

Title: Introduction to quantum gravity - Part 11

Date: Feb 28, 2006 06:30 PM

URL: <http://pirsa.org/06020038>

Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

-Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005 -Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079

-Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048 -Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

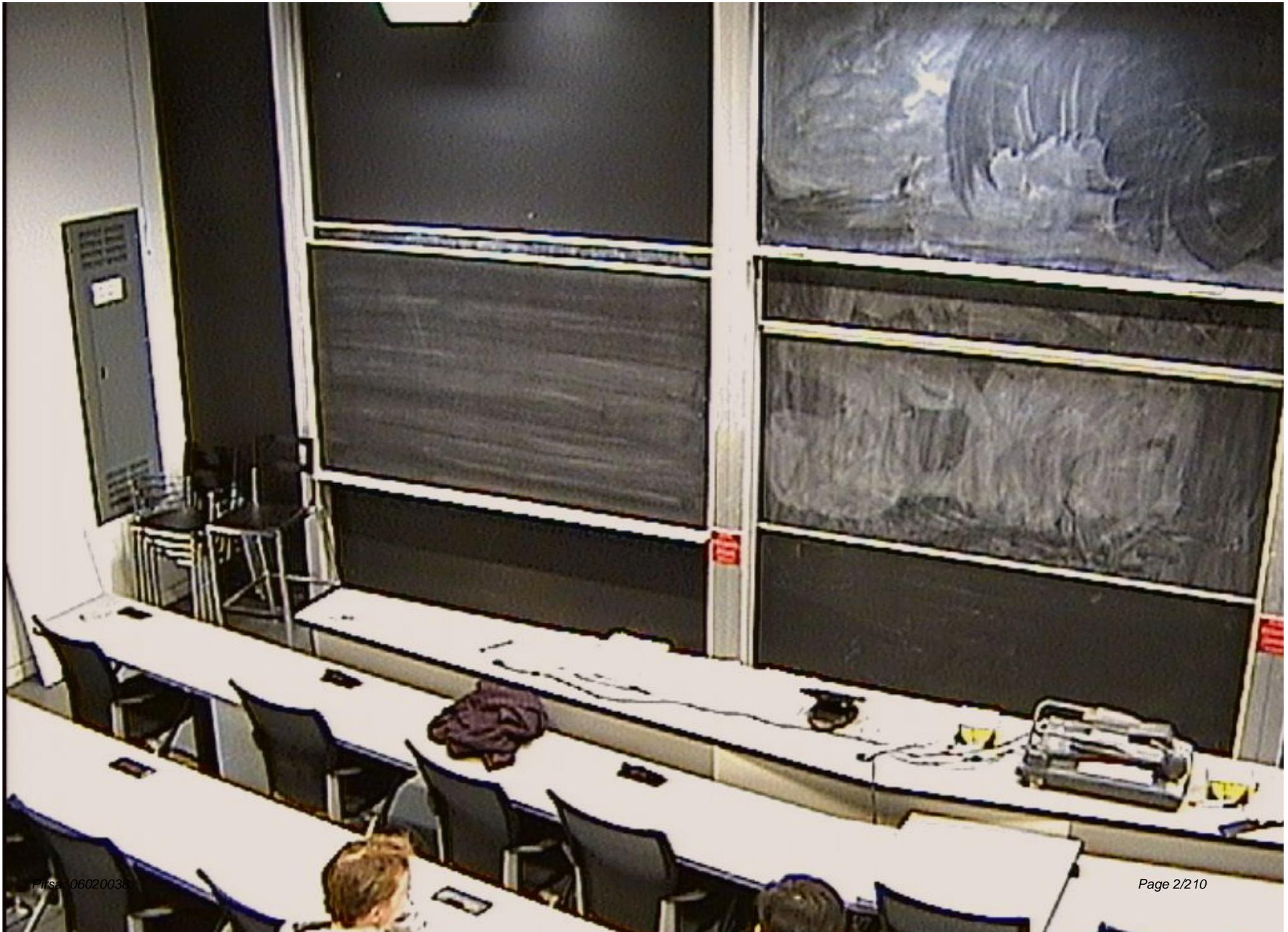
-undergraduate quantum mechanics

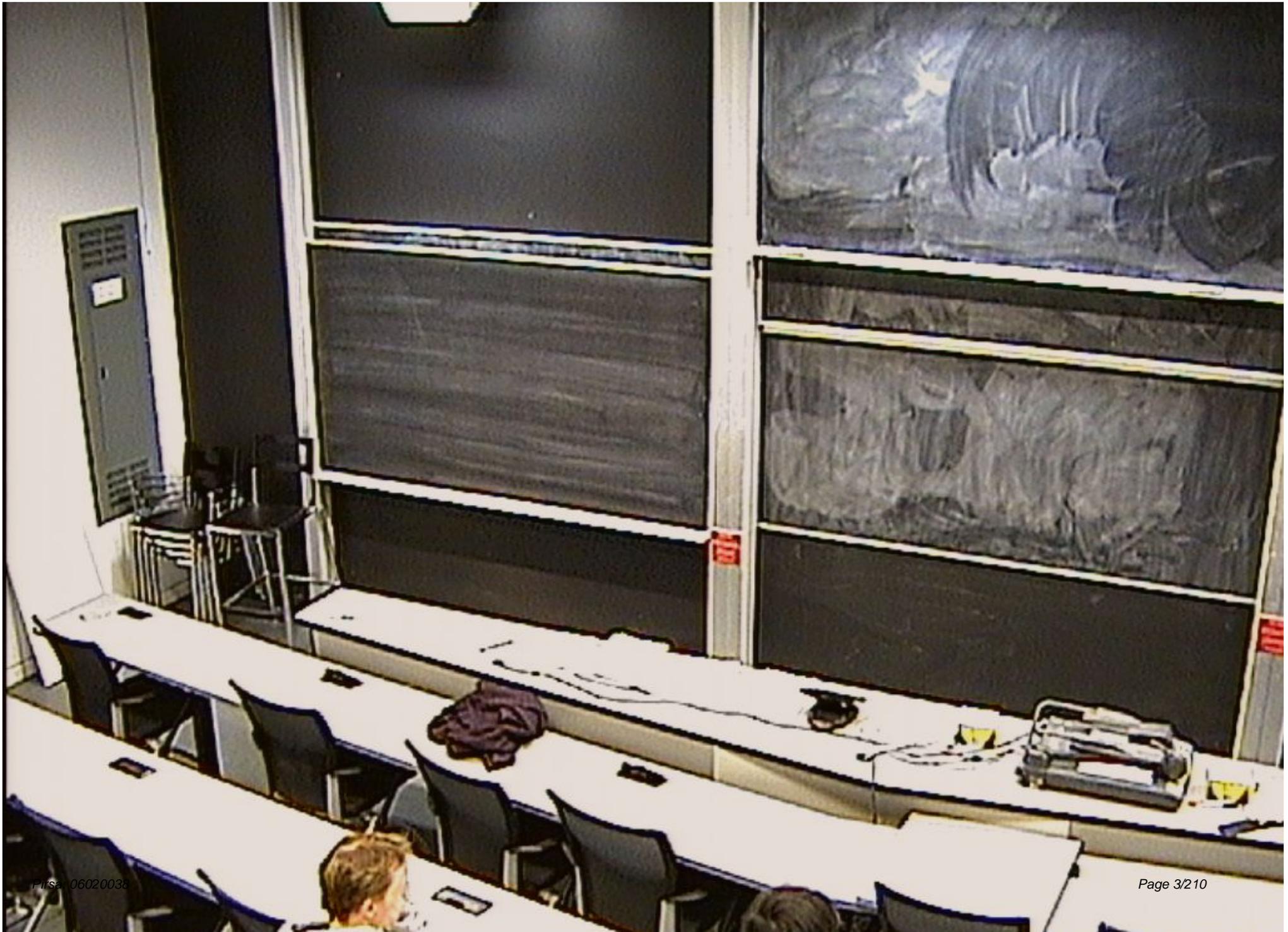
-basics of classical gauge field theories

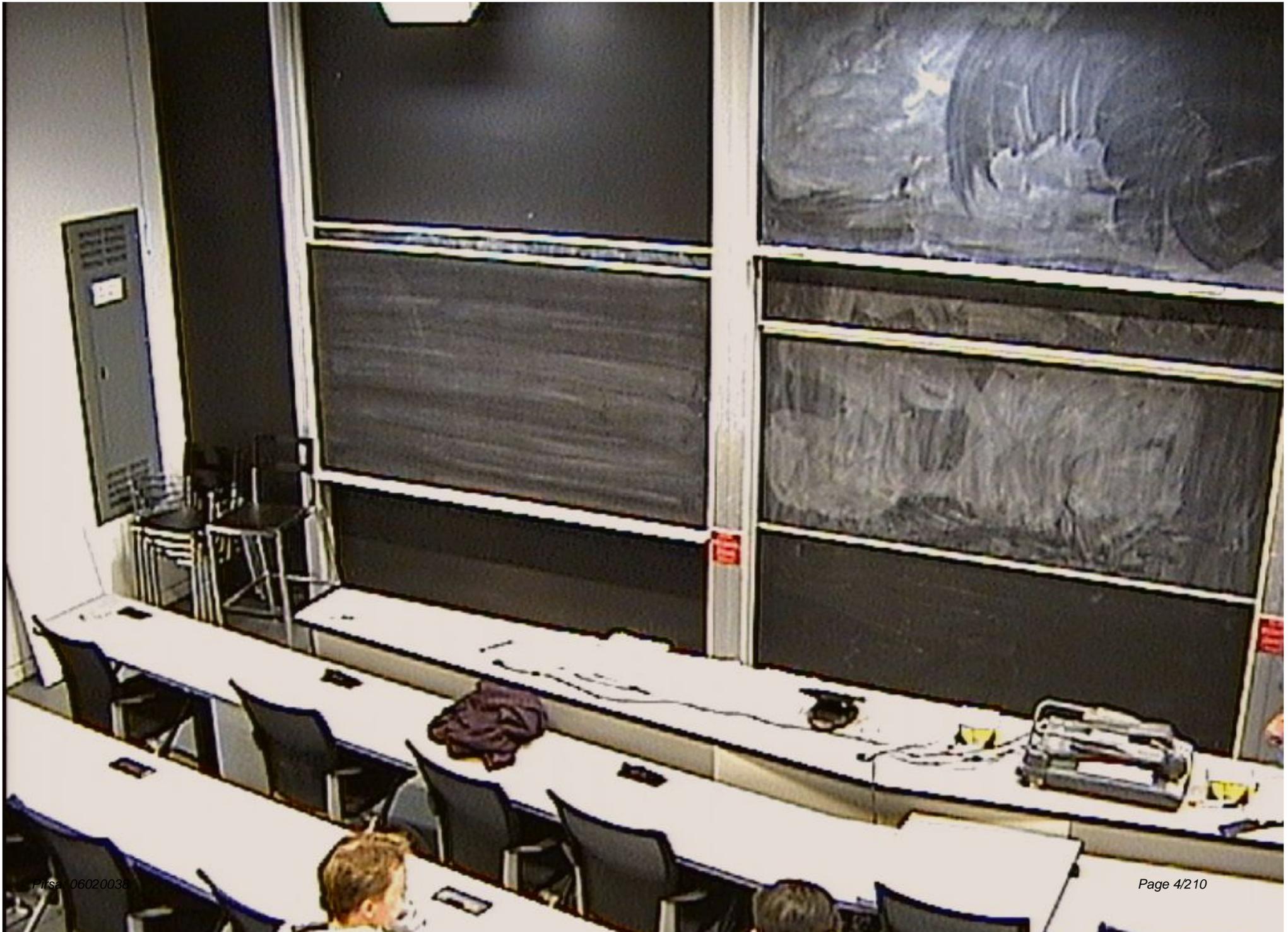
-basic general relativity

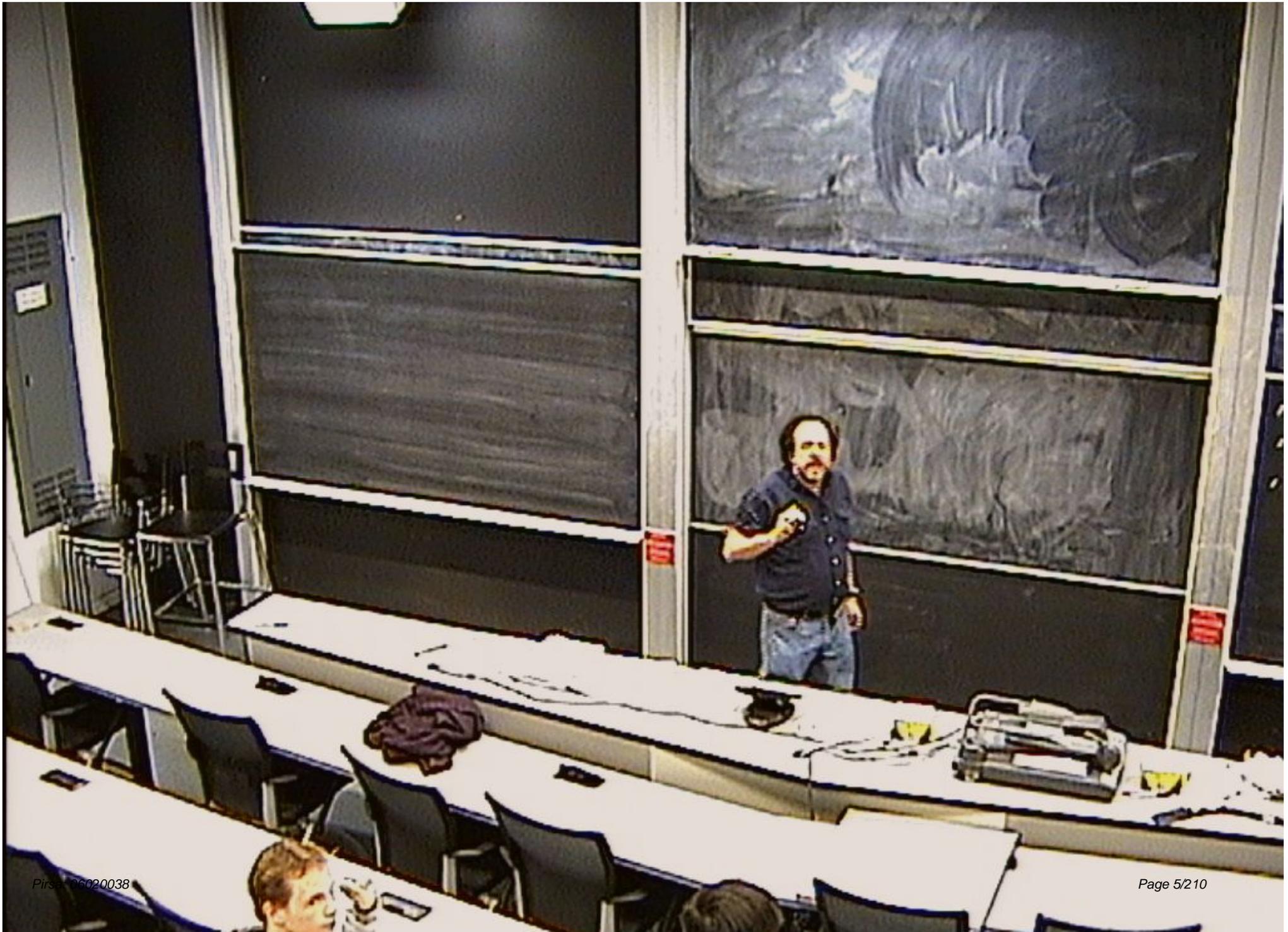
-hamiltonian and lagrangian mechanics

-basics of lie algebras









- Review Relativity
- Review duality = self-duality
-

- Review Palatini
- Review duality & self-duality
-

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekau, TSS formalism...

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekar, TSS formalism
= "classical LQG"

- Review Palatini
- Review duality & self-duality
- Plebanstij / Ashtekar, TSS formalism...
= "classical LQG"
- Laurent-Atrem formalism

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekar, TSS formalism...
= "classical LQG"
- Laurent-Atiyah formalism hep-th/0501191

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekar, TSS formalism.
- "classical LQG"
- Laurent-ARTOM formalism hep-th/0501191

6209079

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$
fields connected to \mathcal{U}^{rev}

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$ $ISO(4)$

fields

metric field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

1-forms

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$ $ISO(4)$

fields connection field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

forms

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

fields $\omega_a^{\mu\nu}$ gauge field
frame fields $e^{\mu\alpha}$ 0123
 $\mu \leftarrow 1 \text{ forms}$

ISO(4)

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$
$$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Euc.} \\ \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \text{Lorentz} \end{cases}$$

Forms

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

fields connection field
 frame fields $e^{\mu}{}_{\alpha}$ 0123
 \uparrow forms

$$g_{\mu\nu} = e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} \eta^{\alpha\beta}$$

ISO(4)

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

\uparrow forms

$$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Euc.} \\ \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Lor.} \end{cases}$$

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

ISO(4)

connection field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

the fields $e^{\mu\alpha} \omega_{\alpha}^{\nu\beta}$

$$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) \\ (-1, 0, 0, 0) \end{cases}$$

forms

$$e^{\mu\alpha} e^{\nu\beta} \eta_{\alpha\beta} \Rightarrow g^{\mu\nu} e^{\mu\alpha} e^{\nu\beta} = \eta^{\alpha\beta}$$

$$\begin{cases} (1, 1, 1, 1) \\ (-1, 0, 0, 0) \end{cases} \text{Euler}$$

$$\begin{cases} (1, 1, 1, 1) \\ (0, 1, 1, 1) \end{cases} \text{Cartan}$$

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

$ISO(4)$

fields connection field

$$\omega_a^{mv} = -\omega_a^{vm}$$

forms

frame fields $e^a \leftarrow 0123$

$$\eta^{mv}$$

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Euc.}$$

$$g_{rs} = e^a{}_{\mu} e^b{}_{\nu} \eta_{ab} \Rightarrow g^{\mu\nu} e_a{}^{\mu} e_b{}^{\nu} = \eta_{ab}$$

$SO(4)$ local symm

$$e_a{}^{\mu} \rightarrow e_a{}^{\mu} + \Lambda^{\mu\nu} e_a{}^{\nu}$$

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{can}$$



Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

ISO(4)

fields connection field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

forms

frame fields $e^{\mu} \leftarrow 0123$

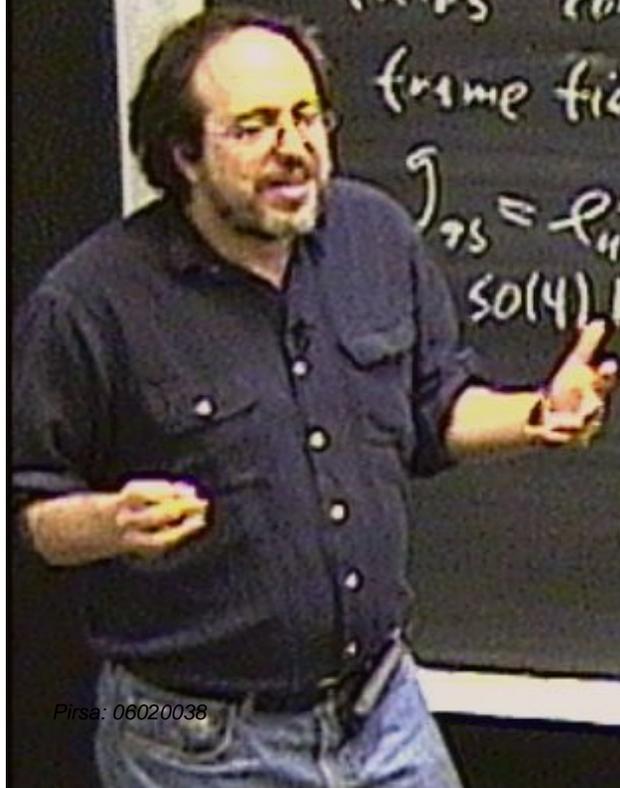
$$\eta^{\mu\nu} = \begin{cases} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \text{Euc.} \\ \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \text{Lor.} \end{cases}$$

$$g_{\mu\nu} = e^{\mu} e^{\nu} \eta_{\alpha\beta} \Rightarrow g^{\mu\nu} e_{\mu} e_{\nu} = \eta^{\alpha\beta}$$

SO(4) local symm $e_{\mu}^{\nu} \rightarrow e_{\mu}^{\lambda} \Lambda_{\lambda}^{\nu}$

$$\Lambda_{\lambda}^{\nu} = \delta_{\lambda}^{\nu} + \omega_{\lambda}^{\mu\nu} e_{\mu}$$

(Lorentz SO(4))



Euclidean 0209079

- Review Palatini
- Review duality & self-duality
- Plebanstvi / Ashtekuv, TSS formalism.
- "classical LQG"
- Laurent-Atiyah formalism hep-th/0501191

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$

ISO(4)

fields connection/gauge

frame fields $e^a{}_\mu$

$$\mathfrak{g}_{\text{ISO}} = \mathfrak{so}(4) \ltimes \mathfrak{t}^4$$

SO(4) local sym

$$\omega_a{}^{bc} = \omega_a{}^{[bc]}$$

$$\mathfrak{so}(4) = \left\{ \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \right\}$$

forms

$$e^a{}_\mu = \eta^{ab} e^b{}_\mu$$

$$\Lambda^2 \mathfrak{t}^4 \cong \mathfrak{so}(4)$$

Palatini $\mathcal{M}^4 = \Sigma^3 \times \mathbb{R}$ ISO(4)

fields connection field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

forms

frame fields $e^{\mu} \in \{0,1,2,3\}$

$$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) & \text{Eucl.} \\ (-1, 0, 0, 0) & \text{Lorentz} \end{cases}$$

$$g_{\mu\nu} = e^{\mu} e^{\nu} \eta_{\mu\nu} \Rightarrow g^{\mu\nu} e_{\mu}^{\alpha} e_{\nu}^{\beta} = \eta^{\alpha\beta}$$

SO(4) local symm $e_{\mu}^{\alpha} \rightarrow e_{\mu}^{\beta} = \Lambda^{\alpha}_{\beta} e_{\mu}^{\alpha}$
(Lorentz SO(4))

Palatini $M^4 = \mathbb{S}^3 \times \mathbb{R}$ $ISO(4)$

fields connection/gauge field
frame fields $e^{\mu} \in 0,1,2,3$

$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ \leftarrow forms

$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) & \text{Eucl.} \\ (-1, 1, 1, 1) & \text{Lorentz } SO(4) \end{cases}$

$g_{\mu\nu} = e^{\mu} e^{\nu} \eta_{\mu\nu} \Rightarrow g^{\mu\nu} e^{\mu} e^{\nu} = \eta^{\mu\nu}$

$SO(4)$ local symm $e^{\mu} \rightarrow e^{\mu} + \Lambda^{\mu\nu} e^{\nu}$ \leftarrow (Lorentz $SO(4)$)

$F_{\mu\nu} =$

Palatini $M^4 = \mathbb{S}^3 \times \mathbb{R}$ $ISO(4)$

fields connection/gauge field $\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ \leftarrow Forms

frame fields $e^a \in 0123$

$$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) & \text{Eucl.} \\ (-1, 0, 0, 0) & \text{Lorentz} \end{cases}$$

$\mathfrak{g}_{\text{FS}} = \mathfrak{so}(4)$ local symm $e_a^\mu \rightarrow e_a^{\mu\prime} = \Lambda_a^\nu e_b^\nu$

$\mathfrak{so}(4)$ local symm $e_a^\mu \rightarrow e_a^{\mu\prime} = \Lambda_a^\nu e_b^\nu$ \leftarrow Lorentz $SO(4)$

$F_{\mu\nu} = dA^{\mu\nu} + A_{\lambda\mu} A^{\lambda\nu}$ $SO(4)$



Palatini $\mathcal{M}^4 = \mathbb{S}^3 \times \mathbb{R}$ $ISO(4)$

fields connection field

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

Forms

frame fields $e^a \in \{0,1,2,3\}$

$$\eta^{\mu\nu} = \begin{cases} (1,1,1,1) & \text{Eucl.} \\ (-1,1,1,1) & \text{Lorentz} \end{cases}$$

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} \Rightarrow g^{\mu\nu} e^a_\mu e^b_\nu = \eta^{ab}$$

$SO(4)$ local symm $e^a_\mu \rightarrow e^b_\mu = \Lambda^b_a e^a_\mu$
 $\Lambda^b_a \in SO(4)$

Field Strength $F^{\mu\nu} = dA^{\mu\nu} + A^\mu_\lambda A^{\lambda\nu}$ $SO(4)$
 = Spacetime Curvature



Palatini $M^4 = \mathbb{S}^3 \times \mathbb{R}$ $ISO(4)$

fields connection field $\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$ 1-forms

frame fields $e^a \in \mathbb{R}^{0123}$

$$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) & \text{Eucl.} \\ (-1, 0, 0, 0) & \text{Lorentz} \end{cases}$$

$\int_{\mathbb{R}^4} \omega^{\mu\nu} \wedge e^{\mu} \wedge e^{\nu} \Rightarrow g^{\mu\nu} e_a^{\mu} e_b^{\nu} = \eta^{ab}$

$SO(4)$ local symm $e_a^{\mu} \rightarrow e_a^{\mu'} = \Lambda^{\mu\nu} e_a^{\nu}$

Field strength $F^{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}_{\lambda} \wedge \omega^{\lambda\nu}$ $SO(4)$



Palatini

R

$SO(4)$

fields connect

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu}$$

forms

frame fields $e^{\mu\alpha}$ 0123
forms

$$\eta^{\mu\nu} = \begin{cases} (1, 1, 1, 1) & \text{Euc.} \\ (-1, 0, 0, 0) & \text{Lorentz} \end{cases}$$

$$g_{\mu\nu} = e^{\mu\alpha} e^{\nu\beta} \eta_{\alpha\beta} \Rightarrow g^{\mu\nu} e^{\mu\alpha} e^{\nu\beta} = \eta^{\alpha\beta}$$

$SO(4)$ local symm $R_a^{\mu\nu} \rightarrow R_a^{\mu\nu} + \Lambda_{\alpha\beta}^{\mu\nu} R_a^{\alpha\beta}$
(Lorentz $SO(4)$)

Field Strength $F_a^{\mu\nu} = d\omega_a^{\mu\nu} + \omega_a^{\mu\lambda} \omega_{\lambda}^{\nu\alpha}$ $SO(4)$
= Spacetime Curvature

$$S_{\text{Palatini}} = \int_M$$

$$\int P_{\text{relativ}} = \int m \rho^{\mu}{}_{\lambda} \rho^{\nu}{}_{\alpha} F^{\lambda\sigma}$$

$$\int \mathcal{P} \psi \psi^\dagger = \int m \psi^\mu \psi^\nu F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma}$$



$$\begin{aligned}
 \int \mathcal{P}_{\text{Poincaré}} &= \int_{\mathcal{M}} \mathcal{L}^{\mu}{}_{\lambda} \mathcal{L}^{\nu}{}_{\alpha} F^{\lambda\sigma} \epsilon_{\mu\nu\alpha\sigma} \\
 &= \int \epsilon^{abcd} \mathcal{L}^{\mu}{}_{\nu\lambda\sigma} \mathcal{L}^{\alpha}{}_{\beta} \mathcal{L}^{\gamma}{}_{\delta} F^{\lambda\sigma}
 \end{aligned}$$

$$\begin{aligned}
 \int \text{Pulsioni} &= \int_M e^{\mu\nu} e^{\lambda\sigma} F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma} \\
 &= \int \epsilon^{abcd} \frac{e^{\mu\nu} e^{\lambda\sigma}}{16\pi} F_{\mu\nu} F_{\lambda\sigma}
 \end{aligned}$$



$$\begin{aligned}
 \int \text{density} &= \int_m \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \partial_\nu \partial_\lambda F^{\rho\sigma} \\
 &= \int_m \epsilon^{abcd} \frac{\partial a \partial b \partial c \partial d}{144 \text{ perm}} \partial_a \partial_b F_{cd}
 \end{aligned}$$



$$\begin{aligned}
 \int \text{Polstini} &= \int_m e^{\mu\nu} e^{\lambda\sigma} F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma} \\
 \text{density} & \\
 &= \int_m \frac{1}{4} \epsilon^{abcd} \underbrace{e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma}_{\text{determinant}} F^{\lambda\sigma}
 \end{aligned}$$



$$\begin{aligned}
 \int \text{Polarini} &= \int_m e^{\mu} \wedge e^{\nu} \wedge F^{\lambda\sigma} \epsilon_{\mu\nu\lambda\sigma} \\
 \text{density} & \\
 &= \int_m \epsilon^{abcd} \underbrace{e_a^{\mu} e_b^{\nu}}_{\text{internal}} F_{cd}^{\lambda\sigma}
 \end{aligned}$$



Euclidean 0209079

• Review Palatini

• Review duality & self-duality

• Plebanski / Ashtekar
- 'classical' TSS formalism.

hep-th/0501191

• Laurent-Atiyah

$$g \eta \eta^t = \eta$$

Euclidean 6209079

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekaru, TSS formalism.
- "classical LQG"

- Laurent-Atiyah formalism hep-th/0501191
 $g \eta \tilde{g}^i = \eta$
 \downarrow
 $g \psi$

spacetime (metric)

$$\begin{aligned}
 \int \text{Poincaré} &= \int_m \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma} E_{\mu\nu} \\
 \text{density} & \\
 &= \int_m \epsilon^{abcd} \underbrace{\epsilon_{\mu\nu\lambda\sigma}}_{\text{interval}} e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma}
 \end{aligned}$$



- spacetime curvature

$$S_{\text{Poincaré}} = \int m \left[\rho^{\mu\nu} \rho_{\mu\nu} F^{\lambda\sigma} \right] \epsilon_{\mu\nu\lambda\sigma}$$

density

$$\int \epsilon_{\mu\nu\lambda\sigma} \rho_a^{\mu\nu} \rho_b^{\lambda\sigma} F_{cd}$$

(integral)



spacetime curvature

$$\begin{aligned}
 S_{\text{Poincaré}} &= \int_M \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\nu} e^{\lambda\sigma} \right] \epsilon_{\mu\nu\lambda\sigma} \\
 \text{density} & \\
 &= \int_M \epsilon^{abcd} \frac{\det(e)}{\det(\eta)} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma \right]
 \end{aligned}$$

- spacetime curvature

$$\begin{aligned}
 \int \text{Poincaré} &= \int m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\alpha\lambda} e^{\nu\beta} e^{\lambda\sigma} \right] \epsilon_{\mu\nu\lambda\sigma} \\
 \text{density} & \\
 &= \int \epsilon^{abcd} \frac{\epsilon_{\nu\lambda\sigma\tau}}{\text{interval}} \left[e_a^\nu e_b^\lambda e_c^\sigma e_d^\tau F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\nu e_b^\lambda e_c^\sigma e_d^\tau \right] \\
 & \qquad \qquad \qquad \det e = \sqrt{\det g_{\mu\nu}}
 \end{aligned}$$



- spacetime curvature

$$\begin{aligned}
 S_{\text{Poincaré}} &= \int_m \left[e^{\mu\lambda} e^{\nu\sigma} F^{\lambda\sigma} - 2\Lambda e^{\mu\lambda} e^{\nu\sigma} e^{\rho\tau} \right] \epsilon_{\mu\nu\lambda\sigma} \\
 \text{density} & \\
 &= \int_m \frac{1}{\sqrt{\det g}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\rho e_d^\sigma \right] \epsilon^{\mu\nu\lambda\sigma} \\
 & \qquad \qquad \qquad \det e = \sqrt{\det g}
 \end{aligned}$$



Spectral curvature

$$\begin{aligned} \int \text{Palatini density} &= \int_M \left[e^{\mu\nu\lambda\sigma} F_{\lambda\sigma} - 2\Lambda e^{\mu\nu\lambda\sigma} \right] \epsilon_{\mu\nu\lambda\sigma} \\ &= \int_M \frac{1}{4} \epsilon^{abcd} \frac{1}{\sqrt{|\det g_a|}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma \right] \frac{1}{\sqrt{|\det g_a|}} \end{aligned}$$

Given anti-Leyser $T^{\mu\nu} = -T^{\nu\mu}$

Spectral curvature

$$\begin{aligned} \int \text{Palatini density} &= \int_m \left[e^\mu{}_\lambda e^\nu{}_\lambda F^{\lambda\sigma} - 2\Lambda e^\mu{}_\lambda e^\nu{}_\lambda e^\lambda{}_\sigma \right] \epsilon_{\mu\nu\lambda\sigma} \\ &= \int_m \frac{1}{\epsilon} \epsilon^{abcd} \frac{1}{|\det e|} \left[e_a{}^\mu e_b{}^\nu F_{cd}{}^{\lambda\sigma} - 2\Lambda e_a{}^\mu e_b{}^\nu e_c{}^\lambda e_d{}^\sigma \right] \end{aligned}$$

$\det e = \sqrt{\det g_n}$

Given anti-Leyser $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

$$\text{dual: } T^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu}{}_{\lambda\beta} T^{\lambda\beta}$$

Palatini

R (SO(4))

fields commut.

(frame fields $e^{\mu} = e^{\mu}_{\alpha} dx^{\alpha}$)

$$\omega_a^{\mu\nu} = -\omega_a^{\nu\mu} \quad \text{1-forms}$$

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \text{Euc.}$$

$$\begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \text{Lorentz}$$

$$g_{\mu\nu} = e^{\alpha}_{\mu} e^{\beta}_{\nu} \eta_{\alpha\beta} \Rightarrow g^{\mu\nu} e^{\alpha}_{\mu} e^{\beta}_{\nu} = \eta^{\alpha\beta}$$

SO(4) local symm $R_i^a \rightarrow R_i^{\prime a} = \Lambda^a_b R_i^b$

Field strength $F_{\mu\nu}^a = d\omega_{\mu\nu}^a + \omega_{\mu}^b \wedge \omega_{\nu}^c \Lambda^a_{bc}$ SO(4)

= Spontaneous curvature

$$S_{\text{Palatini}} = \int_M \left[e^{\mu}_{\alpha} e^{\nu}_{\beta} F^{\alpha\beta} - 2\Lambda e^{\mu}_{\alpha} e^{\nu}_{\beta} e^{\gamma}_{\mu} e^{\delta}_{\nu} \right] \epsilon_{\mu\nu\alpha\beta}$$

$$= \int_M \frac{1}{\det e} \left[e^{\mu}_{\alpha} e^{\nu}_{\beta} F^{\alpha\beta} - 2\Lambda e^{\mu}_{\alpha} e^{\nu}_{\beta} e^{\gamma}_{\mu} e^{\delta}_{\nu} \right] dx^{\alpha} dx^{\beta} dx^{\gamma} dx^{\delta}$$

$\det e = \sqrt{|g|}$

Given anti-symmetric $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\mu\nu}$

R R

$$S_{\text{Petrovi}} = \int_m \left[e^{\mu\nu} e^{\lambda\sigma} F^{\lambda\sigma} - 2\Lambda e^{\hat{\alpha}\beta} e^{\hat{\gamma}\delta} e^{\hat{\epsilon}\zeta} \right] \epsilon_{\mu\nu\lambda\sigma}$$

density

$$= \int_m \frac{\epsilon^{abcd} \int_{\text{interval}} [e_a^\mu e_b^\nu F_{\mu\nu} - 2\Lambda e_{\hat{\alpha}}^{\hat{\beta}} e_{\hat{\gamma}}^{\hat{\delta}} e_{\hat{\epsilon}}^{\hat{\zeta}}]}{\det e = \sqrt{\det g_{\mu\nu}}}$$

even anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

$$T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta} \quad T^{\alpha\beta} = T^{\beta\alpha} \quad T^{\alpha\beta} = T^{\beta\alpha} \text{ and cyclic}$$

$$\begin{aligned}
 \text{density} &= \int m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} \right] \epsilon_{\mu\nu\lambda\sigma} \\
 &= \int m \frac{\epsilon^{abcd}}{\sqrt{\det g_{\mu\nu}}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\gamma e_d^\delta \right]
 \end{aligned}$$

$\det g = \sqrt{\det g_{\mu\nu}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{01} = T^{23}$ and cyclic

$\epsilon^{\alpha\beta\gamma\delta} = +1$ Eul
 -1 Lore

$$\begin{aligned}
 \text{density} &= \int_m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} \right] \epsilon_{\alpha\nu\lambda\sigma} \\
 &= \int_m \frac{1}{\epsilon} \epsilon^{abcd} \frac{1}{\text{interval}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda \frac{e_a^\mu e_b^\nu e_c^\gamma e_d^\delta}{\det e = \sqrt{\det g_n}} \right]
 \end{aligned}$$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $\frac{1}{2} \epsilon^{\mu\nu\alpha\beta} T^{\alpha\beta}$ $T^{01} = T^{23}$ and cyclic

$$\begin{aligned}
 \text{density} &= \int_m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} \right] \epsilon_{\alpha\nu\lambda\sigma} \\
 &= \int_m \frac{1}{\epsilon} \sum_{abcd} \frac{1}{\sqrt{\det g_{\mu\nu}}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\gamma e_d^\delta \right]
 \end{aligned}$$

$\det e = \sqrt{\det g_{\mu\nu}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

1) $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{12} = T^{34}$ and cyclic

+1 Eul
-1 Lore

$$T^{\pm\mu\nu} = \frac{1}{2} (T \pm T^*)^{\mu\nu}$$

$$\begin{aligned}
 \text{density} &= \int_m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\nu} e^{\lambda\sigma} \right] \epsilon_{\alpha\beta\lambda\sigma} \\
 &= \int_m \frac{\epsilon^{abcd}}{\sqrt{-\det g}} \left[e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma \right]
 \end{aligned}$$

$\det g = \sqrt{-\det g_{\mu\nu}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{01} = T^{23}$ and cyclic

1 Eq
- 1 constraint

$$\begin{aligned}
 T^{\pm\alpha\beta} &= \frac{1}{2} (T \pm T^*)^{\alpha\beta} \\
 T^{\pm 0i} &= \frac{1}{2} (T^0 \pm T^{23})
 \end{aligned}$$

R R

$$F_{\mu\nu} = E_1 e_2 e_3$$

R R

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

R R

$$F_{AV} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & B_2 & -B_2 & \\ (-1) & 0 & B_1 & 0 \end{pmatrix}$$

R R

$$F_{01} = E +$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & B_2 & -B_3 & 0 \\ (-1) & 0 & B_1 & 0 \end{pmatrix}$$

R R

$$F_{01}^{\pm} = 'E' \pm 'B'$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_3 & -B_2 & B_1 \\ (-1) & 0 & B_1 & 0 \end{pmatrix}$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spacetime curvature

Palatini
 density = $\int_M [e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} F^{\alpha\beta} - 2\Lambda e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} e^{\gamma}{}_{\delta} e^{\epsilon}{}_{\zeta} \epsilon^{\alpha\beta\gamma\delta}] \epsilon_{\mu\nu\rho\sigma}$
 $= \int_M \frac{1}{4} \epsilon^{abcd} \underbrace{e_a^{\mu} e_b^{\nu} F_{\mu\nu}}_{\text{interval}} \underbrace{e_c^{\lambda} e_d^{\sigma} \epsilon^{\lambda\sigma}}_{\text{deter} = \sqrt{\det g_n}}$

Given anti-Lorentz $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

$T^{\pm\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta} \begin{matrix} \uparrow\beta \\ \uparrow\alpha \end{matrix} T^{\alpha\beta}$
 $T^{01} = T^{23}$ and cyclic

$+1$ Eucl
 -1 Lorentz

$T^{\pm\alpha\beta}$	$(T^{\pm} T^{\alpha})^{\beta\gamma}$
$T^{\pm 01}$	$(T^{01} \pm T^{23})$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ (consists $SO(4)$)
 = Spacetime curvature $SO(4)$

Palatini
 density = $\int_m [e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} e^{\gamma}{}_{\delta} e^{\epsilon}{}_{\zeta}] \epsilon_{\mu\nu\lambda\sigma}$
 $= \int_m \epsilon^{abcd} \frac{1}{\sqrt{\det g}} [e_a^{\mu} e_b^{\nu} F_{cd}{}^{\lambda\sigma} - 2\Lambda e_a^{\mu} e_b^{\nu} e_c^{\gamma} e_d^{\delta}]$
 $\det g = \sqrt{\det g_{\mu\nu}}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{01} = T^{23}$ and cyclic

$T^{\pm 01}$	$=$	$(T^{\pm 23})^{\pm}$
$T^{\pm 02}$	$=$	$(T^{01} \pm T^{34})$

Field strength $F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda}\omega_{\lambda\nu}$ (constant $SO(4)$)
 = Spacetime curvature $SO(4)$

Pelotini density = $\int_m [e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} e^{\lambda}{}_{\gamma} e^{\sigma}{}_{\delta}] \epsilon^{\alpha\beta\gamma\delta}$
 $= \int_m \frac{1}{\sqrt{\det g}} [e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta} e^{\lambda}{}_{\gamma} e^{\sigma}{}_{\delta}] \epsilon^{\alpha\beta\gamma\delta}$
 $\det e = \sqrt{\det g}$

Given anti-symmetric $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{01} = T^{23}$ and cyclic

$\times \times = +1$ Eucl
 -1 Lorentz

$T^{\pm 01} = (T^{\pm 23})^{\times}$	$(T^{\pm 01})^{\times} = \pm T^{\pm 23}$ <u>check</u>
$T^{\pm 02} = (T^{01} \pm T^{23})$	

$\mathbb{R} \oplus \mathbb{R}$
Lie algebra $so(4)$
 $F_{\pm i} = 'E' \pm 'B'$

$$F_{\pm i} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_1 & B_2 & -B_3 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

$\mathbb{R} \oplus \mathbb{R}$
Lie algebra $so(4)$

$$F_{\pm} = E \pm B'$$

$T^{\pm} = T^{\pm}$ transition

$$F_{\pm} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & B_2 & -B_2 & B_1 \\ (-1) & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbb{R} \times \mathbb{R} \quad F_{01}^{\pm} = 'E' \pm B'$$

Lie algebra $SO(4)$

$T^{12} = -T^{21}$ generators

$[T_1, T_2] \in$ not Casimir

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & 0 & -B_2 & B_3 \\ (-1) & 0 & 0 & B_1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{R} \oplus \mathbb{R}$
 Lie algebra $so(4)$
 $[T_1, T_2] \in \mathfrak{so}(4)$

$$F_{01}^{\pm} = 'E' \pm B'$$

$$T^{uv} = -T^{vu}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & 0 & -B_2 & 0 \\ (-1) & 0 & B_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbb{R} \oplus \mathbb{R} \quad F_{01}^{\pm} = 'E' \pm B'$$

Lie algebra $so(4)$

$$[T_1, T_2] \in \dots$$

$$T^{uv} = -T^{vu}$$

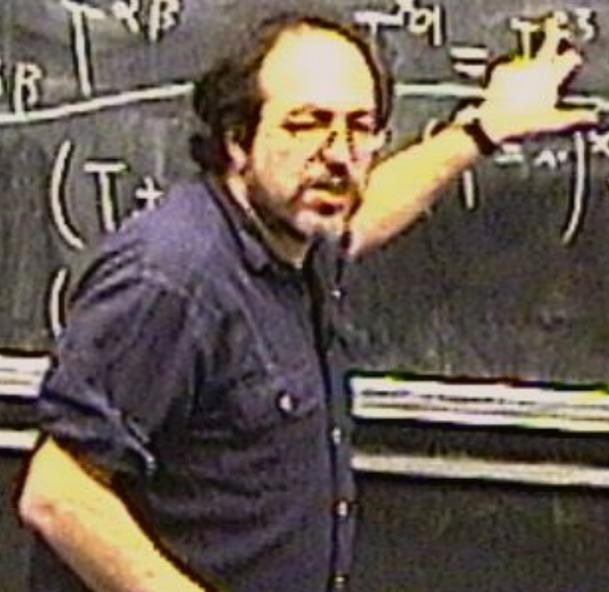
$$[T^+, T^+] \in T^+$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & 0 & -B_2 & B_3 \\ (-1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\lambda\mu} \omega_{\lambda\nu}$ SO(4)
 = Spectral curvature

Action density = $\int_m [e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\lambda\gamma} e^{\sigma\delta}] \epsilon_{\alpha\beta\gamma\delta}$
 $= \int_m \epsilon^{abcd} \frac{1}{\text{interval}} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma] \frac{1}{\det e = \sqrt{\det g_n}}$

Given anti-symmetric $T^{\mu\nu} = -T^{\nu\mu}$ (4d)
 dual: $T^{\star\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} T^{\alpha\beta}$ and cyclic
 $T^{\star\alpha\beta} = T^{\beta\gamma} T^{\gamma\alpha}$
 $T^{\pm\mu\nu} = (T^{\pm\alpha\beta})^{\mu\nu} = \pm T^{\mu\nu\pm}$ check
 $\star\star = +1$ Eul
 -1 Lore



$R \ R$
 Lie algebra (4)
 $\{T_i, \dots\}$

$$F_{01}^{\pm} = 'E \pm B'$$

$$T^+ \dots T^-$$

$$\{T^{\pm}, T^{\pm}\} \in T^{\pm}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & B_2 & -B_3 & 0 \\ (-1) & 0 & B_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\{T^+, T^-\} = 0$$

$\mathbb{R} \oplus \mathbb{R}$

$$\mathfrak{F}_{\mathfrak{sl}(4)} = \{E \pm B\}$$

Lie algebra $\mathfrak{so}(4)$

$$T^+ \dots T^-$$

$$[T_1, T_2] \in \mathfrak{so}(4)$$

$$[T^\pm, T^\pm] \in T^\pm$$

$$F_{\mathfrak{sl}(4)} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_6 & B_3 & -B_2 \\ (-1) & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

$$[T^+, T^-] = 0$$

$$6 \rightarrow 3 + 3$$

$\mathbb{R} \mathbb{R}$

$$F_{\pm 1}^{\pm} = \{E \pm B\}$$

Lie algebra $so(4)$

$$T^{+} \dots T^{-}$$

$$[T_1, T_2] \in \dots$$

$$[T^{\pm}, T^{\pm}] \in T^{\pm}$$

$$[T^+, T^-] = 0$$

$$6 \rightarrow 3 + 3$$

$so(4)$

$$F_{\pm 1}^{\pm} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_1 & B_2 & -B_2 \\ (-1) & & 0 & B_1 \\ & & & 0 \end{pmatrix}$$

R R

$$F_{01}^{\pm} = 'E' \pm B'$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & 0 & -B_2 & B_3 \\ (-1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Lie algebra $so(4)$

$$T^{+} - T^{-}$$

$$[T_1, T_2] \in \text{and}$$

$$[T^{\pm}, T^{\pm}] \in T^{\pm}$$

$$[T^+, T^-] = 0$$

$$6 \rightarrow 3 + 3$$

$$so(4) \quad su(3) + su(3) \\ su(2) + su(2)$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\lambda\mu} \omega_{\lambda\nu}$ (compact $SO(4)$)
 = Spacetime curvature $SO(4)$

Ricci scalar density = $\int_m [e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\lambda\gamma} e^{\delta\epsilon}] \epsilon_{\alpha\nu\lambda\sigma}$
 $= \int_m \frac{1}{\sqrt{|\det g_{\mu\nu}|}} [e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\lambda\gamma} e^{\delta\epsilon}] \epsilon_{\alpha\nu\lambda\sigma}$
 $\det g = \sqrt{|\det g_{\mu\nu}|}$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ $T^{01} = T^{23}$ and cyclic

$\kappa\kappa = +1$ Eucl
 -1 Lorentz

$T^{\pm\mu\nu} = (T \pm T^{\kappa})^{\mu\nu}$

$T^{01} = (T^{01} \pm T^{23})$

$(T^{\pm\mu\nu})^{\kappa} = \pm T^{\mu\nu\pm}$ check

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\lambda\mu} \wedge \omega_{\lambda\nu}$ (Lorentz $SO(4)$)
 = Spacetime curvature

Relativistic density = $\int_M [e^{\mu\alpha} e^{\nu\beta} F_{\alpha\beta} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} e^{\epsilon\zeta}] \epsilon_{\mu\nu\lambda\sigma}$
 $= \int_M \frac{1}{4} \epsilon^{abcd} \frac{1}{\det e} [e_a^\mu e_b^\nu F_{cd}^{\lambda\sigma} - 2\Lambda e_a^\mu e_b^\nu e_c^\lambda e_d^\sigma] \det e = \sqrt{\det g}$

Given anti-Liebrer $T^{\mu\nu} = -T^{\nu\mu}$ (4d) (= Lorentz)

$T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ and cyclic

Euclidean Lorentz	$T^{\pm 4i} = (T^{\pm ij})_{4i}$	$(T^{\pm ij})^{\pm} = \pm i T^{\pm ij}$	<u>check</u>
	$T^{4i} = (T^{ij})_{4i}$		

R R

$$F_{\pm} = \{E \pm B\}$$

Lie algebra $so(4)$

$$T^+ = -T^-$$

$$[T_1, T_2] \in \dots$$

$$[T^{\pm}, T^{\pm}] \in T^{\pm}$$

$$F_{\pm} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_1 & -B_2 & \\ (-1) & 0 & B_1 & \\ & & & 0 \end{pmatrix}$$

$$[T^+, T^-] = 0$$

$$6 \rightarrow 3 + 3$$

$$so(4) \quad \begin{matrix} su(3) + su(3) \\ su(2) + su(2) \end{matrix}$$

$\mathbb{R} \oplus \mathbb{R}$ $F_{\pm} = \{E \pm iB\}$ $T_{\pm} = (T_{\pm})^{-1}$ $F_{\pm} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ B_1 & B_2 & -B_2 & -B_1 \\ (-1) & 0 & 0 & 0 \end{pmatrix}$
 Lie algebra $so(4)$ $T^{+} = -T^{-}$
 $[T_1, T_2] \in \dots$ $\{T^{\pm}, T^{\pm}\} \in T^{\pm}$ $[T^{+}, T^{-}] = 0$
 $6 \rightarrow 3 + 3$
 $so(4) \quad so(3) + so(3)$
 $\quad \quad so(2) + so(2)$

$$\begin{aligned}
 \text{density} &= \int_m \left[e^{\mu\alpha} e^{\nu\beta} F^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} \right] \epsilon_{\mu\nu\lambda\sigma} \\
 &= \int_m \frac{1}{\sqrt{-\det g_{\mu\nu}}} \left[e^{\mu\alpha} e^{\nu\beta} F_{\alpha\beta}^{\lambda\sigma} - 2\Lambda e^{\mu\alpha} e^{\nu\beta} e^{\gamma\delta} \right] \epsilon_{\mu\nu\lambda\sigma}
 \end{aligned}$$

Given anti-tensor $T^{\mu\nu} = -T^{\nu\mu}$ (4d)

dual: $T^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} T^{\gamma\delta}$ and cyclic

$\epsilon\epsilon = +1$ Eul
 -1 Lorentz

$$\begin{aligned}
 T^{\pm\mu\nu} &= (T \pm \star T)^{\mu\nu} \\
 T^{\pm\alpha\beta} &= (T^{\alpha\beta} \pm \star T^{\gamma\delta})
 \end{aligned}$$

$$(T^{\pm\mu\nu})^{\star} = \pm T^{\mu\nu} \quad \text{check}$$

R R

$$F_{0i}^{\pm} = \pm E_i - B_i$$

$$T_{\mu\nu}^{\pm} = (T_{\mu\nu})$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

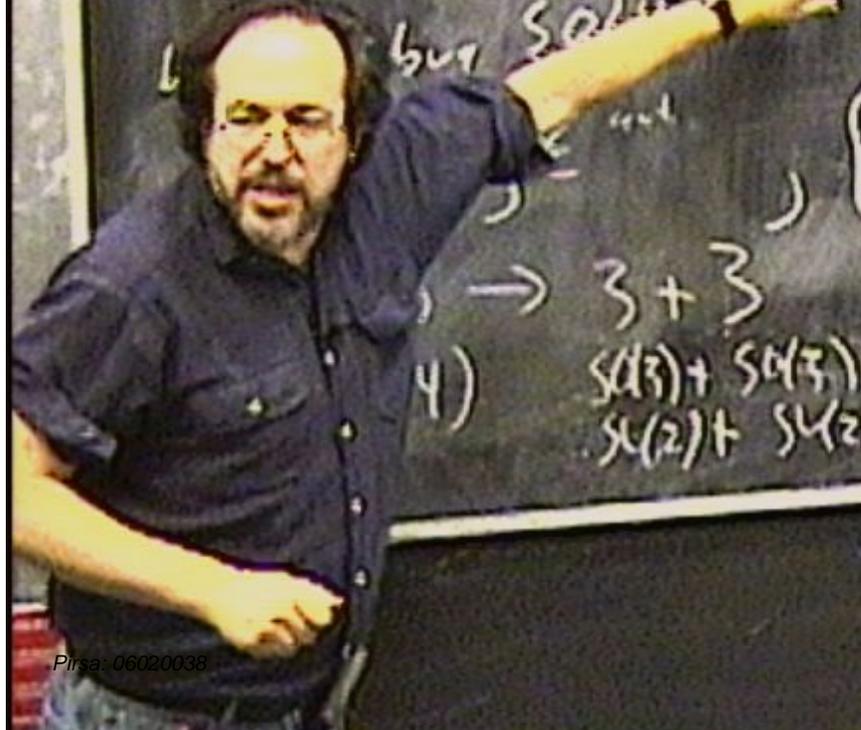
but solutions $T^{++} = T^{--}$

$$([T^{\pm}, T^{\pm}]) \in T^{\pm}$$

$$([T^+, T^-]) = 0$$

$\rightarrow 3 + 3$

$$4) \quad \begin{matrix} SU(3) + SU(3) \\ SU(2) + SU(2) \end{matrix}$$



$\mathbb{R} \mathbb{R}$

$$F_{\pm 1}^{\pm} = \{E \pm iB\}'$$

$$T_{\pm 1}^{\pm} = (T_{\pm 1})'$$

$$F_{\pm 1}^{\pm} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ & B_1 & B_2 & -B_3 \\ (-1) & & 0 & B_3 \\ & & & 0 \end{pmatrix}$$

Lie algebra $so(4)$

$$T^{\pm} = T^{\pm 2}$$

$$[T_1, T_2] \in \dots$$

$$([T^{\pm}, T^{\pm}]) \in T^{\pm}$$

$$= 0$$

$$6 \rightarrow 3 + 3$$

$$\begin{matrix} so(4) & so(3) + so(3) \\ so(3,1) & su(2)_L + su(2)_R \end{matrix}$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spontaneous curvature

$KK = 4 \perp$ Eul \ominus large	$T^{\pm 1/2} = (T^{\pm 1} T^{\pm 1})^{1/2}$ $T^{2/2} = (T^{01} + i T^{23})$	$(T^{\pm 1})^2 = \pm 1$ <u>check</u>
---------------------------------------	--	--------------------------------------

$J^{\mu\nu}$ rotations = $J^{\mu\nu} = \sum_{\lambda} k_{\lambda} J^{\mu\nu\lambda}$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$

= Spin connection SO(4)

$\kappa = \pm 1$ \mathbb{Z}_2	$T^{\pm 1} = (T \pm 1)^{\pm 1}$ $T^{\pm 1} = (T^{\pm 1} \pm 1^{\pm 1})$	$(T^{\pm 1})^2 = \pm 1$
------------------------------------	--	-------------------------

SO(4) $J^{\mu\nu}$ rotations = $J^{\mu\nu} = \epsilon^{\mu\nu\lambda\kappa} J_{\lambda\kappa}$

$[J^{\mu\nu}, J^{\lambda\kappa}] = \epsilon^{\mu\nu\lambda\kappa} J_{\lambda\kappa}$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ (contains $SO(4)$)
 = Spacetime curvature $SO(4)$

$\chi = +1$ (L) ω
 -1 (R) ω

$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha})^{\beta}$
 $T^{\alpha\beta} = (T^{\alpha} \pm T^{\beta})$

$|| \cdot || = \pm || \cdot ||$ (check)

$SO(4)$ J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$

$[J^i, J^j] = \epsilon^{ijk} J^k$ $SO(3)$

B

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spacetime curvature

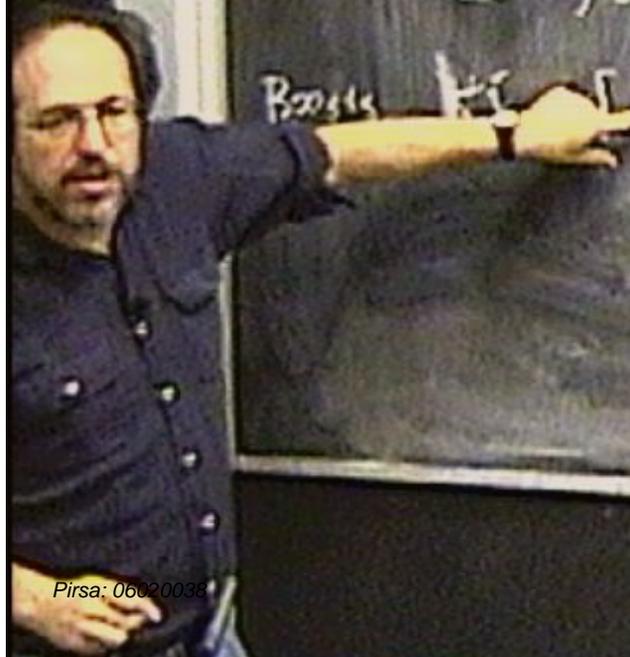
$\chi = \pm 1$ Eucl $\ominus 1$ Lorentz	$T^{\pm\mu\nu} = (T^{\pm 1} T^{\nu})^{\mu}$ $T^{\pm 0i} = (T^{0i} \pm T^{i3})$	$(T^{\pm})^2 = \pm 1$ <u>check</u>
--	---	------------------------------------

SO(4) J^a rotations = $J^k = \epsilon_{ijk} J^i J^j$
 $(T^i, J^j) = \epsilon^{ijk} J^k$ SO(3)
 Basis $\{T^i, J^j\}$

$F_{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}_{\lambda} \wedge \omega^{\lambda\nu}$ (6.4.15) $SO(4)$
 = Spontaneous curvature $SO(4)$

$\kappa\kappa = +1$ Eucl -1 Lorentz	$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha})^{\beta}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})^{\pm}$	$(T^{\pm})^2 = \pm 1$ <u>check</u>
--	--	---

$SO(4)$ J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ $SO(3)$
 Boosts K^i $[K^i, J^j] = -\epsilon^{ijk} K^k$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spontaneous curvature

$\kappa\kappa = +1$ Eucl -1 Lorentz	$T^{\pm\mu\nu} = (T^{\pm} T^{\nu})^{\mu}$ $T^{\pm\alpha} = (T^{\alpha} T^{\pm})^{\alpha}$	$(T^{\pm})^2 = \pm 1$ <u>check</u>
--	--	------------------------------------

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ SO(3)
 Boosts K^{α} $[K^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} K^{\gamma}$ $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spacetime curvature

$\kappa\kappa = +1$ Eucl -1 Lorentz	$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha})^{\beta}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})$	$ T^{\pm}\rangle = \pm T^{\mp}\rangle$ <u>check</u>
--	--	--

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ SO(3)
 Boosts K^{α} $[K^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} K^{\gamma}$ $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$
 $K^{\alpha} = T^{0\alpha}$
 $T^{\pm} = \dots$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spacetime Curvature

$\kappa^{\mu\nu} = +1$ Eucl -1 Lorentz	$T^{\pm\mu\nu} = (T^{\pm} T^{\nu})^{\mu}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})$	$ T^{\pm} = \pm 1$ <u>check</u>
---	--	----------------------------------

SO(4) $J^{\mu\nu}$ rotations = $J^{\mu\nu} = \epsilon^{\mu\nu\lambda\kappa} J^{\lambda\kappa}$
 $[J^{\mu\nu}, J^{\rho\sigma}] = \epsilon^{\mu\nu\lambda\kappa} J^{\lambda\kappa} \quad \text{SO(3)}$
 Boosts K^{μ} $[K^{\mu}, J^{\nu}] = -\epsilon^{\mu\nu\lambda\kappa} K^{\lambda\kappa}$ $[K^{\mu}, K^{\nu}] = \epsilon^{\mu\nu\lambda\kappa} J^{\lambda\kappa}$
 $K^{\mu} = T^{\mu\alpha}$
 $J_{\pm}^{\mu} = J_{\mu} \pm K_{\mu}$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ (for $SO(4)$)
 = Spacetime curvature $SO(4)$

$\kappa = +1$ Euclidean -1 Lorentz	$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha})^{\beta}$ $T^{\alpha\beta} = (T^{\alpha} T^{\beta})$	$ T^{\pm} = \pm 1$ check
---	---	--

$SO(4)$ J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ $SO(3)$
 Boosts K^i $[K^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} K^{\gamma}$ $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} K^{\gamma}$
 $J_{\pm}^{\alpha} = J_{\alpha} \pm K_{\alpha}$ for J^{\pm}

$F_{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}_{\lambda}\omega^{\lambda\nu}$ SO(4)
 = Spontaneous curvature

$\kappa^{\mu} = \pm 1$ (all) ± 1 (large)	$T^{\pm\mu\nu} = (T^{\pm} T^{\nu})^{\mu\nu}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})$	$ T^{\pm} = \pm 1$ check
---	---	--

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ SO(3)
 Basis κ^{α} $[\kappa^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} \kappa^{\gamma}$ $[\kappa^{\alpha}, \kappa^{\beta}] = \epsilon^{\alpha\beta\gamma} \kappa^{\gamma}$
 $\kappa^{\alpha} = T^{\alpha\beta}$
 $J^{\pm}_{\alpha} = J_{\alpha} \pm \kappa_{\alpha}$ $[J^{\pm}_{\alpha}, J^{\pm}_{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\pm}_{\gamma}$ $[J^{\pm}, J^{\mp}] = 0$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spontaneous curvature

$\kappa\kappa = \pm 1$ Euclidean ± 1 Lorentz	$T^{\pm\mu\nu} = (T^{\pm} T^{\nu})^{\mu}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})$	$ T^{\pm} = \pm 1$ check
---	--	--

SO(4) J^a rotations $= J^a = \epsilon^{abc} J^b J^c$
 $[J^a, J^b] = \epsilon^{abc} J^c$ SO(3)
 with $\kappa \in \mathbb{R}$ $[K^a, K^b] = -\epsilon^{abc} \kappa J^c$ [K^a, K^b] = \epsilon^{abc} \kappa J^c
 $\kappa^2 = T^2$
 $J^{\pm} = J^a \pm \kappa K^a$ [J^{\pm}, J^{\pm}] = \epsilon^{abc} J^c [J^+, J^-] = 0
 $J^+ \leftrightarrow J^-$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ (values $SO(4)$)
 = Spinning connection $SO(4)$

$\kappa^{\pm} = \pm 1$ (val) \ominus (arrow)	$T^{\pm}{}^{\mu}{}_{\nu} = (T^{\pm} \Pi^{\mu})_{\nu}$ $T^{\pm}{}^{\mu}{}_{\nu} = (T^{\mu} \pm T^{\nu})$	$ \cdot = \pm 1$ (check)
---	--	--

$SO(4)$ J^{μ} rotations = $J^{\mu} = \epsilon^{\mu\nu\lambda} J^{\nu\lambda}$
 $[J^{\mu}, J^{\nu}] = \epsilon^{\mu\nu\lambda} J^{\lambda}$ $SO(3)$
 Boosts K^{μ} $[K^{\mu}, J^{\nu}] = -\epsilon^{\mu\nu\lambda} K^{\lambda}$ $[K^{\mu}, K^{\nu}] = \epsilon^{\mu\nu\lambda} J^{\lambda}$
 $K^{\mu} = J^{0\mu}$
 $J_{\pm}^{\pm} = J_{\pm} \pm i K_{\pm}$ $[J_{\pm}^{\pm}, J_{\pm}^{\pm}] = \epsilon^{\pm\pm\lambda} J^{\lambda}$ $[J^{\pm}, J^{\mp}] = 0$
 raising $J^{\pm} \leftrightarrow J^{\mp}$

$F_{\mu\nu} = d\omega^{\mu\nu} + \omega^{\mu}_{\lambda} \wedge \omega^{\lambda\nu}$ (6.4.15) $SO(4)$
 = Spacetime curvature $SO(4)$

$\chi = +1$ Eucl
 -1 Lorentz

$$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha})^{\beta}$$

$$T^{\alpha\beta} = (T^{\alpha} \pm T^{\beta})$$

$$|T^{\pm}| = \pm 1 \quad \text{check}$$

$SO(4)$ J^{α} rotations $= J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$

$$[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma} \quad SO(3)$$

Boosts K^{α} $[K^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} K^{\gamma}$ $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$
 $K^{\alpha} = T^{0\alpha}$

\pm
 $J_{\pm} \pm iK_{\pm}$
 reversal $J^+ \leftrightarrow J^-$

$$[J_{\pm}^{\alpha}, J_{\pm}^{\beta}] = \epsilon^{\alpha\beta\gamma} J_{\pm}^{\gamma}$$

$$[J^+, J^-] = 0$$

$$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu\lambda}\omega_{\lambda\nu} \quad \text{SO(4)}$$

= Spontaneous Curvature

$\kappa^2 = \pm 1$ (or ± 1 gauge)	$T^{\pm\mu\nu} = (T^{\pm\mu\nu})^{\kappa}$	$T^{\pm\mu\nu} = \pm T^{\mu\nu}$ (check)
	$T^{\pm\mu\nu} = (T^{\mu\nu} \pm \epsilon^{\mu\nu})$	

SO(4) $J^{\mu\nu}$ generators = $J^{\mu\nu} = \epsilon^{\mu\nu\lambda\kappa} J^{\lambda\kappa}$

$[J^i, J^j] = \epsilon^{ijk} J^k \quad \text{SO(3)}$

Basis K^i $[K^i, J^j] = -\epsilon^{ijk} K^k$ $[K^i, K^j] = \epsilon^{ijk} J^k$

$K^i = J^i$

$J_{\pm}^i = J_i \pm iK_i$ $[J_{\pm}^i, J_{\pm}^j] = \epsilon_{ijk} J^k$ $[J^i, J^j] = 0$

impl. $J^i \leftrightarrow J^j$

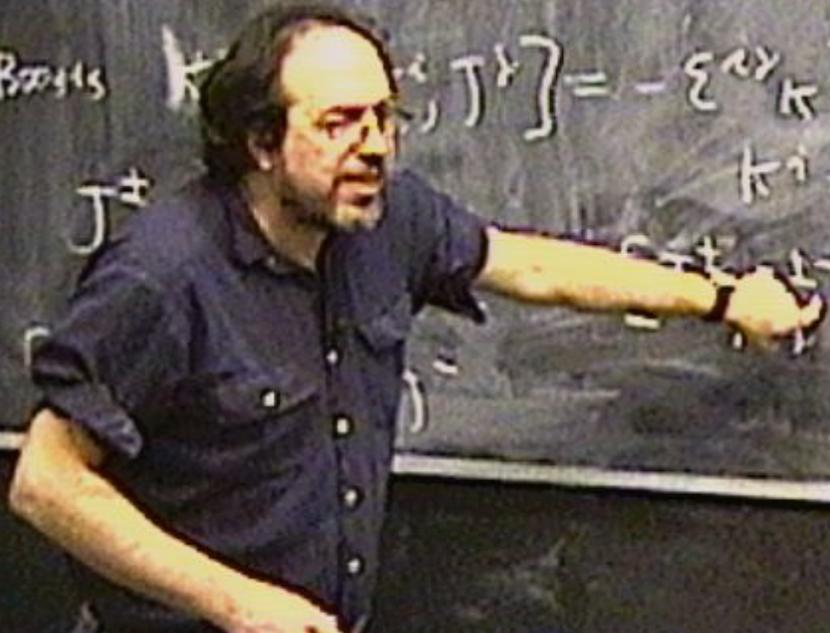


$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ (curvature) SO(4)
 = Spontaneous curvature

$\kappa^{\pm} = \pm 1$ Euclidean ± 1 Lorentz	$T^{\pm\mu\nu} = (T^{\pm} T^{\nu})^{\mu}$ $T^{\pm\alpha\beta} = (T^{\alpha} \pm T^{\beta})$	$ T^{\pm}\rangle = \pm 1 ^{\pm}$ <u>check</u>
---	--	--

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon_{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ SO(3)

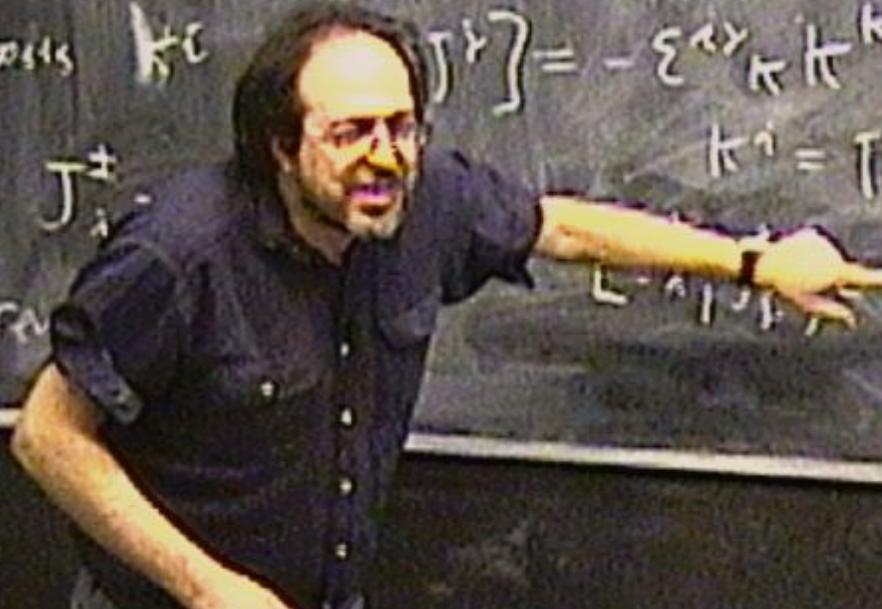
Basis K^{α}, J^{β}
 $[K^{\alpha}, J^{\beta}] = -\epsilon^{\alpha\beta\gamma} K^{\gamma}$ $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$
 $K^{\alpha} = T^{\alpha}$
 $[J^{\pm}, J^{\pm}] = \epsilon_{\alpha\beta\gamma} J^{\alpha\beta}$ $[J^{\pm}, J^{\mp}] = 0$



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spinning Curvature

$\chi\chi = +1$ Eucl -1 Lorentz	$T^{\pm\alpha\beta} = (T^{\pm} T^{\alpha\beta})^{\mu\nu}$ $T^{\pm\alpha\beta} = (T^{\alpha\beta} \pm T^{\beta\alpha})$	$ T^{\pm} = \pm 1$ check
--------------------------------------	---	--

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ SO(3)
 Basis K^{α} $[K^{\alpha}, K^{\beta}] = \epsilon^{\alpha\beta\gamma} K^{\gamma}$
 $K^{\alpha} = J^{\beta\gamma}$
 $[J^{\alpha}, J^{\beta}] = \epsilon^{\alpha\beta\gamma} J^{\gamma}$ [J[±], J[±]] = 0



$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu}^{\lambda} \wedge \omega_{\lambda\nu}$ SO(4)
 = Spinning Curvature

$\kappa\kappa = +1$ Eucl -1 Lorentz	$T^{\pm\alpha\beta} = (T^{\pm} \Pi^{\alpha\beta})_{\mu\nu}$ $T^{\pm 0i} = (T^{\pm i} \pm T^{0i})$	$ T^{\pm} = \pm 1$ check
--	--	--

SO(4) J^{α} rotations = $J^{\alpha} = \epsilon^{\alpha\beta\gamma} J^{\beta\gamma}$
 $[J^i, J^j] = \epsilon^{ijk} J^k$ SO(3)
 Basis K^i $[K^i, J^j] = -\epsilon^{ijk} K^k$ $[K^i, K^j] = \epsilon^{ijk} J^k$
 $K^i = T^{0i}$
 $J_i^{\pm} = J_i \pm iK_i$ $[J_{\alpha}^{\pm}, J_{\beta}^{\pm}] = \epsilon_{\alpha\beta\gamma} J^{\pm\gamma}$ $[J^+, J^-] = 0$
 cancel $J^+ \leftrightarrow J^-$



ω_{IV}

$$A \quad \omega^{HV} \pm \omega^{xHV}$$

$$\bar{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{01} = A^{10} \quad A^{23} = A^{32}$$

$$\bar{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{\mu\nu} = A^{\nu\mu} \quad A^{\bar{i}} = A^{\bar{i}} \quad \bar{i} = 1, 2, 3$$



$$\vec{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu}$$

$$A^{\mu\nu} = -A^{\nu\mu} \quad A^{\mu\mu} = A^{\mu\mu} \quad \mu = 1, 2, 3$$

$$F^{\mu\nu} = F^{\nu\mu}$$

$$\begin{aligned} \bar{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\alpha\mu\nu} & A^{\alpha\beta} &= A^{\beta\alpha} & A^{\alpha\alpha} &= A^{\alpha\alpha} & \alpha &= 1, 2, 3 \\ F^{\mu\nu} &= F^{+\mu\nu} + F^{-\mu\nu} \end{aligned}$$

$$\begin{aligned}
 \bar{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\nu\mu} & A^{\mu\nu} &= A^{\nu\mu} & A^{\mu\mu} &= A^{\mu\mu} & \mu &= 1, 2, 3 \\
 F^{\mu\nu} &= F^{\nu\mu} + F^{\nu\mu} & &= \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)
 \end{aligned}$$

$$\vec{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{\alpha\beta} = -A^{\beta\alpha} \quad A^{\hat{\alpha}} = A^{\alpha\hat{\alpha}} \quad \hat{\alpha} = 1, 2, 3$$

$$F^{\hat{\alpha}} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+\mu\nu} = dA^{\mu\nu} + \frac{1}{2}A^{\tau\mu} \wedge A^{\tau\nu}; \quad F^{\hat{\alpha}} =$$

$$\begin{aligned}
 \vec{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\kappa\lambda\nu} & A^{\mu\nu} &= A^{\nu\mu} & A^{\hat{i}} &= A^{\mu\lambda} & \hat{i} &= 1, 2, 3 \\
 F^{\pm} &= F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*) \\
 F^{+\mu\nu} &= dA^{\mu\nu} + A^{\mu\lambda} A^{\lambda\nu}; & F^{\hat{i}j} &= dA^{\hat{i}j} + \frac{1}{2}\epsilon^{\hat{i}jk} A^{\hat{j}l} A^{\hat{k}l}
 \end{aligned}$$

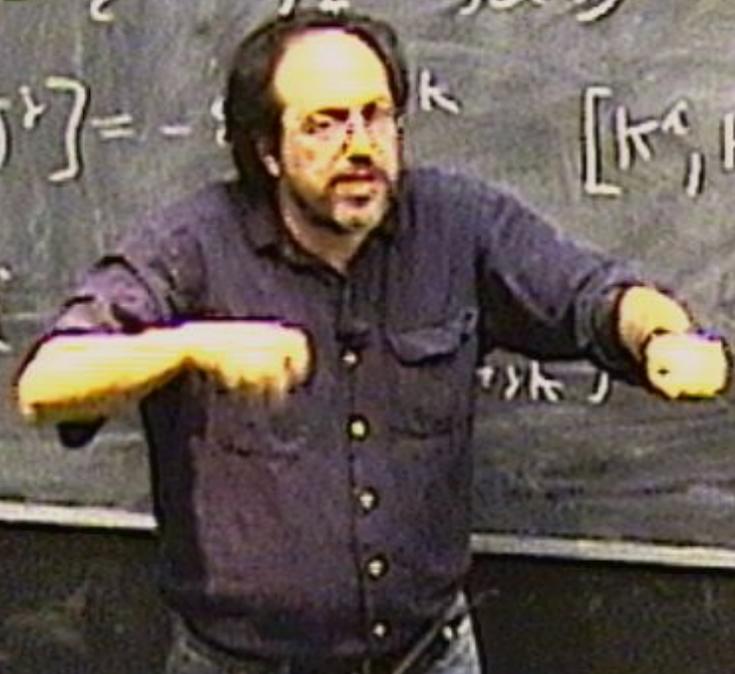


$$\begin{aligned} \vec{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\nu\mu} & A^{\alpha\beta} &= A^{\beta\alpha} & A^{\hat{\alpha}} &= A^{\alpha\hat{\alpha}} & \hat{\alpha} &= 1, 2, 3 \\ F^{\hat{\alpha}} &= F^{+\mu\nu} + F^{-\mu\nu} & &= \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*) \\ F^{+\mu\nu} &= dA^{\mu\nu} + \frac{1}{2}A^{\mu\lambda} \wedge A^{\nu\lambda} & ; & F^{\hat{\alpha}} &= dA^{\hat{\alpha}} + \frac{1}{2}\epsilon^{\hat{\alpha}\gamma\kappa} A^{\gamma} \wedge A^{\kappa} \end{aligned}$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\lambda\mu} \omega_{\lambda\nu}$ (classis $SO(4)$)
 = Spacetime curvature $SO(4)$

$\chi = +1$ Eucl -1 Lorentz	$T^{\pm\mu\nu} = (T^{\pm} T^{\mu\nu})$ $T^{\mu\nu} = (T^{\mu} T^{\nu})$	$(T^{\pm\mu\nu})^{\chi} = \pm T^{\mu\nu \pm}$ <u>check</u>
----------------------------------	--	--

$[J^i, J^j] = \epsilon^{ijk} J^k$ $SO(3)$
 Basis K^i $[K^i, J^j] = -\epsilon^{ijk} K^k$ $[K^i, K^j] = \epsilon^{ijk} J^k$
 $J_{\pm}^i = J^i \pm iK^i$
 raising/lowering $J^+ \leftrightarrow J^-$ $[J^+, J^-] = 0$



Euclidean 6209079

- Review Palatini
- Review duality & self-duality
- Plebanski / Ashtekar, TSS formalism.
- "classical LQG"

- Laurent-Atiyah formalism <http://6561191>

$$g \eta \eta^t = \eta$$
$$\downarrow$$
$$g \psi$$

$$\begin{aligned} \vec{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\nu\mu} & A^{0i} &= -A^{i0} & A^{ij} &= A^{ji} & i, j &= 1, 2, 3 \\ F^{\mu\nu} &= F^{+\mu\nu} + F^{-\mu\nu} & &= \frac{1}{2}(F + F^{\vee}) + \frac{1}{2}(F - F^{\vee}) \\ F^{+\mu\nu} &= dA^{\mu\nu} + A^{\mu\lambda} \wedge A^{\lambda\nu} & F^{+ij} &= dA^{ij} + \frac{1}{2}\epsilon^{ijk} A^j \wedge A^k \\ \Sigma^{\mu\nu} &= e^{\mu\lambda} e^{\lambda\nu} \end{aligned}$$

$$\begin{aligned}
 \vec{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\nu\mu} & A^{\mu\nu} &= -A^{\nu\mu} & A^{\hat{\mu}} &= A^{\mu} & \mu &= 1, 2, 3 \\
 F^{\pm} &= F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{\vee}) + \frac{1}{2}(F - F^{\vee}) \\
 F^{+\mu\nu} &= dA^{\mu\nu} + (A^{\mu\lambda} \wedge A^{\lambda\nu} - A^{\nu\lambda} \wedge A^{\lambda\mu}) & F^{+\hat{\mu}\hat{\nu}} &= dA^{\hat{\mu}\hat{\nu}} + \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} A^{\hat{\rho}} \wedge A^{\hat{\sigma}} \\
 M^{\mu\nu} &= \rho^{\mu\lambda} \rho^{\nu} = -\epsilon^{\mu\nu\lambda}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\kappa\lambda} & A^{\mu\nu} &= -A^{\nu\mu} & A^{\lambda\lambda} &= A^{\mu\mu} & \mu, \nu &= 1, 2, 3 \\
 F^{\pm} &= F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*) \\
 F^{+\mu\nu} &= dA^{\mu\nu} + A^{\mu\lambda} \wedge A^{\lambda\nu} & F^{+\lambda} &= dA^{\lambda} + \frac{1}{2}\epsilon^{\lambda\mu\nu} A^{\mu} \wedge A^{\nu} \\
 M^{\mu\nu} &= \rho^{\mu} \wedge \rho^{\nu} = -\epsilon^{\mu\nu} & \Sigma^{\mu\nu} &= \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}
 \end{aligned}$$



$$\vec{A}^{uv} = \omega^{uv} \pm \omega^{xuv} \quad A^{23} = A^{01} \quad A^i = A^{0i} \quad i = 1, 2, 3$$

$$F^{\pm uv} = F^{+uv} + F^{-uv} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+uv} = dA^{+uv} + \frac{1}{2} \epsilon^{\lambda\mu\nu} A^{+\lambda\mu} A^{+\nu\lambda}$$

$$F^{+i} = dA^i + \frac{1}{2} \epsilon^{ijk} A^j A^k$$

$$\Sigma^{\mu\nu} = \epsilon^{\mu\lambda} \epsilon^{\nu\lambda} = -\epsilon^{\nu\mu}$$

$$S^{Pl.} = \int$$

$$\vec{A}^{uv} = \omega^{uv} \pm \omega^{uv} \quad A^{23} = -A^{01} \quad A^i = A^{i1} \quad i = 1, 2, 3$$

$$F^{uv} = F^{+uv} + F^{-uv} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+uv} = dA^{uv} + \frac{1}{2} \epsilon^{uv\lambda\sigma} A^{+\lambda\sigma}; \quad F^{+\lambda\sigma} = dA^\lambda + \frac{1}{2} \epsilon^{\lambda\mu\nu\kappa} A^\mu A^\nu A^\kappa$$

$$\Sigma^{uv} = \epsilon^{\mu\lambda} \epsilon^{\nu\sigma} = -\epsilon^{\nu\mu}$$

$$\Sigma^{uv} = \Sigma^{+uv} + \Sigma^{-uv}$$

$$S^{Pl.} = \int \Sigma^{uv} \wedge F_{uv}$$

$F_{\mu\nu} = d\omega_{\mu\nu} + \omega_{\mu\lambda}\omega_{\lambda\nu}$ SO(4)
 = Spacetime curvature

} Palatini = $\int_m [e^{\mu\lambda} e^{\nu\sigma} F^{\lambda\sigma} - 2\Lambda e^{\mu\lambda} e^{\nu\sigma} e^{\alpha\beta} e^{\gamma\delta}] \epsilon_{\mu\nu\lambda\sigma}$
 density
 $= \int_m \frac{1}{4} \epsilon^{abcd} [e^{\mu\lambda} e^{\nu\sigma} F_{\mu\nu}^{\lambda\sigma} - 2\Lambda e^{\mu\lambda} e^{\nu\sigma} e^{\alpha\beta} e^{\gamma\delta}]$
 $\det e = \sqrt{\det g_{\mu\nu}}$

Given $T^{\mu\nu} = -T^{\nu\mu}$ (4d) $\lambda = \text{covector}$
 $\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} T^{\alpha\beta}$ $T^{\alpha\beta} = T^{\beta\alpha}$ and cyclic

$T^{\pm\mu\nu} = (T \pm T^{\times})^{\mu\nu}$ $(T^{\pm\mu\nu})^{\times} = \pm T^{\mu\nu\pm}$ (4d)
 $T^{\pm\alpha\beta} = (T^{\alpha\beta} \pm T^{\gamma\delta})$

$$A^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{\mu\nu} = -A^{\nu\mu} \quad A^i = A^{0i} \quad i = 1, 2, 3$$

$$F^{\mu\nu} = F^{\nu\mu} + F^{\nu\mu} = \frac{1}{2}(F + F^{\nu}) + \frac{1}{2}(F - F^{\nu})$$

$$F^{\mu\nu} = dA^{\mu\nu} + [A^{\mu\lambda} A^{\lambda\nu}] \quad F^{ti} = dA^i + \frac{1}{2}\epsilon^{ijk} A^j A^k$$

$$\Sigma^{\mu\nu} = \rho^\mu \wedge \rho^\nu = -\Sigma^{\nu\mu}$$

$$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$



$$\bar{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{\mu\nu} = -A^{\nu\mu} \quad A^i = A^{0i} \quad i = 1, 2, 3$$

$$F^{\pm\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+\mu\nu} = dA^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} A^{\lambda\sigma} \quad F^{+i} = dA^i + \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} A^{\mu\nu} A^{\lambda\sigma}$$

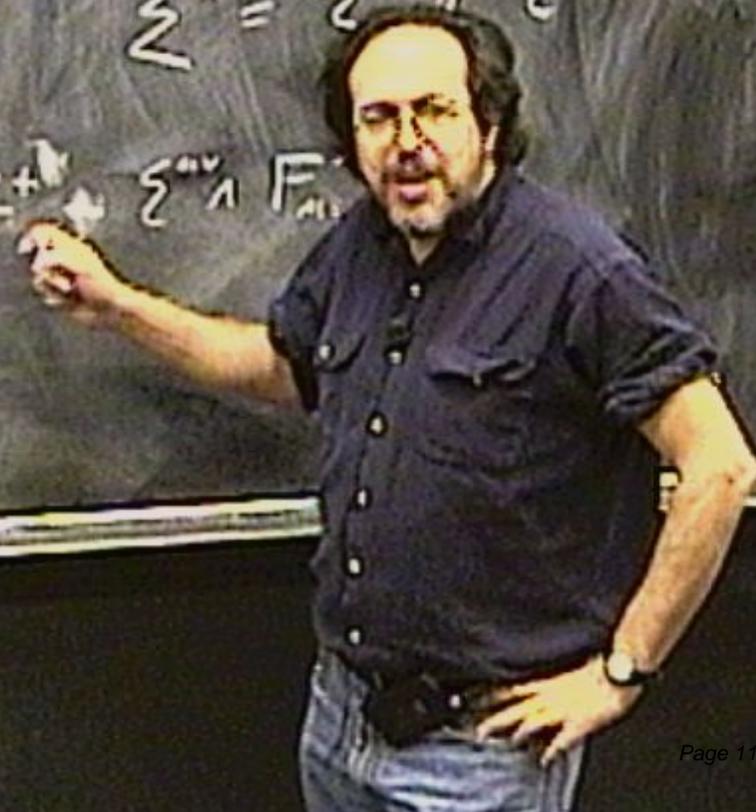
$$\Sigma^{\mu\nu} = \epsilon^{\mu\lambda} \epsilon^{\nu\sigma} = -\epsilon^{\nu\mu}$$

$$\Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$S^{PI} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{+\mu\nu} \wedge F_{\mu\nu} + \Sigma^{-\mu\nu} \wedge F_{\mu\nu}$$

$$\begin{aligned}
\vec{A}^{uv} &= \omega^{uv} \pm \omega^{uv} & A^+ - A^- &= F \\
F^+ &= F^{+uv} + F^{-uv} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*) \\
F^{+uv} &= \lambda A^{\mu\nu} + \lambda A^{\mu\nu} & F^{\pm i} &= \lambda A^i + \frac{1}{2}\epsilon^{ijk} A^j \lambda A^k \\
\Sigma^{uv} &= \epsilon^{\mu\nu\lambda} \epsilon^{\nu\lambda} = -\Sigma^{uv} & \Sigma^{uv} &= \Sigma^{+uv} + \Sigma^{-uv} \\
S^{\text{Pl.}} &= \int \Sigma^{uv} F_{uv} = \int \Sigma^{+uv} F_{uv} + \Sigma^{-uv} F_{uv}
\end{aligned}$$

$$\begin{aligned}
 \vec{A}^{\mu\nu} &= \omega^{\mu\nu} \pm \omega^{\nu\mu} & A^{ij} &= -A^{ji} & A^i &= A^{0i} & i &= 1, 2, 3 \\
 F^{\mu\nu} &= F^{+\mu\nu} + F^{-\mu\nu} & &= \frac{1}{2}(F + F^{\vee}) + \frac{1}{2}(F - F^{\vee}) \\
 F^{+\mu\nu} &= dA^{\mu\nu} + \lambda A^{\mu\lambda} A^{\lambda\nu} & ; & F^{+i} &= dA^i + \frac{1}{2}\epsilon^{ijk} A^j \wedge A^k \\
 \Sigma^{\mu\nu} &= \varrho^\mu \wedge \varrho^\nu = -\Sigma^{\nu\mu} & & \Sigma^{\mu\nu} &= \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu} \\
 S^{Pl.} &= \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{\mu\nu} \wedge F^{+\mu\nu} + \Sigma^{\mu\nu} \wedge F_{\mu\nu}^{-}
 \end{aligned}$$



$\lambda = 1, 2, 3$

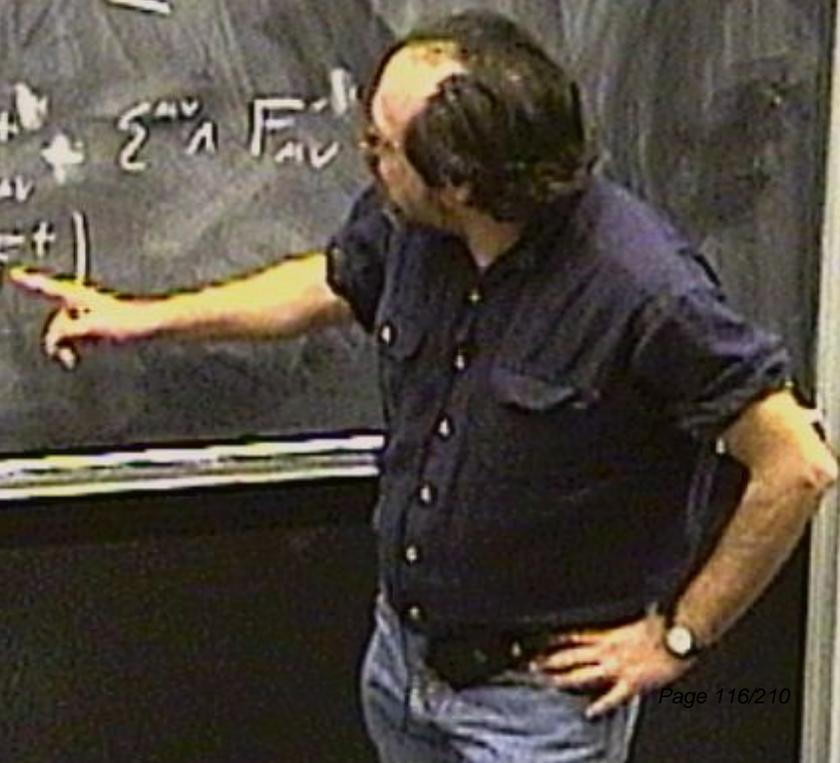
$$\bar{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A^{\mu\nu} = \Lambda^{\mu\nu} \quad A^{\lambda} = A^{\lambda}$$

$$F^{\pm\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+\mu\nu} = dA^{\mu\nu} + \Lambda A^{\mu\lambda} \wedge A^{\lambda\nu}; \quad F^{+\lambda} = d\Lambda^{\lambda} + \frac{1}{2}\epsilon^{\lambda\mu\nu} A^{\mu} \wedge A^{\nu}$$

$$\Sigma^{\mu\nu} = \varrho^{\mu} \wedge \varrho^{\nu} = -\Sigma^{\nu\mu} \quad \Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$S^{Pl.} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int (\Sigma^{+\mu\nu} \wedge F_{\mu\nu} + \Sigma^{-\mu\nu} \wedge F_{\mu\nu})$$



$$\bar{A}^{\mu\nu} = \omega^{\mu\nu} \pm \omega^{\nu\mu} \quad A'^{\mu\nu} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} A^{\alpha\beta} \quad \lambda = 1, 2, 3$$

$$F^{\pm\mu\nu} = F^{+\mu\nu} + F^{-\mu\nu} = \frac{1}{2}(F + F^{\vee}) + \frac{1}{2}(F - F^{\vee})$$

$$F^{+\mu\nu} = dA^{\mu\nu} + \Lambda^{\mu}{}_{\lambda} \Lambda^{\nu}{}_{\kappa} A^{+\lambda\kappa}; \quad F^{+i} = dA^i + \frac{1}{2}\epsilon^{ijk} A^j \wedge A^k$$

$$\Sigma^{\mu\nu} = e^{\mu}{}_{\lambda} e^{\nu}{}_{\kappa} = -\epsilon^{\nu\mu\kappa} \quad \Sigma^{\mu\nu} = \Sigma^{+\mu\nu} + \Sigma^{-\mu\nu}$$

$$S^{Pl.} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu} = \int \Sigma^{\mu\nu} \wedge F_{\mu\nu}^{+} + \Sigma^{\mu\nu} \wedge F_{\mu\nu}^{-}$$

$$= \int \Sigma^{(+)\mu\nu} \wedge F_{\mu\nu}^{(+)} - \Sigma^{(-)\mu\nu} \wedge F_{\mu\nu}^{(-)}$$



$$\int \eta = \int \xi^t \omega \wedge F^+(A')$$

$$S_{III} = \int \Sigma^{\dagger}(\omega) \Lambda F^{\dagger}(\Lambda^{\dagger}) = \int (\Sigma^{\dagger} \Lambda F^{\dagger})$$



$$S_{\text{M}} = \int \Sigma^{\mu\nu} \omega_{\mu\nu} F^{\dagger}(A^{\nu}) = \int (e^{-\Lambda \partial^{\nu}})^{\dagger} \Lambda F^{\dagger}_{\mu\nu}(V)$$

$$S^{(1)} = \int \Sigma^{\mu\nu} \wedge F^{\mu\nu} = \int (e^{\mu}{}_{\alpha} e^{\nu}{}_{\beta}) \wedge F^{\alpha\beta}$$

$$P^{\mu\nu} = \int \left(\frac{1}{2} \Sigma^{\mu\nu} \pm \frac{1}{2} \Sigma^{\mu\nu} \right)$$

P

$$S^{(11)} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(\Lambda^{\mu\nu}) = \int (e^{-\Lambda \theta^{\mu\nu}}) \wedge F^{\dagger}_{\mu\nu}(\nu)$$

$$P^{\mu\nu} = \int \left(I_{\mu\nu} \pm \frac{1}{2} \Sigma_{\mu\nu}^{\mu\nu} \right)$$

$\int_{S^2} \int_{S^3}$

$$(P^{\mu\nu})^2 = P^{\mu\nu}$$

$$P^{\mu\nu} P^{\mu\nu} = 0$$

$$\int \Pi = \int \Sigma^{\mu\nu} \Lambda F^{\mu\nu}(\Lambda') = \int (e^{-\Lambda \theta^{\mu\nu}}) \Lambda F^{\mu\nu}(\Lambda')$$

$$\Lambda^{\mu\nu} = \mathbb{I}_{\mu\nu} \pm \frac{1}{2} \left(\sum_{\alpha\beta} \Lambda^{\alpha\beta} \right)$$

$$\delta \epsilon^{\mu\nu} \delta \omega^{\alpha\beta}$$

$$(\Lambda^{\mu\nu})^2 = \Lambda^{\mu\nu}$$

$$\Lambda^{\mu\nu} \Lambda^{\mu\nu} = 0$$

$$S^{III} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(\Lambda^{\nu}) = \int (e^{-\Lambda \partial^{\nu}}) \wedge F^{\dagger}_{\mu\nu}(x)$$

$$\frac{\delta S^e}{\delta \varrho_a^{\mu}} = \epsilon^{\mu\nu\alpha\beta} \varrho_b^{\nu} F_{cd}^{\dagger}$$

$$S^{(1)} = \int \xi^{\mu\nu} \wedge F^{\dagger}(A^{\nu}) = \int (e^{-\lambda \rho^{\nu}}) \wedge F_{\mu\nu}^{\dagger}(A^{\nu})$$

$$\frac{\delta S^{\mu}}{\delta \rho_a^{\nu}} = \xi^{\alpha\beta\gamma\delta} \rho_b^{\nu} F_{\gamma\delta\alpha\nu}^{\dagger} = 0 = \xi^{\mu\nu} \rho_b^{\nu} (F_{\mu\alpha}^{\dagger})$$

$$S^{(1)} = \int \Sigma^{\mu\nu} \Lambda F^{\mu\nu}(\Lambda) = \int (e^{-\Lambda \partial^\nu}) \Lambda F^{\mu\nu}(x)$$

$$\frac{\delta S^R}{\delta \rho_a^\mu} = \epsilon^{abcd} \rho_b^\nu F_{cd}^{\mu\nu} = 0 = \epsilon^{abcd} \rho_b^\nu (F_{cd}^{\mu\nu})$$



$$S^{(11)} = \int \sum^{\mu\nu} A F^{\mu\nu}(A^{\nu}) = \int (e^{-\lambda\phi^{\nu}}) A F^{\mu\nu}(r)$$

$$\frac{\delta S^e}{\delta \phi_a^{\nu}} = \sum^{\mu\nu} \rho_b^{\nu} F_{cd}^{\mu\nu} \epsilon_{abcd} = 0 = \sum^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\mu\nu} + F_{cd}^{\mu\nu}) \epsilon_{abcd}$$

$$S^{(1)} = \int \Sigma^{\mu\nu} \omega_{\mu\nu} F^{\dagger}(A^{\nu}) = \int (\rho^{\mu\nu} \omega_{\mu\nu}) A F^{\dagger}_{\mu\nu}(r)$$

$$\frac{\delta S^{(1)}}{\delta \omega_a^{\mu}} = \Sigma^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} \epsilon_{abcd} = 0 = \Sigma^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\text{ext}} + F_{cd}^{\text{int}}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta \omega^{\mu}}$$



$$S^{(1)} = \int \xi^{\mu\nu} \Lambda F^{\dagger}(A^{\nu}) = \int (\rho^{\mu\nu} \Lambda) \Lambda F^{\dagger}_{\mu\nu}(x)$$

$$\frac{\delta S^{(1)}}{\delta \rho_b^{\mu\nu}} = \xi^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} \xi_{\mu\nu} = 0 = \xi^{\mu\nu} \rho_b^{\nu} (F_{\mu\nu}^{\text{cl}} + F_{\mu\nu}^{\text{app}}) \xi_{\mu\nu}$$

$$\frac{\delta S}{\delta A^{\dagger}}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_A(\theta \wedge \rho) = 0$$

⇒

$$S^{(1)} = \int \sum^t \omega \wedge F^t(A^t) = \int (e^{-\lambda \rho^{\nu}}) \wedge F_{\mu\nu}^t(x)$$

$$\frac{\delta S^{(1)}}{\delta \rho_a^{\nu}} = \sum^{\mu \nu} \rho_b^{\nu} F_{\mu\nu}^{+10} = 0 = \sum^{\mu \nu} \rho_b^{\nu} (F_{\mu\nu}^{+10} + F_{\mu\nu}^{+10}) \epsilon_{\mu\nu\sigma\rho}$$

$$\frac{\delta S}{\delta A^{\mu}}$$

$$\frac{\delta S}{\delta \omega} = D(\epsilon \eta e) = 0$$

$$\Rightarrow \omega = \omega(-\rho, \rho)$$

Christoffel

$$S^{(1)} = \int \xi^{\mu\nu} \wedge F^{\dagger}(A^{\nu}) = \int (\xi^{\mu\nu} \wedge F^{\dagger}_{\nu\mu}(A^{\nu}))$$

$$\frac{\delta S^{(1)}}{\delta \xi^{\mu\nu}} = \xi^{\alpha\beta\gamma} \ell_{\beta}^{\nu} F_{\gamma\alpha}^{\dagger} = 0 = \xi^{\alpha\beta\gamma} \ell_{\beta}^{\nu} (F_{\gamma\alpha}^{\dagger} + F_{\alpha\gamma}^{\dagger}) \xi_{\alpha\beta\gamma}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}^{\dagger}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}(\omega \wedge \ell) = 0$$

$\Rightarrow \omega = \omega(\rho, \tau, \sigma)$
 Christoffel

$$S^{(1)} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(A^{\nu}) = \int (e^{-\Lambda} \rho^{\nu}) \wedge F^{\dagger}_{ar}(r)$$

$$\frac{\delta S^{(1)}}{\delta \rho_a^{\nu}} = \Sigma^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} \epsilon_{abcd} = 0 = \Sigma^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\dagger} + F_{cd}^{\text{ext}}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta A^{\dagger}} = D_{\mu}^{\dagger} (\rho \wedge \rho)^{\dagger} = 0$$

$$\frac{\delta S}{\delta \omega} = D_{\mu}(\rho \wedge \rho) = 0$$

$\Rightarrow \omega = \omega(\rho, \rho)$
 Christoffel

$$S^{(1)} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(\Lambda^{\mu\nu}) = \int (e^{-\Lambda \rho^{\nu}}) \wedge F^{\dagger}_{\mu\nu}(x)$$

$$\frac{\delta S^{(1)}}{\delta \rho_a^{\nu}} = \Sigma^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} \epsilon_{abcd} = 0 = \Sigma^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\dagger} + F_{cd}^{\dagger}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_a^{\dagger} (\rho \wedge \rho)^{\dagger} = 0 \Rightarrow \mathcal{A}^{\dagger} = (\omega \gamma)^{\dagger}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}(\rho \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\rho, \gamma)$$

Christoffel

$$S^{III} = \int \epsilon^{abcd} \wedge F^+(A^c) = \int (e^a \wedge e^b) \wedge F_{cd}^+(v)$$

$$\frac{\delta S^{III}}{\delta e_a^\mu} = \epsilon^{abcd} e_b^\nu F_{cd}^+ = 0 = \epsilon^{abcd} e_b^\nu (F_{cd}^{ab} + F_{cd}^{ba}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta A^{\mu\nu}} \mathcal{D}_\mu (e^\nu)^\dagger = 0 \Rightarrow (A^\dagger)^\dagger = (\omega^\dagger)^\dagger \rightarrow 0 =$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_\mu (e^\mu) = 0$$

$$\Rightarrow \omega = \omega(e, \partial e)$$

Christoffel

$$S^{(11)} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(A^{\nu}) = \int (e^{-\lambda \rho \nu}) \wedge F_{\rho\nu}^{\dagger}(A^{\nu})$$

$$\frac{\delta S^{(11)}}{\delta \omega^{\mu}} = \epsilon^{\mu\nu\lambda\rho} \rho_{\nu}^{\lambda} F_{\rho\sigma}^{\dagger} \epsilon_{\lambda\mu\sigma\rho} = 0 = \epsilon^{\mu\nu\lambda\rho} \rho_{\nu}^{\lambda} (F_{\rho\sigma}^{\dagger} + F_{\sigma\rho}^{\dagger}) \epsilon_{\lambda\mu\sigma\rho}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_{\lambda}^{\dagger}(\omega \wedge \rho)^{\dagger} = 0 \Rightarrow (A^{\dagger})^{\dagger} = (\omega \wedge \rho)^{\dagger} \rightarrow 0 = \epsilon^{\mu\nu\lambda\rho} \rho_{\nu}^{\lambda} (F_{\rho\sigma}^{\dagger})^{\dagger}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_{\lambda}(\omega \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\rho, \rho)$$

Christoffel

$$S^{III} = \int \xi^{\mu\nu} \Lambda F^{\dagger}(A^{\nu}) = \int (e^{-\Lambda \rho^{\nu}}) \Lambda F^{\dagger}_{\mu\nu}(\nu)$$

$$\frac{\delta S^{III}}{\delta \rho_a^{\nu}} = \xi^{\mu\alpha} \rho_b^{\nu} F_{cd}^{\dagger} \xi_{\mu\nu} = 0 = \xi^{\mu\alpha} \rho_b^{\nu} (F_{cd}^{\text{cl}} + F_{cd}^{\text{qp}}) \xi_{\mu\nu}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_a^{\dagger}(\rho \wedge \rho)^{\dagger} = 0 \Rightarrow \mathcal{A}^{\dagger} = (\omega \rho)^{\dagger} \rightarrow 0 = \xi^{\mu\alpha} \rho_b^{\nu} [F_{cd}^{\text{cl}}(\rho) + F^{\dagger}(\omega)] \xi_{\mu\nu}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_a(\rho \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\rho, \rho)$$

Chern-Simons

$$S^{III} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(A^{\nu}) = \int (e^{-\lambda \rho \nu}) \wedge F_{\mu\nu}^{\dagger}(A^{\nu})$$

$$\frac{\delta S^{III}}{\delta \omega^{\alpha}} = \Sigma^{\alpha\beta\gamma} \mathcal{L}_{\beta}^{\nu} F_{\gamma\mu}^{\dagger} = 0 = \Sigma^{\alpha\beta\gamma} \mathcal{L}_{\beta}^{\nu} (F_{\gamma\mu}^{\dagger} + F_{\gamma\mu}^{\text{ext}}) \Sigma_{\alpha\nu\rho\sigma}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_{\nu}^{\dagger} (\partial_{\lambda} \rho^{\nu}) = 0 \Rightarrow A^{\dagger} = (\omega \rho)^{\dagger} \quad \rightarrow \quad 0 = \Sigma^{\alpha\beta\gamma} \mathcal{L}_{\beta}^{\nu} (F_{\gamma\mu}^{\dagger} + F_{\gamma\mu}^{\text{ext}}) \Sigma_{\alpha\nu\rho\sigma}$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_{\nu} (\partial_{\lambda} \rho^{\nu}) = 0$$

$$\Rightarrow \omega = \omega(\rho)$$

(Christoffel)

$$S^{III} = \int \epsilon^{\mu\nu\alpha\beta} A F^{\dagger}(\Lambda^{\nu}) = \int (\epsilon^{\mu\nu\alpha\beta}) A F^{\dagger}_{\alpha\beta}(\nu)$$

$$\frac{\delta S^{III}}{\delta \omega^{\alpha}} = \epsilon^{\mu\nu\alpha\beta} \partial_{\beta}^{\nu} F_{\mu\nu}^{\dagger} = 0 = \epsilon^{\mu\nu\alpha\beta} \partial_{\beta}^{\nu} (F_{\mu\nu}^{\text{cl}} + F_{\mu\nu}^{\text{qp}}) \epsilon_{\text{aver}}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \underline{D_{\alpha}^{\dagger}(\omega \Lambda^{\alpha})^{\dagger} = 0} \Rightarrow A^{\dagger} = (\omega \Lambda^{\alpha})^{\dagger} \rightarrow 0 = \epsilon^{\mu\nu\alpha\beta} \partial_{\beta}^{\nu} (F_{\mu\nu}^{\text{cl}} + F_{\mu\nu}^{\text{qp}}) \epsilon_{\text{aver}} \rightarrow 0$$

$$\frac{\delta S}{\delta \omega} = D_{\alpha}(\omega \Lambda^{\alpha}) = 0$$

$$\Rightarrow \omega = \omega(\epsilon, \eta, \rho)$$

Christoffel

$$S^{III} = \int \Sigma^{\mu\nu} \Lambda F^{\dagger}(\Lambda^{\dagger}) = \int (e^{-\Lambda \theta^{\nu}}) \Lambda F^{\dagger}_{\mu\nu}(r)$$

$$\frac{\delta S^{III}}{\delta \theta_a^{\nu}} = \Sigma^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} \epsilon_{abcd} = 0 = \Sigma^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\mu\nu} + F_{cd}^{\nu\mu}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta \Lambda^{\dagger}} = D_{\Lambda}^{\dagger}(\theta \wedge \theta)^{\dagger} = 0 \Rightarrow \Lambda^{\dagger} = (\omega \wedge \theta)^{\dagger} \rightarrow 0 = \Sigma^{\mu\nu} \rho_b^{\nu} (F_{cd}^{\mu\nu} + F_{cd}^{\nu\mu}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta \Lambda^{\dagger}}$$

$$\frac{\delta S}{\delta \omega} = D_{\Lambda}(\theta \wedge \theta) = 0$$

$$\Rightarrow \omega = \omega(\theta, \theta)$$

Chern-Simons

$$S^{III} = \int \xi^{\mu\nu} \wedge F^{\dagger}(A^{\mu}) = \int (e^{-\lambda \sigma^{\nu}}) \wedge F^{\dagger}_{\mu\nu}(r)$$

$$\frac{\delta S^{III}}{\delta \sigma_a^{\nu}} = \xi^{\mu\nu} \rho_b^{\nu} F_{cd}^{\dagger} = 0 = \xi^{\mu\nu} \rho_b^{\nu} (F_{cd}^{ab} + F_{cd}^{cb}) \xi_{\mu\nu} \sigma^{\rho}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_a^{\dagger}(\sigma \wedge \rho)^{\dagger} = 0 \Rightarrow (A^{\dagger})^{\dagger} = (\omega \wedge \rho)^{\dagger} \rightarrow 0 = \xi^{\mu\nu} \rho_b^{\nu} (F_{cd}^{ab} + F_{cd}^{cb}) \xi_{\mu\nu} \sigma^{\rho} \quad \text{Re}[\sigma] = 0$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}(\sigma \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\rho, \sigma)$$

(Christoffel)

$$S^{III} = \int \Sigma^{\mu\nu} \wedge F^{\dagger}(\Lambda^{\mu}) = \int (e^{-\Lambda} \wedge \Sigma^{\mu\nu}) \wedge F^{\dagger}_{\mu\nu}(\Lambda)$$

$$\frac{\delta S^{III}}{\delta \Lambda^{\mu}} = \Sigma^{\mu\nu} \wedge \Sigma^{\nu\rho} F^{\dagger}_{\rho\sigma} = 0 = \Sigma^{\mu\nu} \wedge \Sigma^{\nu\rho} (F^{\dagger}_{\rho\sigma} + F^{\dagger}_{\sigma\rho}) \wedge \Sigma^{\rho\sigma}$$

$$\frac{\delta S}{\delta \Lambda^{\dagger}} \quad \mathcal{D}_{\mu}^{\dagger}(\partial \wedge \rho)^{\dagger} = 0 \Rightarrow \Lambda^{\dagger} = (\omega \wedge \rho)^{\dagger} \rightarrow 0 = \Sigma^{\mu\nu} \wedge \Sigma^{\nu\rho} (F^{\dagger}_{\rho\sigma} + F^{\dagger}_{\sigma\rho}) \wedge \Sigma^{\rho\sigma}$$

$R_{\mu\nu} \wedge \Sigma^{\rho\sigma} = 0$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_{\mu}(\partial \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\rho, \rho)$$

Christoffel

$$\vec{A}^{uv} = \omega^{uv} \pm \omega^{*uv} \quad A^{23} = -A^{32} \quad A^i = A^{0i} \quad i = 1, 2, 3$$

$$F^{\pm uv} = F^{+uv} + F^{-uv} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+uv} = dA^{+uv} + A^{+u\lambda} \wedge A^{+v\lambda}$$

$$F^{+i} = dA^i + \frac{1}{2} \epsilon^{ijk} A^j \wedge A^k$$

$$\Sigma^{uv} = e^u \wedge e^v = -\Sigma^{vu}$$

$$\Sigma^{uv} = \Sigma^{+uv} + \Sigma^{-uv}$$

$$S^{Pl.} = \int \Sigma^{uv} \wedge F_{uv} = \int \left[\Sigma^{+uv} \wedge F_{uv}^{+} + \Sigma^{-uv} \wedge F_{uv}^{-} \right]$$

$$S^{III} = \int \xi^{\mu\nu} \wedge F^{\dagger}(\Lambda^{\mu}) = \int (\xi^{\mu\nu} \wedge F^{\dagger}_{\mu\nu}(r))$$

$$\frac{\delta S^{III}}{\delta \xi^{\mu\nu}} = \xi^{\alpha\beta\gamma} \rho_b^{\nu} F_{cd}^{\dagger} = 0 = \xi^{\alpha\beta\gamma} \rho_b^{\nu} (F_{cd}^{\text{cl}} + F_{cd}^{\text{qp}}) \epsilon_{\alpha\beta\gamma}$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_{\mu}^{\dagger}(\theta \wedge \rho) = 0 \Rightarrow \mathcal{A}^{\dagger} = (\omega \wedge \rho)^{\dagger} \rightarrow 0 = \xi^{\alpha\beta\gamma} \rho_b^{\nu} \left[F_{cd}^{\text{cl}}(\theta) + F_{cd}^{\text{qp}}(\omega) \right] \epsilon_{\alpha\beta\gamma}$$

$\text{Re}[\omega] = 0$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_{\mu}(\theta \wedge \rho) = 0$$

$$\Rightarrow \omega = \omega(\theta, \rho)$$

(Christoffel)

$$S^{(11)} = \int \xi^{\dagger(\alpha)} \Lambda F^{\dagger}(\Lambda^{\alpha}) = \int (\xi^{\dagger \alpha} \rho_b^{\nu}) \Lambda F_{\alpha\nu}^{\dagger}(\rho)$$

check

$$\frac{\delta S^{(11)}}{\delta \rho_a^{\nu}} = \xi^{\dagger \alpha} \rho_b^{\nu} F_{\alpha\nu}^{\dagger} = 0 = \xi^{\dagger \alpha} \rho_b^{\nu} (F_{\alpha\nu}^{\text{cl}} + F_{\alpha\nu}^{\text{qp}})$$

$$\frac{\delta S}{\delta A^{\dagger}} \quad \mathcal{D}_n^{\dagger}(\rho_n \rho) = 0 \Rightarrow \mathcal{A}^{\dagger} = (\omega \rho)^{\dagger} \rightarrow 0 = \xi^{\dagger \alpha} \rho_b^{\nu} \left[F_{\alpha\nu}^{\text{cl}}(\rho) + F_{\alpha\nu}^{\text{qp}}(\omega) \right] \xi_{\text{qp}} \quad \text{Re}[\omega] = 0$$

$$\frac{\delta S}{\delta \omega} = \mathcal{D}_n(\rho_n \rho) = 0 \Rightarrow \omega = \omega(\rho, \rho^{\dagger})$$

Christoffel

$$S^{(1)} = \int \sum^{\mu\nu} \Lambda F^{\mu\nu} = \int (e^{-\Lambda} \eta^{\mu\nu}) \Lambda F_{\mu\nu}^{(1)}$$

check

$$\frac{\delta S^{(1)}}{\delta \omega_a^\mu} = \epsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho}^{+\mu} = 0 = \sum^{\mu\nu} \partial_\nu (F_{\lambda\rho}^{+\mu} + F_{\lambda\rho}^{+\nu\rho}) \epsilon_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta A^{\mu\nu}} \quad D_\lambda (\partial_\mu \omega^\nu) = 0 \Rightarrow \omega^\mu = (\omega^\nu)^\dagger \rightarrow 0 = \epsilon^{\mu\nu\lambda\rho} \partial_\nu (F_{\lambda\rho}^{+\mu} + F_{\lambda\rho}^{+\nu\rho}) \epsilon_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta \omega} = D_\lambda (\partial_\mu \omega^\nu) = 0$$

$$\Rightarrow \omega = \omega(\sigma, \tau, \rho)$$

Christoffel

$$\epsilon^{\lambda\mu\nu\rho} \partial_\nu \omega^\sigma =$$



$$S^{(1)} = \int \sum_{\alpha\beta} \omega_{\alpha\beta} F^{\alpha\beta}(A) = \int (\epsilon^{\alpha\beta\gamma\delta} A_{\alpha\beta}^{\gamma\delta}) F_{\alpha\beta}^{\gamma\delta}(A)$$

check

$$\frac{\delta S^{(1)}}{\delta A^{\alpha\beta}} = \epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} F_{\delta\alpha\beta}^{\gamma\delta} = 0 = \epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} (F_{\delta\alpha\beta}^{\gamma\delta} + F_{\delta\alpha\beta}^{\alpha\beta}) \epsilon_{\alpha\beta\gamma\delta}$$

$$\frac{\delta S}{\delta A^{\alpha\beta}} = D_{\alpha} (\partial_{\beta} A^{\alpha}) = 0 \Rightarrow \omega_{\alpha\beta} = (\omega_{\alpha\beta})^{\alpha\beta} \rightarrow 0 = \epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} (F_{\delta\alpha\beta}^{\gamma\delta} + F_{\delta\alpha\beta}^{\alpha\beta}) \epsilon_{\alpha\beta\gamma\delta}$$

$$\frac{\delta S}{\delta \omega} = D_{\alpha} (\partial_{\beta} A^{\alpha}) = 0 \Rightarrow \omega = \omega(\rho, \gamma, \delta)$$

Christoffel

$$\epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} \epsilon_{\alpha\beta\delta\gamma} = \epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} \epsilon_{\alpha\beta\delta\gamma} = \epsilon^{\alpha\beta\gamma\delta} \partial_{\gamma} \epsilon_{\alpha\beta\delta\gamma}$$

$$\vec{A}^{uv} = \omega^{uv} \pm \omega^{*uv} \quad A^{23} = -A^{01} \quad A^i = A^{i2} \quad i=1,2,3$$

$$F_{uv} = F^{+uv} + F^{-uv} = \frac{1}{2}(F + F^*) + \frac{1}{2}(F - F^*)$$

$$F^{+uv} = dA^{+uv} + \frac{1}{2} \epsilon^{\lambda\mu\nu} A^{+uv} A^{\lambda\mu} A^{\nu\lambda}$$

$$\Sigma^{uv} = \epsilon^{\mu\lambda} \epsilon^{\nu\lambda} = -\epsilon^{\nu\mu} \quad \Sigma^{uv} = \Sigma^{+uv} + \Sigma^{-uv}$$

$$S^{Pl.} = \int \Sigma^{uv} \wedge F_{uv} = \int \left[\Sigma^{+uv} \wedge F_{uv}^{+} + \Sigma^{-uv} \wedge F_{uv}^{-} \right]$$

$$= \int \left[\Sigma^{+uv} \wedge F_{uv}^{+} - \Sigma^{-uv} \wedge F_{uv}^{-} \right] + 2 \int \left[\Sigma^{+uv} \wedge \Sigma^{-uv} - \Sigma^{-uv} \wedge \Sigma^{+uv} \right]$$

$$S^{(1)} = \int \sum^{\mu\nu} \Lambda F^{\mu\nu} = \int (e^{-\Lambda} \delta^{\mu\nu} / \Lambda F^{\mu\nu})$$

check

$$\frac{\delta S^{(1)}}{\delta \omega^a} = \epsilon^{abcd} \ell_b^\nu F_{cd}^{\mu\nu} = 0 = \epsilon^{abcd} \ell_b^\nu (F_{cd}^{\mu\nu} + F_{cd}^{\mu\nu}) \epsilon_{\mu\nu\rho\sigma}$$

$$\frac{\delta S}{\delta \Lambda^{\mu\nu}} \quad D_\mu (\partial_\nu \Lambda) = 0 \Rightarrow \Lambda^{\mu\nu} = (\omega^{\mu\nu})^+$$

$$\epsilon^{\mu\nu\rho\sigma} \partial_\mu \partial_\nu \Lambda^{\rho\sigma} = \sum^{\mu\nu} \Lambda^{\mu\nu} - \sum^{\mu\nu} \Lambda^{\mu\nu}$$

$$\frac{\delta S}{\delta \omega} = D_\mu (\partial_\nu \Lambda) = 0 \Rightarrow \omega = \omega(\rho, \sigma) \text{ Christoffel}$$

$$S^+ = \int \Sigma_{ij}^+ F^{ij}$$

$$S^+ = \int \sum_{\lambda}^+ F^{\lambda} - 2\lambda \sum_{\lambda}^+ \lambda S^{\lambda}$$

$$S^+ = \int \sum_{\mathbf{l}}^+ F^+ L - 2\Lambda \sum_{\mathbf{x}}^+ \Lambda S^+$$

$$F = \Lambda \Sigma^+ \partial \Lambda \Sigma^+ \dots$$
$$BF$$

$$S^+ = \int \sum_L^+ F^+ L - 2 \sum_X^+ \Lambda S^+ \Lambda$$

$$F = \Lambda \Sigma^+ \partial \Lambda \Sigma^+ \quad \text{and} \quad \text{sol 2) } BF \quad B^i = \Sigma^+ \Lambda$$

$$S^+ = \int \sum_L^+ F + L - 2\Lambda \sum_x^+ \Lambda S^+$$

BF-Topological S^+ is independent

$$F = \Lambda S^+ \quad \partial \Lambda S^+ = 0$$

SU(2) BF $B^i = \epsilon^{ijk} S^j$

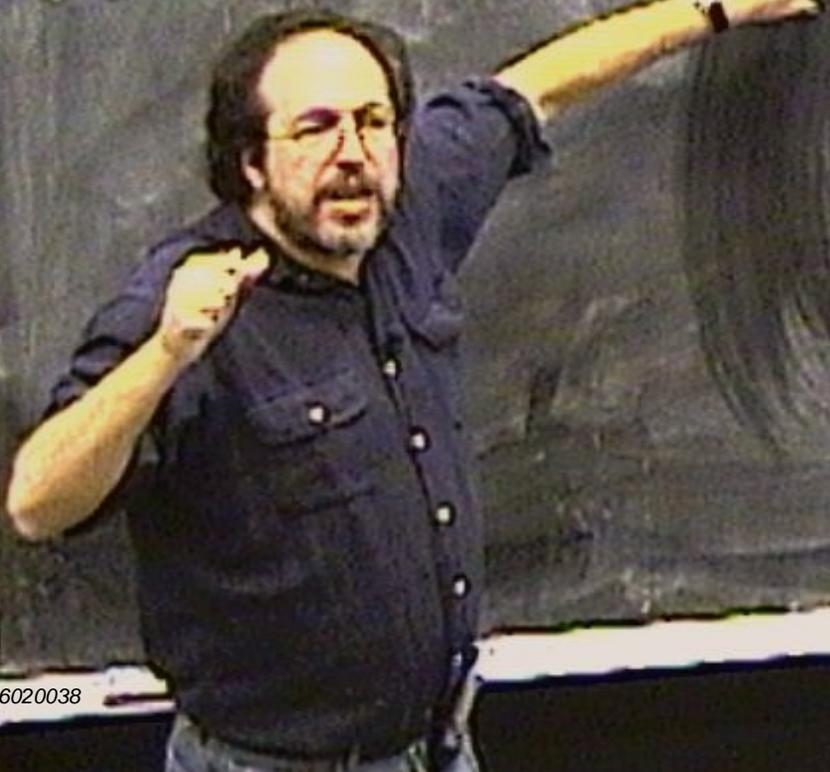
$$S^+ = \int \Sigma^+ \wedge F - 2\Lambda \Sigma^+ \wedge A \Sigma^+$$

BF-Topological Σ^+ is 2+1 dimensional manifold

GR $\Sigma^+ = \mathbb{R}^0 \wedge \mathbb{R}^1 + \mathbb{R}^2 \wedge \mathbb{R}^3$

$$F = A \Sigma^+ \quad D_A \Sigma^+ = 0$$

SV(2) BF $B^2 = \Sigma^+$



$$S^+ = \int \Sigma_L^+ F + L - 2\Lambda \Sigma_L^+ \Lambda \Sigma^+$$

BF-Topological

Σ^+ is 24 independent dof

SU(2) BF

$B^i = \Sigma^{L+}$

GR

$$\Sigma^+ = \rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3 \quad 16 \text{ dof}$$

$$F = \Lambda \Sigma^+ \quad \partial \wedge \Sigma^+ \quad \dots$$

$$S^+ = \int \Sigma_{\mu\nu}^+ F^{\mu\nu} - 2\Lambda \Sigma_{\mu\nu}^+ \Sigma^{\mu\nu}$$

BF-Topological

Σ^+ is 24 independent dof

5(12) BF

$$F = \Lambda \Sigma^+ \quad D\Lambda \Sigma^+ = 0$$

GR

$$\Sigma^+ = \varrho^0 \wedge \varrho^1 + \varrho^2 \wedge \varrho^3 \quad 16 \text{ dof}$$

16 dof

$$\Rightarrow \varrho \wedge F = \dots$$

$$S^+ = \int \Sigma^+ F + i - 2\Lambda \Sigma^+ \Lambda \Sigma^+$$

BF-Topological

GR

Σ^+ is 24 independent dof

$$\Sigma^+ = \rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3 \quad 16 \text{ dof}$$

$$F = \Lambda \Sigma^+ \quad D\Lambda \Sigma^+ = 0$$

SU(2) BF $B^i = \Sigma^{i+}$

$$\Rightarrow \rho \wedge \dots$$

$$S^+ = \int \Sigma_{\Sigma}^+ F + L - 2\Lambda \Sigma_{\Sigma}^+ \Lambda \Sigma^+$$

BF-Topological

Σ^+ is 24 independent dof

SU(2) BF

$$B^i = \Sigma^{\Sigma^+}$$

GR

$$\Sigma^+ = \rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3 \quad 16 \text{ dof}$$

16 dof \Rightarrow

$$\rho \wedge F = \Lambda \rho \wedge \rho$$



$$S^+ = \int \Sigma^+ \wedge F + i - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

BF-Topological

GR

Σ^+ is 24 independent dof

$$\Sigma^+ = \rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3 \quad 16 \text{ dof}$$

$F = \Lambda \Sigma^+ \cdot \partial \wedge \Sigma^+$

5/12) BF $B^i = \Sigma^{i+}$

$$\Rightarrow \rho \wedge F = \Lambda \rho \wedge \rho$$

$$S^+ = \int \Sigma^+ \wedge F + i - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

BF-Topological

GR

Σ^+ is 24 independent dof

$$\Sigma^+ = \rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3 \quad 16 \text{ dof}$$

$F = \Lambda \Sigma^+ \dots$

$SU(2) \text{ BF} \quad B^i = \Sigma^{i+}$

$\Rightarrow \rho^i \wedge F = \Lambda \rho^i \rho^j$

$$\Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$$S^+ = \int \Sigma^+ \wedge F + L - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

BF-Topological Σ^+ is 24 independent dof

$$F = \Lambda \Sigma^+ \quad \partial \wedge \Sigma^+ = 0$$

GR $\Sigma^+ = (\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3)$ 16 dof

$$\Rightarrow \rho^i \wedge F = \Lambda \rho^i \rho^j$$

$$\Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$$S^+ = \int \Sigma_1^+ F + L - 2\Lambda \Sigma_1^+ \Lambda \Sigma^+$$

BF-Topological Σ^+ is 18 independent dof

$$F = \Lambda \Sigma^+ \quad \text{D'Alembert} \quad \text{10 dof}$$

GR $\Sigma^+ = (\Omega^0 \Lambda \Omega^1 + \Omega^2 \Lambda \Omega^3)$ 16 dof

$$\Rightarrow \Omega^1 \Lambda F = \text{No dof}$$

$$\Sigma^{\mu\nu} \Lambda \Sigma^{\gamma\delta} - \frac{1}{3} \delta^{\mu\gamma} \Sigma^{\nu\delta} = 0$$

$$S^+ = \int \Sigma_{11}^+ F^{\mu\nu} \tilde{F}_{\mu\nu} - 2A \Sigma_{11}^+ \tilde{F}^{\mu\nu}$$

$$F = A \tilde{F} \quad \text{and} \quad \tilde{F} = -F$$

BF-Topological

GR

$$\tilde{F}^{\mu\nu} = (R^{\mu\nu} - R A^{\mu\nu})$$

$$\Rightarrow R A F = M_{11}^+$$

$$S^+ = \int \Sigma_{11}^+ A \tilde{F}^{\mu\nu} - \frac{1}{3} S^+ \Sigma_{11}^+ A \tilde{F}^{\mu\nu} = 0 \quad (*)$$

$$S^+ = \int \Sigma^+ \wedge F + i - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

$$F = \Lambda \Sigma^+ \quad \partial \wedge \Sigma^+ = 0$$

BF-Topological

Σ^+ is independent of $S(U(2))$ BF

$$B^i = \Sigma^+$$

GR

$$\Sigma^+ = (e^0 \wedge e^1 + e^2 \wedge e^3) \text{ is dot}$$

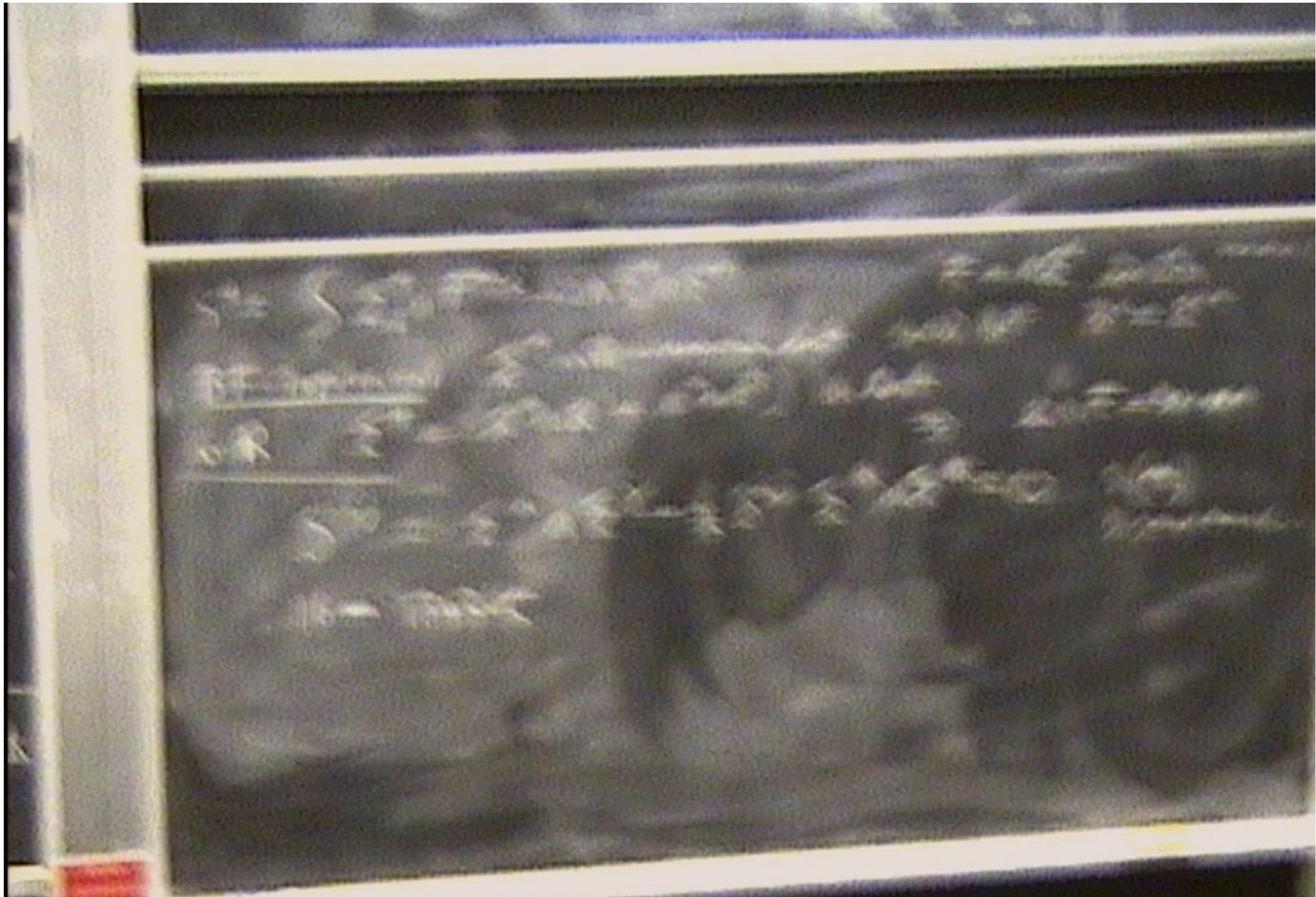
$$\Rightarrow e^i \wedge F = \text{trace}$$

$$S^{ij} = \epsilon^{ij} \wedge \Sigma^k - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0 \quad S(\mathcal{A})$$

$$S^+ = \int \Sigma^+ \wedge F + L - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

BF-Topologie Σ^+ ist 181... $F = \Lambda \Sigma^+ \cdot \partial \Lambda \Sigma^+$
 GR $\Sigma^+ = (\varrho^0 \wedge \varrho^1 + \varrho^2 \wedge \varrho^3)$ 16 dof $\Rightarrow \varrho^i \wedge F = \text{Nur 11}$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0 \quad S(\pi) \quad 3 \text{ gauge transf.}$$



$$S^+ = \int \Sigma_{\perp}^+ F + L - 2\Lambda \Sigma_{\perp}^+ \Lambda S^+$$

BF-Topological Σ^+ is independent of S^+ (12) $B^+ F$ $B^+ = \Sigma^+$
 GR $\Sigma^+ = (Q^0 \Lambda Q^1 + Q^2 \Lambda Q^3)$ 16 dof $\Rightarrow Q^1 \Lambda F = \Lambda Q^1 Q^0$

$$S^{ij} = \Sigma^{\perp} \Lambda \Sigma^{ij} - \frac{1}{3} \delta^{ij} \Sigma^{\perp} \Lambda \Sigma^{\perp} = 0 \quad S(\pi) \quad S \text{ gauge trans...}$$

$$16 \leftarrow 1843 - 5$$

$$S^+ = \int \Sigma^+ \wedge F^+ - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

$$F = \Lambda \Sigma^+ \quad D\Lambda \Sigma^+ \quad \dots$$

BF-Topological

Σ^+ is 18 independent dof

$$S(U(2)) \text{ BF} \quad B^+ = \Sigma^+$$

GR

$$\Sigma^+ = (\mathcal{Q}^0 \wedge \mathcal{Q}^1 + \mathcal{Q}^2 \wedge \mathcal{Q}^3) \quad 16 \text{ dof}$$

$$\Rightarrow \mathcal{Q} \wedge F = \text{No dof}$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0 \quad S(\mathcal{U})$$

3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\Sigma^i \wedge \Sigma^j \quad \Sigma^k \wedge \Sigma^l$$



$$S^+ = \int \Sigma_{i1}^+ F^{i1} - 2\Lambda \Sigma_{\lambda}^+ \Lambda \Sigma^{\lambda+}$$

$$F = \Lambda \Sigma^i \partial \Lambda \Sigma^0$$

$$\text{SU(2) BF} \quad B^i = \Sigma^{i+}$$

BF-Topological Σ^+ is 18 independent dof

GR $\Sigma^+ = (\Omega^0 \wedge \Omega^1 + \Omega^2 \wedge \Omega^3)$ 16 dof

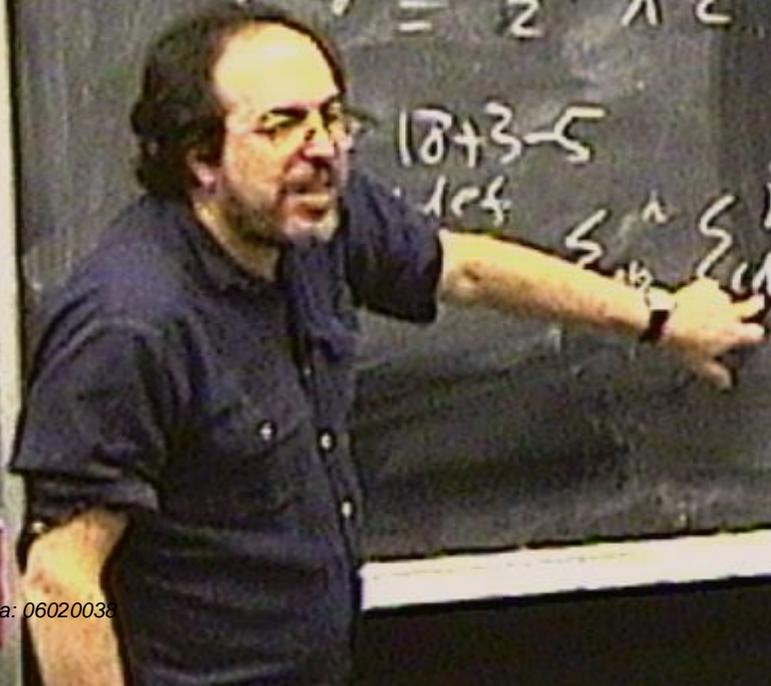
$$\Rightarrow \Omega^i \wedge F = \Lambda \Omega^i \wedge \Omega^j$$

$$\epsilon^{ij} = \Sigma^{\lambda} \wedge \Sigma^{\gamma} - \frac{1}{3} \delta^{ij} \Sigma^{\kappa} \wedge \Sigma^{\kappa} = 0$$

5(m)
3 gauge transf...

18+3-5

$$\sum_{i1} \sum_{i2} \sum_{i3} \sum_{i4}$$



$$S^+ = \int \Sigma_I^+ F + L - 2\Lambda \Sigma_I^+ \Lambda S^+$$

$$F = \Lambda \Sigma^i \partial \Lambda \Sigma^j$$

BF-Topological

Σ^+ is 18 independent dof

sub) BF $B^i = \Sigma^{Li}$

GR

$$\Sigma^+ = (1/2)(\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3)$$

$$\Rightarrow \rho \wedge F = \Lambda g_{ij} dx^i dx^j$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$S(\mathcal{M})$
3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{ac} = \Sigma^{bdef} \Sigma^a \wedge \Sigma^b \wedge \Sigma^c \wedge \Sigma^d \wedge \Sigma^e \wedge \Sigma^f$$

$$S^+ = \int \Sigma_L^+ F^+ L - 2\Lambda \Sigma_L^+ \Lambda S^+$$

$$F = \Lambda \Sigma^+ \partial \Lambda \Sigma^+$$

BF-Topological

Σ^+ is 18 independent dof
 $\Sigma^+ = (1/2)(Q^0 \Lambda Q^0 + Q^i \Lambda Q^i)$ 16 dof

sub) BF $B^i = \Sigma^{Li}$

GR

$\Rightarrow Q \wedge F = \Lambda Q \wedge Q$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0 \quad S(\mathcal{M})$$

3 gauge fields...

$$16 = 18 + 3 - 5$$

\tilde{h}_{ac}

$= \Sigma^{abcd}$

$$\Sigma^a \wedge \Sigma^b \wedge \Sigma^c \wedge \Sigma^d \Rightarrow \tilde{h}_{ac} = \sqrt{5} g_{ac}$$

$$S^+ = \int \Sigma_{\lambda}^+ F^{\lambda\lambda} - 2\Lambda \Sigma_{\lambda}^+ \Lambda \Sigma^{\lambda+}$$

$$F = \Lambda \Sigma^{\lambda} \quad D\Lambda \Sigma^{\lambda} = 0$$

BF-Topological

Σ^+ is 18 independent dof

$$S(12) \quad BF \quad B^{\lambda} = \Sigma^{\lambda+}$$

GR

$$\Sigma^+ = \pm (\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3) \quad 16 \text{ dof}$$

$$\Rightarrow \rho^{\lambda} \wedge F = 0$$

$$S^{ij} = \Sigma^{\lambda} \wedge \Sigma^{\lambda} - \frac{1}{3} \delta^{ij} \Sigma^{\lambda} \wedge \Sigma^{\lambda} = 0$$

$S(\mathbb{R})$
3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{\mu\nu} = \Sigma^{\lambda} \wedge \Sigma^{\lambda}$$

$$\Sigma_{\lambda}^{\mu} \quad \Sigma_{\lambda}^{\nu} \quad \Sigma_{\lambda}^{\kappa} \quad \Sigma_{\lambda}^{\eta}$$

$$\Rightarrow \tilde{h}_{\mu\nu} = \sqrt{5} g_{\mu\nu}$$

$$S^+ = \int \Sigma_{\lambda}^+ F + L - 2\Lambda \Sigma_{\lambda}^+ \Lambda \Sigma^+$$

$$F = \Lambda \Sigma^+ \partial \Lambda \Sigma^+ \dots$$

BF-Topol

Σ^+ (18) independent dof

$$\text{su}(2) \text{ BF} \quad B_i = \Sigma^+$$

GR

$$T = \frac{1}{2} (\dot{Q}^a \dot{Q}^a + e^{\tau} \dot{e}^{\beta}) \quad 16 \text{ dof}$$

$$\Rightarrow Q \wedge F = \text{same}$$

$$\dot{\Sigma}^i = \Sigma^{\lambda} \wedge \dot{\Sigma}^{\lambda} - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \dot{\Sigma}^k = 0 \quad S(\mathcal{M})$$

3 gauge transf...

$$= 18 + 3 - 5$$

$$= \Sigma^{\text{dof}}$$

$$\Sigma_{ab}^{\lambda} \Sigma_{cd}^{\gamma} \Sigma_{ef}^k \Sigma_{gh}^{\mu} \Rightarrow \hat{h}_{\mu\nu} = \sqrt{5} g_{\mu\nu}$$

$$S^+ = \int \Sigma^+ \wedge F - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

$$F = \Lambda \Sigma^+ \quad D\Lambda \Sigma^+ = 0$$

BF-Topological

Σ^+ is independent dot

$$S(12) BF \quad B^i = \Sigma^{i+}$$

GR $\Sigma^+ = (e^0 \wedge e^1 + e^2 \wedge e^3) \wedge \text{dot}$

$$\Rightarrow \hat{e}^i \wedge (F - \Lambda \Sigma) = 0$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$$16 = 10 + 3 - 5$$

$$\tilde{h}_{4c} = \Sigma^{abcd} \quad \Sigma^a \wedge \Sigma^b \wedge \Sigma^c \wedge \Sigma^d \Rightarrow \tilde{h}_{4c} = \sqrt{5} \dots$$

$$S^+ = \int \Sigma_L^+ F + L - 2\Lambda \Sigma_L^+ \Lambda \Sigma^+$$

$$F = \Lambda \Sigma^i \partial_\mu \Sigma^{\mu i}$$

BF-Topological

Σ^+ is 18 independent dof

$$\text{su}(2) \text{ BF} \quad B^i = \Sigma^{Li}$$

GR

$$\Sigma^+ = (1/2 \epsilon^{0123} + \epsilon^{2130}) \text{ 16 dof}$$

$$\Rightarrow \epsilon^i \Lambda (E - \Lambda \Sigma_i) = 0$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$S(\mathcal{M})$
3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{ac} = \epsilon^{abcd} \Sigma^a \wedge \Sigma^b \wedge \Sigma^c \wedge \Sigma^d \Rightarrow \tilde{h}_{ac} = \sqrt{5} g_{ab}$$

$$S^+ = \int \Sigma^+ \wedge F + L - 2\Lambda \Sigma^+ \wedge \Sigma^+$$

$$F = \Lambda \Sigma^i \partial \wedge \Sigma^j$$

$$B^i = \Sigma^{i+}$$

BF-Topological

Σ^+ is 18 independent dof

su(2) BF

GR $\Sigma^+ = \frac{1}{2}(\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3)$ 16 dof

$$e^i \wedge (F - \Lambda \Sigma^i) = 0$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

$S(\mathbb{R}^4)$
3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{ac} = \Sigma^{ab} \wedge \Sigma^{cd} \wedge \Sigma^{ef} \Rightarrow \tilde{h}_{ac} = \sqrt{g} g_{ac}$$

$$\Sigma^i = F^i$$

$$S^+ = \int \Sigma_i^+ \wedge F^{+i} - 2\Lambda \Sigma_i^+ \wedge \Sigma^{i+}$$

$$F = \Lambda \Sigma^i \cdot \partial \wedge \Sigma^j$$

BF-Topological

Σ^+ is 18 independent dof

$$\text{SU}(2) \text{ BF } B^i = \Sigma^{i+}$$

GR

$$\Sigma^+ = (\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3) \text{ 16 dof}$$

$$\rho^i \wedge (E - \Lambda \Sigma_i) = 0$$

$$S^{ij} = \Sigma^i \wedge \Sigma^j - \frac{1}{3} \delta^{ij} \Sigma^k \wedge \Sigma^k = 0$$

5(m)
3 gauge transf...

$$16 = 18 + 3 - 5$$

$\tilde{\rho}_{abc}$

$$= \Sigma^{abcd}$$

$$\Sigma_{ab}^i \Sigma_{cd}^j \Sigma_{ef}^k \Sigma_{gh}^l \Rightarrow$$

$$\tilde{h}_{abc} = \sqrt{3} g_{abc}$$

$$\Sigma^i = F^i$$

$$S^+ = \int \Sigma_{\lambda}^+ F^{\lambda} - 2\Lambda \Sigma_{\lambda}^+ \Lambda S^+$$

$$\delta F = \Lambda \Sigma^{\lambda} \delta \Lambda \Sigma_{\lambda}^+ \dots$$

BF-Topological

Σ^+ is 18 independent dof

su(2) BF $B^i = \Sigma^{\lambda+}$

GR $\Sigma^+ = (1/2)(\rho^0 \rho^1 + \rho^2 \rho^3)$ 16 dof

$$e^{\lambda} \wedge (E - \Lambda \Sigma_{\lambda}) = 0$$

$$\Sigma^{ij} \wedge \Sigma^{\lambda} - \frac{1}{3} \delta^{ij\lambda} \Sigma^{\kappa} \wedge \Sigma^{\kappa} = 0$$

$S(\mathcal{M})$
3 gauge transf...

$$16 = 5$$

$$\tilde{h}_{ac} = \xi$$

$$\sum_{i,j} \Sigma^i \wedge \Sigma^j \Rightarrow \tilde{h}_{ac} = \sqrt{5} g_{ab} \quad \Sigma^i = F^i$$

15

$$S^+ = \int \Sigma_{\mu\nu}^+ F^{\mu\nu} - 2\Lambda \Sigma_{\mu\nu}^+ \wedge \Sigma^{\mu\nu}$$

BF-Topological Σ^+ is 18 independent dof $SU(2)$ BF $B^i = \Sigma^{\mu\nu}$
 GR $\Sigma^+ = (\rho^0 \wedge \rho^1 + \rho^2 \wedge \rho^3)$ 16 dof $\Rightarrow e^{\hat{\alpha}} \wedge (F - \Lambda \Sigma) = 0$

$$S^{ij} = \Sigma^{\mu\nu} \wedge \Sigma^{\rho\sigma} - \frac{1}{3} \delta^{ij} \Sigma^{\kappa\lambda} \wedge \Sigma^{\rho\sigma} = 0 \quad S(\mathcal{M})$$

3 gauge transf...

$$16 = 18 + 3 - 5$$

$$\tilde{h}_{ac} = \Sigma^{bdef} \Sigma_{ab}^{\mu\nu} \Sigma_{cd}^{\rho\sigma} \Sigma_{ef}^{\kappa\lambda} \Sigma_{gh}^{\alpha\beta} \Rightarrow \hat{h}_{ac} = \sqrt{3} g_{ac} \quad \Sigma^i = F^i$$

$$17 \& \Rightarrow \sqrt{3} g_{ab} = F_{ac}^{\mu\nu} F_{bd}^{\rho\sigma} F_{ef}^{\kappa\lambda} \Sigma^{cdef} \Sigma_{ghik}$$

$$178 \Rightarrow \Gamma_{ij} = \Gamma_{ik} \Gamma_{kj} \quad \xi_{ik} \xi_{jk}$$

$$\Phi_{xy} = \Phi_{yx} \quad \downarrow = 0 \Rightarrow \Phi_{xx} = 0$$

$$178 \Rightarrow \Gamma_{ab} = F_{ac} F_{bd} \sum_{c,d} \xi_{cd} \xi_{cd}$$

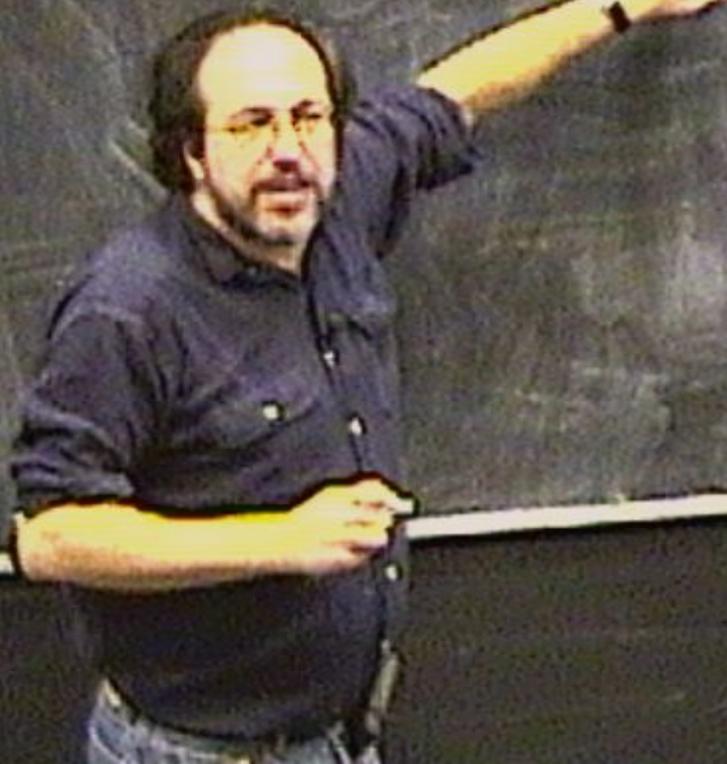
$$\Phi_{xy} = \Phi_{yz} \quad \perp = 0 \Rightarrow \Phi_{xz} = 0$$

$$S = \int \sum_{i,j} \lambda F_{ij} +$$

$$178 \Rightarrow \Gamma_{ij} = F_{ac} F_{bi} F_{cj} \xi^{cd} \xi^{ak}$$

$$\Phi_{xy} = \Phi_{yx} \quad \perp = 0 \Rightarrow \Phi_{xx} = 0$$

$$S = \int \sum^i F_i + \Phi_{xy} \sum^i \xi^i \dot{\xi}^i$$



$$178 \Rightarrow \Gamma g_{ab} = F_{ab} \dot{\Gamma} F_{cd} \dot{\Gamma} F_{ef} \dot{\Gamma} \dots$$

$$\Phi_{xy} = \Phi_{yx} \quad \perp = 0 \Rightarrow \Phi_{xy} = 0$$

$$S = \int \Sigma^i{}_1 F_i + \Phi_{xy} \Sigma^i{}_1 \dot{\Sigma}^j{}_2$$

$$17 \& \Rightarrow \Gamma_{\mu\nu}^{\lambda} = F_{\mu\nu}^{\lambda} \dot{\xi}^{\mu} \dot{\xi}^{\nu} \xi^{\lambda}$$

$$(s.d.s) \Phi_{xy} = \Phi_{ij} \quad \downarrow L=0 \Rightarrow \Phi_{xx} = 0$$

$$S = \int \xi^{\mu} F_{\mu} + \Phi_{xy} \xi^{\mu} \dot{\xi}^{\nu}$$

$$178 \Rightarrow \Gamma g_{ab} = F_{ab}^{\lambda} F_{cd}^{\mu} F_{ef}^{\nu} \epsilon^{abcd} \epsilon^{efgh}$$

$$(s.t.t.) \Phi_{xy} = \Phi_{yx} \quad \downarrow L=0 \Rightarrow \Phi_{xx} = 0$$

$$S = \int \epsilon^{\lambda\mu\nu} F_{\lambda} + \Phi_{xy} \epsilon^{\lambda\mu\nu} \dot{\epsilon}^{\sigma}$$

$$178 \Rightarrow \Gamma_{ij}^k = F_{ij}^k \frac{dx^i}{dt} \frac{dx^j}{dt} \frac{dx^k}{dt}$$

$$(sat) \Phi_{xy} = \Phi_{yx} \quad L=0 \Rightarrow \Phi_{xx} = 0$$

$$S = \int \Sigma^i{}_1 F_i + \Phi_{xy} \Sigma^i{}_1 \dot{\Sigma}^i$$

$$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} = 0 \Rightarrow \exists \lambda \text{ s.t. } \Sigma = (\lambda, \dot{\lambda})$$

$$178 \Rightarrow \Gamma_{ij} = F_{ij}^A F_{ij}^B F_{ij}^C \xi_{ij}^D \xi_{ij}^E$$

$$(s.t.) \Phi_{xy} = \Phi_{yx} \quad \downarrow = 0 \Rightarrow \Phi_{xy} = 0$$

$$S = \int \Sigma^i A F_i + \frac{1}{2} \Phi_{xy} \xi^i \xi^j$$

$$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} \quad \delta \Phi_{xy} = \xi^i \xi^j$$

$$\frac{\delta S}{\delta \xi}$$

$$17 \Rightarrow \Gamma_{\mu\nu}^{\lambda} = F_{\mu\nu}^{\lambda} F_{\mu\nu}^{\lambda} F_{\mu\nu}^{\lambda} \xi^{\mu} \xi^{\nu} \xi^{\lambda}$$

$$(s.t.t.) \Phi_{\mu\nu} = \Phi_{\nu\mu} \quad \perp = 0 \Rightarrow \Phi_{\mu\nu} = 0$$

$$S = \int \xi^{\mu} F_{\mu} + \frac{1}{2} \Phi_{\mu\nu} \xi^{\mu} \xi^{\nu}$$

$$\frac{\delta S}{\delta \Phi_{\mu\nu}} : S_{\mu\nu} = 0 \Rightarrow \exists \lambda \text{ s.t. } \xi = (\lambda, \lambda)$$

$$\frac{\delta S}{\delta \xi} : F^{\mu} = \Phi^{\mu\nu} \xi_{\nu}$$

$$\frac{\delta \xi}{\delta S} = \Delta \lambda \xi = 0$$

$$17 \Rightarrow \Gamma_{ij}^k = F_{ij}^k \frac{\partial x^k}{\partial x^i \partial x^j}$$

(satt) $\Phi_{xy} = \Phi_{yx}$ $\downarrow L=0 \Rightarrow \Phi_{xx}=0$

$$S = \int \Sigma^i F_i + \frac{1}{2} \Phi_{xy} \Sigma^i \Sigma^j$$

$$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} = 0 \Rightarrow \exists \lambda \text{ s.t. } \Sigma = (\lambda, \lambda)$$

$$\frac{\delta S}{\delta \Sigma} : F^i = \Phi^{ij} \Sigma_j$$

$$\frac{\delta S}{\delta A} = \partial_i \Sigma^i = 0$$

$$178 \Rightarrow \Gamma g_{ab} = F_{ac} F_{bd} F_{cd} \Sigma^{ab} \Sigma^{cd}$$

$$(5.44) \Phi_{xy} = \Phi_{yx} \quad \downarrow = 0 \Rightarrow \Phi_{xx} = 0$$

$$S = \int \Sigma^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \Phi_{xy} \Sigma^x \wedge \Sigma^y$$

$$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} = 0 \Rightarrow \exists \theta \text{ s.t. } \Sigma = (\theta \wedge \theta^*)$$

$$\frac{\delta S}{\delta \Sigma} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$

$$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \Sigma = 0$$

$$17 \& \Rightarrow \Gamma_{ij}^k = F_{ik}^j F_{il}^m F_{ml}^k \xi^{il} \xi^{jk}$$

$$(s_{ij}) \Phi_{xy} = \Phi_{ij} \quad \downarrow = 0 \Rightarrow \Phi_{ij} = 0$$

$$S = \int \xi^i \wedge F_i + \frac{1}{2} \Phi_{ij} \xi^i \wedge \xi^j$$

$$\frac{\delta S}{\delta \Phi_{ij}} : S_{xy} = 0 \Rightarrow \exists \theta \text{ s.t. } \xi = (\theta, \theta)$$

$$\frac{\delta S}{\delta \xi^i} : F^i = \Phi^{ij} \xi_j \Rightarrow R_{ij} = 0$$

$$\frac{\delta S}{\delta A} = \mathcal{D} \wedge \xi = 0$$

$$S^{(1)} = \int \sum_{\alpha} \omega_{\alpha} F^{\alpha}(\dot{a}^{\alpha}) - \dots$$

$$\frac{\delta S^{(1)}}{\delta \dot{a}^{\alpha}} = \sum_{\alpha} \omega_{\alpha} \rho_{\alpha}^{\nu} F_{\alpha}^{\nu} = 0 = \sum_{\alpha} \omega_{\alpha} \rho_{\alpha}^{\nu} (F_{\alpha}^{\nu} + F_{\alpha}^{\nu \rho}) \epsilon_{\alpha \nu \rho}$$

$$\frac{\delta S}{\delta A^{\mu}} \quad D_{\mu}(\partial_{\nu} \rho^{\mu\nu}) = 0 \Rightarrow \omega^{\mu} = (\omega^{\mu})^{\dagger} \rightarrow 0 = \sum_{\alpha} \omega_{\alpha} \rho_{\alpha}^{\nu} (F_{\alpha}^{\nu} + F_{\alpha}^{\nu \rho}) \epsilon_{\alpha \nu \rho}$$

$$R_{\alpha \nu \rho}^{\mu} = 0$$

$$\frac{\delta S}{\delta \omega} = D_{\mu}(\partial_{\nu} \rho^{\mu\nu}) = 0$$

$$\Rightarrow \omega = \omega(-\rho, \rho)$$

Christoffel

$$\rho_{\alpha}^{\nu} \rho_{\alpha}^{\rho} \omega^{\mu} \epsilon_{\alpha \nu \rho} =$$

$$\frac{1}{2} \left[\rho_{\alpha}^{\nu} \rho_{\alpha}^{\rho} \omega^{\mu} \epsilon_{\alpha \nu \rho} - \sum_{\alpha} \omega_{\alpha} \sum_{\nu} \rho_{\alpha}^{\nu} \right]$$

$$S^{(1)} = \int \sum^{\mu\nu} \Lambda F^{\mu\nu}(\Lambda^a) - \dots$$

Check

$$\frac{\delta S^{(1)}}{\delta \Lambda^a} = \sum^{\mu\nu} \rho_b^{\nu} F_{cd}^{+\mu} = 0 = \sum^{\mu\nu} \rho_b^{\nu} (F_{cd}^{+\mu} + F_{cd}^{+\mu}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta \Lambda^a} \Rightarrow D_\mu (\rho^{\mu\nu})^+ = 0 \Rightarrow \omega^+ = (\omega^{\mu\nu})^+ \rightarrow 0 = \sum^{\mu\nu} \rho_b^{\nu} (F_{cd}^{+\mu} + F_{cd}^{+\mu}) \epsilon_{abcd}$$

$$\frac{\delta S}{\delta \omega} = D_\mu (\rho^{\mu\nu}) = 0$$

$$\Rightarrow \omega = \omega(\rho, \gamma)$$

Christoffel

$$\rho^{\mu\nu} \rho^{\alpha\beta} \epsilon_{\mu\nu\alpha\beta} =$$

$$\frac{1}{2} \left[\sum^{\mu\nu} \Lambda \Sigma_{\mu\nu}^+ - \sum^{\mu\nu} \Lambda \Sigma_{\mu\nu}^- \right]$$

$$17a \Rightarrow \Gamma g_{ab} = F^a F^b F^c F^d \epsilon^{abcd} \epsilon_{abcd}$$

$$(s.c.) \Phi_{xy} = \Phi_{yx} \quad L=0 \Rightarrow \Phi_{xy} = 0$$

$$S = \int \epsilon^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \Phi_{xy} \epsilon^{\mu\nu} \epsilon^{\alpha\beta} - \Lambda \epsilon^{\mu\nu} \epsilon^{\alpha\beta}$$



$$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} = 0 \Rightarrow \exists \theta \text{ s.t. } \Sigma = (\theta, 0)$$

$$\frac{\delta S}{\delta \Sigma} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$

$$\frac{\delta S}{\delta \Lambda} = \partial \Lambda \Sigma = 0$$

19 July 2014

(5.22) $\Phi_{xy} = \Phi_{yx}$ $L=0 \Rightarrow \Phi_{xy} = 0$

$$S = \int \Sigma^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \Phi_{xy} \Sigma^{\mu\nu} \Sigma^{\mu\nu} - \Lambda \Sigma^{\mu\nu} \Sigma^{\mu\nu}$$

$$\frac{\delta S}{\delta \Phi_{xy}} : \Sigma^{\mu\nu} = 0 \Rightarrow \exists \text{ s.t. } \Sigma = (\epsilon_{10})^{\mu\nu}$$

$$\frac{\delta S}{\delta \Sigma^{\mu\nu}} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma^{\mu\nu} + \Lambda \Sigma^{\mu\nu}$$

$$\frac{\delta S}{\delta \Lambda} = \partial \Lambda \Sigma = 0$$

$\int \dots = \dots$

(2nd) $\Phi_{i,j} = \Phi_{j,i}$ $L=0 \Rightarrow \Phi_{i,i}=0$

$$S = \int \Sigma^\mu \wedge F_\mu \rightarrow \frac{1}{2} \Phi_{i,j} \Sigma^\mu \wedge \Sigma^\nu - \Lambda \Sigma^\mu \wedge \Sigma^\nu$$

$$= \int \Sigma^\mu \wedge F_\mu$$

$$\frac{\delta S}{\delta \Phi_{i,j}} : S_{\Phi_{i,j}} = 0 \Rightarrow \exists \theta = \dots \Sigma = \Phi_{i,j} \theta$$

$$\frac{\delta S}{\delta \Sigma} : F^\mu = \Phi^{i,j} \Sigma_j + \Lambda \Sigma^\mu$$

$$\frac{\delta S}{\delta \Lambda} = \partial \Lambda \Sigma = 0$$



19 July 2013 as the first time

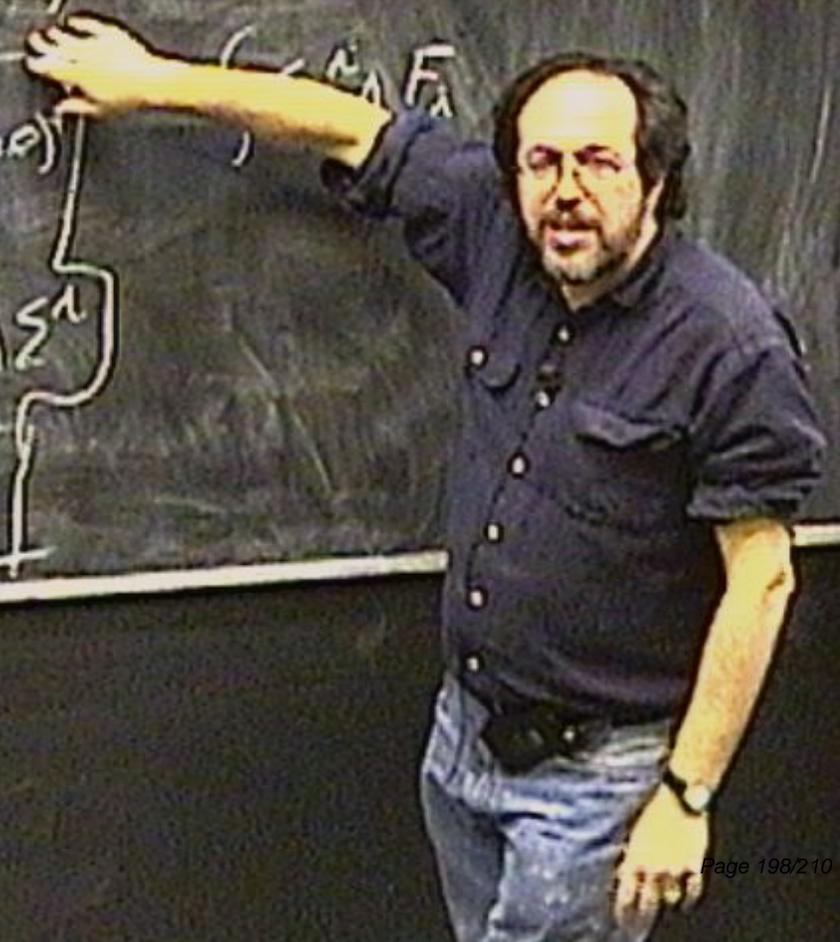
(5.22) $\Phi_{xy} = \Phi_{yx}$ $L=0 \Rightarrow \Phi_{xy} = 0$

$$S = \int \Sigma^\mu \wedge F_\mu \rightarrow \frac{1}{2} \Phi_{xy} \Sigma^\mu \wedge \Sigma^\nu - \Lambda \Sigma^\mu \wedge \Sigma^\nu$$

$\frac{\delta S}{\delta \Phi_{xy}} : \Sigma_{xy} = 0 \Rightarrow \exists \text{ s.t. } \Sigma = (\omega_{10})$

$\frac{\delta S}{\delta \Sigma^\mu} : F^\mu = \Phi^{\nu\rho} \Sigma_\nu + \Lambda \Sigma^\mu$

$\frac{\delta S}{\delta \Lambda} = \mathcal{D} \wedge \Sigma = 0$



PIRSA: 06020038

$$(S_{ij}) \Phi_{xy} = \Phi_{ij}$$

$$L=0 \Rightarrow \Phi_{ij}=0$$

$$\Psi_{xy} = \Phi_{xy} + \Lambda \delta_{xy}$$

$$\Psi_{ij} = 3\Lambda$$

$$S = \int \Sigma^{\mu\nu} \Lambda F_{\mu\nu} + \frac{1}{2} \Phi_{xy} \Sigma^{\mu\nu} \Sigma^{\mu\nu} - \Lambda \Sigma^{\mu\nu} \Sigma^{\mu\nu}$$

$$= \int \left(\Sigma^{\mu\nu} \Lambda F_{\mu\nu} - \frac{1}{2} \Psi_{xy} \Sigma^{\mu\nu} \Sigma^{\mu\nu} \right)$$

$$\frac{\delta S}{\delta \Phi_{xy}} : \Sigma^{\mu\nu} = 0 \Rightarrow \exists \text{ s.t. } \Sigma = (\epsilon_{ij})^{\mu\nu}$$

$$\frac{\delta S}{\delta \Sigma} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma_{\mu\nu} + \Lambda \Sigma^{\mu\nu}$$

$$\frac{\delta S}{\delta \Lambda} = \partial \Lambda \Sigma = 0$$

$$(S.M.) \Phi_{xy} = \Phi_{yx}$$

$$L=0 \Rightarrow \Phi_{xx} = 0$$

$$\Psi_{xy} = \Phi_{xy} + \Lambda \delta_{xy}$$

$$\Psi_{xx} = 3\Lambda$$

$$S = \int \Sigma^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \Phi_{xy} \Sigma^{\mu\nu} \Sigma^{\mu\nu} - \Lambda \Sigma^{\mu\nu} \Sigma^{\mu\nu}$$

$$= \int \left(\Sigma^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \Psi_{xy} \Sigma^{\mu\nu} \Sigma^{\mu\nu} \right)$$

$$H^{\mu\nu} = \Psi^{\mu\nu} \Sigma^{\mu\nu}$$

$$\frac{\delta S}{\delta \Phi_{xy}} : \Psi_{xy} = 0 \Rightarrow \exists \theta \text{ s.t. } \Sigma = (\theta, \theta)$$

$$\frac{\delta S}{\delta \Sigma} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma_{\mu\nu} + \Lambda \Sigma^{\mu\nu}$$

$$\frac{\delta \Sigma}{\delta \Lambda} = \partial \Lambda \Sigma = 0$$

(S.M.) $\Phi_{xy} = \Phi_{yx}$ $L=0 \Rightarrow \Phi_{xx} = 0$

$\Psi_{xy} = \Phi_{xy} + \Lambda \delta_{xy}$
 $\Psi_{xx} = 3\Lambda$

$$S = \int \Sigma^{\lambda} \Lambda F_{\lambda} + \frac{1}{2} \Phi_{xy} \Sigma^{\lambda} \Lambda \dot{\Sigma}^{\lambda} - \Lambda \Sigma^{\lambda} \Lambda \dot{\Sigma}^{\lambda}$$

$$= \int \left(\Sigma^{\lambda} \Lambda F_{\lambda} - \frac{1}{2} \Psi_{xy} \Sigma^{\lambda} \Lambda \dot{\Sigma}^{\lambda} \right)$$

$\frac{\delta S}{\delta \Phi_{xy}} : S_{xy} = 0 \Rightarrow \exists \theta \text{ s.t. } \Sigma = (\theta, \theta, \theta)$

$\frac{\delta S}{\delta \Sigma} : F^{\lambda} = \Phi^{ij} \Sigma_j + \Lambda \Sigma^{\lambda}$

$\frac{\delta S}{\delta \Lambda} = \mathcal{D} \Lambda \Sigma = 0$

$\Pi^{\lambda} = \Psi^{\lambda}_{\gamma} \dot{\Sigma}^{\gamma}$ det.

(s.m.) $\Phi_{xy} = \Phi_{yx}$ $L=0 \Rightarrow \Phi_{xx} = 0$

$\Psi_{xy} = \Phi_{xy} + \Lambda \delta_{xy}$
 $\Psi_{xx} = 3\Lambda$

$S = \int \Sigma^{\mu\nu} \Lambda F_{\mu\nu} \rightarrow \frac{1}{2} \Phi_{xy} \Sigma^x \Lambda \Sigma^y - \Lambda \Sigma^x \Lambda \Sigma^x$

$\frac{\delta S}{\delta \Phi_{xy}} : \Sigma^x \Sigma^y = 0 \Rightarrow \exists 0 \text{ s.t. } \Sigma = (\omega, \omega)$

$\frac{\delta S}{\delta \Sigma} : F^{\mu\nu} = \Phi^{\mu\nu} \Sigma_{\nu} + \Lambda \Sigma^{\mu}$

$\frac{\delta S}{\delta \Lambda} = \partial \Lambda \Sigma = 0$

$= \int \left(\Sigma^{\mu\nu} \Lambda F_{\mu\nu} - \frac{1}{2} \Psi_{xy} \Sigma^x \Lambda \Sigma^y \right)$

$\Pi^{\mu} = \Psi^{\mu\nu} \Sigma_{\nu}$ $\Delta(\Psi) = 0$

$\Rightarrow \Sigma^{\nu} = \Psi^{-1\mu\nu} F^{\mu}$

$$S = \int F^{\dot{i}\dot{\lambda}} F^{\dot{\gamma}\dot{\delta}} \Psi_{\dot{i}\dot{\lambda}\dot{\gamma}\dot{\delta}}^{-1}$$

$$\Psi_{\dot{\lambda}\dot{\gamma}} = \Psi_{\dot{\gamma}\dot{\lambda}} \quad \Psi_{\dot{i}\dot{i}} = 3\Lambda$$

Chiral

$$S[A, \psi] = \int F^i \wedge F^j \psi_{ij}^{-1}$$

$$\psi_{ij} = \psi_{ji} \quad \psi_{ii} = 3\Lambda$$



CHAP 11

$$S[A, \psi] = \int F^i \wedge F^j \psi_{ij}^{-1}$$

$$\psi_{\lambda\gamma} = \psi_{\gamma\lambda} \quad \psi_{\lambda\lambda} = 3\Lambda$$

01/15/11

$$S[A, \psi] = \int F^{\hat{i}} \wedge F^{\hat{j}} \psi_{\hat{i}\hat{j}}^{-1}$$

$$\psi_{\hat{i}\hat{j}} = \psi_{\hat{j}\hat{i}} \quad \psi_{\hat{i}\hat{i}} = 3\Lambda$$

$$\Rightarrow \Gamma_{\alpha\beta} = F_{\alpha\lambda} F_{\beta\lambda} F_{\alpha\lambda} \Sigma^{\alpha\beta\gamma\delta} \Sigma_{\alpha\beta\gamma\delta}$$

$$\Phi_{\lambda\gamma} = \Phi_{\lambda\mu} \quad L=0 \Rightarrow \Phi_{\lambda\gamma} = 0$$

$$\Psi_{\lambda\gamma} = \Phi_{\lambda\gamma} + \Lambda \delta_{\lambda\gamma}$$

$\Psi_{\lambda\lambda} = 3\Lambda$

$$= \int \Sigma^{\lambda\mu} F_{\lambda\mu} + \frac{1}{2} \Phi_{\lambda\gamma} \Sigma^{\lambda\mu} \Sigma^{\nu\sigma} - \Lambda \Sigma^{\lambda\mu} \Sigma_{\lambda\mu}$$

$= \int \Sigma^{\lambda\mu} F_{\lambda\mu} - \frac{1}{2} \Psi_{\lambda\gamma} \Sigma^{\lambda\mu} \Sigma^{\nu\sigma}$

$$\frac{\delta}{\delta \Sigma} : S_{\lambda\gamma} = 0 \Rightarrow \exists \theta \text{ s.t. } \Sigma = (\theta_{\lambda\mu})$$

$$F^{\lambda} = \Phi^{\lambda\gamma} \Sigma_{\gamma} + \Lambda \Sigma^{\lambda}$$

$$\frac{\delta S}{\delta \Sigma} = \partial_{\lambda} \Sigma = 0$$

$$F^{\lambda} = \Psi^{\lambda\gamma} \Sigma_{\gamma} \quad \text{det } \Psi \neq 0$$

$$\Rightarrow \Sigma_{\gamma} = \Psi^{-1\gamma\kappa} F^{\kappa}$$

Christoffel

$$S(\Lambda^{\mu\nu}) = \int F^{\mu\nu} \wedge F^{\rho\sigma} \Psi_{\mu\nu\rho\sigma}$$

$$\Psi_{\lambda\gamma} = \Psi_{\gamma\lambda} \quad \Psi_{\lambda\lambda} = 3\Lambda$$

$$\Psi_{\lambda\gamma} = \Psi_{\lambda\gamma} + \dots$$

$\Psi_{\lambda\lambda} = 3\Lambda$

$$\Sigma^{\bar{j}} = \Lambda \Sigma^{\lambda} \Lambda \Sigma^{\bar{i}}$$

$$\Sigma^{\lambda} \wedge F_{\lambda} = \frac{1}{2} \Psi_{\lambda\gamma} \Sigma^{\lambda} \Lambda \Sigma^{\bar{j}}$$

$$F^{\lambda} = \Psi^{\lambda}_{\gamma} \Sigma^{\bar{j}}$$

$\det \Psi \neq 0$

$$\Rightarrow \Sigma^{\bar{j}} = \Psi^{-1}_{\lambda\gamma} F^{\lambda}$$

$$S(\Lambda, \Psi) = F^{\lambda} \wedge F^{\gamma} \Psi_{\lambda\gamma}$$

$$\Psi_{\lambda\gamma} = \Psi_{\gamma\lambda} \quad \Psi_{\lambda\lambda} = 3\Lambda$$

Christoffel

$$S[A^{\mu\nu}, \Psi_{ij}] = \int F^{\hat{\alpha}} \wedge F^{\hat{\gamma}} \Psi_{ij}^{-1}$$

$$\Psi_{\hat{\alpha}\hat{\gamma}} = \Psi_{\hat{\gamma}\hat{\alpha}} \quad \Psi_{\hat{\alpha}\hat{\alpha}} = 3\Lambda$$