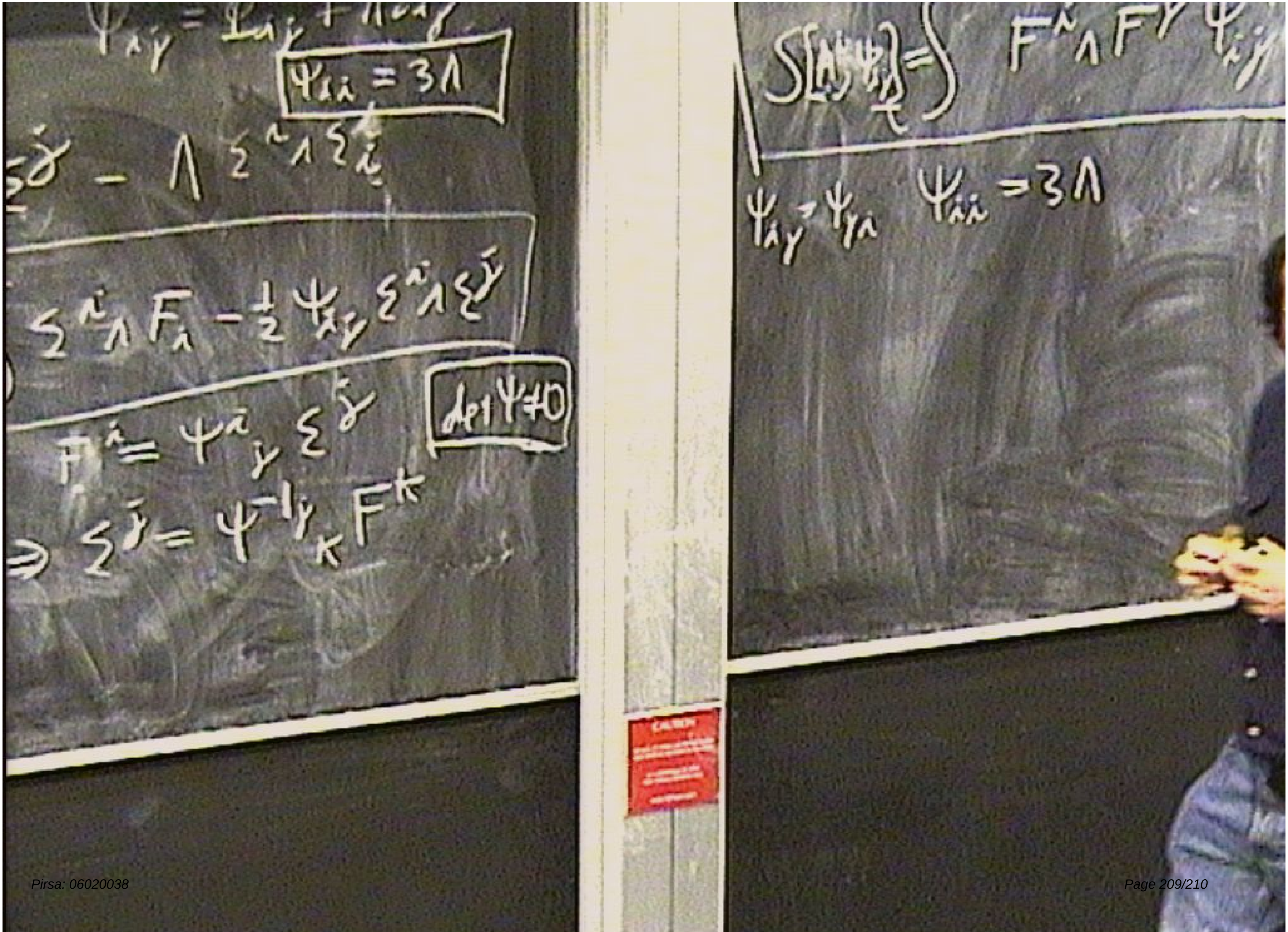
$F^i = \Psi^i{}_j \Sigma^j \quad \det \Psi \neq 0$

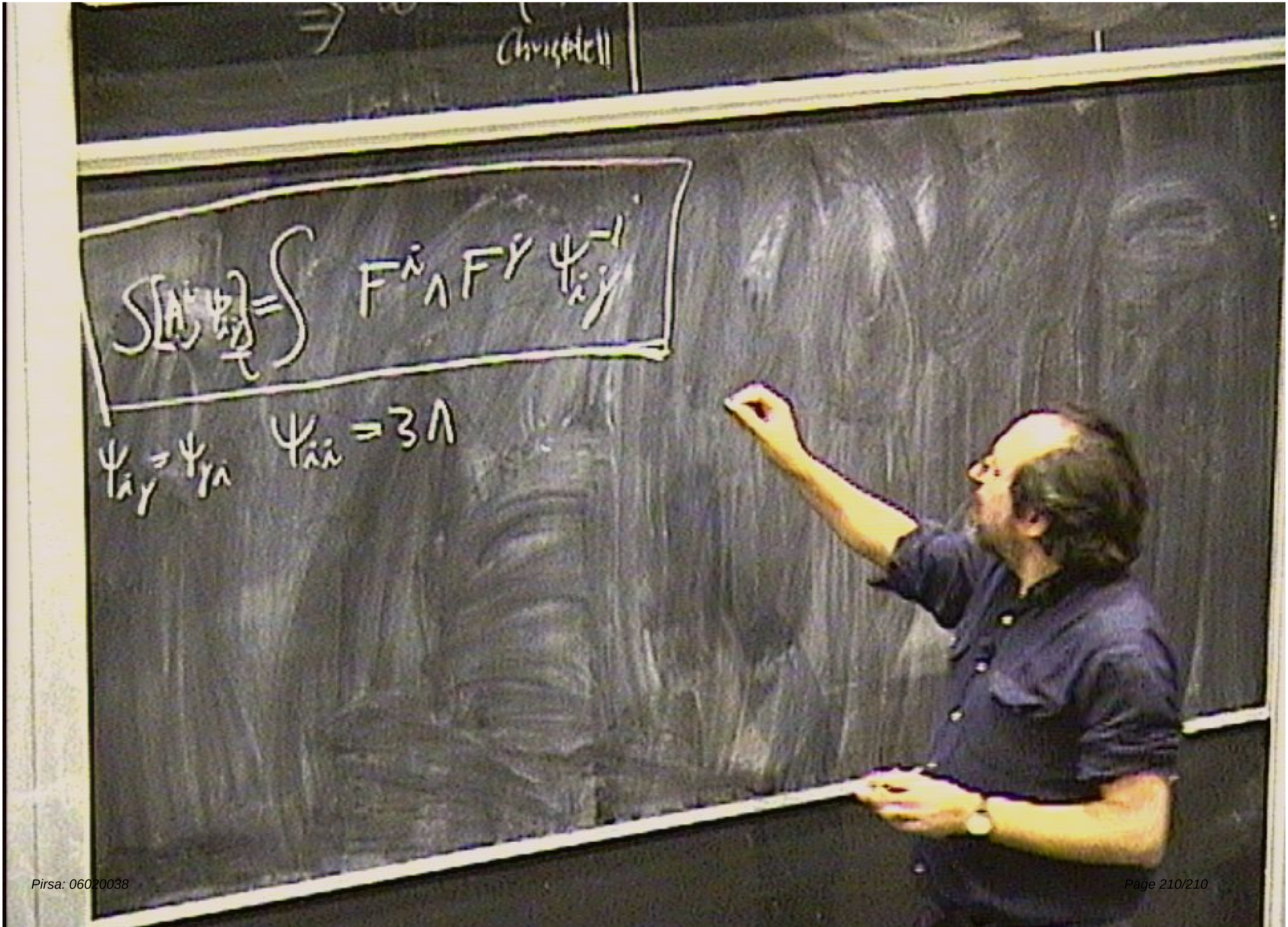
$\frac{\delta S}{\delta \Sigma} : \quad F^i = \Phi^{ij} \Sigma_j + \Lambda \Sigma^i$

$\Rightarrow \Sigma^j = \Psi^{-1\,j}{}_k F^k$

$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$


$S[A^i, \Psi_{ij}] = \int F^i \wedge F^j \Psi^{-1}_{ij}$
$\Psi_{ij} = \Psi_{ji}$
$\Psi_{ii} = 3\Lambda$


Ψ = 3Λ
det Ψ ≠ 0
Ψ → Ψ   Ψ = 3Λ


$S[A^i, \psi_{ij}] = \int F^i \wedge F^j \psi^{-1}_{ij}$
$\psi_{ij} = \psi_{ji}$
$\psi_{ii} = 3\Lambda$