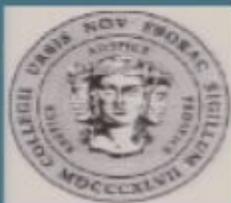


Title: Solving QCD in 2+1 dimensions

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Abstract:



Towards a Solution of QCD in (2+1) Dimensions

Some answers and many questions

V. P. NAIR

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The Perimeter Institute

Waterloo, Canada

February 22, 2006



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Why is YM(2+1) interesting?

- Interesting in its own right

YM(1+1)	YM(2+1)	YM(3+1)
No propagating degrees of freedom	Propagating degrees of freedom,	Highly nontrivial,
Exactly solvable	Nontrivial	Difficult
	Dimensional coupling	
	Super-renormalizable	

- A real physical context for YM(2+1)

Mass gap of YM(2+1) \approx Magnetic screening mass of YM(3+1) at high temperatures

Why is YM(2+1) interesting? (cont'd.)

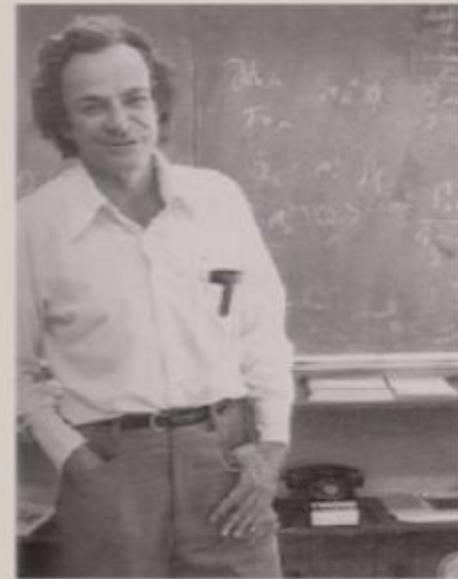
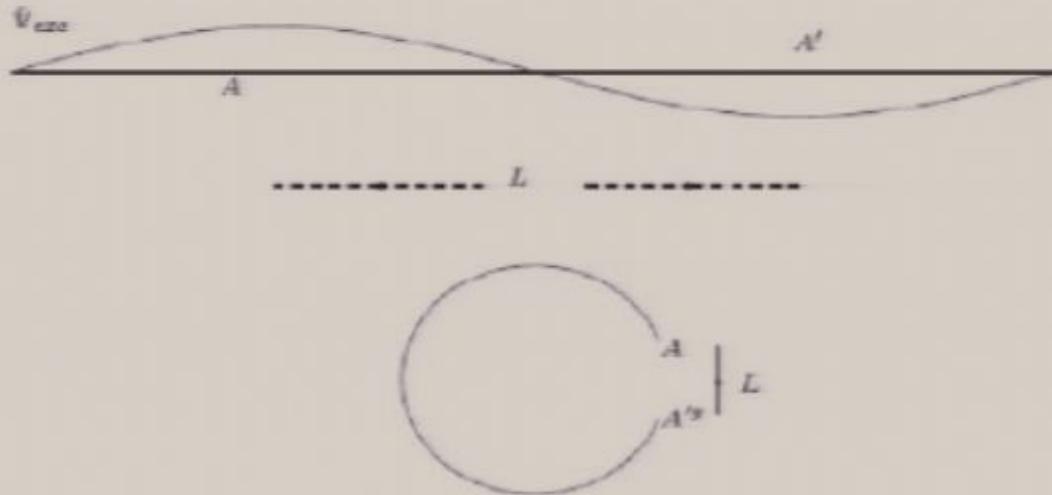
- Magnetic screening is necessary to define perturbation theory at finite temperatures due to magnetic-type infrared divergences.
- At high T , $YM(3 + 1) \approx YM(3)$ because only the lowest Matsubara frequency contributes
- This can be Wick rotated to $YM(2 + 1)$, so by analyzing $YM(2 + 1)$, **we can get an estimate of magnetic mass**

Work in collaboration with [D. Karabali and Chanju Kim](#).

We will use a Hamiltonian approach, some exact results are possible.

Feynman's last problem

- Ψ_0 is real and positive \rightarrow Ψ_{exc} has a node



$$\langle \mathcal{H} \rangle = \frac{1}{2} \int \left[\underbrace{e^2 |\delta\Psi / \delta A|^2}_{\text{gradient energy}} + B^2 |\Psi|^2 / e^2 \right]$$

gradient energy $\sim 1/L^2$

Hamiltonian Analysis

As in any theory, Hamiltonian analysis requires 3 basic ingredients

1. Inner product

- Matrix variables, calculation of gauge-invariant volume
- Identification of proper gauge-invariant variables (CFT argument)

2. Hamiltonian \mathcal{H} in the new variables

- Propagator mass, comparison with resummation techniques,...

3. Solve the Schrödinger equation $\mathcal{H}\Psi = E\Psi$

- Ψ_0 , the vacuum wave function \implies string tension, comparison with lattice estimates

Matrix variables, volume element

- Choose $A_0 = 0$, this leaves A_i , $i = 1, 2$. Gauge transformations act as

$$A_i^g = g A_i g^{-1} - \partial_i g g^{-1}$$

Wave functions are **gauge-invariant** (This is equivalent to imposing Gauss law)

- Choose complex coordinates, $z = x_1 - ix_2$, $\bar{z} = x_1 + ix_2$

$$A \equiv A_z = \frac{1}{2}(A_1 + iA_2), \quad \bar{A} = \frac{1}{2}(A_1 - iA_2)$$

- Parametrize A as

$$A = -\partial M M^{-1} \quad \bar{A} = M^{\dagger-1} \bar{\partial} M^{\dagger}$$

Matrix variables, volume element (cont'd.)

- $G = SU(N) \implies M \in SL(N, \mathbf{C}) = SU(N)^{\mathbf{C}}$

(More generally $G \rightarrow G^{\mathbf{C}}$)

This parametrization is well-known in 2-dimensional YM context, gauged WZW models, etc.

- There is an ambiguity, M and $MV(\bar{z}) \implies$ same A (We will come back to this later)

- The basic advantage of this parametrization is the behaviour under gauge transformation,

$$A \rightarrow A_i^g = g A_i g^{-1} - \partial_i g g^{-1} \implies M \rightarrow M^g = g M$$

- $H = M^\dagger M$ is gauge-invariant

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Matrix variables, volume element (cont'd.)

- Calculation of volume element of the configuration space

$$\begin{aligned}\delta A &= -\partial(\delta M M^{-1}) + [\partial M M^{-1}, \delta M M^{-1}] \\ &= -D(\delta M M^{-1})\end{aligned}$$

$$\delta \bar{A} = \bar{D}(M^{\dagger -1} \delta M^{\dagger})$$

$$\begin{aligned}ds_{\mathcal{A}}^2 &= \int d^2x \operatorname{Tr}(\delta A \delta \bar{A}) \\ &= \int \operatorname{Tr} [(M^{\dagger -1} \delta M^{\dagger})(-\bar{D}D)(\delta M M^{-1})]\end{aligned}$$

$$ds_{SL(N, \mathbf{C})}^2 = \int \operatorname{Tr}(M^{\dagger -1} \delta M^{\dagger} \delta M M^{-1})$$

$$d\mu_{\mathcal{A}} = \det(-\bar{D}D) \underbrace{d\mu(M, M^{\dagger})}$$

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Matrix variables, volume element (cont'd.)

- We can split the $SL(N, \mathbf{C})$ volume element as

$$d\mu(M, M^\dagger) = \underbrace{d\mu(H)}_{\text{Haar for } SL(N, \mathbf{C})/SU(N)} \underbrace{d\mu(U)}_{\text{Haar for } SU(N)}$$

$$d\mu_{\mathcal{A}} = \det(-\bar{D}D) d\mu(H) d\mu(U)$$

A short aside: Parametrize $H = M^\dagger M$ as $H = e^{t^a \varphi^a} \implies$
 $H^{-1} \delta H = \delta \varphi^a R_{ab}(\varphi) t^b \implies d\mu(H) = [d\varphi] \det R$

- For the gauge-invariant configuration space

$$d\mu(\mathcal{C}) = \det(-\bar{D}D) d\mu(H)$$

- The computation of the determinant is as follows.

M =

(unity) (harm)

Matrix variables, volume element (cont'd.)

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Matrix variables, volume element (cont'd.)

- Two remarks

1. YM (2+1) has Gribov problem. But inner product formula has no difficulty due to this, it is exact
2. It shows that matrix elements in YM(2+1) = correlators of a hermitian WZW model

- The Wilson loop operator is given by

$$W(C) = \text{Tr} \mathcal{P} e^{-\oint A} = \text{Tr} \mathcal{P} \exp \left(\frac{\pi}{c_A} \oint J \right)$$
$$J = (c_A/\pi) \partial H H^{-1}$$

All gauge-invariant quantities can be made from J .

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- For the gauge-invariant configuration space

$$\begin{aligned}d\mu(\mathcal{C}) &= \det(-\bar{D}D) d\mu(H) \\ &= d\mu(H) \exp[2 c_A S_{wzw}(H)]\end{aligned}$$

- $S_{wzw}(H)$ is the Wess-Zumino-Witten (WZW) action,

$$S_{wzw}(H) = \frac{1}{2\pi} \int \text{Tr}(\partial H \bar{\partial} H^{-1}) - \frac{i}{12\pi} \int \text{Tr}(H^{-1} dH)^3$$

$$c_A \delta_{ab} = f_{amn} f_{bmn} = N \delta_{ab} \text{ for } SU(N).$$

- The inner product is now given as

$$\langle 1|2\rangle = \int d\mu(H) \exp[2 c_A S_{wzw}(H)] \Psi_1^* \Psi_2$$

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An intuitive argument for mass gap

- The Hamiltonian has the form

$$\mathcal{H} = \int \frac{1}{2} [e^2 E^2 + B^2/e^2]$$

$[E, B] \sim p$ (in momentum space) $\implies \Delta E \Delta B \sim p$, or
 $\Delta E \sim p/\Delta B$

$$\mathcal{E} = \langle \mathcal{H} \rangle \approx \frac{1}{2} \left[e^2 \frac{p^2}{(\Delta B)^2} + \frac{(\Delta B)^2}{e^2} \right]$$

Minimize with respect to $\Delta B \implies (\Delta B)^2 \sim p \implies \mathcal{E} \sim p$.

This is the **photon**.

- For us

$$\langle \mathcal{H} \rangle = \int d\mu(H) \exp [2 c_A S_{wz\bar{w}}(H)] \int \frac{1}{2} [e^2 E^2 + B^2/e^2]$$

An intuitive argument for mass gap (cont'd.)

- Expanding the WZW action

$$\langle \mathcal{H} \rangle \approx \int d\mu(H) \exp \left[-\frac{c_A}{2\pi} \int B \frac{1}{p^2} B + \dots \right] \int \frac{1}{2} \left[e^2 E^2 + \frac{B^2}{e^2} \right]$$

Gaussian $\implies (\Delta B)^2 \sim \pi p^2 / c_A \implies \text{mass gap} \sim e^2 c_A / 2\pi.$

- More detailed analysis \implies

$$m_{\text{mag}} = m = \frac{e^2 c_A}{2\pi} = \frac{g^2 T c_A}{2\pi}$$

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The Hermitian WZW model

- The hermitian model can be analyzed by comparison with the unitary model

Unitary model	Hermitian model
Level k	Level $k + 2 c_A$
$\exp[kS_{wzw}(U)]$	$\exp[(k + 2 c_A)S_{wzw}(H)]$
$\kappa = k + c_A$	$\kappa = -(k + c_A)$
	Compare using $(\kappa \leftrightarrow -\kappa)$
Integrable rep's $\sim \text{spin} \leq k$	
$\langle \text{Nonintegrable...} \rangle = 0$	$\langle \text{Nonintegrable...} \rangle = \infty$

- “Finite norm” $\implies \Psi$'s made of integrable rep's.
 $k = 0$ for us, $\implies \Psi$'s are functions of the current J

Construction of \mathcal{H}

- The Hamiltonian is given by

$$\begin{aligned}\mathcal{H} &= \underbrace{\frac{e^2}{2} \int E^a E^a}_{T} + \underbrace{\frac{1}{2e^2} \int B^a B^a}_{V} \\ &\equiv T + V\end{aligned}$$

- The potential energy is easy to simplify

$$\begin{aligned}V &= \frac{1}{2e^2} \int B^a B^a = \frac{2\pi^2}{e^2 c_A^2} \int : \bar{\partial} J^a \bar{\partial} J^a : \\ &= \frac{\pi}{m c_A} \int : \bar{\partial} J \bar{\partial} J : \\ m &= \frac{e^2 c_A}{2\pi} \quad (\text{This is the basic mass scale})\end{aligned}$$

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Construction of \mathcal{H} (cont'd.)

- The kinetic term is simplified via the chain rule

$$\begin{aligned}
 T \Psi &= -\frac{e^2}{2} \int_x \frac{\delta^2}{\delta A(x) \delta \bar{A}(x)} \Psi \\
 &= -\frac{e^2}{2} \left[\underbrace{\int \frac{\delta J(u)}{\delta A(x)} \frac{\delta J(v)}{\delta \bar{A}(x)}}_{\Omega} \frac{\delta^2 \Psi}{\delta J(u) \delta J(v)} \right. \\
 &\quad \left. + \int \underbrace{\frac{\delta^2 J(u)}{\delta A(x) \delta \bar{A}(x)}}_{\omega} \frac{\delta \Psi}{\delta J(u)} \right] \\
 &= \int \Omega_{ab}(u, v) \frac{\delta^2 \Psi}{\delta J^a(u) \delta J^b(v)} + \int \omega^a(u) \frac{\delta \Psi}{\delta J^a(u)}
 \end{aligned}$$

Construction of \mathcal{H} (cont'd.)

- $\omega^a(u)$ needs regularization

$$\begin{aligned}\omega^a &= -\frac{e^2}{2} \int_x \frac{\delta^2 J^a(u)}{\delta A^b(x) \delta \bar{A}^b(x)} \\ &= (e^2 c_A / 2\pi) M_{am}^\dagger(x) \text{Tr} [t^m \bar{D}_{reg}^{-1}(y, x)]_{y \rightarrow x}\end{aligned}$$

$$\omega^a = m J^a, \quad m = \frac{e^2 c_A}{2\pi}$$

- The kinetic energy is thus given by

$$T = m \left[\int J^a \frac{\delta}{\delta J^a} + \int \Omega_{ab}(u, v) \frac{\delta^2}{\delta J^a(u) \delta J^b(v)} \right]$$

$$\Omega_{ab}(u, v) = \frac{c_A}{\pi^2} \frac{\delta_{ab}}{(u-v)^2} - i \frac{f_{abc} J^c(v)}{u-v} + \mathcal{O}(\epsilon)$$

Can be rechecked, particularly the term $\int J \frac{\delta}{\delta J}$, by self-adjointness of T .

Construction of \mathcal{H} (cont'd.)

- Regularization

$$\overline{\mathcal{G}}_{ma}(x, y) = \frac{1}{\pi(x-y)} \left[\delta_{ma} - e^{-\frac{(x-y)^2}{\epsilon}} [H(x, \bar{y}) H^{-1}(y, \bar{y})] \right]$$

All results checked using regularization.

- T can be written as

$$\langle 1|T|2\rangle = \frac{e^2}{4} \int d\mu(H) e^{2c_A S_{wzw}(H)} \left[\overline{(\mathcal{G}p)_a \psi_1} H_{ab} (\mathcal{G}p)_b \psi_2 + \overline{(\bar{\mathcal{G}}\bar{p})_a \psi_1} H_{ba} (\bar{\mathcal{G}}\bar{p})_b \psi_2 \right]$$

$$[p_a(\vec{x}), M(\vec{y})] = M(\vec{y}) (-it_a) \delta(\vec{y} - \vec{x})$$

$$[\bar{p}_a(\vec{x}), M^\dagger(\vec{y})] = (-it_a) M^\dagger(\vec{y}) \delta(\vec{y} - \vec{x})$$

$$M \circ V(\bar{z}), \quad M$$

$$M =$$

(univ. (hmm))

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(unitary) (herm)

Construction of \mathcal{H} (cont'd.)

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$$[p_a(\vec{x}), M(\vec{y})] = M(\vec{y}) (-it_a) \delta(\vec{y} - \vec{x})$$

$$[\bar{p}_a(\vec{x}), M^\dagger(\vec{y})] = (-it_a) M^\dagger(\vec{y}) \delta(\vec{y} - \vec{x})$$

Construction of \mathcal{H} (cont'd.)

- We use this form for calculations. Self-adjointness is manifest, $\langle 1|T|2\rangle = \langle T|1|2\rangle$.
- A particularly useful result is

$$T V = 2m V$$

$$M \circ V(\bar{z}), \quad M$$

$$M = (\text{unitary})(\text{herm})$$

$$\int \bar{\partial} J \partial J$$

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Vacuum wave function

- Ignore V for the moment. Then we can take $\Psi_0 = 1$, this is okay since $T \Psi_0 = 0$, and since it is normalizable.

$$\int d\mu(H) e^{2c_A S_{wzw}(H)} \Psi_0^* \Psi_0 < \infty$$

- Include V perturbatively, $\Psi_0 = e^P$

$$\begin{aligned}
 P = & -\frac{\pi}{m^2 c_A} \int : \bar{\partial} J \bar{\partial} J : \\
 & - \left(\frac{\pi}{m^2 c_A} \right)^2 \int : \bar{\partial} J \mathcal{D} \bar{\partial} \bar{\partial} J : + \frac{1}{3} \int : \bar{\partial} J [J, \bar{\partial}^2 J] : \\
 & + \dots \qquad \qquad \qquad + \dots
 \end{aligned}$$



sum



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Vacuum wave function (cont'd.)

- The summed-up result is

$$P = -\frac{2}{e^2} \left[\frac{\pi^2}{c_A^2} \int \bar{\partial} J^a(x) \left[\frac{1}{m + \sqrt{m^2 - \nabla^2}} \right]_{x,y} \bar{\partial} J^a(y) + f_{abc} \int J^a(x) J^b(y) J^c(z) f(x, y, z) + \dots \right]$$

$$f(x, y, z) = \int e^{ikx+ipy+iqz} (2\pi)^2 \delta(k + p + q) f(k, p, q)$$

$$f(k, p, q) = \left[\frac{\pi}{2c_A} \right]^3 \frac{(E_k - m)(E_p - m)}{E_k + E_p + E_q} \frac{\bar{k} - \bar{p}}{kp}$$

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$$\begin{aligned}\Psi_0 = e^P &\approx \exp \left[-\frac{1}{2e^2} \int B \frac{1}{\sqrt{-\nabla^2}} B \right] && \frac{k}{m} \gg 1 \\ &\approx \exp \left[-\frac{1}{4e^2 m} \int B^2 \right] && \frac{k}{m} \ll 1\end{aligned}$$

$\mathcal{O}(J^3, J^4)$ terms are small at $k \gg e^2$ and at $k \ll e^2$

- The high k limit agrees with perturbation theory

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String tension

- The expectation value of the Wilson loop can be calculated with Ψ_0

$$\begin{aligned}\langle W_R(C) \rangle &= \langle \text{Tr}_R \mathcal{P} e^{\frac{\pi}{c_A} \oint J} \rangle \\ &\approx (\text{constant}) e^{-\sigma \mathcal{A}_C}\end{aligned}$$

where the string tension σ is given as

$$\sqrt{\sigma} = e^2 \sqrt{\frac{c_A c_R}{4\pi}}$$

c_R = Casimir for representation R

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Comparison with lattice calculations

Compare $\sqrt{\sigma}/e^2$ with numerical (lattice) estimates by Teper et al.
 Our predictions are in black, lattice values are in red.

Group	k=1 Fund.	k=2 antisym	k=3 antisym	k=2 sym	k=3 sym	k=3 mixed
$SU(2)$	0.345 0.335					
$SU(3)$	0.564 0.553					
$SU(4)$	0.772 0.759	0.891 0.883		1.196 1.110		

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Comparison with lattice calculations (cont'd.)

Group	k=1	k=2	k=3	k=2	k=3	k=3
	Fund.	antisym	antisym	sym	sym	mixed
$SU(5)$	0.977 0.966					
$SU(6)$	1.180 1.167	1.493 1.484	1.583 1.569	1.784 1.727	2.318 2.251	1.985 1.921

The difference between predictions and lattice values is $\leq 3\%$.

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Comment on higher corrections

- Are there corrections due to $\mathcal{O}(J^3)$ and higher terms in Ψ_0 ?
- Two types of corrections possible
 - Corrections to coupling, purely numerical. \rightarrow ratios σ_R/σ_F are unaffected
 - Corrections via new diagrams to Wilson line expectation value (under investigation)

Looking ahead

- A prediction for magnetic screening mass,

$$m = \frac{g^2 T c_A}{2\pi}$$

- A number of qualitative features of glueballs
- Predictions for glueball masses in recent work by Leigh-Minic-Yelnikov
- Extension to Yang-Mills-Chern-Simons theory
- Progress on analysis on torus, adjoint string breaking, etc.

Magnetic mass, resummed perturbation theory

- Since $T = m \left[\int J \frac{\delta}{\delta J} + \int \Omega \frac{\delta}{\delta J} \frac{\delta}{\delta J} \right]$, we get $T J^a = m J^a$
- Including the potential energy,

$$(T + V) J^a = \sqrt{p^2 + m^2} J^a + \dots$$

J^a is a “gauge-invariant” definition of a gluon.

- We can bring this out by

$$\Psi \rightarrow e^{-c_A S_{wzw}(H)} \Psi, \quad \mathcal{H} \rightarrow e^{-c_A S_{wzw}(H)} \mathcal{H} e^{-c_A S_{wzw}(H)}$$

$$\mathcal{H} = \frac{1}{2} \int \left[-\frac{\delta^2}{\delta \phi^2} + \phi(-\nabla^2 + m^2)\phi + \dots \right]$$

- Propagator mass for gauge particles (magnetic mass)

$$m = \frac{e^2 c_A}{2\pi} \approx 0.32 e^2 \quad \text{for } SU(2)$$

$$M \circledast V(z), \quad M$$

$$\frac{1}{p^2 + \omega^2}$$

$$\frac{1}{p^2} \quad m_2 \quad \frac{1}{p^2}$$

$$M = \text{(unitary)} \text{(herm)}$$

$$V \sim \frac{1}{e^2} \left(\bar{\partial} J \quad \partial J \right)$$

$$\frac{p}{e^2}$$

Comparison with other methods

$m/e^2 =$	0.35	Common factor for glueball masses (lattice, Philipsen)
	0.51	Max. Abelian gauge (lattice, Karsch et al)
	0.52	Landau gauge (")
	0.44	$\lambda_3 = 2$ gauge (")
	0.38	Resummation of P.T. (Alexanian & Nair)
	0.28	Resummation of P.T. (Buchmuller & Philipsen, Jackiw & Pi)
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Glueballs

- We showed $T J^a = m J^a$, but J^a is not a good state.
- M and $MV(\bar{z})$ give the same A via $A = -\partial M M^{-1}$
- We need invariance (**holomorphic invariance**) under

$$J \rightarrow V J V^{-1} - \partial V V^{-1}, \quad \bar{\partial} J \rightarrow V \bar{\partial} J V^{-1}$$

- A $2J$ - state with holomorphic invariance is given by

$$\Psi_2 = \int f(x, y) : \bar{\partial} J^a(x) [H(x, \bar{y}) H^{-1}(y, \bar{y})]_{ab} \bar{\partial} J^b(y)$$

(This can give 0^{++} states.)

- Take the same $x, y \implies f(x, y) = \sigma(x, y, \epsilon)$

$$T \Psi_2 = 2m \Psi_2$$

Glueballs (cont'd.)

- For higher number of J 's, form the state

$$\Psi_n \sim : \bar{\partial} J^{a_1} \bar{\partial} J^{a_2} \dots \bar{\partial} J^{a_n} : \underbrace{\omega_{a_1 a_2 \dots a_n}}$$

invariant tensor of $SU(N)$

$$T \Psi_n = m_n \Psi_n$$

- Can include center-of-mass motion from $\int B^2/2e^2$. Relative motion \implies higher “radial” excitations \implies Regge trajectory
- Going back to Ψ_2 ,

$$\left\{ 2m - \left[\frac{\nabla_x^2}{2m} + \frac{\nabla_y^2}{2m} \right] + m \log \frac{|x-y|}{2\epsilon} + \dots \right\} f = E f$$

Consistency of approximations is not clear for this equation.

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Glueballs: The Leigh-Minic-Yelnikov approach

- Recent paper by LMY extending our work to glueballs via analysis of correlators.
- The vacuum wave function is written as

$$\Psi_0 = \exp \left[-\frac{\pi}{2m^2 c_A} \int \bar{\partial} J K[L] \bar{\partial} J \right]$$

$$L = \mathcal{D}\bar{\partial}/m^2.$$

- The kernel K is given by

$$K[L] = \frac{J_2(4\sqrt{L})}{\sqrt{L} J_1(4\sqrt{L})}$$

J_1 , J_2 are Bessel functions of orders 1 and 2 respectively.

Glueballs: The Leigh-Minic-Yelnikov approach (cont'd.)

- The two-point function, ignoring a class of interaction terms, is then

$$\begin{aligned} \langle : \bar{\partial} J^a \bar{\partial} J^a :_x : \bar{\partial} J^a \bar{\partial} J^a :_y \rangle &\sim [K^{-1}(|x - y|)]^2 \\ &\sim \frac{1}{|x - y|} \sum_{n,m} (M_n M_m)^{\frac{3}{2}} e^{-(M_n + M_m)|x - y|} \end{aligned}$$

$$M_n = \frac{1}{2} m j_{2,n}$$

zeros of the Bessel function J_2

- A number of glueball masses obtained in this way

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0^{++**}	6.716	7.99 ± 0.22
0^{++***}	7.994	9.44 ± 0.38
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Yang-Mills-Chern-Simons Theory

$$\Psi = \exp \left[k S_{wzw}(M^\dagger) - \frac{k}{4\pi} \int A^a \bar{A}^a \right] \Phi(H)$$

$$\langle 1|2 \rangle = \int d\mu(H) e^{(k+2c_A) S_{wzw}(H)} \Phi_1^* \Phi_2$$

$$T = \frac{e^2}{4\pi} (k + 2c_A) \int J \frac{\delta}{\delta J} + \frac{e^2 c_A}{2\pi} \int \Omega \frac{\delta}{\delta J} \frac{\delta}{\delta J}$$

$$\Phi_0 \approx \exp \left[-\frac{\pi}{m c_A} \int \bar{\partial} J \frac{1}{\tilde{m} + \sqrt{\tilde{m}^2 - \nabla^2}} \bar{\partial} J \right]$$

$$\tilde{m} = \frac{e^2}{4\pi} (k + 2c_A)$$

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Comment on Gribov problem

- The space of gauge potentials has the bundle structure

$$\begin{array}{ccc} \mathcal{G}_* & \rightarrow & \mathcal{A} \\ & & \downarrow \\ & & \mathcal{A}/\mathcal{G}_* \end{array}$$

- This bundle is nontrivial. In particular, $\Pi_2(\mathcal{A}/\mathcal{G}_*) = \mathbf{Z}$ and $\Pi_n(\mathcal{A}) = 0$. There are noncontractible 2-spheres in $\mathcal{A}/\mathcal{G}_*$
- An example of such a configuration is

$$H = \cosh 2f + \mathcal{J} \sinh 2f$$

$$f = \frac{1}{2} \log \left(\frac{z\bar{z} + w\bar{w} + \mu^2}{z\bar{z} + w\bar{w}} \right)$$

Yang-Mills-Chern-Simons Theory

$$\Psi = \exp \left[k S_{wzw}(M^\dagger) - \frac{k}{4\pi} \int A^a \bar{A}^a \right] \Phi(H)$$

$$\langle 1|2 \rangle = \int d\mu(H) e^{(k+2c_A) S_{wzw}(H)} \Phi_1^* \Phi_2$$

$$T = \frac{e^2}{4\pi} (k + 2c_A) \int J \frac{\delta}{\delta J} + \frac{e^2 c_A}{2\pi} \int \Omega \frac{\delta}{\delta J} \frac{\delta}{\delta J}$$

$$\Phi_0 \approx \exp \left[-\frac{\pi}{m c_A} \int \bar{\partial} J \frac{1}{\tilde{m} + \sqrt{\tilde{m}^2 - \nabla^2}} \bar{\partial} J \right]$$

$$\tilde{m} = \frac{e^2}{4\pi} (k + 2c_A)$$

- New integrable operators from CFT \rightarrow screening of $W_F(C)$
- Large $k \rightarrow$ standard perturbation theory
- A number of eigenstates of T can be constructed

Comment on Gribov problem

- The space of gauge potentials has the bundle structure

$$\begin{array}{ccc} \mathcal{G}_* & \rightarrow & \mathcal{A} \\ & & \downarrow \\ & & \mathcal{A}/\mathcal{G}_* \end{array}$$

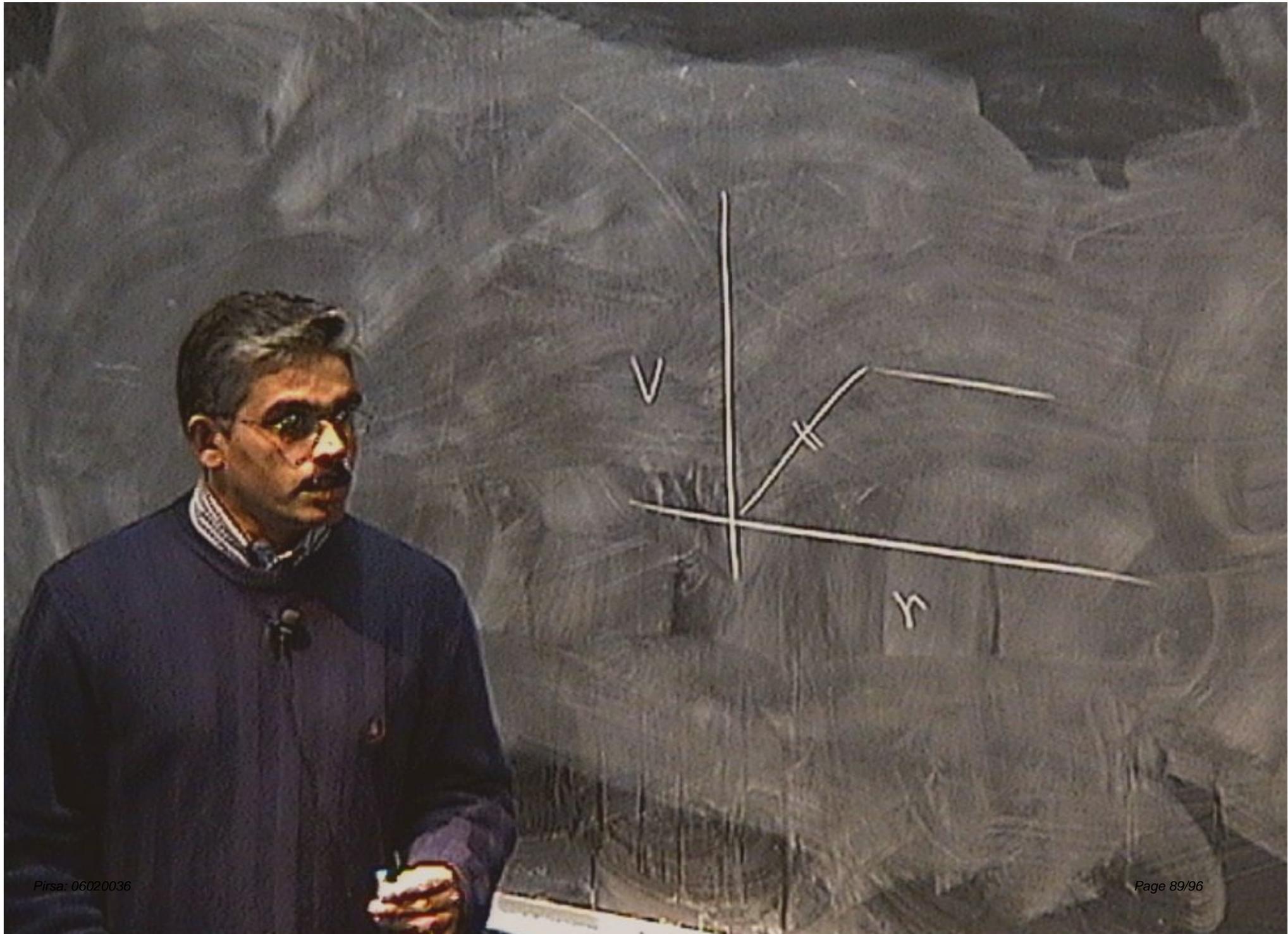
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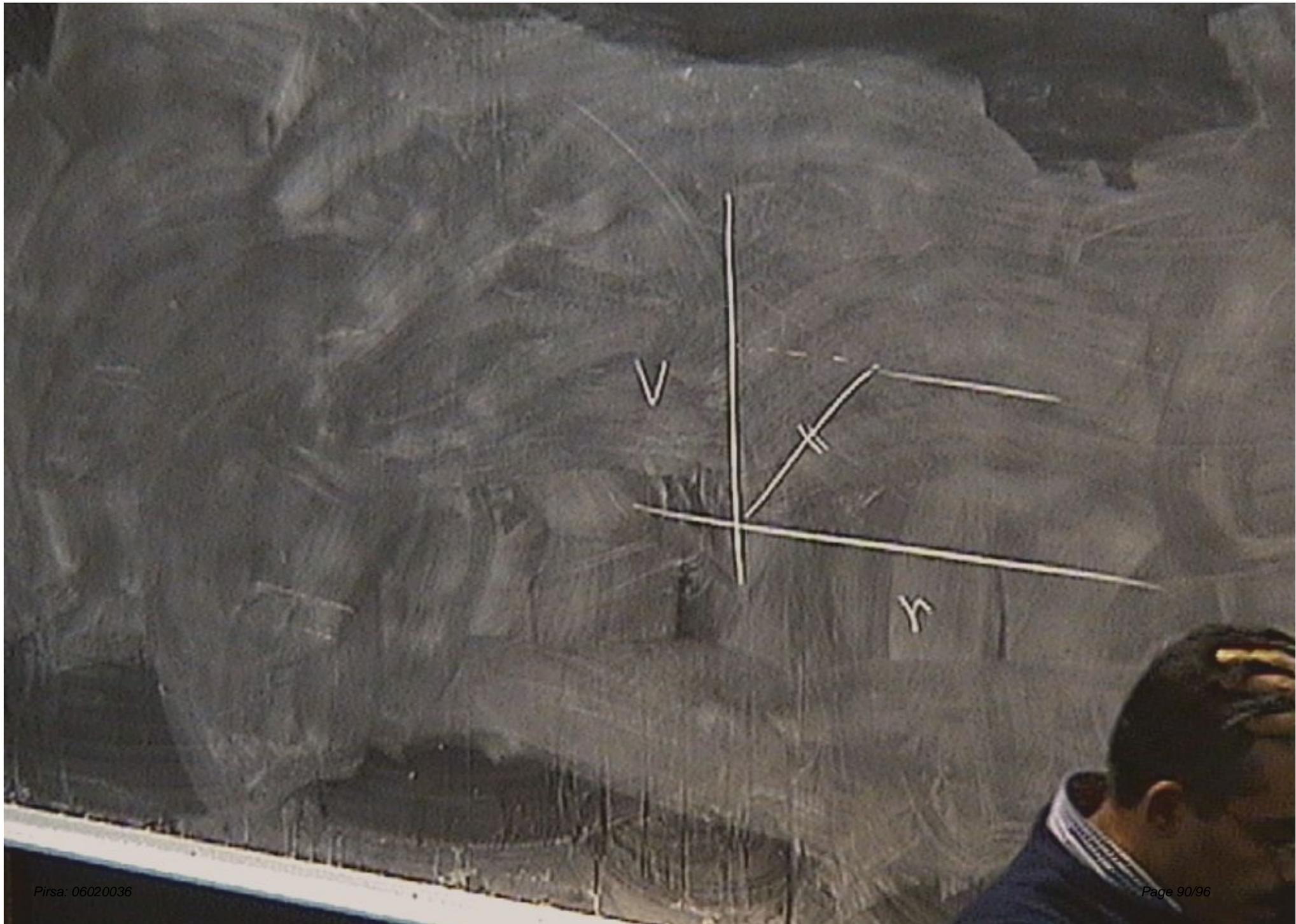
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More questions

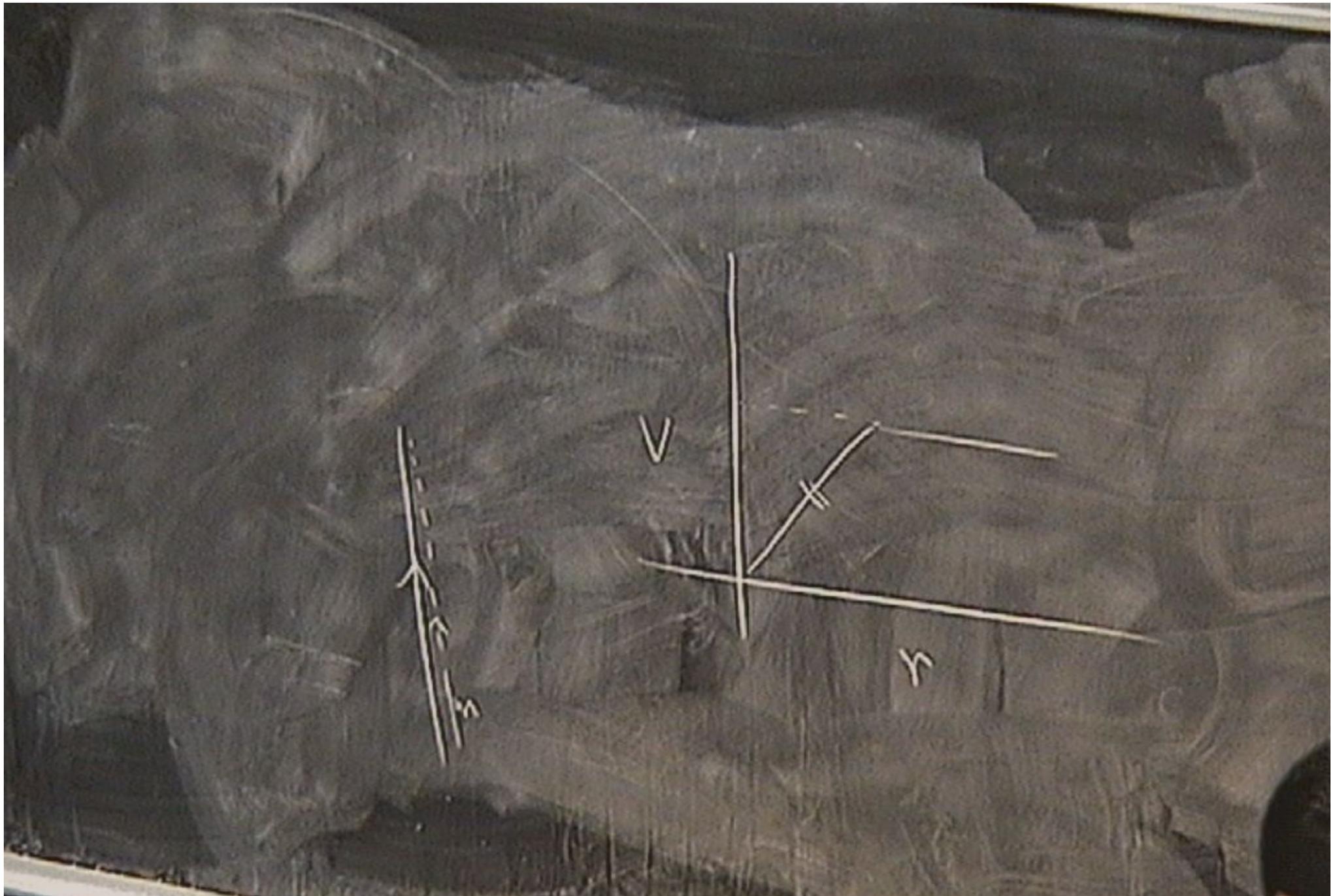
- Is a proof of mass gap possible? (YMCS theory with $k \rightarrow 0$ might be a good starting point?)
- Better handle on glueballs
- Higher order corrections (*)
- Screening of adjoint and string breaking (*)
- Calculations on the torus can help understand the theory at finite temperature (*)
- Connections with the duality-matrix model approach
- Better understanding of the geometrical properties of the configuration space





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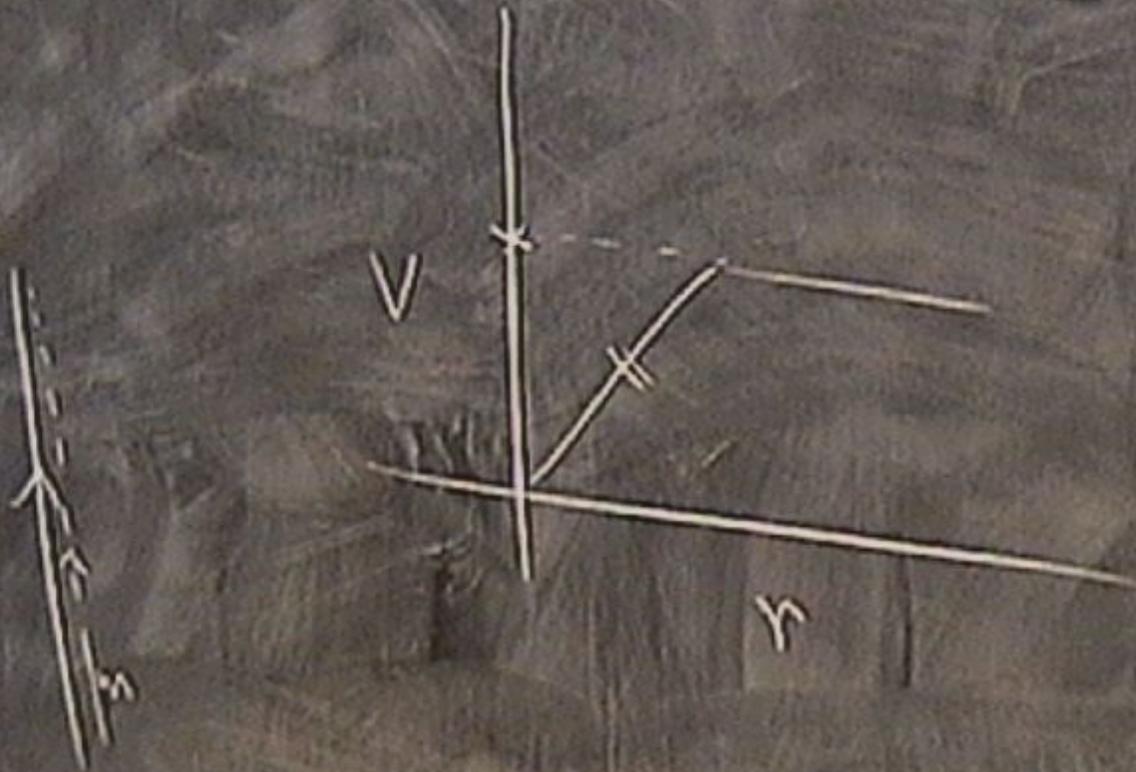
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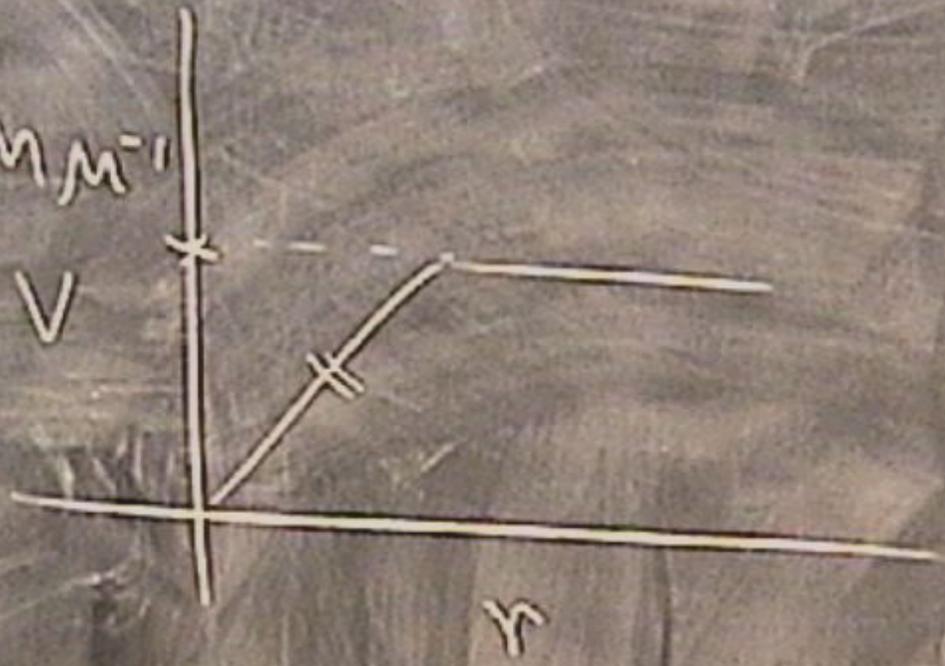
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$$A = -\partial M M^{-1}$$

$$A = \sigma \cdot \partial M M^{-1}$$

S



$$A = -\partial M M^{-1}$$

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$$\frac{e^2 (k + 2c\lambda)}{4\pi}$$

